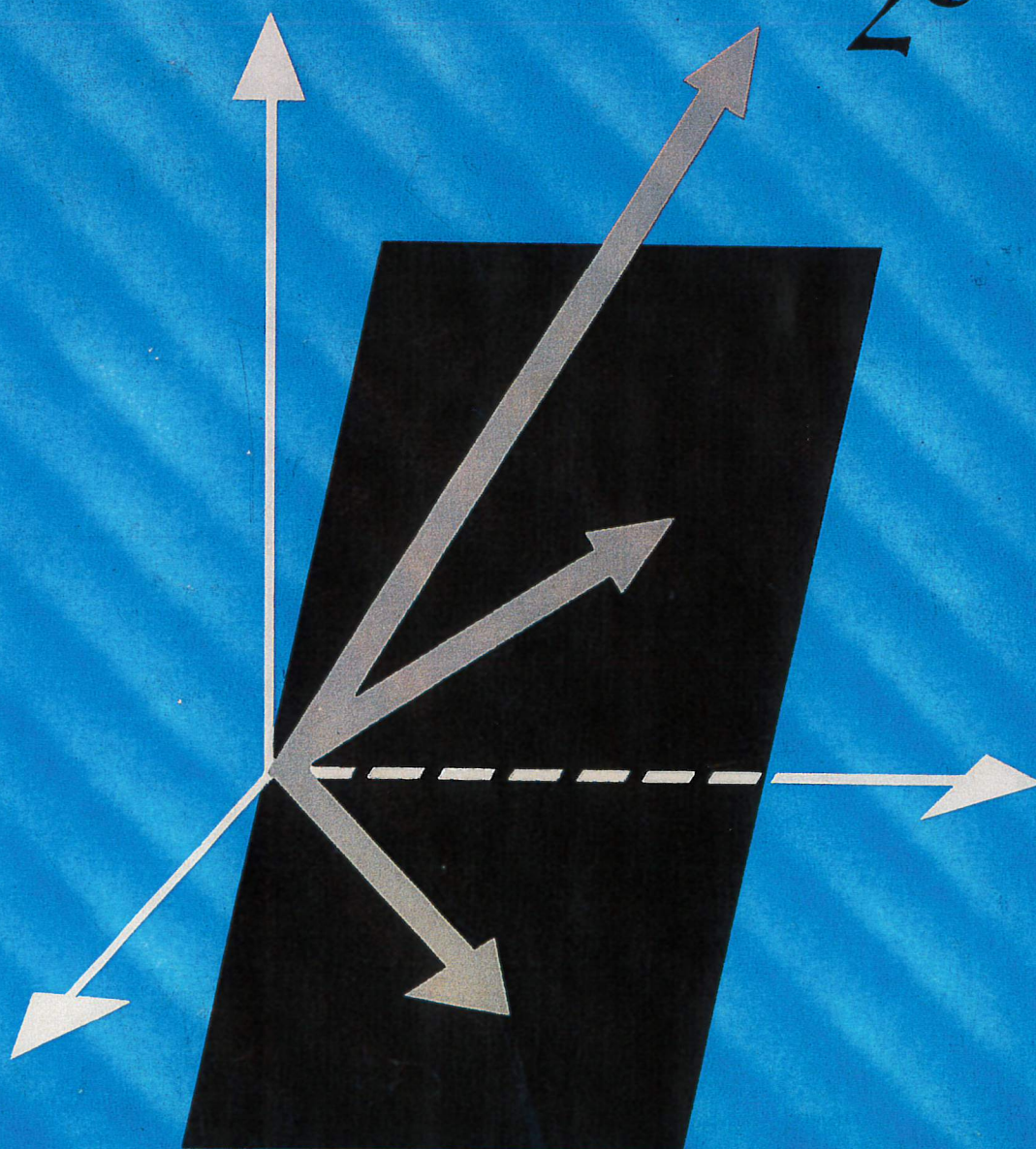


EXHIBIT 1066

Howard Anton

Elementary
Linear
Algebra^{2e}



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SECOND EDITION

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3 Vectors in 2-Space and 3-Space

Readers familiar with the contents of this chapter can go to Chapter 4 with no loss of continuity.

3.1 INTRODUCTION TO VECTORS (GEOMETRIC)

In this section vectors in 2-space and 3-space are introduced geometrically. Arithmetic operations on vectors are defined and some basic properties of these operations are established.

Many physical quantities like area, length, and mass are completely described once a real number representing the magnitude of the quantity is given. Other physical quantities, called *vectors*, are not completely determined until both a magnitude and a direction are specified. Force, displacement, and velocity are examples of vectors.

Vectors can be represented geometrically as directed line segments or arrows in 2-space or 3-space; the direction of the arrow specifies the direction of the vector and the length of the arrow describes its magnitude. The tail of the arrow is called the *initial point* of the vector, and the tip of the arrow the *terminal point*. We shall denote vectors by lower case boldface type like **a**, **k**, **v**, **w** and **x**. When discussing vectors, we shall refer to numbers as *scalars*. All our scalars will be real numbers and will be denoted by ordinary lowercase type like *a*, *k*, *v*, *w* and *x*.

If as in Figure 3.1*a* the initial point of a vector **v** is *A* and the terminal point is *B*, we write

$$\mathbf{v} = \overrightarrow{AB}$$

Vectors having the same length and same direction, like those in Figure 3.1*b*, are called *equivalent*. Since we want a vector to be determined solely by its length

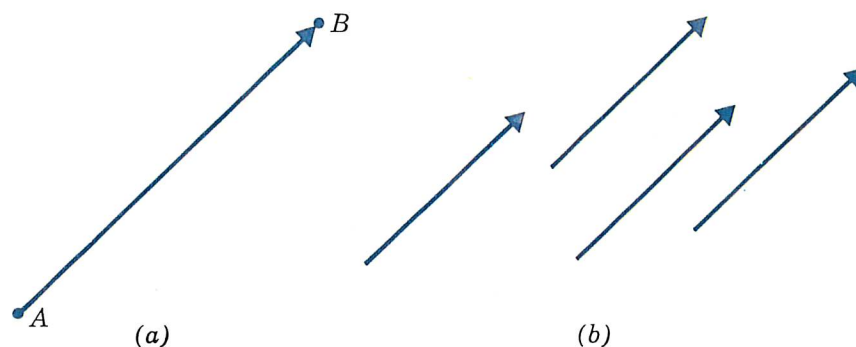


Figure 3.1 (a) The vector $\vec{AB}$. (b) Equivalent vectors.

and direction, equivalent vectors are regarded as *equal* even though they may be located in different positions. If $\mathbf{v}$ and $\mathbf{w}$ are equivalent we write

$$\mathbf{v} = \mathbf{w}$$

Definition. If $\mathbf{v}$ and $\mathbf{w}$ are any two vectors, then the *sum* $\mathbf{v} + \mathbf{w}$ is the vector determined as follows. Position the vector $\mathbf{w}$ so that its initial point coincides with the terminal point of $\mathbf{v}$. The vector $\mathbf{v} + \mathbf{w}$ is represented by the arrow from the initial point of $\mathbf{v}$ to the terminal point of $\mathbf{w}$ (Figure 3.2a).

In Figure 3.3 we have constructed two sums, $\mathbf{v} + \mathbf{w}$ (blue arrows) and $\mathbf{w} + \mathbf{v}$ (white arrows). It is evident that

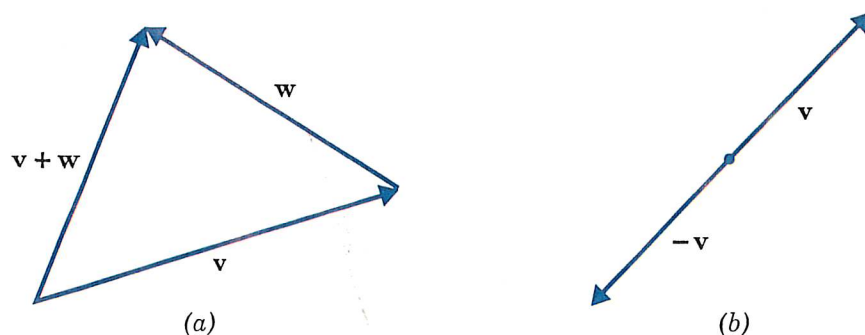
$$\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$$

and that the sum coincides with the diagonal of the parallelogram determined by $\mathbf{v}$ and $\mathbf{w}$ when these vectors are located so they have the same initial point.

A vector of length zero is called a *zero vector* and is denoted by $\mathbf{0}$. We define

$$\mathbf{0} + \mathbf{v} = \mathbf{v} + \mathbf{0} = \mathbf{v}$$

for every vector $\mathbf{v}$. Since there is no natural direction for a zero vector, we shall agree that it can be assigned any direction that is convenient for the problem



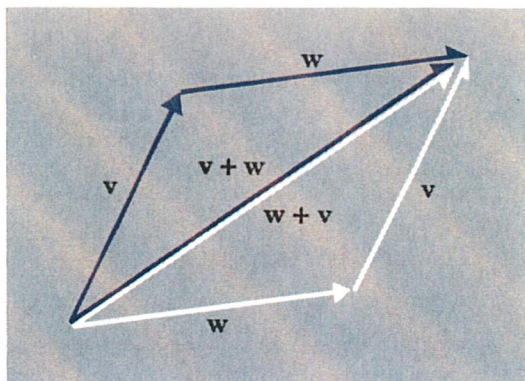


Figure 3.3

being considered. If $\mathbf{v}$ is any nonzero vector, it is obvious that the only vector $\mathbf{w}$ satisfying $\mathbf{v} + \mathbf{w} = \mathbf{0}$ is the vector having the same magnitude as $\mathbf{v}$ but oppositely directed (Figure 3.2b). This vector is called the *negative* (or *additive inverse*) of $\mathbf{v}$ and we write

$$\mathbf{w} = -\mathbf{v}$$

In addition, we define $-\mathbf{0} = \mathbf{0}$.

Definition. If $\mathbf{v}$ and $\mathbf{w}$ are any two vectors, then subtraction is defined by

$$\mathbf{v} - \mathbf{w} = \mathbf{v} + (-\mathbf{w})$$

(Figure 3.4a)

To obtain the difference $\mathbf{v} - \mathbf{w}$ without constructing $-\mathbf{w}$, position $\mathbf{v}$ and $\mathbf{w}$ so their initial points coincide; the vector from the terminal point of $\mathbf{w}$ to the terminal point of $\mathbf{v}$ is then the vector $\mathbf{v} - \mathbf{w}$ (Figure 3.4b).

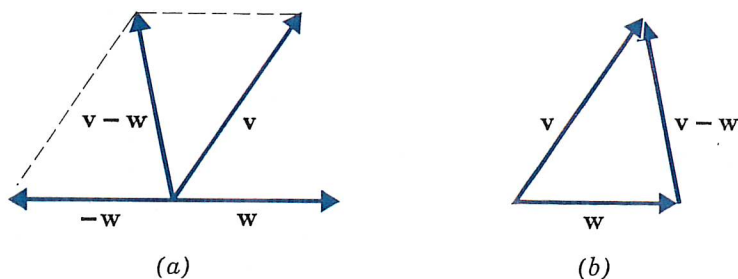


Figure 3.4

Definition. If $\mathbf{v}$ is a vector and k is a real number (scalar), then the *product* $k\mathbf{v}$ is defined to be the vector whose length is $|k|$ times the length of $\mathbf{v}$ and whose direction is the same as that of $\mathbf{v}$ if $k > 0$ and opposite to that of $\mathbf{v}$ if $k < 0$. We define $k\mathbf{v} = \mathbf{0}$ if $k = 0$ or $\mathbf{v} = \mathbf{0}$.

Figure 3.5 illustrates the relation between a vector $\mathbf{v}$ and the vectors $\frac{1}{2}\mathbf{v}$, $(-1)\mathbf{v}$, $2\mathbf{v}$, and $(-3)\mathbf{v}$.

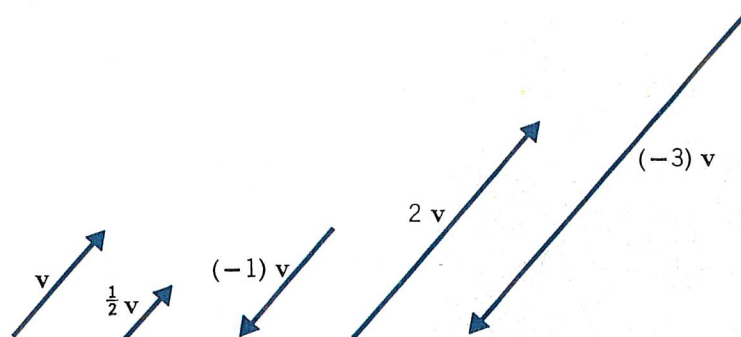


Figure 3.5

Note that the vector $(-1)\mathbf{v}$ has the same length as $\mathbf{v}$ but is oppositely directed. Thus $(-1)\mathbf{v}$ is just the negative of $\mathbf{v}$; that is,

$$(-1)\mathbf{v} = -\mathbf{v}$$

Problems involving vectors can often be simplified by introducing a rectangular coordinate system. For the moment we shall restrict the discussion to vectors in 2-space (the plane). Let $\mathbf{v}$ be any vector in the plane, and assume, as in Figure 3.6, that $\mathbf{v}$ has been positioned so its initial point is at the origin of a rectangular coordinate system. The coordinates (v_1, v_2) of the terminal point of $\mathbf{v}$ are called the *components of $\mathbf{v}$* , and we write

$$\mathbf{v} = (v_1, v_2)$$

If equivalent vectors, $\mathbf{v}$ and $\mathbf{w}$, are located so their initial points fall at the origin, then it is obvious that their terminal points must coincide (since the vectors have the same length and direction). The vectors thus have the same components. It is equally obvious that vectors with the same components must have the same length and same direction, and consequently be equivalent. In summary, two vectors

$$\mathbf{v} = (v_1, v_2) \quad \text{and} \quad \mathbf{w} = (w_1, w_2)$$

are equivalent if and only if

$$v_1 = w_1 \quad \text{and} \quad v_2 = w_2$$

The operations of vector addition and multiplication by scalars are very easy to carry out in terms of components. As illustrated in Figure 3.7, if

$$\mathbf{v} = (v_1, v_2) \quad \text{and} \quad \mathbf{w} = (w_1, w_2)$$

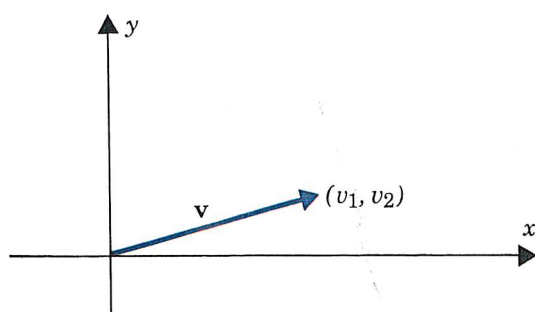


Figure 3.6

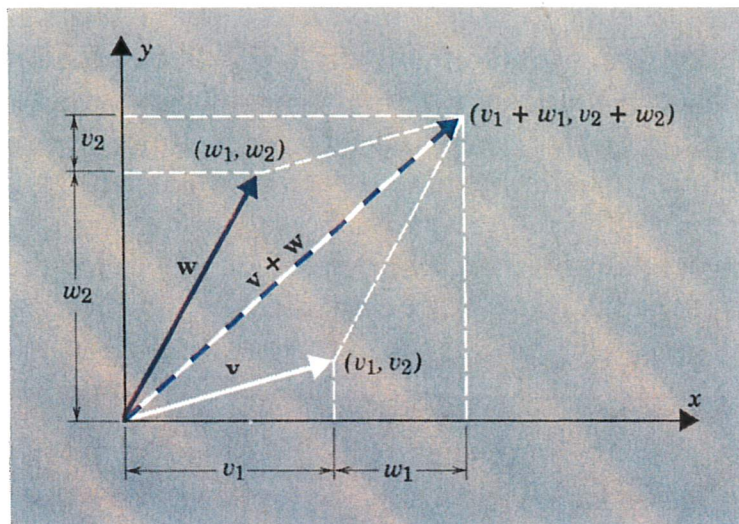


Figure 3.7

then

$$\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2)$$

If $\mathbf{v} = (v_1, v_2)$ and k is any scalar, then by using a geometric argument involving similar triangles, it can be shown (Exercise 14) that

$$k\mathbf{v} = (kv_1, kv_2)$$

(Figure 3.8).

Thus, for example, if $\mathbf{v} = (1, -2)$ and $\mathbf{w} = (7, 6)$, then

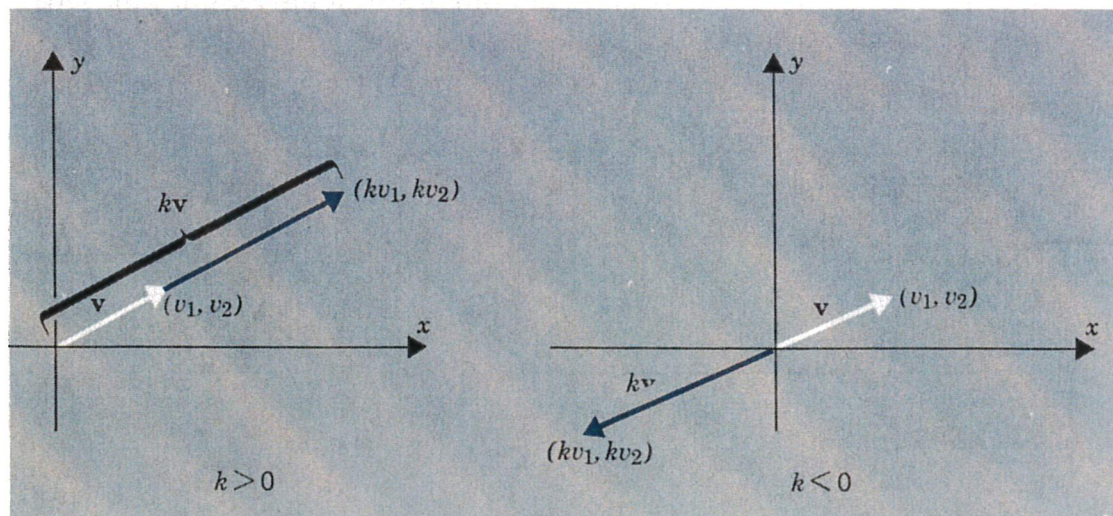
$$\mathbf{v} + \mathbf{w} = (1, -2) + (7, 6) = (1 + 7, -2 + 6) = (8, 4)$$

and

$$4\mathbf{v} = 4(1, -2) = (4(1), 4(-2)) = (4, -8)$$

Just as vectors in the plane can be described by pairs of real numbers, vectors in 3-space can be described by triples of real numbers, by introducing a *rectangular*

Figure 3.8



coordinate system. To construct such a coordinate system, select a point O , called the *origin*, and choose three mutually perpendicular lines, called *coordinate axes*, passing through the origin. Label these axes x , y , and z and select a positive direction for each coordinate axis as well as a unit of length for measuring distances. (Figure 3.9a). Each pair of coordinate axes determines a plane called a *coordinate plane*. These are referred to as the *xy-plane*, the *xz-plane* and the *yz-plane*. To each point P in 3-space we assign a triple of numbers (x, y, z) called the *coordinates of P* as follows. Pass three planes through P parallel to the coordinate planes, and denote the points of intersections of these planes with the three coordinate axes by X , Y , and Z (Figure 3.9b). The coordinates of P are defined to be the signed lengths

$$x = OX \quad y = OY \quad z = OZ$$

In Figure 3.10 we have constructed the points whose coordinates are $(4, 5, 6)$ and $(-3, 2, -4)$.

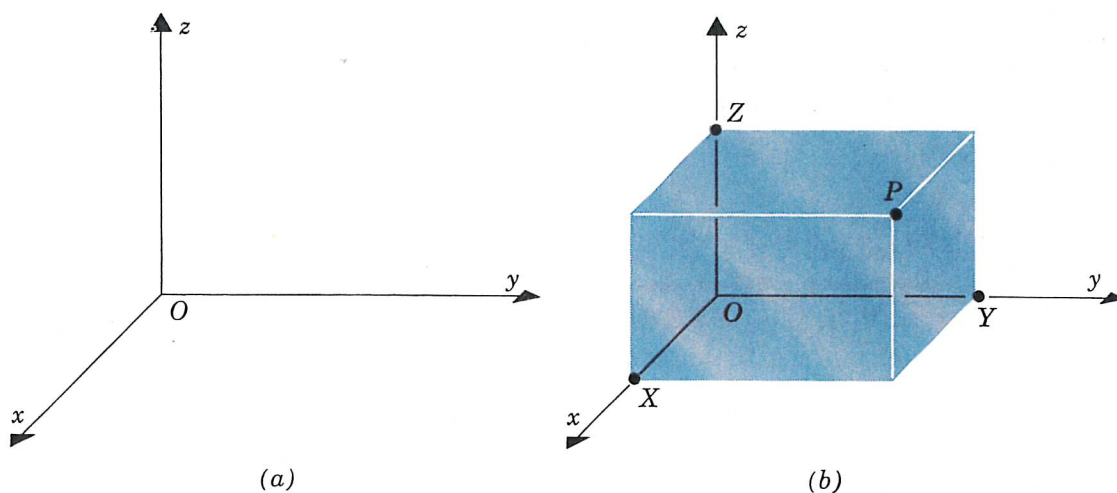
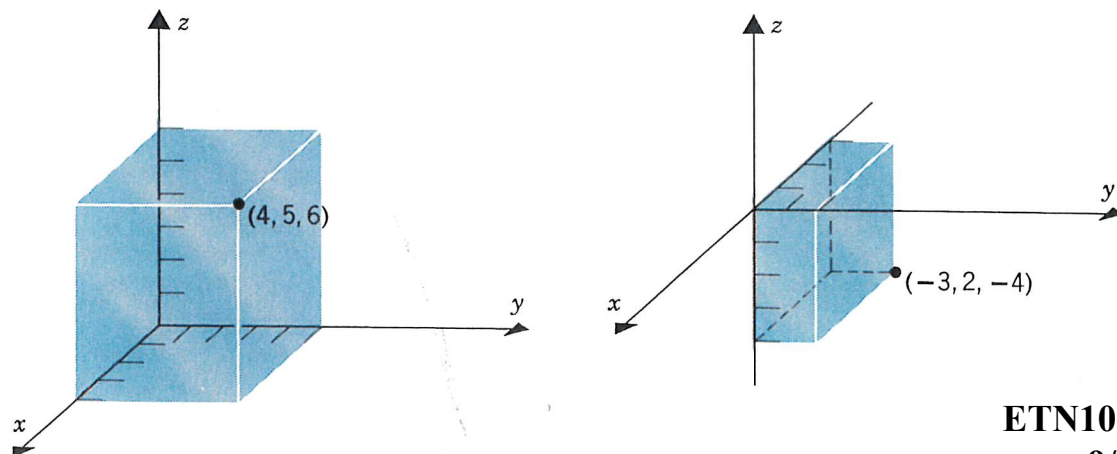


Figure 3.9

Figure 3.10



Rectangular coordinate systems in 3-space fall into two categories, *left-handed* and *right-handed*. A right-handed system has the property that an ordinary screw pointed in the positive direction on the z -axis would be advanced if the positive x -axis is rotated 90° toward the positive y -axis (Figure 3.11a). The system is left-handed if the screw would retract (Figure 3.11b).

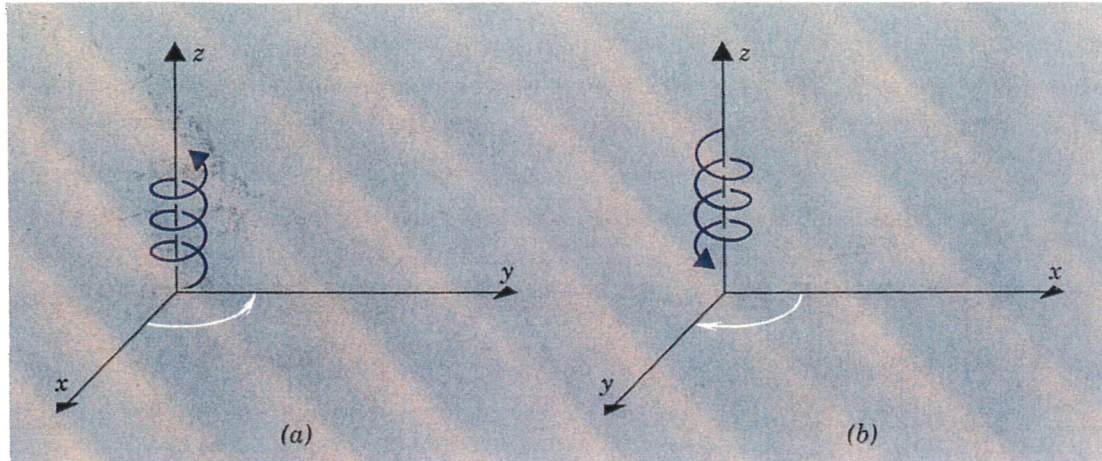


Figure 3.11 (a) Right-handed. (b) Left-handed.

In this book we shall use only right-handed coordinate systems.

If, as in Figure 3.12, a vector $\mathbf{v}$ in 3-space is located so its initial point is at the origin of a rectangular coordinate system, then the coordinates of the terminal point are called the *components* of $\mathbf{v}$ and we write

$$\mathbf{v} = (v_1, v_2, v_3)$$

If $\mathbf{v} = (v_1, v_2, v_3)$ and $\mathbf{w} = (w_1, w_2, w_3)$ are two vectors in 3-space, then arguments similar to those used for vectors in a plane can be used to establish the following results.

- (i) $\mathbf{v}$ and $\mathbf{w}$ are equivalent if and only if $v_1 = w_1$, $v_2 = w_2$ and $v_3 = w_3$
- (ii) $\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2, v_3 + w_3)$
- (iii) $k\mathbf{v} = (kv_1, kv_2, kv_3)$ where k is any scalar

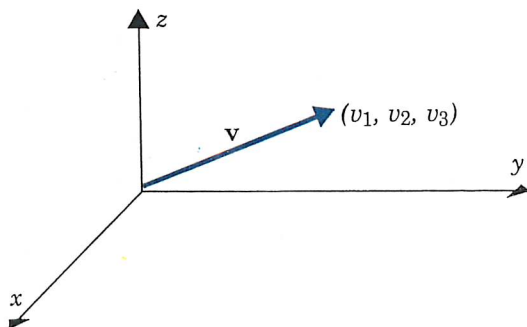


Figure 3.12

Example 1

If $\mathbf{v} = (1, -3, 2)$ and $\mathbf{w} = (4, 2, 1)$, then

$$\mathbf{v} + \mathbf{w} = (5, -1, 3), \quad 2\mathbf{v} = (2, -6, 4), \quad -\mathbf{w} = (-4, -2, -1),$$

$$\mathbf{v} - \mathbf{w} = \mathbf{v} + (-\mathbf{w}) = (-3, -5, 1).$$

Sometimes vectors arise that do not have their initial points at the origin. To find the components of a vector $\mathbf{v}$ having initial point $P_1(x_1, y_1, z_1)$ and terminal point $P_2(x_2, y_2, z_2)$, we construct an equivalent vector whose initial point is at the origin. In Figure 3.13 $\overrightarrow{OQ}$ is such a vector. Therefore, the components of $\mathbf{v} = \overrightarrow{P_1P_2}$ are the coordinates (a, b, c) of the point Q .

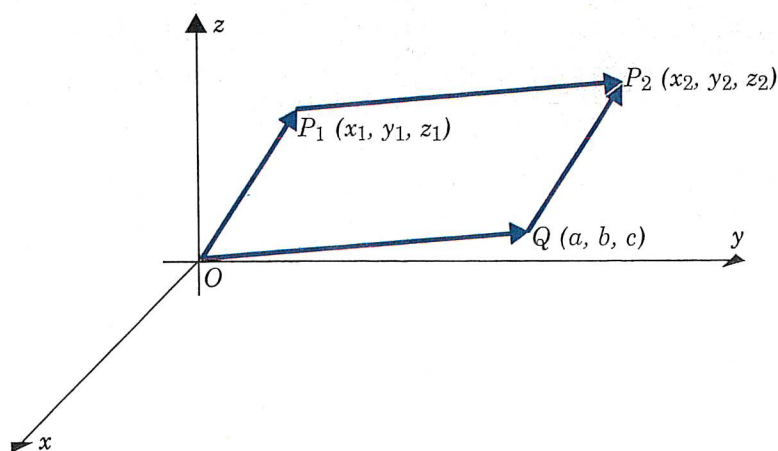


Figure 3.13

From Figure 3.13, $\overrightarrow{OQ} + \overrightarrow{OP_1} = \overrightarrow{OP_2}$, or in terms of components

$$(a, b, c) + (x_1, y_1, z_1) = (x_2, y_2, z_2)$$

or

$$(a + x_1, b + y_1, c + z_1) = (x_2, y_2, z_2)$$

By equating corresponding components and solving for a , b , and c , we find that the components (a, b, c) of $\mathbf{v} = \overrightarrow{P_1P_2}$ are given by

$$a = x_2 - x_1 \quad b = y_2 - y_1 \quad c = z_2 - z_1$$

Example 2

The components of the vector $\mathbf{v} = \overrightarrow{P_1P_2}$ with initial point $P_1(2, -1, 4)$ and terminal point $P_2(7, 5, -8)$ are

$$\mathbf{v} = (7 - 2, 5 - (-1), (-8) - 4) = (5, 6, -12)$$

Analogously, in 2-space the vector $\mathbf{v}$ with initial point $P_1(x_1, y_1)$ and terminal point $P_2(x_2, y_2)$ is $\mathbf{v} = (x_2 - x_1, y_2 - y_1)$.

Example 3

The solutions to many problems can be simplified by translating the coordinate axes to obtain new axes parallel to the original ones.

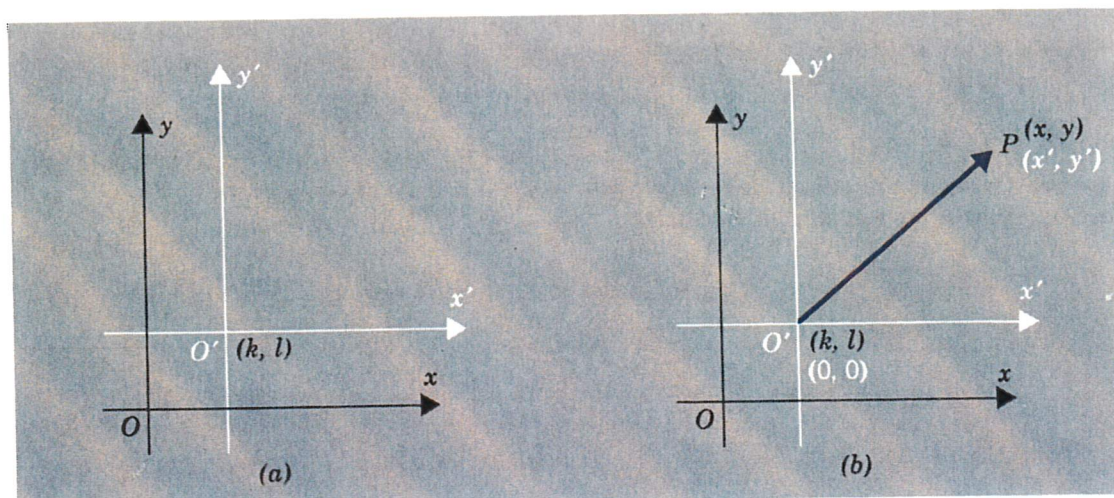


Figure 3.14

In Figure 3.14a we have translated the xy -coordinate axes to obtain an $x'y'$ -coordinate system whose origin O' is at the point $(x, y) = (k, l)$. A point P in 2-space now has both (x, y) coordinates and (x', y') coordinates. To see how the two are related, consider the vector $\overrightarrow{O'P}$ (Figure 3.14b). In the xy -system its initial point is at (k, l) and its terminal point is at (x, y) ; thus $\overrightarrow{O'P} = (x - k, y - l)$. In the $x'y'$ -system its initial point is at $(0, 0)$ and its terminal point is at (x', y') ; thus $\overrightarrow{O'P} = (x', y')$. Therefore

$$x' = x - k \quad y' = y - l$$

These are called the **translation equations**.

To illustrate, if the new origin is at $(k, l) = (4, 1)$ and the xy -coordinates of a point P are $(2, 0)$, then $x'y'$ -coordinates of P are $x' = 2 - 4 = -2$ and $y' = 0 - 1 = -1$.

In 3-space the translation equations are

$$x' = x - k \quad y' = y - l \quad z' = z - m$$

where (k, l, m) are the xyz -coordinates of the new origin.

EXERCISE SET 3.1

- Draw a right-handed coordinate system, and locate the points whose coordinates are:

(a) $(2, 3, 4)$	(b) $(-2, 3, 4)$	(c) $(2, -3, 4)$	(d) $(2, 3, -4)$
(e) $(-2, -3, 4)$	(f) $(-2, 3, -4)$	(g) $(2, -3, -4)$	(h) $(-2, -3, -4)$
(i) $(0, 2, 0)$	(j) $(0, 0, -2)$	(k) $(2, 0, 2)$	(l) $(-2, 0, 0)$
- Sketch the following vectors with the initial points located at the origin:

(a) $\mathbf{v}_1 = (2, 5)$	(b) $\mathbf{v}_2 = (-3, 7)$	(c) $\mathbf{v}_3 = (-5, -4)$
(d) $\mathbf{v}_4 = (6, -2)$	(e) $\mathbf{v}_5 = (2, 0)$	(f) $\mathbf{v}_6 = (0, -8)$
(g) $\mathbf{v}_7 = (2, 3, 4)$	(h) $\mathbf{v}_8 = (2, 0, 2)$	(i) $\mathbf{v}_9 = (0, 0, -2)$