

Lawrence E. Murr

Handbook of Materials Structures, Properties, Processing and Performance

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With 784 Figures and 43 Tables

 **Springer** Reference

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*To Pat, my wife and soul mate
of more than 55 years at this writing,
and Brayden Lawrence Blanks,
our great grandson.*

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Preface

This handbook is an attempt at innovation as it applies to contemporary and evolving materials science and engineering principles and their many applications. It is intended to be interdisciplinary and multidisciplinary as these apply generally to the realm of science and technology encumbered by materials science and engineering in its broadest sense. This work could have been titled many other broader or specific terms, although handbook itself invokes many interrelated definitions which in fact provide a broader context as it applies to materials structures, properties, processing, and performance. For example, consider the following *handbook* definitions:

- A type of reference work or guide book.
- A treatise on a special subject.
- A concise manual or reference book providing specific information or instruction about a subject.
- Comprehensive and detailed work on a particular topic for practitioners, Structured for quick reference and often used as a supplement to a text book.

This handbook ambitiously strives to be all of these. In addition, it can not only serve as a supplement to some specific text, but it can also be configured as a text by selecting specific parts and chapters to be configured for an appropriate course at the graduate or advanced level. While many fundamental topics are briefly reviewed in order for the reader or user (instructor or facilitator) to better appreciate some topic areas, especially what might be considered more advanced materials or materials science and engineering concepts, most of this work is not intended to be used as any configuration of principles of, fundamentals of, or introduction to materials science or materials science and engineering. It is intended to provide specifics as well as an overview of many contemporary materials and materials science and engineering–related topic areas and research areas, especially those which might suggest evolving or future areas relating to advanced materials and manufacturing processes and technologies. In this context, this *handbook* is intended as a concise guide to contemporary materials science and engineering for the technical professional, especially those involved in leading-edge research and development as well as technology development and forecasting in the context

of future product directions involving aspects of advanced materials structures, properties, processing, and performance. These are in fact expanded to ultimately include materials architectures and materials design as it is related to materials simulations and more generally computational issues involving materials science and engineering.

Consistent with these strategies and innovations implicit in the broader context of this Handbook, this work will at first appear as a print and online edition (as an eBook on SpringerLink), and in addition an updated/updatable version will be available on www.SpringerReference.com.

This work in some respects represents an effort encompassing a half-century since it incorporates a wide range of the author's research relating to materials fundamentals dating to the mid-1960s. It incorporates many stages of class notes over at least the past two decades used in graduate courses in materials science and engineering, especially as a discipline-leveling course to accommodate graduate students in a wide range of science and engineering disciplines as well as materials and materials science and engineering. In this sense, I am indebted to many hundreds of students and colleagues who have contributed to this work in various ways, many of whom are acknowledged in the text or in figure credits and references from which much of the work has been adapted. This includes hundreds of undergraduate and graduate student researchers who have contributed to publishing many hundreds of papers in the international literature and who have been a part of my extended academic family for nearly half a century.

I am especially indebted to my administrative assistant of more than two decades, Faye Ekberg, who typed much of the initial manuscript, and Pamela Kuchle and Abelardo Castrejon, undergraduate student assistants who scanned the many illustrations and made many corrections and additions to the final manuscripts, especially those tendered by my Editorial Advisory Board: Drs. Marc Meyers, Naresh Thadhani, Shujun Li, Wenjun Zhu, and Sridhar Pappu. These represent colleagues, students, and long-standing friends who go back more than four decades. Finally, I must acknowledge the help of Michael Luby, Publishing Editor, Engineering, Springer Science + Business Media (New York), Lydia Mueller, Springer Publishing Product Development Senior Editor, Major Reference Works (Heidelberg), and Karin Bartsch, Springer Publishing Product Development Associate Editor MRW (Heidelberg), who helped to configure my initial proposal and provided guidance and support in producing the final Handbook in its various forms.

L. E. Murr

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Lattice Directions and Planes, and Diffraction by Crystals

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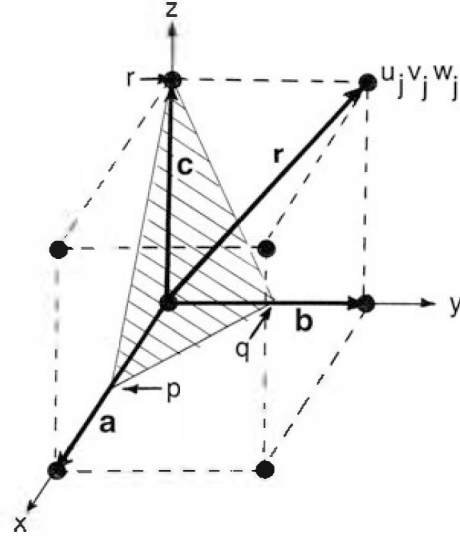
Abstract

Chapter “► [Lattice Directions and Planes, and Diffraction by Crystals](#)” reviews Miller index notation for crystal planes and directions as a precursor to establishing simple principles for diffraction by crystals. Bragg’s law establishes diffraction geometry, while atomic scattering from specific atoms and structure factors determine the amplitudes of diffraction depending upon the atomic species and their crystal or lattice structure. The concept of the Ewald sphere and a reciprocal lattice as it relates to diffraction and diffraction amplitudes is also presented.

Introduction

The notations for lattice (or crystal) directions and planes (correspondingly atomic planes in crystals) are an important feature of crystallography as applied to orientations within crystals or crystal geometries. With reference to Fig. 1 which is somewhat specific to orthogonal lattices, ($\alpha = \beta = \gamma = 90^\circ$) can be generally applied to unit cells in the context of ► [Fig. 1c of chapter “Crystallography Principles.”](#) In Fig. 1, the position vector $\mathbf{r}$ directed from the origin at 000 through any lattice point within the unit cell (dotted) with position coordinates u_j, v_j, w_j is given by

Fig. 1 Notations for directions and planes in a lattice unit cell. The j th atom at the unit cell corner is denoted $u_j v_j w_j$. Shaded plane has intercepts along $\mathbf{a}$ (x), $\mathbf{b}$ (y), and $\mathbf{c}$ (z) denoted p, q, r



$$\mathbf{r} = u_j \mathbf{a} + v_j \mathbf{b} + w_j \mathbf{c} \quad (1)$$

referred to as a real lattice vector, which characterizes a corresponding direction $[uvw]$, which is the direction of a line from the origin to the lattice point, or a unit cell atom position in the crystal unit cell. Because unit cell dimensions are based on $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$ (unity), points or atoms located at fractions of the unit cell dimensions with fractional coordinates are reduced to the least common denominator, and the numerator assumes the designated direction indices: the face-centered atom in the unit cell base in Fig. 1 would have position coordinates $\left[\frac{1}{2} \frac{1}{2} 0\right]$ or $[110]$. Other directions of this form, $[101]$ and $[011]$, are designated directions of a form: $\langle 110 \rangle$. That is, specific directions are denoted $[uvw]$, while directions of this form are denoted $\langle uvw \rangle$.

Planes in the lattice are determined by $(hkl) = h : k : l = \left(\frac{1}{p}, \frac{1}{q}, \frac{1}{r}\right)$ (► Fig. 6 of chapter “Electromagnetic Color and Color in Materials”), and as in the case of directions, for planes outside the unit cell where p, q , and r can be >1 , the indices are reduced to the lowest common denominator, and only the numerator expresses the indices of the plane, (hkl) . This notation was devised by William Miller in 1839, and the indices h, k, l are commonly referred to as the Miller indices. In ► Fig. 6 of chapter “Electromagnetic Color and Color in Materials,” if $p = q = \frac{1}{2}$, $r = 1$, the plane is denoted (Eq. 1). Correspondingly planes of this form $((212), (122)$, and negative index planes) are denoted by planes of a form: $\{hkl\}$, since for cubic crystals implicit in Fig. 1, planes are perpendicular to directions and have equivalent indices. It is common practice to denote both planes and directions by (hkl) : $[hkl]$ and $\{hkl\}$: $\langle hkl \rangle$.

In the case of directions, negative indices can characterize any coordinate in the unit cell equivalents in the four positive quadrants ($+z$) and four negative quadrants ($-z$). For planes and directions in cubic crystals, the indices for negative

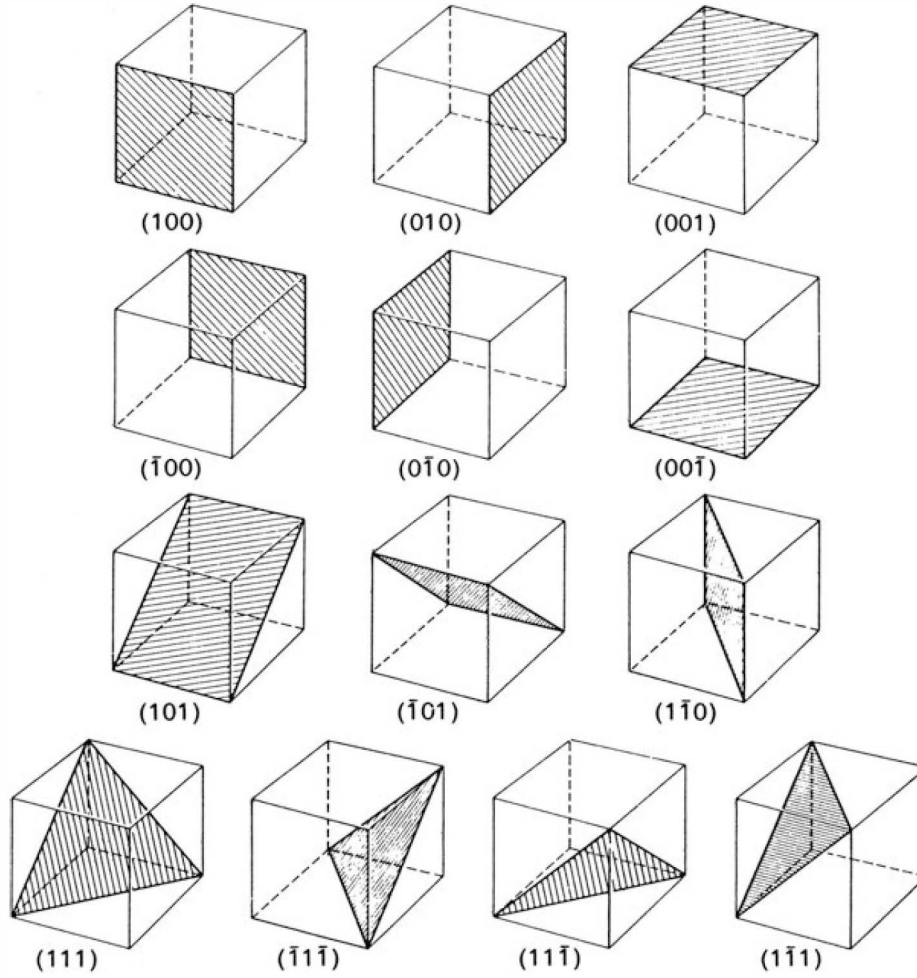


Fig. 2 Some common crystal planes in cubic crystals, including those corresponding to negative indices

coordinates or intercepts are designated with a bar above the index: $[110]$ has a negative (or opposite direction) denoted $[\bar{1}\bar{1}0]$. Some examples of crystal plane designations, including negative indices for cubic unit cells, are illustrated in Fig. 2.

From the cube (orthogonal) geometry illustrated in the convention of Fig. 1, it can be observed that the distance between planes of the same form is given by

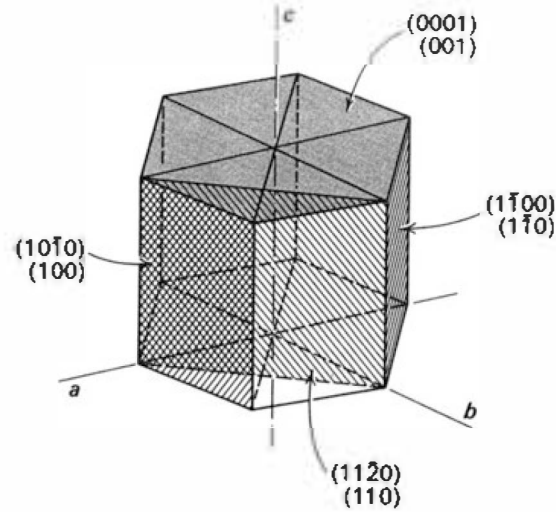
$$d_{hkl}(\text{orthogonal}) = \frac{1}{\sqrt{(h/a)^2 + (k/b)^2 + (l/c)^2}} \quad (2)$$

which for cubic crystals where $a = b = c$ becomes

$$d_{hkl}(\text{cubic}) = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad (3)$$

Similarly, we can find that the angle between directions, θ , is given by

Fig. 3 Crystal planes in the hexagonal system



$$\theta = \cos^{-1} \left[\frac{h_1 h_2 + k_1 k_2 + l_1 l_2}{\sqrt{h_1^2 + k_1^2 + l_1^2} \sqrt{h_2^2 + k_2^2 + l_2^2}} \right] \quad (4)$$

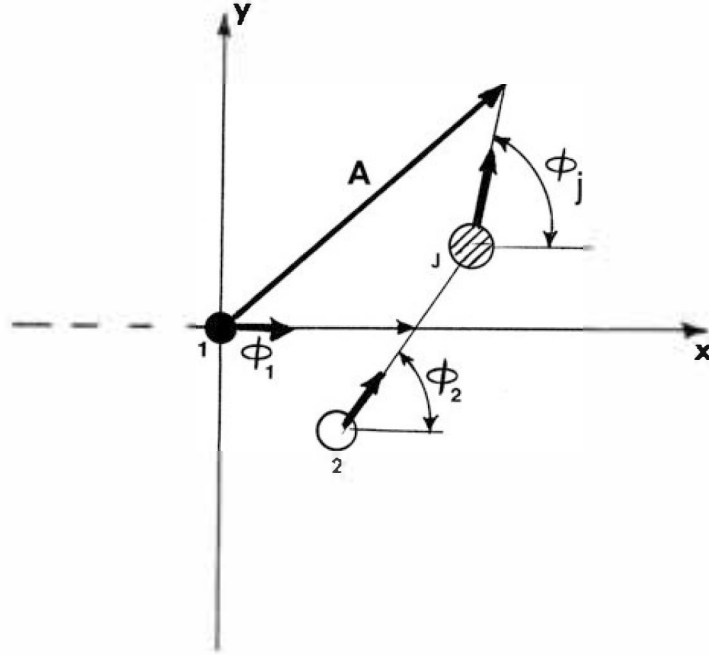
for $[h_1 k_1 l_1]$ and $[h_2 k_2 l_2]$ in a cubic crystal. Since in cubic crystals planes and directions are mutually perpendicular, the angle between two planes, $(h_1 k_1 l_1)$ and $(h_2 k_2 l_2)$, is given by Eq. 4. More general relationships for other crystal unit cells, including other orthogonal unit cells (► Fig. 2 of chapter “Crystallography Principles”), can be found in Murr (1991).

As shown in the periodic table of ► Fig. 4 of chapter “Crystallography Principles,” many elemental metals have the hcp (A3) crystal structure. Indices based on the three axes, a , b , c in Fig. 1 using the Miller index notation are frequently used for hexagonal crystals, but all equivalent planes do not have similar indices, and the same can apply to directions. This can be avoided by using a four-index notation: $(hkil)$, where $i = -(h + k)$. Figure 3 illustrates this index feature for hexagonal crystal unit cells, often called the Miller-Bravais notation.

Principles of Diffraction by Crystals

Prior to the development of diffraction principles by W. H. and W. L. Bragg around 1912, culminating in Bragg’s law (► Eq. 4 of chapter “A Brief Introduction to Quantum Mechanics”) for X-ray reflection from crystal planes of atoms, the geometry and dimensions of crystal unit cells were unknown. But Bragg’s law only relates the angle of reflection (or diffraction) of X-rays and other radiation to the interplanar spacing of atoms (Eq. 2) providing dimensions or lattice parameters but no information regarding atom positions specific to the Bravais lattices (► Fig. 2 of chapter “Crystallography Principles”) and crystal structures unique to specific space groups (► Table 1 of chapter “Crystallography Principles”). The fundamentals of the relationship between atom positions in the unit cells and diffraction intensities are

Fig. 4 Diffraction amplitude from a collection (or volume) of atoms (j) each scattering at a phase angle ϕ_j as represented in a simple 2D (x - y) geometry



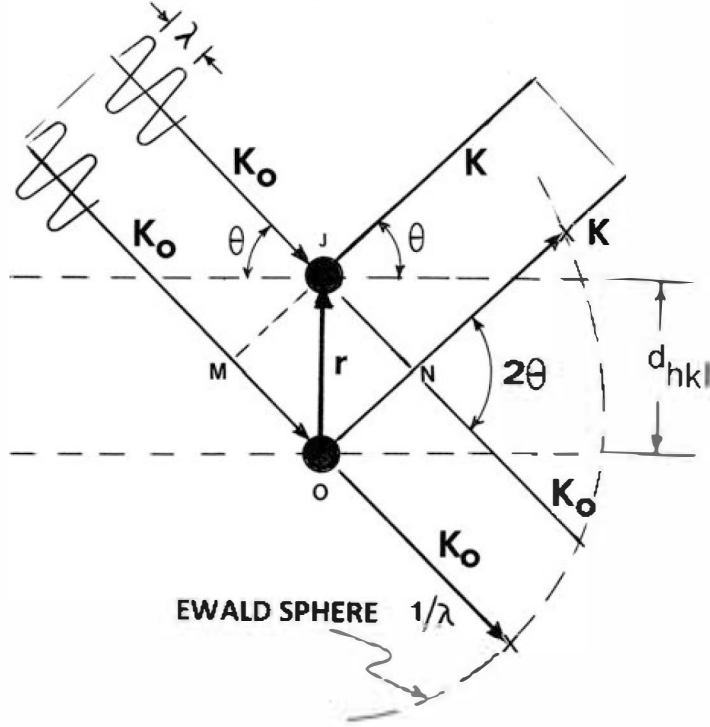
implicit in the structure factor, a mathematical algorithm which relates real lattice points or atom positions in the crystal unit cell with intensity profiles at points representing a reciprocal lattice. This is a mathematical construct which relates the intensities and positions of the diffracted beams to atom positions and crystallography as well as the real sizes of crystalline matter, including the periodicity or range of periodicity of unit cells. In this context, we will present a brief survey of diffraction principles and fundamentals.

Radiation in the form of X-ray photons, electrons, or neutrons composing a beam or diagnostic probe will be selectively scattered by atoms in a diffracting volume of matter. This scattering depends upon the atom (specifically the atomic number, Z), the concentration of specific atoms, and their positions in the unit cell or cells composing the diffracting volume. Each atom scatters some amplitude of radiation determined by an atomic scattering factor, $f_j(\mu)$ for the j th atom represented in a random collection of different atoms shown schematically in Fig. 4. Each atom in Fig. 4 scatters some amplitude at a phase angle represented by ϕ_j for the j th atom. These scattering amplitudes produce diffracted beams having a resultant amplitude represented by

$$A = \sum_j f_j(\mu) \exp(i\phi_j) \quad (5)$$

If we superimpose the Bragg diffraction geometry on Fig. 4, we can consider only two atoms characterizing two crystal planes separated by an interplanar spacing, d_{hkl} , which represents the scalar value of the real lattice vector, $\mathbf{r}$, represented in Fig. 1 (and Eq. 1). This diffraction geometry is represented in Fig. 5. The incident beam of radiation is denoted by a unit vector $\mathbf{K}_0$, while the

Fig. 5 Vector geometry for diffraction from a crystal unit cell (Fig. 1). Note the Ewald sphere boundary having a radius of $1/\lambda$ in reciprocal space



resultant, diffracted beam is denoted by a unit vector, $\mathbf{K}$. Considering the diffraction geometry in Fig. 5, the path difference is given by

$$\Delta = \overline{OM} + \overline{ON} = \mathbf{r} \cdot (\mathbf{K} - \mathbf{K}_o) \quad (6)$$

and the corresponding phase difference is given by

$$\phi_j = \frac{2\pi}{\lambda} \Delta = \frac{2\pi}{\lambda} \mathbf{r} \cdot (\mathbf{K} - \mathbf{K}_o) \quad (7)$$

where it can be recognized from the scalar form of Bragg's law that

$$\Delta = 2d_{hkl} \sin\theta = \lambda \quad (8)$$

Consequently, the condition for diffraction is stipulated for $\Delta = \lambda$ or $\phi_j = 2\pi$. Examination of Eq. 7 with these conditions for diffraction results in

$$\frac{1}{\lambda} (\mathbf{K} - \mathbf{K}_o) = \frac{1}{\mathbf{r}} \quad (9)$$

which implies that the incident beam and diffracted beam have scalar values given by $1/\lambda$ which is self-consistent with the right-hand side of Eq. 9: $1/\mathbf{r}$ whose scalar value is $1/d_{hke}$, implicit in Fig. 5. These reciprocal units necessitate a reciprocal lattice construct which can be fundamentally defined by

$$\mathbf{a}^* = 1/\mathbf{a}; \quad \mathbf{b}^* = 1/\mathbf{b}; \quad \mathbf{c}^* = 1/\mathbf{c} \quad (10)$$

or correspondingly

$$\mathbf{a}^* \cdot \mathbf{a} = 1 = \mathbf{b}^* \cdot \mathbf{b} = \mathbf{c}^* \cdot \mathbf{c}. \quad (11)$$

It follows that if the real crystal unit cell has a volume

$$V_c = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} = (\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b} \quad (12)$$

then the reciprocal lattice vectors can be expressed by

$$\mathbf{a}^* = (\mathbf{b} \times \mathbf{c})/V_c; \quad \mathbf{b}^* = (\mathbf{a} \times \mathbf{c})/V_c; \quad \mathbf{c}^* = (\mathbf{a} \times \mathbf{b})/V_c \quad (13)$$

In the construction of the reciprocal lattice unit cell as shown in ► [Fig. 11 of chapter “Electromagnetic Color and Color in Materials,”](#) the reciprocal lattice vector $(1/\mathbf{r})$ is expressed by

$$1/\mathbf{r} = \mathbf{g} = h\mathbf{a}^* + k\mathbf{b}^* + l\mathbf{c}^* = \mathbf{g}_{hkl} \quad (14)$$

where $\mathbf{g}$ or $\mathbf{g}_{hkl}$ is referred to as a reciprocal lattice vector, and we can rewrite Eq. 9 as

$$(\mathbf{K} - \mathbf{K}_0) = \lambda \mathbf{g} \text{ or } \mathbf{g} = \frac{1}{\lambda} (\mathbf{K} - \mathbf{K}_0). \quad (15)$$

This illustrates that, as shown in [Fig. 5](#), diffraction is satisfied when the Ewald sphere (with radius $1/\lambda$) passes through a reciprocal lattice point (shown by $\mathbf{x}$ in [Fig. 5](#)).

The Structure Factor

The amplitude of diffraction is now considered from a unit cell by considering Eq. 6 in the form

$$F(hkl) = A = \sum_j f_j(\mu) e^{i\phi_j} \quad (16)$$

or from Eqs. 8 and 14

$$F(hkl) = \sum_j f_j(\mu) e^{2\pi i \mathbf{r}_j \cdot \mathbf{g}}, \quad (17)$$

or

$$F(hkl) = A = \sum_j f_j(\mu) e^{2\pi i (h u_j + k v_j + l w_j)} \quad (18)$$

where Eq. 18 is referred to as the crystal structure factor.

For X-rays, the atomic scattering factor is expressed generally by

$$F_{x(j)}(\mu) = \int_0^\infty 4\pi r^2 \rho(r) \frac{\sin \mu r}{\mu r} dr \quad (19)$$

where

$$\mu = \frac{4\pi \sin \theta}{\lambda} = 2\pi |g| = 2\pi g \quad (20)$$

and $\rho(r)$ is related to the electron density of the atom (the atomic number, Z).

For electrons,

$$f_{e(j)}(\mu) = \frac{8\pi^2 m e^2}{h^2} \left(\frac{Z - f_{x(j)}(\mu)}{\mu^2} \right) \quad (21)$$

or

$$f_{e(j)}(\mu) = \frac{m}{2} \left(\frac{e}{h} \right)^2 \left(\frac{Z - f_{x(j)}(\mu)}{(\sin \theta / \lambda)^2} \right). \quad (22)$$

Values of the atomic scattering factor for X-rays ($f_{x(j)}(\mu)$) and electrons ($f_{e(j)}(\mu)$) have been tabulated as a function of $\sin \theta / \lambda$ for atomic number, Z (Barrett and Massalski 1966; Murr 1991; Vainshtein 1964).

The structure factor in Eq. 18 can be interrogated for any crystal structure or crystal unit cell in order to determine amplitudes of diffraction for each diffracting plane (hkl). It can be observed in Fig. 7 that diffraction from crystal planes $\{hkl\}$ in the real crystal is represented by diffraction vectors ideally perpendicular to these planes by points in the reciprocal lattice, which have position coordinates h, k, l in contrast to position coordinates of atoms in the real crystal lattice: u_j, v_j, w_j . This is a consequence of the cross products of real lattice vectors as defined for the reciprocal lattice in Eq. 13, with reference to Fig. 6. Correspondingly, crystal planes, $\{hkl\}$, in the real lattice become points in the reciprocal lattice or vectors perpendicular to these planes, especially for cubic crystals.

It can be easily demonstrated that there are no diffraction extinctions for simple cubic crystals (or unit cells), and all planes will diffract since $F(hkl) = f_j(\mu)$. For bcc crystal unit cells, the body-centered atom has coordinates $\frac{1}{2} \frac{1}{2} \frac{1}{2}$, and the structure factor becomes

$$F(hkl)_{bcc} = f_j(\mu) \left[1 + e^{\pi i(h+k+l)} \right] \quad (23)$$

(using 000 for u_j, v_j, w_j for corner atom symmetry and $8 \times 1/8$ atoms per corner, etc.)

Therefore, it is observed that if $(h + k + l)$ is an odd integer, the structure factor will be zero, and the intensity of diffraction from corresponding crystal planes will be zero:

$$|F(hkl)|^2 = 0 \text{ for bcc} \quad (24)$$

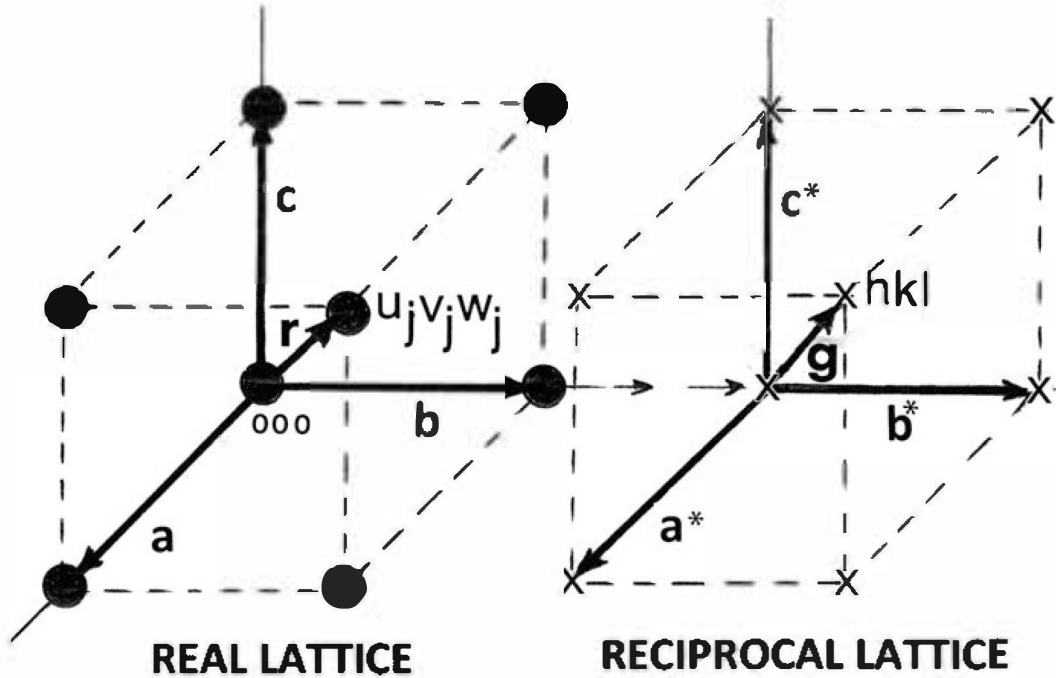


Fig. 6 Comparison of the real (crystal) lattice unit cell with the reciprocal lattice

when $(h + k + l) = 1, 3, 5, \dots$

For fcc crystal unit cells, the structure factor becomes (for corner and face atoms: $\frac{1}{2}, \frac{1}{2}, 0$, etc.)

$$F(hkl)_{\text{fcc}} = f_j(\mu) \left[1 + e^{\pi i(h+l)} + e^{\pi i(k+l)} + e^{\pi i(h+k)} \right]. \quad (25)$$

Consequently, for diffraction to be extinct ($F(hkl)_{\text{fcc}} = 0$), $(h + l)$, $(k + l)$, and $(h + k)$ present mixed even and odd integer conditions.

The diffraction conditions implicit in Eqs. 23 and 25 for bcc and fcc crystal unit cells apply to single atom species. For ionic (bcc) crystal unit cells such as the CsCl unit cell in ► Fig. 3 of chapter “Crystallography Principles,” the structure factor becomes

$$F(hkl)_{\text{CsCl}} = f_{\text{Cs}}(\mu) + f_{\text{Cl}}(\mu)e^{\pi i(h+k+l)} \quad (26)$$

which produces different diffraction intensities for specific, diffracting planes $\{hkl\}$.

Table 1 summarizes the order of diffraction planes for bcc, fcc, diamond cubic, and hcp crystal structures of the same atomic species. These correspond to increasing values of the reciprocal lattice vectors which for cubic crystals would be

$$g = \sqrt{h^2 + k^2 + l^2}/a = 1/d_{hkl} \quad (27)$$

Table 1 Allowed diffraction planes {hkl} for single-species crystal unit cells shown

bcc {hkl}	fcc {hkl}	Diamond {hkl}	hcp {hkl}
110	111	111	100
200	200	220	002
211	220	311	101
220	311	400	102
310	222	331	110
322	400	422	103
321	331	511, 333	200
400	420	440	112
411, 330	422	531	201
420	511, 333	620	004
332	440	533	202
422	531	444	104
510, 431	600, 442	711, 551	203
521	620	642	210
440	533	553	211

Ideally, this sequence of diffracting planes would correspond to diffraction peaks in X-ray diffractometer spectra or the sequence of diffraction rings in electron diffraction patterns for small crystal or particle sizes. These features are illustrated in Fig. 8 for nickel which is fcc with a lattice parameter, $a = 3.52 \text{ \AA}$. Note the correspondence between the X-ray diffraction (XRD) peaks in Fig. 7a (marked 1 to 5) and the electron diffraction rings in Fig. 8b (marked 1 to 5). In Fig. 7a, the diffracting planes and lattice parameter correspond to the interplanar spacing (in Eq. 3) where

$$d_{hkl} = \lambda / 2 \sin \theta \quad (28)$$

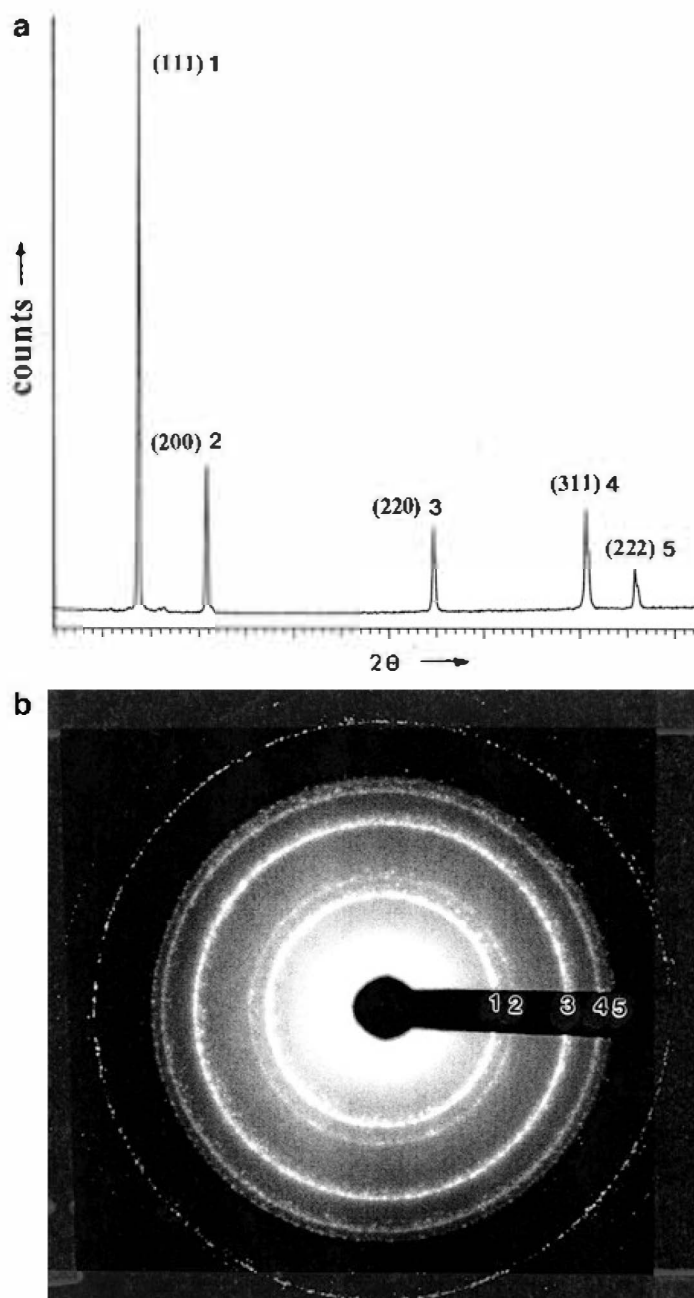
and λ is the X-ray wavelength for X-rays produced from a copper target (Cu-K α X-rays) and equal to $1.54 \text{ \AA}$. In Fig. 7b, the diffracting planes and lattice parameter are related to the same interplanar spacing:

$$d_{hkl} = \lambda L / R, \quad (29)$$

where λ is the electron wavelength in the electron microscope operating at 300 kV and equal to $0.2 \text{ \AA}$. L in Eq. 29 is the electron microscope camera length (in centimeters), and λL is commonly referred to as the *camera constant*, determined by calibrating the instrument with a known standard (such as a nickel thin film). R in Eq. 29 is the radius of the diffraction rings in Fig. 7b measured in centimeters.

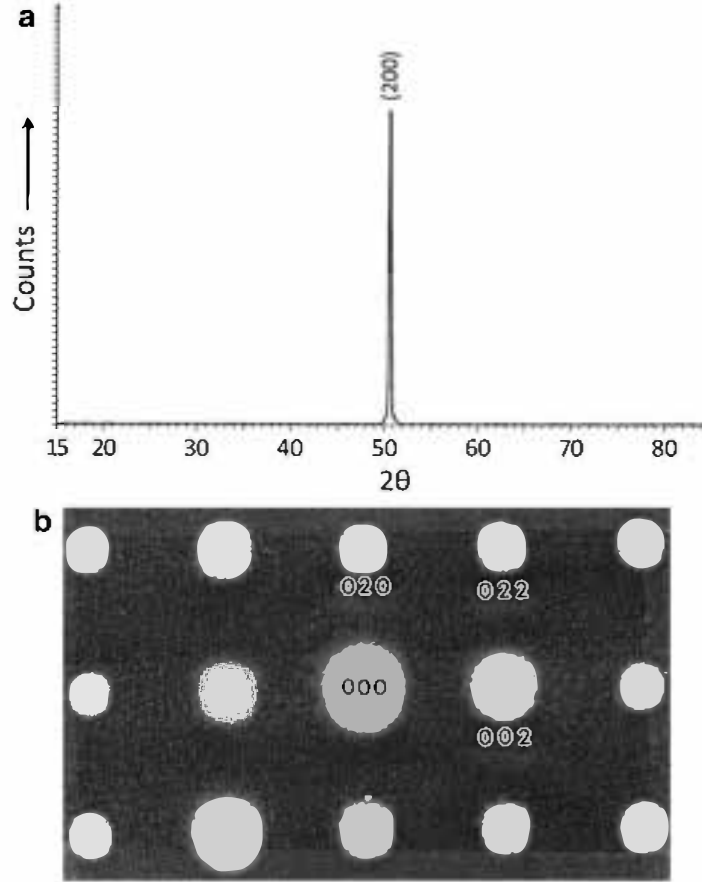
In contrast to diffraction from a powder mixture having random orientations as shown by XRD in Fig. 7a or a polycrystalline thin film as in Fig. 7b for electron diffraction (referred to as selective-area electron diffraction or SAED), diffraction

Fig. 7 Diffraction from random particles and polycrystals of nickel. **(a)** XRD spectrum using Cu-K α x-radiation. **(b)** Electron diffraction pattern from a polycrystalline thin foil in the TEM operating at 300 kV. Diffracting planes are noted 1 to 5 in both **(a)** and **(b)**



from a single crystal or a set of oriented grains all having the same diffracting plane (hkl) will exhibit a single spectral peak in XRD or a specific arrangement of diffraction spots in the SAED pattern. Figure 8 shows diffraction from an fcc NiCr alloy ($a = 3.56 \text{ \AA}$) having a $[100]$ grain surface orientation. Since diffraction from $\{100\}$ is forbidden, the XRD peak in Fig. 8a shows the (200) diffracting plane, parallel to (100), and the half-cell plane. Correspondingly, the SAED pattern in Fig. 8b shows a single-crystal-oriented $[100]$, with diffraction spots from allowed diffracting planes marked (see Table 1). The diffraction spots in Fig. 8b represent

Fig. 8 Diffraction from [100] single crystals of fcc NiCr alloy. (a) XRD spectrum (Cu-K α x-radiation). (b) [100] fcc electron diffraction pattern in TEM at 300 kV accelerating potential



reciprocal lattice vectors referenced to the origin at 000. These are determined from the zone axis criterion

$$hH + kK + lL = 0 \quad (30)$$

where $[HKL]$ is the zone axis, and $[hkl]$ are the reciprocal lattice vectors normal to this axis in the electron microscope. Consequently, for $[HKL] = [100]$, the condition for diffraction is $h = 0$ or $(0kl)$. Therefore, from the listing of the sequence of allowed diffracting planes for fcc in Table 1, we can have 020, 022, and 002 (and the negatives for opposite diffraction spots: $0\bar{2}0$, $0\bar{2}\bar{2}$, and $00\bar{2}$). These are regarded as the first-order diffraction spots.

At this point, it should be recognized that while the intent of this book has focused on more advanced materials issues, we have, as illustrated above, injected fundamentals somewhat strategically. For the reader who may wish to take a more in-depth look at crystallography principles and expand on structure factors or diffraction principles, the International Tables for X-ray Crystallography (1952–1960) continue to be extensive, while the work of Barrett and Massalski (1966) and Murr (1991) can be supplemental and informative. Of course XRD and SAED techniques will be utilized in the continuing development of materials structures and properties that follow.

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