

THE
ART OF
ELECTRONICS

PAUL HOROWITZ
WINFIELD HILL

SECOND EDITION

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Changes in the new edition:

- Totally rewritten chapters on microcomputers and microprocessors (using the IBM PC and the 68008).
- Substantially revised chapters on digital electronics (including PLDs, and the new HC and AC logic families), op-amps and precision design (reflecting the availability of excellent FET-input op-amps), construction techniques (including CAD/CAM).
- Every table has been revised and many new ones added.
- A rewritten chapter on FETs and an all-new chapter on micropower design.
- New sections on active filter designs, switched-capacitor filters, quadrature oscillators, low-dropout regulators, switching supplies, crowbars, isolation amplifiers, SCR latchup, "ground bounce," dynamic power dissipation, optoelectronics, RS-232 interfacing, modems, memory chips, schematic capture, AM detection, battery characteristics, sensor linearization.

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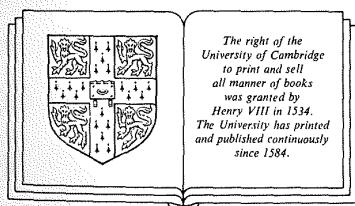
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THE ART OF ELECTRONICS

Second Edition

Paul Horowitz HARVARD UNIVERSITY

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CONTENTS

List of tables	xvi	1.18 Frequency analysis of reactive circuits	30
Preface	xix	1.19 RC filters	35
Preface to first edition	xxi	1.20 Phasor diagrams	39
CHAPTER 1		1.21 "Poles" and decibels per octave	40
FOUNDATIONS	1	1.22 Resonant circuits and active filters	41
Introduction	1	1.23 Other capacitor applications	42
Voltage, current, and resistance	2	1.24 Thévenin's theorem generalized	44
1.01 Voltage and current	2	Diodes and diode circuits	44
1.02 Relationship between voltage and current: resistors	4	1.25 Diodes	44
1.03 Voltage dividers	8	1.26 Rectification	44
1.04 Voltage and current sources	9	1.27 Power-supply filtering	45
1.05 Thévenin's equivalent circuit	11	1.28 Rectifier configurations for power supplies	46
1.06 Small-signal resistance	13	1.29 Regulators	48
Signals	15	1.30 Circuit applications of diodes	48
1.07 Sinusoidal signals	15	1.31 Inductive loads and diode protection	52
1.08 Signal amplitudes and decibels	16	Other passive components	53
1.09 Other signals	17	1.32 Electromechanical devices	53
1.10 Logic levels	19	1.33 Indicators	57
1.11 Signal sources	19	1.34 Variable components	57
Capacitors and ac circuits	20	<i>Additional exercises</i>	58
1.12 Capacitors	20	CHAPTER 2	
1.13 RC circuits: V and I versus time	23	TRANSISTORS	61
1.14 Differentiators	25	Introduction	61
1.15 Integrators	26	2.01 First transistor model: current amplifier	62
Inductors and transformers	28	Some basic transistor circuits	63
1.16 Inductors	28	2.02 Transistor switch	63
1.17 Transformers	28	2.03 Emitter follower	65
Impedance and reactance	29		

2.04 Emitter followers as voltage regulators	68	Basic FET circuits	124
2.05 Emitter follower biasing	69	3.06 JFET current sources	125
2.06 Transistor current source	72	3.07 FET amplifiers	129
2.07 Common-emitter amplifier	76	3.08 Source followers	133
2.08 Unity-gain phase splitter	77	3.09 FET gate current	135
2.09 Transconductance	78	3.10 FETs as variable resistors	138
Ebers-Moll model applied to basic transistor circuits	79	FET switches	140
2.10 Improved transistor model: transconductance amplifier	79	3.11 FET analog switches	141
2.11 The emitter follower revisited	81	3.12 Limitations of FET switches	144
2.12 The common-emitter amplifier revisited	82	3.13 Some FET analog switch examples	151
2.13 Biasing the common-emitter amplifier	84	3.14 MOSFET logic and power switches	153
2.14 Current mirrors	88	3.15 MOSFET handling precautions	169
Some amplifier building blocks	91	Self-explanatory circuits	171
2.15 Push-pull output stages	91	3.16 Circuit ideas	171
2.16 Darlington connection	94	3.17 Bad circuits	171 vskip6pt
2.17 Bootstrapping	96	CHAPTER 4	
2.18 Differential amplifiers	98	FEEDBACK AND OPERATIONAL AMPLIFIERS	175
2.19 Capacitance and Miller effect	102	Introduction	175
2.20 Field-effect transistors	104	4.01 Introduction to feedback	175
Some typical transistor circuits	104	4.02 Operational amplifiers	176
2.21 Regulated power supply	104	4.03 The golden rules	177
2.22 Temperature controller	105	Basic op-amp circuits	177
2.23 Simple logic with transistors and diodes	107	4.04 Inverting amplifier	177
Self-explanatory circuits	107	4.05 Noninverting amplifier	178
2.24 Good circuits	107	4.06 Follower	179
2.25 Bad circuits	107	4.07 Current sources	180
<i>Additional exercises</i>	107	4.08 Basic cautions for op-amp circuits	182
CHAPTER 3		An op-amp smorgasbord	183
FIELD-EFFECT TRANSISTORS	113	4.09 Linear circuits	183
Introduction	113	4.10 Nonlinear circuits	187
3.01 FET characteristics	114	A detailed look at op-amp behavior	188
3.02 FET types	117	4.11 Departure from ideal op-amp performance	189
3.03 Universal FET characteristics	119	4.12 Effects of op-amp limitations on circuit behavior	193
3.04 FET drain characteristics	121	4.13 Low-power and programmable op-amps	210
3.05 Manufacturing spread of FET characteristics	122		

125
9
33
135
istors 138

141
switches 144
vitch

power

171

vskip6pt

ATIONAL

dback 175
ers 176
177
177
177
ifier 178

180
op-amp

d 183
83
187
np behavior 188
eal op-amp
)
limitations on
193
ogrammable

A detailed look at selected op-amp circuits 213

4.14 Logarithmic amplifier 213
4.15 Active peak detector 217
4.16 Sample-and-hold 220
4.17 Active clamp 221
4.18 Absolute-value circuit 221
4.19 Integrators 222
4.20 Differentiators 224

Op-amp operation with a single power supply 224

4.21 Biasing single-supply ac amplifiers 225
4.22 Single-supply op-amps 225

Comparators and Schmitt trigger 229

4.23 Comparators 229
4.24 Schmitt trigger 231

Feedback with finite-gain amplifiers 232

4.25 Gain equation 232
4.26 Effects of feedback on amplifier circuits 233
4.27 Two examples of transistor amplifiers with feedback 236

Some typical op-amp circuits 238

4.28 General-purpose lab amplifier 238
4.29 Voltage-controlled oscillator 240
4.30 JFET linear switch with R_{ON} compensation 241
4.31 TTL zero-crossing detector 242
4.32 Load-current-sensing circuit 242

Feedback amplifier frequency compensation 242

4.33 Gain and phase shift versus frequency 243
4.34 Amplifier compensation methods 245
4.35 Frequency response of the feedback network 247

Self-explanatory circuits 250
4.36 Circuit ideas 250

4.37 Bad circuits 250
Additional exercises 251

CHAPTER 5 ACTIVE FILTERS AND OSCILLATORS 263

Active filters 263

5.01 Frequency response with RC filters 263
5.02 Ideal performance with LC filters 265
5.03 Enter active filters: an overview 266
5.04 Key filter performance criteria 267
5.05 Filter types 268

Active filter circuits 272
5.06 VCVS circuits 273
5.07 VCVS filter design using our simplified table 274
5.08 State-variable filters 276
5.09 Twin-T notch filters 279
5.10 Gyrator filter realizations 281
5.11 Switched-capacitor filters 281

Oscillators 284
5.12 Introduction to oscillators 284
5.13 Relaxation oscillators 284
5.14 The classic timer chip: the 555 286
5.15 Voltage-controlled oscillators 291
5.16 Quadrature oscillators 291
5.17 Wien bridge and LC oscillators 296
5.18 LC oscillators 297
5.19 Quartz-crystal oscillators 300

Self-explanatory circuits 303
5.20 Circuit ideas 303
Additional exercises 303

CHAPTER 6 VOLTAGE REGULATORS AND POWER CIRCUITS 307

Basic regulator circuits with the classic 723 307

6.01 The 723 regulator	307
6.02 Positive regulator	309
6.03 High-current regulator	311
Heat and power design	312
6.04 Power transistors and heat sinking	312
6.05 Foldback current limiting	316
6.06 Overvoltage crowbars	317
6.07 Further considerations in high-current power-supply design	320
6.08 Programmable supplies	321
6.09 Power-supply circuit example	323
6.10 Other regulator ICs	325
The unregulated supply	325
6.11 ac line components	326
6.12 Transformer	328
6.13 dc components	329
Voltage references	331
6.14 Zener diodes	332
6.15 Bandgap (V_{BE}) reference	335
Three-terminal and four-terminal regulators	341
6.16 Three-terminal regulators	341
6.17 Three-terminal adjustable regulators	344
6.18 Additional comments about 3-terminal regulators	345
6.19 Switching regulators and dc-dc converters	355
Special-purpose power-supply circuits	368
6.20 High-voltage regulators	368
6.21 Low-noise, low-drift supplies	374
6.22 Micropower regulators	376
6.23 Flying-capacitor (charge pump) voltage converters	377
6.24 Constant-current supplies	379
6.25 Commercial power-supply modules	382
Self-explanatory circuits	384
6.26 Circuit ideas	384
6.27 Bad circuits	384
<i>Additional exercises</i>	384

CHAPTER 7	
PRECISION CIRCUITS AND LOW-NOISE TECHNIQUES 391	
Precision op-amp design techniques	391
7.01 Precision versus dynamic range	391
7.02 Error budget	392
7.03 Example circuit: precision amplifier with automatic null offset	392
7.04 A precision-design error budget	394
7.05 Component errors	395
7.06 Amplifier input errors	396
7.07 Amplifier output errors	403
7.08 Auto-zeroing (chopper-stabilized) amplifiers	415
Differential and instrumentation amplifiers	421
7.09 Differencing amplifier	421
7.10 Standard three-op-amp instrumentation amplifier	425
Amplifier noise	428
7.11 Origins and kinds of noise	430
7.12 Signal-to-noise ratio and noise figure	433
7.13 Transistor amplifier voltage and current noise	436
7.14 Low-noise design with transistors	438
7.15 FET noise	443
7.16 Selecting low-noise transistors	445
7.17 Noise in differential and feedback amplifiers	445
Noise measurements and noise sources	449
7.18 Measurement without a noise source	449
7.19 Measurement with noise source	450
7.20 Noise and signal sources	452
7.21 Bandwidth limiting and rms voltage measurement	453
7.22 Noise potpourri	454

LOW-NOISE	Interference: shielding and grounding 455	Monostable multivibrators 517
chniques	7.23 Interference 455	8.20 One-shot characteristics 517
nic	7.24 Signal grounds 457	8.21 Monostable circuit example 519
sion amplifier	7.25 Grounding between instruments 457	8.22 Cautionary notes about monostables 519
fset 392	Self-explanatory circuits 466	8.23 Timing with counters 522
or	7.26 Circuit ideas 466	Sequential functions available as ICs 523
395	<i>Additional exercises</i> 466	8.24 Latches and registers 523
; 396	CHAPTER 8	8.25 Counters 524
rs 403	DIGITAL ELECTRONICS 471	8.26 Shift registers 525
r-stabilized)	Basic logic concepts 471	8.27 Sequential PALs 527
ation	8.01 Digital versus analog 471	8.28 Miscellaneous sequential functions 541
r 421	8.02 Logic states 472	Some typical digital circuits 544
ap	8.03 Number codes 473	8.29 Modulo- n counter: a timing example 544
lifier 425	8.04 Gates and truth tables 478	8.30 Multiplexed LED digital display 546
noise 430	8.05 Discrete circuits for gates 480	8.31 Sidereal telescope drive 548
and noise	8.06 Gate circuit example 481	8.32 An n -pulse generator 548
voltage and	8.07 Assertion-level logic notation 482	Logic pathology 551
th	TTL and CMOS 484	8.33 dc problems 551
transistors 445	8.08 Catalog of common gates 484	8.34 Switching problems 552
and feedback	8.09 IC gate circuits 485	8.35 Congenital weaknesses of TTL and CMOS 554
d noise	8.10 TTL and CMOS characteristics 486	Self-explanatory circuits 556
ut a noise	8.11 Three-state and open-collector devices 487	8.36 Circuit ideas 556
noise	Combinational logic 490	8.37 Bad circuits 556
urces 452	8.12 Logic identities 491	<i>Additional exercises</i> 556
and rms voltage	8.13 Minimization and Karnaugh maps 492	CHAPTER 9
154	8.14 Combinational functions available as ICs 493	DIGITAL MEETS ANALOG 565
	8.15 Implementing arbitrary truth tables 500	CMOS and TTL logic interfacing 565
	Sequential logic 504	9.01 Logic family chronology 565
	8.16 Devices with memory: flip-flops 504	9.02 Input and output characteristics 570
	8.17 Clocked flip-flops 507	9.03 Interfacing between logic families 572
	8.18 Combining memory and gates: sequential logic 512	9.04 Driving CMOS and TTL inputs 575
	8.19 Synchronizer 515	9.05 Driving digital logic from comparators and op-amps 577

9.06 Some comments about logic inputs	579	9.33 Feedback shift register sequences	655
9.07 Comparators	580	9.34 Analog noise generation from maximal-length sequences	658
9.08 Driving external digital loads from CMOS and TTL	582	9.35 Power spectrum of shift register sequences	658
9.09 NMOS LSI interfacing	588	9.36 Low-pass filtering	660
9.10 Opto-electronics	590	9.37 Wrap-up	661
Digital signals and long wires	599	9.38 Digital filters	664
9.11 On-board interconnections	599	Self-explanatory circuits	667
9.12 Inter-card connections	601	9.39 Circuit ideas	667
9.13 Data buses	602	9.40 Bad circuits	668
9.14 Driving cables	603	<i>Additional exercises</i>	668
Analog/digital conversion	612		
9.15 Introduction to A/D conversion	612	CHAPTER 10	
9.16 Digital-to-analog converters (DACs)	614	MICROCOMPUTERS	673
9.17 Time-domain (averaging) DACs	618	Minicomputers, microcomputers, and microprocessors	673
9.18 Multiplying DACs	619	10.01 Computer architecture	674
9.19 Choosing a DAC	619	A computer instruction set	678
9.20 Analog-to-digital converters	621	10.02 Assembly language and machine language	678
9.21 Charge-balancing techniques	626	10.03 Simplified 8086/8 instruction set	679
9.22 Some unusual A/D and D/A converters	630	10.04 A programming example	683
9.23 Choosing an ADC	631	Bus signals and interfacing	684
Some A/D conversion examples	636	10.05 Fundamental bus signals: data, address, strobe	684
9.24 16-Channel A/D data-acquisition system	636	10.06 Programmed I/O: data out	685
9.25 $3\frac{1}{2}$ -Digit voltmeter	638	10.07 Programmed I/O: data in	689
9.26 Coulomb meter	640	10.08 Programmed I/O: status registers	690
Phase-locked loops	641	10.09 Interrupts	693
9.27 Introduction to phase-locked loops	641	10.10 Interrupt handling	695
9.28 PLL design	646	10.11 Interrupts in general	697
9.29 Design example: frequency multiplier	647	10.12 Direct memory access	701
9.30 PLL capture and lock	651	10.13 Summary of the IBM PC's bus signals	704
9.31 Some PLL applications	652	10.14 Synchronous versus asynchronous bus communication	707
Pseudo-random bit sequences and noise generation	655	10.15 Other microcomputer buses	708
9.32 Digital noise generation	655	10.16 Connecting peripherals to the computer	711

r
on from
nces 658
ift register
60
667
73
puters, and
ure 674
t 678
and machine
instruction
ample 683
ng 684
signals: data,
84
data out 685
data in 689
status
; 695
ral 697
cess 701
BM PC's bus
as asynchronous
n 707
uter buses 708
erals to the

Software system concepts	714
10.17 Programming	714
10.18 Operating systems, files, and use of memory	716
Data communications concepts	719
10.19 Serial communication and ASCII	720
10.20 Parallel communication: Centronics, SCSI, IPI, GPIB (488)	730
10.21 Local area networks	734
10.22 Interface example: hardware data packing	736
10.23 Number formats	738
CHAPTER 11	
MICROPROCESSORS	743
A detailed look at the 68008	744
11.01 Registers, memory, and I/O	744
11.02 Instruction set and addressing	745
11.03 Machine-language representation	750
11.04 Bus signals	753
A complete design example: analog signal averager	760
11.05 Circuit design	760
11.06 Programming: defining the task	774
11.07 Programming: details	777
11.08 Performance	796
11.09 Some afterthoughts	797
Microprocessor support chips	799
11.10 Medium-scale integration	800
11.11 Peripheral LSI chips	802
11.12 Memory	812
11.13 Other microprocessors	820
11.14 Emulators, development systems, logic analyzers, and evaluation boards	821

CHAPTER 12	
ELECTRONIC CONSTRUCTION TECHNIQUES	827
Prototyping methods	827
12.01 Breadboards	827
12.02 PC prototyping boards	828
12.03 Wire-Wrap panels	828
Printed circuits	830
12.04 PC board fabrication	830
12.05 PC board design	835
12.06 Stuffing PC boards	838
12.07 Some further thoughts on PC boards	840
12.08 Advanced techniques	841
Instrument construction	852
12.09 Housing circuit boards in an instrument	852
12.10 Cabinets	854
12.11 Construction hints	855
12.12 Cooling	855
12.13 Some electrical hints	858
12.14 Where to get components	860
CHAPTER 13	
HIGH-FREQUENCY AND HIGH-SPEED TECHNIQUES	863
High-frequency amplifiers	863
13.01 Transistor amplifiers at high frequencies: first look	863
13.02 High-frequency amplifiers: the ac model	864
13.03 A high-frequency calculation example	866
13.04 High-frequency amplifier configurations	868
13.05 A wideband design example	869
13.06 Some refinements to the ac model	872
13.07 The shunt-series pair	872
13.08 Modular amplifiers	873
Radiofrequency circuit elements	879
13.09 Transmission lines	879

13.10 Stubs, baluns, and transformers	881
13.11 Tuned amplifiers	882
13.12 Radiofrequency circuit elements	884
13.13 Measuring amplitude or power	888
Radiofrequency communications:	
AM	892
13.14 Some communications concepts	892
13.15 Amplitude modulation	894
13.16 Superheterodyne receiver	895
Advanced modulation methods 897	
13.17 Single sideband	897
13.18 Frequency modulation	898
13.19 Frequency-shift keying	900
13.20 Pulse-modulation schemes	900
Radiofrequency circuit tricks 902	
13.21 Special construction techniques	902
13.22 Exotic RF amplifiers and devices	903
High-speed switching 904	
13.23 Transistor model and equations	905
13.24 Analog modeling tools	908
Some switching-speed examples 909	
13.25 High-voltage driver	909
13.26 Open-collector bus driver	910
13.27 Example: photomultiplier preamp	911
Self-explanatory circuits 913	
13.28 Circuit ideas	913
<i>Additional exercises</i> 913	
CHAPTER 14	
LOW-POWER DESIGN 917	
Introduction 917	
14.01 Low-power applications	918

Power sources 920	
14.02 Battery types	920
14.03 Wall-plug-in units	931
14.04 Solar cells	932
14.05 Signal currents	933
Power switching and micropower regulators 938	
14.06 Power switching	938
14.07 Micropower regulators	941
14.08 Ground reference	944
14.09 Micropower voltage references and temperature sensors	948
Linear micropower design techniques 948	
14.10 Problems of micropower linear design	950
14.11 Discrete linear design example	950
14.12 Micropower operational amplifiers	951
14.13 Micropower comparators	965
14.14 Micropower timers and oscillators	965
Micropower digital design 969	
14.15 CMOS families	969
14.16 Keeping CMOS low power	970
14.17 Micropower microprocessors and peripherals	974
14.18 Microprocessor design example: degree-day logger	978
Self-explanatory circuits 985	
14.19 Circuit ideas	985
CHAPTER 15	
MEASUREMENTS AND SIGNAL PROCESSING 987	
Overview 987	
Measurement transducers 988	
15.01 Temperature	988
15.02 Light level	996
15.03 Strain and displacement	1001

15.04 Acceleration, pressure, force, velocity	1004
15.05 Magnetic field	1007
15.06 Vacuum gauges	1007
15.07 Particle detectors	1008
15.08 Biological and chemical voltage probes	1012
Precision standards and precision measurements	1016
15.09 Frequency standards	1016
15.10 Frequency, period, and time-interval measurements	1019
15.11 Voltage and resistance standards and measurements	1025
Bandwidth-narrowing techniques	1026
15.12 The problem of signal-to-noise ratio	1026
15.13 Signal averaging and multichannel averaging	1026
15.14 Making a signal periodic	1030
15.15 Lock-in detection	1031
15.16 Pulse-height analysis	1034
15.17 Time-to-amplitude converters	1035
Spectrum analysis and Fourier transforms	1035
15.18 Spectrum analyzers	1035
15.19 Off-line spectrum analysis	1038
Self-explanatory circuits	1038
15.20 Circuit ideas	1038

APPENDIXES	1043
Appendix A	
The oscilloscope	1045
Appendix B	
Math review	1050
Appendix C	
The 5% resistor color code	1053
Appendix D	
1% Precision resistors	1054
Appendix E	
How to draw schematic diagrams	1056
Appendix F	
Load lines	1059
Appendix G	
Transistor saturation	1062
Appendix H	
LC Butterworth filters	1064
Appendix I	
Electronics magazines and journals	1068
Appendix J	
IC prefixes	1069
Appendix K	
Data sheets	1072
2N4400-1 NPN transistor	1073
LF411-12 JFET operational amplifier	1078
LM317 3-terminal adjustable regulator	1086
Bibliography	1095
Index	1101

SIGNAL

ers	988
38	
cement	1001

generators are provided with logic-level outputs for easy connection to digital circuitry. Like signal generators, these come in the programmable variety.

Function generators

In many ways function generators are the most flexible signal sources of all. You can make sine, triangle, and square waves over an enormous frequency range (0.01 Hz to 10 MHz is typical), with control of amplitude and dc offset (a constant dc voltage added to the signal). Many of them have provision for frequency sweeping, often in several modes (linear or logarithmic frequency variation versus time). They are available with pulse outputs (although not with the flexibility you get with a pulse generator), and some of them have provision for modulation.

Like the other signal sources, function generators come in programmable versions and versions with digital readout of frequency (and sometimes amplitude). The most recent addition to the function-generator family is the synthesized function generator, a device that combines all the flexibility of a function generator with the stability and accuracy of a frequency synthesizer. An example is the HP 8116A, with sine, square, and triangle waves (as well as pulses, ramps, haversines, etc.) from 0.001Hz to 50MHz. Frequency and amplitude (10mV to 16V pp) are programmable, as are linear and logarithmic frequency sweeps. This unit also provides trigger, gate, burst, FM, AM, pulse-width modulation, voltage-controlled frequency, and single cycles. For general use, if you can have only one signal source, the function generator is for you.

CAPACITORS AND AC CIRCUITS

Once we enter the world of changing voltages and currents, or signals, we encounter two very interesting circuit elements that

are useless in dc circuits: capacitors and inductors. As you will see, these humble devices, combined with resistors, complete the triad of passive linear circuit elements that form the basis of nearly all circuitry. Capacitors, in particular, are essential in nearly every circuit application. They are used for waveform generation, filtering, and blocking and bypass applications. They are used in integrators and differentiators. In combination with inductors, they make possible sharp filters for separating desired signals from background. You will see some of these applications as we continue with this chapter, and there will be numerous interesting examples in later chapters.

Let's proceed, then, to look at capacitors in detail. Portions of the treatment that follows are necessarily mathematical in nature; the reader with little mathematical preparation may find Appendix B helpful. In any case, an understanding of the details is less important in the long run than an understanding of the results.

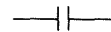


Figure 1.27. Capacitor.

1.12 Capacitors

A capacitor (Fig. 1.27) (the old-fashioned name was *condenser*) is a device that has two wires sticking out of it and has the property

$$Q = CV$$

A capacitor of C farads with V volts across its terminals has Q coulombs of stored charge on one plate, and $-Q$ on the other.

To a first approximation, capacitors are devices that might be considered simply frequency-dependent resistors. They allow you to make frequency-dependent voltage dividers, for instance. For some applications (bypass, coupling) this is

capacitors and, these humble storers, complete circuit elements of all circuitry. They are essential in a wide variety of applications. They are used in combination with inductors, and they differ from resistors for separate applications as well; and there will be examples in later chapters.

Look at capacitors as a mathematical device that is helpful in understanding the details of circuit operation. The old-fashioned capacitor is a device that has capacitance and has the ability to store energy.

The old-fashioned capacitor is a device that has capacitance and has the ability to store energy.

When V volts across a capacitor, the amount of stored energy is $\frac{1}{2}CV^2$. In operation, capacitors are considered similar to resistors. They are energy-dependent devices. For some applications (coupling) this is

almost all you need to know, but for other applications (filtering, energy storage, resonant circuits) a deeper understanding is needed. For example, capacitors cannot dissipate power, even though current can flow through them, because the voltage and current are 90° out of phase.

Taking the derivative of the defining equation above (see Appendix B), you get

$$I = C \frac{dV}{dt}$$

So a capacitor is more complicated than a resistor; the current is not simply proportional to the voltage, but rather to the rate of change of voltage. If you change the voltage across a farad by 1 volt per second, you are supplying an amp. Conversely, if you supply an amp, its voltage changes by 1 volt per second. A farad is very large, and you usually deal in microfarads (μF) or picofarads (pF). (To make matters confusing to the uninitiated, the units are often omitted on capacitor values specified in schematic diagrams. You have to figure it out from the context.) For instance, if you supply a current of 1mA to $1\mu\text{F}$, the voltage will rise at 1000 volts per second. A 10ms pulse of this current will increase the voltage across the capacitor by 10 volts (Fig. 1.28).

Capacitors come in an amazing variety of shapes and sizes; with time, you will come to recognize their more common incarnations. The basic construction is simply two conductors near each other (but not touching); in fact, the simplest capacitors are just that. For greater capacitance, you need more area and closer spacing; the usual approach is to plate some conductor onto a thin insulating material (called a dielectric), for instance, aluminized Mylar film rolled up into a small cylindrical configuration. Other popular types are thin ceramic wafers (disc ceramics), metal foils with oxide insulators (electrolytics), and metallized mica. Each of these types

has unique properties; for a brief rundown, see the box on capacitors. In general, ceramic and Mylar types are used for most noncritical circuit applications; tantalum capacitors are used where greater capacitance is needed, and electrolytics are used for power-supply filtering.

Capacitors in parallel and series

The capacitance of several capacitors in parallel is the sum of their individual capacitances. This is easy to see: Put voltage V across the parallel combination; then

$$\begin{aligned} C_{\text{total}}V &= Q_{\text{total}} = Q_1 + Q_2 + Q_3 + \dots \\ &= C_1V + C_2V + C_3V + \dots \\ &= (C_1 + C_2 + C_3 + \dots)V \end{aligned}$$

or

$$C_{\text{total}} = C_1 + C_2 + C_3 + \dots$$

For capacitors in series, the formula is like that for resistors in parallel:

$$C_{\text{total}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots}$$

or (two capacitors only)

$$C_{\text{total}} = \frac{C_1C_2}{C_1 + C_2}$$

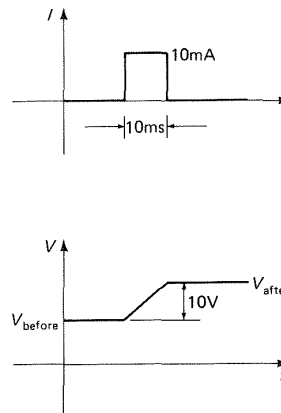


Figure 1.28. The voltage across a capacitor changes when a current flows through it.

CAPACITORS

There is wide variety among the capacitor types available. This is a quickie guide to point out their major advantages and disadvantages. Our judgments should be considered somewhat subjective:

Type	Capacitance range	Maximum voltage	Accuracy	Temperature stability	Leakage	Comments
Mica	1pF–0.01 μ F	100–600	Good		Good	Excellent; good at RF
Tubular ceramic	0.5pF–100pF	100–600		Selectable		Several tempcos (including zero)
Ceramic	10pF–1 μ F	50–30,000	Poor	Poor	Moderate	Small, inexpensive, very popular
Polyester (Mylar)	0.001 μ F–50 μ F	50–600	Good	Poor	Good	Inexpensive, good, popular
Polystyrene	10pF–2.7 μ F	100–600	Excellent	Good	Excellent	High quality, large; signal filters
Polycarbonate	100pF–30 μ F	50–800	Excellent	Excellent	Good	High quality, small
Polypropylene	100pF–50 μ F	100–800	Excellent	Good	Excellent	High quality, low dielectric absorption
Teflon	1000pF–2 μ F	50–200	Excellent	Best	Best	High quality, lowest dielectric absorption
Glass	10pF–1000pF	100–600	Good		Excellent	Long-term stability
Porcelain	100pF–0.1 μ F	50–400	Good	Good	Good	Good long-term stability
Tantalum	0.1 μ F–500 μ F	6–100	Poor	Poor		High capacitance; polarized, small; low inductance
Electrolytic	0.1 μ F–1.6F	3–600	Terrible	Ghastly	Awful	Power-supply filters; polarized; short life
Double layer	0.1F–10F	1.5–6	Poor	Poor	Good	Memory backup; high series resistance
Oil	0.1 μ F–20 μ F	200–10,000			Good	High-voltage filters; large, long life
Vacuum	1pF–5000pF	2000–36,000			Excellent	Transmitters

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EXERCISE 1.12

Derive the formula for the capacitance of two capacitors in series. Hint: Because there is no external connection to the point where the two capacitors are connected together, they must have equal stored charges.

The current that flows in a capacitor during charging ($I = C dV/dt$) has some unusual features. Unlike resistive current, it's not proportional to voltage, but rather to the rate of change (the "time derivative") of voltage. Furthermore, unlike the situation in a resistor, the power (V times I) associated with capacitive current is not turned into heat, but is stored as energy in the capacitor's internal electric field. You get all that energy back when you discharge the capacitor. We'll see another way to look at these curious properties when we talk about *reactance*, beginning in Section 1.18.

1.13 RC circuits: V and I versus time

When dealing with ac circuits (or, in general, any circuits that have changing voltages and currents), there are two possible approaches. You can talk about V and I versus time, or you can talk about amplitude versus signal frequency. Both approaches have their merits, and you find yourself switching back and forth according to which description is most convenient in each situation. We will begin our study of ac circuits in the time domain. Beginning with Section 1.18, we will tackle the frequency domain.

What are some of the features of circuits with capacitors? To answer this question, let's begin with the simple RC circuit (Fig. 1.29). Application of the capacitor rules gives

$$C \frac{dV}{dt} = I = -\frac{V}{R}$$

This is a differential equation, and its solution is

$$V = Ae^{-t/RC}$$

So a charged capacitor placed across a resistor will discharge as in Figure 1.30.

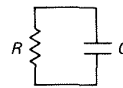


Figure 1.29

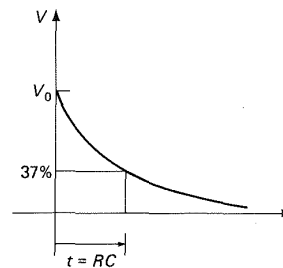


Figure 1.30. RC discharge waveform.

Time constant

The product RC is called the *time constant* of the circuit. For R in ohms and C in farads, the product RC is in seconds. A microfarad across 1.0k has a time constant of 1ms; if the capacitor is initially charged to 1.0 volt, the initial current is 1.0mA.

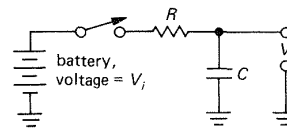


Figure 1.31

Figure 1.31 shows a slightly different circuit. At time $t = 0$, someone connects the battery. The equation for the circuit is then

$$I = C \frac{dV}{dt} = \frac{V_i - V}{R}$$

with the solution

$$V = V_i + Ae^{-t/RC}$$

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Comments

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Small, inexpensive,
very popular

inexpensive,
good, popular

high quality,
large; signal
filters

high quality,
small

high quality,
low dielectric
absorption

high quality,
lowest dielectric
absorption

long-term
stability

Good long-term
stability

high capaci-
tance; polarized,
small; low
inductance

Power-supply
filters; polarized;
short life

Memory backup;
high series
resistance

high-voltage
filters; large,
long life

Transmitters

(Please don't worry if you can't follow the mathematics. What we are doing is getting some important results, which you should remember. Later we will use the results often, with no further need for the mathematics used to derive them.) The constant A is determined by initial conditions (Fig. 1.32): $V = 0$ at $t = 0$; therefore, $A = -V_i$, and

$$V = V_i(1 - e^{-t/RC})$$

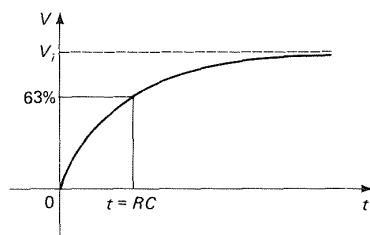


Figure 1.32

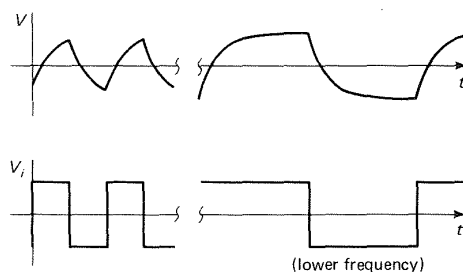


Figure 1.33. Output (top waveform) across a capacitor, when driven by square waves through a resistor.

Decay to equilibrium

Eventually (when $t \gg RC$), V reaches V_i . (Presenting the "5RC rule of thumb": a capacitor charges or decays to within 1% of its final value in 5 time constants.) If we then change V_0 to some other value (say, 0), V will decay toward that new value with an exponential $e^{-t/RC}$. For example, a square-wave input for V_0 will produce the output shown in Figure 1.33.

EXERCISE 1.13

Show that the rise time (the time required to go from 10% to 90% of its final value) of this signal is $2.2RC$.

You might ask the obvious next question: What about $V(t)$ for arbitrary $V_i(t)$? The solution involves an inhomogeneous differential equation and can be solved by standard methods (which are, however, beyond the scope of this book). You would find

$$V(t) = \frac{1}{RC} \int_{-\infty}^t V_i(\tau) e^{-(t-\tau)/RC} d\tau$$

That is, the RC circuit averages past history at the input with a weighting factor $e^{-\Delta t/RC}$

In practice, you seldom ask this question. Instead, you deal in the *frequency domain* and ask how much of each frequency component present in the input gets through. We will get to this important topic soon (Section 1.18). Before we do, though, there are a few other interesting circuits we can analyze simply with this time-domain approach.

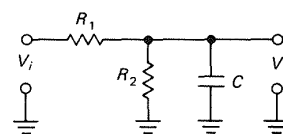


Figure 1.34

Simplification by Thévenin equivalents

We could go ahead and analyze more complicated circuits by similar methods, writing down the differential equations and trying to find solutions. For most purposes it simply isn't worth it. This is as complicated an RC circuit as we will need. Many other circuits can be reduced to it (e.g., Fig. 1.34). By just using the Thévenin equivalent of the voltage divider formed by R_1 and R_2 , you can find the output

$V(t)$ pro
 V_0 .

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A - input

B - RC

C - output

Figure 1.35
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EXERCISE 1.14

$R_1 = R_2 = 10k$, and $C = 0.1\mu F$ in the circuit shown in Figure 1.34. Find $V(t)$ and sketch it.

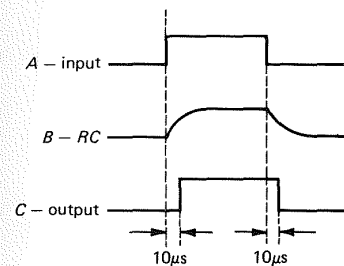
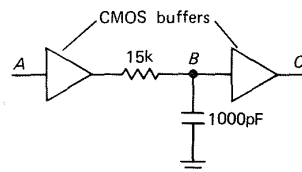


Figure 1.35. Producing a delayed digital waveform with the help of an RC .

Example: time-delay circuit

We have already mentioned logic levels, the voltages that digital circuits live on. Figure 1.35 shows an application of capacitors to produce a delayed pulse. The triangular symbols are "CMOS buffers." They give a HIGH output if the input is HIGH (more than one-half the dc power-supply voltage used to power them), and vice versa. The first buffer provides a replica of the input signal, but with low source resistance, and prevents input loading by the RC (recall our earlier discussion of circuit loading in Section 1.05). The RC output has the characteristic decays and causes the output buffer to switch $10\mu s$ after the input transitions (an RC reaches 50% output in $0.7RC$). In an actual application you would have to consider the effect of the buffer input threshold deviating from

one-half the supply voltage, which would alter the delay and change the output pulse width. Such a circuit is sometimes used to delay a pulse so that something else can happen first. In designing circuits you try not to rely on tricks like this, but they're occasionally handy.

1.14 Differentiators

Look at the circuit in Figure 1.36. The voltage across C is $V_{in} - V$, so

$$I = C \frac{d}{dt}(V_{in} - V) = \frac{V}{R}$$

If we choose R and C small enough so that $dV/dt \ll dV_{in}/dt$, then

$$C \frac{dV_{in}}{dt} \approx \frac{V}{R}$$

or

$$V(t) = RC \frac{d}{dt} V_{in}(t)$$

That is, we get an output proportional to the rate of change of the input waveform.

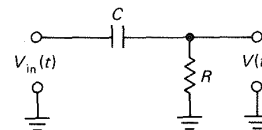
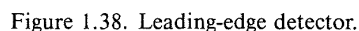
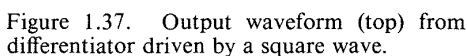


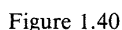
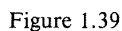
Figure 1.36

To keep $dV/dt \ll dV_{in}/dt$, we make the product RC small, taking care not to "load" the input by making R too small (at the transition the change in voltage across the capacitor is zero, so R is the load seen by the input). We will have a better criterion for this when we look at things in the frequency domain. If you drive this circuit with a square wave, the output will be as shown in Figure 1.37.



Unintentional capacitive coupling

a missing resistor termination on your signal line. If not, you must either reduce the source resistance of the signal line or find a way to reduce capacitive coupling from the offending square wave. The second case is typical of what you might see when you look at a square wave, but have a broken connection somewhere, usually at the scope probe. The very small capacitance of the broken connection combines with the scope input resistance to form a differentiator. *Knowing that you've got a differentiated "something" can help you find the trouble and eliminate it.*



Take a look at the circuit in Figure 1.40. The voltage across R is $V_{\text{in}} - V$, so

If we manage to keep $V \ll V_{\text{in}}$, by keeping the product RC large, then

or

We have a circuit that performs the integral over time of an input signal! You can



Later, when the coefficients and feedback integrators were added, the input was V_{in} . They were added and voltage

Ramp generation

At this point, we see how a ramp circuit is ex-

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see how the approximation works for a square-wave input: $V(t)$ is then the exponential charging curve we saw earlier (Fig. 1.41). The first part of the exponential is a ramp, the integral of a constant; as we increase the time constant RC , we pick off a smaller part of the exponential, i.e., a better approximation to a perfect ramp.

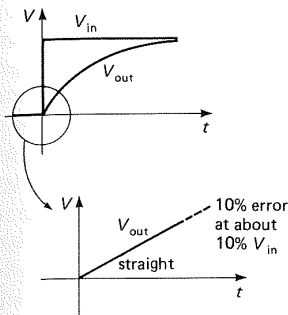


Figure 1.41

Note that the condition $V \ll V_{in}$ is just the same as saying that I is proportional to V_{in} . If we had as input a *current* $I(t)$, rather than a voltage, we would have an exact integrator. A large voltage across a large resistance approximates a current source and, in fact, is frequently used as one.

Later, when we get to operational amplifiers and feedback, we will be able to build integrators without the restriction $V_{out} \ll V_{in}$. They will work over large frequency and voltage ranges with negligible error.

The integrator is used extensively in analog computation. It is a useful subcircuit that finds application in control systems, feedback, analog/digital conversion, and waveform generation.

Ramp generators

At this point it is easy to understand how a ramp generator works. This nice circuit is extremely useful, for example

in timing circuits, waveform and function generators, oscilloscope sweep circuits, and analog/digital conversion circuitry. The circuit uses a constant current to charge a capacitor (Fig. 1.42). From the capacitor equation $I = C(dV/dt)$, you get $V(t) = (I/C)t$. The output waveform is as shown in Figure 1.43. The ramp stops when the current source "runs out of voltage," i.e., reaches the limit of its compliance. The curve for a simple RC , with the resistor tied to a voltage source equal to the compliance of the current source, and with R chosen so that the current at zero output voltage is the same as that of the current source, is also drawn for comparison. (Real current sources generally have output compliances limited by the power-supply voltages used in making them, so the comparison is realistic.) In the next chapter, which deals with transistors, we will design some current sources, with some refinements to follow in the chapters on operational amplifiers (op-amps) and field-effect transistors (FETs). Exciting things to look forward to!

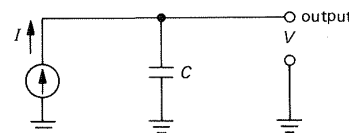


Figure 1.42. A constant current source charging a capacitor generates a ramp voltage waveform.

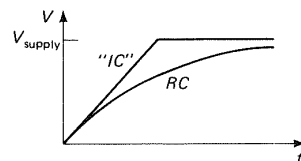


Figure 1.43

EXERCISE 1.15

A current of 1mA charges a $1\mu\text{F}$ capacitor. How long does it take the ramp to reach 10 volts?

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INDUCTORS AND TRANSFORMERS

1.16 Inductors

If you understand capacitors, you won't have any trouble with inductors (Fig. 1.44). They're closely related to capacitors; the rate of current change in an inductor depends on the voltage applied across it, whereas the rate of voltage change in a capacitor depends on the current through it. The defining equation for an inductor is

$$V = L \frac{dI}{dt}$$

where L is called the *inductance* and is measured in henrys (or mH, μ H, etc.). Putting a voltage across an inductor causes the current to rise as a ramp (for a capacitor, supplying a constant current causes the voltage to rise as a ramp); 1 volt across 1 henry produces a current that increases at 1 amp per second.



Figure 1.44. Inductor.

As with capacitive current, inductive current is not simply proportional to voltage. Furthermore, unlike the situation in a resistor, the power associated with inductive current (V times I) is not turned into heat, but is stored as energy in the inductor's magnetic field. You get all that energy back when you interrupt the inductor's current.

The symbol for an inductor looks like a coil of wire; that's because, in its simplest form, that's all it is. Variations include coils wound on various core materials, the most popular being iron (or iron alloys, laminations, or powder) and ferrite, a black, nonconductive, brittle magnetic material. These are all ploys to multiply the inductance of a given coil by the "permeability" of the core material. The core may be in the shape of a rod, a toroid

(doughnut), or even more bizarre shapes, such as a "pot core" (which has to be seen to be understood; the best description we can think of is a doughnut mold split horizontally in half, if doughnuts were made in molds).

Inductors find heavy use in radio-frequency (RF) circuits, serving as RF "chokes" and as parts of tuned circuits (see Chapter 13). A pair of closely coupled inductors forms the interesting object known as a transformer. We will talk briefly about them in the next section.

An inductor is, in a real sense, the opposite of a capacitor. You will see how that works out in the next few sections of this chapter, which deal with the important subject of *impedance*.

1.17 Transformers

A transformer is a device consisting of two closely coupled coils (called primary and secondary). An ac voltage applied to the primary appears across the secondary, with a voltage multiplication proportional to the turns ratio of the transformer and a current multiplication inversely proportional to the turns ratio. Power is conserved. Figure 1.45 shows the circuit symbol for a laminated-core transformer (the kind used for 60Hz ac power conversion).



Figure 1.45. Transformer.

Transformers are quite efficient (output power is very nearly equal to input power); thus, a step-up transformer gives higher voltage at lower current. Jumping ahead for a moment, a transformer of turns ratio n increases the impedance by n^2 . There is very little primary current if the secondary is unloaded.

Transformers serve two important functions in electronic instruments: They

change the (usually low) voltage of the circuit, and they are used as a device for coupling power line, transformer are different. Power from the line is enormous voltage and currents up to several amperes from a transformer. Typical electronic instrument voltages from ratings of 0.1

Transformers are used in radio and television sets. At radio frequencies, use tuned circuits of a wide range of frequencies. Also an intermediate frequency line transformer in Section 1.17 for use at high frequencies. Special core materials to maximize core efficiency. Transformers are often burdened in electronic instruments. The two kinds of transformers are general not

IMPEDANCE

Warning: The mathematics of impedance is not simple; you must be careful to the results.

Circuits with inductors are more complicated than circuits with resistors. Their behavior is different. "Voltage division" with an inductor will be different from voltage division with a resistor. Maintaining these relationships as reactances

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change the ac line voltage to a useful (usually lower) value that can be used by the circuit, and they "isolate" the electronic device from actual connection to the power line, because the windings of a transformer are electrically insulated from each other. *Power transformers* (meant for use from the 110V power line) come in an enormous variety of secondary voltages and currents: outputs as low as 1 volt or so up to several thousand volts, current ratings from a few milliamps to hundreds of amps. Typical transformers for use in electronic instruments might have secondary voltages from 10 to 50 volts, with current ratings of 0.1 to 5 amps or so.

Transformers for use at audiofrequencies and radiofrequencies are also available. At radiofrequencies you sometimes use tuned transformers, if only a narrow range of frequencies is present. There is also an interesting class of transmission-line transformer that we will discuss briefly in Section 13.10. In general, transformers for use at high frequencies must use special core materials or construction to minimize core losses, whereas low-frequency transformers (e.g., power transformers) are burdened instead by large and heavy cores. The two kinds of transformers are in general not interchangeable.

IMPEDANCE AND REACTANCE

Warning: This section is somewhat mathematical; you may wish to skip over the mathematics, but be sure to pay attention to the results and graphs.

Circuits with capacitors and inductors are more complicated than the resistive circuits we talked about earlier, in that their behavior depends on frequency: A "voltage divider" containing a capacitor or inductor will have a frequency-dependent division ratio. In addition, circuits containing these components (known collectively as *reactive* components) "corrupt"

input waveforms such as square waves, as we just saw.

However, both capacitors and inductors are *linear* devices, meaning that the amplitude of the output waveform, whatever its shape, increases exactly in proportion to the input waveform's amplitude. This linearity has many consequences, the most important of which is probably the following: *The output of a linear circuit, driven with a sine wave at some frequency f , is itself a sine wave at the same frequency (with, at most, changed amplitude and phase).*

Because of this remarkable property of circuits containing resistors, capacitors, and inductors (and, later, linear amplifiers), it is particularly convenient to analyze any such circuit by asking how the output voltage (amplitude and phase) depends on the input voltage, *for sine-wave input at a single frequency*, even though this may not be the intended use. A graph of the resulting *frequency response*, in which the ratio of output to input is plotted for each sine-wave frequency, is useful for thinking about many kinds of waveforms. As an example, a certain "boom-box" loudspeaker might have the frequency response shown in Figure 1.46, where the "output" in this case is of course sound pressure, not voltage. It is desirable for a speaker to have a "flat" response, meaning that the graph of sound pressure versus frequency is constant over the band of audible frequencies. In this case the speaker's deficiencies can be corrected by introducing a passive filter with the inverse response (as shown) into the amplifiers of the radio.

As we will see, it is possible to generalize Ohm's law, replacing the word "resistance" with "impedance," in order to describe any circuit containing these linear passive devices (resistors, capacitors, and inductors). You could think of the subject of impedance and reactance as Ohm's law for circuits that include capacitors and inductors. Some important terminology: Impedance is the "generalized resistance"; inductors

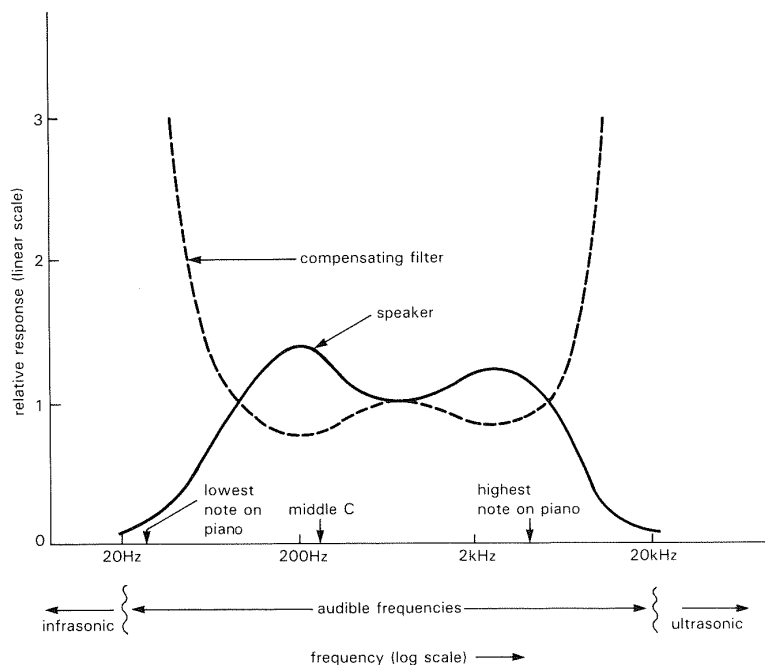


Figure 1.46. Example of frequency analysis: “boom box” loudspeaker equalization.

and capacitors have *reactance* (they are “reactive”); resistors have *resistance* (they are “resistive”). In other words, impedance = resistance + reactance (more about this later). However, you’ll see statements like “the impedance of the capacitor at this frequency is . . .” The reason you don’t have to use the word “reactance” in such a case is that impedance covers everything. In fact, you frequently use the word “impedance” even when you know it’s a resistance you’re talking about; you say “the source impedance” or “the output impedance” when you mean the Thévenin equivalent resistance of some source. The same holds for “input impedance.”

In all that follows, we will be talking about circuits driven by sine waves at a single frequency. Analysis of circuits driven by complicated waveforms is more elaborate, involving the methods we used earlier (differential equations) or decomposition of the waveform into sine

waves (Fourier analysis). Fortunately, these methods are seldom necessary.

1.18 Frequency analysis of reactive circuits

Let’s start by looking at a capacitor driven by a sine-wave voltage source (Fig. 1.47). The current is

$$I(t) = C \frac{dV}{dt} = C\omega V_0 \cos \omega t$$

i.e., a current of amplitude I , with the phase leading the input voltage by 90° . If we consider amplitudes only, and disregard phases, the current is

$$I = \frac{V}{1/\omega C}$$

(Recall that $\omega = 2\pi f$.) It behaves like a frequency-dependent resistance $R = 1/\omega C$, but in addition the current is 90° out of phase with the voltage (Fig. 1.48).

$$V(t) = V_0 \sin \omega t$$

Figure 1



Figure 1

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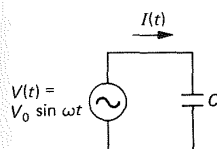


Figure 1.47

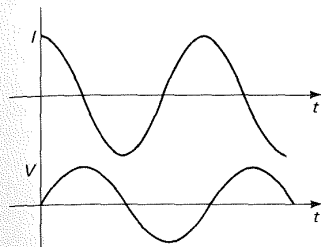


Figure 1.48

For example, a $1\mu\text{F}$ capacitor put across the 110 volt (rms) 60Hz power line draws a current of rms amplitude

$$I = \frac{110}{1/(2\pi \times 60 \times 10^{-6})} = 41.5\text{mA (rms)}$$

Note: At this point it is necessary to get into some complex algebra; you may wish to skip over the math in some of the following sections, taking note of the results as we derive them. A knowledge of the detailed mathematics is not necessary in order to understand the remainder of the book. Very little mathematics will be used in later chapters. The section ahead is easily the most difficult for the reader with little mathematical preparation. Don't be discouraged!

Voltages and currents as complex numbers

As you have just seen, there can be phase shifts between the voltage and current in an ac circuit being driven by a sine wave at some frequency. Nevertheless, as long as the circuit contains only *linear*

elements (resistors, capacitors, inductors), the magnitudes of the currents everywhere in the circuit are still proportional to the magnitude of the driving voltage, so we might hope to find some generalization of voltage, current, and resistance in order to rescue Ohm's law. Obviously a single number won't suffice to specify the current, say, at some point in the circuit, because we must somehow have information about both the magnitude and phase shift.

Although we can imagine specifying the magnitudes and phase shifts of voltages and currents at any point in the circuit by writing them out explicitly, e.g., $V(t) = 23.7\sin(377t + 0.38)$, it turns out that our requirements can be met more simply by using the algebra of complex numbers to *represent* voltages and currents. Then we can simply add or subtract the complex number representations, rather than laboriously having to add or subtract the actual sinusoidal functions of time themselves. Because the actual voltages and currents are real quantities that vary with time, we must develop a rule for converting from actual quantities to their representations, and vice versa. Recalling once again that we are talking about a single sine-wave frequency, ω , we agree to use the following rules:

1. Voltages and currents are *represented* by the complex quantities V and I . The voltage $V_0 \cos(\omega t + \phi)$ is to be represented by the complex number $V_0 e^{j\phi}$. Recall that $e^{j\theta} = \cos \theta + j \sin \theta$, where $j = \sqrt{-1}$.
2. *Actual* voltages and currents are obtained by multiplying their complex number representations by $e^{j\omega t}$ and then taking the real part: $V(t) = \text{Re}(V e^{j\omega t})$, $I(t) = \text{Re}(I e^{j\omega t})$.

In other words,

circuit voltage versus time $V_0 \cos(\omega t + \phi)$	$\longleftrightarrow$ multiply by $e^{j\omega t}$ and take real part	complex number representation $V_0 e^{j\phi} = a + jb$
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(In electronics, the symbol j is used instead of i in the exponential in order to avoid confusion with the symbol i meaning current.) Thus, in the general case the actual voltages and currents are given by

$$\begin{aligned} V(t) &= \operatorname{Re}(V e^{j\omega t}) \\ &= \operatorname{Re}(V) \cos \omega t - \operatorname{Im}(V) \sin \omega t \\ I(t) &= \operatorname{Re}(I e^{j\omega t}) \\ &= \operatorname{Re}(I) \cos \omega t - \operatorname{Im}(I) \sin \omega t \end{aligned}$$

For example, a voltage whose complex representation is

$$V = 5j$$

corresponds to a (real) voltage versus time of

$$\begin{aligned} V(t) &= \operatorname{Re}[5j \cos \omega t + 5j(j) \sin \omega t] \\ &= -5 \sin \omega t \text{ volts} \end{aligned}$$

Reactance of capacitors and inductors

With this convention we can apply complex Ohm's law to circuits containing capacitors and inductors, just as for resistors, once we know the reactance of a capacitor or inductor. Let's find out what these are. We have

$$V(t) = \operatorname{Re}(V_0 e^{j\omega t})$$

For a capacitor, using $I = C(dV/dt)$, we obtain

$$\begin{aligned} I(t) &= -V_0 C \omega \sin \omega t = \operatorname{Re} \left(\frac{V_0 e^{j\omega t}}{-j/\omega C} \right) \\ &= \operatorname{Re} \left(\frac{V_0 e^{j\omega t}}{X_C} \right) \end{aligned}$$

i.e., for a capacitor

$$X_C = -j/\omega C$$

X_C is the reactance of a capacitor at frequency ω . As an example a $1\mu\text{F}$ capacitor has a reactance of $-2653j$ ohms at 60Hz and a reactance of $-0.16j$ ohms at 1MHz. Its reactance at dc is infinite.

If we did a similar analysis for an inductor, we would find

$$X_L = j\omega L$$

A circuit containing only capacitors and inductors always has a purely imaginary impedance, meaning that the voltage and current are always 90° out of phase – it is purely reactive. When the circuit contains resistors, there is also a real part to the impedance. The term “reactance” in that case means the imaginary part only.

Ohm's law generalized

With these conventions for representing voltages and currents, Ohm's law takes a simple form. It reads simply

$$I = V/Z$$

$$V = IZ$$

where the voltage represented by V is applied across a circuit of impedance Z , giving a current represented by I . The complex impedance of devices in series or parallel obeys the same rules as resistance:

$$Z = Z_1 + Z_2 + Z_3 + \cdots \quad (\text{series})$$

$$Z = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots} \quad (\text{parallel})$$

Finally, for completeness we summarize here the formulas for the impedance of resistors, capacitors, and inductors:

$$Z_R = R \quad (\text{resistor})$$

$$Z_C = -j/\omega C = 1/j\omega C \quad (\text{capacitor})$$

$$Z_L = j\omega L \quad (\text{inductor})$$

With these rules we can analyze many ac circuits by the same general methods we used in handling dc circuits, i.e., application of the series and parallel formulas and Ohm's law. Our results for circuits such as voltage dividers will look nearly the same as before. For multiply connected

networks we laws, just as using the $\cos$ and I : The drops around sum of the $\cos$ is zero. The dc circuits, the series circuit

EXERCISE

Use the preceding devices in parallel formulas (Section two capacitors. Hint: In each case have capacitors impedance of then equate it with capacitors

Let's try the simplest circuit applied across considered just look at power laying the ground simple but ex RC filter circuit

Imagine a 110 volt (rms) current flows we have

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Therefore, the

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networks we may have to use Kirchhoff's laws, just as with dc circuits, in this case using the complex representations for V and I : The sum of the (complex) voltage drops around a closed loop is zero, and the sum of the (complex) currents into a point is zero. The latter rule implies, as with dc circuits, that the (complex) current in a series circuit is the same everywhere.

EXERCISE 1.16

Use the preceding rules for the impedance of devices in parallel and in series to derive the formulas (Section 1.12) for the capacitance of two capacitors (a) in parallel and (b) in series. Hint: In each case, let the individual capacitors have capacitances C_1 and C_2 . Write down the impedance of the parallel or series combination; then equate it to the impedance of a capacitor with capacitance C . Find C .

Let's try out these techniques on the simplest circuit imaginable, an ac voltage applied across a capacitor, which we considered just previously. Then, after a brief look at power in reactive circuits (to finish laying the groundwork), we'll analyze some simple but extremely important and useful RC filter circuits.

Imagine putting a $1\mu\text{F}$ capacitor across a 110 volt (rms) 60Hz power line. What current flows? Using complex Ohm's law, we have

$$Z = -j/\omega C$$

Therefore, the current is given by

$$I = V/Z$$

The phase of the voltage is arbitrary, so let us choose $V = A$, i.e. $V(t) = A \cos \omega t$, where the amplitude $A = 110\sqrt{2} \approx 156$ volts. Then

$$I = j\omega C A \approx 0.059 \sin \omega t$$

The resulting current has an amplitude of 59mA (41.5mA rms) and leads the voltage by 90° . This agrees with our previous calculation. Note that if we just wanted to know the magnitude of the current, and

didn't care what the relative phase was, we could have avoided doing any complex algebra: If

$$A = B/C$$

then

$$A = B/C$$

where A , B , and C are the magnitudes of the respective complex numbers; this holds for multiplication, also (see Exercise 1.17). Thus, in this case,

$$I = V/Z = \omega C V$$

This trick is often useful.

Surprisingly, there is no power dissipated by the capacitor in this example. Such activity won't increase your electric bill; you'll see why in the next section. Then we will go on to look at circuits containing resistors and capacitors with our complex Ohm's law.

EXERCISE 1.17

Show that if $\mathbf{A} = \mathbf{B}\mathbf{C}$, then $A = BC$, where A , B , and C are magnitudes. Hint: Represent each complex number in polar form, i.e., $\mathbf{A} = Ae^{i\theta}$.

Power in reactive circuits

The instantaneous power delivered to any circuit element is always given by the product $P = VI$. However, in reactive circuits where V and I are not simply proportional, you can't just multiply them together. Funny things can happen; for instance, the sign of the product can reverse over one cycle of the ac signal. Figure 1.49 shows an example. During time intervals A and C , power is being delivered to the capacitor (albeit at a variable rate), causing it to charge up; its stored energy is increasing (power is the rate of change of energy). During intervals B and D , the power delivered to the capacitor is negative; it is discharging. The average power over a whole cycle for this example is in fact exactly zero, a statement that is always true for any purely reactive circuit element (inductors, capacitors, or any combination

thereof). If you know your trigonometric integrals, the next exercise will show you how to prove this.

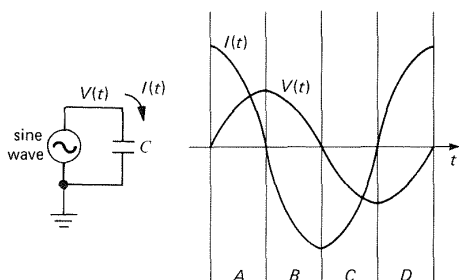


Figure 1.49. When driven by a sine wave, the current through a capacitor leads the voltage by 90° .

EXERCISE 1.18

Optional exercise: Prove that a circuit whose current is 90° out of phase with the driving voltage consumes no power, averaged over an entire cycle.

How do we find the average power consumed by an arbitrary circuit? In general, we can imagine adding up little pieces of VI product, then dividing by the elapsed time. In other words,

$$P = \frac{1}{T} \int_0^T V(t)I(t) dt$$

where T is the time for one complete cycle. Luckily, that's almost never necessary. Instead, it is easy to show that the average power is given by

$$P = \operatorname{Re}(VI^*) = \operatorname{Re}(V^*I)$$

where V and I are complex rms amplitudes.

Let's take an example. Consider the preceding circuit, with a 1 volt (rms) sine wave driving a capacitor. We'll do everything with rms amplitudes, for simplicity. We have

$$V = 1$$

$$I = \frac{V}{-j/\omega C} = j\omega C$$

$$P = \operatorname{Re}(VI^*) = \operatorname{Re}(-j\omega C) = 0$$

That is, the average power is zero, as stated earlier.

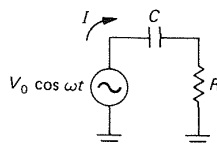


Figure 1.50

As another example, consider the circuit shown in Figure 1.50. Our calculations go like this:

$$Z = R - \frac{j}{\omega C}$$

$$V = V_0$$

$$I = \frac{V}{Z} = \frac{V_0}{R - (j/\omega C)} = \frac{V_0[R + (j/\omega C)]}{R^2 + (1/\omega^2 C^2)}$$

$$P = \operatorname{Re}(VI^*) = \frac{V_0^2 R}{R^2 + (1/\omega^2 C^2)}$$

(In the third line we multiplied numerator and denominator by the complex conjugate of the denominator, in order to make the denominator real.) This is less than the product of the magnitudes of V and I . In fact, the ratio is called the *power factor*:

$$|V| |I| = \frac{V_0^2}{[R^2 + (1/\omega^2 C^2)]^{1/2}}$$

$$\begin{aligned} \text{power factor} &= \frac{\text{power}}{|V| |I|} \\ &= \frac{R}{[R^2 + (1/\omega^2 C^2)]^{1/2}} \end{aligned}$$

in this case. The power factor is the cosine of the phase angle between the voltage and the current, and it ranges from 0 (purely reactive circuit) to 1 (purely resistive). A power factor less than 1 indicates some component of reactive current.

EXERCISE

Show that all the preceding circuit calculations for power, in a capacitor and inductor, are correct for a 110 volt (rms) source.

Power factor is a scale electric reactive current power being the power of $I^2 R$ heating transformers identical user power $[\operatorname{Re}(VI^*)]$ charges inductor power factor yards that you built to cancel industrial machine

EXERCISE

Show that adding a capacitor $C = 1/\omega^2 L$ in a series RL circuit with the word "power factor"

Voltage divider

Our original circuit consisted of a voltage source V_{in} to ground, in series with a resistor R at the junction of a load resistor R_L that simple circuit in which the resistors are replaced by a more complex circuit. In general, the divider is not a frequency divider. The

$$\begin{aligned} I &= \frac{V_{in}}{Z_{total}} \\ Z_{total} &= Z_1 + Z_2 \\ V_{out} &= Z_2 I \end{aligned}$$

= 0

ero, as stated

EXERCISE 1.19

Show that all the average power delivered to the preceding circuit winds up in the resistor. Do this by computing the value of V_R^2/R . What is that power, in watts, for a series circuit of a $1\mu\text{F}$ capacitor and a $1.0\text{k}\Omega$ resistor placed across the 110 volt (rms), 60Hz power line?

Power factor is a serious matter in large-scale electrical power distribution, because reactive currents don't result in useful power being delivered to the load, but cost the power company plenty in terms of I^2R heating in the resistance of generators, transformers, and wiring. Although residential users are only billed for "real" power $[\text{Re}(\mathbf{VI}^*)]$, the power company charges industrial users according to the power factor. This explains the capacitor yards that you see behind large factories, built to cancel the inductive reactance of industrial machinery (i.e., motors).

EXERCISE 1.20

Show that adding a series capacitor of value $C = 1/\omega^2 L$ makes the power factor equal 1.0 in a series RL circuit. Now do the same thing, but with the word "series" changed to "parallel."

Voltage dividers generalized

Our original voltage divider (Fig. 1.5) consisted of a pair of resistors in series to ground, input at the top and output at the junction. The generalization of that simple resistive divider is a similar circuit in which either or both resistors are replaced by a capacitor or inductor (or a more complicated network made from R , L , and C), as in Figure 1.51. In general, the division ratio $V_{\text{out}}/V_{\text{in}}$ of such a divider is not constant, but depends on frequency. The analysis is straightforward:

$$\mathbf{I} = \frac{\mathbf{V}_{\text{in}}}{\mathbf{Z}_{\text{total}}}$$

$$\mathbf{Z}_{\text{total}} = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$\mathbf{V}_{\text{out}} = \mathbf{I} \mathbf{Z}_2 = \mathbf{V}_{\text{in}} \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

Rather than worrying about this result in general, let's look at some simple, but very important, examples.

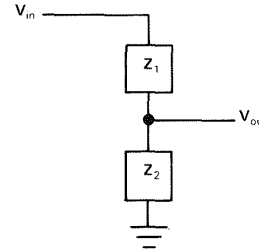


Figure 1.51. Generalized voltage divider: a pair of arbitrary impedances.

1.19 RC filters

By combining resistors with capacitors it is possible to make frequency-dependent voltage dividers, owing to the frequency dependence of a capacitor's impedance $\mathbf{Z}_C = -j/\omega C$. Such circuits can have the desirable property of passing signal frequencies of interest while rejecting undesired signal frequencies. In this section you will see examples of the simplest such RC filters, which we will be using frequently throughout the book. Chapter 5 and Appendix H describe filters of greater sophistication.

High-pass filters

Figure 1.52 shows a voltage divider made from a capacitor and a resistor. Complex Ohm's law gives

$$\mathbf{I} = \frac{\mathbf{V}_{\text{in}}}{\mathbf{Z}_{\text{total}}} = \frac{\mathbf{V}_{\text{in}}}{R - (j/\omega C)}$$

$$= \frac{\mathbf{V}_{\text{in}}[R + (j/\omega C)]}{R^2 + 1/\omega^2 C^2}$$

(For the last step, multiply top and bottom by the complex conjugate of the denominator.) So the voltage across R is just

$$\mathbf{V}_{\text{out}} = \mathbf{I} \mathbf{Z}_R = \mathbf{I} R = \frac{\mathbf{V}_{\text{in}}[R + (j/\omega C)]R}{R^2 + 1/\omega^2 C^2}$$

Most often we don't care about the phase of V_{out} , just its amplitude:

$$V_{out} = (V_{out} V_{out}^*)^{1/2} \\ = \frac{R}{[R^2 + (1/\omega^2 C^2)]^{1/2}} V_{in}$$

Note the analogy to a resistive divider, where

$$V_{out} = \frac{R_1}{R_1 + R_2} V_{in}$$

Here the impedance of the series RC combination (Fig. 1.53) is as shown in Figure 1.54. So the "response" of this circuit, ignoring phase shifts by taking magnitudes of the complex amplitudes, is given by

$$V_{out} = \frac{R}{[R^2 + (1/\omega^2 C^2)]^{1/2}} V_{in} \\ = \frac{2\pi f RC}{[1 + (2\pi f RC)^2]^{1/2}} V_{in}$$

and looks as shown in Figure 1.55. We could have gotten this result immediately by taking the ratio of the *magnitudes* of impedances, as in Exercise 1.17 and the example immediately preceding it; the numerator is the magnitude of the impedance of the lower leg of the divider (R), and the denominator is the magnitude of the impedance of the series combination of R and C .

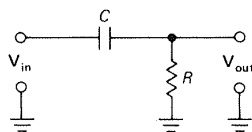


Figure 1.52. High-pass filter.

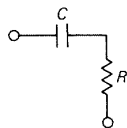


Figure 1.53

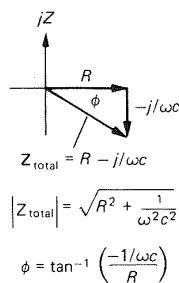


Figure 1.54

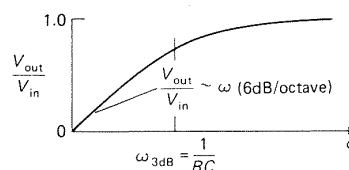


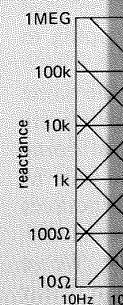
Figure 1.55. Frequency response of high-pass filter.

You can see that the output is approximately equal to the input at high frequencies (how high? $\geq 1/RC$) and goes to zero at low frequencies. This is a very important result. Such a circuit is called a high-pass filter, for obvious reasons. It is very common. For instance, the input to the oscilloscope (Appendix A) can be switched to ac coupling. That's just an RC high-pass filter with the bend at about 10Hz (you would use ac coupling if you wanted to look at a small signal riding on a large dc voltage). Engineers like to refer to the -3 dB "breakpoint" of a filter (or of any circuit that behaves like a filter). In the case of the simple RC high-pass filter, the -3 dB breakpoint is given by

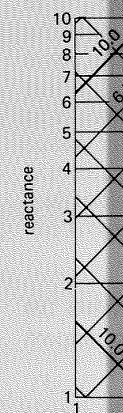
$$f_{3dB} = 1/2\pi RC$$

Note that the capacitor lets no steady current through ($f = 0$). This use as a dc *blocking capacitor* is one of its most frequent applications. Whenever you need to couple a signal from one amplifier to another, you almost invariably use a capacitor. For instance, every hi-fi audio

amplifier coupled, b level its in In such a pick R an interest (i passed wi



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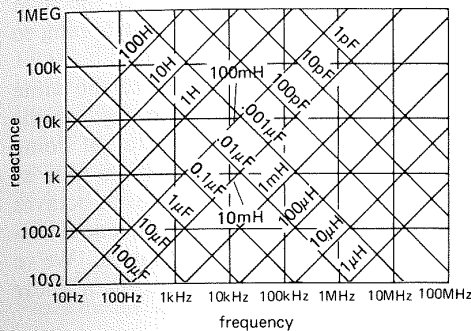


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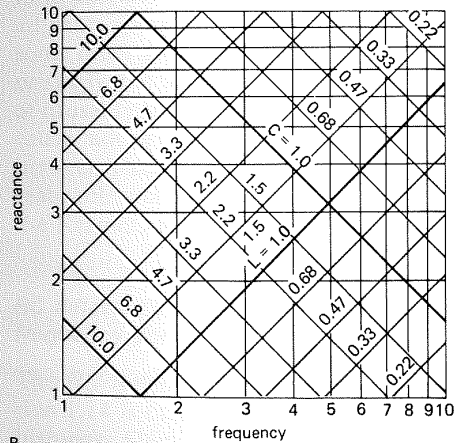
Figure 1.5 capacitors identical, e B. A single standard 2

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amplifier has all its inputs capacitively coupled, because it doesn't know what dc level its input signals might be riding on. In such a coupling application you always pick R and C so that all frequencies of interest (in this case, 20Hz–20kHz) are passed without loss (attenuation).



A



B

Figure 1.56. A. Reactance of inductors and capacitors versus frequency; all decades are identical, except for scale.

B. A single decade from part A expanded, with standard 20% component values shown.

You often need to know the impedance of a capacitor at a given frequency (e.g., for design of filters). Figure 1.56 provides a very useful graph covering large ranges

of capacitance and frequency, giving the value of $|Z| = 1/2\pi fC$.

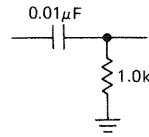


Figure 1.57

As an example, consider the filter shown in Figure 1.57. It is a high-pass filter with the 3dB point at 15.9kHz. The impedance of a load driven by it should be much larger than 1.0k in order to prevent circuit loading effects on the filter's output, and the driving source should be able to drive a 1.0k load without significant attenuation (loss of signal amplitude) in order to prevent circuit loading effects by the filter on the signal source.

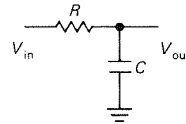


Figure 1.58. Low-pass filter.

Low-pass filters

You can get the opposite frequency behavior in a filter by interchanging R and C (Fig. 1.58). You will find

$$V_{out} = \frac{1}{(1 + \omega^2 R^2 C^2)^{1/2}} V_{in}$$

as seen in Figure 1.59. This is called a low-pass filter. The 3dB point is again at a frequency

$$f = 1/2\pi RC$$

Low-pass filters are quite handy in real life. For instance, a low-pass filter can be used to eliminate interference from nearby radio and television stations (550kHz–800MHz), a problem that plagues audio

amplifiers and other sensitive electronic equipment.

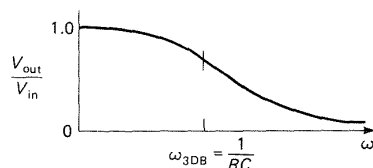


Figure 1.59. Frequency response of low-pass filter.

EXERCISE 1.21

Show that the preceding expression for the response of an RC low-pass filter is correct.

The low-pass filter's output can be viewed as a signal source in its own right. When driven by a perfect ac voltage (zero

source impedance), the filter's output looks like R at low frequencies (the perfect signal source can be replaced by a short, i.e., by its small-signal source impedance, for the purpose of impedance calculations). It drops to zero impedance at high frequencies, where the capacitor dominates the output impedance. The signal driving the filter sees a load of R plus the load resistance at low frequencies, dropping to R at high frequencies.

In Figure 1.60, we've plotted the same low-pass filter response with *logarithmic* axes, which is a more usual way of doing it. You can think of the vertical axis as decibels, and the horizontal axis as octaves (or decades). On such a plot, equal distances correspond to equal ratios. We've also plotted the phase shift, using a linear

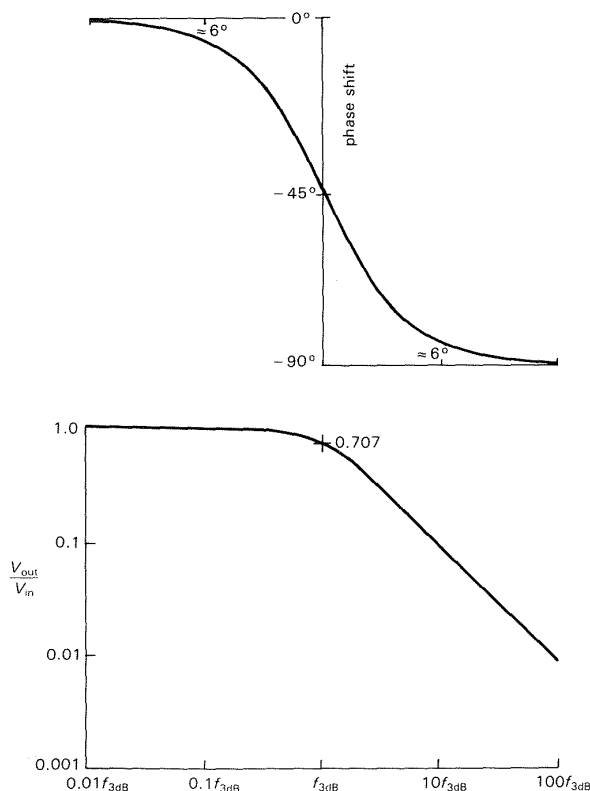


Figure 1.60. Frequency response (phase and amplitude) of low-pass filter, plotted on logarithmic axes. Note that the phase shift is 45° at the 3dB point and is within 6° of its asymptotic value for a decade of frequency change.

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vertical axis (degrees) and the same loga- rithmic frequency axis. This sort of plot is good for seeing the detailed response even when it is greatly attenuated (as at right); we'll see a number of such plots in Chap- ter 5, when we treat active filters. Note that the filter curve plotted here becomes a straight line at large attenuations, with a slope of -20dB/decade (engineers prefer to say " -6dB/octave "). Note also that the phase shift goes smoothly from 0° (at fre- quencies well below the breakpoint) to 90° (well above it), with a value of 45° at the -3dB point. A rule of thumb for single- section RC filters is that the phase shift is $\approx 6^\circ$ from its asymptotic value at $0.1f_{3\text{dB}}$ and $10f_{3\text{dB}}$.

EXERCISE 1.22

Prove the last assertion.

An interesting question is the following: Is it possible to make a filter with some arbitrary specified amplitude response and some other specified phase response? Sur- prisingly, the answer is no: The demands of causality (i.e., that response must follow cause, not precede it) force a relationship between phase and amplitude response of realizable analog filters (known officially as the Kramers-Kronig relation).

RC differentiators and integrators in the frequency domain

The RC differentiator that we saw in Sec- tion 1.14 is exactly the same circuit as the high-pass filter in this section. In fact, it can be considered as either, depending on whether you're thinking of waveforms in the time domain or response in the fre- quency domain. We can restate the ear- lier time-domain condition for its proper operation ($V_{\text{out}} \ll V_{\text{in}}$) in terms of the frequency response: For the output to be small compared with the input, the signal frequency (or frequencies) must be well below the 3dB point. This is easy to check.

Suppose we have the input signal

$$V_{\text{in}} = \sin \omega t$$

Then, using the equation we obtained earlier for the differentiator output,

$$V_{\text{out}} = RC \frac{d}{dt} \sin \omega t = \omega RC \cos \omega t$$

and so $V_{\text{out}} \ll V_{\text{in}}$ if $\omega RC \ll 1$, i.e., $RC \ll 1/\omega$. If the input signal contains a range of frequencies, this must hold for the highest frequencies present in the input.

The RC integrator (Section 1.15) is the same circuit as the low-pass filter; by similar reasoning, the criterion for a good integrator is that the lowest signal frequen- cies must be well above the 3dB point.

Inductors versus capacitors

Inductors could be used, instead of capaci- tors, in combination with resistors to make low-pass (or high-pass) filters. In prac- tice, however, you rarely see RL low- or high-pass filters. The reason is that induc- tors tend to be more bulky and expensive and perform less well (i.e., they depart further from the ideal) than capacitors. If you have a choice, use a capacitor. One exception to this general statement is the use of ferrite beads and chokes in high- frequency circuits. You just string a few beads here and there in the circuit; they make the wire interconnections slightly in- ductive, raising the impedance at very high frequencies and preventing "oscillations," without the added resistance you would get with an RC filter. An RF "choke" is an in- ductor, usually a few turns of wire wound on a ferrite core, used for the same purpose in RF circuits.

□ 1.20 Phasor diagrams

There's a nice graphic method that can be very helpful when trying to understand reactive circuits. Let's take an example, namely the fact that an RC filter atten- uates 3dB at a frequency $f = 1/2\pi RC$,

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which we derived in Section 1.19. This is true for both high-pass and low-pass filters. It is easy to get a bit confused here, because at that frequency the reactance of the capacitor equals the resistance of the resistor; so you might at first expect 6dB attenuation. That is what you would get, for example, if you were to replace the capacitor by a resistor of the same impedance (recall that 6dB means half voltage). The confusion arises because the capacitor is reactive, but the matter is clarified by a phasor diagram (Fig. 1.61). The axes are the real (resistive) and imaginary (reactive) components of the impedance. In a series circuit like this, the axes also represent the (complex) voltage, because the current is the same everywhere. So for this circuit (think of it as an R - C voltage divider) the input voltage (applied across the series R - C pair) is proportional to the length of the hypotenuse, and the output voltage (across R only) is proportional to the length of the R leg of the triangle. The diagram represents the situation at the frequency where the magnitude of the capacitor's reactance equals R , i.e., $f = 1/2\pi RC$, and shows that the ratio of output voltage to input voltage is $1/\sqrt{2}$, i.e., -3dB.

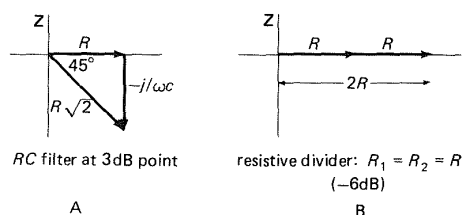


Figure 1.61

The angle between the vectors gives the phase shift from input to output. At the 3dB point, for instance, the output amplitude equals the input amplitude divided by the square root of 2, and it leads by 45° in phase. This graphic method makes it easy to read off amplitude and phase relationships in RLC circuits. For example,

you can use it to get the response of the high-pass filter that we previously derived algebraically.

EXERCISE 1.23

Use a phasor diagram to derive the response of an RC high-pass filter:

$$V_{\text{out}} = \frac{R}{[R^2 + (1/\omega^2 C^2)]^{1/2}} V_{\text{in}}$$

EXERCISE 1.24

At what frequency does an RC low-pass filter attenuate by 6dB (output voltage equal to half the input voltage)? What is the phase shift at that frequency?

EXERCISE 1.25

Use a phasor diagram to obtain the low-pass filter response previously derived algebraically.

In the next chapter (Section 2.08) you will see a nice example of phasor diagrams in connection with a constant-amplitude phase-shifting circuit.

1.21 "Poles" and decibels per octave

Look again at the response of the RC low-pass filter (Fig. 1.59). Far to the right of the "knee" the output amplitude is dropping proportional to $1/f$. In one octave (as in music, one octave is twice the frequency) the output amplitude will drop to half, or -6dB; so a simple RC filter has a 6dB/octave falloff. You can make filters with several RC sections; then you get 12dB/octave (two RC sections), 18dB/octave (three sections), etc. This is the usual way of describing how a filter behaves beyond the cutoff. Another popular way is to say a "3-pole filter," for instance, meaning a filter with three RC sections (or one that behaves like one). (The word "pole" derives from a method of analysis that is beyond the scope of this book and that involves complex transfer functions in the complex frequency plane, known by engineers as the "s-plane.")

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A caution on multistage filters: You can't simply cascade several identical filter sections in order to get a frequency response that is the concatenation of the individual responses. The reason is that each stage will load the previous one significantly (since they're identical), changing the overall response. Remember that the response function we derived for the simple RC filters was based on a zero-impedance driving source and an infinite-impedance load. One solution is to make each successive filter section have much higher impedance than the preceding one. A better solution involves active circuits like transistor or operational amplifier (op-amp) interstage "buffers," or active filters. These subjects will be treated in Chapters 2 through 5.

1.22 Resonant circuits and active filters

When capacitors are combined with inductors or are used in special circuits called active filters, it is possible to make circuits that have very sharp frequency characteristics (e.g., a large peak in the response at a particular frequency), as compared with the gradual characteristics of the RC filters we've seen so far. These circuits find applications in various audiofrequency and radiofrequency devices. Let's now take a quick look at LC circuits (there will be more on them, and active filters, in Chapter 5 and Appendix H).

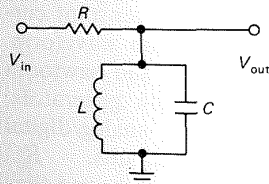


Figure 1.62. LC resonant circuit: bandpass filter.

First, consider the circuit shown in Figure 1.62. The reactance of the LC

combination at frequency f is just

$$\frac{1}{Z_{LC}} = \frac{1}{Z_L} + \frac{1}{Z_C} = \frac{1}{j\omega L} - \frac{\omega C}{j}$$

$$= j\left(\omega C - \frac{1}{\omega L}\right)$$

i.e.,

$$Z_{LC} = \frac{j}{(1/\omega L) - \omega C}$$

In combination with R it forms a voltage divider; because of the opposite behaviors of inductors and capacitors, the impedance of the parallel LC goes to infinity at the *resonant frequency* $f_0 = 1/2\pi\sqrt{LC}$ (i.e., $\omega_0 = 1/\sqrt{LC}$), giving a peak in the response there. The overall response is as shown in Figure 1.63.

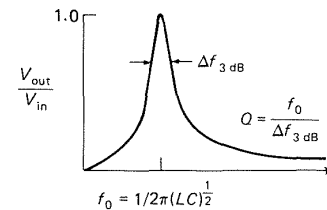


Figure 1.63

In practice, losses in the inductor and capacitor limit the sharpness of the peak, but with good design these losses can be made very small. Conversely, a Q -spoiling resistor is sometimes added intentionally to reduce the sharpness of the resonant peak. This circuit is known simply as a parallel LC resonant circuit or a tuned circuit and is used extensively in radiofrequency circuits to select a particular frequency for amplification (the L or C can be variable, so you can tune the resonant frequency). The higher the driving impedance, the sharper the peak; it is not uncommon to drive them with something approaching a current source, as you will see later. The *quality factor* Q is a measure of the sharpness of the peak. It equals the resonant frequency divided by the width

at the -3dB points. For a parallel RLC circuit, $Q = \omega_0 RC$.

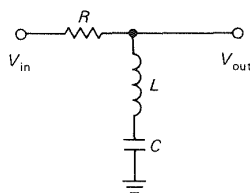


Figure 1.64. LC notch filter ("trap").

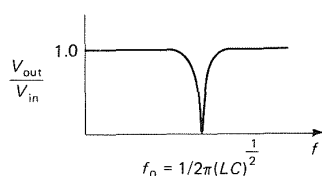


Figure 1.65

Another variety of LC circuit is the series LC (Fig. 1.64). By writing down the impedance formulas involved, you can convince yourself that the impedance of the LC goes to zero at resonance [$f_0 = 1/2\pi(LC)^{1/2}$]; such a circuit is a "trap" for signals at or near the resonant frequency, shorting them to ground. Again, this circuit finds application mainly in radiofrequency circuits. Figure 1.65 shows what the response looks like. The Q of a series RLC circuit is $Q = \omega_0 L/R$.

EXERCISE 1.26

Find the response ($V_{\text{out}}/V_{\text{in}}$ versus frequency) for the series LC trap circuit in Figure 1.64.

1.23 Other capacitor applications

In addition to their uses in filters, resonant circuits, differentiators, and integrators, capacitors are needed for several other important applications. We will treat these in detail later in the book, mentioning them here only as a preview.

Bypassing

The impedance of a capacitor goes down with increasing frequency. This is the basis of another important application: bypassing. There are places in circuits where you want to allow a dc (or slowly varying) voltage, but don't want signals present. Placing a capacitor across that circuit element (usually a resistor) will help to kill any signals there. You choose the capacitor value so that its impedance at signal frequencies is small compared with what it is bypassing. You will see much more of this in later chapters.

Power-supply filtering

Power-supply filtering is really a form of bypassing, although we usually think of it as energy storage. The dc voltages used in electronics are usually generated from the ac line voltage by a process called *rectification* (which will be treated later in this chapter); some residue of the 60Hz input remains, and this can be reduced as much as desired by means of bypassing with suitably large capacitors. These capacitors really are large – they're the big shiny round things you see inside most electronic instruments. You will see how to design power supplies and filters later in this chapter and again in Chapter 6.

Timing and waveform generation

A capacitor supplied with a constant current charges up with a ramp waveform. This is the basis of ramp and sawtooth generators, used in function generators, oscilloscope sweep circuits, analog/digital converters, and timing circuits. RC circuits are also used for timing, and they form the basis of digital delay circuits (monostable multivibrators). These timing and waveform applications are important in many areas of electronics and will be covered in Chapters 3, 5, 8, and 9.

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TABLE 1.1. DIODES

Type	$V_{R(max)}^a$ (V)	$I_{R(max)}^b$ (μ A)	Continuous		Peak		Reverse recovery (ns)	Capacitance (10V) (pF)	Class	Comments
			V_F (V)	I_F (mA)	V_F (V)	I_F (A)				
PAD-1	45	1pA@20V	0.8	5	—	—	—	0.8	lowest I_R	Siliconix
FJT100	30	0.001	—	—	1.1	0.05	—	1.2	very low I_R	1pA@5V, 10pA@15V
ID101	30	10pA@10V	0.8	1	1.1	0.03	—	0.8	very low I_R	Intersil; dual
1N3595	150	3	0.7	10	<1.0	0.2	3000	8.0	low I_R	1nA@125V
1N914	75	5	0.75	10	1.1	0.1	4	1.3	gen purp sig diode	indus std; same as 1N4148
1N6263	60	10	0.4	1	0.7	0.01	0	1.0	Schottky; low V_F	
1N3062	75	50	<1.0	20 ^b	—	—	2	0.6	low cap, sig diode	1pF at 0 volts
1N4305	75	50	0.6	1	—	—	4	1.5	controlled V_F	
1N4002	100	50	0.9	1000	2.3	25	3500	15	1-amp rect	indus std; 7-member fam
1N4007	1000	50	0.9	1000	2.3	25	5000	10		
1N5819	40	10000	0.4	1000	1.1	20	—	50	pwr Schottky	lead mounted
1N5822	40	20000	0.45	3000	1.3	50	—	180	pwr Schottky	lead mounted
1N5625	400	50	1.1	5000	2.0	50	2500	45	5-amp rect	lead mounted
1N1183A	50	1000	1.1	40000	1.3	100	—	—	high curr rect	1N1183RA reverse

(a) $V_{R(max)}$ is repetitive peak reverse voltage, 25°C, 10 μ A leakage.

(b) $I_{R(max)}$ is reverse leakage current at V_R and 100°C ambient temperature.

1.24 Thévenin's theorem generalized

When capacitors and inductors are included, Thévenin's theorem must be restated: Any two-terminal network of resistors, capacitors, inductors, and signal sources is equivalent to a single complex impedance in series with a single signal source. As before, you find the impedance and the signal source from the open-circuit output voltage and the short-circuit current.

DIODES AND DIODE CIRCUITS

1.25 Diodes

The circuit elements we've discussed so far (resistors, capacitors, and inductors) are all *linear*, meaning that a doubling of the applied signal (a voltage, say) produces a doubling of the response (a current, say). This is true even for the reactive devices (capacitors and inductors). These devices are also *passive*, meaning that they don't have a built-in source of power. And they are all two-terminal devices, which is self-explanatory.

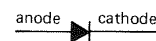


Figure 1.66. Diode.

The diode (Fig. 1.66) is a very important and useful two-terminal passive *non-linear* device. It has the V - I curve shown in Figure 1.67. (In keeping with the general philosophy of this book, we will not attempt to describe the solid-state physics that makes such devices possible.)

The diode's arrow (the anode terminal) points in the direction of forward current flow. For example, if the diode is in a circuit in which a current of 10mA is flowing from anode to cathode, then (from the graph) the anode is approximately 0.5 volt more positive than the cathode; this is called the "forward voltage drop." The reverse current, which is measured in the

nanoamp range for a general-purpose diode (note the different scales in the graph for forward and reverse current), is almost never of any consequence until you reach the reverse breakdown voltage (also called the peak inverse voltage, PIV), typically 75 volts for a general-purpose diode like the 1N914. (Normally you never subject a diode to voltages large enough to cause reverse breakdown; the exception is the zener diode we mentioned earlier.) Frequently, also, the forward voltage drop of about 0.5 and 0.8 volt is of little concern, and the diode can be treated as a good approximation to an ideal one-way conductor. There are other important characteristics that distinguish the thousands of diode types available, e.g., maximum forward current, capacitance, leakage current, and reverse recovery time (see Table 1.1 for characteristics of some typical diodes).

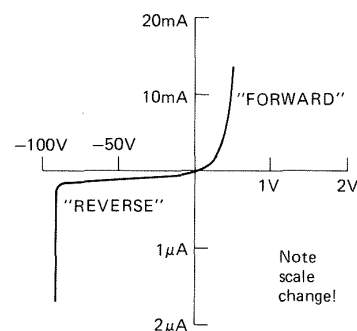


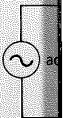
Figure 1.67. Diode V - I curve.

Before jumping into some circuits with diodes, we should point out two things: (a) A diode doesn't actually have a resistance (it doesn't obey Ohm's law). (b) If you put some diodes in a circuit, it won't have a Thévenin equivalent.

1.26 Rectification

A rectifier changes ac to dc; this is one of the simplest and most important applications of diodes (diodes are sometimes

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