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The
BENCHTOP
ELECTRONICS
REFERENCE
MANUAL

VICTOR F.C. VELEY

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ELECTRONICS
REFERENCE
MANUAL**
VICTOR F.C. VELEY



TAB BOOKS Inc.

Blue Ridge Summit, PA 17214

To my beautiful wife, Joyce, who has been my inspiration and strength for over thirty-six years, and to those I hold dear, Jackie, Gill, Phil, Ray, Jaimie, Nicola, Lisa, Katie, Peter, Paul, and Paula.

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Insulator

There is no change in the value of the electric field intensity.
Flux density,

$$\begin{aligned} D &= \mathcal{E} \times \epsilon_0 \epsilon_r \\ &= 5.53 \times 10^{-8} \times 3.5 \\ &= 1.935 \times 10^{-7} \text{ C/m}^2. \end{aligned} \quad (46-8)$$

Charge on each plate,

$$\begin{aligned} Q &= 1.935 \times 10^{-7} \text{ C/m}^2 \times 4 \times 10^{-4} \text{ m}^2 \\ &= 77.4 \text{ pC}. \end{aligned}$$



Chapter 47

Capacitance and the Capacitor

CAPACITANCE IS THAT PROPERTY OF AN ELECTRICAL CIRCUIT THAT PREVENTS ANY *SUDDEN* change in *voltage* and limits the rate of change in the voltage. In other words, the voltage across a capacitance, whose letter symbol is C , cannot change instantaneously.

The property of capacitance is possessed by a capacitor, which is a device for storing charge and consists of two conducting surfaces (copper, silver, aluminum, tin foil, etc.) which are separated by an insulator or dielectric (air, mica, ceramic, etc.).

When a dc voltage, E , is switched across the capacitor, C , in Fig. 47-1, electrons are drawn off the right-hand plate, flow through the voltage source and are then deposited on the left-hand plate. This electron flow is only momentary and ceases when the voltage, V_C , between the plates exactly balances the source voltage, E . The charge then stored by the capacitor directly depends on the value of the applied voltage and also on some constant of the capacitor called the capacitance, which is determined by the component's physical construction. The unit for the capacitance is the farad, F . The ca-

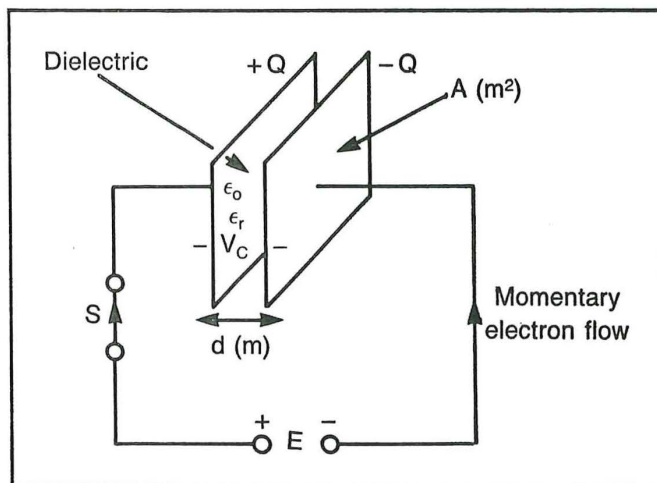


Fig. 47-1.

capacitance is one farad, if when the voltage applied across the capacitor's plates is one volt, the charge stored is one coulomb. Unfortunately, the farad is far too large a unit for practical purposes so that capacitances are normally either measured in microfarads (μF) or picofarads (pF). Note that a picofarad may alternatively be referred to as a micromicrofarad ($\mu\mu\text{F}$).

We already know that a particular electric field intensity (which is inversely proportional to the separation between the plates) produces a certain flux (which is directly proportional to the area of one of the plates). This means that a capacitor's capacitance is directly proportional to the area of the plates, but is inversely proportional to their distance apart. It is true that the capacitance is increased if the conductor plates are brought closer together. However, if the dielectric is made too thin, the voltage between the plates may cause arcing to occur so that the capacitor will be damaged. Each type of insulator has a certain dielectric strength, which is a measure of the insulator's ability to withstand a high electric field intensity. For example, the dielectric strength of mica is typically 50 kV/mm so that in order to avoid breakdown, no more than 50 kV should be applied across a 1 mm thickness of mica. It follows that each capacitor is rated for a particular dc working voltage (WVdc).

MATHEMATICAL DERIVATIONS

For the capacitor in Fig. 47-1,

$$V_C \text{ (volts)} = \frac{-Q \text{ (coulombs)}}{C \text{ (farads)}} \quad (47-1)$$

Using KVL,

$$E + V_C = 0.$$

Therefore

$$E = \frac{Q}{C}, \quad Q = CE, \quad C = \frac{Q}{E} \quad (47-2)$$

Flux Density,

$$D = \frac{Q}{A} \text{ coulombs per square meter} \quad (47-3)$$

Electric field intensity,

$$\mathcal{E} = \frac{E}{d} \text{ volts per meter} \quad (47-4)$$

For a capacitor with a vacuum dielectric,
Permittivity of free space,

$$\begin{aligned} \epsilon_o &= \frac{D}{\mathcal{E}} \\ &= \frac{Q/A}{E/d} \\ &= \frac{Q}{E} \times \frac{d}{A} \end{aligned} \quad (47-5)$$

$$= \frac{C \times d}{A}$$

Equation 47-5 yields,
Capacitance,

$$C = \epsilon_o \times \frac{A}{d} \text{ farads} \quad (47-6)$$

If the dielectric has a relatively permittivity of ϵ_r ,

$$\text{capacitance, } C = \frac{\epsilon_o \epsilon_r A}{d} = \frac{\epsilon A}{d} \text{ farads} \quad (47-7)$$

where ϵ is the dielectric's absolute permittivity, which is measured in farads per meter.
Since $Q = CE$,

$$I = \frac{dQ}{dt} = C \frac{dE}{dt}$$

or

$$\frac{dE}{dt} = \frac{I}{C} \text{ volts per second} \quad (47-8)$$

Because $\frac{dE}{dt}$ is inversely proportional to C , the capacitance will limit the rate of change in the voltage.

Example 47-1

Two parallel aluminum plates are separated by 0.8 mm in air. If the area of one side of each plate is 50 cm², what is the value of the capacitance?

Solution

Capacitance,

$$\begin{aligned} C &= \frac{\epsilon_o \epsilon_r A}{d} & (47-7) \\ &= \frac{8.85 \times 10^{-12} \times 1.0006 \times 50 \times 10^{-4}}{0.8 \times 10^{-3}} \text{ F} \\ &= 55 \text{ pF.} \end{aligned}$$

Example 47-2

A paper capacitor consists of two sheets of tin foil, each side of which has an area of 1500 cm². The sheets are separated by a 0.15 mm thickness of a waxed paper dielectric with a relative permittivity of 2.7. What is the value of the capacitance? If the capacitor is charged to 120 V, what are the values of the electric field intensity, the electric flux density and the charge stored?

Solution

Capacitance,

$$\begin{aligned}
 C &= \frac{\epsilon_o \epsilon_r A}{d} & (47-7) \\
 &= \frac{8.85 \times 10^{-12} \times 2.7 \times 1500 \times 10^{-4}}{0.15 \times 10^{-3}} \text{ F} \\
 &= 0.0239 \mu\text{F}.
 \end{aligned}$$

Charge stored,

$$\begin{aligned}
 Q &= CE \\
 &= 0.0239 \mu\text{F} \times 120 \text{ V} \\
 &= 2.87 \mu\text{C}.
 \end{aligned}$$

Electric flux density,

$$\begin{aligned}
 D &= \frac{\psi}{A} = \frac{Q}{A} = \frac{2.87 \mu\text{C}}{1500 \times 10^{-4} \text{ m}^2} \\
 &= 19.1 \mu\text{C/m}^2.
 \end{aligned}$$

Electric field intensity,

$$\begin{aligned}
 \mathcal{E} &= \frac{V_C}{d} \\
 &= \frac{120 \text{ V}}{0.15 \times 10^{-3} \text{ m}} = 8 \times 10^5 \text{ V/m}.
 \end{aligned}$$

∞

Chapter 48

Energy Stored in the Electric Field of a Capacitor

CONSIDER THAT A CAPACITOR IS BEING CHARGED FROM A CONSTANT CURRENT SOURCE OF I AM-peres over a period of t seconds (Fig. 48-1A). The voltage across the capacitor then increases in a linear manner as shown in the voltage-versus-time graph of Fig. 48-1B. At any time after the switch is closed, one of the plates has acquired a positive charge while the other is negatively charged; the charge stored by the capacitor is given by Q (coulombs) = C (capacitance in farads) $\times V_C$ (volts); see equation 47-2.

When the capacitor is charged, an electric field (ψ), also measured in coulombs, exists between the plates. This represents the stored energy that has been derived from the constant current source.

In another sense, the same energy is related to the force of attraction between the positive and negative plates.

MATHEMATICAL DERIVATIONS

Final Charge,

$$Q = I \times t \text{ coulombs} \quad (48-1)$$

$$\text{Voltage across the capacitor, } V_C = \frac{Q}{C} \text{ volts} \quad (48-2)$$

$$\text{Average voltage across the capacitor} = \frac{V_C}{2} \text{ volts} \quad (48-3)$$

$$\text{Average power at the source} = I \times \frac{V_C}{2} \text{ watts} \quad (48-4)$$

Total energy supplied from the source during the charging process

$$\begin{aligned} &= \text{average power} \times \text{time} = I \times \frac{V_C}{2} \times t \\ &= \frac{1}{2} \times V_C Q = \frac{1}{2} \times \frac{Q^2}{C} = \frac{1}{2} \times C V_C^2 \text{ joules} \end{aligned} \quad (48-5)$$

Alternatively:

The work done in moving a charge through a potential difference, δV_C , is $Q \times \delta V_C$ (joules = coulombs $\times$ volts) = $C V_C \delta V_C$ (48-6)

Integrating over a total period of t seconds,
Total work done or total energy stored

$$\begin{aligned} W &= \int_0^{V_C} C V_C dV_C \\ &= \frac{1}{2} C V_C^2 \text{ joules} \end{aligned} \quad (48-7)$$

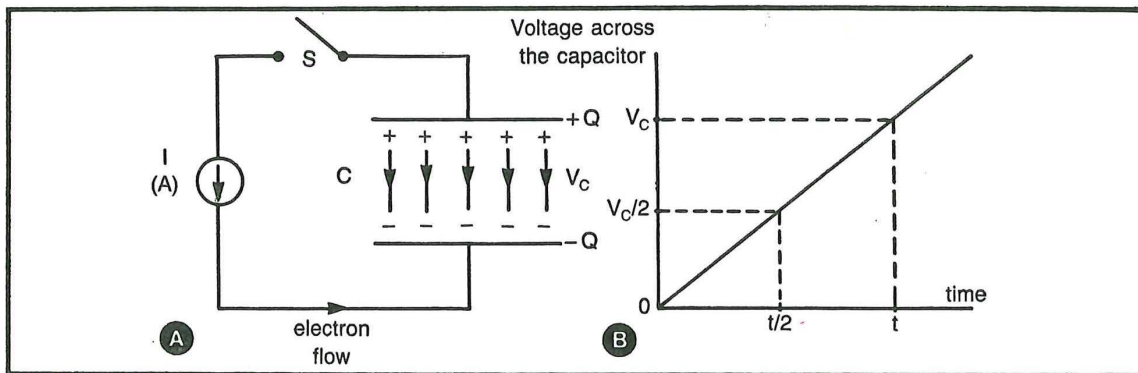


Fig. 48-1.

Example 48-1

In Fig. 48-1, the switch, S, is closed and an $8\mu\text{F}$ capacitor is charged for a period of 7 seconds from a $5\mu\text{A}$ constant current source. At the end of the charging period, what is the amount of charge acquired by the capacitor and what is the corresponding amount of energy stored in the electric field?

Solution

Final charge,

$$Q = I \times t = 5\mu\text{A} \times 7\text{s} = 35\mu\text{C} \quad (48-1)$$

Final voltage across the capacitor,

$$V_C = \frac{Q}{C} = \frac{35\mu\text{C}}{8\mu\text{F}} = 4.75 \text{ V} \quad (48-2)$$

Total energy stored in the capacitor

$$\begin{aligned} &= \frac{1}{2} \times V_C \times Q \\ &= \frac{1}{2} \times 4.75 \text{ V} \times 35 \mu\text{C} = 83.125 \mu\text{J} \end{aligned} \quad (48-5)$$

Notice that the amount of energy stored in a capacitor is normally very small.



Chapter 49

Capacitors in Series

BECAUSE THE CAPACITORS ARE JOINED END-TO-END IN FIG. 49-1, THERE IS ONLY ONE PATH FOR CURRENT flow. Consequently, when S1 is closed in position 1, the charging current for each capacitor is the same and therefore each capacitor must store the same charge, Q (coulombs). But there are two problems that are worth considering. First, since the electron flow cannot pass through the dielectrics, which theoretically are open circuits, how does the middle capacitor become charged? The explanation lies in electrostatic induction. When the electron flow causes the plate, P1, to acquire a positive charge, an equal negative charge is induced on the plate, P2. This causes another positive charge of the same size to appear on the plate, P3, and so on. As the result of this process, the total charge stored between the plates P1 and P6 is Q (not $3Q$). Secondly, we are told that a current can only flow in a closed circuit, but clearly, the dielectrics represent discontinuities in the circuit of Fig. 49-1. James Clerk Maxwell resolved this difficulty by proposing the so-called *displacement current*, which is not an electron flow but is the result of a rate of change in the electric flux. On the SI system the electric flux in a dielectric consists of lines where one line is assumed to emanate from a positive charge of one coulomb and terminate on an equal negative charge. Consequently, the number of lines is the same as the number of coulombs and the electric flux can therefore be measured in coulombs. When the capacitor is being charged, there is a change in the electric flux and its rate of change will be equivalent to coulombs

per second or amperes. This naturally results in the displacement current being measured in the same units as the electron flow.

After the capacitors are charged, there is a voltage across each capacitor and the sum of these voltages must exactly balance the source voltage. Since $Q = VC$ (equation 47-2) and each capacitor carries the same charge, it follows that the *highest* voltage is across the lowest *value* of capacitance and vice-versa.

Each capacitor stores a certain amount of energy in the form of the electric field between its plates. When the switch is moved to position 2, the capacitors discharge and the total energy stored is released in the form of the spark at the contacts of the switch. Incidentally, the spark creates an electromagnetic wave and this was the principle behind the early spark-gap transmitter.

Finally, what is the purpose of connecting capacitors in series? As we shall see in the mathematical derivations, connecting capacitors in series *reduces* the total capacitance so that the total capacitance is less than the value of the smallest capacitance in series. Basically, this is because the series arrangement effectively increases the distance between the end plates, P1 and P6, connected to the source voltage, and the capacitance is inversely proportional to this distance. Because the capacitor has a dc working voltage (WVdc), the series arrangement may be used to distribute the source voltage between the capacitors so that the voltage across an individual capacitor does not exceed its rating.

MATHEMATICAL DERIVATIONS

$$V_1 = \frac{Q}{C_1}, Q = C_1 V_1, C_1 = \frac{Q}{V_1} \quad (49-1)$$

$$V_2 = \frac{Q}{C_2}, Q = C_2 V_2, C_2 = \frac{Q}{V_2} \quad (49-2)$$

$$V_3 = \frac{Q}{C_3}, Q = C_3 V_3, C_3 = \frac{Q}{V_3} \quad (49-3)$$

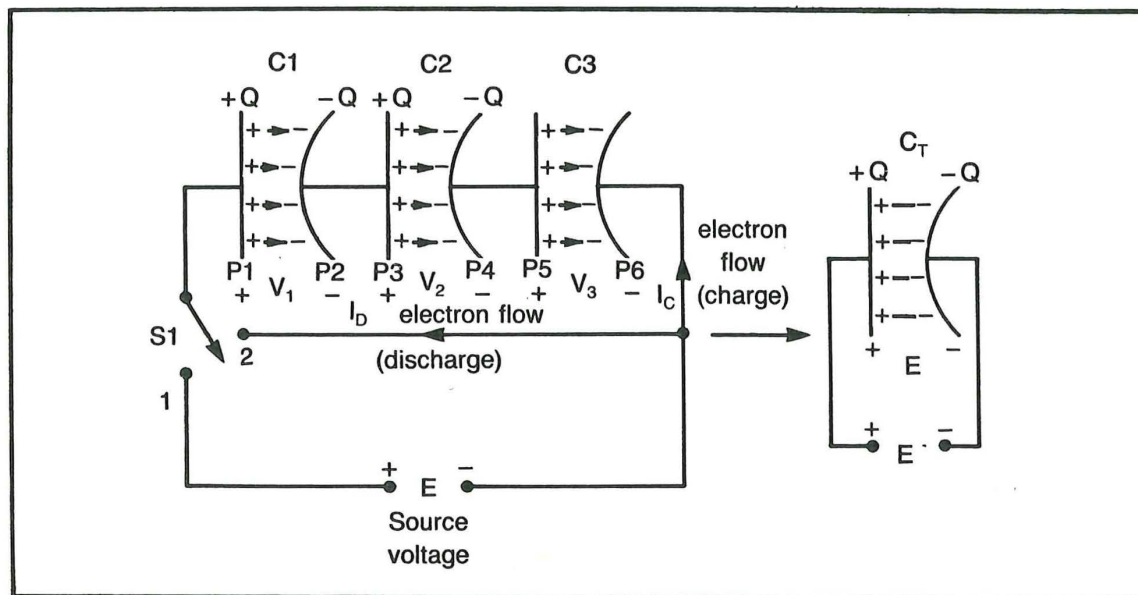


Fig. 49-1.

Source voltage,

$$\begin{aligned}
 E &= V_1 + V_2 + V_3 \\
 &= \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \\
 &= Q \times \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \text{ volts}
 \end{aligned} \tag{49-4}$$

If C_T is the total equivalent capacitance,

$$E = \frac{Q}{C_T} = E \times \frac{1}{C_T} \text{ volts} \tag{49-5}$$

Comparing Equations (49-4), (49-5).

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \tag{49-6}$$

For N capacitors in series,

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}} \text{ farads} \tag{49-7}$$

If the N capacitors are all of equal value C .

$$C_T = \frac{C}{N} \text{ farads} \tag{49-8}$$

The equations for capacitors in series may therefore be compared with those for resistors in parallel. Consequently connecting capacitors in series *reduces* the total capacitance.

Voltage Division

Source voltage,

$$E = Q \times \frac{1}{C_T} = C_1 V_1 \times \frac{1}{C_T} \text{ volts} \tag{49-9}$$

Therefore

$$V_1 = E \times \frac{C_T}{C_1}, V_2 = E \times \frac{C_T}{C_2}, V_3 = E \times \frac{C_T}{C_3} \text{ volts} \tag{49-10}$$

Energy stored in C_1 capacitor,

$$W_1 = \frac{1}{2} C_1 V_1^2$$

$$= \frac{1}{2} \frac{Q^2}{C_1} = \frac{1}{2} Q V_1 \text{ joules} \quad (49-11)$$

Energy stored in C2 capacitor,

$$\begin{aligned} W_2 &= \frac{1}{2} C_2 V_2^2 \\ &= \frac{1}{2} \frac{Q^2}{C_2} = \frac{1}{2} Q V_2 \text{ joules} \end{aligned} \quad (49-12)$$

Energy stored in C3 capacitor,

$$\begin{aligned} W_3 &= \frac{1}{2} C_3 V_3^2 \\ &= \frac{1}{2} \frac{Q^2}{C_3} = \frac{1}{2} Q V_3 \text{ joules} \end{aligned} \quad (49-13)$$

Total energy stored,

$$\begin{aligned} W_T &= \frac{1}{2} Q E = \frac{1}{2} C_T E^2 \\ &= \frac{1}{2} \frac{Q^2}{C_T} \text{ joules} \end{aligned} \quad (49-14)$$

For two capacitors in series,

Total capacitance,

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 \times C_2}{C_1 + C_2} \text{ farads} \quad (49-15)$$

This yields

$$C_1 = \frac{C_2 \times C_T}{C_2 - C_T}, C_2 = \frac{C_1 \times C_T}{C_1 - C_T} \text{ farads} \quad (49-16)$$

$$V_1 = E \times \frac{C_T}{C_1} = E \times \frac{C_2}{C_1 + C_2}, V_2 = E \times \frac{C_1}{C_1 + C_2} \text{ volts} \quad (49-17)$$

Example 49-1

In Fig. 49-1, $C_1 = 20 \mu\text{F}$, $C_2 = 5 \mu\text{F}$, $C_3 = 4 \mu\text{F}$ and $E = 30 \text{ V}$. Calculate the values of C_T , V_1 , V_2 , V_3 , and the energy stored in each capacitor.

Solution

Total capacitance,

$$\begin{aligned}
 C_T &= \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} & (49-4) \\
 &= \frac{1}{\frac{1}{20} + \frac{1}{5} + \frac{1}{4}} = 2 \mu\text{F}
 \end{aligned}$$

Voltage across C1 capacitor,

$$\begin{aligned}
 V_1 &= E \times \frac{C_T}{C_1} = 30 \text{ V} \times \frac{2 \mu\text{F}}{20 \mu\text{F}} & (49-10) \\
 &= 3 \text{ V}.
 \end{aligned}$$

Voltage across C2 capacitor,

$$\begin{aligned}
 V_2 &= E \times \frac{C_T}{C_2} & (49-10) \\
 &= 30 \text{ V} \times \frac{2 \mu\text{F}}{5 \mu\text{F}} = 12 \text{ V}
 \end{aligned}$$

Voltage across C3 capacitor,

$$\begin{aligned}
 V_3 &= E \times \frac{C_T}{C_3} & (49-10) \\
 &= 30 \text{ V} \times \frac{2 \mu\text{F}}{4 \mu\text{F}} = 15 \text{ V}
 \end{aligned}$$

Voltage check:

Source voltage,

$$E = V_1 + V_2 + V_3 = 3 + 12 + 15 = 30 \text{ V} \quad (49-4)$$

Charge stored by each capacitor,

$$\begin{aligned}
 Q &= \text{total charge stored} & (49-5) \\
 &= E \times C_T \\
 &= 30 \text{ V} \times 2 \mu\text{F} \\
 &= 60 \mu\text{C}
 \end{aligned}$$

Energy stored by the C1 capacitor,

$$W_1 = \frac{1}{2} Q V_1 \quad (49-11)$$

$$\begin{aligned}
 &= \frac{1}{2} \times 60 \mu\text{C} \times 3 \text{ V} \\
 &= 90 \mu\text{J}
 \end{aligned}$$

Energy stored by the C2 capacitor,

$$W_2 = \frac{1}{2} Q V_2 \quad (49-12)$$

$$\begin{aligned}
 &= \frac{1}{2} \times 60 \mu\text{C} \times 12 \text{ V} \\
 &= 360 \mu\text{J}
 \end{aligned}$$

Energy stored by the C3 capacitor,

$$W_3 = \frac{1}{2} Q V_3 \quad (49-13)$$

$$\begin{aligned}
 &= \frac{1}{2} \times 60 \mu\text{C} \times 15 \text{ V} \\
 &= 450 \mu\text{J}
 \end{aligned}$$

Total energy stored = $90 + 360 + 450 = 900 \mu\text{J}$.

Energy check:

$$\text{Total energy stored} = \frac{1}{2} Q E = \frac{1}{2} \times 60 \mu\text{C} \times 30 \text{ V} = 900 \mu\text{J} \quad (49-14)$$

Example 49-2

In Fig. 49-2, calculate the values of V_1 and V_2 .

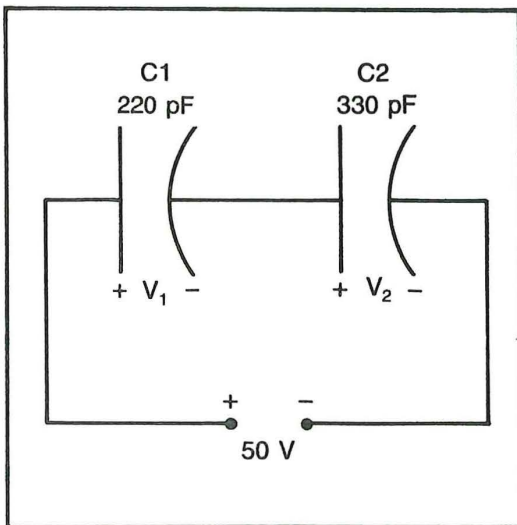


Fig. 49-2.

Solution

Voltage,

$$V_1 = E \times \frac{C_2}{C_1 + C_2} = 50 \text{ V} \times \frac{330 \text{ pF}}{220 \text{ pF} + 330 \text{ pF}} \quad (49-17)$$

$$= 30 \text{ V.}$$

$$V_2 = E \times \frac{C_1}{C_1 + C_2} = 50 \text{ V} \times \frac{220 \text{ pF}}{220 \text{ pF} + 330 \text{ pF}} \quad (49-14)$$

$$= 20 \text{ V.}$$

Voltage check:

$$\text{Source voltage, } E = V_1 + V_2 = 30 + 20 = 50 \text{ V.}$$



Chapter 50

Capacitors in Parallel

CAPACITORS IN PARALLEL ARE CONNECTED ACROSS TWO COMMON POINTS (A, B) OR LINES SO THAT the source voltage is applied across each of the capacitors (Fig. 50-1). Because $Q = C V$, it follows that a different charge must be stored by each capacitor. This is due to the different *momentary* charging currents that exist at the instant the switch, S, is closed. The total charge is then equal to the sum of the individual charges stored by each capacitor.

As we shall see in the mathematical derivations, the purpose of connecting capacitors in parallel is to increase the total capacitance. In Fig. 50-1 we have connected together on one side the plates P1, P2, P3, and, on the other side, the plates P4, P5, P6. The result is to increase the effective surface area to which the capacitance is directly proportional.

Each of the three capacitors in Fig. 50-1, stores energy in the form of the electric field between its plates. The total energy stored is the sum of the capacitor's individual energies.

The principles of capacitors in series (Chapter 49) and capacitors in parallel may be extended to series-parallel arrangements (see example 50-3).

MATHEMATICAL DERIVATIONS

$$\text{Charge } Q_1 = C_1 E, Q_2 = C_2 E, Q_3 = C_3 E \text{ coulombs} \quad (50-1)$$

Total charge stored,

$$\begin{aligned} Q_T &= Q_1 + Q_2 + Q_3 \\ &= C_1 E + C_2 E + C_3 E \\ &= E \times (C_1 + C_2 + C_3) \text{ coulombs} \end{aligned} \quad (50-2)$$

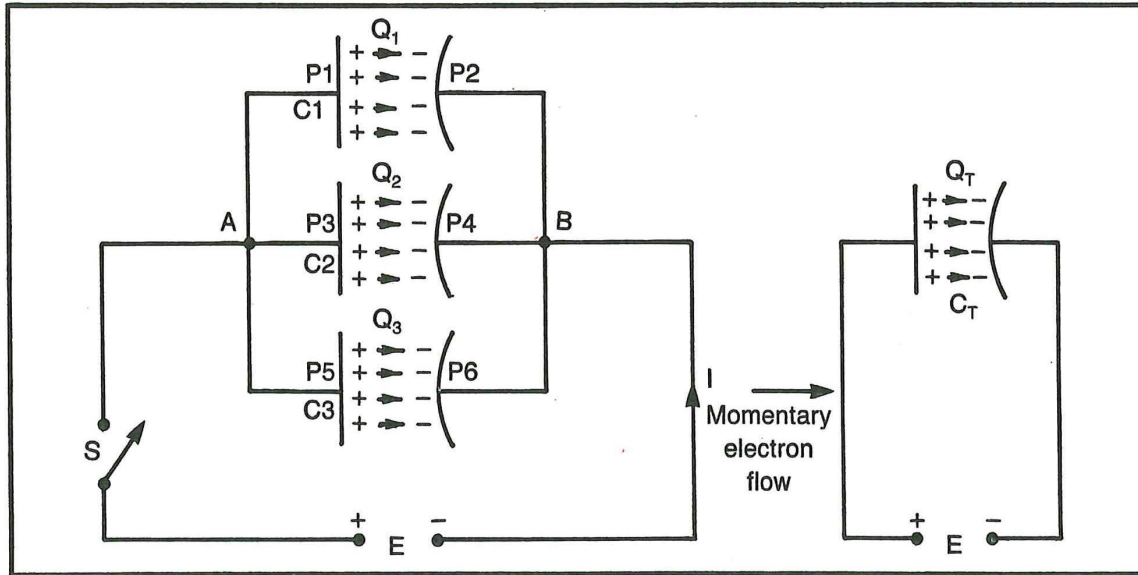


Fig. 50-1.

If C_T is the total equivalent capacitance,

$$\text{Total charge, } Q_T = E \times C_T \text{ coulombs} \quad (50-3)$$

Comparing equations 50-2, 50-3, it follows that

$$\text{Total capacitance, } C_T = C_1 + C_2 + C_3 \text{ farads} \quad (50-4)$$

Consequently, adding capacitors in parallel increases the total capacitance. If a multiplate capacitor is made up of N parallel plates with alternate plates connected as in Fig. 50-2, the total capacitance is

$$C_T = \frac{\epsilon_o \epsilon_r (N - 1) A}{d} \text{ farads} \quad (50-5)$$

where $\epsilon_o = 8.85 \times 10^{-12} \text{ F/m}$, ϵ_r = relative permittivity, A = area of *one* side of *one* plate (m^2) and d = distance between adjacent plates (m).

For N capacitors in parallel,

$$C_T = C_1 + C_2 + C_3 + \dots + C_N \text{ farads} \quad (50-6)$$

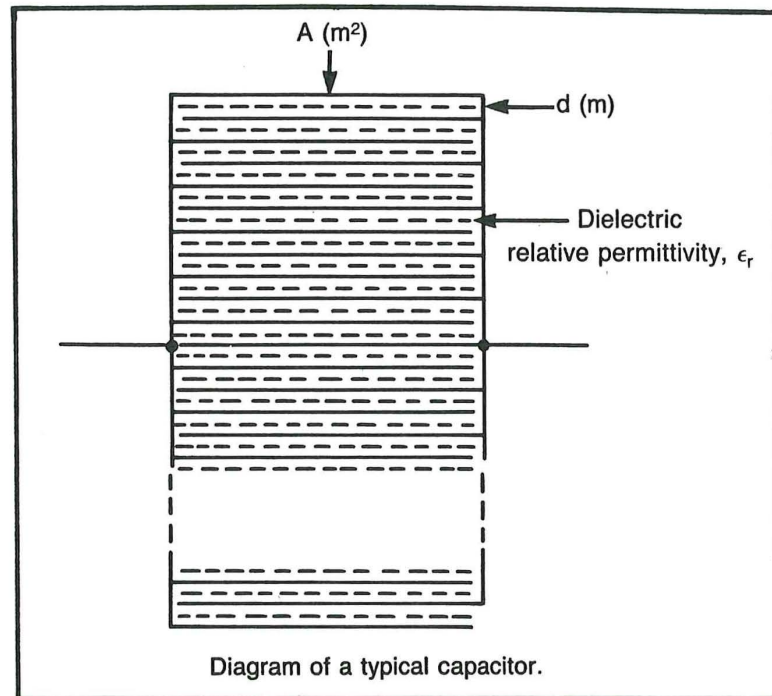
If the N capacitors are all of equal value, C :

$$C_T = NC \text{ farads} \quad (50-7)$$

Energy stored by the C_1 capacitor,

$$W_1 = \frac{1}{2} Q_1 E \quad (50-8)$$

Fig. 50-2.



$$\begin{aligned}
 &= \frac{1}{2} C_1 E^2 \\
 &= \frac{1}{2} Q_1^2 / C_1 \text{ joules}
 \end{aligned}$$

Energy stored by the C2 capacitor,

$$\begin{aligned}
 W_2 &= \frac{1}{2} Q_2 E & (50-9) \\
 &= \frac{1}{2} C_2 E^2 \\
 &= \frac{1}{2} Q_2^2 / C_2 \text{ joules}
 \end{aligned}$$

Energy stored by the C3 capacitor,

$$\begin{aligned}
 W_3 &= \frac{1}{2} Q_3 E & (50-10) \\
 &= \frac{1}{2} C_3 E^2 \\
 &= \frac{1}{2} Q_3^2 / C_3 \text{ joules}
 \end{aligned}$$

Total energy stored,

$$W_T = \frac{1}{2} Q_T E = \frac{1}{2} C_T E^2 = \frac{1}{2} Q_T^2 / C_T \text{ joules} \quad (50-11)$$

Example 50-1

In Fig. 50-1, $C_1 = 20 \mu\text{F}$, $C_2 = 5 \mu\text{F}$, $C_3 = 4 \mu\text{F}$ and $E = 30 \text{ V}$. Calculate the values of Q_1 , Q_2 , Q_3 , Q_T , C_T , the energy stored by each capacitor and the total energy derived from the source voltage. Note: The solution to this example should be compared with the solution to example 49-1, in which the series circuit contains the same values for the capacitors and the source voltage.

Solution

$$\text{Charge, } Q_1 = C_1 E = 20 \mu\text{F} \times 30 \text{ V} = 600 \mu\text{C}. \quad (50-1)$$

$$\text{Charge, } Q_2 = C_2 E = 5 \mu\text{F} \times 30 \text{ V} = 150 \mu\text{C}. \quad (50-1)$$

$$\text{Charge, } Q_3 = C_3 E = 4 \mu\text{F} \times 30 \text{ V} = 120 \mu\text{C}. \quad (50-1)$$

Total charge stored,

$$\begin{aligned} Q_T &= Q_1 + Q_2 + Q_3 = 600 + 150 + 120 \\ &= 870 \mu\text{C}. \end{aligned} \quad (50-2)$$

Total capacitance,

$$\begin{aligned} C_T &= C_1 + C_2 + C_3 = 20 + 5 + 4 \\ &= 29 \mu\text{F}. \end{aligned} \quad (50-4)$$

Charge check:

Total charge,

$$Q_T = C_T \times E = 29 \mu\text{F} \times 30 \text{ V} = 870 \mu\text{C} \quad (50-3)$$

Energy stored by the C_1 capacitor,

$$\begin{aligned} W_1 &= \frac{1}{2} Q_1 \times E \\ &= \frac{1}{2} \times 600 \mu\text{C} \times 30 \text{ V} \\ &= 9000 \mu\text{J} \end{aligned} \quad (50-8)$$

Energy stored by the C_2 capacitor,

$$\begin{aligned} W_2 &= \frac{1}{2} Q_2 \times E \\ &= \frac{1}{2} \times 150 \mu\text{C} \times 30 \text{ V} \\ &= 2250 \mu\text{J} \end{aligned} \quad (50-9)$$

Energy stored by the C3 capacitor,

$$\begin{aligned} W_3 &= \frac{1}{2} Q_3 \times E \\ &= \frac{1}{2} \times 120 \mu\text{C} \times 30 \text{ V} \\ &= 1800 \mu\text{J} \end{aligned} \quad (50-10)$$

Total energy stored,

$$\begin{aligned} W_T &= W_1 + W_2 + W_3 = 9000 + 2250 + 1800 \\ &= 13050 \mu\text{J}. \end{aligned}$$

Energy check:

Total energy stored and therefore derived from the source

$$= \frac{1}{2} Q_T E = \frac{1}{2} \times 875 \mu\text{C} \times 30 \text{ V} = 13050 \mu\text{J} \quad (50-11)$$

Example 50-2

A 40 μF capacitor is charged from a 150 V source. Another 20 μF capacitor is charged from a 100 V source. These capacitors are correctly paralleled immediately after they are disconnected from the sources. What is (a) the voltage across the combination and (b) the total energy stored before and after the parallel connection?

Solution

(a) Charge stored in the 40 μF (C1) capacitor,

$$\begin{aligned} Q_1 &= C_1 E_1 \\ &= 40 \mu\text{F} \times 150 \text{ V} \\ &= 6000 \mu\text{C} \end{aligned} \quad (50-1)$$

Charge stored in the 20 μF (C2) capacitor,

$$\begin{aligned} Q_2 &= C_2 E_2 \\ &= 20 \mu\text{F} \times 100 \text{ V} \\ &= 2000 \mu\text{C} \end{aligned} \quad (50-1)$$

Total charge,

$$Q_T = Q_1 + Q_2 = 6000 + 2000 = 8000 \mu\text{C} \quad (50-2)$$

Total capacitance of the parallel combination,

$$\begin{aligned} C_T &= C_1 + C_2 \\ &= 40 + 20 \\ &= 60 \mu\text{F} \end{aligned} \quad (50-6)$$

Voltage across the parallel combination,

$$\begin{aligned}
 E &= \frac{Q_T}{C_T} \\
 &= \frac{8000 \mu\text{C}}{60 \mu\text{F}} \\
 &= 133.3 \text{ V.}
 \end{aligned} \tag{50-3}$$

(b) Energy stored in the $40 \mu\text{F}$ capacitor,

$$\begin{aligned}
 &= \frac{1}{2} Q_1 E_1 \\
 &= \frac{1}{2} \times 6000 \mu\text{C} \times 150 \text{ V} \\
 &= 0.45 \text{ J}
 \end{aligned} \tag{50-8}$$

Energy stored in the $20 \mu\text{F}$ capacitor,

$$\begin{aligned}
 &= \frac{1}{2} Q_2 E_2 \\
 &= \frac{1}{2} \times 2000 \mu\text{C} \times 100 \text{ V} \\
 &= 0.10 \text{ J}
 \end{aligned} \tag{50-8}$$

Total energy stored *before* the parallel combination = $0.45 + 0.10 = 0.550 \text{ J}$.

Total energy stored *after* the parallel combination = $\frac{1}{2} Q_T E = \frac{1}{2} \times 8000 \mu\text{C} \times 133.3 \text{ V} = 0.533 \text{ J}$.

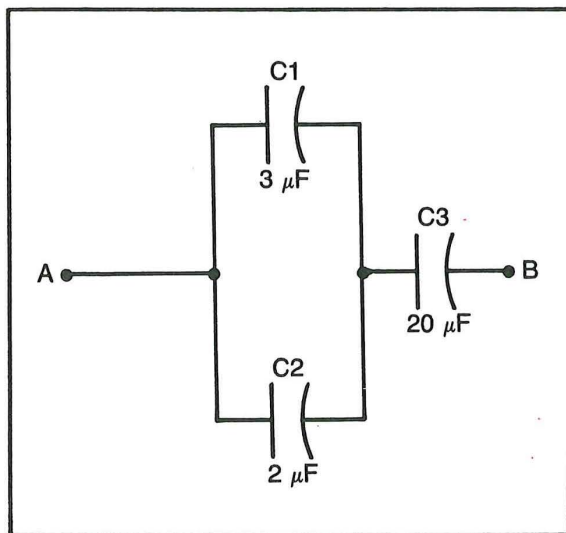


Fig. 50-3.

Notice that energy has been lost as the result of the parallel connection. Neglecting the resistance of any connecting wires, the lost energy appears in the form of the spark that occurs when the parallel connection is made.

Example 50-3

In Fig. 50-3, calculate the value of the total capacitance between the points *A* and *B*.

Solution

$$\text{Capacitance of } 2 \mu\text{F} \parallel 3 \mu\text{F} = 2 + 3 = 5 \mu\text{F} \quad (50-6)$$

Total capacitance between the points *A* and *B*,

$$\begin{aligned} C_T &= \frac{5 \times 20}{5 + 20} \\ &= 4 \mu\text{F} \end{aligned} \quad (49-13)$$



Chapter 51

The RC Time Constant

FROM CHAPTER 47 WE KNOW THAT THE PROPERTY OF CAPACITANCE PREVENTS ANY SUDDEN change of voltage and also limits the rate at which the voltage can change. Consequently, when the switch, *S*, is closed in position 1 (Fig. 51-1A), the capacitor must take a certain time before it acquires its full charge. We are therefore going to discuss the factors that determine the duration of the so-called *transient* or *changing* state. This is the interval between the time at which the switch, *S*, is closed in position 1 and the time when the final or steady-state conditions are reached.

Immediately after the switch is closed in position 1, the capacitor cannot charge instantaneously so that the initial value of V_C must be zero. Consequently, to satisfy KVL, the voltage drop across the resistor, V_R , must balance the source voltage E . The initial current must therefore be at its maximum level of E/R amperes.

As the capacitor charges, its voltage, V_C , rises and therefore the resistor's voltage, V_R , falls because at all times the sum of these voltages exactly balance the source voltage, E . When the voltage drop, V_R , falls, the current is correspondingly less and there is a decrease in the rate at which the capacitor charges. Consequently, we have a situation in which the more the capacitor is charged, the less is its rate of charging. Theoretically, it would take infinite time for the capacitor to charge fully but as we shall see in the mathematical derivations, the capacitor acquires its ultimate charge of CE coulombs to within 1% after a finite time interval, which is determined by the values of the capacitance and the resistance.

Remember that the initial current is E/R amperes (coulombs per second) while the capacitor's final charge is CE coulombs. If the initial current had been maintained, the capacitor would have been fully charged after a time of CE coulombs/ E/R coulombs per second = RC seconds, which is the *time constant* of the circuit. However, because the rate at which the capacitor charges falls off, the capacitor only acquires 63.2% of its final charge after one time constant and it requires *five* time constants before

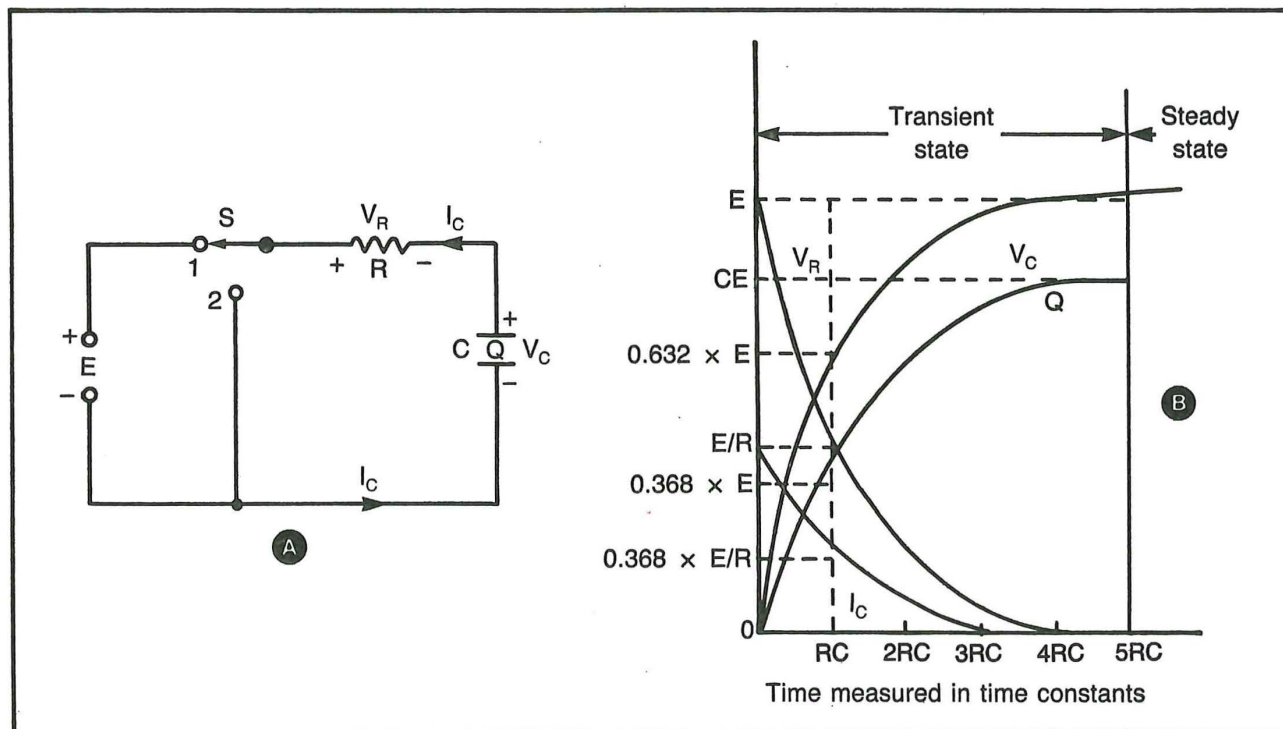


Fig. 51-1.

the capacitor is charged to within 1% of CE coulombs (Fig. 51-1B). Only then can we assume that the transient charging period has been concluded.

Note that the conditions at the start of the transient interval can be predicted by regarding the capacitor as a short circuit while the final steady-state values can be obtained by considering the capacitor to be an open circuit.

Assuming that the steady-state conditions have been reached, let us now move the switch, S , to position 2 (Fig. 51-2A). The capacitor cannot discharge instantaneously so that V_C must still be equal to E volts. By the KVL rule the voltage drop across the resistor must abruptly rise to E volts but its polarity is reversed because the discharge current, I_D , is in the opposite direction to the original charging current, I_C .

As the capacitor discharges, V_C and V_R fall together in order to maintain the voltage balance around the closed loop. When V_R decreases, the current is reduced so that the capacitor discharges more slowly. Theoretically, it would take infinite time for the capacitor to discharge fully but after one time constant the capacitor has *lost* 63.2% of its initial charge and its charge has therefore dropped to 36.8% of CE coulombs. After five time constants we assume that the transient discharge period has been concluded in a sense that I_D , V_C , V_R have all fallen to less than 1% of their original values (Fig. 51-2B).

MATHEMATICAL DERIVATIONS

In Fig. 51-1A the switch, S , is closed in position 1. The initial conditions are

$$Q = 0 \text{ coulombs, } V_C = 0 \text{ volts, } V_R = E \text{ volts, } I_C = \frac{E}{R} \text{ amperes.}$$

At all times,

$$V_R + V_C = E \quad (51-1)$$

Part 2

Alternating Current Principles

Chapter 59

Introduction to Alternating Current (Ac)

PREVIOUS CHAPTERS HAVE BEEN CONCERNED WITH DIRECT CURRENT IN WHICH THE ELECTRON flow has always been in the same direction although the magnitude of the flow has not necessarily been constant.

With alternating current, the current reverses its direction periodically and has an average value of zero. If the average value of the waveform is not zero, it is regarded as a combination of a dc component and an ac component. The alternating current may assume a variety of waveforms some of which are shown in Figs. 59-1 A, B, C, and D. In every case a single complete waveform is known as a cycle and the number of cycles occurring in one second is the frequency, which is therefore the rate at which the waveform repeats.

The letter symbol for the frequency is f and its *SI* unit is the hertz (Hz) which is equivalent to one cycle per second.

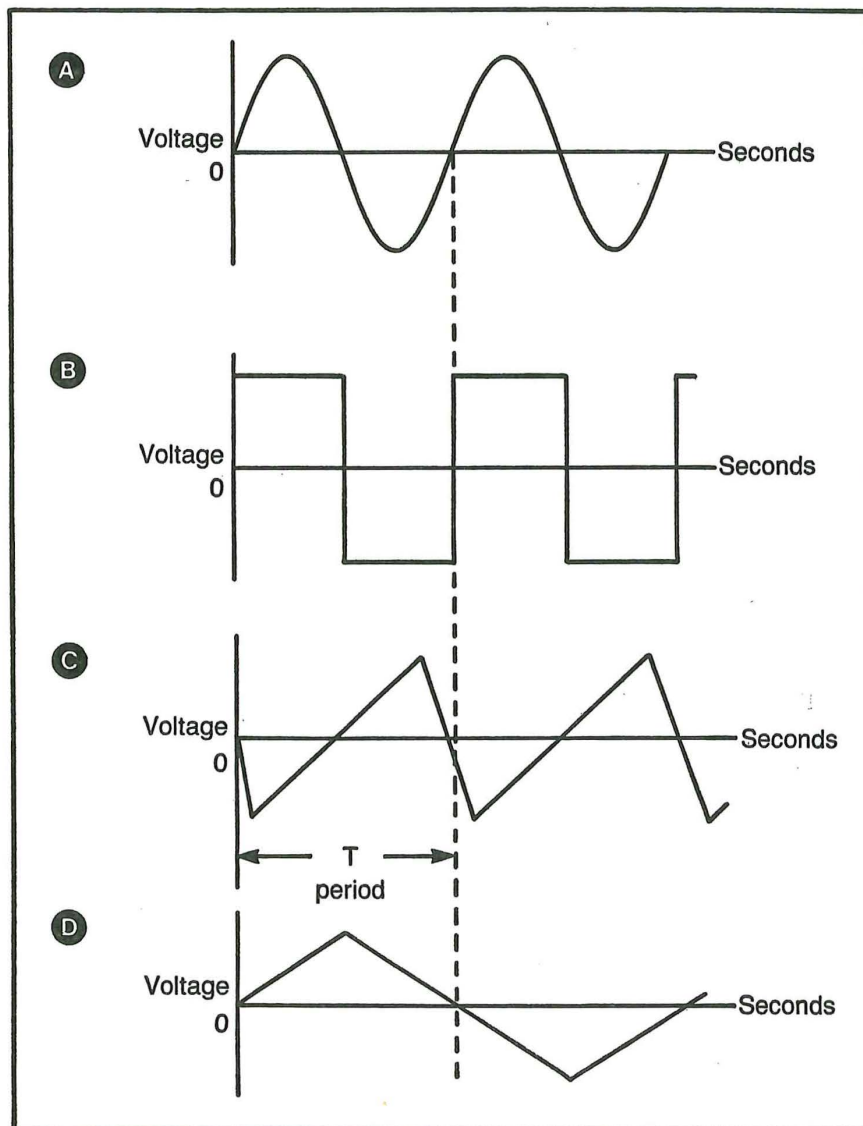
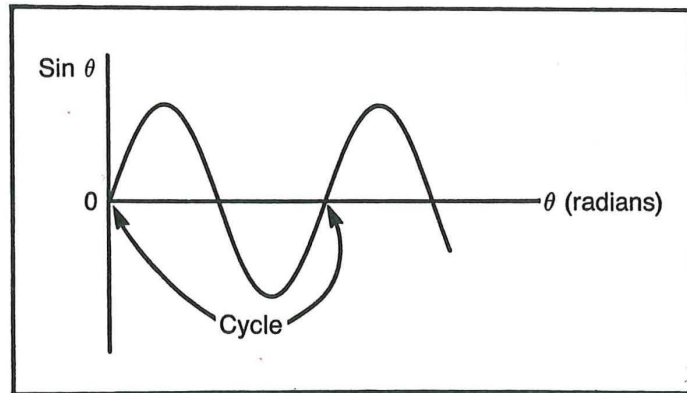


Fig. 59-1.

Fig. 59-2.



The time interval taken by a complete cycle is the period, which is equal to the reciprocal of the frequency. The letter symbol for the period is T and its basic unit is the second (s). As an example the commercial ac line voltage has a frequency of 60 Hz and therefore its period is 1/60th of a second.

The sine wave is not only associated with the commercial line voltage but is also widely used for communications. The curve itself is the result of plotting the mathematical sine function, $\sin \theta$, versus the angle θ , measured in either degrees or radians (Fig. 59-2).

The vertical axis measures only one of a number of electrical parameters such as voltage, current, and power, while the horizontal axis is a time scale. It follows that, for every sine wave, a certain number of radians on the horizontal scale must correspond to a particular time interval. The angular frequency whose letter symbol is the Greek lower-case omega, ω , is found by dividing the number of radians by the equivalent time interval and is therefore measured in radians per seconds (rad/s).

One cycle of a sine wave consists of a "positive" alternation and a "negative" alternation (Fig. 59-3). The maximum excursion from its average or zero line is called the peak or maximum ($E_{\max}$) value while the distance between the wave's crest and trough is a measure of the peak-to-peak (E_{p-p}) value.

Sine waves are of particular importance because Fourier analysis allows the waveforms of Figs. 59-1 B, C, and D, to be broken down into a series of sine waves that consist of a fundamental component together with its harmonics (Fig. 59-4). The harmonic of a sine wave is another sine wave whose frequency is a whole number times the original or fundamental frequency. For example if the fundamental

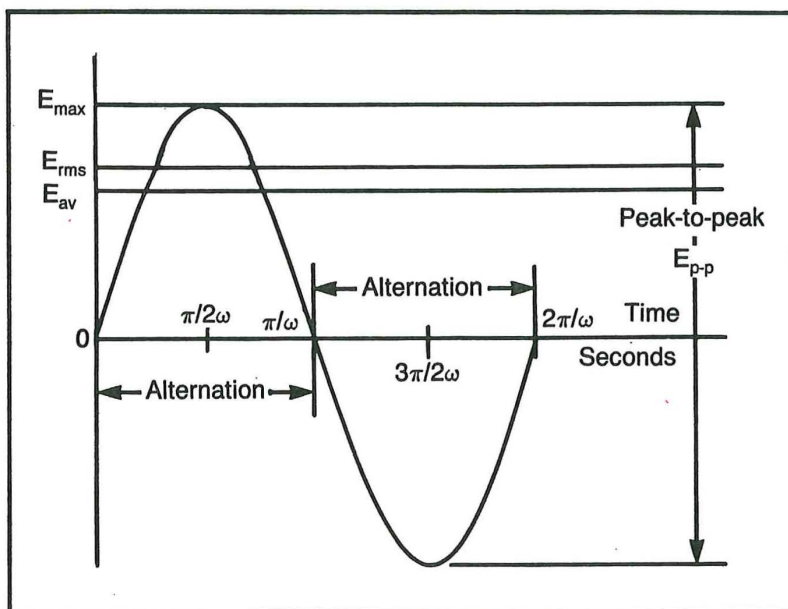


Fig. 59-3.

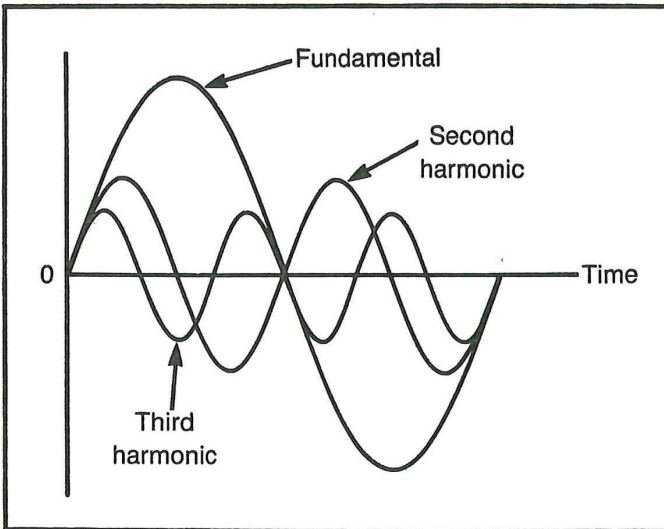


Fig. 59-4.

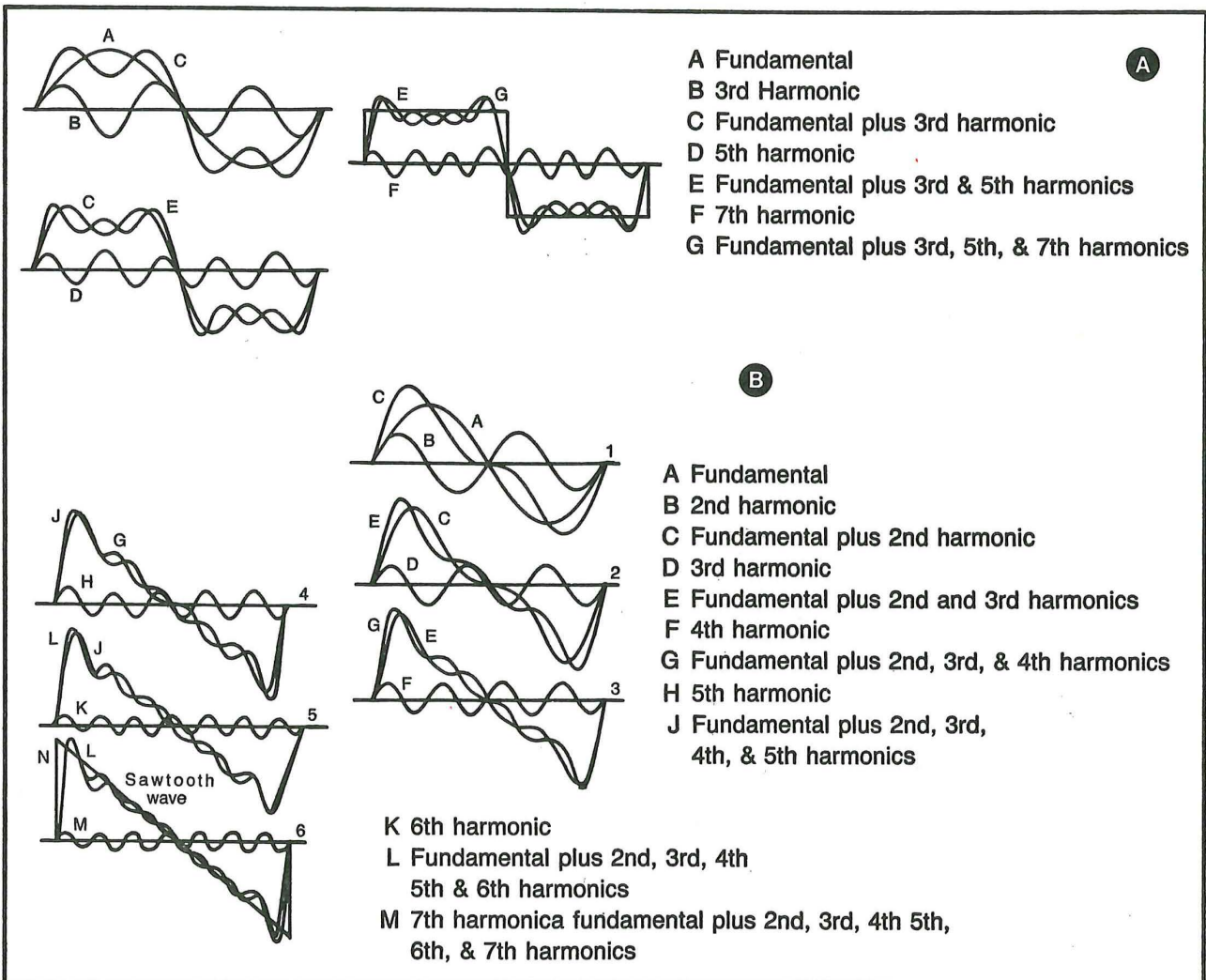


Fig. 59-5.

frequency is 1 kHz (f), the second harmonic has a frequency of 2 kHz ($2f$), the third harmonic frequency is 3 kHz ($3f$) etc. The sine wave synthesis of a square wave and a sawtooth wave are illustrated in Figs. 59-5 A, B.

THE SIMPLE AC GENERATOR (ALTERNATOR)

The ac generator is a machine that is capable of converting mechanical energy into alternating electrical energy. In its simplest form (Fig. 59-6A), a shaft D, D' is driven around by mechanical energy. Attached to the shaft but insulated from it, is a copper loop that rotates between the poles, PP' , of a permanent magnet or an electromagnet (energized by direct current).

The ends of the rotating loop are joined to two separate slip rings, SS' that make continuous contact with the stationary carbon brushes, CC' ; these brushes are then connected to the electrical load.

The poles pieces, PP' , are especially shaped to provide a constant flux density, B , in which the loop rotates (Fig. 59-6B). One complete rotation of the loop will generate one cycle of alternating voltage, which is applied across the load. If the output frequency is 60 Hz, the loop's speed of rotation is $60 \times 60 = 3600$ revolutions per minute. However, if the alternator has four poles, each complete rotation of the loop generates two cycles of alternating voltage and therefore the speed required for a 60 Hz output is only 1800 rpm.

MATHEMATICAL DERIVATIONS

Frequency,

$$f = \frac{1}{T} \text{ Hz} \quad (59-1)$$

Period,

$$T = \frac{1}{f} \text{ seconds} \quad (59-2)$$

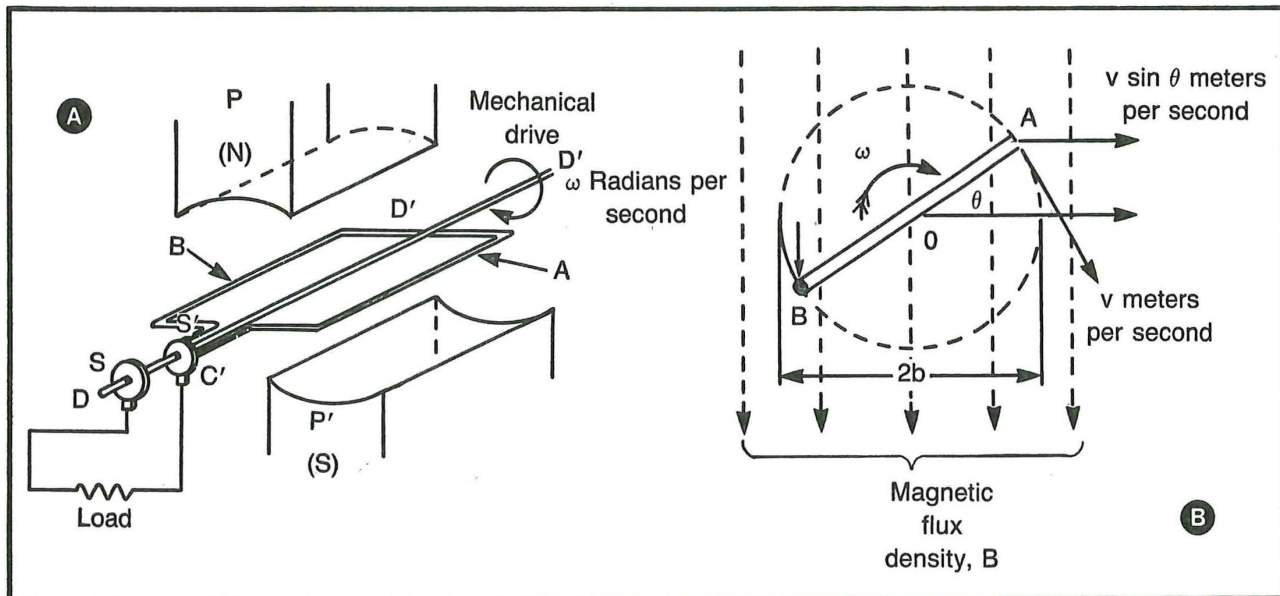


Fig. 59-6.

One radian is the angle at the center of a circle, which subtends (is opposite to) and arc (l) equal in length to the radius (r).

Angle,

$$\theta = \frac{l}{r} \text{ radians} \quad (59-3)$$

Because the total length of the circle's circumference is $2 \pi r$ radians, the angle of 360° must be equivalent to $2 \pi r/r = 2 \pi$ radians. Therefore,

$$\theta \text{ (degrees)} = \frac{360}{2 \pi} \times \theta \text{ radians} = \frac{180}{\pi} \times \theta \text{ radians} \quad (59-4)$$

$$\theta \text{ (radians)} = \frac{\pi}{180} \times \theta \text{ degrees} \quad (59-5)$$

It follows that

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees} = 57.296^\circ = 57^\circ 17' 45'' \quad (59-6)$$

Because the angular velocity or angular frequency, ω , is measured in radians per second, Angular frequency,

$$\omega = \frac{\theta}{t} = \frac{2 \pi}{T} = 2 \pi f \text{ radians per second} \quad (59-7)$$

$$\text{Angle, } \theta = 2 \pi f t \text{ radians} \quad (59-8)$$

where t is the time in seconds.

Peak-to-peak voltage,

$$E_{pp} = 2 \times E_{max} \text{ volts} \quad (59-9)$$

Assume that the loop of N turns in Fig. 59-6B moves with an angular velocity of ω radians per seconds so that the linear velocity of each of the conductors, A , B , is $v = \omega b$ where $2b$ is the width of the loop in meters. The component of the velocity at right angles to the magnetic flux is $v \sin \theta = v \sin \omega t$. From Faraday's law (Equation 36-3), the instantaneous total voltage induced in the loop is:

Instantaneous voltage,

$$\begin{aligned} e &= 2 B l N v \sin \theta \\ &= 2 B l b N \omega \sin \omega t \\ &= B A N \omega \sin \omega t \\ &= E_{max} \sin \omega t = E_{max} \sin 2 \pi f t \text{ volts} \end{aligned} \quad (59-10)$$

where

$$B = \text{uniform flux density (T)}$$

$$\begin{aligned}
 l &= \text{length of each conductor (m)} \\
 A &= \text{area of each turn (m}^2\text{)} \\
 t &= \text{time (s)} \\
 E_{\max} &= B A N \omega = \text{peak or maximum voltage value (V)}
 \end{aligned}$$

$$\text{Generated frequency, } f = \frac{N_p}{60} \text{ hertz} \quad (59-11)$$

$$\text{Speed of rotation, } N = \frac{60 f}{p} \text{ revolutions per minute} \quad (59-12)$$

where

$$\begin{aligned}
 p &= \text{number of pairs of poles} \\
 N &= \text{speed of rotation in rpm.}
 \end{aligned}$$

Example 59-1

A coil of 2000 turns is rotated at 3600 rpm in a uniform magnetic field with a flux density of 0.05 tesla. The average area of each turn is 25 cm² and the axis of rotation is at right angles to the direction of the flux. Calculate the value of (a) the frequency (b) the period (c) the angular frequency and (d) the peak-to-peak voltage. Write down the trigonometrical expression for the instantaneous voltage and calculate its value when the coil has rotated nine radians from the position of zero voltage.

Solution

(a) Frequency,

$$\begin{aligned}
 f &= \frac{Np}{60} \\
 &= \frac{3600 \times 1}{60} \\
 &= 60 \text{ Hz.}
 \end{aligned} \quad (59-11)$$

(b) Period,

$$\begin{aligned}
 T &= \frac{1}{f} \\
 &= 1/60 \text{ s} \\
 &= 16.67 \text{ ms.}
 \end{aligned} \quad (59-2)$$

(c) Angular frequency,

$$\begin{aligned}
 \omega &= 2 \pi f \\
 &= 377 \text{ rad./s.}
 \end{aligned} \quad (59-7)$$

(d) Peak voltage,

$$\begin{aligned} E_{\max} &= B A N \omega \\ &= 0.05 \times 25 \times 10^{-4} \times 2000 \times 377 \\ &= 94.25 \text{ V.} \end{aligned} \quad (59-10)$$

Instantaneous output voltage,

$$\begin{aligned} e &= E_{\max} \sin \omega t \\ &= E_{\max} \sin \theta \\ &= 94.25 \sin 377 t \text{ volts.} \end{aligned} \quad (59-10)$$

If $\theta = 9$ radians,

Instantaneous voltage,

$$\begin{aligned} e &= E_{\max} \sin \theta \\ &= 94.25 \sin (9 \text{ rad.}) \\ &= 38.84 \text{ V.} \end{aligned} \quad (59-10)$$



Chapter 60

The Root-Mean-Square (RMS) or Effective Value of an Ac Voltage or Current

IN CHAPTER 59 WE OBSERVED THAT THE MAGNITUDE OF A SINEWAVE VOLTAGE OR CURRENT COULD be measured in terms of its peak or peak-to-peak value. In fact it is common practice to use a cathode-ray oscilloscope (CRO) to obtain the peak-to-peak value of an alternating voltage. Because the peaks of a sinewave only occur twice instantaneously during a cycle, we would normally prefer to use a value that reflects the whole of the cycle. However, we cannot use the average over the complete cycle since this value is always zero. Instead we turn to the root-mean-square (rms) or effective value. Nonmathematically this is defined as that ac value that will provide the same heating or power effect as its equivalent dc value. In other words, a 100-W 100-V bulb that is connected to a 100-V rms 60-Hz supply will appear to have the same brightness as an identical bulb joined to a 100-Vdc source.

The bulb's tungsten filament, which is operated from the dc source, will glow continuously at a constant brightness. However, when the identical bulb is connected to the ac supply, the filament with its low thermal capacity, alternately glows very brightly and is extinguished at the rate of 120 times per second. Due to the persistence of vision the eye cannot directly observe a flicker effect whose rate exceeds 30 times per second; consequently the brightness is "averaged out" and appears to be the same

Chapter 67

Inductive Reactance

LET US ASSUME IN FIG. 67-1A THAT THE INDUCTOR ONLY POSSESSES THE PROPERTY OF INDUCTANCE and that we can ignore its resistance. When an alternating current is flowing through the coil, the surrounding magnetic field is continually expanding and collapsing so that there is an induced voltage, v_L , whose value must at all times exactly balance the instantaneous value of the source voltage, e . Therefore, when e is instantaneously at its peak value, the value of v_L is also at its peak and since $v_L = -L \frac{di}{dt}$ (equation 39-1), the *rate of change of the current* is at its highest level. This occurs when the slope of the current sine wave is steepest at the point of zero time and zero current. Consequently, when e is at its peak value, i is instantaneously zero and therefore e, i are 90° out of phase (Figs. 67-1 B and C). This may be remembered by the word “ eLi ,” for an inductor, L , the instantaneous voltage, e , leads the instantaneous current, i , by 90° .

We cannot obtain the opposition to the current flow by dividing the instantaneous voltage by the instantaneous current. For example, at the zero degree point, the value of e/i is infinite while at the 90° mark, e/i is zero. However, we can attempt to find the value of E_{rms}/I_{rms} because the effective values are derived from the complete cycles of the voltage and the current.

Assume that the source voltage and the inductance are kept constant but the frequency is raised. The voltage induced in the coil must remain the same but the magnetic field is expanding and collapsing more rapidly so that the required magnetic flux is less. As a result, the current is reduced and is

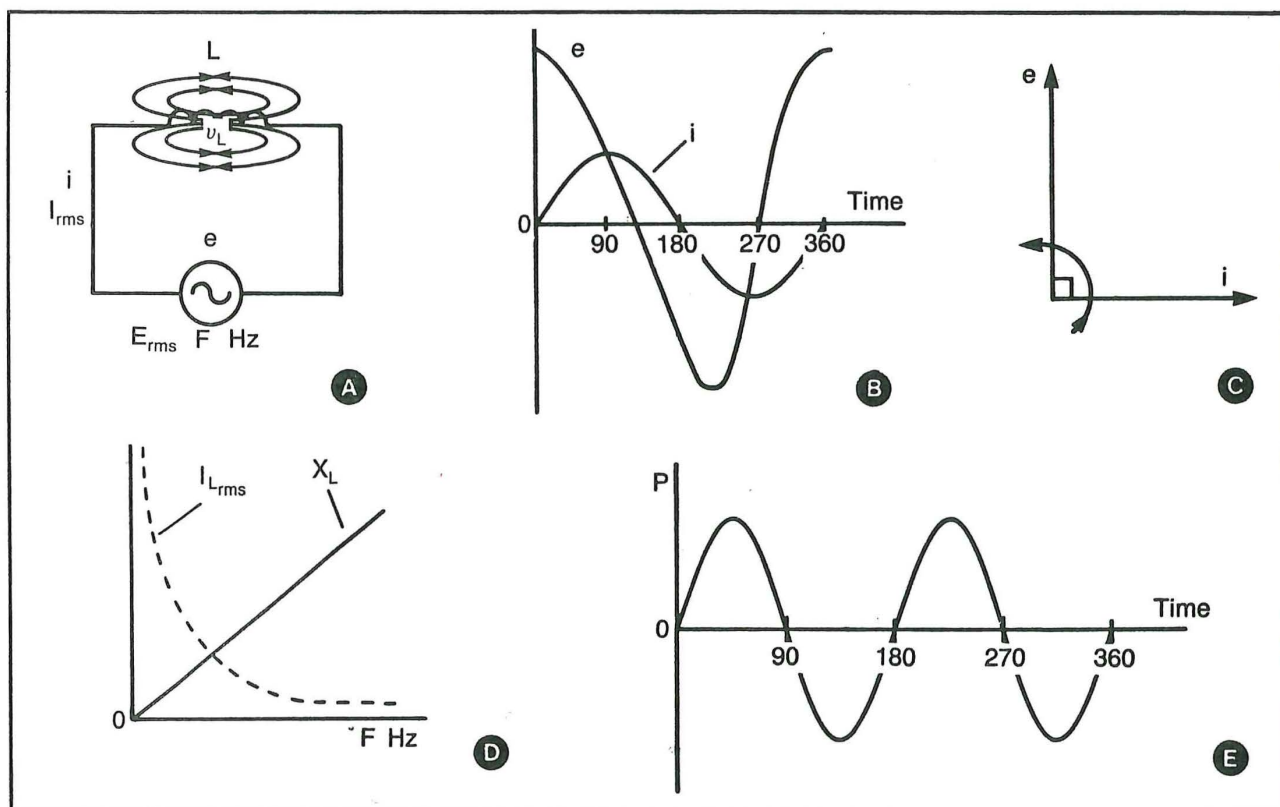


Fig. 67-1.

inversely proportional to the frequency. For example, if the frequency is doubled, the value of the effective current is halved.

Now we will keep the source voltage and the frequency constant but the inductor is replaced by another coil with a higher inductance. Because the induced voltage, $v_L = -L \frac{di}{dt}$ is the same, the value of $\frac{di}{dt}$ must be less and therefore the effective current is again reduced. Finally by raising the value of the source voltage while the frequency and the inductance are unchanged, the induced voltage must be increased and consequently the effective current is greater.

Summarizing, the effective current is directly proportional to the effective voltage, but it is inversely proportional to the frequency and the inductance. The opposition to the alternating current is therefore determined by the product of the frequency and the inductance. This opposition is called the inductive reactance, $X_L = E_{\text{rms}}/I_{\text{rms}}$, and is measured in ohms. As shown in the mathematical derivation, $X_L = 2\pi fL \Omega$ and because X_L is directly proportional to the frequency, the graph of X_L versus frequency is a straight line (Fig. 67-1D).

The instantaneous power, p , is equal to the product of e and i . The graph of the instantaneous power, p , versus time is a second harmonic sine wave (Fig. 67-1E) whose average value is zero over the source voltage's cycle. During the first quarter cycle the magnetic field is established around the inductor and the energy is drawn from the source. However, during the second quarter cycle the magnetic field collapses and the energy is returned to the source. This action is repeated during the third and fourth quarter cycles so that the average power over the complete cycle is zero.

This highlights the difference between resistance and reactance. Both resistance and reactance limit the value of the alternating current but while resistance dissipates (lost) power in the form of heat, reactance does not lose any power at all.

MATHEMATICAL DERIVATIONS

Let a sinewave current, $i = I_{\text{max}} \sin 2\pi ft$ flow through the inductor, L .

Then $v_L = -L \frac{di}{dt}$ and, by Kirchhoff's voltage law, $e + v_L = 0$.

This yields

$$\begin{aligned} e &= L \frac{di}{dt} = 2\pi fL I_{\text{max}} \cos 2\pi ft \\ &= E_{\text{max}} \sin \left(2\pi ft + \frac{\pi}{2} \right) \end{aligned} \quad (67-1)$$

Therefore e leads i by 90° and

$$E_{\text{max}} = 2\pi fL I_{\text{max}} \quad (67-2)$$

$$\frac{E_{\text{max}}}{I_{\text{max}}} = \frac{E_{\text{rms}}}{I_{\text{rms}}} = 2\pi fL = X_L \Omega$$

Then

$$f = \frac{X_L}{2\pi L} \quad \text{and} \quad L = \frac{X_L}{2\pi f} \quad (67-3)$$

Instantaneous power,

$$\begin{aligned}
 p &= e \times i \\
 &= E_{\max} \cos 2 \pi ft \times I_{\max} \sin 2 \pi ft \\
 &= \frac{E_{\max} I_{\max}}{2} \sin (2 \times 2 \pi ft) \\
 &= E_{\text{rms}} I_{\text{rms}} \sin 4 \pi ft.
 \end{aligned}$$

The average value of the instantaneous power over the complete cycle is zero.

Example 67-1

In Fig. 67-1A, $L = 150 \mu\text{H}$, $f = 820 \text{ kHz}$ and $E_{\text{rms}} = 8 \text{ V}$. Find the value of I_{rms} .

Solution

The inductive reactance is given by

$$X_L = 2 \times \pi \times 820 \times 10^3 \times 150 \times 10^{-6} = 773 \Omega \quad (67-2)$$

The current is given by

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{X_L} = \frac{8\text{V}}{773 \Omega} = 10.4 \text{ mA}. \quad (67-2)$$

Example 67-2

In Fig. 67-1A, $f = 15 \text{ kHz}$, $E_{\text{rms}} = 20 \text{ V}$, $I = 12.4 \text{ mA}$. Calculate the value of the inductance, L .

Solution

The inductive reactance is given by

$$X_L = \frac{E_{\text{rms}}}{I_{\text{rms}}} = \frac{20\text{V}}{12.4 \text{ mA}} = 1613 \Omega \quad (67-2)$$

The equation for the inductance is

$$L = \frac{X_L}{2\pi f} = \frac{1613}{2 \times \pi \times 15 \times 10^3} \text{ H} = 17.1 \text{ mH} \quad (67-3)$$



Chapter 68

Capacitive Reactance

WHEN AN ALTERNATING CURRENT, I , IS FLOWING IN THE CIRCUIT OF FIG. 68-1A, THE CAPACITOR is continuously charging and discharging so that the voltage, V_C , across the capacitor must at all times exactly balance the instantaneous value of the source voltage, e . Therefore, when e is momentarily at its peak value, the value of V_C is also at its peak and the capacitor is fully charged. The current is then instantaneously zero and therefore e and i are 90° out of phase (Fig. 68-1 B, C). This may be remembered by the word *iCe*; for an ideal capacitor, C , the instantaneous current, i , leads the instantaneous voltage, e , by 90° . We then combine “iCe” with the word “eLi” for the inductor and create “eLi, the iCe man!”

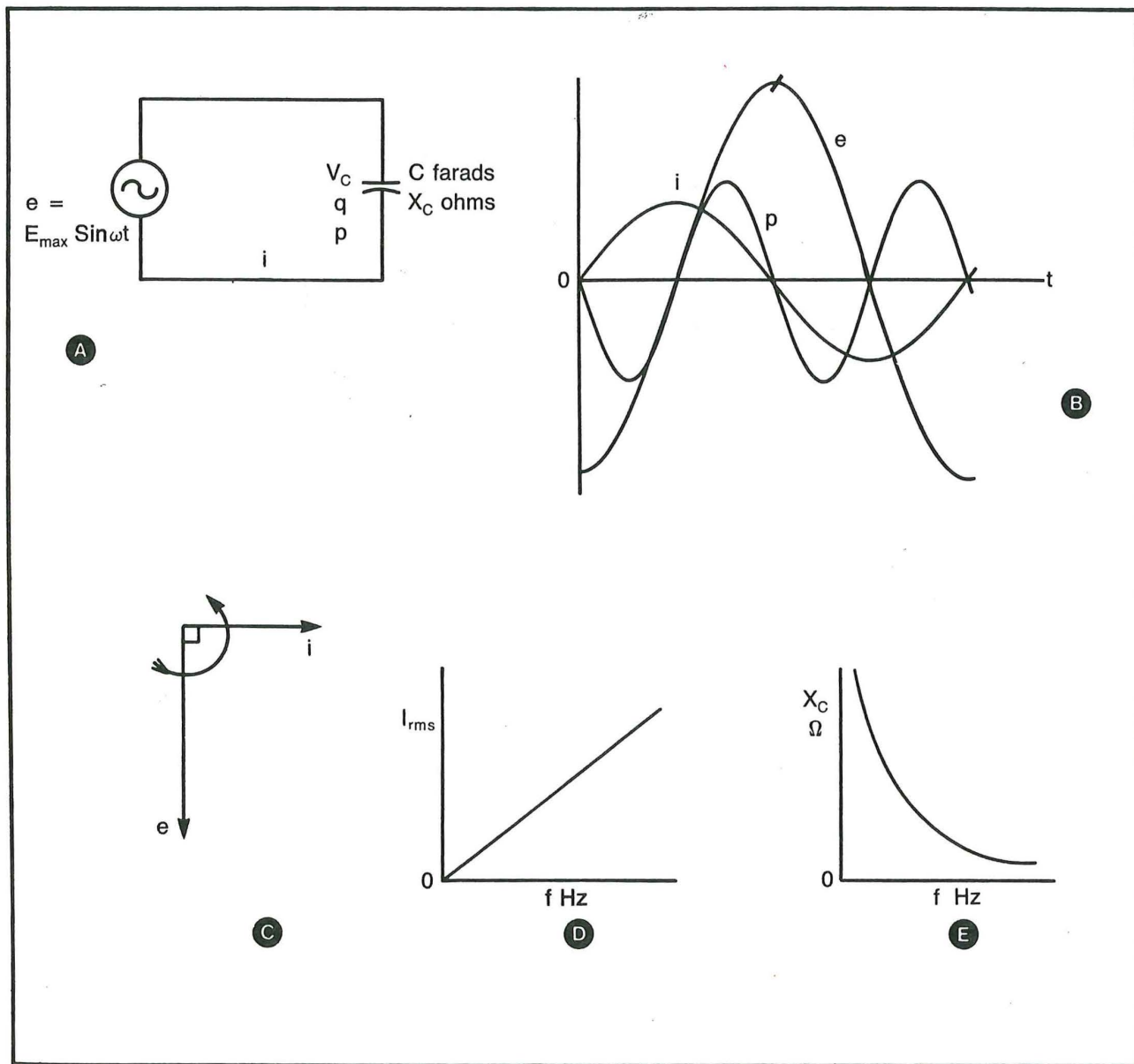


Fig. 68-1.

Let us now derive the factors that determine the capacitor's opposition to the flow of alternating current. Assume that the source voltage and the capacitance are kept constant but the frequency is raised. This reduces the period so that the capacitor must acquire or lose the same amount of charge in a shorter time. The charging or discharging current must therefore be greater and, in fact, the effective current is directly proportional to the frequency, so that if the frequency is doubled, the effective current is also doubled. By contrast, when the frequency is doubled for an *inductor*, the effective current is halved (Chapter 67).

Now we will keep the source voltage and the frequency constant but we will raise the capacitance. The new capacitor will have to store or lose more charge in the same time so that the effective current is again increased. The values of the capacitance and the effective current are directly proportional so that if the capacitance is halved, the effective current is also halved. Finally by raising the source voltage while the frequency and the capacitance are unchanged, the voltage across the capacitor must be greater, the capacitor must store more charge in the same period of time and consequently the effective current is once more increased.

Summarizing, the value of the effective current is directly proportional to the voltage, the capacitance and the frequency. The opposition to the alternating current flow is therefore *inversely* determined by the product of the frequency and the capacitance. This opposition is called the capacitive

reactance, $X_C = \frac{E_{\text{rms}}}{I_{\text{rms}}}$ and is measured in ohms. As shown in the mathematical derivations, $X_C = \frac{1}{2 \pi f C \Omega}$ and because X_C is inversely proportional to the frequency, the graph of X_C versus frequency is the rectangular hyperbolic curve of Fig. 68-1E. By contrast, the effective current is directly

proportional to the frequency so that the graph of I_{rms} versus frequency is a straight line (Fig. 68-1D).

The instantaneous power, p , is equal to the product of e and i . The graph of the instantaneous power, p , versus time is a second harmonic sine wave (Fig. 68-1B) whose average value is zero over the source voltage's cycle. During one quarter cycle the capacitor charges so that energy is drawn from the source and appears in the form of the electric field between the capacitor's plates. However, during the following quarter cycle, the capacitor discharges and the energy is returned to the source.

Like the inductor, the importance of the capacitor lies in the fact that its opposition to alternating from zero to infinity (Fig. 67-1D) while the capacitive reactance decreases from infinity towards zero (Fig. 68-1E). Consequently, for given values of L and C there must always be a (resonant) frequency for which $X_L = X_C$; this fact is used in the L, C tuning circuit which, for example, is capable of selecting a particular station through its ability to distinguish between one frequency and another (Chapters 80 and 82).

MATHEMATICAL DERIVATIONS

Let a sine-wave voltage, $e = E_{\text{max}} \sin 2 \pi f t$, be applied to the capacitor, C .

Then $v_c = \frac{q}{C}$ and, by Kirchhoff's voltage law, $e + v_c = 0$.

This yields

$$e = \frac{q}{C} \text{ or } q = C e.$$

Then

$$\begin{aligned} \frac{dq}{dt} &= i = C \frac{de}{dt} = 2 \pi f C E_{\text{max}} \cos 2 \pi f t \\ &= 2 \pi f C E_{\text{max}} \sin (2 \pi f t + \frac{\pi}{2}) \end{aligned} \quad (68-1)$$

$$= I_{\max} \sin \left(2 \pi f t + \frac{\pi}{2} \right) \text{ amperes}$$

Therefore i leads e by 90° and

$$I_{\max} = 2 \pi f C E_{\max} \quad (68-2)$$

or

$$\frac{E_{\max}}{I_{\max}} = \frac{E_{\text{rms}}}{I_{\text{rms}}} = \frac{1}{2 \pi f C} = X_C \Omega$$

Then

$$f = \frac{1}{2 \pi C X_C} = \frac{0.159}{C X_C} \text{ and } C = \frac{1}{2 \pi f X_C} = \frac{0.159}{f X_C} \quad (68-3)$$

Instantaneous power,

$$\begin{aligned} p &= e \times i \\ &= E_{\max} \sin 2 \pi f t \times I_{\max} \cos 2 \pi f t \\ &= \frac{E_{\max} \times I_{\max}}{2} \sin 4 \pi f t \\ &= E_{\text{rms}} I_{\text{rms}} \sin 4 \pi f t \text{ watts} \end{aligned} \quad (68-4)$$

The average value of the instantaneous power over the complete cycle is zero.

Example 68-1

In Fig. 68-1A, $C = 250 \text{ pF}$, $f = 822 \text{ kHz}$ and $E_{\text{rms}} = 8 \text{ V}$. Find the value of I_{rms} .

Solution

Capacitive reactance,

$$\begin{aligned} X_C &= \frac{1}{2 \times \pi \times 822 \times 10^3 \times 250 \times 10^{-12}} \\ &= 774 \Omega. \end{aligned} \quad (68-2)$$

Current,

$$I = \frac{8 \text{ V}}{774 \Omega} = 10.3 \text{ mA} \quad (68-2)$$

These results should be compared with the results of Example 67-1.

Example 68-2

In Fig. 68-1A, $f = 15 \text{ kHz}$, $E_{\text{rms}} = 20 \text{ V}$, $I_{\text{rms}} = 12.4 \text{ mA}$. Calculate the value of the capacitance.

Solution

Capacitive reactance,

$$X_C = \frac{E_{\text{rms}}}{I_{\text{rms}}} = \frac{20 \text{ V}}{12.4 \text{ mA}} = 1613 \Omega. \quad (68-2)$$

Capacitance,

$$\begin{aligned} C &= \frac{1}{2 \times \pi \times f \times X_C} \quad (68-3) \\ &= \frac{1}{2 \times \pi \times 15 \times 10^3 \times 1613} \text{ F} \\ &= 0.00658 \mu\text{F}. \end{aligned}$$



Chapter 69

The General Ac Circuit—Impedance—Power Factor

PROVIDED THERE IS ONLY A SINGLE SINEWAVE SOURCE, ALL AC CIRCUITS, HOWEVER COMPLEX, MAY ultimately be analyzed into a resistance in series with a reactance, which may be either inductive or capacitive. The total opposition to the flow of alternating current is therefore a resistance/reactance combination that is called the impedance, z . See Fig. 69-1.

The impedance, z , is a phasor that is defined as the ratio of the source voltage phasor to the source current phasor. The magnitude, Z , of the impedance phasor is measured in ohms and is equal to the

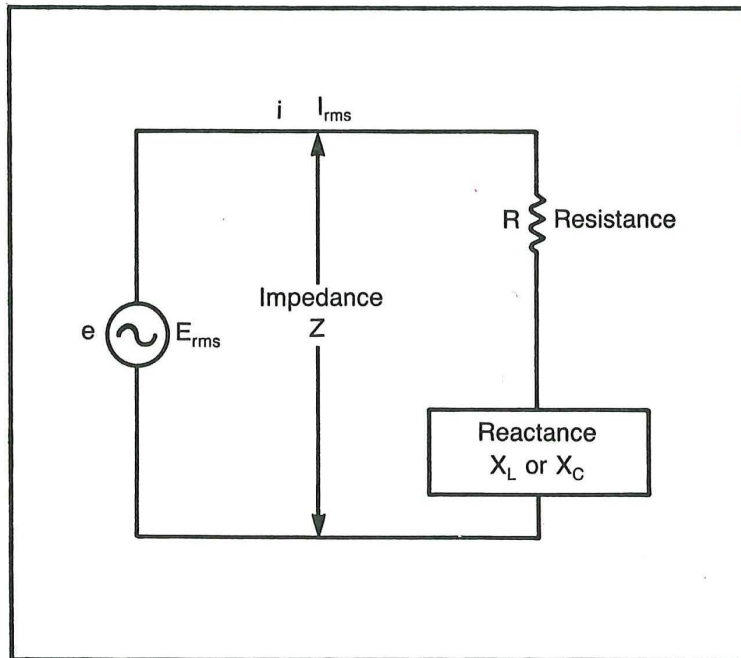


Fig. 69-1.

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Victor F. C. Veley is an experienced electrical and electronics engineer who is presently Dean of Science and High Technology at a Western trade/technical school.

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