

A NOVEL ESTIMATION PROCEDURE FOR THE APERTURE TIME IN ASYNCHRONOUS SINUSOIDAL PWM SYSTEMS

R.A. Guinee, Cork Institute of Technology, Cork, IRELAND, Email: rguinee@cit.ie
C. Lyden, NMRC, University College, Cork, IRELAND

ABSTRACT

A novel analytical technique is presented for the determination of the modulated pulse duration in sinusoidal asynchronous pulse width modulated (PWM) systems. A reasonable level of modulated aperture accuracy can be obtained with little computational effort using the proposed method. The method of evaluation of the pulse edge transitions is based on a second order Taylor series expansion of the sinusoidal reference signal about the zero crossing points of the triangular carrier used as the switching waveform in the comparator modulator. The accuracy of the estimated pulse edge transition times can be gauged from a comparison with values obtained using simulated PWM comparator operation, as a measurement reference, with a very small step size. The simplicity of the proposed solution makes it an attractive feature in an online DSP implementation of PWM operation for adaptive motor control.

1. INTRODUCTION

The application of PWM is generally accepted as an efficient means of power transfer in switch mode power supply DC to DC conversion, uninterruptible power supply (UPS) operation and in the speed and torque control of industrial machine drives. In the latter case the voltage and frequency control of induction and brushless motor drive (BLMD) systems is achieved by means of a sinusoidal reference PWM inverter power stage. PWM for wide bandwidth current loop control in high performance drives [1] relies on asynchronous natural sampling because of its inherent simplicity and ease of implementation using analog circuitry as shown in Figure 1.

The method of analysis used in the study of the width modulation process in these systems is mathematically complex [2] and based on the well known double Fourier series 'wall' model [3]. Alternative methods based on the Fast Fourier Transform (FFT) simulation methods have been applied to avoid this analytical difficulty by determining the PWM spectral content. The FFT approach, however, is not ideally suited to two level width modulated pulse sequences because a large number of sample points is required to accurately resolve the variable PWM waveform edge transitions with a resulting increase in computation time. A more effective method [4] than the FFT in predicting PWM spectra relies on the application of a single Fourier series expansion for periodic output PWM sequences. This is achieved by simulating the action of a uniform sampled all digital PWM circuit for periodic reference waveforms. This method does not however account for natural sampling and switch crossover accuracy is limited by the degree of quantization used for the carrier waveform.

Another consideration besides spectral analysis is the need for a simple model of the sinusoidal PWM process in motor drive

simulation. This is necessary where optimal controller design is an issue and for performance related prediction in wide bandwidth industrial servodrive applications. Model simplification eliminates the need for accurate determination of inverter switching instants from iteration and consequently reduces the excessive computation overhead that would otherwise occur. The application of a single Fourier Series (SFS) representation for the modelling of asynchronous natural sampled PWM inclusive of inverter dead time [5,6] has been used successfully in the simulation of a high performance brushless motor drive system, as shown in Figure 2, with a consequent large reduction in simulation runtime. This approach relied on the attenuation effects of the stator winding to reduce the number of inverter switching harmonics necessary during motor drive simulation, which are responsible for pulse shaping, in addition to the PWM encoded signal content. However no simple analytical methodology is available to accurately determine the modulated pulse duration for the natural sampled sinusoidal case.

In this paper a novel analytical technique is presented for the determination of the modulated pulse duration τ_m in sinusoidal asynchronous PWM systems. The method of evaluation of the pulse edge transitions is based on a second order Taylor series expansion of the sinusoidal reference signal $V_m(t)$ about the zero crossover points of the triangular carrier switching waveform $V_{tri}(t)$. Although such procedures already exist for pulsewidth determination in regular sampled PWM systems [7] as shown in Figure 3 no such method appears to be available for the natural sampled case. The challenge posed in [7] is the solution of a transcendental equation for the natural sampled case, as displayed in Figure 3A, which can be obtained numerically by the many iteration techniques available [8] as for example with the secant and regula-falsi methods. However a reasonable level of modulated aperture accuracy can be obtained with little computational effort using the proposed method. The quality of the estimated pulse edge transition times can be gauged from comparison with the values obtained using simulated PWM comparator operation which is used as a measurement reference. The simplicity of the proposed solution should make this an attractive feature in an online DSP implementation of PWM for adjustable speed drive operation.

2. PWM APERTURE TIME EVALUATION

The aperture times τ_m for sinusoidal PWM inverter operation can be deduced from the following theoretical analysis. The analog comparison of a segment of the sinusoidal reference signal $V_m(t)$ with one cycle of the carrier $V_{tri}(t)$ at the fulcrum nT_s and carrier switching period T_s results in the points of intersection, with magnitude equality given by

$$v_m(t_1) = v_{tri}(t_1) \text{ and } v_m(t_2) = v_{tri}(t_2) \quad (1)$$

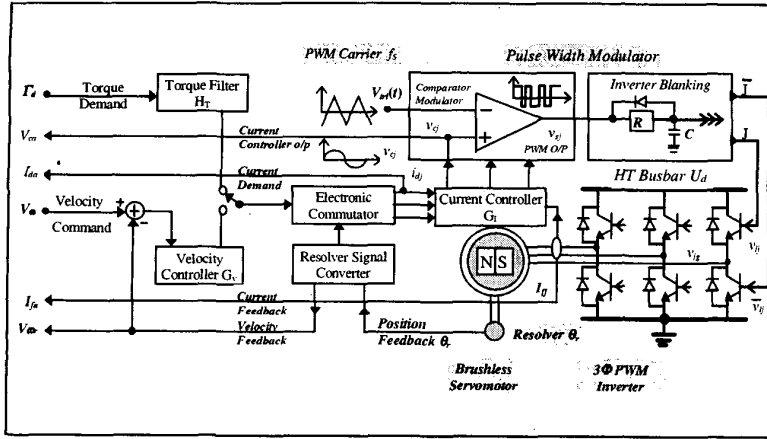


Figure 1. Generic Structure Of A Typical Brushless Motor Drive System

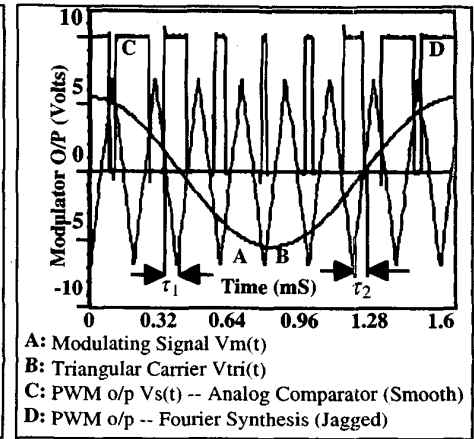


Figure 2. Symmetrical PWM Generation

at instants t_1^* and t_2^* as shown in Figure 4. These natural sampling instants define the modulated pulse transition times. The pulse rising and falling edges are laterally displaced from the zero crossings of $V_{tri}(t)$ by t_2^* and t_1^* or from the carrier axis of symmetry by T_1 and T_2 respectively. The slopes of the positive (s^+) and negative (s^-) going ramps, which define the triangular carrier waveform, are related to these transition instants as follows

$$s^+ = V_{tri} \left((2n+1) \frac{T_s}{2} \right) / \left(\frac{T_s}{4} \right) = V_{tri}(t_1) / \left[t_1 - (4n+1) \frac{T_s}{4} \right] \\ = V_{tri} \left((4n+1) \frac{T_s}{4} + t_1^* \right) / t_1^* \quad (2)$$

$$s^- = V_{tri} \left((2n-1) \frac{T_s}{2} \right) / \left(-\frac{T_s}{4} \right) = V_{tri}(t_2) / \left[t_2 - (4n-1) \frac{T_s}{4} \right] \\ = V_{tri} \left((4n-1) \frac{T_s}{4} - t_2^* \right) / (-t_2^*) \quad (3)$$

From (1) magnitude equality for a p pole pair machine results in

$$V_{tri}(t_1) = s^+ t_1^* = V_m \cos \left[p\omega_r \left\{ (4n+1) \frac{T_s}{4} + t_1^* \right\} + \theta \right] \quad (4)$$

$$V_{tri}(t_2) = -s^- t_2^* = V_m \cos \left[p\omega_r \left\{ (4n-1) \frac{T_s}{4} - t_2^* \right\} + \theta \right] \quad (5)$$

with signal phase shift θ included. Expanding (4) and (5) give

$$\frac{s^+ t_1^*}{V_m} = \cos \left[p\omega_r (4n+1) \frac{T_s}{4} + \theta \right] \cdot \cos(p\omega_r t_1^*) \\ \xleftarrow{V_{cn}^+} \quad (6) \\ - \sin \left[p\omega_r (4n+1) \frac{T_s}{4} + \theta \right] \cdot \sin(p\omega_r t_1^*) \\ \xleftarrow{V_{sn}^+}$$

$$\frac{-s^- t_2^*}{V_m} = \cos \left[p\omega_r (4n-1) \frac{T_s}{4} + \theta \right] \cdot \cos(p\omega_r t_2^*) \\ \xleftarrow{V_{cn}^-} \quad (7) \\ + \sin \left[p\omega_r (4n-1) \frac{T_s}{4} + \theta \right] \cdot \sin(p\omega_r t_2^*) \\ \xleftarrow{V_{sn}^-}$$

The adjusted time varying signal components in (6) and (7) can be represented by a second order Taylor series expansion about the carrier zero crossings using the approximations

$$\sin(p\omega_r t_1^*) = p\omega_r t_1^*; \quad \cos(p\omega_r t_1^*) = 1 - \frac{1}{2!} (p\omega_r t_1^*)^2; \\ \sin(p\omega_r t_2^*) = p\omega_r t_2^*; \quad \cos(p\omega_r t_2^*) = 1 - \frac{1}{2!} (p\omega_r t_2^*)^2; \quad (8)$$

with negligible error. Substituting (8) into (6) and (7) results in the following second order expressions in t^* with

$$\frac{s^+ t_1^*}{V_m} = V_{cn}^+ \cdot \left[1 - \frac{(p\omega_r t_1^*)^2}{2!} \right] - V_{sn}^+ \cdot [p\omega_r t_1^*] \quad (9)$$

$$\frac{-s^- t_2^*}{V_m} = V_{cn}^- \cdot \left[1 - \frac{(p\omega_r t_2^*)^2}{2!} \right] + V_{sn}^- \cdot [p\omega_r t_2^*] \quad (10)$$

The assumption of a truncated power series representation, with elimination of higher order terms in $(p\omega_r t_{1,2}^*)$, can be rationalized by examining the worst case conditions for the argument $(p\omega_r t^*)$ for steady state motor operation as follows. If a 6 pole pair BLMD [6], operating in torque control mode with a PWM switching frequency f_s of 5kHz, is fed with a 100Hz sinusoidal command current i/p with a maximum possible value for t^* of $T_s/4$ then

$$x \equiv \max(p\omega_r t_{1,2}^*) \Big|_{t_{1,2}^* = T_s/4} = 6 \cdot 200\pi \cdot 50 \times 10^{-6} \quad (11A)$$

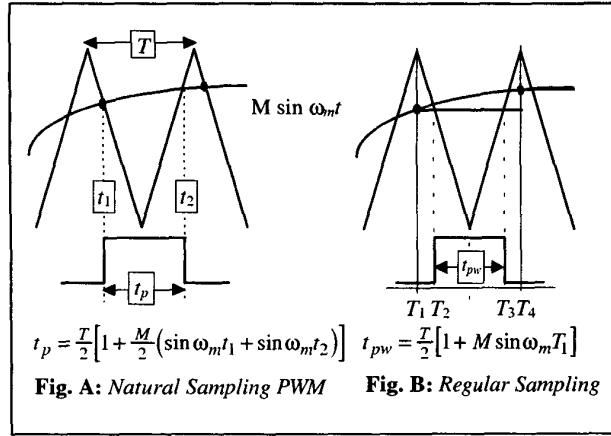


Figure 3: Natural and Regular Sampled PWM

$$\Rightarrow \frac{x^3}{3!} (= 1.116 \times 10^{-3}) \ll \frac{x^2}{2!} (= 17.765 \times 10^{-3}) \quad (11B)$$

which justifies the assumption made. The approximation is improved if the carrier switching frequency is chosen sufficiently higher than the maximum anticipated electrical signal frequency such as $f_s \geq 10pf$, which is generally the case for modern inverter controlled motor drive systems. The benefit of using the zero crossing points as a reference in the polynomial expansion is their proximity to the carrier cycle axis of symmetry, which is manifested as a reduction in the possible interval time span $t_{1,2}^*$ associated with modulated pulse edge occurrence, and to the resultant pulse centre.

Expressions (9) and (10) can be rearranged as a quadratic polynomial in t^* with

$$\left[\frac{V_m V_{cn}^+}{2} (p\omega_r)^2 \right] (t_1^*)^2 + [s^+ + V_m V_{sn}^+ (p\omega_r)] t_1^* - V_m V_{cn}^+ = 0 \quad (12)$$

$$\left[\frac{V_m V_{cn}^-}{2} (p\omega_r)^2 \right] (t_2^*)^2 - [s^- + V_m V_{sn}^- (p\omega_r)] t_2^* - V_m V_{cn}^- = 0 \quad (13)$$

and can be expressed in the generalized form as

$$A \cdot (t^*)^2 + B \cdot t^* + C = 0 \quad (14)$$

with parameter vectors given as equation (15) by

$$\begin{aligned} [A_1 \ B_1 \ C_1]^T &\equiv \left[\frac{V_m V_{cn}^+ (p\omega_r)^2}{2} \quad s^+ + V_m V_{sn}^+ (p\omega_r) \quad -V_m V_{cn}^+ \right]^T \\ [A_2 \ B_2 \ C_2]^T &\equiv \left[\frac{V_m V_{cn}^- (p\omega_r)^2}{2} \quad -[s^- + V_m V_{sn}^- (p\omega_r)] \quad -V_m V_{cn}^- \right]^T \end{aligned}$$

Expression (14) admits to a solution of the form

$$t^* = -\frac{B}{2A} \left[1 \mp \sqrt{1 - \frac{4AC}{B^2}} \right] \quad (16)$$

The discriminant in (16) can be approximated by a binomial expansion with second order terms pending the relative

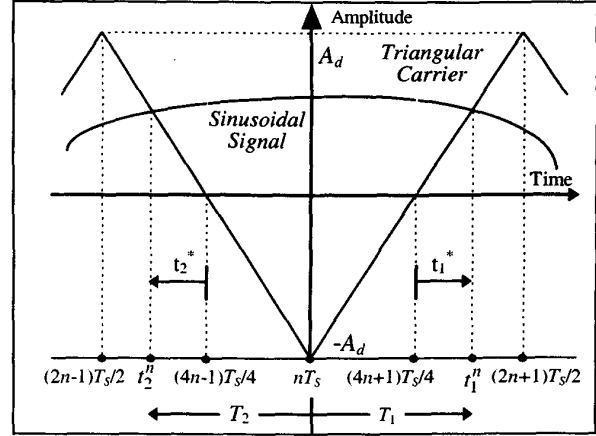


Figure 4: Aperture Time Evaluation for Sinusoidal PWM
magnitude of $(4AC/B^2)$ with unity which can be checked, for the leading edge solution t_1^* , with

$$\left| \frac{4A_1 C_1}{B_1^2} \right| = \left| \frac{-2[m_f V_{cn}^+ (p\omega_r/\omega_s)(\pi/2)]^2}{[1+m_f V_{sn}^+ (p\omega_r/\omega_s)(\pi/2)]^2} \right| \quad (17)$$

$$\text{for } s^+ = \left| s^- \right| = \frac{4A_d}{T_s} = \frac{2A_d}{\pi} \omega_s \quad (18)$$

If worst case conditions are assumed with the following

- (a) From (6) $\max V_{cn}^+ = 1 \Rightarrow V_{sn}^+ = 0$
- (b) $\max m_f = 1$; $\max (p\omega_r/\omega_s) = (600/5000) = 0.12$

then (17) reduces to $|4AC/B^2| \approx 0.071 \ll 1$ which validates the binomial expansion of (16). A similar verification procedure can also be adopted in finding the solution for the trailing edge instant t_2^* in (13) via (16). Expanding (16), using the Binomial theorem, gives

$$t^* = \frac{-B}{2A} + \frac{B}{2A} \left[1 - \frac{1}{2} \left(\frac{4AC}{B^2} \right) - \frac{1}{8} \left(\frac{4AC}{B^2} \right)^2 \right] = -\frac{C}{B} \left[1 + \frac{AC}{B^2} \right] \quad (17)$$

which can be applied to (12) and (13), upon substitution from (15), for solution of the unknown leading and trailing pulse edge transition times $t_{1,2}^*$ as expressions (18) and (19) with

$$\begin{aligned} t_1^* &= \frac{m_f V_{cn}^+}{\left[1+m_f V_{sn}^+ \left(\frac{\pi}{2} \right) \left(\frac{p\omega_r}{\omega_s} \right) \right]} \left[1 - \left(\frac{1}{2} \right) \frac{\left[m_f V_{cn}^+ \left(\frac{\pi}{2} \right) \left(\frac{p\omega_r}{\omega_s} \right) \right]^2}{\left[1+m_f V_{sn}^+ \left(\frac{\pi}{2} \right) \left(\frac{p\omega_r}{\omega_s} \right) \right]^2} \right] \frac{T_s}{4} \\ t_2^* &= \frac{m_f V_{cn}^-}{\left[1-m_f V_{sn}^- \left(\frac{\pi}{2} \right) \left(\frac{p\omega_r}{\omega_s} \right) \right]} \left[1 - \left(\frac{1}{2} \right) \frac{\left[m_f V_{cn}^- \left(\frac{\pi}{2} \right) \left(\frac{p\omega_r}{\omega_s} \right) \right]^2}{\left[1-m_f V_{sn}^- \left(\frac{\pi}{2} \right) \left(\frac{p\omega_r}{\omega_s} \right) \right]^2} \right] \frac{T_s}{4} \end{aligned}$$

The resultant modulated pulse aperture time τ_m is given by the sum of the leading and trailing edge displacement on either side

Table 1: Evaluation of Width Modulated Pulse Transition Times

Carrier: Period $T_s = 200 \mu s$; $A_d = 6.9$ volts; Time Step $\Delta t = 100 ns$;					
Signal: $V_m = 5.52$ volts $\Rightarrow MI = 0.8$; $f_m (\sim pf_r) = 800$ Hz; $\theta = 0$ rad.					
Carrier	Leading Edge time t_1^*		Trailing Edge time t_2^*		
Cycle nT_s Fig 5	Theoretical Estimate Eqn. (18)	Simulation Estimate Fig. 5	Theoretical Estimate Eqn. (19)	Simulation Estimate Fig. 5	Theoretical Pulsewidth Eqn. (20)
$n=0$	37.81 μs	37.8 μs	-	-	-
1	20.89 μs	20.8 μs	36.4 μs	36.4 μs	157.29 μs
2	-4.36 μs	-4.3 μs	11.64 μs	11.6 μs	107.28 μs
3	-27.96 μs	-28 μs	-22.16 μs	-22.2 μs	49.88 μs
4	-40.29 μs	-39.9 μs	-39.45 μs	-39.4 μs	20.26 μs
5	-30.55 μs	-30.6 μs	-33.73 μs	-33.8 μs	35.72 μs
6	0	0.1 μs	-12.9 μs	-12.9 μs	87.1 μs
7	30.55 μs	30.6 μs	12.89 μs	12.9 μs	143.44 μs
8	-	-	33.73 μs	33.8 μs	-

of the carrier axis of symmetry, anchored at nT_s , with

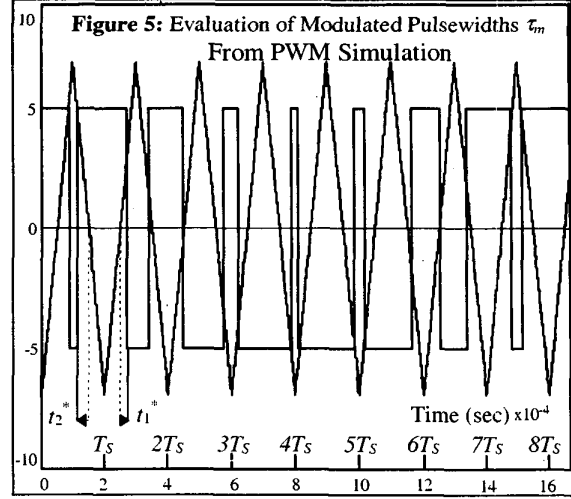
$$\tau_m = \left[\frac{T_s}{4} + t_1^* \right] + \left[\frac{T_s}{4} + t_2^* \right] = (t_1^* + t_2^*) + \frac{T_s}{2} \quad (20)$$

The computational requirement necessary for the evaluation of the leading and trailing edge transition times is a knowledge of the modulating signal reference $V_m(t)$ at the zero crossover times $[(4n \pm 1)T_s/4]$ for each switching period T_s of the triangular carrier $V_m(t)$. All other parameters in the numerical processing of expressions (18) and (19) are known constants for BLMD steady state operation such as the signal amplitude V_m defining m_f , electrical angular frequency $p\omega_r$ and the phase shift θ . The accuracy of the computed estimates $t_{1,2}^*$ is established by comparison with known values obtained from PWM simulation, via an analog comparator modulator with 10ns time steps, used as a measurement reference. The theoretical estimates of modulated aperture times along with those returned from PWM waveform simulation are listed in Table 1. The simulation test strategy, for verification of the theoretical estimates, was based on a natural sampled double edge PWM waveform employing a 600Hz periodic reference signal with a modulation index (MI) of 0.8. This choice gives a reasonable spread in anticipated aperture values ranging from 20 μs to 180 μs , based on [5]

$$\tau_m = \tau[1 + v_m(t)/A_d] = \frac{T_s}{2} [1 + m_f \bar{v}_m(t)] \quad (21)$$

$$\text{with } \frac{T_s}{2} [1 - m_f] \leq \tau_m \leq \frac{T_s}{2} [1 + m_f], \quad (22)$$

for narrow and wide pulsewidth occurrence respectively. The theoretical estimates of the pulse transition times in Table 1 are almost identical to those obtained from PWM simulation with finely resolved time steps ($\sim 0.1 \mu s$) which validates the analysis undertaken and the assumptions made. The width modulated aperture times derived from (20) are also given in Table 1. This analytical approach provides a convenient and rapid means of estimating the pulse duration associated with sinusoidal PWM for steady state inverter operation as in UPS.



3. CONCLUSIONS

A novel theoretical framework has been presented for the prediction of the modulated pulse edge transition times in sinusoidal PWM. The resulting equations for the step time prediction appear simple to compute with low demands on CPU time and consequently could be used online in the sinusoidal PWM control of inverter systems employed in uninterruptable power supply applications. This simple computational feature could also be used in PWM related DSP applications for variable speed motor drives. Results are presented which establish the veracity and substantiate the accuracy of this aperture time estimator in pulse transition time prediction when compared with simulated comparator modulator operation..

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