

An Accurate Calculation of Miller Effect on the Frequency Response and on the Input and Output Impedances of Feedback Amplifiers

S. M. Potirakis and G. E. Alexakis

Abstract—A new method for the calculation of Miller effect on the frequency response and on the input and output impedances of feedback amplifiers is presented. It is based on the Feedback Decomposition Theorem, a recently presented evolution of Miller's Theorem. Application results obtained with this method are compared to those of Miller's Theorem application, through analysis and SPICE simulations, proving its accuracy.

Index Terms—Circuit theory, feedback amplifiers, impedance, poles and zeros, simulation, two-port methods.

I. INTRODUCTION

THE MILLER effect is the most popular application of Miller's Theorem [1], mostly concerning the effect of feedback on input impedance and on frequency response of an amplifier. As known, this was first analyzed by Miller [2] for three-electrode vacuum tubes, but remains valid for all types of amplifiers. The main reason of Miller effect's popularity is its feedback amplifier analysis simplification through its strong intuitive character of the "decomposition" of the feedback network to the known two "feedback reflected" networks, one to the input and one to the output of the main amplifier. However, the feedback amplifier characteristics determined by the common Miller effect approach, based on Miller's Theorem, have been proved inaccurate, leading thus to completely different analysis approaches [3] for attaining accurate calculations.

It has been proved that, "although pole splitting occurs with the insertion of Miller capacitor, under certain conditions both poles could move to lower frequencies together" [4]. It is therefore important to accurately calculate both the dominant pole and the next pole, for confronting issues like transient response analysis and stability analysis of feedback amplifiers, "especially if a strong feedback loop is closed" [3]. The key point, however, is to preserve the strong intuitive character of the "decomposition" of the feedback network into two networks embedded to the input and output of the main amplifier respectively, and be able to carry out calculations

based on such an equivalent circuit. Besides gain frequency response, it is also important to accurately calculate both input and output impedances for impedance matching purposes. Thus, an equivalent circuit yielded from feedback network's "decomposition" should permit such calculations. Several contributions [5]–[8] warn about Miller's Theorem use for the output impedance calculation, leading either to bypassing the problem through other calculation methods or to its calculation back from the original circuit. Output impedance calculation, under the "feedback decomposition" concept, is claimed to be valid when using the Feedback Decomposition Theorem (FDT)[9] and it remains to be confirmed by application. The FDT [9], presented as evolution of Miller's Theorem, yields a true equivalent to the original circuit, retaining the Miller's Theorem concept of decomposing the feedback network.

The present contribution proposes a new method for the accurate calculation of Miller effect on the frequency response (pole and zeros determination) and on the input and output impedances of feedback amplifiers, based on the FDT. First, a brief presentation of the FDT is given for the cases of forward and reverse calculations, both for unilateral and bilateral basic amplifiers (Section II). Then, based on a generic single stage feedback amplifier, Miller effect on the frequency response is for the first time evaluated using both Miller's Theorem and FDT, concluding that the FDT approach is more accurate (Section III). Miller effect on input and output impedances of a feedback amplifier is also considered for both circuits resulting after Miller's Theorem and FDT applications. It is shown, by analysis and SPICE simulation, that the FDT approach provides a more accurate input impedance calculation, while it is the only one of the two approaches that can provide reverse calculations, e.g., output impedance calculation (Section IV). Section V summarizes the conclusions.

II. FEEDBACK DECOMPOSITION THEOREM

The FDT [9] aims to simplify feedback analysis by simplifying the four basic topologies of single loop feedback amplifiers (S-S, P-P, S-P, P-S). It is based on two-port theory and uses the notation of generalized χ -parameters (covering the known z -, y -, h -, and g -parameters) and the performance characteristics that can be defined and derived [10] based on it. The χ -parameters are defined by

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} \chi_{11} & \chi_{12} \\ \chi_{21} & \chi_{22} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}.$$

Manuscript received June 14, 2004; revised November 3, 2004. This work was supported by the Special Research Account of the University of Athens. This paper was recommended by Associate Editor A. Apsel.

S. M. Potirakis is with the Defence Products Development Department, Intracom s.a., Peania, Attica GR19002, Greece (e-mail: spotirak@phys.uoa.gr).

G. E. Alexakis is with the Electronics Laboratory, Physics Department, University of Athens, Panepistimiopolis GR15784, Greece (e-mail: galexaki@phys.uoa.gr).

Digital Object Identifier 10.1109/TCSII.2005.848986

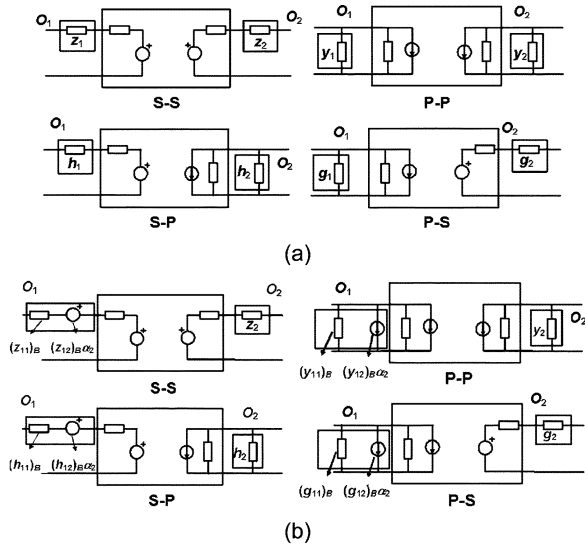


Fig. 1. (a) FDEs special case for forward and reverse signal flow, if the A two-port is bilateral, and for forward signal flow only, if the A two-port is unilateral. (b) FDEs special case for reverse signal flow, if the A two-port is unilateral, also valid for forward calculations. $\chi \in \{z, y, h, g\}$ and its independent variable of the output port $\alpha_2 \in \{I_2, V_2, V_2, I_2\}$, correspondingly.

The symbols ε_1 and ε_2 represent generalized dependent variables for input and output ports, respectively, while α_1 and α_2 are the generalized independent variables, accordingly.

FDT claims that if we have two linear two-port networks, A and B, interconnected in such a way to present the same generalized input independent variables and the same generalized output independent variables, then the interconnection of A and B two-ports is equivalent to the connection of two feedback decomposition one-ports (FDOs), O_1 and O_2 , to the input and output ports of the A two-port, respectively. This equivalent representation is defined as the feedback decomposition equivalent (FDE).

Fig. 1 shows the decomposition to one-ports for the four cases of single loop feedback amplifiers, according to FDT.

Particularly,

if the A two-port network is bilateral, O_1 and O_2 consist of just one passive element (immittance), χ_1 and χ_2 , respectively (as Fig. 1(a) shows). For forward signal flow, i.e., for use in forward performance evaluation, χ_1 and χ_2 are

$$\chi_{i,f} = (\chi_{ii})_B + (\chi_{ij})_B A_f^{(-1)^j}, \quad i, j = 1, 2; \quad i \neq j \quad (1)$$

while for reverse signal flow, i.e., for use in reverse performance evaluation, χ_1 and χ_2 are

$$\chi_{i,r} = (\chi_{ii})_B + (\chi_{ij})_B A_r^{(-1)^i}, \quad i, j = 1, 2; \quad i \neq j \quad (2)$$

If the A two-port network is unilateral, then for forward signal flow, i.e., for use in forward performance evaluation, O_1 and O_2 consist of just one passive element (immittance), χ_1 and χ_2 , respectively [as Fig. 1(a) shows], having the same expression as for the case of the A two-port network being bilateral, i.e., those given by (1). For reverse signal flow, i.e., for use in reverse performance evaluation, O_1 consists of one passive element $(\chi_{11})_B$

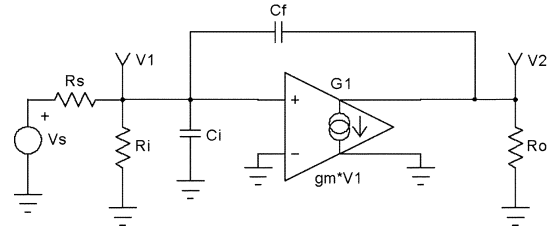


Fig. 2. Generic single-stage feedback amplifier.

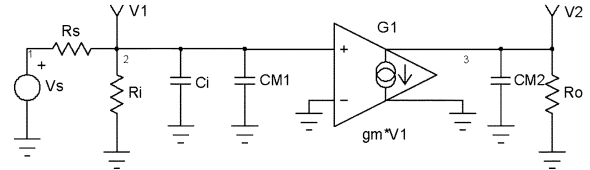


Fig. 3. Circuit of Fig. 2. after the application of Miller's Theorem.

and one dependent active element $[(\chi_{12})_B \cdot \alpha_2]$ and O_2 consists of just one passive element (immittance), χ_2 , [as Fig. 1(b) shows]. The FDO χ_2 has the same expression as for the case of the A two-port network being bilateral, i.e., that given by (2), for $i = 2$ and $j = 1$.

In (1) and (2)

$$A_f = -\chi_{21}/(\chi_{22} + \chi_L), A_r = -\chi_{12}/(\chi_{11} + \chi_S)$$

with

$$\chi_{ij} = (\chi_{ij})_A + (\chi_{ij})_B$$

and $\chi \in \{z, y, h, g\}$.

Note that A_f and A_r are given by FDT and they are part of FDOs expressions, so one does not have to analyze the circuit, prior to FDTs application in order to determine them, as it must be done in the case of Miller's Theorem.

III. MILLER EFFECT ON GAIN TRANSFER FUNCTION

A simple, generic feedback amplifier circuit that is used to study the effect of the feedback capacitance (known also as Miller capacitance), considered between input and output of a feedback amplifier, is shown in Fig. 2.

According to the traditional approach of Miller's Theorem application, Miller capacitance is decomposed into two capacitances C_{M1} and C_{M2} to the input and output, respectively, as shown in Fig. 3, with

$$C_{M1} = C_f \cdot (1 - A_v) \quad (3)$$

$$C_{M2} = C_f \cdot (1 - 1/A_v) \quad (4)$$

where $A_v = V_2/V_1$.

Normally, A_v should be the closed loop forward gain of the feedback amplifier, but this is not clearly stated in Miller's Theorem [5], [1]. However, in most cases of Miller's Theorem application, A_v is assumed to be the dc open loop gain. The reason is that if the closed loop gain is calculated, it clearly depends on the feedback network -the feedback capacitor in this case-

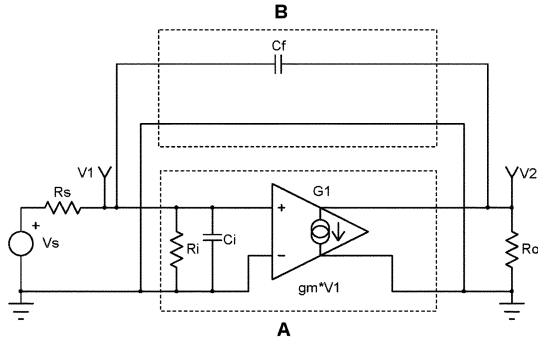


Fig. 4. Circuit of Fig. 2 as a parallel-parallel connection of A and B two-ports.

and this violates the validity condition of Miller's Theorem, requiring a gain independent of the feedback network [1], [11], [12], [4].

Therefore, $A_V = -g_m R_o$ and substituting to (3) we have

$$C_{M1} = C_f(1 + g_m R_o) \quad (5)$$

while from (4) we have

$$C_{M2} = C_f(1 + 1/g_m R_o). \quad (6)$$

Performing simple voltage divider logic calculations on the circuit of Fig. 3, we obtain voltage gain as

$$G_V = \frac{V_2}{V_s} = \frac{V_1}{V_s} \cdot \frac{V_2}{V_1} \quad (7)$$

with

$$\frac{V_1}{V_s} = \frac{1}{1 + \frac{R_s}{R_i} + sR_s(C_i + C_{M1})} \quad (8)$$

and

$$\frac{V_2}{V_1} = \frac{-g_m R_o}{1 + sR_o C_{M2}}. \quad (9)$$

Thus, substituting (8) and (9) to (7) we have

$$G_V = \frac{1}{1 + R_s/R_i + sR_s(C_i + C_{M1})} \cdot \frac{-g_m R_o}{1 + sR_o C_{M2}} \quad (10)$$

or using (5) and (6)

$$G_V = \frac{1}{1 + R_s/R_i + sR_s[C_i + C_f(1 + g_m R_o)]} \cdot \frac{-g_m R_o}{1 + sR_o C_f(1 + 1/g_m R_o)}. \quad (11)$$

From (11), (10), and (9) we conclude that, using Miller's Theorem, the Miller effect on the calculated voltage gain is the shifting of the "input" pole and the appearance of an output pole. Note that (11) has no zeros.

Now, applying FDT to analyze Miller effect on the (forward) voltage gain of the circuit of Fig. 2, we first interpret the circuit as a two-port connection, as shown in Fig. 4.

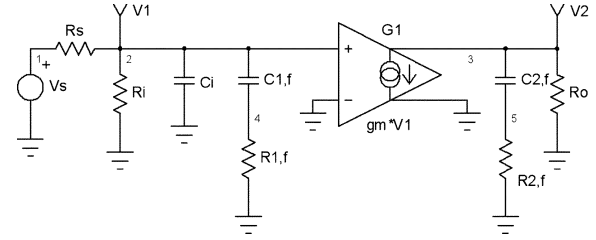


Fig. 5. Circuit of Fig. 2 after the application of FDT, for forward calculations.

Then, applying the FDT for forward calculations, we calculate the FDOs that have to be connected to the input and output of the basic amplifier, as [9]

$$Y_{1,f} = (y_{11})_B + (y_{12})_B A_f \quad \text{and} \quad Y_{2,f} = (y_{22})_B + (y_{21})_B A_f^{-1}$$

with $A_f = (V_2/V_1) = -(y_{21}/y_{22} + y_L)$ and $y_{ij} = (y_{ij})_A + (y_{ij})_B$ (we use y-parameters due to the parallel-parallel connection of A and B two-ports). Applying two-port definitions and properties [10], we easily obtain the Y-matrices for the feedback network and the basic amplifier, respectively

$$\mathbf{Y}_B = sC_f \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \mathbf{Y}_A = \begin{bmatrix} (sC_i R_i + 1)/R_i & 0 \\ g_m & 0 \end{bmatrix}.$$

Then, in a straightforward way, we have

$$A_f = -\frac{g_m - sC_f}{sC_f + 1/R_o}$$

and finally the input one-port is

$$y_{1,f} = \frac{1}{(g_m + R_o^{-1})^{-1} + R_o^{-1}/sC_f(g_m + R_o^{-1})} \quad (12)$$

which is the connection of a resistor $R_{1,f}$ and a capacitor $C_{1,f}$ in series, with $R_{1,f} = (g_m + R_o^{-1})^{-1}$ and $sC_{1,f} = sC_f(g_m + R_o^{-1})/R_o^{-1}$, while the output one-port is

$$y_{2,f} = \frac{1}{-\frac{1}{g_m + R_o^{-1}} + \frac{g_m}{sC_f(g_m + R_o^{-1})}} \quad (13)$$

which is the connection of a resistor $R_{2,f}$ and a capacitor $C_{2,f}$ in series, with $R_{2,f} = -(g_m + R_o^{-1})^{-1}$, and $sC_{2,f} = sC_f(g_m + R_o^{-1})/g_m$.

The equivalent circuit resulting from the application of the FDT, i.e., the FDE, is illustrated in Fig. 5. Following the same analysis method, we calculate the voltage gain by calculating the factors shown in (7), to give a clear idea of the input and output contribution to the transfer function. By simple voltage divider logic calculations on the circuit of Fig. 5, we obtain (14) shown at the bottom of the page and

$$\frac{V_2}{V_1} = -g_m R_o \frac{1 + sR_{2,f}C_{2,f}}{1 + sC_{2,f}(R_{2,f} + R_o)}. \quad (15)$$

Substituting (12) and (13)'s components to (14) and (15), we express (14) and (15) in terms of the components of the original

$$\frac{V_1}{V_s} = (1 + sR_{1,f}C_{1,f}) / \left\{ 1 + \frac{R_s}{R_i} + s \left[C_{1,f} \left(R_{1,f} + \frac{R_s}{R_i} R_{1,f} + R_s \right) + C_i R_s \right] + s^2 C_{1,f} C_i R_{1,f} R_s \right\} \quad (14)$$

circuit (Fig. 2), as in (16), shown at the bottom of the page, and (17)

$$\frac{V_2}{V_1} = -g_m R_o \frac{1 - s(C_f/g_m)}{1 + sC_f R_o}. \quad (17)$$

Finally, from (7), (16), and (17), and after the $1 + sC_f R_o$ factor cancellation from nominator and denominator (pole-zero cancellation), we have (18)

$$\begin{aligned} G_V &= \frac{V_2}{V_s} \\ &= -g_m R_o [1 - s(C_f/g_m)] \left/ \left\{ 1 + \frac{R_s}{R_i} + s[C_f R_o \right. \right. \\ &\quad \left. \left. + C_f R_s (1 + R_o (g_m + R_s^{-1})) + C_i R_s \right] \right. \\ &\quad \left. + s^2 C_f C_i R_o R_s \right\} \end{aligned} \quad (18)$$

which is exactly the voltage gain, if calculated directly from the original circuit of Fig. 2 [3].

Apart from the fact that Miller's Theorem approach ignores the zero of the voltage gain, even the poles of the original circuit are not accurately calculated, as SPICE calculations prove. Using the same values as in [3] ($R_s = 2$ kohms, $R_i = 2$ kohms, $C_i = 100$ pF, $g_m = 0.01$, $R_o = 10$ kohms, and $C_f = 1$ pF), the simulation of the original circuit gives one zero, at 10^{10} rad/s, and two poles, at $-4.85086 \cdot 10^6$ rad/s (dominant pole) and $-2.06149 \cdot 10^8$ rad/s. We also performed a SPICE pole-zero analysis on the original circuit to find the pole-zero contribution of the two factors present in (7), i.e., "input" and "output" contribution to circuit's poles and zeros. We found that the input contributes one zero at $-1 \cdot 10^8$ rad/s and two poles at $-4.85086 \cdot 10^6$ rad/s and $-2.06149 \cdot 10^8$ rad/s, while the output contributes with one zero at $-1 \cdot 10^{10}$ rad/s and one pole at $-1 \cdot 10^8$ rad/s, with an apparent pole-zero cancellation at $-1 \cdot 10^8$ rad/s. We verified these results, by analyzing the original circuit, in terms of the input and output factors contribution, as implied by (7).

On the other hand, SPICE simulation on the circuit of Miller's Theorem (Fig. 3) yields only two poles at $-4.97512 \cdot 10^6$ rad/s (dominant pole) and $-9.90099 \cdot 10^7$ rad/s. Clearly, dominant pole is shifted by 2.6%, while the second pole is wrongly calculated, as it presents 52% deviation from the original circuit's second pole, closer to the dominant pole. Their percentage difference depends on the actual values of the original circuit [8]. But, while the dominant pole results from the Miller effect on the input of the basic amplifier, as referred in (8), simulation on Fig. 3 circuit proved that the second pole does not result from the Miller effect on the output of the basic amplifier, as (9) implies. Moreover, there is a small difference between the simulated ($9.90099 \cdot 10^7$ rad/s) and the calculated from (9) ($1.01 \cdot 10^8$ rad/s) second pole's position. In fact, simulation gives one zero at $-9.90099 \cdot 10^7$ rad/s and two poles, at $-4.97512 \cdot 10^6$ rad/s and $-9.90099 \cdot 10^7$ rad/s, contributed by the input, while

the output contributes one zero at $-9.90099 \cdot 10^7$ rad/s. As expected, the overall gain presented a pole-zero cancellation at $-9.90099 \cdot 10^7$ rad/s.

Finally, the simulation of the FDE circuit (Fig. 5) gives exactly the same results with the original circuit, both for the overall and for the input and output pole-zero analysis. Of course, the simulation verified the pole-zero pair at $-1 \cdot 10^8$ rad/s, which is predicted by (16)–(18), in the same way as in the simulation and analysis of the original circuit. Thus, using the FDT, there is no need to be cautious about any error analysis as when using Miller's Theorems [8].

Based on the above, the FDT approach yields an equivalent to the original circuit that leads to accurate determination of Miller effect on the input, the output and the overall voltage gain, while Miller's Theorem approach does not yield an equivalent circuit. Only the Miller effect on the dominant pole of the basic amplifier is interpreted by Miller's Theorem approach, although with some inaccuracy, while Miller effect on the overall frequency response and on the input and output poles and zeros of the basic amplifier is not successfully interpreted.

IV. MILLER EFFECT ON INPUT AND OUTPUT IMPEDANCE

Following Miller's Theorem approach, we computed the input impedance (Fig. 3) to be

$$Z_{in} = R_s + \frac{1}{R_i^{-1} + s[C_i + C_f(1 + g_m R_o)]} \quad (19)$$

and the output impedance (short circuiting V_s and replacing R_o with an ideal test voltage source, following the common output impedance calculation tactic, e.g., [13]) to be

$$Z_{out} = \frac{1}{sC_f(1 + 1/g_m R_o)}. \quad (20)$$

Based on the equivalent circuit of Fig. 5, we calculated the input impedance as

$$Z_{in} = R_s + \frac{1}{R_i^{-1} + sC_i + \frac{sC_f(g_m + R_o^{-1})}{sC_f + R_o^{-1}}} \quad (21)$$

which we verified it as identical to the Z_{in} calculated directly from the original circuit of Fig. 2.

For output impedance calculation, the circuit of Fig. 5 is not suitable. According to FDT, a reverse calculations equivalent circuit is needed, which in our case of a unilateral amplifier is the one depicted in Fig. 6. If we had used a circuit like the one of Fig. 5, we would have made the input completely independent from the output, destroying the equivalence to the original circuit in reverse calculations [9].

So, applying FDT for reverse calculations to the circuit of Fig. 2, we add the output one-port to the output and leave the input circuit of the B two-port to the input. As described in [9], the output one-port is $y_{2,r} = (y_{22})_B + (y_{21})_B A_r$, with

$$A_r = \left. \frac{V_1}{V_2} \right|_{\substack{\varepsilon_s=0 \\ \chi_s \neq 0}} = -\frac{y_{12}}{y_{11} + y_s} \quad \text{and} \quad y_{ij} = (y_{ij})_A + (y_{ij})_B.$$

$$\frac{V_1}{V_s} = (1 + sC_f R_o) \left/ \left\{ 1 + \frac{R_s}{R_i} + s \left[C_f R_o + C_f R_s (1 + R_o (g_m + R_s^{-1})) + C_i R_s \right] + s^2 C_f C_i R_o R_s \right\} \right. \quad (16)$$

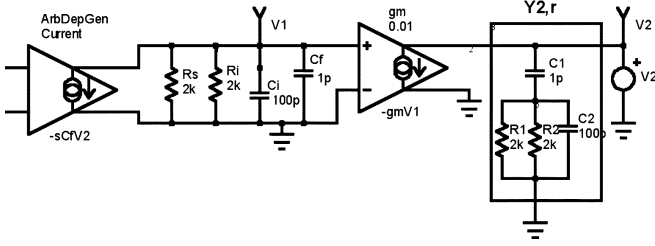


Fig. 6. Circuit of Fig. 2 after the application of FDT, for reverse calculations. This circuit is referred to as the FDE for reverse calculations.

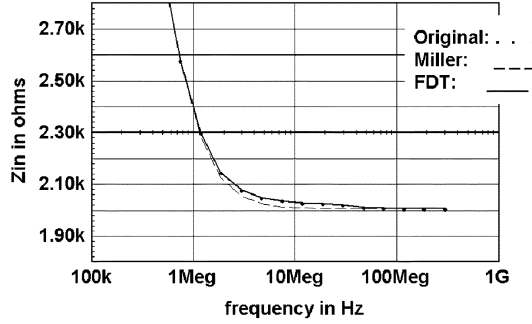


Fig. 7. Input impedance of the circuit of Fig. 2 simulated, using the original circuit and the circuits derived from the application of Miller's Theorem and FDT. Maximum difference of about 1.2% between the original and Miller circuit's input impedances.

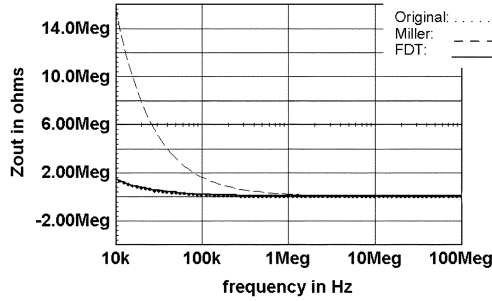


Fig. 8. Output impedance of the circuit of Fig. 2 simulated, using the original circuit and the ones derived from the application of Miller's Theorem and FDT.

Then, in a straightforward way, we have

$$A_r = \frac{sR_i R_s C_f}{R_i + R_s + sR_i R_s (C_i + C_f)}$$

while the output one-port is

$$y_{2,r} = \frac{sC_f (R_i + R_s + sC_i R_i R_s)}{R_i + R_s + sR_i R_s (C_i + C_f)} \quad \text{or} \quad (22)$$

$$y_{2,r} = \frac{1}{\frac{1}{R_i^{-1} + R_s^{-1} + sC_i} + \frac{1}{sC_f}}$$

which is the connection of a capacitor $C_1 = C_{1,f}$ in series to the parallel connection of two resistors $R_1 = R_i$, $R_2 = R_s$ and a capacitor $C_2 = C_i$, as depicted on Fig. 6.

Based on the equivalent circuit of Fig. 6, we calculated the output impedance as follows:

$$Z_{out} = Y_{out}^{-1} = (y_{2,r} + y_{gm})^{-1} \quad \text{with } y_{gm} = g_m V_1 / V_2.$$

But $V_1 = sC_f [R_i^{-1} + R_s^{-1} + s(C_i + C_f)]^{-1} V_2$ and Z_{out} finally yields

$$Z_{out} = \frac{R_i + R_s + sR_i R_s (C_i + C_f)}{sC_f (R_i + R_s + g_m R_i R_s) + s^2 R_i R_s C_i C_f} \quad (23)$$

which we verified it as identical to the Z_{out} calculated directly from the original circuit of Fig. 2.

Comparing the two approaches, it is apparent that (19) and (21) give the same dc input impedances, equal to $R_s + R_i$, but a somewhat different frequency response. Their percentage difference depends on the actual values of the original circuit [8]. From (21) and (23) we can conclude that the output impedance derived by the Miller's Theorem approach has no relevance to the original circuit's output impedance and consequently Miller effect on the output impedance is neither accurately nor qualitatively represented. This is mainly due to the fact that the Miller's Theorem application does not give an equivalent to the original circuit for reverse calculation. In fact, Miller's Theorem does not discriminate between forward and reverse calculation at all. Apart from verifying Z_{in} and Z_{out} to the ones resulting directly from the original circuit of Fig. 2, we performed the corresponding SPICE simulation as well (using again the same values -shown on Fig. 6- as in simulations of Section III). Obtained results confirming the above are shown in Figs. 7 and 8.

V. CONCLUSION

The following conclusions emerge.

- 1) The FDT approach gives an equivalent to the original circuit that leads to accurate determination of Miller effect on the input, output and overall voltage gain.
- 2) Only the Miller effect on the dominant pole of the basic amplifier is interpreted by Miller's Theorem approach, although with some inaccuracy.
- 3) Miller effect on input and output impedances is accurately derived by the FDT approach, providing an equivalent circuit for both forward and reverse calculations. On the other hand, Miller's Theorem approach provides only an approximate calculation of the input impedance, while fails to estimate output impedance.

REFERENCES

- [1] J. Millman and C. C. Halkias, *Electronic Devices and Circuits*. New York: McGraw-Hill, 1967, pp. 348–349.
- [2] J. M. Miller, "Dependence of the input impedance of a three-electrode vacuum tube upon the load in the plate circuit," *Scientific Papers of the National Bureau of Standards (U.S.)*, vol. 15, no. 351, pp. 367–385, 1919.
- [3] A. Filipkowski, "Poles and zeros in transistor amplifier introduced by Miller effect," *IEEE Trans. Educ.*, vol. 42, no. 4, pp. 349–351, Nov. 1999.
- [4] W.-H. Ki, L. Der, and S. Lam, "Re-examination of pole splitting of a generic single stage amplifier," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 44, no. 1, pp. 70–74, Jan. 1997.
- [5] A. B. Macnee, "On the presentation of Miller's theorem," *IEEE Trans. Educ.*, vol. E-28, no. 2, pp. 92–93, May 1985.
- [6] P. E. Gray and C. L. Searle, *Electronic Principles: Physics, Models, and Circuits*. New York: Wiley, 1969, p. 500, 501, 508, 509.
- [7] G. Weiss, "Comment on a theorem dual to Miller's theorem," *IEEE Trans. Educ.*, vol. 33, no. 2, p. 221, 222, May 1990.
- [8] L. Moura, "Error analysis in Miller's theorem's," *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 48, no. 2, pp. 241–249, Feb. 2001.
- [9] S. M. Potirakis and G. E. Alexakis, "The feedback decomposition theorem: The evolution of Miller's theorem," *Int. J. Electron.*, vol. 85, no. 5, pp. 571–587, Nov. 1998.
- [10] S. M. Potirakis, G. E. Alexakis, and G. S. Tombras, "Generalized two-port performance evaluation," in *Proc. ICECS'96 (3rd Int. Conf. Circuits, and Systems)*, 1996, pp. 140–143.
- [11] J. Millman, *Microelectronics: Digital and Analog Circuits and Systems*. New York: McGraw-Hill, 1979.
- [12] A. Grebene, *Bipolar and MOS Analog Integrated Circuit Design*. New York: Wiley Interscience, 1984.
- [13] J. Millman and C. C. Halkias, *Integrated Electronics: Analog and Digital Circuits and Systems*. New York: McGraw-Hill, 1972, p. 248.