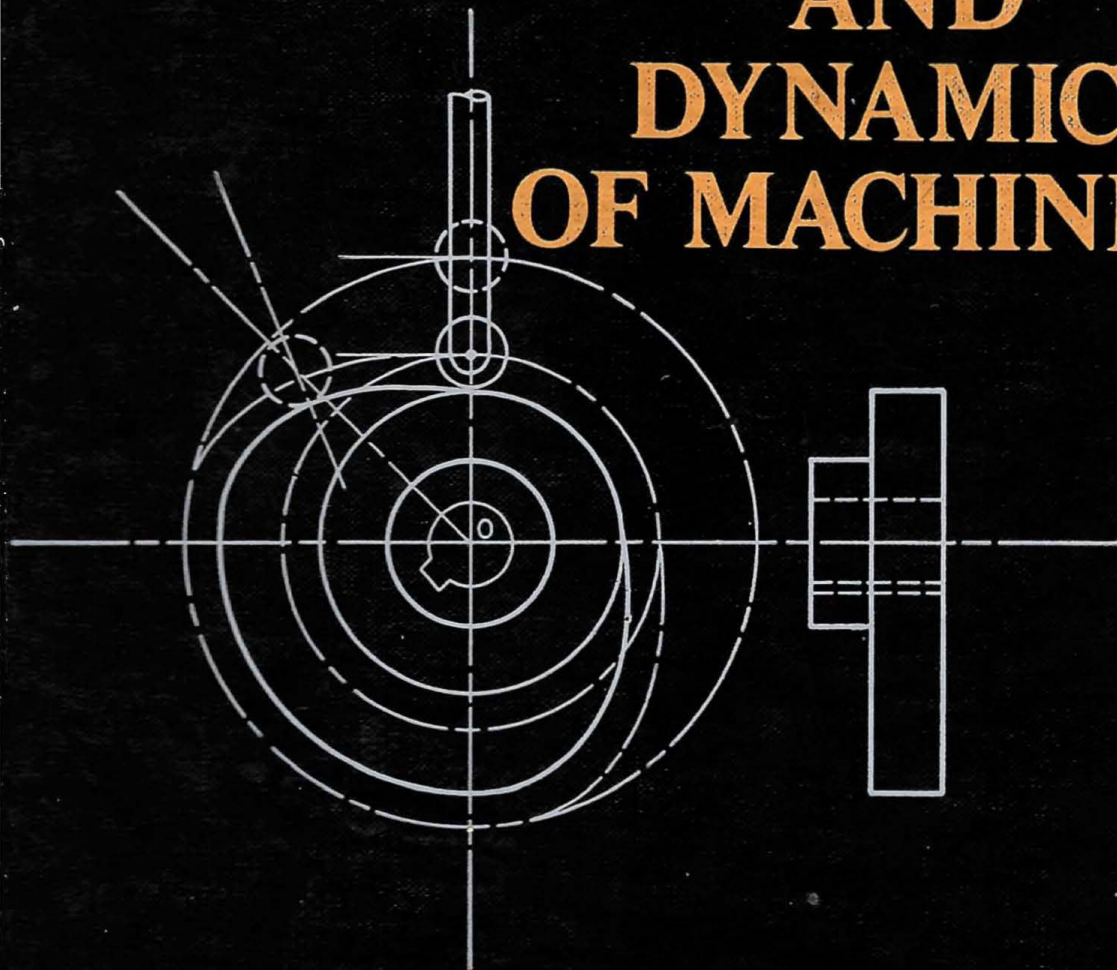


FOURTH EDITION
**MECHANISMS
AND
DYNAMICS
OF MACHINERY**



Hamilton H. Mabie
Charles F. Reinholtz

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whose assistance and forbearance have
made this edition possible.

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Gear Trains

7.1 INTRODUCTION TO GEAR TRAINS

Often, it is necessary to combine several gears and to obtain by so doing what is known as a *gear train*. Given the input angular velocity to a gear train, it is important to be able to determine easily the angular velocity of the output gear and its direction of rotation. The ratio of the input angular velocity to the output angular velocity is known as the *angular-velocity ratio* and is expressed as ω_{in}/ω_{out} .

Figure 7.1 shows a pinion driving an external spur gear and a pinion driving an internal spur gear. In both cases, the angular-velocity ratio is inversely proportional to the number of teeth as indicated. The external gears rotate in opposite directions, and the internal gear rotates in the same direction as its pinion. This is indicated by a minus sign on the velocity ratio in the first case and by a plus sign in the second case. Up to the present time, it has been unnecessary to assign

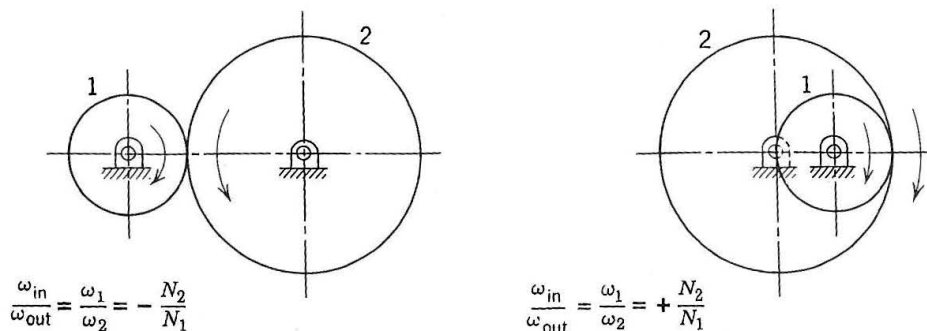


FIGURE 7.1

an algebraic sign to the angular-velocity ratio of a pair of gears. However, when gears are combined to give a gear train, it is important to consider the sign because it indicates direction of rotation. This is especially true in the analysis of planetary gear trains.

Occasionally, it is necessary to change the direction of rotation of a gear without changing its angular velocity. This can be done by placing an *idler gear* between the driven gear and the driver gear. When an idler gear is used, the direction of rotation is changed but the velocity ratio remains the same.

It can be shown that the angular-velocity ratio of a gear train where all gears have fixed axes of rotation is the product of the number of teeth of all the driven gears divided by the product of the number of teeth of all the driving gears. This relation is given in equation form by

$$\frac{\omega_{\text{in}}}{\omega_{\text{out}}} = \frac{\omega_{\text{driver}}}{\omega_{\text{driven}}} = \frac{\text{Product of teeth of driven gears}}{\text{Product of teeth of driving gears}} \quad (7.1)$$

To illustrate the use of Eq. 7.1, consider the gear train of Fig. 7.2, where gear 2 and gear 3 are mounted on the same shaft. The angular-velocity ratio is given by

$$\frac{\omega_{\text{in}}}{\omega_{\text{out}}} = \frac{\omega_1}{\omega_4} = + \frac{N_2 \times N_4}{N_1 \times N_3}$$

The plus sign is determined by observation. That the preceding equation is correct

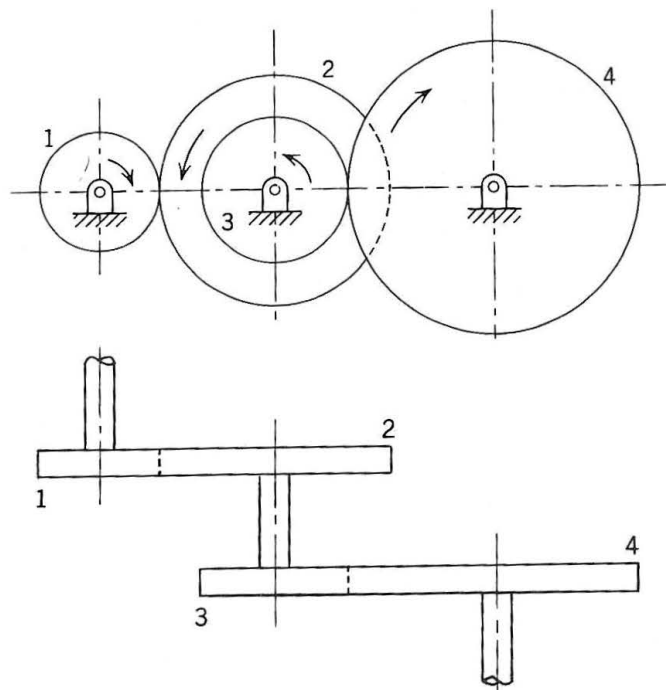


FIGURE 7.2

can be easily shown:

$$\frac{\omega_1}{\omega_2} = -\frac{N_2}{N_1} \quad \text{and} \quad \frac{\omega_3}{\omega_4} = -\frac{N_4}{N_3}$$

$$\frac{\omega_1}{\omega_2} \times \frac{\omega_3}{\omega_4} = +\frac{N_2}{N_1} \times \frac{N_4}{N_3}$$

But

$$\omega_2 = \omega_3$$

Therefore,

$$\frac{\omega_1}{\omega_4} = +\frac{N_2}{N_1} \times \frac{N_4}{N_3}$$

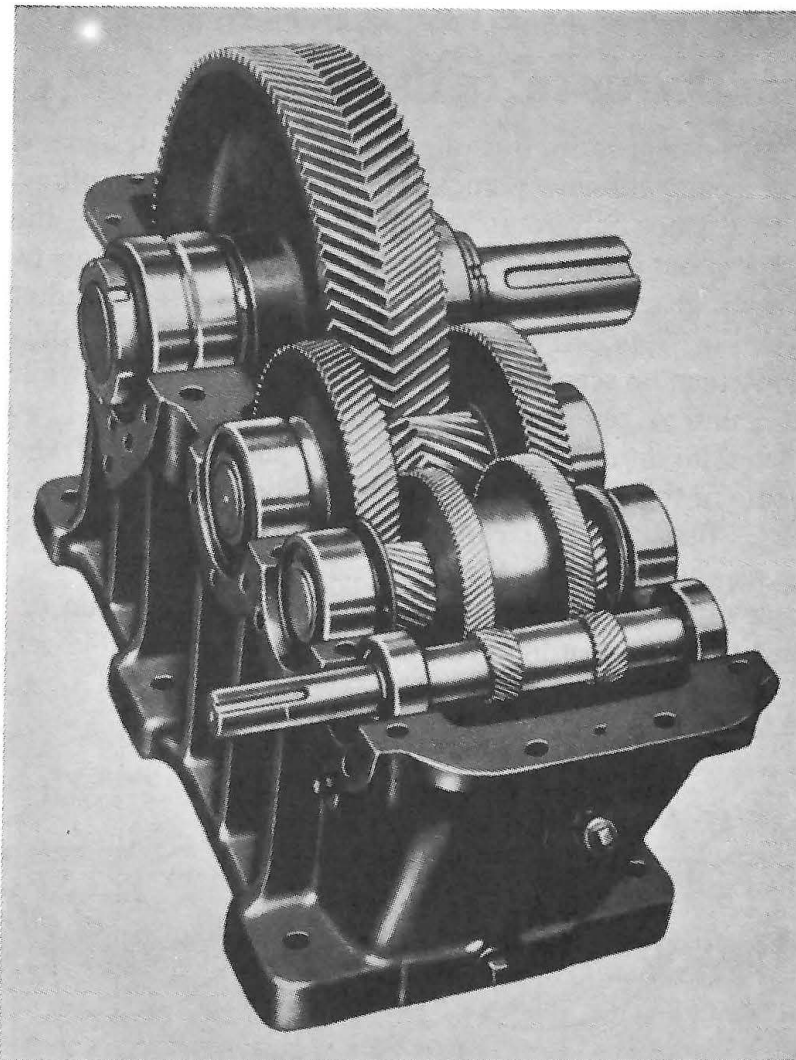


FIGURE 7.3 Triple-reduction speed reducer. (Courtesy of Jones Machinery, Division of Hewitt-Robins, Inc.)

When two gears are fixed to the same shaft as gears 2 and 3 in Fig. 7.2, the gears form a *compound gear*.

Although the angular-velocity ratio is used for calculations involving just one pair of gears, it is more convenient when working with a gear train to use the reciprocal of the angular-velocity ratio. The reason for this is that the angular velocity of the driver will be known from the speed of the motor, and it is necessary only to multiply the speed of the driver by a factor to find the speed of the last gear in the train. This reciprocal is known as the *train value* and is given in equation form by

$$\frac{\omega_{\text{driven}}}{\omega_{\text{driver}}} = \frac{\text{Product of teeth of driving gears}}{\text{Product of teeth of driven gears}} \quad (7.2)$$

In general, gear velocities step down so that this value will be less than 1.00. A typical gear train is illustrated in the triple-reduction speed reducer shown in Fig. 7.3.

7.2 PLANETARY GEAR TRAINS

To obtain a desired gear ratio, it is often advantageous to design a gear train so that one of the gears will have planetary motion. With this motion, a gear will be so driven that it not only rotates about its own center but at the same time its center rotates about another center. Figures 7.4a and 7.4b show two planetary gear trains, where gear 1 is often referred to as the *sun* gear and gear 2 as the *planet* gear. In Fig. 7.4a, arm 3 drives gear 2 about gear 1, which is a fixed external gear. As can be seen, gear 2 rotates about its center *B* while this center rotates about center *A*. As gear 2 rolls on the outside of gear 1, a point on its surface will generate an epicycloid. Figure 7.4b shows the case where gear 1 is an internal gear. In this case, a hypocycloid will be generated by a point on the surface of gear 2. Because of the curves generated, a planetary gear train is often referred to as an *epicyclic*, or *cyclic*, gear train.

It is more difficult to determine the angular-velocity ratio of a planetary gear train than that of an ordinary train because of the double rotation of the

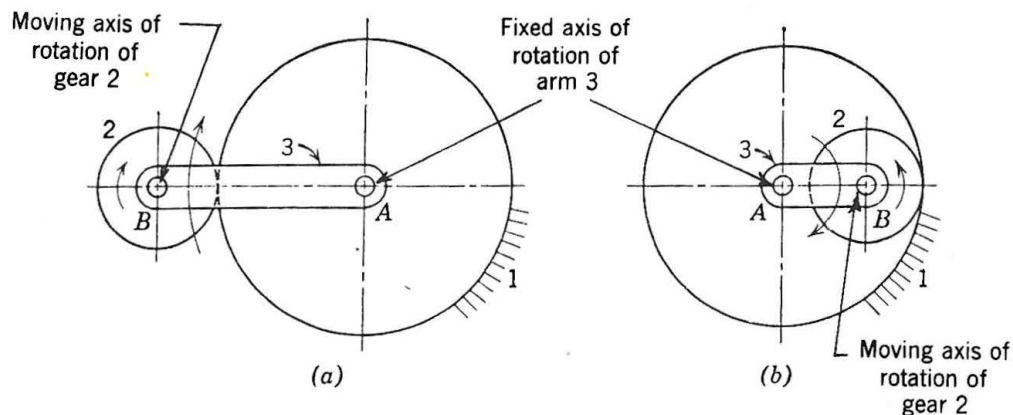


FIGURE 7.4

planet. The angular-velocity ratio may be obtained by the instantaneous center method, the formula method, or the tabulation method. The instantaneous center method will be reserved for Chapter 8, with the other two methods presented here. The formula method will be treated first.

In Fig. 7.4, let it be required to determine ω_{21} given ω_{31} . It should be noted that ω_{21} is defined as the angular velocity of gear 2 relative to gear 1 and ω_{31} is defined as the angular velocity of arm 3 relative to gear 1. Because gear 1 is fixed, this is the same as the angular velocity of gear 2 and of arm 3 relative to the ground. In the solution of the problem, ω_{23}/ω_{13} plays an important part.

Consider the gear train in Fig. 7.4a to be changed so that arm 3 is stationary instead of gear 1. Arm 3 then becomes the ground, and an ordinary gear train results. The ratio ω_{23}/ω_{13} can therefore be evaluated as $-N_1/N_2$. If the mechanism is now returned to its original condition, that is, arm 3 moving and gear 1 fixed, the ratio ω_{23}/ω_{13} will still be $-N_1/N_2$. The reason for this is that when a mechanism is inverted, the relative motion between links does not change. A solution for ω_{21} in terms of the known quantities ω_{31} and ω_{23}/ω_{13} can now be affected by writing an equation of ω_{21} and dividing by ω_{31} as follows:

$$\begin{aligned}\omega_{21} &= \omega_{31} + \omega_{23} \\ \frac{\omega_{21}}{\omega_{31}} &= 1 + \frac{\omega_{23}}{\omega_{31}} = 1 - \frac{\omega_{23}}{\omega_{13}}\end{aligned}$$

Therefore,

$$\omega_{21} = \omega_{31} \left(1 - \frac{\omega_{23}}{\omega_{13}} \right) \quad (7.3)$$

For Fig. 7.4a,

$$\frac{\omega_{23}}{\omega_{13}} = -\frac{N_1}{N_2}$$

and

$$\omega_{21} = \omega_{31} \left(1 + \frac{N_1}{N_2} \right) \quad (7.3a)$$

For Fig. 7.4b,

$$\frac{\omega_{23}}{\omega_{13}} = +\frac{N_1}{N_2}$$

and

$$\omega_{21} = \omega_{31} \left(1 - \frac{N_1}{N_2} \right) \quad (7.3b)$$

TABLE 7.4

	Gear 2	Gear 3	Gear 4	Gear 5	Gear 7	Arm 6
Motion with arm relative to frame	x	x	x	x	x	x
Motion relative to arm	$+x\left(\frac{N_3 \times N_7}{N_2 \times N_4}\right)$			$+x\left(\frac{N_7}{N_5}\right)$	$-x$	0
Total motion relative to frame	60			y	0	x

viewed from the right end. It is required to determine ω_{51} and its direction; ω_{61} is unknown, and $\omega_{71} = 0$.

Let $x = \omega_{61}$ and $y = \omega_{51}$:

$$x + x\left(\frac{N_3 \times N_7}{N_2 \times N_4}\right) = 60$$

$$x\left(1 + \frac{30 \times 76}{18 \times 28}\right) = 60$$

$$x\left(1 + \frac{95}{21}\right) = 60$$

$$x = \frac{21}{116} (60) = +10.86 \text{ rad/s}$$

$\therefore \omega_{61} = +10.86 \text{ rad/s}$ and direction of rotation same as that of ω_{21}

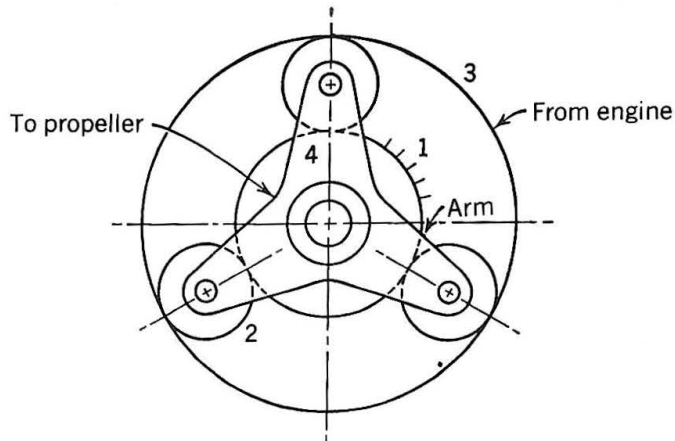
$$\begin{aligned} y &= x + x\left(\frac{N_7}{N_5}\right) \\ &= x\left(1 + \frac{N_7}{N_5}\right) \\ &= 10.86\left(1 + \frac{76}{20}\right) = +52.13 \text{ rad/s} \end{aligned}$$

$\therefore \omega_{51} = +52.13 \text{ rad/s}$ and direction of rotation same as that of ω_{21}

As can be seen, this method of solution is somewhat shorter than that used in the solution of Example 7.2.

7.3 APPLICATIONS OF PLANETARY GEAR TRAINS

Planetary gear trains find many applications in machine tools, hoists, aircraft propeller reduction drives, automobile differentials, automatic transmissions, aircraft servo-drives, and many others. Figure 7.10 shows a diagrammatic sketch of a planetary train used as a reduction between the engine and the propeller in an

**FIGURE 7.10**

aircraft power plant. Figure 7.11 shows a photograph of an actual unit. The earlier aircraft engine reduction drives used bevel gears in the planetary train. These were discarded, however, in favor of spur gears because the spur gear planetary drive could transmit more power in a given space.

In Fig. 7.10, the engine drives the internal gear 3. Gear 2 meshes with the fixed gear 1 and with the gear 3 so that it has planetary motion. Arm 4, or planet

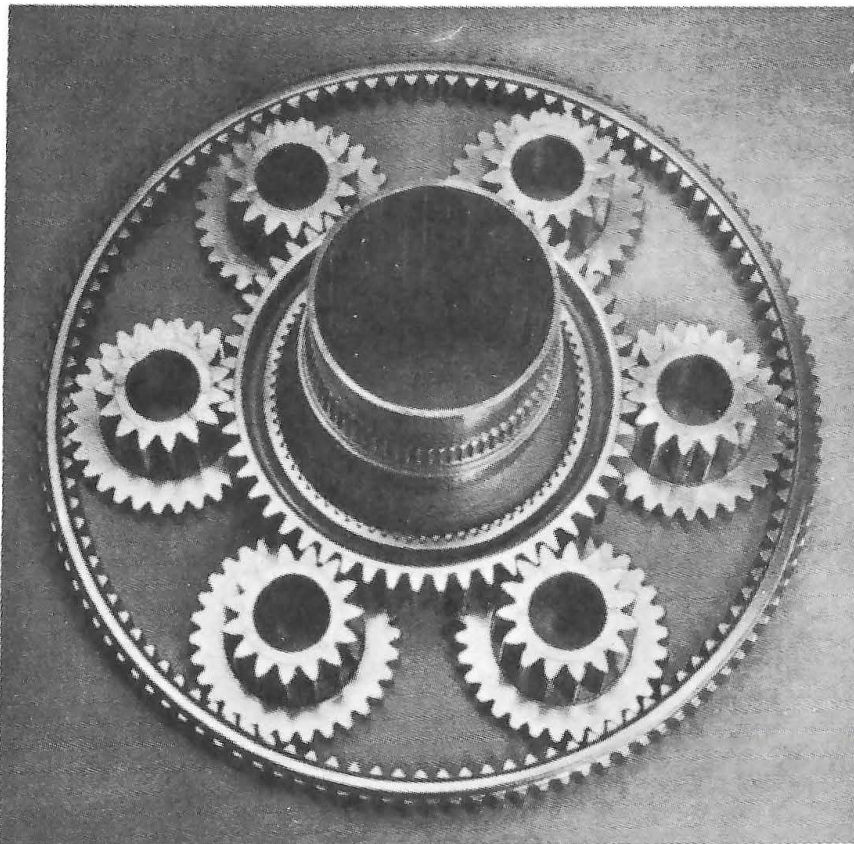


FIGURE 7.11 Planetary reduction unit for aircraft propeller drive.
(Courtesy of Foote Brothers Gear & Machinery Corp.)