

Unbalanced Voltage Control of 3-Phase Loads by the Triggering of Silicon Controlled Rectifiers

WILLIAM SHEPHERD, MEMBER, IEEE, AND JACK STANWAY

Abstract—It is shown that smooth nonreversible control of the unbalanced voltages applied to a 3-phase load may be obtained by using one or two back-to-back SCR pairs. The magnitude of the fundamental components of the nonsinusoidal load voltages can be accurately calculated by Fourier series. Two connections are given which yield smooth reversible voltages, suitable for 3-phase motor control.

INTRODUCTION

IN A PREVIOUS PAPER [1], the response of single-phase circuits controlled by firing-angle adjustment of the back-to-back silicon controlled rectifier (SCR) pair [Fig. 1(a)] was discussed. It has also been shown that by using three SCR pairs switched symmetrically, i.e., identical points of their respective anode voltage cycles, balanced voltage control of 3-phase loads may be obtained. A 3-phase induction motor can be controlled, in this manner, in order to give a range of torque-speed characteristics suitable for fan control [2].

The present investigation begins with a consideration of the type and degree of control that may be exercised over a balanced passive 3-phase load by one or two SCR pairs. In general, the fundamental frequency load voltage excursions that result from adjustable triggering, are similar to the loci that can be obtained by adjusting the direct control current of series connected saturable reactors [3]–[5]. Later in the paper, more complex arrangements suitable for reversible 3-phase motor control are considered.

DELTA CONNECTION WITH SCR PAIRS IN SERIES WITH THE LOAD IMPEDANCES

Consider the connection shown in Fig. 2 with a resistive load. At a firing angle θ of the anode (phase) voltage waveform, the voltage drop across each SCR pair has the form shown by the unshaded part of Fig. 1(b). Across each load impedance Z_L of Fig. 2 the voltage waveshape is shown by the shaded portion of Fig. 1(b). The proportion of fundamental in the load voltage waveform, as the firing angle varies, is given in Fig. 3 which is reproduced, for convenience, from Shepherd and Stanway [1]. It is found that the load current waveform in the circuit of Fig. 2 depends on the firing angle and on the sinusoidal fundamental frequency phase angle of the load impedance [1]. With a resistive load, the currents and voltages both have the form shown in Fig. 1(b).

When the three SCR pairs in Fig. 2 are triggered by sets of signals that are suitably disposed at 120-degree

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The authors are with the Royal Military College of Science, Swindon, Wiltshire, England.

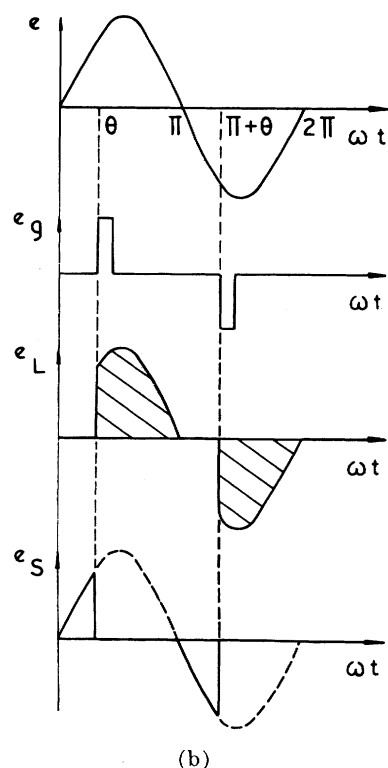
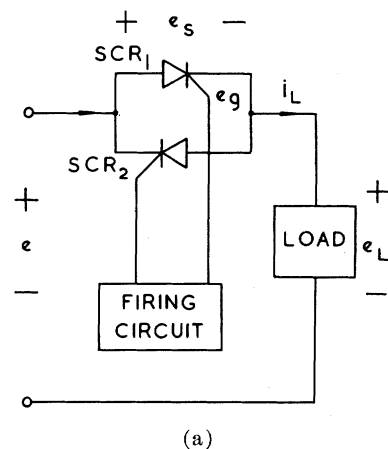


Fig. 1. (a) Back-to-back SCR pair. (b) Voltage waveforms of SCR pair and resistive load.

electrical intervals, balanced control of the load voltages is obtained. Independent triggering of the three SCR pairs results in unbalanced load voltages that are smoothly adjustable over a wide range. Note that, from the viewpoint of motor control, the phase sequence of the load voltages cannot be reversed by triggering of the SCRs.

Symmetrical Components of the Load Voltages

For a phase sequence ABC , the positive, negative, and zero sequence symmetrical components V_1 , V_2 , and V_0 ,

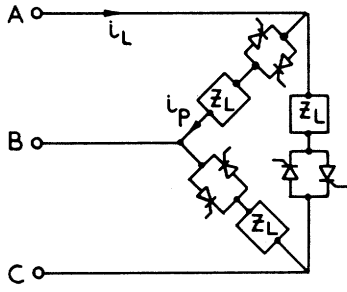


Fig. 2. Delta-connected load with series SCR pair in each phase.

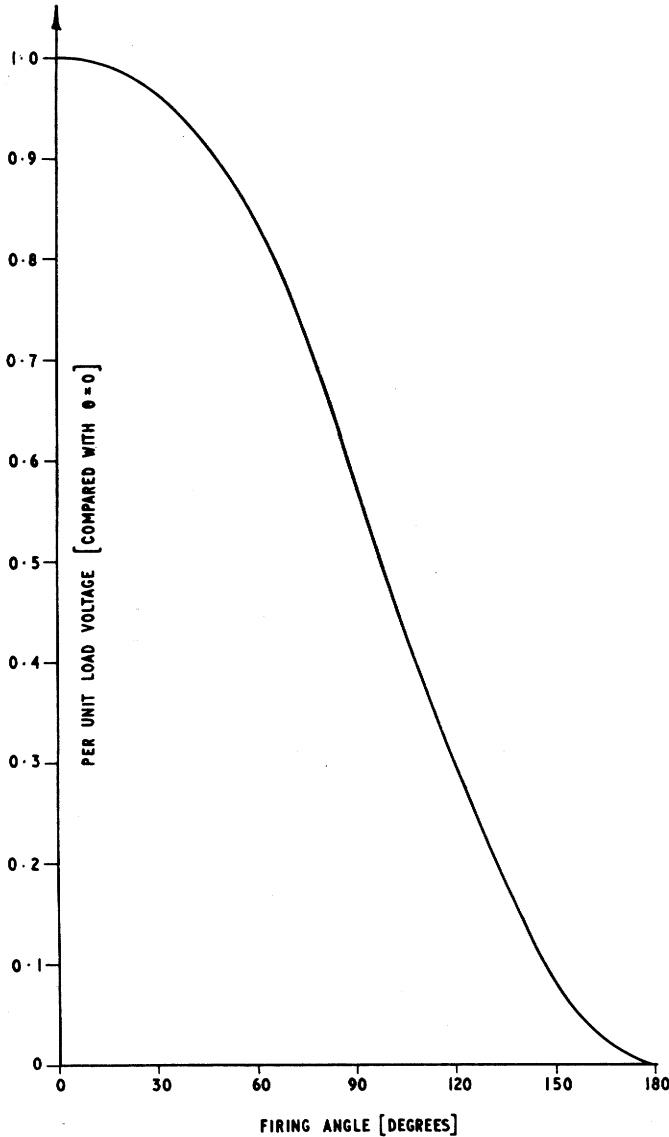


Fig. 3. Fundamental frequency load voltage of circuit of Fig. 2 as firing angle varies; resistive load.

respectively, as referred to phase AB , of the load voltages in Fig. 2, with zero firing angles, are given by

$$\begin{aligned} V_1 &= \frac{1}{3} (V_{AB} + aV_{BC} + a^2V_{CA}) \\ V_2 &= \frac{1}{3} (V_{AB} + a^2V_{BC} + aV_{CA}) \\ V_0 &= \frac{1}{3} (V_{AB} + V_{BC} + V_{CA}) \end{aligned} \quad (1)$$

where a is the vector operator $1/120^\circ$.

By control of the SCR pairs in the three load legs, the three load voltages can be adjusted independently in magnitude between zero and the line-to-line supply voltage. The SCR impedances are negligibly small compared with most load impedances, so that the load voltages may be considered to coincide in time phase with the respective supply voltages.

Now let $V(\alpha)$, $V(\beta)$, and $V(\gamma)$ be the magnitudes of the load voltages in phases AB , BC , and CA , when the firing angles in these phases are α , β and γ , respectively. Since $V(\alpha)$ is in time phase with V_{AB} , $V(\beta)$ with V_{BC} , and $V(\gamma)$ with V_{CA} , (1) becomes

$$\begin{aligned} V_1 &= \frac{1}{3} [V(\alpha) \sin \omega t + aV(\beta) \sin (\omega t - 120) + a^2V(\gamma) \sin (\omega t + 120)] \\ V_2 &= \frac{1}{3} [V(\alpha) \sin \omega t + a^2V(\beta) \sin (\omega t - 120) + aV(\gamma) \sin (\omega t + 120)] \\ V_0 &= \frac{1}{3} [V(\alpha) \sin \omega t + V(\beta) \sin (\omega t - 120) + V(\gamma) \sin (\omega t + 120)]. \end{aligned} \quad (2)$$

In terms of the operator a , (2) becomes

$$\begin{aligned} V_1 &= \frac{1}{3} [V(\alpha) + V(\beta) + V(\gamma)] \\ V_2 &= \frac{1}{3} [V(\alpha) + aV(\beta) + a^2V(\gamma)] \\ V_0 &= \frac{1}{3} [V(\alpha) + a^2V(\beta) + aV(\gamma)]. \end{aligned} \quad (3)$$

Suppose that it is desired now to create a set of unbalanced load voltages such that the magnitude of the negative sequence voltage is some desired fraction k of the magnitude of the positive sequence voltage.

Let

$$|V_2| = k|V_1|. \quad (4)$$

It is found, from (3) and (4), that, for the general case when $V(\alpha) \neq V(\beta) \neq V(\gamma)$,

$$k = \frac{|V_2|}{|V_1|} = \frac{\sqrt{\{V(\alpha) - [V(\beta) + V(\gamma)]/2\}^2 + \{\sqrt{3}/2[V(\beta) - V(\gamma)]\}^2}}{V(\alpha) + V(\beta) + V(\gamma)}. \quad (5)$$

There is clearly an infinite number of combinations of the three variables $V(\alpha)$, $V(\beta)$ and $V(\gamma)$, that will satisfy (5) for any fixed value of k .

Equal Firing of Two SCR Pairs

If the SCR pair in phase AB (Fig. 2) is controlled at any arbitrary firing angle, and the SCR pairs in phases

BC and CA (Fig. 2) are fired at the same angle, with respect to their anode voltages, then $V(\beta) = V(\gamma)$ in (5) which reduces to

$$k = \frac{V(\alpha) - V(\beta)}{V(\alpha) + 2V(\beta)} = \frac{V(\alpha) - V(\gamma)}{V(\alpha) + 2V(\gamma)} \quad (6)$$

The positive sequence voltage magnitude $|V_1|$ and negative sequence voltage magnitude $|V_2|$ that can be obtained with any given value of the adjustable angles $\beta = \gamma$, depend on the magnitude of the fixed firing angle α , as shown in Fig. 4.

An alternative representation, showing $|V_1|$ vs $|V_2|$ is shown in Fig. 5. When $\alpha = 0$ degrees (i.e., when the SCRs in branch AB (Fig. 2) are short-circuited) and when $\alpha = 180$ degrees (i.e., when branch AB in Fig. 2 is open circuited), the symmetrical component loci are found to be linear. The intermediate values $0 \text{ degrees} < \alpha < 180$ degrees are found to give curved loci in the $|V_1|$ vs $|V_2|$ plane.

An analytical expression for the load-voltage-firing-angle characteristic (Fig. 3) was given in Shepherd and Stanway [1]. It is possible, therefore, to express (5) and (6) in terms of the angles α , β , and γ , rather than in terms of the voltages. Since such expressions would become quite unwieldy, the use of (5) or (6), together with Fig. 3, is probably preferable. The method is equally applicable to reactive loads, except that the fundamental voltage-firing-angle, characteristic for the particular load power factor should be used in place of Fig. 3.

Induction Motor Torque-Speed Curves with Asymmetrical SCR Firing

Consider a delta-connected 3-phase induction motor with SCR pairs in series with two stator (primary) phases. The third phase is connected permanently across the supply, as in Fig. 6, so that its voltage and current are continuously sinusoidal.

When the two SCR pairs are triggered symmetrically, the nonsinusoidal voltage waveforms of the two controlled phases are identical to each other. The corresponding current waveforms of the two controlled phases are also identical to each other but differ from the waveform of the voltages. With a given firing angle, both the voltage and current waveshapes depend on the effective load power factor which changes with motor speed [2]. The measured torque-speed characteristics of a 3-hp motor, obtained with symmetrical firing of two SCR pairs, are given in Fig. 6. These torque-speed characteristics indicate that this method is not suitable for speed control purposes.

When only one SCR pair is used in the stator of a delta-connected induction motor, the machine fluxes are sinusoidal in two of the three phases. It may be shown that the fundamental load voltages would then have symmetrical components such as to describe a straight line between the points P and Q in Fig. 5, as the trigger signal varied. The resulting measured torque-speed characteristics for a complete firing-angle excursion from 0 to 135 degrees are shown in Fig. 7.

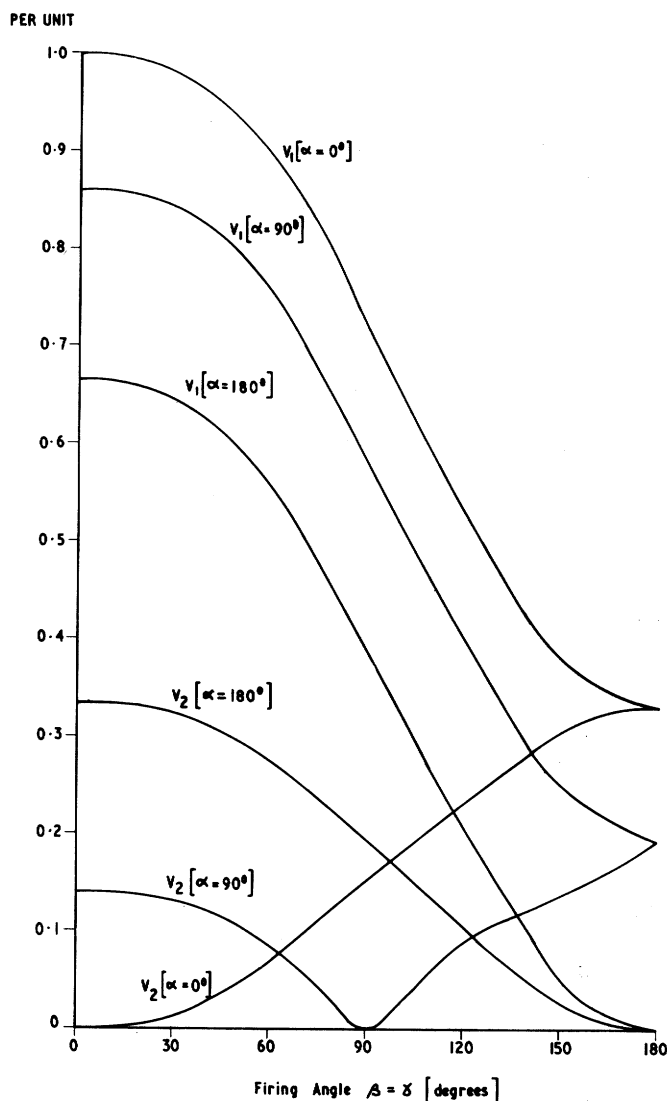


Fig. 4. Positive and negative sequence symmetrical components of fundamental load voltages for resistive circuit of Fig. 2, as firing angle varies.

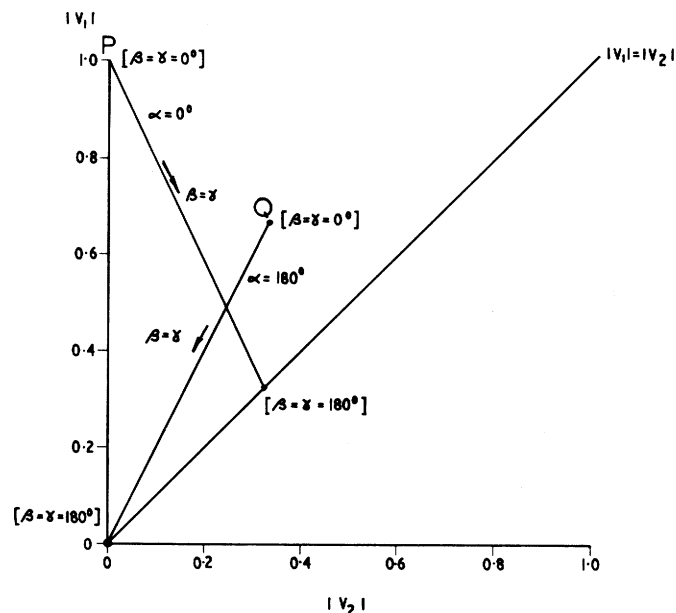


Fig. 5. Positive and negative sequence symmetrical components loci of fundamental load voltages for circuit of Fig. 2; resistive load.

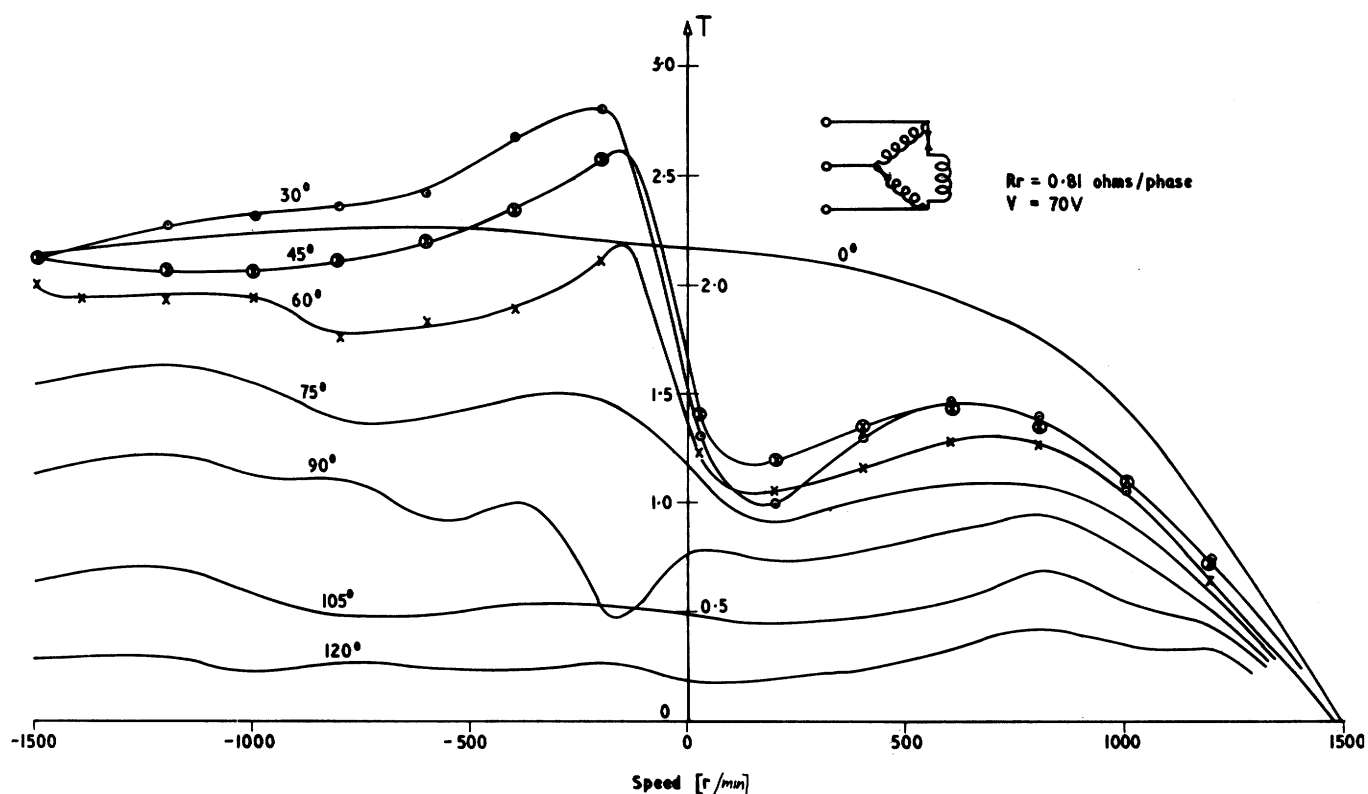


Fig. 6. Measured torque-speed curves with SCR pairs in two stator phases: $R_r = 0.81 \text{ ohm per phase}$, $V = 70 \text{ volts}$, $\alpha = 0 \text{ degrees}$, $\beta = \gamma = \text{variable}$.

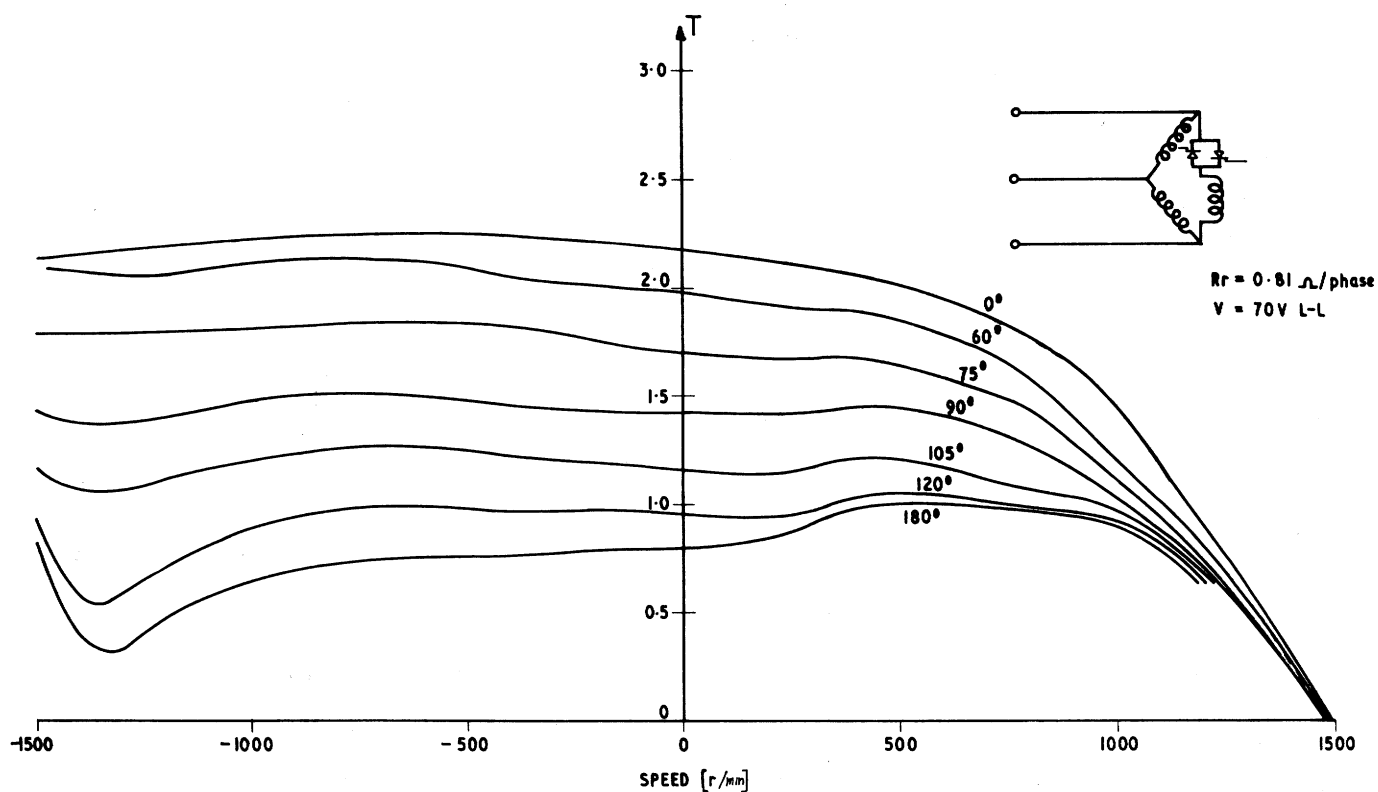


Fig. 7. Measured torque-speed curves with SCR pair in one stator phase: $R_r = 0.81 \text{ ohm per phase}$, $V = 70 \text{ volts}$, $\beta = \gamma = 0 \text{ degrees}$, $\alpha = \text{variable}$.

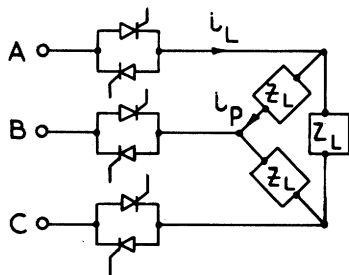


Fig. 8. Delta-connected load with SCR pairs in lines.

DELTA CONNECTION WITH SCR PAIRS IN THE LINES

Balanced voltage control of a 3-phase load may be obtained with SCR pairs in each of the three supply lines, as shown in Fig. 8. The 3-load voltage waveforms, for a resistive load, are then identical to each other but are different from the waveform of Fig. 1(b) [1].

Operation with an SCR Pair in One Supply Line of a Delta-Connected Load

Measured Performance: When the SCR pairs in lines A and C of Fig. 8 fire continuously at zero degrees firing angle, or are replaced by short circuits, the potentials of the load terminals *a* and *c* always coincide with the potentials of the line terminals A and C, respectively. The potential of the load terminal *b* (Fig. 8), varies with respect to the triggering angle of the SCR pair in line Bb. At a triggering angle of zero degrees, terminal *b* coincides in potential with the supply terminal B. With a triggering angle of 180 degrees, terminal *b* is electrically isolated from terminal B and assumes a potential corresponding to the midpoint of the vector potential AC.

The measured fundamental components of the excursions of the potential of terminal *b* are given in Fig. 9 for a resistive and for a highly inductive load. It is known from previous experience, that, if the SCR pair in line Bb (Fig. 9), was replaced by an ideal saturable reactor (i.e., a lossless variable inductor) the theoretical fundamental locus for a resistive load would be the semicircle on the diameter DB. Similarly, with a purely inductive load, the theoretical locus would be the straight line constituting the diameter DB.

Each of the points on the measured loci of Fig. 9 were obtained by three separate measurements with a commercial wave analyser. Points A, C, and N were chosen as bases, since these retained constant displacement from each other. The three fundamental voltages V_{Ab} , V_{Cb} , and V_{Nb} , are shown in Fig. 9 for an arbitrarily selected firing angle, with resistive load.

The possible usefulness of an unbalanced 3-phase controller, such as that shown in Fig. 9, for induction motor speed control can be considered in terms of the obtainable range of symmetrical component load voltages. When the positive sequence and negative sequence symmetrical components of the fundamental load voltages V_{Ab} , V_{bC} , and V_{CA} , are calculated for the resistive load excursion of Fig. 9, the characteristic (c) shown in Fig. 10 is produced. At the triggering condition $\beta = 180$

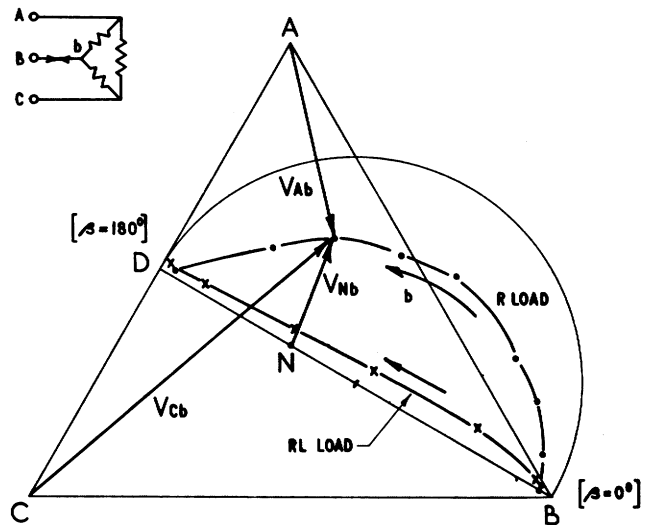


Fig. 9. Measured fundamental load voltages for delta connection with SCR pair in one line.

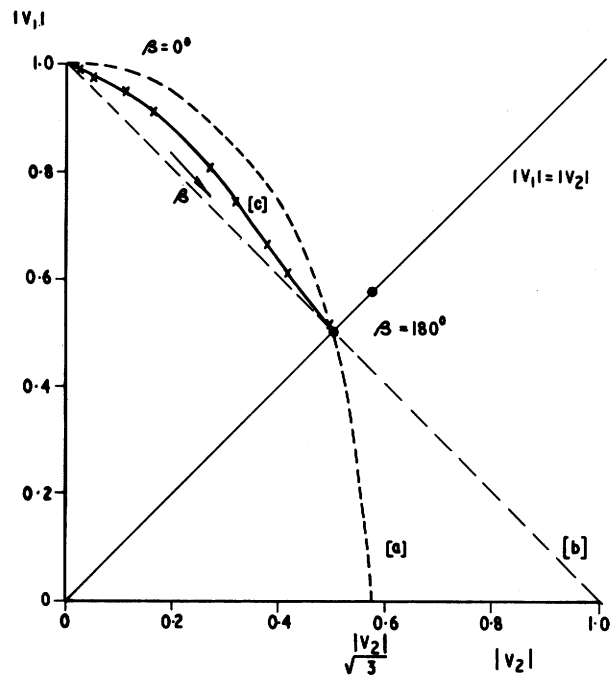


Fig. 10. Positive and negative sequence symmetrical components loci of fundamental load voltages for delta load with series controller in one line, where (a) represents ideal saturable reactor, resistive load; (b), ideal saturable reactor, inductive load; (c) SCR pair, resistive load.

degrees, the potential of load terminal *b* becomes such that $V_{Ab} = V_{bC}$ and the positive sequence and negative sequence voltages are equal. Also shown in Fig. 10 are the theoretical elliptical locus (a), corresponding to the semicircle of Fig. 9, and the theoretical linear locus (b), corresponding to the diameter DB of Fig. 9.

Calculated Performance: The load voltage loci of any asymmetrically controlled 3-phase load may be calculated by performing a Fourier analysis on the three load voltage waveforms.

Consider a resistive delta-connected load with an SCR pair in the supply line Aa. Figure 11 shows the three theoretical load voltage waveforms that would be obtained

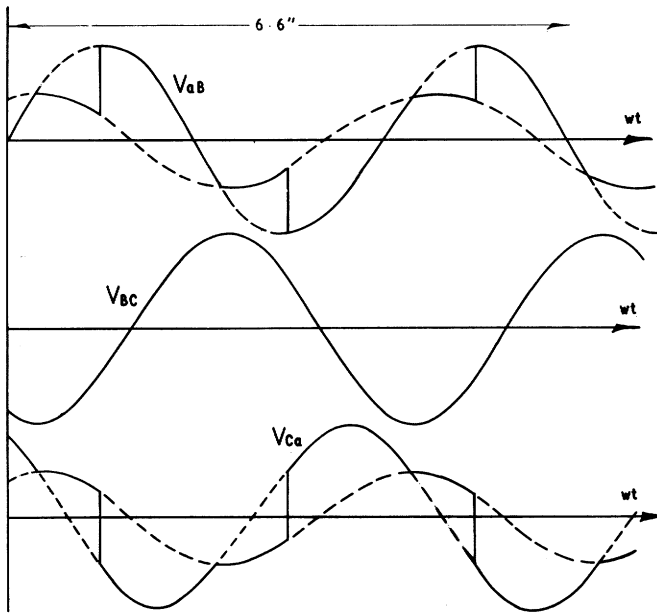


Fig. 11. Load voltage waveforms of delta-connected resistive load with SCR pair in line A; $\alpha = 60$ degrees.

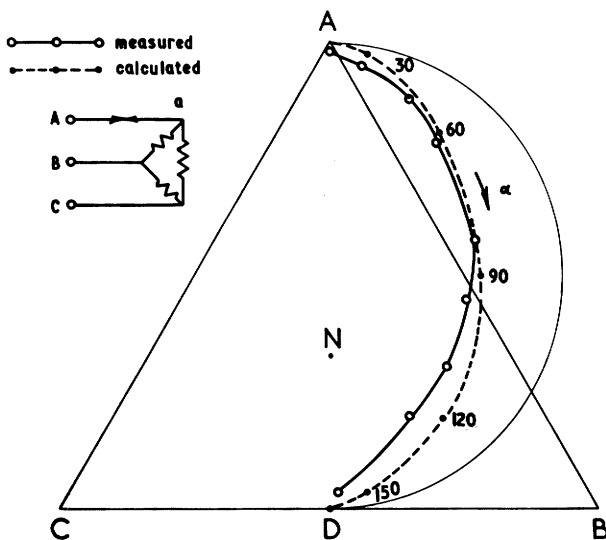


Fig. 12. Fundamental load voltage loci for delta-connected resistive load with SCR pair in line A.

with ideal SCRs. The observed waveforms were extremely close to the ideal, differing only in slight rounding of the discontinuities. Equations (12)–(14) of Appendix I, define the waveforms of Fig. 11. The Fourier coefficients of these waveforms, for fundamental frequency, are calculated in Appendix I and given in (20)–(24).

It is seen in Fig. 12, that the measured and theoretical load voltage loci show good agreement. Some slight discrepancy is due to the small voltage drop across the SCRs during conduction, which was neglected in the calculation.

Operation with SCR Pairs in Two Lines

In the circuit of Fig. 8, let the SCR in line A now be replaced by a short circuit. When the SCR pairs in lines B and C are fired symmetrically, i.e., when the firing angles

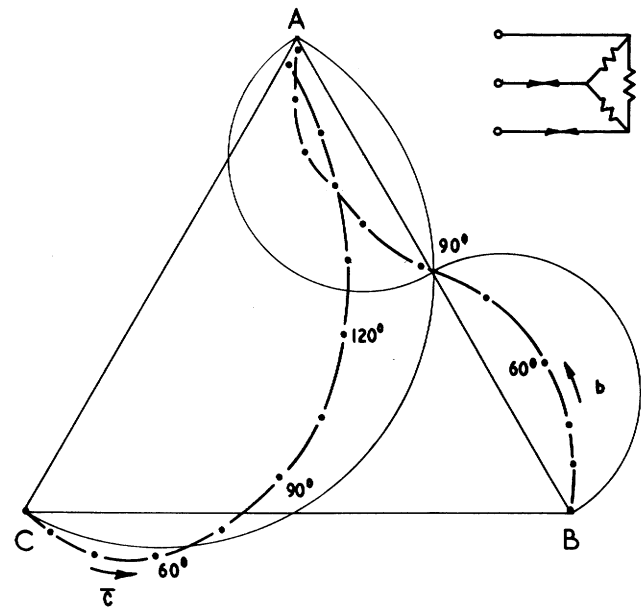


Fig. 13. Fundamental load voltage loci for delta-connected resistive load with symmetrically fired SCR pairs in lines B and C.

are equal with reference to the respective anode voltages, then the fundamental load voltages loci for a resistive load would be the ones given in Fig. 13.

A comparison of the load voltage locus for one SCR pair (Fig. 12) with the loci for two symmetrically fired pairs (Fig. 13) indicates the wide range of unbalanced voltage performance that can be obtained by using two asymmetrically fired pairs. Further investigation might be made, for instance, of the possibility of one SCR pair in a line together with an SCR pair in one phase.

REVERSIBLE VOLTAGE CONTROL USING FOUR SCR PAIRS

A 3-phase load may be smoothly controlled from balanced positive sequence voltages (e.g., sequence ABC) to balanced negative sequence voltages (e.g., sequence BAC) by the use of four SCR pairs as shown in Fig. 14. It is convenient to trigger SCRs Aa and Ab with respect to voltage V_{AN} , and to trigger SCRs Ba and Bb with respect to voltage V_{BN} .

There are an infinite number of possible triggering sequences that may be used, in the circuit of Fig. 14, in order to reverse the phase sequence of the load voltages. A rather conservative sequence is indicated by the five diagrams of Fig. 15. It would be an advantage in motor control to have overlap between the firing of different SCR pairs. In Fig. 14, for instance, it may be desirable to start conduction in branch Ba before conduction ceases in branch Aa. The maximum permissible degree of overlap would depend on the SCR current ratings.

The measured excursions of the fundamental load voltages, with a resistive load, are shown in Fig. 16(a), where the numbered subscripts refer to the various diagrams of Fig. 15. Condition 3 of Fig. 15, for instance, is represented in Fig. 16(a) by the notations a_3 , b_3 , and c_3 , which demonstrated that potential a_3 then coincides with potential B, potential b_3 coincides with potential B.

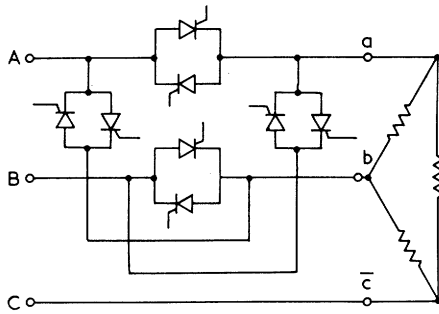


Fig. 14. Reversible voltage control of 3-phase load by use of four SCR pairs.

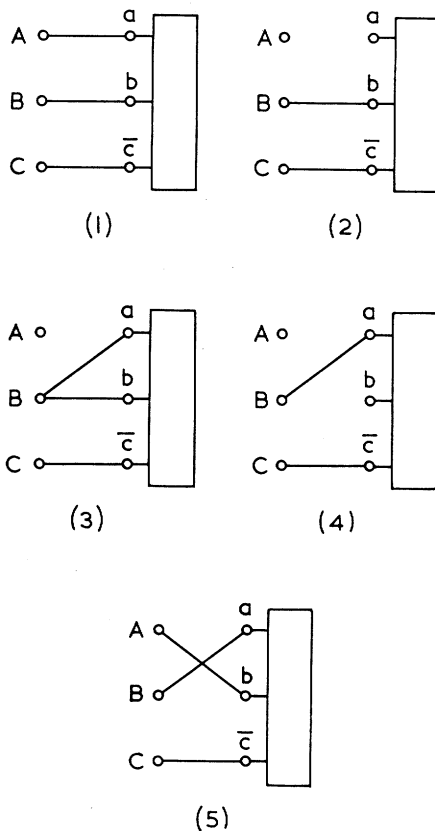


Fig. 15. Possible control sequence for reversing voltages of 3-phase load.

and potential $\bar{e}_3$ coincides with potential C . The symmetrical components of the fundamental load voltages perform the excursions shown in Fig. 16(b), proceeding smoothly from the positive sequence condition 1 of Fig. 15 to the negative sequence condition 5 of Fig. 15. A switching arrangement that would cause overlap of the SCR firings would change the shape of this characteristic without changing the end conditions 1 and 5. With inductive or partially inductive loads, such as the induction motor, the "bow" shaped characteristic of Fig. 16(b) would tend to be flattened into a more linear shape.

An arrangement of four series-connected saturable reactors, using the connection of Fig. 14, has been used for induction motor speed control [6]. It is suggested that

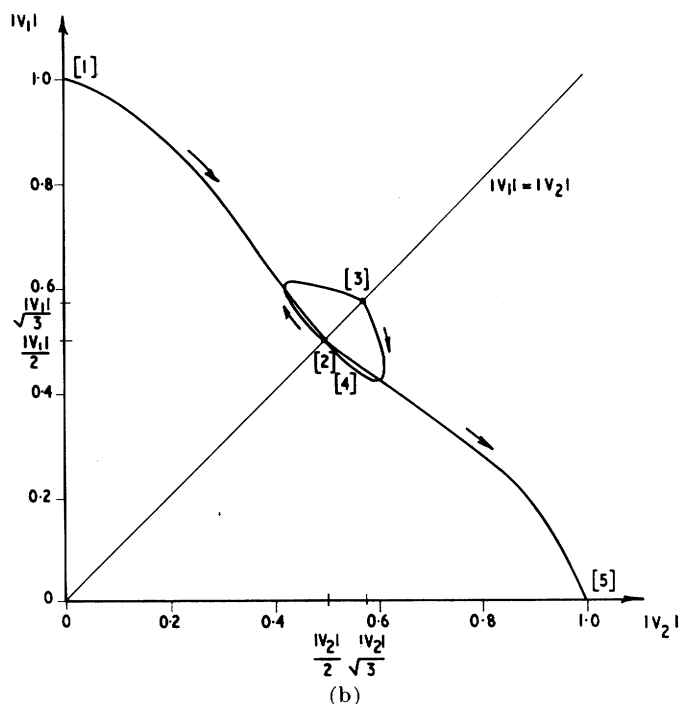
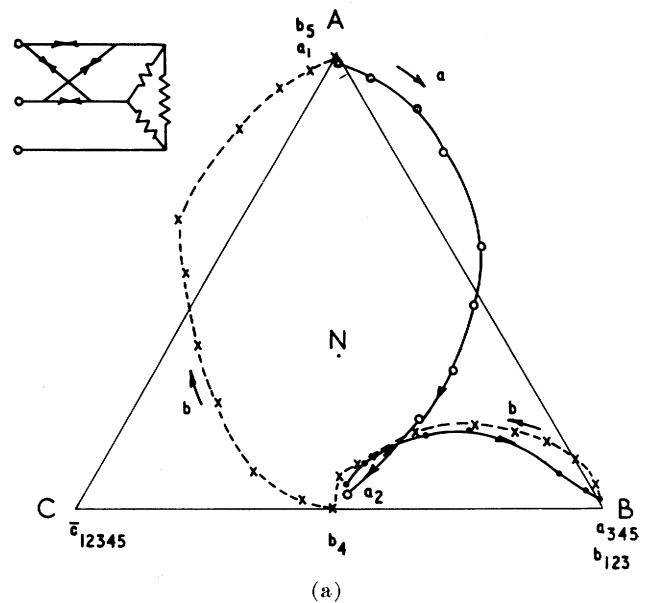


Fig. 16. Measured loci for resistive load with control sequence seen in Fig. 15. (a) Fundamental load voltages. (b) Symmetrical components of Fig. 16(a).

the present arrangement of four SCR pairs gives smooth, reversible control over the same range, with the advantage of less bulky and less dissipative control gear.

REVERSIBLE VOLTAGE CONTROL BY THE USE OF TWO SCR PAIRS AND A TRANSFORMER

The voltages applied to a star-connected 3-phase load can be smoothly and reversibly controlled by use of the arrangement found in Fig. 17. It is seen that the voltage applied to phase aN is constant at all times, and that the load currents I_b and I_c are equal and opposite at all times.

This control scheme cannot be used satisfactorily with delta-connected loads because there would then be no return path for the current in line Aa .

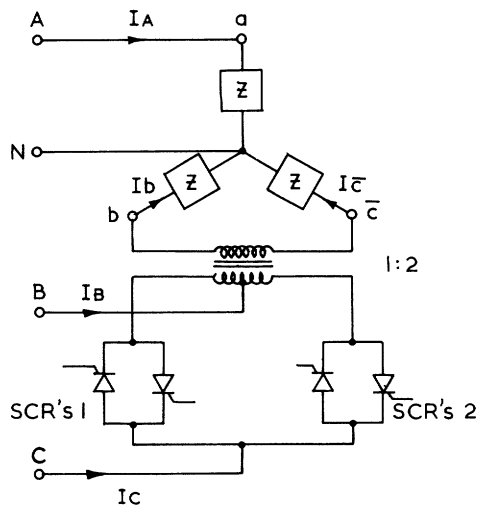


Fig. 17. Transformer-SCR arrangement for reversible control of 3-phase load.

Assume now that the load in Fig. 17 consists of three identical self-impedances Z , each of which is magnetically coupled to the other two by identical mutual impedances Z_m . This is the sort of load that would be presented by an induction motor or by a transformer with balanced load. With SCRs 2 conducting and SCRs 1 nonconducting, it is shown (Appendix II) that the load voltages are given by the expressions

$$V_{bN} = \frac{V_{BC}}{2} + V_{AN} \frac{Z_m}{Z} \quad (7)$$

and

$$V_{cN} = \frac{V_{CB}}{2} + V_{AN} \frac{Z_m}{Z}.$$

Operation with Noncoupled Load Impedances

When the load in Fig. 17 consists of three identical impedances without intermagnetic coupling, and SCRs 2 are fully conducting and SCRs 1 are nonconducting, the load voltages are seen to have the form

$$V_{bN} = \frac{V_{BC}}{2}$$

and

$$V_{cN} = \frac{V_{CB}}{2}. \quad (8)$$

When the SCR firing is such that SCRs 1 are fully conducting and SCRs 2 are nonconducting, the load voltages are then

$$V_{bN} = \frac{V_{CB}}{2}$$

and

$$V_{cN} = \frac{V_{BC}}{2}. \quad (9)$$

At the mid-position, when none of the SCRs are conducting, the load voltages disappear. The load voltage vectors

for the extreme firing conditions and the mid-condition are shown in Fig. 18. Measured loci of the fundamental load voltages, with a resistive load, for a complete firing excursion are shown in Fig. 19. Slight asymmetry of the two halves of the transformer primary winding is responsible for the displacement between the two loci.

The symmetrical components of the load voltage excursion of Fig. 19 are shown in Fig. 20. It is found that the excursion proceeds smoothly from point P , where

$$|V_1| = 0.83, \quad |V_2| = 0.166$$

to point Q , where

$$|V_1| = 0.166, \quad |V_2| = 0.834.$$

Operation with an Induction Motor Load

The balanced star-connected impedances of Fig. 17 are now replaced by the stator (primary) winding of a 3-phase induction motor. The mutual impedances Z_m depend only on the magnetic geometry and are equal and constant, if saturation is negligible. On the other hand, self-impedances Z per phase depend on the effective rotor circuit impedance which, in turn, varies with speed.

At no load, the self-impedance per phase is equal to the magnetizing impedance, if leakage flux is neglected. Also, the magnetizing flux produced by any winding divides equally between the other two windings, since the magnetic circuit is symmetrical with respect to all three phases. At no load, therefore, the self-impedance per phase is twice the mutual impedance;

$$Z = 2Z_m. \quad (10)$$

Combining (7) and (10) gives

$$V_{bN} = \frac{V_{BC}}{2} + \frac{V_{AN}}{2} = V_{BN} \quad (11)$$

$$V_{cN} = \frac{V_{CB}}{2} + \frac{V_{AN}}{2} = V_{CN}.$$

The important conclusion from (11) is that, when the control scheme of Fig. 17 is applied to an induction motor primary circuit, then, ideally, at no load, the motor voltages constitute a balanced 3-phase set equal to the applied voltages. This was confirmed experimentally. A complete firing-angle excursion, at no load, is found to give, very nearly, the reversing motor voltages shown in Fig. 21.

Under load, the relationship between Z and Z_m varies continuously with speed. At rated load, on a 3-hp motor, the load voltages were found to lie about midway between the balanced 3-phase condition found in Fig. 21(a) and the condition found in Fig. 18(a). A set of measured torque-speed characteristics is given in Fig. 22. The effect of the voltage drop across the transformer leakage reactance and the conducting SCRs, is demonstrated by the difference between the torque curves for "direct on-line" operation and SCR controlled operation at a firing angle of zero degrees. The induction motor was found to reverse

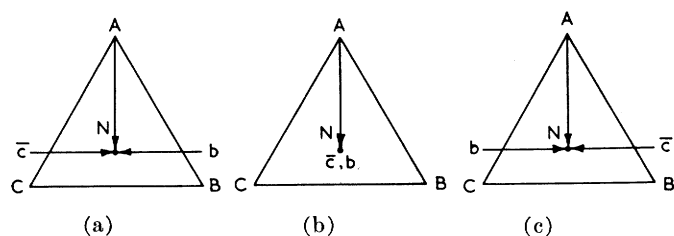


Fig. 18. Load voltage vector diagrams for transformer-SCR controller with noncoupled load impedances. (a) SCRs 1 at 180 degrees; SCRs 2 at 0 degrees. (b) SCRs 1 at 180 degrees; SCRs 2 at 180 degrees. (c) SCRs 1 at 0 degrees; SCRs 2 at 180 degrees.

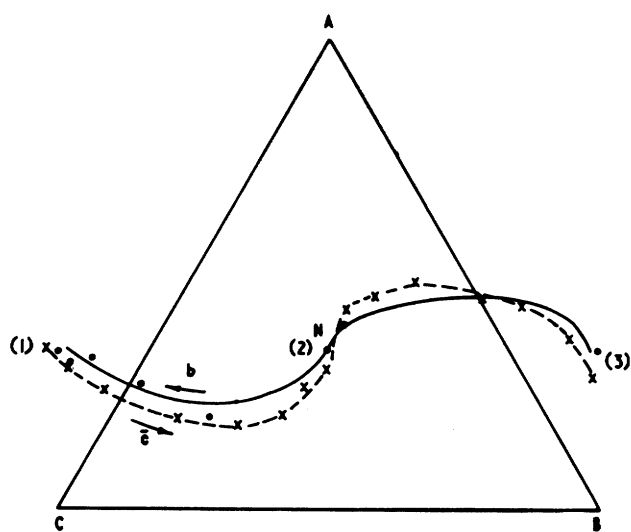


Fig. 19. Measured voltage loci for transformer-SCR controller with resistive load where (1) represents SCRs 1 at 180 degrees and SCRs 2 at 0 degrees; (2) SCRs 1 at 180 degrees and SCRs 2 at 180 degrees; and (3), SCRs 1 at 0 degrees and SCRs 2 at 180 degrees.

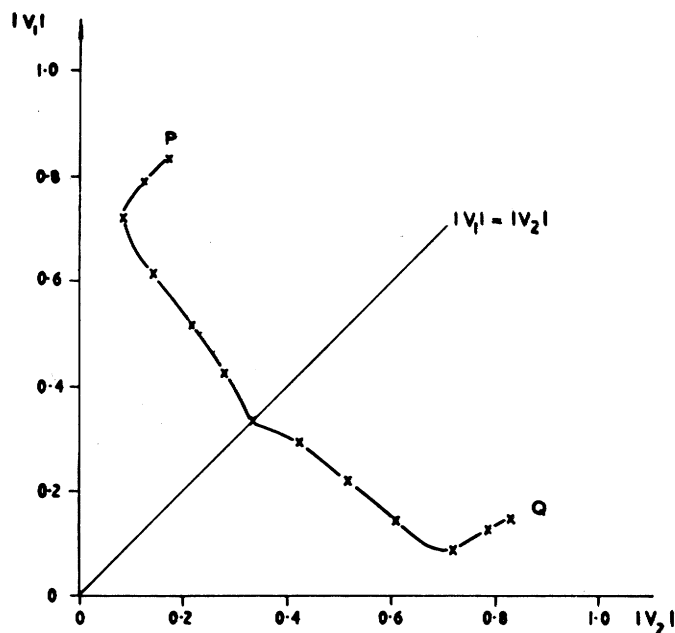


Fig. 20. Positive and negative symmetrical components of load voltages of Fig. 19.

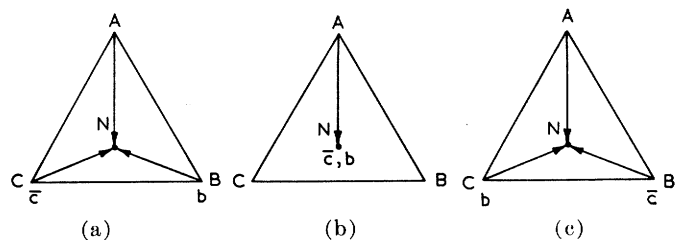


Fig. 21. Load (stator) voltages vector diagrams for transformer-SCR controller connected to unloaded induction motor.

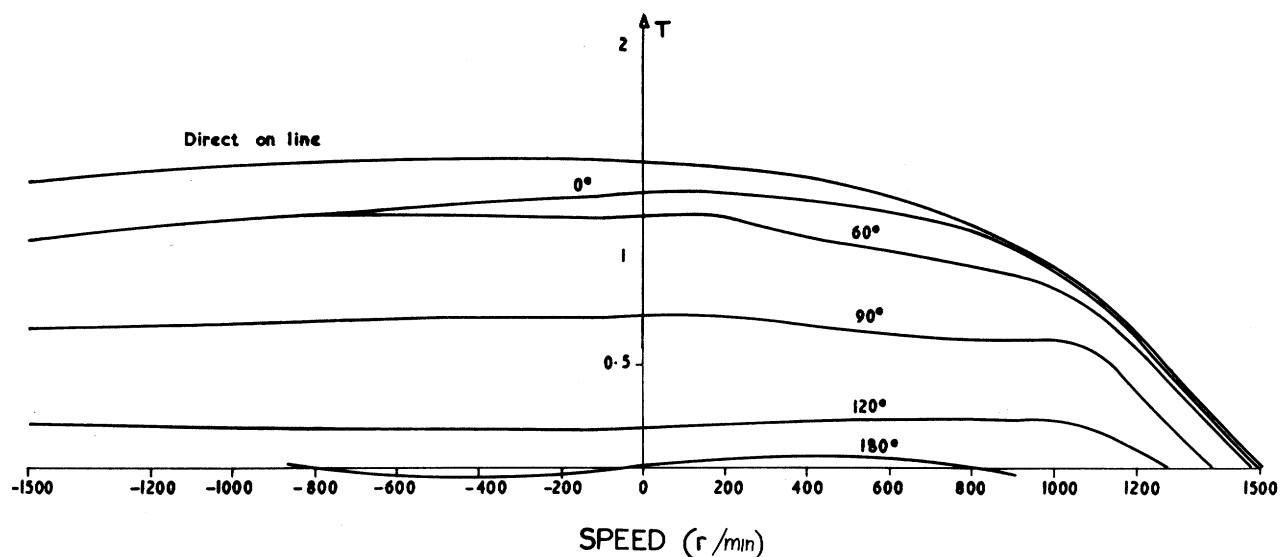


Fig. 22. Measured torque-speed characteristics of star-connected induction motor controlled by transformer-SCR controller: SCRs 1 at 180 degrees, SCRs 2 variable, $V = 100$ volts, and $R_r = 0.81$ ohm per phase.

smoothly in response to SCR trigger control. If the trigger sequence is chosen such that there is no overlap between the firing of the two SCR pairs, the transformer is required to have a rating $1/\sqrt{3}$ times that of the induction motor.

CONCLUSION

Nonreversible control of the load voltages of a balanced delta-connected load was obtained by the use of one or two back-to-back connected SCR pairs. The fundamental components of the nonsinusoidal unbalanced load voltages can be accurately calculated by Fourier series.

Reversible voltages, suitable for 3-phase motor speed control, can be obtained by the use of four SCR pairs with a delta-connected motor or by the use of two SCR pairs and a transformer with a star-connected motor. Experiments showed that both control methods provide motor voltages capable of smooth adjustment from the balanced positive sequence condition to the balanced negative sequence condition.

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APPENDIX I

FOURIER ANALYSIS OF THE LOAD VOLTAGE WAVEFORMS PRODUCED BY ONE SCR PAIR IN THE LINE OF A DELTA-CONNECTED LOAD

The waveforms of Fig. 11 are defined by (12)-(14) when the origin is chosen as the point where V_{AB} passes from negative to positive.

$$V_{AB} = \sin \omega t \left| \begin{array}{l} 30, 210, 360 \\ 0, 30+\theta, 210+\theta \end{array} \right| + \frac{1}{2} \sin (\omega t + 60) \left| \begin{array}{l} 30+\theta, 210+\theta \\ 30, 210 \end{array} \right| \quad (12)$$

$$V_{BC} = \sin (\omega t - 120) \quad (13)$$

$$V_{CA} = \sin (\omega t + 120) \left| \begin{array}{l} 30, 210, 260 \\ 0, 30+\theta, 210+\theta \end{array} \right| + \frac{1}{2} \sin (\omega t + 60) \left| \begin{array}{l} 30+\theta, 210+\theta \\ 30, 210 \end{array} \right| \quad (14)$$

where θ is the firing angle of the SCR pair with reference to its anode voltage V_{AN} .

Now the Fourier series for any function $e(\omega t)$ may be written

$$e(\omega t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t \quad (15)$$

where

$$\frac{a_0}{2} = \frac{1}{2\pi} \int_0^{2\pi} e(\omega t) d\omega t \quad (16)$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} e(\omega t) \cos n\omega t d\omega t \quad (17)$$

$$n = 0, 1, 2, \dots$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} e(\omega t) \sin n\omega t d\omega t \quad (18)$$

$$n = 1, 2, 3, \dots$$

$$c_n = \sqrt{a_n^2 + b_n^2}. \quad (19)$$

When (12)-(14) are substituted into (17) and (18), the following coefficients are obtained for the fundamental component:

For V_{AB} ,

$$a_1 = \frac{1}{8\pi} (3 \cos 2\theta + \sqrt{3}\theta - \sqrt{3} \sin 2\theta - 3) \quad (20)$$

$$b_1 = \frac{1}{4\pi} \left(\frac{\sqrt{3}}{2} \cos 2\theta + \frac{3}{2} \sin 2\theta - \frac{\sqrt{3}}{2} - 3\theta + 4\pi \right). \quad (21)$$

For V_{CA} ,

$$a_1 = \frac{1}{4\pi} \left[\frac{3}{2} + \sqrt{3} (2\pi - \theta) + \frac{\sqrt{3}}{2} \sin 2\theta - \frac{3}{2} \cos 2\theta \right] \quad (22)$$

$$b_1 = \frac{1}{4\pi} \left(3\theta - 2 + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cos 2\theta - \frac{3}{2} \sin 2\theta \right). \quad (23)$$

For V_{BC}

$$c_1 = 1. \quad (24)$$

The voltage locus of V_{AB} , V_{BC} , and V_{CA} , as θ varies, is given in Fig. 12.

APPENDIX II

EQUATIONS OF THE TRANSFORMER-SCR DRIVE

In the sinusoidal steady-state, the branch equations of the load in Fig. 17 are

$$\begin{aligned} V_{aN} &= I_a Z + I_b Z_m + I_c Z_m \\ V_{bN} &= I_a Z_m + I_b Z + I_c Z_m \\ V_{cN} &= I_a Z_m + I_b Z_m + I_c Z. \end{aligned} \quad (25)$$

It is always true that

$$I_b = -I_c. \quad (26)$$

Combining (25) and (26) yields

$$\begin{aligned} V_{aN} &= I_a Z \\ V_{bN} &= I_a Z_m + I_b(Z - Z_m) \\ V_{cN} &= I_a Z_m - I_b(Z - Z_m). \end{aligned} \quad (27)$$

Connection *B* in Fig. 17 is made to the center tap of the primary of a 2 to 1 control transformer. When SCR pair 1 is nonconducting (firing angle 180 degrees) and SCR pair 2 is fully conducting (firing angle 0 degrees), then, neglecting the voltage drop across the conducting SCRs and the transformer leakage reactance,

$$V_{BC} = V_{bN} + V_{cN}. \quad (28)$$

The corresponding expression for the other firing extreme, when SCR pair 1 is fully conducting and SCR pair 2 is nonconducting, is

$$V_{BC} = V_{cN} + V_{bN}. \quad (29)$$

Combining (27) and (28) and, separately, (28) and (29) gives the general expressions for the load voltages.

With voltages V_B and V_C applied to terminals *b* and $\bar{c}$, respectively,

$$V_{bN} = \frac{V_{BC}}{2} + V_{AN} \frac{Z_m}{Z} \quad (7)$$

$$V_{cN} = \frac{V_{CB}}{2} + V_{AN} \frac{Z_m}{Z}.$$

With voltages V_B and V_C applied to terminals $\bar{c}$ and *b*, respectively,

$$\begin{aligned} V_{bN} &= \frac{V_{CB}}{2} + V_{AN} \frac{Z_m}{Z} \\ V_{cN} &= \frac{V_{BC}}{2} + V_{AN} \frac{Z_m}{Z}. \end{aligned} \quad (30)$$

Reversing Thyristor Armature Dual Converter with Logic Crossover Control

DAVID L. DUFF AND ALLAN LUDBROOK, MEMBER, IEEE

Abstract—Several techniques are available for the design of reversible armature power supplies. The approach described in this paper results in a very high performance system and retains the high conversion efficiency of a single converter bridge. A unique firing circuit and regulator design allows the transfer characteristics of the two converter bridges to be coincident. Logic circuitry is used to select the operating bridge dependent on system requirements. Performance characteristics are discussed with particular reference to test results obtained from a 100-hp reversing screw-down supply.

INTRODUCTION

SOLID-STATE ac/dc power conversion equipment is rapidly replacing the motor-generator set as a source of power for dc motor drive systems. This paper describes

an ac/dc solid-state dual converter system with attractive regulator response characteristics which can be used as a "black box" to replace the motor-generator set. It can be specified simply by the systems designer in terms of very few parameters.

RATING PHILOSOPHY

In a thyristor power supply, special attention must be paid to the short-time overload rating. The limit of any rating involving forward blocking is set by the junction temperature. Since the thyristor is a very costly device, the designer must be sure that failure can never occur during any normal or abnormal duty cycle.

There are two solutions to this requirement:

- 1) To rate the thyristors to the specific duty cycle for application on the basis of junction temperature [1]. To protect adequately it would be necessary to supervise the junction temperature or its analog.
- 2) To rate the thyristors to handle maximum current

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The authors are with the Canadian Westinghouse Company, Ltd., Hamilton, Ont., Canada.