

Pareto optimal synthesis of four-bar mechanisms for path generation

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Abstract

In this paper, hybrid multi-objective genetic algorithms (GAs) are used for Pareto optimum synthesis of four-bar linkages considering the minimization of two objective functions simultaneously. The important conflicting objective functions that have been considered in this work are tracking error (TE) and transmission angle's deviation from 90° (TA). The obtained Pareto fronts demonstrate that trade-offs between these two objectives can be recognized so that a designer can optimally compromise for the selection of a desired four-bar linkage. Moreover, the obtained optimum points are also compared with those given in the literature in terms of both tracking error (TE) and transmission angle error (TA).

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1. Introduction

Mechanisms which compose some connected rigid members are exclusively used in the area of mechanical engineering for motion and energy transfer from one or more input members to one or more output members. The concepts involved within the design of mechanisms are analysis and synthesis. In the case of dimensional synthesis, there are three categories, namely, motion generation, function generation, and path or trajectory generation [1]. Both graphical and analytical methods have been used for dimensional synthesis [2–5]. However, such methods are relatively restrictive because of their low precision rates and that they cannot be used in the design of wide variety kinds of mechanisms, particularly when the number of precision points and the complexity of problems increase [1]. In order to circumvent these shortcomings, optimization methods have been increasingly used in recent years for the dimensional synthesis of mechanisms [6–8] especially for path or trajectory generation. In path generation problem, which is the problem of this paper, synthesis is accomplished by using some number of precision points to be traced by the coupler point of the mechanism. Thus, an optimization method is employed to find the set of dimensional or geometrical sizes

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of a mechanism, e.g. lengths, angles, coordinates, so that the error among the precision points (representing the desired trajectory) and the actual traced points (representing the actual trajectory) by the coupler point is minimized. In this way, there are a number of constraints such as the Grashof conditions, the sequence of input angles and the range of design variables that have to be satisfied during the optimization design of the mechanism [8]. There have been different optimization methods applied for the synthesis of mechanisms reported in the literature [7–11]. Among these methods, different types of genetic algorithms have been successfully used for optimal synthesis of different mechanisms because of their high probability in finding the global optimum point and their effectiveness [7,8,10,12–14]. However, all the optimization approaches presented in these references are based on single objective function, namely, the tracking error (TE) between the actual and the desired coupler precision points. In fact, the general optimization syntheses of mechanisms have to be regarded as a multi-objective optimization problem rather than a single-objective optimization problem. In this way, transmission angle (TA) as another important objective function in the synthesis of mechanisms has to be considered together with the tracking error in the optimization process simultaneously. In fact, the minimization of transmission angle's deviation from 90° causes a better torque and energy transfer in the mechanism's motion which, in turn, leads to the smallest actuator possible [6]. However, these objective functions (TE and TA) – to be minimized simultaneously – conflict with each other so that improving one of them will deteriorate another one. Consequently, there is no single optimal solution as the best with respect to these two objectives. Instead, there is a set of optimal solutions or Pareto front [15–21]. Therefore, such presentation of optimum points can collectively exhibit different optimum designs for optimal combinations of both tracking error (TE) and transmission angle error (TA) in the synthesis of mechanisms, and thus are useful for design engineers.

The concept of Pareto front (in the domain of objectives) or set (in the domain of design variables) in multi-objective optimization problems (MOPs) stands for a set of solutions that are non-dominated to each other but are superior to the rest of solutions in the search space. This means that it is not possible to find a *single* solution to be superior to all other solutions with respect to all objectives so that changing the vector of design variables in such a Pareto set consisting of these non-dominated solutions could not lead to the improvement of all objectives simultaneously. Consequently, such a change will lead to deteriorating of at least one objective. Thus, each solution of the Pareto front includes at least one objective inferior to that of another solution in that Pareto front, although both are superior to others in the rest of search space. It must be noted that this non-dominancy does exist in different levels, forming different ranked Pareto fronts, although the first Pareto front is the most important and will be the ultimate solution. The inherent parallelism in evolutionary algorithms makes them suitable for solving MOPs. The early use of evolutionary search is first reported in 1960s by Rosenberg [22]. Since then, there has been a growing interest in devising different evolutionary algorithms for MOPs. Among these methods, the vector evaluated genetic algorithm (VEGA) proposed by Schaffer [23], Fonseca and Fleming's genetic algorithm (MOGA) [24], non-dominated sorting genetic algorithm (NSGA) by Srinivas and Deb [25], the strength Pareto evolutionary algorithm (SPEA) by Zitzler and Thiele [15], the Pareto archived evolution strategy (PAES) by Knowles and Corne [16], and the Niche-Pareto genetic algorithm (NPGA) by Horn et al. [17] are the most important ones. A very good and comprehensive survey of these methods has been presented in [18–20]. Basically, both NSGA and MOGA as Pareto-based approaches use the non-dominated sorting procedure originally proposed by Goldberg [21]. In such Pareto-based approaches, the values of objective functions are used to distinguish the non-dominated solutions in the current population. There are two important issues that have to be considered in such evolutionary multi-objective optimization methods: driving the search towards the true Pareto front and preventing premature convergence or maintaining the genetic diversity within the population [26]. A sharing operation is performed in NSGA to maintain the population diversity that, however, attracted criticisms for being too sensitive to the selection of sharing parameters. Besides, the lack of elitism was also a motivation for the modification of that algorithm to NSGA-II [27] in which a direct elitist mechanism, instead of sharing mechanism, has been introduced to enhance the population diversity. This modified algorithm has been regarded as the state-of-the-art in evolutionary MOPs [28]. A comparison study among SPEA and other evolutionary algorithms on several problems and test functions showed that SPEA clearly outperforms the other multi-objective EAs [29]. Some further investigations developed in [26] demonstrated, however, that the elitist variant of NSGA (NSGA-II) performed similar to SPEA.

In this paper, a hybrid multi-objective genetic algorithm with a new diversity preserving algorithm called ε -elimination diversity algorithm proposed by some of authors [31,32] (modified NSGA-II) is used for optimal dimensional synthesis of four-bar linkages to trace two different trajectories. The conflicting objectives which have been considered for minimization are tracking error (TE) and transmission angle error (TA) which is defined as transmission angle's deviation from 90° . The obtained Pareto fronts in both trajectories demonstrate that trade-offs between these two objectives can be recognized so that a designer can compromise for selection of a desired optimum four-bar linkage. Moreover, the superiority of the obtained optimum points is shown in terms of these objective functions with the designs given in the literature.

2. Multi-objective Pareto optimization

Multi-objective optimization which is also called multi-criteria optimization or vector optimization has been defined as finding a vector of decision variables satisfying constraints to give optimal values to all objective functions [30]. In general, it can be mathematically defined as:

Find the vector $X^* = [x_1^*, x_2^*, \dots, x_n^*]^T$ to optimize

$$F(X) = [f_1(X), f_2(X), \dots, f_k(X)]^T, \quad (1)$$

subject to m inequality constraints

$$g_i(X) \leq 0, \quad i = 1 \text{ to } m, \quad (2)$$

and p equality constraints

$$h_j(X) = 0, \quad j = 1 \text{ to } p, \quad (3)$$

where, $X^* \in \mathfrak{R}^n$ is the vector of decision or design variables, and $F(X) \in \mathfrak{R}^k$ is the vector of objective functions. Without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on the Pareto approach can be conducted using some definitions:

2.1. Definition of Pareto dominance

A vector $U = [u_1, u_2, \dots, u_k] \in \mathfrak{R}^k$ dominates to vector $V = [v_1, v_2, \dots, v_k] \in \mathfrak{R}^k$ (denoted by $U \prec V$) if and only if $\forall i \in \{1, 2, \dots, k\}$, $u_i \leq v_i \wedge \exists j \in \{1, 2, \dots, k\}: u_j < v_j$. It means that there is at least one u_j which is smaller than v_j whilst the rest u 's are either smaller or equal to corresponding v 's.

2.2. Definition of Pareto optimality

A point $X^* \in \Omega$ (Ω is a feasible region in $\mathfrak{R}^n$ satisfying Eqs. (2) and (3)) is said to be Pareto optimal (minimal) with respect to all $X \in \Omega$ if and only if $F(X^*) \prec F(X)$. Alternatively, it can be readily restated as $\forall i \in \{1, 2, \dots, k\}$, $\forall X \in \Omega - \{X^*\}: f_i(X^*) \leq f_i(X) \wedge \exists j \in \{1, 2, \dots, k\}: f_j(X^*) < f_j(X)$. It means that the solution X^* is said to be Pareto optimal (minimal) if no other solution can be found to dominate X^* using the definition of Pareto dominance.

2.3. Definition of Pareto set

For a given MOP, a Pareto set $\mathcal{P}^*$ is a set in the decision variable space consisting of all the Pareto optimal vectors, $\mathcal{P}^* = \{X \in \Omega | \exists X' \in \Omega: F(X') \prec F(X)\}$. In other words, there is no other X' in Ω that dominates any $X \in \mathcal{P}^*$.

2.4. Definition of Pareto front

For a given MOP, the Pareto front $\mathcal{P}^*T$ is a set of vectors of objective functions which are obtained using the vectors of decision variables in the Pareto set $\mathcal{P}^*$, that is, $\mathcal{P}^*T = \{F(X) = (f_1(X), f_2(X), \dots,$

$f_k(X) : X \in \mathcal{P}^*\}$. Therefore, the Pareto front $\mathcal{P}^* T$ is a set of the vectors of objective functions mapped from $\mathcal{P}^*$.

Evolutionary algorithms have been widely used for multi-objective optimization because of their natural properties suited for these types of problems. This is mostly because of their parallel or population-based search approach. Therefore, most difficulties and deficiencies within the classical methods in solving multi-objective optimization problems are eliminated. For example, there is no need for either several runs to find the Pareto front or quantification of the importance of each objective using numerical weights. It is very important in evolutionary algorithms that the genetic diversity within the population be preserved sufficiently [26]. This main issue in MOPs has been addressed by much related research work [31,32]. Consequently, the premature convergence of MOEAs is prevented and the solutions are directed and distributed along the true Pareto front if such genetic diversity is well provided. The Pareto-based approach of NSGA-II [27] has been recently used in a wide range of engineering MOPs because of its simple yet efficient non-dominance ranking procedure in yielding different levels of Pareto frontiers.

Hybrid genetic algorithms in which the exploration scheme of GAs is crossed with the exploitation of another (derivative-based or non-derivative-based) search technique can successfully refine the solution obtained by a global search method like GA. Therefore, multi-dimensional Powell's method [33] as a non-derivative-based method has been used to climb the best hill found by GA. In this work, the hybrid GA stands for the method in which the solutions found in the Pareto front are used as suitable initial guesses for further refinement by Powell's method. This means that the hybrid approach of this work employs multi-objective GA plus multi-dimensional non-derivative based Powell's method after a certain number of generations. It should be noted that there are many different ways of hybridization; but most of them incorporate a local optimizer to the simple GA main loop. Further, a new method called ε -elimination diversity algorithm, recently proposed by some of authors [31,32], is also used in this work to modify NSGA-II so that it can be safely used for any number of objective functions in MOPs.

3. Optimal synthesis of four-bar mechanism

A four-bar mechanism is generally shown in Fig. 1. In order to compare the results, the same notations used in [8] are also adopted here. The synthesis of such mechanism consists of finding the vector of design variables $X = [r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, \theta_0, x_0, y_0, \theta_2^i]$ so that two conflicting objective functions tracking error (TE) and transmission angle error (TA) are minimized simultaneously for a prescribed desired trajectory. It should be noted that θ_2^i is either as the initial input angle ($i = 1$) or as the ordered sequence ($i = 1, 2, \dots, N$) of input angles.

The desired trajectory is usually represented by a set of desired or target points $C_d^i = [C_{xd}^i, C_{yd}^i]^T$ which to be traced by the coupler point C of the mechanism. Therefore, the tracking error (TE), as the first objective function, can be readily computed as

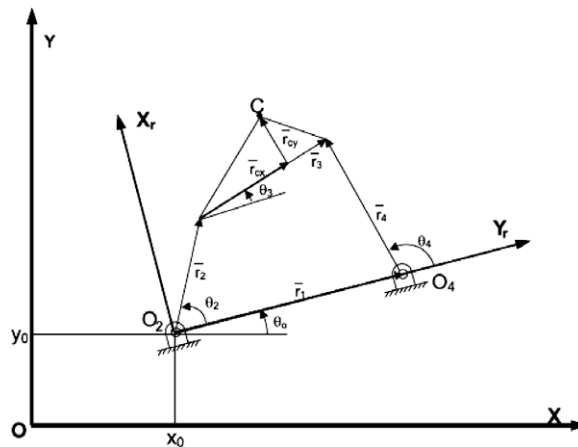


Fig. 1. Coupler point on the coupler link of a four-bar linkage.

$$TE = \sum_{i=1}^N [(C_{xd}^i - C_x^i)^2 + (C_{yd}^i - C_y^i)^2], \quad (4)$$

where $[C_x^i C_y^i]$ are the x – y components of the points traced by the coupler point and N is the number of precision points. The transmission angle error (TA), as the second objective function, is also computed as the sum of the square of the maximum deviations of the transmission angle γ of the mechanism for the whole path and is given by

$$TA = [(\gamma_{\max} - 90^\circ)^2 + (\gamma_{\min} - 90^\circ)^2]. \quad (5)$$

It can be seen that the ideal value of this objective function is zero when both the maximum and the minimum values of the transmission angle γ are 90° during the entire trajectory.

The tracking error (TE) given by Eq. (4) is evaluated when the actual traced points $[C_x^i C_y^i]$ by the coupler point C is computed. In Fig. 1, the position vector of the coupler point C is readily expressed as

$$\vec{r}^c = \vec{R}^0 + \vec{r}^2 + \vec{r}^{cx} + \vec{r}^{cy}, \quad (6)$$

which can be represented in its component as

$$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + r_2 \begin{bmatrix} \cos \varphi_2 \\ \sin \varphi_2 \end{bmatrix} + r_{cx} \begin{bmatrix} \cos \varphi_3 \\ \sin \varphi_3 \end{bmatrix} + r_{cy} \begin{bmatrix} -\sin \varphi_3 \\ \cos \varphi_3 \end{bmatrix}, \quad (7)$$

where

$$\varphi_2 = \theta_2 + \theta_0, \quad (8)$$

$$\varphi_3 = \theta_3 + \theta_0. \quad (9)$$

The coupler angle θ_3 is also computed using the closed vector equation of the four-bar mechanism as shown in Fig. 1

$$\vec{r}^2 + \vec{r}^3 - \vec{r}^4 - \vec{r}^1 = \vec{0}, \quad (10)$$

which can also be expressed in its components with respect to relative coordinates as

$$r_2 \cos \theta_2 + r_3 \cos \theta_3 - r_4 \cos \theta_4 - r_1 = 0, \quad (11a)$$

$$r_2 \sin \theta_2 + r_3 \sin \theta_3 - r_4 \sin \theta_4 = 0. \quad (11b)$$

Consequently, the unknown angles of θ_3 and θ_4 are computed by the equation of Freudenstein [4] when the vector of design variables X is properly (optimally) chosen.

The transmission angle error (TA) given by Eq. (5) can also be evaluated using the following equations [6] for both $\gamma_{\max}$ and $\gamma_{\min}$,

$$\gamma_{\max} = \cos^{-1} \left[\frac{r_4^2 - (r_1 + r_2)^2 + r_3^2}{2r_4r_3} \right], \quad (12a)$$

$$\gamma_{\min} = \cos^{-1} \left[\frac{r_4^2 - (r_1 - r_2)^2 + r_3^2}{2r_4r_3} \right]. \quad (12b)$$

It should be noted that both TE and TA are also penalized due to the violation of some constraints. Such constraints involve the Grashof condition and the sequence of input angles either increasing or decreasing. The ranges of design variables are imposed by the encoding scheme of genetic algorithms.

4. Pareto optimum syntheses of four-bar mechanisms

In this section, the modified NSGA-II with ε -elimination diversity algorithm [31,32] is used for multi-objective optimum design of four-bar linkage for two case studies that have been illustrated in previous section. In order to design a four-bar linkage from the multi-objective optimum point of view, a population of 80 individuals with a crossover probability of 0.95 and mutation probability of 0.1 has been used in 250 generation

for which no further improvement has been achieved. However, in order to refine the results obtained as non-dominated points, the Pareto front is moved to a better one by using a single-objective optimization technique for each non-dominated point. In this scheme, each non-dominated point of the evolved Pareto front is regarded as an initial guess for an optimization technique by keeping one objective fixed whilst improving (if possible) another objective by changing the design vector. Although the individuals in the Pareto front which have been found so far by the evolutionary approach are all non-dominated with respect to each other, they may still be improved, in terms of the values of their objectives, to better individuals of a better Pareto front in which all the newly found individuals are non-dominated again. Such hybrid application of the evolutionary method and a single-objective optimization technique outperforms the sole use of evolutionary approach to find the real optimal Pareto fronts.

Two examples have been adopted from [8] as case studies to present and compare the results using the hybrid GA approach of this paper.

4.1. Case study 1

The first case study is to design a four-bar mechanism with a vector of design variables as $X = [r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, \theta_0, x_0, y_0, \theta_2^1, \theta_2^2, \theta_2^3, \theta_2^4, \theta_2^5, \theta_2^6]$ with a prescribed timing. The desired or target points are six points along a straight line which are specified by $\{C_d^i\} = \{(20, 20), (20, 25), (20, 30), (20, 35), (20, 40), (20, 45)\}$. The range of input variables adopted from [7,8] are set to $r_1, r_2, r_3, r_4 \in [0, 60]$, $r_{cx}, r_{cy}, x_0, y_0 \in [-60, 60]$, and $\theta_0, \theta_2^i \in [0, 2\pi]$.

The obtained Pareto front of TE and TA for the first case study has been shown in Fig. 2. It is clear from this figure that all design points representing different mechanisms are non-dominated with respect to the conflicting objectives.

Fig. 2 depicts the Pareto front of TE and TA representing different non-dominated optimum points for the first case study. In this figure, points A and B stand for the best (TE) and the best (TA), respectively. The corresponding values of objective functions and design variables of these extreme optimum design points are given in Table 1. It should be noted that all the optimum design points (all mechanisms) in this Pareto front are non-dominated and could be chosen by a designer. It is clear from the this figure that choosing a better value for any objective function in these Pareto fronts would cause a worse value of another objective function. However, if the set of decision variables is selected based on those located on the Pareto fronts, it will lead to the best design from the point of view of both of these objective functions (TA, TE). In other words, if any other set of decision variables is chosen, the corresponding values of objective functions will locate a

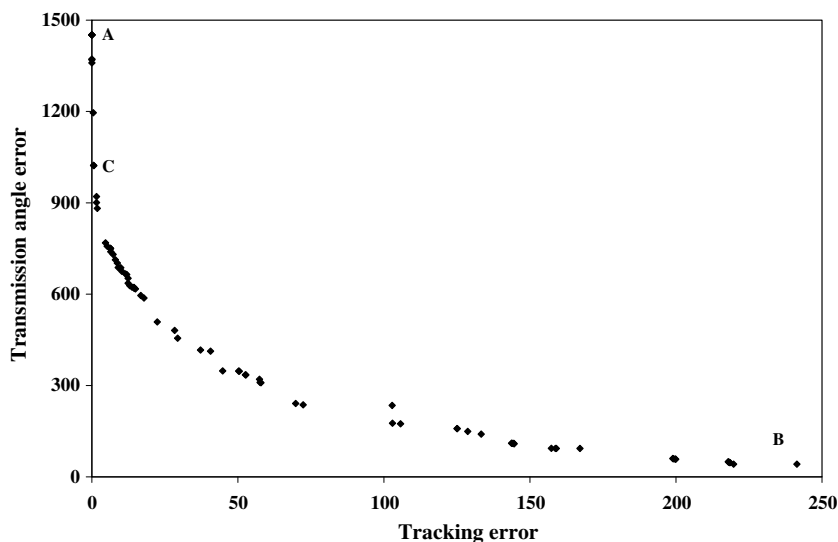


Fig. 2. Pareto front of tracking error and transmission angle error for the first case study.

Table 1

Objective functions and design variables of different points shown on Fig. 2 for the first case study

	Point A	Point B	Point C
r_1	29.1988	53.19662	33.31511
r_2	7.129041	2.127865	6.812921
r_3	26.78071	37.7639	25.80695
r_4	22.82488	37.76393	28.22495
r_{cx}	39.23960	9.93368	37.03012
r_{cy}	28.073533	24.40578	31.12149
θ_0	4.24662	3.629253	−20.0736
x_0	−18.90082	−0.48739	57.44969
y_0	58.46851	48.36552	4.077099
θ_2^1	0.848862	1.465166	1.639698
θ_2^2	2.388157	1.487463	2.564869
θ_2^3	2.907649	1.527999	3.065644
θ_2^4	3.399529	4.574775	3.519768
θ_2^5	3.923457	4.623325	4.080135
θ_2^6	4.969865	4.652822	4.867305
Tracking error (TE)	2.968e−7	241.413	0.543612
Transmission angle error (TA)	1451.455	41.67767	1023.29

point inferior to the corresponding Pareto front. Such inferior area in the space of the two objectives is in fact top/right side of both Fig. 2. Clearly, there are some important optimal design facts between these objective functions that can readily be observed in the Pareto fronts. Such important design facts could not have been found without the use of multi-objective Pareto optimization approach of the mechanism synthesis. From Fig. 2 point C is the point which demonstrate the important optimal design fact. It is evident from Table 3 that the design point A of this paper dominates significantly the optimum design obtained in Ref. [8] in terms of both TE and TA. However, it can be seen that if the designer accepts a tolerable increase in the tracking error, an amount of 30% more improvement in transmission angle can then be achieved by selecting optimum design point C instead of design point A. The mechanism's configuration and the path traced by its coupler points corresponding to points A and C are shown in Figs. 3 and 4, respectively. In order to compare the results, the tracking error (TE) and the transmission angle error (TA) of design points A obtained in this paper are compared with those reported in the literature [8] for the first case study. It is very clear from Table 3 that the design point A presented in this paper is significantly better than that reported in [8] in terms of both objective functions TE and TA.

4.2. Case study 2

The second case study is to design another four-bar mechanism with a vector of design variables as $X = [r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, \theta_0, x_0, y_0, \theta_2^1]$. The desired or target point are 18 points specified by $\{C_d^i\} =$

Table 2

Objective functions and design variables of different points shown on Fig. 3 for the second case study

	Point D	Point E	Point F
r_1	6.305405	7.38815	4.332755
r_2	0.3263703	0.29552	0.33581
r_3	6.425734	5.23982	0.584028
r_4	0.477803	5.29428	4.16409
r_{cx}	1.83112	0.74179	−0.15706
r_{cy}	4.357617	−1.38297	0.401298
x_0	2.589329	−0.61287	0.626482
y_0	−3.434744	−0.66022	0.742331
θ_0	0.855654	1.24508	3.364573
θ_2^1	0.4371914	5.81499	3.448541
Tracking error (TE)	9.8e−3	0.37819	0.01495
Transmission angle error (TA)	5454.51	42.15	3539.2

Table 3

Comparison of results of this work and Ref. [8] for the first case study

	Results of this work		Results of Ref. [8]
	Point A	Point C	
Tracking error (TE)	2.986×10^{-7}	0.543612	0.02617
Transmission angle error (TA)	1451.45	1023.29	4792.02

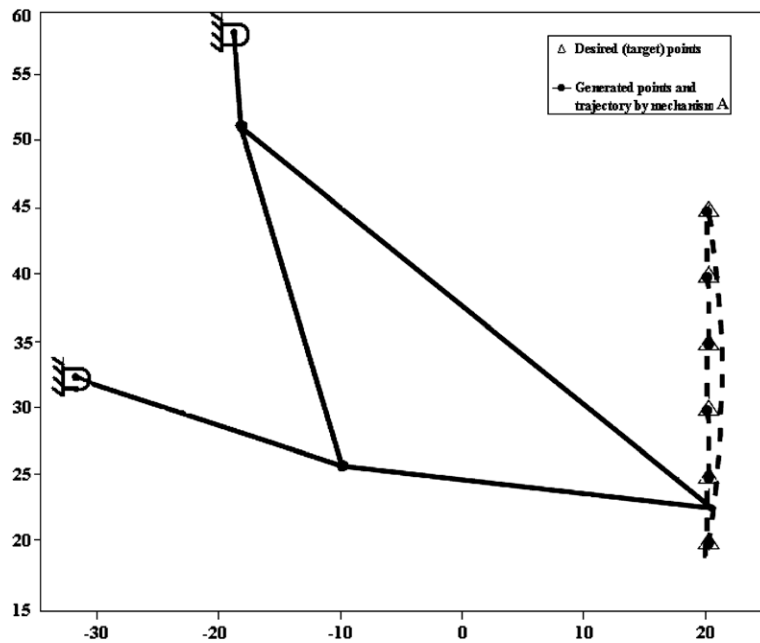


Fig. 3. The mechanism's configuration and the path traced by the coupler point corresponding to the design point A of the Pareto front given in Fig. 2 (case study 1).

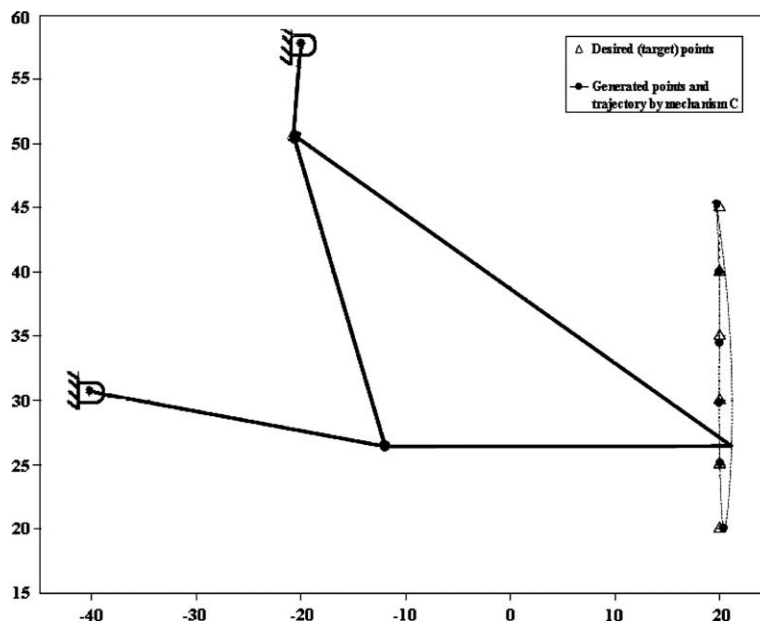


Fig. 4. The mechanism's configuration and the path traced by the coupler point corresponding to the design point C of the Pareto front given in Fig. 2 (case study 1).

$\{(0.5, 1.1), (0.4, 1.1), (0.3, 1.1), (0.2, 1.0), (0.1, 0.9), (0.005, 0.75), (0.02, 0.6), (0.0, 0.5), (0.0, 0.4), (0.03, 0.3), (0.1, 0.25), (0.15, 0.2), (0.2, 0.3), (0.3, 0.4), (0.4, 0.5), (0.5, 0.7), (0.6, 0.9), (0.6, 1.0)\}$. The range of input variables adopted from [7,8] are set to $r_1, r_2, r_3, r_4 \in [0, 50]$, $r_{cx}, r_{cy}, x_0, y_0 \in [-50, 50]$, and $\theta_0, \theta_2^i = [\theta_2^l, \theta_2^u + 20^\circ \times i]$, $i = 1, 2, \dots, 17$.

Fig. 5 depicts the Pareto front of those objective functions for the second case study. In this figure, points D and E stand for the best optimum values obtained for TE and TA, respectively. The corresponding values of the objective functions with their associated design variables are shown in Table 2. From Fig. 5, point F is the point which demonstrate the important optimal design fact. Similarly, optimum design point F obtained in

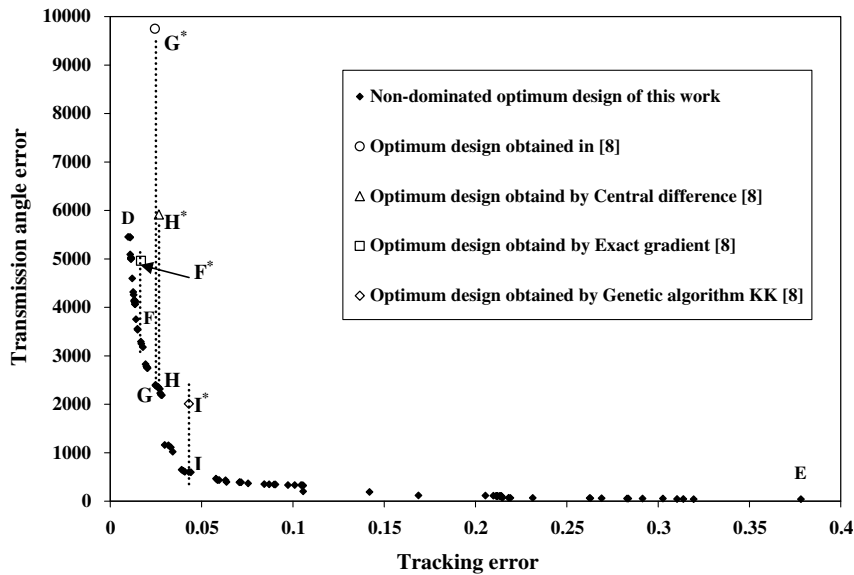


Fig. 5. Pareto front of tracking error and transmission angle error for the second case study.

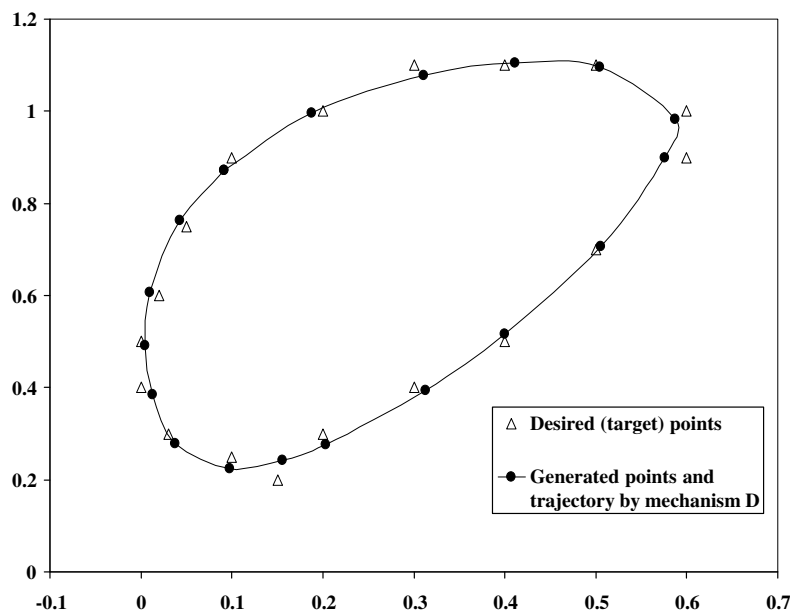


Fig. 6. The path traced by the coupler point corresponding to the design point D of the Pareto front given in Fig. 3 (case study 2).

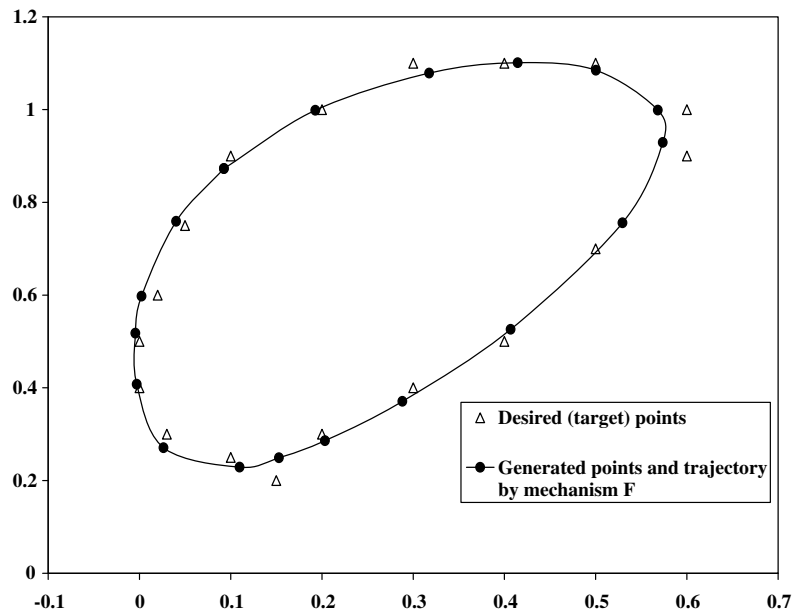


Fig. 7. The path traced by the coupler point corresponding to the design point F of the Pareto front given in Fig. 3 (case study 2).

this paper for the second case study exhibits a small increase in TE in comparison with that of point D (the design with the least tracking error) whilst its TA improves about 35%. The paths traced by its coupler points corresponding to point D and F are also shown in Figs. 6 and 7, respectively. Consequently, such very important trade-offs between conflicting objectives of TE and TA have been achieved by the hybrid multi-objective optimum synthesis of mechanisms.

Table 4 also shows comparisons of the design points D and F obtained in this paper with those given in [7,8] for the second case study. It is evident, from the values of the objective functions TE and TA given in Table 4, that the design point D of this paper dominates the one given in [8] in terms of both objective functions. The trade-off design point F obtained by the methodology of this paper also dominates most of the other designs reported in the literature including the one given in [8]. Such dominancy can also be observed in Fig. 5 where all design points are collectively shown in the Pareto front of TE and TA obtained by the hybrid multi-objective GA approach of this paper. The design points denoted by (G^* , H^* , F^* , I^*) are those obtained in the

Table 4
Comparison of results of several optimization methods for the second case study

	Central difference [8] H^*	Exact gradient [8] F^*	Genetic algorithm [7,8] I^*	Ref. [8] G^*	This work (point D)	This work (point F)
Tracking error (TE)	2.66×10^{-2}	1.68×10^{-2}	4.3×10^{-2}	2.45×10^{-2}	9.80×10^{-3}	1.495×10^{-2}
Transmission angle error (TA)	5914.8	4960.7	2010.7	9746.9	5454.5	3539.2
r_1	2.80000	2.85452	1.879660	3.057878	6.305405	4.33294
r_2	0.36000	0.36355	0.274853	0.237803	0.32637	0.33558
r_3	2.91000	2.91374	1.180253	4.828954	6.425735	0.59119
r_4	0.49845	0.49374	2.138209	2.056456	0.477804	4.16751
r_{cx}	1.023716	1.031223	-0.833592	0.767038	1.831125	-0.15706
r_{cy}	1.718140	1.717471	-0.378770	1.850828	4.357618	0.40129
x_0	0.95000	0.95928	1.132062	1.776808	2.58933	0.62648
y_0	-1.18289	-1.19645	0.663433	-0.641991	-3.43474	0.74233
θ_0	0.75999	0.76398	4.354224	1.002168	0.855655	3.36055
θ_2^1	0.51000	0.51172	2.558625	0.226186	0.437191	3.45760

literature [7,8] which are clearly dominated by those obtained in this paper (G, H, F, I). The design points of this paper (G, H, F, I) represent same value of tracking errors but much less amount of transmission angle errors in comparison with those of (G^* , H^* , F^* , I^*) obtained in literature, respectively.

5. Conclusion

A multi-objective hybrid genetic algorithm with a recently developed diversity preserving ability was used to optimally design four-bar mechanisms. The objective functions which conflict with each other were selected as tracking error and transmission angle error. The multi-objective optimum synthesis of four-bar mechanisms led to the discovering some important trade-offs among those objective functions. It was demonstrated that two effective four-bar mechanisms in terms of both tracking error and transmission angle error could be found among the non-dominated Pareto fronts compromising those conflicting objective functions. The superiority of the obtained mechanisms was shown in comparison with those recently reported in literature. Such hybrid Pareto synthesis of mechanisms could unveil very important design trade-offs between conflicting objective functions which would not have been found otherwise.

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