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Developments in quantitative dimensional synthesis (1970–present): four-bar path and function generation

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ABSTRACT

Dimensional synthesis is a type of inverse problem in linkage kinematics where the objective is to calculate the linkage dimensions required to achieve prescribed linkage output motion. It includes three distinct subcategories: *motion* generation, *path* generation and *function* generation. The authors' first work surveyed developments in quantitative four-bar motion generation. This work concludes the presentation of developments in four-bar dimensional synthesis by surveying the developments made in quantitative four-bar path and function generation since 1970. While early path and function generation methods (like motion generation methods) were primarily *qualitative*, ongoing advancements in computing hardware and software continue to make *quantitative* motion generation more practical.

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1. Dimensional synthesis, four-bar path and function generation

The objective in *dimensional synthesis* is to calculate the linkage dimensions required to achieve prescribed linkage output motion. Prescribed output motion includes link path points, displacements or even higher order quantities such as velocities and accelerations. Calculated linkage dimensions include link lengths, link positions and joint coordinates. In comparison to *kinematic analysis*, where linkage dimensions are known and the resulting linkage output motion is calculated, dimensional synthesis is a type of inverse problem where the output motion is known and the corresponding dimensions are calculated (Figure 1).

Dimensional synthesis includes three distinct subcategories: *motion generation*, *path generation* and *function generation*. Four-bar motion generation was the focus of the authors' first survey paper [1]. This survey paper explores four-bar path and function generation. In four-bar path generation, the objective is to calculate the linkage dimensions required to achieve prescribed coupler link path points (Figure 2). In four-bar function generation, the objective is to calculate the linkage dimensions required to achieve prescribed link displacement angles (Figure 2). In four-bar function generation, the displacement angles of the crank and follower links (angles β and γ , respectively, in Figure 2) are typically defined

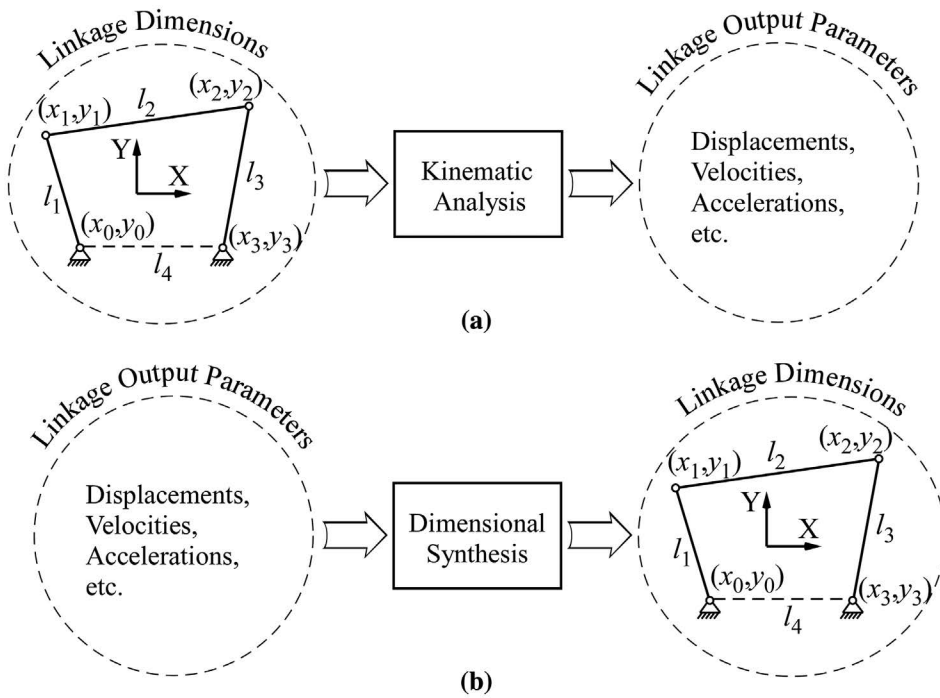


Figure 1. (a) Kinematic analysis vs. (b) dimensional synthesis.

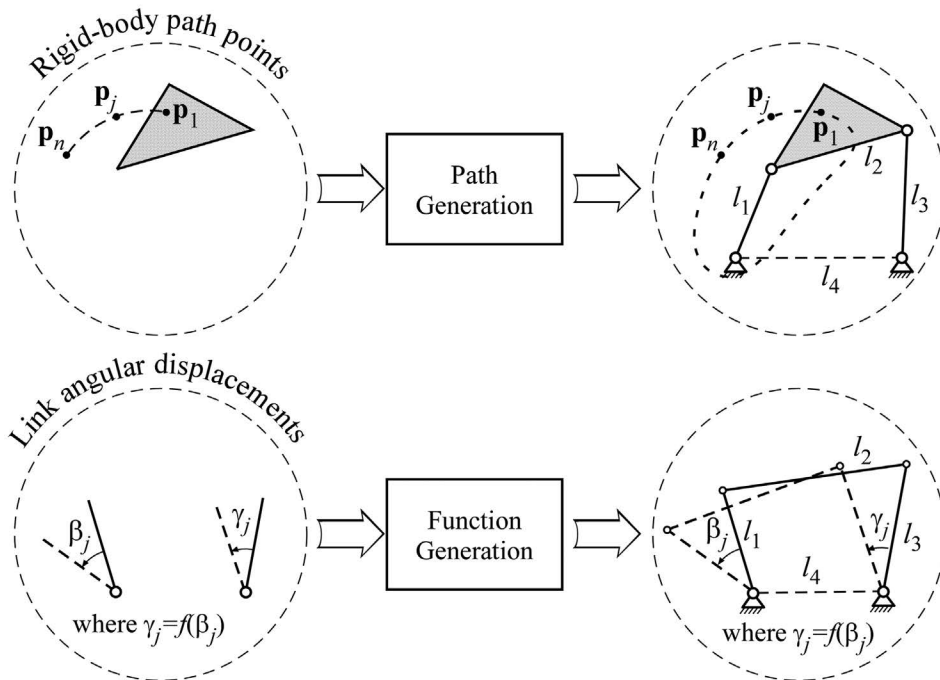


Figure 2. (Top) Four-bar path generation and (bottom) four-bar function generation.

according to a mathematical function (so that $\gamma_j = f(\beta_j)$). In both path and function generation, the prescribed linkage output are called *precision points*.

Hand-drawn geometric-based, graphical methods (which require virtually no computational effort) for dimensional synthesis were most popular when computing technology was in its infancy and are still in use today as introductory approaches to dimensional synthesis [2–4]. In comparison, *quantitative* dimensional synthesis methods include mathematical models that can be solved analytically or numerically by way of a broad assortment of optimization algorithms. Ongoing advancements in computing hardware and software continue to make quantitative four-bar dimensional synthesis methods more practical and more robust. The objective of this work is to provide a survey of the developments in quantitative four-bar path and function generation since 1970.

2. Classical analytical four-bar path and function generation

2.1. Planar four-bar path generation

Coupler path points are prescribed in planar four-bar path generation. In comparison, both coupler path points and coupler displacement angles are prescribed in planar four-bar motion generation. Since the objective in motion generation is to achieve prescribed coupler positions – which are defined by points and displacement angles – motion generation methods can be directly applied to path generation problems. In true path generation however, coupler displacement angles are calculated – not prescribed.

Figure 3 includes a planar four-bar linkage in an arbitrary initial position (position 1) and displaced position (position j) in complex space. From this illustration, vector loops can be formulated for the planar four-bar dyads $\mathbf{W}_1 - \mathbf{Z}_1$ and $\mathbf{U}_1 - \mathbf{S}_1$. Equation (1) is formulated by taking the dyad vector loop sum $\mathbf{W}_1 - \mathbf{Z}_1 - \mathbf{P}_{j1} - \mathbf{Z}_j - \mathbf{W}_j$ and an equation identical in form to Equation (1) is formed by taking the dyad vector loop sum $\mathbf{U}_1 - \mathbf{S}_1 - \mathbf{P}_{j1} - \mathbf{S}_j - \mathbf{U}_j$. In classical quantitative planar four-bar motion generation, Equation (1) forms the basis for a simultaneous equation set to calculate the linkage dimensions (represented by dyad vectors $\mathbf{W}_1$ and $\mathbf{Z}_1$) to achieve four coupler positions (represented by the coupler point $\mathbf{P}_{j1}$ and displacement angle α_j variables) [3]. In the classical four-position model, the dyad displacement angles β_3 and β_4 are calculated (over a prescribed range of β_2 values). In classical quantitative planar four-bar path generation, the same simultaneous equation set is to calculate the same dyad vectors. The difference however is that the dyad displacement angles β_j are prescribed and the coupler displacement angles α_3 and α_4 are calculated (over a prescribed range of α_2 values) [3]. With the latter true quantitative path generation model, planar four-bar linkages are synthesized to precisely achieve four precision points.

$$\mathbf{W}_1(e^{i\beta_j} - 1) + \mathbf{Z}_1(e^{i\alpha_j} - 1) = \mathbf{P}_{j1} \quad (1)$$

2.2. Planar four-bar function generation

Figure 4 includes a planar four-bar linkage in an arbitrary initial position (position 1) and displaced position (position j) in complex space. The classical analytical equation system for function generation is formulated by taking the sum of the vector loop $\mathbf{W}_j - \mathbf{V}_j - \mathbf{U}_j - \mathbf{G}_1$ in Figure 4. Equation (2) is the resulting vector loop equation. Given three displacement angles each for the crank, coupler and follower links (angles β_j , α_j and γ_j respectively), Equation

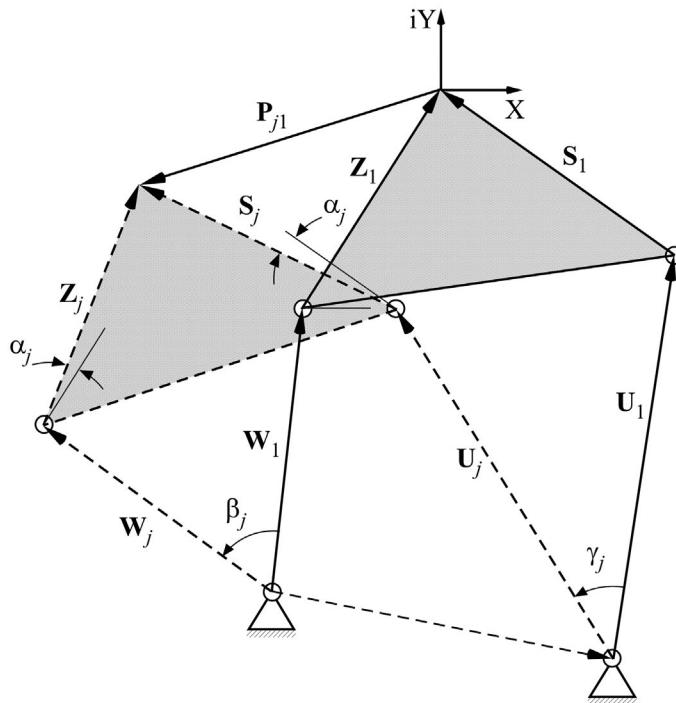


Figure 3. Planar four-bar linkage with dyad vector loop variables.

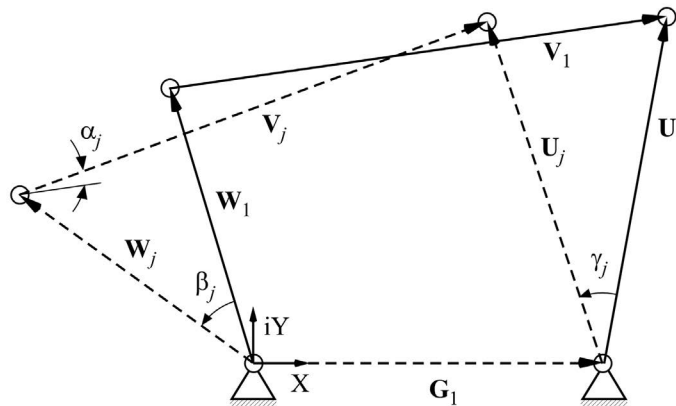


Figure 4. Planar four-bar linkage with vector loop variables.

(2) forms a set of simultaneous equations from which the linkage dimensions (represented by vectors $\mathbf{W}_1$, $\mathbf{V}_1$ and $\mathbf{U}_1$) are calculated [3]. The crank and follower displacement angles (angles β_j and γ_j respectively) are often specified according to a mathematical function so that $\gamma_j = f(\beta_j)$.

$$\mathbf{W}_1 e^{i\beta_j} + \mathbf{V}_1 e^{i\alpha_j} - \mathbf{U}_1 e^{i\gamma_j} - \mathbf{G}_1 = 0 \quad (2)$$

3. Developments in quantitative planar four-bar path generation

3.1. 1970s' developments

Developments during the 1970s include optimum path generation for N precision points. The dimensions of the optimum linkage are determined by minimizing the error in the linkage loop closure equations [5,6]. A survey on the application of analytical path generation techniques was presented [7]. Procedures for both approximate straight and circular path generation were presented [8,9]. General formulations were developed that include a five-precision point approach, a least squares approach and a least squares structural plus mechanical error approach. These approaches consider a maximum of 10 mechanism design parameters [10].

3.2. 1980s' developments

Developments during the 1980s include a constrained optimization model with constraints to minimize the maximum structural error, ensure linkage mobility, ensure an acceptable range of transmission angle and ensure realistic mechanism dimensions [11]. Probability theory was applied to determine the mean and standard deviation of the vector position of a coupler point (and ultimately mean and deviated coupler curves). This work provides insight into effects of manufacturing tolerances on the performance of constrained kinematic chains [12]. A synthesis procedure where tolerances and clearances are stochastically allocated on different members and joints of a linkage so as to restrict the output error in the path of the coupler point within specified limits was developed. This synthesis procedure helps the designer in finding out how much the tolerance or clearance on a particular variable is critical in terms of affecting the output error in the path of a coupler point [13]. The contribution of quasi-static deformation on coupler point error in a path generator with flexible links was investigated [14]. A synthesis procedure was developed to envelope a given curve by a tangent line carried by the coupler link. The approach is based on considering a given curve as the envelope of a moving tangent line and synthesizing the four-bar linkage to envelope the given curve [15]. An unconstrained non-linear least squares model for optimum path generation (with branch defect elimination) was developed. With this model, the number of design variables is 8 – regardless the number of precision points [16]. An algorithm (based on curve parameterization) for coupler curve shape evaluation was developed. This algorithm facilitates the selection of linkage designs for particular path generation tasks [17]. A computer-aided design programme called SYNTRA was applied in a mapping procedure developed for planar four-bar path generation. The procedure maps all possible linkage solutions (given the first grounded pivot of the linkage), linkage type, linkage assembly form and regions where linkage solutions are not found [18]. A technique called *subdomain method* has been used to dimension mechanisms to approximate prescribed, continuous paths. The effectiveness of this method was demonstrated in path generation [19].

3.3. 1990s' developments

Developments during the 1990s include a two-tier pattern matching scheme which presents a designer with a linkage structure having a prescribed coupler curve pattern from a database

of dimensionally parameterized linkages [20]. As an alternative to the Newton–Raphson method for solving systems of non-linear polynomial equations in kinematics, the *continuation method* has also been applied to path generation. This method offers advantages including no initial solution approximations, that full solutions can be obtained (for a system of N polynomials and N unknowns) and that a solution can be obtained for a system of N polynomials and $N + 1$ unknowns [21]. Because no single optimization method exists that is best suited to solve all path generation problems effectively, an approach consisting of an array (called an *orthogonal array*) of qualitative, quantitative and experimental methods was proposed [22]. In a path generation method called the *numerical atlas method*, a library of coupler curves and associated linkage dimension types are produced, stored and searched to determine the best linkage dimension solution for a given desired coupler curve. A method to create and search a library to find the best ‘seed’ mechanisms for subsequent optimization as a path generator was presented [23]. Path generation of a flexible planar four-bar linkage was also explored. Displacement analysis of the flexible linkage is carried out using the *finite element method* and synthesis is formulated and solved as an optimization problem [24]. The *reliability* of a path generator is defined as the probability with which its coupler point will generate the intended path with a specified precision. A reliability-based analysis approach path generator mechanical error was presented [25]. An objective function based on Fourier descriptors that evaluates only the shape differences between two curves was presented as an alternative to objective functions based on structural error [26].

3.4. Developments 2000–2009

Since the year 2000, there has been a substantial increase in the number of developments made in quantitative planar four-bar path generation compared to past decades. Developments during the years 2000–2009 include optimum path generation to achieve a straight path using *natural coordinates* (where coupler motion is described without using angular variables) [27]. An algebraic method was developed for circle path generation [28]. A synthesis method was presented for crank–rocker path generators where structural error is based on the orientation of the grounded link (rather than the path traced by the coupler link) [29].

Through path generation using *neural networks*, approximate solutions are obtained by interpolation over a broad set of kinematic analysis cases [30,31]. *Differential evolution*, an *evolutionary optimization* scheme that can search outside the initial defined bounds for design variables, was applied in path generation. With this optimization scheme, the quality of the linkage solution is not as dependent on the quality of the design variable initial guesses as they are with non-evolutionary optimization schemes [32–34]. An optimization strategy by which a gross-fine search method is employed to obtain the global optimum in approximate path generation was presented [35]. An open-loop path generation method was presented where the open-loop path problem is simplified through a technique known as *geometrical adaptation* [36].

Another type of evolutionary algorithm is the *genetic algorithm*. It is sometimes used in combination with *fuzzy logic* for path generation [37,38]. A genetic algorithm-based method for path generation with joint clearance was also presented [39]. Optimum synthesis using exact gradients (rather than numerically approximated gradients) improves solution calculation efficiency and accuracy in path generation [40]. Path generation with

minimized harmonic content for follower link motion becomes more critical for high-speed path generation applications [41]. The *unsharpened position synthesis* method was extended to produce a method for three- and four-precision point path generation with defect elimination [42].

Optimum path generation using the *tabu-gradient* search algorithm was presented. As the name implies, the tabu-sgradient search employs a tabu search (a *metaheuristic* or random search) to find a solution near the global minimum and follows with a gradient search to move the solution closer to the global minimum [43,44]. A *swarm intelligence*-based pattern matching framework was presented for path generation. With this framework, coupler curves are grouped based on invariants using an *ant-clustering algorithm*, discretized and subjected to pattern matching for down-selection [45]. A probabilistic model for the deviation between the actual coupler curve and the coupler curve generated by the desired linkages (which can accommodate planar four-bar linkages) was presented. This *robust* model simultaneously considers linkage structural error and mechanical error [46].

A computer method using coupler angle function curve for approximate path generation (for more than nine precision points) was presented. This method includes software for path generator visualization [47]. In *mixed exact approximate* path generation, linkages are synthesized to achieve precision points that are specified exactly and approximately [48,49]. A method employing multi-objective genetic algorithms is used for optimum path generation. The two objective functions included are for deviation in the precision point tracking error and the deviation of the transmission angle beyond the 90° optimum value [50].

An optimum synthesis method was developed where both coupler trajectories and velocities are considered (a *bi-objective* problem) in path generation [51]. An analytical synthesis method was developed for straight line path generation. With this method, a four-bar linkage is synthesized to approximate a straight line given three pairs of coincident curve points [52]. While precision points are typically used to define a coupler curve in path generation, an optimal synthesis method was presented where curvature is used to define a coupler curve. In this method, curve curvature is described using normalized Fourier coefficients [53].

3.5. Developments 2010–2017

Developments during the years 2010–2017 include a method that reduces the solution space (which facilitates finding mechanism solutions) for optimum path generation. This method (which was implemented in the commercial software package *Matlab*) applies global optimization methods that do not require the analytical expression of the objective function [54]. A path generation approach for the approximation of open and closed periodic and non-periodic paths was presented. This approach yields a general solution method for planar four-bar path generation for open and closed paths [55,56]. The optimum synthesis of a four-bar Grashof linkage in which the coupler point traces paths comprised of rectilinear segments and circular arcs was also presented [57]. A method which enables classical homotopy continuation solvers to exclude unwanted solutions from a system of polynomials was presented. This method was demonstrated to be capable of eliminating unwanted isolated and positive dimensional solutions in a nine-point path generation problem [58]. A model was presented (and implemented in *Matlab*) for optimum path generation with constraints for circuit, branch and order effect elimination [59]. A *hybrid optimization* approach (where

the benefits of both *stochastic* and *deterministic* optimization are combined) for optimum linkage design was applied to optimum four-bar path generation [60]. The stochastic optimization approach (which utilizes evolutionary optimization) is used to explore the design variable space when searching for the best linkage. The deterministic approach uses a local optimization technique to improve the efficiency by reducing the high computing time (that evolutionary optimization techniques typically require).

An evolutionary algorithm called the *Malaga University mechanism synthesis algorithm* was applied to optimum path generation [61]. The selection of control parameters in differential evolution is not always a simple task. A differential evolution-based algorithm with an operator that simplifies choosing control parameters was presented for optimum path generation [62]. A differential evolution-based algorithm with a modified error function was also applied to optimum path generation. The error function was formulated in a way that makes it suitable for applications where the primary requirement is that the synthesized path should match closely with the prescribed path [63]. An evolutionary algorithm obtained by combining differential evolution with the *real-valued genetic algorithm* was presented for optimum path generation. This algorithm was demonstrated in a group of cases to produce solutions more accurate than those obtained using noted evolutionary methods [64]. A multi-objective genetic algorithm was presented for optimum path generation. The three objective functions included are for deviation in the precision point tracking error, deviation of the transmission angle beyond the 90° optimum value and angular velocity ratio [65]. Differential evolution was applied to *shape synthesis* and *scale-rotation-translation synthesis* for optimum path generation [66]. Differential evolution was also applied for the optimal synthesis of four-bar path generators using a particular objective function called a *circular proximity function* [67].

An approach called *redundancy servomotor* (based on the redundancy function of a servomotor) is applied in path generation. It was demonstrated how path generator design and dynamic performance improvement can be performed in an integrated manner with the redundancy servomotor approach [68]. A comparative study of *Fourier descriptors* (which have been used in the development path generation methods) derived from various shape signatures of planar closed paths was presented [69]. An optimum path generation method that includes joint clearances was developed. Clearances are represented as joint tolerances in the conventional rigid-body model displacement matrix as a design constraint [70]. A computer-aided linkage design system for tracing prescribed open or closed planar curves was presented. With this system, computers are utilized to size a mechanism to meet a set of design requirements while taking advantage of the designers' experience to narrow down the design domain and speed up the design process [71].

Significant control factors for *particle swarm optimization* (a population-based search algorithm for optimization problems) for optimum path generation were identified. Control factors identified include inertial weight, acceleration coefficients, breeding operation and population number [72]. A nested swarm intelligence-based method was presented for optimum path generation. As a nested method, swarm intelligence is used iteratively to reach multiple optimal solutions [73]. Particle swarm optimization was also applied to the problem of path generation with joint clearances and path generation for circular and vertical paths [74,75]. A genetic algorithm-based method was applied in a constrained multi-objective problem to design a three precision point hitch mechanism for a tractor. This method includes objective functions to maximize the mechanical advantage of the

driving link, minimize the deviation of the mechanical advantage from a mean value and obtain a transport pitch in the vicinity of eight inches [76].

A *spring-connected rectangular block model* is a model comprised of three rectangular blocks interconnected with six springs – each spring having two directional stiffness and one orientation parameters. The joint types represented in the model depend on the stiffness and orientation of each spring. A synthesis method was presented that simultaneously carries out path generation and type synthesis of a spring-connected rectangular block model [77]. A constrained optimization method was presented for a linkage with a flexible coupler link. Performance objectives such as path generation error, tangential velocity error and vibrations at the end effector are considered in this method [78].

A comparative study of path-generating four-bar mechanisms using heuristic optimization techniques such as the *imperialist competitive algorithm*, *genetic algorithm*, *particle swarm optimization* and *differential evolution* was presented [79]. An open path generation method was presented using the *Harr wavelet*. This method includes the numerical atlas method [80]. The *modified ants search* technique was applied in optimum mechanism synthesis for a path generation task. Compared to the *ants search* technique, the modified ants search technique offers improvements in the following algorithm aspects: the ant pheromone update and the exploration/exploitation techniques are both modified [81]. In conventional path generation, the number of unknown linkage variables exceeds typically the number of design equations. A system of coupler curve coefficient equations was derived having as many equations as unknowns. With this equation system, linkage parameters are determined exactly [82].

An optimum path generation method was also presented where *structural error* defined as the deviation of the paths of the four-bar moving pivots from their paths in the assembled mechanism (not the deviation of the coupler path from the prescribed trajectory as in conventional methods). [83]. A two-step optimization method was presented for crank-rocker path generation [84]. The first step includes root mean square error minimization to obtain the trajectory shape while the second step includes root mean square minimization to obtain the trajectory size.

A particular type of swarm intelligence algorithm called the *bacterial foraging optimization algorithm* was applied in optimum four-bar path generation [85,86]. This widely accepted global optimization algorithm was inspired by the group foraging behaviour of particular bacteria types (e.g. *E. coli* and *M. xanthus*). The *harmony search algorithm* was applied in optimum four-bar path generation [87]. This algorithm is a population-based meta-heuristic algorithm that mimics the behaviour of a music orchestra when aiming at composing the most harmonious melody (as measured by aesthetic standards). The *hybrid Taguchi random coordinate search algorithm* was applied in optimum path generation. This algorithm combines the *Taguchi method* and the *random coordinate search algorithm* (to efficiently find a global optimal linkage solution) [88]. The *modified krill herd algorithm* was applied in optimum path generation. Compared to the *standard krill herd algorithm* technique, the modified krill herd algorithm offers improvements in the following algorithm aspects: the initialization of fitness functions, the initialization of the vectors which represent the food location and the replacement of the crossover operator with the combination of columns of fitness functions obtained in a single iteration [89]. A software programme was developed for optimum planar four-bar path generation [90]. This programme incorporates *Couenne* – a solver for the global optimization of non-convex *Mixed-Integer Nonlinear*

Programming problems. Optimum four-bar path generation was achieved using a particular type of differential evolution algorithm called the *combined-mutation differential evolution algorithm* [91]. This algorithm (which utilizes three different search modes) possesses better population diversity as well as search ability than the differential evolution algorithm. The *teaching-learning-based optimization* algorithm (as well as the genetic and particle swarm algorithms) was employed for the optimum path generation of defect-free crank–rocker linkages [92]. The *cuckoo search* and teaching–learning-based optimization algorithms were also employed in optimum path generation. The findings show the cuckoo search algorithm to be superior in terms of accuracy [93].

4. Developments in quantitative planar four-bar function generation

4.1. 1970s' developments

Developments during the 1970s include optimum function generation with limiting conditions [94]. A precision position technique for function generation was presented where the geometric inversions of a linkage are not separated. This approach, an alternative to the traditional loop closure method, ensures that the linkage solution will generate the desired characteristics of a function at the precision positions [95]. A general analytical method for approximate function generation over a finite interval was presented. To reduce structural error, a precision point respacing procedure (on the basis of a tenth degree polynomial closely approximating the structural error) is included [96]. The *penalty function technique* was applied to a constrained optimization model for approximate function generation [97]. Centre point and circle point equations were derived (based on relative poles) for analytical five-point function generation [98]. An optimum function generation method for N precision points was presented. The linkage dimensions are determined by minimizing the error in Freudenstein's input–output linkage displacement equation at N design positions [99]. By restating a function generation problem as a motion generation problem, classical Burmester curve theory was applied for function generation [1,100,101]. Optimum function generation (with a transmission angle constraint) was also achieved using *geometric programming* [102].

4.2. 1980s and 1990s' developments

Developments during the 1980 and 1990s include an analysis procedure demonstrated for a function generator with undamped flexible links, distributed mass and operating at a prescribed input rotational speed. The governing equations for link deflections were derived using the Lagrange equation [103]. The penalty function technique was applied to a constrained optimization model for approximate function generation with constraints for maximum velocity and acceleration [104]. Similarly, an optimum synthesis method for function generation satisfying both derivatives of displacements along with the displacements at a discrete set of design positions was demonstrated [105]. An algebraic approach was presented to determine the *function cognates* of a four-bar linkage. Function cognates are linkages that give exactly the same relationship between the displacements of the input and output links [106,107]. A weighted least squares method (using a binary coding concept)

was presented for optimum function generation. With this approach, structural error and mechanical error are considered together [108].

Analytical precision position and optimum synthesis methods for linkages to generate specified force and torque histories were applied to the planar four-bar mechanism. Using the *integration of power equilibrium method*, the torque generation problem is reduced to a function generation problem [109]. The relationship between the three-position function generation problem and the solution space for planar four bar mechanisms was considered and it was shown that two infinities of solutions are possible. These solutions were mapped in a plane as a contour plot to determine the locations where particular types of function generator solutions occur [110].

A third-order polynomial was presented which defines the locus of function generator solutions that have equal deviation of their transmission angle from the ideal of 90° throughout the entire range of motion. When these solutions are mapped into a Cartesian plane, the ground pivot locations produce curves similar to the familiar Burmester curves for four-position synthesis problems [111,112]. Galerkin's method was applied to function generation. Here synthesis is realized by solving the system of equations derived from the conditions where the four-bar displacement function is orthogonal to a set of weighting functions, which are equal, in number, to the number of existing parameters. [113].

Three-position function generation has also been applied in the design of a toggle clamp. A locus of linkage solutions for each toggle position is plotted from a quadratic equation. This approach also offers control over the mechanical advantage at clamping position and defect elimination [114].

4.3. Developments 2000–2009

Since the year 2000, there has also been a substantial increase in the number of developments made in quantitative planar four-bar function generation compared to past decades. Developments during the years 2000–2009 include the optimum synthesis of a planar four-bar function generator for use as an Ackermann steering linkage [115]. A particular case of M_1 motion called T_1 motion (which occurs when the moving body instantaneously experiences a first-order rotational dwell) was applied to produce an algebraic formulation for the function generation of a steering linkage [116]. The *Taguchi method* (a statistical method) was used on the tolerance design of a four-bar function generator to obtain a structural error that is insensitive to variations in manufacturing tolerance and joint clearance [117]. Both the kinematic synthesis and dynamic analysis of a planar four-bar function generator (for use as a quick-return mechanism) were conducted using the commercial mathematical analysis software *Matlab* and its modelling and simulation add-on package *Simulink* [118]. A combination of both parametric computer-aided design and numerical motion generation (called *geometric constraint programming*) provides visual geometric insight into three-, four- and five-precision point function generation problems [119].

4.4. Developments 2010–2017

Developments during the years 2010–2017 include the application of the Taguchi method to determine the optimal design parameter tolerance ranges for function generation with joint clearances [120,121]. A decision support system for function generator design from which

designs are selected, ranked and analysed based on kinematic and non-kinematic criteria was developed [122]. The *generalized reduced gradient method* (a method not previously applied in synthesis) was applied to optimum function generation. An objective function formulated as the difference between the desired and generated functional parameters was minimized using the generalized reduced gradient method [123]. Function generation was applied in the design of a perching micro-air vehicle. Function generators were synthesized to control the pitching and sweeping motions of the micro-air vehicle's wings [124]. The synthesis method presented for path generation and type synthesis of a spring-connected rectangular block model was extended to function generation [77,125]. Both the *artificial immune system algorithm* and the genetic algorithm were utilized in optimum function generation methods for use as Ackerman steering mechanisms [126–128].

A study was conducted where different heuristic optimization methods were compared in an optimum function generation problem. In the function generation problem, the distribution of five precision points and the stroke angles of the crank and follower links were considered as optimization variables [129]. A rotation pole and rotation angle-based analytical method was presented for seven precision point function generation. This analytical method is defined by a system of 20 non-linear equations with 20 unknowns [130]. Using Freudenstein's equation, an optimum synthesis method for five-point function generation was developed. The least square optimization technique is used in this method to minimize structural error between the precision points [131]. Optimal function generation with joint clearances was again considered as an analysis of linkage reliability, but in one study, the reliability analyses are time-dependent and the joint clearances are random [132]. In another study, the reliability analyses are time-dependent and the joint clearances are quantified using a non-probabilistic convex model [133]. A monad vector-based formulation was presented for function generation with finitely, infinitesimally and multiply separated precision points [134]. A Fourier series-based analytical model was presented for function generation for an indefinite number of precision points [135].

5. Developments in quantitative spherical and spatial four-bar path and function generation

5.1. Path generation

Developments in quantitative spherical path generation include a five-point path generation method. With this method spherical dyad solutions (in angular coordinate form) are calculated by solving a fifth-order polynomial [136]. A general method for spherical (and 1 DOF spatial four-bar) coupler curve generation and evaluation was presented. This method was implemented in a computer programme for the *optimum synthesis* (*optimum*, meaning full crank rotatability and branch defect elimination are ensured) of four-bar spatial and spherical path generators [137]. A least squares-based constrained optimization model was presented for spherical path generation for an indefinite number of precision points [138,139]. Though initially formulated for planar path and function generation, the *numerical atlas method* has been extended to spherical path generation [140–142]. Spherical path generation through the use of a differential *evolutionary algorithm* offers advantages including finding the true global minimum of a multi modal search space (regardless of the initial parameter values), fast convergence and the use of fewer control parameters in

comparison to non-evolutionary algorithms [143]. A differential evolutionary algorithm was also applied to the problem of spherical path generation where the acceleration magnitudes of the linkage centre of mass are minimized for shaking force balancing [144]. A constrained non-linear optimization model was developed for spherical path generation with order, branch and circuit defect elimination. This model was codified and solved in the commercial mathematical analysis software *Matlab* (using the interior-point algorithm) [145].

Developments in quantitative spatial path generation include non-linear equation systems (that are solved iteratively) for the synthesis of a skew four-bar spatial linkage type known as *Bennett's mechanism* for approximate path generation [146,147]. On the basis of the theorem that *for any given 1-DOF spatial linkage one can find an infinite number of spatial linkages tracing the same coupler curve*, an indirect cognate of the RSSP linkage was synthesized [148].

5.2. Function generation

Developments in quantitative spherical function generation include a constrained optimization model for minimum structural error. This model was solved using the *interior penalty function method* [149]. An expanded set of computer-generated crank vs. follower displacement angle plots was presented. Such plots are used in a function generation method known as *curve matching* – where a desired plot is matched with the computer-generated plots to determine a spherical four-bar linkage that best approximates the desired plot [150]. A method for *minimum defect* spherical (and planar) function generation was presented. A minimum defect linkage is defined as one having full crank link rotatability and maximum transmission quality [151]. A method that ultimately converts an optimum synthesis problem for spherical function generation to an adaptive curve-fitting problem (using spherical circle point curves) was presented [152]. An analytical method for five-precision point synthesis of spherical function generators was developed. From this method, a five-precision point function generator solution is calculated from a set of 15 linear equations (formulated from an original set of five non-linear equations) [153]. Improvements in spherical three- and four-point function generation include the definition of design coefficients that ultimately result in a design process based on a simple set of linear equations which produce closed-form solutions [154]. A numerical atlas-based method (called the *adaptive numerical atlas method*) was applied for the synthesis of a spherical function generator for use as a pendulum disc-type cutting mechanism in a harvester combine. With the adaptive numerical atlas method, the database of spherical linkage basic dimension types is self-adaptive [155]. A method for optimum spherical function generators for six linkage dimension parameters for at least six precision points was presented. The objective function was derived using quaternion algebra and three approximation approaches (interpolation, least squares and *Chebyshev* approximation) are used in the synthesis procedure [156].

Developments in quantitative spatial function generation include an approach to synthesize RSSR function generators to achieve higher order motion. This is essentially achieved by selecting the linkage's spherical joints from fourth-order curve points (where the fourth-order curves originate from rigid-body motion) [157]. *Function cognates* are those mechanisms that provide exactly the same relationship between the displacements of the input and output links as in the original mechanism. An algebraic method was applied to investigate the function cognates of the RSRC linkage [158]. A method was developed

for the synthesis of RSSR function generators for high-speed applications. This is achieved through the derivation of approximate equations that minimize the higher harmonics of the driving link [159,160].

The *continuous design transmutation method* (a numerical method based on *polynomial continuation theory*) was applied for the synthesis of aRSSR function generator. In the application of this method, the start system and homotopy were based on physical design (not pure mathematical equations) [161].

6. Developments in general quantitative synthesis methods

While the works presented in the prior sections are focused on either path generation or function generation exclusively, methods have been presented that are directly applicable to both synthesis areas. A general optimal synthesis method for spatial linkages was developed. This method includes displacement matrix-derived objective functions and constraints and is implemented by Powell's algorithm [162]. A constrained multifactor optimization problem was applied to RRRR and RRRP path and function generation. In this problem, the contributions of kinematic structural error, mechanical error and link length were considered to indicate design feasibility [163]. A unified method for motion path and function generation was developed where algebraic-geometrical techniques are used to obtain all proper crank-rocker and drag-link linkage solutions with transmission angles in the range $45^\circ \leq \mu \leq 135^\circ$ [164]. Although a geometric construction method was presented for spherical four-bar motion, path and function generation, algebraic expressions for each geometric construction are also provided [165].

A computer-aided design optimization method to satisfy specified kinematic and dynamic conditions was developed. The kinematic conditions consist of coupler point motion and coupler and follower rotation specifications. The dynamic conditions consist of the minimization of the average power consumed by the linkage and a limit on the maximum input torque [166,167]. A numerical optimization method was presented that includes stages for kinematic analysis (via decomposition of the mechanism into Assur groups), linkage assembly (via a penalty function) and optimization (via non-derivative and quasi-Newton methods) [168]. An optimum synthesis method was proposed for planar mechanisms with lower pairs and a configuration of any type. This method begins with a mechanism of dimensions required to satisfy the synthesis conditions (allowing the deformation of its component elements). The mechanism that least needs to be deformed will be the optimum linkage [169].

A formulation that uses exact differentiation (rather than numerical differentiation) to obtain gradient elements was developed for approximate dimensional synthesis [170]. Geometric constraint programming was also applied to general dimension synthesis for finitely, infinitesimally and multiply separated positions [171–173]. A general method was presented for planar, spatial and spherical four-bar path and function generation [174]. After discovering the relationship between harmonic components of the linkage mathematical expressions and the harmonic characteristic parameters of coupler rotation-angle operators, a numerical atlas database is established. A general synthesis method (considering four-bar trajectory, function or velocity generation) was developed that utilizes the *sequential quadratic programming algorithm* for non-linear optimization [175]. The numerical atlas method was applied to RRSS path generation and RSSR function generation [176]. An

optimal dimensional synthesis method for planar mechanisms using differential evolution was demonstrated for several cases including hybrid tasks (tasks involving motion, path and function generation) [177].

The combination of both dimensional synthesis and static force analysis produced what is called *kinetostatic synthesis*. This combined capability enables one to synthesize linkages with flexible links or compliant linkages. A framework for the kinetostatic synthesis of a compliant planar four-bar motion, path and function generator has been established [178]. A generalized approach for the design of compliant mechanisms using a *pseudo rigid-body model* was also presented. The pseudo-rigid-body four-bar mechanism concept considered includes four rigid links with one to four torsional springs located at each revolute joint [179].

7. Developments in quantitative adjustable four-bar path and function generation

A single four-bar path or function generator achieves a single group of precision points with its calculated dimensions. If a four-bar linkage can be designed with links that are adjustable, a single linkage can achieve multiple groups of precision points. This capability gives a four-bar path or function generator a greater range of flexibility.

Developments in adjustable path generation include a planar four-bar linkage with an adjustable fixed pivot to approximate linear, curved and tangent paths [180]. A Grashof double crank linkage with an adjustable fixed pivot offset was considered for adjustable dwell time ratios [181]. A method was presented for a planar four-bar linkage that can be adjusted in real time (by *iterative learning control*) for minimum structural error [182]. A variable coupler curve linkage design was presented that incorporates an adjustable screw-nut for follower link length and/or fixed pivot adjustments [183]. The authors also presented an adjustable four-bar linkage that incorporates a motor-actuated sliding joint on the driving link to achieve variable coupler curves [184]. Variable coupler curve linkages were also presented that incorporated an ionic polymer-metal composite for follower link length adjustments [185–187]. An optimal synthesis method was developed for multi-phase continuous path generation using a planar four-bar linkage with an adjustable length follower dyad [188]. Optimal synthesis methods were presented for a crank-rocker and general four-bar linkage with an adjustable follower link fixed pivot [189,190]. Optimum synthesis methods were presented for adjustable linkages (including follower link fixed pivot-adjustable) to achieve rectangular paths and filleted rectangular paths [191,192]. A method for the inverse kinematic and dynamic analysis of a four-bar mechanism with a continuous one-parameter adjustment was presented [193].

Developments in adjustable function generation include an adjustable function generation method given precision points, velocities, accelerations and higher order quantities [194]. A method to synthesize a four-bar adjustable function generator to achieve two specified functions was presented. This method utilizes five-bar linkage kinematic theory [195]. Incorporating smart materials as actuators produced a function generator that can be adjusted to eliminate the high harmonic output of the follower link (during high speeds) [196]. Optimum synthesis methods for planar linkages with adjustable coupler or follower link lengths or adjustable follower link fixed pivots have also been developed [197,198]. An optimal synthesis method for a mechanism that includes a crank length-adjustable

function generator was also presented as well as a spherical four-bar function generator with adjustable connecting frame poles [199,200].

8. Discussion

While this work presents well over 100 methodologies for quantitative dimensional synthesis, neither it nor the authors' prior survey paper on quantitative motion generation provides any quantitative comparison or ranking of the methodologies in terms of solution accuracy or calculation speed [1]. Rather than compare and rank the methodologies presented in this work based on their respective example problems (which can be highly subjective), a truly objective comparison and ranking should consist of an independent study where these methodologies are codified and used to solve a common array of examples. The authors welcome such an effort from the research community and look forward to either producing such a work or seeing such a work published from the research community.

9. Conclusion

While early path and function generation methods (like early motion generation methods) were primarily *qualitative*, the introduction of computing technology in science and engineering has laid a foundation for *quantitative* path and function generation. This work, a continuation of the authors' first survey on qualitative motion generation, provides a survey of developments in quantitative four-bar path and function generation since 1970 [1]. Ongoing advancements in computing hardware and software continue to make qualitative dimensional synthesis in all of its areas (motion, path and function generation) more robust and practical.

Disclosure statement

No potential conflict of interest was reported by the authors.

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