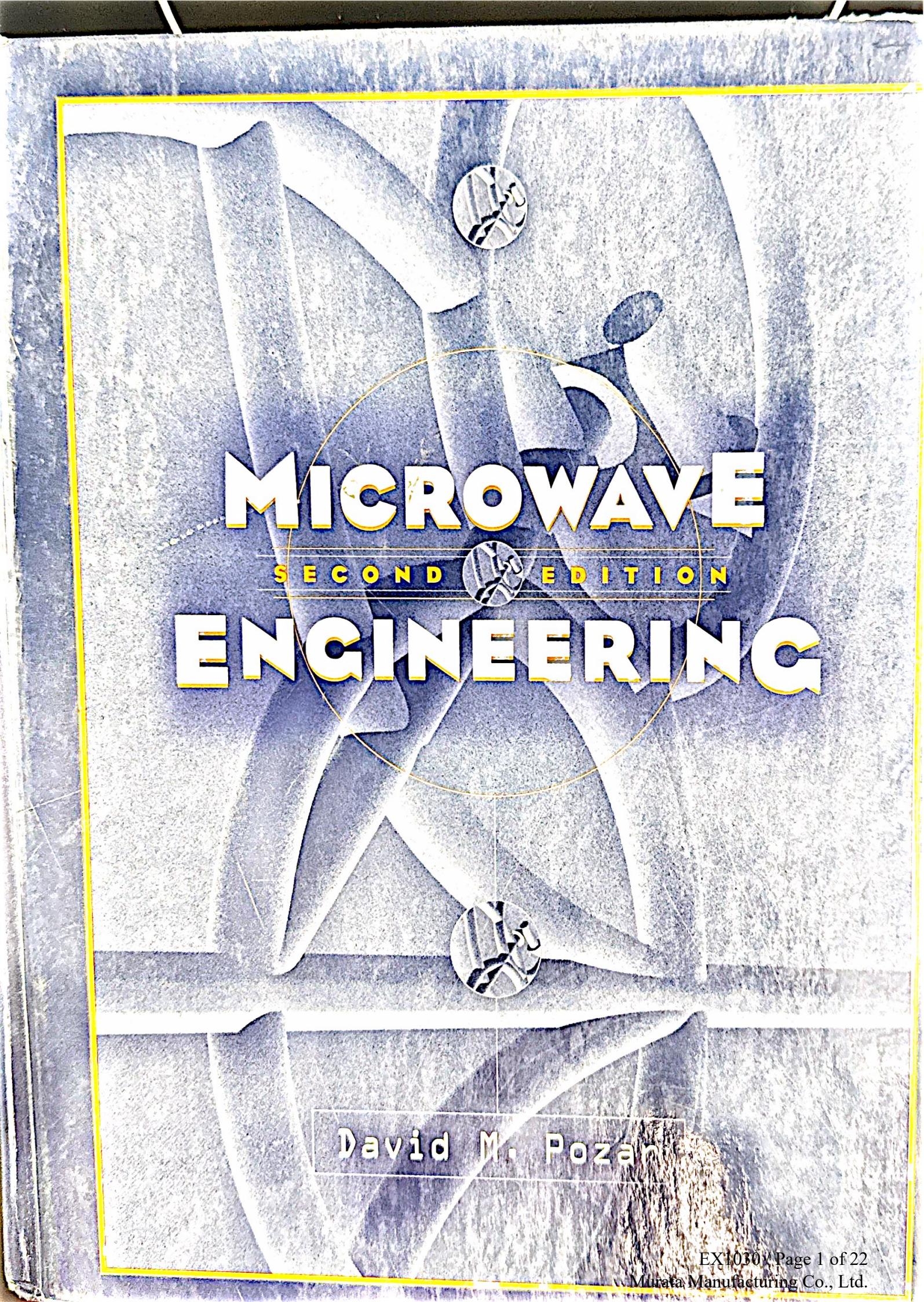




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SECOND EDITION

ENGINEERING

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network is usually cheaper, more reliable, and less lossy than a more complex design.

- **Bandwidth**—Any type of matching network can ideally give a perfect match (zero reflection) at a single frequency. In many applications, however, it is desirable to match a load over a band of frequencies. There are several ways of doing this with, of course, a corresponding increase in complexity.
- **Implementation**—Depending on the type of transmission line or waveguide being used, one type of matching network may be preferable compared to another. For example, tuning stubs are much easier to implement in waveguide than are multisection quarter-wave transformers.
- **Adjustability**—In some applications the matching network may require adjustment to match a variable load impedance. Some types of matching networks are more amenable than others in this regard.

5.1

MATCHING WITH LUMPED ELEMENTS (*L* NETWORKS)

Probably the simplest type of matching network is the *L* section, which uses two reactive elements to match an arbitrary load impedance to a transmission line. There are two possible configurations for this network, as shown in Figure 5.2. If the normalized load impedance, $z_L = Z_L/Z_0$, is inside the $1 + jx$ circle on the Smith chart, then the circuit of Figure 5.2a should be used. If the normalized load impedance is outside the $1 + jx$ circle on the Smith chart, the circuit of Figure 5.2b should be used. The $1 + jx$ circle is the resistance circle on the impedance Smith chart for which $r = 1$.

In either of the configurations of Figure 5.2, the reactive elements may be either inductors or capacitors, depending on the load impedance. Thus, there are eight distinct possibilities for the matching circuit for various load impedances. If the frequency is low enough and/or the circuit size is small enough, actual lumped-element capacitors and inductors can be used. This may be feasible for frequencies up to about 1 GHz or so, although modern microwave integrated circuits may be small enough so that lumped elements can be used at higher frequencies as well. There is, however, a large range of frequencies and circuit sizes where lumped elements may not be realizable. This is a limitation of the *L* section matching technique.

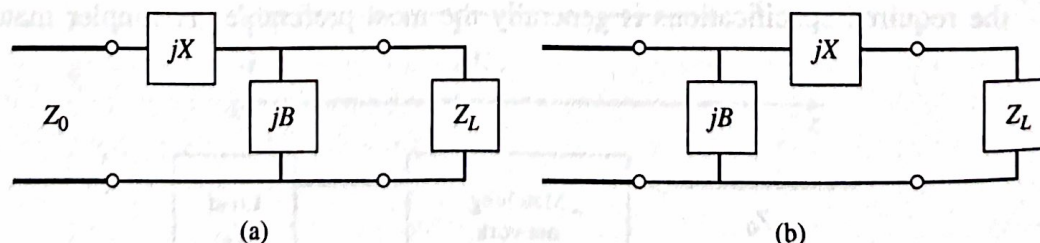


FIGURE 5.2 *L* section matching networks. (a) Network for z_L inside the $1 + jx$ circle. (b) Network for z_L outside the $1 + jx$ circle.

We will now derive the analytic expressions for the matching network elements of the two cases in Figure 5.2, then illustrate an alternative design procedure using the Smith chart.

Analytic Solutions

Although we will discuss a simple graphical solution using the Smith chart, it may be useful to derive expressions for the *L* section matching network components. Such expressions would be useful in a computer-aided design program for *L* section matching, or when it is necessary to have more accuracy than the Smith chart can provide.

Consider first the circuit of Figure 5.2a, and let $Z_L = R_L + jX_L$. We stated that this circuit would be used when $z_L = Z_L/Z_0$ is inside the $1 + jx$ circle on the Smith chart, which implies that $R_L > Z_0$ for this case.

The impedance seen looking into the matching network followed by the load impedance must be equal to Z_0 , for a match:

$$Z_0 = jX + \frac{1}{jB + 1/(R_L + jX_L)}. \quad 5.1$$

Rearranging and separating into real and imaginary parts gives two equations for the two unknowns, X and B :

$$B(XR_L - X_L Z_0) = R_L - Z_0, \quad 5.2a$$

$$X(1 - BX_L) = BZ_0 R_L - X_L. \quad 5.2b$$

Solving (5.2a) for X and substituting into (5.2b) gives a quadratic equation for B . The solution is

$$B = \frac{X_L \pm \sqrt{R_L/Z_0} \sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}. \quad 5.3a$$

Note that since $R_L > Z_0$, the argument of the second square root is always positive. Then the series reactance can be found as

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{BR_L}. \quad 5.3b$$

Equation (5.3a) indicates that two solutions are possible for B and X . Both of these solutions are physically realizable, since both positive and negative values of B and X are possible (positive X implies an inductor, negative X implies a capacitor, while positive B implies a capacitor and negative B implies an inductor.) One solution, however, may result in significantly smaller values for the reactive components, and may be the preferred solution if the bandwidth of the match is better, or the SWR on the line between the matching network and the load is smaller.

Now consider the circuit of Figure 5.2b. This circuit is to be used when z_L is outside the $1 + jx$ circle on the Smith chart, which implies that $R_L < Z_0$. The admittance seen looking into the matching network followed by the load impedance $Z_L = R_L + jX_L$

must be equal to $1/Z_0$, for a match:

$$\frac{1}{Z_0} = jB + \frac{1}{R_L + j(X + X_L)} \quad 5.4$$

Rearranging and separating into real and imaginary parts gives two equations for the two unknowns, X and B :

$$BZ_0(X + X_L) = Z_0 - R_L, \quad 5.5a$$

$$(X + X_L) = BZ_0R_L. \quad 5.5b$$

Solving for X and B gives

$$X = \pm \sqrt{R_L(Z_0 - R_L)} - X_L, \quad 5.6a$$

$$B = \pm \frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}. \quad 5.6b$$

Since $R_L < Z_0$, the arguments of the square roots are always positive. Again, note that two solutions are possible.

In order to match an arbitrary complex load to a line of characteristic impedance Z_0 , the real part of the input impedance to the matching network must be Z_0 , while the imaginary part must be zero. This implies that a general matching network must have at least two degrees of freedom; in the L section matching circuit these two degrees of freedom are provided by the values of the two reactive components.

Smith Chart Solutions

Instead of the above formulas, the Smith chart can be used to quickly and accurately design L section matching networks, a procedure best illustrated by an example.



EXAMPLE 5.1 L -Section Impedance Matching

Design an L section matching network to match a series RC load with an impedance $Z_L = 200 - j100 \Omega$, to a 100Ω line, at a frequency of 500 MHz.

Solution

The normalized load impedance is $z_L = 2 - j1$, which is plotted on the Smith chart of Figure 5.3a. This point is inside the $1 + jx$ circle, so we will use the matching circuit of Figure 5.2a. Since the first element from the load is a shunt susceptance, it makes sense to convert to admittance by drawing the SWR circle through the load, and a straight line from the load through the center of the chart, as shown in Figure 5.3a. Now, after we add the shunt susceptance and convert back to impedance, we want to be on the $1 + jx$ circle, so that we can add a series reactance to cancel the jx and match the load. This means that the shunt susceptance must move us from y_L to the $1 + jx$ circle on the admittance Smith chart. Thus, we construct the rotated $1 + jx$ circle as shown in Figure 5.3a (center at 0.333). (A combined ZY chart is convenient to use here, if it is not too confusing.) Then we see that adding a susceptance of $jb = j0.3$ will move us along a constant conductance circle to $y = 0.4 + j0.5$ (this choice

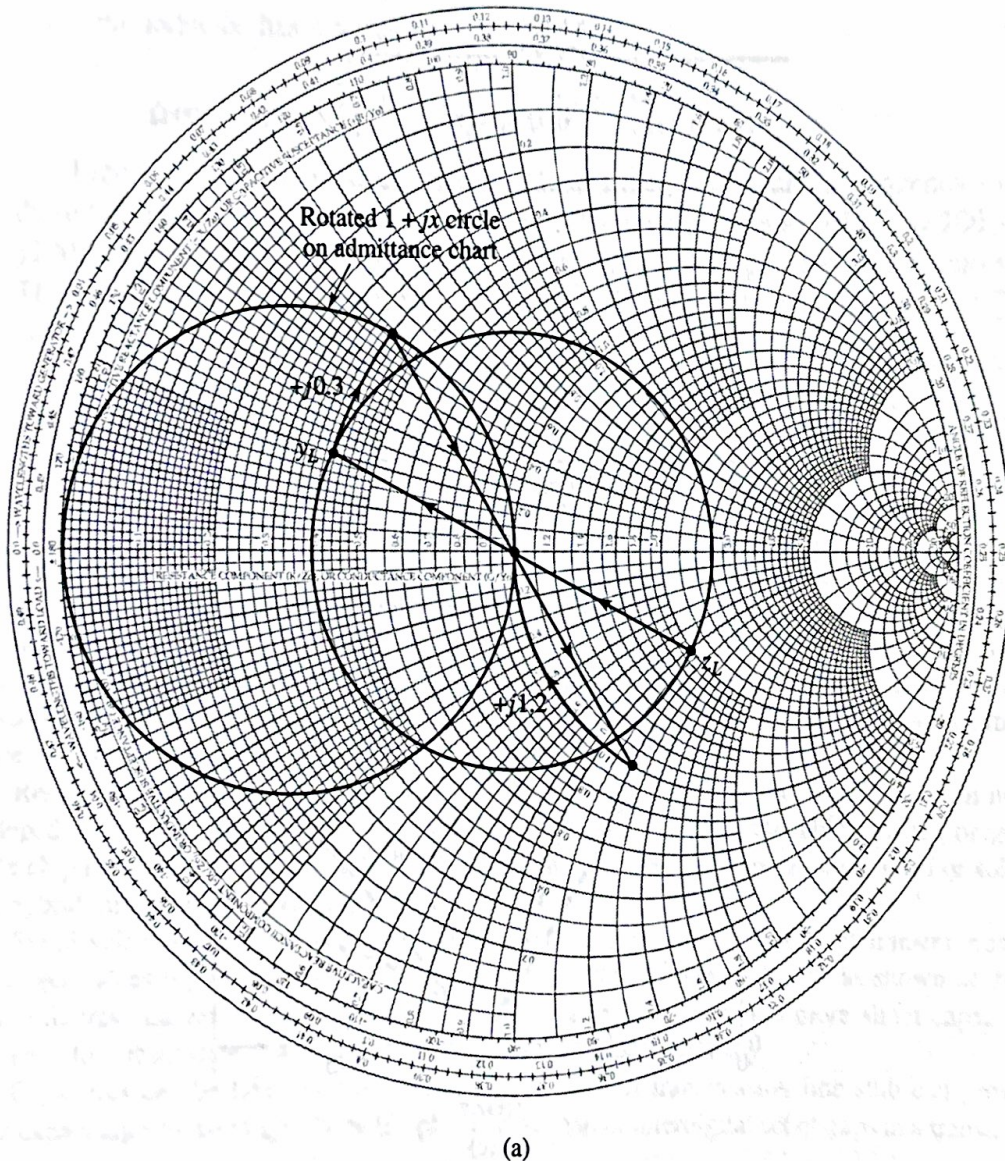


FIGURE 5.3 Solution to Example 5.1. (a) Smith chart for the L section matching networks.

is the shortest distance from y_L to the shifted $1 + jx$ circle). Converting back to impedance leaves us at $z = 1 - j1.2$, indicating that a series reactance $x = j1.2$ will bring us to the center of the chart. For comparison, the formulas of (5.3a,b) give the solution as $b = 0.29$, $x = 1.22$.

This matching circuit consists of a shunt capacitor and a series inductor, as shown in Figure 5.3b. For a frequency of $f = 500$ MHz, the capacitor has a value of

$$C = \frac{b}{2\pi f Z_0} = 0.92 \text{ pF},$$

and the inductor has a value of

$$L = \frac{x Z_0}{2\pi f} = 38.8 \text{ nH}.$$

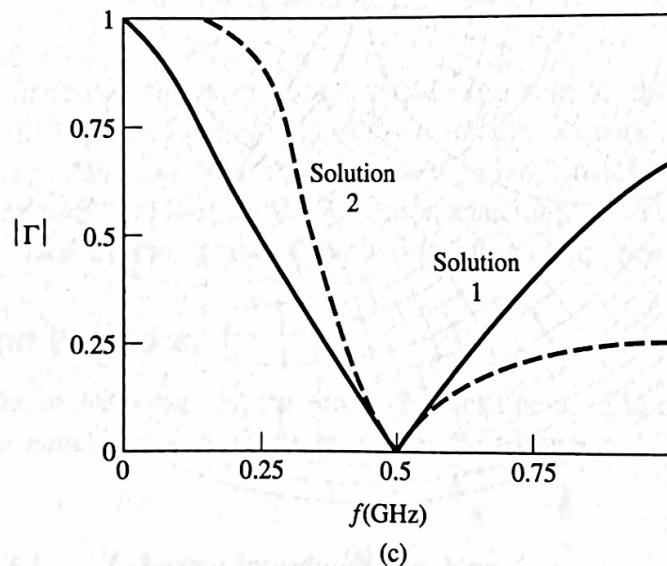
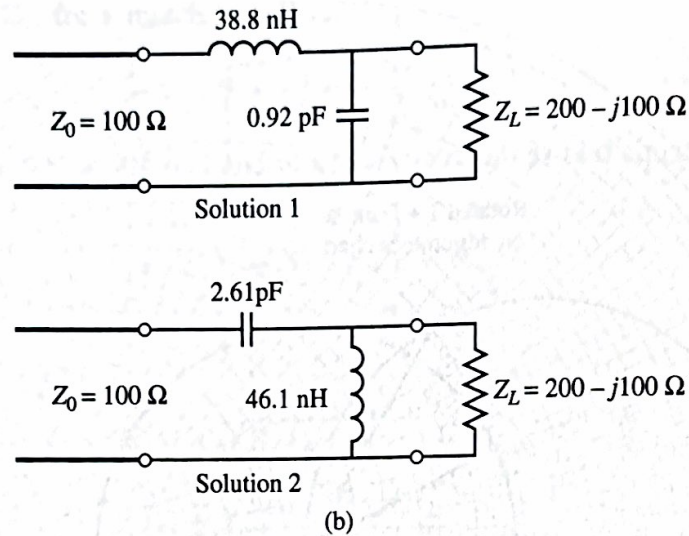


FIGURE 5.3 Continued. (b) The two possible L section matching circuits. (c) Reflection coefficient magnitudes versus frequency for the matching circuits of (b).

It may also be interesting to look at the second solution to this matching problem. If instead of adding a shunt susceptance of $b = 0.3$, we use a shunt susceptance of $b = -0.7$, we will move to a point on the lower half of the shifted $1 + jx$ circle, to $y = 0.4 - j0.5$. Then converting to impedance and adding a series reactance of $x = -1.2$ leads to a match as well. The formulas of (5.3a,b) give this solution as $b = -0.69$, $x = -1.22$. This matching circuit is also shown in Figure 5.3b, and is seen to have the positions of the inductor and capacitor reversed from the first matching network. At a frequency of $f = 500$ MHz, the capacitor has a value of

$$C = \frac{-1}{2\pi f x Z_0} = 2.61 \text{ pF},$$

while the inductor has a value of

$$L = \frac{-Z_0}{2\pi fb} = 46.1 \text{ nH.}$$

Figure 5.3c shows the reflection coefficient magnitude versus frequency for these two matching networks, assuming that the load impedance of $Z_L = 200 - j100 \Omega$ at 500 MHz consists of a 200Ω resistor and a 3.18 pF capacitor in series. There is not a substantial difference in bandwidth for these two solutions. ○



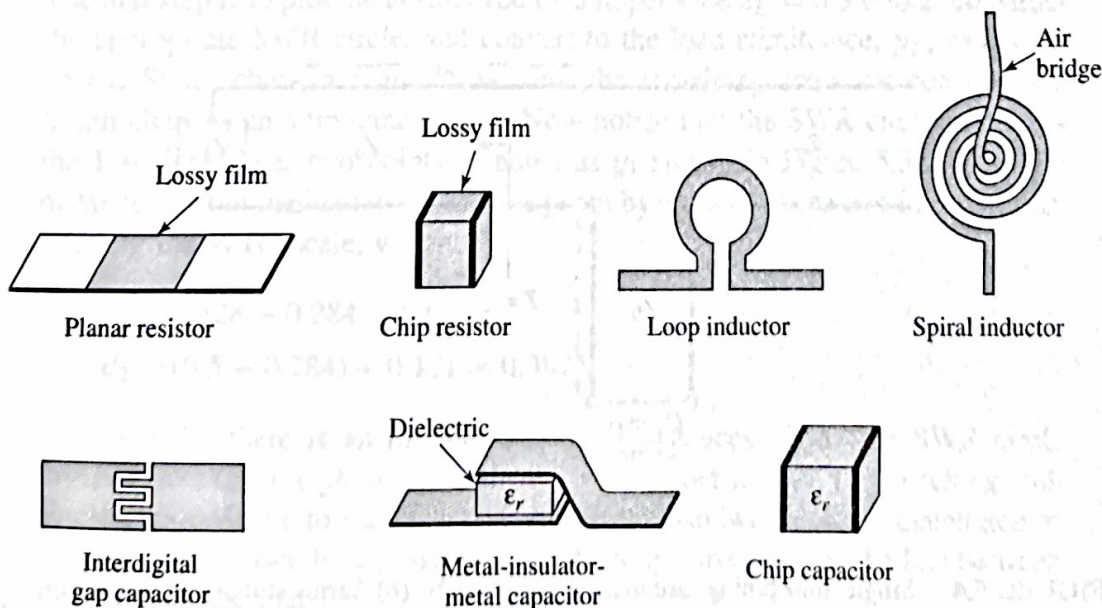
POINT OF INTEREST: Lumped Elements for Microwave Integrated Circuits

Lumped R , L , and C elements can be practically realized at microwave frequencies if the length, ℓ , of the component is very small relative to the operating wavelength. Over a limited range of values, such components can be used in hybrid and monolithic microwave integrated circuits (MICs) at frequencies up to 60 GHz, if the condition that $\ell < \lambda/10$ is satisfied. Usually, however, the characteristics of such an element are far from ideal, requiring that undesirable effects such as parasitic capacitance and/or inductance, spurious resonances, fringing fields, loss, and perturbations caused by a ground plane be incorporated in the design via a CAD model (see the Point of Interest concerning CAD).

Resistors are fabricated with thin films of lossy material such as nichrome, tantalum nitride, or doped semiconductor material. In monolithic circuits such films can be deposited or grown, while chip resistors made from a lossy film deposited on a ceramic chip can be bonded or soldered in a hybrid circuit. Low resistances are hard to obtain.

Small values of inductance can be realized with a short length or loop of transmission line, and larger values (up to about 10 nH) can be obtained with a spiral inductor, as shown in the following figures. Larger inductance values generally incur more loss, and more shunt capacitance; this leads to a resonance that limits the maximum operating frequency.

Capacitors can be fabricated in several ways. A short transmission line stub can provide a shunt capacitance in the range of 0 to 0.1 pF. A single gap or interdigital set of gaps in a transmission



line can provide a series capacitance up to about 0.5 pF. Greater values (up to about 25 pF) can be obtained using a metal-insulator-metal (MIM) sandwich, either in monolithic or chip (hybrid) form.

5.2 SINGLE-STUB TUNING

We next consider a matching technique that uses a single open-circuited or short-circuited length of transmission line (a “stub”), connected either in parallel or in series with the transmission feed line at a certain distance from the load, as shown in Figure 5.4. Such a tuning circuit is convenient from a microwave fabrication aspect, since lumped elements are not required. The shunt tuning stub is especially easy to fabricate in microstrip or stripline form.

In single-stub tuning, the two adjustable parameters are the distance, d , from the load to the stub position, and the value of susceptance or reactance provided by the shunt or series stub. For the shunt-stub case, the basic idea is to select d so that the admittance, Y , seen looking into the line at distance d from the load is of the form $Y_0 + jB$. Then

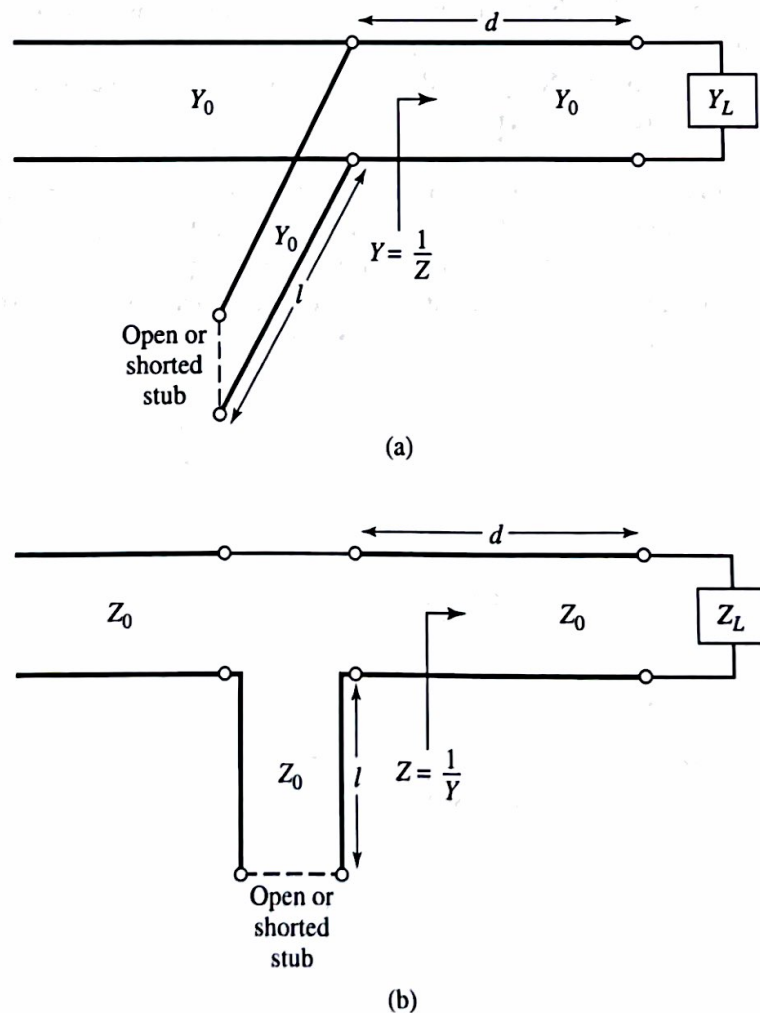


FIGURE 5.4 Single-stub tuning circuits. (a) Shunt stub. (b) Series stub.

the stub susceptance is chosen as $-jB$, resulting in a matched condition. For the series stub case, the distance d is selected so that the impedance, Z , seen looking into the line at a distance d from the load is of the form $Z_0 + jX$. Then the stub reactance is chosen as $-jX$, resulting in a matched condition.

As discussed in Chapter 2, the proper length of open or shorted transmission line can provide any desired value of reactance or susceptance. For a given susceptance or reactance, the difference in lengths of an open- or short-circuited stub is $\lambda/4$. For transmission line media such as microstrip or stripline, open-circuited stubs are easier to fabricate since a via hole through the substrate to the ground plane is not needed. For lines like coax or waveguide, however, short-circuited stubs are usually preferred, because the cross-sectional area of such an open-circuited line may be large enough (electrically) to radiate, in which case the stub is no longer purely reactive.

Below we discuss both Smith chart and analytic solutions for shunt and series stub tuning. The Smith chart solutions are fast, intuitive, and usually accurate enough in practice. The analytic expressions are more accurate, and useful for computer analysis.

Shunt Stubs

The single-stub shunt tuning circuit is shown in Figure 5.4a. We will first discuss an example illustrating the Smith chart solution, and then derive formulas for d and ℓ .



EXAMPLE 5.2 Single-Stub Shunt Tuning

For a load impedance $Z_L = 15 + j10 \, \Omega$, design two single-stub shunt tuning networks to match this load to a $50 \, \Omega$ line. Assuming that the load is matched at 2 GHz, and that the load consists of a resistor and inductor in series, plot the reflection coefficient magnitude from 1 GHz to 3 GHz for each solution.

Solution

The first step is to plot the normalized load impedance $z_L = 0.3 + j0.2$, construct the appropriate SWR circle, and convert to the load admittance, y_L , as shown on the Smith chart in Figure 5.5a. For the remaining steps we consider the Smith chart as an admittance chart. Now notice that the SWR circle intersects the $1 + jb$ circle at two points, denoted as y_1 and y_2 in Figure 5.5a. Thus the distance d , from the load to the stub, is given by either of these two intersections. Reading the WTG scale, we obtain

$$d_1 = 0.328 - 0.284 = 0.044\lambda,$$

$$d_2 = (0.5 - 0.284) + 0.171 = 0.387\lambda.$$

Actually, there is an infinite number of distances, d , on the SWR circle that intersect the $1 + jb$ circle. Usually, it is desired to keep the matching stub as close as possible to the load, to improve the bandwidth of the match and to reduce losses caused by a possibly large standing wave ratio on the line between the stub and the load.

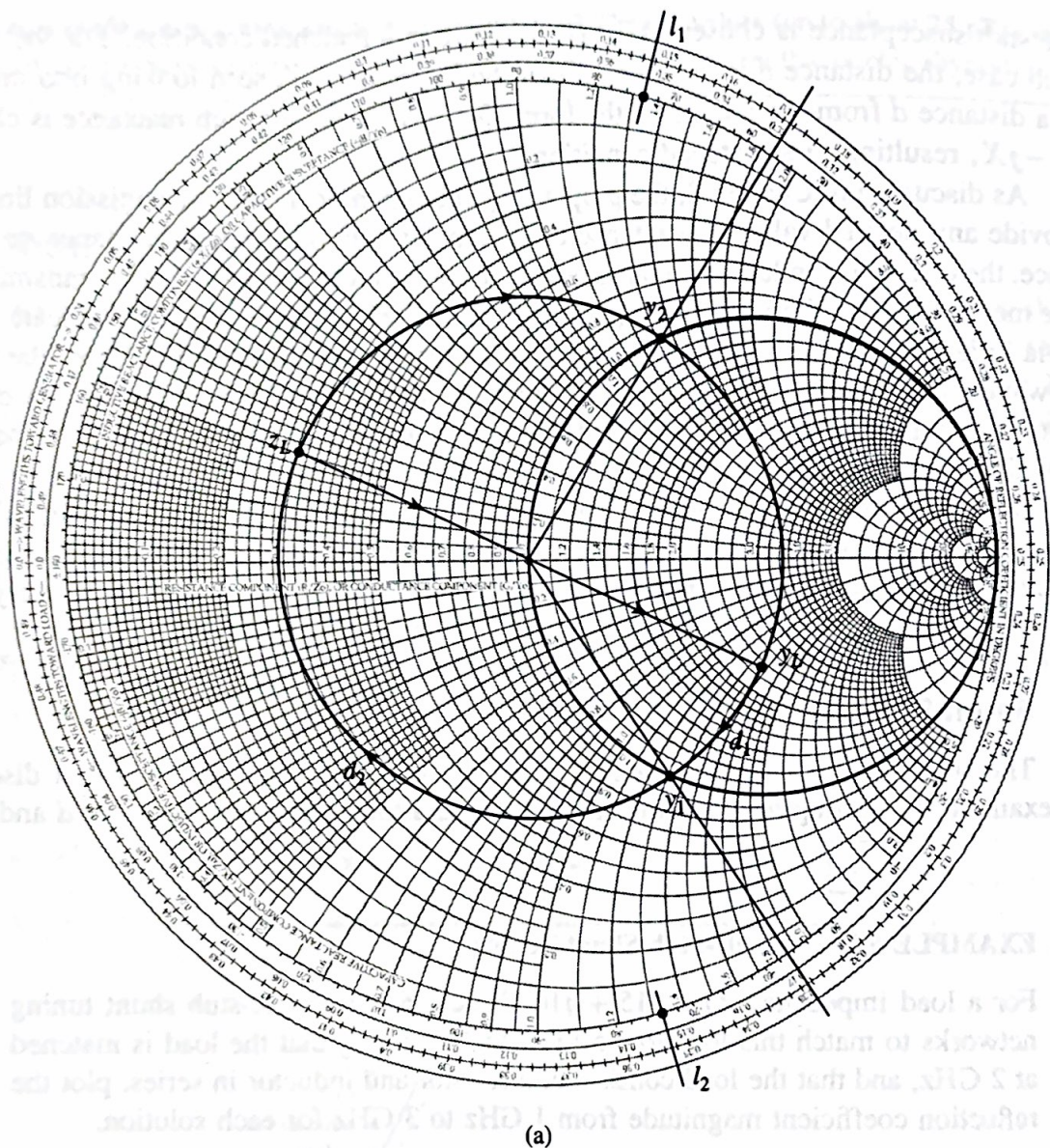


FIGURE 5.5 Solution to Example 5.2. (a) Smith chart for the shunt-stub tuners.

At the two intersection points, the normalized admittances are

$$y_1 = 1 - j1.33,$$

$$y_2 = 1 + j1.33.$$

Thus, the first tuning solution requires a stub with a susceptance of $j1.33$. The length of an open-circuited stub that gives this susceptance can be found on the Smith chart by starting at $y = 0$ (the open circuit) and moving along the outer edge of the chart ($g = 0$) toward the generator to the $j1.33$ point. The length is then

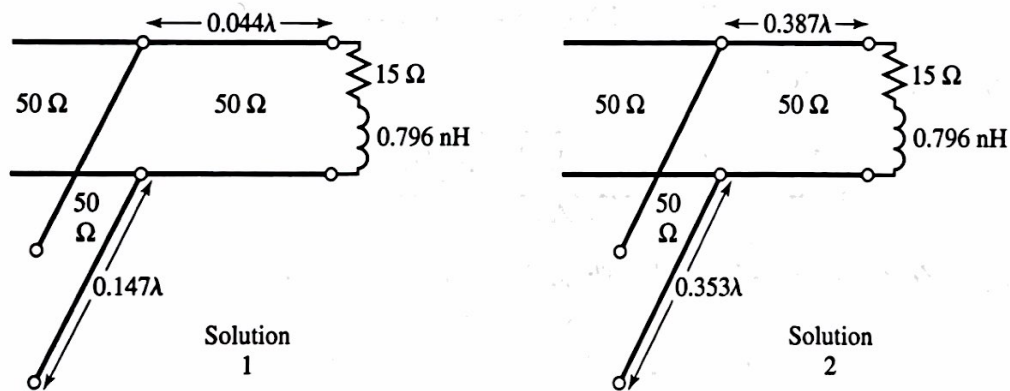
$$\ell_1 = 0.147\lambda.$$

Similarly, the required open-circuit stub length for the second solution is

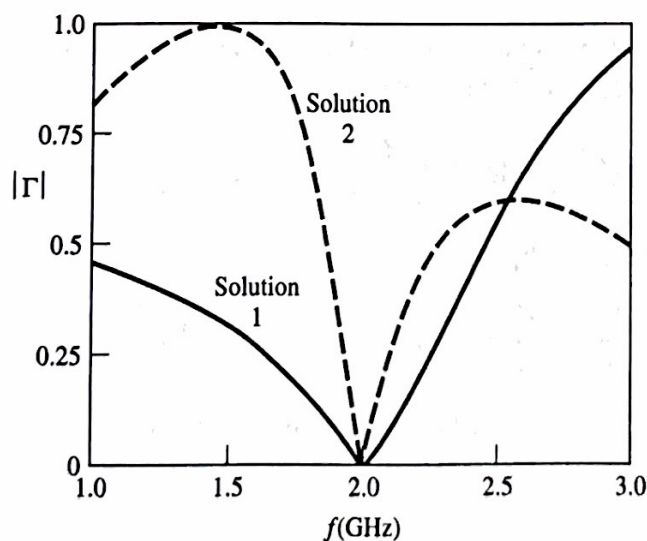
$$\ell_2 = 0.353\lambda.$$

This completes the tuner designs.

To analyze the frequency dependence of these two designs, we need to know the load impedance as a function of frequency. The series- RL load impedance is $Z_L = 15 + j10 \Omega$ at 2 GHz, so $R = 15 \Omega$ and $L = 0.796 \text{ nH}$. The two tuning circuits are shown in Figure 5.5b. Figure 5.5c shows the calculated reflection coefficient magnitudes for these two solutions. Observe that solution 1 has a significantly better bandwidth than solution 2; this is because both d and ℓ are shorter for solution 1, which reduces the frequency variation of the match. ○



(b)



(c)

FIGURE 5.5 Continued. (b) The two shunt-stub tuning solutions. (c) Reflection coefficient magnitudes versus frequency for the tuning circuits of (b).

To derive formulas for d and ℓ , let the load impedance be written as $Z_L = 1/Y_L = R_L + jX_L$. Then the impedance Z down a length, d , of line from the load is

$$Z = Z_0 \frac{(R_L + jX_L) + jZ_0 t}{Z_0 + j(R_L + jX_L)t}, \quad 5.7$$

where $t = \tan \beta d$. The admittance at this point is

$$Y = G + jB = \frac{1}{Z},$$

$$\text{where } G = \frac{R_L(1 + t^2)}{R_L^2 + (X_L + Z_0 t)^2}, \quad 5.8a$$

$$B = \frac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0[R_L^2 + (X_L + Z_0 t)^2]}. \quad 5.8b$$

Now d (which implies t) is chosen so that $G = Y_0 = 1/Z_0$. From (5.8a), this results in a quadratic equation for t :

$$Z_0(R_L - Z_0)t^2 - 2X_L Z_0 t + (R_L Z_0 - R_L^2 - X_L^2) = 0.$$

Solving for t gives

$$t = \frac{X_L \pm \sqrt{R_L[(Z_0 - R_L)^2 + X_L^2]/Z_0}}{R_L - Z_0}, \quad \text{for } R_L \neq Z_0. \quad 5.9$$

If $R_L = Z_0$, then $t = -X_L/2Z_0$. Thus, the two principal solutions for d are

$$\frac{d}{\lambda} = \begin{cases} \frac{1}{2\pi} \tan^{-1} t, & \text{for } t \geq 0 \\ \frac{1}{2\pi} (\pi + \tan^{-1} t), & \text{for } t < 0. \end{cases} \quad 5.10$$

To find the required stub lengths, first use t in (5.8b) to find the stub susceptance, $B_s = -B$. Then, for an open-circuited stub,

$$\frac{\ell_o}{\lambda} = \frac{1}{2\pi} \tan^{-1} \left(\frac{B_s}{Y_0} \right) = \frac{-1}{2\pi} \tan^{-1} \left(\frac{B}{Y_0} \right), \quad 5.11a$$

while for a short-circuited stub,

$$\frac{\ell_s}{\lambda} = \frac{-1}{2\pi} \tan^{-1} \left(\frac{Y_0}{B_s} \right) = \frac{1}{2\pi} \tan^{-1} \left(\frac{Y_0}{B} \right). \quad 5.11b$$

If the length given by (5.11a) or (5.11b) is negative, $\lambda/2$ can be added to give a positive result.

Series Stubs

The series stub tuning circuit is shown in Figure 5.4b. We will illustrate the Smith chart solution by an example, and then derive expressions for d and ℓ .

**EXAMPLE 5.3** Single-Stub Series Tuning

Match a load impedance of $Z_L = 100 + j80$ to a $50\ \Omega$ line using a single series open-circuit stub. Assuming that the load is matched at 2 GHz, and that the load consists of a resistor and inductor in series, plot the reflection coefficient magnitude from 1 GHz to 3 GHz.

Solution

The first step is to plot the normalized load impedance, $z_L = 2 + j1.6$, and draw the SWR circle. For the series-stub design, the chart is an impedance chart. Note that the SWR circle intersects the $1 + jx$ circle at two points, denoted as z_1 and z_2 in Figure 5.6a. The shortest distance, d_1 , from the load to

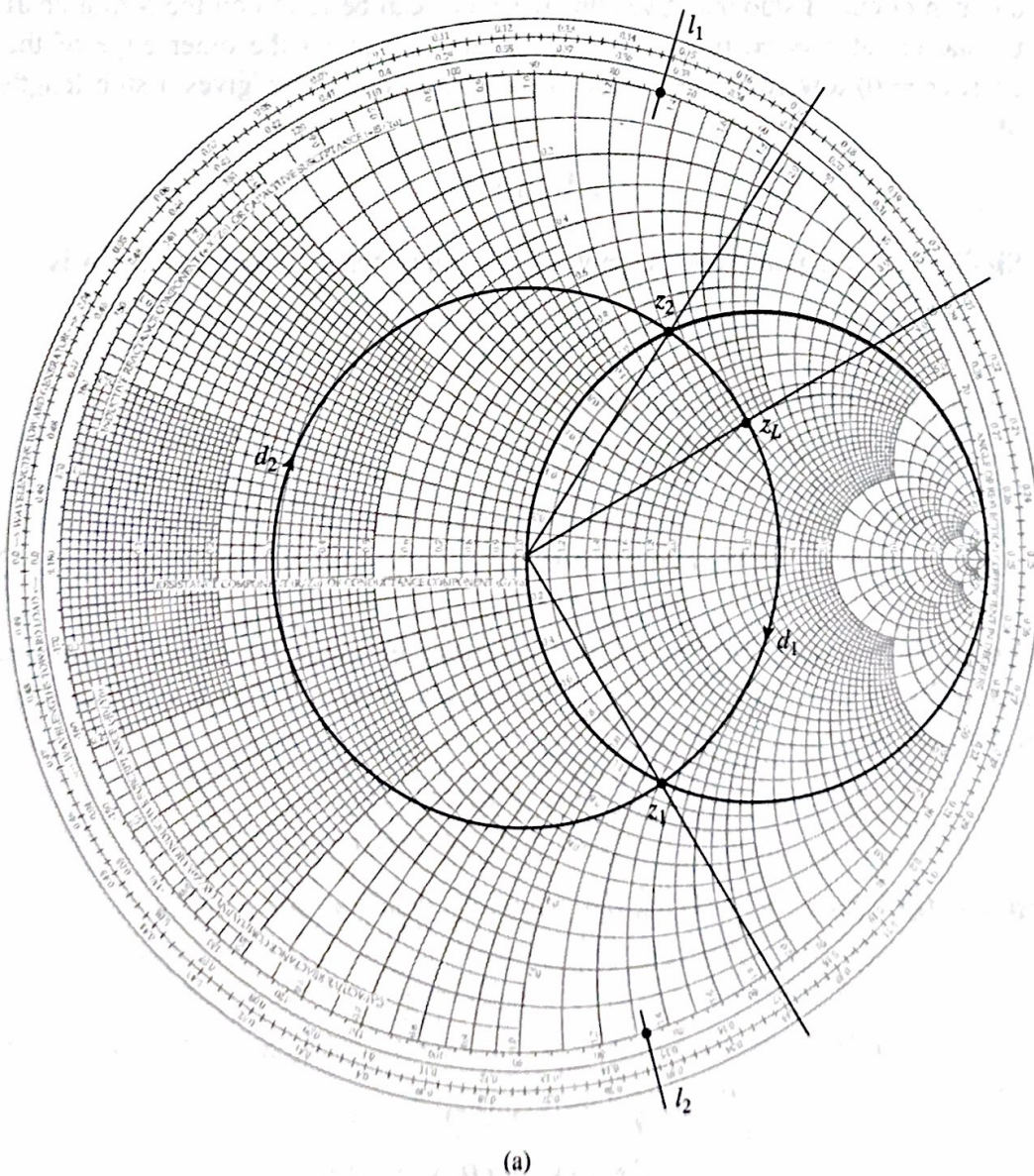


FIGURE 5.6 Solution to Example 5.3. (a) Smith chart for the series-stub tuners.

the stub is, from the WTG scale,

$$d_1 = 0.328 - 0.208 = 0.120\lambda,$$

while the second distance is

$$d_2 = (0.5 - 0.208) + 0.172 = 0.463\lambda.$$

As in the shunt-stub case, additional rotations around the SWR circle lead to additional solutions, but these are usually not of practical interest.

The normalized impedances at the two intersection points are

$$z_1 = 1 - j1.33,$$

$$z_2 = 1 + j1.33.$$

Thus, the first solution requires a stub with a reactance of $j1.33$. The length of an open-circuited stub that gives this reactance can be found on the Smith chart by starting at $z = \infty$ (open circuit), and moving along the outer edge of the chart ($r = 0$) toward the generator to the $j1.33$ point. This gives a stub length of

$$\ell_1 = 0.397\lambda.$$

Similarly, the required open-circuited stub length for the second solution is

$$\ell_2 = 0.103\lambda.$$

This completes the tuner designs.

If the load is a series resistor and inductor with $Z_L = 100 + j80 \Omega$ at 2 GHz, then $R = 100 \Omega$ and $L = 6.37$ nH. The two matching circuits are shown in Figure 5.6b. Figure 5.6c shows the calculated reflection coefficient magnitudes versus frequency for the two solutions. $\bigcirc$

To derive formulas for d and ℓ for the series-stub tuner, let the load admittance be written as $Y_L = 1/Z_L = G_L + jB_L$. Then the admittance Y down a length, d , of line from the load is

$$Y = Y_0 \frac{(G_L + jB_L) + jtY_0}{Y_0 + jt(G_L + jB_L)}, \quad 5.12$$

where $t = \tan \beta d$, and $Y_0 = 1/Z_0$. Then the impedance at this point is

$$Z = R + jX = \frac{1}{Y},$$

where

$$R = \frac{G_L(1 + t^2)}{G_L^2 + (B_L + Y_0 t)^2}, \quad 5.13a$$

$$X = \frac{G_L^2 t - (Y_0 - tB_L)(B_L + tY_0)}{Y_0[G_L^2 + (B_L + Y_0 t)^2]}. \quad 5.13b$$

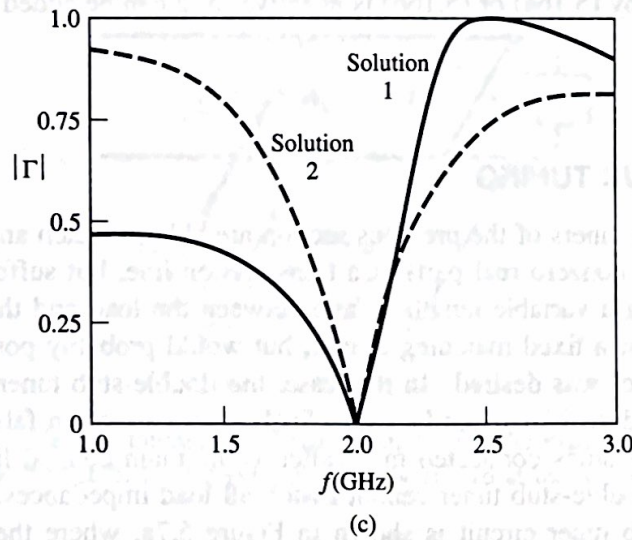
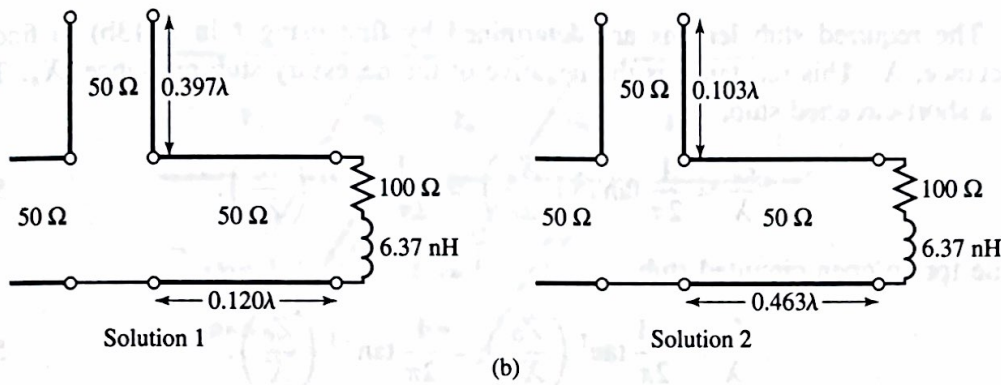


FIGURE 5.6 Continued. (b) The two series-stub tuning solutions. (c) Reflection coefficient magnitudes versus frequency for the tuning circuits of (b).

Now d (which implies t) is chosen so that $R = Z_0 = 1/Y_0$. From (5.13a), this results in a quadratic equation for t :

$$Y_0(G_L - Y_0)t^2 - 2B_L Y_0 t + (G_L Y_0 - G_L^2 - B_L^2) = 0.$$

Solving for t gives

$$t = \frac{B_L \pm \sqrt{G_L(Y_0 - G_L)^2 + B_L^2}/Y_0}{G_L - Y_0}, \quad \text{for } G_L \neq Y_0. \quad 5.14$$

If $G_L = Y_0$, then $t = -B_L/2Y_0$. Then the two principal solutions for d are

$$d/\lambda = \begin{cases} \frac{1}{2\pi} \tan^{-1} t & \text{for } t \geq 0 \\ \frac{1}{2\pi} (\pi + \tan^{-1} t) & \text{for } t < 0. \end{cases} \quad 5.15$$

The required stub lengths are determined by first using t in (5.13b) to find the reactance, X . This reactance is the negative of the necessary stub reactance, X_s . Thus, for a short-circuited stub,

$$\frac{\ell_s}{\lambda} = \frac{1}{2\pi} \tan^{-1} \left(\frac{X_s}{Z_0} \right) = \frac{-1}{2\pi} \tan^{-1} \left(\frac{X}{Z_0} \right), \quad 5.16a$$

while for an open-circuited stub,

$$\frac{\ell_o}{\lambda} = \frac{-1}{2\pi} \tan^{-1} \left(\frac{Z_0}{X_s} \right) = \frac{1}{2\pi} \tan^{-1} \left(\frac{Z_0}{X} \right). \quad 5.16b$$

If the length given by (5.16a) or (5.16b) is negative, $\lambda/2$ can be added to give a positive result.

5.3 DOUBLE-STUB TUNING

The single-stub tuners of the previous section are able to match any load impedance (as long as it has a nonzero real part) to a transmission line, but suffer from the disadvantage of requiring a variable length of line between the load and the stub. This may not be a problem for a fixed matching circuit, but would probably pose some difficulty if an adjustable tuner was desired. In this case, the double-stub tuner, which uses two tuning stubs in fixed positions, can be used. Such tuners are often fabricated in coaxial line, with adjustable stubs connected in parallel to the main coaxial line. We will see, however, that the double-stub tuner cannot match all load impedances.

The double-stub tuner circuit is shown in Figure 5.7a, where the load may be an arbitrary distance from the first stub. Although this is more representative of a practical situation, the circuit of Figure 5.7b, where the load Y'_L has been transformed back to the position of the first stub, is easier to deal with and does not lose any generality. The stubs shown in Figure 5.7 are shunt stubs, which are usually easier to implement in practice than are series stubs; the latter could be used just as well, in principle. In either case, the stubs can be open-circuited or short-circuited.

Smith Chart Solution

The Smith chart of Figure 5.8 illustrates the basic operation of the double-stub tuner. As in the case of the single-stub tuners, two solutions are possible. The susceptance of the first stub, b_1 (or b'_1 , for the second solution), moves the load admittance to y_1 (or y'_1). These points lie on the rotated $1 + jb$ circle; the amount of rotation is d wavelengths toward the load, where d is the electrical distance between the two stubs. Then transforming y_1 (or y'_1) toward the generator through a length, d , of line leaves us at the point y_2 (or y'_2), which must be on the $1 + jb$ circle. The second stub then adds a susceptance b_2 (or b'_2), which brings us to the center of the chart, and completes the match.

Notice from Figure 5.8 that if the load admittance, y_L , were inside the shaded region of the $g_0 + jb$ circle, no value of stub susceptance b_1 could ever bring the load point

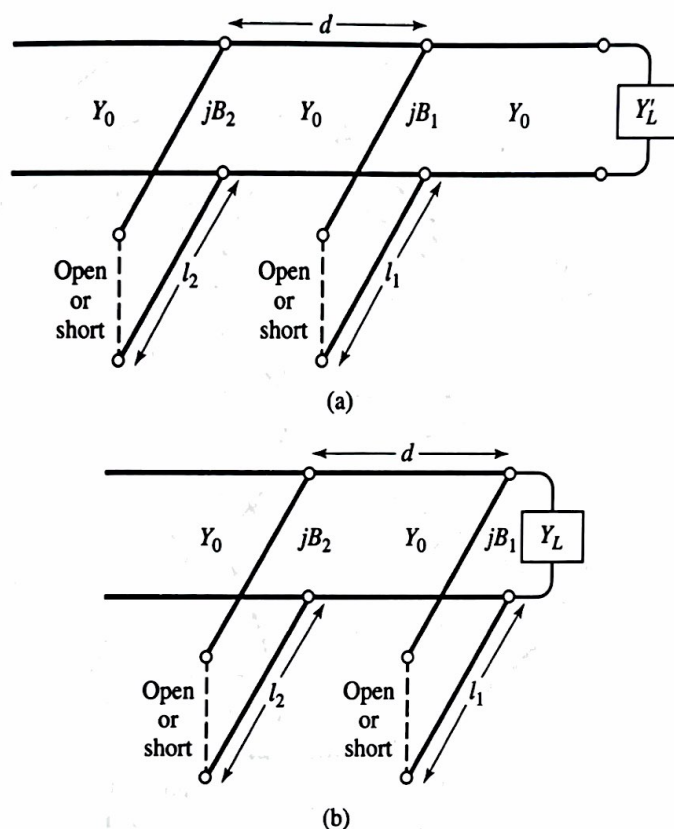


FIGURE 5.7 Double-stub tuning. (a) Original circuit with the load an arbitrary distance from the first stub. (b) Equivalent circuit with load at the first stub.

to intersect the rotated $1 + jb$ circle. This shaded region thus forms a forbidden range of load admittances, which cannot be matched with this particular double-stub tuner. A simple way of reducing the forbidden range is to reduce the distance, d , between the stubs. This has the effect of swinging the rotated $1 + jb$ circle back toward the $y = \infty$ point, but d must be kept large enough for the practical purpose of fabricating the two separate stubs. In addition, stub spacings near 0 or $\lambda/2$ lead to matching networks that are very frequency sensitive. In practice, stub spacings are usually chosen as $\lambda/8$ or $3\lambda/8$. If the length of line between the load and the first stub can be adjusted, then the load admittance y_L can always be moved out of the forbidden region.



EXAMPLE 5.4 Double-Stub Tuning

Design a double-stub shunt tuner to match a load impedance $Z_L = 60 - j80 \, \Omega$ to a $50 \, \Omega$ line. The stubs are to be short-circuited stubs, and are spaced $\lambda/8$ apart. Assuming that this load consists of a series resistor and capacitor, and that the match frequency is 2 GHz, plot the reflection coefficient magnitude versus frequency from 1 GHz to 3 GHz.

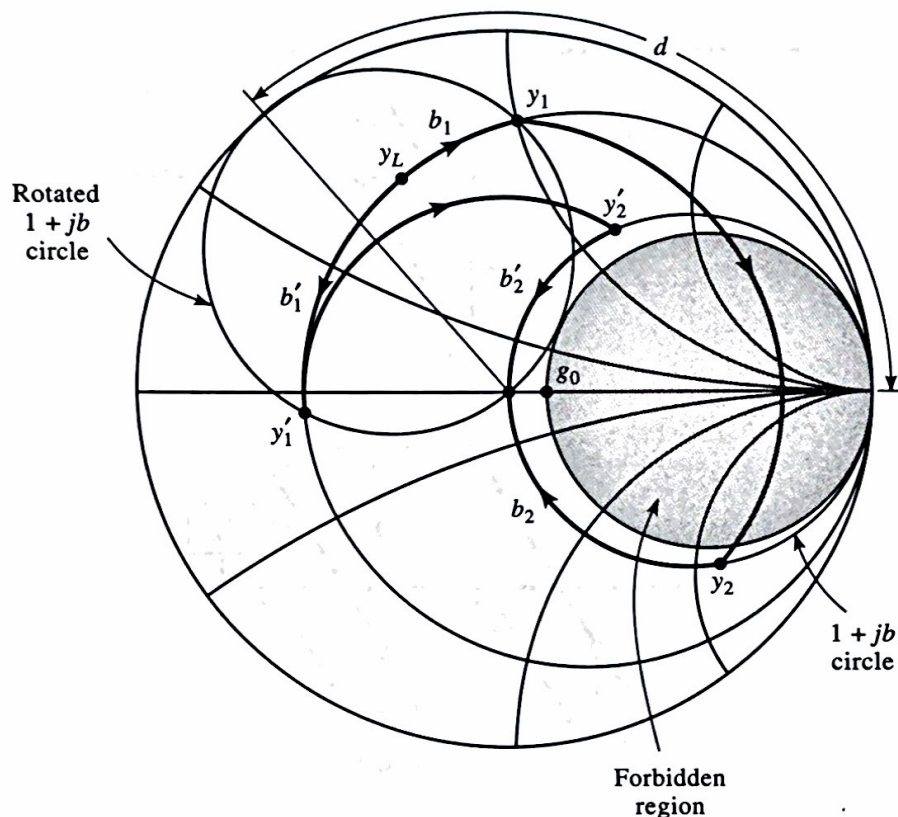


FIGURE 5.8 Smith chart diagram for the operation of a double-stub tuner.

Solution

The normalized load admittance is $y_L = 0.3 + j0.4$, which is plotted on the Smith chart of Figure 5.9a. Next we construct the rotated $1 + jb$ conductance circle, by moving every point on the $g = 1$ circle $\lambda/8$ toward the load. We then find the susceptance of the first stub, which can be one of two possible values:

$$b_1 = 1.314,$$

or

$$b'_1 = -0.114.$$

We now transform through the $\lambda/8$ section of line by rotating along a constant radius (SWR) circle $\lambda/8$ toward the generator. This brings the two solutions to the following points:

$$y_2 = 1 - j3.38,$$

or

$$y'_2 = 1 + j1.38.$$

Then the susceptance of the second stub should be

$$b_2 = 3.38,$$

or

$$b'_2 = -1.38.$$

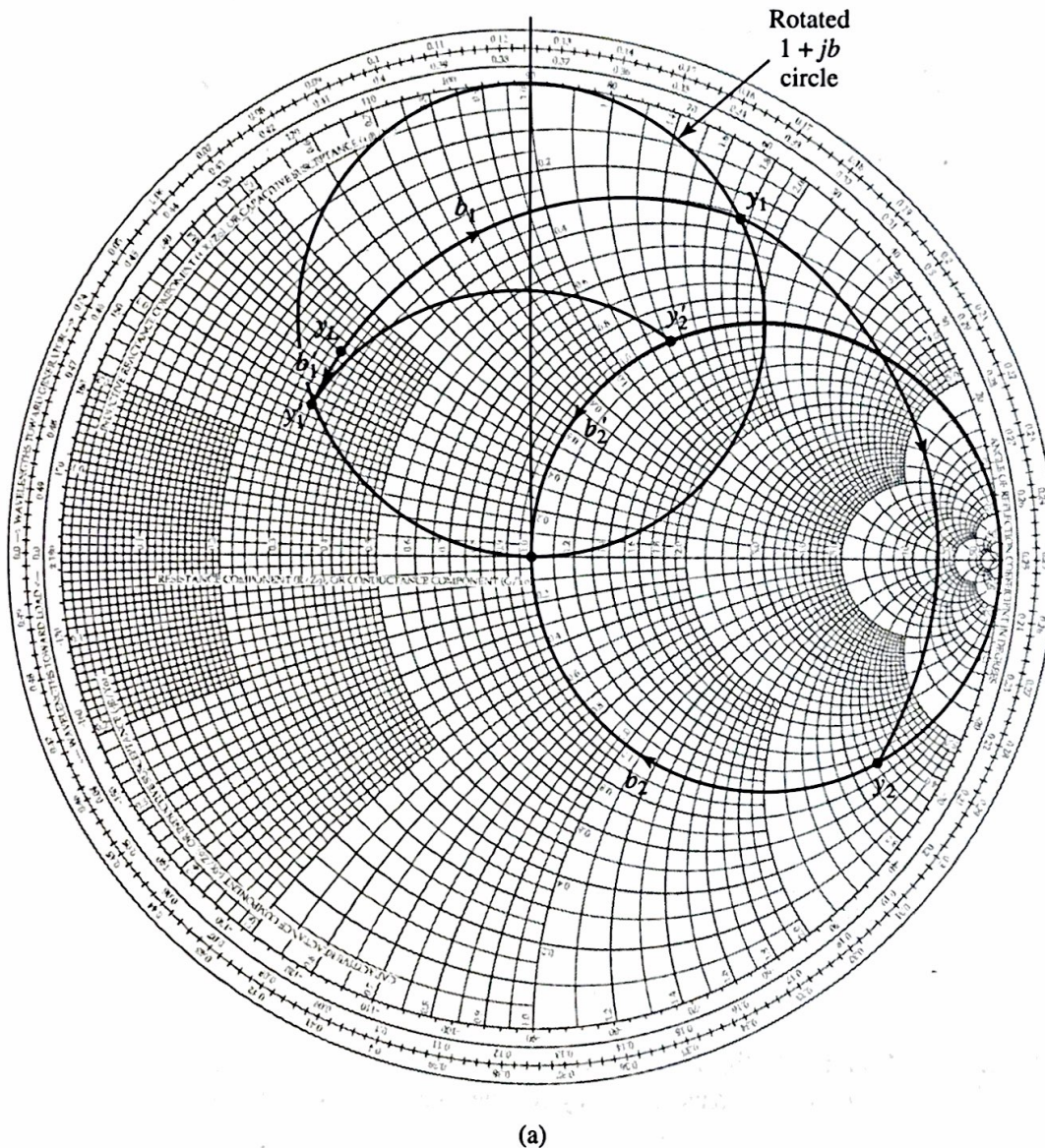


FIGURE 5.9 Solution to Example 5.4. (a) Smith chart for the double-stub tuners.

The lengths of the short-circuited stubs are then found as

$$\ell_1 = 0.396\lambda, \quad \ell_2 = 0.454\lambda,$$

or

$$\ell'_1 = 0.232\lambda, \quad \ell'_2 = 0.100\lambda.$$

This completes both solutions for the double-stub tuner design.

Now if the resistor-capacitor load $Z_L = 60 - j80 \, \Omega$ at $f = 2 \, \text{GHz}$, then $R = 60 \, \Omega$ and $C = 0.995 \, \text{pF}$. The two tuning circuits are then shown in Figure 5.9b, and the reflection coefficient magnitudes are plotted versus frequency in Figure 5.9c. Note that the first solution has a much narrower bandwidth than the second (primed) solution, due to the fact that both stubs for the first solution are somewhat longer (and closer to $\lambda/2$) than the stubs of the second solution.



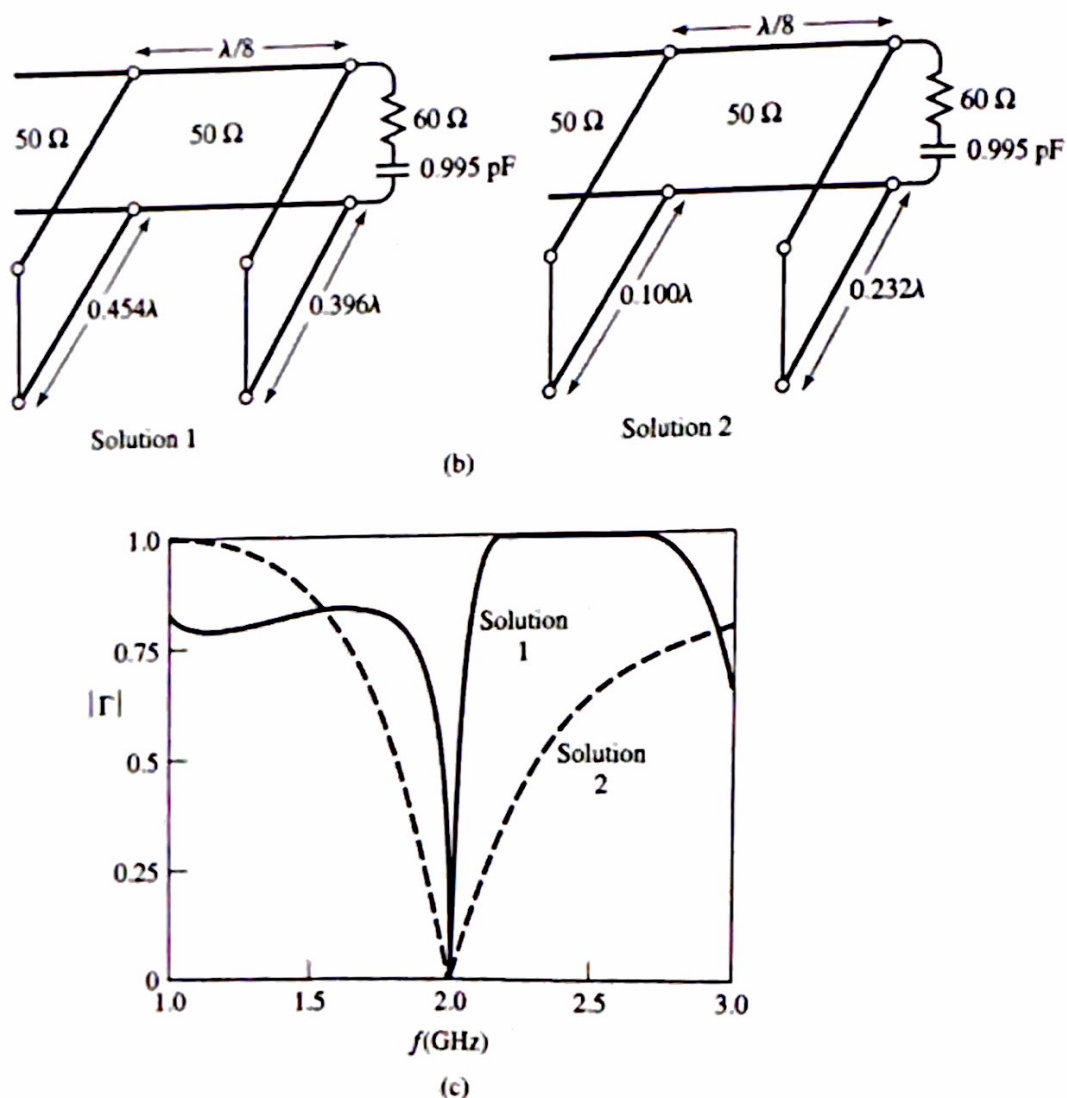


FIGURE 5.9 Continued. (b) The two double-stub tuning solutions. (c) Reflection coefficient magnitudes versus frequency for the tuning circuits of (b).

Analytic Solution

Just to the left of the first stub in Figure 5.7b, the admittance is

$$Y_1 = G_L + j(B_L + B_1), \quad 5.17$$

where $Y_L = G_L + jB_L$ is the load admittance and B_1 is the susceptance of the first stub. After transforming through a length d of transmission line, the admittance just to the right of the second stub is

$$Y_2 = Y_0 \frac{G_L + j(B_L + B_1 + Y_0 t)}{Y_0 + jt(G_L + jB_L + jB_1)}, \quad 5.18$$

where $t = \tan \beta d$ and $Y_0 = 1/Z_0$. At this point, the real part of Y_2 must equal Y_0 , which leads to the equation

$$G_L^2 - G_L Y_0 \frac{1+t^2}{t^2} + \frac{(Y_0 - B_L t - B_1 t)^2}{t^2} = 0. \quad 5.19$$

Solving for G_L gives

$$G_L = Y_0 \frac{1+t^2}{2t^2} \left[1 \pm \sqrt{1 - \frac{4t^2(Y_0 - B_L t - B_1 t)^2}{Y_0^2(1+t^2)^2}} \right]. \quad 5.20$$

Since G_L is real, the quantity within the square root must be nonnegative, and so

$$0 \leq \frac{4t^2(Y_0 - B_L t - B_1 t)^2}{Y_0^2(1+t^2)^2} \leq 1.$$

This implies that

$$0 \leq G_L \leq Y_0 \frac{1+t^2}{t^2} = \frac{Y_0}{\sin^2 \beta d}, \quad 5.21$$

which gives the range on G_L that can be matched for a given stub spacing, d . After d has been fixed, the first stub susceptance can be determined from (5.19) as

$$B_1 = -B_L + \frac{Y_0 \pm \sqrt{(1+t^2)G_L Y_0 - G_L^2 t^2}}{t}. \quad 5.22$$

Then the second stub susceptance can be found from the negative of the imaginary part of (5.18) to be

$$B_2 = \frac{\pm Y_0 \sqrt{Y_0 G_L (1+t^2) - G_L^2 t^2} + G_L Y_0}{G_L t}. \quad 5.23$$

The upper and lower signs in (5.22) and (5.23) correspond to the same solutions. The open-circuited stub length is found as

$$\frac{\ell_o}{\lambda} = \frac{1}{2\pi} \tan^{-1} \left(\frac{B}{Y_0} \right), \quad 5.24a$$

while the short-circuited stub length is found as

$$\frac{\ell_s}{\lambda} = \frac{-1}{2\pi} \tan^{-1} \left(\frac{Y_0}{B} \right), \quad 5.24b$$

where $B = B_1, B_2$.

5.4 THE QUARTER-WAVE TRANSFORMER

As discussed in Section 2.5, the quarter-wave transformer is a simple and useful circuit for matching a real load impedance to a transmission line. An additional feature of the quarter-wave transformer is that it can be extended to multisection designs in a methodical manner, for broader bandwidth. If only a narrow band impedance match is required, a single-section transformer may suffice. But, as we will see in the next few