

# Electric Machinery

**Sixth Edition**

**A. E. Fitzgerald**

*Late Vice President for Academic Affairs  
and Dean of the Faculty  
Northeastern University*

**Charles Kingsley, Jr.**

*Late Associate Professor of Electrical  
Engineering, Emeritus  
Massachusetts Institute of Technology*

**Stephen D. Umans**

*Principal Research Engineer  
Department of Electrical Engineering and  
Computer Science  
Laboratory for Electromagnetic and  
Electronic Systems  
Massachusetts Institute of Technology*



Boston Burr Ridge, IL Dubuque, IA Madison, WI New York San Francisco St. Louis  
Bangkok Bogotá Caracas Kuala Lumpur Lisbon London Madrid Mexico City  
Milan Montreal New Delhi Santiago Seoul Singapore Sydney Taipei Toronto

# McGraw-Hill Higher Education

A Division of The McGraw-Hill Companies

## ELECTRIC MACHINERY, SIXTH EDITION

Published by McGraw-Hill, a business unit of The McGraw-Hill Companies, Inc., 1221 Avenue of the Americas, New York, NY 10020. Copyright © 2003, 1990, 1983, 1971, 1961, 1952 by The McGraw-Hill Companies, Inc. All rights reserved. Copyright renewed 1980 by Rosemary Fitzgerald and Charles Kingsley, Jr. All rights reserved. No part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written consent of The McGraw-Hill Companies, Inc., including, but not limited to, in any network or other electronic storage or transmission, or broadcast for distance learning.

Some ancillaries, including electronic and print components, may not be available to customers outside the United States.

This book is printed on acid-free paper.

International 1 2 3 4 5 6 7 8 9 0 DOC/DOC 0 9 8 7 6 5 4 3 2  
Domestic 1 2 3 4 5 6 7 8 9 0 DOC/DOC 0 9 8 7 6 5 4 3 2

ISBN 0-07-366009-4

ISBN 0-07-112193-5 (ISE)

Publisher: *Elizabeth A. Jones*

Developmental editor: *Michelle L. Flomenhoft*

Executive marketing manager: *John Wannemacher*

Project manager: *Rose Koos*

Production supervisor: *Sherry L. Kane*

Media project manager: *Jodi K. Banowetz*

Senior media technology producer: *Phillip Meek*

Coordinator of freelance design: *Rick D. Noel*

Cover designer: *Rick D. Noel*

Cover image courtesy of: *Rockwell Automation/Reliance Electric*

Lead photo research coordinator: *Carrie K. Burger*

Compositor: *Interactive Composition Corporation*

Typeface: *10/12 Times Roman*

Printer: *R. R. Donnelley & Sons Company/Crawfordsville, IN*

### Library of Congress Cataloging-in-Publication Data

Fitzgerald, A. E. (Arthur Eugene), 1909–

Electric machinery / A. E. Fitzgerald, Charles Kingsley, Jr., Stephen D. Umans. —6th ed.

p. cm. —(McGraw-Hill series in electrical engineering. Power and energy)

Includes index.

ISBN 0-07-366009-4—ISBN 0-07-112193-5

I. Electric machinery. I. Kingsley, Charles, 1904–. II. Umans, Stephen D. III. Title.

IV. Series.

TK2181 .F5 2003

621.31'042—dc21

2002070988

CIP

INTERNATIONAL EDITION ISBN 0-07-112193-5

Copyright © 2003. Exclusive rights by The McGraw-Hill Companies, Inc., for manufacture and export. This book cannot be re-exported from the country to which it is sold by McGraw-Hill. The International Edition is not available in North America.

www.mhhe.com

## **Speed and Torque Control**

**T**he objective of this chapter is to discuss various techniques for the control of electric machines. Since an in-depth discussion of this topic is both too extensive for a single chapter and beyond the scope of this book, the presentation here will necessarily be introductory in nature. We will present basic techniques for speed and torque control and will illustrate typical configurations of drive electronics that are used to implement the control algorithms. This chapter will build upon the discussion of power electronics in Chapter 10.

Note that the discussion of this chapter is limited to steady-state operation. The steady-state picture presented here is quite adequate for a wide variety of electric-machine applications. However, the reader is cautioned that system dynamics can play a critical role in some applications, with concerns ranging from speed of response to overall system stability. Although the techniques presented here form the basis for dynamic analyses, the constraints of an introductory textbook are such that a more extensive discussion, including transient and dynamic behavior, is not possible.

In the discussion of torque control for synchronous and induction machines, the techniques of field-oriented or vector control are introduced and the analogy is made with torque control in dc motors. This material is somewhat more sophisticated mathematically than the speed-control discussion and requires application of the  $dq0$  transformations developed in Appendix C. The chapter is written such that this material can be omitted at the discretion of the instructor without detracting from the discussion of speed control.

### **11.1 CONTROL OF DC MOTORS**

Before the widespread application of power-electronic drives to control ac machines, dc motors were by far the machines of choice in applications requiring flexibility of control. Although in recent years ac drives have become quite common, the ease of control of dc machines insure their continued use in many applications.

### 11.1.1 Speed Control

The three most common speed-control methods for dc motors are adjustment of the flux, usually by means of field-current control, adjustment of the resistance associated with the armature circuit, and adjustment of the armature terminal voltage.

**Field-Current Control** In part because it involves control at a relatively low power level (the power into the field winding is typically a small fraction of the power into the armature of a dc machine), *field-current control* is frequently used to control the speed of a dc motor with separately excited or shunt field windings. The equivalent circuit for a separately excited dc machine is found in Fig. 7.4a and is repeated in Fig. 11.1. The method is, of course, also applicable to compound motors. The shunt field current can be adjusted by means of a variable resistance in series with the shunt field. Alternatively, the field current can be supplied by power-electronic circuits which can be used to rapidly change the field current in response to a wide variety of control signals.

Figure 11.2a shows in schematic form a switching scheme for pulse-width modulation of the field voltage. This system closely resembles the pulse-width modulation system discussed in Section 10.3.2. It consists of a rectifier which rectifies the ac input voltage, a dc-link capacitor which filters the rectified voltage, producing a dc voltage  $V_{dc}$ , and a pulse-width modulator.

In this system, because only a unidirectional field current is required, the pulse-width modulator consists of a single switch and a free-wheeling diode rather than the more complex four-switch arrangement of Fig. 10.45. Assuming both the switch and diode to be ideal, the average voltage across the field winding will be equal to

$$V_f = DV_{dc} \quad (11.1)$$

where  $D$  is the duty cycle of the switching waveform; i.e.,  $D$  is the fraction of time that the switch  $S$  is on.

Figure 11.2b shows the resultant field current. Because in the steady-state the average voltage across the inductor must equal zero, the average field current  $I_f$  will thus be equal to

$$I_f = \frac{V_f}{R_f} = D \left( \frac{V_{dc}}{R_f} \right) \quad (11.2)$$

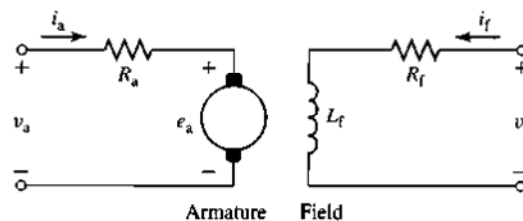


**Figure 11.1** Equivalent circuit for a separately excited dc motor.

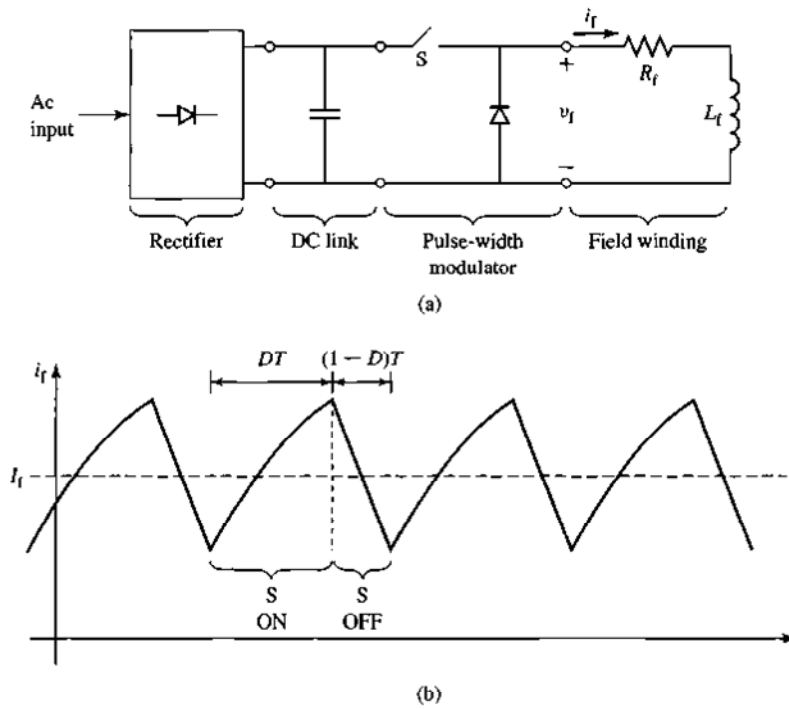


**Figure 11.2** (a) Pulse-width modulation system for a dc-machine field winding. (b) Field-current waveform.

Thus, the field current can be controlled simply by controlling the duty cycle of the pulse-width modulator. If the field-winding time constant  $L_f/R_f$  is long compared to the switching time, the ripple current  $\Delta i_f$  will be small compared to the average current  $I_f$ .

#### EXAMPLE 11.1

A 25-kW, 3600 r/min, 240-V dc motor has an armature resistance of  $47 \text{ m}\Omega$  and a shunt-field with a resistance of  $187 \Omega$  and an inductance of 4.2 H. Calculate (a) the average field current  $I_f$  and (b) the magnitude of the current ripple  $\Delta i_f$  when the field winding is supplied from a 240 V dc source by pulse-width modulation with a duty cycle  $D = 0.75$  and a switching period of 1 msec.

#### ■ Solution

a. The average field current is readily found from Eq. 11.2

$$I_f = D \left( \frac{V_{dc}}{R_f} \right) = 0.75 \left( \frac{240}{187} \right) = 0.96 \text{ A}$$

- b. The field time constant  $\tau = L_f/R_f = 22.5$  msec is much longer than the switching period of 1 msec. Thus, the ripple current can be calculated using Eq. 10.32 as

$$\begin{aligned}\Delta i_f &= \left( \frac{2V_{dc}}{R_f} \right) \left( \frac{T}{\tau} \right) D(1-D) \\ &= \left( \frac{2 \times 240}{187} \right) \left( \frac{1}{22.5} \right) 0.75 \times (1 - 0.75) \\ &= 21.4 \text{ mA}\end{aligned}$$

### Practice Problem 11.1

The duty-cycle  $D$  of the dc-motor controller of Example 11.1 is suddenly switched from 0.75 to 1.0. Calculate (a) the resultant steady-state field current and (b) the time constant for the change from the initial value of 1.08 A to the new final value.

#### Solution

- a. 1.28 A  
b. 22.5 msec

To examine the effect of field-current control, let us begin with the case of a dc motor driving a load of constant torque  $T_{load}$ . From Eqs. 7.9 and 7.14, the generated voltage of a dc motor can be written as

$$E_a = K_f I_f \omega_m \quad (11.3)$$

where  $I_f$  is the average field current,  $\omega_m$  is the angular velocity in rad/sec, and  $K_f = K_a \mathcal{P}_d N_f$  is a geometric constant which depends upon the dimensions of the motor, the properties of the magnetic material used to construct the motor, as well as the number of turns in the field winding. Note that strictly speaking,  $K_f$  is not constant since it is proportional to the direct-axis permeance, which typically varies as the flux-level in the motor increases to the point that the effects of magnetic saturation become significant.

The electromagnetic torque is given by Eq. 7.16 as

$$T_{mech} = \frac{E_a I_a}{\omega_m} = K_f I_f I_a \quad (11.4)$$

and the armature current can be seen from the equivalent circuit of Fig. 11.1 to be given by

$$I_a = \frac{(V_a - E_a)}{R_a} \quad (11.5)$$

Setting the motor torque equal to  $T_{\text{load}}$ , Eqs. 11.3 through 11.5 can be solved for  $\omega_m$

$$\omega_m = \frac{(V_a - I_a R_a)}{K_f I_f} = \frac{(V_a - \frac{T_{\text{load}} R_a}{K_f I_f})}{K_f I_f} \quad (11.6)$$

From Eq. 11.6, recognizing that the armature resistance voltage drop  $I_a R_a$  is generally quite small in comparison to the armature voltage  $V_a$ , we see that for a given load torque, the motor speed will increase with decreasing field current and decrease as the field current is increased. The lowest speed obtainable is that corresponding to maximum field current (the field current is limited by heating considerations); the highest speed is limited mechanically by the mechanical integrity of the rotor and electrically by the effects of armature reaction under weak-field conditions giving rise to poor commutation.

Armature current is typically limited by motor cooling capability. In many dc motors, cooling is aided by a shaft-driven fan whose cooling capacity is a function of motor speed. To examine in an approximate fashion the limitations on the allowable continuous motor output as the speed is changed, we will neglect the influence of changing ventilation and assume that the armature current  $I_a$  cannot exceed its rated value, in order to insure that the motor will not overheat. In addition, in our approximate argument we will neglect the effect of rotational losses (which of course also change with motor speed). Because the voltage drop across the armature resistance is relatively small, the speed voltage  $E_a$  will remain essentially constant at a value slightly below the applied armature voltage; any change in field current will be compensated for by a change in motor speed.

Thus under constant-terminal-voltage operation with varying field current, the  $E_a I_a$  product, and hence the allowable motor output power, remain substantially constant as the speed is varied. A dc motor controlled in this fashion is referred to as a *constant-power drive*. Torque, however, varies directly with field flux and therefore has its highest allowable value at the highest field current and hence lowest speed. Field-current control is thus best suited to drives requiring increased torque at low speeds. When a motor so controlled is used with a load requiring constant torque over the speed range, the rating and size of the machine are determined by the product of the torque and the highest speed. Such a drive is inherently oversized at the lower speeds, which is the principal economic factor limiting the practical speed range of large motors.

#### EXAMPLE 11.2

With an armature terminal voltage of 240 V and with a shunt-field current of 0.34 A, the no-load speed of the dc motor of Example 11.1 is found to be 3600 r/min. In this example, the motor is assumed to be driving a load which varies with speed as

$$P_{\text{load}} = 22.4 \left( \frac{n}{3600} \right)^3 \text{ kW}$$

where  $n$  is the motor speed in r/min. A rheostat is to be installed in series with the shunt field to vary the speed. Assuming the armature terminal voltage to remain constant at 240 V, calculate

the required resistance range if the speed is to be varied between 1800 and 3600 r/min. The effect of rotational losses can be ignored.

### ■ Solution

The load torque is equal to the load power divided by the motor speed  $\omega_m$  expressed in rad/sec. First expressing the power in terms of  $\omega_m$

$$P_{\text{load}} = 22.4 \left( \frac{\omega_m}{120\pi} \right)^3 \text{ kW}$$

The load torque is then given by

$$T_{\text{load}} = \frac{P_{\text{load}}}{\omega_m} = 22.4 \left( \frac{\omega_m^2}{(120\pi)^3} \right) = 4.18 \times 10^{-4} \omega_m^2 \text{ N} \cdot \text{m}$$

Thus, at 1800 r/min,  $\omega_m = 60\pi$  and  $T_{\text{load}} = 14.9 \text{ N} \cdot \text{m}$ . At 3600 r/min,  $\omega_m = 120\pi$  and  $T_{\text{load}} = 59.4 \text{ N} \cdot \text{m}$ .

Before solving for  $I_f$ , we must find the value of  $K_f$ , which can be found from the no-load data. Specifically, we see that with a terminal voltage of 240-V and at a no-load speed of 3600 r/min ( $\omega_m = 120\pi$ ), the corresponding field current is 0.34 A. Since under no-load conditions  $E_a \approx V_a$ , we can find  $K_f$  from Eq. 11.3 as

$$K_f = \frac{E_a}{I_f \omega_m} = \frac{240}{0.34 \times 120\pi} = 1.87 \text{ V/(A} \cdot \text{rad/sec)}$$

To find the required field current, we can solve Eq. 11.6 for  $I_f$

$$I_f = \frac{V_a}{2K_f \omega_m} \left( 1 \pm \sqrt{1 - \frac{4\omega_m T_{\text{load}} R_a}{V_a^2}} \right)$$

Recognizing that  $R_a$  is small and hence that  $I_f \approx V_a/(K_f \omega_m)$  we see that the positive sign should be used and thus

$$I_f = \frac{V_a}{2K_f \omega_m} \left( 1 + \sqrt{1 - \frac{4\omega_m T_{\text{load}} R_a}{V_a^2}} \right)$$

Once the field current has been found, the total field resistance can be found as

$$(R_f)_{\text{total}} = \frac{V_a}{I_f} = \frac{240}{I_f}$$

and the required added rheostat resistance can be found by subtracting the resistance of the shunt-field winding (187  $\Omega$ ) from  $(R_f)_{\text{total}}$ .

This leads to the following table:

| r/min | $T_{\text{load}}$ [N · m] | $I_f$ [A] | $(R_f)_{\text{total}}$ [ $\Omega$ ] | $R_{\text{rheostat}}$ [ $\Omega$ ] |
|-------|---------------------------|-----------|-------------------------------------|------------------------------------|
| 1800  | 14.9                      | 0.678     | 354                                 | 167                                |
| 3600  | 59.4                      | 0.334     | 719                                 | 532                                |

Thus, the rheostat must be able to cover the range from 166  $\Omega$  to 532  $\Omega$ .

**Practice Problem 11.2**

The rheostat of Example 11.2 is to be replaced by a duty cycle controller operating from the 240-V dc supply. Calculate the duty-cycle range required to achieve operation over the speed range of 1800–3600 r/min as specified in Example 11.2.

**Solution**

$$0.26 \leq D \leq 0.53$$

**Armature-Circuit Resistance Control** *Armature-circuit resistance control* provides a means of obtaining reduced speed by the insertion of external series resistance in the armature circuit. It can be used with series, shunt, and compound motors; for the last two types, the series resistor must be connected between the shunt field and the armature, not between the line and the motor. It is a common method of speed control for series motors and is generally analogous in action to wound-rotor-induction-motor control by the addition of external series rotor resistance.

Depending upon the value of the series armature resistance, the speed may vary significantly with load, since the speed depends on the voltage drop in this resistance and hence on the armature current demanded by the load. For example, a 1200-r/min shunt motor whose speed under load is reduced to 750 r/min by series armature resistance will return to almost 1200-r/min operation if the load is removed because the no-load current produces a voltage drop across the series resistance which is insignificant. The disadvantage of poor speed regulation may not be important in a series motor, which is used only where varying-speed service is required or can be tolerated.

A significant disadvantage of this method of speed control is that the power loss in the external resistor is large, especially when the speed is greatly reduced. In fact, for a constant-torque load, the power input to the motor plus resistor remains constant, while the power output to the load decreases in proportion to the speed. Operating costs are therefore comparatively high for lengthy operation at reduced speeds. Because of its low initial cost however, the series-resistance method (or the variation of it discussed in the next paragraph) will often be attractive economically for applications which require only short-time or intermittent speed reduction. Unlike field-current control, armature-resistance control results in a *constant-torque drive* because both the field-flux and, to a first approximation, the allowable armature current remain constant as speed changes.

A variation of this control scheme is given by the *shunted-armature method*, which may be applied to a series motor, as in Fig. 11.3a, or a shunt motor, as in Fig. 11.3b. In effect, resistors  $R_1$  and  $R_2$  act as a voltage divider applying a reduced voltage to the armature. Greater flexibility is possible because two resistors can now be adjusted to provide the desired performance. For series motors, the no-load speed can be adjusted to a finite, reasonable value, and the scheme is therefore applicable to the production of slow speeds at light loads. For shunt motors, the speed regulation in the low-speed range is appreciably improved because the no-load speed is definitely lower than the value with no controlling resistors.

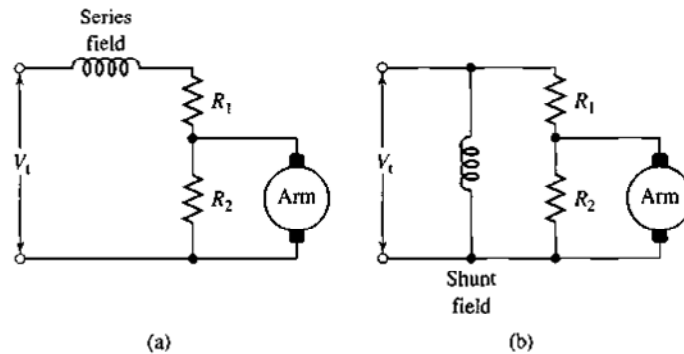


**Figure 11.3** Shunted-armature method of speed control applied to (a) a series motor and (b) a shunt motor.

**Armature-Terminal Voltage Control** *Armature-terminal voltage control* can be readily accomplished with the use of power-electronic systems such as those discussed in Chapter 10. Figure 11.4 shows in somewhat schematic form three possible configurations. In Fig. 11.4a, a phase-controlled rectifier in combination with a dc-link filter capacitor can be used to produce a variable dc-link voltage which can be applied directly to the armature terminals of the dc motor.

In Fig. 11.4b, a constant dc-link voltage is produced by a diode rectifier in combination with a dc-link filter capacitor. The armature terminal voltage is then varied by a pulse-width modulation scheme in which switch  $S$  is alternately opened and closed. When switch  $S$  is closed, the armature voltage is equal to the dc-link voltage  $V_{dc}$ , and when the switch is opened, current transfers to the freewheeling diode, essentially setting the armature voltage to zero. Thus the average armature voltage under this condition is equal to

$$V_a = DV_{dc} \quad (11.7)$$

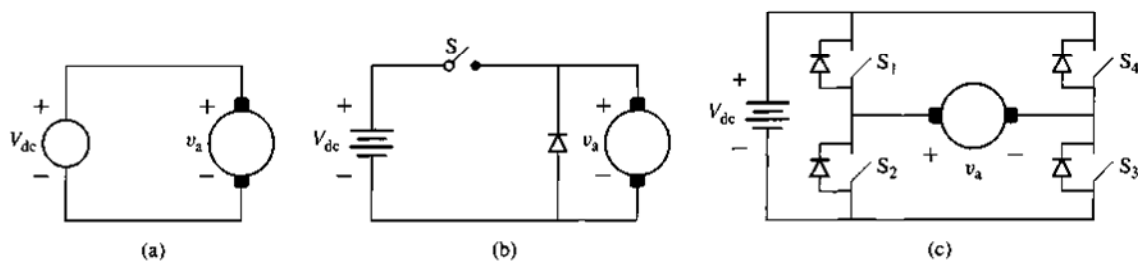


**Figure 11.4** Three typical configurations for armature-voltage control. (a) Variable dc-link voltage (produced by a phase-controlled rectifier) applied directly to the dc-motor armature terminals. (b) Constant dc-link voltage with single-polarity pulse-width modulation. (c) Constant dc-link voltage with a full H-bridge.

where

$V_a$  = average armature voltage (V)

$V_{dc}$  = dc-link voltage (V)

$D$  = PWM duty cycle (fraction of time that switch S is closed)

Figure 11.4c shows an H-bridge configuration as is discussed in the context of inverters in Section 10.3.3. Note that if switch S3 is held closed while switch S4 remains open, this configuration reduces to that of Fig. 11.4b. However, the H-bridge configuration is more flexible because it can produce both positive- and negative-polarity armature voltage. For example, with switches S1 and S3 closed, the armature voltage is equal to  $V_{dc}$  while with switches S2 and S4 closed, the armature voltage is equal to  $-V_{dc}$ . Clearly, using such an H-bridge configuration in combination with an appropriate choice of control signals to the switches allows this PWM system to achieve any desired armature voltage in the range  $-V_{dc} \leq V_a \leq V_{dc}$ .

Armature-voltage control takes advantage of the fact that, because the voltage drop across the armature resistance is relatively small, a change in the armature terminal voltage of a shunt motor is accompanied in the steady state by a substantially equal change in the speed voltage. With constant shunt field current and hence field flux, this change in speed voltage must be accompanied by a proportional change in motor speed. Thus, motor speed can be controlled directly by means of the armature terminal voltage.

### EXAMPLE 11.3

A 500-V, 100-hp, 2500 r/min, separately excited dc motor has the following parameters:

|                      |                                                           |
|----------------------|-----------------------------------------------------------|
| Field resistance:    | $R_f = 109 \, \Omega$                                     |
| Rated field voltage: | $V_{f0} = 300 \, \text{V}$                                |
| Armature resistance: | $R_a = 0.084 \, \Omega$                                   |
| Geometric constant:  | $K_f = 0.694 \, \text{V}/(\text{A} \cdot \text{rad/sec})$ |



Assuming the field voltage to be held constant at 300 V, use MATLAB<sup>†</sup> to plot the motor speed as a function of armature voltage with the motor operating under no-load and also under rated full-load torque as the armature voltage is varied from 250 V to 500 V.

### ■ Solution

From Eq. 11.4

$$I_a = \frac{T_{\text{mech}}}{K_f I_f}$$

and from Eq. 11.5

$$I_a = \frac{V_a - E_a}{R_a} = \frac{V_a - K_f I_f \omega_m}{R_a}$$

<sup>†</sup> MATLAB is a registered trademark of The MathWorks, Inc.

Hence we can solve for  $\omega_m$

$$\omega_m = \frac{V_a - \left( \frac{T_{\text{mech}} R_a}{K_f I_f} \right)}{K_f I_f}$$

and the speed in r/min as

$$n = \left( \frac{30}{\pi} \right) \omega_m$$

Finally, the field current is

$$I_f = \frac{V_f}{R_f} = \frac{300}{109} = 2.75 \text{ A}$$

and the rated full-load torque is given by

$$T_{\text{rated}} = \frac{P_{\text{rated}}}{(\omega_m)_{\text{rated}}} = \frac{100 \times 746}{2500 \times \left( \frac{\pi}{30} \right)} = 285 \text{ N} \cdot \text{m}$$

Figure 11.5 is the desired plot. Notice that the speed drops approximately 63 r/min as the torque is increased from zero to full-load, independent of the armature voltage and machine speed.

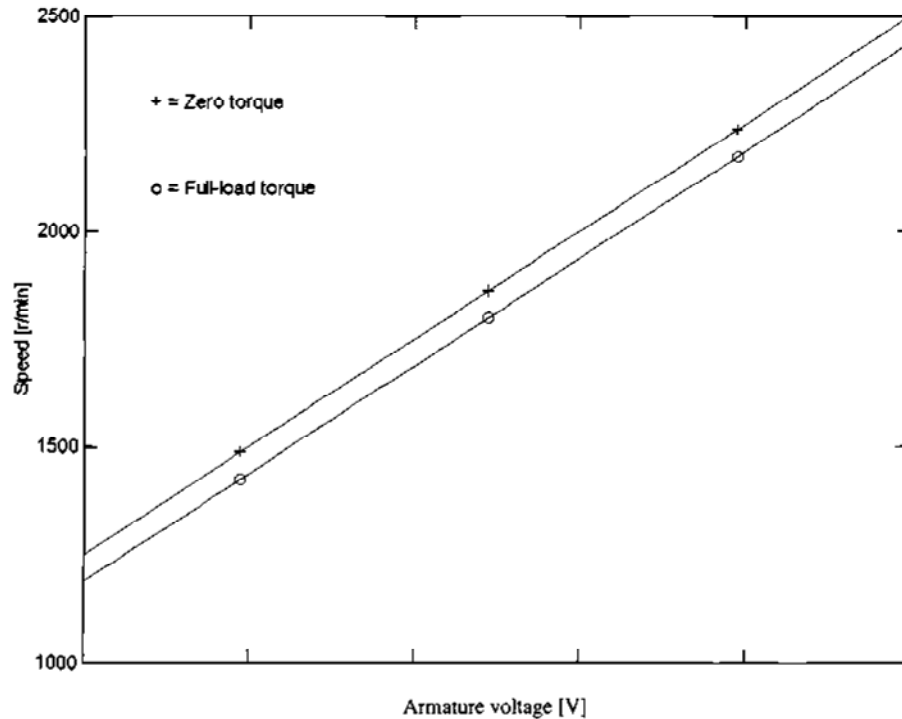


**Figure 11.5** A plot of speed versus armature voltage for the dc motor of Example 11.3.

Here is the MATLAB script:

```

clc
clear

% Motor parameters
Rf = 109;
Ra = 0.084;
Kf = 0.694;

% Constant field voltage
Vf = 300;

% Resulting field current
If = Vf/Rf;

% Rated speed in rad/sec
omegarated = 2500*(pi/30);

% Rated power in Watts
Prated = 100*746;

% Rated torque in N-m
Trated = Prated/omegarated;

% Vary the armature voltage from 250 to 500 V
% and calculate speed.
for n=1:101
    Va(n) = 250 * (1 + (n-1)/100);

    % Zero torque
    T = 0;
    omega = (Va(n) - T*Ra/(Kf*If))/(Kf*If);
    NoLoadRPM(n) = omega*30/pi;

    % Full-load torque
    T = Trated;
    omega = (Va(n) - T*Ra/(Kf*If))/(Kf*If);
    FullLoadRPM(n) = omega*30/pi;
end

plot(Va,NoLoadRPM)
hold
plot(Va(20),NoLoadRPM(20),'+')
plot(Va(50),NoLoadRPM(50),'+')
plot(Va(80),NoLoadRPM(80),'+')
plot(Va,FullLoadRPM)
plot(Va(20),FullLoadRPM(20),'o')
plot(Va(50),FullLoadRPM(50),'o')
plot(Va(80),FullLoadRPM(80),'o')
hold
xlabel('Armature voltage [V]')

```

```

ylabel('Speed [r/min]')
text(270,2300,'+ = Zero torque')
text(270,2100,'o = Full-load torque')

```

**Practice Problem 11.3**

Calculate the change in armature voltage required to maintain the motor of Example 11.3 at a speed of 2000 r/min as the load is changed from zero to full-load torque.

**Solution**

12.5 V

Frequently the control of motor voltage is combined with field-current control in order to achieve the widest possible speed range. With such dual control, base speed can be defined as the normal-armature-voltage, full-field speed of the motor. Speeds above base speed are obtained by reducing the field current; speeds below base speed are obtained by armature-voltage control. As discussed in connection with field-current control, the range above base speed is that of a constant-power drive. The range below base speed is that of a constant-torque drive because, as in armature-resistance control, the flux and the allowable armature current remain approximately constant. The overall output limitations are therefore as shown in Fig. 11.6a for approximate allowable torque and in Fig. 11.6b for approximate allowable power. The constant-torque characteristic is well suited to many applications in the machine-tool industry, where many loads consist largely of overcoming the friction of moving parts and hence have essentially constant torque requirements.

The speed regulation and the limitations on the speed range above base speed are those already presented with reference to field-current control; the maximum speed thus does not ordinarily exceed four times base speed and preferably not twice base speed. For conventional machines, the lower limit for reliable and stable operation is

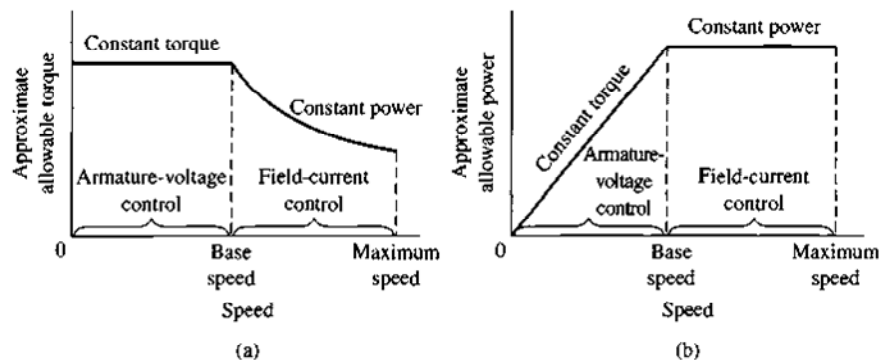


**Figure 11.6** (a) Torque and (b) power limitations of combined armature-voltage and field-current methods of speed control.

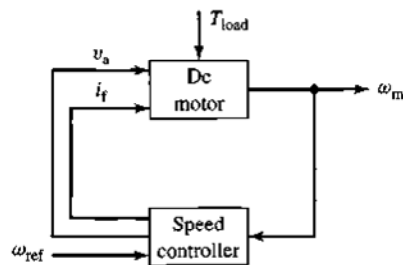


**Figure 11.7** Block diagram for a speed-control system for a separately excited or shunt-connected dc motor.

about one-tenth of base speed, corresponding to a total maximum-to-minimum range not exceeding 40:1.

With armature reaction ignored, the decrease in speed from no-load to full-load torque is caused entirely by the full-load armature-resistance voltage drop in the dc generator and motor. This full-load armature-resistance voltage drop is constant over the voltage-control range, since full-load torque and hence full-load current are usually regarded as constant in that range. When measured in r/min, therefore, the speed decrease from no-load to full-load torque is a constant, independent of the no-load speed, as we saw in Example 11.3. The torque-speed curves accordingly are closely approximated by a series of parallel straight lines for the various motor-field adjustments. Note that a speed decrease of, say, 40 r/min from a no-load speed of 1200 r/min is often of little importance; a decrease of 40 r/min from a no-load speed of 120 r/min, however, may at times be of critical importance and require corrective steps in the layout of the system.

Figure 11.7 shows a block diagram of a feedback-control system that can be used to regulate the speed of a separately excited or shunt-connected dc motor. The inputs to the dc-motor block include the armature voltage and the field current as well as the load torque  $T_{load}$ . The resultant motor speed  $\omega_m$  is fed back to a controller block which represents both the control logic and power electronics and which controls the armature voltage and field current applied to the dc motor, based upon a reference speed signal  $\omega_{ref}$ . Depending upon the design of the controller, with such a scheme it is possible to control the steady-state motor speed to a high degree of accuracy independent of the variations in the load torque.

#### EXAMPLE 11.4

Figure 11.8 shows the block diagram for a simple speed control system to be applied to the dc motor of Example 11.3. In this controller, the field voltage is held constant (not shown) at its rated value of 300 V. Thus, the control is applied only to the armature voltage and takes the form

$$V_a = V_{a0} + G(\omega_{ref} - \omega_m)$$

where  $V_{a0}$  is the armature voltage when  $\omega_m = \omega_{ref}$  and  $G$  is a multiplicative constant.

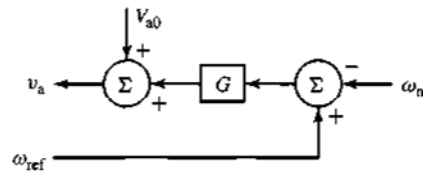


**Figure 11.8** Simple dc-motor speed controller for Example 11.4.

With the reference speed set to 2000 r/min ( $\omega_{ref} = 2000 \times \pi/30$ ), calculate  $V_{a0}$  and  $G$  so that the motor speed is 2000 r/min at no load and drops only by 25 r/min when the torque is increased to its rated full-load value.

### ■ Solution

As was found in Example 11.3, the field current under this condition will be 2.75 A. At no load, 2000 r/min,

$$V_a \approx E_a = K_f I_f \omega_m = 0.694 \times 2.75 \times 2000 \left( \frac{\pi}{30} \right) = 400 \text{ V}$$

and thus  $V_{a0} = 400 \text{ V}$ .

The full load torque was found in Example 11.3 to be  $T_{rated} = 285 \text{ N}\cdot\text{m}$  and thus the armature current required to achieve rated full-load torque can be found from Eq. 11.4

$$I_a = \frac{T_{rated}}{K_f I_f} = \frac{285}{0.694 \times 2.75} = 149 \text{ A}$$

At a speed of 1975 r/min,  $E_a$  will be given by

$$E_a = K_f I_f \omega_m = 0.694 \times 2.75 \times 1975 \left( \frac{\pi}{30} \right) = 395 \text{ V}$$

and thus

$$V_a = E_a + I_a R_a = 395 + 149 \times 0.084 = 408 \text{ V}$$

Solving for  $G$  gives

$$G = \frac{V_a - V_{a0}}{\omega_{ref} - \omega_m} = \frac{408 - 400}{(2000 - 1975) \left( \frac{\pi}{30} \right)} = 3.06 \text{ V} \cdot \text{sec/rad}$$

### Practice Problem 11.4

If the load torque in Example 11.4 is equal to half of the rated full-load torque, calculate (a) the speed of the motor and (b) the corresponding load power.

### Solution

- 1988 r/min
- 29.6 kW

In the case of permanent-magnet dc motors, the field flux is, of course, fixed by the permanent magnet (with the possible exception of any effects of temperature changes on the magnet properties as the motor heats up). From Eqs. 11.3 and 11.4, we see that the voltage generated can be written in the form

$$E_a = K_m \omega_m \quad (11.8)$$

and that the electromagnetic torque can be written as

$$T_{\text{mech}} = K_m I_a \quad (11.9)$$

Comparison of Eqs. 11.8 and 11.9 with Eqs. 11.3 and 11.4 show that the analysis of a permanent-magnet dc motor is identical to that of a shunt or separately excited dc motor with the exception that the torque-constant  $K_m$  must be substituted for the term  $K_f I_f$ .

### EXAMPLE 11.5

The permanent-magnet dc motor of Example 7.9 has an armature resistance of  $1.03 \, \Omega$  and a torque constant  $K_m = 0.22 \, \text{V/(rad/sec)}$ . Assume the motor to be driving a constant power load of  $800 \, \text{W}$  (including rotational losses), and calculate the motor speed as the armature voltage is varied from  $40$  to  $50 \, \text{V}$ .

#### ■ Solution

The motor power output (including rotational losses) is given by the product  $E_a I_a$  and thus we can write

$$P_{\text{load}} = E_a I_a = K_m \omega_m I_a$$

Solving for  $\omega_m$  gives

$$\omega_m = \frac{P_{\text{load}}}{K_m I_a}$$

The armature current can be written as

$$I_a = \frac{(V_a - E_a)}{R_a} = \frac{(V_a - K_m \omega_m)}{R_a}$$

These two equations can be combined to give an equation for  $\omega_m$  of the form

$$\omega_m^2 - \left( \frac{V_a}{K_m} \right) \omega_m + \frac{P_{\text{load}} R_a}{K_m^2} = 0$$

from which we can find

$$\omega_m = \frac{V_a}{2K_m} \left[ 1 \pm \sqrt{1 - \frac{4P_{\text{load}} R_a}{V_a^2}} \right]$$

Recognizing that, if the voltage drop across the armature resistance is small,  $V_a \approx E_a = K_m \omega_m$ , we pick the positive sign and thus

$$\omega_m = \frac{V_a}{2K_m} \left[ 1 + \sqrt{1 - \frac{4P_{\text{load}} R_a}{V_a^2}} \right]$$

Substituting values, we find that for  $V_a = 40$  V,  $\omega_m = 169.2$  rad/sec (1616 r/min) and for  $V_a = 50$  V,  $\omega_m = 217.5$  rad/sec (2077 r/min).

### Practice Problem 11.5

Calculate the speed variation (in r/min) of the permanent-magnet dc motor of Example 11.5 if the armature voltage is held constant at 50 V and the load power varies from 100 W to 500 W.

### Solution

2077 r/min to 1540 r/min

### 11.1.2 Torque Control

As we have seen, the electromagnetic torque of a dc motor is proportional to the armature current  $I_a$  and is given by

$$T_{\text{mech}} = K_t I_f I_a \quad (11.10)$$

in the case of a separately excited or shunt motor and

$$T_{\text{mech}} = K_m I_a \quad (11.11)$$

in the case of a permanent-magnet motor.

From these equations we see that torque can be controlled directly by controlling the armature current. Fig. 11.9 shows three possible configurations. In Fig. 11.9a, a phase-controlled rectifier, in combination with a dc-link filter inductor, can be used to create a variable dc-link current which can be applied directly to the armature terminals of the dc motor.

In Fig. 11.9b, a constant dc-link current is produced by a diode rectifier. The armature terminal voltage is then varied by a pulse-width modulation scheme in which switch S is alternately opened and closed. When switch S is opened, the current  $I_{dc}$  flows into the dc-motor armature while when switch S is closed, the armature is

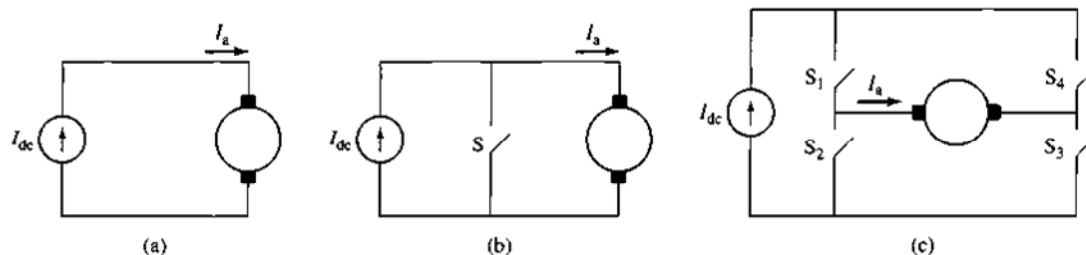


**Figure 11.9** Three typical configurations for armature-current control. (a) Variable dc-link current (produced by a phase-controlled rectifier) applied directly to the dc-motor armature terminals. (b) Constant dc-link current with single-polarity pulse-width modulation. (c) Constant dc-link current with a full H-bridge.

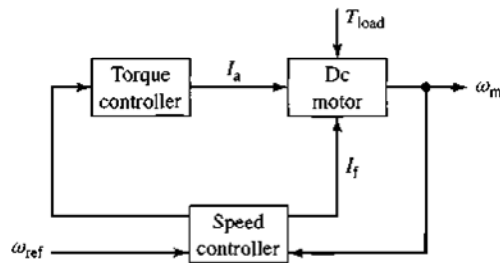


**Figure 11.10** Block diagram of a dc-motor speed-control system using direct-control of motor torque.

shorted and  $I_a$  decays. Thus, the duty cycle of switch  $S$  will control the average current into the armature.

Finally, Fig 11.9c shows an H-bridge configuration as is discussed in the context of inverters in Section 10.3.2. Appropriate control of the four switches  $S1$  through  $S4$  allows this PWM system to achieve any desired armature average current in the range  $-I_{dc} \leq I_a \leq I_{dc}$ .

Note that in each of the PWM configurations of Fig. 11.9b and c, rapid changes in instantaneous current through the dc machine armature can give rise to large voltage spikes, which can damage the machine insulation as well as give rise to flashover and voltage breakdown of the commutator. In order to eliminate these effects, a practical system must include some sort of filter across the armature terminals (such as a large capacitor) to limit the voltage rise and to provide a low-impedance path for the high-frequency components of the drive current.

Figure 11.10 shows a typical configuration in which the torque control is surrounded by a speed-feedback loop. This looks similar to the speed control of Fig. 11.7. However, instead of controlling the armature voltage, in this case the output of the speed controller is a torque reference signal  $T_{ref}$  which in turn serves as the input to the torque controller. One advantage of such a system is that it automatically limits the dc-motor armature current to acceptable levels under all operating conditions, as is shown in Example 11.6.

#### EXAMPLE 11.6

Consider the 100-hp dc motor of Examples 11.3 and 11.4 to be driving a load whose torque varies linearly with speed such that it equals rated full-load torque (285 N·m) at a speed of 2500 r/min. We will assume the combined moment of inertia of the motor and load to equal 0.92 kg·m<sup>2</sup>. The field voltage is to be held constant at 300 V.

- Calculate the armature voltage and current required to achieve speeds of 2000 and 2500 r/min.
- Assume that the motor is operated from an armature-voltage controller and that the armature voltage is suddenly switched from its 2000 r/min to its 2500 r/min value. Calculate the resultant motor speed and armature current as a function of time.

- c. Assume that the motor is operated from an armature-current controller and that the armature current is suddenly switched from its 2000 r/min to its 2500 r/min value. Calculate the resultant motor speed as a function of time.

### ■ Solution

- a. Neglecting any rotational losses, the armature current can be found from Eq. 11.4 by setting  $T_{\text{mech}} = T_{\text{load}}$

$$I_a = \frac{T_{\text{load}}}{K_f I_f}$$

Substituting

$$T_{\text{load}} = \left(\frac{n}{n_f}\right) T_{f1}$$

where  $n$  is the motor speed in r/min,  $n_f = 2500$  r/min and  $T_{f1} = 285$  N·m gives

$$I_a = \frac{n T_{f1}}{n_f K_f I_f}$$

Solving for  $V_a = E_a + I_a R_a$  then allows us to complete the following table:

| r/min | $\omega_m$ [rad/sec] | $V_a$ [V] | $I_a$ [A] | $T_{\text{load}}$ [N·m] |
|-------|----------------------|-----------|-----------|-------------------------|
| 2000  | 209                  | 410       | 119       | 228                     |
| 2500  | 262                  | 513       | 149       | 285                     |

- b. The dynamic equation governing the speed of the motor is

$$J \frac{d\omega_m}{dt} = T_{\text{mech}} - T_{\text{load}}$$

Substituting  $\omega_m = (\pi/30)n$  and  $\omega_r = (\pi/30)n_f$  we can write

$$T_{\text{load}} = \left(\frac{T_{f1}}{\omega_r}\right) \omega_m$$

Under armature-voltage control,

$$\begin{aligned} T_{\text{mech}} &= K_f I_f I_a = K_f I_f \left( \frac{V_a - E_a}{R_a} \right) \\ &= K_f I_f \left( \frac{V_a - K_f I_f \omega_m}{R_a} \right) \end{aligned}$$

and thus the governing differential equation is

$$J \frac{d\omega_m}{dt} = K_f I_f \left( \frac{V_a - K_f I_f \omega_m}{R_a} \right) - \left( \frac{T_{f1}}{\omega_r} \right) \omega_m$$

or

$$\begin{aligned} \frac{d\omega_m}{dt} + \frac{1}{J} \left( \frac{T_{f1}}{\omega_r} + \frac{(K_f I_f)^2}{R_a} \right) \omega_m &= \frac{K_f I_f V_a}{J R_a} \\ &= \frac{d\omega_m}{dt} + 48.4 \omega_m - 24.7 V_a = 0 \end{aligned}$$

From this differential equation, we see that with the motor initially at  $\omega_m = \omega_i = 209$  rad/sec, if the armature voltage  $V_a$  is suddenly switched from  $V_i = 413$  V to  $V_f = 513$  V, the speed will rise exponentially to  $\omega_m = \omega_f = 262$  rad/sec as

$$\begin{aligned}\omega_m &= \omega_f + (\omega_i - \omega_f)e^{-t/\tau} \\ &= 262 - 53e^{-t/\tau} \text{ rad/sec}\end{aligned}$$

where  $\tau = 1/48.4 = 20.7$  msec. Expressed in terms of r/min

$$n = 2500 - 50e^{-t/\tau} \text{ r/min}$$

The armature current will decrease exponentially with the same 20.7 msec time constant from an initial value of  $(V_f - V_i)/R_a = 1190$  A to its final value of 149 A. Thus,

$$I_a = 149 + 1041e^{-t/\tau} \text{ A}$$

Notice that it is unlikely that the supply to the dc motor can supply this large initial current (eight times the rated full-load armature current) and, in addition, the high current and corresponding high torque could potentially cause damage to the dc motor commutator, brushes, and armature winding. Hence, as a practical matter, a practical controller would undoubtedly limit the rate of change of the armature voltage to avoid such sudden steps in voltage, with the result that the speed change would not occur as rapidly as calculated here.

- c. The dynamic equation governing the speed of the motor remains the same as that in part (b) as does the equation for the load torque. However, in this case, because the motor is being operated from a current controller, the electromagnetic torque will remain constant at  $T_{\text{mech}} = T_f = 285$  N·m after the current is switched from its initial value of 119 A to its final value of 149 A.

Thus

$$J \frac{d\omega_m}{dt} = T_{\text{mech}} - T_{\text{load}} = T_f - \left( \frac{T_f}{\omega_f} \right) \omega_m$$

or

$$\begin{aligned}\frac{d\omega_m}{dt} + \left( \frac{T_f}{J\omega_f} \right) \omega_m &= \frac{T_f}{J} \\ &= \frac{d\omega_m}{dt} + 1.18\omega_m - 310 = 0\end{aligned}$$

In this case, the speed will rise exponentially to  $\omega_m = \omega_f = 262$  rad/sec as

$$\begin{aligned}\omega_m &= \omega_f + (\omega_i - \omega_f)e^{-t/\tau} \\ &= 262 - 53e^{-t/\tau} \text{ rad/sec}\end{aligned}$$

where now the time constant  $\tau = 1/1.18 = 845$  msec.

Clearly, the change in motor speed under the current controller is much slower. However, at no point during this transient do either the motor current or the motor torque exceed their rated value. In addition, should faster response be desired, the armature current (and hence motor torque) could be set temporarily to a fixed value higher than the rated value (e.g., two or three times rated as compared to the factor of 8 found in part (b)), thus limiting the potential for damage to the motor.

### Practice Problem 11.6

Consider the dc motor/load combination of Example 11.6 operating under current (torque) control to be operating in the steady-state at a speed of 2000 r/min at an armature current of 119 A. If the armature current is suddenly switched to 250 A, calculate the time required for the motor to reach a speed of 2500 r/min.

#### Solution

0.22 sec

## 11.2 CONTROL OF SYNCHRONOUS MOTORS

### 11.2.1 Speed Control

As discussed in Chapters 4 and 5, synchronous motors are essentially constant-speed machines, with their speed being determined by the frequency of the armature currents as described by Eqs. 4.40 and 4.41. Specifically, Eq. 4.40 shows that the synchronous angular velocity is proportional to the electrical frequency of the applied armature voltage and inversely proportional to the number of poles in the machine

$$\omega_s = \left( \frac{2}{\text{poles}} \right) \omega_e \quad (11.12)$$

where

$\omega_s$  = synchronous spatial angular velocity of the air-gap mmf wave [rad/sec]

$\omega_e = 2\pi f_e$  = angular frequency of the applied electrical excitation [rad/sec]

$f_e$  = applied electrical frequency [Hz]

Clearly, the simplest means of synchronous motor control is speed control via control of the frequency of the applied armature voltage, driving the motor by a polyphase voltage-source inverter such as the three-phase inverter shown in Fig. 11.11. As is discussed in Section 10.3.3, this inverter can either be used to supply stepped ac voltage waveforms of amplitude  $V_{dc}$  or the switches can be controlled to produce pulse-width-modulated ac voltage waveforms of variable amplitude. The dc-link voltage  $V_{dc}$  can itself be varied, for example, through the use of a phase-controlled rectifier.

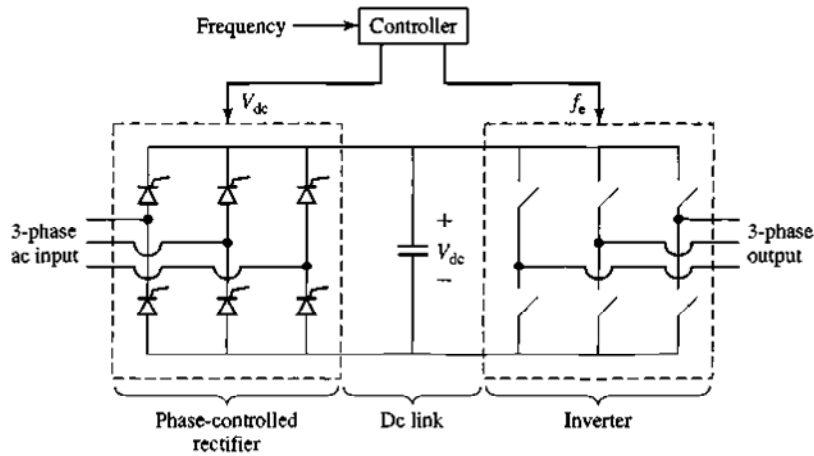


**Figure 11.11** Three-phase voltage-source inverter.

The frequency of the inverter output waveforms can of course be varied by controlling the switching frequency of the inverter switches. For ac-machine applications, coupled with this frequency control must be control of the amplitude of the applied voltage, as we will now see.

From Faraday's Law, we know that the air-gap component of the armature voltage in an ac machine is proportional to the peak flux density in the machine and the electrical frequency. Thus, if we neglect the voltage drop across the armature resistance and leakage reactance, we can write

$$V_a = \left( \frac{f_e}{f_{\text{rated}}} \right) \left( \frac{B_{\text{peak}}}{B_{\text{rated}}} \right) V_{\text{rated}} \quad (11.13)$$

where  $V_a$  is the amplitude of the armature voltage,  $f_e$  is the operating frequency, and  $B_{\text{peak}}$  is the peak air-gap flux density.  $V_{\text{rated}}$ ,  $f_{\text{rated}}$ , and  $B_{\text{rated}}$  are the corresponding rated-operating-point values.

Consider a situation in which the frequency of the armature voltage is varied while its amplitude is maintained at its rated value ( $V_a = V_{\text{rated}}$ ). Under these conditions, from Eq. 11.13 we see that

$$B_{\text{peak}} = \left( \frac{f_{\text{rated}}}{f_e} \right) B_{\text{rated}} \quad (11.14)$$

Equation 11.14 clearly demonstrates the problem with constant-voltage, variable-frequency operation. Specifically, for a given armature voltage, the machine flux density is inversely proportional to frequency and thus as the frequency is reduced, the flux density will increase. Thus for a typical machine which operates in saturation at rated voltage and frequency, any reduction in frequency will further increase the flux density in the machine. In fact, a significant drop in frequency will increase

the flux density to the point of potential machine damage due both to increased core loss and to the increased machine currents required to support the higher flux density.

As a result, for frequencies less than or equal to rated frequency, it is typical to operate a machine at constant flux density. From Eq. 11.13, with  $B_{\text{peak}} = B_{\text{rated}}$

$$V_a = \left( \frac{f_e}{f_{\text{rated}}} \right) V_{\text{rated}} \quad (11.15)$$

which can be rewritten as

$$\frac{V_a}{f_e} = \frac{V_{\text{rated}}}{f_{\text{rated}}} \quad (11.16)$$

From Eq. 11.16, we see that constant-flux operation can be achieved by maintaining a constant ratio of armature voltage to frequency. This is referred to as *constant-volts-per-hertz* (constant V/Hz) operation. It is typically maintained from rated frequency down to the low frequency at which the armature resistance voltage drop becomes a significant component of the applied voltage.

Similarly, we see from Eq. 11.13 that if the machine is operated at frequencies in excess of rated frequency with the voltage at its rated value, the air-gap flux density will drop below its rated value. Thus, in order to maintain the flux density at its rated value, it would be necessary to increase the terminal voltage for frequencies in excess of rated frequency. In order to avoid insulation damage, it is common to maintain the machine terminal voltage at its rated value for frequencies in excess of rated frequency.

The machine terminal current is typically limited by thermal constraints. Thus, provided the machine cooling is not affected by rotor speed, the maximum permissible terminal current will remain constant at its rated value  $I_{\text{rated}}$ , independent of the applied frequency. As a result, for frequencies below rated frequency, with  $V_a$  proportional to  $f_e$ , the maximum machine power will be proportional to  $f_e V_{\text{rated}} I_{\text{rated}}$ . The maximum torque under these conditions can be found by dividing the power by the rotor speed  $\omega_s$ , which is also proportional to  $f_e$  as can be seen from Eq. 11.12. Thus, we see that the maximum torque is proportional to  $V_{\text{rated}} I_{\text{rated}}$ , and hence it is constant at its rated-operating-point value.

Similarly, for frequencies in excess of rated frequency, the maximum power will be constant and equal to  $V_{\text{rated}} I_{\text{rated}}$ . The corresponding maximum torque will then vary inversely with machine speed as  $V_{\text{rated}} I_{\text{rated}} / \omega_s$ . The maximum operating speed for this operating regime will be determined either by the maximum frequency which can be supplied by the drive electronics or by the maximum speed at which the rotor can be operated without risk of damage due to mechanical concerns such as excessive centrifugal force or to the presence of a resonance in the shaft system.

Figure 11.12 shows a plot of maximum power and maximum torque versus speed for a synchronous motor under variable-frequency operation. The operating regime below rated frequency and speed is referred to as the *constant-torque regime* and that above rated speed is referred to as the *constant-power regime*.

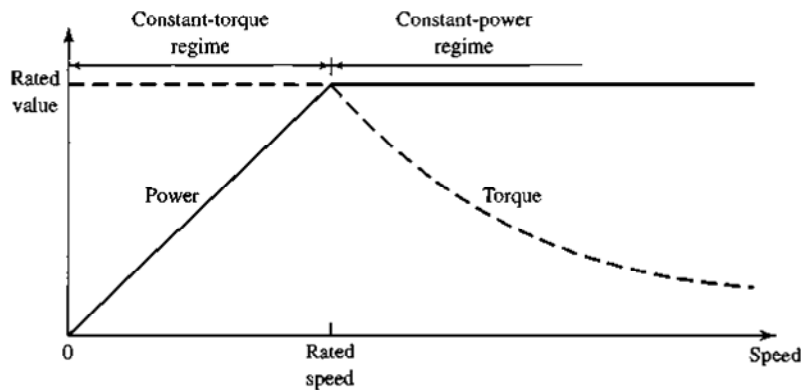


**Figure 11.12** Variable-speed operating regimes for a synchronous motor.

### EXAMPLE 11.7

The 45-kVA, 220-V, 60-Hz, six-pole, three-phase synchronous machine of Example 5.4 is to be operated as a motor and driven from a variable-frequency, three-phase voltage-source inverter which provides 220 V at 60 Hz and which maintains constant V/Hz as the frequency is reduced. The machine has a saturated synchronous reactance of 0.836 per unit and achieves rated open-circuit voltage at a field current of 2.84 A. For the purposes of this example, assume that the motor losses are negligible.

- With the motor operating at 60 Hz, 220 V and at rated power, unity power factor, calculate (i) the motor speed in r/min and (ii) the motor field current.
- If the inverter frequency is reduced to 50 Hz and the motor load adjusted to rated torque, calculate the (i) the resulting motor speed and (ii) the motor field current required to again achieve unity power factor.

#### ■ Solution

- (i) The motor will operate at its synchronous speed which can be found from Eq. 4.41

$$n_s = \left( \frac{120}{\text{poles}} \right) f_e = \left( \frac{120}{6} \right) 60 = 1200 \text{ r/min}$$

- (ii) As seen in Chapter 5, the field current can be determined from the generated voltage. For motor operation,

$$\hat{E}_{af} = \hat{V}_a - jX_s \hat{I}_a = 1.0 - j0.836 \times 1.0 = 1.30 \angle -39.9^\circ$$

where  $V_a$  has been chosen as the reference phasor. Thus the field current is

$$I_f = 1.30 \times 2.84 = 3.69 \text{ A}$$

Note that we have chosen to solve for  $E_{af}$  in per unit. A solution in real units would have of course produced the same result.

- (i) When the frequency is reduced from 60 Hz to 50 Hz, the motor speed will drop from 1200 r/min to 1000 r/min.

(ii) Let us again consider the equation for the generated voltage

$$\hat{E}_{af} = \hat{V}_a - jX_s \hat{I}_a$$

where here we will assume that the equation is written in real units.

As the inverter frequency is reduced from 60 Hz, the inverter voltage will drop proportionally since the inverter maintains constant V/Hz. Thus we can write

$$V_a = \left( \frac{\omega_m}{\omega_{m0}} \right) V_{a0}$$

where the subscript 0 is used to indicate a 60-Hz value as found in part (a). Reactance is also proportional to frequency and thus

$$X_s = \left( \frac{\omega_m}{\omega_{m0}} \right) X_{s0}$$

The generated voltage is proportional to both the motor speed (and hence the frequency) and the field current, and thus we can write

$$E_{af} = \left( \frac{\omega_m}{\omega_{m0}} \right) \left( \frac{I_f}{I_{f0}} \right) E_{af0}$$

Finally, if we recognize that, to operate at rated torque and unity power factor under this reduced frequency condition, the motor armature current will have to be equal to the value found in part (a), i.e.,  $I_a = I_{a0}$ , we can write the generated voltage equation as

$$\left( \frac{\omega_m}{\omega_{m0}} \right) \left( \frac{I_f}{I_{f0}} \right) \hat{E}_{af0} = \left( \frac{\omega_m}{\omega_{m0}} \right) \hat{V}_{a0} - j \left( \frac{\omega_m}{\omega_{m0}} \right) X_{s0} \hat{I}_{a0}$$

or

$$\left( \frac{I_f}{I_{f0}} \right) \hat{E}_{af0} = \hat{V}_{a0} - jX_{s0} \hat{I}_{a0}$$

Since the subscripted quantities correspond to the solution of part (a), they must satisfy

$$\hat{E}_{af0} = \hat{V}_{a0} - jX_{s0} \hat{I}_{a0}$$

and thus we see that we must have  $I_f = I_{f0}$ . In other words, the field current for this operating condition is equal to that found in part (a), or  $I_f = 3.69$  A.

### Practice Problem 11.7

Consider 50-Hz operation of the synchronous motor of Example 11.7, part (b). If the load torque is reduced to 75 percent of rated torque, calculate the field current required to achieve unity power factor.

#### Solution

3.35 A

Although during steady-state operation the speed of a synchronous motor is determined by the frequency of the drive, speed control by means of frequency control is of limited use in practice. This is due in most part to the fact that it is difficult for the rotor of a synchronous machine to track arbitrary changes in the frequency

of the applied armature voltage. In addition, starting is a major problem, and, as a result, the rotors of synchronous motors are often equipped with a squirrel-cage winding known as an *amortisseur* or *dampers winding* similar to the squirrel-cage winding in an induction motor, as shown in Fig. 5.3. Following the application of a polyphase voltage to the armature, the rotor will come up almost to synchronous speed by induction-motor action with the field winding unexcited. If the load and inertia are not too great, the motor will pull into synchronism when the field winding is subsequently energized.

Problems with changing speed result from the fact that, in order to develop torque, the rotor of a synchronous motor must remain in synchronism with the stator flux. Control of synchronous motors can be greatly enhanced by control algorithms in which the stator flux and its relationship to the rotor flux are controlled directly. Such control, which amounts to direct control of torque, is discussed in Section 11.2.2.

### 11.2.2 Torque Control

Direct torque control in an ac machine, which can be implemented in a number of different ways, is commonly referred to as *field-oriented control* or *vector control*. To facilitate our discussion of field-oriented control, it is helpful to return to the discussion of Section 5.6.1. Under this viewpoint, which is formalized in Appendix C, stator quantities (flux, current, voltage, etc.) are resolved into components which rotate in synchronism with the rotor. *Direct-axis* quantities represent those components which are aligned with the field-winding axis, and *quadrature-axis* components are aligned perpendicular to the field-winding axis.

Section C.2 of Appendix C derives the basic machine relations in dq0 variables for a synchronous machine consisting of a field winding and a three-phase stator winding. The transformed flux-current relationships are found to be

$$\lambda_d = L_d i_d + L_{af} i_f \quad (11.17)$$

$$\lambda_q = L_q i_q \quad (11.18)$$

$$\lambda_f = \frac{3}{2} L_{af} i_d + L_{ff} i_f \quad (11.19)$$

where the subscripts d, q, and f refer to armature direct-, quadrature-axis, and field-winding quantities respectively. Note that throughout this chapter we will assume balanced operating conditions, in which case zero-sequence quantities will be zero and hence will be ignored.

The corresponding transformed voltage equations are

$$v_d = R_a i_d + \frac{d\lambda_d}{dt} - \omega_{me} \lambda_q \quad (11.20)$$

$$v_q = R_a i_q + \frac{d\lambda_q}{dt} + \omega_{me} \lambda_d \quad (11.21)$$

$$v_f = R_f i_f + \frac{d\lambda_f}{dt} \quad (11.22)$$

where  $\omega_{me} = (\text{poles}/2)\omega_m$  is the electrical angular velocity of the rotor.

Finally, the electromagnetic torque acting on the rotor of a synchronous motor is shown to be (Eq. C.31)

$$T_{\text{mech}} = \frac{3}{2} \left( \frac{\text{poles}}{2} \right) (\lambda_d i_q - \lambda_q i_d) \quad (11.23)$$

Under steady-state, balanced-three-phase operating conditions,  $\omega_{\text{me}} = \omega_e$  where  $\omega_e$  is the electrical frequency of the armature voltage and current in rad/sec. Because the armature-produced mmf and flux waves rotate in synchronism with the rotor and hence with respect to the dq reference frame, under these conditions an observer in the dq reference frame will see constant fluxes, and hence one can set  $d/dt = 0$ .<sup>1</sup>

Letting the subscripts F, D, and Q indicate the corresponding constant values of field-, direct- and quadrature-axis quantities respectively, the flux-current relationships of Eqs. 11.17 through 11.19 then become

$$\lambda_D = L_d i_D + L_{af} i_F \quad (11.24)$$

$$\lambda_Q = L_q i_Q \quad (11.25)$$

$$\lambda_F = \frac{3}{2} L_{af} i_D + L_{ff} i_F \quad (11.26)$$

Armature resistance is typically quite small, and, if we neglect it, the steady-state voltage equations (Eqs. 11.20 through 11.22) then become

$$v_D = -\omega_e \lambda_Q \quad (11.27)$$

$$v_Q = \omega_e \lambda_D \quad (11.28)$$

$$v_F = R_f i_F \quad (11.29)$$

Finally, we can write Eq. 11.23 as

$$T_{\text{mech}} = \frac{3}{2} \left( \frac{\text{poles}}{2} \right) (\lambda_D i_Q - \lambda_Q i_D) \quad (11.30)$$

From this point on, we will focus our attention on machines in which the effects of saliency can be neglected. In this case, the direct- and quadrature-axis synchronous inductances are equal and we can write

$$L_d = L_q = L_s \quad (11.31)$$

where  $L_s$  is the synchronous inductance. Substitution into Eqs. 11.24 and 11.25 and then into Eq. 11.30 gives

$$\begin{aligned} T_{\text{mech}} &= \frac{3}{2} \left( \frac{\text{poles}}{2} \right) [(L_s i_D + L_{af} i_F) i_Q - L_s i_Q i_D] \\ &= \frac{3}{2} \left( \frac{\text{poles}}{2} \right) L_{af} i_F i_Q \end{aligned} \quad (11.32)$$

<sup>1</sup> This can easily be derived formally by substituting expressions for the balanced-three-phase armature currents and voltages into the transformation equations.

Equation 11.32 shows that torque is produced by the interaction of the field flux (proportional to the field current) and the quadrature-axis component of the armature current, in other words the component of armature current that is orthogonal to the field flux. By analogy, we see that the direct-axis component of armature current, which is aligned with the field flux, produces no torque.

This result is fully consistent with the generalized torque expressions which are derived in Chapter 4. Consider for example Eq. 4.73 which expresses the torque in terms of the product of the stator and rotor mmfs ( $F_s$  and  $F_r$  respectively) and the sine of the angle between them.

$$T = -\left(\frac{\text{poles}}{2}\right) \left(\frac{\mu_0 \pi D l}{2g}\right) F_s F_r \sin \delta_{sr} \quad (11.33)$$

where  $\delta_r$  is the electrical space angle between the stator and rotor mmfs. This shows clearly that no torque will be produced by the direct-axis component of the armature mmf which, by definition, is that component of the stator mmf which is aligned with that of the field winding on the rotor.

Equation 11.32 shows the torque in a nonsalient synchronous motor is proportional to the product of the field current and the quadrature-axis component of the armature current. This is directly analogous to torque production in a dc machine for which Eqs. 7.10 and 7.13 can be combined to show that the torque is proportional to the product of the field current and the armature current.

The analogy between a nonsalient synchronous machine and dc machine can be further reinforced. Consider Eq. 5.21, which expresses the rms value of the line-to-neutral generated voltage of a synchronous generator as

$$E_{af} = \frac{\omega_e L_{af} i_F}{\sqrt{2}} \quad (11.34)$$

Substitution into Eq. 11.32 gives

$$T_{\text{mech}} = \frac{3}{2} \left(\frac{\text{poles}}{\sqrt{2}}\right) \frac{E_{af} i_Q}{\omega_e} \quad (11.35)$$

This is directly analogous to Eq. 7.16 ( $T_{\text{mech}} = E_a I_a / \omega_m$ ) for a dc machine in which the torque is proportional to the product of the generated voltage and the armature current.

The brushes and commutator of a dc machine force the commutated armature current and armature flux along the quadrature axis such that  $I_d = 0$  and it is the interaction of this quadrature-axis current with the direct-axis field flux that produces the torque.<sup>2</sup> A field-oriented controller which senses the position of the rotor and controls the quadrature-axis component of armature current produces the same effect in a synchronous machine.

<sup>2</sup> In a practical dc motor, the brushes may be adjusted away from this ideal condition somewhat to improve commutation. In this case, some direct-axis flux will be produced, corresponding to a small direct-axis component of armature current.

Although the direct-axis component of the armature current does not play a role in torque production, it does play a role in determining the resultant stator flux and hence the machine terminal voltage, as can be readily shown. Specifically, from the transformation equations of Appendix C,

$$v_a = v_D \cos(\omega_e t) - v_Q \sin(\omega_e t) \quad (11.36)$$

and thus the rms amplitude of the armature voltage is equal to<sup>3</sup>

$$\begin{aligned} V_a &= \sqrt{\frac{v_D^2 + v_Q^2}{2}} = \omega_e \sqrt{\frac{\lambda_D^2 + \lambda_Q^2}{2}} \\ &= \omega_e \sqrt{\frac{(L_s i_D + L_{af} i_F)^2 + (L_s i_Q)^2}{2}} \end{aligned} \quad (11.37)$$

Dividing  $V_a$  by the electrical frequency  $\omega_e$ , we get an expression for rms armature flux linkages

$$(\lambda_a)_{rms} = \sqrt{\frac{\lambda_D^2 + \lambda_Q^2}{2}} = \sqrt{\frac{(L_s i_D + L_{af} i_F)^2 + (L_s i_Q)^2}{2}} \quad (11.38)$$

Similarly, the transformation equations of Appendix C can be used to show that the rms amplitude of the armature current is equal to

$$I_a = \sqrt{\frac{i_D^2 + i_Q^2}{2}} \quad (11.39)$$

From Eq. 11.32 we see that torque is controlled by the product  $i_F i_Q$  of the field current and the quadrature-axis component of the armature current. Thus, simply specifying a desired torque is not sufficient to uniquely determine either  $i_F$  or  $i_Q$ . In fact, under the field-oriented-control viewpoint presented here, there are actually three independent variables,  $i_F$ ,  $i_Q$  and  $i_D$ , and, in general, three constraints will be required to uniquely determine them. In addition to specifying the desired torque, a typical controller will implement additional constraints on terminal flux-linkages and current using the basic relationships found in Eqs. 11.38 and 11.39.

Figure 11.13a shows a typical field-oriented torque-control system in block-diagram form. The torque-controller block has two inputs,  $T_{ref}$ , the reference value or set point for torque and  $(i_F)_{ref}$ , the reference value or set point for the field current, which is also supplied to the power supply which supplies the current  $i_F$  to the motor field winding.  $(i_F)_{ref}$  is determined by an auxiliary controller which also determines

<sup>3</sup> Strictly speaking, armature resistance should be included in the voltage expression, in which case the rms amplitude of the armature voltage would be given by the expression

$$V_a = \sqrt{\frac{v_D^2 + v_Q^2}{2}} = \sqrt{\frac{(R_a i_D - \omega_e \lambda_Q)^2 + (R_a i_Q + \omega_e \lambda_D)^2}{2}}$$

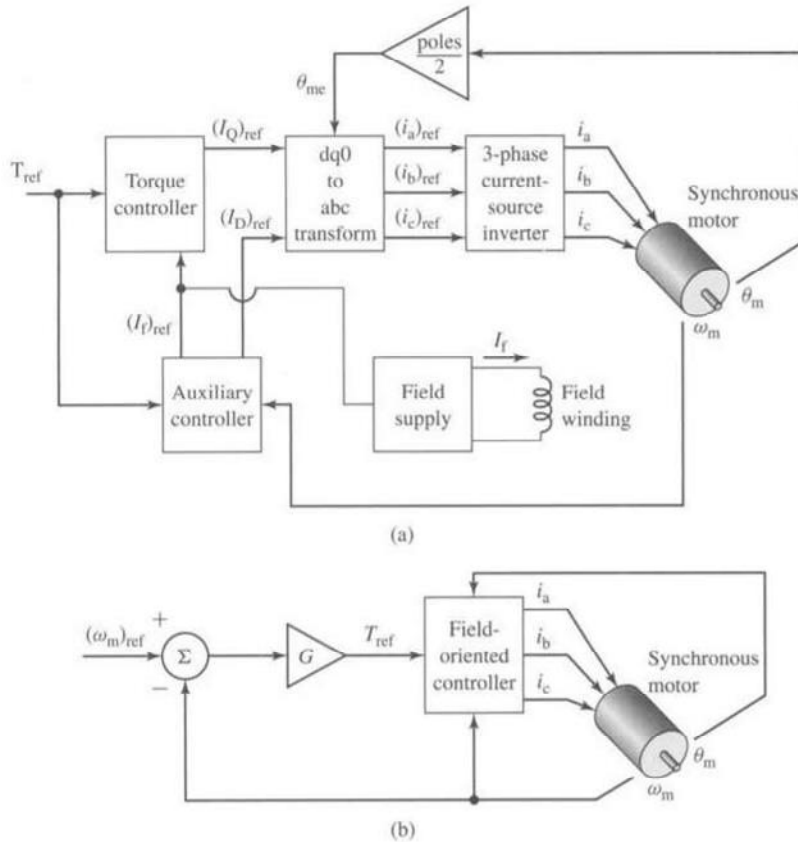


**Figure 11.13** (a) Block diagram of a field-oriented torque-control system for a synchronous motor. (b) Block diagram of a synchronous-motor speed-control loop built around a field-oriented torque control system.

the reference value  $(i_D)_{ref}$  of the direct-axis current, based upon desired values for the armature current and voltage. The torque controller calculates the desired quadrature-axis current from Eq. 11.32 based upon  $T_{ref}$  and  $(i_F)_{ref}$

$$(i_Q)_{ref} = \frac{2}{3} \left( \frac{2}{\text{poles}} \right) \frac{T_{ref}}{L_{af}(i_F)_{ref}} \quad (11.40)$$

Note that a position sensor is required to determine the angular position of the rotor in order to implement the dq0 to abc transformation.

In a typical application, the ultimate control objective is not to control motor torque but to control speed or position. Figure 11.13b shows how the torque-control system of Fig. 11.13a can be used as a component of a speed-control loop, with speed feedback forming an outer control loop around the inner torque-control loop.

**EXAMPLE 11.8**

Consider again the 45-kVA, 220-V, six-pole synchronous motor of Example 11.7 operating at 60 Hz with a field current of 2.84 A. If the motor is loaded to rated torque and operating under a field-oriented control system such that  $i_D = 0$ , calculate (a) the phase currents  $i_a(t)$ ,  $i_b(t)$ , and  $i_c(t)$  as well as the per-unit value of the armature current and (b) the motor terminal voltage in per unit. Assume that  $\theta_{me} = 0$  at time  $t = 0$  (i.e., the rotor direct axis is aligned with phase  $a$  at  $t = 0$ ).

**■ Solution**

- a. We must first calculate  $L_{af}$ . From Example 11.7, we see that the motor produces rated open-circuit voltage (220-V rms, line-to-line) at a field current of 2.84 A. From Eq. 11.34

$$L_{af} = \frac{\sqrt{2}E_{af}}{\omega_e i_F}$$

where  $E_{af}$  is the rms, line-to-neutral generated voltage. Thus

$$L_{af} = \frac{\sqrt{2}(220/\sqrt{3})}{120\pi \times 2.84} = 0.168 \text{ H}$$

Rated torque for this six-pole motor is equal to

$$\begin{aligned} T_{\text{rated}} &= \frac{P_{\text{rated}}}{(\omega_m)_{\text{rated}}} = \frac{P_{\text{rated}}}{(\omega_e)_{\text{rated}}(2/\text{poles})} \\ &= \frac{45 \times 10^3}{120\pi(2/6)} = 358 \text{ N} \cdot \text{m} \end{aligned}$$

Thus, setting  $T_{\text{ref}} = T_{\text{rated}} = 358 \text{ N} \cdot \text{m}$  and  $(i_F)_{\text{ref}} = 2.84 \text{ A}$ , we can find  $i_Q$  from Eq. 11.40 as

$$i_Q = \frac{2}{3} \left( \frac{2}{\text{poles}} \right) \frac{T_{\text{ref}}}{L_{af}(i_F)_{\text{ref}}} = \frac{2}{3} \left( \frac{2}{6} \right) \frac{358}{0.168 \times 2.84} = 167 \text{ A}$$

Using the fact that  $\theta_{me} = \omega_e t$  and setting  $i_D = 0$ , the transformation from dq0 variables to abc variables (Eq. C.2 of Appendix C) gives

$$i_a(t) = i_D \cos(\omega_e t) - i_Q \sin(\omega_e t) = -167 \sin(\omega_e t) = -\sqrt{2}(118) \sin(\omega_e t) \text{ A}$$

where  $\omega_e = 120\pi \approx 377 \text{ rad/sec}$ . Similarly

$$i_b(t) = -\sqrt{2}(118) \sin(\omega_e t - 120^\circ) \text{ A}$$

and

$$i_c(t) = -\sqrt{2}(118) \sin(\omega_e t + 120^\circ) \text{ A}$$

The rms armature current is 118 A and the machine base current is equal to

$$I_{\text{base}} = \frac{P_{\text{base}}}{\sqrt{3}V_{\text{base}}} = \frac{45 \times 10^3}{\sqrt{3}220} = 118 \text{ A}$$

Thus the per-unit machine terminal current is equal to  $I_a = 118/118 = 1.0$  per unit.

- b. The motor terminal voltage can most easily be found from the rms phasor relationship

$$\hat{V}_a = jX_s \hat{I}_a + \hat{E}_{af}$$

In part (a) we found that  $i_a = -\sqrt{2}(118) \sin(\omega_e t)$  A and thus

$$\hat{I}_a = j118 \text{ A}$$

We can find  $E_{af}$  from Eq. 11.34 as

$$E_{af} = \frac{\omega_e L_{af} i_F}{\sqrt{2}} = \frac{120\pi \times 0.168 \times 2.84}{\sqrt{2}} = 127 \text{ V line-to-neutral}$$

and, thus, since  $(\hat{E}_{af})_{rms}$  lies along the quadrature axis, as does  $\hat{I}_a$ ,

$$\hat{E}_{af} = j127 \text{ V}$$

The base impedance of the machine is equal to

$$Z_{base} = \frac{V_{base}^2}{P_{base}} = \frac{220^2}{45 \times 10^3} = 1.076 \Omega$$

and the synchronous reactance of 0.836 pu is equal to  $X_s = 0.836 \times 1.076 = 0.899 \Omega$ .

Thus the rms line-to-neutral terminal voltage

$$\begin{aligned} \hat{V}_a &= jX_s \hat{I}_a + \hat{E}_{af} = j0.899(j118) + j127 \\ &= -106 + j127 = 165 \angle 129.9^\circ \text{ V line-to-neutral} \end{aligned}$$

or  $V_a = 287 \text{ V rms line-to-line} = 1.30$  per unit.

Note that the terminal voltage for this operating condition is considerably in excess of the rated voltage of this machine, and hence such operation would not be acceptable. As we shall now discuss, a control algorithm which takes advantage of the full capability to vary  $i_F$ ,  $i_D$ , and  $i_Q$  can achieve rated torque while not exceeding rated terminal voltage.

### Practice Problem 11.8

Calculate the per-unit terminal voltage and current of Example 11.8 if the field-oriented controller maintains  $i_D = 0$  while reducing  $i_F$  to 2.6 A.

#### Solution

$V_a = 1.29$  per unit and  $I_a = 1.09$  per unit.

As we have discussed, a practical field-oriented control must determine values for all three currents  $i_F$ ,  $i_D$ , and  $i_Q$ . In Example 11.8 two of these values were chosen relatively arbitrarily ( $i_F = 2.84$  and  $i_D = 0$ ) and the result was a control that achieved the desired torque but with a terminal voltage 30 percent in excess of the motor-rated voltage. In a practical system, additional constraints are required to achieve an acceptable control algorithm.

One such algorithm would be to require that the motor operate at rated flux and at unity terminal power factor. Such an algorithm can be derived with reference to the

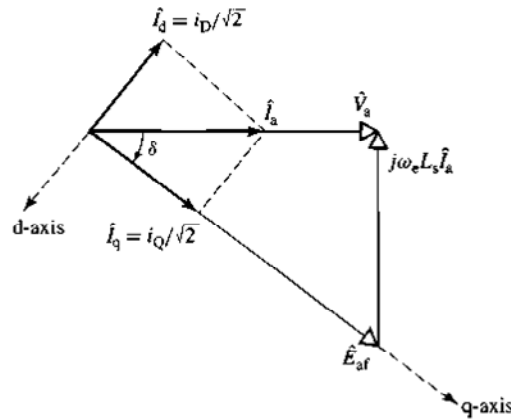


**Figure 11.14** Phasor diagram for unity-power-factor field-oriented-control algorithm.

phasor diagram of Fig 11.14 and can be implemented using the following steps:

Step 1. Calculate the line-to-neutral armature voltage corresponding to rated flux as

$$V_a = (V_a)_{\text{rated}} \left( \frac{\omega_m}{(\omega_m)_{\text{rated}}} \right) \quad (11.41)$$

where  $(V_a)_{\text{rated}}$  is the rated line-to-neutral armature voltage at rated motor speed,  $\omega_m$  is the desired motor speed, and  $(\omega_m)_{\text{rated}}$  is its rated speed.

Step 2. Calculate the rms armature current from the desired torque  $T_{\text{ref}}$  as

$$I_a = \frac{P_{\text{ref}}}{3V_a} = \frac{T_{\text{ref}} \omega_m}{3V_a} \quad (11.42)$$

where  $P_{\text{ref}}$  is the mechanical power corresponding to the desired torque.

Step 3. Calculate the angle  $\delta$  based upon the phasor diagram of Fig 11.14

$$\delta = -\tan^{-1} \left( \frac{\omega_e L_s I_a}{V_a} \right) \quad (11.43)$$

where  $\omega_e = \omega_{me} = (\text{poles}/2)\omega_m$  is the electrical frequency corresponding to the desired motor speed.

Step 4. Calculate  $(i_Q)_{\text{ref}}$  and  $(i_D)_{\text{ref}}$

$$(i_Q)_{\text{ref}} = \sqrt{2} I_a \cos \delta \quad (11.44)$$

$$(i_D)_{\text{ref}} = \sqrt{2} I_a \sin \delta \quad (11.45)$$

Step 5. Calculate  $(i_F)_{\text{ref}}$  from Eq. 11.32

$$(i_F)_{\text{ref}} = \frac{2}{3} \left( \frac{2}{\text{poles}} \right) \frac{T_{\text{ref}}}{L_{af}(i_Q)_{\text{ref}}} \quad (11.46)$$

This algorithm is illustrated in Example 11.9.

**EXAMPLE 11.9**

The 45-kVA, 220-V synchronous motor of Example 11.8 is to be again operated at rated torque and speed from a field-oriented control system. Calculate the required motor field current and the per-unit terminal voltage and current if the unity-power-factor algorithm described above is implemented.

**■ Solution**

We will follow the individual steps outlined above.

Step 1. Since the motor is operating at rated speed, the desired terminal voltage will be the rated line-to-neutral voltage of the machine.

$$V_a = \frac{220}{\sqrt{3}} = 127 \text{ V} = 1.0 \text{ per unit}$$

Step 2. Setting  $T_{\text{ref}} = 358 \text{ N}\cdot\text{m}$  and  $\omega_m = (2/\text{poles})\omega_e = 40\pi$ , the rms armature current can be calculated from Eq. 11.42

$$I_a = \frac{T_{\text{ref}} \omega_m}{3(V_a)} = \frac{358 \times (40\pi)}{3 \times 127} = 118 \text{ A}$$

As calculated in Example 11.8,  $I_{\text{base}} = 118 \text{ A}$  and thus  $I_a = 1.0$  per unit. This is as expected, since we want the motor to operate at rated torque, speed and terminal voltage, and at unity power factor.

Step 3. Next calculate  $\delta$  from Eq. 11.43. This calculation requires that we determine the synchronous inductance  $L_s$ .

$$L_s = \frac{X_s}{(\omega_e)_{\text{rated}}} = \frac{0.899}{120\pi} = 2.38 \text{ mH}$$

Thus

$$\begin{aligned} \delta &= -\tan^{-1} \left( \frac{\omega_e L_s I_a}{V_a} \right) \\ &= -\tan^{-1} \left( \frac{120\pi \times 2.38 \times 10^{-3} \times 118}{127} \right) = -0.695 \text{ rad} = -39.8^\circ \end{aligned}$$

Step 4. We can now calculate the desired values of  $i_Q$  and  $i_D$  from Eqs. 11.44 and 11.45.

$$(i_Q)_{\text{ref}} = \sqrt{2} I_a \cos \delta = 128 \text{ A}$$

and

$$(i_D)_{\text{ref}} = \sqrt{2} I_a \sin \delta = -107 \text{ A}$$

Step 5.  $(i_F)_{\text{ref}}$  is found from Eq. 11.46

$$(i_F)_{\text{ref}} = \frac{2}{3} \left( \frac{2}{\text{poles}} \right) \frac{T_{\text{ref}}}{L_{af}(i_Q)_{\text{ref}}} = \frac{2}{3} \left( \frac{2}{6} \right) \frac{358}{0.168 \times 128} = 3.70 \text{ A}$$

**Practice Problem 11.9**

Repeat Example 11.9 for the motor operating at rated torque and half of rated speed. Calculate (a) the desired motor field current, (b) the line-to-neutral terminal voltage (in volts), and (c) the armature current (in amperes).

**Solution**

- $(i_F)_{ref} = 3.70 \text{ A}$
- $V_a = 63.5 \text{ V line-to-neutral}$
- $I_a = 118 \text{ A}$

The discussion of this section has focused upon synchronous machines with field windings and the corresponding capability to control the field excitation. The basic concept, of course, also applies to synchronous machines with permanent magnets on the rotor. However, in the case of permanent-magnet synchronous machines, the effective field excitation is fixed and, as a result, there is one less degree of freedom for the field-oriented control algorithm.

For a permanent-magnet synchronous machine, since the effective equivalent field current is fixed by the permanent magnet, the quadrature-axis current is determined directly by the desired torque. Consider a three-phase permanent-magnet motor whose rated rms, line-to-neutral open-circuit voltage is  $(E_{af})_{rated}$  at electrical frequency  $(\omega_e)_{rated}$ . From Eq. 11.34 we see that the equivalent  $L_{af}I_f$  product for this motor, which we will refer to by the symbol  $\Lambda_{PM}$ , is

$$\Lambda_{PM} = \frac{\sqrt{2}(E_{af})_{rated}}{(\omega_e)_{rated}} \quad (11.47)$$

Thus, the direct-axis flux-current relationship for this motor, corresponding to Eq. 11.24, becomes

$$\lambda_D = L_d i_D + \Lambda_{PM} \quad (11.48)$$

and the torque expression of Eq. 11.32 becomes

$$T_{mech} = \frac{3}{2} \left( \frac{\text{poles}}{2} \right) \Lambda_{PM} i_Q \quad (11.49)$$

From Eq. 11.49 we see that, for a permanent-magnet synchronous machine under field-oriented control, the quadrature-axis current is uniquely determined by the desired torque and Eq. 11.40 becomes

$$(i_Q)_{ref} = \frac{2}{3} \left( \frac{2}{\text{poles}} \right) \frac{T_{ref}}{\Lambda_{PM}} \quad (11.50)$$

Once  $(i_Q)_{ref}$  has been specified, the only remaining control choice remains to determine the desired value for the direct-axis current,  $(i_D)_{ref}$ . One possibility is simply to set  $(i_D)_{ref} = 0$ . This will clearly result in the lowest possible armature current for a given torque. However, as we have seen in Example 11.8, this is likely to result in terminal voltages in excess of the rated voltage of the machine. As a

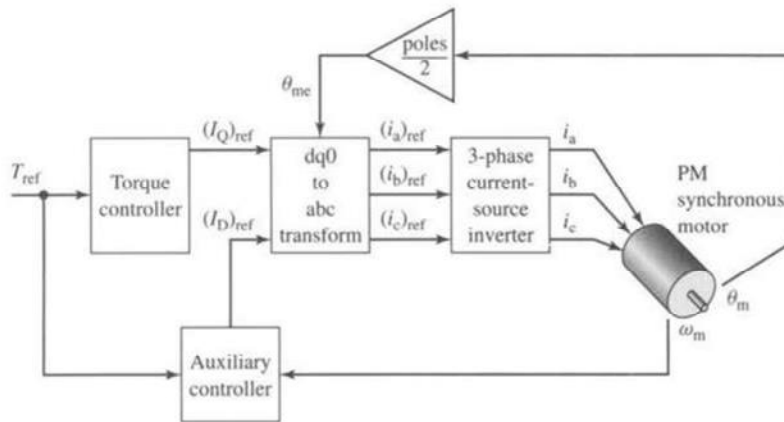


**Figure 11.15** Block diagram of a field-oriented torque-control system for a permanent-magnet synchronous motor.

result, it is common to supply direct-axis current so as to reduce the direct-axis flux linkage of Eq. 11.48, which will in turn result in reduced terminal voltage. This technique is commonly referred to as *flux weakening* and comes at the expense of increased armature current.<sup>4</sup> In practice, the chosen operating point is determined by a trade-off between reducing the armature voltage and an increase in armature current. Figure 11.15 shows the block diagram for a field-oriented-control system for use with a permanent-magnet motor.

#### EXAMPLE 11.10

A 25-kW, 4000-rpm, 220-V, two-pole, three-phase permanent-magnet synchronous motor produces rated open-circuit voltage at a rotational speed of 3200 r/min and has a synchronous inductance of 1.75 mH. Assume the motor is to be operated under field-oriented control at 2800 r/min and 65 percent of rated torque.

- Calculate the required quadrature-axis current.
- If the controller is designed to minimize armature current by setting  $i_D = 0$ , calculate the resultant armature flux linkage in per-unit.
- If the controller is designed to maintain the armature flux-linkage at its rated value, calculate the corresponding value of  $i_D$  and the corresponding rms and per-unit values of the armature current.

#### ■ Solution

- The rated speed of this machine is

$$(\omega_m)_{\text{rated}} = 4000 \left( \frac{\pi}{30} \right) = 419 \text{ rad/sec}$$

<sup>4</sup> See T. M. Jahns, "Flux-Weakening Regime Operation of an Interior Permanent Magnet Synchronous Motor Drive," *IEEE Transactions on Industry Applications*, Vol. 23, pp. 681–689.

and the rated torque is

$$T_{\text{rated}} = \frac{P_{\text{rated}}}{(\omega_m)_{\text{rated}}} = \frac{25 \times 10^3}{419} = 59.7 \text{ N} \cdot \text{m}$$

This motor achieves its rated-open-circuit voltage of  $220/\sqrt{3} = 127 \text{ V rms}$  at a speed of  $n = 3200 \text{ r/min}$ . The corresponding electrical frequency is

$$\omega_e = \left( \frac{\text{poles}}{2} \right) \left( \frac{\pi}{30} \right) n = \left( \frac{\pi}{30} \right) 3200 = 335 \text{ rad/sec}$$

From Eq. 11.47,

$$\Lambda_{PM} = \frac{\sqrt{2}(E_{af})_{\text{rated}}}{\omega_e} = \frac{\sqrt{2}127}{335} = 0.536 \text{ Wb}$$

Thus, setting  $T_{\text{ref}} = 0.65T_{\text{rated}} = 38.8 \text{ N} \cdot \text{m}$ , from Eq. 11.50 we find that

$$(i_Q)_{\text{ref}} = \frac{2}{3} \left( \frac{2}{\text{poles}} \right) \frac{T_{\text{ref}}}{\Lambda_{PM}} = \frac{2}{3} \left( \frac{38.8}{0.536} \right) = 48.3 \text{ A}$$

b. With  $(i_D)_{\text{ref}} = 0$ ,

$$\lambda_D = \Lambda_{PM} = 0.536 \text{ Wb}$$

and

$$\lambda_Q = L_s i_Q = (1.75 \times 10^{-3})48.3 = 0.0845 \text{ Wb}$$

Thus, the rms line-to-neutral armature flux is equal to

$$\lambda_a = \sqrt{\frac{\lambda_D^2 + \lambda_Q^2}{2}} = \sqrt{\frac{0.536^2 + 0.0845^2}{2}} = 0.384 \text{ Wb}$$

The base rms line-to-neutral armature flux can be determined from the base line-to-neutral voltage  $(V_a)_{\text{base}} = 127 \text{ V}$  and the base frequency  $(\omega_e)_{\text{base}} = 419 \text{ rad/sec}$  as

$$(\lambda_a)_{\text{base}} = \frac{(V_a)_{\text{base}}}{(\omega_e)_{\text{base}}} = 0.303 \text{ Wb}$$

Thus, the per-unit armature flux is equal to  $0.384/0.303 = 1.27$  per unit. From this calculation we see that the motor is significantly saturated at this operating condition. In fact, the calculation is probably not accurate because such a degree of saturation will most likely give rise to a reduction in the synchronous inductance as well as the magnetic coupling between the rotor and the stator.

c. In order to maintain rated armature flux linkage, the control will have to produce a direct-axis component of armature current to reduce the direct-axis flux linkage such that the total armature flux linkage is equal to the rated value  $(\lambda_a)_{\text{base}}$ . Specifically, we must have

$$\lambda_D = \sqrt{2(\lambda_a)_{\text{base}}^2 - \lambda_Q^2} = \sqrt{2 \times 0.303^2 - 0.0844^2} = 0.420 \text{ Wb}$$

We can now find  $(i_D)_{\text{ref}}$  from Eq. 11.48 (setting  $L_d = L_s$ )

$$(i_D)_{\text{ref}} = \frac{\lambda_D - \Lambda_{\text{PM}}}{L_s} = \frac{0.420 - 0.536}{1.75 \times 10^{-3}} = -66.3 \text{ A}$$

The corresponding rms armature current is

$$I_a = \sqrt{\frac{(i_D)_{\text{ref}}^2 + (i_Q)_{\text{ref}}^2}{2}} = \sqrt{\frac{66.3^2 + 48.3^2}{2}} = 58.0 \text{ A}$$

The base rms armature current for this machine is equal to

$$I_{\text{base}} = \frac{P_{\text{base}}}{\sqrt{3} V_{\text{base}}} = \frac{25 \times 10^3}{\sqrt{3} 220} = 65.6 \text{ A}$$

and hence the per-unit armature current is equal to  $58.0/65.6 = 0.88$  per unit.

Comparing the results of parts (b) and (c) we see how flux weakening by the introduction of direct-axis current can be used to control the terminal voltage of a permanent-magnet synchronous motor under field-oriented control.

#### Practice Problem 11.10

Consider again the motor of Example 11.10. Repeat the calculations of parts (b) and (c) of Example 11.10 for the case in which the motor is operating at 80 percent of rated torque at a speed of 2500 r/min.

#### Solution

For part (b),  $\lambda_a = 1.27$  per unit.

For part (c),  $I_a = 0.98$  per unit.

## 11.3 CONTROL OF INDUCTION MOTORS

### 11.3.1 Speed Control

Induction motors supplied from a constant-frequency source admirably fulfill the requirements of substantially constant-speed drives. Many motor applications, however, require several speeds or even a continuously adjustable range of speeds. From the earliest days of ac power systems, engineers have been interested in the development of adjustable-speed ac motors.

The synchronous speed of an induction motor can be changed by (a) changing the number of poles or (b) varying the line frequency. The operating slip can be changed by (c) varying the line voltage, (d) varying the rotor resistance, or (e) applying voltages of the appropriate frequency to the rotor circuits. The salient features of speed-control methods based on these five possibilities are discussed in the following five sections.

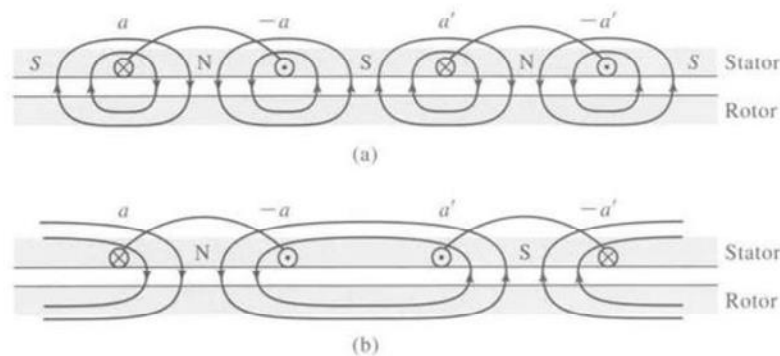


**Figure 11.16** Principles of the pole-changing winding.

**Pole-Changing Motors** In pole-changing motors, the stator winding is designed so that, by simple changes in coil connections, the number of poles can be changed in the ratio 2 to 1. Either of two synchronous speeds can then be selected. The rotor is almost always of the squirrel-cage type, which reacts by producing a rotor field having the same number of poles as the inducing stator field. With two independent sets of stator windings, each arranged for pole changing, as many as four synchronous speeds can be obtained in a squirrel-cage motor, for example, 600, 900, 1200, and 1800 r/min for 60-Hz operation.

The basic principles of the pole-changing winding are shown in Fig. 11.16, in which  $aa$  and  $a'a'$  are two coils comprising part of the phase-a stator winding. An actual winding would, of course, consist of several coils in each group. The windings for the other stator phases (not shown in the figure) would be similarly arranged. In Fig. 11.16a the coils are connected to produce a four-pole field; in Fig. 11.16b the current in the  $a'a'$  coil has been reversed by means of a controller, the result being a two-pole field.

Figure 11.17 shows the four possible arrangements of these two coils: they can be connected in series or in parallel, and with their currents either in the same direction (four-pole operation) or in the opposite direction (two-pole operation). Additionally, the machine phases can be connected either in Y or  $\Delta$ , resulting in eight possible combinations.

Note that for a given phase voltage, the different connections will result in differing levels of air-gap flux density. For example, a change from a  $\Delta$  to a Y connection will reduce the coil voltage (and hence the air-gap flux density) for a given coil arrangement by  $\sqrt{3}$ . Similarly, changing from a connection with two coils in series to two in parallel will double the voltage across each coil and therefore double the magnitude of the air-gap flux density. These changes in flux density can, of course, be compensated for by changes in the applied winding voltage. In any case, they must be considered, along with corresponding changes in motor torque, when the configuration to be used in a specific application is considered.

**Armature-Frequency Control** The synchronous speed of an induction motor can be controlled by varying the frequency of the applied armature voltage. This method

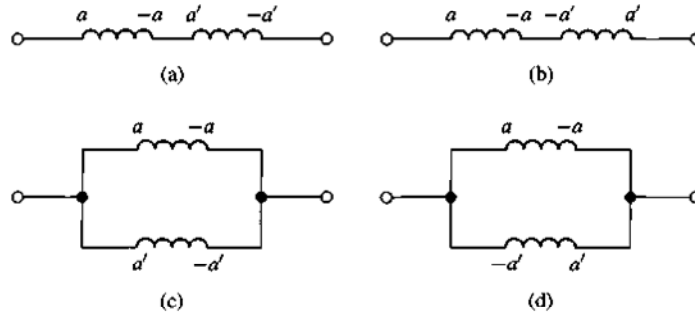


**Figure 11.17** Four possible arrangements of phase-a stator coils in a pole-changing induction motor: (a) series-connected, four-pole; (b) series-connected, two-pole; (c) parallel-connected, four-pole; (d) parallel-connected, two-pole.

of speed control is identical to that discussed in Section 11.2.1 for synchronous machines. In fact, the same inverter configurations used for synchronous machines, such as the three-phase voltage-source inverter of Fig. 11.11, can be used to drive induction motors. As is the case with any ac motor, to maintain approximately constant flux density, the armature voltage should also be varied directly with the frequency (constant-volts-per-hertz).

The torque-speed curve of an induction motor for a given frequency can be calculated by using the methods of Chapter 6 within the accuracy of the motor parameters at that frequency. Consider the torque expression of Eq. 6.33 which is repeated here.

$$T_{\text{mech}} = \frac{1}{\omega_s} \left[ \frac{n_{\text{ph}} V_{1,\text{eq}}^2 (R_2/s)}{(R_{1,\text{eq}} + (R_2/s))^2 + (X_{1,\text{eq}} + X_2)^2} \right] \quad (11.51)$$

where  $\omega_s = (2/\text{poles})\omega_e$  and  $\omega_e$  is the electrical excitation frequency of the motor in rad/sec,

$$\hat{V}_{1,\text{eq}} = \hat{V}_1 \left( \frac{jX_m}{R_1 + j(X_1 + X_m)} \right) \quad (11.52)$$

and

$$R_{1,\text{eq}} + jX_{1,\text{eq}} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} \quad (11.53)$$

To investigate the effect of changing frequency, we will assume that  $R_1$  is negligible. In this case,

$$\hat{V}_{1,\text{eq}} = \hat{V}_1 \left( \frac{X_m}{X_1 + X_m} \right) \quad (11.54)$$

$$R_{1,\text{eq}} = 0 \quad (11.55)$$

and

$$X_{1,eq} = \frac{X_m X_l}{X_l + X_m} \quad (11.56)$$

Let the subscript 0 represent rated-frequency values of each of the induction-motor parameters. As the electrical-excitation frequency is varied, we can then write

$$(X_{1,eq} + X_2) = \left( \frac{\omega_e}{\omega_{e0}} \right) (X_{1,eq} + X_2)_0 \quad (11.57)$$

Under constant-volts-per-hertz control, we can also write the equivalent source voltage as

$$\hat{V}_1 = \left( \frac{\omega_e}{\omega_{e0}} \right) (\hat{V}_1)_0 \quad (11.58)$$

and hence, since  $\hat{V}_{1,eq}$  is equal to  $\hat{V}_1$  multiplied by a reactance ratio which stays constant with changing frequency,

$$\hat{V}_{1,eq} = \left( \frac{\omega_e}{\omega_{e0}} \right) (\hat{V}_{1,eq})_0 \quad (11.59)$$

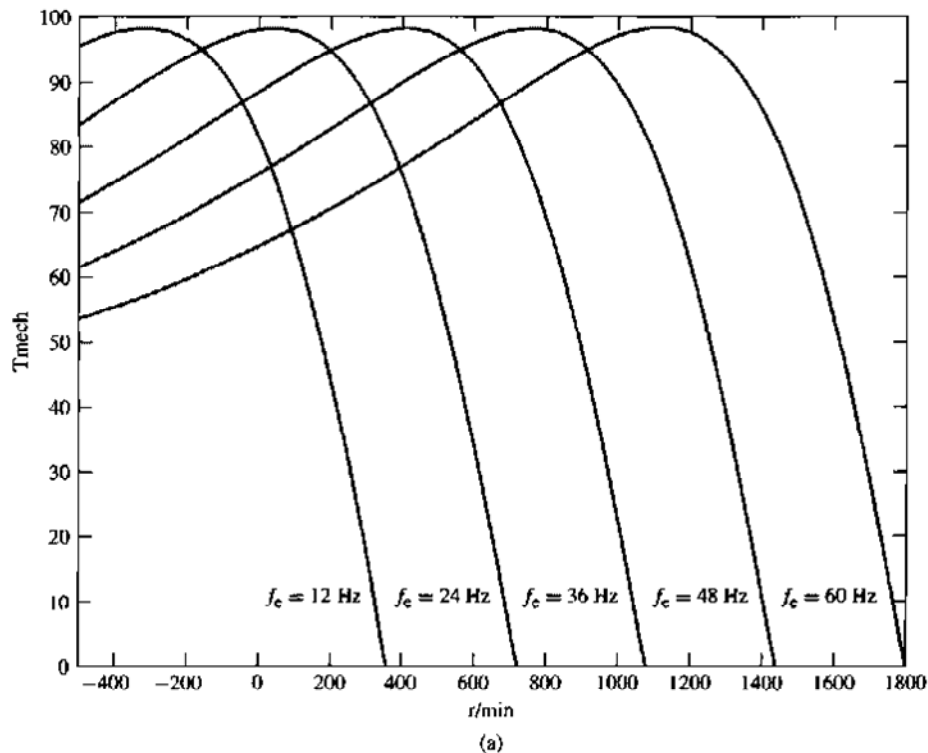


**Figure 11.18** A family of typical induction-motor speed-torque curves for a four-pole motor for various values of the electrical supply frequency. (a)  $R_l$  sufficiently small so that its effects are negligible. (b)  $R_l$  not negligible.

Finally, we can write the motor slip as

$$s = \frac{\omega_s - \omega_m}{\omega_s} = \frac{\text{poles}}{2} \left( \frac{\Delta\omega_m}{\omega_e} \right) \quad (11.60)$$

where  $\Delta\omega_m = \omega_s - \omega_m$  is the difference between the synchronous and mechanical angular velocities of the motor.

Substitution of Eqs. 11.57 through 11.60 into Eq. 11.51 gives

$$T_{\text{mech}} = \frac{n_{\text{ph}}[(V_{1,\text{eq}})_0]^2(R_2/\Delta\omega)}{\left[\left(\frac{2\omega_{\text{poles}}}{\text{poles}}\right)(R_2/\Delta\omega)\right]^2 + [(X_{1,\text{eq}} + X_2)_0]^2} \quad (11.61)$$

Equation 11.61 shows the general trend in which we see that the frequency dependence of the torque-speed characteristic of an induction motor appears only in the term  $R_2/\Delta\omega$ . Thus, under the assumption that  $R_1$  is negligible, as the electrical supply frequency to an induction motor is changed, the shape of the speed-torque curve as a function of  $\Delta\omega$  (the difference between the synchronous speed and the motor speed) will remain unchanged. As a result, the torque-speed characteristic will simply shift along the speed axis as  $\omega_e(f_e)$  is varied.

A set of such curves is shown in Fig. 11.18a. Note that as the electrical frequency (and hence the synchronous speed) is decreased, a given value of  $\Delta\omega$  corresponds to a

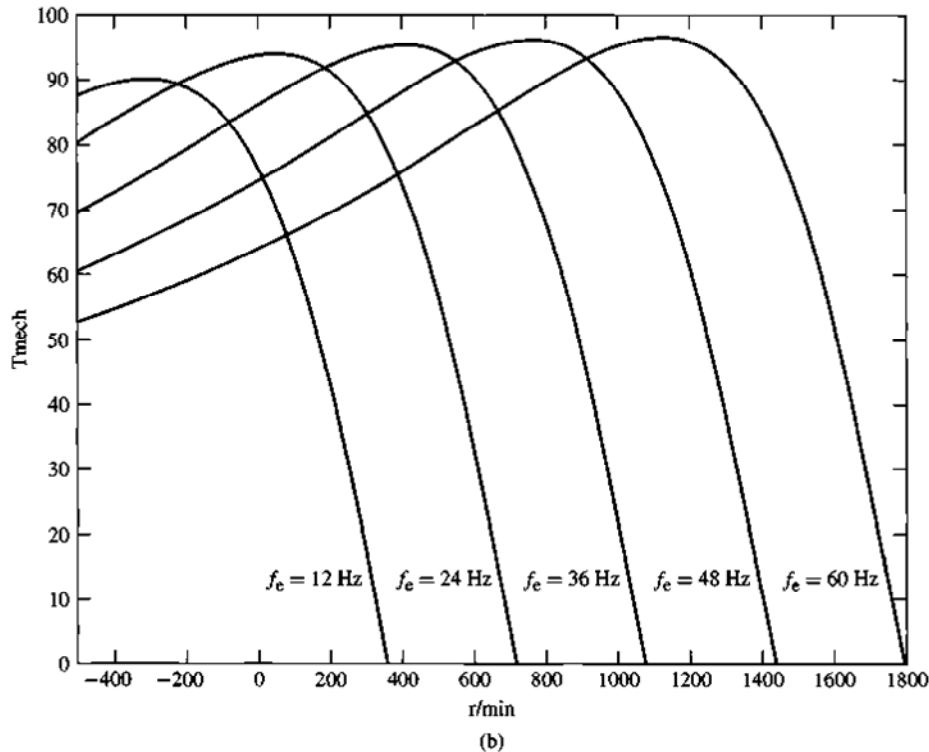


**Figure 11.18** (Continued)

larger slip. Thus, for example, if the peak torque of a four-pole motor driven at 60 Hz occurs at 1638 r/min, corresponding to a slip of 9 percent, when driven at 30 Hz, the peak torque will occur at 738 r/min, corresponding to a slip of 18 percent.

In practice, the effects of  $R_1$  may not be fully negligible, especially for large values of slip. If this is the case, the shape of the speed-torque curves will vary somewhat with the applied electrical frequency. Figure 11.18b shows a typical family of curves for this case.

**EXAMPLE 11.11**

The three-phase, 230-V, 60-Hz, 12-kW, four-pole induction motor of Example 6.4 (with  $R_2 = 0.2 \Omega$ ) is to be operated from a variable-frequency, constant-volts-per-hertz motor drive whose terminal voltage is 230 V at 60 Hz. The motor is driving a load whose power can be assumed to vary as

$$P_{\text{load}} = 10.5 \left( \frac{n}{1800} \right)^3 \text{ kW}$$

where  $n$  is the load speed in r/min. Motor rotational losses can be assumed to be negligible.

Write a MATLAB script to find the line-to-line terminal voltage, the motor speed in r/min, the slip and the motor load in kW for (a) a source frequency of 60 Hz and (b) a source frequency of 40 Hz.

**■ Solution**

As the electrical frequency  $f_e$  is varied, the motor reactances given in Example 6.4 must be varied as

$$X = X_0 \left( \frac{f_e}{60} \right)$$

where  $X_0$  is the reactance value at 60 Hz. Similarly, the line-to-neutral armature voltage must be varied as

$$V_1 = \frac{220}{\sqrt{3}} \left( \frac{f_e}{60} \right) = 127 \left( \frac{f_e}{60} \right) \text{ V}$$

From Eq. 4.40, the synchronous angular velocity of the motor is equal to

$$\omega_s = \left( \frac{4\pi}{\text{poles}} \right) f_e = \pi f_e \text{ rad/sec}$$

and, at any given motor speed  $\omega_m$ , the corresponding slip is given by

$$s = \frac{\omega_s - \omega_m}{\omega_s}$$

Using Eqs. 11.51 through 11.53, the motor speed can be found by searching over  $\omega_m$  for that speed at which  $P_{\text{load}} = \omega_m T_{\text{mech}}$ . If this is done, the result is:

a. For  $f_e = 60$  Hz:

Terminal voltage = 230 V line-to-line

Speed = 1720 r/min

$$\text{Slip} = 4.4\%$$

$$P_{\text{load}} = 9.17 \text{ kW}$$

b. For  $f_e = 40 \text{ Hz}$ :

$$\text{Terminal voltage} = 153 \text{ V line-to-line}$$

$$\text{Speed} = 1166 \text{ r/min}$$

$$\text{Slip} = 2.8\%$$

$$P_{\text{load}} = 2.86 \text{ kW}$$

Here is the MATLAB script:

```
clc
clear

%Here are the 60-Hz motor parameters
V10 = 230/sqrt(3);
Nph = 3;
poles = 4;
fe0 = 60;
R1 = 0.095;
R2 = 0.2;
X10 = 0.680;
X20 = 0.672;
Xm0 = 18.7;

% Two frequency values
fe1 = 60;
fe2 = 40;
for m = 1:2,
    if m == 1
        fe = fe1;
    else
        fe = fe2;
    end

% Calculate the reactances and the voltage
X1 = X10*(fe/fe0);
X2 = X20*(fe/fe0);
Xm = Xm0*(fe/fe0);
V1 = V10*(fe/fe0);

%Calculate the synchronous speed
omegas = 4*pi*fe/poles;
ns = 120*fe/poles;

%Calculate stator Thevenin equivalent
V1eq = abs(V1*j*Xm/(R1 + j*(X1+Xm)));
```

```

Z1eq = j*Xm*(R1+j*X1)/(R1 + j*(X1+Xm));
R1eq = real(Z1eq);
X1eq = imag(Z1eq);

%Search over the slip until the Pload = Pmech
slip = 0.;
error = 1;
while error >= 0;
    slip = slip + 0.00001;
    rpm = ns*(1-slip);
    omegam = omegas*(1-slip);
    Tmech = (1/omegas)*Nph*V1eq^2*(R2/slip);
    Tmech = Tmech/((R1+R2/slip)^2 + (X1+X2)^2);
    Pmech = Tmech*omegam;
    Pload = 10.5e3*(rpm/1800)^3;
    error = Pload - Pmech;
end %End of while loop
fprintf('\nFor fe = %g [Hz]:',fe)
fprintf('\n    Terminal voltage = %g [V l-l]',V1*sqrt(3))
fprintf('\n    rpm = %g',rpm)
fprintf('\n    slip = %g [percent] ',100*slip)
fprintf('\n    Pload = %g [kW]',Pload/1000)
fprintf('\n\n')
end

```

---

**Practice Problem 11.11**

Repeat Example 11.11 for a source frequency of 50 Hz.

**Solution**

Terminal voltage = 192 V line-to-line

Speed = 1447 r/min

Slip = 3.6%

$P_{\text{load}} = 5.45 \text{ kW}$

---

**Line-Voltage Control** The internal torque developed by an induction motor is proportional to the square of the voltage applied to its primary terminals, as shown by the two torque-speed characteristics in Fig. 11.19. If the load has the torque-speed characteristic shown by the dashed line, the speed will be reduced from  $n_1$  to  $n_2$ . This method of speed control is commonly used with small squirrel-cage motors driving fans, where cost is an issue and the inefficiency of high-slip operation can be tolerated. It is characterized by a rather limited range of speed control.

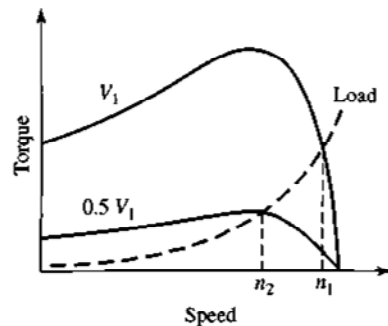


**Figure 11.19** Speed control by means of line voltage.

**Rotor-Resistance Control** The possibility of speed control of a wound-rotor motor by changing its rotor-circuit resistance has already been pointed out in Section 6.7.1. The torque-speed characteristics for three different values of rotor resistance are shown in Fig. 11.20. If the load has the torque-speed characteristic shown by the dashed line, the speeds corresponding to each of the values of rotor resistance are  $n_1$ ,  $n_2$ , and  $n_3$ . This method of speed control has characteristics similar to those of dc shunt-motor speed control by means of resistance in series with the armature.

The principal disadvantages of both line-voltage and rotor-resistance control are low efficiency at reduced speeds and poor speed regulation with respect to change in load. In addition, the cost and maintenance requirements of wound-rotor induction motors are sufficiently high that squirrel-cage motors combined with solid-state drives have become the preferred option in most applications.

### 11.3.2 Torque Control

In Section 11.2.2 we developed the concept of field-oriented-control for synchronous machines. Under this viewpoint, the armature flux and current are resolved into two components which rotate synchronously with the rotor and with the air-gap flux wave.

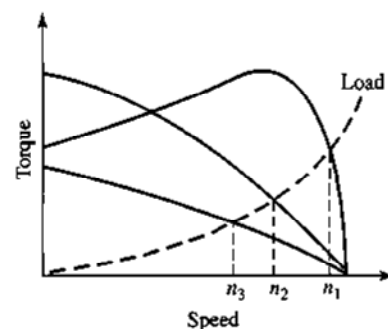


**Figure 11.20** Speed control by means of rotor resistance.

The components of armature current and flux which are aligned with the field-winding are referred to as *direct-axis components* while those which are perpendicular to this axis are referred to as *quadrature-axis components*.

It turns out that the same viewpoint which we applied to synchronous machines can be applied to induction machines. As is discussed in Section 6.1, in the steady-state the mmf and flux waves produced by both the rotor and stator windings of an induction motor rotate at synchronous speed and in synchronism with each other. Thus, the torque-producing mechanism in an induction machine is equivalent to that of a synchronous machine. The difference between the two is that, in an induction machine, the rotor currents are not directly supplied but rather are induced as the induction-motor rotor slips with respect to the rotating flux wave produced by the stator currents.

To examine the application of field-oriented control to induction machines, we begin with the dq0 transformation of Section C.3 of Appendix C. This transformation transforms both the stator and rotor quantities into a synchronously rotating reference frame. Under balanced-three-phase, steady-state conditions, zero-sequence quantities will be zero and the remaining direct- and quadrature-axis quantities will be constant. Hence the flux-linkage current relationships of Eqs. C.52 through C.58 become

$$\lambda_D = L_S i_D + L_m i_{DR} \quad (11.62)$$

$$\lambda_Q = L_S i_Q + L_m i_{QR} \quad (11.63)$$

$$\lambda_{DR} = L_m i_D + L_R i_{DR} \quad (11.64)$$

$$\lambda_{QR} = L_m i_Q + L_R i_{QR} \quad (11.65)$$

In these equations, the subscripts D, Q, DR, and QR represent the constant values of the direct- and quadrature-axis components of the stator and rotor quantities respectively. It is a straight-forward matter to show that the inductance parameters can be determined from the equivalent-circuit parameters as

$$L_m = \frac{X_{m0}}{\omega_{e0}} \quad (11.66)$$

$$L_S = L_m + \frac{X_{10}}{\omega_{e0}} \quad (11.67)$$

$$L_R = L_m + \frac{X_{20}}{\omega_{e0}} \quad (11.68)$$

where the subscript 0 indicates the rated-frequency value.

The transformed voltage equations Eqs. C.63 through C.68 become

$$v_D = R_S i_D - \omega_e \lambda_Q \quad (11.69)$$

$$v_Q = R_S i_Q + \omega_e \lambda_D \quad (11.70)$$

$$0 = R_{aR} i_{DR} - (\omega_e - \omega_{me}) \lambda_{QR} \quad (11.71)$$

$$0 = R_{aR} i_{QR} + (\omega_e - \omega_{me}) \lambda_{DR} \quad (11.72)$$

where one can show that the resistances are related to those of the equivalent circuit as

$$R_a = R_1 \quad (11.73)$$

and

$$R_{aR} = R_2 \quad (11.74)$$

For the purposes of developing a field-oriented-control scheme, we will begin with the torque expression of Eq. C.70

$$T_{\text{mech}} = \frac{3}{2} \left( \frac{\text{poles}}{2} \right) \left( \frac{L_m}{L_R} \right) (\lambda_{DR} i_q - \lambda_{QR} i_d) \quad (11.75)$$

For the derivation of the dq0 transformation in Section C.3, the angular velocity of the reference frame was chosen to the synchronous speed as determined by the stator electrical frequency  $\omega_e$ . It was not necessary for the purposes of the derivation to specify the absolute angular location of the reference frame. It is convenient at this point to choose the direct axis of the reference frame aligned with the rotor flux.

If this is done

$$\lambda_{QR} = 0 \quad (11.76)$$

and the torque expression of Eq. 11.75 becomes

$$T_{\text{mech}} = \frac{3}{2} \left( \frac{\text{poles}}{2} \right) \left( \frac{L_m}{L_R} \right) \lambda_{DR} i_Q \quad (11.77)$$

From Eq. 11.71 we see that

$$i_{DR} = 0 \quad (11.78)$$

and thus

$$\lambda_{DR} = L_m i_D \quad (11.79)$$

and

$$\lambda_D = L_S i_D \quad (11.80)$$

From Eqs. 11.79 and 11.80 we see that by choosing set  $\lambda_{QR} = 0$  and thus aligning the synchronously rotating reference frame with the axis of the rotor flux, the direct-axis rotor flux (which is, indeed, the total rotor flux) as well as the direct-axis flux are determined by the direct-axis component of the armature current. Notice the direct analogy with a dc motor. In a dc motor, the field- and direct-axis armature fluxes are determined by the field current and in this field-oriented control scheme, the rotor and direct-axis armature fluxes are determined by the direct-axis armature current. In other words, in this field-oriented control scheme, the direct-axis component of armature current serves the same function as the field current in a dc machine.

The torque equation, Eq. 11.77, completes the analogy with the dc motor. We see that once the rotor direct-axis flux  $\lambda_{DR}$  is set by the direct-axis armature current, the

torque is then determined by the quadrature-axis armature current just as the torque is determined by the armature current in a dc motor.

In a practical implementation of the technique which we have derived, the direct- and quadrature-axis currents  $i_D$  and  $i_Q$  must be transformed into the three motor phase currents  $i_a(t)$ ,  $i_b(t)$ , and  $i_c(t)$ . This can be done using the inverse dq0 transformation of Eq. C.48 which requires knowledge of  $\theta_s$ , the electrical angle between the axis of phase  $a$ , and the direct-axis of the synchronously rotating reference frame.

Since it is not possible to measure the axis of the rotor flux directly, it is necessary to calculate  $\theta_s$ , where  $\theta_s = \omega_e t + \theta_0$  as given by Eq. C.46. Solving Eq. 11.72 for  $\omega_e$  gives

$$\omega_e = \omega_{me} - R_{aR} \left( \frac{i_{QR}}{\lambda_{DR}} \right) \quad (11.81)$$

From Eq. 11.65 with  $\lambda_{QR} = 0$  we see that

$$i_{QR} = - \left( \frac{L_m}{L_R} \right) i_Q \quad (11.82)$$

Eq. 11.82 in combination with Eq. 11.79 then gives

$$\omega_e = \omega_{me} + \frac{R_{aR}}{L_R} \left( \frac{i_Q}{i_D} \right) = \omega_{me} + \frac{1}{\tau_R} \left( \frac{i_Q}{i_D} \right) \quad (11.83)$$

where  $\tau_R = L_R/R_{aR}$  is the rotor time constant.

We can now integrate Eq. 11.83 to find

$$\hat{\theta}_s = \left[ \omega_{me} + \frac{1}{\tau_R} \left( \frac{i_Q}{i_D} \right) \right] t + \theta_0 \quad (11.84)$$

where  $\hat{\theta}_s$  indicates the calculated value of  $\theta_s$  (often referred to as the *estimated value* of  $\theta_s$ ). In the more general dynamic sense

$$\hat{\theta}_s = \int_0^t \left[ \omega_{me} + \frac{1}{\tau_R} \left( \frac{i_Q}{i_D} \right) \right] dt' + \theta_0 \quad (11.85)$$

Note that both Eqs. 11.84 and 11.85 require knowledge of  $\theta_0$ , the value of  $\theta_s$  at  $t = 0$ . Although we will not prove it here, it turns out that in a practical implementation, the effects of an error in this initial angle decay to zero with time, and hence it can be set to zero without any loss of generality.

Figure 11.21a shows a block diagram of a field-oriented torque-control system for an induction machine. The block labeled "Estimator" represents the calculation of Eq. 11.85 which calculates the estimate of  $\theta_s$  required by the transformation from dq0 to abc variables.

Note that a speed sensor is required to provide the rotor speed measurement required by the estimator. Also notice that the estimator requires knowledge of the rotor time constant  $\tau_R = L_R/R_{aR}$ . In general, this will not be known exactly, both due to uncertainty in the machine parameters as well as due to the fact that the rotor

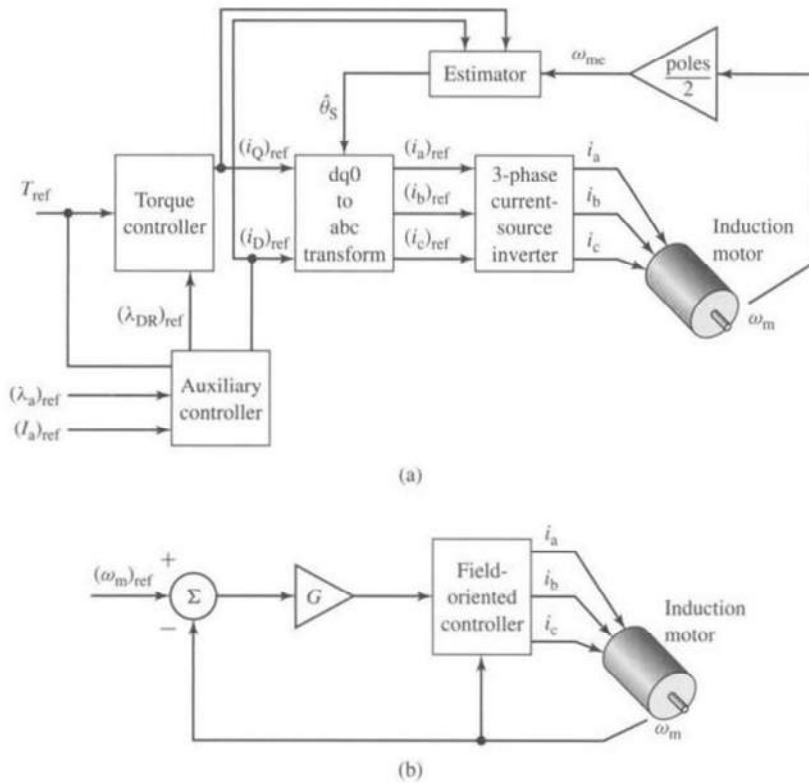


**Figure 11.21** (a) Block diagram of a field-oriented torque-control system for an induction motor. (b) Block diagram of an induction-motor speed-control loop built around a field-oriented torque control system.

resistance  $R_{aR}$  will undoubtedly change with temperature as the motor is operated. It can be shown that errors in  $\tau_R$  result in an offset in the estimate of  $\theta_s$ , which in turn will result in an error in the estimate for the position of the rotor flux with the result that the applied armature currents will not be exactly aligned with the direct- and quadrature-axes. The torque controller will still work basically as expected, although there will be corresponding errors in the torque and rotor flux.

As with the synchronous motor, the rms armature flux-linkages can be found from Eq. 11.38 as

$$(\lambda_a)_{rms} = \sqrt{\frac{\lambda_D^2 + \lambda_Q^2}{2}} \quad (11.86)$$

Combining Eqs. 11.63 and 11.82 gives

$$\lambda_Q = L_S i_Q + L_m i_{QR} = \left( L_S - \frac{L_m^2}{L_R} \right) i_Q \quad (11.87)$$

Substituting Eqs. 11.80 and 11.87 into Eq. 11.86 gives

$$(\lambda_a)_{\text{rms}} = \sqrt{\frac{L_S^2 i_D^2 + \left(L_S - \frac{L_m^2}{L_R}\right)^2 i_Q^2}{2}} \quad (11.88)$$

Finally, as discussed in the footnote to Eq. 11.37, the rms line-to-neutral armature voltage can be found as

$$\begin{aligned} V_a &= \sqrt{\frac{v_D^2 + v_Q^2}{2}} = \sqrt{\frac{(R_a i_D - \omega_e \lambda_Q)^2 + (R_a i_Q + \omega_e \lambda_D)^2}{2}} \\ &= \sqrt{\frac{\left(R_a i_D - \omega_e \left(L_S - \frac{L_m^2}{L_R}\right) i_Q\right)^2 + (R_a i_Q + \omega_e L_S i_D)^2}{2}} \quad (11.89) \end{aligned}$$

These equations show that the armature flux linkages and terminal voltage are determined by both the direct- and quadrature-axis components of the armature current. Thus, the block marked “Auxiliary Controller” in Fig. 11.21a, which calculates the reference values for the direct- and quadrature-axis currents, must calculate the reference currents  $(i_D)_{\text{ref}}$  and  $(i_Q)_{\text{ref}}$  which achieve the desired torque subject to constraints on armature flux linkages (to avoid saturation in the motor), armature current,  $(I_a)_{\text{rms}} = \sqrt{(i_D^2 + i_Q^2)}/2$  (to avoid excessive armature heating) and armature voltage (to avoid potential insulation damage).

Note that, as we discussed with regard to synchronous machines in Section 11.2.2, the torque-control system of Fig. 11.21a is typically imbedded within a larger control loop. One such example is the speed-control loop of Fig. 11.21b.

### EXAMPLE 11.12

The three-phase, 230-V, 60-Hz, 12-kW, four-pole induction motor of Example 6.7 and Example 11.11 is to be driven by a field-oriented speed-control system (similar to that of Fig. 11.21b) at a speed of 1740 r/min. Assuming the controller is programmed to set the rotor flux linkages  $\lambda_{DR}$  to the machine rated peak value, find the rms amplitude of the armature current, the electrical frequency, and the rms terminal voltage, if the electromagnetic power is 9.7 kW and the motor is operating at a speed of 1680 r/min.

#### ■ Solution

We must first determine the parameters for this machine. From Eqs. 11.66 through Eq. 11.74

$$\begin{aligned} L_m &= \frac{X_{m0}}{\omega_{e0}} = \frac{18.7}{120\pi} = 49.6 \text{ mH} \\ L_S &= L_m + \frac{X_{10}}{\omega_{e0}} = 49.6 \text{ mH} + \frac{0.680}{120\pi} = 51.41 \text{ mH} \\ L_R &= L_m + \frac{X_{20}}{\omega_{e0}} = 49.6 \text{ mH} + \frac{0.672}{120\pi} = 51.39 \text{ mH} \\ R_s &= R_1 = 0.095 \Omega \\ R_{aR} &= R_2 = 0.2 \Omega \end{aligned}$$

The rated rms line-to-neutral terminal voltage for this machine is  $230/\sqrt{3} = 132.8$  V and thus the peak rated flux for this machine is

$$(\lambda_{\text{rated}})_{\text{peak}} = \frac{\sqrt{2}(V_a)_{\text{rated}}}{\omega_e} = \frac{\sqrt{2} \times 132.8}{120\pi} = 0.498 \text{ Wb}$$

For the specified operating condition

$$\omega_m = n \left( \frac{\pi}{30} \right) = 1680 \left( \frac{\pi}{30} \right) = 176 \text{ rad/sec}$$

and the mechanical torque is

$$T_{\text{mech}} = \frac{P_{\text{mech}}}{\omega_m} = \frac{9.7 \times 10^3}{176} = 55.1 \text{ N} \cdot \text{m}$$

From Eq. 11.77, with  $\lambda_{\text{DR}} = \lambda_{\text{rated}} = 0.498$  Wb

$$\begin{aligned} i_Q &= \frac{2}{3} \left( \frac{2}{\text{poles}} \right) \left( \frac{L_R}{L_m} \right) \left( \frac{T_{\text{mech}}}{\lambda_{\text{DR}}} \right) \\ &= \frac{2}{3} \left( \frac{2}{4} \right) \left( \frac{51.39 \times 10^{-3}}{49.6 \times 10^{-3}} \right) \left( \frac{55.1}{0.498} \right) = 38.2 \text{ A} \end{aligned}$$

From Eq. 11.79,

$$i_D = \frac{\lambda_{\text{DR}}}{L_m} = \frac{0.498}{49.6 \times 10^{-3}} = 10.0 \text{ A}$$

The rms armature current is thus

$$I_a = \sqrt{\frac{i_D^2 + i_Q^2}{2}} = \sqrt{\frac{10.0^2 + 38.2^2}{2}} = 27.9 \text{ A}$$

The electrical frequency can be found from Eq. 11.81

$$\omega_e = \omega_{me} + \frac{R_{sR}}{L_R} \left( \frac{i_Q}{i_D} \right)$$

With  $\omega_{me} = (\text{poles}/2)\omega_m = 2 \times 176 = 352$  rad/sec

$$\omega_e = 352 + \left( \frac{0.2}{51.39 \times 10^{-3}} \right) \left( \frac{38.2}{10.0} \right) = 367 \text{ rad/sec}$$

and  $f_e = \omega_e/(2\pi) = 58.4$  Hz.

Finally, from Eq. 11.89, the rms line-to-neutral terminal voltage

$$\begin{aligned} V_s &= \sqrt{\frac{\left( R_s i_D - \omega_e \left( L_s - \frac{L_m^2}{L_R} \right) i_Q \right)^2 + (R_s i_Q + \omega_e L_s i_D)^2}{2}} \\ &= 140.6 \text{ V line-to-neutral} = 243.6 \text{ V line-to-line} \end{aligned}$$

**Practice Problem 11.12**

Consider again the induction motor and field-oriented-control system of Example 11.12. Assume that the speed is readjusted to 1700 r/min and that the electromagnetic power is known to increase to 10.0 kW. Find the rms amplitude of the armature current, the electrical frequency, and the rms terminal voltage for this new operating condition.

**Solution**

$$\text{Armature current} = 28.4 \text{ A}$$

$$f_e = 59.1 \text{ Hz}$$

$$\text{Terminal voltage} = 142.5 \text{ V line-to-neutral} = 246.9 \text{ V line-to-line}$$

The ability to independently control the rotor flux and the torque has important control implications. Consider, for example, the response of the direct-axis rotor flux to a change in direct-axis current. Equation C.66, with  $\lambda_{qR} = 0$ , becomes

$$0 = R_{aR} i_{dR} + \frac{d\lambda_{dR}}{dt} \quad (11.90)$$

Substituting for  $i_{dR}$  in terms of  $\lambda_{dR}$

$$i_{dR} = \frac{\lambda_{dR} - L_m i_d}{L_R} \quad (11.91)$$

gives a differential equation for the rotor flux linkages  $\lambda_{dR}$

$$\frac{d\lambda_{dR}}{dt} + \left( \frac{R_{aR}}{L_R} \right) \lambda_{dR} = \left( \frac{L_m}{L_R} \right) i_d \quad (11.92)$$

From Eq. 11.92 we see that the response of the rotor flux to a step change in direct-axis current  $i_d$  is relatively slow;  $\lambda_{dR}$  will change exponentially with the rotor time constant of  $\tau_R = L_R / R_{aR}$ . Since the torque is proportional to the product  $\lambda_{dR} i_q$  we see that fast torque response will be obtained from changes in  $i_q$ . Thus, for example, to implement a step change in torque, a practical control algorithm might start with a step change in  $(i_q)_{ref}$  to achieve the desired torque change, followed by an adjustment in  $(i_D)_{ref}$  (and hence  $\lambda_{dR}$ ) to readjust the armature current or terminal voltage as desired. This adjustment in  $(i_D)_{ref}$  would be coupled with a compensating adjustment in  $(i_q)_{ref}$  to maintain the torque at its desired value.

**EXAMPLE 11.13**

Consider again the induction motor of Example 11.12. Assuming the motor speed and electromagnetic power remain constant (at 1680 r/min and 9.7 kW), use MATLAB to plot the per-unit armature current  $I_a$  and terminal voltage  $V_a$  as a function of  $i_D$  as  $(\lambda_{dR})_{ref}$  is varied between 0.8 and 1.2 per unit, where 1.0 per unit corresponds to the rated peak value.

**■ Solution**

The desired plot is given in Fig. 11.22. Note that the armature current decreases and the terminal voltage increases as  $\lambda_{dR}$  is increased. This clearly shows how  $i_D$ , which controls  $\lambda_{dR}$ , can be

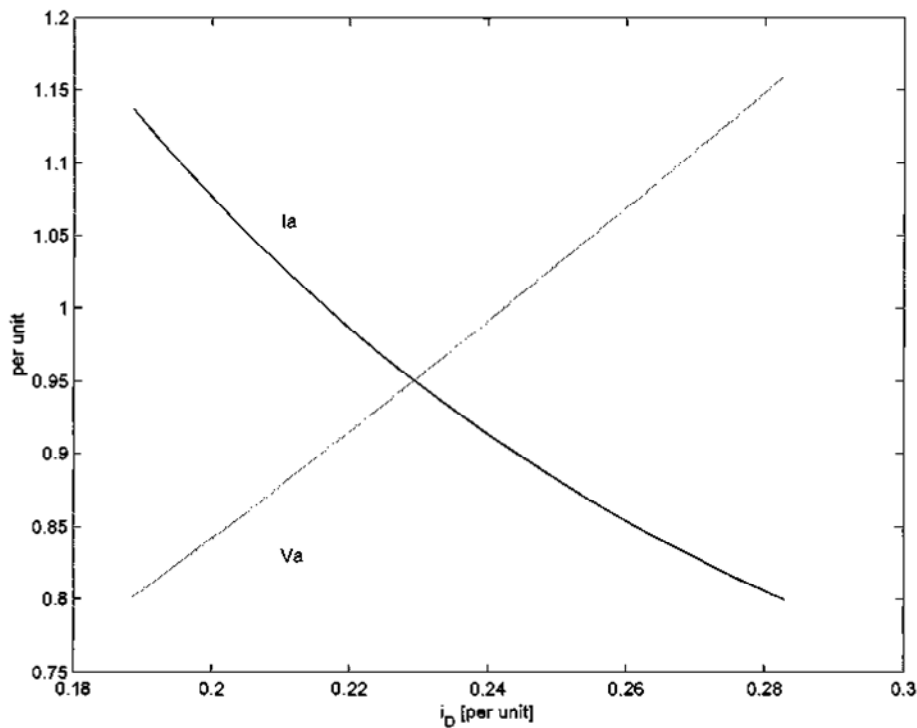


**Figure 11.22** MATLAB plot for Example 11.13 showing the effect of the direct-axis current  $i_D$  on the armature voltage and current for an induction motor at constant speed and load.

chosen to optimize the tradeoff between such quantities as armature current, armature flux linkages, and terminal voltage.

Here is the MATLAB script:

```
clc
clear

%Motor rating and characteristics
Prated = 12e3;
Vrated = 230;
Varated = 230/sqrt(3);
ferated = 60;
omegaerated = 2*pi*ferated;
Lambdarated = sqrt(2)*Varated/omegaerated;
Irated = Prated/(sqrt(3)*Vrated);
Ipeakbase = sqrt(2)*Irated;
poles = 4;

%Here are the 60-Hz motor parameters
```

```

V10 = Vrated/sqrt(3);
X10 = 0.680;
X20 = 0.672;
Xm0 = 18.7;
R1 = 0.095;
R2 = 0.2;

%Calculate required dq0 parameters
Lm = Xm0/omegaerated;
LS = Lm + X10/omegaerated;
LR = Lm + X20/omegaerated;
Ra = R1;
RaR = R2;

% Operating point
n = 1680;
omegam = n*pi/30;
omegame = (poles/2)*omegam;
Pmech = 9.7e3;
Tmech = Pmech/omegam;

% Loop to plot over lambdaDR
for n = 1:41
    lambdaDR = (0.8 + (n-1)*0.4/40)*Lambdarated;
    lambdaDRpu(n) = lambdaDR/Lambdarated;
    iQ = (2/3)*(2/poles)*(LR/Lm)*(Tmech/lambdaDR);
    iD = (lambdaDR/Lm);
    iDpu(n) = iD/Ipeakbase;
    iQR = - (Lm/LR)*iQ;
    Ia = sqrt((iD^2 + iQ^2)/2);
    Iapu(n) = Ia/Irated;
    omegae = omegame - (RaR/LR)*(iQ/iD);
    fe(n) = omegae/(2*pi);
    Varms = sqrt(((Ra*iD-omegae*(LS-Lm^2/LR)*iQ)^2 + ...
        (Ra*iQ+ omegae*LS*iD)^2)/2);
    Vapu(n) = Varms/Varated;
end

%Now plot
plot(iDpu,Iapu)
hold
plot(iDpu,Vapu,':')
hold
xlabel('i_D [per unit]')
ylabel('per unit')
text(.21,1.06,'Ia')
text(.21,.83,'Va')

```

---

## 11.4 CONTROL OF VARIABLE-RELUCTANCE MOTORS

Unlike dc and ac (synchronous or induction) machines, VRMs cannot be simply “plugged in” to a dc or ac source and then be expected to run. As is discussed in Chapter 8, the phases must be excited with (typically unipolar) currents, and the timing of these currents must be carefully correlated with the position of the rotor poles in order to produce a useful, time-averaged torque. The result is that although the VRM itself is perhaps the simplest of rotating machines, a practical VRM drive system is relatively complex.

VRM drive systems are competitive only because this complexity can be realized easily and inexpensively through power and microelectronic circuitry. These drive systems require a fairly sophisticated level of controllability for even the simplest modes of VRM operation. Once the capability to implement this control is available, fairly sophisticated control features can be added (typically in the form of additional software) at little additional cost, further increasing the competitive position of VRM drives.

In addition to the VRM itself, the basic VRM drive system consists of the following components: a rotor-position sensor, a controller, and an inverter. The function of the rotor-position sensor is to provide an indication of shaft position which can be used to control the timing and waveform of the phase excitation. This is directly analogous to the timing signal used to control the firing of the cylinders in an automobile engine.

The controller is typically implemented in software in microelectronic (microprocessor) circuitry. Its function is to determine the sequence and waveforms of the phase excitation required to achieve the desired motor speed-torque characteristics. In addition to set points of desired speed and/or torque and shaft position (from the shaft-position sensor), sophisticated controllers often employ additional inputs including shaft-speed and phase-current magnitude. Along with the basic control function of determining the desired torque for a given speed, the more sophisticated controllers attempt to provide excitations which are in some sense optimized (for maximum efficiency, stable transient behavior, etc.).

The control circuitry consists typically of low-level electronics which cannot be used to directly supply the currents required to excite the motor phases. Rather its output consists of signals which control an inverter which in turn supplies the phase currents. Control of the VRM is achieved by the application of an appropriate set of currents to the VRM phase windings.

Figures 11.23a to c show three common configurations found in inverter systems for driving VRMs. Note that these are simply H-bridge inverters of the type discussed in Section 10.3. Each inverter is shown in a two-phase configuration. As is clear from the figures, extension of each configuration to drive additional phases can be readily accomplished.

The configuration of Fig. 11.23a is perhaps the simplest. Closing switches  $S_{1a}$  and  $S_{1b}$  connects the phase-1 winding across the supply ( $v_1 = V_0$ ) and causes the winding current to increase. Opening just one of the switches forces a short across

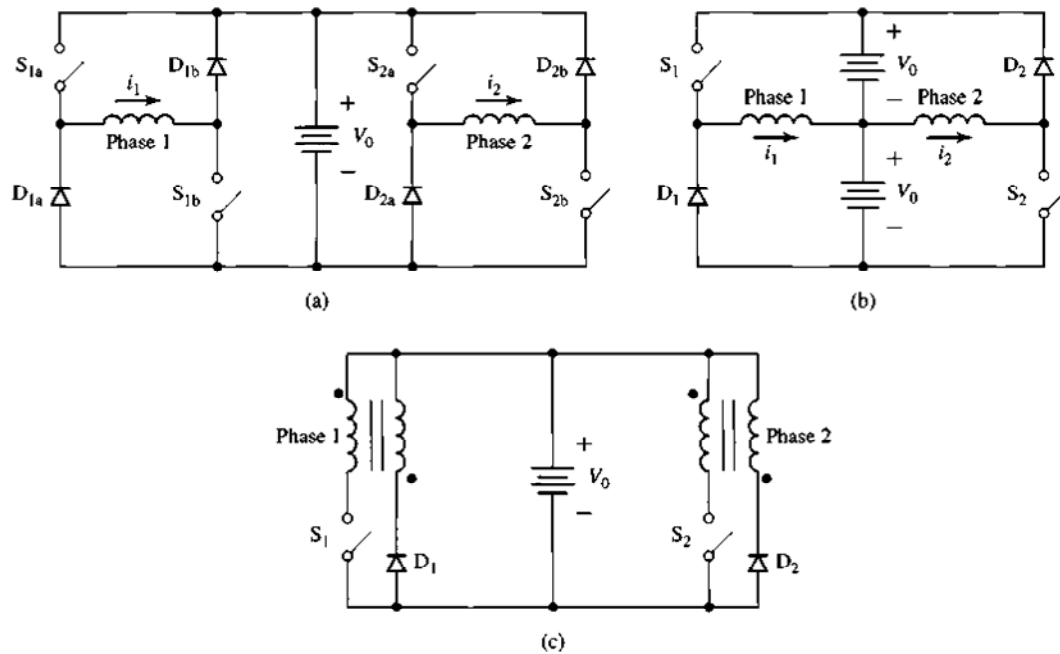


**Figure 11.23** Inverter configurations. (a) Two-phase inverter which uses two switches per phase. (b) Two-phase inverter which uses a split supply and one switch per phase. (c) Two-phase inverter with bifilar phase windings and one switch per phase.

the winding and the current will decay, while opening both switches connects the winding across the supply with negative polarity through the diodes ( $v_1 = -V_0$ ) and the winding current will decay more rapidly. Note that this configuration is capable of regeneration (returning energy to the supply), but not of supplying negative current to the phase winding. However, since the torque in a VRM is proportional to the square of the phase current, there is no need for negative winding current.

As discussed in Section 10.3.2, the process of pulse-width modulation, under which a series of switch configurations alternately charge and discharge a phase winding, can be used to control the average winding current. Using this technique, an inverter such as that of Fig. 11.23a can readily be made to supply the range of waveforms required to drive a VRM.

The inverter configuration of Fig. 11.23a is perhaps the simplest of H-bridge configurations which provide regeneration capability. Its main disadvantage is that it requires two switches per phase. In many applications, the cost of the switches (and their associated drive circuitry) dominates the cost of the inverter, and the result is that this configuration is less attractive in terms of cost when compared to other configurations which require one switch per phase.

Figure 11.23b shows one such configuration. This configuration requires a split supply (i.e., two supplies of voltage  $V_0$ ) but only a single switch and diode per phase.

Closing switch S1 connects the phase-1 winding to the upper dc source. Opening the switch causes the phase current to transfer to diode D1, connecting the winding to the bottom dc source. Phase 1 is thus supplied by the upper dc source and regenerates through the bottom source. Note that to maintain symmetry and to balance the energy supplied from each source equally, phase 2 is connected oppositely so that it is supplied from the bottom source and regenerates into the top source.

The major disadvantages of the configuration of Fig. 11.23b are that it requires a split supply and that when the switch is opened, the switch must withstand a voltage of  $2V_0$ . This can be readily seen by recognizing that when diode D1 is forward-biased, the switch is connected across the two supplies. Such switches are likely to be more expensive than the switches required by the configuration of Fig. 11.23a. Both of these issues will tend to offset some of the economic advantage which can be gained by the elimination of one switch and one diode as compared with the inverter circuit of Fig. 11.23a.

A third inverter configuration is shown in Fig. 11.23c. This configuration requires only a single dc source and uses only a single switch and diode per phase. This configuration achieves regeneration through the use of *bifilar* phase windings. In a bifilar winding, each phase is wound with two separate windings which are closely coupled magnetically (this can be achieved by winding the two windings at the same time) and can be thought of as the primary and secondary windings of a transformer.

When switch S1 is closed, the primary winding of phase 1 is energized, exciting the phase winding. Opening the switch induces a voltage in the secondary winding (note the polarity indicated by the dots in Fig. 11.23c) in such a direction as to forward-bias D1. The result is that current is transferred from the primary to the secondary winding with a polarity such that the current in the phase decays to zero and energy is returned to the source.

Although this configuration requires only a single dc source, it requires a switch which must withstand a voltage in excess of  $2V_0$  (the degree of excess being determined by the voltage developed across the primary leakage reactance as current is switched from the primary to the secondary windings) and requires the more complex bifilar winding in the machine. In addition, the switches in this configuration must include snubbing circuitry (typically consisting of a resistor-capacitor combination) to protect them from transient overvoltages. These overvoltages result from the fact that although the two windings of the bifilar winding are wound such that they are as closely coupled as possible, perfect coupling cannot be achieved. As a result, there will be energy stored in the leakage fields of the primary winding which must be dissipated when the switch is opened.

As is discussed in Section 10.3, VRM operation requires control of the current applied to each phase. For example, one control strategy for constant torque production is to apply constant current to each phase during the time that  $dL/d\theta_m$  for that phase is constant. This results in constant torque proportional to the square of the phase-current magnitude. The magnitude of the torque can be controlled by changing the magnitude of the phase current.

The control required to drive the phase windings of a VRM is made more complex because the phase-winding inductances change both with rotor position and

with current levels due to saturation effects in the magnetic material. As a result, it is not possible in general to implement an open-loop PWM scheme based on a precalculated algorithm. Rather, pulse-width-modulation is typically accomplished through the use of current feedback. The instantaneous phase current can be measured and a switching scheme can be devised such that the switch can be turned off when the current has been found to reach a desired maximum value and turned on when the current decays to a desired minimum value. In this manner the average phase current is controlled to a predetermined function of the rotor position and desired torque.

This section has provided a brief introduction to the topic of drive systems for variable-reluctance machines. In most cases, many additional issues must be considered before a practical drive system can be implemented. For example, accurate rotor-position sensing is required for proper control of the phase excitation, and the control loop must be properly compensated to ensure its stability. In addition, the finite rise and fall times of current buildup in the motor phase windings will ultimately limit the maximum achievable rotor torque and speed.

The performance of a complete VRM drive system is intricately tied to the performance of all its components, including the VRM, its controller, and its inverter. In this sense, the VRM is quite different from the induction, synchronous, and dc machines discussed earlier in this chapter. As a result, it is useful to design the complete drive system as an integrated package and not to design the individual components (VRM, inverter, controller, etc.) separately. The inverter configurations of Fig. 11.23 are representative of a number of possible inverter configurations which can be used in VRM drive systems. The choice of an inverter for a specific application must be made based on engineering and economic considerations as part of an integrated VRM drive system design.

## 11.5 SUMMARY

This chapter introduces various techniques for the control of electric machines. The broad topic of electric machine control requires a much more extensive discussion than is possible here so our objectives have been somewhat limited. Most noticeably, the discussion of this chapter focuses almost exclusively on steady-state behavior, and the issues of transient and dynamic behavior are not considered.

Much of the control flexibility that is now commonly associated with electric machinery comes from the capability of the power electronics that is used to drive these machines. This chapter builds therefore on the discussion of power electronics in Chapter 10.

The starting point is a discussion of dc motors for which it is convenient to subdivide the control techniques into two categories: speed and torque control. The algorithm for speed control in a dc motor is relatively straight forward. With the exception of a correction for voltage drop across the armature resistance, the steady-state speed is determined by the condition that the generated voltage must be equal to the applied armature voltage. Since the generated voltage is proportional to the field

flux and motor speed, we see that the steady-state motor speed is proportional to the armature voltage and inversely proportional to the field flux.

An alternative viewpoint is that of torque control. Because the commutator/brush system maintains a constant angular relationship between the field and armature flux, the torque in a dc motor is simply proportional to the product of the armature current and the field flux. As a result, dc motor torque can be controlled directly by controlling the armature current as well as the field flux.

Because synchronous motors develop torque only at synchronous speed, the speed of a synchronous motor is simply determined by the electrical frequency of the applied armature excitation. Thus, steady-state speed control is simply a matter of armature frequency control. Torque control is also possible. By transforming the stator quantities into a reference frame rotating synchronously with the rotor (using the dq0 transformation of Appendix C), we found that torque is proportional to the field flux and the component of armature current in space quadrature with the field flux. This is directly analogous to the torque production in a dc motor. Control schemes which adopt this viewpoint are referred to as *vector* or *field-oriented* control.

Induction machines operate asynchronously; rotor currents are induced by the relative motion of the rotor with respect to the synchronously rotating stator-produced flux wave. When supplied by a constant-frequency source applied to the armature winding, the motor will operate at a speed somewhat lower than synchronous speed, with the motor speed decreasing as the load torque is increased. As a result, precise speed regulation is not a simple matter, although in most cases the speed will not vary from synchronous speed by an excessive amount.

Analogous to the situation in a synchronous motor, in spite of the fact that the rotor of an induction motor rotates at less than synchronous speed, the interaction between the rotor and stator flux waves is indeed synchronous. As a result, a transformation into a synchronously rotating reference frame results in rotor and stator flux waves which are constant. The torque can then be expressed in terms of the product of the rotor flux linkages and the component of armature current in quadrature with the rotor flux linkages (referred to as the *quadrature-axis component* of the armature current) in a fashion directly analogous to the field-oriented viewpoint of a synchronous motor. Furthermore, it can be shown that the rotor flux linkages are proportional to the direct-axis component of the armature current, and thus the direct-axis component of armature current behaves much like the field current in a synchronous motor. This field-oriented viewpoint of induction machine control, in combination with the power-electronic and control systems required to implement it, has led to the widespread applicability of induction machines to a wide range of variable-speed applications.

Finally, this chapter ends with a brief discussion of the control of variable-reluctance machines. To produce useful torque, these machines typically require relatively complex, nonsinusoidal current waveforms whose shape must be controlled as a function of rotor position. Typically, these waveforms are produced by pulse-width modulation combined with current feedback using an H-bridge inverter of the type

discussed in Chapter 10. The details of these waveforms depend heavily upon the geometry and magnetic properties of the VRM and can vary significantly from motor to motor.

## 11.6 BIBLIOGRAPHY

Many excellent books are available which provide a much more thorough discussion of electric-machinery control than is possible in the introductory discussion presented here. This bibliography lists a few of the many textbooks available for readers who wish to study this topic in more depth.

- Boldea, I., *Reluctance Synchronous Machines and Drives*. New York: Clarendon Press-Oxford, 1996.
- Kenjo, T., *Stepping Motors and Their Microprocessor Controls*. New York: Clarendon Press-Oxford, 1984.
- Leonhard, W., *Control of Electric Drives*. Berlin: Springer, 1996.
- Miller, T. J. E., *Brushless Permanent-Magnet and Reluctance Motor Drives*. New York: Clarendon Press-Oxford, 1989.
- Miller, T. J. E., *Switched Reluctance Motors and Their Controls*. New York: Magna Press Publishing and Clarendon Press-Oxford, 1993.
- Mohan, N., *Advanced Electric Drives: Analysis, Control and Modeling Using Simulink*. Minneapolis: MNPERE (<http://www.MNPERE.com>), 2001.
- Mohan, N., *Electric Drives: An Integrative Approach*. Minneapolis: MNPERE (<http://www.MNPERE.com>), 2001.
- Murphy, J. M. D., and F. G. Turnbull, *Power Electronic Control of AC Motors*. New York: Pergamon Press, 1988.
- Novotny, D. W., and T. A. Lipo, *Vector Control and Dynamics of AC Drives*. New York: Clarendon Press-Oxford, 1996.
- Subrahmanyam, V., *Electric Drives: Concepts and Applications*. New York: McGraw-Hill, 1996.
- Trzynadlowski, A. M., *Control of Induction Motors*. San Diego, California: Academic Press, 2001.
- Vas, P., *Sensorless Vector and Direct Torque Control*. Oxford: Oxford University Press, 1998.

## 11.7 PROBLEMS

- 11.1** When operating at rated voltage, a 3-kW, 120-V, 1725 r/min separately excited dc motor achieves a no-load speed of 1718 r/min at a field current of 0.70 A. The motor has an armature resistance of 145 m $\Omega$  and a shunt-field resistance of 104  $\Omega$ . For the purposes of this problem you may assume the rotational losses to be negligible.

This motor will control the speed of a load whose torque is constant at 15.2 N·m over the speed range of 1500–1800 r/min. The motor will be operated at a constant armature voltage of 120 V. The field-winding will be supplied from the 120-V dc armature supply via a pulse-width modulation

system, and the motor speed will be varied by varying the duty cycle of the pulse-width modulation.

- Calculate the field current required to achieve operation at 15.2 N·m torque and 1800 r/min. Calculate the corresponding PWM duty cycle  $D$ .
- Calculate the field current required to achieve operation at 15.2 N·m torque and 1500 r/min. Calculate the corresponding PWM duty cycle.
- Plot the required PWM duty cycle as a function of speed over the desired speed range of 1500 to 1800 r/min.

**11.2** Repeat Problem 11.1 for a load whose torque is 15.2 N·m at 1600 r/min and which varies as the speed to the 1.8 power.

**11.3** The dc motor of Problem 11.1 has a field-winding inductance  $L_f = 3.7$  H and a moment of inertia  $J = 0.081$  kg·m<sup>2</sup>. The motor is operating at rated terminal voltage and an initial speed of 1300 r/min.

- Calculate the initial field current  $I_f$  and duty cycle  $D$ .  
At time  $t = 0$ , the PWM duty cycle is suddenly switched from the value found in part (a) to  $D = 0.60$ .
- Calculate the final values of the field current and motor speed after the transient has died out.
- Write an expression for the field-current transient as a function of time.
- Write a differential equation for the motor speed as a function of time during this transient.

**11.4** A shunt-connected 240-V, 15-kW, 3000 r/min dc motor has the following parameters

|                      |                             |
|----------------------|-----------------------------|
| Field resistance:    | $R_f = 132 \Omega$          |
| Armature resistance: | $R_a = 0.168 \Omega$        |
| Geometric constant:  | $K_f = 0.422$ V/(A·rad/sec) |



When operating at rated voltage, no-load, the motor current is 1.56 A.

- Calculate the no-load speed and rotational loss.
- Assuming the rotational loss to be constant, use MATLAB to plot the motor output power as a function of speed. Limit your plot to a maximum power output of 15 kW.
- Armature-voltage control is to be used to maintain constant motor speed as the motor is loaded. For this operating condition, the shunt field voltage will be held constant at 240-V. Plot the armature voltage as a function of power output required to maintain the motor at a constant speed of 2950 r/min.
- Consider that the situation for armature-voltage control is applied to this motor while the field winding remains connected in shunt across the armature terminals. Repeat part (c) for this operating condition. Is such operation feasible? Why is the motor behavior significantly different from that in part (c)?



**11.5** The data sheet for a small permanent-magnet dc motor provides the following parameters:

|                     |                                                     |
|---------------------|-----------------------------------------------------|
| Rated voltage:      | $V_{\text{rated}} = 3 \text{ V}$                    |
| Rated output power: | $P_{\text{rated}} = 0.28 \text{ W}$                 |
| No-load speed:      | $n_{\text{nl}} = 12,400 \text{ r/min}$              |
| Torque constant:    | $K_m = 0.218 \text{ mV/(r/min)}$                    |
| Stall torque:       | $T_{\text{stall}} = 0.094 \text{ oz}\cdot\text{in}$ |

- Calculate the motor armature resistance.
- Calculate the no-load rotational loss.
- Assume the motor to be connected to a load such that the total shaft power (actual load plus rotational loss) is equal 0.25 W at a speed of 12,000 r/min. Assuming this load to vary as the square of the motor speed, write a MATLAB script to plot the motor speed as a function of terminal voltage for  $1.0 \text{ V} \leq V_a \leq 3.0 \text{ V}$ .

**11.6** The data sheet for a 350-W permanent-magnet dc motor provides the following parameters:

|                      |                                      |
|----------------------|--------------------------------------|
| Rated voltage:       | $V_{\text{rated}} = 24 \text{ V}$    |
| Armature resistance: | $R_a = 97 \text{ m}\Omega$           |
| No-load speed:       | $n_{\text{nl}} = 3580 \text{ r/min}$ |
| No-load current:     | $I_{a,\text{nl}} = 0.47 \text{ A}$   |

- Calculate the motor torque-constant  $K_m$  in  $\text{V}/(\text{rad}/\text{sec})$ .
- Calculate the no-load rotational loss.
- The motor is supplied from a 30-V dc supply through a PWM inverter. Table 11.1 gives the measured motor current as a function of the PWM duty cycle  $D$ .

Complete the table by calculating the motor speed and the load power for each value of  $D$ . Assume that the rotational losses vary as the square of the motor speed.

**Table 11.1** Motor-performance data for Problem 11.6.

| $D$  | $I_a \text{ (A)}$ | r/min | $P_{\text{load}} \text{ (W)}$ |
|------|-------------------|-------|-------------------------------|
| 0.80 | 13.35             |       |                               |
| 0.75 | 12.70             |       |                               |
| 0.70 | 12.05             |       |                               |
| 0.65 | 11.40             |       |                               |
| 0.60 | 10.70             |       |                               |
| 0.55 | 10.05             |       |                               |
| 0.50 | 9.30              |       |                               |

- 11.7** The motor of Problem 11.5 has a moment of inertia of  $6.4 \times 10^{-7}$  oz-in-sec<sup>2</sup>. Assuming it is unloaded and neglecting any effects of rotational loss, calculate the time required to achieve a speed of 12,000 r/min if the motor is supplied by a constant armature current of 100 mA.
- 11.8** An 1100-W, 150-V, 3000-r/min permanent-magnet dc motor is to be operated from a current-source inverter so as to provide direct control of the motor torque. The motor torque constant is  $K_m = 0.465$  V/(rad/sec); its armature resistance is  $1.37 \Omega$ . The motor rotational loss at a speed of 3000 r/min is 87 W. Assume that the rotational loss can be represented by a constant loss torque as the motor speed varies.
- Calculate the rated armature current of this motor. What is the corresponding mechanical torque in N·m?
  - The current source supplies a current of 6.2 A to the motor armature, and the motor speed is measured to be 2670 r/min. Calculate the load torque and power.
  - Assume the load torque of part (b) to vary linearly with speed and the motor and load to have a combined inertia of  $2.28 \times 10^{-3}$  kg·m<sup>2</sup>. Calculate the motor speed as a function of time if the armature current is suddenly increased to 7.0 A.
- 11.9** The permanent-magnet dc motor of Problem 11.8 is operating at its rated speed of 3000 r/min and no load. If rated current is suddenly applied to the motor armature in such a direction as to slow the motor down, how long will it take the motor to reach zero speed? The inertia of the motor alone is  $1.86 \times 10^{-3}$  kg·m<sup>2</sup>. Ignore the effects of rotational loss.
- 11.10** A 1100-kVA, 4600-V, 60-Hz, three-phase, four-pole synchronous motor is to be driven from a variable-frequency, three-phase, constant V/Hz inverter rated at 1250-kVA. The synchronous motor has a synchronous reactance of 1.18 per unit and achieves rated open-circuit voltage at a field current of 85 A.
- Calculate the rated speed of the motor in r/min.
  - Calculate the rated current of the motor.
  - With the motor operating at rated voltage and speed and an input power of 1000-kW, calculate the field current required to achieve unity-power-factor operation.  
The load power of part (c) varies as the speed to the 2.5 power. With the motor field-current held fixed, the inverter frequency is reduced such that the motor is operating at a speed of 1300 r/min.
  - Calculate the inverter frequency and the motor input power and power factor.
  - Calculate the field current required to return the motor to unity power factor.



- 11.11** Consider a three-phase synchronous motor for which you are given the following data:

Rated line-to-line voltage (V)  
 Rated volt-amperes (VA)  
 Rated frequency (Hz) and speed (r/min)  
 Synchronous reactance in per unit  
 Field current at rated open-circuit voltage (AFNL) (A)

The motor is to be operated from a variable-frequency, constant V/Hz inverter at speeds of up to 120 percent of the motor-rated speed.

- Under the assumption that the motor terminal voltage and current cannot exceed their rated values, write a MATLAB script which calculates, for a given operating speed, the motor terminal voltage, the maximum possible motor input power, and the corresponding field current required to achieve this operating condition. You may consider the effects of saturation and armature resistance to be negligible.
- Exercise your program on the synchronous motor of Problem 11.10 for motor speeds of 1500 and 2000 r/min.



- 11.12** For the purposes of performing field-oriented control calculations on non-salient synchronous motors, write a MATLAB script that will calculate the synchronous inductance  $L_s$  and armature-to-field mutual inductance  $L_{af}$ , both in henries, and the rated torque in N·m, given the following data:

Rated line-to-line voltage (V)  
 Rated (VA)  
 Rated frequency (Hz)  
 Number of poles  
 Synchronous reactance in per unit  
 Field current at rated open-circuit voltage (AFNL) (A)

- 11.13** A 100-kW, 460-V, 60-Hz, four-pole, three-phase synchronous machine is to be operated as a synchronous motor under field-oriented torque control using a system such as that shown in Fig. 11.13a. The machine has a synchronous reactance of 0.932 per unit and AFNL = 15.8 A. The motor is operating at rated speed, loaded to 50 percent of its rated torque at a field current of 14.0 A with the field-oriented controller set to maintain  $i_D = 0$ .
- Calculate the synchronous inductance  $L_s$  and armature-to-field mutual inductance  $L_{af}$ , both in henries.
  - Find the quadrature-axis current  $i_Q$  and the corresponding rms magnitude of the armature current  $i_a$ .
  - Find the motor line-to-line terminal voltage.
- 11.14** The synchronous motor of Problem 11.13 is operating under field-oriented torque control such that  $i_D = 0$ . With the field current set equal to 14.5 A and

with the torque reference set equal to 0.75 of the motor rated torque, the motor speed is observed to be 1475 r/min.

- a. Calculate the motor output power.
  - b. Find the quadrature-axis current  $i_Q$  and the corresponding rms magnitude of the armature current  $i_a$ .
  - c. Calculate the stator electrical frequency
  - d. Find the motor line-to-line terminal voltage.
- 11.15** Consider the case in which the load on the synchronous motor in the field-oriented torque-control system of Problem 11.13 is increased, and the motor begins to slow down. Based upon some knowledge of the load characteristic, it is determined that it will be necessary to raise the torque set point  $T_{\text{ref}}$  from 50 percent to 80 percent of the motor-rated torque to return the motor to its rated speed.
- a. If the field current were left unchanged at 14.0 A, calculate the values of quadrature-axis current, rms armature current, and motor line-to-line terminal voltage (in V and in per unit) which would result in response to this change in reference torque.
  - b. To achieve this operating condition with reasonable armature terminal voltage, the field-oriented control algorithm is changed to the unity-power-factor algorithm described in the text prior to Example 11.9. Based upon that algorithm, calculate
    - (i) the motor terminal line-to-line terminal voltage (in V and in per unit).
    - (ii) the rms armature current.
    - (iii) the direct- and quadrature-axis currents,  $i_D$  and  $i_Q$ .
    - (iv) the motor field current.
- 11.16** Consider a 500-kW, 2300-V, 50-Hz, eight-pole synchronous motor with a synchronous reactance of 1.18 per unit and  $\text{AFNL} = 94$  A. It is to be operated under field-oriented torque control using the unity-power-factor algorithm described in the text following Example 11.8. It will be used to drive a load whose torque varies quadratically with speed and whose torque at a speed of 750 r/min is 5900 N·m. The complete drive system will include a speed-control loop such as that shown in Fig. 11.13b.
- Write a MATLAB script whose input is the desired motor speed (up to 750 r/min) and whose output is the motor torque, the field current, the direct- and quadrature-axis currents, the armature current, and the line-to-line terminal voltage. Exercise your script for a motor speed of 650 r/min.
- 11.17** A 2-kVA, 230-V, two-pole, three-phase permanent magnet synchronous motor achieves rated open-circuit voltage at a speed of 3500 r/min. Its synchronous inductance is 17.2 mH.
- a. Calculate  $\Lambda_{\text{PM}}$  for this motor.
  - b. If the motor is operating at rated voltage and rated current at a speed of 3600 r/min, calculate the motor power in kW and the peak direct- and



quadrature-axis components of the armature current,  $i_D$  and  $i_Q$  respectively.

- 11.18** Field-oriented torque control is to be applied to the permanent-magnet synchronous motor of Problem 11.18. If the motor is to be operated at 4000 r/min at rated terminal voltage, calculate the maximum torque and power which the motor can supply and the corresponding values of  $i_D$  and  $i_Q$ .
- 11.19** A 15-kVA, 230-V, two-pole, three-phase permanent-magnet synchronous motor has a maximum speed of 10,000 r/min and produces rated open-circuit voltage at a speed of 7620 r/min. It has a synchronous inductance of 1.92 mH. The motor is to be operated under field-oriented torque control.
- Calculate the maximum torque the motor can produce without exceeding rated armature current.
  - Assuming the motor to be operated with the torque controller adjusted to produce maximum torque (as found in part (a)) and  $i_D = 0$ , calculate the maximum speed at which it can be operated without exceeding rated armature voltage.
  - To operate at speeds in excess of that found in part (b), flux weakening will be employed to maintain the armature voltage at its rated value. Assuming the motor to be operating at 10,000 r/min with rated armature voltage and current, calculate
    - the direct-axis current  $i_D$ .
    - the quadrature-axis current  $i_Q$ .
    - the motor torque.
    - the motor power and power factor.



- 11.20** The permanent magnet motor of Problem 11.17 is to be operated under vector control using the following algorithm.

Terminal voltage not to exceed rated value

Terminal current not to exceed rated value

$i_D = 0$  unless flux weakening is required to avoid excessive armature voltage

Write a MATLAB script to produce plots of the maximum power and torque which this system can produce as a function of motor speed for speeds up to 10,000 r/min.

- 11.21** Consider a 460-V, 25-kW, four-pole, 60-Hz induction motor which has the following equivalent-circuit parameters in ohms per phase referred to the stator:

$$R_1 = 0.103 \quad R_2 = 0.225 \quad X_1 = 1.10 \quad X_2 = 1.13 \quad X_m = 59.4$$

The motor is to be operated from a variable frequency, constant-V/Hz drive whose output is 460-V at 60-Hz. Neglect any effects of rotational loss. The motor drive is initially adjusted to a frequency of 60 Hz.

- a. Calculate the peak torque and the corresponding slip and motor speed in r/min.
  - b. Calculate the motor torque at a slip of 2.9 percent and the corresponding output power.
  - c. The drive frequency is now reduced to 35 Hz. If the load torque remains constant, estimate the resultant motor speed in r/min. Find the resultant motor slip, speed in r/min, and output power.
- 11.22** Consider the 460-V, 250-kW, four-pole induction motor and drive system of Problem 11.21. 
- a. Write a MATLAB script to plot the speed-torque characteristic of the motor at drive frequencies of 20, 40, and 60 Hz for speeds ranging from -200 r/min to the synchronous speed at each frequency.
  - b. Determine the drive frequency required to maximize the starting torque and calculate the corresponding torque in N·m.
- 11.23** A 550-kW, 2400-V, six-pole, 60-Hz three-phase induction motor has the following equivalent-circuit parameters in ohms-per-phase-Y referred to the stator: 

$$R_1 = 0.108 \quad R_2 = 0.296 \quad X_1 = 1.18 \quad X_2 = 1.32 \quad X_m = 48.4$$

The motor will be driven by a constant-V/Hz drive whose voltage is 2400 V at a frequency of 60 Hz.

The motor is used to drive a load whose power is 525 kW at a speed of 1138 r/min and which varies as the cube of speed. Using MATLAB, plot the motor speed as a function of frequency as the drive frequency is varied between 20 and 60 Hz.

- 11.24** A 150-kW, 60-Hz, six-pole, 460-V three-phase wound-rotor induction motor develops full-load torque at a speed of 1157 r/min with the rotor short-circuited. An external noninductive resistance of 870 mΩ is placed in series with each phase of the rotor, and the motor is observed to develop its rated torque at a speed of 1072 r/min. Calculate the resistance per phase of the original motor.
- 11.25** The wound rotor of Problem 11.24 will be used to drive a constant-torque load equal to the rated full-load torque of the motor. Using the results of Problem 11.24, calculate the external rotor resistance required to adjust the motor speed to 850 r/min.
- 11.26** A 75-kW, 460-V, three-phase, four-pole, 60-Hz, wound-rotor induction motor develops a maximum internal torque of 212 percent at a slip of 16.5 percent when operated at rated voltage and frequency with its rotor short-circuited directly at the slip rings. Stator resistance and rotational losses may be neglected, and the rotor resistance may be assumed to be constant, independent of rotor frequency. Determine
- a. the slip at full load in percent.
  - b. the rotor  $I^2R$  loss at full load in watts.

- c. the starting torque at rated voltage and frequency N·m.

If the rotor resistance is now doubled (by inserting external series resistance at the slip rings), determine

- d. the torque in N·m when the stator current is at its full-load value.  
e. the corresponding slip.

- 11.27** A 35-kW, three-phase, 440-V, six-pole wound-rotor induction motor develops its rated full-load output at a speed of 1169 r/min when operated at rated voltage and frequency with its slip rings short-circuited. The maximum torque it can develop at rated voltage and frequency is 245 percent of full-load torque. The resistance of the rotor winding is  $0.23 \Omega/\text{phase}$ . Neglect rotational and stray-load losses and stator resistance.

- a. Compute the rotor  $I^2R$  loss at full load.  
b. Compute the speed at maximum torque.  
c. How much resistance must be inserted in series with the rotor to produce maximum starting torque?

The motor is now run from a 50-Hz supply with the applied voltage adjusted so that, for any given torque, the air-gap flux wave has the same amplitude as it does when operated 60 Hz at the same torque level.

- d. Compute the 50-Hz applied voltage.  
e. Compute the speed at which the motor will develop a torque equal to its rated value at 60-Hz with its slip rings short-circuited.

- 11.28** The three-phase, 2400-V, 550-kW, six-pole induction motor of Problem 11.23 is to be driven from a field-oriented speed-control system whose controller is programmed to set the rotor flux linkages  $\lambda_{DR}$  equal to the machine rated peak value. The machine is operating at 1148 r/min driving a load which is known to be 400 kW at this speed. Find:

- a. the value of the peak direct- and quadrature-axis components of the armature currents  $i_D$  and  $i_Q$ .  
b. the rms armature current under this operating condition.  
c. the electrical frequency of the drive in Hz.  
d. the rms line-to-line armature voltage.



- 11.29** A field-oriented drive system will be applied to a 230-V, 20-kW, four-pole, 60-Hz induction motor which has the following equivalent-circuit parameters in ohms per phase referred to the stator:

$$R_1 = 0.0322 \quad R_2 = 0.0703 \quad X_1 = 0.344 \quad X_2 = 0.353 \quad X_m = 18.6$$

The motor is connected to a load whose torque can be assumed proportional to speed as  $T_{\text{load}} = 85(n/1800) \text{ N}\cdot\text{m}$ , where  $n$  is the motor speed in r/min.

The field-oriented controller is adjusted such that the rotor flux linkages  $\lambda_{DR}$  are equal to the machine's rated peak flux linkages, and the motor speed is 1300 r/min. Find

- a. the electrical frequency in Hz.  
b. the rms armature current and line-to-line voltage.

- c. the motor input kVA.

If the field-oriented controller is set to maintain the motor speed at 1300 r/min, write a MATLAB script to plot the rms armature V/Hz as a percentage of the rated V/Hz as a function of  $\lambda_{DR}$  as  $\lambda_{DR}$  is varied between 80 and 120 percent of the machine's rated peak flux linkages.

- 11.30** The 20-kW induction motor-drive and load of Problem 11.29 is operating at a speed of 1450 r/min with the field-oriented controller adjusted to maintain the rotor flux linkages  $\lambda_{DR}$  equal to the machine's rated peak value.



- Calculate the corresponding values of the direct- and quadrature-axis components of the armature current,  $i_D$  and  $i_Q$ , and the rms armature current.
- Calculate the corresponding line-to-line terminal voltage drive electrical frequency.

The quadrature-axis current  $i_Q$  is now increased by 10 percent while the direct-axis current is held constant.

- Calculate the resultant motor speed and power output.
  - Calculate the terminal voltage and drive frequency.
  - Calculate the total kVA input into the motor.
  - With the controller set to maintain constant speed, determine the set point for  $\lambda_{DR}$ , as a percentage of rated peak flux linkages, that sets the terminal V/Hz equal to the rated machine rated V/Hz. (Hint: This solution is most easily found using a MATLAB script to search for the desired result.)
- 11.31** A three-phase, eight-pole, 60-Hz, 4160-V, 1250-kW squirrel-cage induction motor has the following equivalent-circuit parameters in ohms-per-phase-Y referred to the stator:

$$R_1 = 0.212 \quad R_2 = 0.348 \quad X_1 = 1.87 \quad X_2 = 2.27 \quad X_m = 44.6$$

It is operating from a field-oriented drive system at a speed of 805 r/min and a power output of 1050 kW. The field-oriented controller is set to maintain the rotor flux linkages  $\lambda_{DR}$  equal to the machine's rated peak flux linkages.

- Calculate the motor rms line-to-line terminal voltage, rms armature current, and electrical frequency.
- Show that steady-state induction-motor equivalent circuit and corresponding calculations of Chapter 6 give the same output power and terminal current when the induction motor speed is 828 r/min and the terminal voltage and frequency are equal to those found in part (a).