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HISTORICAL NOTE

Poiseuille and his law

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The formula known as Poiseuille's Law states that for laminar flow of a fluid (liquid or gas) along a pipe

$$\dot{V} = \frac{\pi p r^4}{8\eta l}$$

where $\dot{V}$ = the flow rate; p = the pressure gradient between the two ends of the pipe; r = the radius of the pipe; l = the length of the pipe; η = the viscosity of the fluid.

Poiseuille's Law is widely known, and frequently applied and referred to; but who was Poiseuille, and how did he conceive this relatively complex formula?

History

Jean Leonard Marie Poiseuille (1797–1869) was born and died in Paris. Remarkably little seems to be known about his life.[1] He studied medicine for a considerable time and submitted a thesis for his Doctorate in 1828 (aged 30–31 years). Where he carried out his early experimental studies, and how they were financed, is obscure.

His published work includes '*Studies on the force of the aortic heart*' (thesis, in 1828), '*Studies on the action of the arteries in arterial circulation*' (in 1829), '*Studies on the causes of the movement of the blood in the veins*' (in 1830), '*On the causes of the movement of the blood in the capillaries*' (in 1835), and '*Experimental studies on the movement of liquids in tubes of very small diameter*' (his most famous paper, completed in 1842 and published in 1846).[1] For his work '*On the causes of the movement of the blood in the capillaries*' he was awarded the Paris Académie des Sciences prize for experimental physiology. In later life he became a foundation member of the Académie de Médecine of Paris.[1]

Some believe that he was the first to use a mercury manometer to measure blood pressure[2] and to transmit the blood pressure of dogs and horses to the manometer he used connecting tubes filled with water saturated with bicarbonate of soda to prevent coagulation.[1]

His interest in the circulation and the factors governing capillary blood flow led to the series of experiments which culminated in the formula known today as *Poiseuille's Law*.

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Poiseuille's experiments

His experiments on the factors influencing flow through tubes of very fine bore were carried out in 1842, and published in the *Mémoires des Savants Etrangers*, Vol. IX pp. 435–544, in 1846. They were translated in full into English and published in *Rheological Memoirs*, Vol. 1, No. 1, in 1940.[3]

In the first series of experiments Poiseuille measured the rate at which a given volume of distilled water at 10°C passed through a range of fine bore glass tubes (0·0133–0·616 mm internal diameter) when different pressures were applied at one end. For a given tube he determined that $Q = KP$ (where Q = the flow rate; P = the pressure producing the flow; and K = a constant for the particular tube).

The next measurements were of the rates at which a given volume of distilled water at 10°C passed through tubes of different length, L, when a fixed pressure gradient (775 mmHg) was applied. He found that

$$Q = \frac{K'P}{L}.$$

He next measured the volume of distilled water which passed in a given time (500 seconds) through tubes of different diameter, D, when a fixed pressure gradient (775 mmHg) was applied, and derived

$$Q = \frac{K''PD^4}{L}.$$

K″ in the last formula was constant whatever the dimensions of the tube and whatever the pressure gradient, but it varied markedly with temperature. In a fourth set of measurements he calculated values for K″ at water temperatures ranging from 0·5°C to 45°C.

The relationship $Q = KP$ did not of course hold if the flow-rate became excessive. Excerpts translated from section 80[3] read 'This relation between efflux and the pressure does not hold at certain limits of speed of the liquid molecules . . . . . the tubes should have a length in excess of a certain determined limit . . . . . this limit decreases with a decrease in the diameter of the tube itself . . . . .'

He seemed at the time to have no concept of viscosity nor did he appear to understand why the formula ceased to hold at higher flow-rates. From his *in vivo* studies of capillary blood flow under the microscope, he did however have some idea of laminar flow. Paragraph 133 translates in part '. . . . . the velocity of different fluid filaments, from the axis of the vessel to the walls, is far from being constant. This velocity is at its maximum at the axis of the vessel; it decreases proportionally as the walls are approached: thus the velocity very near the walls has an extreme slowness . . . . .'

Though he did not appreciate the nature of viscosity, nor differentiate between laminar and turbulent flow, he had succeeded in deriving a correct formula. The German hydraulics engineer Gotthilf Hagen made somewhat similar measurements earlier than Poiseuille, and it has been suggested that the formula should in fact be called the Hagen–Poiseuille Law. Others, mathematicians and physicists, deduced

$$\dot{V} = \frac{\pi pr^4}{8\eta l}.$$

The electrical analogue

The relationship between laminar flow-rate, driving pressure, fluid viscosity and length and radius of a tube, can probably be appreciated most easily by drawing an analogy

between laminar flow of a fluid (liquid or gas) along a tube, and flow of electrical current along a conductor.[4]

A. Ohm's law governs the flow of electrical current along a conductor: $I \propto \frac{V}{R}$ (when I = current flow; V = potential difference; and R = resistance).

Since electrical resistance is proportional to the resistivity of the conductor and to its length, and inversely proportional to its cross sectional area: $I \propto V \frac{\pi \alpha^2}{r.l}$ (where r = resistivity of the conductor; l = length of the conductor; and x = radius of the conductor).

B. For laminar flow of a fluid along a tube: $\dot{V} \propto \frac{\text{the force producing the flow}}{\text{the resistance to flow}}$.

Since resistance to fluid flow is proportional to the viscosity* of the fluid and to the length of the tube, and inversely proportional to its cross sectional area: $\dot{V} \propto$ the force producing the flow $\frac{\pi x^2}{\eta.l}$ (where $\dot{V}$ = fluid flow; η = viscosity of the fluid; l = length of the tube; and x = radius of the tube).

Furthermore since the force producing flow is proportional to the pressure gradient† acting on the fluid column, and to the area on which the pressure acts:

$$\dot{V} \propto p\pi x^2 . \frac{\pi x^2}{\eta l}$$

$$\propto \frac{p x^4}{\eta l} \text{ (where p = the pressure gradient acting on the fluid column).}$$

$$\propto \frac{K'' P D^4}{L} \text{ (Poiseuille's findings).}$$

Summary

Comparatively little is known of the life of Jean Leonard Marie Poiseuille (1797–1869) of Paris. He made important contributions to the experimental study of circulatory dynamics but it can hardly be said that Poiseuille knowingly described the law which governs laminar flow. He did however establish, in a series of meticulously executed experiments, that at a given temperature water flow through tubes of very fine bore is inversely proportional to the length of the tube and directly proportional to the pressure gradient and to the fourth power of the tube diameter.

References

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* Dynamic viscosity by definition is that property of a fluid which offers resistance to its non-accelerated laminar flow.

† Pressure is force per unit area.