Figure 19.3 have equations $x-y=7$ and $x-y=23$. The two constants thus found are the end-points of the 95% confidence interval for the difference between median percentages of cars breaking down. Note that the interval does not include 0, which is of course consistent with the fact that the Mann–Whitney test rejected the null hypothesis of no difference at the $\alpha_2=5\%$ significance level (and also at much more stringent levels).

ARITHMETICAL METHOD

It is quite easy to simulate the graphical method without actually drawing the graph. This is achieved by constructing a table of differences between the A and B values (i.e. B−A) as shown below. The B values are shown in the bottom row of the table, going from smallest to largest; the A values are shown in the left-hand column, with the smallest at the bottom and the largest at the top. Note that tied values are duplicated as appropriate. In the body of the table are displayed the $n_A-n_B=8\times12=96$ differences between the A and B values. (These differences correspond to the n_An_B comparisons made in the Mann–Whitney test.)

A												
15	−7	−4	−1	+2	+4	+6	+8	+10	+10	+21	+23	+26
10	−2	+1	+4	+7	+9	+11	+13	+15	+15	+26	+28	+31
7	+1	+4	+7	+10	+12	+14	+16	+18	+18	+29	+31	+34
7	+1	+4	+7	+10	+12	+14	+16	+18	+18	+29	+31	+34
6	+2	+5	+8	+11	+13	+15	+17	+19	+19	+30	+32	+35
4	+4	+7	+10	+13	+15	+17	+19	+21	+21	+32	+34	+37
4	+4	+7	+10	+13	+15	+17	+19	+21	+21	+32	+34	+37
3	+5	+8	+11	+14	+16	+18	+20	+22	+22	+33	+35	+38
	8	*11*	*14*	*17*	*19*	*21*	*23*	*25*	*25*	*36*	*38*	*41* B

We now start marking off the differences from the top left-hand corner, beginning with the −7 and then in natural ascending order, i.e. −7 first, then −4, −2 and −1, then the three +1's, the the two +2's, . . . , and so on until we have marked off 23 (one more than the tabulated value of 22) numbers. You will find that there are 18 differences of +6 or less and five of +7. So the 23rd difference marked off is +7; this is then the lower limit of the confidence interval. Next we carry out a similar operation starting from the +38 at the bottom right-hand corner, and then marking off 23 numbers in descending order: +38, +37, +37, +35, +35, +34, +34, +34, +34, +33, etc. You should find the 23rd number marked off in this sequence is +23. The required confidence interval is thus (+7, +23), the same result as obtained by the graphical method.

Discussion

If you compare the graphical and the arithmetical methods, it should be apparent why they are equivalent. In short, the differences in the arithmetical method are marked off in precisely the same order that the 45° lines cross the corresponding

points in the graphical method. The arithmetical method is quickest to carry out if the sample sizes are fairly small, but the graphical method is probably the better choice otherwise. However, you will see that even the graphical method is quite time-consuming when the samples are at all large, since the number $n_A n_B$ of points to consider, and the number (found from the tables) of points to count in from the extremes, both increase sharply with increasing sample sizes.

Theory

The theory of this method may not be quite so easy to understand as that of the one-sample method based on the sign test. This is particularly so because, whereas in the one-sample case it is common to think of testing an H_0 that ϕ equals any specified value, in the two-sample case it is most natural to think of testing just the H_0 that the difference between the population medians is zero. However, we need to broaden our minds here to consider similarly testing the H_0 that the median difference (say, median of B minus median of A) equals any specified value, d. In the Mann–Whitney test, where we were testing that the median difference is zero, the basic argument was that, given a random observation from each process, there is a probability of $\frac{1}{2}$ each that the A is greater or less than the B, i.e. that the B−A difference is positive or negative. Now suppose we are testing H_0 that the median difference is $d=10$, say. If this is true, obviously it is no longer the case that positive and negative differences are equally likely. Instead there are equal probabilities ($\frac{1}{2}$ each) that the difference is greater than or less than 10; equivalently, that *(B value)−(A value)−10 is positive or negative*. If you think for a moment, you will see that a clever way to test this hypothesis is to subtract 10 from all the B values (or, alternatively, add 10 to all the A values), and then carry out the ordinary Mann–Whitney test on the resulting data.

As an exercise, carry out this operation on the car breakdowns data, by adding 10 to the A values. (This will incidentally give you some practice at dealing with ties!) The answer is $U=30$, and there is thus no reason for rejecting this H_0 since the $\alpha_2=10\%$ critical region is $U\leqslant 26$. Hence a median percentage difference of 10% is quite consistent with these data (as opposed to the median difference of 0 which was conclusively rejected).

The same operation can clearly be carried out with any desired hypothesized d. What we need therefore is to find the minimum and maximum values of d which will just permit non-rejection of H_0 (say, at the $\alpha_2=5\%$ level). The table below shows the values of U, calculated as in the above exercise, for several values of d.

d	U	d	U
$d=5$	16	⋮	⋮
$5<d<6$	17	$d=21$	$27\frac{1}{2}$
$d=6$	$17\frac{1}{2}$	$21<d<22$	25
$6<d<7$	18	$d=22$	24
$d=7$	$20\frac{1}{2}$	$22<d<23$	23
$7<d<8$	23	$d=23$	$22\frac{1}{2}$
$d=8$	$24\frac{1}{2}$	$23<d<26$	22
$8<d<9$	26	$d=26$	21
$d=9$	$26\frac{1}{2}$	$26<d<28$	20
⋮	⋮	$d=28$	$19\frac{1}{2}$

Thus, in fact, the range of values of d which would not be rejected at the $\alpha_2=5\%$ level (i.e. which do not result in the value U lying in the critical region $U\leqslant 22$) is $7<d<23$. Notice that there may be end-point discrepancies (e.g. should 7 be included in the confidence interval or not?) between the method as presented above and the theory as presented here but, particularly with continuous distributions, this is of negligible importance. A refinement could be made to the method to produce an exact correspondence.)

The general justification of the method is thus as follows. The 45° lines in the graphical method have equations of the form $x-y=d$, each line corresponding to a different value of d (for example, given by the x coordinate of the point where the line intersects the x axis). As a 45° line moves into the picture from either extreme, the number of points crossed by the stage at which its equation is $x-y=d$ is the value of the U statistic when testing the H_0 that the median difference is d. Before the line starts moving into the picture, no points have been crossed, i.e. $U=0$, so any corresponding d is obviously an unreasonable value of the median difference. As the line moves into the picture, U steadily increases, and the process is stopped as soon as U exceeds the critical value, i.e. as soon as the test would yield a non-significant result.

Large sample sizes

Critical values may be calculated very accurately from the normal approximation given in Chapter 5. Once obtained, they may then be used as above to provide confidence intervals. As already mentioned, both the graphical and arithmetical methods are rather unwieldy with large samples, with the graphical method being the lesser of the two evils. So in general, for large samples you will need to use the computer program given at the end of this chapter (Program 19.1).

CONFIDENCE INTERVALS FOR A MATCHED-PAIRS DIFFERENCE

INTRODUCTION

Finally, we present a confidence interval method based on Wilcoxon's signed-rank test (see Ch. 8). This is particularly relevant therefore for providing an interval for the average change in a matched-pairs situation. However, it may also be used as an alternative to the first method in this chapter for producing confidence intervals for ϕ, *as long as* the population distribution may be regarded as symmetric. The method is considerably more involved than that based on the sign test, but should generally give narrower intervals.

PRACTICAL PROBLEM

Again for illustration purposes, we shall use the relevant practical problem from earlier in the book; in this case, it is the one in Chapter 8 concerning the improvement in BOD levels after the implementation of anti-pollution controls. The data for this problem are given below.

Site	Before controls	Four years later	Change
1	17.4	13.6	3.8
2	15.7	10.1	5.6
3	12.9	10.3	2.6
4	9.8	9.2	0.6
5	13.4	11.1	2.3
6	18.7	20.4	−1.7
7	13.9	10.4	3.5
8	11.0	11.4	−0.4
9	5.4	4.9	0.5
10	10.4	8.9	1.5
11	16.4	11.2	5.2
12	5.6	4.8	0.6

METHOD

Denoting the n differences by $X_1, X_2, \ldots, X_n$, we calculate the $\frac{1}{2}n(n+1)$ pairwise averages $u_{ij}=\frac{1}{2}(X_i+X_j)$ for $i=1,2,\ldots,n$ and $j=i,i+1,\ldots,n$. Note that these include $X_1, X_2, \ldots, X_n$ themselves, corresponding to $u_{11}, u_{22}, \ldots, U_{nn}$. We then look up a value in the Wilcoxon table (Table D or the second table on page 29 in *EST*), exactly as we did when forming a confidence interval based on the sign test using Table C (or the first table on page 29 in *EST*). Thus here, with $n=12$ and $\gamma=99\%$, say, we enter the table at $\alpha_2=1\%$ and obtain the value $T=7$. Now we proceed precisely as with the previous two techniques in this chapter, but operating on the set of u_{ij} averages defined above. That is, we delete the seven smallest u_{ij} values and the seven largest u_{ij} values. The required confidence interval is then the range of the remaining averages, i.e. from the eighth smallest to the eighth largest average.

The work may be conveniently set out in the triangular table given below. Listed down the left-hand side and in the top row are the 12 changes in BOD levels; the body of the table contains the 78 u_{ij} values. For ease in identifying the smallest and largest u_{ij} for deletion, we have arranged the 12 differences in ascending order before forming the table.

	−1.7	*−0.4*	*0.5*	*0.6*	*0.8*	*1.5*	*2.3*	*2.6*	*3.5*	*3.8*	*5.2*	*5.6*
5.6	1.95	2.60	3.05	3.10	3.20	3.55	3.95	4.10	4.55	4.70	5.40	5.60
5.2	1.75	2.40	2.85	2.90	3.00	3.35	3.75	3.90	4.35	4.50	5.20	
3.8	1.05	1.70	2.15	2.20	2.30	2.65	3.05	3.20	3.65	3.80		
3.5	0.90	1.55	2.00	2.05	2.15	2.50	2.90	3.05	3.50			
2.6	0.45	1.10	1.55	1.60	1.70	2.05	2.45	2.60				
2.3	0.30	0.95	1.40	1.45	1.55	1.90	2.30					
1.5	−0.10	0.55	1.00	1.05	1.15	1.50						
0.8	−0.45	0.20	0.65	0.70	0.80							
0.6	−0.55	0.10	0.55	0.60								
0.5	−0.60	0.05	0.50									
−0.4	−1.05	−0.40										
−1.7	−1.70											

Then the smallest (or rather, largest negative) u_{ij} values will be found at the bottom of the table and the largest positive towards the right-hand corner. You may check that the eighth smallest u_{ij} is 0.05, and the eighth largest is 4.10; the required confidence interval for the average decrease in BOD is therefore (0.05, 4.10). Note that 0 is just outside this confidence interval, reflecting the fact that the null hypothesis of no change was rejected in Chapter 8 at the $\alpha_1 = \frac{1}{2}\%$.

Discussion

As with the previous method related to the Mann–Whitney test, this technique becomes very laborious with large sample sizes, and so the computer routine (Program 19.2) will be particularly useful here. Note, however, that it is not necessary in practice to complete the whole of the table of u_{ij} values; having seen where the largest and smallest u_{ij} are found, one can, with care, concentrate on calculating just the u_{ij} values in those two parts of the table.

Theory

The theory of this technique is perhaps a little tricky to follow. Recall that the statistic for Wilcoxon's signed-rank test was the sum of the ranks of either the positive or negative differences, the differences being ranked irrespective of sign. Let us first establish the link between these ranks and the u_{ij} used in forming the confidence interval.

Consider the rank of a typical difference (irrespective of sign) in our example, say the 1.5; its rank is 5. Now focus attention on all those u_{ij} which involve 1.5. The averages of 1.5 with the four smaller differences (−0.4, 0.5, 0.6 and 0.8) are 0.55, 1.00, 1.05 and 1.15; note, in particular, that they must all be non-negative and less than 1.5. The other u_{ij} involving the 1.5 must either be negative or be greater than 1.5 (except the u_{ij} involving 1.5 twice, which must of course be equal to 1.5). Therefore we see that there are precisely five u_{ij} values involving 1.5 which lie between 0 and 1.5 inclusive; and five happens to be the rank of 1.5. This rule is immediately generalizable. The rank of any observation, x say, in the current context (i.e. ignoring signs) is between 0 and x inclusive. (The rule needs some refinement in the case of tied observations, but we will ignore that complication here as it turns out to be irrelevant to the main argument.)

Now refer to the table of u_{ij} values. Notice first that all the u_{ij} in any column must be greater than or equal to the difference at the top of the column. In particular, if we consider a column headed by a *negative* difference (either of the first two columns in this example), it follows that the rank of that difference is simply given by the number of negative u_{ij} in that column (so the rank of −1.7 is 6 and the rank of −0.4 is 1). The remaining columns cannot contain any negative differences. Therefore we obtain the very important result that *the sum of the ranks of the negative differences is equal to the total number of negative* u_{ij} in the table. A similar argument involving rows instead of columns shows that the sum of the ranks of the positive differences is equal to the total number of positive u_{ij} in the table. This is consistent with the fact that these two sums of ranks must add up to $1+2+3+\ldots+n=\frac{1}{2}n(n+1)$, which is of course equal to the total number of u_{ij} that have been defined. The two rank sums in the examples are respectively 7 and 71, which agrees with the value of $T=7$ found in Chapter 8.

Now, as with the previous method based on the Mann–Whitney test, we must

broaden our minds to consider what would happen if we wished to test the null hypothesis that the average value of the change (B value minus A value, say) were equal to some non-zero value d. The obvious way to adjust the test procedure is to replace the 'B minus A' differences used previously by 'B minus A minus d', and then proceed as usual. Suppose we take $d=2.5$. Then, retaining the natural ordering of the differences used above, they become:

$$-4.2 \quad -2.9 \quad -2.0 \quad -1.9 \quad -1.7 \quad -1.0 \quad -0.1 \quad +0.1 \quad +1.0 \quad +1.3 \quad +2.7 \quad +3.1$$

If you carry out the signed-rank test on these data, you will find that T is now $29\frac{1}{2}$ (given by the sum of the ranks of the positive differences; the sum of the ranks of the negative differences is $48\frac{1}{2}$), which is not significant. Therefore, an average improvement in BOD of 2.5 is consistent with the data.

However, of more importance in the current context, let us now construct the table of u_{ij} values for these new differences (B−A−d). These are given in the table below.

	−4.2	*−2.9*	*−2.0*	*−1.9*	*−1.7*	*−1.0*	*−0.2*	*0.1*	*1.0*	*1.3*	*2.7*	*3.1*
3.1	−0.55	0.10	0.55	0.60	0.70	1.05	1.45	1.60	2.05	2.20	2.90	3.10
2.7	−0.75	−0.10	0.35	0.40	0.50	0.85	1.25	1.40	1.85	2.00	2.70	
1.3	−1.45	−0.80	−0.35	−0.30	−0.20	0.15	0.55	0.70	1.15	1.30		
1.0	−1.60	−0.95	−0.50	−0.45	−0.35	0	0.40	0.55	1.00			
0.1	−2.05	−1.40	−0.95	−0.90	−0.80	−0.45	−0.05	0.10				
−0.2	−2.20	−1.55	−1.10	−1.05	−0.95	−0.60	−0.20					
−1.0	−2.60	−1.95	−1.50	−1.45	−1.35	−1.00						
−1.7	−2.95	−2.30	−1.85	−1.80	−1.70							
−1.9	−3.05	−2.40	−1.95	−1.90								
−2.0	−3.10	−2.45	−2.00									
−2.9	−3.55	−2.90										
−4.2	−4.2											

Note first that there are altogether 48 negative u_{ij}, 29 positive u_{ij} and one 0. We shall count the 0 as half a plus and half a minus, so obtaining $48\frac{1}{2}$ negatives and $29\frac{1}{2}$ positives, consistent with the test result above.

Now the crux of the matter is to observe that we *could* very easily have computed these new sums of ranks directly from the previous table of u_{ij}. It is apparent that the new u_{ij} are simply 2.5 ($=d$) less than the original u_{ij}. Thus the *criterion of whether a new u_{ij} is positive or negative is identical to the criterion of whether the corresponding old u_{ij} is greater or less than 2.5 (or d, in general).*

So if we wanted to compute the statistic for several different values of d, the hypothesized average change, this gives us a much more efficient method than having to recalculate and rerank the differences (or form a new table of u_{ij}) each time. For any given value of d, all we do is to count the number of original u_{ij} which are less than, or greater than, d.

We can now apply this principle to justify the confidence interval technique. As usual, the confidence interval of level γ should contain those values of d which would not be rejected by a two-sided test of level $\alpha_2=100\%-\gamma$. It is easy to see that any value of d less than 0.5 would result in T being less than or equal to 7 (because there are seven u_{ij} values less than 0.5). But as soon as d gets to 0.5, T reaches 8 and becomes non-significant at the $\alpha_2=1\%$ level. Similarly, any value

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of d exceeding 4.10 gives $T \leqslant 7$, because there are just seven u_{ij} values greater than 4.0. Thus we indeed have $0.5 \leqslant d \leqslant 4.10$ as the range of values of d which are not rejected by a two-sided test with $\alpha_2 = 1\%$.

The generalization to all other n and γ is immediate.

Large sample sizes

For n beyond the range of the tables, we may find critical values using the normal approximation illustrated in Chapter 8; these critical values are then used in the same way as those read directly from the tables for smaller values of n.

Assignments

Construct appropriate 90% confidence intervals for: (a) assignments 1 and 2 of Chapter 3; (b) assignment 3 of Chapter 5.

1 Pain-relieving drugs

A pharmaceutical company sponsored an experiment to compare the effects of two pain-relieving drugs (identified as A and B) for the treatment of a certain disease. Ten patients took part in the experiment, each of them being given both drugs at different times. The number of hours of relief gained by each patient after the administration of each drug is given in the following table.

Patient	Drug A	Drug B
1	2.8	3.8
2	4.6	3.5
3	3.7	5.7
4	5.1	4.8
5	5.6	5.2
6	1.2	2.1
7	6.3	8.6
8	1.4	2.0
9	3.4	5.3
10	4.8	5.4

Task: Construct a 90% confidence interval for the average difference in performance between the drugs.

Program 19.1 Confidence interval for difference between two samples

```
100 REM *** CONFIDENCE INTERVAL FOR DIFFERENCE BETWEEN TWO SAMPLES ***

120 REM ** INPUT SAMPLES **
130 INPUT "SIZE OF SAMPLE A ? ";NA
140 DIM A(NA)
150 PRINT "INPUT SAMPLE A"
160 FOR I=1 TO NA
170 INPUT A(I)
180 NEXT I
190 PRINT
200 INPUT "SIZE OF SAMPLE B ? ";NB
210 LET N=NA*NB
220 DIM B(NB),D(N)
230 PRINT "INPUT SAMPLE B"
240 FOR I=1 TO NB
250 INPUT B(I)
260 NEXT I

270 REM ** CALCULATION OF THE DIFFERENCES **
280 LET L=0
290 FOR I=1 TO NB
300 FOR J=1 TO NA
310 LET L=L+1
320 LET D(L)=B(I)-A(J)
330 NEXT J
340 NEXT I
350 GO SUB 2000

360 REM ** CALCULATION OF THE INTERVAL **
370 PRINT
380 INPUT "CRITICAL VALUE ? ";C
390 PRINT
400 PRINT "CONFIDENCE INTERVAL IS "; D(C+1),D(N-C)
410 STOP

2000 REM ** SORT THE DIFFERENCES - USE LINES 2000-2160 OF PROGRAM 10.1 **
```

Program 19.2 Confidence interval for matched-pairs differences

```
100 REM *** CONFIDENCE INTERVAL FOR MATCHED PAIRS ***

120 REM ** INPUT SAMPLES **
130 INPUT "NUMBER OF PAIRS ? ";N
140 LET N2=N*(N+1)/2
150 DIM A(N),B(N),X(N),D(N2)
160 PRINT "INPUT THE PAIRS OF OBSERVATIONS"
170 FOR I=1 TO N
180 INPUT A(I),B(I)
190 NEXT I
200 PRINT

210 REM ** CALCULATION OF THE DIFFERENCES **
220 FOR I=1 TO N
230 LET X(I)=A(I)-B(I)
240 NEXT I
250 LET L=0
260 FOR I=1 TO N
270 FOR J=I TO N
280 LET L=L+1
290 LET D(L)=(X(I)+X(J))/2
300 NEXT J
310 NEXT I
320 GO SUB 2000

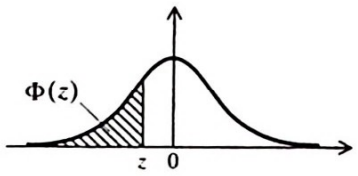
330 REM ** CALCULATION OF THE INTERVAL **
340 PRINT
350 INPUT "CRITICAL VALUE ? ";C
360 PRINT
370 PRINT "CONFIDENCE INTERVAL IS "; D(C+1),D(N-C)

2000 REM ** SORT THE DIFFERENCES - USE LINES 2000-2160 OF PROGRAM 10.1 **
```

PART VII
Tables

Table A Probabilities and percentage points of the standard normal distribution.

$$\Phi(z) = \int_{-\infty}^{z} (1/\sqrt{2\pi})\exp(-u^2/2)\,du = P(Z \leq z)$$



Probabilities

z	0	1	2	3	4	5	6	7	8	9
−3.0	0.0013	0.0010	0.0007	0.0005	0.0003	0.0002	0.0002	0.0001	0.0001	0.0000
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0020	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0238	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0300	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0570	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Continued overleaf

Table A *Continued*

Probabilities

z	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7974	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9648	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

Percentage points

	10%	5%	1%	0.1%	0.01%	0.001%
One-sided	1.2816	1.6449	2.3263	3.0902	3.7190	4.2649
Two-sided	1.6449	1.9600	2.5758	3.2905	3.8906	4.4172

Table B Upper-tail probabilities of the chi-squared (χ^2) distribution.

	α			α	
ν	5%	1%	ν	5%	1%
1	3.841	6.635	26	38.885	45.642
2	5.991	9.210	27	40.113	46.963
3	7.815	11.345	28	41.337	48.278
4	9.488	13.277	29	42.557	49.588
5	11.070	15.086	30	43.773	50.892
6	12.592	16.812	31	44.985	52.191
7	14.067	18.475	32	46.194	53.486
8	15.507	20.090	33	47.400	54.776
9	16.919	21.666	34	48.602	56.061
10	18.307	23.209	35	49.802	57.342
11	19.675	24.725	36	50.998	58.619
12	21.026	26.217	37	52.192	59.893
13	22.362	27.688	38	53.384	61.162
14	23.685	29.141	39	54.572	62.428
15	24.996	30.578	40	55.758	63.691
16	26.296	32.000	45	61.656	69.957
17	27.587	33.409	50	67.505	76.154
18	28.869	34.805	60	79.082	88.379
19	30.144	36.191	70	90.531	100.43
20	31.410	37.566	80	101.88	112.33
21	32.671	38.932	90	113.15	124.12
22	33.924	40.289	100	124.34	135.81
23	35.172	41.638			
24	36.415	42.980			
25	37.652	44.314			

Table C Critical values for the sign test.

n	α_1 = α_2 =	5% 10%	2½% 5%	1% 2%	½% 1%	n	α_1 = α_2 =	5% 10%	2½% 5%	1% 2%	½% 1%
1		–	–	–	–	26		8	7	6	6
2		–	–	–	–	27		8	7	7	6
3		–	–	–	–	28		9	8	7	6
4		–	–	–	–	29		9	8	7	7
5		0	–	–	–	30		10	9	8	7
6		0	0	–	–	31		10	9	8	7
7		0	0	0	–	32		10	9	8	8
8		1	0	0	0	33		11	10	9	8
9		1	1	0	0	34		11	10	9	9
10		1	1	0	0	35		12	11	10	9
11		2	1	1	0	36		12	11	10	9
12		2	2	1	1	37		13	12	10	10
13		3	2	1	1	38		13	12	11	10
14		3	2	2	1	39		13	12	11	11
15		3	3	2	2	40		14	13	12	11
16		4	3	2	2	41		14	13	12	11
17		4	4	3	2	42		15	14	13	12
18		5	4	3	3	43		15	14	13	12
19		5	4	4	3	44		16	15	13	13
20		5	5	4	3	45		16	15	14	13
21		6	5	4	4	46		16	15	14	13
22		6	5	5	4	47		17	16	15	14
23		7	6	5	4	48		17	16	15	14
24		7	6	5	5	49		18	17	15	15
25		7	7	6	5	50		18	17	16	15

Table D Critical values for the Wilcoxon signed-rank test.

n	α_1 = 5% / α_2 = 10%	2½% / 5%	1% / 2%	½% / 1%	n	α_1 = 5% / α_2 = 10%	2½% / 5%	1% / 2%	½% / 1%
1	–	–	–	–	26	110	98	84	75
2	–	–	–	–	27	119	107	92	83
3	–	–	–	–	28	130	116	101	91
4	–	–	–	–	29	140	126	110	100
5	0	–	–	–	30	151	137	120	109
6	2	0	–	–	31	163	147	130	118
7	3	2	0	–	32	175	159	140	128
8	5	3	1	0	33	187	170	151	138
9	8	5	3	1	34	200	182	162	148
10	10	8	5	3	35	213	195	173	159
11	13	10	7	5	36	227	208	185	171
12	17	13	9	7	37	241	221	198	182
13	21	17	12	9	38	256	235	211	194
14	25	21	15	12	39	271	249	224	207
15	30	25	19	15	40	286	264	238	220
16	35	29	23	19	41	302	279	252	233
17	41	34	27	23	42	319	294	266	247
18	47	40	32	27	43	336	310	281	261
19	53	46	37	32	44	353	327	296	276
20	60	52	43	37	45	371	343	312	291
21	67	58	49	42	46	389	361	328	307
22	75	65	55	48	47	407	378	345	322
23	83	73	62	54	48	426	396	362	33
24	91	81	69	61	49	446	415	379	35
25	100	89	76	68	50	466	434	397	37

Table E Critical values for the Kolmogorov–Smirnov goodness-of-fit test (for completely specified distributions).

n	α_1 = 5% / α_2 = 10%	2½% / 5%	1% / 2%	½% / 1%	n	α_1 = 5% / α_2 = 10%	2½% / 5%	1% / 2%	½% / 1%
1	0.9500	0.9750	0.9900	0.9950	16	0.2947	0.3273	0.3657	0.3920
2	0.7764	0.8419	0.9000	0.9293	17	0.2863	0.3180	0.3553	0.3809
3	0.6360	0.7076	0.7846	0.8290	18	0.2785	0.3094	0.3457	0.3706
4	0.5652	0.6239	0.6889	0.7342	19	0.2714	0.3014	0.3369	0.3612
5	0.5094	0.5633	0.6272	0.6685	20	0.2647	0.2941	0.3287	0.3524
6	0.4680	0.5193	0.5774	0.6166	21	0.2586	0.2872	0.3210	0.3443
7	0.4361	0.4834	0.5384	0.5758	22	0.2528	0.2809	0.3139	0.3367
8	0.4096	0.4543	0.5065	0.5418	23	0.2475	0.2749	0.3073	0.3295
9	0.3875	0.4300	0.4796	0.5133	24	0.2424	0.2693	0.3010	0.3229
10	0.3687	0.4092	0.4566	0.4889	25	0.2377	0.2640	0.2952	0.3166
11	0.3524	0.3912	0.4367	0.4677	26	0.2332	0.2591	0.2896	0.3106
12	0.3382	0.3754	0.4192	0.4490	27	0.2290	0.2544	0.2844	0.3050
13	0.3255	0.3614	0.4036	0.4325	28	0.2250	0.2499	0.2794	0.2997
14	0.3142	0.3489	0.3897	0.4176	29	0.2212	0.2457	0.2747	0.2947
15	0.3040	0.3376	0.3771	0.4042	30	0.2176	0.2417	0.2702	0.2899

Table F Critical values for Lilliefors' normality test.

n	α_1 = 5% / α_2 = 10%	2½% / 5%	1% / 2%	½% / 1%	n	α_1 = 5% / α_2 = 10%	2½% / 5%	1% / 2%	½% / 1%
1	–	–	–	–	16	0.1954	0.2129	0.2332	0.2476
2	–	–	–	–	17	0.1901	0.2071	0.2270	0.2410
3	0.3666	0.3758	0.3812	0.3830	18	0.1852	0.2017	0.2212	0.2349
4	0.3453	0.3753	0.4007	0.4131	19	0.1807	0.1968	0.2158	0.2292
5	0.3189	0.3431	0.3755	0.3970	20	0.1765	0.1921	0.2107	0.2238
6	0.2972	0.3234	0.3523	0.3708	21	0.1725	0.1878	0.2060	0.2188
7	0.2802	0.3043	0.3321	0.3509	22	0.1688	0.1838	0.2015	0.2141
8	0.2652	0.2880	0.3150	0.3332	23	0.1653	0.1800	0.1974	0.2097
9	0.2523	0.2741	0.2999	0.3174	24	0.1620	0.1764	0.1936	0.2056
10	0.2411	0.2619	0.2869	0.3037	25	0.1589	0.1730	0.1899	0.2018
11	0.2312	0.2514	0.2754	0.2916	26	0.1560	0.1699	0.1865	0.1981
12	0.2225	0.2420	0.2651	0.2810	27	0.1533	0.1670	0.1833	0.1947
13	0.2148	0.2336	0.2559	0.2714	28	0.1507	0.1642	0.1802	0.1915
14	0.2077	0.2261	0.2476	0.2627	29	0.1483	0.1615	0.1773	0.1884
15	0.2013	0.2192	0.2401	0.2549	30	0.1460	0.1589	0.1746	0.1855

Table G Critical values for the Mann–Whitney test.

n_1	n_2	α_1 = α_2 =	5% 10%	2½% 5%	1% 2%	½% 1%	n_1	n_2	α_1 = α_2 =	5% 10%	2½% 5%	1% 2%	½% 1%
2	2		–	–	–	–	4	17		15	11	8	6
2	3		–	–	–	–	4	18		16	12	9	6
2	4		–	–	–	–	4	19		17	13	9	7
2	5		0	–	–	–	4	20		18	14	10	8
2	6		0	–	–	–							
2	7		0	–	–	–	5	5		4	2	1	0
2	8		1	0	–	–	5	6		5	3	2	1
2	9		1	0	–	–	5	7		6	5	3	1
2	10		1	0	–	–	5	8		8	6	4	2
2	11		1	0	–	–	5	9		9	7	5	3
2	12		2	1	–	–	5	10		11	8	6	4
2	13		2	1	0	–	5	11		12	9	7	5
2	14		3	1	0	–	5	12		13	11	8	6
2	15		3	1	0	–	5	13		15	12	9	7
2	16		3	1	0	–	5	14		16	13	10	7
2	17		3	2	0	–	5	15		18	14	11	8
2	18		4	2	0	–	5	16		19	15	12	9
2	19		4	2	1	0	5	17		20	17	13	10
2	20		4	2	1	0	5	18		22	18	14	11
							5	19		23	19	15	12
3	3		0	–	–	–	5	20	25	20	16	13	
3	4		0	–	–	–							
3	5		1	0	–	–	6	6		7	5	3	2
3	6		2	1	–	–	6	7		8	6	4	3
3	7		2	1	0	–	6	8		10	8	6	4
3	8		3	2	0	–	6	9		12	10	7	5
3	9		4	2	1	0	6	10		14	11	8	6
3	10		4	3	1	0	6	11		16	13	9	7
3	11		5	3	1	0	6	12		17	14	11	9
3	12		5	4	2	1	6	13		19	16	12	10
3	13		6	4	2	1	6	14		21	17	13	11
3	14		7	5	2	1	6	15		23	19	15	12
3	15		7	5	3	2	6	16		25	21	16	13
3	16		8	6	3	2	6	17		26	22	18	15
3	17		9	6	4	2	6	18		28	24	19	16
3	18		9	7	4	2	6	19		30	25	20	17
3	19		10	7	4	3	6	20		32	27	22	18
3	20		11	8	5	3							
							7	7		11	8	6	4
4	4		1	0	–	–	7	8		13	10	7	6
4	5		2	1	0	–	7	9		15	12	9	7
4	6		3	2	1	0	7	10		17	14	11	9
4	7		4	3	1	0	7	11		19	16	12	10
4	8		5	4	2	1	7	12		21	18	14	12
4	9		6	4	3	1	7	13		24	20	16	13
4	10		7	5	3	2	7	14		26	22	17	15
4	11		8	6	4	2	7	15		28	24	19	16
4	12		9	7	5	3	7	16		30	26	21	18
4	13		10	8	5	3	7	17		33	28	23	19
4	14		11	9	6	4	7	18		35	30	24	21
4	15		12	10	7	5	7	19		37	32	26	22
4	16		14	11	7	5	7	20		39	34	28	24

Continued overleaf

Table G *Continued*

n_1	n_2	$\alpha_1 =$ $\alpha_2 =$	5% 10%	2½% 5%	1% 2%	½% 1%
8	8		15	13	9	7
8	9		18	15	11	9
8	10		20	17	13	11
8	11		23	19	15	13
8	12		26	22	17	15
8	13		28	24	20	17
8	14		31	26	22	18
8	15		33	29	24	20
8	16		36	31	26	22
8	17		39	34	28	24
8	18		41	36	30	26
8	19		44	38	32	28
8	20		47	41	34	30
9	9		21	17	14	11
9	10		24	20	16	13
9	11		27	23	18	16
9	12		30	26	21	18
9	13		33	28	23	20
9	14		36	31	26	22
9	15		39	34	28	24
9	16		42	37	31	27
9	17		45	39	33	29
9	18		48	42	36	31
9	19		51	45	38	33
9	20		54	48	40	36
10	10		27	23	19	16
10	11		31	26	22	18
10	12		34	29	24	21
10	13		37	33	27	24
10	14		41	36	30	26
10	15		44	39	33	29
10	16		48	42	36	31
10	17		51	45	38	34
10	18		55	48	41	37
10	19		58	52	44	39
10	20		62	55	47	42
11	11		34	30	25	21
11	12		38	33	28	24
11	13		42	37	31	27
11	14		46	40	34	30
11	15		50	44	37	33
11	16		54	47	41	36
11	17		57	51	44	39
11	18		61	55	47	42
11	19		65	58	50	45
11	20		69	62	53	48
12	12		42	37	31	27
12	13		47	41	35	31
12	14		51	45	38	34
12	15		55	49	42	37
12	16		60	53	46	41
12	17		64	57	49	44
12	18		68	61	53	47
12	19		72	65	56	51
12	20		77	69	60	54
13	13		51	45	39	34
13	14		56	50	43	38
13	15		61	54	47	42
13	16		65	59	51	45
13	17		70	63	55	49
13	18		75	67	59	53
13	19		80	72	63	57
13	20		84	76	67	60
14	14		61	55	47	42
14	15		66	59	51	46
14	16		71	64	56	50
14	17		77	69	60	54
14	18		82	74	65	58
14	19		87	78	60	63
14	20		92	83	73	67
15	15		72	64	56	51
15	16		77	70	61	55
15	17		83	75	66	60
15	18		88	80	70	64
15	19		94	85	75	69
15	20		100	90	80	73
16	16		83	75	66	60
16	17		89	81	71	65
16	18		95	86	76	70
16	19		101	92	82	74
16	20		107	98	87	79
17	17		96	87	77	70
17	18		102	92	82	75
17	19		109	99	88	81
17	20		115	105	93	86
18	18		109	99	88	81
18	19		116	106	94	87
18	20		123	112	100	92
19	19		123	113	101	93
19	20		130	119	107	99
20	20		138	127	114	105

Table H Critical values for the Tukey test for location-differences.

$\alpha_1 = 5\%$, $\alpha_2 = 10\%$

n_2=	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	
n_1= 2	11	11	11	11	11	11	10	10	10	10	9	9	8	9	8	7	–	–	–	
3	10	10	10	9	9	9	9	9	8	8	8	7	7	7	6	6	7	6		
4	9	9	9	8	8	8	8	8	8	7	7	7	7	6	6	6	6			
5	8	8	8	8	8	8	7	7	7	7	7	6	6	6	6	6			9	20
6	8	8	8	7	7	7	7	7	7	7	6	6	6	6	6			9	9	19
7	7	7	7	7	7	7	7	7	6	6	6	6	6	6			9	9	9	18
8	7	7	7	7	7	7	7	6	6	6	6	6	6			9	9	9	9	17
9	7	7	7	7	7	7	6	6	6	6	6	6			9	9	9	9	9	16
10	7	7	7	7	6	6	6	6	6	6	6			9	9	9	9	9	9	15
11	7	7	7	6	6	6	6	6	6	6			9	9	9	9	9	9	9	14
12	7	6	6	6	6	6	6	6	6			9	9	9	9	9	9	9	9	13
13	6	6	6	6	6	6	6	6			9	9	9	9	9	9	9	9	10	12
14	6	6	6	6	6	6	6			9	9	9	9	9	9	9	10	10	10	11
15	6	6	6	6	6	6			9	9	9	9	9	9	9	10	10	10	10	10
16	6	6	6	6	6			9	9	9	9	9	9	9	10	10	10	10	11	9
17	6	6	6	6			8	9	9	9	9	9	10	10	10	10	11	11	11	8
18	6	6	6			8	8	9	9	9	9	10	10	10	11	11	11	11	12	7
19	6	6			8	8	9	9	9	9	10	10	10	11	11	12	12	12	13	6
20	6			8	8	9	9	9	10	10	10	11	11	12	12	12	13	13	14	5
			–	9	8	9	9	10	10	11	11	12	12	13	13	14	14	15	15	4
		–	–	–	–	10	11	10	11	12	12	13	14	14	15	15	16	17	17	3
	–	–	–	–	–	–	–	–	–	–	–	15	16	17	18	19	20	19	20	2=n_1
n_2=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	

$\alpha_1 = 1\%$, $\alpha_2 = 2\%$

Continued overleaf

Table H *Continued*

$\alpha_1 = 2\frac{1}{2}\%$, $\alpha_2 = 5\%$

$n_2=$	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	
$n_1=$ 2	17	16	16	15	15	14	13	13	12	13	12	11	10	–	–	–	–	–	–	
3	14	13	13	13	12	12	11	11	10	10	9	9	8	8	7	8	–	–		
4	12	12	11	11	11	10	10	10	9	9	9	8	8	7	7	7	8			
5	11	11	10	10	10	9	9	9	9	8	8	8	7	7	7	7			10	20
6	10	10	10	9	9	9	9	8	8	8	8	7	7	7	7			10	10	19
7	9	9	9	9	9	8	8	8	8	8	7	7	7	7			10	10	10	18
8	9	9	9	8	8	8	8	8	8	7	7	7	7			10	10	10	10	17
9	9	8	8	8	8	8	8	8	7	7	7	7			10	10	10	10	10	16
10	8	8	8	8	8	8	8	7	7	7	7			10	10	10	10	10	10	15
11	8	8	8	8	8	8	7	7	7	7			10	10	10	10	10	10	10	14
12	8	8	8	8	8	7	7	7	7			10	10	10	10	10	10	10	11	13
13	8	8	8	8	7	7	7	7			10	10	10	10	10	10	10	11	11	12
14	8	8	8	7	7	7	7			10	10	10	10	10	10	11	11	11	11	11
15	8	8	7	7	7	7			10	10	10	10	10	10	11	11	11	11	12	10
16	8	7	7	7	7			10	10	10	10	10	10	11	11	11	11	12	12	9
17	7	7	7	7			9	10	10	10	10	10	11	11	11	12	12	12	13	8
18	7	7	7			9	9	10	10	10	10	11	11	11	12	12	13	13	13	7
19	7	7			9	9	10	10	10	11	11	11	12	12	13	13	13	14	14	6
20	7			10	9	10	10	10	11	11	12	12	12	13	13	14	14	15	15	5
			–	–	10	11	10	11	11	12	12	13	13	14	15	15	16	16	17	4
		–	–	–	–	–	–	12	13	14	13	14	15	15	16	17	18	18	19	3
	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	21	22	$2=n_1$

$n_2=$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	

$\alpha_1 = \frac{1}{2}\%$, $\alpha_2 = 1\%$

Table I Critical values for the Tukey-type test for dispersion differences.

$\alpha_1=5\%$, $\alpha_2=10\%$

	n_2																		
n_1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	–	–	–	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	–	–	–	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2
4	–	–	4	4	4	3	3	3	3	3	2	2	2	2	2	2	2	2	2
5	–	–	5	5	4	4	4	3	3	3	3	3	3	3	2	2	2	2	2
6	–	–	6	5	5	5	4	4	4	4	3	3	3	3	3	3	3	2	2
7	–	7	7	6	6	5	5	4	4	4	4	4	3	3	3	3	3	3	3
8	–	8	8	7	6	6	5	5	5	4	4	4	4	4	3	3	3	3	3
9	–	9	8	7	7	6	6	5	5	5	5	4	4	4	4	4	4	3	3
10	–	10	9	8	7	7	6	6	5	5	5	5	4	4	4	4	4	4	4
11	–	11	10	9	8	7	7	6	6	6	5	5	5	5	4	4	4	4	4
12	–	12	11	9	9	8	7	7	6	6	6	5	5	5	5	4	4	4	4
13	–	13	11	10	9	8	8	7	7	6	6	6	5	5	5	5	5	4	4
14	–	14	12	11	10	9	8	8	7	7	6	6	6	5	5	5	5	5	5
15	–	14	13	11	10	9	9	8	8	7	7	6	6	6	6	5	5	5	5
16	–	15	14	12	11	10	9	8	8	7	7	7	6	6	6	6	5	5	5
17	–	16	14	13	12	10	10	9	8	8	7	7	7	6	6	6	6	5	5
18	–	17	15	13	12	11	10	9	9	8	8	7	7	7	6	6	6	6	5
19	–	18	16	14	13	12	11	10	9	9	8	8	7	7	7	6	6	6	6
20	–	19	17	15	13	12	11	10	10	9	8	8	8	7	7	7	6	6	6

Continued overleaf

Table I *Continued*

$\alpha_1 = 2\frac{1}{2}\%$, $\alpha_2 = 5\%$

	n_2																		
n_1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	–	–	–	–	–	–	2	2	2	2	2	2	2	2	2	2	2	2	2
3	–	–	–	–	3	3	3	3	3	3	3	2	2	2	2	2	2	2	2
4	–	–	–	4	4	4	4	4	3	3	3	3	3	3	3	3	3	2	2
5	–	–	–	5	5	5	4	4	4	4	4	3	3	3	3	3	3	3	3
6	–	–	–	6	6	5	5	5	4	4	4	4	4	4	3	3	3	3	3
7	–	–	7	7	6	6	5	5	5	5	4	4	4	4	4	4	4	3	3
8	–	–	8	7	7	6	6	6	5	5	5	5	4	4	4	4	4	4	4
9	–	–	9	8	8	7	7	6	6	6	5	5	5	5	5	4	4	4	4
10	–	–	10	9	8	8	7	7	6	6	6	5	5	5	5	5	5	4	4
11	–	–	11	10	9	8	8	7	7	6	6	6	6	5	5	5	5	5	5
12	–	12	11	10	10	9	8	8	7	7	7	6	6	6	5	5	5	5	5
13	–	13	12	11	10	9	9	8	8	7	7	7	6	6	6	6	5	5	5
14	–	14	13	12	11	10	9	9	8	8	7	7	7	6	6	6	6	6	5
15	–	15	14	13	11	11	10	9	9	8	8	7	7	7	6	6	6	6	6
16	–	16	15	13	12	11	10	10	9	9	8	8	7	7	7	7	6	6	6
17	–	17	15	14	13	12	11	10	10	9	9	8	8	7	7	7	7	6	6
18	–	18	15	15	13	12	11	11	10	9	9	8	8	8	7	7	7	7	6
19	–	19	17	15	14	13	12	11	10	10	9	9	8	8	8	7	7	7	7
20	–	20	18	15	15	14	12	12	11	10	10	9	9	8	8	8	7	7	7

$\alpha_1=1\%$, $\alpha_2=2\%$

	n_2																		
n_1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	–	–	–	–	–	–	–	–	–	–	–	2	2	2	2	2	2	2	2
3	–	–	–	–	–	–	–	3	3	3	3	3	3	3	3	3	3	3	3
4	–	–	–	–	–	4	4	4	4	4	4	4	3	3	3	3	3	3	3
5	–	–	–	–	5	5	5	5	5	4	4	4	4	4	4	4	4	3	3
6	–	–	–	–	6	6	6	5	5	5	5	5	4	4	4	4	4	4	4
7	–	–	–	7	7	7	6	6	6	5	5	5	5	5	5	4	4	4	4
8	–	–	–	8	8	7	7	6	6	6	6	5	5	5	5	5	5	5	4
9	–	–	–	9	8	8	7	7	7	6	6	6	6	5	5	5	5	5	5
10	–	–	10	10	9	9	8	8	7	7	7	6	6	6	6	6	5	5	5
11	–	–	11	11	10	9	9	8	8	7	7	7	7	6	6	6	6	6	5
12	–	–	12	11	11	10	9	9	8	8	8	7	7	7	6	6	6	6	6
13	–	–	13	12	11	11	10	9	9	8	8	8	7	7	7	7	6	6	6
14	–	–	14	13	12	11	10	10	9	8	8	8	7	7	7	7	7	7	6
15	–	–	15	14	13	12	11	10	10	9	9	9	8	8	8	7	7	7	7
16	–	–	16	14	13	12	12	11	10	10	9	9	9	8	8	8	7	7	7
17	–	–	17	15	14	13	12	12	11	10	10	9	9	9	8	8	8	7	7
18	–	–	17	16	15	14	13	12	11	11	10	10	9	9	9	8	8	8	8
19	–	–	18	17	16	14	13	13	12	11	11	10	10	9	9	9	8	8	8
20	–	–	19	18	16	15	14	13	12	12	11	11	10	10	9	9	9	8	8

Continued overleaf

Table I *Continued*

$\alpha_1 = \frac{1}{2}\%$, $\alpha_2 = 1\%$

	n_2																		
n_1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2	2
3	–	–	–	–	–	–	–	–	–	–	3	3	3	3	3	3	3	3	3
4	–	–	–	–	–	–	–	4	4	4	4	4	4	4	4	4	4	3	3
5	–	–	–	–	–	–	5	5	5	5	5	4	4	4	4	4	4	4	4
6	–	–	–	–	–	6	6	6	6	5	5	5	5	5	5	4	4	4	4
7	–	–	–	–	7	7	7	6	6	6	6	5	5	5	5	5	5	5	5
8	–	–	–	–	8	8	7	7	7	6	6	6	6	6	5	5	5	5	5
9	–	–	–	9	9	8	8	8	7	7	7	6	6	6	6	6	6	5	5
10	–	–	–	10	10	9	9	8	8	8	7	7	7	6	6	6	6	6	6
11	–	–	–	11	10	10	9	9	8	8	8	7	7	7	7	7	6	6	6
12	–	–	–	12	11	11	10	9	9	9	8	8	8	7	7	7	7	7	6
13	–	–	–	13	12	11	11	10	10	9	9	8	8	8	8	7	7	7	7
14	–	–	14	14	13	12	11	11	10	10	9	9	9	8	8	8	7	7	7
15	–	–	15	14	13	13	12	11	11	10	9	9	9	8	8	8	8	8	7
16	–	–	16	15	14	13	13	12	11	11	10	10	9	9	9	8	8	8	8
17	–	–	17	16	15	14	13	12	12	11	11	10	10	9	9	9	9	8	8
18	–	–	18	17	16	15	14	13	12	12	11	11	10	10	10	9	9	9	8
19	–	–	19	18	16	15	14	14	13	12	12	11	11	10	10	10	9	9	9
20	–	–	20	18	17	16	15	14	13	13	12	12	11	11	10	10	10	9	9

Table J Critical values for the Rosenbaum two-sample test.

$\alpha_1=5\%$, $\alpha_2=10\%$

	n_1																			
n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	1
2	–	–	–	–	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	–	–	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2
4	–	–	4	4	3	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2
5	–	5	5	4	4	4	3	3	3	3	3	3	3	3	3	2	2	2	2	2
6	–	6	5	5	4	4	4	4	3	3	3	3	3	3	3	3	3	3	2	2
7	–	7	6	5	5	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3
8	–	8	7	6	5	5	5	4	4	4	4	3	3	3	3	3	3	3	3	3
9	–	9	7	6	6	5	5	5	4	4	4	4	4	3	3	3	3	3	3	3
10	–	9	8	7	6	6	5	5	5	4	4	4	4	4	4	3	3	3	3	3
11	–	10	9	8	7	6	6	5	5	5	4	4	4	4	4	4	4	3	3	3
12	–	11	9	8	7	6	6	6	5	5	5	4	4	4	4	4	4	4	3	3
13	–	12	10	9	8	7	6	6	5	5	5	5	4	4	4	4	4	4	4	4
14	–	12	11	9	8	7	7	6	6	5	5	5	5	4	4	4	4	4	4	4
15	–	13	11	10	9	8	7	6	6	6	5	5	5	5	4	4	4	4	4	4
16	–	14	12	10	9	8	7	7	6	6	6	5	5	5	5	5	4	4	4	4
17	–	15	12	11	9	8	8	7	7	6	6	6	5	5	5	5	5	4	4	4
18	–	16	13	11	10	9	8	7	7	6	6	6	6	5	5	5	5	5	4	4
19	19	16	14	12	10	9	8	8	7	7	6	6	6	5	5	5	5	5	5	4
20	20	17	14	12	11	10	9	8	7	7	7	6	6	6	5	5	5	5	5	5

Continued overleaf

Table J *Continued*

$\alpha_1 = 2\frac{1}{2}\%$, $\alpha_2 = 5\%$

	n_1																			
n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
2	–	–	–	–	–	–	–	2	2	2	2	2	2	2	2	2	2	2	2	2
3	–	–	–	–	3	3	3	3	3	3	3	3	2	2	2	2	2	2	2	2
4	–	–	–	4	4	4	3	3	3	3	3	3	3	3	3	3	3	3	2	2
5	–	–	5	5	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3	3
6	–	–	6	5	5	5	4	4	4	4	4	3	3	3	3	3	3	3	3	3
7	–	–	7	6	6	5	5	5	4	4	4	4	4	4	3	3	3	3	3	3
8	–	8	7	7	6	6	5	5	5	4	4	4	4	4	4	4	3	3	3	3
9	–	9	8	7	7	6	6	5	5	5	5	4	4	4	4	4	4	4	4	3
10	–	10	9	8	7	7	6	6	5	5	5	5	4	4	4	4	4	4	4	4
11	–	11	10	9	8	7	6	6	6	5	5	5	5	5	4	4	4	4	4	4
12	–	12	10	9	8	7	7	6	6	6	5	5	5	5	5	4	4	4	4	4
13	–	13	11	10	9	8	7	7	6	6	6	5	5	5	5	5	5	4	4	4
14	–	13	12	10	9	8	8	7	7	6	6	6	5	5	5	5	5	5	4	4
15	–	14	12	11	10	9	8	8	7	7	6	6	6	6	5	5	5	5	5	5
16	–	15	13	12	10	9	9	8	7	7	7	6	6	6	6	5	5	5	5	5
17	–	16	14	12	11	10	9	8	8	7	7	7	6	6	6	6	5	5	5	5
18	–	17	15	13	11	10	9	9	8	8	7	7	7	6	6	6	6	5	5	5
19	–	18	15	13	12	11	10	9	8	8	7	7	7	6	6	6	6	6	5	5
20	–	19	16	14	12	11	10	9	9	8	8	7	7	7	6	6	6	6	6	5

$\alpha_1=1\%$, $\alpha_2=2\%$

	n_1																			
n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
2	–	–	–	–	–	–	–	–	–	–	–	–	2	2	2	2	2	2	2	2
3	–	–	–	–	–	–	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	–	–	–	–	4	4	4	4	4	4	3	3	3	3	3	3	3	3	3	3
5	–	–	–	5	5	5	5	4	4	4	4	4	4	4	3	3	3	3	3	3
6	–	–	–	6	6	5	5	5	5	4	4	4	4	4	4	4	4	3	3	3
7	–	–	7	7	6	6	6	5	5	5	5	4	4	4	4	4	4	4	4	4
8	–	–	8	8	7	6	6	6	5	5	5	5	5	4	4	4	4	4	4	4
9	–	–	9	8	8	7	7	6	6	6	5	5	5	5	5	4	4	4	4	4
10	–	–	10	9	8	8	7	7	6	6	6	5	5	5	5	5	5	5	4	4
11	–	–	11	10	9	8	8	7	7	6	6	6	6	5	5	5	5	5	5	5
12	–	–	11	10	9	9	8	8	7	7	6	6	6	6	5	5	5	5	5	5
13	–	13	12	11	10	9	9	8	8	7	7	6	6	6	6	6	5	5	5	5
14	–	14	13	12	11	10	9	8	8	7	7	7	7	6	6	6	6	5	5	5
15	–	15	14	12	11	10	10	9	8	7	7	7	7	6	6	6	6	6	6	5
16	–	16	15	13	12	11	10	9	9	8	8	7	7	7	7	6	6	6	6	6
17	–	17	15	14	12	11	10	10	9	9	8	8	7	7	7	7	6	6	6	6
18	–	18	16	14	13	12	11	10	10	9	9	8	8	7	7	7	7	6	6	6
19	–	19	17	15	14	12	11	11	10	9	9	8	8	8	7	7	7	7	6	6
20	–	20	18	16	14	13	12	11	10	10	9	9	8	8	8	7	7	7	7	7

Continued overleaf

Table J *Continued*

$\alpha_1 = \frac{1}{2}\%$, $\alpha_2 = 1\%$

	n_1																			
n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2	2
3	–	–	–	–	–	–	–	–	3	3	3	3	3	3	3	3	3	3	3	3
4	–	–	–	–	–	4	4	4	4	4	4	4	4	3	3	3	3	3	3	3
5	–	–	–	–	5	5	5	5	4	4	4	4	4	4	4	4	4	4	3	3
6	–	–	–	6	6	6	5	5	5	5	5	4	4	4	4	4	4	4	4	4
7	–	–	–	7	7	6	6	6	5	5	5	5	5	5	4	4	4	4	4	4
8	–	–	–	8	7	7	7	6	6	6	5	5	5	5	5	5	5	4	4	4
9	–	–	9	9	8	8	7	7	6	6	6	6	5	5	5	5	5	5	5	5
10	–	–	10	9	9	8	8	7	7	7	6	6	6	6	5	5	5	5	5	5
11	–	–	11	10	9	9	8	8	7	7	7	6	6	6	6	6	5	5	5	5
12	–	–	12	11	10	9	9	8	8	7	7	7	7	6	6	6	6	6	5	5
13	–	–	13	12	11	10	9	9	8	8	7	7	7	7	6	6	6	6	6	6
14	–	–	14	12	11	11	10	9	9	8	8	8	7	7	7	7	6	6	6	6
15	–	–	14	13	12	11	10	10	9	9	8	8	8	7	7	7	7	6	6	6
16	–	–	15	14	13	12	11	10	10	9	9	8	8	8	7	7	7	7	6	6
17	–	–	16	15	13	12	11	11	10	10	9	9	8	8	8	7	7	7	7	7
18	–	–	17	15	14	13	12	11	11	10	9	9	9	8	8	8	7	7	7	7
19	–	19	18	16	15	14	13	12	11	10	10	9	9	9	8	8	8	7	7	7
20	–	20	19	17	15	14	13	12	11	11	10	10	9	9	9	8	8	8	8	7

Table K Critical values for the Kolmogorov–Smirnov two-sample test.

n_1	n_2	$\alpha_1 =$ $\alpha_2 =$	5% 10%	2½% 5%	1% 2%	½% 1%
2	2		–	–	–	–
2	3		–	–	–	–
2	4		–	–	–	–
2	5		10	–	–	–
2	6		12	–	–	–
2	7		14	–	–	–
2	8		16	16	–	–
2	9		18	18	–	–
2	10		18	20	–	–
2	11		20	22	–	–
2	12		22	24	–	–
2	13		24	26	26	–
2	14		24	26	28	–
2	15		26	28	30	–
2	16		28	30	32	–
2	17		30	32	34	–
2	18		32	34	36	–
2	19		32	36	38	38
2	20		34	38	40	40
2	21		36	38	42	42
2	22		38	40	44	44
2	23		38	42	44	46
2	24		40	44	46	48
2	25		42	46	48	50
3	3		9	–	–	–
3	4		12	–	–	–
3	5		15	15	–	–
3	6		15	18	–	–
3	7		18	21	21	–
3	8		21	21	24	–
3	9		21	24	27	27
3	10		24	27	30	30
3	11		27	30	33	33
3	12		27	30	33	36
3	13		30	33	36	39
3	14		33	36	39	42
3	15		33	36	42	42
3	16		36	39	45	45
3	17		36	42	45	48
3	18		39	45	48	51
3	19		42	45	51	54
3	20		42	48	54	57
3	21		45	51	54	57
3	22		48	51	57	60
3	23		48	54	60	63
3	24		51	57	63	66
3	25		54	60	66	69
4	4		16	16	–	–
4	5		16	20	20	–
4	6		18	20	24	24
4	7		21	24	28	28
4	8		24	28	32	32
4	9		27	28	32	36
4	10		28	30	36	36
4	11		29	33	40	40
4	12		36	36	40	44
4	13		35	39	44	48
4	14		38	42	48	48
4	15		40	44	48	52
4	16		44	48	52	56
4	17		44	48	56	60
4	18		46	50	56	60
4	19		49	53	57	64
4	20		52	60	64	68
4	21		52	59	64	72
4	22		56	62	66	72
4	23		57	64	69	76
4	24		60	68	76	80
4	25		63	68	75	84
5	5		20	25	25	25
5	6		24	24	30	30
5	7		25	28	30	35
5	8		27	30	35	35
5	9		30	35	36	40
5	10		35	40	40	45
5	11		35	39	44	45
5	12		36	43	48	50
5	13		40	45	50	52
5	14		42	46	51	56
5	15		50	55	60	60
5	16		48	54	59	64
5	17		50	55	63	68
5	18		52	60	65	70
5	19		56	61	70	71
5	20		60	65	75	80
5	21		60	69	75	80
5	22		63	70	78	83
5	23		65	72	82	87
5	24		67	76	85	90
5	25		75	80	90	95
6	6		30	30	36	36
6	7		28	30	35	36
6	8		30	34	40	40
6	9		33	39	42	45
6	10		36	40	44	48
6	11		38	43	49	54
6	12		48	48	54	60
6	13		46	52	54	60
6	14		48	54	60	64
6	15		51	57	63	69
6	16		54	60	66	72
6	17		56	62	68	73
6	18		66	72	78	84
6	19		64	70	77	83
6	20		66	72	80	88
6	21		69	75	84	90
6	22		70	78	88	92
6	23		73	80	91	97
6	24		78	90	96	102
6	25		78	88	89	107

Continued overleaf

Table K *Continued*

n_1	n_2	α_1 = 5% α_2 = 10%	2½% 5%	1% 2%	½% 1%
7	7	35	42	42	42
7	8	34	40	42	48
7	9	36	42	47	49
7	10	40	46	50	53
7	11	44	48	55	59
7	12	46	53	58	60
7	13	50	56	63	65
7	14	56	63	70	77
7	15	56	62	70	75
7	16	59	64	73	77
7	17	61	68	77	84
7	18	65	72	83	87
7	19	69	76	86	91
7	20	72	79	91	93
7	21	77	91	98	105
7	22	77	84	97	103
7	23	80	89	101	108
7	24	84	92	105	112
7	25	86	97	108	115
8	8	40	48	48	56
8	9	40	46	54	55
8	10	44	48	56	60
8	11	48	53	61	64
8	12	52	60	64	68
8	13	54	62	67	72
8	14	58	64	72	76
8	15	60	67	75	81
8	16	72	80	88	88
8	17	68	77	85	88
8	18	72	80	88	94
8	19	74	82	93	98
8	20	80	88	100	104
8	21	81	89	102	107
8	22	84	94	106	112
8	23	89	98	107	115
8	24	96	104	120	128
8	25	95	104	118	125
9	9	54	54	63	63
9	10	50	53	61	63
9	11	52	59	63	70
9	12	57	63	69	75
9	13	59	65	73	78
9	14	63	70	80	84
9	15	69	75	84	90
9	16	69	78	87	94
9	17	74	82	92	99
9	18	81	90	99	108
9	19	80	89	99	107
9	20	84	93	104	111
9	21	90	99	111	117
9	22	91	101	113	122
9	23	94	106	117	126
9	24	99	111	123	132
9	25	101	114	125	135
10	10	60	70	70	80
10	11	57	60	69	77
10	12	60	66	74	80
10	13	64	70	78	84
10	14	68	74	84	90
10	15	75	80	90	100
10	16	76	84	94	100
10	17	79	89	99	106
10	18	82	92	104	108
10	19	85	94	104	113
10	20	100	110	120	130
10	21	95	105	118	126
10	22	98	108	120	130
10	23	101	114	127	137
10	24	106	118	130	140
10	25	110	125	140	150
11	11	66	77	88	88
11	12	64	72	77	86
11	13	67	75	86	91
11	14	73	82	90	96
11	15	76	84	95	102
11	16	80	89	100	106
11	17	85	93	104	110
11	18	88	97	108	118
11	19	92	102	114	122
11	20	96	107	118	127
11	21	101	112	124	134
11	22	110	121	143	143
11	23	108	119	132	142
11	24	111	124	139	150
11	25	117	129	143	154
12	12	72	83	96	96
12	13	71	81	92	95
12	14	78	86	94	104
12	15	84	93	102	108
12	16	88	96	108	116
12	17	90	100	112	119
12	18	96	108	120	126
12	19	99	108	121	130
12	20	104	116	128	140
12	21	108	120	132	141
12	22	110	124	138	148
12	23	113	125	138	149
12	24	132	144	156	168
12	25	120	138	153	165

Table K *Continued*

n_1	n_2	$\alpha_1 =$ 5% $\alpha_2 =$ 10%	2½% 5%	1% 2%	½% 1%
13	13	91	91	104	117
13	14	78	89	102	104
13	15	87	96	107	115
13	16	91	101	112	121
13	17	96	105	118	127
13	18	99	110	123	131
13	19	104	114	130	138
13	20	108	120	135	143
13	21	113	126	140	150
13	22	117	130	143	156
13	23	120	135	152	161
13	24	125	140	155	166
13	25	131	145	160	172
14	14	98	112	112	126
14	15	92	98	111	123
14	16	96	106	120	126
14	17	100	111	125	134
14	18	104	116	130	140
14	19	110	121	135	148
14	20	114	126	142	152
14	21	126	140	154	161
14	22	124	138	152	164
14	23	127	142	159	170
14	24	132	146	164	176
14	25	136	150	169	182
15	15	105	120	135	135
15	16	101	114	120	133
15	17	105	116	131	142
15	18	111	123	138	147
15	19	114	127	142	152
15	20	125	135	150	160
15	21	126	138	156	168
15	22	130	144	160	173
15	23	134	149	165	179
15	24	141	156	174	186
15	25	145	160	180	195
16	16	112	128	144	160
16	17	109	124	139	143
16	18	116	128	142	154
16	19	120	133	151	160
16	20	128	140	156	168
16	21	130	145	162	173
16	22	136	150	168	180
16	23	141	157	175	187
16	24	152	168	184	200
16	25	149	167	186	199
17	17	136	136	153	170
17	18	118	133	150	164

n_1	n_1	$\alpha_1 =$ 5% $\alpha_2 =$ 10%	2½% 5%	1% 2%	½% 1%
17	19	126	141	158	166
17	20	130	146	163	175
17	21	136	151	168	180
17	22	142	157	176	187
17	23	146	163	181	196
17	24	151	168	187	203
17	25	156	173	196	207
18	18	144	162	180	180
18	19	133	142	160	176
18	20	136	152	170	182
18	21	144	159	177	189
18	22	148	164	184	196
18	23	152	170	189	204
18	24	162	180	198	216
18	25	162	180	202	216
19	19	152	171	190	190
19	20	144	160	171	187
19	21	147	163	184	199
19	22	152	169	190	204
19	23	159	177	197	209
19	24	164	183	204	218
19	25	168	187	211	224
20	20	160	180	200	220
20	21	165	173	193	199
20	22	160	176	196	212
20	23	164	184	205	219
20	24	172	192	212	228
20	25	180	200	220	235
21	21	168	189	210	231
21	22	163	183	205	223
21	23	171	189	213	227
21	24	177	198	222	237
21	25	182	202	225	244
22	22	198	198	242	242
22	23	173	194	217	237
22	24	182	204	228	242
22	25	189	209	234	250
23	23	207	230	253	253
23	24	183	205	228	249
23	25	195	216	243	262
24	24	216	240	264	288
24	25	204	225	254	262
25	25	225	250	275	300

Table L Critical values for Spearman's rank correlation coefficient.

$$r_S = 1 - \frac{6D^2}{n^3 - n}$$

n	α_1^R = 5% α_2 = 10%	2½% 5%	1% 2%	½% 1%	n	α_1^R = 5% α_2 = 10%	2½% 5%	1% 2%	½% 1%
1	–	–	–	–	26	0.3306	0.3901	0.4571	0.5009
2	–	–	–	–	27	0.3242	0.3828	0.4487	0.4915
3	–	–	–	–	28	0.3180	0.3755	0.4401	0.4828
4	1.0000	–	–	–	29	0.3118	0.3685	0.4325	0.4749
5	0.9000	1.0000	1.0000	–	30	0.3063	0.3624	0.4251	0.4670
6	0.8286	0.8857	0.9429	1.0000	31	0.3012	0.3560	0.4185	0.4593
7	0.7143	0.7857	0.8929	0.9286	32	0.2962	0.3504	0.4117	0.4523
8	0.6429	0.7381	0.8333	0.8810	33	0.2914	0.3449	0.4054	0.4455
9	0.6000	0.7000	0.7833	0.8333	34	0.2871	0.3396	0.3995	0.4390
10	0.5636	0.6485	0.7455	0.7939	35	0.2829	0.3347	0.3936	0.4328
11	0.5364	0.6182	0.7091	0.7545	36	0.2788	0.3300	0.3882	0.4268
12	0.5035	0.5874	0.6783	0.7273	37	0.2748	0.3253	0.3829	0.4211
13	0.4835	0.5604	0.6484	0.7033	38	0.2710	0.3209	0.3778	0.4155
14	0.4637	0.5385	0.6264	0.6791	39	0.2674	0.3168	0.3729	0.4103
15	0.4464	0.5214	0.6036	0.6536	40	0.2640	0.3128	0.3681	0.4051
16	0.4294	0.5029	0.5824	0.6353	41	0.2606	0.3087	0.3636	0.4002
17	0.4142	0.4877	0.5662	0.6176	42	0.2574	0.3051	0.3594	0.3955
18	0.4014	0.4716	0.5501	0.5996	43	0.2543	0.3014	0.3550	0.3908
19	0.3912	0.4596	0.5351	0.5842	44	0.2513	0.2978	0.3511	0.3865
20	0.3805	0.4466	0.5218	0.5699	45	0.2484	0.2945	0.3470	0.3822
21	0.3701	0.4364	0.5091	0.5558	46	0.2456	0.2913	0.3433	0.3781
22	0.3608	0.4252	0.4975	0.5438	47	0.2429	0.2880	0.3396	0.3741
23	0.3528	0.4160	0.4862	0.5316	48	0.2403	0.2850	0.3361	0.3702
24	0.3443	0.4070	0.4757	0.5209	49	0.2378	0.2820	0.3326	0.3664
25	0.3369	0.3977	0.4662	0.5108	50	0.2353	0.2791	0.3293	0.3628

Table M Critical values for Kendall's rank correlation coefficient.

$$\tau = \frac{N_c - N_D}{\frac{1}{2}n(n-1)}$$

n	α_1^R = 5% α_2 = 10%	2½% 5%	1% 2%	½% 1%	n	α_1^R = 5% α_2 = 10%	2½% 5%	1% 2%	½% 1%
1	–	–	–	–	26	0.2369	0.2800	0.3292	0.3600
2	–	–	–	–	27	0.2308	0.2707	0.3219	0.3561
3	–	–	–	–	28	0.2275	0.2646	0.3122	0.3439
4	1.0000	–	–	–	29	0.2217	0.2611	0.3103	0.3399
5	0.8000	1.0000	1.0000	–	30	0.2184	0.2552	0.3011	0.3333
6	0.7333	0.8667	0.8667	1.0000	31	0.2129	0.2516	0.2946	0.3247
7	0.6190	0.7143	0.8095	0.9048	32	0.2097	0.2460	0.2903	0.3226
8	0.5714	0.6429	0.7143	0.7857	33	0.2045	0.2424	0.2879	0.3144
9	0.5000	0.5556	0.6667	0.7222	34	0.2014	0.2371	0.2799	0.3119
10	0.4667	0.5111	0.6000	0.6444	35	0.1966	0.2336	0.2773	0.3042
11	0.4182	0.4909	0.5636	0.6000	36	0.1937	0.2317	0.2730	0.3016
12	0.3939	0.4545	0.5455	0.5758	37	0.1922	0.2282	0.2673	0.2973
13	0.3590	0.4359	0.5128	0.5641	38	0.1892	0.2233	0.2632	0.2916
14	0.3626	0.4066	0.4725	0.5165	39	0.1876	0.2200	0.2605	0.2874
15	0.3333	0.3905	0.4667	0.5048	40	0.1846	0.2179	0.2564	0.2846
16	0.3167	0.3833	0.4333	0.4833	41	0.1805	0.2146	0.2537	0.2805
17	0.3088	0.3676	0.4265	0.4706	42	0.1777	0.2125	0.2497	0.2753
18	0.2941	0.3464	0.4118	0.4510	43	0.1761	0.2093	0.2470	0.2735
19	0.2865	0.3333	0.3918	0.4386	44	0.1734	0.2072	0.2431	0.2685
20	0.2737	0.3263	0.3789	0.4211	45	0.1717	0.2040	0.2404	0.2667
21	0.2667	0.3143	0.3714	0.4095	46	0.1691	0.2019	0.2386	0.2638
22	0.2641	0.3074	0.3593	0.3939	47	0.1674	0.1989	0.2359	0.2599
23	0.2569	0.2964	0.3518	0.3913	48	0.1667	0.1968	0.2323	0.2571
24	0.2464	0.2899	0.3406	0.3768	49	0.1633	0.1956	0.2296	0.2534
25	0.2400	0.2867	0.3333	0.3667	50	0.1624	0.1918	0.2278	0.2506

Table N Critical values for the Kruskal–Wallis test (small sample sizes). Unequal sample sizes

$$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1)$$

$k = 3$			$k = 4$		
Sample sizes	$\alpha =$ 5%	1%	Sample sizes	$\alpha =$ 5%	1%
2 2 2	–	–	2 1 1 1	–	–
			2 2 1 1	–	–
3 2 1	–	–	2 2 2 1	5.679	–
3 2 2	4.714	–	2 2 2 2	6.167	6.667
3 3 1	5.143	–			
3 3 2	5.361	–	3 1 1 1	–	–
3 3 3	5.600	7.200	3 2 1 1	–	–
			3 2 2 1	5.833	–
4 1 1	–	–	3 2 2 2	6.333	7.133
4 2 1	–	–	3 3 2 1	6.244	7.200
4 2 2	5.333	–	3 3 2 2	6.527	7.636
4 3 1	5.208	–	3 3 3 1	6.600	7.400
4 3 2	5.444	6.444	3 3 3 2	6.727	8.015
4 3 3	5.791	6.745	3 3 3 3	7.000	8.538
4 4 1	5.455	7.036			
4 4 3	5.598	7.144	4 1 1 1	–	–
4 4 4	5.692	7.564	4 2 1 1	5.833	–
			4 2 2 1	6.133	7.000
5 1 1	–	–	4 2 2 2	6.545	7.391
5 2 1	5.000	–	4 3 1 1	6.178	7.067
5 2 2	5.160	6.533	4 3 2 1	6.309	7.455
5 3 1	4.960	–	4 3 2 2	6.621	7.871
5 3 2	5.251	6.909	4 3 3 1	6.545	7.758
5 3 3	5.648	7.079	4 3 3 2	6.795	8.333
5 4 1	4.985	6.955	4 3 3 3	6.984	8.659
5 4 2	5.273	7.205	4 4 1 1	5.945	7.909
5 4 3	5.656	7.445	4 4 2 1	6.386	7.909
5 4 4	5.657	7.760	4 4 2 2	6.731	8.346
5 5 1	5.127	7.309	4 4 3 1	6.635	8.231
5 5 2	5.338	7.338	4 4 3 2	6.874	8.621
5 5 3	5.705	7.578	4 4 3 3	7.038	8.876
5 5 4	5.666	7.823	4 4 4 1	6.725	8.588
5 5 5	5.780	8.000	4 4 4 2	6.957	8.871
6 1 1	–	–			
6 2 1	4.822	–			
6 2 2	5.345	6.655			
6 3 1	4.855	6.873			
6 3 2	5.348	6.970			
6 3 3	5.615	7.410			
6 4 1	4.947	7.106			
6 4 2	5.340	7.340			
6 4 3	5.610	7.500			
6 4 4	5.681	7.795			
6 5 1	4.990	7.182			
6 5 2	5.338	7.376			
6 5 3	5.602	7.590			
6 5 4	5.661	7.936			
6 5 5	5.729	8.028			
6 6 1	4.945	7.121			
6 6 2	5.410	7.467			
6 6 3	5.625	7.725			
6 6 4	5.724	8.000			
6 6 5	5.765	8.124			
6 6 6	5.801	8.222			

$k = 5$			$k = 6$		
Sample sizes	$\alpha =$ 5%	1%	Sample sizes	$\alpha =$ 5%	1%
1 1 1 1 1	–	–	1 1 1 1 1 1	–	–
2 1 1 1 1	–	–	2 1 1 1 1 1	–	–
2 2 1 1 1	–	–	2 2 1 1 1 1	–	–
2 2 2 1 1	6.750	–	2 2 2 1 1 1	7.600	–
2 2 2 2 1	7.133	7.533	2 2 2 2 1 1	8.018	8.618
2 2 2 2 2	7.418	8.291	2 2 2 2 2 1	8.455	9.227
			2 2 2 2 2 2	8.846	9.846
3 1 1 1 1	–	–			
3 2 1 1 1	6.583	–	3 1 1 1 1 1	–	–
3 2 2 1 1	6.800	7.600	3 2 1 1 1 1	7.467	–
3 2 2 2 1	7.309	8.127	3 2 2 1 1 1	7.945	8.509
3 2 2 2 2	7.682	8.682	3 2 2 2 1 1	8.348	9.136
3 3 1 1 1	7.111	–	3 2 2 2 2 1	8.731	9.692
3 3 2 1 1	7.200	8.073	3 2 2 2 2 2	9.033	10.22
3 3 2 2 1	7.591	8.576	3 3 1 1 1 1	7.909	8.564
3 3 2 2 2	7.910	9.115	3 3 2 1 1 1	8.303	9.045
3 3 3 1 1	7.576	8.424	3 3 2 2 1 1	8.615	9.628
3 3 3 2 1	7.769	9.051	3 3 2 2 2 1	8.923	10.15
3 3 3 2 2	8.044	9.505	3 3 3 1 1 1	8.461	9.564
3 3 3 3 1	8.000	9.451	3 3 3 2 1 1	8.835	10.08
4 1 1 1 1	–	–	4 1 1 1 1 1	7.333	–
4 2 1 1 1	6.733	–	4 2 1 1 1 1	7.827	8.400
			4 2 2 1 1 1	8.205	9.000
			4 2 2 2 1 1	8.558	9.538
			4 2 2 2 2 1	8.868	10.07
			4 3 1 1 1 1	8.053	9.023
			4 3 2 1 1 1	8.429	9.506
			4 3 2 2 1 1	8.742	10.01
			4 3 3 1 1 1	8.654	9.934
			4 4 1 1 1 1	8.231	9.538
			4 4 2 1 1 1	8.571	9.940
			5 1 1 1 1 1	7.909	–
			5 2 1 1 1 1	7.891	8.682

Continued overleaf

Table N *Continued*

Equal sample sizes

$$H = \frac{12}{n^2k(nk+1)}\sum_{i=1}^{k} R_i^2 - 3(nk+1)$$

	$k = 3$		$k = 4$		$k = 5$		$k = 6$	
n	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%
2	–	–	6.167	6.667	7.418	8.291	8.846	9.846
3	5.600	7.200	7.000	8.538	8.333	10.20	9.789	11.82
4	5.692	7.654	7.235	9.287	8.685	11.07	10.14	12.72
5	5.780	8.000	7.377	9.789	8.876	11.57	10.36	13.26
6	5.801	8.222	7.453	10.09	9.002	11.91	10.50	13.60
7	5.819	8.378	7.501	10.25	9.080	12.14	10.59	13.84
8	5.805	8.465	7.534	10.42	9.126	12.29	10.66	13.99
9	5.831	8.529	7.557	10.53	9.166	12.41	10.71	14.13
10	5.853	8.607	7.586	10.62	9.200	12.50	10.75	14.24
11	5.885	8.648	7.623	10.69	9.242	12.58	10.76	14.32
12	5.872	8.712	7.629	10.75	9.274	12.63	10.79	14.38
13	5.901	8.735	7.645	10.80	9.303	12.69	10.83	14.44
14	5.896	8.754	7.658	10.84	9.307	12.74	10.84	14.49
15	5.902	8.821	7.676	10.87	9.302	12.77	10.86	14.53
16	5.909	8.822	7.678	10.90	9.313	12.79	10.88	14.56
17	5.915	8.856	7.682	10.92	9.325	12.83	10.88	14.60
18	5.932	8.865	7.698	10.95	9.334	12.85	10.89	14.63
19	5.923	8.887	7.701	10.98	9.342	12.87	10.90	14.64
20	5.926	8.905	7.703	10.98	9.353	12.91	10.92	14.67
∞	5.991	9.210	7.815	11.34	9.488	13.28	11.07	15.09

Table O Critical values for Friedman's test.

$$M = \frac{12}{nk(k+1)} \sum_{i=1}^{k} R_i^2 - 3n(k+1)$$

	$k = 3$		$k = 4$		$k = 5$		$k = 6$	
n	$\alpha = 5\%$	1%	$\alpha = 5$	1%	$\alpha = 5$	1%	$\alpha = 5\%$	1%
2	–	–	6.000	–	7.600	8.000	9.143	9.714
3	6.000	–	7.400	9.000	8.533	10.13	9.857	11.76
4	6.500	8.000	7.800	9.600	8.800	11.20	10.29	12.71
5	6.400	8.400	7.800	9.960	8.960	11.68	10.49	13.23
6	7.000	9.000	7.600	10.20	9.067	11.87	10.57	13.62
7	7.143	8.857	7.800	10.54	9.143	12.11	10.67	13.86
8	6.250	9.000	7.650	10.50	9.200	13.20	10.71	14.00
9	6.222	9.556	7.667	10.73	9.244	12.44	10.78	14.14
10	6.200	9.600	7.680	10.68	9.280	12.48	10.80	14.23
11	6.545	9.455	7.691	10.75	9.309	12.58	10.84	14.32
12	6.500	9.500	7.700	10.80	9.333	12.60	10.86	14.38
13	6.615	9.385	7.800	10.85	9.354	12.68	10.89	14.45
14	6.143	9.143	7.714	10.89	9.371	12.74	10.90	14.49
15	6.400	8.933	7.720	10.92	9.387	12.80	10.92	14.54
16	6.500	9.375	7.800	10.95	9.400	12.80	10.96	14.57
17	6.118	9.294	7.800	10.05	9.412	12.85	10.95	14.61
18	6.333	9.000	7.733	10.93	9.422	12.89	10.95	14.63
19	6.421	9.579	7.863	11.02	9.432	12.88	11.00	14.67
20	6.300	9.300	7.800	11.10	9.400	12.92	11.00	14.66
∞	5.991	9.210	7.815	11.34	9.488	13.28	11.07	15.09

Table P Critical values for the M_1 match test.

	$k = 3$		$k = 4$		$k = 5$		$k = 6$	
n	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%
2	–	–	4	–	5	5	4	6
3	9	–	8	12	7	9	7	9
4	12	18	12	15	11	14	11	14
5	18	20	17	20	17	20	17	20
6	23	29	23	28	23	27	23	27
7	31	39	30	36	30	35	30	35
8	39	48	39	45	39	44	38	44
9	48	57	48	55	48	54	48	54
10	59	67	58	66	58	65	58	65

Critical values for $k \geq 3$ and $n \geq 11$

	α	
n	5%	1%
11	70	79
12	83	93
13	97	107
14	112	122
15	127	139
16	144	156
17	162	175
18	180	194
19	200	214
20	220	236

Table Q Critical values for the M_2 match test.

	$k = 3$		$k = 4$		$k = 5$		$k = 6$	
n	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%
2	–	–	8.0	–	8.0	10.0	8.0	10.0
3	9.0	–	10.0	12.0	9.0	11.0	9.0	10.0
4	15.0	18.0	15.0	18.0	15.5	17.5	15.5	17.5
5	24.0	26.0	23.0	26.5	24.0	26.0	24.0	26.5
6	32.0	36.0	33.0	35.5	34.0	36.5	34.5	37.5
7	43.0	51.0	45.0	48.0	48.0	49.0	47.0	50.0
8	56.0	61.0	58.0	61.5	59.5	63.5	61.0	64.5
9	70.0	76.0	73.5	77.5	75.0	79.5	76.5	81.0
10	81.0	92.0	90.0	94.5	92.5	97.0	94.5	99.0
11	105.0	110.0	109.0	114.0	112.0	117.0	114.0	119.0
12	125.0	130.0	129.0	134.5	132.5	138.5	135.5	141.0
13	146.0	152.0	151.5	157.5	155.5	161.5	158.5	164.5
14	169.0	176.0	175.5	181.5	180.5	186.5	183.5	190.5
15	193.0	201.0	201.0	208.0	206.5	213.5	210.5	217.5
16	219.0	227.0	228.5	235.5	234.5	242.0	239.0	247.0
17	247.0	256.0	257.5	265.5	264.5	272.5	269.5	278.5
18	277.0	286.0	288.5	297.0	296.5	305.0	302.0	310.5
19	308.0	318.0	321.0	330.0	330.0	339.0	336.0	345.5
20	341.0	351.0	355.5	365.0	365.5	375.0	372.0	382.0

Table R Critical values for the Terpstra–Johnckheere test.

Unequal sample sizes ($k = 3$)

Sample sizes	α 5%	α 1%	Sample sizes	α 5%	α 1%	Sample sizes	α 5%	α 1%
2 2 2	1	2	3 6 8	26	19	5 6 9	41	31
2 2 3	2	0	3 6 9	29	21	5 6 10	44	34
2 2 4	3	1	3 6 10	32	24	5 7 7	37	28
2 2 5	4	1	3 7 7	26	19	5 7 8	41	32
2 2 6	5	2	3 7 8	30	22	5 7 9	46	35
2 2 7	6	3	3 7 9	34	25	5 7 10	50	39
2 2 8	7	3	3 7 10	37	28	5 8 8	46	36
2 3 3	3	1	3 8 8	33	25	5 8 9	51	40
2 3 4	5	2	3 8 9	38	28	5 8 10	56	44
2 3 5	6	3	3 8 10	42	32	5 9 9	56	44
2 3 6	8	4	3 9 9	42	32	5 9 10	61	49
2 3 7	9	5	3 9 10	47	36	5 10 10	67	53
2 3 8	10	6	3 10 10	51	40			
2 4 4	6	3				6 6 6	33	25
2 4 5	8	5	4 4 4	12	8	6 6 7	37	28
2 4 6	10	6	4 4 5	14	10	6 6 8	41	32
2 4 7	12	7	4 4 6	17	12	6 6 9	46	36
2 4 8	14	9	4 4 7	20	14	6 6 10	51	39
2 5 5	10	6	4 4 8	22	16	6 7 7	42	32
2 5 6	13	8	4 4 9	25	18	6 7 8	46	36
2 5 7	15	10	4 4 10	28	20	6 7 9	52	41
2 5 8	17	12	4 5 5	17	12	6 7 10	57	45
2 6 6	15	10	4 5 6	20	15	6 8 8	52	41
2 6 7	18	12	4 5 7	24	17	6 8 9	57	45
2 6 8	21	14	4 5 8	27	20	6 8 10	63	50
2 7 7	21	15	4 5 9	30	22	6 9 9	63	50
2 7 8	24	17	4 5 10	33	24	6 9 10	69	55
2 8 8	27	20	4 6 6	24	17	6 10 10	75	60
			4 6 7	27	20			
3 3 3	5	2	4 6 8	31	23	7 7 7	47	37
3 3 4	7	4	4 6 9	35	26	7 7 8	52	41
3 3 5	9	5	4 6 10	38	29	7 7 9	58	46
3 3 6	11	6	4 7 7	31	23	7 7 10	63	50
3 3 7	12	8	4 7 8	35	27	7 8 8	58	46
3 3 8	14	9	4 7 9	40	30	7 8 9	64	51
3 3 9	17	11	4 7 10	44	33	7 8 10	70	56
3 3 10	18	12	4 8 8	40	30	7 9 9	70	56
3 4 4	9	5	4 8 9	44	34	7 9 10	76	62
3 4 5	11	7	4 8 10	49	38	7 10 10	83	67
3 4 6	14	9	4 9 9	49	38			
3 4 7	16	11	4 9 10	54	42	8 8 8	64	52
3 4 8	18	12	4 10 10	59	47	8 8 9	70	57
3 4 9	21	14				8 8 10	77	62
3 4 10	23	16	5 5 5	21	15	8 9 9	77	63
3 5 5	14	9	5 5 6	24	18	8 9 10	84	68
3 5 6	17	11	5 5 7	28	21	8 10 10	91	74
3 5 7	19	13	5 5 8	31	24			
3 5 8	22	15	5 5 9	35	26	9 9 9	84	69
3 5 9	25	18	5 5 10	39	29	9 9 10	91	75
3 5 10	28	20	5 6 6	28	21	9 10 10	98	81
3 6 6	20	14	5 6 7	32	24			
3 6 7	23	16	5 6 8	36	28	10 10 10	106	88

Equal sample sizes, n

Number of samples, k	n	α 5%	α 1%
4	2	5	2
	3	14	10
	4	29	23
	5	50	40
	6	75	62
	7	106	90
	8	143	122
	9	184	160
	10	231	203
5	2	10	7
	3	28	21
	4	54	44
	5	90	76
	6	134	116
	7	188	164
	8	250	222
	9	322	288
	10	403	364
6	2	17	13
	3	45	37
	4	86	73
	5	141	123
	6	210	186
	7	292	261
	8	388	351
	9	498	454
	10	623	570

Table S Critical values for Page's test.

	$k = 3$		$k = 4$		$k = 5$		$k = 6$	
n	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%
2	28	–	58	60	103	106	166	173
3	41	42	84	87	150	155	244	252
4	54	55	111	114	197	204	321	331
5	66	68	137	141	244	251	397	409
6	79	81	163	167	291	299	474	486
7	91	93	189	193	338	346	550	563
8	104	106	214	220	384	393	640	625
9	116	119	240	246	431	441	718	701
10	128	131	266	272	477	487	793	777
11	141	144	292	298	523	534	869	852
12	153	156	317	324	570	581	946	928
13	165	169	343	350	616	628	1022	1003
14	178	181	369	376	662	674	1098	1078
15	190	194	394	402	708	721	1174	1153
16	202	206	420	428	754	767	1249	1229
17	215	218	446	453	800	814	1325	1304
18	227	231	471	479	846	860	1401	1379
19	239	243	497	505	892	906	1477	1454
20	251	256	522	531	938	953	1529	1552

Table T Critical values for the L_1 match test for ordered alternatives.

	$k = 3$		$k = 4$		$k = 5$		$k = 6$	
n	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%
2	–	–	6	8	6	8	6	7
3	–	–	7	9	7	9	7	9
4	9	12	9	10	9	10	9	10

$n \geq 5$, k any value

	α	
n	5%	1%
5	10	12
6	11	13
7	13	15
8	14	16
9	15	18
10	16	19
11	18	20
12	19	22
13	20	23
14	21	24
15	23	26
16	24	27
17	25	28
18	26	30
19	27	31
20	29	32

Table U Critical values for the L_2 match test for ordered alternatives.

	$k = 3$		$k = 4$		$k = 5$		$k = 6$	
n	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%	$\alpha = 5\%$	1%
2	6	–	7.0	8.0	6.5	7.5	6.5	8.0
3	8	9	8.5	9.5	8.5	10.0	9.0	10.5
4	10	11	10.5	12.0	11.0	12.5	11.5	13.0
5	12	13	12.5	14.0	13.0	15.0	13.5	15.0
6	14	15	14.5	16.5	15.0	17.0	15.5	17.5
7	16	17	16.5	18.5	17.5	19.5	18.0	20.0
8	18	19	18.5	20.5	19.5	21.5	20.0	22.0
9	20	21	20.5	22.5	21.5	23.5	22.0	24.5
10	22	23	22.5	24.5	23.5	26.0	24.0	26.5
11	24	25	24.5	27.0	25.5	28.0	26.5	28.5
12	25	27	26.5	29.0	27.5	30.0	28.5	31.0
13	27	29	28.5	31.0	29.5	32.0	30.5	33.0
14	29	31	30.5	33.0	31.5	34.5	32.5	35.0
15	31	33	32.5	35.0	33.5	36.5	34.5	37.5
16	33	35	34.5	37.0	35.5	38.5	36.5	39.5
17	35	37	36.5	39.0	37.5	40.5	38.5	41.5
18	36	39	38.0	41.0	39.5	42.5	40.5	43.5
19	38	40	40.0	43.0	41.5	44.5	42.5	45.5
20	40	42	42.0	45.0	43.5	46.5	44.5	47.5

Table V Critical values for the S_1 match test for interaction.

		α	
n_1	n_2	5%	1%
3	3	0	0
3	4	4	3
3	5	7	6
3	6	11	10
3	7	16	12
3	8	22	18
3	9	29	24
3	10	37	32
4	4	6	5
4	5	9	7
4	6	13	11
4	7	18	15
4	8	24	20
4	9	31	27
4	10	39	34
5	5	12	10
5	6	17	14
5	7	22	18
5	8	28	24
5	9	35	30
5	10	43	38
6	6	21	18
6	7	26	22
6	8	32	28
6	9	39	34
6	10	47	42
7	7	31	27
7	8	37	33
7	9	45	39
7	10	53	47
8	8	44	39
8	9	51	45
8	10	59	53
9	9	58	52
9	10	66	60
10	10	74	68

Table W Critical values for the S_2 match test for interaction.

		α	
n_1	n_2	5%	1%
3	3	7.0	6.0
3	4	11.5	11.0
3	5	17.5	16.5
3	6	25.5	24.5
3	7	32.5	29.0
3	8	43.0	39.0
3	9	55.5	51.0
3	10	69.0	64.5
4	4	16.0	15.0
4	5	22.5	21.0
4	6	30.5	29.0
4	7	40.0	36.5
4	8	51.0	47.0
4	9	64.0	60.0
4	10	79.0	74.0
5	5	29.0	27.5
5	6	37.5	35.5
5	7	47.5	44.0
5	8	59.5	55.0
5	9	73.0	68.0
5	10	88.0	83.0
6	6	46.0	42.5
6	7	56.5	52.0
6	8	68.5	64.5
6	9	82.5	64.5
6	10	98.0	92.5
7	7	67.5	62.5
7	8	79.5	74.5
7	9	94.0	88.5
7	10	109.5	104.5
8	8	92.5	87.0
8	9	106.5	101.0
8	10	123.0	116.5
9	9	121.5	115.0
9	10	137.5	131.0
10	10	154.5	147.5

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Table X Critical values for the runs test.

Lower critical values

α_1=5%, α_2=10%

n_1 =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
	–	–	–	–	–	–	2	2	2	2	2	2	2	2	2	2	2	2	2	$2 = n_2$
		–	–	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	
n_1 = 2	–		2	2	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4
3	–	–		3	3	3	3	4	4	4	4	4	5	5	5	5	5	5	5	5
4	–	–	–		3	4	4	4	5	5	5	5	5	6	6	6	6	6	6	6
5	–	–	2	2		4	4	5	5	5	6	6	6	6	6	7	7	7	7	7
6	–	2	2	3	3		5	5	6	6	6	6	7	7	7	7	8	8	8	8
7	–	2	2	3	3	3		6	6	6	7	7	7	8	8	8	8	8	9	9
8	–	2	3	3	3	4	4		6	7	7	8	8	8	8	9	9	9	9	10
9	–	2	3	3	4	4	5	5		7	8	8	8	9	9	9	10	10	10	11
10	–	2	3	3	4	5	5	5	6		8	9	9	9	10	10	10	10	11	12
11	–	2	3	4	4	5	5	6	6	7		9	9	10	10	10	11	11	11	13
12	2	2	3	4	4	5	6	6	7	7			10	10	11	11	11	12	12	14
13	2	2	3	4	5	5	6	6	7	7	8	8		11	11	11	12	12	12	15
14	2	2	3	4	5	5	6	7	7	8	8	9	9		11	12	12	13	13	16
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10		12	13	13	13	17
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11		13	14	14	18
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11		14	14	19
18	2	4	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12		15	20
19	2	4	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13		
20	2	4	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14	
n_2 =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	

α_1=2½%, α_2=5%

Continued overleaf

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Table X *Continued*

n_1=	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2	$2 = n_2$
		–	–	–	–	–	–	2	2	2	2	2	2	2	2	2	2	2	2	3
			–	–	–	–	2	2	2	2	3	3	3	3	3	3	3	3	3	4
n_1 = 2	–				2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	5
3	–	–				2	3	3	3	3	4	4	4	4	4	5	5	5	5	6
4	–	–	–					3	3	4	4	4	5	5	5	5	5	6	6	7
5	–	–	–	–					4	4	4	5	5	5	5	6	6	6	6	8
6	–	–	–	2	2					4	5	5	5	6	6	6	6	7	7	9
7	–	–	–	2	2	3					5	5	6	6	6	7	7	7	8	10
8	–	–	2	2	3	3	3					6	6	6	7	7	7	8	8	11
9	–	–	2	2	3	3	3	4					7	7	7	8	8	8	9	12
10	–	–	2	2	3	3	4	4	5					7	8	8	8	9	9	13
11	–	–	2	3	3	4	4	5	5	5					8	8	9	9	10	14
12	–	2	2	3	3	4	4	5	5	6	6					9	9	10	10	15
13	–	2	2	3	3	4	5	5	5	6	6	7					10	10	10	16
14	–	2	2	3	4	4	5	5	6	6	7	7	7					10	11	17
15	–	2	3	3	4	4	5	6	6	7	7	7	8	8					11	18
16	–	2	3	3	4	5	5	6	6	7	7	8	8	9	9					19
17	–	2	3	3	4	5	5	6	7	7	8	8	8	9	9	10				20
18	–	2	3	4	4	5	6	6	7	7	8	8	9	9	10	10	10			
19	–	2	3	4	4	5	6	6	7	8	8	9	9	10	10	10	11	11		
20	–	2	3	4	4	5	6	7	7	8	8	9	9	10	10	11	11	12	12	
n_2 =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	

$\alpha_1 = \frac{1}{2}\%$, $\alpha_2 = 1\%$

Upper critical values

α_1=5%, α_2=10%

n_1 =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
2	–		–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$2 = n_2$
3	–	–			–	7	–	–	–	–	–	–	–	–	–	–	–	–	–	3
4	–	–	–			8	9	9	9	–	–	–	–	–	–	–	–	–	–	4
5	–	–	9	10			9	9	10	11	11	11	–	–	–	–	–	–	–	5
6	–	–	9	10	11			11	11	12	12	12	13	13	13	13	–	–	–	6
7	–	–	–	11	12	13			12	13	13	13	14	14	14	14	15	15	15	7
8	–	–	–	11	12	13	14			13	14	14	15	15	15	16	16	16	16	8
9	–	–	–	–	13	14	14	15			14	15	15	16	16	17	17	17	17	9
10	–	–	–	–	13	14	15	16	16			16	16	17	17	17	18	18	18	10
11	–	–	–	–	13	14	15	16	17	17			17	17	18	18	19	19	19	11
12	–	–	–	–	13	14	16	16	17	18	19			18	18	19	19	20	20	12
13	–	–	–	–	–	15	16	17	18	19	19	20			19	20	20	21	21	13
14	–	–	–	–	–	15	16	17	18	19	20	20	21			20	21	21	22	14
15	–	–	–	–	–	15	16	18	18	19	20	21	22	22			21	22	22	15
16	–	–	–	–	–	–	17	18	19	20	21	21	22	23	23			23	23	16
17	–	–	–	–	–	–	17	18	19	20	21	22	23	23	24	25			24	17
18	–	–	–	–	–	–	17	18	19	20	21	22	23	24	25	25	26			18
19	–	–	–	–	–	–	17	18	20	21	22	23	23	24	25	26	26	27		19
20	–	–	–	–	–	–	17	18	20	21	22	23	24	25	25	26	27	27	28	20
n_2 =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	

α_1=2½%, α_2=5%

Continued overleaf

Table X *Continued*

n_1 =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
2	–			–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	$2 = n_2$
3	–	–			–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	3
4	–	–	–			9	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	4
5	–	–	–	–			10	11	11	–	–	–	–	–	–	–	–	–	–	–	–	5
6	–	–	–	11	12			12	12	13	13	–	–	–	–	–	–	–	–	–	–	6
7	–	–	–	–	13	13			13	14	14	15	15	15	–	–	–	–	–	–	–	7
8	–	–	–	–	13	14	15			14	15	15	16	16	17	17	17	17	–	–	–	8
9	–	–	–	–	–	15	15	16			16	16	17	17	18	18	18	18	19	19	19	9
10	–	–	–	–	–	15	16	17	17			17	18	18	19	19	19	20	20	20	20	10
11	–	–	–	–	–	15	16	17	18	19			18	19	19	20	20	21	21	21	22	11
12	–	–	–	–	–	–	17	18	19	19	20			19	20	21	21	22	22	22	23	12
13	–	–	–	–	–	–	17	18	19	20	21	21			21	21	22	22	23	23	24	13
14	–	–	–	–	–	–	17	18	19	20	21	22	23			22	23	23	24	24	24	14
15	–	–	–	–	–	–	–	19	20	21	22	22	23	24			23	24	24	25	25	15
16	–	–	–	–	–	–	–	19	20	21	22	23	24	24	25			24	25	26	26	16
17	–	–	–	–	–	–	–	19	20	22	22	23	24	25	26	26			26	26	27	17
18	–	–	–	–	–	–	–	–	21	22	23	24	25	25	26	27	28			27	27	18
19	–	–	–	–	–	–	–	–	21	22	23	24	25	26	27	27	28	29			28	19
20	–	–	–	–	–	–	–	–	21	22	24	24	25	26	27	28	29	29	30			20
n_2 =	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20			

$\alpha_1 = \frac{1}{2}\%$, $\alpha_2 = 1\%$

Table Y Critical values for the runs up and down test.

	$\alpha_1=5\%$ $\alpha_2=10\%$		$\alpha_1=2\frac{1}{2}\%$ $\alpha_2=5\%$		$\alpha_1=1\%$ $\alpha_2=2\%$		$\alpha_1=\frac{1}{2}\%$ $\alpha_2=1\%$	
n	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
3	–	–	–	–	–	–	–	–
4	–	–	–	–	–	–	–	–
5	1	–	1	–	–	–	–	–
6	1	–	1	–	1	–	1	–
7	2	–	2	–	1	–	1	–
8	2	–	2	–	2	–	1	–
9	3	8	3	–	2	–	2	–
10	3	9	3	–	3	–	2	–
11	4	10	4	10	3	–	3	–
12	4	11	4	11	4	–	3	–
13	5	12	5	12	4	12	4	–
14	6	12	5	13	5	13	4	13
15	6	13	6	14	5	14	4	14
16	7	14	6	14	6	15	5	15
17	7	15	7	15	6	16	6	16
18	8	15	7	16	7	16	6	17
19	8	16	8	17	7	17	7	18
20	9	17	8	17	8	18	7	18
21	10	18	9	18	8	19	8	19
22	10	18	10	19	9	20	8	20
23	11	19	10	20	10	20	9	21
24	11	20	11	20	10	21	10	22
25	12	21	11	21	11	22	10	22

Table Z Critical values for Fisher's exact test.

N	m	M	C	α(%)	N	m	M	C	α(%)	N	m	M	C	α(%)	N	m	M	C	α(%)
6	3	3	0	10.00	13	5	6	0	3.3	16	4	9	0	3.8	18	3	12	0	4.9
						5	7	1	17.2		4	10	0	1.6		3	13	0	2.5−
7	2	5	0	9.5+		5	8	1	6.4		4	11	1	12.6		3	14	1	21.6
	3	4	0	5.7		6	6	1	15.5+		4	12	1	5.4		3	15	1	11.3
						6	7	1	5.0+		5	7	0	5.8		4	9	0	8.2
8	2	6	0	7.1							5	8	0	2.6		4	10	0	4.6
	3	5	0	3.6	14	2	11	0	6.6		5	9	1	15.4		4	11	0	2.3
	4	4	0	2.9		2	12	0	2.2		5	10	1	7.1		4	12	1	16.7
						3	9	0	5.5−		5	11	1	2.6		4	13	1	8.8
9	2	7	0	5.6		3	10	0	2.2		6	6	0	5.2		4	14	1	3.7
	3	5	0	9.5+		3	11	1	18.7		6	7	0	2.1		5	8	0	5.9
	3	6	0	2.4		4	7	0	7.0		6	8	1	11.9		5	9	0	2.9
	4	4	0	7.9		4	8	0	3.0		6	9	1	4.9		5	10	0	1.3
	4	5	0	1.6		4	9	1	19.0		6	10	1	1.5+		5	11	1	9.5−
						4	10	1	8.2		7	7	1	10.9		5	12	1	4.3
10	2	8	0	4.4		5	6	0	5.6		7	8	1	4.1		5	13	1	1.5+
	3	6	0	6.7		5	7	0	2.1		7	9	1	1.1		6	6	0	10.0−
	3	7	0	1.7		5	8	1	12.6		8	8	1	1.0+		6	7	0	5.0−
	4	5	0	4.8		5	9	1	4.6							6	8	0	2.3
	4	6	1	23.8		6	6	0	1.9	17	2	13	0	8.8		6	9	1	13.1
	5	5	1	20.6		6	7	1	10.3		2	14	0	4.4		6	10	1	6.3
						6	8	1	3.3		2	15	0	1.5−		6	11	1	2.6
11	2	9	0	3.6		7	7	1	2.9		3	11	0	5.9		6	12	2	11.5−
	3	7	0	4.8							3	12	0	2.9		7	7	0	2.1
	3	8	0	1.2	15	2	12	0	5.7		3	13	0	1.2		7	8	1	11.3
	4	5	0	9.1		2	13	0	1.9		3	14	1	12.6		7	9	1	5.0−
	4	6	0	3.0		3	9	0	8.8		4	9	0	5.9		7	10	1	1.8
	4	7	1	17.6		3	10	0	4.4		4	10	0	2.9		7	11	2	7.7
	5	5	0	2.6		3	11	0	1.8		4	11	0	1.3		8	8	1	4.6
	5	6	1	13.4		3	12	1	16.3		4	12	1	10.5+		8	9	1	1.5+
						4	8	0	5.1		4	13	1	4.5−		8	10	2	6.1
12	2	9	0	9.1		4	9	0	2.2		5	7	0	8.1		9	9	2	5.7
	2	10	0	3.0		4	10	1	15.4		5	8	0	4.1					
	3	7	0	9.1		4	11	1	6.6		5	9	0	1.8	19	2	15	0	7.0
	3	8	0	3.6		5	6	0	8.4		5	10	1	12.0		2	16	0	3.5+
	3	9	1	25.5−		5	7	0	3.7		5	11	1	5.5+		2	17	0	1.2
	4	6	0	6.1		5	8	0	1.4		5	12	1	2.0−		3	12	0	7.2
	4	7	0	2.0+		5	9	1	9.4		6	6	0	7.5−		3	13	0	4.1
	4	8	1	13.3		5	10	1	3.4		6	7	0	3.4		3	14	0	2.1
	5	5	0	5.3		6	6	0	3.4		6	8	0	1.4		3	15	1	19.4
	5	6	0	1.5+		6	7	0	1.1		6	9	1	8.6		3	16	1	10.1
	5	7	1	9.1		6	8	1	7.0		6	10	1	3.5+		4	10	0	6.5+
	6	6	1	8.0		6	9	1	2.2		6	11	1	1.1		4	11	0	3.6
						7	7	1	6.3		7	7	0	1.2		4	12	0	1.8
13	2	10	0	7.7		7	8	1	1.8		7	8	1	7.3		4	13	1	14.2
	2	11	0	2.6							7	9	1	2.7		4	14	1	7.5−
	3	8	0	7.0	16	2	12	0	10.00		7	10	2	10.4		4	15	1	3.1
	3	9	0	2.8		2	13	0	5.00		8	8	1	2.4		5	8	0	7.9
	3	10	1	21.7		2	14	0	1.7		8	9	2	8.9		5	9	0	4.3
	4	6	0	9.8		3	10	0	7.1							5	10	0	2.2
	4	7	0	4.2		3	11	0	3.6	18	2	14	0	7.8		5	11	1	14.2
	4	8	0	1.4		3	12	0	1.4		2	15	0	3.9		5	12	1	7.6
	4	9	1	10.3		3	13	1	14.3		2	16	0	1.3		5	13	1	3.5−
	5	5	0	8.7		4	8	0	7.7		3	11	0	8.6		5	14	1	1.2

Table Z *Continued*

N	m	M	C	α(%)	N	m	M	C	α(%)	N	m	M	C	α(%)	N	n	N	C	α(%)
19	6	7	0	6.8	20	8	9	1	5.0−	21	9	12	2	1.7	22	9	11	2	8.0
	6	8	0	3.4		8	10	1	2.0−		10	10	2	4.5+		9	12	2	3.4
	6	9	0	1.5+		8	11	2	8.0		10	11	2	1.5−		9	13	2	1.2
	6	10	1	9.9		8	12	2	3.1							10	10	2	7.6
	6	11	1	4.7		9	9	1	1.8	22	1	21	0	9.1		10	11	2	3.0
	6	12	1	1.9		9	10	2	7.0		2	17	0	8.7		10	12	3	9.1
	6	13	2	9.2		9	11	2	2.5−		2	18	0	5.2		11	11	3	8.6
	7	7	0	3.1		10	10	2	2.3		2	19	0	2.6					
	7	8	0	1.3							2	20	1	35.5−	23	1	22	0	8.7
	7	9	1	8.0	21	1	20	0	9.5+		3	14	0	7.3		2	18	0	7.9
	7	10	1	3.5−		2	16	0	9.5+		3	15	0	4.5+		2	19	0	4.7
	7	11	1	1.3		2	17	0	5.7		3	16	0	2.6		2	20	0	2.4
	7	12	2	5.8		2	18	0	2.9		3	17	0	1.3		2	21	1	34.0
	8	8	1	7.4		2	19	1	37.1		3	18	1	14.5+		3	14	0	9.5−
	8	9	1	3.0		3	13	0	8.4		3	19	1	7.5+		3	15	0	6.3
	8	10	2	11.0		3	14	0	5.3		4	11	0	9.0		3	16	0	4.0
	8	11	2	4.3		3	15	0	3.0		4	12	0	5.7		3	17	0	2.3
	9	9	2	10.3		3	16	0	1.5+		4	13	0	3.4		3	18	0	1.1
	9	10	2	3.7		3	17	1	15.9		4	14	0	1.9		3	19	1	13.3
						3	18	1	8.3		4	15	1	15.3		3	20	1	6.9
20	1	19	0	10.00		4	11	0	7.0		4	16	1	9.2		4	12	0	7.5−
	2	16	0	6.3		4	12	0	4.2		4	17	1	4.8		4	13	0	4.7
	2	17	0	3.2		4	13	0	2.3		4	18	1	2.0−		4	14	0	2.8
	2	18	0	1.1		4	14	0	1.2		5	9	0	9.8		4	15	0	1.6
	3	12	0	9.8		4	15	1	10.5+		5	10	0	6.0		4	16	1	13.4
	3	13	0	6.1		4	16	1	5.5+		5	11	0	3.5+		4	17	1	8.0
	3	14	0	3.5+		4	17	1	2.3		5	12	0	1.9		4	18	1	4.2
	3	15	0	1.8		5	9	0	7.8		5	13	1	13.4		4	19	1	1.7
	3	16	1	17.5+		5	10	0	4.5+		5	14	1	7.9		5	10	0	7.6
	3	17	1	9.1		5	11	0	2.5−		5	15	1	4.1		5	11	0	4.7
	4	10	0	8.7		5	12	0	1.2		5	16	1	1.9		5	12	0	2.7
	4	11	0	5.2		5	13	1	9.5−		5	17	2	11.0		5	13	0	1.5−
	4	12	0	2.9		5	14	1	5.0+		6	8	0	8.0		5	14	1	11.2
	4	13	0	1.4		5	15	1	2.3		6	9	0	4.6		5	15	1	6.6
	4	14	1	12.2		5	16	2	12.6		6	10	0	2.5−		5	16	1	3.5
	4	15	1	6.4		6	8	0	6.3		6	11	0	1.2		5	17	1	1.6
	4	16	1	2.7		6	9	0	3.4		6	12	1	8.7		5	18	2	9.6
	5	9	0	6.0		6	10	0	1.7		6	13	1	4.6		6	8	0	9.9
	5	10	0	3.3		6	11	1	11.0		6	14	1	2.2		6	9	0	5.9
	5	11	0	1.6		6	12	1	5.9		6	15	2	10.7		6	10	0	3.4
	5	12	1	11.6		6	13	1	2.8		6	16	2	5.1		6	11	0	1.8
	5	13	1	6.1		6	14	1	1.1		7	7	0	7.5+		6	12	1	11.9
	5	14	1	2.8		6	15	2	6.1		7	8	0	4.0		6	13	1	6.9
	5	15	2	14.5+		7	7	0	5.9		7	9	0	2.0+		6	14	1	3.7
	6	7	0	8.9		7	8	0	3.0		7	10	1	11.8		6	15	1	1.7
	6	8	0	4.8		7	9	0	1.4		7	11	1	6.3		6	16	2	9.0
	6	9	0	2.4		7	10	1	8.5+		7	12	1	3.1		6	17	2	4.2
	6	10	0	1.1		7	11	1	4.2		7	13	1	1.3		7	7	0	9.3
	6	11	1	7.6		7	12	1	1.8		7	14	2	6.4		7	8	0	5.2
	6	12	1	3.6		7	13	2	8.2		7	15	2	2.7		7	9	0	2.8
	6	13	1	1.4		7	14	2	3.5−		8	8	0	1.9		7	10	0	1.4
	6	14	2	7.5−		8	8	0	1.3		8	9	1	10.5−		7	11	1	8.9
	7	7	0	4.4		8	9	1	7.5−		8	10	1	5.3		7	12	1	4.8
	7	8	0	2.0+		8	10	1	3.4		8	11	1	2.4		7	13	1	2.3
	7	9	1	11.6		8	11	1	1.3		8	12	2	9.6		7	14	2	10.3
	7	10	1	5.7		8	12	2	5.9		8	13	2	4.4		7	15	2	5.1
	7	11	1	2.5−		8	13	2	2.2		8	14	2	1.7		7	16	2	2.1
	7	12	2	10.4		9	9	1	3.2		9	9	1	4.9		8	8	0	2.6
	7	13	2	4.5−		9	10	1	1.2		9	10	1	2.1					
	8	8	1	10.8		9	11	2	4.8										

Continued overleaf

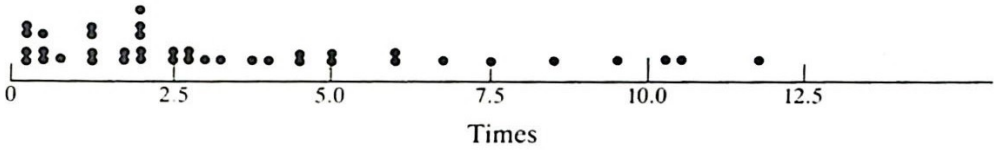
Table Z *Continued*

N	m	M	C	α(%)	N	m	M	C	α(%)	N	m	M	C	α(%)	N	m	M	C	α(%)
23	8	9	0	1.2	24	5	15	1	9.5−	24	11	11	2	3.4	25	6	16	1	2.4
	8	10	1	7.5+		5	16	1	5.5+		11	12	2	1.2		6	17	1	1.1
	8	11	1	3.8		5	17	1	2.9		11	12	3	10.0−		6	18	2	6.5−
	8	12	1	1.7		5	18	1	1.3		11	13	3	4.1		6	19	2	3.0
	8	13	2	7.3		5	19	2	8.5−		12	12	3	3.9		7	8	0	8.1
	8	14	2	3.3		6	9	0	7.4							7	9	0	4.8
	8	15	2	1.2		6	10	0	4.5−	25	1	24	0	8.0		7	10	0	2.7
	9	9	1	7.1		6	11	0	2.5+		2	19	0	10.00		7	11	0	1.4
	9	10	1	3.3		6	12	0	1.4		2	20	0	6.7		7	12	1	9.3
	9	11	1	1.4		6	13	1	9.6		2	21	0	4.0		7	13	1	5.3
	9	12	2	5.8		6	14	1	5.6		2	22	0	2.00		7	14	1	2.8
	9	13	2	2.4		6	15	1	2.9		2	23	1	31.3		7	15	1	1.4
	9	14	3	8.3		6	16	1	1.4		3	16	0	7.3		7	16	2	6.9
	10	10	1	1.3		6	17	2	7.6		3	17	0	4.9		7	17	2	3.4
	10	11	2	5.2		6	18	2	3.6		3	18	0	3.0		7	18	2	1.4
	10	12	2	2.0+		7	8	0	6.6		3	19	0	1.7		8	8	0	4.5−
	10	13	3	6.6		7	9	0	3.7		3	20	1	18.3		8	9	0	2.4
	11	11	2	1.9		7	10	0	2.0−		3	21	1	11.3		8	10	0	1.2
	11	12	3	5.9		7	11	1	11.9		3	22	1	5.8		8	11	1	7.5+
						7	12	1	6.9		4	13	0	7.8		8	12	1	4.0
24	1	23	0	8.3		7	13	1	3.7		4	14	0	5.2		8	13	1	2.0−
	2	19	0	7.2		7	14	1	1.8		4	15	0	3.3		8	14	2	8.7
	2	20	0	4.3		7	15	2	8.4		4	16	0	2.0−		8	15	2	4.4
	2	21	0	2.2		7	16	2	4.1		4	17	0	1.1		8	16	2	2.0−
	2	22	1	32.6		7	17	2	1.7		4	18	1	10.5+		8	17	3	7.8
	3	15	0	8.3		8	8	0	3.5−		4	19	1	6.2		9	9	0	1.1
	3	16	0	5.5+		8	9	0	1.7		4	20	1	3.2		9	10	1	6.8
	3	17	0	3.5−		8	10	1	10.1		4	21	1	1.3		9	11	1	3.4
	3	18	0	2.0−		8	11	1	5.5−		5	11	0	7.5+		9	12	1	1.6
	3	19	1	19.8		8	12	1	2.7		5	12	0	4.8		9	13	2	6.7
	3	20	1	12.3		8	13	1	1.2		5	13	0	3.0		9	14	2	3.2
	3	21	1	6.3		8	14	2	5.7		5	14	0	1.7		9	15	2	1.3
	4	12	0	9.3		8	15	2	2.5+		5	15	1	12.8		9	16	3	5.0+
	4	13	0	6.2		8	16	3	9.5−		5	16	1	8.1		10	10	1	3.2
	4	14	0	4.0		9	9	1	9.6		5	17	1	4.7		10	11	1	1.4
	4	15	0	2.4		9	10	1	4.9		5	18	1	2.5−		10	12	2	5.7
	4	16	0	1.3		9	11	1	2.3		5	19	1	1.1		10	13	2	2.5+
	4	17	1	11.9		9	12	2	8.9		5	20	2	7.5+		10	14	3	8.3
	4	18	1	7.1		9	13	2	4.3		6	9	0	9.0		10	15	3	3.6
	4	19	1	3.7		9	14	2	1.8		6	10	0	5.7		11	11	2	5.5−
	4	20	1	1.5+		9	15	3	6.4		6	11	0	3.4		11	12	2	2.3
	5	10	0	9.4		10	10	1	2.1		6	12	0	1.9		11	13	3	7.2
	5	11	0	6.1		10	11	2	8.0		6	13	0	1.0+		11	14	3	2.9
	5	12	0	3.7		10	12	2	3.6		6	14	1	7.8		12	12	3	6.8
	5	13	0	2.2		10	13	2	1.4		6	15	1	4.5+		12	13	3	2.6
	5	14	0	1.2		10	14	3	4.9										

Outline Solutions to the Assignments

Chapter 3

1.
Dot diagram:



H_0: Median time for the new store is 1.2 min.
H_1: Median time for the new store is greater than 1.2 min.

Test: sign text.
Decision rule: at $\alpha_1=5\%$, reject H_0 in favour of H_1 if $S\leqslant 13$ ($n=37$).
$S=8$ and this is significant at $\alpha_1=\frac{1}{2}\%$.

2.
H_0: Population median level of sodium is 140 ppm.
H_1: Population median level of solution is not 140 ppm.

Test: Wilcoxon's signed-rank test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $T\leqslant 46$ ($n=19$).
$T=79$ and so H_0 cannot be rejected.

Chapter 4

1.
H_0: The mutations are in the ratio 27 : 9 : 9 : 9 : 3 : 3 : 3 : 1.
H_1: The mutations are not in the ratio 27 : 9 : 9 : 9 : 3 : 3 : 3 : 1.

Test: chi-squared goodness-of-fit test.
Decision rule: at $\alpha=5\%$, reject H_0 in favour of H_1 if $\chi^2\geqslant 14.067$ ($\nu=7$).
$\chi^2=33.225$, which is significant at $\alpha=1\%$, examining the differences shows that the theory breaks down with the 'long, red and ebony' mutation.

2.

H_0: The arrival times are normally distributed.
H_1: The arrival times are not normally distributed.

Test: Lilliefors' test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $D_{40}\geqslant 0.1305$.
$D_{40}=0.0994$ and so H_0 cannot be rejected.

Chapter 5

1.

H_0: The median VC is the same for the two populations.
H_1: The median VC is different for the two populations.

Test: Mann–Whitney test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $U\leqslant 8$ ($n_1=n_2=7$).
$U=15.5$ and so H_0 cannot be rejected.

H_0: The median FEV is the same for the two populations.
H_1: The median FEV is different for the two populations.

Test: Mann–Whitney test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $U\leqslant 8$ ($n_1=n_2=7$).
$U=19$ and so H_0 cannot be rejected.

H_0: The median TLC is the same for the two populations.
H_1: The median TLC is different for the two populations.

Test: Mann–Whitney test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $U\leqslant 8$ ($n_1=n_2=7$).
$U=24$ and so H_0 cannot be rejected.

H_0: The median IGV is the same for the two populations.
H_1: The median IGV is different for the two populations.

Test: Mann–Whitney test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $U\leqslant 8$ ($n_2=n2=7$).
$U=24$ and so H_0 cannot be rejected.

2.

H_0: On average, there is no difference in results from the machines.
H_1: On average, the results from the two machines are different.

Test: Tukey's location test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $Ty\geqslant 7$.

$Ty=8$ and so H_0 is rejected in favour of H_1.
Test: Mann–Whitney test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $U\leqslant 119$.
$U=90$ and so H_0 is rejected in favour of H_1; in fact the result is significant at $\alpha_2=1\%$ and so with this test (not surprisingly) we have strong evidence to support H_1.

3.

H_0: There is no difference between the average heights of the populations of A and B sects.
H_1: There is some difference between the average heights of the populations of A and B sects.

Test: Mann–Whitney test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $U\leqslant 26$ ($n_1=9$, $n_2=12$).
$U=32$ and so H_0 cannot be rejected, i.e. there is no evidence to support H_1.

Chapter 6

1.

H_0: There is no difference in spread between the clay and silt contents.
H_1: There is some difference in spread between the clay and silt contents.

Test: Siegel–Tukey test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $U\leqslant 30$ ($n_1=n_2=11$).
Site 1: $U=45$ and so H_0 cannot be rejected.
Site 2: $U=28$, which is significant at $\alpha_2=5\%$ but not at $2\frac{1}{2}\%$ and so there is some evidence to support H_1 at this site.
Site 3: $U=37$ and so H_0 cannot be rejected.
Site 4: $U=48.5$ and so H_0 cannot be rejected.
Site 5: $U=52$ and so H_0 cannot be rejected.
Site 6: $U=45$ and so H_0 cannot be rejected.
Site 7: $U=60$ and so H_0 cannot be rejected.
Site 8: $U=55$ and so H_0 cannot be rejected.

2.

H_0: Each method produces the same dispersion.
H_1: The methods produce different amounts of spread (i.e. one is more erratic than the other).

Test: Tukey–type test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $Ts \leqslant 6$ ($n_1=n_2=10$).
$Ts=8$ and so H_0 cannot be rejected.

Chapter 7

1.

H_0: There is no difference between the clay and silt contents.

H_1: There is some general difference between the clay and silt contents.

Test: Kolmogorov–Smirnov test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $D^* \geqslant 77$ ($n_1=n_2=11$).
Site 1: $D^*=77$ and so H_0 is rejected in favour of H_1.
Site 2: $D^*=88$, which is significant at $\alpha_2=1\%$ and so there is strong evidence to support H_1 at this site.
Site 3: $D^*=88$, which is significant at $\alpha_2=1\%$ and so there is strong evidence to support H_1 at this site.
Site 4: $D^*=55$ and so H_0 cannot be rejected.
Site 5: $D^*=77$ and so H_0 is rejected in favour of H_1.
Site 6: $D^*=99$, which is significant at $\alpha_2=1\%$ and so there is strong evidence to support H_1 at this site.
Site 7: $D^*=55$ and so H_0 cannot be rejected.
Site 8: $D^*=88$, which is significant at $\alpha_2=1\%$ and so there is strong evidence to support H_1 at this site.

2.

H_0: The distributions of goals for England and Scotland are the same.

H_1: The distributions are not the same.

Test: Kolmogorov–Smirnov test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $D^* \geqslant 2000$ ($n_1=n_2=100$).
$D^*=800$ and so H_0 cannot be rejected.

3.

Dot diagram:

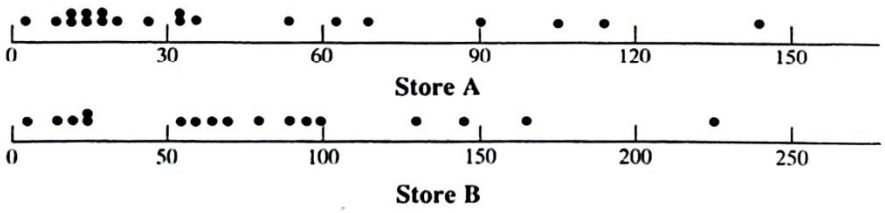


Exhibit 1006
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H_0: Both stores are as busy as each other.
H_1: One store is busier than the other.

Test: Rosenbaum's test.
Decision rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $R\geqslant 6$ ($n_1=17$, $n_2=20$).
$R=2\frac{1}{2}$ and so H_0 cannot be rejected.

Chapter 8

1.

H_0: The average gain in weight is the same for both schemes.
H_1: Scheme B produces, on average, greater gain in weight than scheme A.

Test: Wilcoxon signed–rank test.
Decision rule: at $\alpha_1=5\%$, reject H_0 in favour of H_1 if $T\leqslant 21$ ($n=13$).
$T=9\frac{1}{2}$ and so H_0 can be rejected; in fact the result is significant at $\alpha_1=1\%$, giving strong evidence in support of H_1.

2.

H_0: The average percentages of sand and silt are the same.
H_1: There is a difference between the average percentages of sand and silt.

Test: Wilcoxon signed-rank test.
Decision-rule: at $\alpha_2=5\%$, reject H_0 in favour of H_1 if $T\leqslant 13$ ($n=12$).
$T=26$ and so H_0 cannot be rejected.

Chapter 9

1.

H_0: There is no correlation between relative weight and insulin resistance.
H_1: There is positive correlation between relative weight and insulin resistance.

Test: Spearman's test.
Decision rule: at $\alpha_1=5\%$, reject H_0 in favour of H_1 if $r_S\geqslant 0.3805$ ($n=20$).
$r_S=0.3756$ and so H_0 cannot be rejected.

2.

H_0: There is no correlation between the number of awards and the tuberculosis death rate.

H_1: There is negative correlation between the number of awards and the tuberculosis death rate.

Test: Kendall's test.
Decision rule: at $\alpha_1=5\%$, reject H_0 in favour of H_1 if $\tau \leqslant -0.4182$ ($n=11$).
$\tau = -0.9636$, which very significant (more than $\frac{1}{2}\%$) and so there is strong evidence of negative correlation between the variables; but see note (d) on page 184.

Chapter 10

1.

Wind speed (mph)	Gradient	Intercept
5	1.056	−4.611
10	1.211	−21.278
15	1.333	−31.667
20	1.422	−38.889
25	1.489	−44.556
30	1.511	−48.444
35	1.556	−51.111
40	1.600	−53.000
45	1.567	−54.500
50	1.600	−55.000

2.
Rank regression equation is $r(Y) = 13.753 + 0.3603r(X)$.

3.
Rank regression equation is $r(Y) = 0.1154 + 0.984r(X)$.
(a) 67°C, (b) 78°C, (c) 134°C.

Chapter 11

1.
H_0: There is no correlation between blood pressure (X) and heart weight (Y).
H_1: There is positive correlation between X and Y.

Test: Olmstead–Tukey test.
Decision rule: at $\alpha_1=5\%$, reject H_0 in favour of H_1 if $C \geqslant 9$.
$C=29$, which is significant even at $\alpha_1=\frac{1}{2}\%$ and so there is strong evidence of positive correlation between X and Y.

$\tilde{X}_1=94.5$, $\tilde{Y}_1=385$, $\tilde{X}_2=131.5$, $\tilde{Y}_2=477.5$, giving a first approximate regression line of $Y=2.5X+148.75$.

Chapter 12

1.

H_0: Opinion on physical recreation is independent of region.
H_1: Opinion on physical recreation depends on the region.

Test: chi-squared test.
Decision rule: at $\alpha=5\%$, reject H_0 in favour of H_1 if $\chi^2 \geqslant 16.919$ (9 df).
$\chi^2=15.56$ and so H_0 cannot be rejected.

H_0: Opinion on leisure facilities is independent of region.
H_1: Opinion on leisure facilities depends on the region.

Test: chi-squared test.
Decision rule: at $\alpha=5\%$, reject H_0 in favour of H_1 if $\chi^2 \geqslant 16.919$ (9 df).
$\chi^2=14.05$ and so H_0 cannot be rejected.

H_0: Opinion on sport and health is independent of region.
H_1: Opinion on sport and health depends on the region.

Test: chi-squared test.
Decision rule: (because of low expected frequencies, rows 2 and 3 combined) at $\alpha=5\%$, reject H_0 in favour of H_1 if $\chi^2 \geqslant 7.815$ (3 df).
$\chi^2=7.5$ and so H_0 cannot be rejected.

Chapter 13

1.

H_0: The average copper concentrations is the same at the seven locations.
H_1: At least one location has an average concentration different from the rest.

Test: Kruskal–Wallis test.
Decision rule: at $\alpha=5\%$, reject H_0 in favour of H_1 if $H \geqslant 12.592$ (from chi-squared distribution with 6 df).
$H=9.657$ and so H_0 cannot be rejected.

2.

H_0: The average increase in weight is the same for the three brands.
H_1: At least one brand has an average different from the rest.

Test: Kruskal–Wallis test.
Decision rule: at $\alpha=5\%$, reject H_0 in favour of H_1 if $H \geq 5.991$ (from chi-squared with 2 df).
$H=4.7965$ and so H_0 cannot be rejected.

or
Test: median test.
Decision rule: at $\alpha=5\%$, reject H_0 in favour of H_1 if $\alpha_2 \geq 5.991$ (2 df).
$\chi^2=3.8$ and so H_0 cannot be rejected.

Chapter 14

1.

H_0: There is no difference in average pulse readings between the patients.
H_1: At least one patient has an average pulse different from the rest.

Test: Friedman's test.
Decision rule: at $\alpha=5\%$, reject H_0 in favour of H_1 if $M \geq 6.500$ ($k=3$, $n=4$).
$M=3.375$ and so H_0 cannot be rejected.

H_0: There is no difference in average pulse readings between the times.
H_1: At least one time has an average pulse different from the rest.

Test: Friedman's test.
Decision rule: at $\alpha=5\%$, reject H_0 in favour of H_1 if $M \geq 7.400$ ($k=4$, $n=3$).
$M=8.5$, which is significant at $\alpha=5\%$ but not at 1%, and so there is some, but not strong, evidence to support H_1. Examining the ranked data shows that the average pulse is lowest at 0.600 h and greatest at 1800 h; there is probably no significant difference between times 1000 and 2200 h.

2.

H_0: The average amount of iron absorbed is the same for the four treatments.
H_1: At least one treatment gives an average amount of iron absorbed different from the rest.

Test: the match tests.
Decision rules: (1) at $\alpha=5\%$, reject H_0 in favour of H_1 if $M_1 \geq 39$ ($k=4$, $n=8$),
(2) at $\alpha=5\%$, reject H_0 in favour of H_1 if $M_2 \geq 58$ ($k=4$, $n=8$).

$M_1=51$ and $M_2=72$, both of which are significant at $\alpha=1\%$ and so there is strong evidence to support H_1. Dunn's test shows there to be a significant difference between EO and CN and between CO and CN.

3.

H_0: There are no differences in averages between the students.
H_1: At least one average is different from the rest.

Test: the match tests.
Decision rules: (a) at $\alpha=5\%$, reject H_0 in favour of H_1 if $M_1 \geq 12$ ($k=4$, $n=4$),
(2) at $\alpha=5\%$, reject H_0 in favour of H_1 if $M_2 \geq 15$ ($k=4$, $n=4$).
$M_1=8$ and $M_2=13$ and so using either test means that H_0 cannot be rejected.

H_0: There are no differences in averages between the specimens.
H_1: At least one average is different from the rest.

Test: the match tests.
Decision rules: (1) at $\alpha=5\%$, reject H_0 in favour of H_1 if $M_1 \geq 12$ ($k=4$, $n=4$),
(2) at $\alpha=5\%$, reject H_0 in favour of H_1 if $M_2 \geq 15$ ($k=4$, $n=4$).
$M_1=12$ and $M_2=15\frac{1}{2}$ and so both tests provide some evidence in support of H_1.

Chapter 15

1.

H_0: The average times to complete the jobs are the same for all light conditions.
H_1: The average time to complete the job increases with worsening light conditions.

Test: Tersptra–Jonckheere test.
Decision rule: at $\alpha=5\%$, reject H_0 in favour of H_1 if $W \leq 39$ (from the approximation).
$W=60$ and so H_0 cannot be rejected.

2.

(i)

H_0: The average amount of food eaten is the same for all deprivation levels.
H_1: At least one average amount is different from the rest.