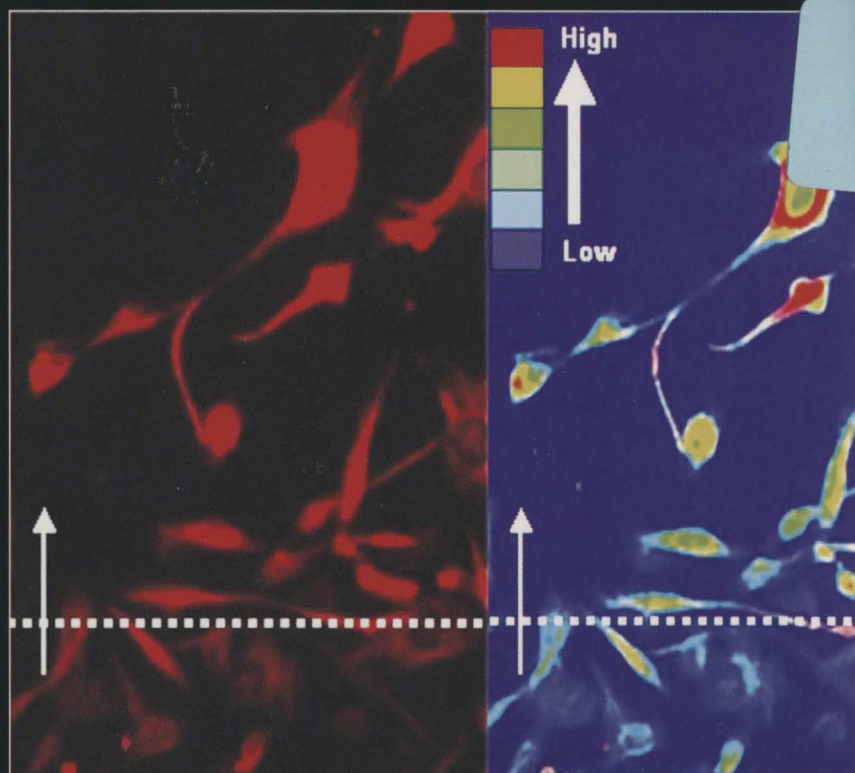


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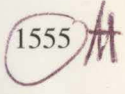
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Estimation of Time-Varying Coherence Function Using Time-Varying Transfer Functions

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Abstract—We introduce a new method to estimate reliable time-varying coherence functions (TVCF) for causal systems. The technique is based on our previously developed method to estimate time-varying transfer functions (TVTF), known as the time-varying optimal parameter search algorithm [Zou, R., H. Wang, and K. H. Chon. A robust time-varying identification algorithm using basis functions. *Ann. Biomed. Eng.* 31: 840–853, 2003]. The TVCF is estimated by the multiplication of two TVTFs. The two TVTFs are obtained using signal x as the input and signal y as the output to produce the first TVTF, and signal y as the input and signal x as the output to produce the second TVTF. Demonstration of the feasibility and efficacy of the proposed approach is provided with both simulation examples and application to renal blood flow and pressure data. The proposed approach provides higher time-frequency resolution TVCF than afforded by the short time Fourier transform based TVCF.

Keywords—Time-varying transfer function, Time-varying coherence function, ARMA, Time-varying, Renal autoregulation.

INTRODUCTION

Applications of the time-frequency distribution (TFD) have had a tremendous impact in understanding dynamic processes of various physiological systems. Physiological systems are inherently time varying (TV), thus, immediate acceptance of the TFD-based methods by the biomedical and biological communities was readily expected. Recently, new methods were introduced to compute time-varying transfer functions (TVTF) that have the potential to further advance the understanding of the dynamic processes underlying physiological systems.^{13,14} The characterization of physiological systems with the TVTF is important because the admittance gain between input and output signals is correctly characterized to be transient, and not stationary, as is assumed with the time-invariant transfer function.

In this paper, we utilize a parametric (model based, e.g., autoregressive moving average (ARMA) model) TVTF to estimate the TV coherence function (TVCF). Currently, the

estimation of the TVCF is mainly based on the use of the short-time Fourier transform (STFT) where the ratios between the magnitudes of the TV cross spectrum and the TV auto spectra of the two signals are calculated. Two disadvantages of the STFT are the assumption that time segments are stationary and the limited time-frequency resolution, which preclude the STFT being the method of choice in obtaining accurate and high resolution TVCF. Furthermore, the STFT suffers from high variance. Consequently, the likelihood of obtaining statistical significance of the coherence value is diminished. Recent separate works by Xu *et al.*,¹¹ and Lovette and Ropella⁸ have alleviated the high variance problem of the STFT by introducing multiple window time-frequency (TF) analysis. However, the inherent problem of limited resolution with the STFT was not improved by this approach, but in fact the variance reduction scheme had the undesired effect of further reducing the TF resolution.

As in the time-invariant case, high-resolution TF spectra, TVTF, and TVCF can be obtained via parametric approaches. Arnold *et al.*¹ have developed a parametric TV spectrum and TVCF by means of Kalman filtering. In their work, a parametric TV cross spectrum was obtained by reformulating an ARMA process into a vector AR model, from which the vector TV autospectrum between two signals were obtained. The accuracy of any time-invariant and TV parametric approaches greatly hinges on the proper choice of the model order. In the work by Arnold *et al.*,¹ determination of a model order was not discussed.

An accurate approach for estimating the TVCF is greatly needed, because even today many investigators still resort to time-invariant coherence function analysis.^{2,3,10} The coherence function is important because it provides information as to how two signals are phase-coupled or “coherent” with each other. For most physiological systems, it is expected that the coherence between two signals is not the same throughout the time duration of a data set. Thus, the TVCF is expected to reveal more accurate insights into the time-varying nature of the coupling between two signals. In this work, we propose a different parametric method to estimate TVCF than the approach proposed by Arnold *et al.*¹

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Our approach is based on the algorithm termed time-varying optimal parameter search (TVOPS), which we have recently developed for estimating TVTF.^{13,14} The TVOPS has been shown to be effective in selecting only the significant time-varying model terms, and is more accurate than the two most well-established model order criteria: the Akaike Information criterion (AIC) and the minimum description length (MDL).⁹

The proposed approach is presented in the Methods section. Illustrative simulation examples as well as the application of the proposed method to experimental data are presented in the Results section to demonstrate the feasibility and efficacy of the proposed approach.

METHODS

In this section, we demonstrate that the TVCF can be obtained by using the TVTF relationships. To demonstrate the use of the TVTF in obtaining the TVCF, we first define the TVCF via the nonparametric time-frequency spectra:

$$|\gamma(t, f)|^4 = \frac{|S_{xy}(t, f)|^2}{S_{xx}(t, f)S_{yy}(t, f)} \frac{|S_{yx}(t, f)|^2}{S_{yy}(t, f)S_{xx}(t, f)} \quad (1)$$

where $S_{xy}(t, f)$ and $S_{yx}(t, f)$ represent the time-frequency cross spectrum, and $S_{xx}(t, f)$ and $S_{yy}(t, f)$ denote the autospectra of the two signals x and y , respectively. The above expression $\frac{|S_{xy}(t, f)|^2}{S_{xx}(t, f)S_{yy}(t, f)}$ is the coherence function when x is considered as the input and y as the output while $\frac{|S_{yx}(t, f)|^2}{S_{yy}(t, f)S_{xx}(t, f)}$ is the coherence function when y is considered as the input and x as the output. We note that for a linear time-varying system with x and y as the input and output signals, respectively, the following TVTF in terms of time-frequency spectra can be obtained:

$$H_{x \rightarrow y}(t, f) = \frac{S_{xy}(t, f)}{S_{xx}(t, f)} \quad (2)$$

where $H_{x \rightarrow y}(t, f)$ denotes the TVTF from the input x to the output y signals. Similarly, if we reversed the input and output relationship such that the variables y and x represent input and output signals, respectively, as Fig. 1 depicts, then the following TVTF can be obtained:

$$H_{y \rightarrow x}(t, f) = \frac{S_{yx}(t, f)}{S_{yy}(t, f)} \quad (3)$$

The desired relationship of Eq. (1) can be obtained by multiplying the two TVTF relationships of Eqs. (2) and (3), which yields:

$$\begin{aligned} & |H_{x \rightarrow y}(t, f)H_{y \rightarrow x}(t, f)|^2 \\ &= \left| \left(\frac{S_{xy}(t, f)}{S_{xx}(t, f)} \right) \left(\frac{S_{yx}(t, f)}{S_{yy}(t, f)} \right) \right| \left| \left(\frac{S_{yx}(t, f)}{S_{yy}(t, f)} \right) \left(\frac{S_{xy}(t, f)}{S_{xx}(t, f)} \right) \right| \\ &= \left[\frac{|S_{xy}(t, f)|^2}{S_{xx}(t, f)S_{yy}(t, f)} \right] \left[\frac{|S_{yx}(t, f)|^2}{S_{yy}(t, f)S_{xx}(t, f)} \right] \\ &= |\gamma(t, f)|^4 \end{aligned} \quad (4)$$

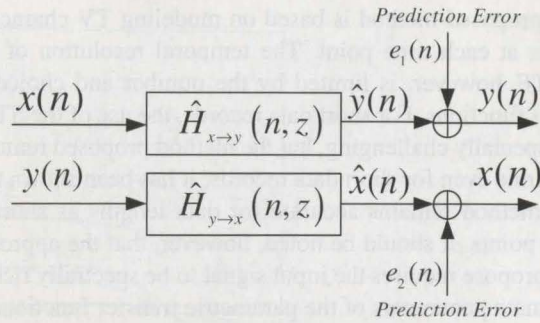


FIGURE 1. Block diagram of two procedures involved in calculation of time-varying coherence function.

Thus, time-varying magnitude squared coherence, $|\gamma(t, f)|^2$, is then obtained by multiplying the two transfer functions, $|H_{x \rightarrow y}(t, f)H_{y \rightarrow x}(t, f)|$, together.

Given the relationship of Eq. (4), we can obtain a high resolution TVCF via the parametric TVTF. Specifically, each of the two transfer functions in Eq. (4) can be obtained using autoregressive moving average (ARMA) models:

$$\begin{aligned} y(n) &= -\sum_{i=1}^{P_1} a(n, i)y(n-i) + \sum_{j=0}^{Q_1} b(n, j)x(n-j) \\ x(n) &= -\sum_{i=1}^{P_2} \alpha(n, i)x(n-i) + \sum_{j=0}^{Q_2} \beta(n, j)y(n-j) \end{aligned} \quad (5)$$

where the top expression in Eq. (5) represents $y(n)$ and $x(n)$ as the output and input, respectively, and the reverse for the bottom expression of Eq. (5). Our definition of the ARMA model refers to two measured signals of $y(n)$ and $x(n)$. Given the ARMA models of Eq. (5), the two transfer functions of Eq. (4) can be obtained by the following:

$$\begin{aligned} H_{x \rightarrow y}(n, e^{j\omega}) &= \frac{B(n, e^{j\omega})}{A(n, e^{j\omega})} = \frac{\sum_{l=0}^{Q_1} b(n, l)e^{-j\omega l}}{1 + \sum_{i=1}^{P_1} a(n, i)e^{-j\omega i}} \\ H_{y \rightarrow x}(n, e^{j\omega}) &= \frac{\beta(n, e^{j\omega})}{\alpha(n, e^{j\omega})} = \frac{\sum_{l=0}^{Q_2} \beta(n, l)e^{-j\omega l}}{1 + \sum_{i=1}^{P_2} \alpha(n, i)e^{-j\omega i}} \end{aligned} \quad (6)$$

In this paper, we obtain an accurate and high resolution TVTF of Eq. (6) by the previously-developed approach, termed the TVOPS.^{13,14}

In two references^{13,14}, it was demonstrated that more accurate estimate of the TVTF can be obtained than of the STFT, including an example concerning TV amplitude modulated signals. Furthermore, because the method proposed does not utilize segmentation of the data records, as is the case with the STFT, neither time nor frequency resolutions are compromised. Note that for the proposed approach, the TVTF, and consequently the TVCF, are obtained for every time point and are based on a few parameters chosen by the TVOPS from the initially-selected model orders P_1, Q_1, P_2 and Q_2 in Eq. (6). Therefore, while the STFT assumes that each segment of data is stationary,

the proposed method is based on modeling TV characteristics at each time point. The temporal resolution of the TVTF, however, is limited by the number and choice of basis functions. For short data records, the use of the STFT is especially challenging, but the method proposed remains accurate even for short data records; it has been shown that the method remains accurate for data lengths as short as 500 points. It should be noted, however, that the approach we propose requires the input signal to be spectrally rich to estimate parameters of the parametric transfer functions.

An abbreviated version of the algorithm for estimating TVTF via the TVOPS is outlined in the Appendix section. For full details of the TVOPS algorithm, the reader is referred to.^{13,14}

In the proceeding section, we show that theoretical values of TVCF are bounded in the interval of zero to one. To illustrate, consider two signals modeled by the following equations:

$$\begin{aligned} y(n) &= \hat{h}_{x \rightarrow y}(n) * x(n) + e_1(n) \\ x(n) &= \hat{h}_{y \rightarrow x}(n) * y(n) + e_2(n) \end{aligned} \quad (7)$$

Here, $e_1(n)$ and $e_2(n)$ are prediction error terms that are uncorrelated to $y(n)$ and $x(n)$, respectively, and $*$ denotes the convolution operator. Taking the Fourier transform on both sides of Eq. (7) yields:

$$\begin{aligned} S_{yy} &= |\hat{H}_{x \rightarrow y}|^2 S_{xx} + S_{e_1} = |\hat{H}_{x \rightarrow y}|^2 S_{xx} + \sigma_{e_1}^2 \\ S_{xx} &= |\hat{H}_{y \rightarrow x}|^2 S_{yy} + S_{e_2} = |\hat{H}_{y \rightarrow x}|^2 S_{yy} + \sigma_{e_2}^2 \end{aligned} \quad (8)$$

Note that:

$$\begin{aligned} |S_{xy}| &= |\hat{H}_{x \rightarrow y} S_{xx}| \\ |S_{yx}| &= |\hat{H}_{y \rightarrow x} S_{yy}| \end{aligned} \quad (9)$$

Substituting the expressions in Eq. (8) and Eq. (9) into Eq. (1):

$$\begin{aligned} |\gamma(t, f)|^4 &= \frac{|S_{xy}|^2 |S_{yx}|^2}{S_{xx} S_{yy} S_{yy} S_{xx}} \\ &= \frac{|\hat{H}_{x \rightarrow y}|^2 S_{xx}^2}{S_{xx} (|\hat{H}_{x \rightarrow y}|^2 S_{xx} + \sigma_{e_1}^2)} \frac{|\hat{H}_{y \rightarrow x}|^2 S_{yy}^2}{S_{yy} (|\hat{H}_{y \rightarrow x}|^2 S_{yy} + \sigma_{e_2}^2)} \\ &= \frac{|\hat{H}_{x \rightarrow y}|^2 S_{xx}^2 |\hat{H}_{y \rightarrow x}|^2 S_{yy}^2}{(|\hat{H}_{x \rightarrow y}|^2 S_{xx}^2 + S_{xx} \sigma_{e_1}^2) (|\hat{H}_{y \rightarrow x}|^2 S_{yy}^2 + S_{yy} \sigma_{e_2}^2)} \\ &= \frac{|\hat{H}_{x \rightarrow y}|^2 S_{xx}^2 |\hat{H}_{y \rightarrow x}|^2 S_{yy}^2}{S_{xx}^2 S_{yy}^2 |\hat{H}_{x \rightarrow y}|^2 |\hat{H}_{y \rightarrow x}|^2 + S_{xx}^2 S_{yy} |\hat{H}_{x \rightarrow y}|^2 \sigma_{e_2}^2 + S_{yy}^2 S_{xx} |\hat{H}_{y \rightarrow x}|^2 \sigma_{e_1}^2 + S_{xx} S_{yy} \sigma_{e_1}^2 \sigma_{e_2}^2} \\ &= \frac{1}{1 + \frac{\sigma_{e_2}^2}{S_{yy} |\hat{H}_{y \rightarrow x}|^2} + \frac{\sigma_{e_1}^2}{S_{xx} |\hat{H}_{x \rightarrow y}|^2} + \frac{\sigma_{e_1}^2 \sigma_{e_2}^2}{S_{yy} |\hat{H}_{y \rightarrow x}|^2 S_{xx} |\hat{H}_{x \rightarrow y}|^2}} \\ &= \frac{1}{1 + \frac{1}{SNR_1} + \frac{1}{SNR_2} + \frac{1}{SNR_1 SNR_2}} < 1 \end{aligned} \quad (10)$$

For simplicity, the time and frequency arguments have been omitted in Eq. (10).

As Eq. (10) indicates, the value of $|\gamma(t, f)|^4$ is bounded by 1. Note that when signal-to-noise ratios SNR_1 and SNR_2 approach ∞ , $|\gamma(t, f)|^4$ tends to 1, and when SNR_1 and SNR_2 approach zero, $|\gamma(t, f)|^4$ tends to zero.

As shown above, theoretically $|\gamma(t, f)|^4$ is bounded by 1. However, when the estimated transfer functions have poles lying on the unit circle, the value of $|\gamma(t, f)|^4$ may be greater than one. Note that because the system is time-varying, the poles are also naturally time-varying. When a pole is initially stable but becomes unstable (or vice versa), it will cross the unit circle periphery, as it moves from interior to exterior or vice versa. If this occurs, the magnitude of the transfer function becomes infinite. As a consequence, $|\gamma(t, f)|^4$ will have a value greater than one. Under this circumstance, the coherence relationship between two signals is undefined because the system undergoes a transition from stable to unstable or vice versa.

Stability only places restriction on the poles and not the zeros. For the inverse system (one with system function $1/H(z)$), the poles become zeros and vice versa, thus, for the minimum phase system where both poles and zeros are inside the unit circle, the maximum TVCF values will be no larger than a unit value. Thus, in the event where the estimated transfer functions do not satisfy a minimum phase system, the TVCF will also have values greater than one.

The top panel of Fig. 2 shows an example of the time course of poles, which drift from instability into stability, and the resultant coherence function estimate is provided in the bottom panel of Fig. 2. During the first 390 s, the magnitudes of the three poles have values outside the unit circle and consequently, coherence values are greater than one during this time period, as shown in the bottom panel of Fig. 2(b). As the poles cross over from the unstable to stable condition (at the instant poles cross the unit circle), even higher coherence values at two distinct frequencies are observed at time points centered around 400 s.

It should be noted that, in certain circumstances, the method might yield greater than unit coherence values beyond the frequency band of interest. This situation may arise because signal dynamics are missing in these frequency bands, which can lead to erroneous coherence values. However, this is trivial and can be effectively ignored since

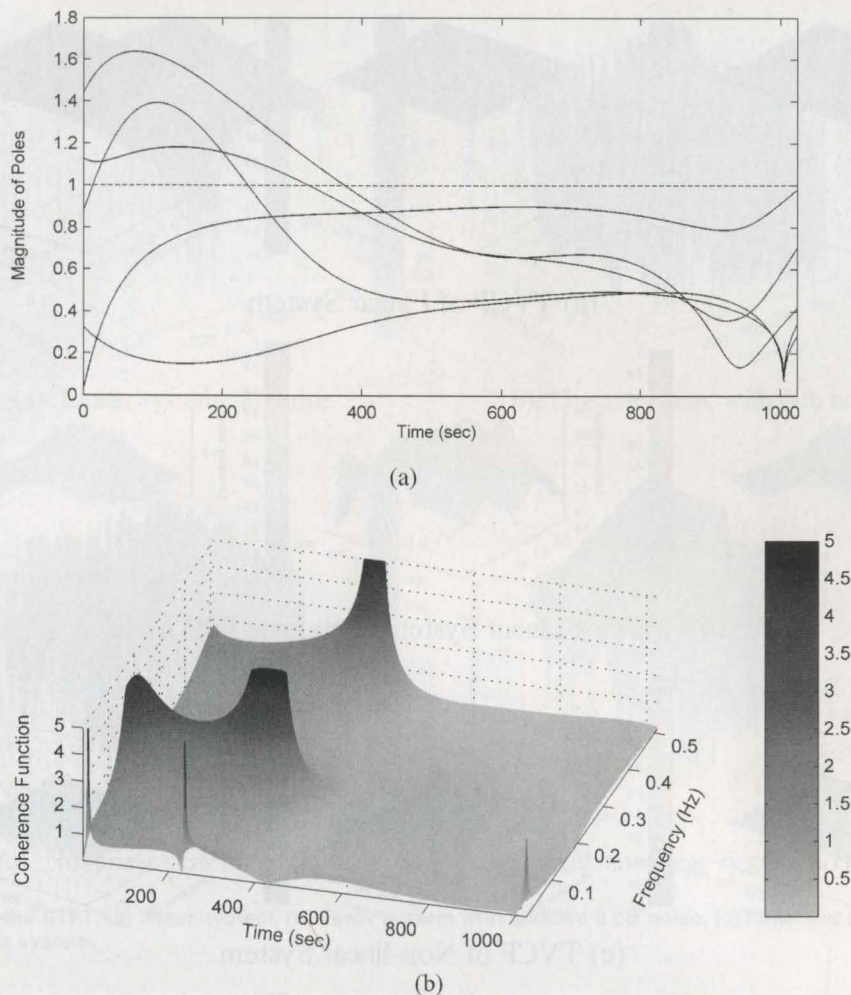


FIGURE 2. Poles crossing over the unit circle introduces high values in the estimated coherence function. (a) Magnitude of poles of the estimated transfer functions (b) Estimated time-varying coherence function.

the coherence values are not in the frequency range of interest.

RESULTS

Simulation examples are used to demonstrate both the feasibility and efficacy of the proposed method for obtaining the TVCF. In particular, the expected decrease of TV coherence values with noise and nonlinearity are demonstrated with 100 Monte Carlo realizations. The plots shown in Figs. 3–5 are results of one single realization, while the averaged results are reported in Table 1. With the use of the previously-developed TVOPS algorithm to compute the TVTF, three parameters that need to be preset are the number of Legendre functions, the model order and the threshold value. Based on these three preset parameters, TVOPS determines only those model orders that are deemed to be significant among initial candidate model terms. Details regarding this procedure are provided in the Appendix section as well as in references.^{13,14} For

both simulation examples and application of the method to the experimental data, we used the Legendre basis functions based on our *a priori* knowledge that underlying dynamics do not change abruptly. For fast-changing dynamics, more appropriate basis functions to use would be the Walsh functions. The threshold value is used for the linear independent candidate term search, as this value is dependent on the signal-to-noise ratio as well as on whether the signal is colored or white. Since *a priori* knowledge of the aforementioned conditions is unknown, we normally set the threshold value to 0.0001 and 0.001 for clean and noise-corrupted signals, respectively. Thus, for simulation examples and experimental data analyses to follow, we set the number of Legendre functions to 5 and the threshold value to 0.0001 for a noiseless simulation example and 0.001 for those cases (including experimental data) that are contaminated by noise. To determine the proper number of basis functions, we have modified the Akaike Information criterion to:

$$AIC = N \log \sigma^2 + 2[p \cdot k + 1] \quad (11)$$

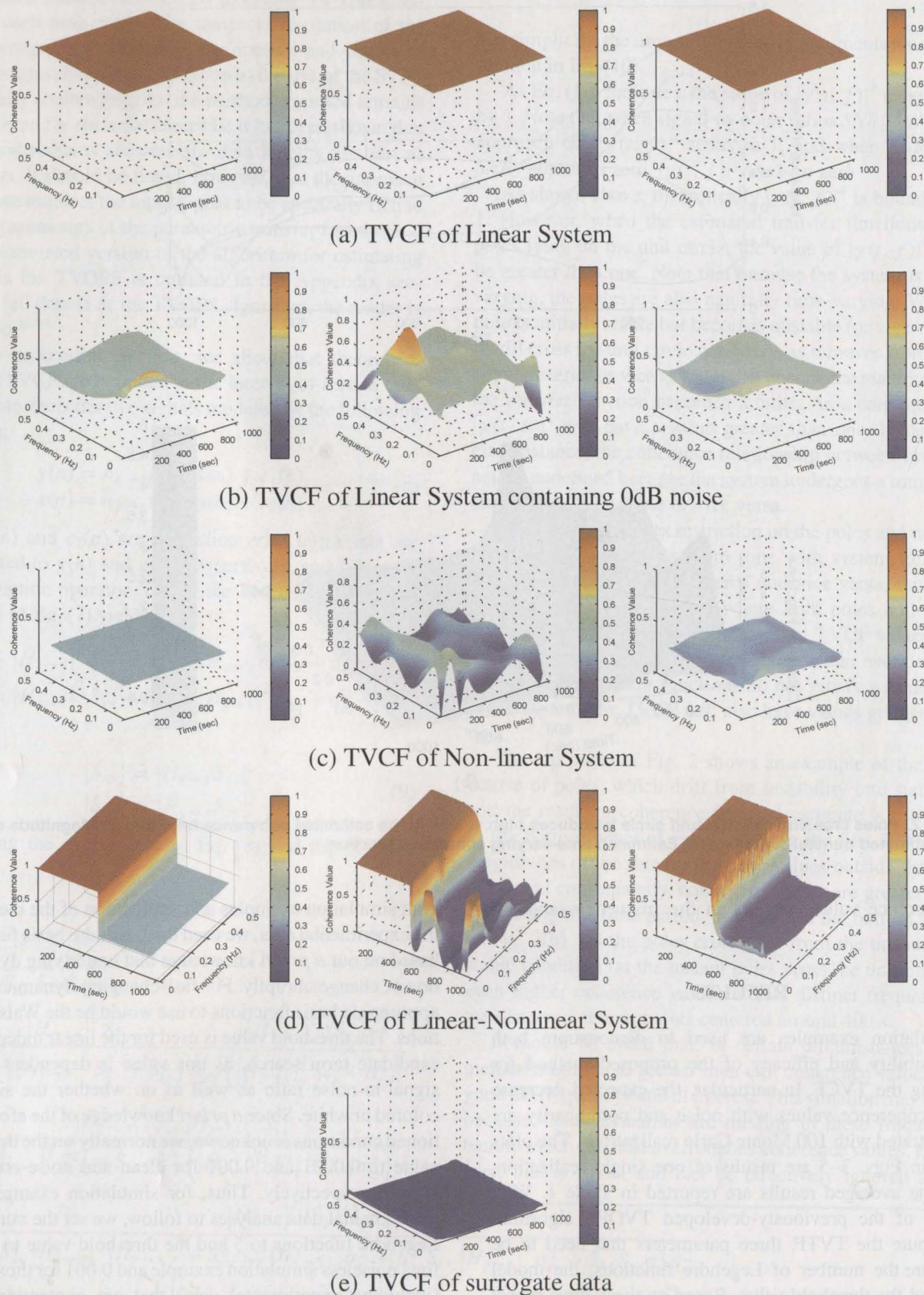


FIGURE 3. The left panels are the theoretical TVCF values; the middle panels are the TVCFs via TVOPS based on a single realization; and the right panels are the averaged TVCFs via TVOPS based on 100 realizations. (a) linear system, (b) linear system with additive 0 dB noise, (c) nonlinear system, (d) linear system followed by nonlinear system, (e) surrogate data of combined linear and nonlinear systems shown in (d).

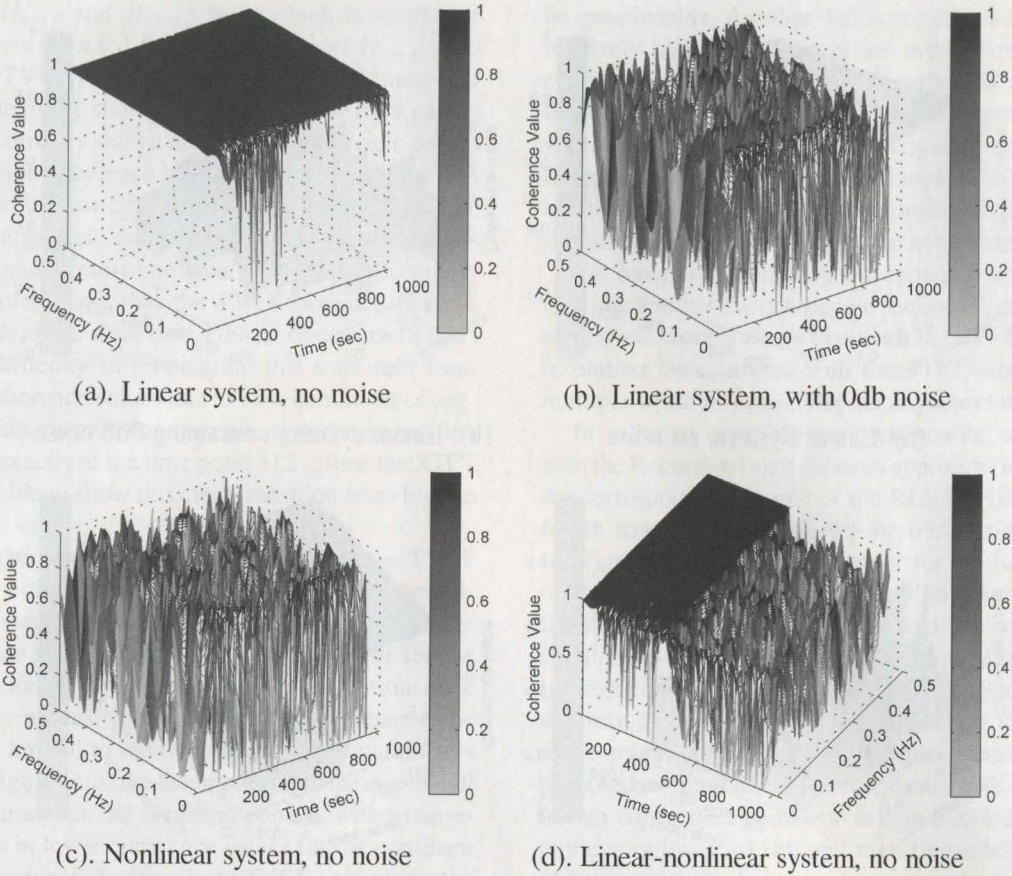


FIGURE 4. TVCF via the STFT: (a) linear system, (b) linear system with additive 0 dB noise, (c) nonlinear system, (d) linear system followed by nonlinear system.

where N is the data length, σ^2 is the prediction error, p is the number of significant terms (not to be confused with the maximum model order), k denotes the number of basis functions. The minimum value of the AIC is used as the criterion for determining the proper number of basis functions among the initially-chosen V sets of basis functions.

Simulation Examples

For the first simulation example, the following TV expression ($N = 1024$) was generated with a sampling rate of 1 Hz:

$$y(n) = a_1(n)y(n-1) + a_2(n)y(n-2) + x(n) \quad (12)$$

with

$$\begin{aligned} a_1(n) &= 0.3 \sin(2\pi n/N) + 0.2 \\ a_2(n) &= 0.09 \sin(2\pi n/N) + 0.15 \end{aligned} \quad (13)$$

where x and y are the input and output signals, respectively. TVOPS was employed to estimate the transfer functions (both $H_{x \rightarrow y}$ and $H_{y \rightarrow x}$) with an initial model order of ARMA(6, 6) from which only 4 (out of 13) and 3 (out of 13) significant model terms were selected by the TVOPS for the estimation of $H_{x \rightarrow y}$ and $H_{y \rightarrow x}$, respectively. For

the above expression, because it is linear, TV, and without noise corruption, we expect the TVCF to be of unit value for all times, as shown in the left panel of Fig. 3(a). Indeed, the TVCF values, shown in the middle (based on a single realization) and right (averaged over 100 realizations) panels of Fig. 3(a), have near unit values across all times and frequencies (see Table 1). For all plots in Fig. 3, the left, middle and right panels represent, in order, the theoretical values, a single realization estimated via the TVOPS, and the averaged estimates from applying the TVOPS to multiple realizations. The middle and right panels of Fig. 3(b) show the decrease of coherence values across all times and frequencies, as expected, when Eq. (12) is contaminated with 0 dB Gaussian White noise (GWN) that is time invariantly added to the output of Eq. (12). There is a good agreement with the estimated TVCF values and the theoretical value (0.44) shown in the left panel of Fig. 3(b). To further demonstrate another scenario of decrease in TVCF values, the following nonlinear expression was generated:

$$y(n) = a_1(n)y(n-1) + a_2(n)y(n-2) + x(n) + x^2(n) \quad (14)$$

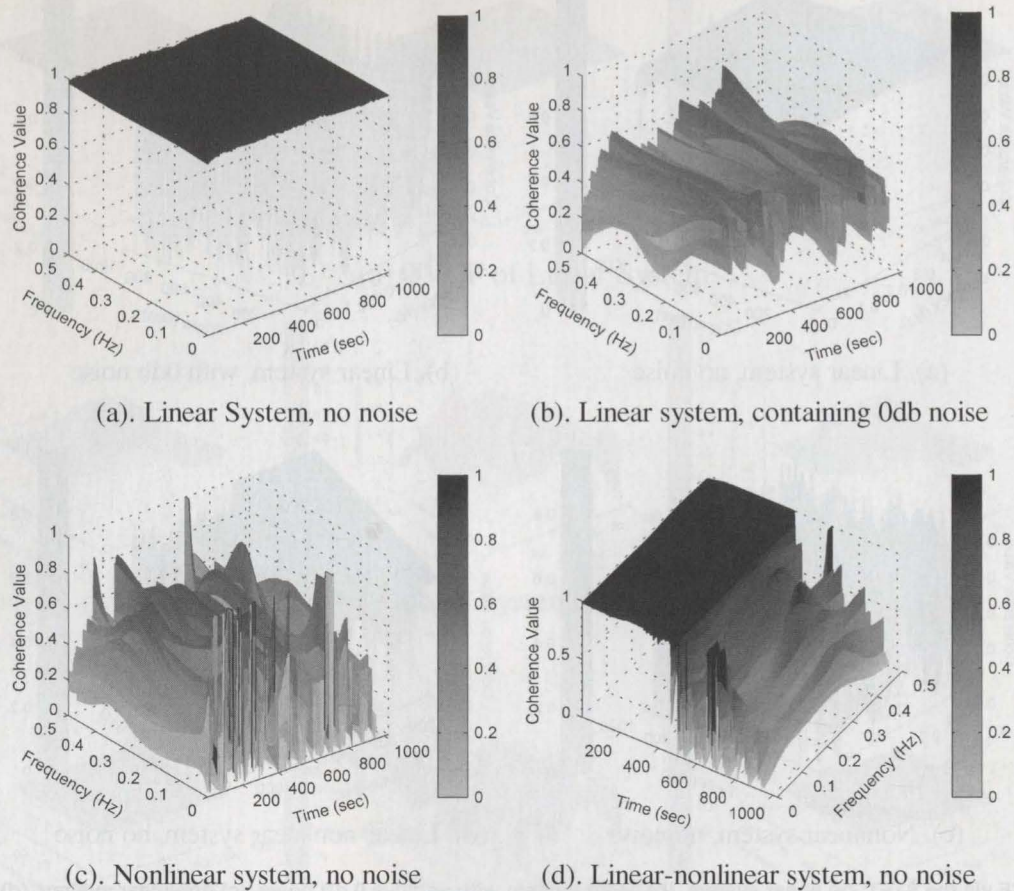


FIGURE 5. TVCF via the RLS: (a) linear system, (b) linear system with additive 0 dB noise, (c) nonlinear system, (d) linear system followed by nonlinear system.

The initial model order was set to ARMA(6, 6) (for both $H_{x \rightarrow y}$ and $H_{y \rightarrow x}$) from which a total of 4 and 6 significant model terms were selected for the estimation of $H_{x \rightarrow y}$ and $H_{y \rightarrow x}$, respectively. The linear terms in Eq. (14) are the same as shown in Eq. (12). Note in Fig. 3(c) the significantly lower coherence values than in the linear case, for all times and frequencies, due to the overall TV nonlinearity of the system response of Eq. (14). The estimated TVCF values

shown in the middle and right panels correspond well to the theoretically expected TVCF value (0.33) shown in the left panel of Fig. 3(c).

The next simulation example considers the time course of a noiseless TV linear and nonlinear system as described by Eqs. (12–14). The linear dynamic (Eqs. (12, 13)) occurs from 0 to 512 s and the nonlinear dynamic (Eq. (14)) occurs thereafter. The initial model order was set to ARMA(6,

TABLE 1. Mean and standard deviations of coherence function.

Methods	Cases				
	Linear	0 db noise	Nonlinear	Linear-nonlinear	
				$t = 1$ to 512	$t = 513$ to 1024
TVOPS	1.00 ± 0.01	0.44 ± 0.04	$0.32 \pm 0.01^*$	0.97 ± 0.03	$0.14 \pm 0.01^*$
STFT	0.99 ± 0.02	$0.62 \pm 0.07^*$	$0.59 \pm 0.05^*$	0.98 ± 0.07	$0.60 \pm 0.06^*$
RLS	0.99 ± 0.01	0.43 ± 0.02	$0.36 \pm 0.03^*$	0.97 ± 0.01	$0.39 \pm 0.03^*$

Note. No statistically significant differences were observed between the three methods under the conditions reported in columns 2 (linear, noise free) and 5 (before the onset of nonlinearity). Student-Newman-Keuls test was performed; * denotes $p < 0.05$.

6) (for both $H_{x \rightarrow y}$ and $H_{y \rightarrow x}$) from which 6 significant parameters were selected for both $H_{x \rightarrow y}$ and $H_{y \rightarrow x}$. The results of the TVCF for this combination of TV linear and nonlinear systems are shown in the middle and right panels of Fig. 3(d). Correctly shown in the middle and right panels of Fig. 3(d), high coherence values are observed from 0 to 512 s followed by low coherence values thereafter. Thus, the proposed approach, due to its high-resolution capability, is able to show transition from high coherence to low coherence values. Note that the TVCF obtained using a technique such as the short time Fourier transform (STFT) would have difficulty in pinpointing this transition from high to low coherence values due to its requirement of segmenting the data record. For example, if the segmentation did not occur exactly at the time point 512 s, then the STFT would not be able to show this clear transition from high to low coherence values.

For the middle and right panels of Fig. 3(d), the TVCF values for the nonlinear period are lower than the theoretical value, shown in the left panel in Fig. 3(d). This lower coherence value is due to greater prediction error for the nonlinear portion of the signal. Greater prediction error results because we restrict the estimated parameters to be linear and are limited to the 6 most significant parameters for the entire signal, thus, the linear portion of the signal will be better captured than the nonlinear portion, which consequently results in lower coherence values for the nonlinear portion of the signal. In Fig. 3(c), TVCF values via the TVOPS are very close to the theoretical value, since all the selected parameters were solely devoted to fitting nonlinear data. The RLS estimated TVCF values, shown in Fig. 5(d), are closer to the theoretical value because RLS uses far more parameters than does the TVOPS. For example, the RLS uses 6 parameters at each time instant, resulting in a total of 6144 parameters, whereas the TVOPS uses only 72 parameters for the entire data.

To further demonstrate the efficacy of the proposed method, we applied the method to surrogate data, which was generated from the input and output of Eqs. (12–14) (a linear and nonlinear system). The initial model order was set to ARMA(6, 6) (for both $H_{x \rightarrow y}$ and $H_{y \rightarrow x}$) from which 4 significant parameters were selected for both $H_{x \rightarrow y}$ and $H_{y \rightarrow x}$. Surrogate data technique destroys any linear and nonlinear correlations that may exist between the input and output data. Therefore we would expect low coherence values across all times. This is exactly what Fig. 3(e) shows.

Figures 4(a–d) show comparative results based on the STFT for the same simulations as described above with the proposed method. As compared to Figs. 3(a–d), the STFT has a very low resolution both in time and frequency domains. This is an unavoidable and well-documented limitation of the STFT. While these particular simulation examples involve 1024 data points, had there been an even smaller number of data points, the utility of the STFT would

be questionable. Another deficiency that is apparent is the relatively high coherence values even when the signal is contaminated with noise as well as when the system is nonlinear. The third row of Table 1 provides mean TVCF values via the STFT for all four cases considered. As compared to the second row of Table 1, coherence values are significantly higher with the two aforementioned cases, which should have resulted in low coherence values.

We have also compared the proposed approach to the multiple windows using Slepian sequences approach.¹¹ The results are slightly better than with the STFT, but a similar resolution limitation as with the STFT remains with the multiple windows using Slepian sequences method.

In order to compare the performance of our method with the Recursive Least Squares approach (RLS), we show the corresponding results of the RLS in Figs. 5(a–d). The fourth row of Table 1 shows the mean coherence values for Figs. 5(a–d). The model order for the RLS was based on the minimum description length followed by the coefficient estimates using the RLS, and the subsequent calculation of the TVTF and TVCF using Eq. (6) and Eq. (4), respectively. As the figures show, RLS also gives consistently better time-frequency resolution than the STFT while providing similar time-frequency resolution to the proposed method. It is interesting to note that the RLS has an edge effect similar to a high amplitude oscillation at the very low (~ 0 Hz) and high frequencies (~ 0.5 Hz), and they occur only when the system is nonlinear (Figs. 5(c) and (d)). The estimated parameters are unstable during the nonlinear case (not shown). The recursive nature of the RLS, which may lead to over-parameterization and unstable model parameters in nonlinear systems, can be the source of the high amplitude oscillations seen in Figs. 5(c) and (d).

Resolution Comparison

The following simulation example explores time and frequency resolution of the TVCF using comparison between the proposed method and the STFT. The simulation is based on a linear complex frequency modulation signal. For this case, we consider coherences between real and complex portions of the signal. A GWN signal was added to the real portion of the signal and another GWN signal, independent from the first, was added to the complex portion of the signal. In both cases, the resultant signal-to-noise ratio was equal to 20 dB. It is expected that high coherences should be observed only during linear increasing frequencies and time. Figs. 6(a) and (b) are the result of the proposed method and the STFT, respectively. It is apparent that only the proposed method is able to correctly discriminate high coherence values only during the linearly-increasing frequencies and time. Due to the limited time-frequency resolution of the STFT, it is not as accurate as the proposed method in delineating high coherence values

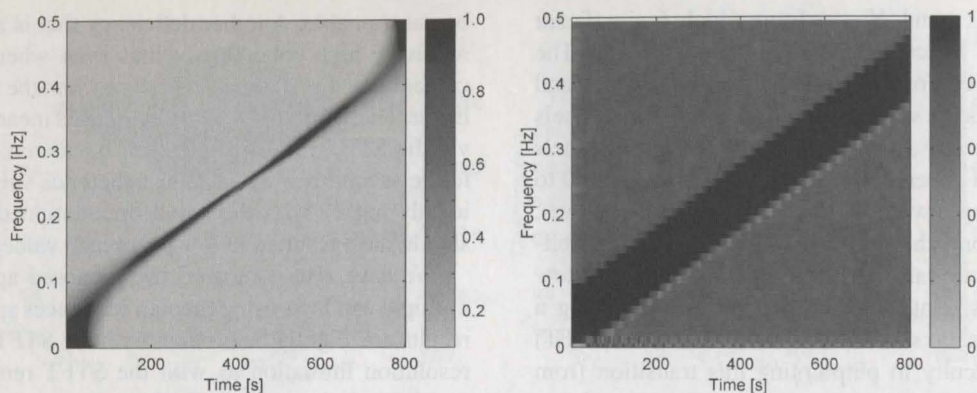


FIGURE 6. Comparison of TVOPS (left panel) and the STFT (right panel) for a complex linear chirp signal.

appearing only at frequencies that are linearly increasing with time.

Application of the Method to Renal System Data

The proposed method is applied to study the nonstationary features of the renal autoregulatory mechanisms and how these features might be different between time-invariant and TV correlation functions for normotensive rats. Renal autoregulation is the process in which fluctuations in renal blood flow caused by fluctuations in blood pressure are minimized. The myogenic mechanism and tubuloglomerular feedback (TGF) are the two mechanisms known to be responsible for renal autoregulation. The myogenic mechanism and TGF are shown to oscillate in the frequency range of 0.1–0.2 Hz and 0.03–0.05 Hz, respectively.^{5–7}

To estimate the TVCF of the renal autoregulatory system, the renal blood flow data were recorded under broadband forced arterial blood pressure fluctuations. The detailed experimental data collection procedures can be found in reference.⁷

Data analysis is based on 4 recordings from normotensive rats. Each of the experimental data records used for analysis was 256 s long, with a sampling rate of one sample per second, after digital low-pass filtering to avoid aliasing. Each data record, containing 256 data points, was subjected to second-degree polynomial trend removal (which included demeaning) and was normalized to unit variance. Representative time series of the blood pressure (top panel) and blood flow (bottom panel) are shown in Fig. 7.

The estimated TVCFs for the 4 normotensive rats are shown in the left panels of Fig. 8. The top right panels of Fig. 8 show a portion of the TV coherence values of the left panels from 0.03 to 0.05 Hz. These curves represent the frequency range where the TGF mechanism is known to operate and is shown for the readers' convenience. The bottom-most curves represent 0.03 Hz and the top-most curves represent 0.05 Hz. The bottom right panels represent the time-invariant coherence functions, shown for convenient comparison to the TVCFs.

For the myogenic mechanism (0.1–0.2 Hz), high coherence values are observed for all times for all four rats. Time invariant coherence functions also show high coherence

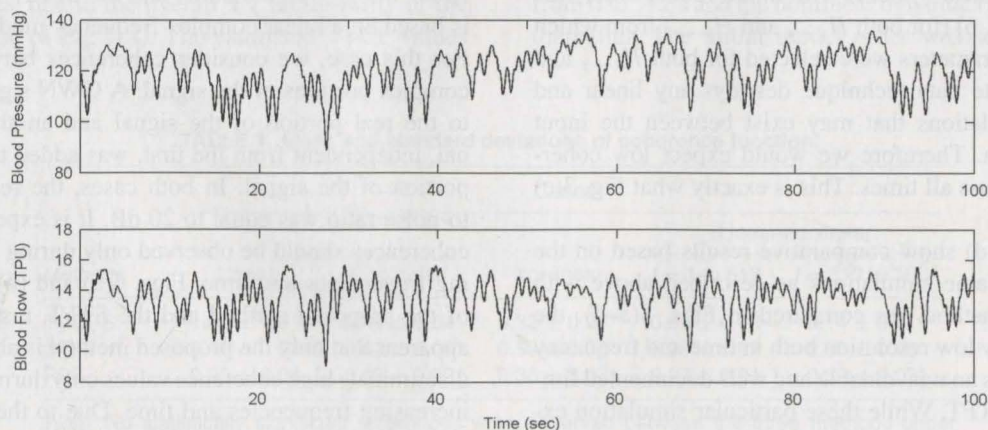
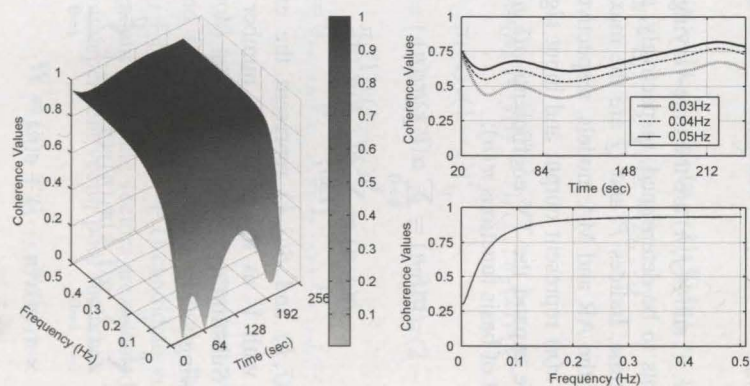
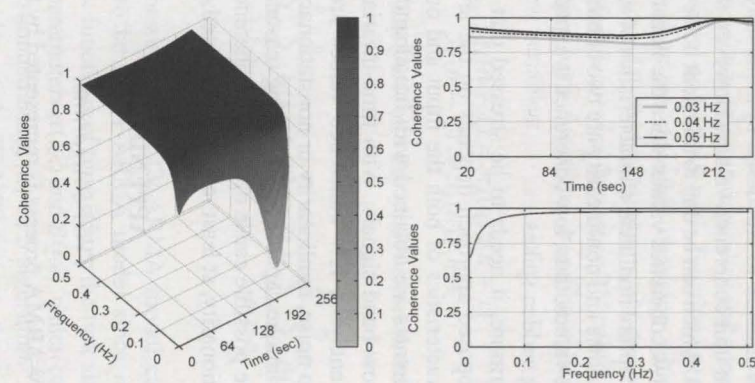


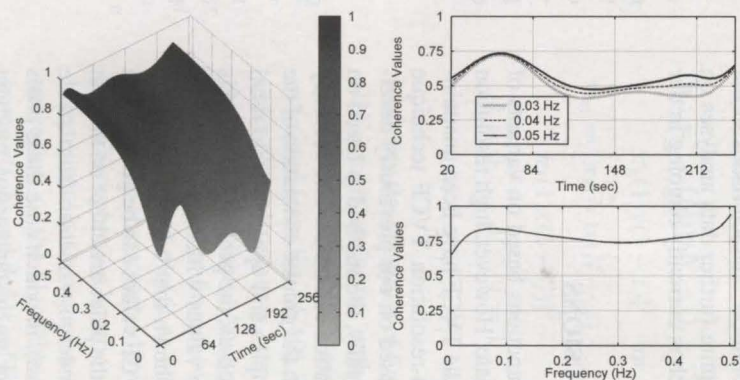
FIGURE 7. Representative blood pressure (top panel) and blood flow (bottom panel) data.



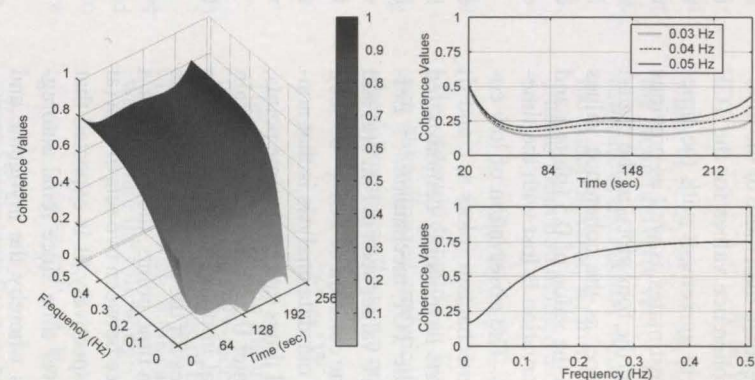
(a)



(b)



(c)



(d)

FIGURE 8. (a)–(d) TVCFs on renal blood flow and pressure data obtained from 4 rats. Top right panels show a portion of the TV coherence values of the left panels from 0.03 to 0.05 Hz. Bottom-right panels represent time-invariant coherence values.

values for all four rats at the myogenic mechanism's frequency range. The TGF, however, shows lower coherence values that are more time-dependent than those of the myogenic mechanism. The low coherence values for the TGF with the TVCF estimate are in agreement with the time-invariant coherence function estimates as well as previous other data analyses.⁵⁻⁷ Two of the four plots, in the region of TGF show time dependence, as the coherence values fluctuate above or lower than the value of 0.5 (plots a and c). Note that this type of observation is lost with the time-invariant coherence functions. The observation of low coherence values for the TGF mechanism (plots a, c and d) is expected, as previous reports have amply demonstrated nonlinear characteristics of the TGF mechanism.^{4,12} Furthermore, fluctuations of TVCF values seen in Fig. 8(a) and 8(c) are all greater than the time constant of TGF (~33 sec). The new insight gained from our data analysis is that nonlinearity does not occur for all times (at certain time points, TV coherence values are greater 0.5), but rather at certain time points. The physiological implication of the increase or decrease of coherence values for the renal blood pressure and flow data is that at certain time points, there is either a high or low degree of coherence between two sets of data (at certain frequency ranges), respectively. It is expected that coherence values fluctuate over time since renal autoregulation is a dynamic process whereby the myogenic and TGF mechanisms work in concert to maintain steady renal blood flow while blood pressure varies over a wide pressure range. For a complete understanding of the physiological basis of nonstationary and nonlinear characteristics of the TGF mechanism, we will require further data analyses on more experimental data, which is currently ongoing in our laboratory.

CONCLUSIONS

Time-varying coherence functions based on variants of the STFT are simple to calculate. However, high resolution and meaningful estimates of the TVCF have been unattainable. A simple and yet high-resolution TVCF technique has been introduced and is based on our previously developed TVTF estimate. Specifically, the method is based on multiplication of transfer function estimates. The accuracy of the TVCF, thus, depends on the precise estimation of the TVTF. Both simulation examples and application to renal data have shown that in most cases, the proposed method is reliable and yields coherence values within the bounds of 0 and 1. However, we have shown that with the proposed approach, in certain cases, TVCF values can be greater than 1. This scenario occurs when the poles are either on the unit circle periphery or as they undergo transition from stable to unstable poles (or vice versa), in which case these poles will lie on the unit circle for an instantaneous moment of time. Poles on the unit circle will result in coherence values greater than 1. In certain biological data, the afore-

mentioned scenario may occur. For example, signals that bifurcate from stable to unstable dynamics or bifurcation from deterministic to stochastic systems may all result in larger than unit coherence values. However, sudden spikes in the data, which can be due to transient noise glitches, will result in low coherence values since the linear TVTF is not suited to capture nonlinear characteristics of sudden spikes. Note that noise and nonlinearity are two contributors to low coherence values, thus, low coherence is expected with data exhibiting sudden spikes.

Furthermore, it needs to be stressed that the accuracy of the proposed approach depends on the level of broadband characteristics of both the input and output signals; better estimates are obtained with broadband signals than with narrow band signals. With time-invariant coherence function analysis, low coherence values are usually attributed to noise, nonlinearity or nonstationarity. However, with a technique capable of estimating reasonably accurate TVCF, the possible cause of the low coherence values due to nonstationarity, at least, can be eliminated.

APPENDIX

TVOPS Algorithm

The TV-ARMA process is represented by the following equation:

$$y(n) = \sum_{i=1}^P a(i, n)y(n-i) + \sum_{j=0}^Q b(j, n)x(n-j) + e(n) \quad (\text{A.1})$$

where $a(i, n)$ and $b(j, n)$ are the time-varying AR and MA coefficients to be determined, respectively, and are functions of time. Indices P and Q are the maximum model orders of the AR and MA models, respectively. Variables $y(n)$ and $x(n)$ represent output and input signals, respectively. We expand the TV coefficients $a(i, n)$ and $b(j, n)$ onto a set of basis functions $\pi(n)$:

$$\begin{aligned} a(i, n) &= \sum_{k=0}^V \alpha(i, k)\pi_k(n) \\ b(j, n) &= \sum_{k=0}^V \beta(j, k)\pi_k(n) \end{aligned} \quad (\text{A.2})$$

where $\alpha(i, k)$ and $\beta(j, k)$ represent the expansion parameters with V as the maximum number of basis sequences. Substituting Eq. (A.2) into Eq. (A.1), we obtain the following:

$$\begin{aligned} y(n) &= \sum_{i=1}^P \sum_{k=0}^V \alpha(i, k)\pi_k(n)y(n-i) + \sum_{j=0}^Q \sum_{k=0}^V \beta(j, k) \\ &\quad \times \pi_k(n)x(n-j) + e(n) \end{aligned} \quad (\text{A.3})$$

The next step is to select proper basis functions so that TV coefficients can be estimated. We have experimented

with three different types of basis functions: Legendre polynomial, discrete prolate spheroidal sequence (DPSS), and Walsh. Our experience has shown that different basis functions show their own unique tractability and accuracy. Legendre polynomial and DPSS basis functions perform well if the coefficients are smoothly changing with time, for example, sinusoidal. Walsh functions, on the other hand, behave well for piece-wise stationary TV coefficients.

Once proper basis functions, $\pi(n)$, have been chosen, we create new variables:

$$\begin{aligned} y_k(n-i) &= \pi_k(n)y(n-i) \\ x_k(n-j) &= \pi_k(n)x(n-j) \end{aligned} \quad (\text{A.4})$$

Substituting Eq. (A.4) into Eq. (A.3) results in the following expression:

$$\begin{aligned} y(n) &= \sum_{i=1}^P \sum_{k=0}^V \alpha(i, k) y_k(n-i) + \sum_{j=0}^Q \sum_{k=0}^V \beta(j, k) x_k \\ &\quad \times (n-j) + e(n) \end{aligned} \quad (\text{A.5})$$

Eq. (A.5) shows that the TV-ARMA model can now be considered to be a TIV-ARMA model since $\alpha(i, k)$ and $\beta(j, k)$ are not functions of time. Thus, the task simplifies to solving for parameters $\alpha(i, k)$ and $\beta(j, k)$, which can be more effectively estimated using the OPS algorithm.⁹

The algorithm of the TVOPS follows the similar two-step procedure of OPS. There are two main differences, however. The first is that we arrange the pool of candidate vectors in the following matrix form:

$$\begin{aligned} \mathbf{M} &= (\mathbf{Y}_0(n-1), \dots, \mathbf{Y}_V(n-1), \mathbf{X}_0(n), \dots, \mathbf{X}_V(n), \\ &\quad \mathbf{Y}_0(n-2), \dots, \mathbf{Y}_V(n-2), \dots) \end{aligned} \quad (\text{A.6})$$

where

$$\begin{aligned} \mathbf{Y}_k(n-i) &= [\pi_k(N)y(N-i), \dots, \pi_k(2)y(2-i), \\ &\quad \pi_k(1)y(1-i)]^T \\ \mathbf{X}_k(n-j) &= [\pi_k(N)x(N-j), \dots, \pi_k(2)x(2-j), \\ &\quad \pi_k(1)x(1-j)]^T \\ k &= 0, \dots, V; \quad i = 1, \dots, P; \quad j = 0, \dots, Q \end{aligned}$$

Note that bold and unbold letters represent vectors (or matrices) and scalars, respectively. Among the pool of $P \times (V+1) + (Q+1) \times (V+1)$ candidate vectors, linearly independent candidate vectors are selected to form a new linearly independent vector pool such that:

$$\mathbf{W} = (w_1, w_2, \dots, w_S)$$

where S is the number of linearly-independent candidate vectors.

With the new candidate pool of linearly-independent vectors, least-squares analysis is performed:

$$\mathbf{Y} = \mathbf{W}\Theta + \mathbf{E} \quad (\text{A.7})$$

where $\Theta = [\theta_1, \theta_2, \dots, \theta_S]^T$, $\mathbf{Y}$ is the signal vector and $\mathbf{E}$ is the prediction error vector. In Eq. (A.7), θ_i is the coefficient estimate of the ARMA model. The objective is to minimize the equation error, $\mathbf{E}$, in the least-squares sense using the criterion function defined as follows:

$$J_N(\Theta) = [\mathbf{Y} - \mathbf{W}\Theta]^T [\mathbf{Y} - \mathbf{W}\Theta] \quad (\text{A.8})$$

The criterion function in Eq. (A.8) is quadratic in Θ , and can be minimized analytically with respect to Θ , yielding the following well-known equation:

$$\hat{\Theta} = [\mathbf{W}^T \mathbf{W}]^{-1} \mathbf{W}^T \mathbf{Y} \quad (\text{A.9})$$

The next stage of the algorithm involves further pruning of insignificant linearly-independent candidate vectors obtained from the first stage of the algorithm (Eqs. (A.4–A.6)). To determine which of the candidate vectors are significant, we calculate the following:

$$C_m = \frac{1}{N} \hat{\theta}_m^2 \mathbf{w}_m^T \mathbf{w}_m \quad m = 1 \sim S \quad (\text{A.10})$$

where S represents the number of linearly-independent candidate vectors and $\hat{\theta}_m$ represents the coefficient obtained from Eq. (A.9). Note that the vector $\mathbf{w}_m$ is calculated using Eq. (A.11):

$$\begin{aligned} \mathbf{w}_m &= [\pi_k(N)y(N-i), \dots, \pi_k(2)y(2-i), \\ &\quad \pi_k(1)y(1-i)]^T \quad \text{or} \\ \mathbf{w}_m &= [\pi_k(N)x(N-j), \dots, \pi_k(2)x(2-j), \\ &\quad \pi_k(1)x(1-j)]^T \\ k &= 0, \dots, V; \quad i = 1, \dots, P; \quad j = 0, \dots, Q; \\ n &= 1, \dots, N; \quad m = 1, \dots, S \end{aligned} \quad (\text{A.11})$$

Note that the operation of Eq. (A.10) involves the adding of all C_m together for $\mathbf{w}_m$ with the same $y(n-i)$ or $x(n-j)$ but with different π_k . A magnitude value of each C_m , as estimated from Eq. (A.10), is then arranged in descending order. We only retain $\mathbf{w}_m$ terms that significantly reduce the estimation residuals. An approach that can be adopted to determine the significance of C_m terms is to calculate $(C_m - C_{m+1})/C_m$ and plot it as a function of m . The significant term can be characterized by the first maximum value in the plot. This operation has the added benefit of also discarding insignificant basis sequences, π_k , at the same time. The final procedure of the algorithm is to estimate ARMA model terms $\alpha(i, k)$ and $\beta(j, k)$ of Eq. (A.3) using the least-squares method and the calculation of TV coefficients $a(i, n)$ and $b(j, n)$ using Eq. (A.2).

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