

FORMATION FLYING SATELLITE CONTROL UTILIZING INPUT SHAPING

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The precise control of spacecraft with flexible appendages, such as solar panels or antennas, is extremely difficult. The complexity of this task is magnified many times when several of these spacecraft must be controlled in unison as in formation flying. Formation flying (FF) requires that individual spacecraft in the group maintain relative position and velocities with respect to each other. One of the difficulties with formation flying is that the satellites must be autonomous and able to generate and correct their own orbit and attitude profiles without continual guidance from the ground. The objective of the current work is to enhance the existing formation flying technologies by integrating input shaping techniques to reduce vibration in flexible appendages. Specifically, input shaping will be used with a virtual structure PD formation flying control algorithm to control motion

INTRODUCTION

Formation Flying Architectures

The precise control of spacecraft with flexible appendages, such as solar panels or antennas, is extremely difficult. The complexity of this task is magnified many times when several of these spacecraft must be controlled in unison as in formation flying. Formation flying (FF) requires that individual spacecraft in the group maintain relative position and velocities with respect to each other. One of the biggest difficulties with formation flying is that the satellites must be autonomous and able to generate and correct their own orbit and attitude profiles without continual guidance from the ground. There are many different ways to control the motion of the entire group of satellites. Some examples are Leader Following,¹ Bang-Bang,² linearized feedback coupled with model predictive control,³ and PD control.⁴ The control architectures can be divided into three main groups: centralized control, decentralized control, and virtual structure. The work

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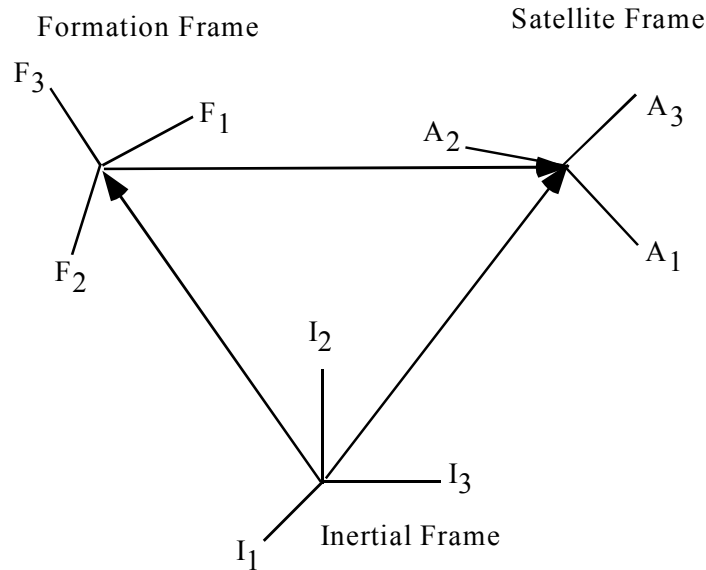


Figure 1: Coordinate Frame Geometry.

presented in this paper addresses a formation flying algorithm that contains aspects of each of these three categories.

Although many control algorithms model satellites as a point mass, more accurate models contain flexibility. The flexibility of the satellite can come from many sources with the most recognizable being solar panels and communications antennae. Many flexible satellites are positioned and oriented using on-off thruster jets. The on-off nature of the spacecraft's maneuvering and positioning thrusters increases the complexity of the controlling scheme. In addition, every time the thrusters fire, vibration is introduced into the flexible modes. Vibration induced by the reference command can be effectively addressed with the use of input shaping.⁵⁻⁹ Input shaping has been extended to both time-optimal and fuel-efficient commands and deflection-limiting commands.^{10, 11} In formation flying, however, the additional performance requirement of coordinated satellite motion becomes extremely important.

The objective of the current paper is to enhance the existing formation flying technologies by integrating input shaping techniques. Specifically, input shaping will be used with the coordination architecture proposed by Beard.¹² The architecture proposed by Beard contains elements of centralized control, decentralized control and virtual structures. The overall formation supervisor knows the position and attitude of each of the satellites in the constellation. Each individual satellite however, does not know the position and attitude of any other satellites. The overall formation is described using a virtual structure that is completely rigid. From this virtual structure, the desired locations and attitudes for each of the satellites can be computed. The coordinate geometry is shown in Figure 1, and is composed of the inertial, formation and relative satellite

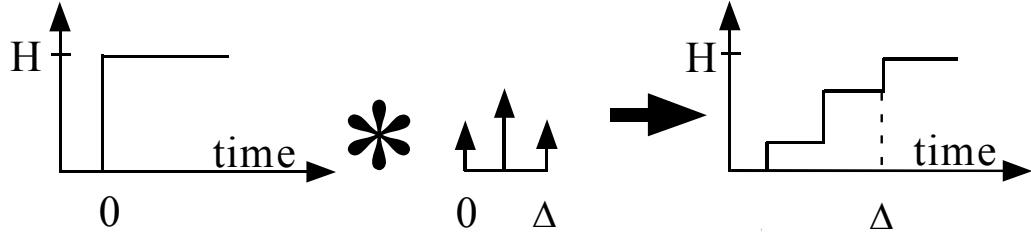


Figure 2: Input Shaping a Step Input.

frames. The formation states represent the virtual structure and are modeled as point masses.

Input Shaping

Input shaping is a command generation scheme that reduces vibration of flexible systems. Input shaping is implemented by convolving a sequence of impulses, called the input shaper, with any system command.⁵ The convolved signal is then used as the reference command. Figure 2 demonstrates the process with a step input and an input shaper that contains three positive impulses. The resulting shaped input is a three-step staircase command.

The amplitudes and time locations of the impulses in an input shaper are determined by satisfying constraint equations. The constraint on residual vibration amplitude can be expressed as the ratio of residual vibration amplitude with shaping to that without shaping. The percentage vibration can be determined by using the expression for residual vibration of a second-order harmonic oscillator of frequency ω and damping ratio ζ .¹³ The vibration from a series of impulses is divided by the vibration from a single impulse to get the percentage vibration:

$$V(\omega, \zeta) = e^{-\zeta \omega t_n} \sqrt{[C(\omega, \zeta)]^2 + [S(\omega, \zeta)]^2} \quad (1)$$

where,

$$C(\omega, \zeta) = \sum_{i=1}^n A_i e^{\zeta \omega t_i} \cos\left(\omega \sqrt{1 - \zeta^2} t_i\right) \quad (2)$$

and

$$S(\omega, \zeta) = \sum_{i=1}^n A_i e^{\zeta \omega t_i} \sin\left(\omega \sqrt{1 - \zeta^2} t_i\right) \quad (3)$$

If $V(\omega, \zeta)$ is set equal to zero at the modeling parameters, (ω_m, ζ_m) , then a shaper that satisfies the equation is called a Zero Vibration (ZV) shaper. Most of the constraints that are used are composed of residual vibration constraints; however, other constraints can be used in addition. These constraints can ensure time-optimality or fuel-efficiency.¹⁴

In order to generate an input shaper for a given system, only the system's natural frequencies and damping ratios are required. The simplest input shaper to construct and implement is the Zero Vibration (ZV) shaper.⁵ This simple shaper is capable of

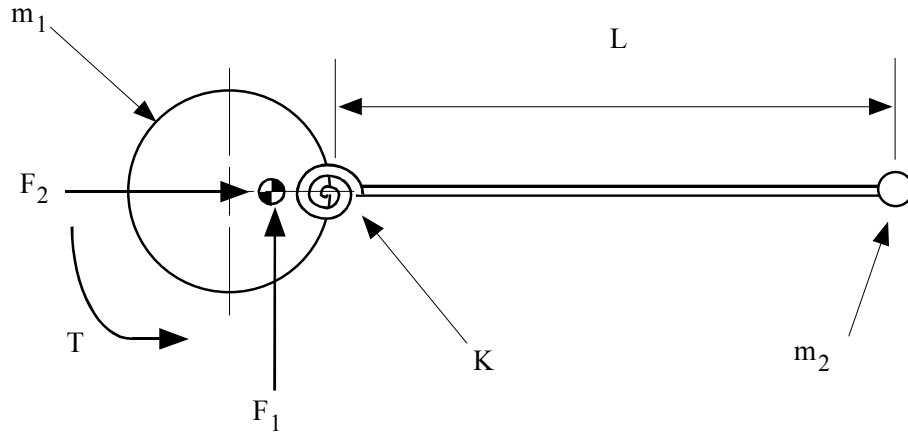


Figure 3: Flexible Satellite Model.

eliminating the residual vibration of one mode. However, the ZV shaper is also the least robust shaper available.¹⁵ Although there are many shapers with higher robustness to modeling errors, this research utilizes the ZV shaper because the focus is not on robustness, but rather on combining Input Shaping and Formation Flying.

SPACECRAFT DYNAMICS

Flexible Model

The satellites studied here are modeled as two mass-spring systems. This represents a rigid satellite with a single flexible appendage. The flexible appendage can be used to represent a variety of structures such as an antennae or solar panel. The two mass-spring system representing the satellites is shown in Figure 3. As seen in the figure, the satellite is composed of two primary bodies: main satellite body (m_1) and flexible appendage (m_2). The flexible appendage is modeled as a massless rod-point mass with a total length given by L . The flexibility of the appendage is modeled using a rotational spring. The rotational spring connects the main body of the satellite and the appendage. The actuator forces for the satellite's thrusters are modeled as two body fixed forces, F_1 and F_2 , and a Torque, T . F_1 , F_2 and T are equipollent representations of the actual thruster forces. The forces F_1 , F_2 are always applied in the inertial x and y direction.

The four generalized coordinates used to describe the system are: x , y , θ , φ . The rotation of the main satellite body is described by θ , and the deflection of the rotational spring is described by φ . The equations of motion for the system are:

$$\ddot{x} = \frac{F_1}{m_{total}} \quad (4)$$

$$\ddot{y} = \frac{F_2}{m_{total}} \quad (5)$$

$$\ddot{\theta} = \frac{T + k\varphi}{J_{CG}} \quad (6)$$

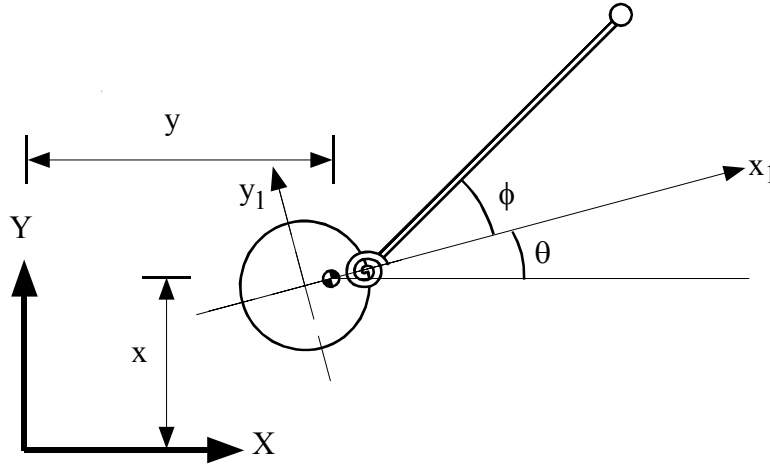


Figure 4: Reference Frame.

$$\ddot{\phi} = \frac{-k\phi(\text{}_{CG}^1J + \text{}_{CG}^2J) - \text{}_{CG}^2J \cdot T}{\text{}_{CG}^1J \cdot \text{}_{CG}^2J} \quad (7)$$

where x_t is the location to the satellite system's center of mass relative to the body fixed x_1y_1 axes shown in Figure 4. $\text{}_{CG}^1J$ and $\text{}_{CG}^2J$ are the mass moments of inertia for the main satellite body and flexible antennae respectively. These mass moments of inertia are about the center of mass for the entire system.

A PD control law is used to control the motion of the satellite and has the following form:

$$\begin{aligned} \frac{F_1}{m_{total}} &= -K_p(x - x_d) - K_v(\dot{x} - \dot{x}_d) \\ \frac{F_2}{m_{total}} &= -K_p(y - y_d) - K_v(\dot{y} - \dot{y}_d) \\ \frac{T}{\text{}_{CG}^1J} &= -K_p(\theta - \theta_d) - K_v(\dot{\theta} - \dot{\theta}_d) \end{aligned} \quad (8)$$

where x , y , and θ are the current state values and x_d , y_d , and θ_d are the desired values. K_p , and K_v represent the proportional and derivative controller gains. Although many actual satellites are positioned using on-off thruster jets, this preliminary work utilizes continuous actuator effort.

Experimental Verification of Input Shaping

Experiments verify the utility of applying input shaping to a rigid satellite with a flexible appendage were conducted at the Two-Dimensional Spacecraft Dynamics Simulator at the Tokyo Institute of Technology.¹⁶ The system consists of a 3m x 5m flat glass floor, three Dynamics and Intelligent control Simulators for satellite Clusters

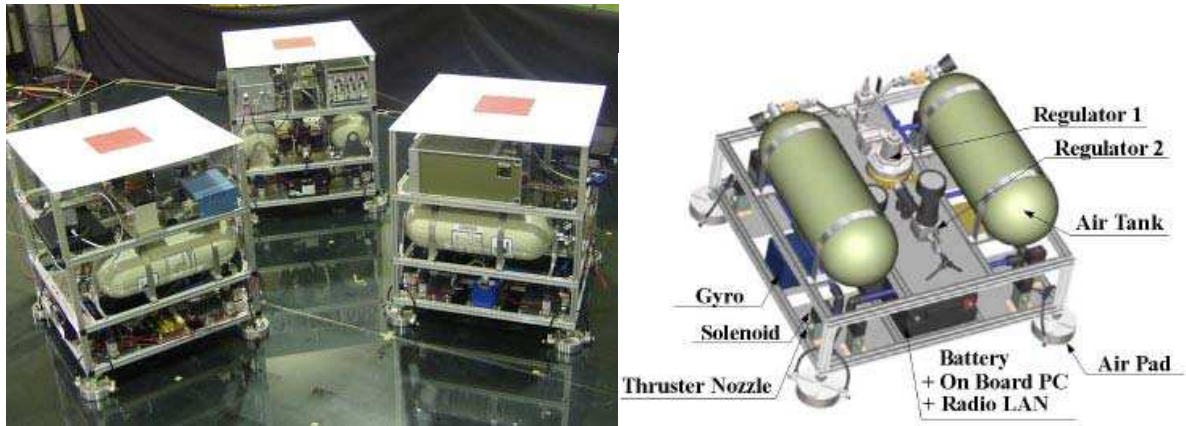


Figure 5: Dynamics and Intelligent Control Simulators.

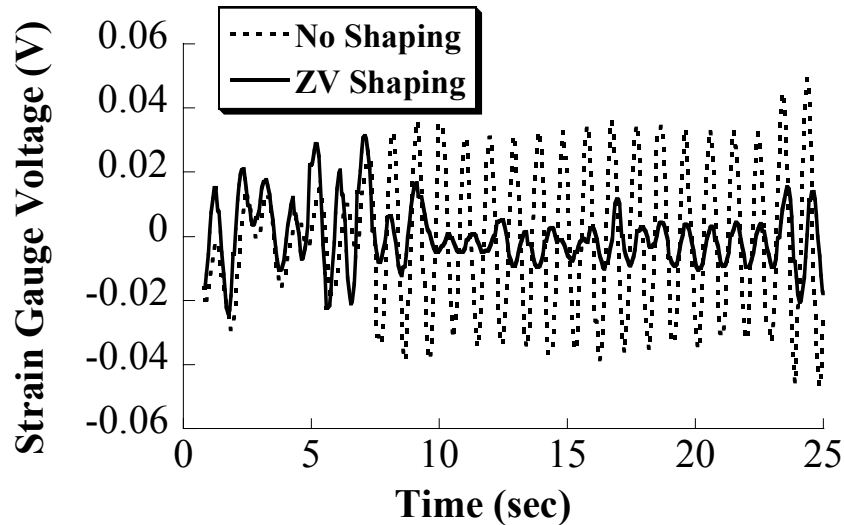


Figure 6: Flexible Satellite System Response to a Translation Maneuver.

(DISC), and an image processing system measuring two-dimensional position. The DISCs float on compressed air pads. Position and attitude are controlled by air thrusters whose commands are sent via a wireless local area network. Figure 5 shows the DISC units. For these experiments, the DISC units were modified by attaching a flexible beam to the top of the units to simulate a solar panel or a flexible antenna.

Figure 6 shows the experimental results of a flexible satellite performing a straight line translation maneuver. The satellite is controlled by a PD feedback controller and has a single flexible appendage. A strain gauge is attached at the midpoint of the flexible arm. Figure 3 represents a simple yet adequate model of the DISC unit that was used to design the input shaper. As seen in Figure 6, the translation maneuver that

utilizes input shaping has a much better system response than the translation maneuver that does not utilize input shaping.

FORMATION FLYING SIMULATION

For the formation flying architecture used here, there are two sets of state variables that are necessary to adequately describe the motion of the satellites: formation and constellation states.¹² The formation is described by a single virtual structure that has an inertial position, $\mathbf{r}_f$, inertia velocity $\mathbf{v}_f$, inertial attitude $\mathbf{q}_f$, and inertial angular velocity $\boldsymbol{\omega}_f$. The attitude, $\mathbf{q}_f$, is a unit quaternion that represents the orientation of the formation frame. The coordination frame C_f is located at this position and given attitude. The relative position and attitude of the i^{th} spacecraft with respect to the formation frame is given by $\mathbf{r}_{if}$, $\mathbf{v}_{if}$, $\mathbf{q}_{if}$, and $\boldsymbol{\omega}_{if}$ respectively. The formation state describes the position and orientation of the entire formation and the positions and orientation of each of the satellites with respect to the formation frame and is given by

$$\mathbf{X}_f = \begin{pmatrix} \mathbf{r}_f & \mathbf{v}_f & \mathbf{q}_f & \boldsymbol{\omega}_f \\ \mathbf{r}_{if} & \mathbf{v}_{if} & \mathbf{q}_{if} & \boldsymbol{\omega}_{if} \\ \dots & & & \\ \mathbf{r}_{Nf} & \mathbf{v}_{Nf} & \mathbf{q}_{Nf} & \boldsymbol{\omega}_{Nf} \end{pmatrix} \quad (9)$$

where N is the number of spacecraft in the constellation.

The constellation states describes the position and orientation of each satellite related to the formation frame by the coordinate geometry shown in shown in Figure 1. The satellite states are referenced from the inertial frame and are represented by the position, x_i , and y_i , the attitude of the main satellite body, θ_i , and the deflection of the rotational spring, ϕ_i . The state of each spacecraft is defined to be

$$\mathbf{X}_i = (\mathbf{x}_i \quad \mathbf{y}_i \quad \theta_i \quad \phi_i) \quad (10)$$

and the state of the constellation is defined to be

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}_1 & \mathbf{y}_1 & \theta_1 & \phi_1 \\ \mathbf{x}_2 & \mathbf{y}_2 & \theta_2 & \phi_2 \\ \mathbf{x}_3 & \mathbf{y}_3 & \theta_3 & \phi_3 \end{pmatrix} \quad (11)$$

The angle of rotation of the main satellite body θ , can be used to determine the appropriate unit quaternion for each of the satellites. Similarly, the desired angle θ_d can be computed from a given unit quaternion.¹⁷

The motion of the formation and satellites is composed of two regimes: Formation Initialization and Formation Configuration. The goal of the Formation Initialization maneuver is to move the satellites from their starting positions to the desired a desired initial position and orientation. The goal of the Formation Configuration maneuver is to reorient that formation to the desired location. For example, the Formation Configuration would be used to point the satellites to a desired star. During both of these maneuvering sequences, the formation frames and the satellites converge to their respective desired locations and attitudes. The initialization of the satellites and the formation states

happens during Formation Initialization. For this motion sequence, the satellites are given their desired locations independent of the formation frame. Consequently, the satellites and the formation frame converge to their desired locations independently. During Formation Configuration, the desired location of the satellites depends on the convergence of the formation states to their desired locations.

Formation Control

The convergence of the formation frame is dependent on the PD gains used in the simulation. For Formation Initialization, the PD gains for the formation ($K_r^{(1)}$, $K_v^{(1)}$, $K_q^{(1)}$, $K_\omega^{(1)}$) were picked so that the formation states quickly converged to the desired locations. Because the satellites converge to their desired locations independent of the formation frame for this sequence it is less important for the formation frame to have a small peak overshoot. For Formation Configuration, the PD gains of the formation ($K_r^{(2)}$, $K_v^{(2)}$, $K_q^{(2)}$, $K_\omega^{(2)}$) were picked so that the formation states quickly converged to their desired locations with a minimal amount of peak overshoot and a small settling time.

Local Satellite Control

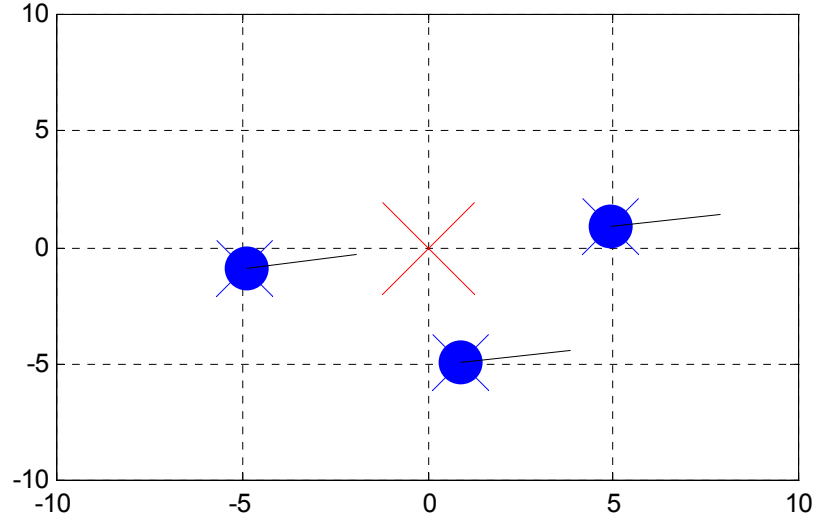
The local control of the satellites was achieved using a standard PD controller. The gains for Formation Initialization and Formation Configuration were picked so as to avoid any unnecessary maneuvering of the satellites. The lifespan of any deep-space mission depends heavily on the amount of fuel available to each satellite; consequently fuel minimization will play an important role in any control algorithm. Although fuel minimization was not directly programmed for in this simulation, it will be considered in future research. Following the description of the PD gains proposed by Beard the PD gains for the local satellite control are described by $K^{(5)}$ for the Formation Initialization maneuver, and $K^{(6)}$ for the Formation Configuration maneuver. For Formation Initialization, the PD gains for the satellites ($K_r^{(5)}$, $K_v^{(5)}$, $K_\theta^{(5)}$, $K_\omega^{(5)}$) were chosen so that the satellites converged to the desired locations within second and with minimal overshoot. For Formation Configuration, the PD gains for the satellites ($K_r^{(6)}$, $K_v^{(6)}$, $K_\theta^{(6)}$, $K_\omega^{(6)}$) were chosen so that the states quickly converge, while avoiding a large amount of overshoot in the desired position. This is imperative since the satellites are moving to the desired locations while the formation states are still converging. As a result, the values of the derivative gains have been increased so as to minimize peak overshoot and reduce the settling time for the formation states. Table 1 shows the values of the parameters used in this study.

SIMULATION RESULTS

Simulations were created in MATLAB and encompassed formation initialization and formation configuration maneuvers. Formation Initialization, initializes the formation and instructs each of the satellites to move from their starting location to the

Table 1: Model Parameters.

Simulation Model Parameters	Value
m_1	1 kg
m_2	.05 kg
r_1	1m
L	3m
K	100 N/m
$K_v^{(1)}, K_v^{(5)}, K_v^{(6)}, K_\omega^{(1)}, K_r^{(5)}, K_r^{(6)}, K_\theta^{(5)}, K_\theta^{(6)}, K_\omega^{(5)}, K_\omega^{(6)}$	10
$K_r^{(1)}, K_q^{(1)}, K_v^{(2)}, K_r^{(2)}, K_q^{(2)}, K_\omega^{(2)}$	100

**Figure 7: Desired Initialized Positions (Formation Initialization).**

desired formation initialization positions. When the error between the desired and actual states is less than a specified value (ε_I), the simulation moves into Formation Configuration. Formation Configuration calculates the desired positions of the satellite based on the current position of the formation frame. The desired position and orientation of each of the satellites is a function of time because the satellites are moving to the designated positions while the formation frame is converging to its desired location.

The desired motion is as follows. First, Formation Initialization initializes the satellites from the starting position to the desired position and orientation shown in Figure 7. The formation states are represented by the four “x’s” shown in the figure. The large “X” shows the position of the coordination frame C_f given in (9). The positions of the constellation states given in (11) are represented by the colored circles. The black thin line extending out from the circle represents the flexible appendage and shows the orientation and of the flexible appendage. The orientation of the flexible appendage is

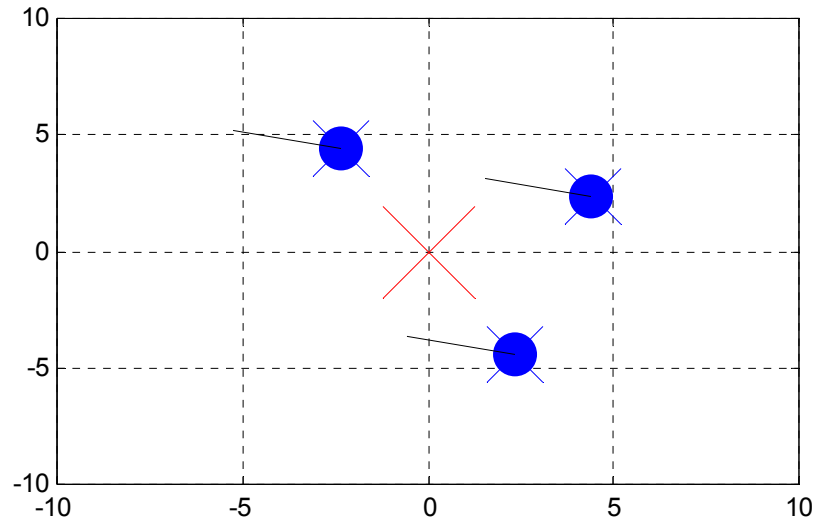


Figure 8: Desired Positions (Formation Configuration).

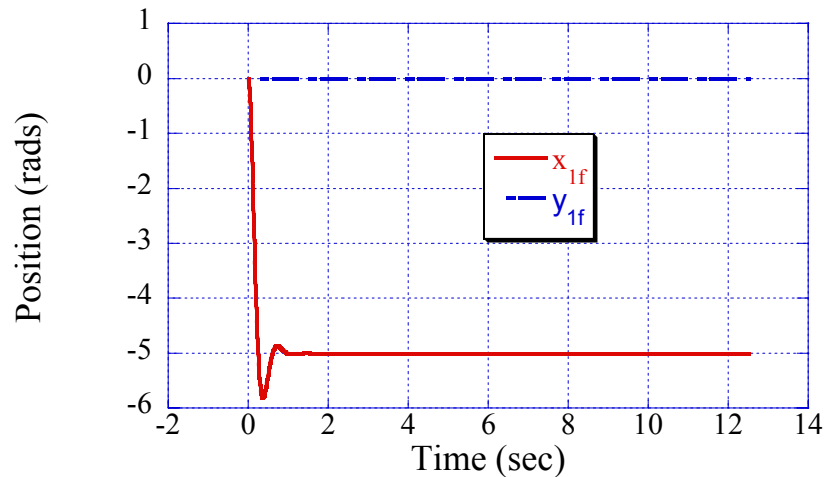


Figure 9: Convergence of Formation States for Satellite 1.

given by the addition of θ and φ . Next, during Formation Configuration, the formation frame C_f rotates 120 degrees. Relative to this formation frame the satellites rotate an additional 50 degrees. The desired position and orientation of the satellites is shown in Figure 8.

The position of satellite 1 for Formation Initialization and Formation Configuration is shown in Figure 10. As seen in the figure, the position of the satellite does not show any vibration. This is because the small mass of the flexible appendage is not able to back-drive the main satellite body. However, because the satellites have a flexible mode, transient and residual vibration will be present throughout the maneuverings of the satellites. If the vibration of the flexible appendage is large enough, damage to the satellite is a distinct possibility. As a result, it is important to mitigate the vibration

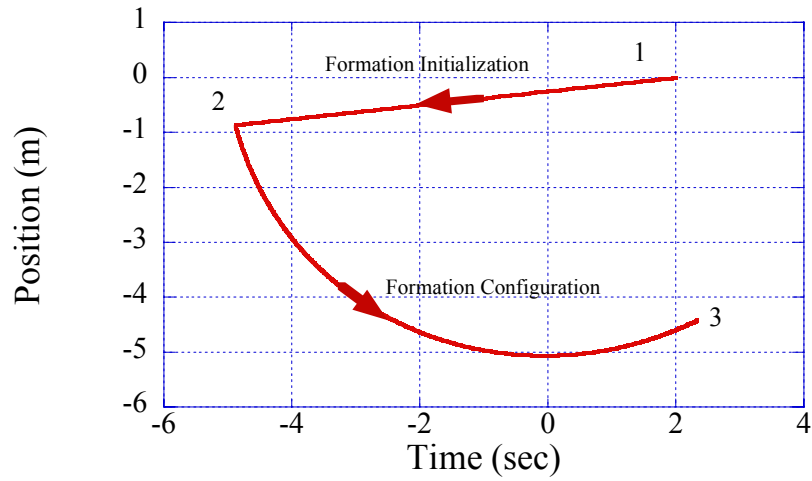


Figure 10: Satellite 1 Response.

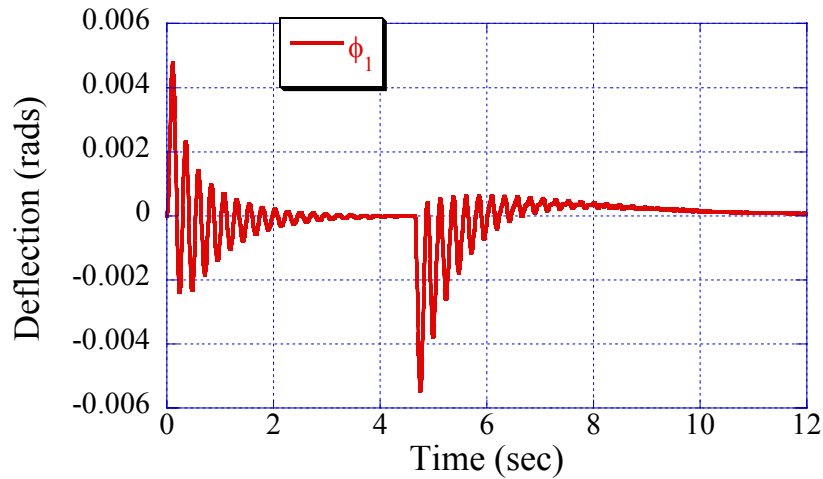


Figure 11: Spring Angle for Satellite 1.

effect. Figure 11 shows the response of the spring angle ϕ as a function of time. As seen, residual vibration is present during Formation Initialization and Formation Configuration. Formation Configuration begins at approximately 4.65 seconds.

A ZV shaper was added to the local control scheme for each of the satellites. The desired satellite states are passed directly to each satellite during the Formation Initialization stage as step changes in position and velocity. The desired step change in position is convolved with the ZV shaper to produce the new desired position. The ZV shaper was determined for the closed-loop frequency of 4.2 Hz and a damping ratio of 0.053. Figure 12 shows the spring angle response for satellite 1 during Formation Initialization for both the unshaped and ZV shaped cases. As seen in the figure, the ZV shaping completely eliminates the residual vibration for the slewing maneuver. Figure 13

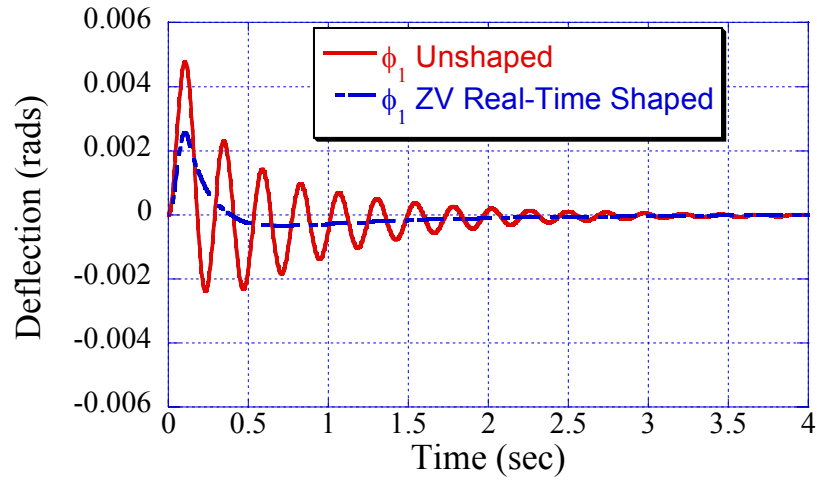


Figure 12: Spring Angle during Formation Initialization for Satellite 1.

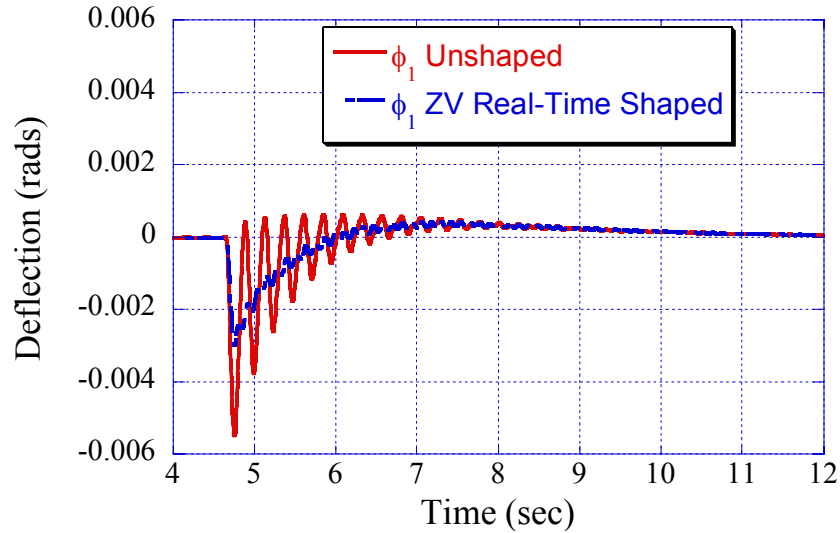


Figure 13: Formation Configuration for Satellite 1.

shows the spring angle response for satellite 1 during Formation Configuration. As seen in the figure, the ZV shaper is able to significantly reduce the vibration during the motion.

The dynamics of the simulation changes slightly when the system enters the Formation Configuration. At this time the formation states are converging to the desired locations and are calculated for a small time interval. At the end of a designated time interval, the desired position and velocities for each of the satellites is passed directly to the satellite local control. Instead of passing a continuous trajectory to the satellites, the satellite are given desired step changes in position and velocity. These step changes

cause a small amount of transient deflection during the maneuver. However, the vibration is eliminated once the satellites reach their final destination. Input shaping definitely provides a large reduction in the vibration during both Formation Initialization and Formation Configuration.

CONCLUSION

This paper demonstrated the benefits of utilizing input shaping with a formation flying control architecture. If the desired FF control scheme is utilized with a flexible spacecraft model, residual vibration of the flexible body will be present after most slewing maneuvers. The amount of residual vibration present is extremely important as it has the potential to damage delicate solar panels or antennae. Input shaping significantly reduces the amount of vibration present in the system. Input shaping is effective at reducing the residual vibration during maneuvers when the formation frame is converging its desired location at the same time that the satellites are moving. The price for utilizing input shaping is a small increase in the rise time, however, the vibration reduction and potential fuel savings makes this an extremely attractive technique.

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