

## Detection of Salient Curvature Features on Polygonal Surfaces

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### Abstract

*We develop an approach for stable detection of perceptually salient curvature features on surfaces approximated by dense triangle meshes. The approach explores an “area degenerating” effect of the focal surface near its singularities and combines together a new approximations of the mean and Gaussian curvatures, nonlinear averaging of curvature maps, histogram-based curvature extrema filtering, and an image processing skeletonization procedure adapted for triangular meshes. Finally we use perceptually significant curvature extrema triangles to enhance the Garland-Heckbert mesh decimation method.*

### 1. Introduction

Surface creases defined via extrema of the principal curvatures along their curvature lines are important for many computer graphics and geometric modeling applications. Besides their mathematical beauty<sup>19,24,13</sup>, the curvature extrema have been intensively studied in connection with research on quality control of free-form surfaces<sup>17</sup>, image and data analysis<sup>8</sup>, face recognition<sup>13</sup>, medical image analysis<sup>23</sup>, and human perception<sup>15</sup> (see also references therein).

Practical extraction of the curvature extrema and their subsets involves estimation of high-order surface derivatives and, therefore, is not a simple task. Various numerical methods were proposed for surfaces given parametrically<sup>17,14,8</sup>, in implicit form<sup>22,27,28,4,3</sup>, or as the graph of a function defined on a rectangular grid (i.e., the grey-level intensity surface of a 2D image)<sup>25,12,21,1</sup>. Detection of curvature extrema on surfaces approximated by triangular meshes has received less attention. A “focusing” effect of the focal surface near its singularities was explored in<sup>20</sup>. A method to detect feature lines defined via second order derivatives of the surface normal on a triangulated surface was developed in<sup>18</sup>. An image processing approach for curvature extrema detection on polygonal surfaces was proposed in<sup>2</sup>.

In this paper, we develop an approach for detection of salient extrema of the principal curvatures along their curvature lines on surfaces approximated by dense triangle meshes. Fig. 1 demonstrates salient curvature extrema triangles detected on the Stanford bunny and Cyberware Venus head models. The detected triangle strips correspond well to perceptually significant ridges and ravines.

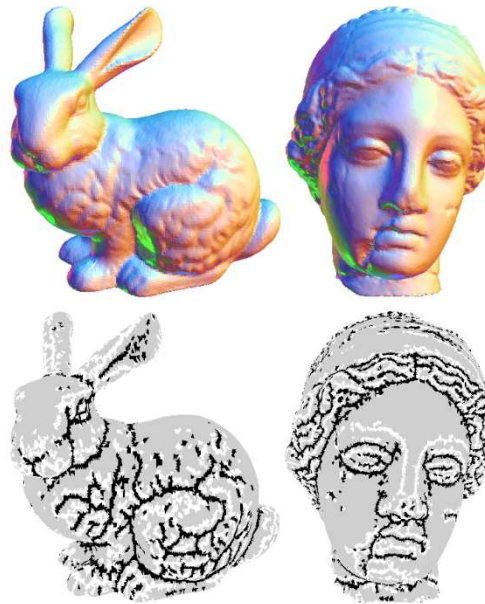


**Figure 1:** Salient curvature extrema triangles detected on the Stanford bunny and Cyberware Venus head model. White triangles are situated along convex creases (ridges) and black triangles are situated along concave creases (ravines).

Our approach explores an “area degenerating” effect of the focal surface near its singularities. The effect was used in<sup>20</sup> for detection of the curvature extrema along their curvature lines. In order to achieve a robust extraction of the curvature extrema we propose a new simple method for estimating the principal curvatures of a surface approximated by a dense triangle mesh. The method fits our needs better than

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the renowned methods developed by Taubin<sup>26</sup> and Desbrun and co-workers<sup>7,6</sup>. To improve the estimates of the surface normals and principal curvatures we employ simple nonlinear diffusion processes. We use curvature statistics in order to ignore insignificant curvature extrema. Finally we adapt an image processing technique for triangle meshes and use it for bridging gaps between connected chains of curvature extrema triangles.

Perceptually salient curvature extrema triangles can be used for mesh decimation purposes. We modify the Garland-Heckbert quadric error metric approach<sup>10,9</sup> by assigning higher weights to salient curvature extrema triangles and providing the edges connecting the vertices of the triangles with higher collapsing costs. Fig. 2 compares the decimation of the Stanford Buddha statue (left) by the Garland-Heckbert method (middle) and our modification of the method (right). Note that small face features are better preserved by the modified method.



**Figure 2:** Left: Stanford Buddha statue consisting of 1,087,716 triangles. Middle and right: simplified meshes consisting of 17,400 triangles (98.4% reduction). Middle: decimation according to the Garland-Heckbert quadric error metric method. Right: modified Garland-Heckbert method was used.

The rest of the paper is organized as follows. In Section 2, the main section of this paper, we describe our curvature extrema detection approach. Section 3 is devoted to our modification of the Garland-Heckbert method. We conclude in Section 4.

## 2. Detection of Salient Curvature Extrema Triangles

### 2.1. Focal Surface and Curvature Extrema

Consider a smooth generic surface. Denote by  $k_{\max}$  and  $k_{\min}$  the largest and the smallest principal curvatures, respectively,  $k_{\max} \geq k_{\min}$ . The principal centers of curvature are the points situated on the surface normals at the distances  $1/k_{\max}$  and  $1/k_{\min}$  from the surface. The loci of the principal centers form the so-called focal surface. The focal surface consists of two sheets corresponding to the maximal and minimal principal curvatures. The focal surface has singularities consisting of space curves called the *focal ribs*. Fig. 3 demonstrates a part of a focal sheet with a typical focal rib. The focal ribs themselves may have singularities.

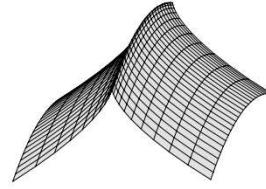


**Figure 3:** A part of a focal sheet with a rib.

The surface curves corresponding to the focal ribs are natural generalization of the curve vertices for surfaces.

For a surface given parametrically  $\mathbf{s} = \mathbf{s}(u, v)$ , its focal surface is described by:

$$\mathbf{f}(u, v) = \mathbf{s}(u, v) + \frac{\mathbf{n}(u, v)}{k(u, v)}, \quad k = k_{\max}, k_{\min},$$

where  $\mathbf{n}(u, v)$  is the oriented normal. When a principal curvature vanishes,  $k = 0$ , the corresponding sheet of the focal surface goes to infinity.

It turns out (see, for example, <sup>19,24</sup>) that the focal ribs correspond to the lines on the surface where the principal curvatures have extrema along their associated principal directions and the points where the principal curvatures are equal (umbilical points). The method proposed in <sup>20</sup> for detection of the curvature extrema triangles is based on the following observation. Consider a small triangle, say  $PQR$  on a surface  $\mathbf{s}(u, v)$  and the associated, “lifted”, triangle  $ABC$  on  $\mathbf{f}(u, v)$

$$\begin{aligned} A &= \mathbf{s}(P) + \mathbf{n}(P)/k(P), \\ B &= \mathbf{s}(Q) + \mathbf{n}(Q)/k(Q), \\ C &= \mathbf{s}(R) + \mathbf{n}(R)/k(R), \end{aligned}$$

as seen in Fig. 4. Assume now that  $P \rightarrow O$ ,  $Q \rightarrow O$ , and  $R \rightarrow O$ , where  $O$  is a surface point. Then  $O$  is a curvature extremum,  $\partial k / \partial \mathbf{t} = 0$  where  $\mathbf{t}$  is a principal directions, if and only if  $\text{Area}(ABC) / \text{Area}(PQR) \rightarrow 0$ <sup>20</sup>.

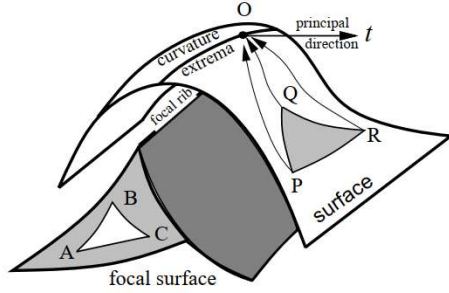


Figure 4: Surface and Focal Surface.

## 2.2. Estimation of Principal Curvatures

In this section, we describe a new simple method for estimating the principal curvatures of a surface approximated by a dense triangle mesh. According to our experience, the Desbrun et al. method<sup>7,6</sup> is best suited for estimation of the mean curvature vector while the Taubin method<sup>26</sup> provides with the best estimation of the curvature tensor (the principal curvatures and directions). However, since we do not use the principal directions in our approach, below we propose a simple method for estimating the principal curvatures only. According to our numerical experiments, it allows us to achieve a better detection of the curvature extrema triangles.

Consider an oriented surface. Let  $\mathbf{t}$  be a tangent vector and  $\mathbf{n}$  be the unit normal at a surface point  $P$ . The normal section curve associated with  $\mathbf{t}$  at  $P$  is defined as the intersection between the surface and plane passing through  $P$  and generated by  $\mathbf{t}$  and  $\mathbf{n}$ . Let  $\mathbf{t}_{\max}$  and  $\mathbf{t}_{\min}$  be the principal directions at  $\mathbf{x}$  associated with the principal curvatures  $k_{\max}$  and  $k_{\min}$ , respectively, and  $k_n(\varphi)$  be the curvature of the normal section curve, where  $\varphi$  is the angle between  $\mathbf{t}$  and  $\mathbf{t}_{\max}$ . According to Euler's formula

$$k_n(\varphi) = k_{\max} \cos^2 \varphi + k_{\min} \sin^2 \varphi.$$

Simple calculations show that

$$\frac{1}{2\pi} \int_0^{2\pi} k_n(\varphi) d\varphi = \frac{k_{\max} + k_{\min}}{2} = H, \quad (1)$$

$$\frac{1}{2\pi} \int_0^{2\pi} k_n(\varphi)^2 d\varphi = \frac{3}{2} H^2 - \frac{1}{2} K, \quad (2)$$

where  $H$  is the mean curvature and  $K = k_{\max} \cdot k_{\min}$  is the Gaussian curvature. These integral formulas allow us to estimate the mean and Gaussian curvatures of a triangulated surface.

Let a smooth surface be approximated by a triangle mesh. Consider a mesh vertex  $P$ . Let us estimate the normal at  $P$  as the normalized weighted sum of the normals of the incident faces, with weights proportional to the surface areas of the

faces:

$$\mathbf{n}(P) = \frac{\sum A_k \mathbf{n}_k}{\|\sum A_k \mathbf{n}_k\|}.$$

Let us put the origin of coordinates at  $P$  and consider a normal section curve  $\mathbf{r} = \mathbf{r}(s)$  obtained as the intersection of the surface and the plane generated by  $\mathbf{n}(P)$  and  $\overrightarrow{PQ}$ , where  $Q$  belongs to the first ring of neighbors of  $P$ , as seen in Fig. 5, and  $s$  is arclength.

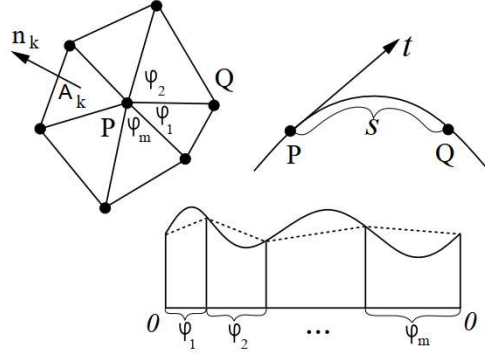


Figure 5: The first ring of neighbors of vertex P

The Taylor series expansion of  $\mathbf{r}(s)$  gives

$$\mathbf{r}(s) = \mathbf{r}(0) + s\mathbf{r}'(0) + \frac{s^2}{2}\mathbf{r}''(0) + \dots$$

Thus

$$\mathbf{n}(P) \cdot \overrightarrow{PQ} \approx k \frac{s^2}{2}, \quad s \approx \|PQ\|, \quad k \approx \frac{2\mathbf{n}(P) \cdot \overrightarrow{PQ}}{\|PQ\|^2}$$

where  $k$  is the curvature of  $\mathbf{r}(s)$  at  $P$ .

The trapezoid approximation of (1) leads to

$$2\pi H = \int_0^{2\pi} k_n(\varphi) d\varphi \approx k_1 \left( \frac{\varphi_1 + \varphi_m}{2} \right) + k_2 \left( \frac{\varphi_2 + \varphi_3}{2} \right) + \dots + k_m \left( \frac{\varphi_{m-1} + \varphi_m}{2} \right).$$

A similar trapezoid approximation of (2) allows us to estimate the Gaussian curvature  $K$ . Now the principal curvatures are obtained by

$$k_{\max} = H + \sqrt{H^2 - K}, \quad k_{\min} = H - \sqrt{H^2 - K}.$$

Fig. 6 exposes a rounded octahedron (left) and two sheets of its focal surface (middle and right). The sheets of the focal surface are well reconstructed.



**Figure 6:** Left: a rounded octahedron. Middle and right: the sheets of the focal surface of the octahedron.

### 2.3. Curvature Extrema Detection

To detect curvature extrema triangles associated with a principal curvature  $k$  we first compute the area-ratio  $\text{Area}(ABC)/\text{Area}(PQR)$  for every mesh triangle  $PQR$  and its associated triangle  $ABC$  on the  $k$ -sheet of the focal surface. We mark the triangle  $PQR$  as a curvature extrema triangle if its ratio falls in the smallest 30% of all the area-ratios.

Fig. 7 shows the curvature extrema triangles detected on the rounded octahedron with the principal curvatures estimated by the Desbrun et al. <sup>6</sup> method (left), Taubin <sup>26</sup> method (middle), and our method described above (right).

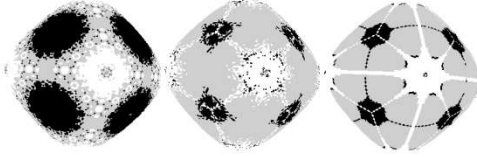


**Figure 7:** Detection of the curvature extrema triangles on rounded octahedron. Left: Desbrun et al. <sup>6</sup> method was used. Middle: Taubin <sup>26</sup> method was used. Right: method described in Section 2.2 was used.

However the comparison with the curvature extrema curves detected analytically on a rounded octahedron defined as an implicit function (see the left image of Fig. 8) shows that the initial curvature estimates are not enough and smoothing and filtering procedures are required. The procedures are described in the subsequent subsections. The right image of Fig. 8 presents the curvature extrema triangles detected on the rounded octahedron after a skeletonization step has been applied (see Subsection 2.6 for details).

### 2.4. Smoothing Normals and Curvatures

For complex triangle meshes the initial estimates of the principal centers of curvature are usually poor and additional smoothing is required. We use the following repeated local averaging schemes for smoothing the mesh normals  $\mathbf{n}$  and principal curvatures  $k$ :

$$\mathbf{n}(P) \leftarrow \frac{\mathbf{n}(P) + \sum w_i \mathbf{n}(Q_i)}{|\mathbf{n}(P) + \sum w_i \mathbf{n}(Q_i)|}, \quad k(P) \leftarrow \frac{k(P) + \sum w_i k(Q_i)}{1 + \sum w_i},$$

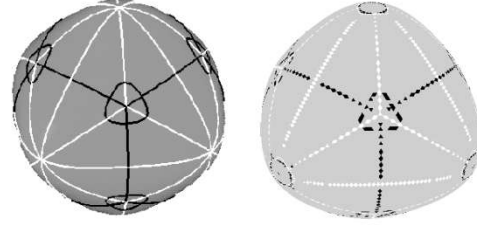


**Figure 8:** Right: the curvature extrema detected analytically on the implicit function  $x^4 + y^4 + z^4 + 5x^2y^2 + 5y^2z^2 + 5z^2x^2 = 1$ . Left: the curvature extrema triangles detected on the rounded octahedron.

where  $P$  is a mesh vertex, vertices  $Q_i$  form the first ring neighbors of  $P$ ,  $w_i$  are positive weights, and the summations are taken over the first ring of neighbors.

Simple averaging with equal weights does not improve curvature extrema detection substantially. A much better result is achieved when use nonlinear averaging with weights defined by

$$w_i = \exp \left( -c \left| \frac{\mathbf{n}(P) - \mathbf{n}(Q_i)}{d_i} \right|^2 \right)$$

for averaging the normals and by

$$w_i = \exp \left( -c \left| \frac{k(P) - k(Q_i)}{d_i} \right|^2 \right)$$

for averaging the principal curvatures. Here  $d_i$  is the distance between  $P$  and  $Q_i$  and  $c$  is a positive constant. According to our experiments, smoothing with  $c = 0.01$  produces very satisfactory results.

However, simple averaging with equal weights is much more useful for visualization of focal surface features. Actually, smoothing the principal curvatures by simple averaging blends the focal sheets with offset surfaces. Fig. 9 exposes the “focal sheets” associated with  $k_{\min}$  for the bunny and Venus head models oriented by their outer normals. Note that sharp features of the “focal sheets” are in a good correspondence with convex creases (ridges) of the original models.

### 2.5. Curvature Extrema Filtering

Not all curvature extrema are perceptually salient since the surface flattens at some of them. So we consider the triangles corresponding to the so-called *crest lines* <sup>27, 28</sup> consisting of the extrema of the maximal in absolute value principal curvature  $k$

$$k = \begin{cases} k_{\max} & \text{if } |k_{\max}| > |k_{\min}| \\ k_{\min} & \text{if } |k_{\min}| > |k_{\max}| \end{cases}$$

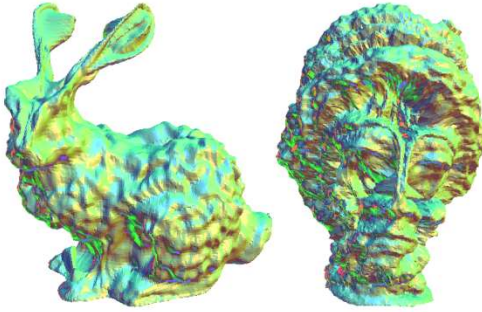


**Figure 9:** “Focal sheets” associated with  $k_{\min}$  for the bunny and Venus head models oriented by their outer normals after 5 passes of smoothing the principal curvatures and normals by simple averaging.

along its corresponding curvature line.

Fig. 10 shows the triangles corresponding to the crest lines. For the top images simple averaging was used while for the bottom images nonlinear averaging was used.

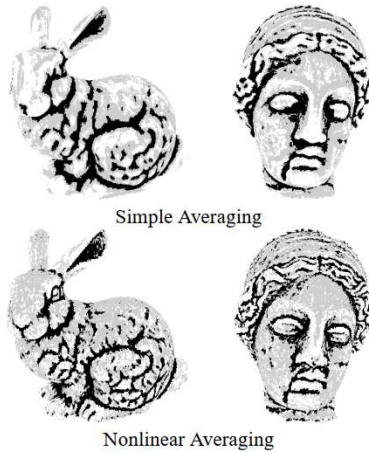


**Figure 10:** Detection of curvature extrema triangles corresponding to crest lines. Top: the normals and principal curvatures were smoothed by three iterations of simple averaging. Bottom: the normals and principal curvatures were smoothed by three iterations of nonlinear averaging.

However some curvature extrema triangles corresponding to crest lines are located at flat surface regions. So we keep only those triangles for which

$$|k_{\max}| + |k_{\min}| > T,$$

where  $T$  is a threshold. A good result is obtained when  $T$

is chosen at the 50th percentile of the “curvedness” data  $|k_{\max}| + |k_{\min}|$  of the entire surface.

Fig. 11 demonstrates advantages of this filtering scheme.



**Figure 11:** Salient curvature extrema triangles detected on complex triangular meshes.

## 2.6. Skeletonization

Skeletonization (thinning) is a powerful image processing technique used for line feature extraction. We adapt the famous Zhang-Suen algorithm<sup>30</sup> for triangular meshes and use it for thinning of the set of the detected curvature extrema triangles. Instead of the cityblock metric we use its triangular analog, see Fig. 12.

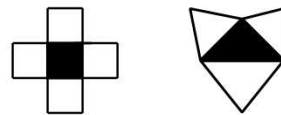


**Figure 12:** Left: a pixel and its neighbors according to the cityblock metric. Right: a triangle and its neighbors according to the triangular analog of the cityblock metric.

The thinning procedure was applied to the curvature extrema triangles detected on the rounded octahedron model, see the right image of Fig. 7. The result is demonstrated in the right image of Fig. 8. Unfortunately the above simple modification of the Zhang-Suen method may disconnect chains of triangles, as it is seen in the right image of Fig. 8.

To restore connectivity, one extra layer of the surrounding triangles is added to every skeletal triangle. The results for the Stanford bunny and Cyberware Venus head models are presented in Fig. 1.

### 3. Application to Mesh Decimation

Since curvature extrema triangles are perceptually significant, it is worth to take them into account during a mesh decimation procedure. We enhance the Garland-Heckbert quadric-error metric approach<sup>9,10</sup> by assigning higher weights to the salient curvature extrema triangles and providing the edges connecting the vertices of the triangles with higher collapsing costs. In our current implementation<sup>29</sup>, we increase the weight of each salient curvature extrema triangle in one and a half times, the collapsing cost of each edge connecting two vertices of the salient curvature extrema triangles in three times, and the collapsing cost of an edge incident with only one such vertex in two times. The curvature extrema triangles associated with the maximal and minimal principal curvatures are treated separately.

Fig. 13 displays simplified Venus head models after 99% and 99.5% decimation by the pure Garland-Heckbert (left) and our modification of the method (right).



**Figure 13:** Left: Garland-Heckbert method was used. Right: our modification of the method was used. Top: 99% reduction. Bottom: 99.5% reduction.

Fig. 13 and Fig. 2 demonstrate advantages of our modification.

It is interesting that both the Garland-Heckbert method and our modification of it provide with almost the same  $L^2$

accuracy (we use Metro<sup>5</sup> and the Venus head model), as seen in Fig. 14. Probably an error metric which takes into account the mesh normals would reveal better approximation properties of our modification of the Garland-Heckbert method.

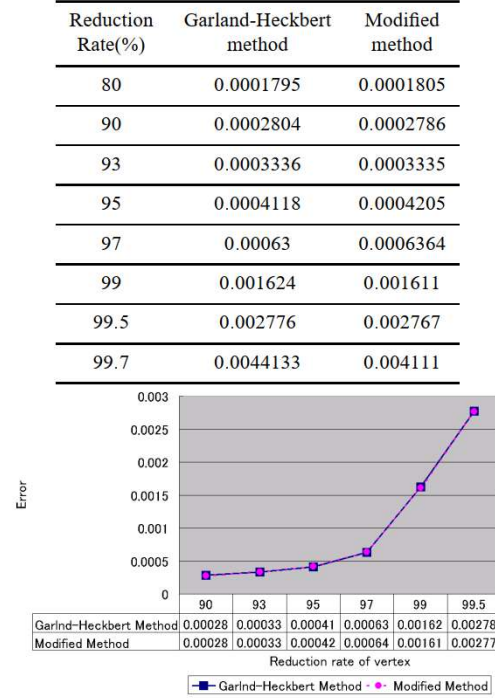


**Figure 14:** Metro<sup>5</sup> error analysis of the original Garland-Heckbert method and our modification of it.

According to our experiments, our modification of the Garland-Heckbert scheme performs better than the original method only for 99% or higher reduction rates. Since modern 3D scanner systems can produce highly detailed meshes with hundreds of millions of triangles and current computers can not display meshes of even much smaller size in real time, our modification has many potential applications.

In our modification of the Garland-Heckbert quadric-error metric approach we treat the salient curvature extrema triangles in a very simple way. Possibly much more can be achieved by adapting quadric-error metric schemes developed in<sup>11,16</sup>.

### 4. Conclusion

In this paper, we presented an approach for stable detection of perceptually salient curvature extrema on surfaces

approximated by dense triangle meshes. The approach explores an “area degenerating” effect of the focal surface near its singularities and combines together a new approximations of the mean and Gaussian curvatures, nonlinear averaging of curvature maps, histogram-based curvature extrema filtering, and an image processing skeletonization procedure adapted for triangular meshes. Finally perceptually significant curvature extrema triangles are used to enhance the Garland-Heckbert mesh decimation method.

All methods described in this paper, including the original Garland-Heckbert mesh decimation method and focal surface visualization, are implemented in a mesh decimation toolkit developed by the first author<sup>29</sup>. The toolkit is written in Java 3D and available on the Web for evaluation<sup>29</sup>. Fig. 15 shows the main GUI window of the toolkit.

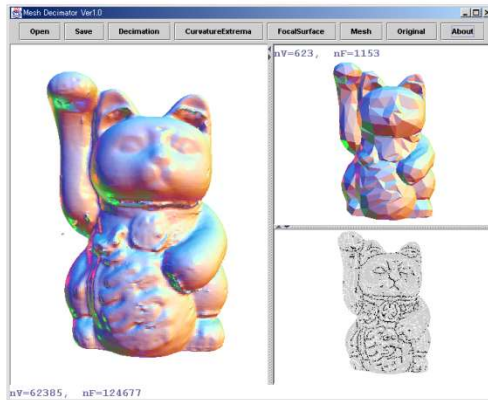


**Figure 15:** GUI of a mesh decimator toolkit developed by the first author. A model of “Maneki-neko”, a Japanese good luck talisman, is used for demonstration.

As we mentioned in Section 2.1, the surface curves formed by the extrema of the principal curvatures along their principal directions correspond to the cuspidal edges of the focal surface. Thus a possible approach to detect the curvature extrema curves can be based on extraction of focal surface edges with small dihedral angles. Fig. 16 shows the curvature extrema edges detected on the rounded octahedron model.

Our current implementation of this idea does not work well for complex polygonal models and work is in progress to achieve stable detection of perceptually salient curvature extrema edges.

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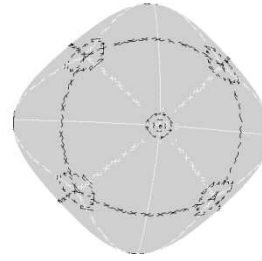


**Figure 16:** Curvature extrema edges detected on rounded octahedron.

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