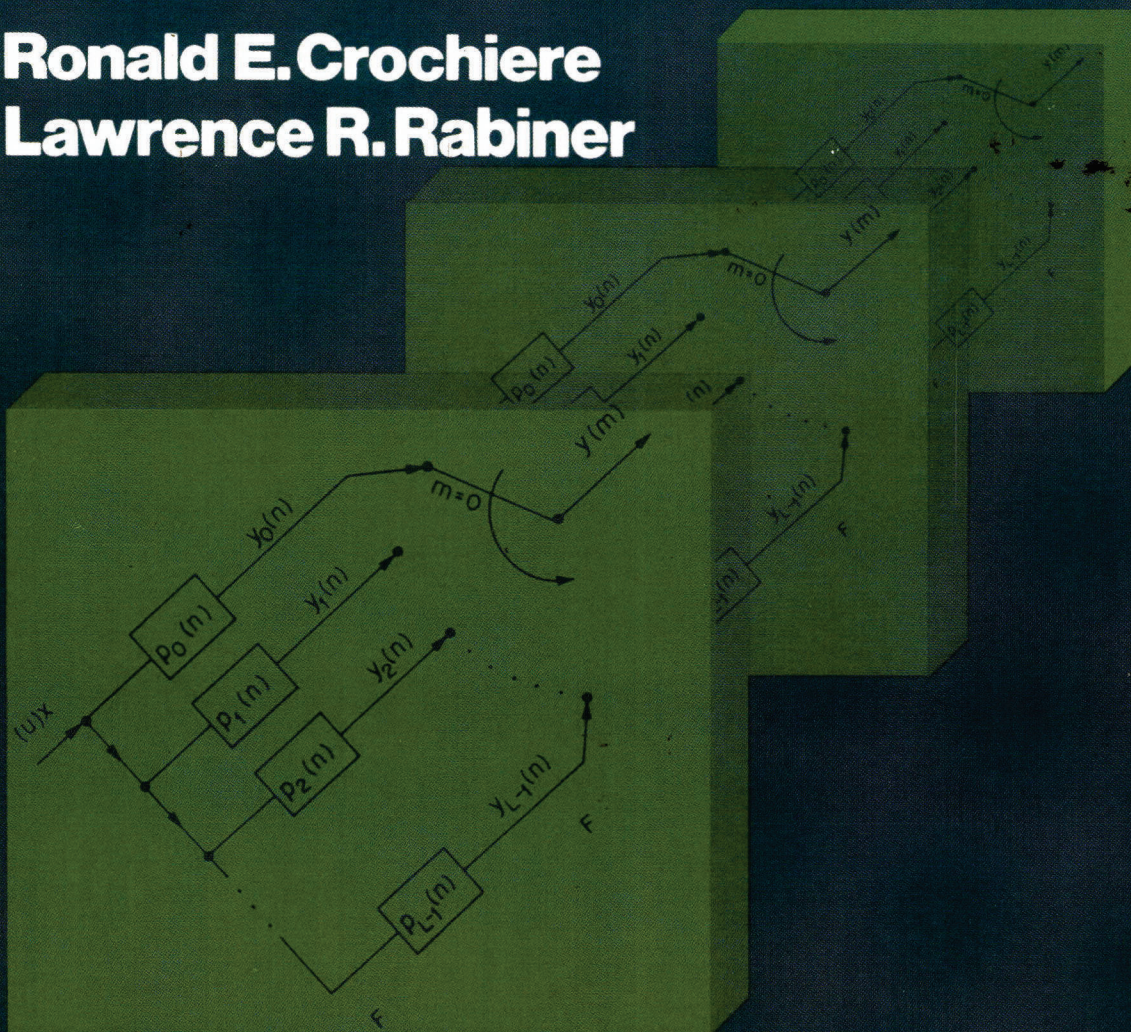
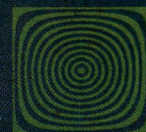


Multirate Digital Signal Processing

Ronald E. Crochiere
Lawrence R. Rabiner



PRENTICE-HALL SIGNAL PROCESSING SERIES
ALAN V. OPPENHEIM, SERIES EDITOR





MULTIRATE DIGITAL SIGNAL PROCESSING

RONALD E. CROCHIERE

LAWRENCE R. RABINER

*Acoustics Research Department
Bell Laboratories
Murray Hill, New Jersey*

Prentice-Hall, Inc., Upper Saddle River, New Jersey 07458

Library of Congress Cataloging in Publication Data

Crochiere, Ronald E.

Multirate digital signal processing.

(Prentice-Hall signal processing series)

Includes index.

1. Signal processing—Digital techniques.

I. Rabiner, Lawrence R. II. Title.

III. Series.

TK5102.5.C76 1983 621.38'043 82-23113

ISBN 0-13-605162-6

Text processing: *Donna Manganelli*

Editorial/production and supervision by *Barbara Cassel* and *Mary Carnis*

Cover design: *Mario Piazza*

Manufacturing buyer: *Anthony Caruso*

© 1983 by Prentice-Hall, Inc., Upper Saddle River, NJ 07458

All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher.

Printed in the United States of America

10 9 8 7

ISBN 0-13-605162-6

Prentice-Hall International (UK) Limited, London
Prentice-Hall of Australia Pty. Limited, Sydney
Prentice-Hall Canada Inc., Toronto
Prentice-Hall Hispanoamericana, S.A., Mexico
Prentice-Hall of India Private Limited, New Delhi
Prentice-Hall of Japan, Inc., Tokyo
Pearson Education Asia Pte. Ltd., Singapore
Editora Prentice-Hall do Brasil, Ltda., Rio de Janeiro

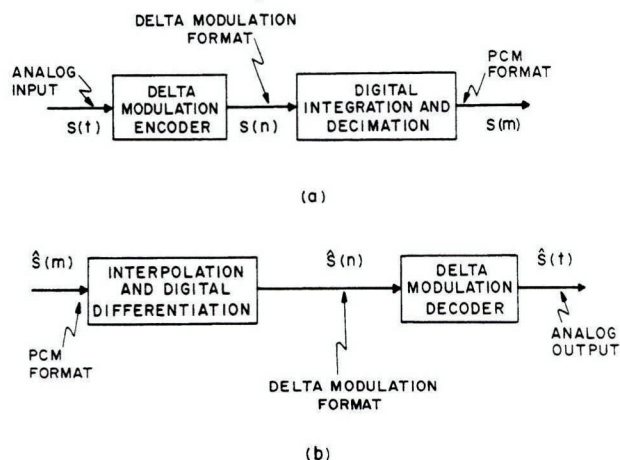


Figure 1.3 Illustration of (a) a delta modulation-to-PCM conversion; (b) a PCM-to-delta modulation conversion

Fig. 1.4a illustrates the basic process of translating a series of 12 TDM digital speech signals, $s_1(n)$, $s_2(n)$, ..., $s_{12}(n)$, to a single FDM signal $r(m)$ and Fig. 1.4b illustrates the reverse (FDM-to-TDM) translation process. The sampling rate of the TDM speech signals is 8 kHz whereas the sampling rate of the FDM signal is much higher to accommodate the increased total bandwidth. In each channel of the TDM-to-FDM translator the sampling rate is effectively increased (by interpolation) to the higher FDM sampling rate. The signal is then modulated by single-sideband modulation techniques to its appropriate frequency band location in the range 56 to 112 kHz as illustrated in Fig. 1.5. The interpolated and modulated channel signals are then digitally summed to give the desired FDM signal. In the FDM-to-TDM translator the reverse process takes place.

As seen from Fig. 1.4 the process of translation between TDM and FDM formats involves sampling rate conversion and therefore these systems are inherently multirate systems. In practice it is often convenient to integrate the signal processing operations of modulation and interpolation (or decimation) and share the processing that is common to all of the channels in a way that reduces the computation, in the overall translator, substantially. Thus the block diagrams in Fig. 1.4 only serve as conceptual models of the system whereas the actual computational structures for efficient implementation of these systems may take on considerably different forms. The methods for design of efficient multirate signal processing structures are also covered in this book.

1.2.4 Sub-Band Coding of Speech Signals

A fourth example of a practical multirate digital system is that of sub-band coding [1.10]. Sub-band coding is a technique that is used to efficiently encode speech

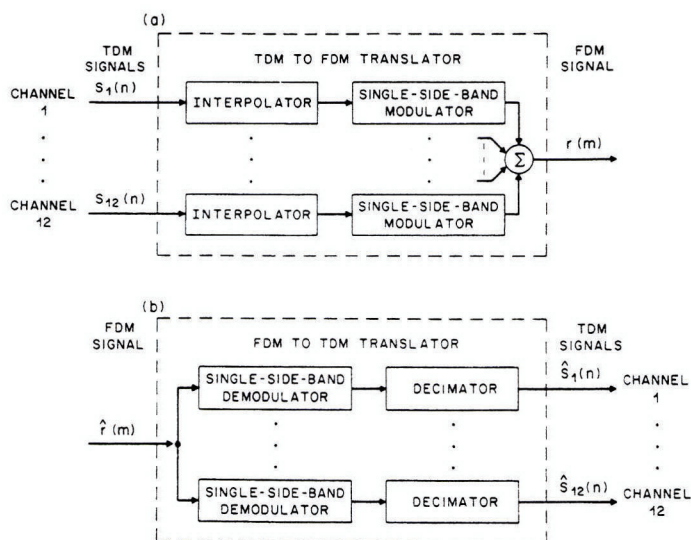


Figure 1.4 Illustration of (a) a TDM-to-FDM translator; and (b) an FDM-to-TDM translator.

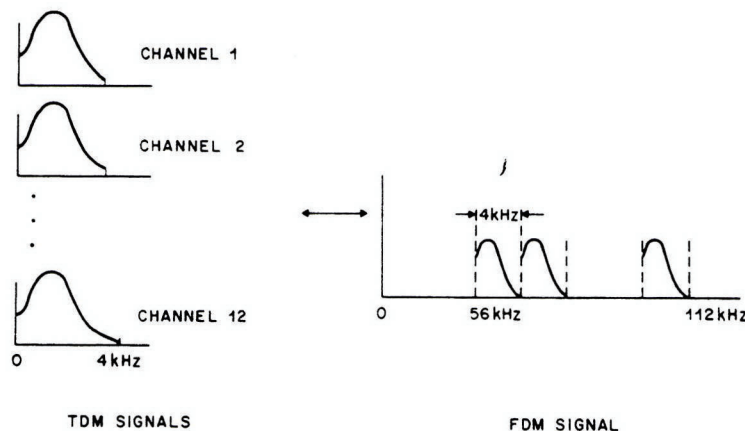


Figure 1.5 Spectral interpretation of TDM-to-FDM signal translation.

signals at low bit-rates by taking advantage of the time varying properties of the speech spectrum as well as some well known properties of speech perception. It is based on the principle of decomposing the speech signal into a set of sub-band signals and separately encoding each of these signals with an adaptive PCM quantizer. By carefully selecting the number of bits/sample used to quantize each sub-band, according to perceptual criteria, an efficient encoding of the speech can be achieved.

Fig. 1.6a shows a block diagram of a sub-band coder-decoder, and Fig. 1.6b shows a typical 5-band filter bank arrangement for sub-band coding. A key element in this design is the implementation of the filter bank analysis and synthesis systems. Each sub-band in the filter bank analyzer is effectively obtained by a process of bandpass filtering, modulation to zero frequency, and decimation in a manner similar to that of the FDM-to-TDM translator described in Section 1.2.3.

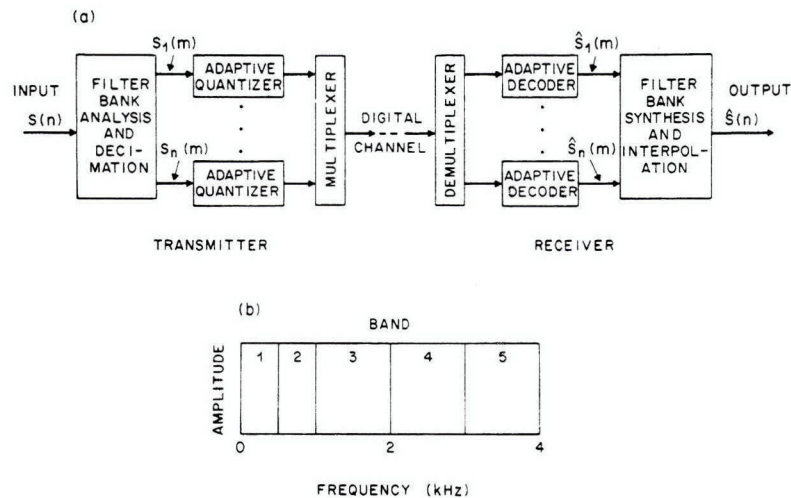


Figure 1.6 (a) Illustration of an N-band sub-band coder; and (b) an example of the frequency bands for a 5-band sub-band coder design.

In the receiver the reverse process takes place with the filter bank synthesizer reconstructing an output signal from the decoded sub-band signals. This process is similar to that of the TDM-to-FDM translator. Again, the concepts and the computational techniques for efficiently designing and implementing filter bank analyzers and synthesizers of this type are covered in this book.

1.2.5 Short-Time Spectral Analysis and Synthesis

A final example of multirate signal processing systems is a short-time spectral analysis and synthesis system [1.16]. Such systems are widely used in the areas of speech processing, antenna systems, and radar systems. Fig. 1.7 gives a simplified explanation of a spectrum analysis-synthesis system. The signal $s(n)$ is periodically windowed with a sliding window (shown in Fig. 1.7b) to form a short-time, finite duration, piece of the signal at each time slot. This short-time signal is then transformed with a fast DFT (discrete Fourier transform) algorithm to form a short-time spectral estimate of the signal for that time slot. Based on the spectral information, the signal may be modified in a variety of ways (or the short-time spectrum may be an end result in itself). The modified short-time spectrum is then inverse transformed to form a modified short-time segment of the signal. Finally, a

7.1 GENERAL ISSUES AND DEFINITIONS

Figure 7.1(a) illustrates the basic framework for a K -channel filter bank *analyzer* and Fig. 7.1(b) illustrates a similar framework for a K -channel *synthesizer*. In the analyzer the input signal $x(n)$ is divided into a set of K -channels where $X_k(m)$, $k = 0, 1, \dots, K-1$, denotes the channel signals. In the synthesizer the corresponding signals will be denoted by the “hat” notation. Thus $\hat{X}_k(m)$, $k = 0, 1, \dots, K-1$, denotes the input channel signals to the synthesizer and $\hat{x}(n)$ denotes the output signal, where the sampling rates for the channel signals and the input and output signals will, in general, be different. The signals $X_k(m)$ and $\hat{X}_k(m)$ may also, in general, be different depending on the application. It should be noted that in this chapter we will refer to systems of the form of Fig. 7.1 *interchangeably* as filter banks or spectrum analyzers (or synthesizers) since the basic mathematical models and structures can be applied to both types of applications.

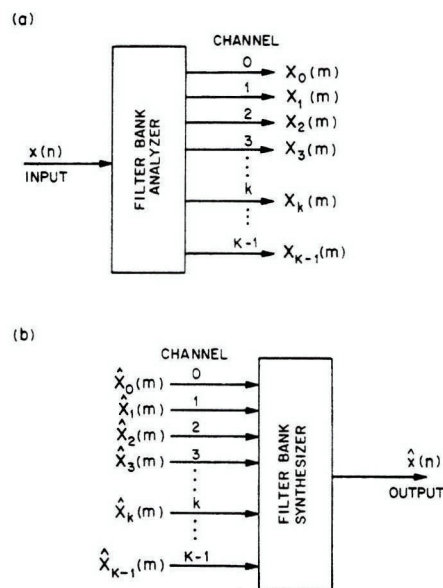


Figure 7.1 Basic framework for; (a) a K -channel filter bank analyzer; and (b) a K -channel filter bank synthesizer.

There are many ways to define the channel signals in the configuration above. For example, one important characteristic that distinguishes different classes of filter banks is the manner in which the channel signals are modulated. In this chapter we consider mainly two types of modulation as discussed in Chapters 2 and 6: *complex* (or *quadrature*) *modulation* and *single-sideband (SSB) modulation*.

Figure 7.2 briefly reviews these two forms of modulation, where ω_Δ denotes the width of the frequency band of interest for a particular filter bank channel centered at frequency ω_k . In the case of complex modulation [Fig. 7.2(a)] it is clear from the discussion in Chapter 2 that the channel signals $X_k(m)$ are complex-valued signals, whereas for single-sideband modulation [Fig. 7.2(b)] they are real valued. Complex modulation is generally preferred in systems such as spectrum analyzers [7.1, 7.2, 7.6, 7.10-7.12, 7.19, 7.20], whereas single-sideband modulation is generally preferred in systems such as communication systems [7.4, 7.5, 7.8, 7.9, 7.16-7.18] and coding systems [7.13-7.15, 7.24].

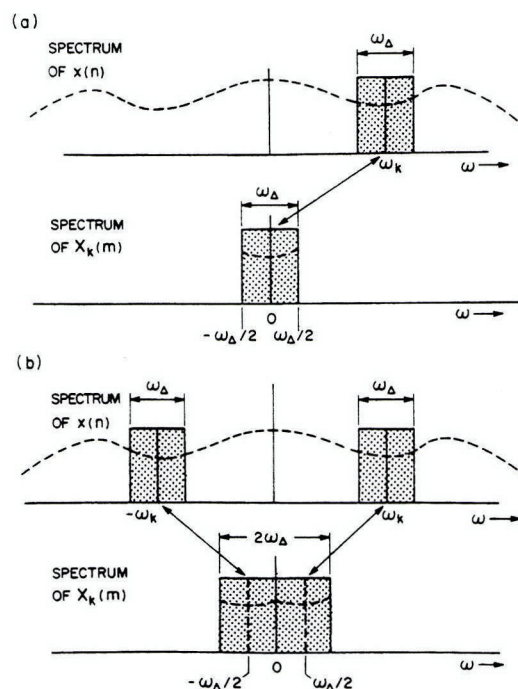


Figure 7.2 Comparison of filter bank modulation techniques; (a) complex (quadrature) modulation; (b) single-sideband modulation.

Another major difference in filter bank systems results from the manner in which the filters are designed for the channels. Figure 7.3 illustrates three examples of filter designs which result in nonoverlapping [part (a)], slight overlapping [part (b)], and substantial overlapping [part (c)] of the bands of the filter bank. Nonoverlapping designs are of interest, for example, in time division to frequency division (TDM-FDM) transmultiplexers for communication systems. In such applications the channel signals $X_k(m)$ and $\hat{X}_k(m)$ may be sets of telephone signals and $x(n)$ and $\hat{x}(n)$ may be frequency-multiplexed forms of these telephone

signals [7.4, 7.5, 7.8, 7.9, 7.16-7.18]. In an application such as this, it is desired to keep the channel signals completely separated with guard bands between channels to avoid *crosstalk*. Crosstalk also occurs due to the fact that the stopband regions of the filter designs can never have infinite attenuation. Figure 7.4(a) illustrates this fact: namely, that crosstalk is directly affected by the degree of stopband attenuation in the filter design.

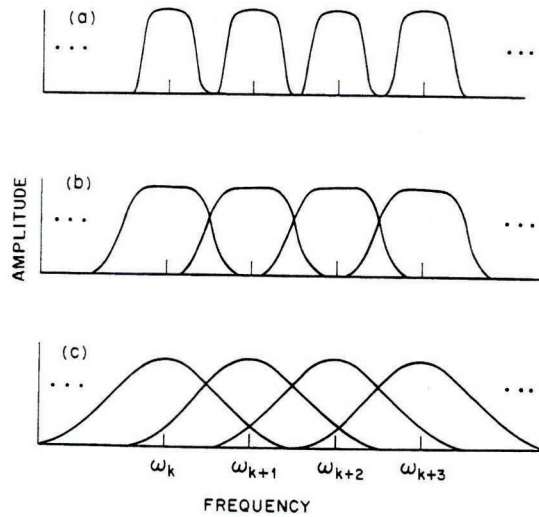


Figure 7.3 Comparison of filter bank designs; (a) nonoverlapping; (b) slight amount of overlapping; (c) substantial amount of overlapping.

Slightly overlapping filter bank designs are sometimes desired in systems, such as bandwidth compression systems, where an input signal $x(n)$ is analyzed in terms of its spectral components $X_k(m)$ for encoding into a compressed digital form [7.2, 7.3, 7.6, 7.7, 7.10, 7.13-7.15, 7.19, 7.24]. In reconstruction, the spectral components, $\hat{X}_k(m)$, are decoded and used to synthesize an output signal $\hat{x}(n)$ which is intended to be a replica of the input signal $x(n)$. Thus, in the absence of encoding, it is desired to have an analyzer/synthesizer system such that if the output of the analyzer is applied directly (i.e., without any channel distortion) to the input of the synthesizer, that is, if

$$\hat{X}_k(m) = X_k(m) \text{ for all } k \text{ and } m \quad (7.1a)$$

then

$$\hat{x}(n) = x(n) \quad (7.1b)$$