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DIGITAL AND ANALOG COMMUNICATION SYSTEMS

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illustrated and analyzed. In addition to the theory, practical aspects of transmitter and receiver design will be emphasized.

First, we will study the mathematical basis of modulation theory.

4-2

REPRESENTATION OF BANDPASS WAVEFORMS AND SYSTEMS

What is a general representation for bandpass digital and analog signals? How do we represent a modulated signal? How do we represent bandpass noise? These are some of the questions that are answered in this section.

Definitions: Baseband, Bandpass, and Modulation

Definition. A *baseband* waveform has a spectral magnitude that is nonzero for frequencies in the vicinity of the origin (i.e., $f = 0$) and negligible elsewhere.

Definition. A *bandpass* waveform has a spectral magnitude that is nonzero for frequencies in some band concentrated about a frequency $f = \pm f_c$, where $f_c \gg 0$. The spectral magnitude is negligible elsewhere. f_c is called the *carrier frequency*.

For bandpass waveforms the value of f_c may be arbitrarily assigned for mathematical convenience in *some* problems. In others, namely, modulation problems, f_c is the frequency of an oscillatory signal in the transmitter circuit and is the assigned frequency of the transmitter, such as, for example, 850 kHz for an AM broadcasting station.

In communication problems, the information source signal is usually a baseband signal: for example, a transistor-transistor logic (TTL) waveform from a digital circuit, an audio signal from a microphone, or a video signal from a television camera. As described in Chapter 1, the communication engineer has the job of building a system that will transfer the information in this source signal $m(t)$ to the desired destination. As shown in Fig. 4-1, this usually requires the use of a bandpass signal, $s(t)$, which has a bandpass spectrum that is con-

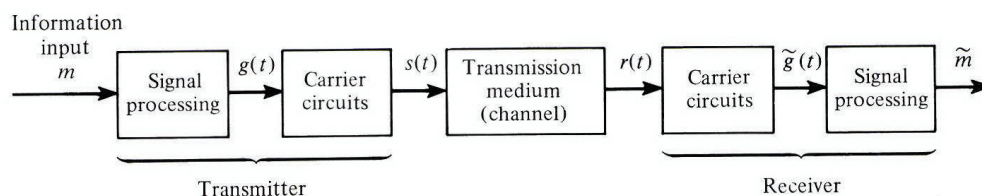


FIGURE 4-1 A communication system.

centrated at $\pm f_c$ so that it will propagate across the communication channel (either a software or a hardware channel).

Definition. *Modulation* is the process of encoding the source information onto a bandpass signal with a carrier frequency f_c . This bandpass signal is called the *modulated* signal $s(t)$, and the baseband source signal is called the *modulating* signal $m(t)$.

Examples of exactly how modulation is accomplished are given later in this chapter. The definition indicates that modulation may be visualized as a mapping operation that maps the source information onto the bandpass signal $s(t)$ that will be transmitted over the channel.

As the modulated signal passes through the channel, noise corrupts it. The result is a bandpass signal-plus-noise waveform that is available at the receiver input, $r(t)$ (see Fig. 4-1). The receiver has the job of trying to recover the information that was sent from the source. $\tilde{m}$ denotes the corrupted version of m .

Complex Envelope Representation

It is very interesting that *all* bandpass waveforms, whether they arise due to a modulated signal, interfering signals, or noise, may be represented in a convenient form given by the following theorem. $v(t)$ will be used to denote the bandpass waveform canonically; specifically, it can represent the signal when $s(t) \equiv v(t)$, the noise when $n(t) \equiv v(t)$, the filtered signal plus noise at the channel output when $r(t) \equiv v(t)$, or any other type of bandpass waveform.

Theorem. Any bandpass waveform may be represented by

$$v(t) = \text{Re}\{g(t)e^{j\omega_c t}\} \quad (4-1a)$$

where $\text{Re}\{\cdot\}$ denotes the real part of $\{\cdot\}$, $g(t)$ is called the *complex envelope* of $v(t)$, and f_c is the associated carrier frequency, $\omega_c = 2\pi f_c$. Furthermore, two other equivalent representations are[†]

$$v(t) = R(t) \cos[\omega_c t + \theta(t)] \quad (4-1b)$$

and

$$v(t) = x(t) \cos \omega_c t - y(t) \sin \omega_c t \quad (4-1c)$$

where

$$\begin{aligned} g(t) &= x(t) + jy(t) = |g(t)|e^{j\angle g(t)} \\ &\equiv R(t)e^{j\theta(t)} \end{aligned} \quad (4-2)$$

[†]The symbol $\equiv$ denotes an equivalence and the symbol $\triangleq$ denotes a definition.

and

$$x(t) = \operatorname{Re}\{g(t)\} \equiv R(t) \cos \theta(t) \quad (4-3a)$$

$$y(t) = \operatorname{Im}\{g(t)\} \equiv R(t) \sin \theta(t) \quad (4-3b)$$

$$R(t) \triangleq |g(t)| \equiv \sqrt{x^2(t) + y^2(t)} \quad (4-4a)$$

$$\theta(t) \triangleq \angle g(t) = \tan^{-1} \left(\frac{y(t)}{x(t)} \right) \quad (4-4b)$$

The waveforms $g(t)$, $x(t)$, $y(t)$, $R(t)$, and $\theta(t)$ are *all* baseband waveforms and, *except* for $g(t)$, they are all real waveforms. $R(t)$ is a nonnegative real waveform.

Proof. Any physical waveform (it does *not* have to be periodic) may be represented over all time, $T_o \rightarrow \infty$, by the complex Fourier series:

$$v(t) = \sum_{n=-\infty}^{n=\infty} c_n e^{jn\omega_o t}, \quad \omega_o = 2\pi/T_o \quad (4-5)$$

Furthermore, since the physical waveform is real $c_{-n} = c_n^*$ and using $\operatorname{Re}\{\cdot\} = \frac{1}{2}\{\cdot\} + \frac{1}{2}\{\cdot\}^*$,

$$v(t) = \operatorname{Re} \left\{ c_o + 2 \sum_{n=1}^{\infty} c_n e^{jn\omega_o t} \right\} \quad (4-6)$$

Introducing the arbitrary parameter f_c , this becomes[†]

$$v(t) = \operatorname{Re} \left\{ \left(c_o e^{-j\omega_c t} + 2 \sum_{n=1}^{n=\infty} c_n e^{j(n\omega_o - \omega_c)t} \right) e^{j\omega_c t} \right\} \quad (4-7)$$

so that (4-1a) follows where

$$g(t) \equiv c_o e^{-j\omega_c t} + 2 \sum_{n=1}^{\infty} c_n e^{j(n\omega_o - \omega_c)t} \quad (4-8)$$

Since $v(t)$ is a bandpass waveform with nonzero spectrum concentrated near $f = f_c$, the Fourier coefficients c_n are nonzero only for values of n in the range $\pm nf_o \approx f_c$. Therefore, from (4-8) $g(t)$ has a spectrum that is concentrated near $f = 0$. That is, $g(t)$ is a *baseband* waveform. It is also obvious from (4-8) that $g(t)$ may be a complex function of time. Representing the complex envelope in terms of two real functions in Cartesian coordinates, we have

$$g(t) \equiv x(t) + jy(t)$$

[†]Since the frequencies involved in the argument of $\operatorname{Re}\{\cdot\}$ are all positive, it can be shown that the complex function $c_o + 2 \sum_{n=1}^{\infty} c_n e^{jn\omega_o t}$ is analytic in the upper-half complex t plane. Many interesting properties result because this function is an analytic function of a complex variable.

where $x(t) = \text{Re}\{g(t)\}$ and $y(t) = \text{Im}\{g(t)\}$. $x(t)$ is said to be the *in phase modulation* associated with $v(t)$ and $y(t)$ is said to be the *quadrature modulation* associated with $v(t)$. Alternatively, the polar form of $g(t)$, represented by $R(t)$ and $\theta(t)$, is given by (4-2), where the identities between Cartesian and polar coordinates are given by (4-3) and (4-4). $R(t)$ and $\theta(t)$ are real waveforms and, in addition, $R(t)$ is always nonnegative. $R(t)$ is said to be the *amplitude modulation* (AM) on $v(t)$ and $\theta(t)$ is said to be the *phase modulation* (PM) on $v(t)$. It is also realized that if $v(t)$ is a deterministic waveform, $x(t)$, $y(t)$, $R(t)$, and $\theta(t)$ are also deterministic. If $v(t)$ is stochastic, for example representing bandpass noise, $x(t)$, $y(t)$, $R(t)$, and $\theta(t)$ are stochastic baseband processes. Thus, in general, bandpass noise includes both AM, $R(t)$, and PM, $\theta(t)$, noise components. This will be discussed further in Chapter 6. The usefulness of the complex envelope representation for bandpass waveforms cannot be overemphasized. In modern communication systems, the bandpass signal is often partitioned into two channels, one for $x(t)$ called the *I* (in-phase) channel and one for $y(t)$ called the *Q* (quadrature-phase) channel.

Bandpass Filtering

In Sec. 2-6 the general transfer function technique was described for the treatment of linear filter problems. Now a shortcut technique will be developed for bandpass filters. It is emphasized that this technique is applicable only for bandpass filters and waveforms. Here the complex envelope can also be used to describe bandpass filter characteristics. Since the impulse response of a bandpass filter $h(t)$ is a (deterministic) bandpass waveform, it may be represented by a complex envelope $k(t)$ as shown in Fig. 4-2.

Theorem. Referring to Fig. 4-2, the complex envelope out of a bandpass filter is given by

$$g_2(t) = \frac{1}{2}g_1(t)*k(t) \quad (4-9)$$

where $g_1(t)$ is the complex envelope of the input and $k(t)$ is the complex envelope of the impulse response. It also follows that

$$G_2(f) = \frac{1}{2}G_1(f)K(f) \quad (4-10)$$

This theorem may be proved by using the properties of linear systems (Sec. 2-6) and is given as a homework problem at the end of this chapter.

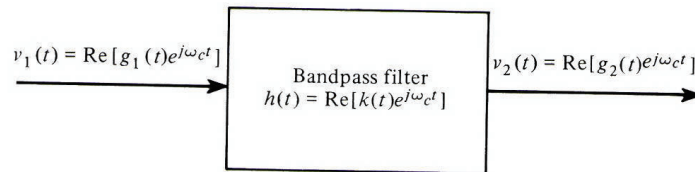


FIGURE 4-2 Filtering bandpass waveforms.