

frequency bands. As discussed in Section 7.8, the presence of surfaces near a source effectively increases its sound power output. Depending on its physical characteristics and frequency, a source may radiate uniformly in all directions (omnidirectional source) or more or less in certain directions (directional source). Source directivity is quantified by the directivity factor Q , which is the ratio of the mean-square sound pressure radiated in the receiver direction to the average of that radiated in all directions (see also Chapter 4). Small sources with three similar major dimensions can often be modeled as compact sources. Most real sources (large machines, conveyors, building walls) are more complex than a compact source, being extended in space in one or more dimensions. Readers wishing to do further reading on more fundamental aspects of room acoustics should consult reference 1. Throughout this chapter, references are provided for the benefit of readers who wish to do more in-depth reading on the topic under discussion.

7.1 ROOM SOUND FIELDS AND CONTROLLING FACTORS

Propagation of Sound

Sound waves propagate away from a source, being reflected (or scattered) by room boundaries, barriers, and furnishings. The resulting sound field at the receiver is composed of two parts. Sound which propagates directly from the source to the receiver is the direct sound. Its amplitude decreases with increasing distance from the source, as in a free field (discussed in Chapters 1 and 2). Sound which reaches the receiver after reflection from room surfaces or furnishings is the reverberant sound. Its amplitude decreases with increasing source distance at a rate which depends on the source geometry and on the acoustical properties of the room and its contents, as will be discussed below. To examine sound propagation in rooms, independent of the source sound power, to which it is directly related, we define the sound propagation function $SPF(r) = L_p(r) - L_w$ in decibels as the variable characterizing the effect of the room alone on the sound field. Sound pressure levels may then be obtained using $L_p(r) = SPF(r) + L_w$. Thus, from Eqs. (1.27)–(1.31), for a compact source with $Q = 1$ in a free field, $SPF(r) = -20 \log_{10} r - 11$ dB. In the case of multiple sources, the individual source contributions are obtained from the source sound power levels and the room sound propagation function values at the relevant source/receiver distances. The total sound pressure level at the receiver position is obtained by summing the individual levels on a power (p^2) basis.

Sound Decay

Shortly after a steady-state sound source begins to radiate, an equilibrium (or steady state) is established between the rate of energy absorption in the room and the rate of energy emission by the source. If the source then ceases to radiate, the energy in the room decreases with time at a rate determined by the rate of energy absorption. This is the room sound decay. It is usually characterized by

the reverberation time T_{60} , which is the time in seconds required for the sound pressure level to decrease by 60 dB. Calculations of T_{60} are based on the average rate of decay over some part of the decay curve—usually the -5 - to -35 -dB part. Also of interest, since it quantifies perceived reverberance, is the early-decay time (EDT), based on the average rate of decay over the first 10 dB.

Air Absorption

Energy is absorbed continuously as sound propagates in air. This process follows an exponential law, $E(r) = E_0 e^{-2mr}$, where the constant $2m$ in the exponent is called the energy air absorption exponent, expressed in nepers per meter (1 Np = 8.69 dB), with $e = 2.7173$. Air absorption depends on air temperature, relative humidity, ambient pressure, and sound frequency. Table 7.1 presents some typical values calculated using reference 2.

Surface Absorption and Reflection

The ability of a surface to absorb incident sound energy is characterized by the energy absorption coefficient α . This is the fraction of the energy striking the surface that is not reflected, as discussed in more detail in Chapters 8 and 11. It is usually measured in octave- or third-octave bands. If E_i is the energy incident on a surface with absorption coefficient α , the reflected energy is $E_r = E_i(1 - \alpha)$. Ignoring transmission, acoustical energy not absorbed by a surface that it strikes is reflected. If the surface is flat, hard, and homogeneous, reflection is specular; that is, the angle of reflection is equal to the incident angle. In practice, because of finite absorber size, surface roughness, as well as physical or impedance discontinuities, the energy is scattered or diffused (reflected into a range of angles). Such diffuse reflections usually result in a more rapid decay of sound and lower reverberation times in the room.

TABLE 7.1 Energy Air Absorption Exponents ($2m$ in Terms of 10^{-3} Np/m) Predicted Using Reference 2 Assuming Ambient Pressure of 101.3 kPa

Temperature, °C	Relative Humidity, %	Air Absorption Exponents at Selected Frequencies						
		125 Hz	250 Hz	500 Hz	1000 Hz	2000 Hz	4000 Hz	8000 Hz
10	25	0.1	0.2	0.6	1.7	5.8	18.7	43.0
	50	0.1	0.2	0.5	1.0	2.8	9.8	33.6
	75	0.1	0.2	0.5	0.9	2.0	6.5	23.6
20	25	0.1	0.3	0.6	1.2	3.5	12.4	41.6
	50	0.1	0.3	0.7	1.2	2.3	6.5	22.4
	75	0.1	0.2	0.6	1.3	2.2	5.0	15.8
30	25	0.1	0.4	0.9	1.5	3.0	8.3	28.8
	50	0.1	0.2	0.8	1.7	3.0	5.8	16.4
	75	0.0	0.2	0.6	1.7	3.3	5.8	13.4

Furnishings

Many rooms are not empty, containing furnishings (the various obstacles, such as desks in a classroom or machines in a workshop, also called fittings). Often furnishings occupy the lower region of a room and there are fewer objects at higher elevations. Of course, the density and horizontal distribution of room furnishings may vary considerably. Acoustical energy propagating in furnished regions is scattered as well as partially absorbed, significantly modifying the sound field. In particular, it increases the rate of sound decay, reducing reverberation times. It also causes a redistribution of steady-state sound energy toward sources, due to back scattering, resulting in higher sound pressure levels near sources and a higher rate of decrease of levels with distance from sources.

7.2 DIFFUSE-FIELD THEORY

By far the best known theoretical models for predicting room sound fields are based on diffuse-field theory. Diffuse-field theory is widely applied because of its simplicity. Often forgotten is the fact that it may be of limited applicability because of its restrictive assumption of a diffuse field. The sound field in a room is diffuse if it has the following attributes:

1. At any position in the room, energy is incident from all directions with equal intensities (and random phase, though we are ignoring phase in this chapter).
2. The reverberant sound does not vary with receiver position.

These conditions are approximated only in specially designed acoustical test rooms called reverberation chambers, discussed in Section 7.8.

Average Diffuse-Field Surface Absorption Coefficient

Diffuse-field theory uses the average rate of random-incidence surface sound absorption averaged over all of the room surfaces. Thus, we define the average diffuse-field surface absorption coefficient as

$$\bar{\alpha}_d = \frac{\sum S_i \bar{\alpha}_{di}}{\sum S_i} \quad (7.1)$$

where S_i and α_d are the surface area and diffuse-field absorption coefficient of the i th surface. If α_d is to be useful, it is necessary that no part of the room be strongly absorbing, since in this case a diffuse sound field cannot exist. Absorbing objects such as seats, tables, and people must be included when calculating α_d , despite the fact that such objects have ill-defined surface areas. In such cases it is common practice to assign an absorption A_i in square meters to each object, where $A_i = \alpha_d S_i$, with S_i the surface area. The absorptions of all objects are

summed, and the total absorption is included in the calculation of α_d , with no modification made to the total area. In other words, the total area $\sum S_i$ in Eq. (7.1) is taken to be that of the room boundaries, excluding objects and people. In the case of closely spaced absorbers, such as auditorium seats or suspended ceiling baffles, caution is necessary, since the total absorption may depend on the total area covered and may not be the sum of the absorptions of the individual objects. Also, the absorption coefficients of small patches of absorbing material (e.g., as measured in a reverberation room) are higher than those of a large area of the material (e.g., covering all of a room surface), due to the effects of diffraction at the material's edges. In the former case, the absorption coefficients of highly absorptive materials can exceed 1.0 (see also Section 7.7).

Sabine and Eyring Approaches

Several diffuse-field approaches exist; two are considered here. Both are expressed by the following equations, where Eq. (7.3) is applicable only for a compact source:

$$T_{60} = \frac{55.3V}{cA} \quad (\text{sound decay}) \quad (7.2)$$

$$L_p(r) = L_w + 10 \log_{10} \left(\frac{Q}{4\pi r^2} + \frac{4}{R} \right) \quad (\text{steady-state conditions}) \quad (7.3)$$

where T_{60} = reverberation time, s

L_w = source sound power level, dB re 10^{-12} W

L_p = sound pressure level, dB re 2×10^{-5} N/m²

c = speed of sound in air, ≈ 344 m/s at standard temperature and pressure (STP), m/s

R = room constant, $= A/(1 - \alpha_d)$, m²

A = total room absorption, $= -S \ln(1 - \alpha_d) + 4mV$ (Eyring)
 $\approx \alpha_d S + 4mV$ (Sabine), m²

r = source/receiver distance, m

Q = directivity factor

$2m$ = energy air absorption coefficient, Np/m

V = room volume, m³

α_d = average diffuse-field surface absorption coefficient

S = total room (surface and barrier) surface area, m²

The Eyring approach is applicable to rooms with arbitrary average surface sound absorption coefficient. It should be used, for example, to determine absorption coefficients of room surfaces from measured reverberation times. If, however, the average surface absorption coefficient is sufficiently low (say, $\alpha_d < 0.30$), it is accurate to use the Sabine approach; that is, Eqs. (7.2) and (7.3) with $-\ln(1 - \alpha_d)$ replaced by α_d . Thus, the Sabine approach can be used when determining the

sound absorption coefficients of materials in reverberation rooms. Of course, the approach applied in a particular sound field prediction must be consistent with that used to obtain the sound absorption coefficient data used in the prediction. Refer to reference 1 for further discussion of the relative merits of the Sabine, Eyring, and other diffuse-field theories.

Diffuse-field theory predicts the following characteristics of the sound field:

1. *Sound Decay/Reverberation Time.* After a sound source ceases to radiate, the mean-square pressure at the receiver decays exponentially with time. The corresponding sound pressure level decreases linearly with time. To a first approximation, the reverberation time is directly proportional to V/S and inversely proportional to α_d .
2. *Steady State.* The total field is the sum of direct and reverberant components described, respectively, by the first and second terms in parentheses of Eq. (7.3). The direct field, which dominates near the source, is independent of the room's properties. Its sound pressure level decreases at 6 dB/dd (dd = distance doubling). The reverberant field, which dominates far from the source, does not vary with source/receiver distance. Its sound pressure level is, to a first approximation (when both α_d and m are small), inversely proportional to α_d and S . The sound propagation function is as shown in Fig. 7.1.

Note that diffuse-field theory accounts for only some of the relevant room acoustical parameters and for some of these in an approximate manner. In particular, room geometry and source directivity are modeled only approximately. Neither the distribution of the surface absorption nor the presence of barriers or furnishings is modeled. This and the related fact that the theory is based on restrictive hypotheses seriously limit its applicability. To give one concrete example, the constant

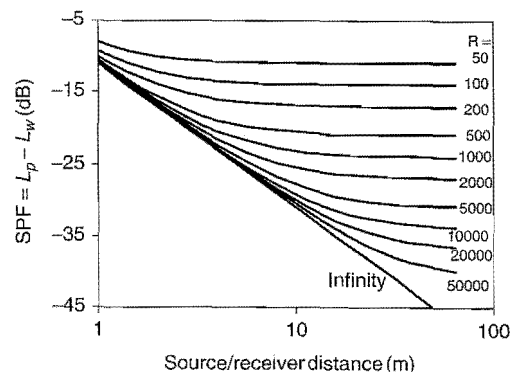


FIGURE 7.1 Sound propagation function curves predicted by diffuse-field theory for various values of the room constant R in m^2 .

sound pressure level at sufficiently large distances from sources that is predicted by diffuse-field theory is only (approximately) found in practice in small rooms with low surface absorption (e.g., reverberation rooms, discussed in Section 7.8). Noise levels due to a single source in most rooms—especially large or absorptive ones—decrease monotonically with distance from the source (see, e.g., Fig. 7.4b below). Reference 3 discusses the applicability of diffuse-field theory more fully.

Diffuse-field theory, when applicable, can be used to estimate the reductions of reverberation time and noise level that occur when absorptive material is added to a room. Let A_b and A_a be the total room absorptions before and after treatment; $A_a = A_b + \Delta A$, where ΔA is the increase in room absorption. If A_b and ΔA are known, values of A_b and A_a can be used directly in Eqs. (7.2) and (7.3). Alternatively, if $T_{60,b}$ before treatment is known, $T_{60,a}$ after treatment can be determined from Eq. (7.2) as follows: since $T_{60,b} = 55.3V/(cA_b)$, then $A_b = 55.3V/(cT_{60,b})$. Now, $T_{60,a} = 55.3V/(cA_a) = 55.3V/[c(A_b + \Delta A)]$, and thus

$$T_{60,a} = \frac{55.3V}{c \{ [55.3V/(cT_{60,b})] + \Delta A \}} \quad (7.4)$$

The steady-state sound pressure level $L_{p,a}$ after treatment can be calculated from that before treatment, $L_{p,b}$, as follows:

$$L_{p,a}(r) = L_{p,b}(r) + 10 \log_{10} \left[\frac{Q/(4\pi r^2) + 4/R_a}{Q/(4\pi r^2) + 4/R_b} \right] \quad (7.5)$$

At positions near sources, where the direct field dominates, the reduction in sound pressure level is small. At large distances from all sources, where the reverberant field dominates, the reduction approaches that of the reverberant sound pressure level alone. This, to a first approximation (when both α_d and m are small), is given by $10 \log_{10}(A_a/A_b)$. Doubling the room absorption reduces the sound pressure level of the reverberant field by 3 dB. Similarly, using Eq. (7.2), the reverberation time after treatment is $T_{60,a} = T_{60,b}(A_b/A_a)$. Doubling the sound absorption halves the reverberation time.

As an example, consider a room with dimensions $10 \text{ m} \times 5 \text{ m} \times 2.4 \text{ m}$ (volume $V = 120 \text{ m}^3$, surface area $S = 172 \text{ m}^2$) and all concrete surfaces with $\alpha_b = 0.05$. Thus, ignoring air absorption and using the Sabine approach, we have $A_b = \alpha_b S = 0.05 \times 172 = 8.6 \text{ m}^2$ and $T_{60,b} = 55.3V/(cA_b) = 55.3 \times 120/(344 \times 8.6) = 2.2 \text{ s}$. A suspended acoustical ceiling with $\alpha_c = 0.9$ and surface area of 50 m^2 is installed to improve the acoustical environment. After treatment, $A_a = 8.6 + (0.9 - 0.05) \times 50 = 51.1 \text{ m}^2$, so $T_{60,a} = 2.2 \times 8.6/51.1 = 0.4 \text{ s}$.

7.3 OTHER PREDICTION APPROACHES

When diffuse-field theory cannot be applied, alternative approaches can be used to predict the steady-state sound pressure level, sound decay, and reverberation times in a room. These models apply only to compact sources. Extended

sources must be approximated by an array of compact sources and the prediction model applied to each source. Two main approaches are available to accomplish this objective: computer algorithms (the method of image sources, ray or beam tracing, radiosity, hybrid models, etc.) and empirical models. The different approaches make assumptions which define and limit their applicabilities and accuracies. Furthermore, they take the various room acoustical parameters into account to a greater or lesser extent. To take advantage of the strengths of the different approaches and avoid their weaknesses, hybrid models combining several approaches into one model have been developed. Empirical prediction models are usually limited in application to specific building types.

Method of Image Sources

The method of image sources is based on the assumption that reflections from surfaces, assumed specular, can be replaced by the direct sound contributions of image sources. Figure 7.2 shows the simple example of a partial two-dimensional array of image rooms and sources. The simplest implementation of the method of image sources applies to empty, rectangular-parallelepiped rooms with no barriers.⁴ In this case, the image sources corresponding to the infinite number of reflections are located on a three-dimensional grid. The total mean-square pressure is the sum of the contributions of all of the image sources, allowing for spherical divergence and energy losses due to absorption by the air and at each surface encountered by the ray from the image source. For decaying sound, the sound pressure at time t after the cessation of sound generation is calculated by summing over all image sources located at distances greater than ct from the receiver, where c is the speed of sound. The method of image sources can be extended to empty rooms of arbitrary shape bounded by planes; however, calculation times are greatly increased and become impracticable. The method of image sources can also account for the presence of furnishings, under the assumption that they are isotropically distributed in random fashion throughout

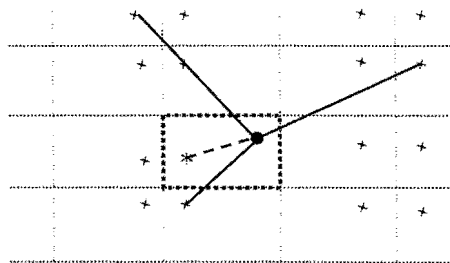


FIGURE 7.2 Image space of an empty two-dimensional room, showing the source (*), and receiver (●), the image sources (*), the image surface planes (.....), the direct sound (- - -), and the propagation paths of sound from three of the image sources (—).

the room.⁵ In this case, the furnishings, as well as the sources, are mirrored in the room surfaces. The steady-state and decaying sound fields are formed as before. However, the contribution of each image source is modified due to scattering and absorption by the furnishings; absorption at surfaces is also increased due to scattering.

Ray and Beam Tracing

Ray-tracing techniques⁶ can be used to predict sound fields in rooms of arbitrary shape with arbitrary surface absorption distributions, different surface reflection properties, and variable furnishing density. Source directivity can also be modeled. A computer program simulates the emission of a large number of rays (or beams) from each source in either random or deterministic fashion. Each ray is followed as it propagates in the room, being reflected and scattered by surfaces, barriers, and furnishings until it reaches a receiver position. The energy of a ray is attenuated according to spherical divergence as well as surface, furnishing, and air absorption. In principle, any surface reflection law (specular, diffuse, etc.) can be modeled. Ondet and Barbry⁷ proposed an algorithm for accounting for quasi-arbitrary furnishing distributions in a ray-tracing model. Figure 7.3, taken from reference 8, illustrates the potential of ray-tracing techniques. It shows the contour map of the predicted reductions of total A-weighted noise levels over the floor of a furnished workshop containing eight machinery sources, which result from the introduction of an acoustical barrier around a noise-sensitive assembly bench and a partial ceiling absorption treatment suspended over the region of the sources. Of course, the program must be run for each frequency band (i.e.,

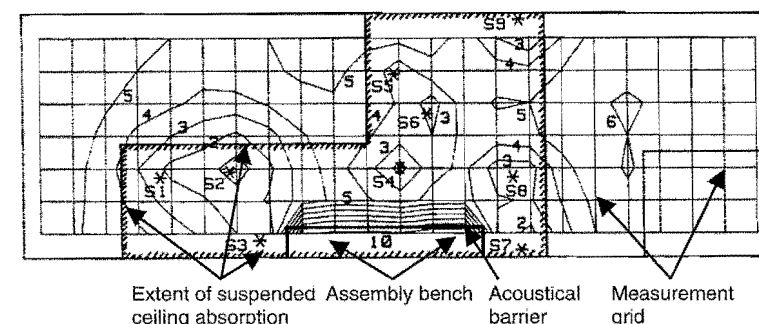


FIGURE 7.3 Floor layout of a furnished workshop (dimensions 46.0 × 15.0 × 7.2 m) showing a contour map of the predicted reductions of total A-weighted noise levels which result from the introduction of an acoustical barrier around a noise-sensitive assembly bench and of an absorptive treatment suspended below the ceiling over the region of the eight machinery sources (the shaded, L-shaped, enclosed area). The map was produced from noise levels predicted before and after treatment at positions on an imaginary 2 × 2-m grid, 1.5 m above the floor, as shown. Also shown are the noise sources (e.g., S4*) and the contour lines and their values (e.g., 5).

octave- or third-octave bands) to account for the frequency-dependent nature of the sound absorption coefficient of the room surfaces and that of the barrier insertion loss. This kind of detailed analysis is not possible using diffuse-field theory. It would only be possible to estimate noise reductions from the increase in absorption. These reductions could then be corrected as a function of the proximity of the receiver to the acoustical treatment and of the estimated insertion loss of the barrier.

Empirical Models

Empirical models are developed from experimental data. They usually focus on a particular category of room—models for offices, classrooms, and industrial workshops are discussed below. Measurements are made, in a number of rooms of some category, of the acoustical quantities to be predicted. Empirical algorithms or equations which predict the experimental results from appropriate predictor parameters describing the rooms are developed. This can, for example, involve multivariable regression modeling, whereby statistical modeling software is used to find the minimum set of mutually statistically independent predictors, each of which is highly correlated with the data to be predicted, that most accurately predicts the measured data. For further information on multivariable regression modeling, consult reference 9.

Auralization

Auralization is the process of rendering audible sounds “played” in a virtual room for which the acoustical response has been predicted. It allows subjective evaluation of the room. If done accurately, auralization allows full three-dimensional sound perception, with source externalization (i.e., the sound is perceived to be outside the head), spatial localization, and spaciousness. Auralization involves predicting the combined response of a virtual room and a virtual listener in it. The listener response is quantified by head-related transfer functions (HRTFs) quantifying how the acoustical response of the human external auditory system varies with frequency and angle. The room response is predicted using models, such as ray tracing, which predict the individual sound path contributions arriving at the receiver; since impulse responses involving phase effects are required, the room’s phase response must also be considered. The result is the binaural impulse response (comprising the impulse responses at the left and right eardrums) of the room–listener combination. This is convolved with a sound signal for replay to a real listener. Readers interested in further details of this technique should consult reference 10.

7.4 DOMESTIC ROOMS AND CLOSED OFFICES

Schultz¹¹ investigated the prediction of sound pressure levels in domestic rooms and closed offices. He measured the variation of sound pressure level with distance from a source in a variety of small rooms, observing that levels never

become constant at larger distances as predicted by diffuse-field theory. In fact, he found that the curves always had a slope of about -3 dB/dd, though the absolute levels of the curves varied considerably. He further found that existing models developed for predicting sound propagation in, for example, workshops and corridors could not predict his experimental results. Schultz therefore proposed the following empirical formula:

$$L_p(r) = L_w - 10 \log_{10} r - 5 \log_{10} V - 3 \log_{10} f + 12 \text{ dB} \quad (7.6)$$

where r is the source–receiver distance in meters, V is the room volume in cubic meters, and f is the frequency in hertz. Note that this formula does not explicitly contain a room absorption term. Such behavior contrasts markedly with that predicted by diffuse-field theories and is an excellent example of their limitations.

7.5 CLASSROOMS

Relevance and Characteristics

Classrooms are acoustically critical spaces in which verbal communication is crucial for teaching and learning. Acoustically, conference rooms, gymnasiums, and other rooms for speech have much in common with classrooms. Many classrooms are built with little or no attention paid to the acoustical design. Studies have shown that nonoptimal acoustical conditions in classrooms result in impaired verbal communication between teachers and students, impaired student language development and learning, and teacher voice and other problems.^{12,13} The problems are particularly acute for “acoustically challenged” listeners, including young and hearing-impaired people and those using a second language.

Classrooms vary from small seminar rooms for a few occupants to school classrooms for several tens of children to larger university lecture rooms and auditoria accommodating hundreds of listeners. Smaller classrooms are usually of rectangular shape. Larger lecture rooms can have, for example, fan plan shape, inclined seating, and nonflat ceiling profiles. In smaller classrooms, talkers and listeners can be anywhere in the classroom, and source–receiver distances can vary from less than a meter to several meters. In lecture rooms, the talker is usually at one end of the room, with the listeners spread out in front; source–receiver distances can vary from several meters to several tens of meters. For hygiene reasons, classrooms may have hard, nonabsorptive surfaces, though carpets and wall and ceiling absorption are not uncommon. Lecture and conference rooms can have nonabsorptive or padded, sound-absorptive seating. Of course, the occupants themselves contribute significant absorption to the classroom. This and the fact that classroom occupancy can vary considerably must be considered in the acoustical design. A detailed discussion of the physical and acoustical characteristics of (university) classrooms is found in reference 14. Of particular interest is the rate at which speech and noise levels decrease with distance from their sources,

which is small in small, nonabsorptive classrooms. It increases with classroom size and absorption and can be large in large, absorptive (e.g., occupied) classrooms. One consequence of this is that speech levels in large classrooms may decrease by 10 or more decibels from the front to the back of the seating area.

Speech Intelligibility

In classrooms, as in other rooms for speech, quality and ease of verbal communication are prime concerns (see also Section 7.7). The quality can be quantified in human terms by speech intelligibility, the percentage of words correctly identified by the listener. Here, in general, we consider the effect of the room on the accurate transmission of speech signals from a talker to a listener and not, for example, the effects of individual talker and listener characteristics. Verbal communication is considered to be affected by two main factors—the classroom reverberation and the speech-signal-to-background-noise-level difference (often called the signal-to-noise ratio). The effect of reverberation can be quantified by the early-to-late energy ratio C_{50} (which is highly correlated with early decay time and reverberation time in many cases). This frequency-dependent ratio is usually measured in octave bands. The signal-to-noise level difference depends on the levels of speech and the levels of noise at the listener position. Total A-weighted or octave- or third-octave-band equivalent continuous levels are often considered. If the voice levels remain constant, the speech intelligibility decreases with increased reverberation and increases with increased signal-to-noise level difference. A number of physical metrics exist which combine these two quantities into a single speech intelligibility measure; these include the useful-to-detrimental energy ratio U_{50} , articulation index AI (discussed in Section 7.7), speech transmission index STI, and speech intelligibility index SII.

Numerous studies have found that many classrooms have nonoptimal acoustical conditions, with excessive noise levels and reverberation. A review of published data and of the many complex issues associated with speech intelligibility in classrooms is contained in reference 15. The classroom speech sources are the teachers' and the students' voices. Classroom noise sources include mechanical services (e.g., ventilation outlets), classroom equipment (projectors, computers), and the teachers' or students' voices when another person is generating the signal to be heard. Noise breaking into the classroom from outside can be significant when the classroom is located near transportation (such as highways and airports) or in cases when children are active in nearby corridors or play areas. Finally, classroom activity itself generates significant noise, including impact noise from furniture, toys, and so on. Data on the typical sound pressure levels generated by human talkers and classroom noise sources are available.¹⁶ It is likely that a number of complex factors, including room acoustics, affect what these levels are at listener positions in a given classroom at a given time.

It is generally considered that, for excellent speech intelligibility, background noise levels should not exceed about 40 dBA for acoustically unchallenged listeners and 30 dBA for acoustically challenged listeners. The question of what is

the optimal reverberation for speech intelligibility is a complex one. One body of opinion considers that reverberation should always be minimized. However, this is based on the results of speech intelligibility tests that were done under conditions that did not account for the acoustical behavior of classrooms. Investigations involving the useful-to-detrimental energy ratio speech intelligibility metric and realistic room acoustical modeling show that some reverberation can be beneficial (since it increases reverberant speech levels). The results indicate that the reverberation should be optimized as a function of the classroom volume and background noise. Optimal reverberation times vary from low values in small, quiet classrooms to over 1 s in large, noisy ones. For a full discussion of this issue, refer to reference 17. In any case, reverberation should be minimized in classrooms with high signal-to-noise level differences (e.g., resulting from voice amplification). Moreover, it is likely that less reverberation can be tolerated by acoustically challenged listeners. It has been shown that speech intelligibility is not very sensitive to variations in reverberation, and it has been suggested that a classroom reverberation time of about 0.5 s is usually appropriate.¹⁸

Standards and guidelines exist to help design professionals achieve high acoustical quality in classrooms and other educational spaces. They specify the acoustical conditions to be achieved in new and renovated rooms. They discuss reverberation times and background noise levels to be achieved in the unoccupied room. They also provide advice on how to achieve the design targets. Two recent examples of such standards are ANSI S12.60—2002 in the United States¹⁹ and Building Bulletin 93 in the United Kingdom.²⁰

Predicting Classroom Acoustics

Predicting acoustical quality in a classroom involves predicting room reverberation and steady-state sound pressure levels generated by the speech and noise sources, including student activity. Prediction should take into account the absorption contributed by the room occupants and furnishings (e.g., absorptive seating). This involves an appropriate prediction model and accurate input data. In the case of small, untreated classrooms, it is likely that diffuse-field theory is reasonably accurate. For treated and larger classrooms, comprehensive techniques such as ray tracing can be used. Empirical models are also available and inherently incorporate realistic data. Based on measurements of reverberation times and steady-state levels generated by a speech source in a wide variety of typical university classrooms and on published information, empirical models for predicting octave-band reverberation times and total A-weighted speech levels have been developed.²¹ Approximate values of $T_{60,u}$ in the unoccupied classroom can be determined using Eq. (7.2), with the average surface absorption coefficient calculated as the sum of contributions (presented in Table 7.2) to the room average value, which are associated with the sound-absorbing features present in the classroom. An alternative empirical formula for predicting the 1-kHz octave-band $T_{60,u}$ is

$$T_{60,u} = 0.874 + 0.0021(LW) + 0.303\text{refl} + 0.412\text{basic} \\ - 0.384\text{absorb} - 0.804\text{upseat} \quad (7.7)$$

TABLE 7.2 Typical Octave-Band Contributions of Classroom Sound-Absorptive Features to Room-Average Surface Absorption Coefficient

Surface Feature	125 Hz	250 Hz	500 Hz	1000 Hz	2000 Hz	4000 Hz	8000 Hz
Basic	0.12	0.10	0.10	0.09	0.10	0.09	0.10
Carpeted floor	0.00	0.00	0.02	0.04	0.07	0.09	0.17
Wall/ceiling absorption	0.01	0.04	0.08	0.10	0.10	0.10	0.10
Upholstered seats	0.20	0.20	0.16	0.12	0.09	0.07	0.05

TABLE 7.3 Octave-Band Values of Average Absorption per Person

Band, Hz	125	250	500	1000	2000	4000	8000
A_p , m ²	0.25	0.45	0.67	0.81	0.82	0.83	1.14

where the parameters are defined following Eq. (7.10). The absorption of the classroom occupants can be included using the data in Table 7.3, and the number of occupants can be used to estimate the associated increase in classroom absorption. For an average talker speaking between a normal and a raised voice (total A-weighted sound power level ≈ 74 dBA), total A-weighted speech levels in the unoccupied classroom can be predicted from

$$SLA_u(r) = I_u + s_u \frac{\log_{10}(r)}{\log_{10}(2)} \quad (7.8)$$

with

$$I_u = 65.8 - 0.0105(LW) + 1.52\text{fwdist} - 1.41\text{absorb} - 4.32\text{upseat} \quad (7.9)$$

and

$$s_u = -1.21 - 0.088 L + 1.14\text{basic} \quad (7.10)$$

where $T_{60,u}$ = 1-kHz octave-band reverberation time, s

SLA_u = total A-weighted speech level, dBA

I_u = intercept of total A-weighted sound propagation curve, dBA

s_u = slope of total A-weighted sound propagation curve, dBA/dd

r = source-receiver distance, m

L = average classroom length, m

W = average classroom width, m

fwdist = distance from source to nearest wall, m

refl = 1 if classroom contains beneficial reflectors, = 0 if not

absorb = factor quantifying extent of sound-absorptive wall or ceiling treatment

absorb = 1 corresponds to full-coverage wall or ceiling treatment; in other cases, values should be scaled proportionately

upseat = 0 for nonabsorptive seating; = 1 for padded, sound-absorptive seating

basic = 1 if all classroom surfaces and seats are not sound absorptive; = 0 otherwise

As an example, consider a large classroom with average dimensions of $L = 20$ m long, $W = 10$ m wide, and 6 m high (volume $V = 1200$ m³, surface area $S = 760$ m²). The distance from the typical lecturing position to the nearest (front) wall is fwdist = 2.5 m. The classroom has a profiled ceiling directing sound to the back (i.e., beneficial reflectors, refl = 1). Consider the cases where N_p is 50 or 200 occupants. Before treatment, the classroom has no sound-absorptive surfaces (absorb = 0, basic = 1) and nonabsorptive seating (upseat = 0). Thus, the average surface absorption coefficient is that corresponding to the "basic" configuration in Table 7.2 ($\alpha = 0.09$). Equation (7.2) gives the corresponding $T_{60,u}$'s. Assuming air absorption exponents for a temperature of 20°C and a relative humidity of 50% ($m = 0.0012$ Np/m; see Table 7.1) and using the Eyring approach, the total absorption in the unoccupied room at 1 kHz is $A_u = -S \ln(1 - \alpha) + 4mV = -760 \ln(1 - 0.09) + 4 \times 0.0012 \times 1200 = 77.4$ m². From Eq. (7.2) the 1-kHz $T_{60,u} = 55.3V/(cA_u) = 55.3 \times 1200/(344 \times 77.4) = 2.49$ s. With 50 occupants, the total room absorption $A_o = A_u + N_p A_p = 77.4 + 50 \times 0.81 = 117.9$ m², and $T_{60,o} = 55.3V/(cA_o) = 55.3 \times 1200/(344 \times 117.9) = 1.64$ s. Similarly, with 200 occupants $T_{60,o} = 0.81$ s. Here, T_{60} is excessive with low occupancy and decreases sharply with the number of occupants. To control the reverberation, the classroom is treated with full-coverage wall absorption (absorb = 1) and padded, sound-absorptive seating (upseat = 1); thus, basic = 0. The average surface absorption coefficients are now the sum of the values for basic, wall/ceiling absorption, and upholstered seats given in Table 7.2 (e.g., $0.09 + 0.10 + 0.12 = 0.31$ at 1 kHz). The 1-kHz T_{60} 's in the classroom when unoccupied and with 50 or 200 occupants are 0.67, 0.63, and 0.52 s. The treatment significantly reduces T_{60} —in the unoccupied classroom by as much as 200 occupants do—and decreases the variation of T_{60} with the number of occupants.

As for speech levels, consider listeners seated at the front and back of the classroom at distances of 3 and 12 m from the talker. Before treatment, from Eq. (7.9), $I_{u,b} = 65.8 - 0.0105LW + 1.52\text{fwdist} - 1.41\text{absorb} - 4.32\text{upseat} = 65.8 - 0.0105 \times 20 \times 10 + 1.52 \times 2.5 - 1.41 \times 0 - 4.32 \times 0 = 67.5$ dBA and, from Eq. (7.10), $s_{u,b} = -1.21 - 0.088L + 1.14\text{basic} = -1.21 - 0.088 \times 20 + 1.14 \times 1 = -1.83$ dBA/dd. Using Eq. (7.8), the speech level at any position is $SLA_{u,b}(r) = 67.5 - 1.83 \log_{10}(r)/\log_{10}(2)$; thus, at 3 and 12 m speech levels are 64.6 and 61.0 dBA. Levels in the occupied classroom can be calculated using Eq. (7.5). Assuming $Q = 2$ to the front of a talker, at the front and back of the classroom speech levels are 63.1 and 59.2 dB with 50 occupants

and 60.9 and 56.2 dBA with 200 occupants. After treatment, $I_{u,a} = 65.8 - 0.0105 \times 20 \times 10 + 1.52 \times 2.5 - 1.41 \times 0 - 4.32 \times 1 = 63.2$ dBA and $s_{u,a} = -1.21 - 0.088 \times 20 + 1.14 \times 0 = -3.0$ dBA/dd, giving speech levels of 57.1 and 51.1 dBA unoccupied at the front and back seats; occupants decrease levels by no more than 1 dB. Acoustical treatment causes speech levels at the front and back of the unoccupied classroom to decrease by 7.5 and 9.9 dBA for 50 occupants and with 200 occupants by 4.5 and 6.1 dB. These results must be interpreted with care, as they do not account for certain relevant nonacoustical factors. For example, they assume that the talker's vocal output is invariant, as would be the case for a loudspeaker. In fact, research suggests that a talker's vocal output varies with the prevailing classroom acoustical conditions; a preliminary empirical model for predicting this phenomenon is presented in reference 16.

Noise levels generated by classroom noise sources (e.g., projectors or ventilation outlets) can be estimated from their output power levels and receiver distances using the above empirical model after adjusting for the difference between the actual and the assumed output power levels. Student activity noise should also be considered; a preliminary empirical model for predicting this is presented in reference 16. Then signal-to-noise level differences can be determined. Classroom early-decay times and signal-to-noise level differences can then be used to determine values of speech intelligibility metrics.

Controlling Classroom Sound

Controlling and optimizing the acoustical conditions in a classroom or other rooms for speech involve three fundamental considerations:

1. *Promoting High Speech Levels.* Avoid excessive classroom cubic volume due, for example, to high and vaulted ceilings. Use room geometries which direct sound to the back of the room. In large lecture rooms, this can include angled reflectors around teaching areas and profiled ceilings. Keep at least the central part of the ceiling sound reflective to promote the reflection of speech sounds to the back of the classroom. Use approximately square floor plans to avoid long and wide rooms. Amplification by a speech reinforcement system or a sound field enhancement system may be an option; their design is beyond the scope of this chapter; see reference 22. One important issue to consider at the classroom design stage is that the optimal acoustical conditions for unaided speech may not be the same as with a speech reinforcement system.

2. *Controlling Background Noise.* Avoid open-plan design. Control the noise and vibration of mechanical services (see Chapter 16). Locating ventilation outlets toward the front of lecture rooms where speech levels are highest helps optimize signal-to-noise level differences throughout the room. Avoid high terminal velocities of supply air terminal devices and place air-volume control devices at distances of 0.5 m or more upstream to minimize noise generated by turbulent flow. Choose quiet equipment for use in the classroom. Impact noise due to student activity can be reduced by the use of carpets and cushioning materials

(split tennis balls on desk, table, and chair legs are widely used). The partitions bounding the classroom must provide adequate sound isolation (see Chapter 10); in critical cases, this might require the use of nonopenable windows.

3. *Optimizing Reverberation.* Apply appropriate sound-absorptive materials to the room surfaces. Avoid applying sound absorption to the central part of the ceiling, which provides useful reflections between talkers and listeners. Using sound-absorptive seating allows the ceiling to be left reflective and reduces the sensitivity of the classroom's acoustical conditions to the number of occupants.

7.6 INDUSTRIAL WORKSHOPS

Relevance and Characteristics

Industrial workshops often have serious acoustical problems due to excessive noise and reverberation. Most jurisdictions have regulations aimed at limiting the risk of hearing damage by limiting the noise exposure of industrial workers. Excessive reverberation can also lead to poor verbal communication and a reduced ability to identify warning signals, and thus danger, as well as to stress and fatigue.

Industrial buildings come in every shape and size. However, many are rectangular in floorplan (with widely varying dimension ratios and floorplan sizes), with flat or nonflat (e.g., pitched or sawtooth) roofs. The floors of most workshops are made of concrete. The walls are often of brick or blockwork, sometimes of metal cladding. Workshop roofs are usually of suspended-panel construction, consisting of metal or other panels supported by metal trusswork or portal frames. A common modern construction is the steel deck, consisting of, for example, profiled metal as the internal surface, a vapor barrier, several centimeters of thermal insulation, tar paper, and gravel ballast as the external surface, again supported by metal trusswork. Acoustical steel decks exist; the inner metal layer is perforated, and its profiles are filled with sound-absorptive material, providing high sound absorption over a broad range of frequencies. Acoustical decks can support loads almost as great as normal decks, cost only about 10% more, and should always be considered at the design stage. The average surface absorption coefficients of untreated industrial buildings are 0.08–0.16 in the 125- and 250-Hz octave bands, varying with construction, and 0.06–0.08 in the 500–4000-Hz octave bands.

Figures 7.4a and b show the reverberation times and 1-kHz sound propagation function curves measured in an empty, untreated workshop with average dimensions of 45 m × 42.5 m and height of 4 m and a double-panel roof.²³ It is often the case that the sound propagation function— $L_p(r) - L_w$ as defined in Eq. (7.3)—at most source–receiver distances and the reverberation time are found to be highest at midfrequencies. The values of both quantities decrease at low and high frequencies due to increased panel and air absorption.

The effects of workshop furnishings are also illustrated in Figs. 7.4a and b, showing the reverberation times and sound propagation function curves after first 25 and then an additional 25 printing machines were introduced. These

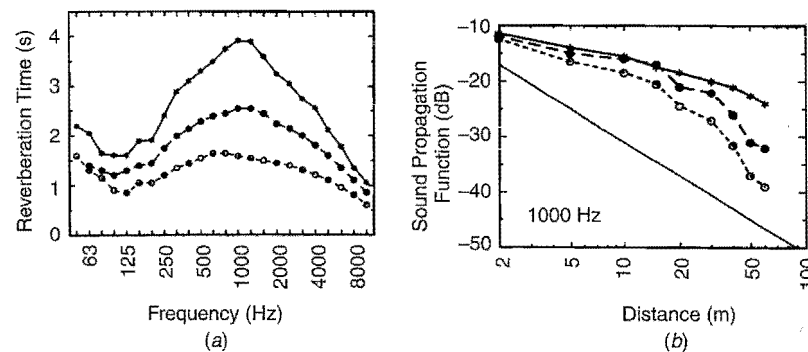


FIGURE 7.4 Measured (a) third-octave-band reverberation times and (b) 1-kHz sound propagation curves in an industrial workshop when empty ($\times$ — $\times$) and containing 25 ($\bullet$ — $\bullet$) and 50 ($\circ$ — $\circ$) metal machines; (—) free-field sound.

metal machines had average dimensions of 3 m $\times$ 3 m and height of 2 m. Introduction of the furnishings significantly decreased reverberation times and sound propagation function values. According to diffuse-field theory, the decreases of reverberation time for this particular workshop correspond to the introduction of as much as 900 m² of acoustical absorption.²³ In fact, they are related to increased diffusion, not increased absorption. The percentage changes of reverberation time and the magnitudes of the changes of the sound propagation function with increasing furnishing density vary little with frequency. These results are supported by similar measurements in other workshops. They again illustrate the limitations of diffuse-field theory, which cannot accurately predict the effects of complex room geometry or furnishings.

Workshop Noise Prediction

Models developed for predicting noise levels and reverberation times in industrial workshops fall into two categories—*comprehensive* models, based on approaches such as ray tracing and the method of image sources, and *simplified*, based on simple theoretical or empirical approaches. Comprehensive models have been evaluated experimentally, concluding that the ray-tracing model of Ondet and Barbry⁷ was most inherently accurate.²⁴ Ray tracing can account for complex room shape, barriers, and nonisotropic absorption and furnishing distributions. Simplified models have also been evaluated experimentally, concluding that a model developed by Kuttruff was inherently accurate in furnished workshops.²⁶

Kuttruff²⁶ developed a simplified model for long and wide furnished workshops based on radiosity theory; the numerous furnishings on the floor and the ceiling were modeled as diffusely reflecting surfaces. According to this model, in a workshop of height h and with average absorption coefficient α , the sound pressure level $L_p(r)$ generated by an omnidirectional compact source of sound power level L_w at a distance r is given by

$$L_p(r) = L_w + 10 \log_{10} \left\{ \frac{1}{4\pi r^2} + \frac{1-\alpha}{\pi h^2} \left[\left(1 + \frac{r^2}{h^2} \right)^{-1.5} + \frac{\beta(1-\alpha)}{\alpha} \left(\beta^2 + \frac{r^2}{h^2} \right)^{-1.5} \right] \right\} \quad (7.11)$$

where $\beta \approx 1.5\alpha^{-0.306}$. Despite the assumption of infinite length and width, the model performed well in comparison with measurements made in workshops of a wide range of lengths and widths.²⁵ One disadvantage of the model is that it does not include a parameter that can be varied to account for variations in the number of furnishings. However, as can be seen from Fig. 7.4b, the variations are small at the smaller source–receiver distances that dominate noise levels in many practical cases. An alternative is empirical models for predicting workshop noise levels and reverberation times.²⁷

When dealing with multiple sources in workshops, the effect of noise control measures may be determined to a first approximation by considering what happens at a distance corresponding to the average source–receiver distance. Changes in total noise levels due to noise control measures are approximated by changes in the sound propagation function at this distance. For machine operator positions, the average source–receiver distance is small—typically 1–2 m. For a room with a more or less square floorplan and with sound sources uniformly distributed over the floor, the average source–receiver distance is about one-half the average horizontal dimension.

As an example, let us use the Kuttruff model to calculate the 1-kHz noise level in the workshop illustrated in Fig. 7.3 (but without the acoustical screen and partial suspended ceiling shown) before and after introduction of full-coverage suspended ceiling absorption. Consider a position midway along the assembly bench and 1.5 m above the floor. The workshop dimensions are length 48.0 m, width 16.0 m, and height 7.2 m. Assume that the average midfrequency absorption coefficients of the workshop surfaces are 0.07 (untreated) and 0.4 (treated). The workshop contains nine noise sources located as shown in Fig. 7.3, with 1-kHz sound power levels and source–receiver distances as shown in Table 7.4. Table 7.4 shows the individual noise-level contributions of the sources before and after treatment. Total levels are determined by energy-based addition of the individual source contributions. The result is 82.4 dB before treatment and 78.7 dB after treatment. Introduction of the ceiling treatment is predicted to reduce the 1-kHz noise level at the assembly bench by 3.7 dB (confirming that the treatment illustrated in Fig. 7.3 is highly cost-effective).

Accurate prediction relies on an inherently accurate model and accurate input data. Workshop surface absorption coefficients were discussed above. Many workshop prediction models also include a parameter describing the workshop furnishing density. It is not known how to determine this quantity accurately. In theory, it can be estimated as the total surface area of the furnishings (or, more practically, of imaginary boxes that would fit around the individual objects)

TABLE 7.4 Details of Calculation of 1-kHz Noise Levels in Workshop before and after Acoustical Treatment Using Kuttruff Simplified Prediction Model^a

Source Number	L_w , dB	Distance, r (m)	Before $L_{p,a}(r)$, dB	After $L_{p,b}(r)$, dB
1	94	13.5	70.7	65.3
2	101	9.8	79.1	74.8
3	92	7.6	71.1	67.5
4	94	3.9	75.6	73.5
5	86	9.5	64.2	60.0
6	90	7.6	69.1	65.5
7	91	8.7	69.6	65.6
8	89	8.5	67.6	63.8
9	93	15.5	69.0	63.2
Total noise level			82.4	78.7

^aReference 26.

divided by 4 times the volume of the furnished region.²⁴ However, there is some evidence that this procedure underestimates the correct value.⁸ Research has shown that average workshop furnishing densities may attain 0.2 m^{-1} or higher, those of a workshop's furnished regions 0.5 m^{-1} or higher.²⁸ In any case, no simplified model will ever predict noise levels in workshops of complex shape or furnishing distributions or that contain, for example, barriers with high accuracy. In such cases, ray tracing may be the only viable option. Finally, a problem shared by all prediction models is the accurate estimation of the sound power levels of the noise sources. This problem is discussed in Chapter 4.

Workshop Noise Control

To achieve the reduction of workshop noise levels (as required by modern occupational noise regulations) in a cost-effective manner, the application of noise control principles should be incorporated into new designs and renovation in the following order of priority:

1. *Control at Source.* Reduce the sound power outputs of the equipment by design or by retrofit acoustical treatment.
2. *Control of Direct Field.* Isolate receiver positions from noisy sources by increasing the distance between them, by the use of source enclosures (see Chapter 12), and by surrounding the receiver by a cabin or screen. Barriers and screens are often not practical or cost-effective alternatives in workshops. Often separation can be achieved by appropriate planning of the workshop layout, taking full advantage of the building geometry, natural barriers such as stockpiles, and furnishings at the design stage.
3. *Control of Reverberant Field.* Apply sound-absorptive materials to the room surfaces. Such treatments should be accorded lower priority since they tend to be

expensive and not very effective near noise sources, such as at operator positions. In practice, reductions of 0–6 dBA are possible. The best surfaces to treat are those closest to noise sources and/or receiver positions. In low-height industrial workshops, this usually implies the ceiling. Consider an acoustical steel deck at the design stage or acoustical treatments consisting of sound-absorptive materials applied to or suspended from the ceiling—for example, acoustical baffles (rectangular pieces of absorptive material) hung in appropriate patterns at appropriate densities. A particularly cost-effective treatment consists of suspending sound absorbers directly above noise sources. Difficulties may arise due to interference with overhead cranes and lighting and sprinkler systems and with respect to fire regulations and hygiene requirements. The treatment of walls may be warranted in more regularly shaped enclosures and where noise sources are located close to walls when it is important to absorb the strong wall reflections. Note that in workshops of any shape surface absorption, even when it has little effect on the total noise levels, may significantly reduce reverberation, reducing the perceived “noisiness” of the work environment and improving verbal communication.

Readers interested in the low-noise design of industrial workshops should be aware of relevant ISO standards available to help in the task. ISO 11688^{29,30} provides advice on the design of low-noise machinery and equipment. Part 1 discusses planning, and Part 2 discusses the principles of low-noise design. ISO 14257³¹ defines criteria for evaluating the acoustical quality of workshops with respect to noise control and describes how to perform the measurements to evaluate an existing workshop.

7.7 OPEN-PLAN OFFICES

Open-plan offices are one of the most common modern work environments. In these large spaces, many seated workers are separated by low barriers that provide partial visual and acoustical separation between workstations. The barriers may be free-standing but today are usually in the form of integrated furniture or cubicles. The ceiling and the carpeted floor form two large, extended sound-absorbing planes; their horizontal dimensions are much greater than the height of the ceiling. The space between these surfaces is filled with office furniture and barriers that are usually sound absorbing. Diffuse-field theory certainly does not apply in such a space; the sound pressure level decreases continuously with distance from sources (an example is shown in reference 3). The principal problem in an open-plan office is not propagation to large distances but the provision of privacy between neighboring workstations. The important sound paths are, therefore, the short-range ones. Sound from one work position reflects from extended surfaces (ceiling, walls, and windows) and diffracts over or around the edges of barriers, as illustrated in Fig. 7.5. These sound paths must be controlled to provide acoustical privacy for workers.

Open-plan offices can provide a reasonable degree of acoustical privacy if they are carefully designed as a complete system and if adjacent work functions are

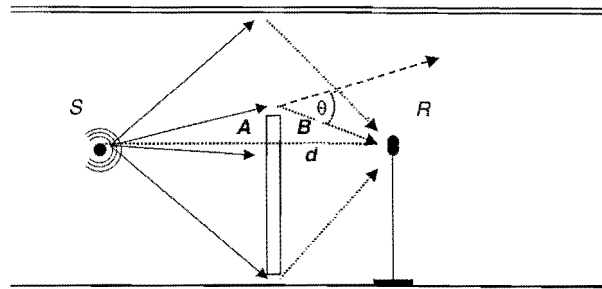


FIGURE 7.5 Possible sound paths between workstations in an open-plan office cross section. Reflection from the ceiling is reduced by using highly absorptive ceiling materials, transmission through the barrier by selecting a sufficiently heavy panel with adequate sound transmission loss, and diffraction over the barrier by increasing the barrier height.

compatible and not too close together. If any aspect of the system is neglected, then adequate acoustical privacy will not be achieved. Obtaining adequate acoustical privacy is first a question of attenuating unwanted speech sounds from adjacent work spaces, because speech sounds are much more distracting than most other types of noises. It is usually also necessary to include masking sound to render intruding speech sounds less intelligible while not being a source of annoyance. Special protection should be provided against noisy devices such as printers and copiers—for example, by locating them in a shielded area. Conference rooms with full-height partitions, providing good sound insulation, should be available for activities which require low background noise levels and particularly high privacy.

Open-Plan Office Barriers

Barriers (also called screens, partial-height partitions, workstation panels, or office dividers) provide sound attenuation and visual privacy between workstations for seated persons. They are the basic component of systems furniture (usually forming cubicles) that combines the functions of a barrier and supporting amenities such as storage compartments, lighting, power, communications, and work surfaces into a single unit. A barrier should attenuate sound that passes through it so that the transmitted sound is negligible.

Sound diffracts over the top of the barrier to reach the next workstation. For an infinitely wide barrier between a source and a receiver position, the insertion loss IL relative to the level at the receiver in the absence of the barrier can be approximated by³²

$$IL = 13.9 + 7 \log_{10} N + 1.4(\log_{10} N)^2 \text{ dB} \quad \text{for } N \geq 0.001 \quad (7.12)$$

where the Fresnel number $N = 2f(A + B - d)/c$, d is the straight-line distance from the source to the receiver in meters, A is the distance from the source to

the top of barrier in meters, B is the distance from the top of the barrier to the receiver in meters, and c is the sound speed (Fig. 7.5 shows A , B , and d). The greater the angle that sound has to bend to reach the receiver position on the other side of the barrier (θ in Fig. 7.5), the greater is the insertion loss of the barrier. Thus, higher barriers are more effective than lower ones, and barriers placed close to the talker or listener are more effective than those equidistant from both.

The effects of diffraction around and transmission through a barrier can be combined to determine the total insertion loss of a barrier. Sound transmitted through the screen should be negligible relative to the sound diffracted around it, especially at those frequencies important for speech intelligibility. Specifying that, at 1000 Hz, the normal-incidence sound transmission loss should be 6 dB greater than the theoretical insertion loss due to diffraction is a satisfactory criterion. This criterion leads to the requirement that the minimum mass per unit area of the barrier, ρ_s in kilograms per square meter, should be $\rho_s \geq 2.7(A + B - d)$. For an isolated screen, the total effect of the barrier can be calculated by applying Eq. (7.12) to each edge in turn and then summing the acoustical energies.

Maximum barrier dimensions are usually limited by physical convenience, possible interference with airflow, and preservation of the open look. Recommended minimum dimensions are height 1.7 m and width 1.8 m. Of course, a complete cubicle is equivalent to a long barrier, for which there is negligible propagation around the ends. If the attenuation of a high barrier is desired but visual openness must be maintained, a plate of glass or transparent plastic panel can be fitted to the top of a low barrier to increase its "acoustical height." The gap between the bottom edge of the barrier and the floor should be small; otherwise sound can reflect under the barrier to the opposite side. This is not critical, because sound following this path tends to be diffused or absorbed by furniture and carpets; a gap of up to 100 mm can be left when the floor is carpeted.

Measures for Rating Acoustical Privacy

Acoustical privacy is usually referred to as speech privacy because it is speech sounds that are usually the most disturbing (see also Section 7.5). Speech privacy is essentially the opposite of speech intelligibility. That is, the lower the intelligibility of the speech, the greater the speech privacy. Speech privacy (and speech intelligibility) is related to the level of the intruding speech sounds relative to the less meaningful ambient noise. Hence, speech privacy is related to measures involving the signal-to-noise level difference, where speech is the signal and the general ambient noise is the noise component.

Articulation index (AI) is a frequency-weighted signal-to-noise level difference measure which indicates the expected speech intelligibility in particular conditions. The signal-to-noise level differences in each frequency band are weighted according to their relative importance to the intelligibility of speech. These weighted signal-to-noise level differences are summed to obtain the AI value, between 0 and 1. An AI of 1 is intended to indicate conditions in which

near-perfect intelligibility is expected. An AI close to 0 is expected to indicate conditions of near-perfect speech privacy. The AI has been widely used as a measure of speech privacy; $AI \leq 0.15$ is considered to provide *normal* or *acceptable* speech privacy in open-plan office situations. The AI has now been replaced by the SII. It is similar to the AI but has slightly larger values, so the criterion for acceptable speech privacy becomes $SII \leq 0.20$. Both have been described in ANSI standards.^{33,34}

Figure 7.6 illustrates the suitability of this criterion. In this figure, median speech intelligibility scores are plotted versus SII for situations simulating conditions in open-plan offices. It is seen that $SII = 0.2$ represents the point below which privacy increases rapidly with decreasing SII. One can think of it as the point at which improvements to the acoustical design start to improve speech privacy. In practice, it is also a practically achievable goal if all aspects of the acoustical design are carefully considered.

Acoustical Design of Complete Workstations

Sound propagation between workstations in an open-plan office with modular workstations (cubicles) involves many different sound paths, not just propagation over a simple screen as described above. There are reflections from the ceiling and the other panels of the workstation as well as from nearby walls. These consist of not only simple first-order reflections but also paths with multiple reflections, such as those involving the floor and the ceiling or the vertical surfaces of the workstation. The problem of sound propagation between workstations has been addressed using the method of image sources (see Section 7.3) and implemented as a computer algorithm. Diffraction over the screen was modeled using Maekawa's results described in Eq. (7.12). The method of image sources assumes that all reflections are specular, so the angle of incidence equals the angle of reflection. However, the model includes an empirical correction to the

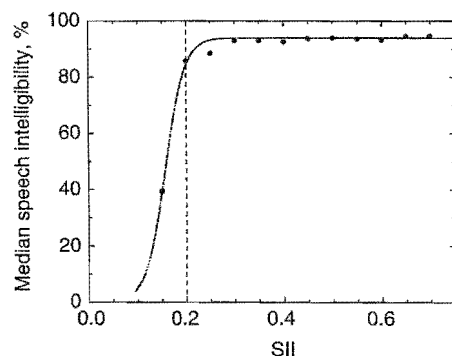


FIGURE 7.6 Variation of median speech intelligibility score with SII for situations simulating a range of conditions in open-plan offices.³⁵

absorption coefficients of ceiling materials (obtained from standard reverberation room tests) to account for the limited range of angles of incidence for sound propagation between workstations, compared to that in a diffuse reverberant test room. The model has been validated with respect to a wide range of measurement conditions and is used here to illustrate the effects of various design parameters. See references 36–39 for details of the model.

Speech and Noise Levels

Two factors that have very large effects on speech privacy are the speech levels of the talkers in the open offices and the general ambient noise levels. These cannot be ignored when striving for acceptable speech privacy. Fortunately, people do not usually talk with *normal* vocal effort in open-plan offices. Measurements of talkers in open-plan offices have found mean voice levels close to those described as *casual* vocal effort. Using normal voice levels to calculate expected speech privacy greatly exaggerates the lack of speech privacy. The intermediate office speech level (IOSL) shown in Fig. 7.7 is recommended as the speech source level for calculating speech privacy in open-plan offices. It was obtained by averaging the mean speech levels measured in open-plan offices and the casual voice data and then adding 3 dB, corresponding to one standard deviation above the average talker level. The IOSL spectrum is therefore representative of louder talkers in open-office situations and corresponds to a speech source level at a distance of 1 m in a free field of 53.1 dBA. References 40–42 provide more details of speech levels.

One important aspect of achieving speech privacy is to encourage the use of lower voice levels as a form of open-plan office etiquette. Extended discussions should not be held in open-office areas but should be moved to closed meeting rooms.

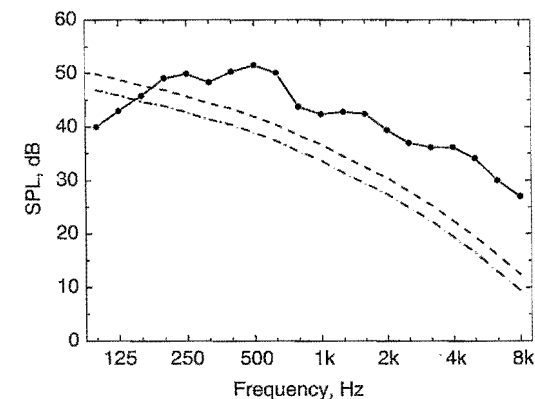


FIGURE 7.7 The IOSL spectrum and noise-masking spectra for open-office design calculations: (—●—) IOSL, (---) optimum masking; (·····) maximum masking.

It is difficult to obtain acceptable speech privacy if the general ambient noise level is low. On the other hand, if ambient noise levels are high, they can become annoying and cause people to talk louder. There is a narrow range of ambient noise levels that are expected to be acceptable and that can also partially mask intruding speech sounds from adjacent workstations. (The masking of speech by noise is a well-researched field, and ANSI S3.5³⁴ includes many references on this topic.) For this reason, electronic sound-masking systems are often included in successful open-office designs. They can be designed to provide nearly ideal noise levels to mask speech sounds and enhance privacy without being unduly disturbing. The masking noise should be adjusted to sound like natural ventilation system noise and to be evenly distributed throughout the office. Figure 7.7 includes an example of a noise-masking spectrum that was judged to be at an optimal level, corresponding to 45 dBA. It has been found that the maximum acceptable level for masking sound is approximately 48 dBA. Hence, the maximum masking spectrum in Fig. 7.7 represents an example of such a maximum noise-masking spectrum. These two noise spectra are indicative of the narrow range of masking noises that provides *acceptable* speech privacy. Spatial variations of masking noise should be less than 3 dBA.

Conventional sound-masking systems have included those with centrally located electronics and others with distributed units. Distributed systems, with many small independent units, have the advantage of avoiding correlated sources, which can lead to annoying spatial variations in the masking sound. Manufacturers of both types claim various practical advantages. Although propagation into the ceiling void may aid the homogeneity of the masking sound in the office below, the masking sound will be modified by propagation through the ceiling tiles, and transmission through lighting fixtures can lead to localized areas of higher sound levels below. More recently, sound-masking systems with loudspeakers mounted in the ceiling tiles and on the panels of workstations have been introduced to provide better control over the resulting masking sound. The installation of noise-masking systems is best left to experienced professionals.

Important Design Parameters

In the design of an open-plan office, it is first assumed that the transmission loss of the workstation panels is sufficient to attenuate, to an insignificant level, the sound propagating directly through the panels. The requirement for surface density given by Eq. (7.12) usually leads to a minimum value corresponding to $STC \geq 20$ (STC is the sound transmission class, obtained from a standard sound transmission loss test⁴³; see also Chapter 10). After this, the two most important parameters are the sound absorption of the ceiling and the height of the separating barriers or panels. If these are not adequate, it will not be possible to achieve acceptable speech privacy.

The shaded area of Fig. 7.8 indicates combinations of ceiling absorption and panel height that can provide $SII \leq 0.20$ if the other details considered below are also acceptable. Ceiling absorption is described in terms of the sound absorption

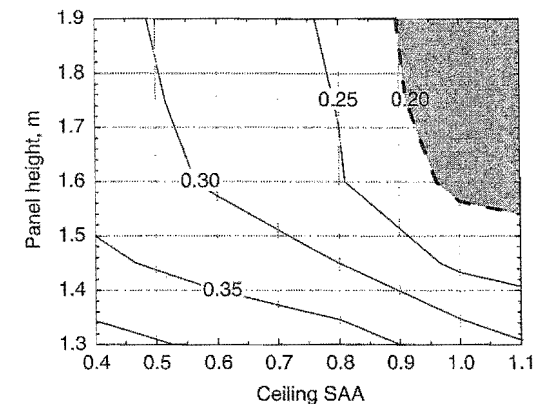


FIGURE 7.8 Shaded area shows combinations of workstation panel height and ceiling absorption that lead to *acceptable* speech privacy corresponding to $SII \leq 0.20$.⁴⁴ Contours in unshaded part indicate degrees of unacceptable speech privacy. SAA is the average of the absorption coefficients in the third-octave bands from 250 Hz to 2.5 kHz.

average (SAA), which is the average of the absorption coefficients in the third-octave frequency bands from 250 Hz to 2.5 kHz. It replaces the older noise reduction coefficient (NRC) and has very similar values. Values of SAA greater than 1.0 are included in Fig. 7.8 because such values can result from the standard test procedure, due to the effects of edge diffraction (as mentioned in Section 7.2). It is seen that the sound absorption of the ceiling should correspond to $SAA \geq 0.90$ and the separating panels should be ≥ 1.7 m high. Since the results in Fig. 7.8 were calculated with other details set to nearly ideal values, combinations of ceiling absorption and screen height outside of the shaded area make it impossible to achieve acceptable speech privacy.

A third key design parameter is the size of the workstation plan. The results in Fig. 7.8 are for a workstation with dimensions of 3 m $\times$ 3 m. If the workstation length and width each decrease by 1 m, then the resulting SII increases by 0.05. That is, for a 2 m $\times$ 2 m workstation plan, meeting the $SII \leq 0.20$ criterion would require very high screens and the most absorptive ceiling materials.

Other Design Parameters

The sound-absorptive properties of the workstation panels and the floor have smaller effects on speech privacy. The height of the ceiling and the presence of lighting fixtures in the ceiling can also influence speech privacy.

The results shown in Fig. 7.8 were calculated for workstation panels with $SAA = 0.9$. If this were reduced to 0.75, the corresponding SII values would decrease by 0.01. If the panel absorption were decreased to 0.6, SII values would decrease by 0.02. These seem to be small degradations in the overall performance, but one must always remember that most designs will at best be on

the edge of just providing acceptable speech privacy; even these small improvements may help to meet the $SII \leq 0.20$ criterion. Of course, if the panels were not highly absorptive (i.e., $SAA < 0.60$), there would be a more significant increase in the resulting SII values. For example, if the workstation panels were non-absorptive in an otherwise ideal workstation design, SII would increase from 0.2 to 0.3.

Varying floor absorption and varying ceiling height generally have very small effects on the resulting SII values. In most cases, varying these parameters changes SII by no more than about 0.01. On the other hand, lighting fixtures in the ceiling can degrade the speech privacy of the open-office design. The effect of a lighting fixture depends on the type of lighting unit and its location. Lights with a flat plastic or glass surface produce the strongest unwanted reflections and are most troublesome when located over the separating panel between two workstations. They will change SII values most when installed in a highly absorptive ceiling. Evaluation of the effects of several types and locations of lighting fixtures has shown that, for a ceiling with $SAA = 0.90$, SII values could increase by up to 0.08. Open-grill lighting fixtures have smaller effects but still reduce the effectiveness of a highly absorptive ceiling.

Practical Issues and Other Problems

Sound propagating in the horizontal plane can bypass barriers by reflection from vertical surfaces and hence reduce the sound attenuation between work positions. To minimize these effects, surfaces such as walls, office barriers, square columns, backs of cabinets, and systems furniture, as well as bookcases, should be covered with sound-absorptive material having SAA values of 0.7 or higher. A thickness of 2.5 cm or more of glass fiber with a porous fabric cover will satisfy this requirement. The application of carpets directly to hard surfaces is not an effective solution, since typical carpets have low sound absorption coefficients. Round columns with diameters of 0.5 m or less may be left uncovered. A simple way to avoid wall reflections is to avoid gaps between the wall and barriers. To prevent reflections from their surfaces, office barriers should be covered with sound-absorptive material on both faces.

Where workstations are adjacent to windows, there are often large gaps between the workstation panels and the window. To provide adequate acoustical privacy between these workstations, this gap must be filled with additions to the workstation panels. This problem cannot easily be solved using drapes, which would need to be heavy and closed; furthermore, most slatted blinds do not reduce reflections. Using the area next to the windows as a corridor is another solution to this problem.

Standard procedures exist for the evaluation of the degree of speech privacy in an existing office, in a mock-up of a proposed office,⁴⁵ or for the evaluation of open-plan noise-masking systems.⁴⁶ It is very important to evaluate the speech and noise in the office to completely determine the existing degree of speech privacy.

7.8 REVERBERATION ROOMS

Reverberation rooms are designed and equipped to give a close approximation to a diffuse sound field. Measurements in these rooms are used to characterize the sound-absorptive properties of materials, the sound power of sources, and the sound transmission through building elements, among other things, under standard diffuse-field conditions according to various standards.^{43,47-52} A typical reverberation room has a volume of about 200 m³; some are constructed with nonparallel walls. The walls and surfaces of the room are made highly reflective so that reverberation times are high and the region dominated by the direct field of a source is as small as possible.

At low frequencies, the frequency response to wide-band noise shows peaks corresponding to individual room modes. As frequency increases, the spacing between modes becomes less, the modes begin to overlap, and the individual modes are less obvious. At some transition frequency the room response for bands of noise becomes approximately constant, the properties of the sound field become more uniform, and the room response may be described in statistical terms. This transition point is usually defined by the Schroeder frequency⁵³ $f_s = 2000(T_{60}/V)^{1/2}$ hertz. For a $V = 250$ m³ room with $T_{60} = 5$ s, $f_s = 282$ Hz.

To make the response of the room more uniform at low frequencies, it is usually advisable to add low-frequency sound-absorbing elements. Even in rooms of approximately 200 m³ volume, correctly chosen dimensions, and the recommended amount of sound absorption, the spatial variations of pressure and the sound decay rate are often too large to satisfy the precision requirements of standards. It is therefore common to add fixed panels suspended at random positions and orientations throughout the room to perturb the room modes and to create more diffuse conditions. In many cases rotating diffusers are also used for this purpose.

Fixed diffusers are most important in rectangularly shaped rooms used for sound absorption measurements. They may not be necessary in rooms that are sufficiently nonrectangular in shape. They help to create a more diffuse sound field during the measurement of sound decays by ensuring that, throughout the sound decay, a portion of the decaying sound energy is redirected toward the sound-absorbing sample. When commissioning a new reverberation room for sound absorption measurements, it is necessary to systematically increase the number of diffusers until measured absorption coefficients just reach maximum values due to the increasingly diffuse conditions in the chamber. Of course, adding too many diffusers can limit the number of valid positions for microphones, which must always be located more than a half wavelength (at the lowest frequency of interest) from reflecting surfaces such as walls and diffusing panels.

Rotating diffusers are particularly useful in reverberation rooms used for measuring the sound power levels of devices with strong tonal components. These types of sound sources lead to large spatial variations in sound levels that can be effectively reduced using a rotating diffuser. By continuously changing the geometry of the room, they shift the modal patterns and hence average out some of the

spatial variations of the sound levels. Making the rotating diffuser in the form of panels of revolution can reduce the required drive power and aerodynamic noise.

In addition to such measures, it is still necessary to sample the room volume in order to measure mean-squared sound pressures and decay rates accurately. To provide statistically independent samples of the sound field, microphone positions must be more than one-half wavelength apart (at the lowest frequency of interest) as well as be located more than one-half wavelength away from reflecting objects to avoid nonrepresentative measures of the room average sound level.

It has been shown that close to one, two, or three infinite reflecting planes the sound pressure and energy increase. The mean-squared pressure p^2 can be expressed as

$$p^2 = 1 + \sum_{n=1}^{N_{im}} \frac{\sin(kr_n)}{kr_n} \quad (\text{N/m}^2)^2 \quad (7.13)$$

where $k = 2\pi/\lambda$ (λ = wavelength)

p^2 = mean-square pressure normalized to unity in absence of images or far from reflecting surfaces

r_n = distances from images of measurement point to measurement point

N_{im} = number of images: 1 for measurement near a plane surface, 3 near an edge, and 7 near a corner

Closer than $\lambda/2$ to highly reflecting surfaces, sound pressure increases significantly because of these positive interference effects. To account for the increase in sound energy close to surfaces, an adjustment term $(1 + S\lambda/8V)$ is included in the calculation of the sound power W from the space-averaged mean-square sound pressure. Thus, the relationship used to determine the sound power W of a source from the space-averaged mean-square sound pressure p^2 that it creates in the room is⁵⁴

$$W = \frac{55.3 p^2 V}{4 \rho c^2 T_{60}} (1 + S\lambda/8V) \quad \text{N} \cdot \text{m/s} \quad (7.14)$$

where ρ = density of air, kg/m^3

c = speed of sound in air, m/s

V = room volume, m^3

T_{60} = reverberation time, s

It is assumed that sampling of the sound field in the room is confined to the central regions away from room surfaces. The Sabine formula, Eq. (7.2), relating reverberation time to room absorption is assumed to hold and is the basis for determinations of sound absorption in reverberation rooms.^{47,48}

The planning and qualification of reverberation rooms are complex and best done by experienced professional acousticians.

7.9 ANECHOIC AND HEMI-ANECHOIC CHAMBERS

An anechoic chamber is a room with all of its interior surfaces highly absorptive such that a source in the room radiates in essentially free-field conditions. The resulting sound field has only a direct component. Hemi-anechoic chambers have a hard floor with all other interior surfaces highly sound absorptive. They are used for measuring the sound power level and radiation pattern of equipment (such as road vehicles, appliances, etc.) that are operated over a hard surface. The acoustical performance of an anechoic or hemi-anechoic room can be evaluated by determining how closely conditions in the room approximate those in a free field. This is usually done by measuring the variation of sound levels with distance from an approximately omnidirectional source; it should ideally be -6 dB per doubling of distance; see Eq. (7.3) and Fig. 7.1.

Tests are made in an anechoic room when it is necessary to accurately measure the unperturbed sound radiated by a source—for example, when measuring its radiation pattern (directivity) or sound power. The surfaces of anechoic chambers are made highly absorptive by lining them with deep, sound-absorptive materials. The lining typically consists of wedges of mineral wool or glass fiber.⁵⁵ All anechoic chambers are more anechoic at high than at low frequencies. The lowest frequency at which an anechoic chamber can be used depends primarily on the chamber volume and the depth of the wedges. This cutoff frequency, at which the wedges absorb 99% of the incident sound energy, is usually achievable with a wedge depth (including any air space between the base of the wedge and the hard wall) of approximately one-quarter wavelength. A large chamber with 1-m-deep wedges may be effective down to 80–100 Hz. To provide a walking surface to facilitate setting up experiments in an anechoic chamber, an open-metal-floor grid that is removed after the setup or a permanent wire-mesh floor can be provided. The perimeter frame of such a wire-mesh floor and to a lesser extent the wire mesh itself may degrade the high-frequency performance of the chamber.

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CHAPTER 8

Sound-Absorbing Materials and Sound Absorbers*

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8.1 INTRODUCTION

One of the most frequent problems faced by noise control engineers is how to design sound absorbers that provide the desirable sound absorption coefficient as a function of frequency in a manner that minimizes the size and cost, does not introduce any environmental hazards, and stands up to hostile environments such as high temperatures, high-speed turbulent flow, or contamination. The designer of sound absorbers must know how to choose the proper sound-absorbing material, the geometry of the absorber, and the protective facing. The theory of sound-absorbing materials and sound absorbers has progressed considerably during the last 10 years. Much of this progress is documented in K. U. Ingard's *Notes on Sound Absorption Technology*¹ and in F. P. Mechel's *Schall Absorber (Sound Absorbers)*² and the separately sold computer program on CD-ROM.³ Ingard's book is a paperback edition of modest length and price and comes with a CD-ROM that allows the reader to make easy numerical predictions from almost all of the equations used in the book. The underlying physical processes are explained with great clarity and the mathematical treatment is kept simple. Derivations and difficult explanation are presented in appendices. The books cover all aspects of sound absorption in great detail. To derive full benefit, it helps if the reader has a reasonably strong mathematical background. However, even those with no such background will find the figures, which give the

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difficult mathematical results in graphical form, useful and practical. Mechel's book consists of three hard-covered volumes totaling 2866 pages. The associated computer program is available in MAC and Window versions but is expensive. The classical book of H. Kuttruff, *Room Acoustics*,⁴ deals predominantly with the room acoustics aspects of sound absorption and should be studied by acousticians designing buildings for the performing arts. Beranek's recently updated classic book *Acoustic Measurements*⁵ describes the methods and experimental hardware for measuring all acoustical descriptors of sound-absorbing materials and sound absorbers. The reader with serious interest in sound absorption technology is advised to study all of these texts.

How Sound Is Absorbed

Sound is the organized superposition of particle motion on the random thermal motion of the molecules. The speed of the organized particle motion in air is typically six orders of magnitude smaller than that of the thermal motion. All sound absorbers facilitate the conversion of the energy carried by the organized particle motion into random motion. All forces, other than those that compress and accelerate the fluid, caused by the oscillatory particle flow in the presence of solid material result in loss of acoustical energy. The most important contribution to the conversion is associated with the drag forces caused by friction between the interface of a rigid or flexible wall or the skeleton of the porous or fibrous sound absorber material and the fluid in the thin acoustical boundary layer. For porous and fibrous sound-absorbing materials the acoustical flow speed is low and the flow remains laminar, and the drag force is proportional to the acoustical particle velocity. At high velocities, which occur at the mouth of resonators, the flow separates, turbulence is created, and the friction force becomes proportional to the square of the velocity. Other loss mechanisms, discussed in more detail elsewhere,^{1,2} include the isothermal compression of air at low frequencies, direct conversion of acoustical energy into heat owing to a time lag between compression, and heat flow that takes place in closed cell foams. In the case of plate and foil (very thin plate) absorbers the acoustical energy is converted into heat in the vibrating flexible plate and radiated as sound from the rear of the plate or is transmitted in the form of vibration energy into connected structures.

A sound absorber can absorb only that part of the incident sound energy not reflected at its surface. Consequently, it is important to keep the reflection at the surface as low as possible. Essentially, the part of the incident acoustical energy that enters the absorber should be dissipated before it returns to the surface after traversing the absorber and reflecting from a rigid backing. Otherwise, the absorber gives back acoustical energy to the fluid on the receiver side that is in addition to the initial reflection. This requires a sufficient thickness. The challenge in sound absorber design is to keep the absorber thickness to a minimum.

Sound Absorption Coefficients

The acoustical performance of flat sound absorbers is characterized by their sound absorption coefficient α , defined as the ratio of the sound power, W_{tr} , that is not

reflected (i.e., dissipated in the absorber, transmitted through the absorber into a room to its rear, or conducted, in the form of vibration energy, to a connected structure) and the sound power incident on the face of the absorber, W_{inc} :

$$\alpha \equiv \frac{W_{\text{tr}}}{W_{\text{inc}}} \quad (8.1)$$

For convenience in analyses, the absorption coefficient is defined in terms of sound pressure reflection factor R of the absorber interface, namely

$$\alpha = 1 - |R|^2 \quad (8.2)$$

The vertical lines bracketing R indicate the absolute value. The reflection factor R is usually a function of the angle of sound incidence, the frequency, the material, and the geometry of the absorber. The absorber is characterized by its wall impedance (otherwise known as surface impedance) Z_w , defined as

$$Z_w = \frac{p}{v_n} \quad \text{N} \cdot \text{s/m}^3 \quad (8.3)$$

where p is the sound pressure and v_n is the normal component of the particle velocity, both evaluated at the interface. A substantial part of this chapter concerns the prediction of the wall impedance offered by a large variety of sound absorbers. The sound absorption coefficient as defined in Eqs. (8.1) and (8.2) is further differentiated according to the angular composition of the incident sound field, such as *normal incidence*, *oblique incidence*, and *random incidence*, and whether the absorber is *locally reacting* (sound cannot propagate in it parallel to the interface) or *non-locally reacting* (where sound can propagate in it parallel to the interface). At normal incidence there is no difference between locally and non-locally reacting materials.

Measurement of Normal-Incidence Sound Absorption Coefficient

α_0 . The normal incidence sound absorption coefficient α_0 can be measured in an impedance tube according to the American Society for Testing and Materials (ASTM) standard C384-98 as $\alpha_0 = 4\zeta/(\zeta + 1)^2$, where $\zeta = p_{\text{max}}/p_{\text{min}}$ is the ratio of the maximum and minimum standing-wave sound pressure pattern in the tube upstream of the sample. Normal-incidence sound absorption coefficients, measured in an impedance tube, never exceed unity.

Measurement of Random-Incidence Sound Absorption Coefficient

$\alpha_R(\text{rev})$ in Reverberation Room. The random-incidence sound absorption coefficient $\alpha_R(\text{rev})$, which is also referred to as the *Sabine absorption coefficient*, can be measured directly in a reverberation room according to ASTM C423-02. It is defined as $\alpha_R(\text{rev}) \equiv (55.3V/S)[(1/T_S) - (1/T_0)]$, where V is the volume of the reverberation chamber in m^3 , $S = 6.7 \text{ m}^2$ is the standardized surface area of the test sample, and T_S and T_0 are the reverberation times in seconds (see

Chapter 7) measured with and without the sample, respectively. It is important to distinguish between the two random-incidence sound absorption coefficients $\alpha_R(\text{rev})$ and α_R . The coefficient $\alpha_R(\text{rev})$, measured in the reverberation room, can yield values which exceed unity (measured values up to 1.2, as can be seen in Fig. 8.10). Such values obviously violate the theoretical definition of α given in Eq. (8.36) implying that the panel absorbs more energy than is incident on it. This fact that measured values of $\alpha_R(\text{rev}) > 1$ results from the finite size of the test sample, which means that there is diffraction at the edges of the sample. In design calculations, it is customary to replace all values of $\alpha_R(\text{rev}) > 1$ with unity. The random-incidence sound absorption coefficient α_R is computed from the wall impedance and never yields absorption coefficients that exceed unity. Based on the analyses of $\alpha_R(\text{rev})$ measured according ASTM C423-02 for a large variety of thicknesses and flow resistivity of Owens Corning series 700 fiberglass boards, Godfrey⁶ has proposed an empirical prediction scheme to relate $\alpha_R(\text{rev})$ of a standard size (6.7 m²) sample to α_R predicted theoretically from the computed wall impedance of the samples as

$$\alpha'_R(\text{rev}) = \left(\frac{21.3}{f^{0.5}} + 0.73 \right) \alpha_R$$

where $\alpha'_R(\text{rev})$ is the empirically predicted Sabine absorption coefficient of a standard size sample and f is the frequency in hertz. This formula provides reasonably accurate predictions for low flow resistivities (in the range of 9000 N·m/s⁴ = 0.56 $\rho_0 c_0$ /in. to 17,000 N·m/s⁴ = 1.1 $\rho_0 c_0$ /in.) and small layer thicknesses from 1 to 3 in. (2.5–7.5 cm). For flow resistivities greater than 32,000 N·m/s⁴ = 2 $\rho_0 c_0$ /in. and layer thicknesses ≥ 76 mm = 3 in., the analytical method⁷ yields the best prediction.

8.2 SOUND ABSORPTION BY NON-SOUND ABSORBERS

Although inefficiently, all rigid and flexible structures absorb sound. If efficient sound absorbers are present in the room, the contribution of these marginal sound absorbers may be neglected. However, if they represent the only sound-absorbing mechanism (such as is in the case of reverberation rooms without low-frequency absorbers), they, and at high frequencies the air absorption, account for the total sound absorption and set an upper limit for the maximum achievable reverberation time and for the buildup of reverberant sound pressure in these special rooms.

Sound Absorption of Rigid Nonporous Wall

The general assumption that a rigid, nonporous wall, however massive, gives rise to total reflection of the incident sound is not valid. In the immediate vicinity of the interface there are two phenomena that cause small but finite dissipation of sound energy. The first is that the wall-parallel component of the particle velocity of the incident sound results in shear forces in the acoustical boundary

layer. The acoustical boundary layer is identical to that which would develop at the interface between a stationary fluid and a plane, rigid wall oscillating with the frequency and velocity amplitude of the wall-parallel component of the sound-induced particle motion. The second component of the unavoidable dissipation results from the large thermal capacity of the wall, which makes it impossible to fully recover the heat energy in the rarefaction phase that was built up in the compression phase. According to reference 4, for randomly incident sound, the combined effect of these two dissipation processes results in a lower limit of the sound absorption coefficient of

$$\alpha_{\min} = 1.8 \times 10^{-4} (f)^{1/2} \quad (8.4)$$

where f is the frequency in hertz. For frequencies of 1 and 10 kHz, Eq. (8.4) yields values of $\alpha_{\min}$ of 0.006 and 0.018, respectively.

Sound Absorption of Flexible Nonporous Wall

Building partitions such as walls, windows, and doors absorb low-frequency sound. The sound power lost, W^{Loss} , when the incident sound interacts with a nonporous, flexible, homogeneous, and isotropic single partition that has another room or the outdoors on the receiver side is made up of three components:

$$W^{\text{Loss}} = W^{\text{ForcedTrans}} + W^{\text{ResTrans}} + W^{\text{ResDiss}} \quad \text{N} \cdot \text{m/s} \quad (8.5)$$

where $W^{\text{ForcedTrans}}$ is the sound power transmitted by the forced bending waves (i.e., mass law portion radiated from the receiver side by the forced bending waves), W^{ResTrans} is the sound power transmitted by the free resonant bending waves, and W^{ResDiss} is the sound power dissipated in the partition by the free resonant bending waves. The first two terms in Eq. (8.5) can be expressed in the form of sound transmission loss, TL, of the partition and the third term as a function of the space-time mean-square value of the sound-induced resonant vibration velocity $\langle v^2 \rangle$ of the plate, yielding

$$W^{\text{Loss}} = W^{\text{inc}} 10^{-\text{TL}/10} + \langle v^2 \rangle \rho_s \omega \eta S \quad \text{N} \cdot \text{m/s} \quad (8.6)$$

where η is the composite loss factor of the plate accounting for energy loss through dissipation in the plate as well as for that lost through structural coupling to neighboring structures at the plate boundaries and S is the surface area of the plate in square meters. The sound absorption coefficient is defined as

$$\alpha = \frac{W^{\text{Loss}}}{W^{\text{inc}}} \quad (8.7)$$

where W^{inc} is the sound power incident on the source side of the partition. As shown in Chapter 10, for random incidence, where the sound energy has equal

probability of angle of incidence, the incident sound power is

$$W^{mc} = S \left(\frac{\langle p^2 \rangle}{4\rho_0 c_0} \right) \quad \text{N} \cdot \text{m/s} \quad (8.8)$$

where S is the surface area of the partition (one side) in square meters, $\langle p^2 \rangle$ (N^2/m^4) is the mean-square space-time average sound pressure in the source room, ρ_0 is the density of air in kilograms per cubic meter, and c_0 is the speed of sound of the gas at design temperature in meters per second. Using the results of the analyses in Chapter 10, for the power dissipated in the partition by the free bending waves, one obtains the following approximate formula for the random-incidence sound absorption coefficient α^{rand} of a single, flexible, homogeneous, isotropic, nonporous partition:

$$\alpha^{\text{rand}} \approx 10^{(-\text{TL}_{\text{rand}}/10)} + \frac{2\pi\sqrt{12}\rho_0 c_0^3 \sigma_{\text{rad}}}{\rho_M h^2 c_L \omega^2} \quad (8.9)$$

where TL_{rand} = random-incidence sound transmission loss of panel (measured or predicted according to Chapter 10)

ω = angular frequency, $= 2\pi f$

f = frequency, Hz

σ_{rad} = radiation efficiency of free bending waves (see Chapter 10)

ρ_M = density of plate material, kg/m^3

h = plate thickness, m

c_L = speed sound in plate material, in m/s

The second term on the right side of Eq. (8.9) is strictly valid only if the ratio of the power loss through dissipation and sound radiation by the free bending waves exceeds unity, $(h\rho_M\omega\eta_C/\rho_0 c_0\sigma_{\text{rad}}) > 1$, which is almost always the case because σ_{rad} strongly decreases with decreasing frequency below the coincidence frequency of the plate. In Eq. (8.9) the first term represents the loss of energy by the transmission of sound into the receiver space as defined in Chapter 10. The second term represents the combined contribution of the dissipation of the free resonant bending waves in the plate and energy loss in the form of vibration energy transmission to neighboring structures at the boundaries. It is interesting to note that this second term does not depend of the composite loss factor η_C of the plate (the TL of the plate of course depends on η_C in the frequency range at and above the critical frequency where the free bending waves control the sound radiation). The physical reason for this is that in the case of a small loss factor the plate vibrates more vigorously and the combination of the increased vibration and decreased loss factor results in the same dissipation as happens in the case of a larger loss factor and less vigorous vibration response.

While the first term in Eq. (8.9) is valid for any type of partition (including, e.g., double walls and inhomogeneous and non isotropic plates), the second term is valid only for single, homogeneous, isotropic, platelike partitions. In case of

double walls or double windows the first term will exhibit a peak at the double-wall resonance frequency where the TL has a sharp minimum. In the case of double partitions made of two isotropic, homogeneous plates with air space in between, the second term in Eq. (8.9) can be crudely approximated by entering the parameters of the source-side plate into the second term.

8.3 SOUND ABSORBERS USING THIN LAYERS

For the purposes of this chapter, we call those constructions "sound absorbers" which have been designed on purpose to yield high sound absorption in a narrow or wide frequency band. The rest of this chapter deals with the design of such absorbers. We start with those absorber configurations which can be described by a few directly measurable parameters, such as a thin, flow-resistive layer in front of an air space backed by a rigid wall that can be fully characterized by its flow resistance and mass per unit surface area, and will proceed to configurations utilizing thick porous layers and to tuned absorbers.

Thin, Flow-Resistive Layer in Front of Rigid Wall

A sound absorber that consists of a thin, flow-resistive material in front of a rigid wall with an air space in between, such as shown in Fig. 8.1, is the configuration for which it is the easiest to predict acoustical performance. Accordingly, it is logical to treat it first. The flow-resistive layer might be rigid or flexible.

The sketch on the left in Fig. 8.1 depicts a configuration where the air space between the rigid porous layer and the rigid wall is partitioned in a honeycomb pattern to prohibit propagation of sound parallel to the plane of the porous layer. This absorber configuration is called *locally reacting* because the sound field in the absorber depends only on the sound pressure on the interface at the location of the particular honeycomb cell. The sound in the absorber can propagate only perpendicular to the plane of the interface. The sketch on the right in Fig. 8.1 has no partitioning of the air space and the sound can propagate within the absorber parallel to the plane of the interface. The sound pressure at any particular location in the air space depends on the sound pressure on all the locations on the sound-exposed face of the absorber. This configuration is called a *non-locally reacting* sound absorber.

Rigid Porous Layer

A thin, rigid, flow-resistive layer can be fully characterized by a single parameter, namely its flow resistance R_f or its normalized impedance $z' = R_f/(\rho_0 c_0)$, where ρ_0 is the density and c_0 is the speed of sound of the fluid. The normalized impedance of the partitioned air space is $z'' = j \cot(kt)$, where $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength of the sound, and t is the depth of the air space.

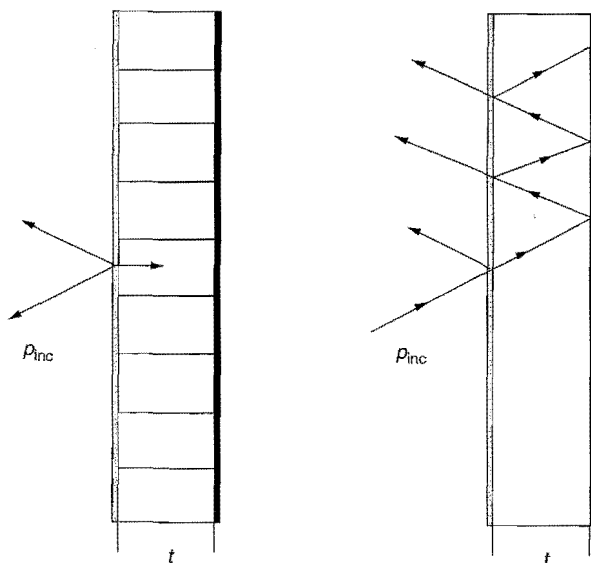


FIGURE 8.1 Thin porous sound-absorbing layer in front of an air space backed by a rigid wall. Left: Partitioned, locally reacting air space. Right: Not partitioned, non-locally reacting air space.

The normalized wall impedance for normal incidence, $\theta = 0$, is then given by

$$z_{w0} = z' + jz'' = \frac{R_f}{\rho c} + j \cot(kt) \quad (8.10)$$

The impedance of the air space is zero when $\cot(kt) = 0$, which is the case at those frequencies where the depth of the air space is an odd number of quarter wavelengths. The maximum value of the normal-incidence sound absorption coefficient is

$$\alpha_{0,\max} = \frac{4z'}{(1+z')^2} \quad (8.11)$$

indicating that total absorption occurs for $z' = 1$. According to reference 4, the normal-incidence sound absorption coefficient is given by

$$\alpha_0(f) = \left\{ \left[\left(\frac{R_f}{\rho_0 c_0} \right)^{0.5} + \left(\frac{\rho_0 c_0}{R_f} \right)^{0.5} \right]^2 + \frac{\rho_0 c_0}{R_f} \cot^2 \left(\frac{2\pi f t}{c_0} \right) \right\}^{-1} \quad (8.12)$$

Ingard¹ has computed the sound absorption coefficient of a rigid, flow-resistive layer in front of a locally reacting (partitioned) and non-locally reacting

(unpartitioned) air space backed by a rigid wall for both normal and random incidence, as shown in Fig. 8.2.

Figure 8.2 shows that for a locally reacting air space the maximum normal-incidence sound absorption coefficient occurs at those frequencies where the depth of the air space corresponds to an odd multiple of quarter wavelengths [$t = (n+1)\lambda/4$, where $n = 0, 1, 2, \dots$]. The physical reason for this is that the sound pressure in the partitioned air space just behind the porous layer is zero at these frequencies (the sound that is fully reflected from the hard wall is 180° out of phase with the incident sound). Consequently, the sound pressure gradient across the porous layer, Δp , reaches its maximum value that is numerically equal to the sound pressure on the incident side of the layer, namely $\Delta p = p_{\text{inc}}(1+R)$, where p_{inc} is the amplitude of the incident sound and R is the reflection factor:

$$R = \frac{R_f - \rho_0 c_0}{R_f + \rho_0 c_0}$$

This pressure gradient produces a particle velocity $v = \Delta p / R_f$ through the porous layer and the sound power dissipated per unit surface area of the absorber,

$$W_{\text{diss}} = v \Delta p = \frac{(p_{\text{inc}})^2 (1+R)^2}{R_f}$$

In the case of $R_f = \rho_0 c_0$, $R = 0$ and $W_{\text{diss}} = (p_{\text{inc}})^2 / (\rho_0 c_0) = W_{\text{inc}}$, indicating that all of the incident sound energy is dissipated in the porous layer. For values of $R_f \neq \rho_0 c_0$, the sound absorption coefficient still is a maximum but less than unity. The more R_f differs from $\rho_0 c_0$, the smaller is the sound absorption coefficient.

The maxima of the random-incidence sound absorption coefficient for a partitioned air space occur at the same frequencies as those at normal incidence. However, there is no value of R_f that would yield total absorption.

Another significant feature to be noted from Fig. 8.2 is that for a partitioned, locally reacting air space curves for both the normal and random-incidence sound absorption coefficients versus frequency have notches (zero absorption) at frequencies that correspond to an even multiple of half wavelengths. This occurs because at the corresponding frequencies the sound reflected from the hard wall combines at the rear of the porous layer with the incident sound in such a manner that there is no sound pressure gradient across the porous layer and consequently no sound is absorbed at these frequencies. This does not affect the random-incidence sound absorption coefficient obtained for the not partitioned (non-locally reacting) air space, as shown in the curves marked *c*. The advantage of not having notches in curves of the random-incidence sound absorption coefficient versus frequency must be "paid for" by having substantially lower sound absorption at low frequencies. The most important observation from Fig. 8.2 is that it is not possible to achieve a high sound absorption coefficient with a rigid porous layer in front of an air space backed by a rigid wall unless the thickness

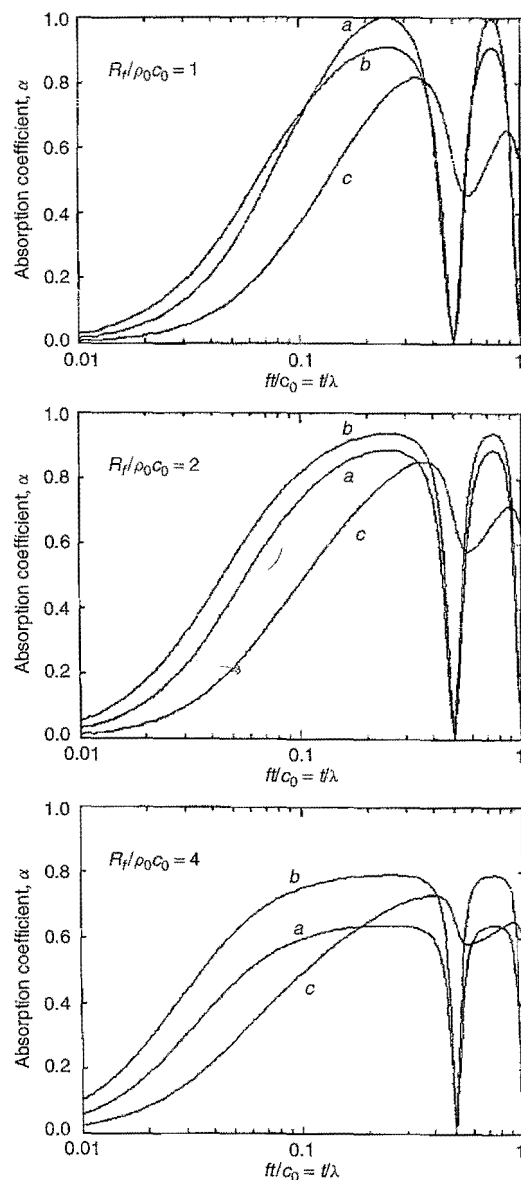


FIGURE 8.2 Sound absorption coefficient of a single, rigid, flow-resistive layer in front of an air space of thickness t backed by a hard wall as a function of the normalized frequency in the form of air space thickness–wavelength ratio $t/\lambda = tf/c_0$ with the normalized flow resistance of the layer, $R_f/\rho_0 c_0$, as the parameter: *a*, normal incidence; *b*, random-incidence, locally reacting air space; *c*, random-incidence, non-locally reacting air space. (After Ref. 1.)

of the air space exceeds one-eighth of the wavelength. For example, an air space thickness of 0.4 m (17 in.) or more is necessary to achieve a high degree of sound absorption at 100 Hz.

As we will discuss below, substantial low-frequency sound absorption can be achieved with an air space thickness of less than one-eighth of the wavelength if the porous layer is not rigid.

Limp Porous Layer

It is little known that sound absorbers using a limp, flow-resistive layer can provide high sound absorption at much lower frequencies than those with a rigid layer. However, caution should be exercised in designing such absorbers for applications where the sound pressure is very high. The limp, flow-resistive layer (e.g., heavy glass fiber cloth or stainless steel wire mesh) should be able to withstand the stresses owing to the high-amplitude sound-induced motion without fatigue. If the porous sound-absorbing layer is limp instead of being rigid, the mass per unit area of the layer and the stiffness per unit area of the air space between the layer and the hard wall result in a resonant system. Near the resonance frequency the limp porous layer will exhibit large-amplitude motion. Approximately, the resonance frequency f_{res} is given by¹

$$f_{res} \approx \frac{1}{2\pi} \left(\frac{\kappa \rho_0 (c_0)^2}{t m''} \right)^{1/2} \text{ Hz} \quad (8.13)$$

where $\kappa = 1.4$ is the adiabatic compression coefficient, t is the thickness of the air space in meters, and m'' is the mass per unit area of the limp porous layer in kilograms per square meter. The analytical model to predict the sound absorption coefficient of sound absorbers utilizing a limp porous screen is documented in reference 1. The computer program on the CD included in reference 1 allows prediction of the normal- and random-incidence sound absorption coefficient of such absorbers with locally reacting and non-locally reacting air space. The data in Figs. 8.3–8.5 were computed this way and show the sound absorption coefficient as a function of the normalized frequency, $f_n = t/\lambda = tf/c_0$, with the normalized flow resistance of the limp porous layer ($R_f/\rho_0 c_0$) as a parameter for various ratios of the mass of the limp layer and the mass of air in the air space behind, $MR = m''/t\rho_0$.

Figure 8.3 shows that the first maximum of the normal-incidence sound absorption coefficient occurs close to the mass–spring resonance frequency given by Eq. (8.13). The higher is the mass ratio $MR = m''/\rho_0 t$, the lower is the frequency of the peak sound absorption. For $MR = 16$ the frequency of the peak occurs at a frequency that corresponds to a normalized frequency $f_n = 0.03$. This is more than a factor of 8 lower than the peak at $f_n = 0.25$ one would obtain with a rigid porous layer. Also note that the first peak of the normal-incidence sound absorption coefficient is unity (100% absorption) if the mass ratio equals the normalized flow resistance (i.e., when $MR = R_f/\rho_0 c_0$). To achieve a relatively high

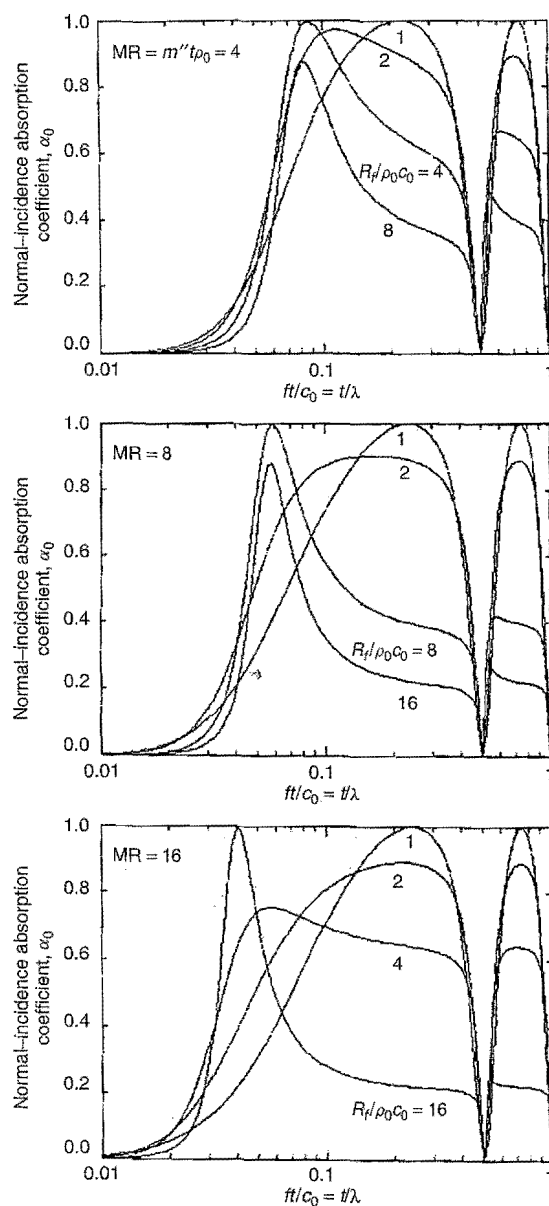


FIGURE 8.3 Normal-incidence absorption coefficient of a single, limp, flow-resistive layer in front of an air space of thickness t backed by a rigid wall as a function of the normalized frequency in the form of the thickness-wavelength ratio $t/\lambda = tf/c_0$ with the normalized flow resistance of the layer, $R_f/\rho_0 c_0$, as the parameter for mass ratios MR. (After Ref. 1.)

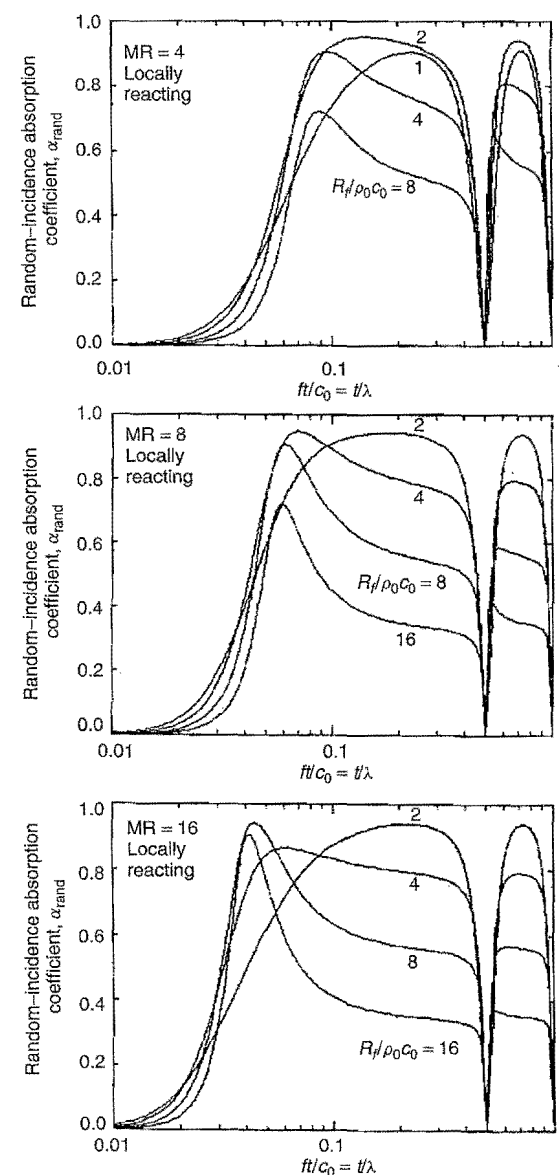


FIGURE 8.4 Random-incidence absorption coefficient of a single, limp, flow-resistive layer in front of a partitioned (locally reacting) air space of thickness t backed by a rigid wall as a function of the normalized frequency in the form of the thickness-wavelength ratio $t/\lambda = tf/c_0$ with the normalized flow resistance of the layer, $R_f/\rho_0 c_0$, as the parameter for mass ratios MR. (After Ref. 1.)

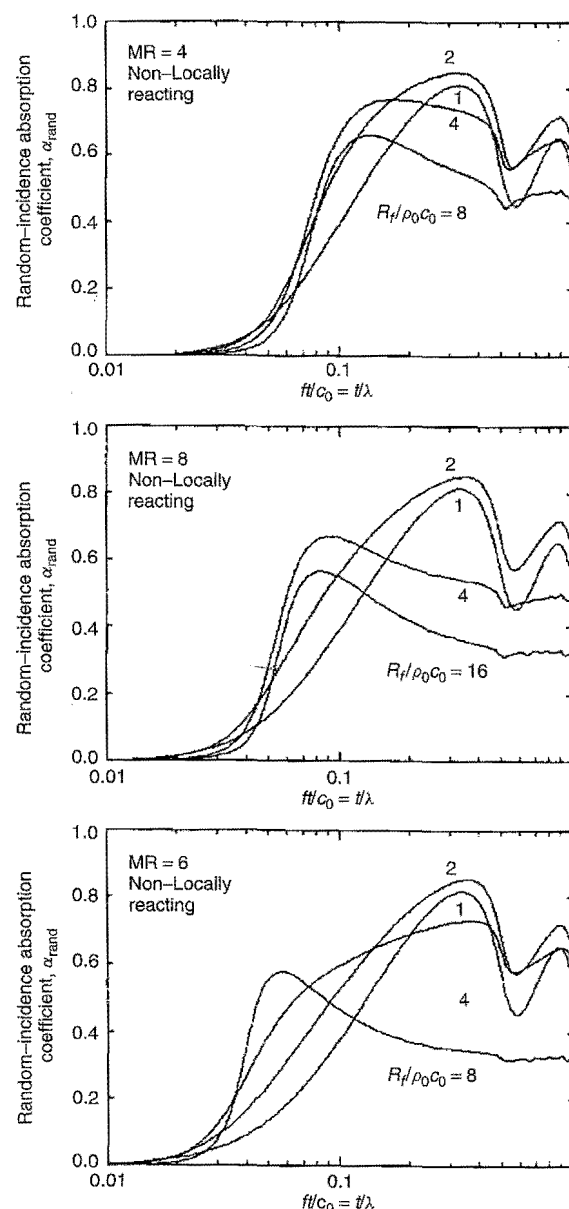


FIGURE 8.5 Random-incidence absorption coefficient of a single, limp, flow-resistive layer in front of a not-partitioned (non-locally reacting) air space of thickness t backed by a rigid wall as a function of the normalized frequency in the form of the thickness-wavelength ratio $t/\lambda = tf/c_0$ with the normalized flow resistance of the layer, $R_f/\rho_0 c_0$, as the parameter for mass ratios MR. (After Ref. 1.)

low-frequency absorption in a wide frequency band, a normalized flow resistance $R_f/\rho_0 c_0$ of between 2 and 3 seems to be a good choice.

Figure 8.4 shows the random-incidence sound absorption coefficient as a function of the normalized frequency computed for a locally reacting (partitioned) air space. Sufficient distance must be provided between the limp porous layer and the honeycomb configuration used to partition the air space to prevent the limp layer from hitting the honeycomb layer at the largest expected displacement amplitude. Curves of the absorption coefficient versus the normalized frequency have similar shape as those in Fig. 8.3 for normal incidence, except that no combination of the design parameters results in total absorption. To obtain substantial low-frequency sound absorption, the mass ratio $MR \approx 2$ seems to be a good choice.

Figure 8.5 shows the random-incidence sound absorption coefficients for a non-locally reacting (unpartitioned) air space. Note the very different behavior from that observed for the partitioned air space in Figs. 8.2 and 8.3. The first, and undesirable, difference is that no combinations of parameters yield random-incidence sound absorption coefficients much above $\alpha_{\text{rand}} = 0.8$. The second, and desirable, attribute of the nonpartitioned air space is that there are no notches in the frequency response where the absorption coefficient would go to zero, as was the case for the partitioned air space.

Multiple Limp Porous Layers

As documented in reference 1, very high sound absorption at low frequencies can be achieved over nearly four octaves by placing a large number (up to 16) of limp, porous layers of low-flow resistance in front of a rigid wall. The computer program supplied with reference 1 allows the prediction of the normal- and random-incidence sound absorption coefficients of such absorbers and the optimization of performance by trial and error varying the flow resistance, mass per unit area, and number and position of the layers.

8.4 POROUS BULK SOUND-ABSORBING MATERIALS AND ABSORBERS

Porous bulk sound-absorbing materials are utilized in almost all areas of noise control engineering. This section deals with the following aspects:

1. description of the key physical attributes and parameters that cause a porous material to absorb sound,
2. description of the acoustical performance of porous sound absorbers used to perform specific noise control functions,
3. compilation of acoustical parameters that allow the quantitative design of sound-absorbing configurations on the bases of material and geometric parameters, and

4. experimental methods for measuring the acoustical parameters of porous sound-absorbing materials and the acoustical performance of porous sound absorbers.

Porous materials used for sound absorption may be fibrous, cellular, or granular. Fibrous materials may be in the form of mats, boards, or preformed elements manufactured of glass, mineral, or organic fibers (natural or man made) and include felts and felted textiles. Applications of fibrous materials in silencers require some form of protective covering. The most common covering consists of perforated metal with a fiberglass cloth behind. Sometimes metal screens are inserted between the perforated metal and fiberglass cloth. Cellular materials include polymer foams of varying degrees of rigidity and porous metals. For special applications increasing use is being made of porous metals (e.g., aluminum foams) as sound absorbers. During manufacture, before solidification, metal foams have closed cells, that is, without interconnections. At this stage such materials are poor sound absorbers. However, as liquid-metal foams solidify, thermal stresses occur. The solidified foams usually have cracked cell walls, which significantly increases sound absorption. In addition, by slightly rolling thin sheets of foam from, say, a thickness of 10 to 9 mm, further mechanical cracking occurs, and the interconnections between adjacent cells widen. This increases sound absorption even more. The result is an absorption coefficient versus frequency that has a maximum between 1 and 5 kHz and a peak value of up to 95%. By placing an air gap between metal foam and a rigid wall, one can shift the frequency curve to lower frequencies. In comparison with other materials, glass wool, for example, which gives high absorption over a wide frequency range, metal foams are not very good sound absorbers. However, the high weight-specific stiffness, good crash-energy absorption ability, and fire resistance of porous metals make them suitable for sound absorption panels in the aircraft and automotive industries.

Granular materials can be regarded as an alternative to fibrous and foam absorbers in many indoor and outdoor applications.⁸ Sound-absorbing granular materials combine good mechanical strength and very low manufacturing costs. Granular materials may be unconsolidated (loose) or consolidated through use of some form of binder on the particles as in wood-chip panels, porous concrete, and pervious road surfaces. As well as man-made granular materials, there are many naturally occurring granular materials, including sands, gravel, soils, and snow. The acoustical properties of such materials are important for outdoor sound propagation.

A common feature of porous sound-absorbing materials is that the pores are interconnected and have typical dimensions below 1 mm, that is, much less than the wavelengths of the sounds of interest in noise control.

Porous materials must be considered to be elastic if they are in the form of flexible sheets. However, in many cases, the stiffness of the solid frame of the material is much greater than that of air so that the material may be treated as if it were rigid over a wide range of frequencies. This means that such materials

can be treated as lossy, homogeneous media. This chapter will be confined to porous materials that may be treated as rigid framed.

How Rigid Porous Materials Absorb Sound

When excited by an incident sound wave, the air molecules in the pores of a porous material are made to oscillate. The proximity of the surrounding solid means that the oscillations result in frictional losses. An important factor that determines the relative contribution of frictional losses is the size (width) of the pores relative to the thickness of the viscous boundary layer. At low frequencies the viscous boundary layer thickness might be comparable with the pore width and the viscous loss is high. At high frequencies the viscous boundary layer thickness may be significantly less than the pore size and the viscous loss is small. At such high frequencies the oscillating flow is "pluglike." However, the presence of the solid causes changes in the flow direction and expansions and contractions of the flow through irregular pores, resulting in loss of momentum in the direction of wave propagation. This mechanism is relatively important at higher frequencies. In the larger pores and at lower frequencies heat conduction also plays a part in energy loss. During the passage of a sound wave, the air in the pores undergoes periodic compression and decompression and an accompanying change of temperature. If the solid part of the material is relatively heat conducting, then the large surface-to-volume ratio means that during each half-period of oscillation there is heat exchange and the compressions are essentially isothermal. At high frequencies the compression process is adiabatic. In the frequency range between isothermal and adiabatic compression the heat exchange process results in further loss of sound energy. In a fibrous material this loss is especially high if the sound propagates parallel to the plane of the fibers and may account for up to 40% of sound attenuation (energy lost per meter of propagation). The losses from forced mechanical oscillations of the skeleton of a porous material are generally so low that it is reasonable to neglect them.

Physical Characteristics of Porous Materials and Their Measurement

Porosity. The gravimetric measurement of porosity requires the weighing of a known volume of dry material. Since in inexpensive fibrous materials, such as mineral wool, the large droplets (or shot with diameter over 100 μm) may constitute up to 30% of the weight, their contribution to the acoustical porosity should be disregarded. Shot can be separated from fiber by a centrifuge process. The dry weight can be used together with the sample volume to calculate the bulk density ρ_B . Subsequently an assumed solid density is used to calculate the porosity h from

$$h = 1 - \frac{\rho_A}{\rho_B} \quad (8.14)$$

For glass fiber and mineral wool products the density of the fiber material is $\rho_B = 2450 \text{ kg/m}^3$. For silica sand, the density of the mineral grains is 2650 kg/m^3 . A

gravimetric method which may be used with some consolidated granular materials is to saturate the sample with water and deduce the porosity from the relative weights of the saturated and unsaturated samples. Mercury has been used as the pore-filling fluid in some applications (e.g., soils), but for many materials the introduction of liquids affects the pores. Following a proposal of Cremer and Hubert,⁹ Champoux et al.¹⁰ have developed a *dry* method of porosity determination which is based on the measurements of the change in pressure within a sample container subject to a small known change in volume. The lid of the container is a plunger, which is driven by a precise micrometer. The pressure inside the chamber is monitored by a sensitive pressure transducer and an air reservoir connected to the container through a valve serves to isolate the system from fluctuations in atmospheric pressure. The system has been estimated to deliver values of porosity accurate to within 2%. An important feature of the method from the point of view of acoustical properties is that it measures the porosity of connected air-filled pores. However, the gravimetric methods do not differentiate between sealed pores and connected pores. Recently a new method for determining porosity based on a simple measurement of displaced air volume has been proposed.¹¹ This has the advantage that it does not require compensation for temperature, as does the method of Champoux et al.¹⁰

An alternative method which may be used with some consolidated materials is to saturate the sample with water and deduce the porosity from the relative weights of the saturated and unsaturated samples.

An acoustical (ultrasonic) impulse method for measuring porosity using the impulse reflected at the first interface of a slab of air-saturated porous material has been proposed and has been shown to give good results for plastic foams¹²⁻¹⁴ and random bead packing.¹⁵

The porosity of a stack of spherical particles depends on the form of the packing, ranging from 0.26 for the densest packing (face-centered-cubic) to 0.426 for simple-cubic packing.¹⁶ A random packing of spheres has a porosity of 0.356.¹⁷ An approximate value that may be assumed for the porosity of a granular material is 0.4.

Typical ranges of values of porosity are listed in Table 8.1.

Tortuosity. The tortuosity of a porous solid is a measure of the irregularity of the fluid-filled paths through the solid matrix. At very high frequencies, it is responsible for the difference between the speed of sound in air and the speed of sound through a rigid porous material. Tortuosity is related to the formation factor used to describe the electrical conductivity of a porous solid saturated with a conducting fluid. Indeed, tortuosity can be measured using an electrical conduction technique in which the electrical resistivity of such a saturated porous sample is compared to the resistivity of the saturating fluid alone. Thus

$$T = \frac{F}{h} \quad (8.15)$$

TABLE 8.1 Measured and Calculated Values of Porosity and Tortuosity

Material	Porosity, h	Method of Determination	Tortuosity, T	Method of Determination
Lead shot, 3.8 mm particle size	0.385	Measured by weighing	1.6	Estimated as $\alpha = 1/\phi^{0.5}$ (fits acoustic data)
Gravel, 10.5 mm grain size	0.45	Measured by weighing	1.799	Cell model predictions ^a
Gravel, 5-10 mm grain size	0.4	Measured by weighing	1.55	Deduced by fitting surface admittance data
Glass beads, 0.68 mm ^b	0.375	Unspecified	1.46	Deduced by fitting surface admittance data
Coustone ^b	0.4	Unspecified	1.742	Measured ^c
Foam YB10 ^b	0.61	Unspecified	1.833	Cell model predictions ^a
Porous concrete	0.312	Measured	1.664	Measured ^c
Clay granulate, Laterlite, 1-3 mm grain size	0.52		1.918	Measured ^c
Olivine sand	0.444	Measured	1.8	Deduced by fitting surface admittance data
Aluminum foam	0.93	Measured by weighing	1.25	Deduced by fitting surface admittance data (assuming porosity of 0.52)
Polyurethane foam (Recticel Wetter, Belgium)			1.626	Cell model predictions ^a
Sample w1 ^d	0.98	Measured	1.1	Ultrasonic measurements (laser-generated pulses)
Sample w2 ^d	0.97	Measured	1.07	Deduced by fitting surface admittance data

^aFrom Ref. 20.

^bFrom Ref. 26.

^cFrom Ref. 11.

^dFrom Ref. 27.

where F is the formation factor defined by

$$F = \frac{\sigma_s}{\sigma_f} \quad (8.16)$$

where σ_f and σ_s are the electrical conductivities of the fluid and the fluid-saturated sample, respectively. These in turn are defined by

$$\sigma = \frac{GL}{A} \quad (8.17)$$

where L is the length of the sample, A is the area of the end of the sample, and G is the ratio of the resulting current to the voltage applied across the sample.

To measure the formation factor, first a cylindrical sample of the material is saturated with a conducting fluid (brine solution is convenient). Saturation is achieved by drawing the fluid through the sample after forming a vacuum above it. Agitation of the sample is also required if the pore sizes are small. A voltage is applied across the saturated sample placed between two similarly shaped electrodes at a known separation. The conductivity of the fluid is measured at similar voltages within a separate fluid-tight unit. The use of separate current and voltage probes assures a good contact between the end of the sample and the electrodes, eliminates problems associated with voltage drop at the current electrodes, and allows the simultaneous measurement of the electrical resistivities of the fluid and the saturated porous material.

The tortuosity of a random stacking of glass spheres is given by $1/\sqrt{h}$.¹⁸ This has been verified for a range of porosities from 0.33 to 0.38 and is a special case of the relationship

$$T = h^{-n'} \quad (8.18)$$

where n' depends on grain shape and has the value 0.5 for spheres.¹⁹

An alternative derivation for stacked identical spheres²⁰ gives

$$T = 1.5 + \frac{1-h}{2h} \quad (8.19)$$

The equivalent formula for the tortuosity of a system of identical parallel fibers is

$$T = \frac{1}{h} \quad (8.20)$$

This means that the tortuosity of a typical fibrous material used in noise control is a little larger than 1 since the porosity is close to (but never greater than) 1. In a rigid porous material, tortuosity is one of the properties that contribute to the "structure factor" used in classical descriptions and is related to the "added mass" that is used in the theory of propagation of sound in porous and elastic materials.²¹ The structure factor, introduced in classical texts, is intended to include frequency-dependent thermal effects (complex bulk modulus) and frequency-dependent effects due to fiber motion. This means it can *only* be determined by acoustical means. For characterizing the acoustical properties of a bulk porous material, tortuosity is preferred over the structure factor since it has clear physical meaning and can be deduced by nonacoustical means in many cases.

At the highest frequencies, the speed of sound inside the rigid porous material is equal to the speed of sound in air divided by the square root of tortuosity. Since it is primarily responsible for the acoustical properties of porous materials at high audible or ultrasonic frequencies, tortuosity can be deduced from ultrasonic measurements.²²

Some representative values of tortuosity are listed in Table 8.1.

Flow Resistance and Flow Resistivity. The most important parameter in determining the acoustical behavior of thin porous materials is the airflow resistance R_f . For bulk-, blanket-, or board-type porous materials the flow resistivity (specific flow resistance per unit thickness) $R_1 = R_f/\Delta x$, where Δx is the thickness of the layer, is the key acoustical parameter. The flow resistivity is a measure of the resistance per unit thickness inside the material experienced when a steady flow of air moves through the test sample. Flow resistance R_f represents the ratio of the applied pressure gradient to the induced volume flow rate and has units of pressure divided by velocity. If a material has a high flow resistivity (high flow resistance per unit thickness), it means that it is difficult for air to flow through the surface. In disciplines other than noise control engineering (e.g., geophysics), it is more common to refer to air permeability (k), which is related to the inverse of flow resistivity ($k = \eta/R_1$, where η is the dynamic coefficient of air viscosity). Since flow resistivity is related to the inverse of permeability, high flow resistivity implies low permeability. Typically, low permeability results from very low surface porosity.

Flow resistance is measured as

$$R_f = \frac{\Delta p}{v} = \frac{TS \Delta p}{V} \quad \text{N} \cdot \text{s} \cdot \text{m}^{-3} \text{ or } \text{Pa} \cdot \text{s} \cdot \text{m}^{-1} \text{ (mks rayls)} \quad (8.21)$$

From this the flow resistivity is obtained as

$$R_1 = \frac{R_f}{\Delta x} \quad \text{N} \cdot \text{s} \cdot \text{m}^{-4} \text{ or } \text{Pa} \cdot \text{s} \cdot \text{m}^{-2} \text{ (mks rayls/m)} \quad (8.22)$$

where Δp is the static pressure differential across a homogeneous layer of thickness Δx , v is the velocity of the steady flow through the material, V is the volume of air passing through the test sample during the time period T , and S is face area (one side) of the sample.

Since R_f generally depends on the velocity v , it is customary to measure it at a number of different flow rates and extrapolate measured R_f versus v to $R_f(v = 0.05 \text{ cm/s})$ because below this particle velocity the flow resistance of most fibrous materials does not depend any more on the velocity. The fact that, in general, flow resistivity depends on particle velocity becomes important when considering the behavior of porous absorbers at high sound levels.

The measurement of the flow resistance and flow resistivity of porous building materials has been standardized on a compressed-air apparatus.²³ A similar apparatus may be used to determine the flow resistivity of soils or granular materials (Fig. 8.6). In this measurement the pressure gradient across the sample in a fixed sample holder is monitored together with various (low) flow rates. Compressed air is passed through a series of regulating valves and a very narrow opening into chamber E. This creates an area of low pressure immediately in front of the three tubes connected to the rest of the system. Air is drawn from the environment through the sample as a result of the pressure differential. The rate of airflow through the system is controlled by three flowmeters, giving a total measurement

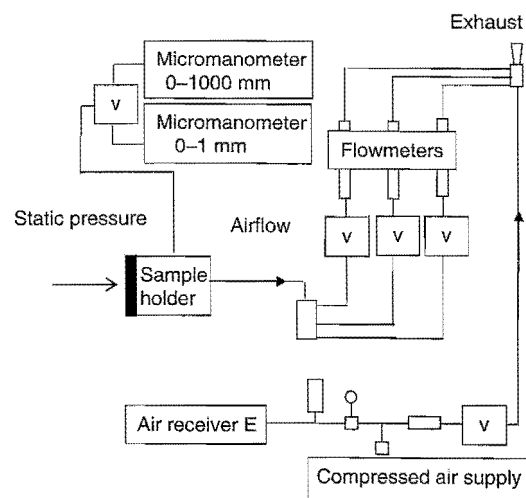


FIGURE 8.6 Concept sketch of a compressed-air apparatus for laboratory measurement of flow resistance.

range between 8.7 and 0.1 L/min. Typically the flow rate must be kept below 3 L/min to avoid structural damage to the sample.

A comparative method²⁴ makes use of a calibrated known resistance (a laminar-flow element) placed in series with the test sample. Variable-capacitance pressure transducers are used to measure pressure differences across both the test sample and the calibrated resistance. For steady, nonpulsating flow, the ratio of flow resistances equals the ratio of measured pressure differences. The airflow may also be controlled electronically. A unique method of measuring flow resistance by Ingard¹ does not require any flow-moving device, flow-rate meter, or static pressure differential sensor, just a stop watch. In this test setup the airflow (after the piston reaches its terminal velocity) is forced through the sample, located at the open bottom end of the tube, at a constant rate driven by the static pressure differential $\Delta p = Mg/S$, determined by the weight, Mg , of a tightly sliding piston in the sample holder tube and by the cross-sectional area S of the piston. The factor $g = 9.81 \text{ m/s}^2$ is the acceleration due to gravity. The flow resistance of the sample is then inversely proportional to the time T_L required for the piston to travel a specific distance L . The flow resistance of the sample is determined as $R_f = C[Mg \cos(\Phi)/LA^2] ST_L$, where A is the free surface area of the sample and Φ is the angle of tube axis with the vertical. For vertical tube orientation $\Phi = 0$. The small leakage flow between the piston and the tube wall, which is of consequence only if the sample has a very high flow resistance, is accounted for by the correction factor

$$C = \frac{1 + T_{L0}/T_L}{1 - T_L/T_{\text{Closed}}}$$

where T_{L0} and T_{Closed} are respectively the piston travel times with the sample removed and when the end of the tube is air tightly closed. The variation of the flow rate through the sample (needed to extrapolate the flow resistance to $5 \times 10^{-4} \text{ m/s}$) can be accomplished by either changing the tube axis angle Φ or increasing the mass of the piston.

Empirical Prediction of Flow Resistivity. According to private communications with Richard Godfrey⁶ of Owens Corning Corporation, the flow resistivity of fibrous sound-absorbing materials can be approximately predicted by a slightly modified version of Eq. 10.4 of the 1971 edition of this book. The modified equation is

$$R_1 \approx \frac{3450}{d^2} \left[\left(\frac{\text{SpGrGlass}}{\text{SpGrFiber}} \right) \rho_{\text{bulk}} \right]^{1.53} \text{ N} \cdot \text{m/s}^4 \quad (8.23)$$

where d is the average fiber diameter in micrometers (10^{-6} m), SpGrGlass and SpGrFiber are the specific gravities of glass and that of the actual fiber material, and ρ_{bulk} is the bulk density (with shot contribution deducted) of the fibrous material in kilograms per cubic meter. Note that $1 \mu\text{m} = 4 \times 10^{-5} \text{ in.}$ and that $\rho_{\text{bulk}} \approx \rho_M h$, where ρ_M is the density of the fiber material and h is the porosity. It has been found that flow resistivity values measured according to the American Society of Mechanical Engineers (ASME) Standard E 522 for the entire glass fiber-based product line of different bulk densities and fiber diameters are successfully predictable by Eq. (8.23) with a correlation coefficient of 0.93. Equation (8.23) also can be used for predicting the flow resistivity of polymer-based fibrous sound-absorbing materials. For the same bulk density, the polymer fibers have approximately 2.5 times as much surface area and flow resistivity as glass fibers.

Theoretical Prediction of Flow Resistivity. For almost all granular or fibrous sound-absorbing materials, the flow resistivity cannot be predicted analytically on the basis of the geometry of the skeleton. It must be measured. For a few idealized absorber materials, such as materials made of identical spheres or parallel identical fibers, it is possible to make such a prediction. Since the prediction formulas identify important parameters that might be used to scale flow resistivity data measured for a specific material composition to other compositions in cases where direct measurement of the flow resistivity is not feasible, it is instructive to consider at least one idealized case where theoretical prediction is possible.

For stacked identical spheres with radius r it can be shown that²⁰

$$R_1 = \frac{\eta}{k} = \frac{9\eta(1-h)}{2r^2h^2} \frac{5(1-\Theta)}{5-9\Theta^{1/3}+5\Theta-\Theta^2} \quad (8.24)$$

where h is the porosity and

$$\Theta = \frac{3}{\sqrt{2\pi}}(1-h) \cong 0.675(1-h) \quad (8.25)$$

The equivalent formula for the flow resistivity of a system of identical parallel fibers of radius r is²⁵

$$R_1 = \frac{16\eta h(1-h)}{r^2 h^2 [(1-8\Gamma) + 8\Gamma(1-h) - (1-h)^2 - 2\ln(1-h)]} \quad (8.26)$$

where $\Gamma \approx 0.577$. This formula overestimates the flow resistivity of fibrous materials as a consequence of shot content, random fiber orientation, and distribution of fiber diameters in a real material.

Analytical Characterization of Porous Granular or Fibrous Sound-Absorbing Materials. For certain idealized microstructures such as parallel, identical cylindrical or slitlike pores, it is possible to make straightforward analytical predictions of the acoustical properties.^{1,19,21} Although analytical characterization of porous granular or fibrous sound-absorbing materials with arbitrary microstructures is possible, it is a relatively complex procedure.²⁷⁻³¹ Moreover, at present, many of the required parameters are not routinely available to practicing noise control engineers. Consequently, the analysis is not reviewed in this chapter. For a comprehensive introduction to recent analytical treatments, the reader should consult reference 21. Semiempirical formulas for the acoustical properties of fibrous materials have been derived based on such a sophisticated analytical treatment.³² It should be noted, also, that empirical formulas have been derived that predict the acoustical properties of granular media from knowledge only of porosity, grain density, and mean grain size.³³

Acoustical Properties of Porous Materials. The acoustical properties of porous sound-absorbing materials are characterized by their complex propagation constant k_a and complex characteristic impedance Z_{Ca} , which are defined by

$$p(x, t) = \hat{p} e^{-jk_a x} e^{j\omega t} \quad \text{N/m}^2 \quad (8.27)$$

$$\langle p(x, t) \rangle_y = Z_{Ca} \langle v_x(x, t) \rangle_y \quad \text{N/m}^2 \quad (8.28)$$

where $\hat{p}$ represents the amplitude at $x = 0$ and $\langle \dots \rangle$ represents an average perpendicular to the propagation direction (x) over an area that is small compared with the wavelength but large compared with the size of pores. The propagation constant

$$k_a = \beta - j\alpha$$

includes the attenuation constant α , which may be obtained by using a probe tube microphone to measure the decrease of the sound pressure level (in nepers per meter) of a plane sound wave propagating in a very thick layer of material. The phase constant β is obtained by measuring the change of phase with distance. It is equal to the angular frequency divided by the frequency-dependent sound speed within the material. The complex characteristic impedance $Z_{Ca} = R_{Ca} - jX_{Ca}$ is obtained by measuring the surface impedance of a thick layer (sufficiently thick

that reflection from the end is not detectable) of the absorber material placed in an impedance tube.

An alternative to use of a thick sample with a practically "infinite length" is to use a sample of known finite length and to measure the transfer function between two microphones located in an impedance tube with the loudspeaker source at one end and the sample at the other (closed) end.³⁴ A broadband input signal such as white noise or a sine sweep can be used. By means of a frequency analysis, output signals from the microphones are used to calculate the transfer function, which is converted to the surface impedance of a sample. The characteristic impedance and propagation constant of a sample can be obtained as long as two distinct sets of surface impedance are measured. This can be realized by using either two samples of different thicknesses³⁵ or a single sample backed with two different lengths of air cavities.³⁶ For the former, the two-thickness method, it is convenient if the length of the second sample is double that of the first. In the two-cavity method, the difference in the lengths of the air cavities needs to be tuned to fit the frequency range of interest together with the tube diameter and the microphone spacing.

Empirical Predictions from Regression Analyses of Measured Data

There are two families of parameters that determine the sound absorption coefficient. The characterization would be simplest if both k and Z_C could be expressed in terms of a single parameter. Figure 8.7a shows the measured³⁷ normal-incidence sound absorption coefficient α_0 of different Rockwool materials of practically infinite thickness (between 0.5 and 1 m, depending on the bulk density of the absorber material) as a function of frequency with bulk density of the material as parameter. Figure 8.7a clearly indicates that the bulk density is not the parameter that would collapse the measured data points into a single curve.

In 1970, Delany and Bazley³⁸ used many measurements on fibrous materials to deduce semiempirical formulas based on the dimensioned parameter (f/R_1) . Equations (8.29a) and (8.29b) are valid only if the frequency f is entered in hertz (s^{-1}) and the flow resistivity R_1 in mks rayls per meter ($\text{N} \cdot \text{s} \cdot \text{m}^{-4}$). Although they have been used widely and successfully, Delany and Bazley's formulas give rise to unphysical results at low frequencies. In particular, the real part of the surface impedance of a rigid-backed layer goes negative. Miki's model³⁹ represents a modification of these formulas based on Delany-Bazley's data and an electrical analogy for the acoustical properties of porous materials. Miki's formulas are as follows:

$$k_{an} = \frac{k}{k_0} = 1 + 0.109 \left(\frac{\rho_0 f}{R_1} \right)^{-0.618} - j0.160 \left(\frac{\rho_0 f}{R_1} \right)^{-0.618} \quad (8.29a)$$

$$Z_{Cn} = \frac{Z_C}{Z_0} = 1 + 0.070 \left(\frac{\rho_0 f}{R_1} \right)^{-0.632} - j0.107 \left(\frac{\rho_0 f}{R_1} \right)^{-0.632} \quad (8.29b)$$

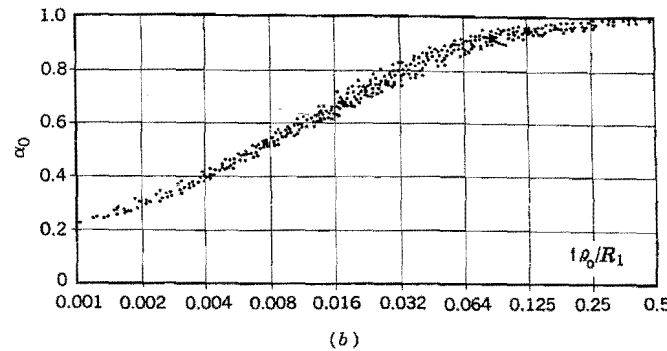
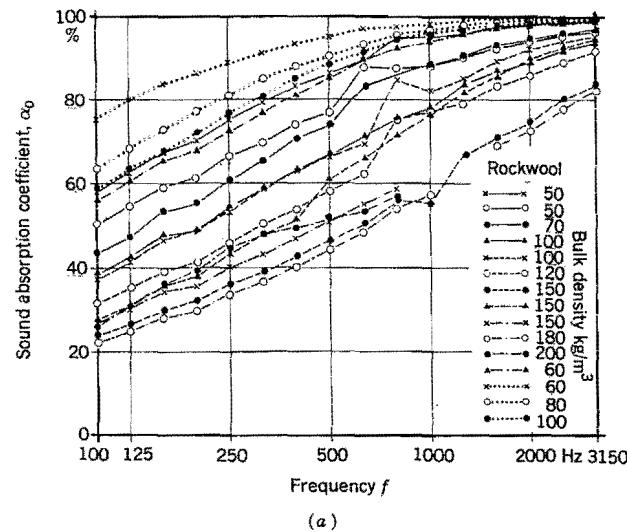


FIGURE 8.7 Normal-incidence sound absorption coefficients of different rock wool materials of practically infinite thickness (0.5–1 m) measured in an impedance tube plotted as a function of (a) frequency with bulk density ρ_A as parameter and (b) nondimensional frequency parameter $\rho_0 f/R_1 = \rho_0 c_0/(R_1 \lambda)$.

Figure 8.7b shows the same data points as Fig. 8.7a but now as a function of the dimensionless variable $E = \rho_0 f/R_1$ on the horizontal scale, where ρ_0 is the density of air, f is the frequency, and R_1 is the flow resistivity of the bulk material at the density at which α_0 was measured. Clearly $E = \rho_0 f/R_1 = \rho_0 c_0/\lambda R_1$ is the single parameter that collapses all measured data. According to Fig. 8.7b, even for a layer of practically infinite thickness, very high sound absorption is obtainable only in the frequency range where $(\lambda/4)R_1 < \rho_0 c_0$. To fulfill this requirement at low frequencies, where λ is large, requires the use of a sound-absorbing material of low flow resistivity. The normalized, dimensionless frequency variable

$E = \rho_0 f/R_1$ is useful in describing not only the sound-absorbing capability of the semi-infinite layer of fibrous material but also the propagation constant and characteristic impedance of the bulk material.

It is useful to present the propagation constant and characteristic impedance in a dimensionless manner as

$$k_{an} = \frac{k}{k_0} = \frac{a - jb}{k_0} = a_n - jb_n \quad (8.30a)$$

$$Z_{Cn} = \frac{Z_C}{Z_0} = R - jX \quad (8.30b)$$

where $k_0 = \omega/c_0$ is the wavenumber in air and $Z_0 = \rho_0 c_0$ is the characteristic impedance of the gas filling the voids between the fibers for plane waves.

Figures 8.8 and 8.9 show plots of the real and imaginary parts of the normalized propagation constant k_{an} and that of the normalized characteristic impedance Z_{Cn} for a large variety of mineral wool sound-absorbing materials plotted as a function of the normalized frequency parameter, indicating that indeed the normalized frequency parameter $E = \rho_0 f/R_1$ is a universal descriptor of fibrous porous sound-absorbing materials. The data presented in Figs. 8.8 and 8.9 and similar curves for glass fiber materials were obtained by careful measurements of the acoustical and material characteristics (a , b , R , X , and R_1) of over 70 different types of materials.³⁸

The solid lines in Figs. 8.8 and 8.9 result from the regression analyses of the data and have the form

$$k_{an} = \frac{k}{k_0} = (1 + a''E^{-\alpha''}) - ja'E^{-\alpha'} \quad (8.31a)$$

$$Z_{Cn} = \frac{Z_C}{Z_0} = (1 + b'E^{-\beta'}) - jb''E^{-\beta''} \quad (8.31b)$$

The regression parameters a' , a'' , b' , b'' , α' , α'' , β' , and β'' in Eqs. (8.31) are compiled in Table 8.2. There are different regression parameters for the normalized frequency regions below and above $E = 0.025$. It was found that measured fibrous materials could be divided into two categories: (1) mineral wool and basalt wool and (2) glass fiber.

Polyester Fiber Materials

Polyester fiber materials are used increasingly to replace glass and mineral fiber materials in situations where there is concern to keep the air completely free of fibers suspected to have an adverse influence on health. An example material consists of a mix of two kinds of fibers: (1) polyethyleneterephthalate and (2) a core of polyethyleneterephthalate and a lining of copolyester. The raw mix is treated at 150°C to melt the external lining of the “bicomponent” fibers and hence form a skeleton of thermally bound fibers. The fiber diameters are between

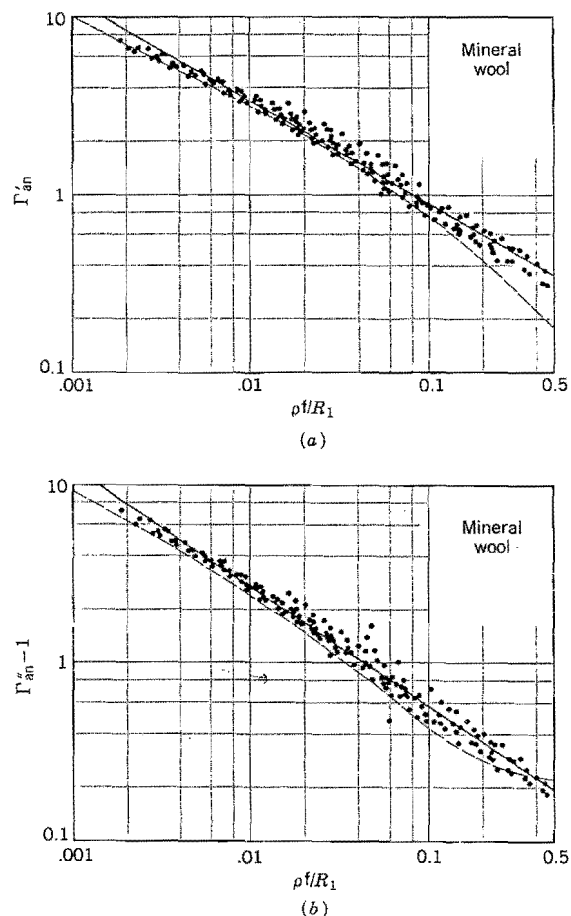


FIGURE 8.8 Measured normalized propagation constant $k_{an} = k/k_0 = a_n + ib_n$ for mineral wool as a function of normalized frequency parameter $E = \rho_0 f/R_1$: (—) regression line; Eq. (8.31a) and Table 8.2; (---) prediction for cylindrical pores.^{19,21,27}

TABLE 8.2 Regression Coefficients for Predicting Propagation Constant and Characteristic Impedance of Fibrous Sound-Absorbing Materials

Material	E Region	b'	β'	b''	β''	a''	α''	a'	α'
Mineral and basalt wool	$E \leq 0.025$	0.081	0.699	0.191	0.556	0.136	0.641	0.322	0.502
	$E > 0.025$	0.0563	0.725	0.127	0.655	0.103	0.716	0.179	0.663
Glass fiber	$E \leq 0.025$	0.0668	0.707	0.196	0.549	0.135	0.646	0.396	0.458
	$E > 0.025$	0.0235	0.887	0.0875	0.770	0.102	0.705	0.179	0.674

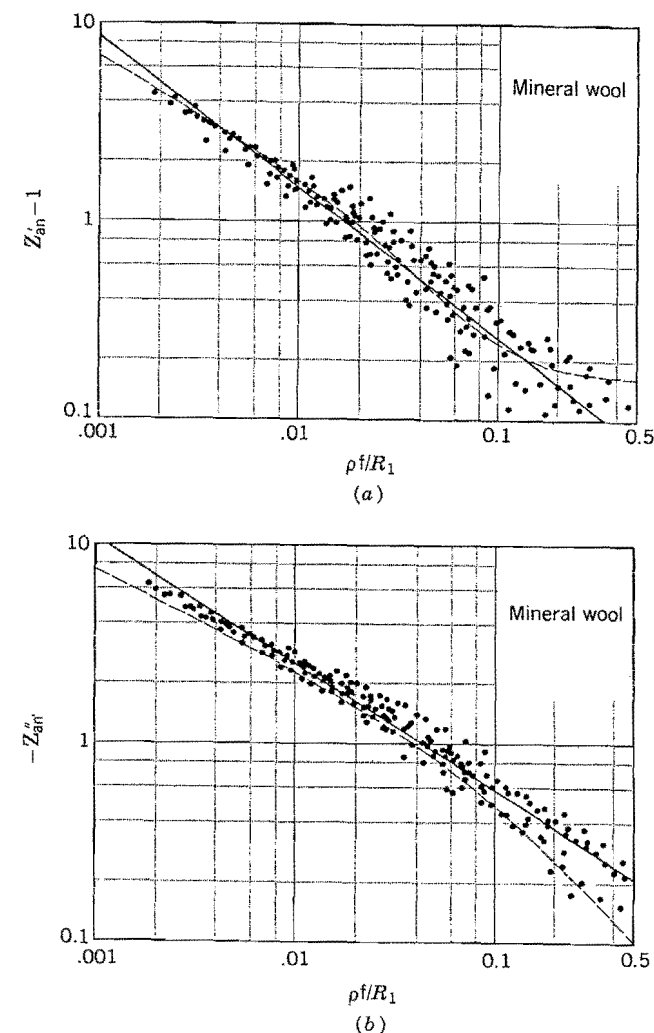


FIGURE 8.9 Measured normalized characteristic impedance $Z_{Cn} = Z_C/Z_0 = R + iX$ for mineral wool as a function of normalized frequency parameter $E = \rho_0 f/R_1$: (—) regression line; Eq. (8.29b); (---) prediction for cylindrical pores.

17.9 and 47.8 μm (mean 33 μm) and have a mean length of 55 mm. Based on measurements of 38 samples, the flow resistivity has been found to obey the relationship⁴⁰

$$R_1 D^2 = 26 \rho_A^{1.404} \quad (8.32)$$

where D is the mean diameter in micrometers and ρ_A the bulk density in kg/m^3 .

TABLE 8.3 Regression Coefficients for Predicting Propagation Constant and Characteristic Impedance of Polyester Fiber Materials

Material	b'	β'	b''	β''	α'	α'	α''	α''
Polyester fiber	0.159	0.571	0.121	0.530	0.078	0.623	0.074	0.660

Regression coefficients for predicting acoustical properties of polyester fiber materials are given in Table 8.3.³⁸

Plastic Foams

Since concerns about fire hazard and release of toxic combustion products can be overcome largely by suitable treatments, plastic foams are used increasingly for noise control applications. A large variety of plastic foams are available with several different types of physical structure. Polyurethane foams, based on polyester or polyether polyols, are used most commonly. These foams can be fully or partially reticulated, that is, with all membranes or with varying proportions of membranes between cells removed. Formulas (8.31) with similar regression coefficients to those for fibrous materials are applicable to plastic foams.^{41,42} However, unlike with fibrous materials, it has not been found necessary to distinguish between E-regions. The relevant regression coefficients are listed in Table 8.4.

The first row of Table 8.4 also shows the comparable values obtained from the empirical formulas of Delany and Bazley (ref. 38). Although these formulas

TABLE 8.4 Regression Coefficients for Predicting Propagation Constant and Characteristic Impedance of Plastic Foams

Material	b'	β'	b''	β''	α'	α'	α''	α''
Mineral and glass fiber	0.0571	0.754	0.087	0.732	0.0978	0.700	0.189	0.595
Fully reticulated polyurethane foam, $60 \leq R_1 \leq 6229$ (Cummings/Beadle)	0.0953	0.491	0.0986	0.665	0.174	0.372	0.167	0.636
Mixed plastic foams, ⁴² $2900 \leq R_1 \leq 24300$	0.209	0.548	0.105	0.607	0.188	0.554	0.163	0.592
Fully reticulated polyurethane foam, $380 \leq R_1 \leq 3200^a$	0.114	0.369	0.0985	0.758	0.136	0.491	0.168	0.795
Partly reticulated polyurethane foam, $R_1 = 10,100^b$	0.279	0.385	0.0881	0.799	0.267	0.461	0.158	0.700

^aFrom ref. 43.

^bFrom ref. 44.

have been superseded by those due to Mechel (ref. 37) and Miki (ref. 39), they serve to indicate that the coefficient values for plastic foams are significantly different and that the Delany and Bazley relationships would not generally give satisfactory predictions for the bulk properties of plastic foams.

Effects of Temperature

In sound absorbers or silencers designed to operate at high temperatures, it is important to know the acoustical characteristics of the fibrous porous sound-absorbing materials at design temperature. Fortunately, it is not necessary to measure the propagation constant k and characteristic impedance Z_C at design temperature T because the values of these acoustical characteristics measured at room temperature, T_0 , can be scaled. The dimensionless frequency variable $E = \rho_0 f / R_1$ must be evaluated at design temperature. Consider that the influence of temperature on ρ and η is

$$\rho(T) = \rho(T_0) \frac{T_0}{T} \quad (8.33)$$

$$\eta(T) = \eta(T_0) \left(\frac{T}{T_0} \right)^{0.5} \quad (8.34)$$

where T and T_0 are the design temperature and room temperature, respectively, measured on an absolute scale (i.e., Kelvin and Rankine; $K = ^\circ C + 273$). Considering that $\rho = \rho_0 (T/T_0)^{-1}$ and $c = c_0 (T/T_0)^{1/2}$, the acoustical characteristics at design temperature T (in Kelvin) are determined as follows:

$$b(T) = b[E(T)] \frac{\omega}{c_0} \left(\frac{T}{T_0} \right)^{-1/2} \quad (8.35a)$$

$$Z_C(T) = Z_{Cn}[E(T)] (\rho c_0) \left(\frac{T}{T_0} \right)^{-1/2} \quad (8.35b)$$

where $b[E(T)]$ and $Z_{Cn}[E(T)]$ are the normalized attenuation constant and normalized characteristic impedance according to Eqs. (8.30) computed for $E = E(T)$ determined from Eqs. (8.34) and (8.35). When fibrous sound-absorbing materials are first used in high-temperature applications, the material undergoes changes. For mats and boards the binder burns off. This decreases the bulk density to a negligible amount. The burning off of the binder does not appreciably change the flow resistivity. After the first high-temperature exposure the glass or mineral fibers thicken. Their virgin diameter d_v increases to the burned-off diameter $d_{BOF} = d_v + 0.5$. This increase is approximately $0.5 \mu m$. This increase in fiber diameter and corresponding decrease in flow resistivity can be accounted for by correcting flow resistivity measured at room temperature by multiplying it by the factor $(1 + 0.5/d_v)^{-2}$ when calculating $E_{BOF}(T_0) = f \rho_0 / R_1 (1 + 0.5/d_v)^{-2}$.

Example 8.1. Compute the real and imaginary parts of the normalized propagation constant and characteristic impedance of a very (practically infinite) thick layer of glass fiber at a frequency of 100 Hz at 20°C and for 500°C and determine α_N using the regression parameter values in Table 8.2. The flow resistivity of the fibrous material at 20°C is $R_1(T_0) = 16,000 \text{ N} \cdot \text{s/m}^4$ and $\rho_0 = 1.2 \text{ kg/m}^3$.

Solution

$$T_0 = 273 + 20 = 293 \text{ K} \quad T = 273 + 500 = 773 \text{ K}$$

$$E(T_0) = \frac{\rho_0 f}{R_1} = 1.2 \times \frac{100}{16,000} = 0.0075, \quad \text{which is } < 0.025$$

$$E(T) = E(T_0) \left(\frac{T}{T_0} \right)^{-1.65} = 0.0075 \left(\frac{773}{293} \right)^{-1.65} = 1.5 \times 10^{-3},$$

which is < 0.025

The regression parameters from Table 8.2 are

$$\begin{aligned} a' &= 0.396 & a'' &= 0.135 & \alpha' &= 0.458 & \alpha'' &= 0.646 \\ b' &= 0.0668 & b'' &= 0.196 & \beta' &= 0.707 & \beta'' &= 0.549 \end{aligned}$$

According to Eqs. (8.35), we obtain the following:

Parameter	At 20°C	At 500°C
$b = a' E^{-\alpha'}$	3.72	7.78
$a = 1 + a'' E^{-\alpha''}$	4.18	10.0
$R = 1 + b' E^{-\beta'}$	3.12	7.62
$X = b'' E^{-\beta''}$	2.88	6.9

The normal-incidence sound absorption coefficient according to Eq. (8.36) is

$$\alpha_N = 4 \frac{Z'_{an}}{Z'^2_{an} + 2Z'_{an} + 1 + Z''^2_{an}}$$

yielding

$$\alpha_N = \begin{cases} 0.49 & \text{for } 20^\circ\text{C} \\ 0.25 & \text{for } 500^\circ\text{C} \end{cases}$$

8.5 SOUND ABSORPTION BY LARGE FLAT ABSORBERS

When considering sound absorption, the portion of a sound wave incident on the absorber that is absorbed is a quantity of interest. This is easiest to define when

the surface of the absorber is flat and sufficiently large that sound waves scattered at the edges of the absorber can be neglected. For a plane incident sound wave, it is possible to assign a sound energy absorption coefficient α for each point on the absorber surface given by

$$\alpha = \frac{\text{absorbed energy}}{\text{incident energy}} = 1 - |R|^2 \quad (8.36)$$

where R is the reflection factor, which is defined as the ratio of the reflected and incident sound pressures at the interface. A high sound absorption coefficient ($\alpha \rightarrow 1$) requires that $|R| \rightarrow 0$. Note that $|R| = 0.1$ corresponds to $\alpha = 0.99$. In this chapter we shall deal only with infinitely large, flat, homogeneous absorbers. Edge effects manifest themselves in increased sound absorption with increasing perimeter-surface area ratio of the absorber.⁴⁵ Edge effects are involved also when numerical values greater than 1 are obtained for random-incidence absorption coefficients measured in reverberation rooms.

Plane Sound Waves at Normal Incidence

For plane waves of sound incident perpendicularly to an absorber it is sufficient to know the complex normal specific surface impedance $Z_1 = Z'_1 - jZ''_1$ of the absorber, which is the ratio of sound pressure to the normal component of the particle velocity at the interface (see Chapter 1). The reflection factor and absorption coefficient are related by

$$R = \frac{Z_1 - Z_0}{Z_1 + Z_0} \quad \alpha = \frac{4Z'_1 Z_0}{(Z'_1 + Z_0)^2 + Z''^2_1} \quad (8.37)$$

where $Z_0 = \rho_0 c_0$ is the characteristic impedance of air for plane waves and ρ_0 is the density and c_0 the speed of sound in air.

For sound absorbers consisting of highly porous layers with perforated or cloth facings of smaller porosity, a modified air-side surface impedance must be used in Eq. (8.37). This is obtained by taking the air-side volume velocity averaged over the face area of the absorber. If an absorber with surface impedance Z_1 is covered by a perforated facing with porosity h , then $Z_1 = Z_i/h$.

If $|R| \rightarrow 0$, Eq. (8.37) indicates that $Z_1 \rightarrow Z_0$. This means that the ideal absorber should have a surface impedance similar to that of unbounded air. For a thick (effectively semi-infinite) layer, this requires that the characteristic impedance of porous sound-absorbing materials, Z_C , should be only slightly above that for air, Z_0 , and so it is imperative to keep the porosity high. For fibrous sound-absorbing materials, the porosity should be in the range of 0.95–0.99 and for perforated facings the porosity should be above 0.25.

This is illustrated in Fig. 8.10, which shows random-incidence absorption coefficients $\sigma_R(\text{rev})$ for various configurations with a glass fiber fabric or perforated plate covering measured in a reverberation room. Note the relatively

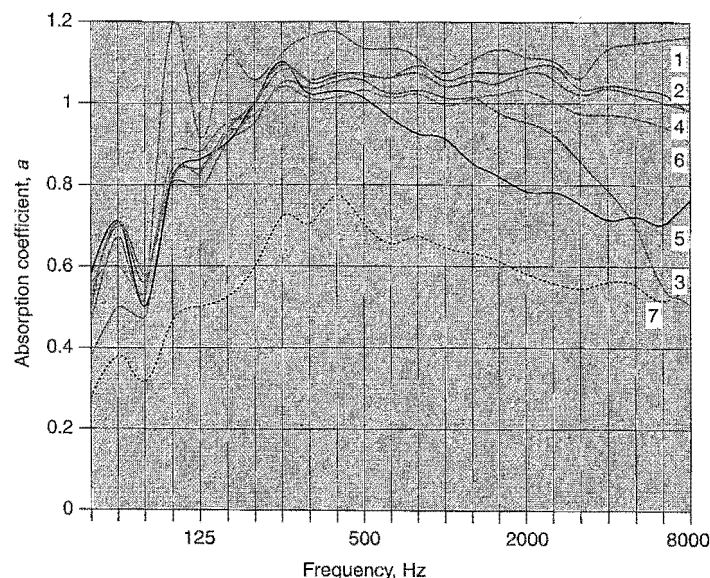


FIGURE 8.10 Random-incidence sound absorption coefficients of 200-mm-thick exhaust silencer pillows (basalt fiber core, density 125 kg/m^3) as a function of frequency measured in a reverberation room: (1) high-porosity fabric covering, equivalent flow resistivity $12 \text{ kPa} \cdot \text{s} \cdot \text{m}^{-2}$; (2) perforated sheet (32% open area) over high-porosity fabric covering; (3) perforated sheet (17% open area) over high-porosity fabric covering; (4) high-porosity fabric covering and fiberglass facing over basalt fiber core; (5) low-porosity fabric covering, equivalent flow resistivity $43 \text{ kN} \cdot \text{m} \cdot \text{s}^{-4}$; (6) perforated sheet (32% open area) and fiberglass facing over basalt fiber core; (7) perforated sheet (17% open area), low-porosity fabric covering and fiberglass facing. (Courtesy of John Bowman, Lancaster Glass Fiber, Ltd.)

poor results for the configurations of low-percentage open-area perforated-plate (17%), low-porosity, high-flow-resistivity fabric covering ($43,000 \text{ mks rays/m} = 100 \rho_0 c_0$ per meter $= 2.4 \rho_0 c_0$ per inch). Further note that the measured values of $\alpha_R(\text{rev})$ exceed unity. The reasons for this behavior have been discussed in the introductory part of this chapter.

Normal Incidence on a Porous Layer in Front of a Rigid Wall

A rigid wall represents a nearly [see Eq. (8.4)] perfect acoustical reflector. It is an acoustically hard surface. When a porous layer is placed on such an acoustically hard backing, the surface impedance is controlled by the combination of the incident and (multiple) reflected sound waves in the layer, yielding

$$Z_1 = Z_{Ca} \coth(jk_a d) = -jZ_{Ca} \cot(k_a d) \quad \text{N} \cdot \text{s/m}^3 \quad (8.38a)$$

where d is the layer thickness and Z_{Ca} is the characteristic impedance.

Equation (8.38a) may be expressed as

$$Z_1 = Z_{Ca} \frac{\sinh(bd) \cosh(bd) - j \sin(ad) \cos(ad)}{\cosh^2(bd) - \cos^2(ad)} \quad \text{N} \cdot \text{s/m}^3 \quad (8.38b)$$

The impedance given by Eqs. (8.38) exhibits the following behavior:

- When $d \ll \frac{1}{4} \lambda_a$ [$\lambda_a = 2\pi/a$ is the wavelength in the absorbing material, and a is defined in Eq. (8.30a)], which is the case when the layer is thin or the frequency is low, the magnitude of $\coth(jk_a d)$ is always large and the lack of impedance matching between Z_1 and Z_0 leads to a small sound absorption coefficient. This is the reason there is no “sound-absorbing paint” and that rugs absorb sound only modestly.
- When d is sufficiently large compared with the wavelength inside the material and the attenuation constant is not too small, the acoustical behavior of the layer approximates that of an “infinitely thick” layer. In this case $\coth(jk_a d) \rightarrow 1$ and the surface impedance is the same as the characteristic impedance.
- When $\coth(jk_a d)$ is minimum, the surface impedance of the hard-backed layer is minimum and the absorption coefficient takes its maximum value. Consideration of the expanded form in Eq. (8.38b) indicates that, for $bd > 0$, which is the only situation of interest, the real part is never zero. So the normalized surface impedance is minimum approximately when the imaginary part is zero. In fact, this condition gives a slight underestimate of the frequency of maximum absorption, whereas $\cos(ad) = 0$, corresponding to $d = \frac{1}{4} \lambda_a$, gives an overestimate.

In general the absorption coefficient is maximum at a frequency that is somewhat less than the frequency at which the layer is a quarter-wavelength thick. Doubling the layer thickness halves the frequency of maximum absorption.

For practical purposes a layer of thickness d can be considered “infinitely thick” if $bd > 2$. For such conditions $Z_1 = Z_{Ca}$. The higher the flow resistivity of the porous material, the more Z_1 exceeds Z_0 . Even in the extreme case of practically infinite layer thickness (which at low frequencies and low flow resistivity becomes very large), the sound absorption coefficient will be small. As frequency increases, the general tendency is $Z_{Ca} \rightarrow Z_0/h$, and for highly porous absorbers where $h \approx 1$, the good impedance matching results in a high absorption coefficient.

Sound-Absorbing Layer Separated from a Rigid Wall by an Air Space. The sound-absorbing configuration depicted in Fig. 8.11 is frequently used in practical applications (e.g., hung acoustical ceilings). The impedance of a layer of air of thickness t in front of a rigid wall is

$$Z_2 = -jZ_0 \cot(k_0 t) \quad \text{N} \cdot \text{s/m}^3 \quad (8.39)$$

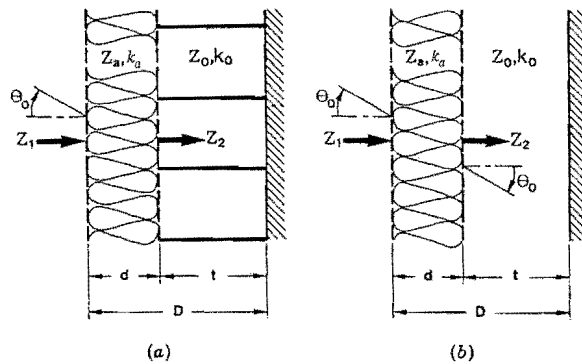


FIGURE 8.11 Combination of a bulk-reacting absorber layer with (a) an air gap such that sound can only travel perpendicular to the hard wall and (b) an air gap such that sound can travel parallel to the hard wall.

where $k_0 = \omega/c_0 = 2\pi\lambda_0/c_0$ is the acoustical wavenumber for air. The internal reflection coefficient at the back of the layer is $R_B = (Z_2 - Z_{Ca})/(Z_2 + Z_{Ca})$ and the surface impedance at the front of the layer is

$$Z_1 = Z_C \left(\frac{Z_2 + jZ_{Ca} \tan(k_a d)}{Z_{Ca} + jZ_2 \tan(k_a d)} \right) \text{ N} \cdot \text{s/m}^3 \quad (8.40a)$$

where Z_C is the normalized characteristic impedance of the porous layer, k_a is the (not-normalized) propagation constant in the porous layer, and d is the layer thickness. Note that Z_2 is zero when $t = \frac{1}{4}\lambda_0$; that is, the impedance Z_s of the absorbing layer backed by zero impedance is given by $Z_s = jZ_C \tan(k_a d) = Z_C \tanh(jk_a d)$. If the impedance of the absorbing layer backed by a hard wall [i.e., with $Z_2 \rightarrow \infty$ in Eq. (8.40a) or from Eq. (8.38a)] is denoted by Z_h , then $Z_{Ca} = \sqrt{Z_h Z_s}$. So Eq. (8.40a) can be written in the form

$$Z_1 = \frac{Z_h(Z_2 + Z_s)}{Z_2 + Z_s} = Z_h \frac{1 + Z_s/Z_2}{1 + Z_s/Z_h} \text{ N} \cdot \text{s/m}^3 \quad (8.40b)$$

The reflection factor and absorption coefficient α are computed by using Z_1 in Eq. (8.37). For small air space thickness such that $t/\lambda_0 < \frac{1}{8}$, $|Z_h| \gg |Z_s|$ and the following approximation is valid:

$$Z_1 \approx \frac{Z_h}{1 + Z_h/Z_2} \text{ N} \cdot \text{s/m}^3 \quad (8.41)$$

When the thickness of the absorbing layer is not too small, that is, $|Z_h| \ll |Z_2|$, Eq. (8.41) results in $Z_1 \approx Z_h$. Consequently at low frequencies a very thin air space between the absorbing layer and the rigid wall is ineffective.

As long as the layer thickness is small compared with the wavelength ($d < \frac{1}{8}\lambda_a$), the air space behind the absorbing layer results in a reduction of the surface impedance and at low frequencies the magnitude of the surface impedance shifts toward the characteristic impedance of air Z_0 . This results in a decrease of the reflection factor R and a corresponding increase of the normal-incidence sound absorption coefficient. The largest improvement is observed when the thickness of the air space is a quarter of a wavelength, $t = \frac{1}{4}\lambda_0$.

When the air space thickness corresponds to a multiple of $\frac{1}{2}\lambda_0$, $\sin(k_0 t) \rightarrow 0$ and Z_2 is very large [see Eq. (8.39)]. At such frequencies, the air space becomes totally ineffective.

Oblique Sound Incidence

For oblique sound incidence one must distinguish between locally reacting and bulk-reacting absorbers. Locally reacting absorbers are those where sound propagation parallel to the absorber surface is prohibited, as, for example, in partitioned porous layers, porous layers backed by a partitioned air space as depicted on the left in Fig. 8.1, in Helmholtz resonators with partitioned volumes, and in small plate absorbers. Local reaction is a good approximation to absorber behavior also when the flow resistivity is relatively high, causing the transmitted wave to bend toward the normal to the surface. This is true, for example, at high densities or in granular media consisting of small particles. In bulk-reacting absorbers, such as a low-flow-resistivity porous layer, possibly with an unpartitioned air space behind it (Fig. 8.11b), sound propagation in the direction parallel to the absorber surface is possible in the sound-absorbing layer or in the air space behind it. The term "locally reacting" results from the fact that the particle velocity at the interface depends only on the *local* sound pressure. For bulk-reacting absorbers, the particle velocity at the interface depends not only on the local sound pressure but also on the particular distribution of the sound pressure in the entire absorber volume. Certain materials are inherently anisotropic; that is, the properties vary with direction through the material. This is true, for example, of fibrous materials where the fibers lie in planes parallel to the surface. Strictly, in such materials the propagation constant and characteristic impedance are a function of angle. However, this complication will not be considered further here.

Oblique-Incidence Sound on Locally Reacting Absorber. Locally reacting absorbers are characterized by a surface impedance Z_1 that is independent of the angle of incidence θ_0 . The reflection factor $R(\theta_0)$ for locally reacting absorbers is determined by how well the surface impedance Z_1 matches the field impedance of $Z_0/\cos \theta_0$ and is given by

$$R(\theta_0) = \frac{Z_1 \cos \theta_0 - Z_0}{Z_1 \cos \theta_0 + Z_0} \quad (8.42)$$

Equation (8.42) indicates that best impedance match, and correspondingly the lowest reflection factor for a given incidence angle θ_0 , is achieved by $|Z_1| > Z_0$

and that such "overmatched" surface impedances will yield absorption maxima at a specific angle of incidence.

Oblique-Incidence Sound on Bulk-Reacting Absorber. Mineral-fiber and open-pore foams are the most frequently used bulk-reacting sound-absorbing materials. As long as the materials may be treated as isotropic, the acoustical behavior of these materials can be fully characterized by their propagation constant k_a and characteristic acoustical impedance Z_a . Methods for measuring and predicting these key acoustical parameters are given in Section 8.4 under Physical Characteristics of Porous Materials and their Measurement.

Semi-Infinite Layer. Figure 8.12 shows the incident, reflected, and transmitted waves at the interface. The combination of these waves must satisfy the following boundary conditions: (1) equal normal components of the impedances and (2) equality of the wavenumber components parallel to the interface. The second of these requirements yields the refraction law

$$\frac{\sin \theta_1}{\sin \theta_0} = \frac{k_0}{k_a} \quad (8.43)$$

where θ_1 is the complex propagation angle of the sound inside the absorber. The complex reflection factor is given by

$$R(\theta_0) = \frac{1 - (Z_0/Z_1) \cos \theta_0}{1 + (Z_0/Z_1) \cos \theta_0} \quad (8.44a)$$

and

$$Z_1 = \frac{Z_{Ca}}{[1 - (k_0/k_a)^2 \sin^2 \theta_0]^{1/2}} \quad \text{N} \cdot \text{s/m}^3 \quad (8.44b)$$

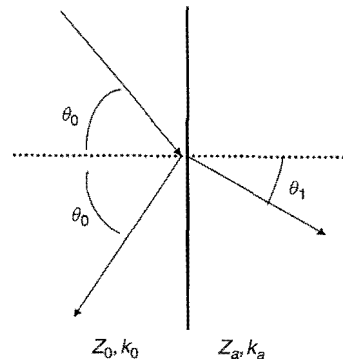


FIGURE 8.12 Reflection and transmission of an oblique-incidence sound wave by a semi-infinite, bulk-reacting absorber.

Finite-Layer Thickness. For a finite layer of bulk-absorbing material of thickness d in front of a hard wall, the reflection factor is

$$R(\theta_0) = \frac{1 - (Z_0/Z_{1d})/\cos \theta_0}{1 + (Z_0/Z_{1d}) \cos \theta_0} \quad (8.45)$$

where $Z_{1d} = (Z_{Ca}/\cos \theta_1) \coth(jk_a d \cos \theta_1)$ and θ_1 is defined in Eq. (8.43),

Multiple Layers. Absorbers may consist of a porous sound-absorbing layer with an air space behind, as illustrated in Fig. 8.11, or a number of porous layers of different thicknesses and different acoustical characteristics (Fig. 8.13). Analytical expressions describing the absorber functions and design charts to predict the sound absorption coefficients for those multiple-layer absorbers are provided in reference 46.

Random Incidence. In a diffuse sound field where the intensity $I(\theta) = I$ is independent of the incident angle θ , the sound power incident on a small surface area dS of an absorber is given as

$$dW_{\text{inc}} = I dS \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos \theta \sin \theta d\theta = \pi I dS \quad (8.46)$$

and the absorbed portion as

$$\begin{aligned} dW_a &= I dS \int_0^{2\pi} d\phi \int_0^{\pi/2} \alpha(\theta) \cos \theta \sin \theta d\theta \\ &= 2\pi I dS \int_0^{\pi/2} \alpha(\theta) \cos \theta \sin \theta d\theta \end{aligned} \quad (8.47)$$

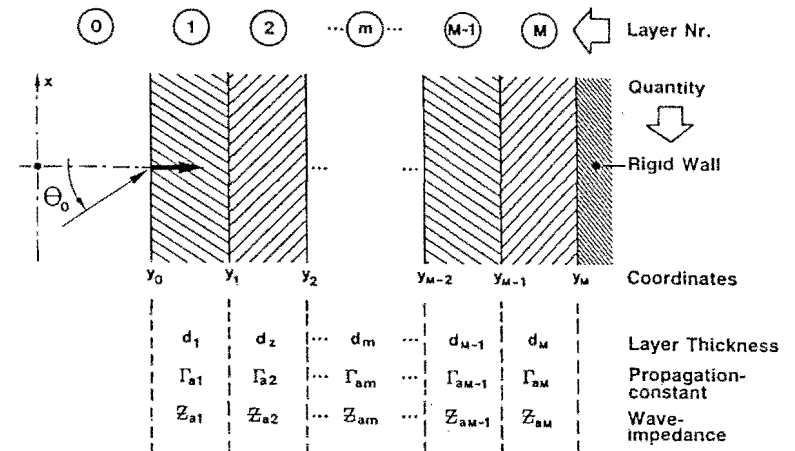


FIGURE 8.13 Multiple-layer absorber.

So the random-incidence sound absorption coefficient is given by

$$\alpha_R = \frac{dW_d}{dW_{inc}} = 2 \int_0^{\pi/2} \alpha(\theta) \cos \theta \sin \theta d\theta \quad (8.48)$$

Random-Incidence Sound Absorption Coefficient of Locally Reacting Absorbers. When the constituent materials of a multiple-layer absorber are locally reacting, the combination of Eqs. (8.2), (8.42), and (8.48) will yield the random-incidence sound absorption coefficient. The surface impedance Z_1 that is required for the computation of the random-incidence absorption coefficient can be measured in an impedance tube or computed, and the true random-incidence sound absorption coefficient α_R can be determined as⁴

$$\alpha_R = \frac{8}{|z|} \cos(\beta) \left\{ 1 + \frac{\cos 2\beta}{\sin \beta} \arctan \left(\frac{|z| \sin \beta}{1 + |z| \cos \beta} \right) - \frac{\cos \beta}{|z|} \ln(1 + 2|z| \cos \beta + |z|^2) \right\} \quad (8.49)$$

where $z = Z/\rho_0 c_0$ is the normalized wall impedance and $\beta = \arctan[\text{Im}(z)/\text{Re}(z)]$.

The maximum value of the true $\alpha_R(\text{max}) = 0.955$ that occurs at a normalized wall impedance of $|z| = 1.6$ and $\beta \approx \pm 30^\circ$ indicates that no locally reacting sound absorber can achieve total absorption or even the 99% absorption that is required as a minimum for the walls of anechoic chambers. The random-incidence sound absorption coefficient measured in a reverberation room, $\alpha_R(\text{rev})$, as described in Chapter 7, can yield physically impossible values above unity (sometimes in excess of 1.2). To avoid confusion, the computed physically correct random-incidence coefficient represented by α_R and that measured with the reverberation room method should have a different symbol, namely $\alpha_R(\text{rev})$.

Random-Incidence Sound Absorption Coefficient of Bulk-Reacting Absorbers. For bulk-reacting absorbers the integration in Eq. (8.48) can only be evaluated numerically. It is more appropriate to use the geometric and acoustical characteristics of the porous liner than its impedance to generate design charts for general use.

8.6 DESIGN CHARTS FOR FIBROUS SOUND-ABSORBING LAYERS

For design use it is useful to have charts where the sound absorption coefficient is plotted as a function of the first-order parameters of the absorber, such as thickness d , flow resistivity R_1 , and frequency f . This section contains such design charts. Design charts for a multiple-layered absorber, such as shown in Fig. 8.13, are given in reference 46.

Monolayer Absorbers

As shown in Fig. 8.7b, the key acoustical parameters k_a and Z_{Ca} of fibrous sound-absorbing materials depend only on a single material parameter, namely the flow resistivity R_1 (specific flow resistance per unit thickness). Consequently, the absorption coefficient can be computed and plotted as a function of two dimensionless variables:

$$F = \frac{d}{\lambda} = \frac{fd}{c_0} \quad \text{and} \quad R = \frac{R_1 d}{Z_0} \quad (8.50)$$

Then $k_0 d = 2\pi F$ and $E = \rho_0 f / R_1 = F/R$. At oblique incidence, the angle θ is the third input variable. In the design charts contour lines of constant values of α are plotted on a log-log chart of R versus F . A variation of frequency f produces a horizontal path, an increase of the flow resistivity R_1 causes an upward move, and an increase in layer thickness d leads to a diagonally upward shift to the right. Frequency curves for an absorber layer may be derived by a horizontal intersection, with a starting point determined by the individual parameter values.

Figure 8.14 shows lines of constant absorption of a monolayer absorber for *normal incidence* (the absorber may be bulk or locally reacting). The maximum absorption is reached at about $R = 1.2$ for $F = 0.25$, that is, at a thickness d equal to a quarter of a free-field wavelength and at the flow resistance of the layer of $1.2Z_0$. Higher resonances at odd multiples of a quarter wavelength are visible for a small flow resistance. The "summit line" of the first maximum, which goes through the points of relative maxima of α at the first resonance, is inclined so

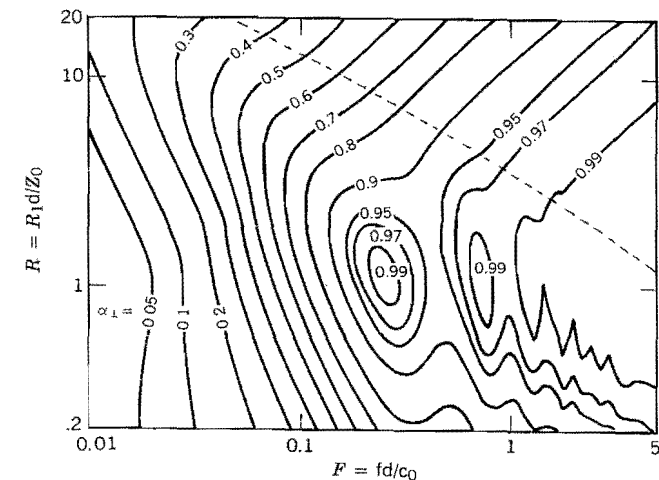


FIGURE 8.14 Lines of constant absorption at normal-incidence sound; absorption coefficient for fibrous absorbers of thickness d and flow resistivity R_1 ; no air space.

that the first relative maximum will occur below $F = 0.25$ for flow resistances such that $R > 1$, and the first relative maximum of the frequency curve will occur at somewhat higher frequencies than $F = 0.25$ for flow resistances such that $R < 1$. The higher orders of resonances are relatively stationary in frequency as R varies.

At *oblique incidence* the sound absorption for small angles, below about $\theta = 30^\circ$, is only modified slightly. The modifications in $\alpha(\theta)$ are pronounced for larger angles with locally reacting absorbers. The curves on the left in Figs. 8.15 *a-c* were computed for a locally reacting absorber for θ values of 30° , 45° , and 60° . The resonance structure of the contour plots becomes very distinct at low flow resistances and high frequencies.

Curves on the right in Figs. 8.15 *a-c* show the analogous contour plots of $\alpha(\theta)$ for the same values of θ but now for bulk-reacting (isotropic) absorber layers. Compared to those for a local-reacting absorber, the contours are relatively continuous. The maxima of absorption are shifted downward toward smaller R values for larger angles.

The *random-incidence* absorption coefficients α_R for locally reacting and bulk-reacting absorbers are plotted on the left and right, respectively, in Fig. 8.16. For locally reacting absorbers the absolute maximum of $\alpha_R = 0.95$ is attained for a matched flow resistance $R = 1$ at $F = 0.367$, that is, at a thickness $d = 0.367\lambda_0$, which is larger than a free-field quarter wavelength $\frac{1}{4}\lambda_0$. With homogeneous isotropic absorbers, the absorption coefficient α_R becomes larger than with locally reacting absorbers. Only a weak resonance maximum (belonging to the second resonance) is observed with homogeneous absorbers. The contour lines of absorption are smooth and steady.

Key Information Contained in Design Charts. Many of the general conclusions that were known qualitatively can be answered now in a quantitative manner by studying Figs. 8.14–8.16.

Determining Thickness of Absorber. What is the thickness d_∞ from which an absorber layer starts to behave acoustically as infinitely thick and yields no further increase in absorption? This thickness starts where the contour lines become 45° diagonally straight lines. The exact position of the curve for thickness d_∞ will depend on the criterion chosen and on the tolerance allowed. If, at this limit, the final value of α is supposed to be reached with a deviation between 1 and 3%, then the limit curve for normal incidence (in Fig. 8.14) is defined by

$$F = 7.45R^{-1.67} \quad (8.51)$$

The layer becomes practically infinite when the sound attenuation during propagation of the sound wave through the layer from the surface to the rigid rear wall is $8.68\Gamma'_\alpha d_\infty = 24$ dB, as first derived in reference 2. This is a much higher attenuation than usually assumed as sufficient (about 6–10 dB) for neglecting the influence of the reflection from the rigid wall on the sound absorption at the front side of the absorber layer.

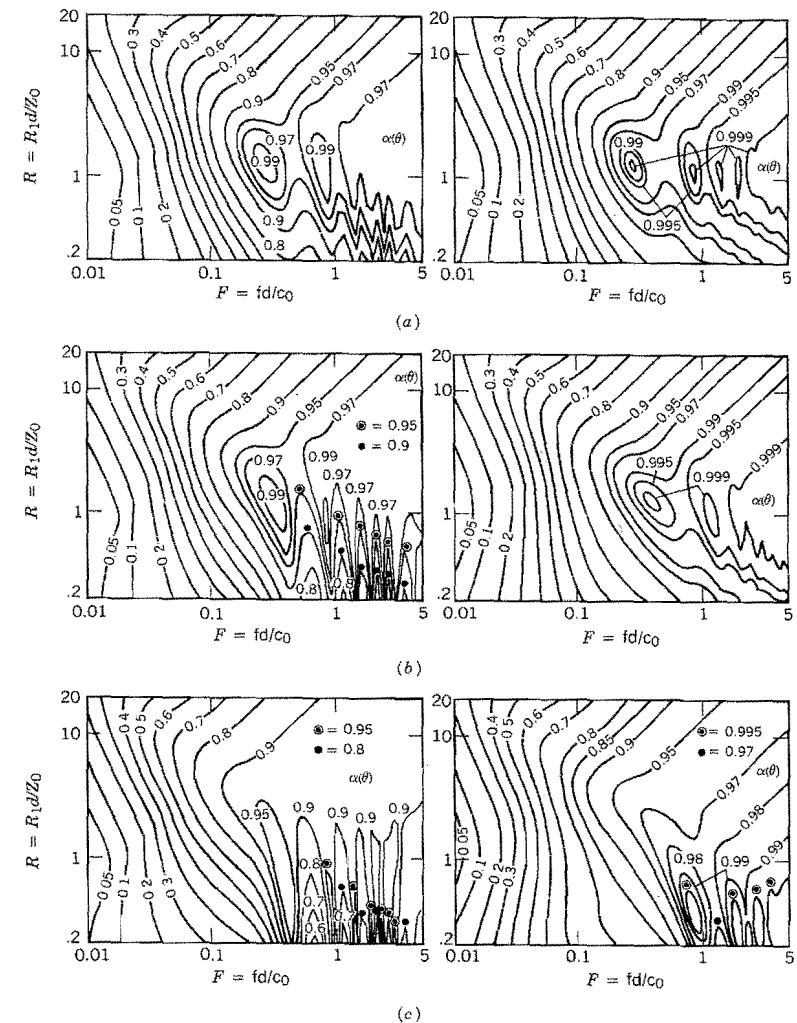


FIGURE 8.15 Lines of constant absorption $\alpha(\theta)$ for fibrous absorbers of thickness d and flow resistivity R_1 for discrete angles of incidences θ ; no air space. Left: locally reacting. Right: bulk reacting. (a) $\theta = 30^\circ$; (b) $\theta = 45^\circ$; (c) $\theta = 60^\circ$.

The relationships for “practically infinite” layer thickness d_∞ for which $\Delta L(d_\infty) = 24$ dB can be formulated as

$$fd_\infty^{2.67} R_1^{1.67} = 59 \times 10^6 \quad \text{for } \theta = 0^\circ \quad (8.52a)$$

$$fd_\infty^{2.56} R_1^{1.56} = 34.3 \times 10^6 \quad \text{for } \theta = 45^\circ, \quad \text{locally reacting and random incidence} \quad (8.52b)$$

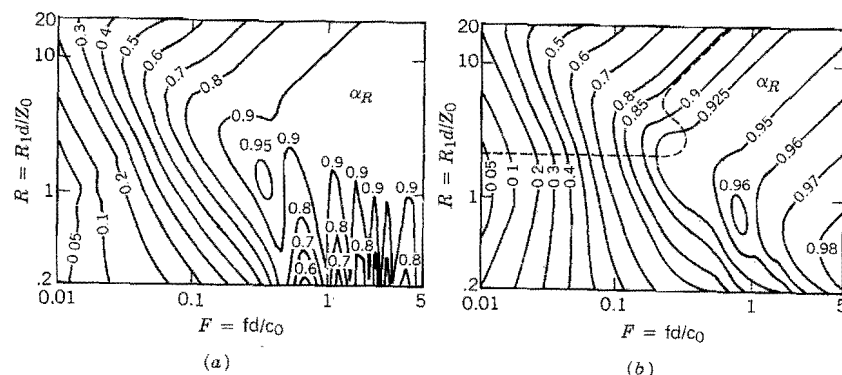


FIGURE 8.16 Lines of constant random-incidence absorption coefficient α_R for fibrous absorbers of thickness d ; no air space: (a) locally reacting liner; (b) bulk-reacting liner.

$$f d_{\infty}^{2.4} R_1^{1.4} = 7.7 \times 10^6 \quad \text{for } \theta = 45^\circ, \text{ bulk reacting} \quad (8.52c)$$

$$f d_{\infty}^{2.2} R_1^{1.2} = 1.1 \times 10^6 \quad \text{for random-incidence bulk reacting} \quad (8.52d)$$

where f = frequency, Hz

d_{∞} = layer thickness, m

R_1 = flow resistivity, $\text{N} \cdot \text{s/m}^4$

Optimal Choice of Flow Resistance. Optimal values for the normalized specific flow resistance of the layer, $R = R_1 d / Z_0$, plotted on the vertical scale of Figs. 8.14–8.16, depend on what the designer wants to accomplish.

If the aim is to maximize the sound absorption coefficient, then the optimal choice of normalized specific flow resistance R is in the range of 1–2 for both locally reacting and bulk-reacting absorbers and for normal, oblique, or random angles of sound incidence with the exception of bulk-reacting absorbers for $\theta > 45^\circ$ incidence where $R = 0.7$ yields the best results.

The absorption coefficient depends strongly on flow resistivity R_1 , where the curves in Figs. 8.14–8.16 have nearly horizontal contours. The design charts reveal that the typical orientation of the curves of constant α is nearly vertical, indicating only a slight dependence on R_1 . This is fortunate because our ability to determine R_1 and our accurate control over its value in the manufacturing process are limited.

Bulk-Reacting versus Locally Reacting Absorbers. A comparison of two graphs in Fig. 8.16 indicates that the random-incidence sound absorption coefficient of locally reacting and bulk-reacting absorbers of the same thickness is practically identical (within 10%) in the left upper quadrant, where $R > 2$ and $F = fd/c_0 < 0.25$, above the dotted line in the graph on the right in Fig. 8.16.

A comparison of $\theta = 45^\circ$ curves in Fig. 8.15b with the random-incidence curves presented in Fig. 8.16 indicates that the $\theta = 45^\circ$ curves for both locally

reacting and bulk-reacting absorbers agree with the random-incidence curves within an error of 5%, except in the range where $\alpha > 0.9$, which is of little practical interest. This close agreement between $\alpha(45^\circ)$ and α_R indicates that it is permissible to compute $\alpha(45^\circ)$ instead of the much more difficult α_R .

Two-Layer Absorbers

The two-layer absorbers shown in Fig. 8.11 have a layer of fibrous sound-absorbing material of thickness d backed by an air space of thickness t , with total thickness $D = d + t$. Figure 8.11a shows a partitioned (e.g., in a honeycomb pattern) air space resulting in a locally reacting absorber, and Fig. 8.11b shows an unpartitioned air space resulting in a bulk-reacting absorber. The sound absorber types shown in Fig. 8.11 are usually employed when it is impractical to fill the entire absorber thickness with porous material because no fibrous material of sufficiently low flow resistivity is available to keep the normalized flow resistance $R = R_1 d / Z_0 < 2$. Based on the analyses described in reference 5, design charts similar to those presented in a preceding section for monolayer absorbers were computed.

Figure 8.17 shows contour maps of the random-incidence sound absorption coefficient α_R in a diffuse sound field for a bulk absorber layer of thickness d in front of a locally reacting air gap of thickness t (see Fig. 8.11a) for three fractions, $d/D = 0.25, 0.5, 0.75$, absorber thickness d , and total thickness $D = d + t$. Because the two-layer design curves in Fig. 8.17 are not plotted as functions of the total layer thickness D , they are not directly comparable with the monolayer design curves presented in Fig. 8.16. Some differences may be noticed by comparing these figures with Fig. 8.16, which applies to the bulk monolayer absorber.

First, the variable $F = fd/c_0$ for maximum absorption in the first resonance maximum is shifted toward smaller values with decreasing d/D , which results in higher absorption at low frequencies for less absorber material thickness. Generally, however, the reduction in weight of the material is not as large. The horizontal shift of the absorption maximum toward lower F values, when realized by a reduction in d , makes a larger flow resistivity necessary on the ordinate in order to hold R on a constant value. The increase in R_1 is achieved mostly by an increase of the material bulk density ρ_A . For most fibrous absorber materials R_1 is proportional to about $\rho_A^{1.5}$. Hence, only a small net reduction in absorber material is possible by the addition of an air gap.

For thick absorber layers, that is, for large d , the air gap has no effect, as expected. The limit for the onset of an “infinite” thickness is the same as with the monolayer absorber.

The resonance structure of the layered absorber with small d/D resembles more that of the locally reacting monolayer absorber (see Fig. 8.16a). The character of the bulk-reacting monolayer absorber plot (see Fig. 8.16b), which must be the asymptotic limit for increasing d/D , becomes dominant for $d/D \geq 0.5$.

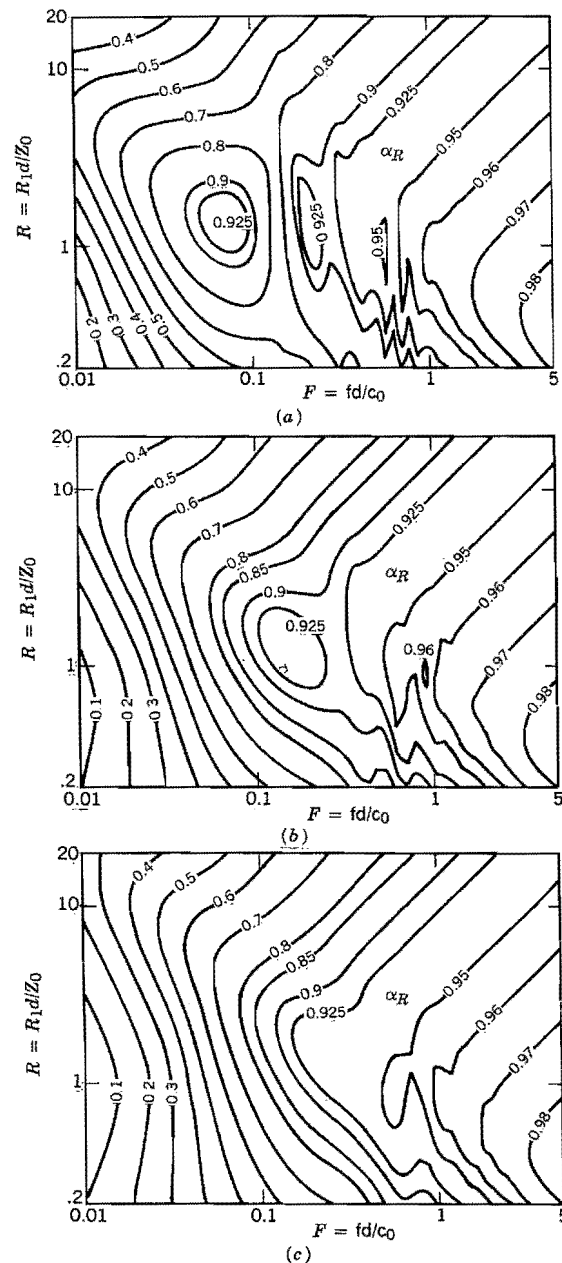


FIGURE 8.17 Lines of constant random-incidence absorption coefficient α_R for a bulk-reacting absorber layer of thickness d in front of a locally reacting air gap of thickness t for thickness ratios $d/D = d/(d+t)$ of (a) 0.25, (b) 0.5, and (c) 0.75.

The sound absorption coefficient of a porous layer of thickness d with an air gap behind it is generally higher than that of a monolayer with equal d . However, the absorption of the layered absorber is smaller than that of a monolayer absorber with equal total thickness $d' = D$. In the range $R > 1$, the absorption of a layered absorber with $d/D = 0.75$ corresponds to that of a monolayer absorber with thickness $d' = 1.25d$ and the absorption of a layered absorber with $d/D = 0.5$ corresponds to that of a monolayer having a thickness $d' = 1.67d$. The quite different character of the plot for $d/D = 0.25$ excludes a similar equivalence.

Finally, the lines of constant α_R of the layered absorber have strong deflections toward the left for values of α_R between about 0.7 and 0.9. As a consequence, the optimum flow resistance $R_1 d$ (which is defined by the leftmost point of a curve) has a more distinct meaning than with monolayer absorbers.

Design graphs for layered absorbers with bulk-reacting absorber layers in front of a bulk-reacting air gap (see Fig. 8.7b) are given in reference 45.

Multilayer Absorbers

Multilayer absorbers, such as shown in Fig. 8.13, have been treated in reference 4. Best results are obtained if the flow resistivity of the layers, R_1 , increases from the interface toward the rigid wall. These results obtained with such multilayer absorbers are somewhat better than those obtained with a monolayer absorber of equal thickness. However, the improvements seldom justify the added cost and complexity. Of course, Fig. 8.13 represents an idealization of practical constructions (see Fig. 8.18) which combine multiple porous layers with perforations and slots.

It should be noted, however, that even the most elaborate multiple-layer absorber cannot match the random-incidence sound absorption performance of anechoic wedges.

Thin Porous Surface Layers

Thin porous surface layers such as mineral wool felt sprayed on plastic, steel wool, mineral wool, or glass fiber cloth; wire mesh cloth; and thin perforated metal are frequently used to provide mechanical protection. They also reduce the

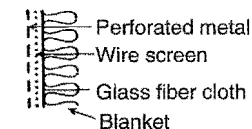


FIGURE 8.18 Multilayered absorber with perforated porous layer structure.

loss of fibers. The acoustical properties of such thin layers are characterized by their flow resistance $R_s = \Delta p/v$ and their mass per unit area ρ_s . The surface impedance is

$$Z_s = \begin{cases} R_s & \text{(fixed)} \\ \frac{(j\omega\rho_s)R_s}{j\omega\rho_s + R_s} & \text{(free)} \end{cases} \quad \begin{matrix} (8.53a) \\ (8.53b) \end{matrix}$$

which must be added to the wall impedance of the absorber. If the porous surface layer is not free to move, Eq. (8.53a) is used. For surface layers that are free to move due to the pressure differential produced by their flow resistance, Eq. (8.53b) is used.

Tables 8.5 and 8.6 provide design information on wire mesh cloth and glass fiber cloth, respectively. The flow resistance values listed in these tables represent the linear part of the flow resistance, which is appropriate for design use only if the particle velocity is low. At high sound pressure levels (above 140 dB), the nonlinear behavior of the flow resistance must be determined by measuring the flow resistance as a function of the face velocity.

Sintered porous metals have been developed for silencers in jet engine inlets. If backed by a honeycomb-partitioned air space, they provide good sound absorption and remain linear up to high sound pressure levels and for high-Mach-number grazing flow. Their additional advantage is that they do not require any surface protection. Table 8.7 gives the flow resistance R_s and mass per unit area ρ_s for some commercially available porous metal sheets.

Steel wool mats typically have a thickness from 10 to 45 mm, mass per unit area ρ_s from 1 to 3.8 kg/m², and a specific flow resistance R_s from 100 to 500 N · s/m³. Recently, such steel wool mats needled on mineral wool of specified thickness and density have become available on special order.

Perforated metal facings, when they rest directly on a porous sound-absorbing layer, can be accounted for by a series impedance Z_s given by

$$Z_s = \frac{j\omega\rho_0}{\varepsilon} \left\{ l + \Delta l \left[H(1 - |v_h|) - j \frac{\Gamma_a}{k_0} \frac{Z_a}{Z_0} \right] \right\} \quad (8.54)$$

TABLE 8.5 Mechanical Characteristics and Flow Resistance R_s of Wire Mesh Cloths

Wires/cm	Wires/in.	Wire Diameter		Mass per Unit Area		Flow Resistance R_s	
		μm (10^{-6} m)	mils (10^{-3} in.)	kg/m ²	lb/ft ²	N · s/m ³	$\rho_0 c_0$
12	30	330	13.0	1.6	0.32	5.7	0.014
20	50	220	8.7	1.2	0.25	5.9	0.014
40	100	115	4.5	0.63	0.13	9.0	0.022
47	120	90	3.6	0.48	0.1	13.5	0.033
80	200	57	2.25	0.31	0.63	24.6	0.06

TABLE 8.6 Mechanical Characteristics and Flow Resistance R_s of Glass Fiber Cloth

Manufacturer ^b	Cloth Number	Surface Density ^a		Construction, Ends × Picks	Flow Resistance, mks rayls (N · s/m ³)
		oz/yd ²	g/m ²		
1, 2, 3	120	3.16	96	60 × 58	300
1, 2, 3	126	5.37	164	34 × 32	45
1, 2, 3	138	6.70	204	64 × 60	2200
1, 2, 3	181	8.90	272	57 × 54	380
3	1044	19.2	585	14 × 14	36
2	1544	17.7	535	14 × 14	19
3	3862	12.3	375	20 × 38	350
1	1658	1.87	57	24 × 24	10
1	1562	1.94	59	30 × 16	<5
1	1500	9.60	293	16 × 14	13
1	1582	14.5	442	60 × 56	400
1	1584	24.6	750	42 × 36	200
1	1589	12.0	366	13 × 12	11

^aAveraged over a large sample.

^bCode numbers for manufacturers are as follows: (1) Burlington Glass Fabrics Company; (2) J. P. Schwebel and Company; (3) United Merchants Industrial Fabrics.

TABLE 8.7 Specific (Unit-Area) Flow Resistance R_s , Thickness, and Mass per Unit Area of Sintered Porous Metals Manufactured by the Brunswick Corporation^a

$\rho_0 c_0$	Resistance Air, 70°F N · s/m ³	Specific Flow Resistance NLF ^b 500/20	Designation	Thickness		Mass per Unit Area	
				mm	in.	kg/m ²	lb/ft ²
0.25	100	3.6	FM 125	1.0	0.04	3.9	0.79
		5.0	FM 127	0.76	0.03	3.3	0.67
		2.6	FM 185	0.5	0.02	2.0	0.4
		2.0	347-10-20-AC3A-A	0.5	0.02	1.32	0.27
		2.0	347-10-30-AC3A-A	0.76	0.03	1.1	0.23
		2.0	FM 802	0.5	0.02	1.3	0.27
0.88	350	4.7	FM 134	0.89	0.035	3.8	0.77
1.25	500	1.8	FM 122	0.76	0.03	1.4	0.28
		3.6	FM 126	0.66	0.026	3.7	0.76
		3.3	FM 190	0.41	0.016	2.0	0.4
		2.0	347-50-30-AC3A-A	0.76	0.03	1.4	0.29

^aExcept FM 802, which is made of Hastelloy X, all materials are type 347 stainless steel. (Courtesy of Brunswick Corporation.)

^bNonlinearity factor, calculated as the ratio of flow resistances obtained at flow velocities of 500 and 20 cm/s, respectively.

where ε is the fractional open area of the perforated facing, l is the plate thickness, Δl is the end correction length of the perforations given in Section 8.5, H is a step function that is unity for $|v_h| \leq 1$ m/s and zero for $|v_h| > 1$ m/s, and v_h is the particle velocity in the holes of the perforations. For thin, unrestrained perforated surface protection plates, it is necessary to take the parallel combination of Z_s and $j\omega\rho_s$ according to Eq. (8.53b).

8.7 RESONANCE ABSORBERS

In building acoustics the most frequently used type of resonant absorber is the Helmholtz resonator, consisting of the mass of an air volume in a cross-sectional area restriction (such as holes or slits in a covering plate) and the compliance of the air volume behind the covering plate. In the narrow frequency range, centered at their resonance frequency, the resonators "soak in" the acoustical energy from a large room volume in the vicinity of the neck. This high energy concentration results in high local sound pressures at the neck (and in the internal volume) which are much higher than that one would register with the resonator opening closed. On the room side, this near-field sound pressure decays exponentially with increasing distance. In architectural application of resonance absorbers care should be exercised not to locate them too near to any member of the audience to avoid distortion of the auditory experience. The open bottles embedded in the vertical portion of the seats in ancient Greek amphitheatres indicate that this local amplification of the sound field in the vicinity of acoustical resonators was known more than 2000 years ago. What was not known, which misguided their application, was that the amplification occurs only in a narrow frequency band and cannot locally amplify such broadband signals as speech. Such a Helmholtz resonator (named after the nineteenth-century German physicist Ludwig Helmholtz, who was first to utilize its narrow-band tuning to analyze the spectral composition of complex sounds) is shown in Fig. 8.19. In the following treatment it is assumed that all dimensions of a single resonator are small compared with the acoustical wavelength (except in the case of two-dimensional resonators with slits) and that the skeleton of the resonator is rigid.

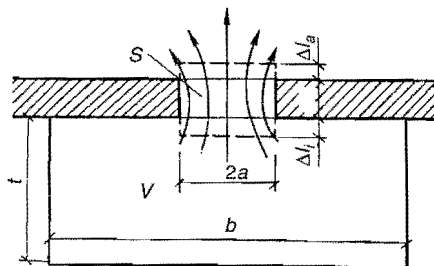


FIGURE 8.19 Key geometric parameters of a Helmholtz resonator.

Acoustical Impedance of Resonators

The specific acoustical impedance of the resonator opening Z_R is the sum of the impedance of the enclosed air volume Z_v and that of the air volume that oscillates in and around the resonator mouth Z_m , namely,

$$Z_R = Z_v + Z_m = (Z'_v + jZ''_v) + (Z'_m + jZ''_m) \quad \text{N} \cdot \text{s/m}^3 \quad (8.55)$$

The volume impedance $Z_v = jZ''_v$ is purely imaginary and predominantly of spring character while the mouth impedance has a real part Z'_m and an imaginary part Z''_m and is predominantly of mass character.

The impedance of a rectangular resonator volume is

$$Z_v = jZ''_v = -j\rho_0 c_0 \cot(k_0 t) \frac{S_a}{S_b} \quad \text{N} \cdot \text{s/m}^3 \quad (8.56a)$$

where S_b is the surface area of the resonator cover plate, $S_a = \pi a^2$ is the area of the resonator mouth, t is the depth of the resonator cavity, and $V = S_b t$ is the resonator volume. If the resonator dimensions are small compared with the wavelength ($k_0 t \ll 1$), Eq. (8.56a) yields

$$Z_v = jZ''_v = -j \frac{\rho_0 c_0^2}{\omega} \frac{S_a}{V} \quad \text{N} \cdot \text{s/m}^3 \quad (8.56b)$$

Thus, when all the dimensions are smaller than the wavelength of interest, the shape of the air cavity does not matter.

The impedance of the resonator mouth Z_m consists of components related to the oscillations of air internal to the mouth, within the mouth, and external to the mouth of the resonator plus the impedance of the screen (Z_s) that may be placed across the resonator mouth to provide resistance. For circular resonator openings of radius a and orifice plate thickness l , the resulting total impedance is given by

$$Z'_m = \rho_0 \left[\sqrt{8\nu\omega} \left(1 + \frac{l}{2a} \right) + \frac{(2\omega a)^2}{16c_0} \right] + Z_s \quad \text{N} \cdot \text{s/m}^3 \quad (8.57a)$$

$$Z''_m = \omega\rho_0 \left[l + \left(\frac{8}{3\pi} \right) 2a \right] + \left(l + \frac{1}{2a} \right) \sqrt{8\nu\omega} \quad \text{N} \cdot \text{s/m}^3 \quad (8.57b)$$

where ν is the kinematic viscosity ($\nu = 1.5 \times 10^{-6}$ m²/s for air at room temperature). The quantity $0.5(8\nu/\omega)^{1/2}$ is the viscous boundary layer thickness.¹

In general, the first term in Eq. (8.57a) applies only for smooth orifice edges. For sharp orifice edges it can be many times higher. Also note that the second term in Eq. (8.57b) is usually small compared with the first term and can be neglected.

The results of Eqs. (8.57) for circular orifices and for a single resonator can be generalized for other orifice shapes and for situations where the orifice is located

on a large wall or at a two- or three-dimensional corner. This generalized form is

$$Z'_m = \frac{P(l + P/2\pi)}{4S_a} \rho_0 \sqrt{8v\omega} + \rho_0 c_0 \frac{k_0^2 S_a / \Omega}{1 + k_0^2 S_a / \Omega} + Z_s \quad \text{N} \cdot \text{s/m}^3 \quad (8.58a)$$

$$Z''_m = j\omega\rho_0 \left[l + 2\Delta l + \frac{P(l + P/2\pi)}{4S_a} \sqrt{8v/\omega} \right] \quad \text{N} \cdot \text{s/m}^3 \quad (8.58b)$$

where P is the perimeter of the orifice, S_a is its surface area, and Ω is the spatial angle the resonator "looks into":

$$\Omega = \begin{cases} 4\pi & \text{for resonator away from all walls} \\ 2\pi & \text{flush mounted on wall far from corners} \\ \pi & \text{flush mounted on wall at two-dimensional corners} \\ \frac{1}{2}\pi & \text{flush mounted on wall at three-dimensional corners} \end{cases}$$

The quantity $\Delta l = 16a/3\pi$ represents the combined internal and external end corrections.

Resonance Frequency

The resonance frequency f_0 of the Helmholtz resonator occurs where Z''_m and Z'_v are equal in magnitude. Combination of Eqs. (8.56b) and (8.57b) yields

$$f_0 = \frac{c_0}{2\pi} \sqrt{\frac{S_a}{V(\dots)}} \approx \frac{c_0}{2\pi} \sqrt{\frac{S_a}{V(l + 16a/3\pi)}} \quad (8.59)$$

where $(\dots) = l + \Delta l$ represents the quantity in square brackets in Eq. (8.58b).

The viscous terms in (8.57b) are ignored in (8.59). However, this is usual when calculating a resonance frequency because the viscous contribution is small. For resonators where the area of the mouth, S_a , is not much smaller than the area of the top plate of the resonator, the proximity of the side walls of the cavity influences the flow in the resonator mouth. In this case it is necessary to determine the combined end correction from $\Delta l = 8a/3\pi + \Delta l_{\text{int}}$.⁴⁷

Absorption Cross Section of Individual Resonators

For individual resonators (or groups of resonators where the distance between the individual resonators is large enough that interaction is negligible), the absorption coefficient α is meaningless. In this case, the sound-absorbing performance must be characterized by the absorption cross section A of the individual resonator. The absorption cross section is defined as that surface area (perpendicular to the direction of sound incidence) through which, in the undisturbed sound wave (resonator not present), the same sound power would flow through as the sound power absorbed by the resonator.

The power dissipated in the mouth of a resonator is

$$W_m = 0.5|v_a|^2 S_a R_T = \frac{0.5 \times 2^n |p_{\text{inc}}|^2 S_a R_T}{|Z_R|^2} \quad \text{W} \quad (8.60)$$

Accordingly, the absorption cross section is

$$A = \frac{2^n \rho_0 c_0 S_a R_T}{|Z_R|^2} \quad \text{m}^2 \quad (8.61)$$

where $n = 0$ for resonators placed in free space, $n = 1$ if the resonators are flush mounted in a wall, $n = 2$ if the resonators are at the junction of two planes, and $n = 3$ if the resonators are in a corner; $R_T = Z'_m + Z_{\text{rad}}$ is the total resistance.

At the resonance frequency f_0 , where $Z''_v + Z''_m = 0$, the absorption cross section reaches its maximum value A_0 :

$$A = 2^n \frac{S_a \rho_0 c_0}{R_{\text{rad}}} \left[\frac{R_T / R_{\text{rad}}}{|1 + R_T / R_{\text{rad}}|^2} \right] \quad \text{m}^2 \quad (8.62a)$$

For a flush-mounted ($n = 1$) circular resonator area $S_a = \pi a^2$, the radiation resistance is $R_{\text{rad}} = 2(\pi a)^2 \rho_0 c_0 / \lambda_0^2$, and Eq. (8.62a) yields

$$A = \frac{1}{\pi} \lambda_0^2 \left[\frac{R_T / R_{\text{rad}}}{|1 + R_T / R_{\text{rad}}|^2} \right] \quad \text{m}^2 \quad (8.62b)$$

where λ_0 is the wavelength at the resonance frequency of the resonator. The maximum value of the absorption cross section is obtained by matching the internal resistance to the radiation resistance at the resonance frequency, yielding ($R_T = R_{\text{rad}}$)

$$A_0^{\text{max}} = \frac{\lambda_0^2}{4\pi} \quad \text{m}^2 \quad (8.63)$$

According to Eq. (8.63), a matched resonator tuned to 100 Hz can achieve absorption cross section $A_0^{\text{max}} = 0.92 \text{ m}^2$.

According to Eq. (8.61), the frequency dependence of A can be presented in generalized form as

$$\frac{A(f)}{A_0} = \frac{1}{1 + Q^2 \phi^2} \quad (8.64)$$

where the Q of the resonator and the normalized frequency parameter ϕ are given as

$$Q = \frac{Z''_m(\omega_0)}{Z'_m(\omega_0)} \quad \phi = \frac{f}{f_0} - \frac{f_0}{f}$$

Figure 8.20 shows the normalized absorption cross section A/A_0 as a function of the normalized frequency ϕ with Q as the parameter, indicating that the

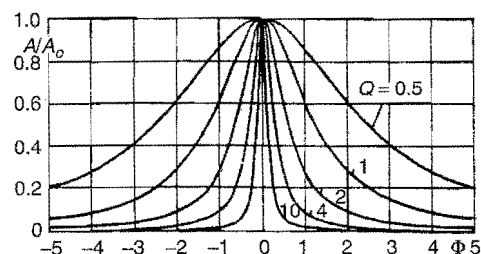


FIGURE 8.20 Normalized absorption cross section A/A_0 as function of normalized frequency ϕ with Q as parameter.

normalized bandwidth of the absorption cross section f_0/Q corresponds to the relative bandwidth $\Delta f/f_0 = 1/Q$.

Consequently, the choice of Q determines the relative shape of the absorption curve and the factor in the square brackets in Eq. (8.62b) determines the height of the curve at resonance. To obtain optimal absorption characterized by high absorption at resonance A_0 and wide bandwidth requires resistance matching $R_T = R_{rad}$ and a relatively low value of Q .

Nonlinearity and Grazing Flow

The oscillating air mass (that includes the end-correction term) is a measure for the reversible kinetic energy of the acoustical resonator. Reversibility and spatial coherence diminish with increasing turbulence due to jet flow through the orifice caused by high-amplitude sound and by the grazing flow of velocity U_∞ . Both reduce the air mass that participates in the oscillating motion and consequently increase resonance frequency and increase the losses of the resonator.

Nonlinearity and grazing flow affect only the mouth impedance Z_m of the resonator orifice. Table 8.8 is a compilation of the nonlinear effects owing to high sound pressure level and grazing flow for a perforated plate. The round holes are regularly distributed. The porosity of the perforated plate is $\varepsilon = \pi a^2/b^2$, where a is the hole radius and b the hole spacing. The amplitude nonlinearity is characterized by the particle-velocity-based Mach number $M_0 = v/c_0$, which is the ratio of the particle velocity in the resonator orifice and the speed of sound in air. The grazing flow nonlinearity is characterized by the flow-velocity-based Mach number $M_\infty = U_\infty/c_0$, where U_∞ is the velocity of the grazing flow far from the wall. The formulas presented in Table 8.8 are analytical results adjusted to yield agreement with measured data.⁴⁸ The impedances in Table 8.8 are values averaged over the plate surface.

Internal Resistance of Resonators

The most difficult part of resonator design is the prediction of the internal resistance. In the earlier section Acoustical Impedance of Resonators (page 265)

formulas were given to predict the friction loss resistance [first term in Eq. (8.57a)], which is valid only for rounded orifice perimeter. Additional resistance due to sharp edges cannot be predicted analytically. Table 8.8 contains formulas to predict resonator resistance due to nonlinear effects owing to high sound pressure level and grazing flow.

Should it be desirable to obtain higher loss resistance than provided by friction and nonlinear effects (e.g., obtaining larger absorption bandwidth), porous materials such as a screen, felt, or layer of fibrous sound-absorbing material must be placed in (or behind) the resonator orifice. Figure 8.21 shows some of the more frequently used ways to increase resonator resistance. In Fig. 8.21, R_1 is the flow resistivity of the porous material, d is its thickness, t_1 is its distance from the back side of the cover plate, t is the depth of the resonator cavity, $2a$ is the diameter of the holes, Δl is the end correction, and ε is the porosity of the cover plate.

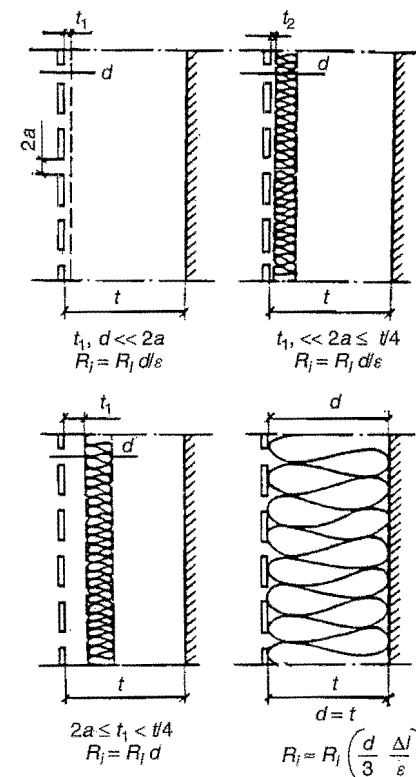


FIGURE 8.21 Methods to increase resonator resistance and the achieved specific acoustical resistance R_i (R_1 = flow resistivity of porous material, d is its thickness, and $\Delta l = 16a/3\pi$).

TABLE 8.8 Nonlinearity of Orifice Impedance $Z_m = Z'_m + jZ''_m$ of Perforated-Plate Resonator Covers Caused by High Sound Pressure Level L_p and Grazing Flow U_∞^a

Grazing Flow	Low SPL, $L_p < L_{ql}$	Medium SPL, $L_{ql} \leq L_p \leq L_{oh}$	High SPL, $L_p > L_{oh}$
No flow or very low flow, $M_\infty \leq 0.025$, $U_\infty \leq 8$ m/s	$L_{ql} = 107 + 27 \log[4(1 - \epsilon^2)\omega\rho_0 v(1 + l/2a)^2]$ dB	$L_{oh} = L_{ql} + 30$ dB	
	$Z'_m = R_0$ $Z''_m = X_0(\delta)$	$Z'_m = \sqrt{R_h^2 - R_0^2}$ $Z''_m = X_0(\delta)$	$Z'_m = R_h$ $Z''_m = X_0(\delta)$
	$R_0 = \frac{\rho_0}{\epsilon} \sqrt{8\nu\omega} \left(1 + \frac{l}{2a}\right) + (\rho_0/8\epsilon c_0)(2a\omega)^2$		$R_h = \frac{1}{\epsilon} \sqrt{2\rho_0(1 - \epsilon^2)} \times 10^{(-2.25 + 0.018L_p)}$
	$X_0 = \frac{\omega\rho_0}{\epsilon} \left[\sqrt{\frac{8\nu}{\omega}} \left(1 + \frac{l}{2a}\right) + l + \delta \right]$		
	$\delta = \delta_0 = 0.85(2a)\phi_0(\epsilon)$	$\delta = \delta_0\phi_1(M_0)$	$\delta = \delta_0\phi_1(M_0)$
	$\phi_0(\epsilon) = 1 - 1.47\sqrt{\epsilon} + 0.47\sqrt{\epsilon^3}$	$M_0 = \frac{1}{\sqrt{0.5\rho_0 c_0^2(1 + \epsilon^2)}}$	$\phi_1(M_0) = \frac{1 + 5 \times 10^3 M_0^2}{1 + 10^4 M_0^2}$
With flow, $M_\infty \geq 0.025$, $U_\infty \geq 10$ m/s	$L_p \leq L_{ml}$ $L_{ml} = 175 + 40 \log M_\infty$ dB	$L_{ml} \leq L_p \leq L_{mh}$	$L_p > L_{mh}$ $L_{mh} = L_{ml} + 18$ dB
	$Z'_m = R_m$ $Z''_m = X_0(\delta)$	$Z'_m = \sqrt{R_m^2 + R_h^2}$ $Z''_m = X_0(\delta)$	$Z'_m = R_h$ $Z''_m = X_0(\delta)$
	$R_m = 0.6\rho_0 c_0 \frac{1 - \epsilon^2}{\epsilon} (M_\infty - 0.025) - 40R_0(M_\infty - 0.05)$; $M_\infty \leq 0.05$		
	$R_m = 0.3\rho_0 c_0 \frac{1 - \epsilon^2}{\epsilon} M_\infty$	$M_\infty > 0.05$	
	$\delta = \delta_0\phi_2(M_\infty)$	$\delta = \delta_0\phi_2(M_\infty)$	$\delta = \delta_0\phi_1(M_\infty)$
	$\phi_2(M_\infty) = 1/(1 + 305M_\infty^2)$		

^a L_p is octave-band SPL in dB re 20 μ Pa at the peak of the spectrum.

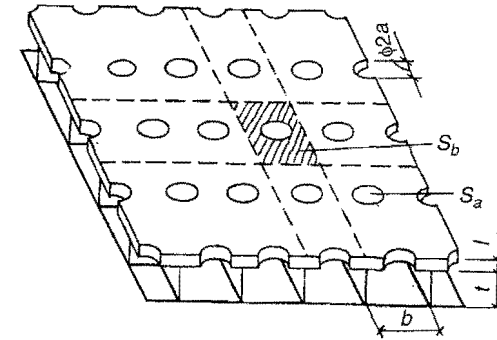


FIGURE 8.22 Resonators arranged on a raster to form a surface array.

Placing of flow-resistive materials in or behind the resonator opening reduces nonlinearity and sensitivity to grazing flow.

Spatial Average Impedance of Resonator Arrays

Locally reacting absorbers such as resonators are frequently used in the form of a surface array, shown in Fig. 8.22, where the individual resonators are arranged periodically. If the dimension b or $\sqrt{S_b}$ in Fig. 8.22 is much smaller than the acoustical wavelength, it is not necessary to take into account the interaction of the individual elements (resonators) with the sound field. To characterize the absorber, it is sufficient in this case to compute the spatial average wall impedance Z_1 :

$$Z_1 = \frac{\langle p \rangle}{\langle v \rangle} = \frac{S_b}{S_a} \frac{p_a}{v_a} = \frac{S_b}{S_a} Z_R = \frac{Z_R}{\epsilon} \quad \text{N} \cdot \text{s/m}^3 \quad (8.65)$$

where Z_R is the impedance of the resonator as given in Eqs. (8.55) "measured" in the orifice of the resonator and $\epsilon = S_a/S_b$ is the surface porosity. The effective wall impedance Z_1 is then used in Eq. (8.37) to determine the absorption coefficient. The impedances Z_m in Table 8.8 are effective impedances averaged over the plate surface (resonator mass impedance divided by porosity).

8.8 PLATE AND FOIL ABSORBERS

Plate absorbers are used in absorbing low-frequency tonal noise in situations where Helmholtz resonators are not feasible. The resonator consists of the mass of a thin plate and usually the stiffness of the air space between the plate and the elastic mounting at the perimeter of the plate. Foils are a special case of a thin limp plate.

Limp Thin Plate or Foil Absorber

The resonance frequency of a foil absorber is

$$f_0 = \frac{\sqrt{\rho_0 c_0^2 / m'' t}}{2\pi} \quad (8.66)$$

where t is the depth of the partitioned air space in meters and m'' is the mass per unit area of the limp foil in kilograms per square meter. If the air space depth is not small ($t > \frac{1}{8}\lambda$), there will be many resonance frequencies ω_n ($n = 0, 1, 2, \dots$) given by

$$\frac{\omega_n t}{c_0} \tan \frac{\omega_n t}{c_0} = \frac{\rho_0 t}{m''} \quad (8.67)$$

For an unpartitioned air space behind the foil the resonance frequency depends on the angle of sound incidence θ_0 . In this case t must be replaced in Eqs. (8.66) and (8.67) by $t \cos \theta_0$. If the air space is filled (without obstructing the movement of the foil) with a porous sound-absorbing material, then $(t/c_0)\Gamma_a''$ is used instead of t/c_0 in Eq. (8.67).

Foil-Wrapped Porous Absorber

Sound absorbers consisting of a protective layer of thin foil, a porous layer, and an air space behind are frequently used where the porous material must be protected from dust, dirt, and water. The effect of the thin foil can be taken into account by a series impedance $Z_s = j\omega m''$ that is added to the surface impedance Z_1 [see Eq. (8.37)]. It is essential that the foil is not stretched so that its inertia and not its membrane stress controls its response. Inserting a large-mesh (≥ 1 cm) wire cloth between the porous sound-absorbing material and the protective foil is a practical way to assure this.

Elastically Supported Stiff Plate

A form of resonant plate absorber that can be easily treated analytically is the elastically supported stiff plate. The elastic support may be localized by discrete points to occur along the perimeter in the form of a resilient gasket strip. The effective stiffness of such a resonator is

$$s'' = \frac{s_e}{S_p} + \frac{\rho_0 c_0^2}{t} \quad \text{N/m}^3$$

and the resonance frequency is

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{s''}{m''}} = \frac{1}{2\pi} \sqrt{\frac{s_e/S_p + \rho_0 c_0^2/t}{m''}} \quad \text{Hz} \quad (8.68)$$

where $s_e = S_p \Delta p / \Delta x$ is the dynamic stiffness of resilient plate mounting and S_p is the surface area of the plate. Note that the elastic mounting increases the

resonance frequency (compared with that obtainable with a limp foil of the same m''). This can be compensated for by an appropriate increase of m'' . However, an increase of m'' increases the Q of the resonator ($Q = \sqrt{s''/m''}/R$) resulting in a narrower bandwidth.

Elastic Foil Absorber

Elastic foil absorbers consist of small air volumes enclosed in thin (200–400- μm) foil coffers. The coffers have typical dimensions of a few centimeters. A typical

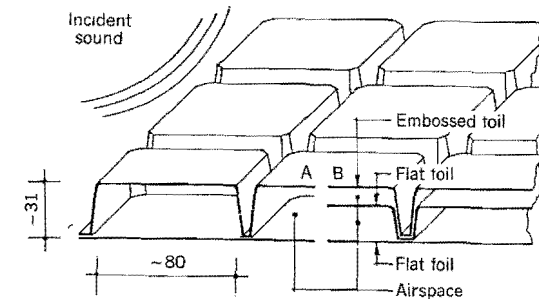


FIGURE 8.23 Construction of a foil absorber made of cold-drawn PVC foil.

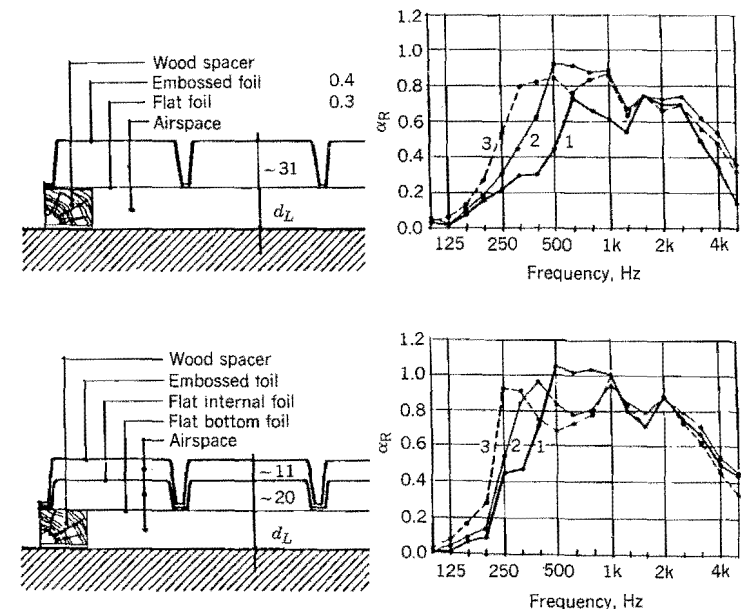


FIGURE 8.24 Random-incidence sound absorption coefficient α_R of absorbers made of cold-drawn PVC foil: (1) $d_L = 0$; (2) $d_L = 25$ mm; (3) $d_L = 50$ mm.

foil absorber is shown in Fig. 8.23. The wavelength of the incident sound is large compared with the typical dimensions of the coffers. Consequently, the incident sound periodically compresses the air in the individual coffers by exciting all of the volume-displacing modes of vibration of the various coffer walls. Because of the small thickness of the coffer walls, these resonances fall into the frequency range of 200–3150 Hz, which is of primary interest in building applications.⁴⁹ The foil material can be plastic (PVC) or metal. It is beneficial to emboss the foil because this leads to a more even distribution of the resonance frequencies. Figure 8.24 shows the measured random-incidence sound absorption coefficient obtained with two different foil absorber configurations, indicating that significant broadband sound absorption is achievable.

The key advantage of these foil absorbers is that they are nonporous, do not support bacterial growth, are lightweight, and can be made light transparent. Their main application is in breweries, packaging plants, hospitals, and computer chip manufacturing areas, where high emphasis is placed on hygiene and dust cannot be tolerated. They also lend themselves as sound absorbers in high-moisture environments such as swimming pools and as muffler baffles in cooling towers. Their light transparency is a distinct advantage in industrial halls. Considerable experience has been gained in such applications in Europe.⁴⁹

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Passive Silencers

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9.1 INTRODUCTION

Chapter Organization

This chapter is organized in seven sections. The present section reviews some of the significant milestones in silencer design, citing relevant references. Section 9.2 reviews the acoustical criteria commonly used to determine silencer performance. Section 9.3 discusses the decomposition of a silencer into basic acoustical transmission elements, presents transmission matrix expressions for some elements used in the Section 9.4 examples, and provides references for additional elements not explicitly covered in this chapter. Section 9.4 discusses the predicted performance of several silencer designs and provides selected examples of designs aimed at achieving specific acoustical goals. Section 9.5 provides a more qualitative discussion of perforated-element mufflers along with a few muffler design examples. Section 9.6 considers dissipative silencers most widely used to attenuate broadband noise in ducts with gas flow with a minimum of pressure drop across the silencer. Section 9.7 discusses designs that combine reactive silencer low-frequency tonal performance with passive silencer high-frequency broadband performance. A reader solely interested in designing a silencer for a specific application may proceed directly to Sections 9.3–9.7.

Background Information

The noise generated by air/gas handling/consuming equipment, such as fans, blowers, and internal combustion engines, is controlled through the use of two

types of devices: (1) passive silencers and lined ducts whose performance is a function of the geometric and sound-absorbing properties of their components and (2) active noise control silencers whose noise cancellation features are controlled by various electromechanical feed-forward and feedback techniques. This chapter focuses on passive silencers while active noise control is discussed in Chapter 17 and lined ducts are covered in Chapter 16.

In the remainder of this chapter, the term *silencer* is often used generically to refer to any type of passive noise control device. Specific names are introduced primarily in cases where the use of "silencer" may lead to ambiguities.

Over the last four decades silencers have been the focus of many research programs that improved the understanding of the basic phenomena and resulted in more accurate design methods. The groundwork for the behavior of lined ducts with no flow was laid even earlier by Morse¹ and Cremer,² and the first systematic evaluation of mufflers with no flow was conducted by Davis et al.³ Those works were followed by numerous theoretical and experimental investigations which addressed additional important issues such as uniform flow, temperature gradient, and the behavior of new silencer components that are common in modern applications. Reviews of the major accomplishments in this area have been presented periodically in the form of individual articles in professional journals, chapters in engineering handbooks,⁴⁻⁷ and, more recently, books by Munjal^{8,9} solely devoted to mufflers.

The analytical work on this subject benefited substantially from the early adaptation of the transfer matrix approach to silencer modeling. This method, which was promoted for the description of mechanical systems early on,¹⁰ was used along with electrical analogs to describe the behavior of basic silencer components¹¹ for plane-wave sound propagation in the absence of flow. Subsequent investigations began addressing the effects of flow on the response of elements consisting of area discontinuities^{12,13} and generating transfer matrix models for silencer elements. The predicted performance of various reactive silencers using transfer matrices with convective and dissipative effects in the presence of flow agreed well with measured data.¹⁴

Concurrently, systematic studies were initiated on the behavior of perforated plates (perforates) to take advantage of their dissipative properties¹⁵⁻¹⁷ for general applications in the transportation industry. Such perforates are used in automotive applications as part of the two- and three-pipe elements featuring one and two perforated pipes, respectively, contained within a larger diameter rigid-wall cylindrical cavity. Such configurations were investigated¹⁸⁻²³ by combining an orifice model with the transfer matrices of the axially segmented perforated tube or tubes and the unpartitioned cavity.

At higher frequencies or for large mufflers, three-dimensional effects become significant. The approaches used in three-dimensional silencer analysis include the finite-element, boundary-element, and acoustical-wave finite-element methods. Each has some inherent advantages at higher frequencies where higher order modes will start propagating, but on the average they are much more complex and time consuming to implement. Therefore, the plane-wave transfer matrix

method is presently the most widely used approach, particularly for synthesis of an initial configuration of a passive silencer.²⁴⁻²⁶

Experimental work conducted in parallel with the analysis validated the developed models and pointed out areas requiring further investigation. Methods for the direct measurement of transfer matrices were developed²⁷⁻³¹ to validate/modify the models of existing silencer elements. Substantial attention was also paid to the source impedance, which was expected to influence the insertion loss of a silencer and the net radiated acoustical power of the system. The source impedance of a six-cylinder engine was measured with an impedance tube using pure tones,^{32,33} and the impedance of electroacoustical drivers and multicylinder engines was subsequently measured³⁴⁻³⁷ using the much quicker two-microphone methods.³⁸⁻⁴⁰ More recent indirect methods characterized the source impedance through measurements using two, three, or four different acoustical loads.⁴¹⁻⁴⁶

The research performed and the knowledge developed in the field of passive silencers are very extensive and cannot be adequately covered in a single chapter. Accordingly, this chapter reviews the basic background information, summarizes the methodologies for silencer design, demonstrates these approaches through selected examples, and discusses additional topics and references useful to general applications. A reader wishing to acquire more information on internal combustion engine silencers is urged to review references 47-50 for a summary of a general automotive silencer development approach and of typical predicted and measured results, references 51 and 52 for condensed qualitative reviews of the accomplishments in this field, and references 8 and 9 for an extensive bibliography on and a detailed discussion of most silencer-related topics.

9.2 SILENCER PERFORMANCE METRICS

The majority of noise sources have an intake and an exhaust and, generally, they require both intake and exhaust silencers. The two cases are characterized by different flow direction, back pressure, average gas temperature, and average sound pressure levels, but the corresponding hardware is developed using the same principles and design methods. The following discussion is tailored to exhaust silencers but it is equally applicable to intake silencers.

Silencer Selection Factors

The use of a silencer is prompted by the need to reduce the radiated noise of a source but, in most applications, the final selection is based on trade-offs between the predicted acoustical performance, mechanical performance, volume/weight, and cost of the resulting system.

During development, the acoustical performance (insertion loss) of a candidate silencer is determined from the free-field sound pressure levels measured at the same relative locations with respect to the noise outlet of the unsilenced and silenced sources. If the noise outlet is inserted into a reverberant space, then the difference can be measured anywhere within the reverberant field, and the difference sound pressure level represents the power-based insertion loss.

The impact of the silencer on the mechanical performance of the source is determined from the change in the silencer back pressure. For a continuous-flow source, such as a fan or a gas turbine, the impact is determined from the increase in the average back pressure; by contrast, for an intermittent-flow source, such as a reciprocating engine, the impact is a function of the increase in the exhaust manifold pressure when an exhaust valve is open.

Most silencers are subject to volume/weight constraints, which also influence the silencer design process. In addition, the initial purchase/installation cost and the periodic maintenance cost are other important factors that influence the silencer selection process.

Since noise is the root cause of silencer design, the remainder of this chapter focuses on the prediction of a silencer's acoustical performance and also on the estimation of back pressure. The additional performance criteria associated with hardware volume, weight, and cost are trade-off parameters to be determined separately for each application.

Factors Influencing Acoustical Performance

Reactive and Dissipative Silencers. The net change ΔW in the acoustical power radiated from a source can be expressed as

$$\Delta W = W_1 - W_2 = W_1 - (W'_1 - W_d) = (W_1 - W'_1) + W_d \quad \text{N} \cdot \text{m/s} \quad (9.1)$$

where W_1 and W_2 correspond to the unsilenced and silenced sources (Fig. 9.1), $W_1 - W'_1$ is the net change in the acoustical power output of the source resulting from changes in the silencer's reflection coefficient, and W_d is attributable to the dissipative properties of the silencer.

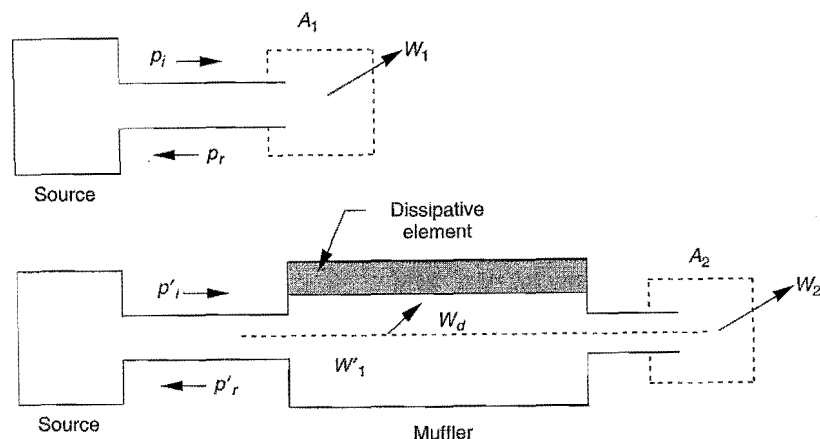


FIGURE 9.1 Radiated noise reduction mechanisms of a silencer.

The reflection coefficient is manipulated primarily by introducing cross-sectional discontinuities in the piping system, and the dissipated part W_d depends largely on the dissipative properties of the silencer elements. In practice, all silencer elements reflect and dissipate acoustical energy to some extent. However, traditionally, they have been categorized into reactive and dissipative silencers when their insertion loss is dominated by reflective and dissipative mechanisms, respectively. This convention has also been adopted in this chapter, but the reader should be aware that most common applications involve various degrees of overlap between the two extremes.

Flanking Paths and Secondary Source Mechanisms. The acoustical power radiated from a silenced source, such as an engine, includes airborne noise (W_{EX}) from the exhaust pipe outlet, silencer shell noise (W_{SH}) radiated from the vibrating walls of the silencer and exhaust line pipes, and a contribution (W_{AD}) from additional sources, such as source shell and source intake, located upstream of the silencer.

The shell noise results from the excitation of silencer components by the vibrating engine, by the intense internal acoustical pressure fields, and by aerodynamic forces. These contributions can be reduced substantially through various methods, but they cannot be totally eliminated and, eventually, they set the limit for the achievable insertion loss for high-performance silencers.

The acoustical power contributions W_{SH} and W_{AD} are excluded from further consideration in the remainder of the chapter; that is, the discussion is focused on the impact of silencers on the airborne noise contributions W_{EX} . However, one should be aware of their existence and significance, particularly in experimental applications where limited resources may lead to the measurement of W_{EX} in the presence of W_{SH} and W_{AD} .

Silencer System Modeling

Silencer Component Representation. The basic components of a typical silencer system comprise the noise source, silencer, connecting pipes, and surrounding medium (Fig. 9.2a). The pipe connecting the source to the silencer is treated as part of the source. Figure 9.2b shows the corresponding electrical analog of the acoustical system, which is widely used to facilitate the representation and handling of acoustical transmission lines. This analog model, which uses acoustical pressure p and mass velocity $\rho_0 S u$ in kg/s, instead of voltage and current, represents the noise source with a source pressure p_s and an internal impedance Z_s , the silencer and pipe segment with four-pole elements (T_{ij} and D_{ij}), and the surrounding medium with a termination (or radiation load) impedance Z_T . Analytical expressions for the transfer matrices of pipes and selected silencer elements are presented in references 8 and 9. Some of them are given later in Section 9.3.

Let $p_i, u_i, i = 1, 2, 3$, designate the acoustical pressure and particle velocity at the interfaces of acoustical components of the source-silencer-load system illustrated in Fig. 9.2b. These quantities may be obtained by solving the system

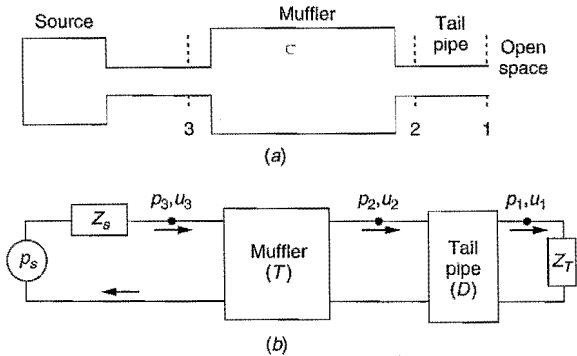


FIGURE 9.2 (a) Acoustical and (b) electrical analog components of a silencer.

of equations describing the response of the components depicted in Fig. 9.2b, namely,

$$p_s = p_3 + \rho_0 Z_s S_3 u_3 \quad (9.2)$$

$$\begin{bmatrix} p_3 \\ \rho_0 S_3 u_3 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_2 \\ \rho_0 S_2 u_2 \end{bmatrix} \quad (9.3)$$

$$\begin{bmatrix} p_2 \\ \rho_0 S_2 u_2 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ \rho_0 S_1 u_1 \end{bmatrix} \quad (9.4)$$

$$p_1 = Z_T \rho_0 S_1 u_1 \quad (9.5)$$

where ρ_0 is the mean gas density and S_i is the duct cross-sectional area at the i th location. The calculated quantities p_i , $\rho_0 S_i u_i$, $i = 1, 2, \dots$, can then be combined with the selected performance criterion to determine the effectiveness of the silencer.

Acoustical Performance Criteria. The most frequently used performance criteria are the insertion loss (IL), noise reduction (NR), and transmission loss (TL). All three use a sound-pressure-level difference as a performance indicator; therefore, they require no explicit knowledge of the source strength p_s , of Eq. (9.2). The number of required system parameters (source impedance Z_s , transfer matrix elements D_{ij} , and termination impedance Z_T *) other than the silencer matrix T_{ij} can be further reduced by the specific choice of the performance criterion.

The IL is defined as the change in the radiated sound pressure level resulting from the insertion of the muffler, that is, the replacement of a pipe segment of length l_1 (Fig. 9.3a) located downstream from the source by the silencer and a new tail-pipe segment of length l_2 (Fig. 9.3b). Insertion loss is expressed as

$$IL = L_b - L_a = 20 \log \left| \frac{p_b}{p_a} \right| \quad (9.6)$$

*The impedance Z in this chapter is defined as $p_i/(\rho_0 S_i u_i)$.

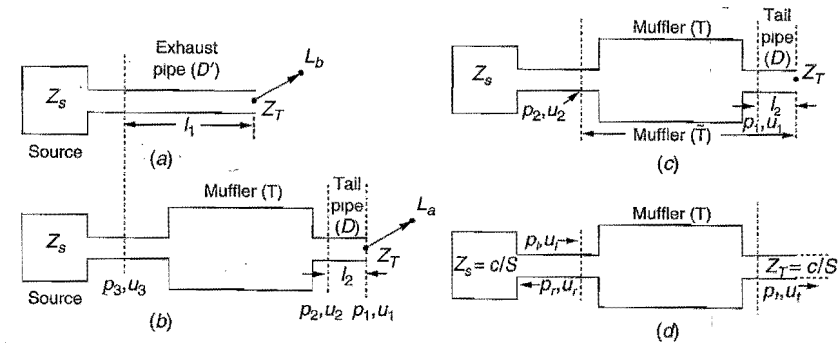


FIGURE 9.3 Quantities used for the determination of (a,b) insertion loss, (c) noise reduction, and (d) transmission loss.

where p_b , p_a and L_b , L_a are the measured sound pressures and sound pressure levels at the same relative location (distance and orientation) with respect to the exhaust outlet (Fig. 9.3a) before and after the installation of the muffler and "log" refers to a logarithm with respect to the base 10. Mathematically,

$$IL = 20 \log \left| \frac{\tilde{T}_{11} Z_T + \tilde{T}_{12} + \tilde{T}_{21} Z_s Z_T + \tilde{T}_{22} Z_s}{D_{11} Z_T + D_{12} + D_{21} Z_s Z_T + D_{22} Z_s} \right| \quad (9.7)$$

where $\tilde{T}_{ij}$ is the combined transfer matrix element of the silencer and its tail pipe ($\tilde{T} = TD$), and D'_{ij} is the transfer matrix element of the replaced exhaust pipe segment. If the silencer is simply added to the end of the source ($l_1 = l_2 = 0$ in Figs. 9.3a and b), then both D' and D are identity matrices (see Transfer Matrices for Reactive Elements in Section 9.3) and Eq. (9.7) is reduced to

$$IL(l_1 = l_2 = 0) = 20 \log \left| \frac{T_{11} Z_T + T_{12} + T_{21} Z_s Z_T + T_{22} Z_s}{Z_T + Z_s} \right| \quad (9.8)$$

The noise reduction is given by

$$NR = L_2 - L_1 = 20 \log \left| \frac{p_2}{p_1} \right| \quad (9.9)$$

that is, it is the difference between the sound pressure levels measured at two points 1 and 2 (Fig. 9.3c) of the silenced system located upstream and downstream of the silencer, respectively. If D_{ij} and $\tilde{T}_{ij}$ represent the transfer matrix elements for the silencer portions downstream of points 1 and 2, respectively, the noise reduction is given by

$$NR = 20 \log \left| \frac{\tilde{T}_{11} Z_T + \tilde{T}_{12}}{D_{11} Z_T + D_{12}} \right| \quad (9.10)$$

The TL is the acoustical power-level difference between the incident and transmitted waves of an anechoically terminated silencer (see Fig. 9.3d). In terms of the corresponding transfer matrix, the transmission loss is given by

$$TL = 20 \log \left| \frac{T_{11} + (S/c)T_{12} + (c/S)T_{21} + T_{22}}{2} \right| \quad (9.11)$$

where c is the speed of sound at design temperature and S is the cross-sectional area of the reference pipe.

This expression assumes that (a) the area of cross section of the exhaust pipe upstream of the silencer is the same as that of the tailpipe, that is, S ; (b) the silencer is simply added at the end of the source ($l_1 = l_2 = 0$); and (c) the tail pipe end (or radiation end) is anechoic ($Z_T = c/S$).

The IL is the most appropriate indicator of a silencer's performance because it is the level difference of the acoustical power radiated from the unsilenced and silenced systems. It is easy to quantify from pre- and postsilencing data, but it is hard to predict because it depends on Z_s and Z_T , which vary from one application to another. By contrast, the TL is easy to predict but is only an approximation of the silencer's actual performance because it does not account for the source impedance and it models all silencer outlets with anechoic terminations. Equations (9.7) and (9.11) show that the TL and IL of a silencer become identical when the noise source and silencer termination are anechoic, that is, $Z_s = Z_T = c/S$. Similarly, it can readily be shown⁸ that for a constant-pressure source ($Z_s = 0$), the NR and IL become identical, provided the cross-sectional area of the exhaust pipe upstream of the silencer is the same as that of the tailpipe.

The ultimate selection of an evaluation criterion is based on trade-offs between the desired accuracy in the predictions and the amount of available resources. For example, Z_s , required for IL predictions, can be determined experimentally through various methods,³²⁻⁴⁶ but the procedure is too costly for the majority of applications. As a result, the silencer design is, generally, based on predicted TL, with a clear understanding of the associated approximations, while the final evaluation of the hardware during field tests is based on the measured IL.

9.3 REACTIVE SILENCER COMPONENTS AND MODELS

Reactive silencers consist typically of several pipe segments that interconnect a number of larger diameter chambers. These silencers reduce the radiated acoustical power primarily through impedance mismatch, that is, through the use of acoustical impedance discontinuities to reflect sound back toward the source. In essence, the more pronounced the discontinuities, the higher the amount of reflected power. Acoustical impedance discontinuities are commonly achieved through (a) sudden cross-sectional changes (i.e., expansions or contractions), (b) wall property changes (i.e., transition from a rigid-wall pipe to an equal-diameter absorbing wall pipe), or (c) any combination thereof.

Silencer Representation by Basic Silencer Elements

Every silencer can be divided into a number of segments or elements each represented by a transfer matrix. The transfer matrices can then be combined to obtain the system matrix, which may then be substituted into Eqs. (9.7), (9.10), or (9.11) to predict the corresponding acoustical performance for the silencer system.

The procedure is illustrated by considering the silencer of Fig. 9.4, which is divided into the basic elements, labeled 1–9, indicated by the dashed lines. Elements 1, 3, 5, 7, and 9 are simple pipes of constant cross section. Element 2 is a simple area expansion, element 4 is an area contraction with an extended outlet pipe, element 6 is an area expansion with an extended inlet pipe, and element 8 is a simple area contraction. The nine elements are characterized by the transfer matrices $T^{(1)}$ through $T^{(9)}$; therefore, the system matrix $T^{(S)}$ for the entire silencer is obtained from

$$T^{(S)} = T^{(1)} \cdot T^{(2)} \cdot \dots \cdot T^{(9)} \quad (9.12)$$

through matrix multiplication. The matrices for each of the above elements may be derived from the formulas presented later in this section.

A few of the most common reactive muffler elements are discussed in this section and some are used in illustrative design examples. Additional elements that were the subject of previous investigations include gradually varying area ducts (such as horns), uniform-area compliant-wall ducts (such as hoses), Quincke tubes, inline cavities, inline bellows, catalytic converter elements, branch sub systems, side inlets/outlets, cavity-backed resistive-wall elements, and perforated-duct elements with and without mean flow through the perforations. Transfer matrices for each of these elements have been derived over the last three decades by different researchers,⁵³⁻⁷⁰ and explicit expressions for their four-pole parameters are given in detail in references 8 and 9.

The variety of silencer elements and the multitude of elements per silencer result in numerous silencer configurations; therefore, a comprehensive study of all possible silencer applications is beyond the scope of this work. For this

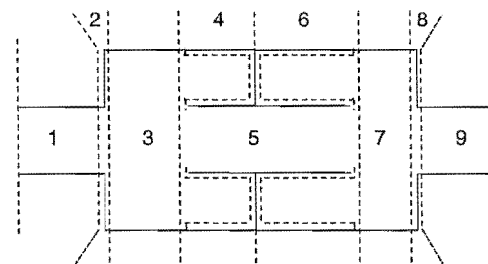


FIGURE 9.4 Decomposition of a silencer into basic elements.

reason, the remainder of this chapter uses selected silencer configurations to demonstrate

- the typical process for modeling a basic silencer element with a (four-pole parameter) transmission matrix,
- the process for predicting the acoustical performance of a given silencer configuration from the properties (transmission matrices) of its constituent elements, and
- the process for selecting a silencer design to achieve approximately a specified acoustical performance.

Furthermore, Section 9.5 provides additional information and sketches for perforated-tube elements and analyzes the performance of selected perforated-tube muffler configurations. The reader could also apply the demonstrated design process to any other silencer configurations by obtaining and utilizing the relevant transmission matrix information from the corresponding references 8, 9, 53–69.

Transfer Matrices for Reactive Elements

The transfer matrix of a silencer element is a function of the element geometry, state variables of the medium, mean flow velocity, and properties of duct liners, if any. The results presented below correspond to the linear sound propagation of a plane wave in the presence of superimposed flow. In certain cases, the matrix may also be influenced by nonlinear effects, higher order modes, and temperature gradients; these latter effects, which can be included in special cases, are discussed qualitatively later in this section, but they are excluded from the analytical procedure described below. The following is a list of variables and parameters that appear in most transfer matrix relations of reactive elements:

- p_i = acoustical pressure at i th location of element
- u_i = particle velocity at i th location of element
- ρ_0 = mean density of gas, kg/m³
- c = sound speed m/s, $= 331\sqrt{\theta/273}$
- θ = absolute temperature, K = °C + 273
- S_i = cross section of element at i th location, m²
- $Y_i = c/S_i$
- A, B = amplitudes of right- and left-bound fields
- $k_c = k_0/(1 - M^2)$ assuming negligible frictional energy loss along straight pipe segments
- $k_0 = \omega/c = 2\pi f/c$
- $\omega = 2\pi f$
- f = frequency, Hz
- $M = V/c$
- V = mean flow velocity through S
- T_{ij} = (ij) th element of transmission matrix or transfer matrix

Symbols without subscripts, such as V , c , and M , describe quantities associated with the reference duct.

Pipe with Uniform Cross Section. The acoustical pressure and mass velocity fields p , $\rho_0 Su$ in a pipe with uniform cross section S and a mean flow V_0 are given by

$$p(x) = (Ae^{-jk_c x} + Be^{jk_c x})e^{j(Mk_c x + \omega t)} \quad (9.13a)$$

$$\rho_0 Su(x) = (Ae^{-jk_c x} - Be^{jk_c x}) \frac{e^{j(Mk_c x + \omega t)}}{Y} \quad (9.13b)$$

where S is the cross section of the pipe. Equations (9.13a) and (9.13b) can be evaluated at $x = 0$ and $x = l$ to obtain the corresponding fields p_2 , $\rho_0 Su_2$ and p_1 , $\rho_0 Su_1$, respectively. Upon elimination of the constants A and B , one obtains^{8,13}

$$\begin{bmatrix} p_2 \\ \rho_0 Su_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_1 \\ \rho_0 Su_1 \end{bmatrix} \quad (9.14)$$

where the transmission matrix T_{pipe} is given by

$$T_{\text{pipe}} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_{\text{pipe}} = e^{-jMk_c l} \begin{bmatrix} \cos k_c l & jY_0 \sin k_c l \\ \frac{j}{Y} \sin k_c l & \cos k_c l \end{bmatrix} \quad (9.15)$$

In the transfer matrix of Eq. (9.15) the acoustical energy dissipation that may result from friction between the gas and the rigid wall as well as from turbulence is neglected. These effects, which would result in a slightly different matrix,⁸ may be noticeable in very long exhaust systems but they are negligible for most silencer applications.

Cross-Sectional Discontinuities. The transition elements used to model most cross-sectional discontinuities are shown in Figs. 9.5b,c,e,f and in the first column of Table 9.1. Using decreasing element-subscript values with distance from the noise source, the cross-sectional areas upstream, at, and downstream of the transition (S_3 , S_2 , and S_1) are related through⁸

$$C_1 S_1 + C_2 S_2 + S_3 = 0 \quad (9.16)$$

where the constants C_1 and C_2 (Table 9.1) are selected so as to satisfy the compatibility of the cross-sectional areas across the transition.

For each configuration, Table 9.1 also shows the pressure loss coefficient K , which accounts for the conversion of some mean-flow energy and acoustical field energy into heat at the discontinuities. As indicated, $K \leq 0.5$ for area contractions, while $K \rightarrow (S_1/S_3)^2$ for area expansions at large values of S_1/S_3 .

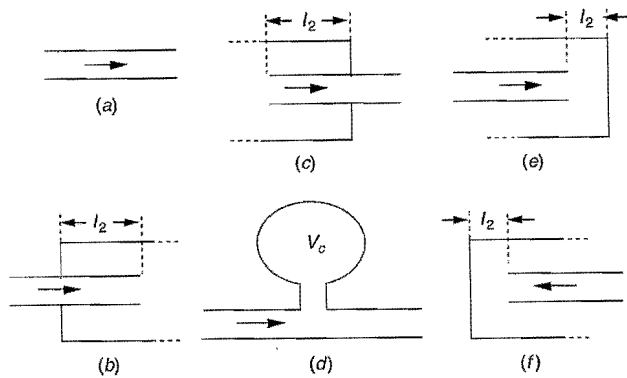


FIGURE 9.5 Basic silencer element: (a) plain pipe; (b,c) extended tube inlet/outlet; (d) resonator; (e,f) reversal expansion/contraction.

TABLE 9.1 Parameter Values of Transition Elements

Element Type	C_1	C_2	K
	-1	-1	$\frac{1 - S_1/S_3}{2}$
	-1	1	$\left(\frac{S_1}{S_3} - 1\right)^2$
	1	-1	$\left(\frac{S_1}{S_3}\right)^2$
	1	-1	0.5

Transfer matrices for cross-sectional discontinuities (csd) in the presence of mean flow that include terms proportional up to the fourth power (M^4) of the Mach number are presented in reference 8. However, in most silencer design applications, $M \ll 1$; therefore, terms of the form $1 + M^n$, $n \geq 2$, are set to unity. This simplification reduces the matrix T_{csd} , which relates the upstream and

downstream acoustical fields p_3 , $\rho S u_3$ and p_1 , $\rho S u_1$ through

$$\begin{bmatrix} p_3 \\ \rho_0 S_3 u_3 \end{bmatrix}_{\text{upstream}} = T_{csd} \begin{bmatrix} p_1 \\ \rho_0 S_1 u_1 \end{bmatrix}_{\text{downstream}} \quad (9.17)$$

to

$$[T_{csd}] = \begin{bmatrix} 1 & k M_1 Y_1 \\ \frac{C_2 S_2}{C_1 S_2 Z_2 + S_2 M_3 Y_3} & \frac{C_2 S_2 Z_2 - M_1 Y_1 (C_1 S_1 + S_3 K)}{C_2 S_2 Z_2 + S_3 M_3 Y_3} \end{bmatrix} \quad (9.18)$$

where $Z_2 = -j(c/S_2) \cot k_0 l_2$

l_2 = length of extended inlet/outlet pipe, m

Letting length l_2 tend to zero (as in the case of sudden expansion and contraction shown in Fig. 9.5) yields the transfer matrix

$$\begin{bmatrix} 1 & K M_1 Y_1 \\ 0 & 1 \end{bmatrix} \quad (9.20)$$

Resonators. A resonator is a cavity-backed opening in the sidewall of a pipe Fig. (9.5d). The opening may consist of a single hole on the pipe wall (Fig. 9.6a) or a closely distributed group of holes (Fig. 9.6b). The volume behind this opening can comprise a throat of length l_t and cross section S_u terminated by a straight pipe of cross-sectional area S_c and depth l_c (Fig. 9.6c), a concentric cylinder extending a length l_u and l_d upstream and downstream of the opening (Fig. 9.6d), an odd-shaped chamber of total volume V_c (Fig. 9.6e), or an extended-tube (quarter-wave) resonator of length l_2 (Figs. 9.5b,c,e,f).

The resonator opening in the wall of the main duct is assumed to be well localized; that is, the axial dimension of the perforated section is much smaller than the wavelength and typically smaller than a duct diameter. This requirement ensures that the duct-cavity interaction is in phase over the entire surface of the connecting opening. Transmission matrices for multihole openings with substantial axial dimensions requiring modeling by perforated-tube elements may be found in references 8 and 9.

The transfer matrix for a resonator for a stationary medium is given by⁹

$$T_r = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_r} & 1 \end{bmatrix} \quad (9.21)$$

where $Z_r = Z_t + Z_c$, Z_t is the impedance of the throat connecting the pipe to the cavity, and Z_c is the impedance of the cavity. Here, Z_c is independent of the flow in the main duct and is given by one of the following expressions:

$$Z_c = \begin{cases} Z_{tc} = -j \frac{c}{S_c} \cot k_0 l_c \\ Z_{cc} = -j \frac{c}{S_c} \frac{1}{\tan k_0 l_u + \tan k_0 l_d} \quad (\text{m} \cdot \text{s})^{-1} \\ Z_{gv} = -j \frac{c}{k_0 V_c} \end{cases} \quad (9.22)$$

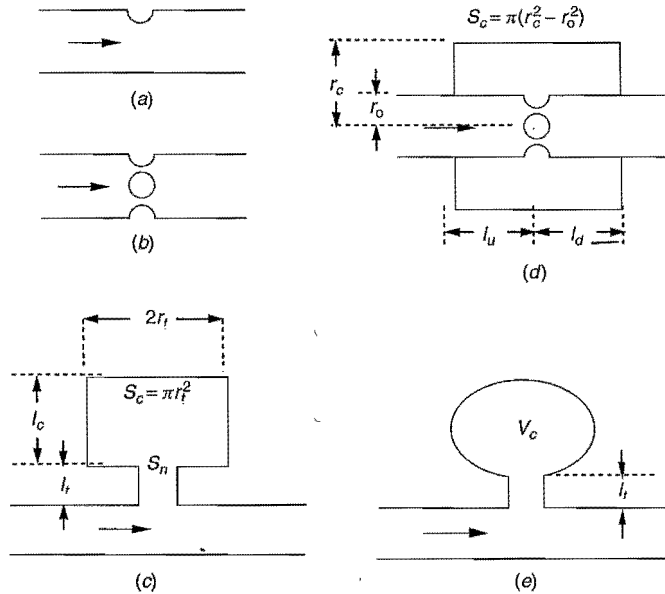


FIGURE 9.6 Resonator components and configurations.

where the subscripts tt, cc, and gv refer to the transverse-tube (quarter-wave resonator), concentric-cylinder, and general-volume cavities. These quantities and the lengths l_u , l_d , and l_c , the cross section S_c , and the volume V_c are illustrated in Fig. 9.6. It should be noted that at low frequencies ($k_0 l_c \ll 1$ and $k_0 l_u, k_0 l_d \ll 1$) both Z_{tt} and Z_{cc} are reduced to the expression of Z_{gv} .

The impedance Z_t of the throat connecting the duct to the cavity changes dramatically with grazing flow; therefore, it is characterized by two sets of values. For $M = 0$, this quantity is given by

$$Z_t^{[M=0]} = \frac{1}{n_h} \left(\frac{ck_0^2}{\pi} + j \frac{ck_0(l_t + 1.7r_o)}{S_0} \right) \quad (\text{m} \cdot \text{s})^{-1}, \quad (9.23)$$

where l_t is the length of the connecting throat, r_o the orifice radius, S_o the area of a single orifice, and n_h the total number of perforated holes. In the absence of mean flow, this expression has yielded good agreement between predicted and measured results for single-hole and well-localized multihole resonators.

The presence of grazing flow ($M \neq 0$ in the duct) has a strong effect on the impedance Z_t of the resonator throat. Measurements conducted using single- and multihole throats^{15,57} have led to the empirical expression

$$Z_t^{[M \neq 0]} = \frac{c}{\sigma S_0} [7.3 \times 10^{-3}(1 + 72M) + j2.2 \times 10^{-5}(1 + 51l_t)(1 + 408r_o)f] \quad (\text{m} \cdot \text{s})^{-1} \quad (9.24)$$

where the parameters l_t and r_o in Eq. (9.23) are in meters and σ is the porosity. The expression of Eq. (9.24) is more appropriate for predictions in the presence of grazing flow.

Transmission Matrices for Other Elements. The mass and momentum conservation laws used to derive the transmission matrices for a pipe segment, area discontinuities, and resonators can be similarly deployed to obtain similar expressions for other elements. The reader may find additional information on transmission matrices in references 8, 9, and 53–69.

9.4 PERFORMANCE PREDICTION AND DESIGN EXAMPLES FOR EXPANSION CHAMBER MUFFLERS

Simple Expansion Chamber Muffler (SECM)

This silencer consists of an exhaust pipe, an expansion chamber, and a tail pipe of different transverse dimensions as shown in Fig. 9.7. The TL for this simple configuration may be obtained by substituting into Eq. (9.11) the transmission matrices for these elements obtained from Eqs. (9.15) and (9.18).

In a simplified case, where the flow is or is assumed to be insignificant, the product of the three matrices is reduced to the expression

$$TL = 10 \log \left[1 + 0.25 \left(N - \frac{1}{N} \right)^2 \sin^2 kL \right] \quad \text{dB} \quad (9.25)$$

where k is the wavenumber, L is the chamber length, and N is the area ratio given by $N = S_2/S_1$, where S_2 is the area of cross section of the chamber and S_1 that of the tail pipe or exhaust pipe (assumed to be equal).

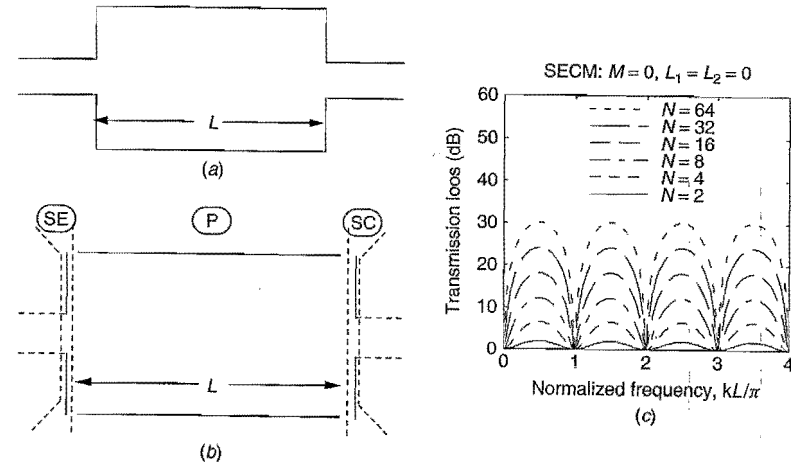


FIGURE 9.7 Simple expansion chamber muffler: (a) typical cross section; (b) muffler elements; (c) predicted transmission loss for different area ratios.

The predicted TL is presented in Fig. 9.7c for a few values of the area ratio N and displayed versus the dimensionless quantity $q = kL/\pi$. The troughs of the TL curve occur at $kL/\pi = n$, while the peaks occur at $kL/\pi = (2n - 1)/2$, where $k = \omega/c$ is the wave number and n is an integer.

Example 9.1. The predicted curves of Fig. 9.7c can be used to develop a starting design of a SECM with a given performance goal. For example, assume that we need a 10-dB reduction for a 180-Hz tone generated by the exhaust of a small engine venting through a pipe with a diameter $d = 2$ in. (5.1 cm) at a temperature of 50°C and at a negligible flow speed.

The typical design process aligns the 180-Hz tone with one of the predicted acoustical performance peaks. Generally, alignment with a peak on a low-area-ratio (S_2/S_1) curve will minimize the muffler diameter and alignment with a peak at a low value of kL/π will minimize the length of the resulting muffler. In the present example, the design may proceed along the following steps:

- Step 1: The 180-Hz tone is aligned with the predicted 12-dB peak of the $S_2/S_1 = 8$ curve at $kL/\pi = 0.5$ (Fig. 9.7c).
- Step 2: At 50°C (323 K), the sound speed $c = 331$ radical $\sqrt{323/273} = 360$ m/s.
- Step 3: The corresponding wavenumber $k = 2\pi f/c = 2\pi \times 180/360 = 3.14$ rad/m.
- Step 4: The required muffler length L (given by $kL/\pi = 0.5$) $= 0.5\pi/k = 0.5\pi/3.14 = 0.5$.
- Step 5: The required muffler diameter $D = d\sqrt{S_2/S_1} = 5.1\sqrt{8} = 14.4$ cm.

One important implication of antilog addition and subtraction is that raising the trough levels of the IL (or, for that matter, the TL) by an amount DL has a more favorable impact than raising the peaks by the same amount DL . Therefore, it is desirable to try to raise the troughs (particularly, the first few at the lower end frequency range) through design adjustments, even at the expense of some reduction in the peaks of the revised muffler configuration.

A major disadvantage of the simple expansion chamber silencer is that in certain applications time-varying tones and their harmonics may align simultaneously with the periodic troughs and cause a severe deterioration in acoustical performance. This problem may be resolved to varying degrees by using extended-tube (inlet and/or outlet) elements.

Extended-Outlet Mufflers

An extended-outlet muffler (Fig. 9.8a) represents the first incremental step toward improving the performance of a single-chamber muffler. This configuration is decomposed into the three basic elements illustrated in Fig. 9.8b:

- a sudden expansion element (SE) at the inlet, flush with the left wall of the chamber;

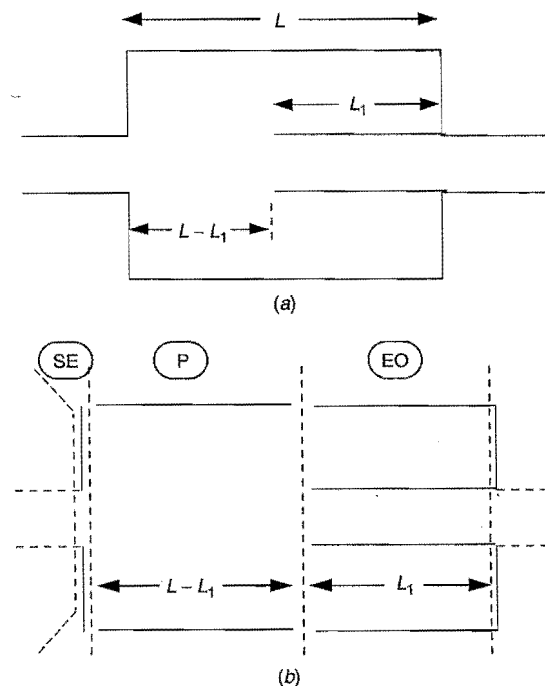


FIGURE 9.8 Extended-outlet expansion chamber: (a) typical configuration; (b) constituent muffler elements.

- a pipe element (P); and
- an extended-outlet element (EO) at the right end of the chamber.

The extended-outlet muffler design introduces a new length, L_1 , which increases by 1 the number of dimensionless parameters (L/d , S_2/S_1 , and L_1/L) that can be used to optimize the muffler's TL.

Transmission matrices for the silencer elements shown in Fig. 9.8b are obtained from Eq. (9.15) (for the pipe element P), Eq. (9.18) (extended-outlet element EO), and Eq. (9.20) (sudden expansion element SE).

It should be noted that at certain frequencies the impedance Z_2 , introduced by the transmission matrix of Eq. (9.19) for the extended-outlet element, would approach zero, and the branch element would generate a pressure release condition. Under these conditions, the incident wave would appear to interact with a closed-end cavity, and no acoustical power would be transmitted downstream. For rigid end plates, this condition would occur when $\cot(kL_1) = 0$, corresponding to $kL_1/\pi = (2n - 1)/2$. Accordingly, a dominant peak in the source spectrum can be reduced significantly by proper selection of the tube length extended into the chamber.

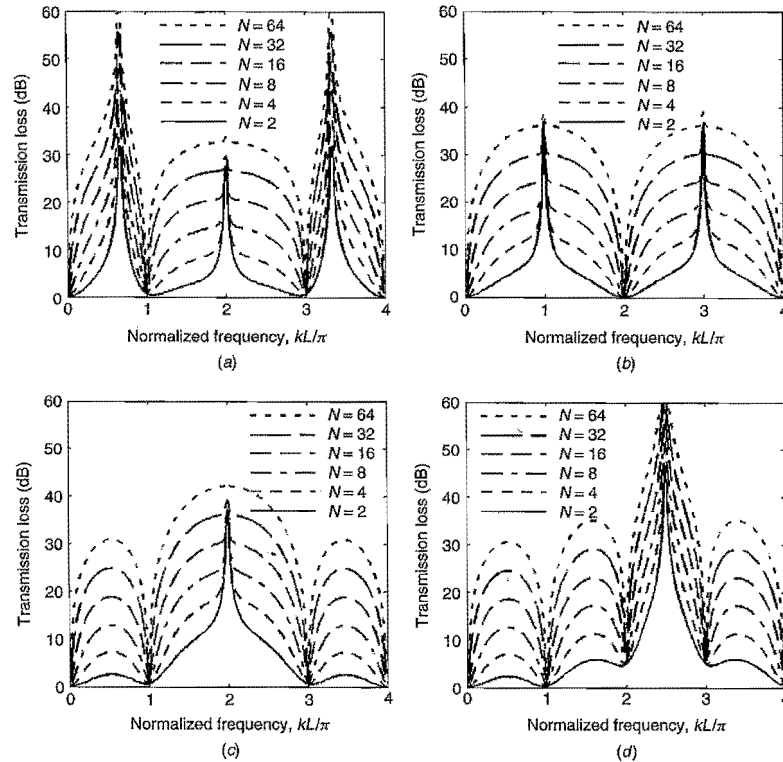


FIGURE 9.9 Transmission loss of extended-outlet muffler for different outlet pipe lengths: $M = 0$; L_1/L equals (a) 0.75, (b) 0.5, (c) 0.25, and (d) 0.2.

Figures 9.9a–9.9d illustrate the variation of the TL for an extended-outlet expansion chamber for four values of the dimensionless parameter L_1/L . As expected, the maximum peak of TL shifts toward higher frequencies as L_1/L decreases. Furthermore, a suitable choice of L_1/L , resulting in $kL_1/\pi = (2n - 1)/2$, n being an integer, relocates some performance peaks so as to eliminate some of the troughs that would otherwise occur at $kL = n\pi$ (Fig. 9.7c) for the simple expansion chamber muffler. This behavior is exploited in the following two design examples.

Example 9.2. Consider the application requiring a 15-dB reduction of a steady 120-Hz tone generated by the exhaust of a small engine venting through a pipe with a diameter $d = 1.5$ in. (3.8 cm) at a temperature of 100°C and at a negligible flow speed.

Since the tone is steady, that is, it does not vary appreciably versus time, we can consider aligning it with one of the sharp peaks in the predicted TL. This

suggests selection of the Fig. 9.9a configuration, characterized by $L_1/L = 0.75$, which has a family of relatively sharp peaks at $kL/\pi = 0.75$.

In the present example, the design may proceed along the following steps:

- Step 1: The 120-Hz tone is aligned with the predicted >12-dB peak of the $S_2/S_1 = 8$ curve at $kL/\pi = 0.75$ (Fig. 9.9a).
- Step 2: At 100°C (373 K), the sound speed $c = 331\sqrt{373/273} = 387$ m/s.
- Step 3: The corresponding wavenumber $k = 2\pi f/c = 2\pi \times 120/387 = 1.95$ rad/m.
- Step 4: The required muffler length (given by $kL/\pi = 0.75$) $L = 0.75\pi/1.95 = 1.2$ m.
- Step 5: The required muffler diameter $D = d\sqrt{S_2/S_1} = 3.8\sqrt{8} = 10.8$ cm.

Example 9.3. Consider the application requiring a 10-dB reduction of a time-varying 150-Hz tone generated by the exhaust of a small engine venting through a pipe with diameter $d = 2.5$ in. (6.3 cm) at a temperature of 80°C and at a negligible flow speed. Furthermore, assume that the a variable load in the engine causes its frequency to vary $\pm 15\%$, that is, operating anywhere in the $127 < f < 172$ -Hz range.

Since the frequency of the tone is unsteady, we can consider aligning it with one of the “broad” peaks in the predicted TL. This suggests selection of the Fig. 9.9b configuration, characterized by $L_1/L = 0.5$, which has a family of relatively “broad hill peaks” around the spikes occurring at $kL/\pi = 1$.

All curves with $S_2/S_1 \geq 4$ appear to have TL > 10 dB within 15% of $kL/\pi = 1$. For a conservative result, we select the curve with $S_2/S_1 = 8$ and complete the muffler design using the following steps:

- Step 1: The 150-Hz tone is aligned with the predicted >10-dB peak of the $S_2/S_1 = 8$ curve at $kL/\pi = 1$ (Fig. 9.9b).
- Step 2: At 80°C (353 K), the sound speed $c = 331\sqrt{353/273} = 376$ m/s.
- Step 3: The corresponding wavenumber $k = 2\pi f/c = 2\pi \times 150/376 = 2.5$ rad/m.
- Step 4: The required muffler length (given by $kL/\pi = 1$) $L = 1.0\pi/2.5 = 1.25$ m.
- Step 5: The required muffler diameter $D = d\sqrt{S_2/S_1} = 6.3\sqrt{8} = 18$ cm.

The TL for extended-inlet silencers can be derived in a similar manner. For negligible values of flow, their acoustical performance is similar to that of the extended-outlet silencers; in other words, for negligible inflow, either side of the muffler can be used as the inlet with no appreciable change in the TL.

Double-Tuned Expansion Chamber (DTEC)

The “filling up” of the troughs achieved through the extended-outlet muffler can be further enhanced through the simultaneous deployment of an extended-inlet

pipe of length L_2 (double tuning). An extended inlet introduces an additional parameter, L_2/L , which used in conjunction with the extended-outlet parameter L_1/L can fill up additional troughs and further improve the achievable TL over a broader frequency range. For optimum results, L_1 and L_2 are selected so as to neutralize different sets of troughs to the extent possible.

The new double-tuned expansion chamber (DTEC) silencer design (Fig. 9.10a) is decomposed into three basic elements (Fig. 9.10b):

- an extended-inlet element (EI) at the left end of the chamber,
- a pipe element (P), and
- an extended-outlet element (EO) at the right end of the chamber

Thus, the introduction of the new length, L_2 , for the extended inlet enhances the optimization of the DTEC's transmission loss through an increased number of dimensionless parameters, which now include d/L , S_2/S_1 , L_1/L , and L_2/L .

Transmission matrices for the Fig. 9.10b silencer elements are obtained from Eq. (9.15) (for the pipe element P) and from Eq. (9.18) (extended-inlet and extended-outlet elements EI and EO).

Of the infinite number of $(L_2/L, L_1/L)$ combinations, a selection of $L_1 = L/2$ and $L_2 = L/4$ leads to a particularly favorable TL. Specifically, since the extended-inlet element includes a term proportional to $\cot(kL_2)$ [see Eqs. 9.18 and 9.19], it produces stop bands at $kL_2 = (2n - 1)\pi/2$ or, equivalently, at $kL = 2(2n - 1)\pi$, filling up the troughs at $kL/\pi = 2, 6, 10, \dots$. Similarly, since the EO element includes a term proportional to $\cot(kL_1)$, it produces stop bands at

$kL_1 = (2n - 1)\pi/2$ or, equivalently, at $kL = (2n - 1)\pi$, filling up the troughs at $kL/\pi = 1, 3, 5, \dots$. In other words, this design features troughs only at $kL/\pi = 0, 4, 8, 12, \dots$

The corresponding predicted TL for a few area ratio values is shown in Fig. 9.10c. Its most prominent feature is the lack of troughs at locations other than at $kL/\pi = 4n$, n being an integer. Accordingly, this design offers a better solution for broadband performance than any of the previous configurations. High reduction limited to a narrow band (like Fig. 9.9a) can be achieved by adjusting the inlet/outlet lengths away from their $(L_1/L, L_2/L) = (\frac{1}{2}, \frac{1}{4})$ values.

Example 9.4. Consider the application requiring a 10-dB reduction of 200–600-Hz broadband noise generated by a noise source venting through a pipe with diameter $d = 2$ in. (5.1 cm) at a temperature of 120°C and at a negligible flow speed. Because of the broadband requirements, a DTEC muffler design is selected through the following steps:

- Step 1: The 400-Hz center of the subject frequency band is aligned with the center, $kL/\pi = 2$ of the broad TL peaks of Fig. 9.10c; this places the upper and lower frequency bounds (200 and 600 Hz) at kL/π values of 1 and 3, respectively.
- Step 2: The $S_2/S_1 = 4$ curve (Fig. 9.10c) is selected because it provides > 10 dB TL in the $1 < kL/\pi < 3$ range.
- Step 3: The required muffler diameter $D = d\sqrt{S_2/S_1} = 5.1\sqrt{4} = 10.1$ cm.
- Step 4: At 120°C (393 K), the sound speed $c = 331\sqrt{393/273} = 397$ m/s.
- Step 5: Given that $kL/\pi = 1$ corresponds to 200 Hz, the required muffler length $L = k\pi c/(2\pi f) = 397/(2 \times 200) = 0.99$ m, $L_1 = L/2 = 0.495$ m and $L_2 = L/4 = 0.247$ m.

General Design Guidelines for Expansion Chamber Mufflers

Transmission loss curves with higher levels and fewer or less pronounced troughs can be obtained with more complex muffler configurations. Specifically, one can cascade two or more DTECs to further optimize performance through an increased number of system parameters. However, an increasing system complexity is not without limits, since muffler size and weight are important design parameters in practical applications. For example, increasing the number of elements reduces the average length/diameter of individual elements, which reduces low-frequency performance, and increases the number of partitions, which increases weight. These competing factors improve certain features while degrading others and lead to design trade-offs that are specific to each application.

A close examination of a number of designs (not detailed here) featuring one to three DTECs leads to the following observations or design considerations:

- The TL of a SECM features nulls (troughs) at $kL/\pi = 1, 2, 3, 4, 5$.

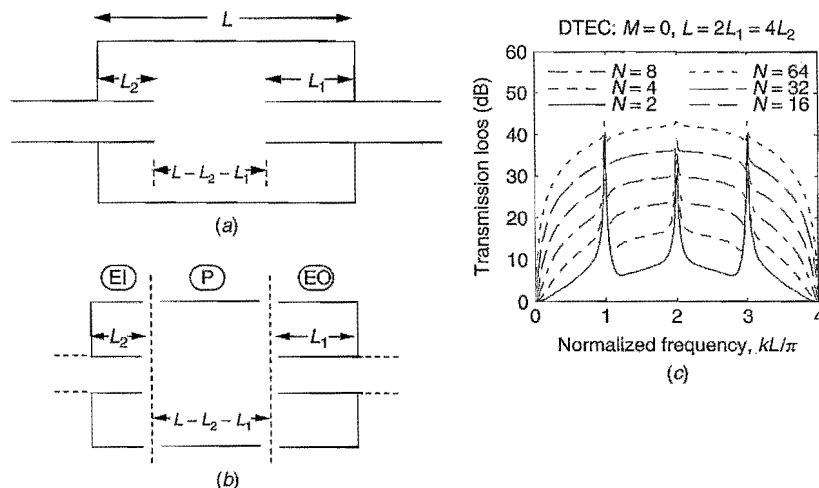


FIGURE 9.10 Double-tuned expansion chamber: (a) typical configuration; (b) constituent elements; (c) predicted TL for $L_2 = L/4$, $L_1 = L/2$.

- If an SECM is augmented with an extended-inlet (or outlet, for that matter) pipe equal in length to half the chamber length, then the troughs corresponding to $kL/\pi = 1, 3, 5, 7, 9, \dots$ are eliminated.
- If an SECM is augmented with an extended-inlet (or outlet, for that matter) pipe equal in length to quarter the chamber length, then the troughs corresponding to $kL/\pi = 2, 6, 10, 14, \dots$ are eliminated.
- If an SECM is augmented with extended-inlet and extended-outlet pipes equal in length to half and quarter the chamber length, respectively (DTEC), then the TL retains troughs only at $kL/\pi = 4, 8, 12, 16, \dots$
- The TL of two identical cascaded DTECs occupying the same envelope as a single DTEC features less pronounced troughs and generally higher levels, except at low frequencies ($kL/\pi < 0.5$), where some degradation is observed.
- The TL of two unequal cascaded DTECs can be further improved over the TL of two identical cascaded DTECs occupying the same envelope.

Additional TL improvements can be achieved through the use of three or more cascaded DTECs occupying the same total envelope as one DTEC. However, such improvements lead to further degradation of low-frequency performance; therefore, commercial application mufflers generally include no more than two or three cascaded DTEC chambers. Nevertheless, a designer who is forced to use a larger number of chambers by the needs of a specific application may recover low-frequency performance through the use of other types of elements, as discussed in the next section.

9.5 REVIEW OF PERFORATED-ELEMENT MUFFLERS

Perforated-element silencers have long been known to be acoustically more efficient than the corresponding simple tubular-element silencers. However, a systematic aeroacoustical analysis of perforated elements began only in late 1970s, when Sullivan introduced his segmentation model.¹⁸⁻²⁰ This was followed by the distributed-parameter model of Munjal et al., which produced experimentally verified four-pole parameters for perforated elements, namely, concentric-tube resonators (Fig. 9.11a), plug chambers (Fig. 9.11b), and three-duct cross-flow (Fig. 9.11c) and reverse-flow chambers.^{8,9,21-23}

This section conducts a parametric study on some perforated-element mufflers to identify representative trends and to develop basic design guidelines.²⁵ Again, TL has been selected as an appropriate performance index. Explicit expressions for the impedance and the transfer matrix parameters of perforated muffler elements in terms of the geometric and operating variables and formulas for TL and IL are given in Chapter 3 of reference 8.

Range of Variables

Perforated-tube mufflers share several parameters with the expansion chamber mufflers discussed earlier, but they also include additional parameters accounting

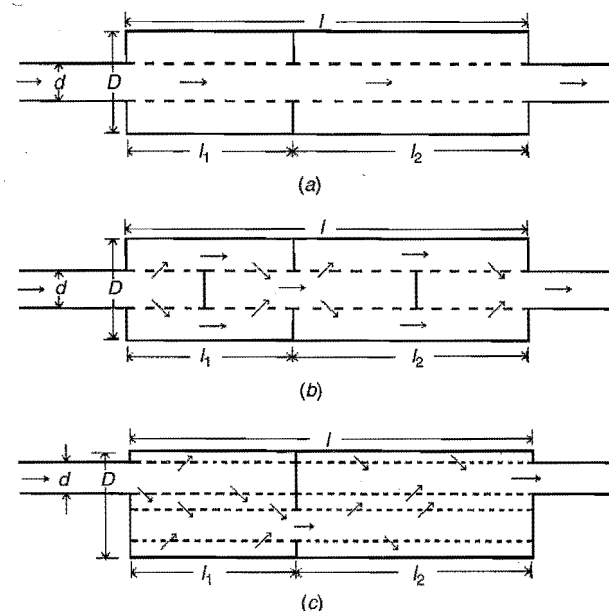


FIGURE 9.11 Schematic of the two-chamber configurations of three types of perforated-tube mufflers: (a) Concentric-tube resonator; (b) plug muffler; (c) three-duct cross-flow muffler.

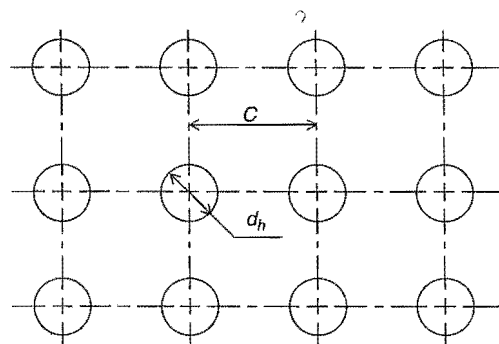
for the additional features of the perforated tubes. Thus, the physical parameters influencing the performance of the Fig. 9.11 perforated mufflers include the following:

- M = mean-flow Mach number in exhaust pipe
- l = total length of muffler shell
- D = internal diameter of muffler shell
- d = internal diameter of exhaust pipe and tail pipe
- d_h = diameter of perforated holes (Fig. 9.12)
- C = center-to-center distance between consecutive holes (Fig. 9.12)
- N_c = number of chambers within fixed-length muffler shell
- t_w = wall thickness of perforated tube

Another parameter, often used instead of explicit values of d_h or C , is the porosity (or open-area ratio) of the perforated-tube wall, which is defined by

$$\sigma = \frac{\pi d_h^2}{4C^2} \quad (9.26)$$

Extensive simulations conducted for representative configurations encountered in most practical applications showed that the performance of perforated-tube

FIGURE 9.12 Parameters d_h and C of perforated-tube walls.

mufflers is relatively insensitive (within ± 1 dB) to the hole diameter d_h and wall thickness t_w . For this reason, all of the following predictions have been conducted at the typically encountered values of $d_h = 4$ mm and $t_w = 1$ mm.

Furthermore, to reduce the simulations to a manageable number, the pipe diameter was fixed to a commonly encountered value of $d = 30$ mm for all cases considered in this section. This constraint does not influence the corresponding predicted TL curves, because they are invariant with respect to the specific value of d when plotted against the nondimensional frequency parameter kR ($R = D/2$ is the shell radius and $k = \omega/c$).

Despite the simplification resulting from the fixed values of d , d_h , and t_w , the number of perforated-tube muffler configurations is still too large for a comprehensive study. Under the circumstances, the influence of each parameter on muffler performance is demonstrated by selecting a reference (default) configuration and then by changing individual parameters (one at a time) about the default values. Table 9.2 shows the range of parameter values investigated including the default values (last column).

The TL was calculated as a function of the dimensionless frequency kR to extend the validity of the predictions over all geometrically similar hardware, that is, mufflers differing only by a length scale. The nondimensional frequency parameter kR was varied in steps of 0.025 from 0.025 to 3, which is well

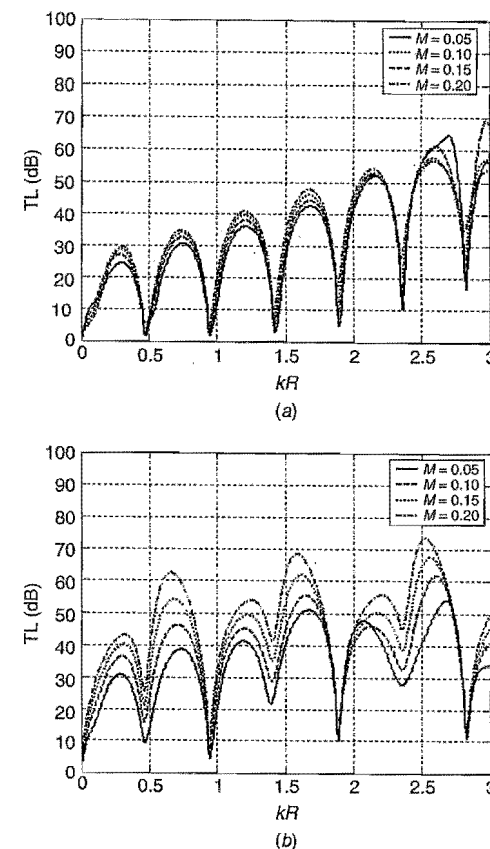
TABLE 9.2 Parameter Values Used in Perforated Tube Muffler Performance Predictions

Parameter Name	Description	Range	Default
M	Mach number	0.05, 0.1, 0.15, 0.2	0.15
D/d	Expansion ratio	2, 3, 4	3
C/d_h	Center-to-center distance	3, 4, 5, 6	4
N_c	Number of chambers	1, 2, 3	2
L_2/L	Chamber size	$1, \frac{1}{2}, \frac{1}{3}$	$\frac{1}{2}$
L/d	Muffler length-pipe diameter	15, 20, 25	20

within the plane-wave cutoff limit of 3.83 for the axisymmetric mufflers of Figs. 9.11a,b. By comparison, the cutoff limit of kR for the asymmetric configuration of Fig. 9.11c is 1.84.

Acoustical Performance

Figures 9.13–9.17 show the computed TL for the Fig. 9.11 mufflers as a function of the normalized frequency kR . The three plots (a, b, and c) in each figure display the TL of a concentric-tube resonator muffler, plug muffler, and three-duct cross-flow muffler, respectively, and the different traces within each plot correspond to different combinations of muffler parameter values listed in Table 9.2.

FIGURE 9.13 Effect three of mean flow Mach number on TL of (a) CTR, (b) plug muffler, and (c) duct muffler: (—) $M = 0.05$; (---) $M = 0.1$; (...) $M = 0.15$; (-·-·-) $M = 0.2$.

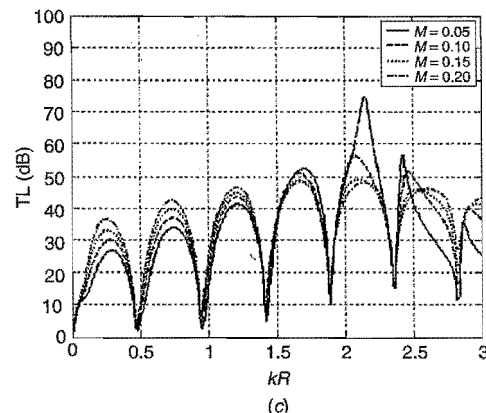


FIGURE 9.13 (continued)

Figure 9.13 shows the effect of the mean-flow Mach number on the acoustical performance of three perforated-element muffler designs. These predictions show a general increase in TL when the mean-flow Mach number increases. The effect is minimum for the concentric-tube resonator (CTR) and maximum for the plug muffler; this is not surprising given that (because of their intrinsic geometric features) the former and latter configurations present the minimum and maximum “blockages,” respectively, to the mean flow through the muffler.

Clearly, the choice of the Mach number is not in the designer's control, as it is determined by the engine displacement, speed, and exhaust pipe diameter. Nevertheless, the designer may combine the information in Fig. 9.13 with back-pressure information (discussed in the next section) in the selection of a muffler design appropriate for the specific application.

The effect of the expansion ratio, or the diameter ratio D/d , is illustrated in Fig. 9.14. The TL of all three perforated-element mufflers improves considerably with higher diameter ratio.

In most practical applications, the shell diameter D is directly related to muffler volume, weight, and cost and influences the choice of the expansion ratio D/d and the type of muffler. The alternate troughs in the TL curves are higher for a plug muffler (Fig. 9.14b) than for the other two designs; therefore, when space is constrained, the plug muffler typically offers a higher acoustical performance. Unfortunately, it also leads to a typically higher back pressure, as discussed later. As a result, the final choice is usually based on a trade-off between acoustical and mechanical performance.

The effect of the center-to-center spacing between consecutive holes (which determines the porosity) on the TL of the mufflers is illustrated in Fig. 9.15. As can be seen, the performance of all perforated mufflers tends to that of the simple expansion chamber muffler for sufficiently high values of porosity ($\sigma > 0.1$) but is considerably better for lower values of porosity, particularly for the plug

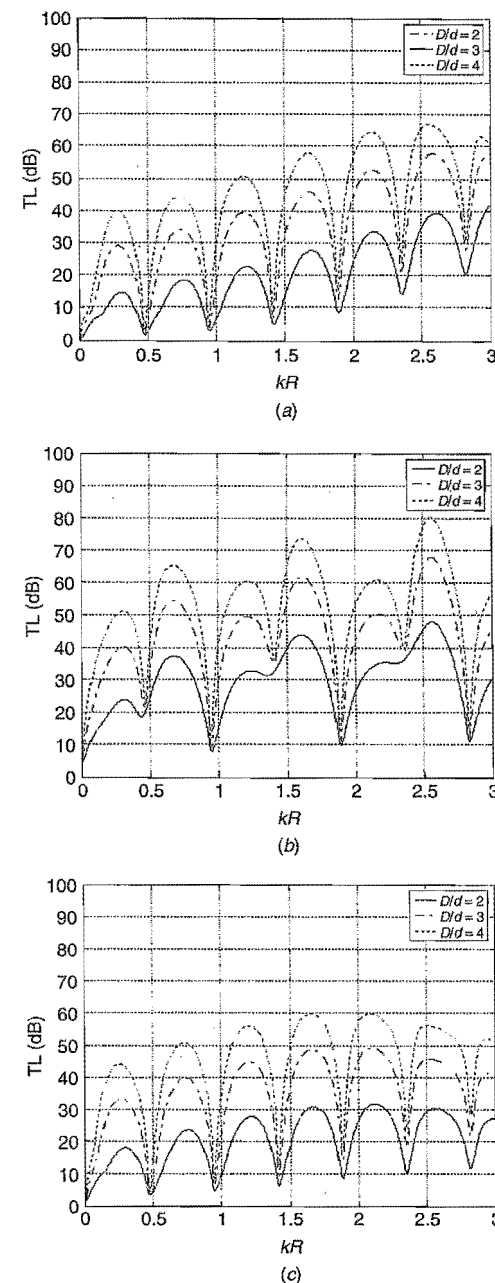


FIGURE 9.14 Effect of expansion ratio on TL of (a) CTR, (b) plug muffler, and (c) three-duct muffler: (—) $D/d = 2$; (---) $D/d = 3$; (...) $D/d = 4$.

muffler and the three duct muffler configurations. On the other hand, a reduced porosity raises back pressure, except for the CTR, where the back pressure is nearly independent of porosity, as demonstrated later in this section. Consequently, the porosity provides another parameter that can be used to trade off between acoustical and mechanical performance.

Another, and perhaps more meaningful, parameter often used instead of the porosity is the open area ratio x , defined as

$$x = \frac{\text{total area of perforations}}{\text{cross-sectional area of pipe}} \quad (9.27)$$

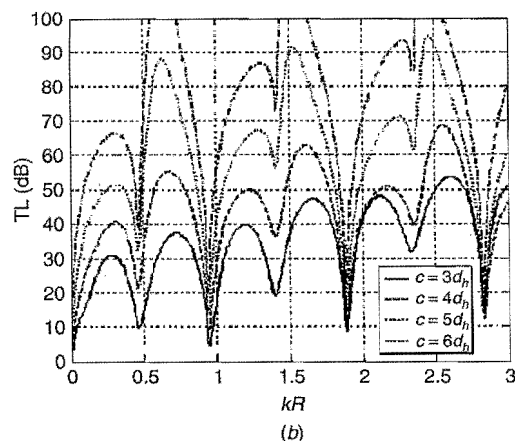
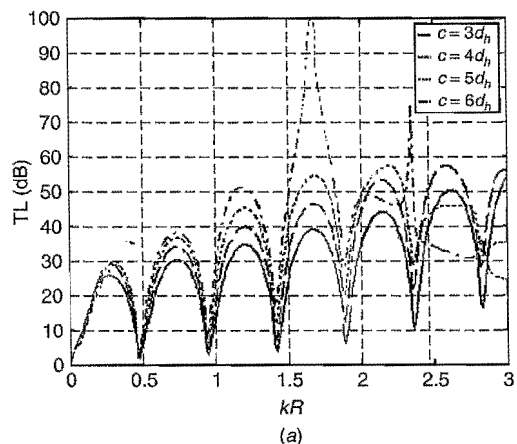


FIGURE 9.15 Effect of center-to-center distance between consecutive holes on TL of (a) CTR, (b) plug muffler, and (c) three-duct muffler: (—) $C = 3d_h$; (---) $C = 4d_h$; (...) $C = 5d_h$; (- - -) $C = 6d_h$.

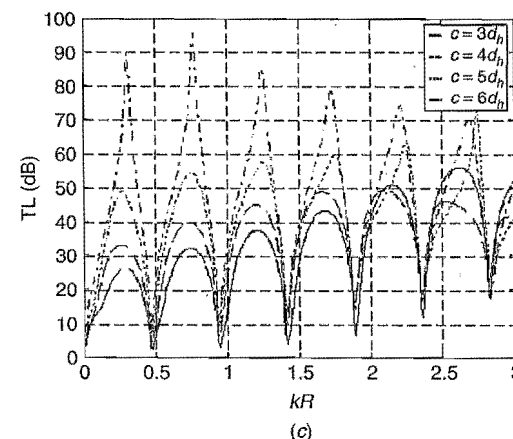


FIGURE 9.15 (continued)

The denominator refers to the inlet/outlet pipe cross section that accommodates the entire mean flow. For example, the CTR (Fig. 9.15a) and three-duct muffler (Fig. 9.15c), may be described in terms of either the porosity associated with the indicated $C = 3d_h, 4d_h, 5d_h, 6d_h$ or in terms of the corresponding open-area ratio values of 3.49, 1.78, 1.25, and 0.87, respectively. The corresponding values of x for the plug muffler (Fig. 9.15b) are half as much.

The plot in Fig. 9.16 shows the effect of multiple equal partitioning of a muffler of overall length l on the TL. The normalized frequency spacing between the consecutive troughs is proportional to the number of partitions and, as in the case of the simple expansion chamber mufflers, the troughs occur at frequencies given by

$$\sin kl_{1,2} = 0 \quad kl_{1,2} = n\pi \quad n = 1, 2, 3 \dots \quad (9.28)$$

In general, the partitioning improves the TL of all three mufflers, except at the low-frequency end of the CTR curves, where the TL decreases. An increased number of partitions N_c would be particularly desirable for a CTR, where an increase in partitions would not increase the back pressure. However, the back pressure will be proportional to the cube of the number of chambers for the other two types of mufflers. Therefore, increased partitioning must be closely considered with an increased porosity to optimize acoustical performance while maintaining the back pressure within acceptable levels.

Figure 9.17 illustrates the effect of unequal partitioning of a muffler of overall length l on the TL by comparing the results of an equally and an unequally partitioned muffler for each of the three designs. One can observe that unequal partitioning (a) has no significant impact on the envelope of the TL peaks, (b) influences significantly the location of individual peaks and troughs, and (c) raises noticeably the level of each alternate trough. On average, the additional

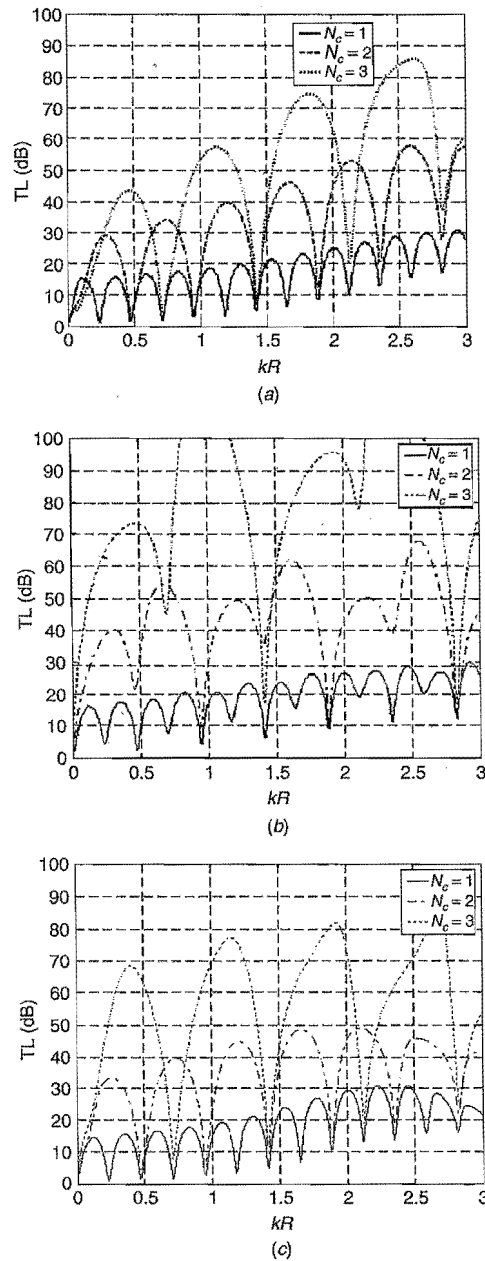


FIGURE 9.16 Effect of number of chambers within the same overall length on TL of (a) CTR, (b) plug muffler, and (c) three-duct muffler: (—) $N_c = 1$; (---) $N_c = 2$; (...) $N_c = 3$.

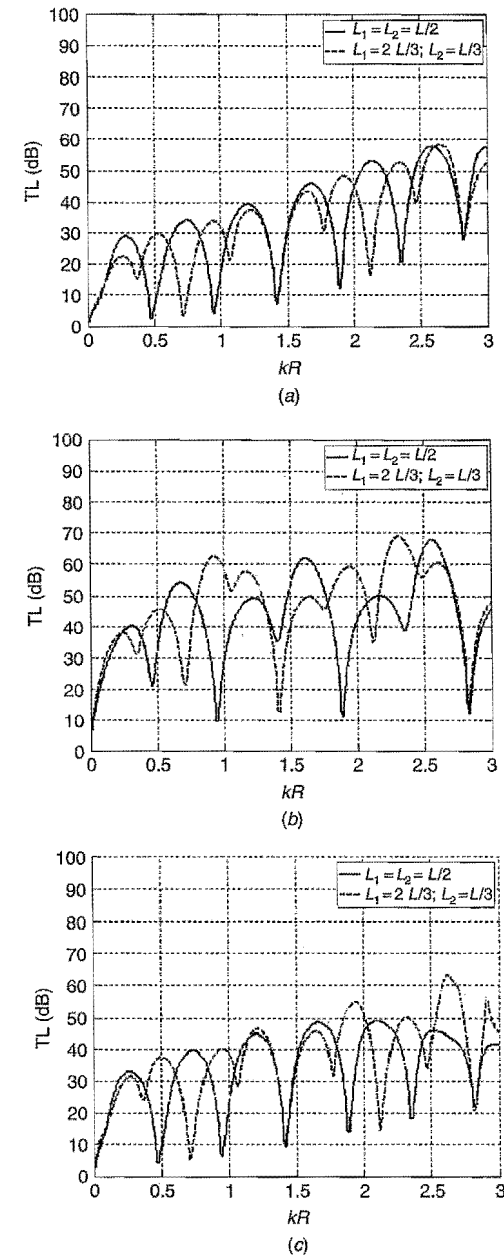


FIGURE 9.17 Effect of unequal partitioning of (a) CTR, (b) plug muffler, and (c) three-duct muffler: (—) $l_1 = l_2 = \frac{1}{2}$; (---) $l_1 = 2l/3, l_2 = l/3$.

tuning enabled by the unequal partitioning will enhance the overall acoustical performance of mufflers.

Back Pressure

The static pressure drop resulting from energy dissipation in sheared flow regions of the muffler elements results in a substantial back pressure on the piston of a reciprocating engine. This generally has an adverse effect on the volumetric efficiency, power, and specific fuel consumption of a multicylinder engine.

A systematic experimental study was undertaken²⁵ to derive empirical expressions for pressure drop or head loss across all the three types of perforated-element chambers of Fig. 9.11. The results were expressed as a coefficient that normalized the pressure drop with respect to the incoming dynamic head in the pipe, namely,

$$y = \frac{\Delta p}{H} \quad H = \frac{1}{2} \rho U^2 \quad (9.29)$$

The parameter y was dependent on the open-area ratio x of the perforate defined by Eq. (9.27), and its value was determined to vary as follows:

Concentric-tube resonator:

$$y_{ctr} = 0.06x \quad 0.6 < x \quad (9.30)$$

Plug muffler:

$$y_{pm} = 5.6e^{-0.23x} + 67.3e^{-3.05x} \quad 0.25 < x < 1.4 \quad (9.31)$$

Three-duct cross-flow chamber:

$$y_{cfc} = 4.2e^{-0.06x} + 16.7e^{-2.03x} \quad 0.4 < x < 5.8 \quad (9.32)$$

During the measurements of the parameter y ,²⁵ the desired variation in the open-area ratio x was achieved by varying the length l_2 (see Fig. 9.11). The measured y [Eqs. (9.30)–(9.32)] was found to be independent of the mean-flow velocity U (for Mach number $M \leq 0.2$) and very weakly dependent on the expansion ratio D/d and lengths l_1 and l_3 for values of x in the range indicated by Eqs. (9.30)–(9.32). However, the expressions for y in these equations are least-squares fits and therefore are not limited to the indicated range; they cover the entire practical range ($0.2 < x < 6.0$).

The back-pressure coefficient y_{ec} for the expansion chamber mufflers of Section 9.3 is analogously given by⁸

$$y_{ec} = (1 - n)^2 + \frac{1}{2}(1 - n) \quad n = \left(\frac{d}{D}\right)^2 \quad (9.33)$$

Equations (9.30)–(9.32) show that the static pressure drop across a CTR, which involves no net flow through the perforated-tube walls, is much smaller than that across the corresponding plug muffler and the three-duct cross-flow chamber. Similarly, Equations (9.30) and (9.33) indicate that the pressure drop across a CTR is less than that across a simple expansion chamber of identical shell size and pipe diameter.

9.6 DISSIPATIVE SILENCERS

Dissipative silencers are the most widely used devices to attenuate the noise in ducts through which gas flows and in which the broadband sound attenuation must be achieved with a minimum of pressure drop across the silencer. They are frequently used in the intake and exhaust ducts of gas turbines, air conditioning and ventilation ducts connected to small and large industrial fans, cooling-tower installations, and the ventilation and access openings of acoustical enclosures, and they have an allowed pressure drop that typically ranges from 125 to 1500 Pa (0.5–6 in. H₂O). Unlike reactive silencers, which mostly reflect the incident sound wave toward the source, dissipative silencers attenuate sound by converting the acoustical energy propagating in the passages into heat caused by friction in the voids between the oscillating gas particles and the fibrous or porous sound-absorbing materials, as described in detail in Chapter 8.

The theories of dissipative silencers were developed long ago,^{1,2,71–74} and they are highly complex. This chapter provides design information in a form that can be readily used by engineers not thoroughly trained in acoustics. Though there are a large variety of geometries used, the most common configurations include parallel-baffle silencers, round silencers, and lined ducts.

Lined Ducts

The sound attenuation of lined and unlined ducts and lined and unlined bends are treated in Chapter 16. The geometry of frequently used dissipative silencers is shown schematically in Fig. 9.18.

Figure 9.19 shows some of the baffle constructions that have been used in typical applications. Only the acoustically significant features are shown. Protective treatments such as perforated facing, fiberglass cloth, and porous screens are omitted. If the same concepts depicted in Figs. 9.19a–i are applied as a duct lining, one-half of the baffles depicted in Fig. 9.19 constitutes the lining.

Figure 9.19 illustrates the cross sections of frequently used silencer baffle configurations.

The full-depth porous baffle depicted in Fig. 9.19a is the most frequently used in parallel-baffle silencers. The other baffle configurations are used in special-purpose silencers custom designed to yield high attenuation in specific

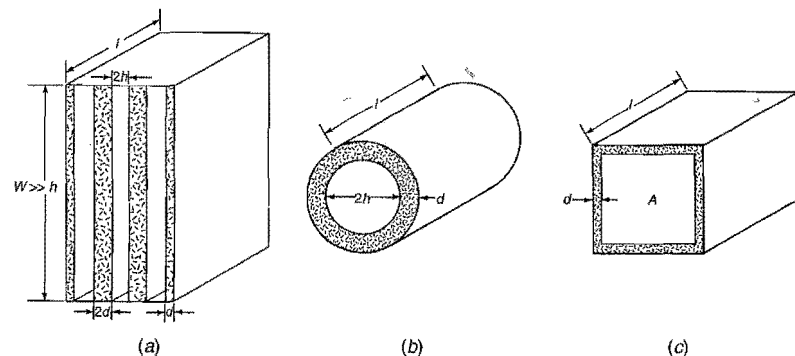


FIGURE 9.18 Geometry of frequently used silencer types: (a) parallel baffle; (b) round; (c) lined ducts.

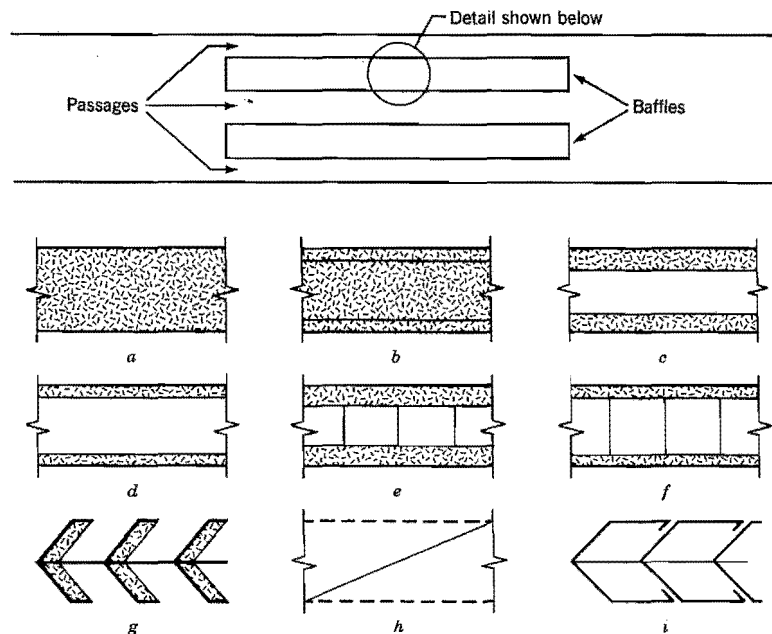


FIGURE 9.19 Cross section of frequently used silencer baffle configurations: (a) full-depth porous; (b) porous center layer with thin resistive facing on both sides; (c) thick porous surface layer with unpartitioned center air space; (d) thin porous surface layers with unpartitioned air space; (e) thick porous layer with partitioned center air space; (f) thin porous surface layers with partitioned center air space; (g) tuned cavity (Christmas tree), porous material in cavities protected from flow; (h) small percentage open-area perforated surface plates exposed to grazing flow, partitioned center air space; (i) Helmholtz resonators.

frequency ranges or to work in hostile environments of contaminated flow or high temperatures.

Key Performance Parameters

The key design parameters of silencers are acoustical insertion loss (IL), pressure drop (Δp), flow-generated noise, size cost, and life expectancy. The challenge of silencer design is to obtain the needed IL without exceeding the allowable pressure drop and size for a minimum of cost. These are frequently opposing requirements, and the optimal design represents a balance compromise between them.

Insertion Loss. The IL of a silencer is defined as

$$IL = 10 \log \frac{W_0}{W_M} \quad (9.34)$$

where W_0 and W_M represent the sound power in the duct without and with the silencer, respectively. Provided that structure-borne flanking along the muffler casing and sound radiation from the casing is kept low, the sound power in the duct with the silencer in place is given by

$$W_M = W_0 \times 10^{-(\Delta L_i + \Delta L_{ENT} + \Delta L_{EX})/10} + W_{SG} \quad (9.35)$$

where W is in watts (N·m/s); ΔL_i represents the attenuation of the silencer of length l ; ΔL_{ENT} and ΔL_{EX} are the entrance and exit losses, respectively; and W_{SG} is the sound power generated by the flow exiting the silencer.

Combining Eqs. (9.34) and (9.35) yields

$$IL = -10 \log \left(\frac{W_{SG}}{W_0} + 10^{-(\Delta L_i + \Delta L_{ENT} + \Delta L_{EX})/10} \right) \quad (9.36)$$

In the extreme case of very high silencer attenuation, the second term on the right-hand side of Eq. (9.36) becomes comparable to the first, and the achievable IL is affected by the self-generated noise of the silencer, and in this case Eq. (9.36) is nonlinear. When the flow velocity in the silencer passages is sufficiently low, flow noise is negligible. In this case, Eq. (9.36) is linear and simplifies to

$$IL \cong \Delta L_i + \Delta L_{ENT} + \Delta L_{EX} \quad \text{dB} \quad (9.37)$$

Entrance Loss ΔL_{ENT} . In most dissipative silencers, the entrance losses ΔL_{ENT} are small if the incident sound energy is in the form of a plane wave normally incident on the silencer entrance. This is always the case for straight ducts at low frequencies. The small entrance loss should be considered as a safety factor.

However, if the cross dimensions of the duct are much larger than the wavelength, the incident sound field usually contains a very large number of higher order modes. The conversion of the semidiffuse sound field in the entrance duct into a plane-wave field in the narrow silencer passages typically results in an entrance loss of 3–6 dB. The engineer can also assign any entrance loss between 0 dB at low frequencies and up to 8 dB at high frequencies on the basis of prior experience with similar situations or on the basis of scale model measurements. If no such information is available, Fig. 9.20 can be used to estimate the entrance loss.

Exit Loss ΔL_{EX} . Most exit losses ΔL_{EX} are generated when the silencer is located at the open end of a duct and the typical cross dimensions of the opening are small compared with the wavelength. In this case, the exit loss is predominantly determined by the end reflection. Exit losses for silencers inserted in ducts are usually small and can either be neglected or considered as part of the safety margin.

It should be noted that the relative importance of the entrance and exit losses diminishes as the silencer length increases because both quantities are independent of this length. Figure 9.21 shows qualitatively a typical sound pressure level versus distance recorded by a microphone traveling through the silencer and indicates the three components of IL.

Silencer Attenuation ΔL_l . The silencer attenuation ΔL_l is proportional to its length (tail and nose of baffle not included) and to the lined perimeter of the passage, P , and inversely proportional to the cross-sectional area of the passage,

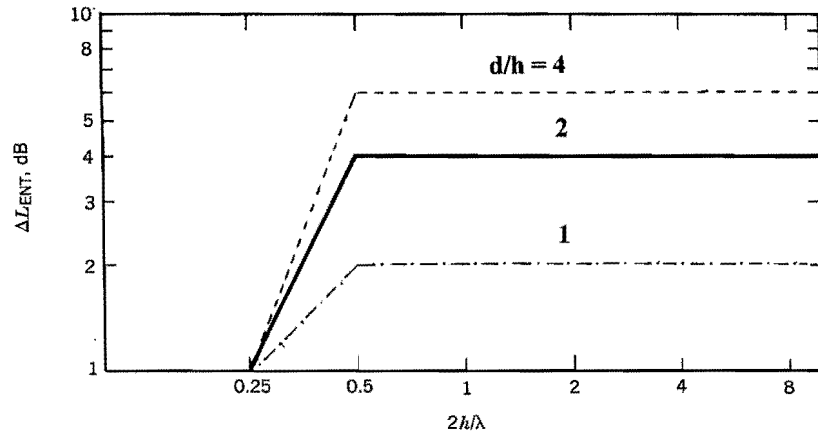


FIGURE 9.20 Acoustical entrance loss coefficient, ΔL_{ENT} , of silencers in a large duct with a semireverberant sound field in the entrance duct: $2h$ = silencer passage cross dimension.

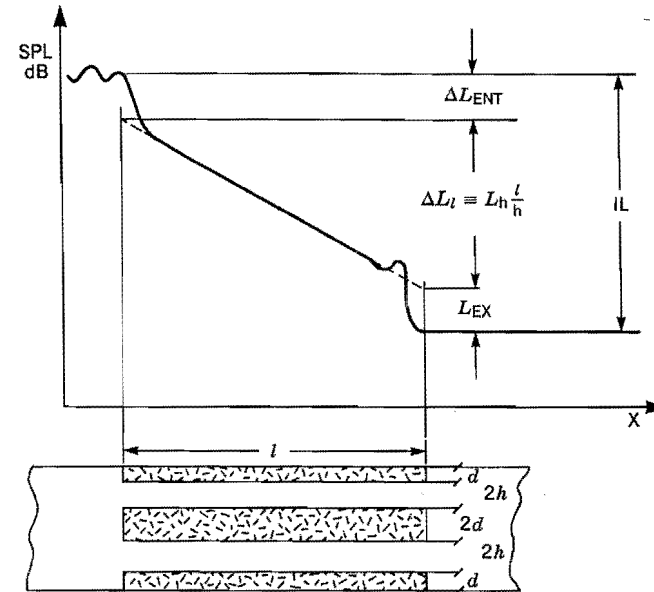


FIGURE 9.21 Typical SPL-versus-distance curve obtained when microphone traverses through a silencer.

A. It can be expressed as

$$\Delta L_l = \left(\frac{P}{A} \right) l L_h \quad (9.38)$$

where L_h is the parameter that depends in a complex manner on the geometry of the passage and the baffle, acoustical characteristics of the porous sound-absorbing material filling the baffles, frequency, and temperature. The quantity L_h , which also depends on the velocity of the flow in the passage, is usually referred to as the attenuation per channel height. The major part of this section is devoted to the determination of this important normalized sound attenuation parameter.

Pressure Drop Δp . The total pressure drop Δp_T across a muffler is made up of entrance, exit, and friction losses:

$$\Delta p_T = 1/2 \rho v_p^2 \left[K_{ENT} + K_{EX} + \left(\frac{P}{A} \right) l K_F \right] = \Delta p_{ENT} + \Delta p_{EX} + \Delta p_F \quad \text{N/m}^2 \quad (9.39)$$

where ρ is the density of the gas and v_p its face velocity in the passage of the silencer. The constants K_{ENT} and K_{EX} are the entrance and exit head loss coefficients, which depend only on the geometry of the baffle/passage configuration.

The third term on the right-hand side of Eq. (9.39) represents the friction losses, given by

$$\Delta p_F = \frac{P}{A} l (K_F \frac{1}{2} \rho v_p^2) \quad \text{N/m}^2 \quad (9.40)$$

Comparing Eq. (9.38) with Eq. (9.40), one notes that baffle configurations that tend to yield high acoustical attenuation ΔL_i also tend to yield high frictional pressure losses Δp_F . Both ΔL_i and Δp_F are proportional to $(P/A)l$, indicating that high silencer attenuation and low frictional pressure drop are contradictory requirements. This finding emphasizes the need to optimize L_h by beneficial choice of the acoustical parameters of the baffles before one resorts to obtaining increased sound attenuation by increasing the factor $(P/A)l$.

Parallel-Baffle Silencers

The parallel-baffle-type silencer shown in Fig. 9.18a is the most frequently used because of its good acoustical performance and low cost. The attenuation of such a silencer is proportional to the perimeter-area ratio P/A , the length l , and L_h . Therefore, it is maximized by maximizing $(P/A)L_h$. The largest perimeter-area ratio obtained for narrow passages is $1/h$. Allowing for entrance losses, Eq. (9.38) yields the following simple formula for silencer attenuation:

$$\Delta L_i = L_h \frac{l}{h} + \Delta L_{\text{ENT}} \quad \text{dB} \quad (9.41)$$

where ΔL_{ENT} can be approximated from Fig. 9.20. The following discussion shows how to obtain L_h from the geometric and acoustical parameters of the silencer.

Prediction of Attenuation. The sound energy traveling in the passages of a parallel-baffle silencer, depicted in Fig. 9.19, is attenuated effectively in a wide bandwidth if (1) the sound enters the porous sound-absorbing material in the baffles and (2) a substantial part of the energy of the sound wave entering the baffle is dissipated before it can reenter the passage. Formulas for wall impedance that yield maximum attenuation in a narrow-frequency band are given in reference 2.

Requirement 1 is fulfilled if the passage height is small compared with wavelength (i.e., $2h < \lambda$) and the porous sound-absorbing material is open enough and has sufficiently low flow resistivity so that the sound wave enters the baffle rather than being reflected at the interface. This requires a "fluffy" material of low flow resistivity. Requirement 2 is fulfilled by a porous material of moderate flow resistivity. The requirements of easy sound penetration and high dissipation are contradictory unless the baffle is very thick and is packed with porous sound-absorbing material of low flow resistivity. Consequently, the choice of baffle thickness and flow resistivity of the porous sound-absorbing material is always a compromise.

Generally, the silencer geometry is controlled by the shape of the attenuation-versus-frequency curve we aim to achieve. To provide reasonable attenuation at the low end of the frequency spectrum, the baffle thickness $2d$ must be on the order of one-eighth of the wavelength. To provide reasonable attenuation at the high end of the frequency spectrum, the passage height $2h$ must be not much larger than the wavelength. To allow reasonable penetration of the sound and yield the needed dissipation, the total flow resistance $R_1 d$ of a baffle of thickness $2d$ must be 2–6 times the characteristic impedance of the gas in the silencer passages at design temperature.

Quantitative Considerations. The normalized attenuation constant L_h is obtained by solving the coupled wave equation in the passage and in the porous material of the baffle and requiring that (1) the coupled wave, which propagates axially in both the passage and baffle, has a common propagation constant Γ_c and (2) both particle velocity and the sound pressure are continuous at the passage baffle interface.

The coupled wave equation^{1,71–74} can be solved by numerical iteration methods to yield the common propagation constant Γ_c . The normalized attenuation L_h is then obtained from

$$L_h = 8.68h \operatorname{Re}\{\Gamma_c\} \quad \text{dB/m} \quad (9.42)$$

where Γ_c depends on the characteristic impedance ρc of the gas in the passage, the characteristic impedance Z_a and the propagation constant Γ_c of the porous material in the baffle, and the geometry.

The characteristic impedance Z_a and the propagation constant Γ_c of the porous sound-absorbing materials (which are complex quantities and vary with frequency) are generally not available. As discussed in Chapter 8, one can approximate these important parameters with reasonable accuracy if the flow resistivity of the porous sound-absorbing material R_1 is known. For fibrous sound-absorbing materials the characteristic impedance Z_a and propagation constant Γ_a in the bulk porous material can be estimated using the empirical formulas presented in Chapter 8.

The most accurate characterization of the porous materials is achieved by measuring the characteristic impedance and propagation constant on a sufficiently large number of samples as a function of frequency at room temperature and by scaling these data to design temperature. Note that this attenuation, which is computed on the basis of the acoustical parameters of the porous material predicted from flow resistivity by employing the formulas given in Chapter 8, compares very well with experimental data if the porous sound-absorbing material is homogeneous.

Normalized Graphs to Predict Acoustical Performance of Parallel-Baffle Silencers. The normalized attenuation L_h has been computed for various percentages of open area of the silencer cross section (i.e., for various h/d) and for various values of the normalized flow resistance $R = R_1 d / \rho c$ of isotropic porous sound-absorbing material in the baffles. It is presented in Figs. 9.22a–c

for open-area ratios of 66, 50, and 33%, respectively. The effect of nonisotropic material is covered in reference 72. The vertical axis in Fig 9.22 represents the normalized attenuation L_h in decibels on a logarithmic scale. The lower horizontal scale represents the normalized frequency parameter $\eta = 2h/c$. It is valid for all temperatures and gases provided that the speed of sound c is taken at the actual temperature. The upper horizontal scale, which is valid only for air at room temperature, represents the product of the half-passage height h in centimeters and the frequency f in kilohertz.

Figures 9.22a-c show that the attenuation starts to decrease rapidly above the frequency where the passage height $2h$ becomes large compared to the wavelength (i.e., $\eta > 1$). The attenuation in this frequency region can be increased by up to 10 dB by utilizing a two-stage silencer with staggered-baffle arrangement. Note that the bandwidth of appreciable attenuation increases with decreasing percentage of open area of the silencer cross section.

Figure 9.22 shows that in the range of $R = R_1 d / \rho c$ from 1 to 5, the attenuation does not depend strongly on the specific choice of the flow resistance; this coincidence is welcome because the present lack of adequate knowledge of and control over the material characteristics represents the weakest link in the prediction process. Note that if the normalized flow resistance becomes too large

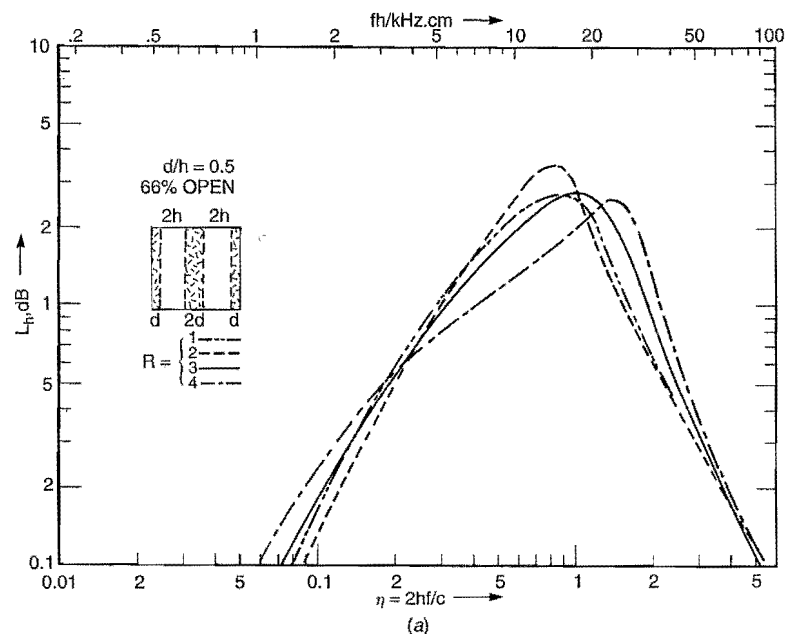


FIGURE 9.22 Normalized attenuation-versus-frequency curves for parallel-baffle silencers with normalized baffle flow resistance $R = R_1 d / \rho c$ as parameter: (a) 66% open; (b) 50% open; (c) 33% open.

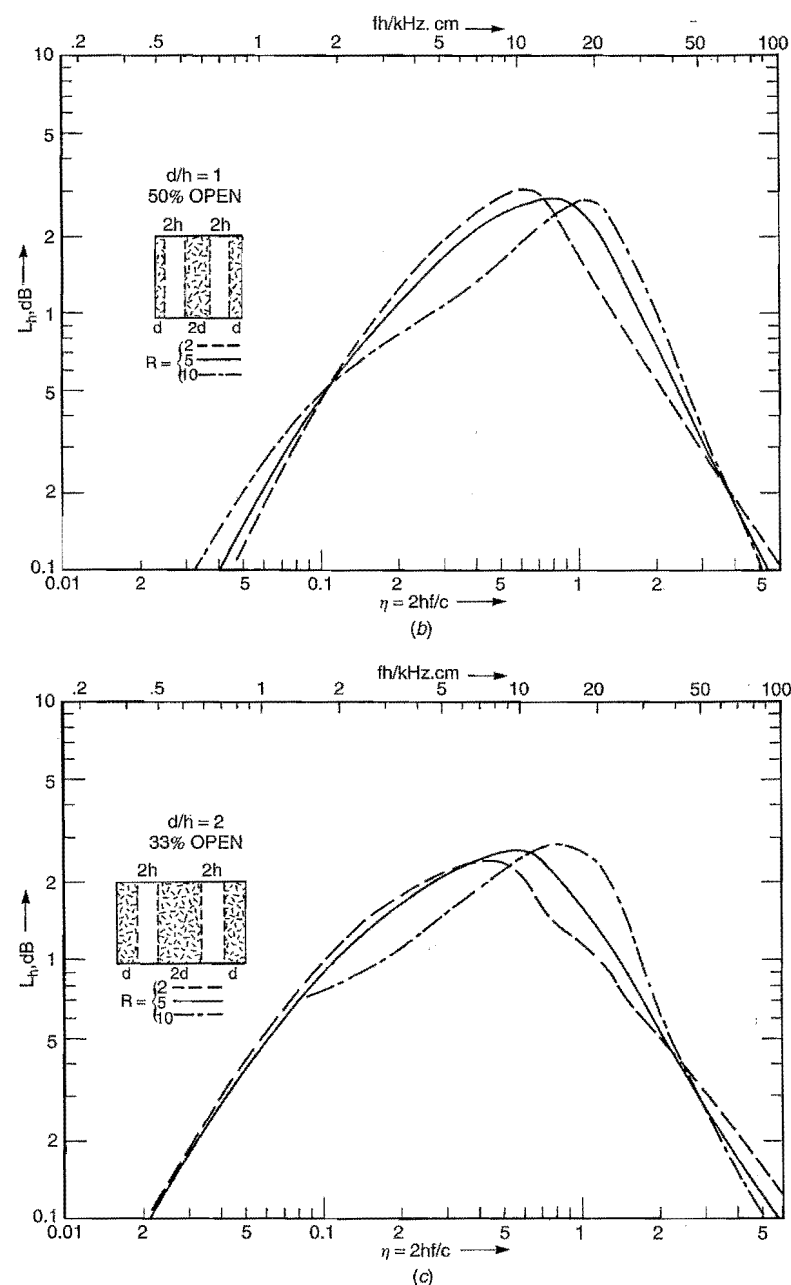


FIGURE 9.22 (continued)

($R \geq 10$), a substantial decrease of attenuation occurs in the frequency region from $\eta = 0.2$ to $\eta = 1$ accompanied by a modest increase of attenuation at very low and at high frequencies.

The normalized attenuation-versus-frequency curves presented in Figs. 9.22a–c correspond to zero flow; corrections to account for flow are presented in a later section.

The use of the design information presented in Figs. 9.22a–c is illustrated by a few examples.

Example 9.5. Predict the attenuation-versus-frequency curve of a parallel-baffle silencer consisting of 200-mm- (8-in.-) thick parallel baffles 1 m (40 in.) long spaced 400 mm (16 in.) off center when the flow resistance of the baffle $R_1 d = 5\rho c$ and the duct carries a very low velocity air at 20°C; $h = 0.1$ m; $d = 0.1$ m; $L = 1$ m; $c = 340$ m/s; $\rho = 1.2$ kg/m³; $R = R_1 d / \rho c = 5$.

Solution

1. Determine frequency f^* , which corresponds to $\eta = 1$:

$$f^* = \frac{c}{2h} = \frac{340 \text{ m/s}}{0.2 \text{ m}} = 1700 \text{ Hz}$$

2. Determine $1/h = 1 \text{ m}/0.1 \text{ m} = 10$.
3. Determine the applicable normalized attenuation-versus-frequency curve for $d/h = 1$ and $R = 5$; the solid curve in Fig. 9.22b is applicable.
4. Mark the frequency $f^* = 1700$ Hz on the horizontal scale of a sheet of transparent graph paper that has the same horizontal and vertical scales as Fig. 9.22b and align it with $\eta = 1$ in Fig. 9.22b.
5. Shift the transparent graph paper vertically until the mark $L_h = 1$ in Fig. 9.22b corresponds to 10 ($1/h = 10$) on the transparent overlay.
6. Copy the solid curve in Fig. 9.22b that corresponds to $R = 5$ on the overlay.
7. The copied curve then corresponds to the attenuation-versus-frequency curve of the silencer according to $\Delta L_t = L_h(1/h)$; the above procedure is sketched in Fig. 9.23.

Example 9.6. Design a parallel-baffle silencer that yields the attenuation listed below:

f , Hz	100	200	500	1000	2000	4000
ΔL , dB	4	9	19	26	10	5

Design Steps

1. Find the graph in Figs. 9.22a–c that best matches the shape of the desired attenuation-versus-frequency curve plotted in Fig. 9.23. Overlay the transparent paper of Fig. 9.23 on the curve that yields the best match and shift the transparent overlay horizontally and vertically until all of the

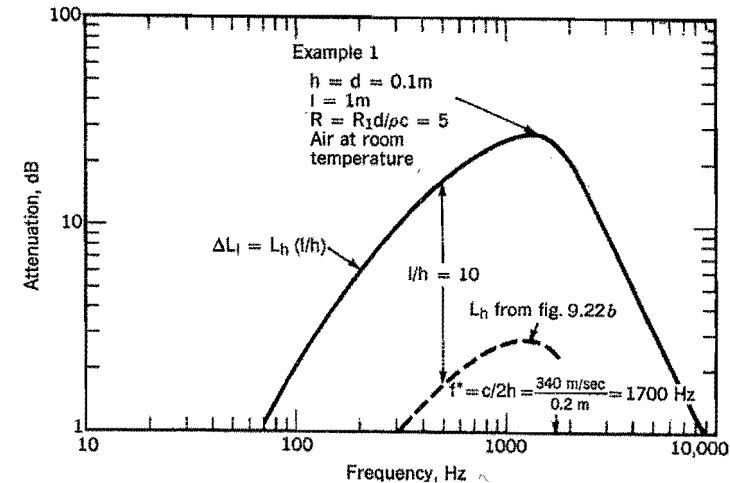


FIGURE 9.23 Attenuation prediction for parallel-baffle silencer, Example 9.5.

desired attenuation-versus-frequency points fall below the chosen normalized attenuation-versus-frequency curve. In this case, the solid curve in Fig. 9.22c gives the best match.

2. On the overlay, mark the frequency f^* that corresponds to $\eta = 1$ on the horizontal scale of the appropriate design curve under the overlay, and on the vertical scale, mark the attenuation ΔL^* , which corresponds to $L_h = 1$ on the design curve below the overlay, as shown in Fig. 9.24. In this case, these will be $f^* = 2000$ Hz and $\Delta L^* = 10$ dB.
3. From the design curve below the overlay, note the values of the parameters d/h and R that correspond to the curve that provides the best match. In this case, these are $d/h = 2$ and $R = R_1 d / \rho c = 5$.
4. On the basis of the information obtained in steps 3 and 4, one obtains the geometric and acoustical parameters of the silencer that will yield the specified attenuation as follows:

- Passage height $2h$:

$$\eta = 1 = \frac{2hf^*}{c} \quad \text{yields} \quad 2h = \frac{340 \text{ m/s}}{2000 \text{ s}^{-1}} = 0.17 \text{ m}$$

- Baffle thickness $2d$:

$$2d = 2(2h) = 2 \times 0.17 \text{ m} = 0.34 \text{ m}$$

- Silencer length:

$$\Delta L^* = 10 = \frac{l}{h} \quad \text{yields} \quad l = \Delta L^* h = 10 \times \frac{0.17}{2} \text{ m} = 0.85 \text{ m}$$

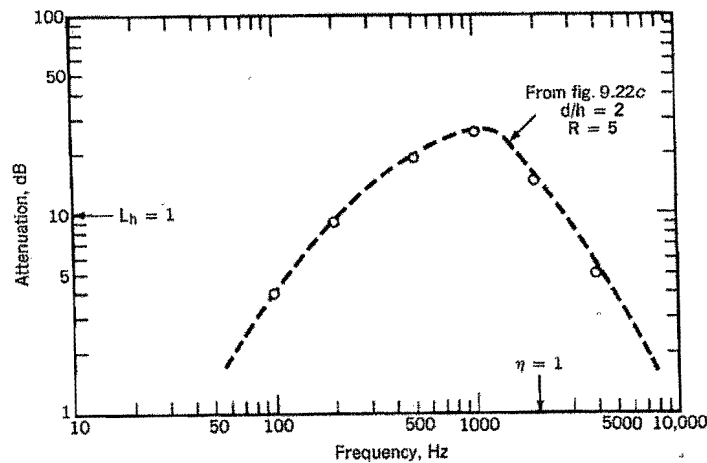


FIGURE 9.24 Acoustical design of a parallel-baffle silencer, Example 9.6: (—) design requirements; (---) curve of matching attenuation versus frequency from Fig. 9.22c.

- Flow resistance per unit thickness of porous material:

$$\frac{R_1 d}{\rho c} = 5 \quad \text{yields} \quad R_1 = \frac{5 \rho c}{d} = \frac{5}{0.17 \text{ m}} \rho c = 29.4 \rho c / \text{m}$$

or

$$R_1 = 0.7 \rho c / \text{in.} \quad \text{or} \quad R_1 = 1.2 \times 10^4 \text{ N} \cdot \text{s} / \text{m}^4$$

Cross-Sectional Area. The cross-sectional area of the muffler is determined by the maximum allowable pressure drop and self-generated noise as discussed later.

Effect of Temperature. The design temperature affects the acoustical and aerodynamic performance of the silencers because the following key parameters depend on the temperature: (1) speed of sound, (2) density of the gas, and (3) viscosity. The effects of temperature are taken into account as follows:

$$c(T) = c_0 \sqrt{\frac{273 + T}{293}} \quad \text{m/s} \quad (9.43)$$

$$\rho(T) = \rho_0 \frac{293}{273 + T} \quad \text{kg/m}^3 \quad (9.44)$$

$$R(T) = R_0 \left(\frac{273 + T}{293} \right)^{1/2} \quad (9.45)$$

where T is the design temperature in degrees Celsius, c_0 is the speed of sound, ρ_0 is the density of the gas (usually air) at 20°C , and

$$R_0 = \frac{R_1(20^\circ\text{C})d}{\rho_0 c_0} \quad (9.46)$$

where $R_1(20^\circ\text{C})$ is the flow resistivity of the porous bulk material at 20°C . This material parameter is usually provided by the manufacturer or is measured.

Example 9.7. To illustrate how to account for the effect of temperature, let us predict the attenuation provided by the silencer of Example 9.5 at $T = 260^\circ\text{C}$ (500°F).

Solution The effect of temperature must be accounted for in the flow resistivity and in the speed of sound according to Eqs. (9.45) and (9.43), yielding $c(T) = 457 \text{ m/s}$ and $R(T) = 10$. From now on, the solution proceeds according to the same steps followed in Example 9.5.

1. $f^* = c(T)/2h = (457 \text{ m/s})/0.2 \text{ m} = 2285 \text{ Hz}$; $l/h = (1 \text{ m}/0.1 \text{ m}) = 10$.
2. The applicable normalized attenuation curve that corresponds to $d/h = 1$ and $R = 10$ is the short/long-dashed curve in Fig. 9.22b.

This results in the attenuation-versus-frequency curve shown as the solid line in Fig. 9.25. The dashed curve in Fig. 9.25 is the attenuation of the same silencer at room temperature (20°C), as determined in Example 9.5.

Comparing the solid curve obtained for 260°C with the dashed curve obtained for 20°C , one notes a shift in the attenuation-versus-frequency curve toward higher frequencies with increasing temperature. This shift is mainly due to the increase of propagation speed of sound with increasing temperature. In addition, one also observes distortion in the shape of the attenuation-versus-frequency curve. This is caused by the increase in the flow resistivity of the porous material,

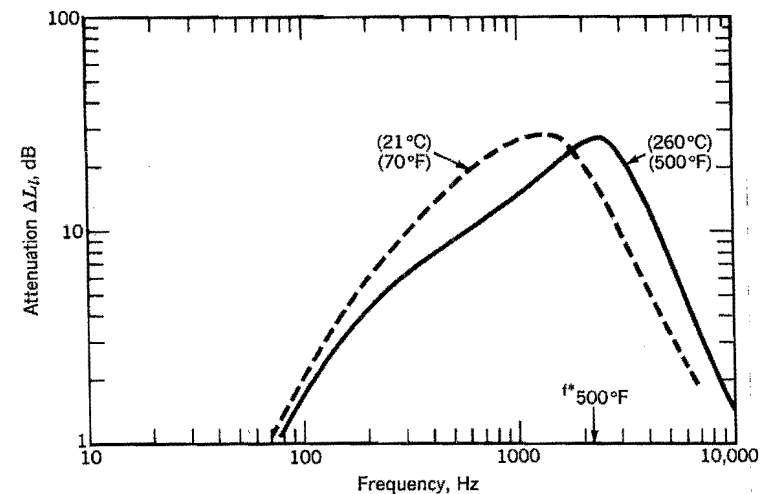


FIGURE 9.25 Attenuation of silencer of Example 9.5 at 260°C (500°F), Example 9.7.

which in turn is due to the increasing viscosity with increasing temperature. This effect has also been predicted by Mechel.⁷⁵

Baffle Thickness Considerations. A particular percentage of open area of a silencer can be accomplished either by a small number of thick baffles or by a large number of thin baffles. Figure 9.26 shows the attenuation-versus-frequency curves computed for a 2-m- (6.5-ft-) long silencer of 50% open area with baffle thickness $2d = 2h$ of 152 mm (5 in.), 203 mm (8 in.), 254 mm (10 in.), and 305 mm (12 in.). The baffles are filled with a fibrous sound-absorbing material that has a flow resistivity $R_1 = 51,500 \text{ N} \cdot \text{s/m}^4$ at 500°C ($1\rho\text{c/in.}$ at 20°C). The outstanding feature of the data presented in Fig. 9.26 is that in the midfrequency region, the sound attenuation decreases with increasing baffle thickness. At 500 Hz, the 152-mm- (6-in.-) thick baffles yield 25 dB attenuation while the 305-mm- (12-in.-) thick baffles yield only 11 dB. This is because the sound does not fully penetrate into the fibrous sound-absorbing material in the thick baffles. Consequently, the material and the space in the center of the thick baffles are wasted.

Figure 9.27 shows the predicted sound attenuation of a silencer of the same geometry as that in Fig. 9.26, but the baffles in this case are filled with fibrous sound-absorbing material of low flow resistivity such that the total normalized flow resistance of each baffle was $R = R_1 d / \rho c = 2$, which allows full penetration of the sound into even the thickest baffle. Consequently, the sound attenuation at

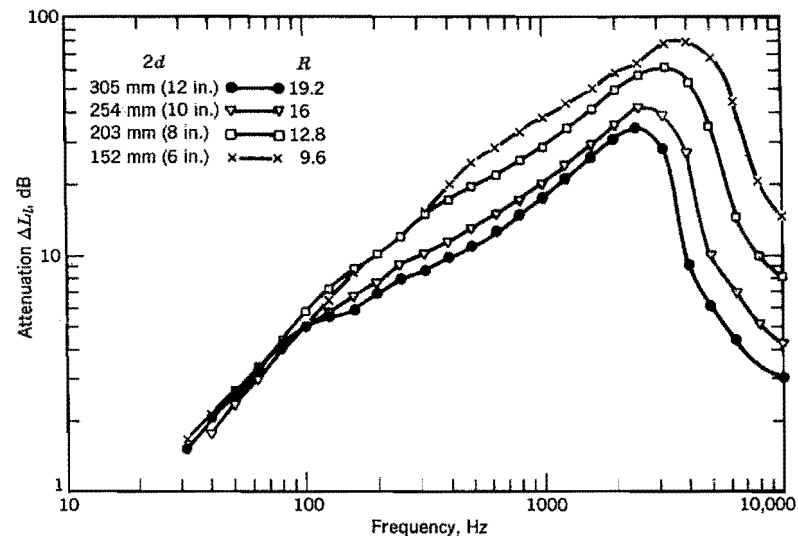


FIGURE 9.26 Computed attenuation-versus-frequency curves for a 2-m- (6.6-ft-) long parallel-baffle silencer of 50% open area with baffle thickness $2d$ as parameter.

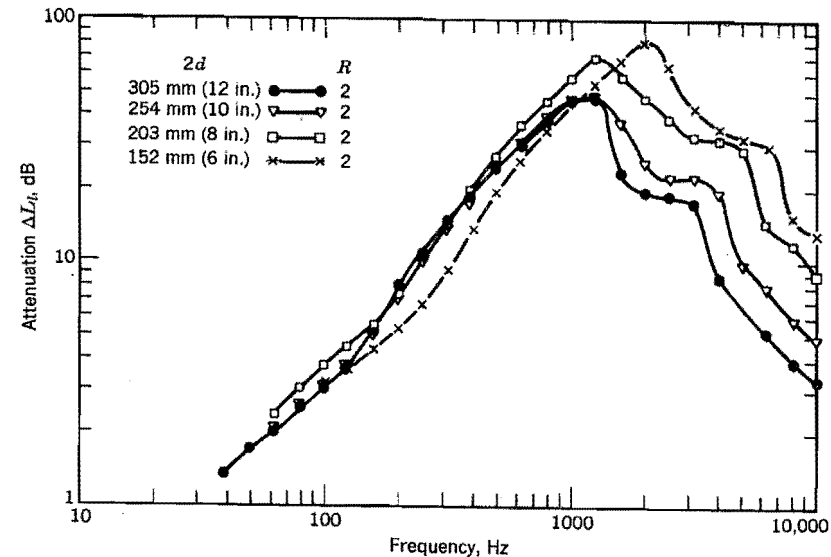


FIGURE 9.27 Computed attenuation-versus-frequency curves of the same silencer as Fig. 9.26 but fibrous fill is chosen to yield $R = R_1 d / \rho c = 2$ for all baffle thicknesses.

low and midfrequencies depends only slightly on the baffle thickness. Comparing Figs. 9.26 and 9.27 reveals that using a few thick baffles (which is more economical than using many thinner baffles) results in decreased sound attenuation at midfrequencies unless the baffles are filled with a sound-absorbing material of low enough resistivity so that the normalized baffle flow resistance at design temperature, $R = R_1 d / \rho c$, is much less than 10. Fibrous sound-absorbing materials that fulfill this requirement for thick baffles used in elevated temperatures may not be readily available. Consequently, silencer baffles of traditional design in high-temperature applications should be kept at a thickness that seldom exceeds $2d = 200 \text{ mm}$ (8 in.).

Note that the attenuation-versus-frequency curve of parallel-baffle silencers increases monotonically up to a frequency $f = c/2d$, where the passage width corresponds to a wavelength, and decreases sharply above it.

Round Silencers

Round silencers, as depicted in Fig. 9.18b, are used in connection with round ducts. The curvature of the duct casing results in a high form stiffness that yields high sound transmission loss of the silencer wall at low frequencies. The acoustical performance of round silencers with respect to normalized attenuation L_R is very similar to that of parallel-baffle silencers. The diameter of the round passage, $2h$, and the thickness of the homogeneous isotropic lining, d , resemble

the passage width $2h$ and the half-baffle thickness d of a parallel-baffle silencer. The silencer attenuation $\Delta L'(l)$ of a round muffler of length l is obtained from

$$\Delta L'(l) = L_h^r \frac{l}{h} \quad (9.47)$$

The sound attenuation in round ducts has been studied by Scott⁷¹ and Mechel.⁷⁴ Their work forms the theoretical foundations for computing their performance.

Figure 9.28 shows curves of normalized attenuation L_h^r versus normalized frequency $\eta = 2hf/c$ for round silencers with homogeneous isotropic lining for thickness–passage radius ratios d/h of 0.5, 1, and 2, respectively, with the normalized lining flow resistance $R = R_1 d / \rho c$ as parameter. The lower horizontal scale is valid for all temperatures, while the upper scale is valid only for air at room temperature. Figure 9.28 shows that the normalized attenuation increases monotonically with frequency until the wavelength corresponds to the diameter of the passage ($\eta = 2hf/c = 2h/\lambda = 1$). Above this frequency the attenuation decreases rapidly with increasing frequency, as was the case for parallel-baffle silencers. Note that the maximum normalized attenuation is about $L_{h,\max}^r \approx 6$ dB

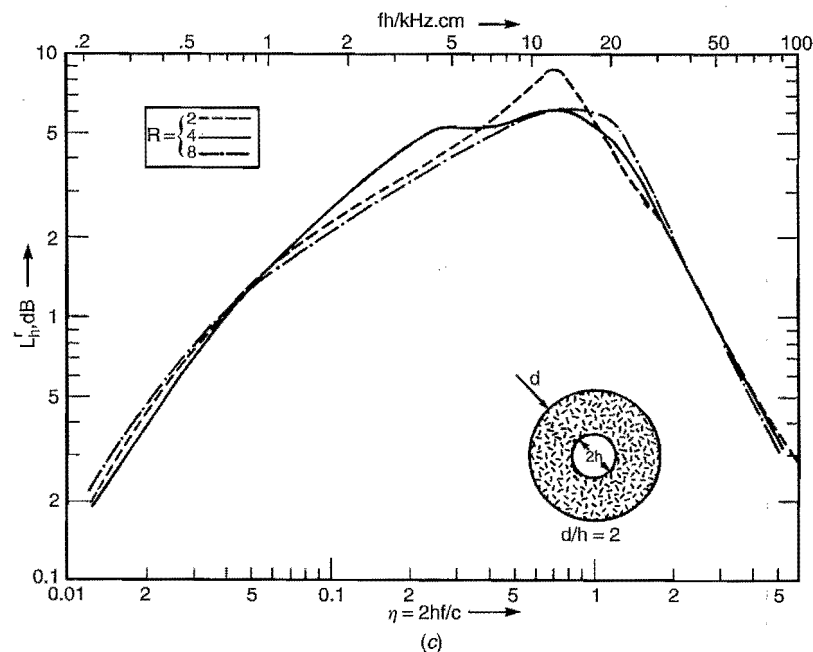
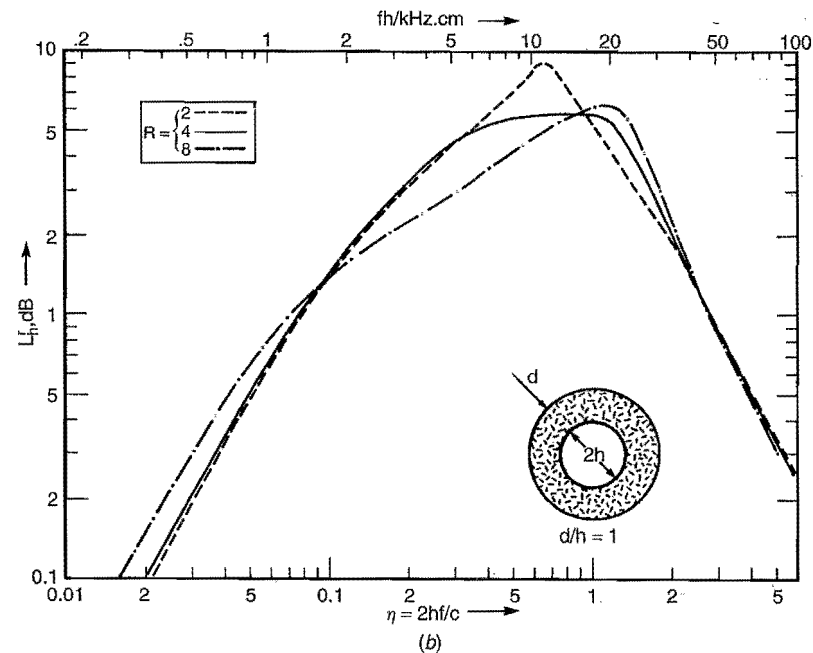
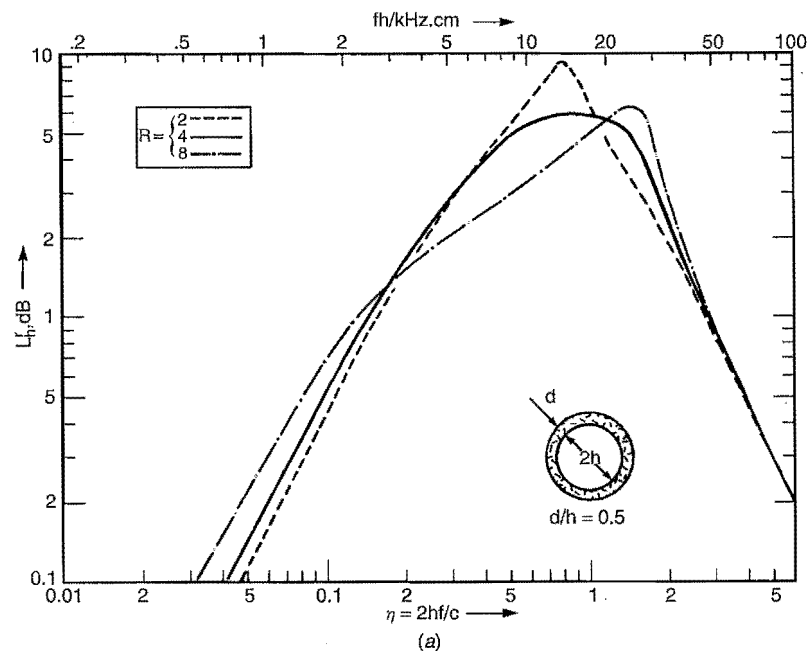


FIGURE 9.28 Curves of normalized attenuation L_h^r versus frequency for round silencers with normalized lining flow resistance $R = R_1 d / \rho c$ as parameter: (a) $d/h = 0.5$; (b) $d/h = 1$; (c) $d/h = 2$.

FIGURE 9.28 (continued)

for round silencers compared to $L_{h,\max}^r \approx 3$ dB for parallel-baffle silencers. This is because a round passage has twice as high perimeter-area ratio ($2/h$) than a narrow passage of a parallel-baffle silencer ($1/h$). As expected, the attenuation bandwidth increases toward low frequencies with increasing lining thickness d . The curves presented in Fig. 9.28 are used in the same way as the corresponding curves for parallel-baffle silencers by applying the design steps listed earlier in the Examples 9.5 and 9.7. Incidentally, similar curves and trends have been predicted by Mechel.⁷⁵

Pod Silencers

Round silencers have a generic disadvantage of providing poor high-frequency performance when the passage diameter is large compared with the wavelength. This disadvantage can be overcome by inserting a center body or pod into the passage⁷⁶. With the center body in place, the round silencer has a narrow annular passage, just like a parallel-baffle silencer, and the attenuation will continue to increase monotonically until the wavelength of the sound becomes equal to the width of the narrow annular passage, as illustrated in the example of Fig. 9.29. Both the rigid and absorbing center bodies result in a modest increase in attenuation at low and midfrequencies, which is mostly due to reduction of the passage cross-sectional area. The attenuation of this silencer without center body

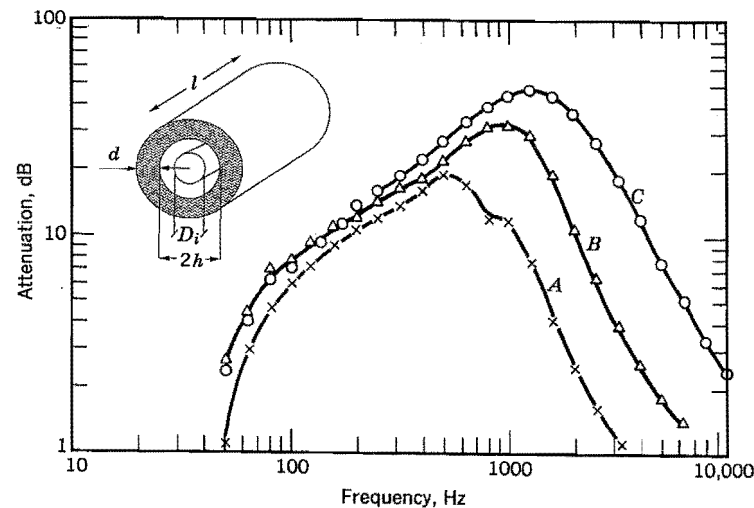


FIGURE 9.29 Effect of center body on the attenuation-versus-frequency curve of a round silencer, passage diameter $2h = 0.6$ m (24 in.), center body diameter $D_i = 0.3$ m (12 in.), lining thickness $d = 0.2$ m (8 in.), length $l = 1.2$ m (48 in.), $T = 20^\circ\text{C}$ (68°F), $R_1 = 16,000 \text{ N} \cdot \text{s/m}^4$ ($1\rho c/\text{in.}$): A, no center body; B, rigid center body; C, absorbing center body.

decreases sharply above 560 Hz, the frequency where the wavelength equals the diameter of the passage. With the rigid center body the attenuation continues to increase up to 1130 Hz and with the porous center body up to 2260 Hz.

If center bodies are impractical, the high-frequency attenuation of round silencers can also be increased by inserting parallel baffles into the round passage. Similarly, the high-frequency attenuation of silencers with rectangular cross section can be increased by inserting parallel baffles (oriented perpendicular to the plane of the thick, low-frequency liner, or side branches on the sidewalls) into the rectangular passage. As described in detail by Kurze and Ver⁷⁷, the inserted parallel baffles also increase the low-frequency attenuation by a beneficial interaction with the low-frequency part of the silencer located at the walls. The mechanism of this interaction is that the reduction of the wave speed owing to the structure factor of the parallel baffles increases the attenuation performance of the low-frequency liner on the sidewalls. Similarly, the structure factor of the low-frequency liner on the sidewalls increases the attenuation of the parallel baffles. Consequently, the beneficial interaction results in increased sound attenuation at both low and high frequencies. This type of insert silencer also has the benefit of requiring substantially less total silencer length than the traditional series combination of a low- and high-frequency silencer section.

Effect of Flow on Silencer Attenuation

The flow affects the attenuation of sound in silencers in three ways:

1. It changes slightly the effective propagation speed of the sound.
2. By creating a velocity gradient near the passage boundaries, it refracts the sound propagating in the passage toward the lining if the propagation is in the flow direction (exhaust silencers) and "focuses" the sound toward the middle of the passage if the propagation is opposite to the flow direction (intake silencers).
3. It increases the effective flow resistance of the baffle.

Figure 9.30a shows the effect of flow on the attenuation performance of an exhaust silencer when the flow direction coincides with the direction of sound propagation. Note that the attenuation is decreased at low frequencies and is increased slightly at high frequencies. In most cases, a Mach number $M > 0.1$ is not permissible because of material deterioration or self-noise.

Figure 9.30b shows what happens when the sound propagates against the flow. In this case the attenuation at low frequencies increases because it takes the sound a longer time to traverse through the muffler passage. The high-frequency attenuation decreases because the velocity gradients in the passage "channel" the sound toward the center of the passage. The effect of flow on attenuation can be approximated by appropriate shift of the attenuation-versus-frequency curve obtained without flow ($M = 0$).

The attenuation in the presence of flow is obtained by shifting the no-flow ($M = 0$) attenuation curve as illustrated in Fig. 9.31a when the direction of sound

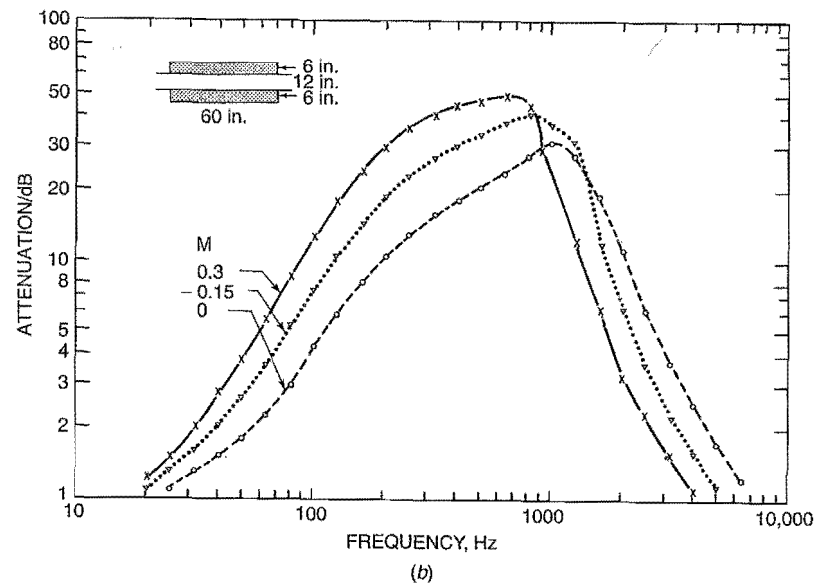
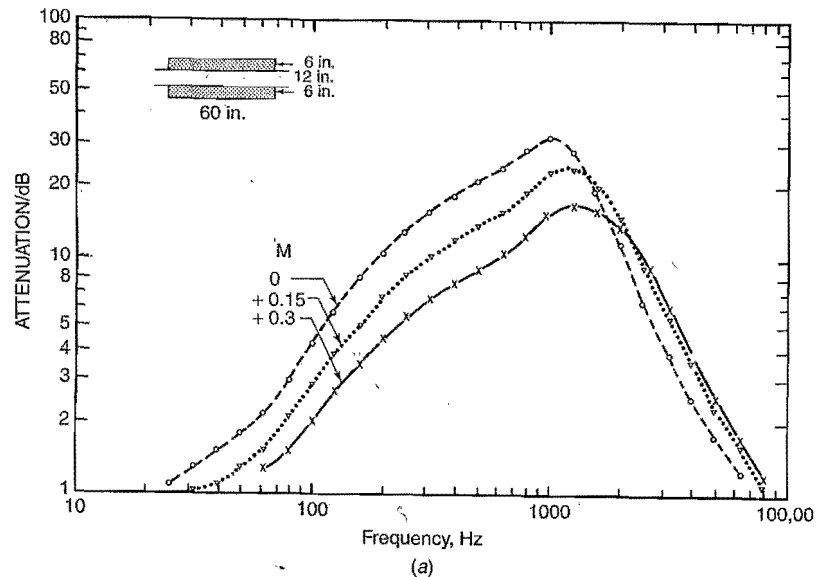


FIGURE 9.30 Effect of flow on sound attenuation for Mach numbers $M = 0, 0.15, 0.3$: sound propagation (a) in flow direction and (b) against flow direction.

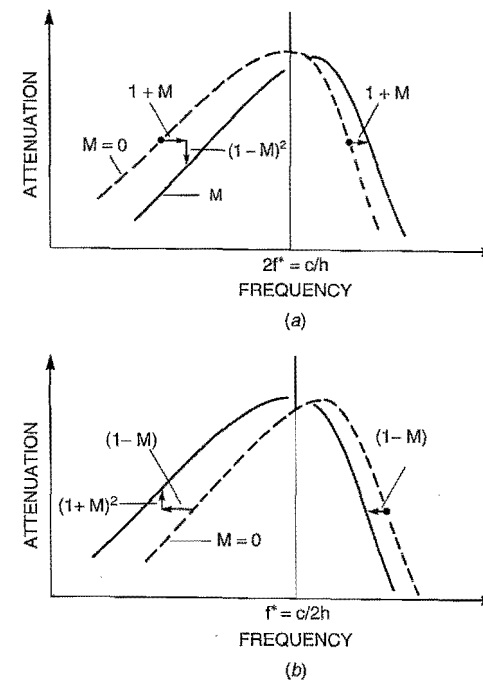


FIGURE 9.31 Rules for shifting the no-flow ($M = 0$) attenuation-versus-frequency curve to account for flow effects: sound propagation (a) in flow direction and (b) against flow direction.

propagation and the gas flow are the same and in Fig. 9.31b when they are opposite.

Figure 9.32 shows the attenuation-versus-frequency curve of a silencer with $M = 0.15$ flow in the direction of sound propagation. The values represented by the crosses are those directly computed for $M = +0.15$ (dotted line in Fig. 9.30a) and those represented by the open circles were obtained by shifting the $M = 0$ curve in Fig. 9.30a according to the guidelines discussed in conjunction with Fig. 9.31. The agreement between the curve obtained by shifting and by computations (which takes into account flow effects in the wave equation) is good.

The empirical flow correction procedure illustrated in Fig. 9.31 is based on experience with parallel-baffle silencers. We have no experience at present to gauge its applicability and accuracy for other silencer geometries.

Flow-Generated Noise

At present there is no universally accepted method for predicting the flow-generated noise of silencers. Information provided by silencer manufacturers shall be used wherever available. The empirical predictive scheme presented here

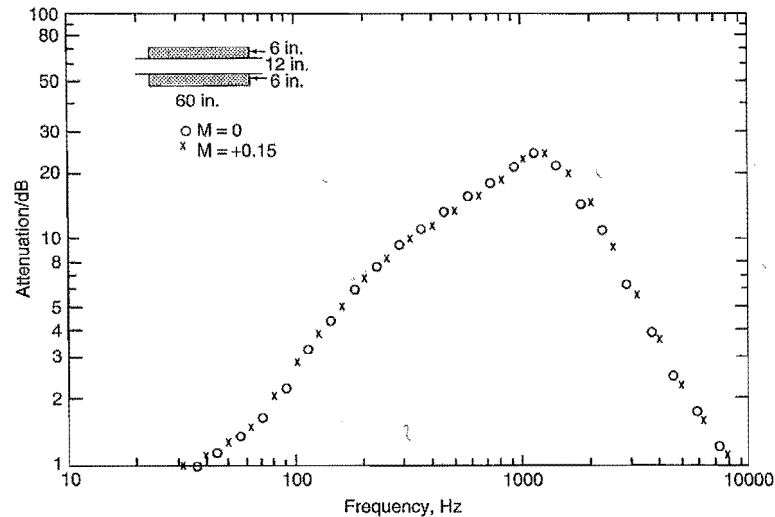


FIGURE 9.32 Attenuation-versus-frequency curve of a silencer for sound propagation in flow direction, $M = +0.15$: (x) computed; (o) obtained by shifting the $M = 0$ curve according to Fig. 9.31a.

is based on a broad range of experimental data on flow-generated noise of duct silencers and is reproduced from ISO 14163:1998(E).⁷⁸

An estimate for the octave-band sound power level of regenerated sound can be obtained from Eq. (9.48):

$$L_{W,oct} = B + \left\{ 10 \lg \frac{PcSn}{W_0} + 60 \lg Ma + 10 \lg \left[1 + \left(\frac{c}{2fH} \right)^2 \right] - 10 \lg \left[1 + \left(\frac{f\delta}{v} \right)^2 \right] \right\} \quad (9.48)$$

where B = a value depending on type of silencer and frequency, dB

v = flow velocity in narrowest cross section of silencer, m/s

c = speed of sound in medium, m/s

Ma = Mach number ($Ma = v/c$)

P = static pressure in duct, Pa

S = area of narrowest cross section of passage, m²

n = number of passages

f = octave-band center frequency, Hz

H = maximum dimension of duct perpendicular to baffles, m

δ = length scale characterizing high-frequency spectral content of regenerated noise, m

$W_0 = 1 \text{ W} = 1 \text{ N} \cdot \text{m/s}$

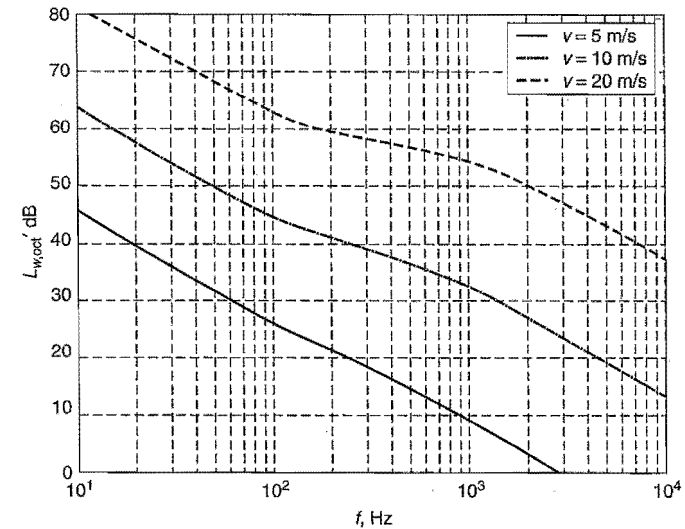


FIGURE 9.33 Octave-band sound power level $L_{W,oct}$ of regenerated sound versus frequency f for air under ambient conditions in a ducted silencer with narrowest cross section $S = 0.5 \text{ m}^2$, maximum transverse dimension $H = 1 \text{ m}$, and different flow velocities v .

The sound power level of regenerated sound will vary with the temperature T approximately with $-25 \lg(T/T_0)$ decibels. For smooth-walled dissipative splitter silencers used in heating, ventilation, and air-conditioning equipment, an approximation is given by $B = 58$ and $\delta = 0.02 \text{ m}$. For this case, a graph of Eq. (9.48) is shown in Fig. 9.33, and the A-weighted sound power level of a duct cross section of 1 m^2 is then calculated from

$$L_{WA} = \left(-23 + 67 \lg \left[\frac{v}{v_0} \right] \right) \text{ dB} \quad (9.49)$$

where $v_0 = 1 \text{ m/s}$. For other types of silencers, particularly for resonator silencers, B may be larger in certain frequency bands. However, no general information can be given on the values of B and δ .

Prediction of Silencer Pressure Drop

The maximum permissible pressure drop at design flow rates and at design temperatures, together with the flow-generated noise, determines the cross-sectional dimensions of a silencer. It is important that the silencer designer makes good use of all the available pressure drop, though with an adequate factor of safety. The maximum permissible pressure drop Δp_{max} must be carefully allocated among the pressure drop of transition ducts (Δp_{trans}) and the total pressure drop of the silencer, Δp_{tot} :

$$\Delta p_{max} \geq F_s (\Delta p_{trans} + \Delta p_{tot}) \text{ N/m}^2 \quad (9.50)$$

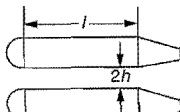
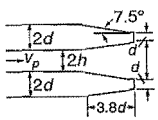
The specific choice of the safety factor ($F_s > 1$) is influenced by the degree of inhomogeneity of the inflow and by guaranty obligations regarding pressure drop performance of the silencer system. The silencer system usually includes the transition ducts both upstream and downstream of the silencer.

Detailed information on how to predict the pressure drop of inlet and exit transitions has been compiled by Idel'chik.⁷⁹ The silencer pressure losses are expressed as a product of the dynamic head in the muffler passage, $0.5\rho v_p^2$, and a loss coefficient. The terms K_{ENT} , K_F , and K_{EX} represent the entrance, friction, and exit loss coefficients. They are predicted according to the formulas given in Table 9.3

The required silencer face area is determined by an iteration process. First, Eq. (9.39) is solved for the passage velocity v_p . The initial face area A'_F is obtained as

$$A'_F = \frac{Q}{v_p} \left(\frac{100}{\text{POA}} \right) \quad \text{m}^2 \quad (9.51)$$

TABLE 9.3 Pressure Loss Coefficient for Parallel Baffle Silencers

Geometry	Loss Coefficient
Square-edge nose 	$K_{ENT} = \frac{0.5}{1 + h/d}$
Rounded nose 	$K_{ENT} \approx \frac{0.05}{1 + h/d}$
Typical perforated metal facing 	$K_F \approx 0.0125$ $\ell = \text{baffle length, tail and nose not included}$
Square tail 	$K_{EX} = \left(\frac{1}{1 + h/d} \right)^2$
Rounded tail 	$K_{EX} = 0.7 \left(\frac{1}{1 + h/d} \right)^2$
Faired tail, 7.5° 	$K_{EX} = 0.6 \left(\frac{1}{1 + h/d} \right)^2$

where Q is the volume flow rate through the silencer (in cubic meters per second) v_p is the passage velocity (in meters per second), and POA is the percentage of open area of the silencer cross section (e.g., for parallel-baffle mufflers POA = 50 means that $h/d = 1$; the passage height is the same as the baffle thickness). Based on this initial value of $A_F = A'_F$, the pressure loss of the inlet and exit transitions is calculated and added to the muffler pressure drop. The total $F_s(\Delta p_{trans} + \Delta p_T)$ is compared with maximum permissible pressure drop Δp_{max} according to Eq. (9.50). If $F_s(\Delta p_{trans} + \Delta p_T) > \Delta p_{max}$, the face area of the silencer must be increased. This results in a decrease of Δp_T and a relatively smaller increase in Δp_{trans} , and the iteration is continued until the inequality, expressed in Eq. (9.50), is satisfied.

Economic Considerations

For large silencers such as used in electric power plants, the product of the pressure drop and volume flow rate, $Q\Delta p$, represents a substantial power that is lost (converted to heat). The cost to produce this power during the entire design life of the installation usually far exceeds the purchase cost of the silencer. Therefore, it is important to specify a silencer pressure drop that yields the lowest total cost. The total cost includes the present worth of revenue requirements to purchase and install the silencer (which decreases with increasing pressure drop allowance) and the present worth of revenue requirements of the energy cost caused by operating the silencer (which increases with increasing pressure drop allowance). The optimal pressure drop is that which yields the lowest total cost. Information on how to predict the optimal silencer pressure drop is given in reference 80.

9.7 COMBINATION MUFFLERS

It is clear from Sections 9.1–9.3 that the TL of reactive mufflers is generally characterized by several troughs that limit the overall value of TL notwithstanding the peaks, owing to the arithmetic of antilog summation. The acoustically lined ducts and parallel-baffle mufflers (or splitter silencers) do not suffer from this characteristic; they are characterized by a wide-band TL curve. However, they have rather poor performance (attenuation) at low frequencies where the reactive mufflers have a relative edge. One way out of these limitations is to combine reflective or reactive elements and dissipative elements into a combination muffler (or hybrid muffler). An acoustically lined plenum chamber is one obvious example of a combination muffler. Figures 9.34–9.37 illustrate the concept and present some design guidelines in the process.⁸¹

Figure 9.34a is an unlined simple axisymmetric expansion chamber. Figures 9.34b,c show the chamber with lining (with resistivity of $5000 \text{ Pa} \cdot \text{s/m}^2$) of 18.2 and 36.4 mm thickness, respectively. The plots in Fig. 9.34 compare the computed values of their axial transmission loss TL_a (assuming rigid shell). Clearly, the role of lining is to raise and even out the troughs, particularly at the medium and high frequencies.

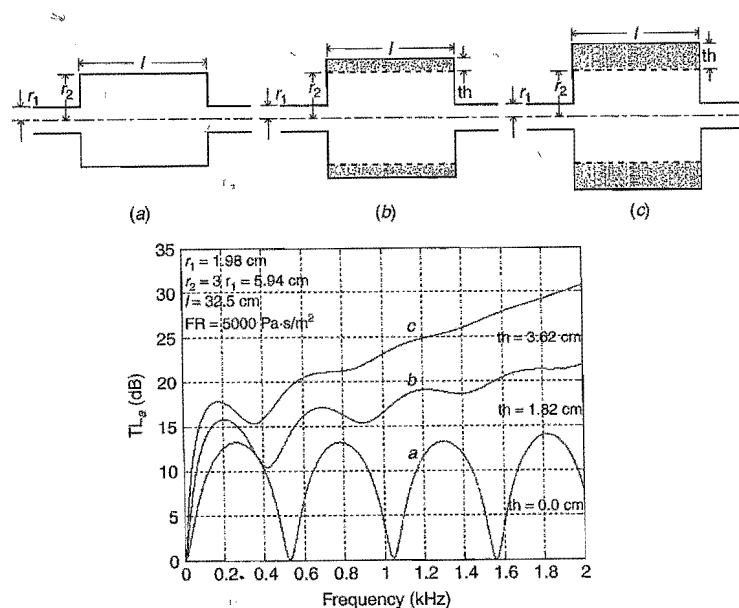


FIGURE 9.34 Use of acoustical lining in combination with a simple expansion chamber or plenum; th = lining thickness in centimeters.

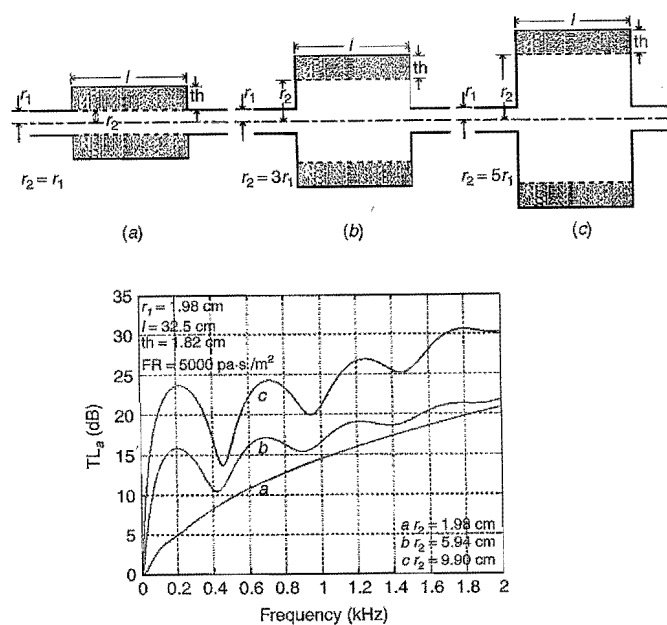


FIGURE 9.35 Use of sudden area discontinuities in combination with lined duct.

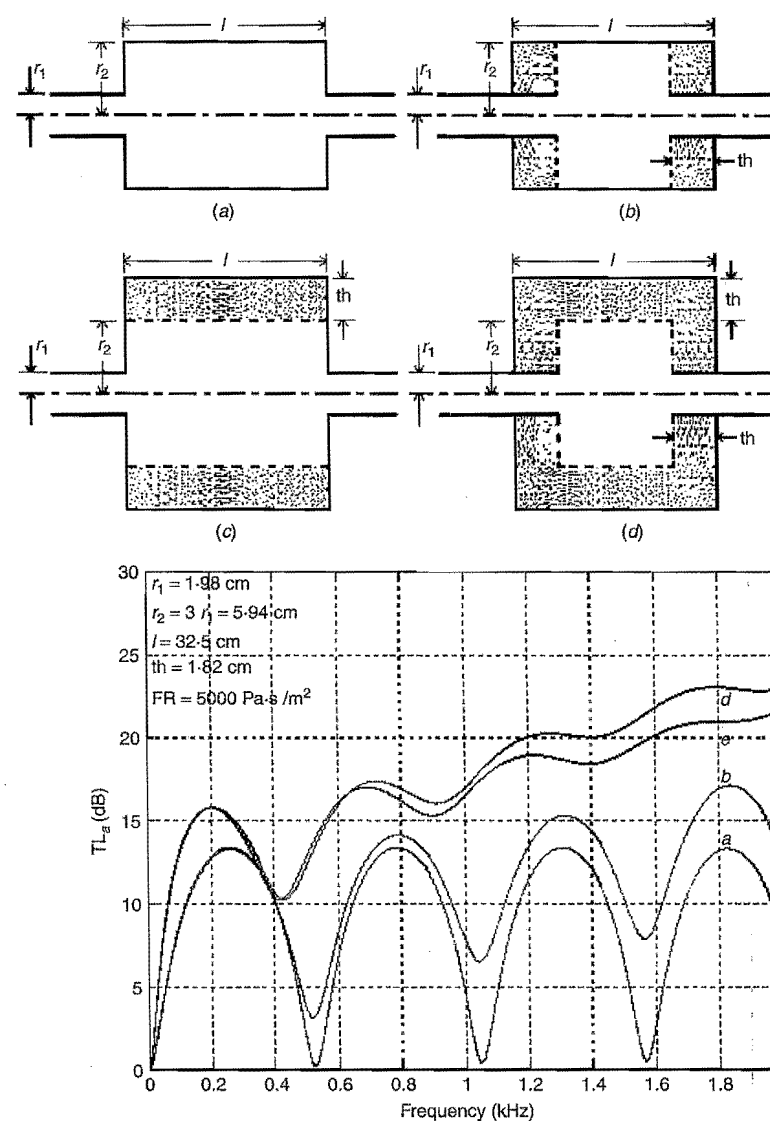


FIGURE 9.36 Effect of lined walls and lined shell.

Figure 9.35 shows the effect of building a sudden expansion and contraction into an acoustically lined circular duct, keeping the radial thickness of the lining constant. As can be noticed, the area discontinuities (sudden impedance mismatch) increase TL, particularly at the low frequencies (≤ 400 Hz).

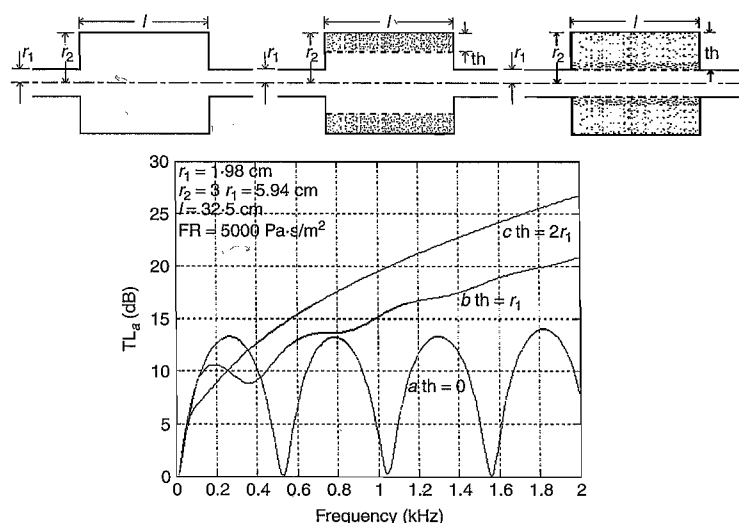


FIGURE 9.37 Reactive elements versus dissipative elements within the same overall shell radius.

A plenum chamber has lining not only on the shell but also on the side-walls/plates. Figure 9.36 shows (a) a simple expansion chamber (plenum), (b) one with lined sidewalls, (c) one with lined shell only, and (d) one with lined shell as well as lined sidewalls (or end plates). It may be noticed that while the sidewall lining has considerable effect (in raising the troughs), the effect of shell lining is much more pronounced. In fact, a comparison of curves *b* and *c* of Fig. 9.36 shows that the relative effect of sidewall lining is only marginal.

In all the three foregoing figures, provision of lining has increased the overall radius of the muffler shell. Sometimes, it may not be feasible or advisable to increase the shell radius. Then, the internal lining of the shell would decrease the ratio of sudden expansion and sudden contraction. Figure 9.37 shows the effect of this compromise (between reaction and dissipation), keeping the overall shell radius constant. It may be observed that the performance of lined ducts is relatively poor at very low frequencies, when a simple unlined chamber has an edge. The two can supplement each other in a combination muffler (configuration *b* in Fig. 9.37).

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Sound Generation

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Acoustical engineers can be grouped into two categories: those who try to *minimize* the efficiency of sound radiation and are called *noise control engineers* and those who try to *maximize* it and are called *audio engineers*. This book addresses the first group exclusively.

The main duty of noise control engineers is to know how to reduce, in a cost-effective manner, the noise produced by equipment at the location of an observer. The noise reduction can be achieved (1) *at the source*; (2) *along the propagation path* by building extensive barriers between the source and receiver, installing silencers, and so on, as discussed in Chapters 5 and 9; and (3) *at the receiver* by enclosing it, as discussed in Chapter 12, or by creating a limited “zone of silence” around it by active noise control, as described in Chapter 18.

The most effective, and by far the least expensive, noise control can be achieved at the source by reducing its sound radiation efficiency. A reduction of the efficiency of noise generation of the source results in a commensurate *reduction of the noise at all observer locations*. In comparison, the erection of barriers, enclosing the receiver, and creating a localized “zone of silence” at the receiver are *effective only in limited spatial regions*. Putting silencers on the noisy equipment is cumbersome and usually increases the size and weight and reduces the mechanical efficiency of the equipment. The last two measures are often referred to as the *brute-force* methods.

The reduction of the efficiency of noise radiation at the source can be achieved passively by changing the shape of the body if its vibration velocity is given and by changing both the shape and mass of the body if the vibratory force acting on it is given. Reducing the efficiency of sound radiation by active means is generally accomplished by placing a secondary sound source of the same volume displacement magnitude but opposite phase in the immediate vicinity of the body, as discussed in detail in Chapter 18, or by a combination of these passive and active measures.

If constructional changes are possible, always the passive measures should be implemented first, even if the needed additional reduction will be achieved by active means. Even if they do not achieve all the needed noise reduction, passive noise control measures reduce the demand on the power-handling capability of the noise-canceling loudspeakers. In addition, passive noise control measures remain effective in case the active control fails. The most desirable, but also the most difficult to achieve, passive noise control measures are those which require constructional changes of the equipment. Not infrequently, the same constructional changes that reduce the efficiency of sound radiation also may improve the mechanical efficiency of the equipment. The primary purpose of every machine is to perform a specific, usually mechanical, function. The noise is an unwanted by-product. The reduction of the efficiency of sound radiation at the source must always be accomplished without interfering with the primary function of the equipment. As we will show later, the reduction in the efficiency of sound radiation of any noise source is always achieved by reducing the force or forces that act on the surrounding fluid.

10.1 BASICS OF SOUND RADIATION

This chapter deals with the sound radiation of small pulsating and oscillating rigid bodies. Throughout this chapter the adjective *small* means that the dimensions of the body are small compared with the acoustical wavelength. Sound radiation of large bodies is treated in Chapter 11. An exhaustive treatment of the sound radiation of all types of bodies is given in reference 1.

Historical Overview

Technical acoustics is a branch of applied physics. All acoustical phenomena can be derived from Newton's basic laws. During the early part of the last century the science of technical acoustics has been developed mainly by physicists with a strong background in electromagnetism or by electronics engineers. The reason for this is that only they had the mathematical background to deal with dynamic phenomena in general and, most importantly, with the wave equation. In those days, the education of mechanical engineers was almost exclusively in static deformation of structures and *slow* phenomena in hydrodynamics. Physicists often describe the acoustical phenomena by mathematical formalisms that facilitate the mathematical analyses. Unfortunately, these formalisms, such as *monopoles*, *dipoles*, *quadrupoles*, "*equivalent electric circuits*," and "*short circuiting*," were often not familiar to mechanical engineers. This built artificial barriers for the mechanical engineers which have been overcome only in the last few decades due to the dramatic increase in the education of mechanical engineers in dynamic phenomena.

This chapter attempts to explain the important phenomena of sound generation in a form, the author believes, that might have been done if the theory of sound generation had been developed by mechanical engineers. As we will show in the

remainder of the chapter, the sound radiation phenomena can be fully described without knowing anything about monopoles, dipoles, quadrupoles, and equivalent electric circuits.

The added advantage of characterizing the sound sources by the force and the moment they exert on the fluid rather than assigning abstract descriptors such as *equivalent point sound source*, *dipole strength*, and *quadrupole strength* means that such characterization gives a better understanding of the physical phenomena involved and identifies the actual physical parameters that control all types of sound generation processes.

Qualitative Description of Sound Radiation of Small Rigid Bodies

Small rigid bodies oscillating in an unbounded fluid experience three types of reaction forces; (1) inertia force needed to accelerate the fluid to move out of the way of the body, (2) force to compress the fluid, (3) friction forces owing to the viscosity of the fluid.

The inertia force, which is by far the largest of them, is 90° out of phase with the velocity of the body. The compressive force is in phase with the velocity and is much smaller than the inertia force. However, it has to be considered because it determines the sound radiation. The friction force is usually small enough compared with the two other components that it can be neglected by assuming that the fluid has no viscosity.

Sound is generated by phenomena that cause a localized compression of the fluid (gases or liquids). This chapter deals with the most common sources of sound generated by pulsation and oscillatory motion of small rigid bodies and by pointlike forces and moments acting on the fluid.

Anyone who has operated a bicycle pump can attest to it that it takes considerable force to maintain a repetitive pumping motion of the piston when the valve is closed. The compression of the air in the closed cavity generates a large reaction force that opposes the motion of the piston. The opposing force depends only on the magnitude of the stroke and does not depend on the rapidity of the pumping.

It takes considerably less force to perform the same repetitive pumping motion when the valve is open and the hose is not connected to the tire. The force that resists the stroke in this case is the inertia force needed to push the air mass (stroke times surface area of the piston times the density of the air) through the open valve. The inertia force is proportional to the acceleration (i.e., the rapidity of the pumping motion) of this mass. The reaction force is very small if the rate of repetition is low and increases with increasing repetition rate. At sufficiently high repetition rate it becomes harder to accelerate the air mass that must be pushed through the open valve during increasingly shorter time periods than to compress the air in the pump's cavity. In this case the force needed to operate the leaky pump approaches that needed to operate the pump with the valve closed.

As we will see, similar behavior is observed in the sound radiation of small rigid bodies where the surrounding fluid is compressed in earnest only when the force needed to move the fluid in the vicinity of the pulsating or oscillating

body out of the way becomes the same order of magnitude as that required to compress it.

Pulsating Small Rigid Bodies. If a small body, exemplified by a pulsating sphere, operates at low frequencies where the wavelength of the sound is large compared with the acoustical wavelength, the pressure has two components. One is 90° out of phase with the velocity of the pulsating surface and a much smaller component is in phase with it. The former component is owing to the inertia of the surrounding fluid that is pushed into a larger spherical area during the outward motion of the pulsating surface and pulled back during the inward movement without much compression. The latter component of the pressure, which is in phase with the surface velocity, is owing to the compression of the surrounding fluid.

The force needed to overcome the inertia of this back-and-forth sloshing fluid volume increases with increasing frequency and so does the in-phase component of the surface pressure. Above a frequency where the wavelength becomes smaller than six times the radius of the sphere, it becomes easier to compress the fluid than to accelerate the fluid volume and the pulsating sphere becomes an efficient radiator of sound. Both the in-phase and 90° out-of-phase components of the pressure are evenly distributed over the surface of the sphere and the pulsating sphere radiates sound omnidirectionally.

At low frequencies not only the pulsating sphere but also all pulsating small bodies have an omnidirectional sound-radiating pattern and both the far-field sound pressure and radiated sound power depend only on the net volume flux and the frequency irrespective of how the vibration pattern is distributed over their surface. Because of these unique properties, pulsating small bodies are referred to as monopoles.

Oscillating Small Rigid Bodies. In the case of an oscillating rigid body, exemplified by an oscillating sphere, the fluid pushed aside by the forward-moving half of the sphere is "sucked in" by the void created by the receding half of the sphere. The fluid moves back and forth around the sphere between the two poles. The velocity of the fluid is highest near the surface of the sphere and decreases exponentially with increasing radial distance. This exponentially decreasing velocity field is referred to as the near field. The kinetic energy flux of the near field depends on the geometry of the body, the direction of the oscillatory motion, and the density of the fluid surrounding the body. The quantity *added mass/fluid density* is a property of the body and its motion. It is referred to in the acoustical literature as the *added volume*. In the hydrodynamic literature the *added mass coefficient* is defined as the ratio of the added volume, V_{ad} , and volume of the body, V_b .

Push-pull action between the poles (located in the direction of the oscillatory motion) makes it easy to move the fluid back and forth and the surface of the

oscillating small rigid sphere, for the same surface velocity, experiences much smaller reaction pressure than the pulsating sphere and radiates substantially less sound. The magnitude of the reaction pressure on the surface of the oscillating sphere has its maxima at the poles where the radial velocity has its maxima and zero at the equator where the radial velocity is zero. Consequently, the sound radiation pattern is maximum in the direction of the oscillatory motion and zero in the direction perpendicular to it. This behavior is not restricted to the oscillating sphere but holds for small oscillating rigid bodies of any shape. In analogy to the similar radiation pattern of the magnetic field caused by an oscillating charge, oscillating small rigid bodies are referred to as dipole sound sources.

The net oscillatory force exerted on the fluid is obtained by integrating the pressure distribution over the surface of the sphere. The force points in the direction of the oscillatory motion. As we will show later, with respect to sound radiation, a small, oscillating rigid body is equivalent to a pointlike force (i.e., pressure exerted by a small surface) acting on the fluid. The radiated sound power is obtained as the product of the body's velocity and the component of the net reaction force that is in phase with the velocity.

The distribution of the reaction pressure on the surface is known only for a few bodies. Consequently, the net oscillatory force they exert on the fluid (and consequently the radiated sound field) cannot be obtained directly. As we will show later, the force exerted on the fluid by such bodies—and consequently also their sound radiation pattern—can be predicted on the bases of the oscillatory velocity and geometry of the body and its added mass in the direction of the oscillatory motion. The latter can be found frequently in the hydrodynamic literature or can be determined experimentally by measuring the resonance frequency of the body supported on a spring; first in a vacuum then immersed in a fluid (preferably a liquid).

Moment Excitation: Lateral Quadrupole. Two closely spaced identical small rigid bodies oscillating in the same direction but 180° out of phase, or the equivalent two closely spaced parallel pointlike forces acting on the fluid 180° out of phase, are referred to as a *lateral quadrupole*. The forces constituting the moment are frequently due to shear strain. Both of these descriptions are helpful in understanding the sound radiation properties.

If we consider the representation of the two closely spaced small rigid bodies oscillating in opposite directions, it becomes apparent that the fluid pushed aside by the pole of one of the bodies does not need to be moved around to the opposite pole but is easily moved into the void created by the receding pole of the second body. The reaction pressure on the surface of each of the bodies, and consequently the sound field they produce, is substantially smaller than would result when only one of the bodies would oscillate.

If we consider the model of two parallel closely spaced pointlike forces, we realize that the lateral quadrupole represents excitation of the fluid by a moment.

The source strength is characterized by the moment, regardless of whether the moment is constituted by large forces spaced very close to each other or by smaller forces spaced at a larger distance apart, provided that the distance is still much smaller than the wavelength.

The sound radiation pattern of a lateral quadrupole is zero in two planes. The first of this is perpendicular to the plane defined by the two parallel forces and cuts the plane of the forces halfway between them. All points located in this plane have an equal distance from the two bodies moving in opposite directions and their contribution to the sound field cancels each other.² The second plane of zero sound pressure is perpendicular to the direction of the motion of the oscillating bodies and cuts through their center. The sound pressure is zero at all points on this plane because the two oscillating bodies constituting the lateral dipole have zero radial velocity in these directions.

The sound radiation pattern has maxima in two planes. Both of these planes cut the zero planes at 45° angles. In two dimensions the radiation pattern resembles an old-fashioned, four-bladed propeller where the four maxima are represented by the four blades and the minima by the void between the blades.

If the two opposing forces are of different magnitude, the sound radiation pattern is the superposition of the quadrupole radiation pattern associated with a moment constituted by two opposing forces having the same magnitude as the smaller force and a dipole pattern associated with a single pointlike force that equals the difference between the two forces.

Because the maxima of the dipole pattern “fill” the zero planes of the quadrupole pattern, the resulting radiation pattern has no zero value in any direction.

Sound Excitation by Two Opposing Forces Aligned Along a Line: Longitudinal Quadrupole. If two equal-magnitude but opposing dynamic forces are placed on a line a small distance apart (or the equivalent of two identical small rigid bodies are oscillating 180° out of phase), the sound radiation pattern resembles that of a single pointlike force. However, the two maximum lobes, pointing in the direction of the forces, are much narrower and the radiated sound power is much smaller than if only one of the forces acted on the fluid. In analogy to the directivity pattern of the magnetic field generated by two closely spaced charges oscillating in the opposite direction, this type of sound source is referred to as a longitudinal quadrupole. This type of radiation pattern occurs when a small rigid body oscillates in water near to and perpendicular to the water–air interface. In this case the mirror image represents the identical second body that oscillates 180° out of phase.

If the two opposing forces are not equal, then the resulting radiation pattern is the superposition of the pattern of a longitudinal quadrupole and a dipole. Because both radiation patterns have zero value in a plane that is perpendicular to the forces and cuts through the center of the force pair, the resulting radiation pattern has zero value in this plane. The contribution of the dipole component widens the two maximum lobes.

Radiated Sound Power. The sound power radiated by all of the above-discussed sound sources can also be determined by integrating the intensity $p_{rms}^2(r, \vartheta, \theta)/\rho_0 c_0$ over a large radius ($kr \gg 1$) sphere centered at the acoustical center of the source, where $p_{rms}^2(r)$ is the mean-square value of the sound pressure at distance r in the direction designated by the angles ϑ and θ .

Acoustical Parameters of Sound Radiation

This section contains a brief introductory discussion of the acoustical parameters of fluids, including the speed of sound, density, and plane-wave acoustical impedance.

Propagation Speed of Sound in Fluids. The molecules of fluids and gases in equilibrium undertake a random motion with equal probability of direction, c_t , and the propagating speed in a stationary fluid, c_0 , given by²

$$c_0 = c_t \sqrt{\gamma} = \sqrt{\frac{\gamma RT}{M}} \quad \text{m/s} \quad (10.1)$$

where $\gamma = c_p/c_v$ is the specific heat ratio with $\gamma \approx 1.4$ for air and $\gamma \approx 1.3$ for steam, R is the universal gas constant ($R = 8.9 \text{ J/K}$), T is the absolute temperature in degrees Kelvin (K), ($0 \text{ K} = -273^\circ\text{C}$), and M is the molar mass equal to 0.029 kg/mole for air and 0.018 kg/mole for steam. For air Eq. (10.1) yields

$$c_0 = 20.02 \sqrt{T(\text{K})} = 342.6 \sqrt{\frac{T(\text{K})}{293}} = 342.6 \sqrt{\frac{T(^{\circ}\text{C}) + 273}{293}} \quad \text{m/s} \quad (10.2)$$

For air at $20^\circ\text{C} = 68^\circ\text{F} = 293 \text{ K}$ the propagating speed is $c_0 = 342.6 \text{ m/s}$, at which a sound wave travels in air. The individual molecules collide with each other in a random fashion exchanging momentum with each other. If they impinge on a solid impervious wall, they rebound back from it. Sound is generated when dynamic forces acting on the fluid superimpose a very small but organized dynamic velocity on the very large random velocity of the molecules. The word “organized” means that in the case of periodic motion the superimposed velocity has a specific direction and a specific frequency and in the case of random motion it has a direction and its frequency composition can be described either by its power spectrum or autocorrelation function, as described in Chapter 3. In a stationary fluid the kinetic energy of the molecules acquired through the excitation causes compression and the compression then accelerates the molecules. In a stationary fluid the kinetic energy and potential energy are in balance in any small volume of the fluid. If the sound source acts on a moving media, the kinetic energy depends on the velocity and the direction of the fluid movement. In this chapter we will deal with stationary media only. Sound radiation in moving media is covered in Chapter 15.

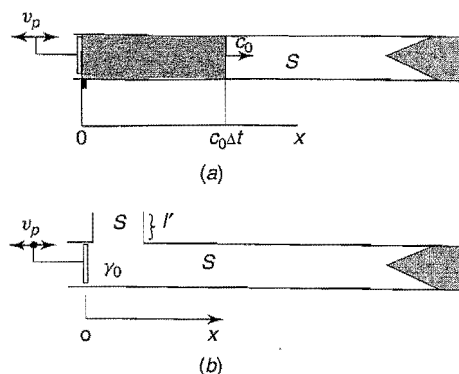


FIGURE 10.1 Oscillating plane rigid piston radiating sound into a small-diameter rigid pipe that is either infinitely long or anechoically terminated: (a) Hypothetical experiment to determine the plane wave impedance of fluids, No side branch; (b) open side branch of the same cross section area S as the face area of the piston and effective length l' .

The superimposed organized particle velocity of sound we experience is usually in the range 10^{-8} m/s (corresponding to a sound pressure level of 14 dB re 20 μ Pa) to 10^{-4} m/s (corresponding to 94 dB re 20 μ Pa).

Plane-Wave Impedance. The plane-wave impedance can be obtained by performing a hypothetical experiment similar to that suggested by Ingard,² as shown in Fig. 10.1a.

Hypothetical Experiment 1. Assume, as sketched in Fig. 10.1a, that a plane, rigid piston of surface area S is located on the left end of a small-diameter tube with rigid walls filled with a fluid of density ρ_0 and speed of sound c_0 . If the piston moves to the right with a constant speed of v_p , the face of the piston experiences a reaction pressure. To determine the plane-wave impedance defined as $Z_0 = p(x=0)/v_p$, we need to find that particular pressure $p(x=0)$ at the piston-fluid interface (at position $x=0$) that results from the piston velocity v_p . At the time Δt , the shaded volume of the fluid on the left, $S_0 c_0 \Delta t$, has acquired the particle velocity v_0 and the rest of the fluid on the right is motionless. The momentum M acquired by the fluid during the time Δt is then $M = v_0 \rho S_0 c_0 \Delta t$. According to Newton's law the force on the interface is

$$F(x=0) = Sp(x=0) = \frac{d}{dt} M \Delta t = v_p \rho_0 S c_0 \quad \text{N} \quad (10.3)$$

yielding the plane wave impedance Z_0 ,

$$Z_0 \equiv \frac{p(x=0)}{v_p} = \rho_0 c_0 \quad \text{N} \cdot \text{s/m}^3 \quad (10.4)$$

Performing the same thought experiment on a solid bar yields the plane-wave impedance of a solid as

$$Z_0 \approx \rho_M c_L \quad \text{N} \cdot \text{s/m}^3 \quad (10.5)$$

where c_L = propagation speed of longitudinal waves in bar, $(E/\rho)^{1/2}$ m/s

ρ_M = density of solid material, kg/m³

E = Young's modulus, N/m²

If the piston oscillates with a velocity of the form $v_p e^{j\omega t}$, starting at $t=0$, the sound pressure in the tube filled with a fluid (air) is

$$p(x, t) = v_p \rho_0 c_0 e^{-jkx} e^{j\omega t} \quad \text{N/m}^2 \quad (10.6)$$

and the sound power radiated by the piston, W_{rad} , is

$$W_{\text{rad}} = \frac{1}{2} \hat{v}_p \text{Re}[\hat{p}_p(x=0)] S \rho_0 c_0 \quad \text{N} \cdot \text{m/s} \quad (10.7)$$

where in Eqs. (10.6) and (10.7) $\omega = 2\pi f$, f is the frequency in hertz, $k = \omega/c_0$ is the wavenumber, and the hat above the symbol signifies the peak value. Observing Eqs. (10.6) and (10.7), note that the higher is the product of density and speed of sound, the larger are the sound pressure and the radiated power that a given excitation velocity v will generate.

For example, when plane waves are generated in a column of air, water, and steel, an enforced velocity $v = 1$ mm/s = 10^{-3} m/s will produce the sound pressure p , sound pressure level SPL (in dB re 2×10^{-5} N/m²) listed below. Assuming that the cross-sectional area is 10^{-4} m², we also can predict the radiated sound power W_{rad} in watts:

Material	ρ (kg/m ³)	c (m/s)	p (N/m ²)	SPL (dB)	W_{rad} (W)
Air inside rigid tube	1.21	344	0.42	86	4.2×10^{-8}
Water inside rigid tube	998	1481	1478	157	1.5×10^{-4}
Steel bar in air	7700	5050	3.9×10^4	186	3.9×10^{-3}

In plane waves all of the volume displacement produced by the vibratory excitation at the interface is converted into compression and the resulting sound pressure at the interface is in phase with the excitation velocity. In this case, the sound radiation is that which is the maximum achievable. In other situations, where the volume displaced by the vibratory excitation is mainly used for "pushing aside" rather compressing the fluid, the resulting sound pressure at the interface is much smaller. That is, the pressure has only a small component that is in phase with the vibration velocity of the excitation, which is the reason for the inefficient low-frequency sound radiation of small bodies. For such small sound

sources, maintaining the same velocity with increasing frequency means that it is increasingly more difficult to push aside the fluid. When it becomes much easier to compress the fluid than to push it aside, the pressure produced at the interface reaches its maximum and is nearly in phase with the excitation velocity and the efficiency of sound radiation approaches that of a plane wave. The remaining very small but finite phase difference is essential because it determines the radius of curvature of the wave front.

Piston Generating Sound in Rigid Tube with Open Side Branch. To illustrate the effect in a quantitative manner that the efficiency of sound generation is decreased if it is easier to push the fluid aside than to compress it, consider the situation shown in Fig. 10.1b. It differs from the situation depicted in Fig. 10.1a by the addition of an open side branch to the pipe in the immediate vicinity of the piston. Let assume that the piston, the main pipe, and the side-branch pipe all have the same cross-sectional area S and that the effective length of the open-ended side branch is l' . The mass of the air in the side branch is $\rho_0 S l'$. The force needed to push this mass aside, $j\omega v_p \rho_0 S l'$, is smaller than the force $v_p \rho_0 c_0 S$ needed to compress the fluid, and the presence of the open side branch will reduce the ability of the piston to exert force on the fluid.

The sound pressure that the oscillating piston with peak velocity amplitude $\hat{v}_p$ produces is given by

$$\hat{p}(x=0) = \hat{v}_p \frac{1}{1/(j\omega\rho_0 l') + 1/(\rho_0 c_0)} \quad \text{N/m}^2 \quad (10.8)$$

Separating the real and imaginary parts in Eq. (10.7) yields

$$\hat{p}(x=0) = \hat{v}_p \frac{(\omega\rho_0 l')^2 \rho_0 c_0 + (j\omega\rho_0 l')(\rho_0 c_0)^2}{(\rho_0 c_0)^2 + (\omega\rho_0 l')^2} \quad \text{N/m}^2 \quad (10.9)$$

At low frequencies where $\omega\rho_0 l' \ll \rho_0 c_0$, Eq. (10.8) yields

$$\hat{p}(x=0) \approx \hat{v}_p (j\omega\rho_0 l') \quad \text{N/m}^2 \quad (10.10)$$

At high frequencies where $\omega\rho_0 l' \gg \rho_0 c_0$, Eq. (10.8) yields

$$\hat{p} \approx \hat{v}_p \rho_0 c_0 \quad \text{N/m}^2 \quad (10.11)$$

Inspecting Eqs. (10.9) and (10.10), note that the sound pressure produced by the piston is small, is nearly 90° out of phase of the velocity of the piston, and increases linearly the frequency ω . Using the vocabulary of electric engineers, the low impedance of the mass in the open side branch, $j\omega\rho_0 l'$, "shunts" $\rho_0 c_0$, the plane-wave impedance of the fluid in the tube.

More specifically at low frequencies most of the volume displaced by the oscillating piston flows into the open side branch, where it encounters little "resistance," and only a very small portion flows into the main tube. Consequently,

the magnitude of the reaction force exerted on the piston owing to the presence of both the side branch and the main pipe is only a minute amount larger than the small force needed to keep the fluid mass in the side branch oscillating. The same small force is exerted on the fluid in the main pipe. This force is substantially smaller than it would be if the entrance of the side branch were closed. Accordingly, the sound pressure in the main tube is also substantially smaller.

The sound power that the piston is radiating into the main tube is the product of the velocity of the piston and the small component of the reaction force that is in phase with the velocity and is given by

$$W_{\text{rad}} = \frac{1}{2} \hat{v}_p \text{Re}[\hat{p}(x=0)] S = \frac{1}{2} \hat{v}_p^2 S \frac{\rho_0 c_0 (\omega\rho_0 l')^2}{(\rho_0 c_0)^2 + (\omega\rho_0 l')^2} \quad \text{N m/s} \quad (10.12)$$

The hat above the variables indicates peak amplitude. If the rms value of the piston' velocity were used, the factor $\frac{1}{2}$ would be omitted in Eq. (10.12).

Inspecting Eq. (10.12), note that at low frequencies where $\omega\rho_0 l' \ll \rho_0 c_0$, the radiated sound power approaches zero with decreasing frequency. At high frequencies where $\omega\rho_0 l' \gg \rho_0 c_0$, the radiated sound power becomes independent of frequency and approaches that obtained in Eq. (10.7) for the pipe without an open side branch.

Pipe with Open Side Branch Representing Sound Radiation of Pulsating Sphere. Following the qualitative suggestion by Cremer and Hubert,³ we are assigning the specific values for S and l' in Eqs. (10.9) and (10.12), namely, $S = 4\pi a^2$, $\hat{v}_p = \hat{v}_s$, and $l' = a$, and considering that the wavenumber $k = \omega/c_0$ and $\hat{v}_p S = \hat{v}_s 4\pi a^2 = \hat{q}$ is the peak volume velocity of our sound source, Eq. (10.9) yields the well-known³ formula for the peak value of the sound pressure on the surface of the sphere:

$$\hat{p}(a) = \hat{v}_s \frac{j\omega\rho_0 a}{1 + jka} \quad \text{N/m}^2 \quad (10.13)$$

Also note that the volume of fluid in the side branch is $V_{\text{ad}} = 4\pi a^3$ and the added mass of the fluid is $M_{\text{ad}} = 4\pi a^3 \rho_0$. For a pulsating sphere the volume V_{ad} is three times the volume of the sphere and is called the *added volume* in the acoustical literature and in M_{ad} is called the *added mass*. The added mass plays a central role in the sound radiation of small pulsating and oscillating rigid bodies. The added mass is a measure of the ability of these bodies to exert a dynamic force on the surrounding fluid.

Substituting the same values into Eq. (10.12) yields the well-known formula⁴ for the sound power radiated by the a pulsating sphere of radius a and peak surface velocity $\hat{v}_s$:

$$W_{\text{rad}} = \hat{v}_s^2 4\pi a^2 \left(\frac{\rho_0 c_0}{2} \right) \frac{(ka)^2}{1 + (ka)^2} = \hat{q}^2 \left(\frac{\rho_0 c_0}{2} \right) \frac{(ka)^2}{1 + (ka)^2} \quad \text{N/m}^2 \quad (10.14)$$

Sound Radiation by Volume-Displacing Sound Sources

To describe the sound radiation of small volume-displacing sound sources, we take the unusual approach of starting with the case where the source operates in the most restrained environment and then gradually weakening the restraints to arrive at the unrestrained case where the sound source operates in an undisturbed infinite fluid.

On the top of Figure 10.2 is shown the starting point where the volume-displacing sound source is represented by a piston that vibrates with an axial velocity v_p in the mouth of a rigid linear horn. The horn represents the restraint. The linear horn is a rigid hollow cone with the tip cut off. The cutoff end becomes the mouthpiece of the horn. The diameter of the cross-sectional area of the linear horn increases linearly with increasing distance r from the tip and its cross-sectional area increases with r^2 . The cross-sectional area of the horn at its mouth, $l\theta$, is chosen to fit the mouth of a human speaker. The symbol θ is the solid angle (θ equals π for a quarter space, 2π for a half space, and 4π for a full space, as illustrated in Fig. 10.3).

The cross-sectional area at the large end of the horn is chosen to be $S_{\text{End}} \geq \lambda^2 \geq c_0^2/f_{\text{int}}^2$, where f_{int} is the lowest frequency of interest (usually 500 Hz for voice communication) so that practically no sound is reflected from the far end of the horn. If this condition is fulfilled, the use of the linear horn is equivalent to increasing the size of the mouth of the human speaker by the ratio of the end and mouthpiece cross-sectional areas of the horn, as illustrated by the lower sketch in Fig. 10.11.

The cross-sectional area of the horn can be expressed as

$$S(r) = \theta r^2 \quad \text{m}^2 \quad (10.15)$$

According to Cremer and Hubert,³ the sound pressure inside the horn as a function of distance from the tip can be expressed as

$$\bar{p}(r) = \frac{\tilde{v}_p (l/r)}{1/(\rho_0 c_0) + 1/(j\omega\rho_0 l)} \quad \text{N/m}^2 \quad (10.16)$$

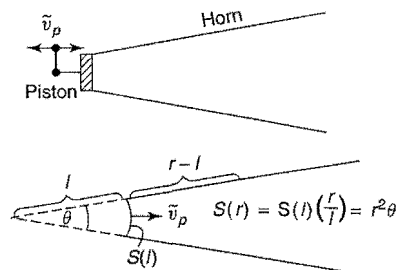


FIGURE 10.2 Vibrating piston sound source driving a linear acoustical horn (see text for explanation of symbols).

where v_p is the vibration velocity of the piston at the mouth of the horn and l is the radial length from the cutoff tip of the cone to the mouthpiece end, as illustrated in Fig. 10.2. Observing Eq. (10.16), note the following:

1. The term $j\omega\rho_0 l$ represents the added mass that is the consequence of the expansion of the cross section, which makes it possible to push the fluid off axis. In the extreme case when $\theta \rightarrow 0$ the horn turns into a rigid pipe and the added mass becomes infinite. This means that the near field in the pipe extends to infinity.
2. The sound propagates in the linear horn at all frequencies (which is not the case in an exponential horn, where no sound propagates below its cutoff frequency).
3. At low frequencies when $kl = (\omega/c_0)l \ll 1$ the sound pressure increases linearly with frequency as $p(r) \cong j\omega\rho_0(l^2/r)\tilde{v}_p$.
4. Above the frequency $\omega \geq c_0/l$ the linear horn works at full efficiency and the sound pressure as a function of distance becomes $p(r) \cong v_p(l/r)\rho_0 c_0$.

Considering that the volume velocity at the mouth of the horn is $q = v_p \theta l^2$, the radiated sound power takes the form

$$W_{\text{rad}} = q^2 \rho_0 c_0 \left(\frac{k^2}{\theta} \right) \frac{1}{1 + (kl)^2} = q^2 \frac{\rho_0 \omega^2}{c_0 \theta} \left[\frac{1}{1 + \pi(S/\lambda^2)(4\pi/\theta)} \right] \quad \text{N} \cdot \text{m/s} \quad (10.17)$$

Figure 10.3 shows what happens if we gradually weaken the restraint by increasing the solid angle θ from zero to 4π . In the extreme case, when the solid angle

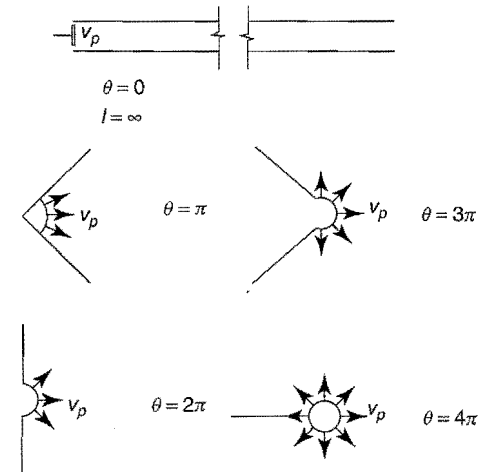


FIGURE 10.3 Increasing the solid angle θ of the horn; Top left: $\theta = 0$, pipe. Middle left: $\theta = \pi$, quarter space. Bottom left: $\theta = 2\pi$, half space. Middle right: $\theta = 3\pi$, three-quarter space. Bottom right: pulsating sphere in unbounded space.

of the horn θ is zero and tip length l approaches infinity, the horn becomes a cylindrical pipe, as shown in the top sketch. In this extreme case the sound pressure in the pipe is

$$p(r, t) = (q/S) e^{-jkr} e^{j\omega t} \text{ N/m}^2 \quad (10.18)$$

indicating that theoretically the amplitude of the sound pressure would not decrease with distance. A practical use of this attribute is shown on the left in Fig. 10.11.

Consider now the effect of increasing the solid angle of the horn from $\theta = 0$ by an infinitesimally small $\Delta\theta$. Consequence of this is a phenomenological change in the behavior of the sound field, namely the sound pressure approaches zero at an infinitely large axial distance while for the pipe it would remain just as high as on the surface of the piston. Such quantum jumps are not allowed in Newtonian physics. The change in the behavior of the sound field is rapid but continuous. In the constant cross-sectional area of the pipe the near field extends infinitely. In the linear horn with infinitesimally small solid angle the near field extends nearly infinitely. Since in practice there are only finite-length pipes and horns, a microphone near the end of the anechoically terminated finite-length horn with an infinitesimally small solid angle would practically sense the same sound pressure as it would in a pipe.

Let us now explore the behavior of our linear horn given in Eq. (10.17) at solid angles of practical interest. Equation (10.17) yields

$$W_{\text{rad}} = q^2 (\rho_0 c_0) \left(\frac{k^2}{4\pi} \right) n \text{ N} \cdot \text{m/sec} \quad (10.19)$$

where $n = 1$ when the volume source is located in an infinite, undisturbed fluid;

$n = 2$ when the volume source is flash mounted in an infinite, hard, flat baffle and radiates into a half space;

$n = 4$ when the sound source is located in a two-dimensional corner and radiates into a quarter space; and

$n = 8$ when located in a three-dimensional corner and radiating into an eighth space.

If there is a net volume displacement (as far as the far-field sound pressure and the radiated sound power are concerned), the specific distribution of the velocity over the vibrating surface is unimportant. This is because the non-volume-displacing vibration patterns (dipole, quadrupole, etc.) radiate low-frequency sound much less efficiently than volume-displacing patterns.

Figure 10.4 shows volume sources with different distributions of the vibration velocity but having the same net volume velocity. The solid and dashed lines represent the location of the vibrating surface at two time instances. The solid line shows the maximum and the dashed line the minimum volume displacement positions, respectively.

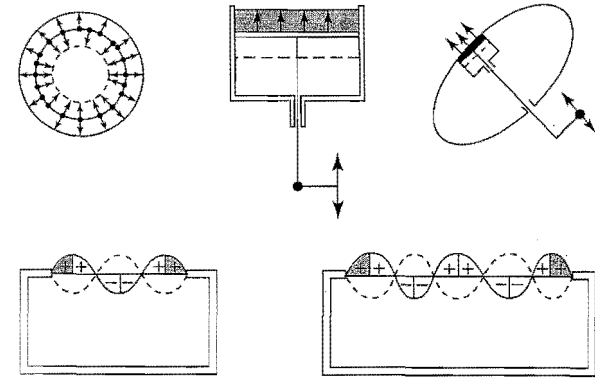


FIGURE 10.4 Small volume sources with different distributions of the vibration velocity but the same net volume velocity.

Limits of Maximum Achievable Volume Velocity of Pulsating Sphere. The maximum peak volume velocity $\hat{q}_{\text{max}}$ achievable with a small pulsating sphere of volume $V_{\text{sphere}} = 4\pi a^3/3$ is limited by (1) the impossibility of having a larger volume displacement than the volume of the sphere, (2) the desire of not having supersonic surface velocity, and (3) the desire of keeping nonlinear distortion within acceptable limits. These requirements are accounted for in Eq. (10.20) below.

$$\frac{\hat{q}_{\text{max}}}{V_{\text{sphere}}} \leq \frac{n}{a/(3c_0) + 1/(12\pi\omega)} \text{ s}^{-1} \quad (10.20)$$

where the first summand in the denominator accounts for the first limit, the second for the second limit, and the factor $n \ll 1$ in the numerator for the third limit.

Combining Eqs. (10.19) and (10.20) and solving for $\hat{W}_{\text{rad}}$ yield the maximum peak acoustical power a pulsating sphere of radius a can produce:

$$\begin{aligned} \hat{W}_{\text{rad}}(\text{max}) &\leq 2\pi \left(\frac{\rho_0 c_0^3 a^4 k^2 n^2}{\{1 + [3/(12\pi)][1/(ka)]\}^2} \right) \\ &= 2\pi \left(\frac{\rho_0 \omega^2 c_0 a^4 n^2}{1 + [3/(12\pi)][c_0/(\omega a)]} \right) \text{ N} \cdot \text{m/s} \end{aligned} \quad (10.21)$$

If we use the pulsating sphere as a loudspeaker to reproduce voice and music, it would be appropriate to choose n in the range of 0.001–0.01 to keep distortion at peak events within tolerable limits.

A dodecahedron-shaped enclosure with a loudspeaker flash mounted on each of the 12 faces of the enclosure is often used to simulate an omnidirectional volume sound source. The top photograph in Fig. 10.5 shows such a special reciprocity transducer⁵⁻⁷ (loudspeaker) that the author has developed for the NASA

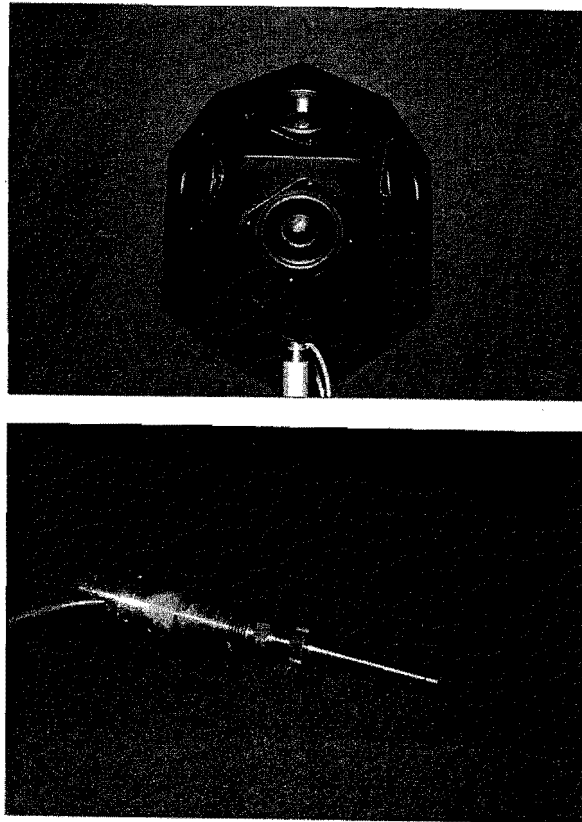


FIGURE 10.5 Photographs of a 4-in. edge-length dodecahedron loudspeaker to simulate a pulsating sphere as an omnidirectional sound source with volume velocity calibration and compensation for acoustical loading. Top: View of the loudspeaker. Bottom: Removed internal $\frac{1}{4}$ -in. microphone used for volume velocity calibration and for correcting the acoustical loading when the source is placed in a confined environment.

Langley Research Center. The lower photograph in Fig. 10.5 shows the internal microphone removed from the center of the speaker. The internal microphone is used for volume velocity calibration at low frequencies, for measuring and compensating for the effect of acoustical loading (when the loudspeaker is placed in a small, confined environment such as inside an automobile or in the cockpit of an airplane or placed near a boundary) on the volume velocity, and for measuring nonlinear distortion at low frequencies. The volume velocity calibration of the transducer at mid- and high frequencies was accomplished experimentally by calibrating the transducer as a microphone and utilizing the principle of

reciprocity to obtain the volume velocity as a function of the applied voltage to the loudspeakers.

Example 10.1. determine the minimum radius $a_{\min}$ of a pulsating sphere that should radiate 1 W peak acoustical power at 100 Hz for a distortion parameter $n = 0.005$ and $n = 0.01$.

Solution solving Eq. (10.21) for the radius a yields

$$a_{\min} = \begin{cases} 0.088 \text{ m} \cong 3.5 \text{ in.} & \text{for } n = 0.005 \\ 0.075 \text{ m} \cong 3 \text{ in.} & \text{for } n = 0.01 \end{cases}$$

The investigations above indicates that to radiate a practically meaningful amount of sound power at low frequencies requires a sizable sound source. Consequently, a *point sound source* is a practical impossibility. Furthermore, if the point sound source does not exist, then there are no physically meaningful acoustic dipoles and quadrupoles.

Consequently, it would be physically more meaningful to substitute the following in the engineering acoustical vocabulary:

small-volume sound source instead of *monopole* or *point source*,
force (acting on the fluid) instead of *dipole*, and
moment (acting on the fluid) instead of *quadrupole*.

Sound Radiation by Non-Volume-Displacing Sound Sources

Parts (a) and (b) in Figure 10.6 show the response of an unbounded fluid to rigid, non-volume-displacing sound sources and parts (c) and (d) that of volume-displacing sound sources. All of the sources are small compared with the acoustical wavelength. The upper row depicts the body and shows the variables that are responsible for the sound generation. The lower row shows, in a schematic manner, how the fluid particles are pushed aside, creating a near field. The symbol $\tilde{v}_b$ represents the dynamic velocity of the body and $\tilde{v}_n$ the particle velocity in the near field. As we will show latter, the kinetic energy flux of this near field, $\omega(0.5\rho_0\tilde{v}_n)$, integrated over the entire fluid volume outside the body represents the added mass, which is a measure for the vibrating body's ability to exert a dynamic force on the fluid ("grab onto it").

In Fig. 10.6 (a) and (b) are examples of non-volume-displacing sound sources and (c) and (d) of volume-displacing sound sources.

In Fig. 10.6a the upper sketch depicts a small rigid body that exhibits random motion in one dimension with a velocity $\tilde{v}_b$ and the lower sketch depicts the direction and path of the particle velocity in the near field at a given moment in time. Note that the fluid "pushed aside" by the forward face of the body is

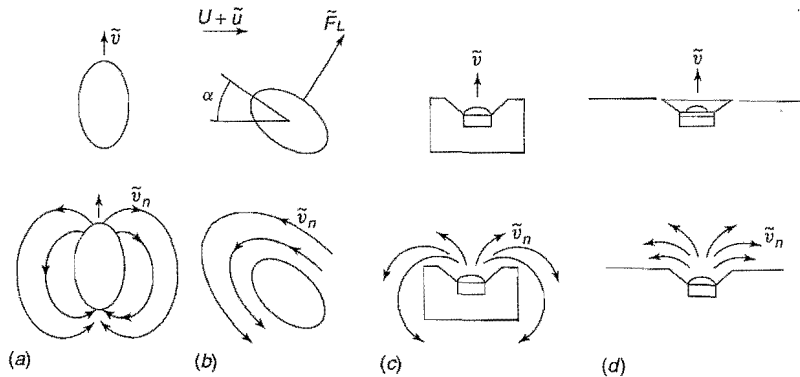


FIGURE 10.6 Schematic representation of various non-volume-displacing and volume-displacing sound sources and the associated particle velocity in the near field. The symbol $\tilde{v}$ is the vibration velocity of the body and $\tilde{v}_n$ is the particle velocity in the near field. (a) Non-volume-displacing sound source; small, rigid body of arbitrary shape moving with a velocity $\tilde{v}$ in an unbounded fluid. (b) Stationary, small, rigid body exposed to a flow field of stationary velocity U with a fluctuating component $\tilde{u}$. The angle of attack is α and the resulting dynamic force is $\tilde{F}_L = C_L (\frac{1}{2} \rho_0 \tilde{u}^2)$ and C_L is the lift coefficient of the body. (c) Volume-displacing small sound source typified by a loudspeaker mounted on the face of an airtight, rigid enclosure. (d) Volume-displacing small sound source flash mounted in an infinitely large, plane, rigid baffle.

“sucked in” by the void created by the receding backward face of the body. This “push-pull” situation results in a very small force. Borrowing from the vocabulary of electronic engineers, this situation is referred to as an “acoustical short circuit.”

In Fig. 10.6b the upper sketch depicts a small stationary body exposed to a flow field of average velocity U and a fluctuating component $\tilde{u}$. The fluctuating component of the flow produces a fluctuating lift force $\tilde{F}_L$ and a fluctuating drag force $\tilde{F}_D$ (the drag force is not shown to preserve clarity).

In Fig. 10.6c the upper sketch shows a loudspeaker mounted on a face of a rigid-walled enclosure. In this case the rigid enclosure prevents the “pulling” by the receding back face of the speaker. Moving aside the fluid by “push” alone, without the help of the “pull,” requires more force. More force acting on the fluid translates into higher sound radiation. This is the reason that a small oscillating surface radiates sound more efficiently if its back side is enclosed than the same surface would radiate if the back side is not enclosed.

Sound sources with an enclosed back side displace the same volume in the back side as on the front side. However, the volume displaced by the back side is prevented from “communicating” with the surrounding fluid by the rigid enclosure. If a dynamic force of constant amplitude drives the vibration velocity (which is the case for our loudspeaker shown in the upper sketch in Fig. 10.6c),

the obtainable vibration velocity is limited either by the volume stiffness of the air in the enclosure volume or by the mass of the moving part of loudspeaker.

If the enclosure is small compared with the acoustical wavelength, the sound pressure that is produced at the surface of the active part has time to spread around the outside wall of the enclosure during each vibration cycle. As a direct consequence of this, all small volume-displacing sound sources, independent of their geometry, have an omnidirectional sound radiation pattern in the far field.

In Fig. 10.6d the upper sketch shows a vibrating surface (in the form of a loudspeaker membrane) flush mounted in a flat infinite rigid baffle. This is a situation that basically resembles the configuration in Fig. 10.6c, except that the sound radiation must take place in a hemisphere (half space). Consequently, for the same volume displacement the mean-square sound pressure p_{rms}^2 at a given radial distance r will be twice as large as an omnidirectionally radiating source would produce.

Response of Unbounded Fluid to Excitation by Oscillating Small Rigid Body. The dynamic “near-point” force acting on the fluid is always in the form of a dynamic pressure over a small area. By *small*, we mean here and in the rest of this chapter that the body is small compared with the acoustic wavelength ($\lambda = c_0/f$).

The quintessential question is, “How does the organized motion that a small rigid body imparts on the fluid spread out, that is, diffuse in various directions with increasing distance through the molecular collision process.

Diffusion of Sound Energy in Response to Excitation of Fluid by Oscillating Small Rigid Sphere. This section deals with the prediction of the sound radiation pattern and the radiated sound power of small, rigid, oscillating bodies for one-dimensional, harmonic, and random motion.

Harmonic Excitation. The sound field that the harmonic, one-dimensional, oscillatory velocity $v(\omega) = \hat{v}_b e^{j\omega t}$ of a small rigid sphere generates is well known and for $r \geq a$ is given by⁴

$$\hat{p}(r, \varphi, \omega) = \hat{v}_b(\omega) \cos(\varphi) \left(\frac{j\omega\rho_0 a^3}{2 - k^2 a^2 + j2ka} \right) \left(\frac{1 + jkr}{r^2} \right) e^{-jk(r-a)} e^{j\omega t} \text{ N/m}^2 \quad (10.22)$$

where $\hat{p}(r, \varphi, \omega)$ = peak sound pressure (N/m²) at distance r (m) in direction φ

$\hat{v}_b$ = peak velocity of body in $\varphi = 0$ direction, m/s

ω = radian frequency, $2\pi f$ Hz

f = frequency of oscillation, Hz

ρ_0 = density fluid, kg/m³

a = radius of sphere, m

k = wavenumber, ω/c_0 , m⁻¹

c_0 = propagation speed of sound, m/s

It is instructive to evaluate Eq. (10.22) at the two extreme cases, namely on the surface of the sphere ($r = a$) and in the geometric far field where $kr \gg 1$.

On the surface of the oscillating sphere Eq. (10.22) yields the reaction pressure of the fluid

$$\hat{p}(a, \varphi, \omega) = \hat{v}_b(\omega) \cos(\varphi) e^{j\omega t} \frac{k^4 a^4 + jka(2 + k^2 a^2)}{4 + k^4 a^4} \quad \text{N/m}^2 \quad (10.23)$$

The radiation impedance of the sphere, defined as the ratio of the surface pressure and the radial velocity $\hat{v}_b \cos(\varphi)$, is

$$Z_{\text{rad}} = \left\{ \begin{array}{l} \rho_0 c_0 \left\{ \frac{k^4 a^4 + jka(2 + k^2 a^2)}{4 + k^4 a^4} \right\} \quad \text{Ns/m}^3 \end{array} \right. \quad (10.24a)$$

$$\left\{ \begin{array}{l} \rho_0 c_0 \left(\frac{k^4 a^4}{4} + \frac{jka}{2} \right) \quad \text{for } ka \ll 1 \end{array} \right. \quad (10.24b)$$

$$\left\{ \begin{array}{l} \rho_0 c_0 \quad \text{for } ka \gg 1 \end{array} \right. \quad (10.24c)$$

The dynamic force exerted on the fluid in the direction of the oscillation, F_{ax} , is obtained by integrating the axial component of the elementary radial force, $dF_x = \cos(\varphi) dF_{\text{rad}} = p(a, \varphi) \cos(\varphi) dS$ over the surface of the sphere, yielding

$$\hat{F}_{\text{ax}} = \left\{ \begin{array}{l} \frac{2}{3} \hat{v}_b \rho_0 c_0 (2\pi a^2) \frac{k^4 a^4 + jka(2 + k^2 a^2)}{4 + k^4 a^4} \quad \text{N} \end{array} \right. \quad (10.25a)$$

$$\left\{ \begin{array}{l} j\omega \rho_0 \hat{v}_b \frac{2\pi a^2}{3} = j\omega \hat{v}_b \rho_0 V_{\text{ad}} \quad \text{for } ka \ll 1 \end{array} \right. \quad (10.25b)$$

$$\left\{ \begin{array}{l} \frac{4\pi a^2}{3} \rho_0 c_0 \hat{v}_b \quad \text{for } ka \gg 1 \end{array} \right. \quad (10.25c)$$

where V_{ad} is the added volume and $\rho_0 V_{\text{ad}}$ is the added mass (as we will define latter on). Observing Eq. (10.24b) note that at low frequencies $\hat{F}_{\text{ax}}$ is the force needed to produce the same acceleration as that of the sphere ($j\omega \hat{v}_b$) on a concentrated mass that is equal the added mass.

The radiated sound power, W_{rad} , is obtained as a product of the oscillatory velocity and the component of the axial force that is in-phase with it, yielding

$$W_{\text{rad}} = \left\{ \begin{array}{l} \frac{1}{2} |\hat{v}_b| \text{Re}\{F_{\text{ax}}\} = \frac{1}{2} |\hat{v}_b|^2 \rho_0 c_0 (4\pi a^2) \frac{k^4 a^4}{4 + k^4 a^4} \quad \text{Nm/s} \end{array} \right. \quad (10.26a)$$

$$\left\{ \begin{array}{l} \frac{1}{2} |\hat{v}_b|^2 \rho_0 c_0 (\pi a^2/3) k^4 a^4 \quad \text{for } ka \ll 1 \end{array} \right. \quad (10.26b)$$

$$\left\{ \begin{array}{l} \frac{1}{2} |\hat{v}_b|^2 \rho_0 c_0 (4\pi a^2/3) \quad \text{for } ka \gg 1 \end{array} \right. \quad (10.26c)$$

Equation (10.25) indicates that at low frequencies ($ka \ll 1$) the radiated sound power is small and increases with the fourth power of the frequency and at high frequencies ($ka \gg 1$) reaches a frequency-independent value.

In the far field ($kr \gg 1$) Eq. (10.22) takes the form

$$\hat{p}(r, \varphi, \omega) = \left\{ \begin{array}{l} -\frac{\hat{v}_b}{r} \cos(\varphi) \rho_0 c_0 k^3 a^3 \frac{(2 - k^2 a^2) + j2ka}{4 + k^4 a^4} \quad \text{N/m}^2 \end{array} \right. \quad (10.27a)$$

$$\left\{ \begin{array}{l} -\frac{\hat{v}_b(\omega) \cos(\varphi)}{4\pi r c_0} \omega^2 \rho_0 2\pi a^3 \quad \text{for } ka \ll 1 \end{array} \right. \quad (10.27b)$$

$$\left\{ \begin{array}{l} -\hat{v}_b \rho_0 c_0 \left(\frac{a}{r} \right) \cos(\varphi) \quad \text{for } ka \gg 1 \end{array} \right. \quad (10.27c)$$

Sound Radiation of Oscillating Small Rigid Nonspherical Bodies. Realizing that $2\pi a^3/2 = 4\pi a^3/3 + 2\pi a^3/3$ and that $V_b = 4\pi a^3/3$ is the volume of the sphere and $V_{\text{ad}} = 2\pi a^3/3$ is the added mass of an oscillating rigid sphere, we can write Eq. (10.24) in the form

$$\hat{p}(r, \varphi, \omega) \approx -\frac{\omega^2 \rho_0}{4\pi r c} \hat{v}_b(\omega) \cos(\varphi) [V_b + V_{\text{ad}}(\square, \Psi)] e^{j(\omega t - kr)} \quad \text{N/m}^2 \quad (10.28)$$

where the symbols $\square$ and Ψ in the argument of V_{ad} signify that the added mass depends on the geometry of the body and the specific direction of its oscillatory motion.

While Eq. (10.27b) is valid only for a small oscillating rigid sphere, Eq. (10.28) is exact for an oscillating sphere and is a good engineering approximation for small rigid bodies of any shape. It is instructive to rewrite Eq. (10.28) in the form

$$\hat{p}(r, \varphi, \omega) \approx -\frac{\omega^2 \rho_0}{4\pi r c_0} \hat{v}_b(\omega) \cos(\varphi) (S_p l_v) e^{j(\omega t - kr)} \quad \text{N/m}^2 \quad (10.29)$$

where S_p is the projected area of the body on a plane perpendicular to the oscillatory motion and

$$l_v = \frac{V_b + V_{\text{ad}}(\square, \Psi)}{S_p} \quad \text{m} \quad (10.30)$$

is a length defined by the author in Eq. (10.30), where the angle Ψ corresponds to the direction of the oscillation of the body. The importance of the length l_v is that it defines the frequency range to $ka \ll 1$, where the body can be considered small and where Eq. (10.29) is valid.

The $\cos(\varphi)$ dependence of the directivity pattern in the vicinity of the oscillating body is strictly valid only for spheres. However, in the far field of any oscillating small rigid body (i.e., $kr \gg 1$ and $kl_v \ll 1$) the same $\cos(\varphi)$ dependence applies provided that the fluid displaced by the forward-moving face of the body is free to move to the void left by the opposite face. A detailed mathematical proof of this is given by Koopmann and Fahline.⁸

Prediction of l_v . The length l_v of various three-dimensional and two-dimensional bodies are listed in Table 10.1. The l_v values in Table 10.1 were computed utilizing the formulas for V_{ad} given by Newman⁹

For the spheroid with major axis $2a$ and minor axes $2b$ and aspect ratio b/a , the *Ver-lengths* l_{vx} , l_{vy} , and l_{vz} are obtained by finding $m'_{11} = V_{ad}(x)/V_b$ and $m'_{22} = V_{ad}(y)/V_b = V_{ad}(z)/V_b$ in Fig. 10.7 at the desired b/a ratio and performing the operation indicated in Table 10.1. The increase in moment of inertia (in addition to that measured in a vacuum) when the ellipsoid body rotates around any of the minor axes is obtained by finding the numerical value of m'_{55} that belongs to the appropriate b/a ratio and performing the operation indicated in Table 10.1.

Predicting Radiated Sound Power. The sound power radiated by small oscillating rigid bodies is obtained by integrating the far-field intensity $I = p^2(r, \varphi, \omega)/\rho_0 c_0$ over the surface of a large sphere of radius r centered at the oscillating small rigid body, namely

$$W_{rad} \simeq \int_0^\pi \frac{|\hat{p}|^2(r, \varphi, \omega)}{\rho_0 c_0} 2\pi r^2 \sin(\varphi) d\varphi \quad \text{N} \cdot \text{m/s} \quad (10.31)$$

Combining Eqs. (10.29) and (10.31) yields

$$W_{rad} \simeq \frac{k^4 \rho_0 c_0 S_p^2 (l_v)^2}{12\pi} v_{b,rms}^2 \quad \text{N} \cdot \text{m/s} \quad (10.32)$$

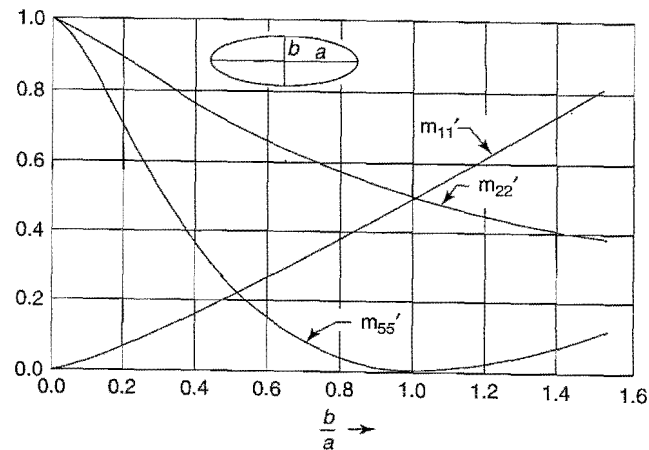
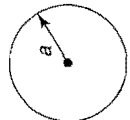
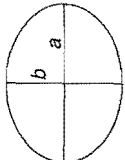
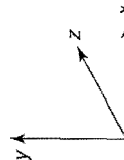


FIGURE 10.7 Added-mass coefficient V_{ad}/V_b for a spheroid, of length $2a$ and maximum diameter $2b$. The added-mass coefficient m'_{11} corresponds to longitudinal acceleration, m'_{22} corresponds to lateral acceleration in the equatorial plane, and m'_{55} denotes the added moment of inertia coefficient for rotation about an axis in the equatorial plane representing the ratio of the added moment of inertia and the moment of inertia of the displaced volume of the fluid. After Ref. 9. Reproduced by permission of the MIT Press.

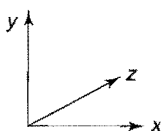
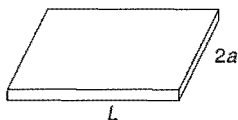
TABLE 10.1 Added Volume, V_{ad} , Ver-length, l_v , and Added Moment of Inertia M of Oscillating and Rotating Rigid Bodies

Name and Sketch	V_{ad} and l_v	M
Sphere of Radius a 	$V_{ad}(x) = V_{ad}(y) = V_{ad}(z) = 2\pi a^3/3$ $l_{vx} = l_{vy} = l_{vz} = 2a$	$M_x = M_y = M_z = 0$
Spheroid of aspect ratio a/b  	$V_{ad}(x) = (\frac{4}{3})\pi ab^2 m'_{11}$ $V_{ad}(y) = V_{ad}(z) = (\frac{4}{3})\pi ab^2 m'_{22}$ $l_{vx} = (\frac{4}{3})a(1 + m'_{11})$ $l_{vy} = l_{vz} = (\frac{4}{3})\left(\frac{b^2}{a}\right)(1 + m'_{22})$	$M_x = 0$ $M_y = M_z = (\frac{4}{15})\pi \rho_0 ab^2(a^2 + b^2)$

(continued overleaf)

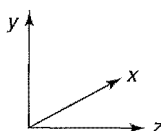
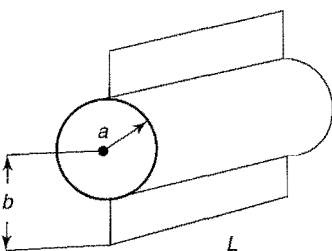
TABLE 10.1 (continued)

Name and Sketch	V_{ad} and I_v	M
Long thin plate strip $L \gg 2a; kL \ll 1$	$V_{ad}(x) = V_{ad}(z) = 0$ $V_{ad}(y) = \pi a^2 L$ $I_{vx} = I_{vz} = 0$ $I_{vy} = (\pi/2)a$	$M_x = (\frac{1}{8})\pi\rho_0 a^4 L$

Long round finned rod $L \gg 2a; kL \ll 1$

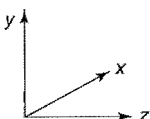
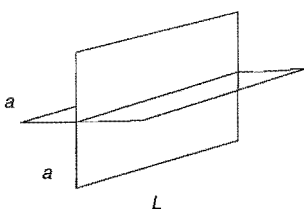
$$\begin{aligned}
 V_{ad}(x) &= N\pi I^a \\
 V_{ad}(y) &= \pi a^2 L \\
 V_{ad}(z) &= \pi[a^2 + (b^2 - a^2)^2/b^2]L \\
 I_{vx} &= N\pi I^a \\
 I_{vy} &= \pi a \\
 I_{vz} &= \pi[a^2/b + (b^2 - a^2)^2/2b^3]
 \end{aligned}$$

$$M_x = \text{see footnote } b$$

Long rectangular plates crossing perpendicularly $L \gg 2a; kL \ll 1$

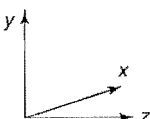
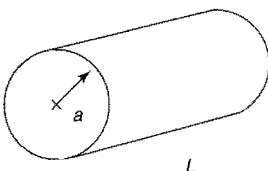
$$\begin{aligned}
 V_{ad}(x) &= N\pi I^a \\
 V_{ad}(y) &= V_{ad}(z) = \pi a^2 L \\
 I_{vx} &= N\pi I^a \\
 I_{vy} &= I_{vz} = (\pi/2)a
 \end{aligned}$$

$$M_x = (2/\pi)a^4 L$$

Long cylindrical rod of radius a $L \gg 2a; kL \ll 1$

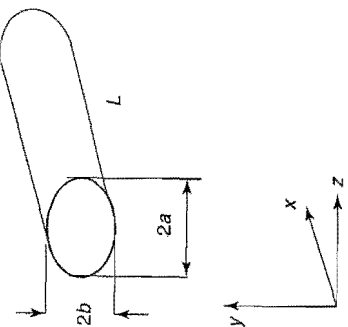
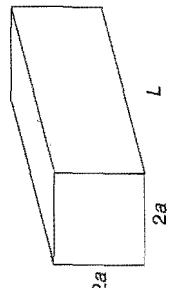
$$\begin{aligned}
 V_{ad}(x) &= N\pi I^a \\
 V_{ad}(y) &= V_{ad}(z) = \pi a^2 L \\
 I_{vx} &= N\pi I^a \\
 I_{vy} &= I_{vz} = \pi a
 \end{aligned}$$

$$M_x = 0$$



(continued overleaf)

TABLE 10.1 (continued)

Name and Sketch	V_{ad} and I_v	M
Long rod with elliptical cross section $L \gg 2a; kL \ll 1$	 $V_{ad}(x) = NPl^a$ $V_{ad}(y) = \pi a^2 L$ $V_{ad}(z) = \pi b^2 L$ $I_{vx} = NPl^a$ $I_{vy} = I_{vz} = (\pi/2)(a+b)$	$M_x = (\frac{1}{8})\pi\rho_0 a^4 L$
Long rod of square cross section $L \gg 2a; kL \ll 1$	 $V_{ad}(x) = NPl^a$ $V_{ad}(y) = V_{ad}(z) = 4.754a^2 L$ $I_{vx} = NPl^a$ $I_{vy} = I_{vz} = 2.38a$	$M_x = 0.725a^4 L$

^a NPl = not of practical interest
^b $M_x = \{\pi^{-1} \csc^4(\alpha) [2a^2 - \alpha \sin(4\alpha) + (1/2) \sin^2(2\alpha) - \pi/2], \sin(\alpha) = 2ab/(a^2 + b^2), \text{ and } \pi/2 < \alpha < \pi$

For a oscillating, small, rigid spherical body of radius a the area projected on the plane perpendicular to the direction of the oscillatory motion $S_p = \pi a^2$ and $l_v = 2a$ and Eq. (10.32) yield

$$W_{\text{rad}}(\text{Sphere}) = \frac{\rho_0 c_0 (4\pi a^2) (ka)^4}{24} |v_b|^2 \quad \text{N} \cdot \text{m/s} \quad (10.33)$$

This is the well-known^{3,4} formula for the sound power radiated by a small ($ka \ll 1$) sphere of radius a oscillating with an rms velocity amplitude of $v_{b,\text{rms}}$.

Random Excitation. Let us investigate the case where a small rigid sphere exhibits random motion in one dimension (e.g., in the x direction). As described in Chapter 3, the random motion is characterized by the power spectrum of its velocity in the x direction, $S_{vv}(x)$.

The power spectrum, as defined in Chapter 3, represents the quantity obtained by filtering the random time signal in a 1-Hz bandwidth multiplying by itself and time averaging the product. In this chapter we use the symbol S with a double subscript to represent a power spectrum or cross spectrum. The double-letter subscript of S identifies which of the two filtered signal are multiplied with the other before time averaging. For example the symbol $S_{vv}(\omega)$ represents the autospectrum of the random velocity signal filtered in a 1-Hz bandwidth centered at the angular frequency $\omega = 2\pi f$, where f is the frequency in hertz. The symbol $S_{xy}(\omega)$ represents a cross spectrum of two random signals $x(t)$ and $y(t)$ obtained by filtering both signals in a 1-Hz bandwidth, multiplying them, and time averaging the product. The power spectrum of a signal resulting from the superposition of two signals $z(t) = x(t) + y(t)$ is computed as $S_{zz}(\omega) = S_{xx}(\omega) + S_{yy}(\omega) + 2S_{xy}(\omega)$. If the two random processes $x(t)$ and $y(t)$ are not correlated, $S_{xy}(\omega) = 0$. If the two random processes of the same power spectral density are fully correlated (i.e., they have the same cause), the cross spectrum is identical to the autospectrum, $S_{xy}(\omega) = S_{xx}(\omega) = S_{yy}(\omega)$ and $S_{zz}(\omega) = 4S_{xx}(\omega) = 4S_{yy}(\omega)$.

If the velocity of the vibrating body is exclusively in the x direction and is a random function of time, with a power spectrum of $S_{v_x v_x}(\omega)$ the power spectrum of the sound at distance r in the direction ϕ from the positive x axis, $S_{pp}(r, \phi, \omega)$, is

$$S_{pp}(\omega, r, \phi) = S_{v_x v_x}(\omega) |h_1(\omega, \rho_0, k, a, r, \phi)|^2 \quad \text{N}^2/\text{m}^4 \quad (10.34)$$

where

$$h_1(\omega, \rho_0, k, l_v, r, \phi) = \frac{\omega^2 \rho_0}{4\pi r c_0} \cos(\phi) (S_p l_v) e^{jkr} \quad (10.35)$$

Hypothetical Experiment 2: Force Acting on Fluid. To find the dynamic force acting on the fluid F_f , let us perform the thought experiment shown in Fig. 10.8. Let a dynamic force $\mathbf{F}_b$ act on a small, rigid, hollow body of arbitrary shape at its center of gravity and determine the oscillatory velocity $\mathbf{v}_b$ of the

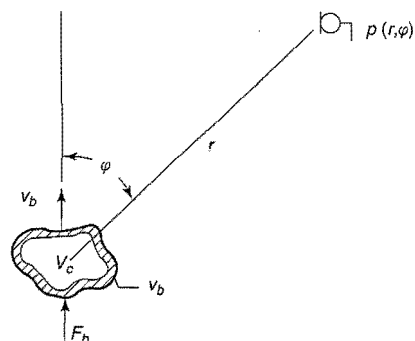


FIGURE 10.8 Hypothetical experiment to determining the force F_f that acts on the fluid and the sound field it produces by letting the wall thickness of an oscillating, hollow rigid body approach zero.

body responding to the force as

$$\mathbf{F}_b = \mathbf{v}_b(j\omega m_b + Z_{\text{rad}}) \quad \text{N} \quad (10.36)$$

where m_b is the mass of the body and $Z_{\text{rad}} = F_f/v_b$ is the radiation impedance:

$$\mathbf{F}_b = j\omega \mathbf{v}_b[\rho_M(V_b - V_c) + \rho_0 V_c + \rho_0 V_{\text{ad}}(\square, \Psi)] \quad \text{N} \quad (10.37)$$

where V_b is the volume of the oscillating, hollow, rigid body and V_c is the volume of the inside cavity, filled with the same fluid that surrounds the body. Both volumes are in cubic meters. The symbol ρ_M represents the density of the solid part of the body in kilograms per cubic meter. The symbol V_{ad} , in cubic meters, is the added volume, or in the hydrodynamic vocabulary the added mass coefficient. The symbols $\square$ and Ψ in the argument of V_{ad} are to remind us that it is a function of the body's geometry and the particular angle of its oscillatory motion to one of the designated axes of the body.

Let us now investigate the very important extreme case of Eq. (10.37) where the wall thickness of the hollow body approaches zero (and still remains rigid) by setting $V_c \rightarrow V_b$. In this case the entire applied dynamic excitation force $F_b = F_f$ is acting directly on the fluid, yielding

$$\mathbf{F}_f = v_b j\omega \rho_0 V_b + \mathbf{v}_b j\omega \rho_0 V_{\text{ad}}(\square, \Psi) \quad \text{N} \quad (10.38)$$

The first term in Eq. (10.38) represents the flux of the momentum of the fluid volume displaced by the body, $\rho_0 V_b[\partial(v_b)/\partial t]$, and the second term, $\rho_0 V_{\text{ad}}(\square, \Psi)[\partial(v_b)/\partial t]$, the flux of the momentum of the near field obtained by integrating the component of the particle velocity that is out phase with the velocity of the body over the entire volume outside of the body. Fortunately, we do not need to perform this integration to obtain $V_{\text{ad}}(\square, \Psi)$ because

- hydrodynamists have computed it for many body geometries of interest (see Table 10.1 and Fig. 10.7),
- for most bodies of practical interest it can be well approximated by spheroids of the appropriate aspect ratio (see Fig. 10.7), and
- it can be determined experimentally.

Equation (10.38) is of fundamental importance. It signifies the following:

- The force acting on the fluid acts in the direction of the oscillatory motion.
- Two-dimensional bodies, which have no volume ($V_b = 0$) can exert force on the fluid provided their velocity has a component perpendicular to their plane and, consequently, $V_{\text{ad}} \neq 0$.
- No force can be exerted on the fluid without a finite added mass (i.e., $V_{\text{ad}} \neq 0$).

The far-field sound pressure as a function of distance r , direction φ , and frequency ω produced by a pointlike force F_f is obtained by solving Eq. (10.38) for v_b and inserting this value into Eq. (10.25), yielding

$$p(r, \varphi, \omega) = -j \frac{k F_f \cos(\varphi)}{4\pi r} e^{j(\omega t - kr)} = -j \frac{F_f \cos(\varphi)}{2r\lambda} e^{j(\omega t - kr)} \quad \text{N/m}^2 \quad (10.39)$$

The radiated sound power W_{rad} is obtained by integrating the intensity $I = p^2(r, \varphi, \omega)/\rho_0 c_0$ over the surface of a sphere of radius $r \gg (1/k)$ centered on the excitation point. Carrying out the integration yields

$$W_{\text{rad}} = \frac{k^2 F_{f,\text{rms}}^2}{12\pi \rho_0 c_0} = \frac{\omega^2 F_{f,\text{rms}}^2}{12\pi \rho_0 c_0^3} \quad \text{N} \cdot \text{m/s} \quad (10.40)$$

If the excitation is random, the excitation and response parameters have to be replaced by their respective power spectra.

Equation (10.39) describes the diffusion law of acoustics. It gives the full answer to the question "How does this organized motion—that is, in the direction of the force at the application point—diffuse with increasing distance in other directions through the molecular collision process?"

Equation (10.39) signifies that:

- The sound pressure in any particular direction is proportional the projection of the force vector in that direction, $F_f \cos(\phi)$, indicating that the strength of the diffusion varies as $\cos(\phi)$. The radiated sound pressure has its maximum in the direction of the force and it is zero perpendicular to it. This is consistent with the fact that the oscillatory motion of the body exerts on the fluid the maximum force in the motion direction and no force at all in the direction perpendicular to it. The directivity pattern $\cos(\phi)$ in three dimensions is represented by two spheres of unit diameter aligned in the

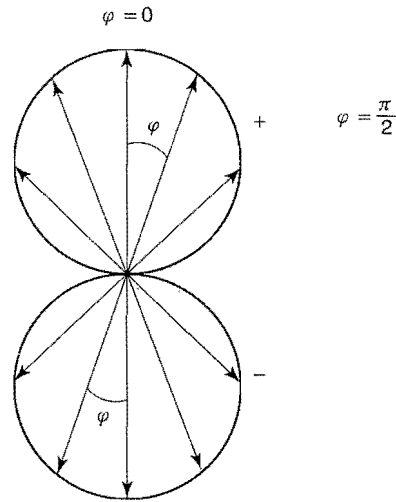


FIGURE 10.9 Far-field sound radiation pattern of a small rigid body oscillating in the $\phi = 0$ direction or an oscillatory force acting on the fluid in the $\phi = 0$ direction.

direction of the oscillatory motion or the direction of the force and centered on the line of the oscillating body or on the point where the force acts on the fluid, as shown (in two dimensions) in Fig. 10.9.

2. The sound pressure in any direction is inversely proportional to the source-receiver distance r .
3. Independent of what causes the dynamic force F_f which acts on the fluid, the sound pressure fully determines the sound radiation pattern and sound power output of small rigid bodies. The force can be, for example, the reaction force to the oscillatory movement of the body or that caused by fluctuating lift or drag forces generated on small stationary bodies by the fluctuation of the inflow velocity or by vortex shedding at the trailing edge of airfoils. Fluctuating inflow is caused by incoming turbulence of a scale that is large compared with the size of the body. If the scale of the convected pressure fluctuations (turbulence) is large compared with the size of the rigid body, the force would be identical to that obtained by moving the body in a stationary fluid in the direction and with the velocity of the fluctuating component of the unsteady inflow and predicting lift and drag forces.

However, a rigid stationary body exposed to fluctuating aerodynamic forces radiates sound into a moving media. Consequently, the radiated sound pressure and sound power is different from that an oscillating rigid body, that exerts the same dynamic force on the fluid, would produce in a stationary media. The effect of moving media on sound radiation is treated in Chapter 15.

4. The sound pressure is independent of the density of the surrounding fluid. This is because the acceleration of the fluid caused by the force is inversely proportional to the density and the sound pressure is proportional to the product of the density and acceleration.

If the force applied to the F_f is of random nature, it is characterized by its power spectral density $S_{FF}(\omega)$ and the sound pressure it produces by its power spectral density $S_{pp}(r, \phi, \omega)$, as defined in Chapter 3. In this case Eq. (10.39) takes the form

$$S_{pp}(\omega, r, \phi) = S_{FF}(\omega) \left| \frac{\omega \cos(\phi)}{4\pi r c_0} \right|^2 \quad (\text{N/m}^2)^2 \quad (10.41)$$

and the power spectrum of the radiated sound power takes the form

$$S_w(\omega) = S_{FF}^2(\omega) \left| \frac{\omega^2}{12\pi \rho_0^2 c_0^3} \right|^2 \quad (\text{N} \cdot \text{m/s})^2 \quad (10.42)$$

Response of Bounded Fluid to Point Force Excitation. It is instructive to investigate briefly what effects nearby boundaries have on the sound radiation pattern and on the sound power radiated by a point force. We will consider

1. the most severe restricting boundaries, surrounding the oscillating rigid body by an infinitely long (or finite-length but at both ends anechoically terminated) small-diameter rigid pipe;
2. rigid plane boundaries; and
3. yielding plane boundaries.

Rigid Pipe Surrounding Point Force. Figure 10.10 shows the situation where a point force of arbitrary orientation is acting in a fluid inside of a pipe of diameter d and cross-sectional area $S = d^2\pi/4 = r^2\pi$ that is anechoically terminated at both ends. Let the axis of the pipe coincide with the x axis of the Cartesian coordinate system and let the point force, which acts at $x = 0$, have components F_x , F_y , and F_z . [If we place a small rigid oscillating body with known velocity v_b into the tube, we must first determine the oscillatory force F_f according to Eq. (10.38).] Let restrict our analysis to the frequency range where the acoustical wavelength is large compared with the internal diameter of the pipe (i.e., $\lambda \gg d$) when only plain sound waves can propagate in the pipe and assume that

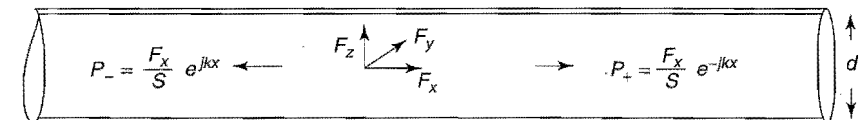


FIGURE 10.10 Force exerted on the fluid inside of a small-diameter, infinitely long, or on both ends anechoically terminated rigid pipe.

the near field of the body giving rise to the force is not influenced by the presence of the pipe (i.e., $kl_b \ll d$). This is not an unreasonable assumption because we can always choose a reasonably small size body and compensate for it by appropriately increasing its oscillatory velocity (as long as $v_b \ll c_0$) to produce the same force on the fluid. With these minor restrictions, the sound pressure at a distance $x \gg d$ the sound pressure $p(x)$ and the power spectrum S_{pp} for any $x \gg d$ will be

$$p(x) = -p(-x) = \left(\frac{F_x}{S} \right) e^{-kx} e^{j\omega t} \quad \text{N/m}^2 \quad (10.43)$$

$$S_{pp}(f) = \left(\frac{S_{FF}(f)}{S} \right) \quad \text{N/m}^2 \quad (10.44)$$

Note that Eq. (10.40) signifies that the amplitude of the sound propagating inside the rigid pipe does not diminish with increasing distance, as was the case when the point force was acting on an unbounded fluid [see Eq. (10.39)]. Before electroacoustical communication was invented, sea captains communicated with distant crew in the belly of a ship through pipes, as shown in Fig. 10.11a, and on the deck by using an acoustical horn, as depicted in Fig. 10.11b.

Equal sound power is radiated toward the right and left and the total sound power is

$$W'_{\text{rad}} = \frac{2[(F_x)^2/S]}{\rho_0 c_0} \quad \text{N/m}^2 \quad (10.45)$$

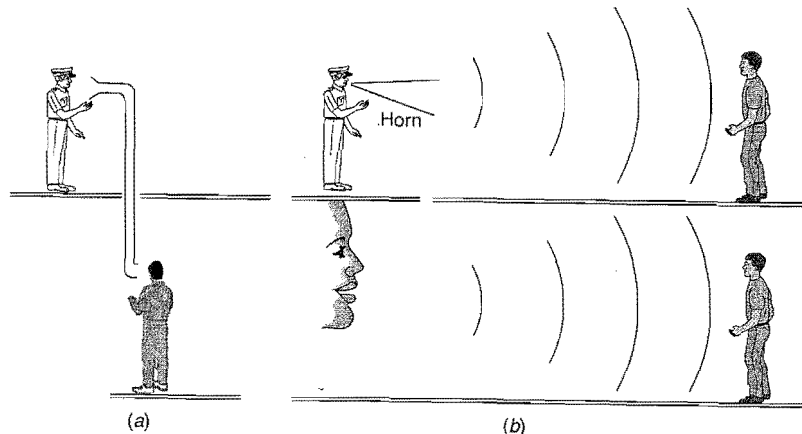


FIGURE 10.11 Passive devices used for voice communication to large distances before the invention of electroacoustical devices. (a) Captain gives orders to crew member located in a lower deck. (b) Top: Captain gives orders to distant crew member on deck using an acoustical horn. Bottom: Using a horn is equivalent to enlarging the mouth of the speaker to the size of the downstream cross-sectional area of the horn.

$$S'_{ww}(f) = \frac{S_{F_x F_x}^2(f)}{|S(\rho_0 c_0)|^2} \quad \text{N/m}^2 \quad (10.46)$$

where $S'_{ww}(f)$ is the power spectrum density of the radiated sound power. The prime notation signifies that the parameter is obtained when the point force is acting inside a small-diameter ($kr \ll 1$) rigid pipe.

The ratio $W'_{\text{rad}}/W_{\text{rad}}$ is a measure of the effect that enclosing the force with a rigid pipe has on the radiated sound power. It is obtained by dividing Eq. (10.45) by Eq. (10.39), yielding

$$\frac{W'_{\text{rad}}}{W_{\text{rad}}} = \left(\frac{6\pi}{S} \right) \left(\frac{1}{k^2} \right) \left\{ \frac{1}{1 + (F_y/F_x)^2 + (F_z/F_x)^2} \right\} \quad (10.47)$$

Equation (10.47) indicates that no sound power is radiated if $F_x = 0$, independent of how large F_y and F_z might be. The effect of placing the force inside the pipe reaches its maximum if the force is aligned with the axis of the tube (i.e., $F_y = F_z = 0$) and the value inside the curly brackets is unity. In this case Eq. (10.47) simplifies to

$$\frac{W'_{\text{rad}}}{W_{\text{rad}}} = \left(\frac{3}{2\pi} \right) \frac{c_0^2}{sf^2} \quad (10.48)$$

Example 10.2. Calculate the increase in radiated sound power and the sound power level when an axially oriented force located in a rigid pipe of $a = 2.5 \text{ cm} = 0.025 \text{ m}$ internal radius at a frequency $f = 100 \text{ Hz}$ at a temperature of $20^\circ\text{C} = 68^\circ\text{F}$.

Solution Utilizing Eq. (10.2), we find $c_0 = 342.6 \text{ m/s}$ and with the above values Eq. (10.48) yields

$$\frac{W'_{\text{rad}}}{W_{\text{rad}}} = \frac{(3/2\pi)(342.6)^2}{\pi(0.025)^2(100)^2} = 2854$$

indicating that the same point force oriented axially in this rigid tube radiates 2854 times as much sound power than it does in a unbounded fluid. This corresponds to a $10 \log_{10}(2854) = 34.6 \text{ dB}$ increase of the sound power level.

10.2 EFFECT OF NEARBY PLANE BOUNDARIES ON SOUND RADIATION

In this section we only consider entirely rigid and fully yielding plane boundaries. Other boundaries of practical interest, such as lining the rigid boundary with a porous sound-absorbing material of finite thickness, is beyond the scope of this chapter. For guidance in this situation see Chapter 6. The infinitely rigid plane boundary is approached by an infinitely large, nonporous wall and the fully yielding plane boundary by the undisturbed horizontal air–water interface. With

regard to their effect on the sound power radiated by a point force (or by an oscillating small rigid body), the plane boundaries do not need to be infinitely large. For practical purposes it is sufficient that they extend, in each direction from the position of the force, many acoustical wavelengths.

A water tank with practically all boundaries pressure releasing (pressure reflection coefficient $R \simeq -1$) is realized by lining the retaining sidewalls and the bottom of the pool with a thin impervious foil with an elastic layer behind. These pools are the equivalent of reverberation chambers with the important exception that in this case the sound pressure reflection coefficient $R \approx -1$ instead of $R \approx +1$ in the case of the (air-filled) reverberation chamber. In both cases the walls have theoretically zero sound absorption coefficient, $\alpha \cong [1 - |R|^2]$ (see Chapter 8). As we will show later, the vicinity of a rigid boundary increases the sound power output and that of a yielding boundary decreases the sound power output provided that the sound source is less than a half wavelength from the plain boundary.

Effect of Nearby Plane Rigid Boundaries. Rigid plane boundaries are places where the particle velocity is zero and the sound pressure doubles. If the sound source is a small, oscillating, rigid body or a dynamic force acting on the fluid, the directivity pattern of the image source must have the same strength but a mirrored directivity pattern of the actual sound source to produce zero particle velocity in the direction perpendicular to the plane of the boundary and, at the same time, double the sound pressure at all locations in the plane of the rigid boundary, as illustrated in Fig. 10.12.

The sound field in the receiver side of the rigid plane boundary can be constructed as the superposition of the sound fields that would result from the real sound source and its mirror image if both were placed in an infinite fluid. Then, as the sound source approaches the plane rigid interface, the sound field on the source side approaches that of a single source with doubled source strength.

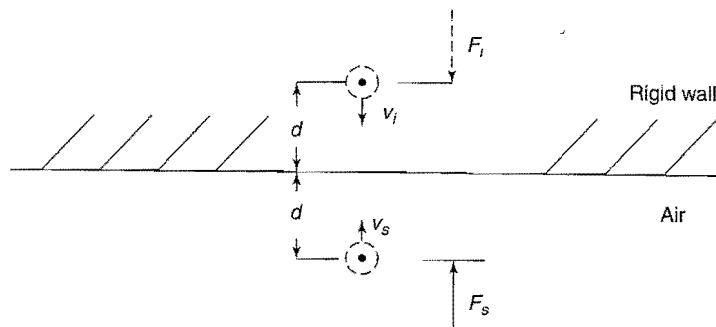


FIGURE 10.12 Representation of a plane rigid boundary, such as at air-hard wall interface, by mirroring the original sound source and superimposing the sound field of the real and in-phase image source in the source-side half space.

If it approaches a rigid two-dimensional corner (i.e., the intersection of two perpendicular plane rigid surfaces), there will be three collapsing mirror images and the sound field in the source-side quarter space approaches that of a single source with 4 times larger source strength. Finally, if the sound source approaches a three-dimensional corner (the intersection of three perpendicular rigid plane surfaces), there will be seven collapsing mirror images and the sound field in the source-side one-eighth space approaches that of a single source with 8 times the source strength.

Effect of Nearby Plane Pressure-Release Boundaries. The pressure-release boundary is well approximated by the plane water-air interface shown schematically in Fig. 10.13 for the force acting perpendicular to the interface and Fig. 10.14 when it acts parallel to the interface. For an oblique-incidence force, the force vector can be decomposed into a component that is perpendicular to the interface and into components that are parallel to the interface. Consequently, we will investigate the perpendicular and parallel cases only. If the sound source is a small, oscillating, rigid body or a dynamic force acting on the fluid, the directivity pattern of the image source must have the same strength but opposite phase and a mirrored directivity pattern of the actual sound source to produce zero sound pressure on the entire interface and, at the same time, double the sound particle velocity at all locations in the plane of the pressure release boundary.

Excitation Is Perpendicular to Pressure-Release Boundary. For the case shown in Fig. 10.13, where the distance to the interface is small compared to the wavelength ($kd \ll 1$) and the body exerting the force is small ($kl_v \ll 1$) and the direction of motion and the force exerted on the fluid are perpendicular to the plane pressure-release interface, the superposition of the sound pressures produced by the real source and that produced by its negative mirror image yields

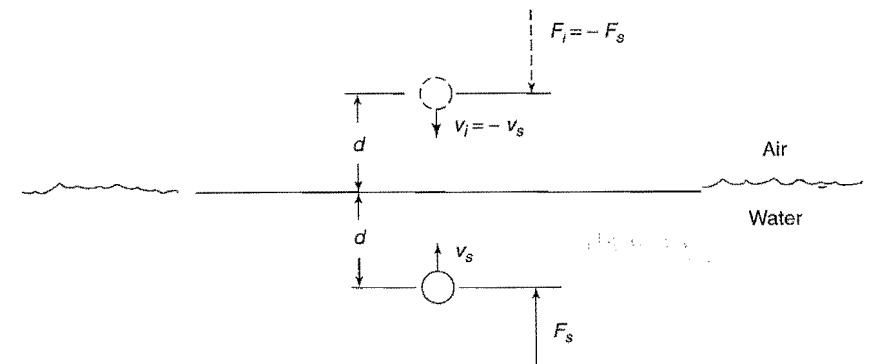


FIGURE 10.13 Representation of a plane pressure-release boundary, such as the water-air interface, by mirroring the original sound source and the 180° out-of-phase mirror image superimposing the sound field of the real and image sources in the source-side half space. Motion is perpendicular to the interface.

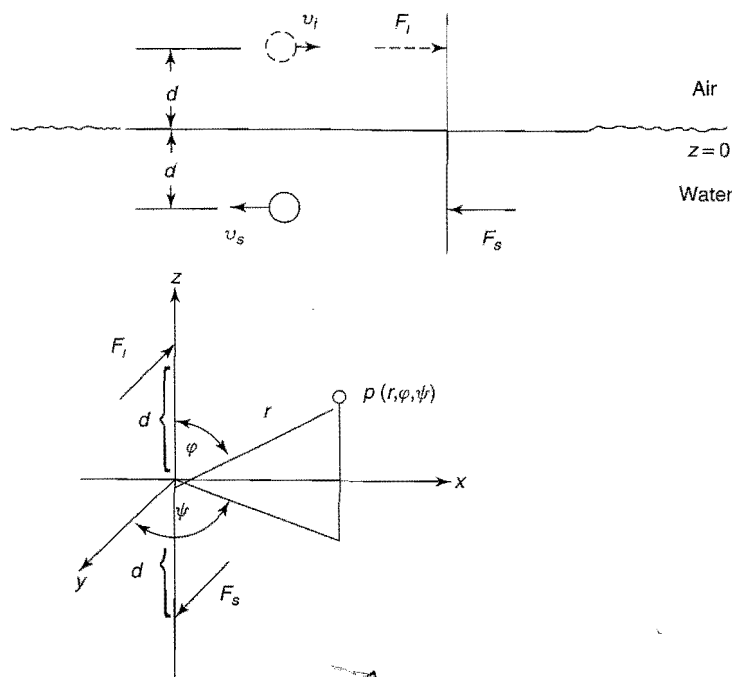


FIGURE 10.14 Representation of a plane pressure-release boundary, such as the water-air interface, by mirroring the original sound source 180° out of phase and superimposing the sound field of the original sound source and the image in the source-side half space. Motion is parallel to the interface.

the sound pressure, intensity, and radiated sound power in the liquid obtained by substituting $F = \rho_0 \omega Q h$ in the respective formulas given by Bies and Hansen¹⁰ for the sound radiation of longitudinal quadrupoles constituted from two dipoles of strength Qh , where Q is the volume velocity of the monopoles that make up the dipole and $2h$ is the distance between them:

$$p(r, \varphi, \omega) = \left(\frac{1}{\pi r} \right) \left(\frac{\omega^2}{c_0^2} \right) F d \cos^2(\varphi) = \left(\frac{1}{\pi r} \right) k^2 F d \cos^2(\varphi) \quad \text{N/m}^2 \quad (10.49)$$

$$I(r, \varphi, \omega) = \left(\frac{F^2 d^2 k^4 \cos^4(\varphi)}{(\pi r)^2 \rho_0 c_0} \right) = \frac{F^2 d^2 \omega^4 \cos^4(\varphi)}{(\pi r)^2 \rho_0 c_0^5} \quad \text{N/m} \cdot \text{s} \quad (10.50)$$

$$W_{\text{rad}} = \left(\frac{4}{5\pi} \right) \frac{F^2 d^2 k^4}{\rho_0 c_0} = \left(\frac{4}{5\pi} \right) F^2 d^2 \omega^4 \cos^4(\varphi) \quad \text{N} \cdot \text{m/s} \quad (10.51)$$

where $p(r, \varphi, \omega)$ is the sound pressure at distance r at an angle φ from the direction of the motion or from the force at the angular frequency $\omega = 2\pi f$, $I(r, \varphi, \omega)$ is the sound intensity in the radial direction from the excitation point,

W_{rad} is the radiated sound power, F is the rms value of the force, and d is the distance between the excitation point and the plain pressure-release boundary (i.e., the depth of the excitation point in the water).

Note that Eqs. (10.49)–(10.51) are valid for both cases:

- A single excitation source is at a distance d from the plane pressure-release boundary.
- Two identical small bodies at a distance $2d$ oscillate with the same amplitude but opposite phase in an infinite fluid (for practical purposes far enough from the interface).

Observing Eq. (10.49), note that (1) the radiation pattern is much more directional than that obtained when the body radiates into an infinite fluid [$\cos^2(\varphi)$ vs. $\cos(\varphi)$]; (2) here again the sound pressure is independent of the density of the fluid; and (3) comparing Eq. (10.49) to Eq. (10.39) reveals that the vicinity of the pressure-release boundary substantially reduces the efficiency of the sound radiation at low frequencies. Those familiar with quadrupoles will realize that Eqs. (10.49)–(10.51) describe a longitudinal quadrupole constituted either by two opposing forces of equal magnitude acting on the fluid a short distance $2d$ apart or by two equal-size small rigid bodies oscillating in opposite directions in a small distance $2d$ apart.

Excitation Is Parallel to Pressure-Release Boundary. Figure 10.14 shows the situation where the excitation is parallel to the plane pressure-release boundary. For this case the sound field descriptors, obtained in a similar manner as for the longitudinal quadrupole above, are

$$p(r, \varphi, \omega) = \left(\frac{1}{\pi c_0^2 r} \right) \omega^2 F d \cos(\varphi) \sin^2(\psi) \quad \text{N/m}^2 \quad (10.52)$$

$$I(r, \varphi, \psi, \omega) = \left(\frac{1}{\rho_0 c_0^5} \right) \left(\frac{1}{\pi r} \right)^2 \omega^4 (F d)^2 \cos^2(\varphi) \sin^4(\psi) \quad \text{N/m} \cdot \text{s} \quad (10.53)$$

$$W_{\text{rad}} = \left(\frac{4}{15\pi} \right) \left[\frac{(F d)^2 k^4}{\rho_0 c_0} \right] = \left(\frac{4}{15\pi} \right) \left[\frac{(F d)^2 \omega^4}{\rho_0 c_0^5} \right] \quad \text{N} \cdot \text{m/s} \quad (10.54)$$

Those familiar with quadrupoles will realize that Eqs. (10.52)–(10.54) describe a lateral quadrupole.

Sound Radiation of Oscillating Moment in Infinite Liquid. An oscillating moment consists of a pair of oscillating forces which (a) act in the same plane, (b) have the same magnitude but opposite direction and phase, and (c) are at a short distance apart.

Observing Fig. 10.14, we realize that the force pair has all the above attributes. Consequently, if we substitute M in place of Fd in Eqs. (10.52)–(10.54), the