

FIGURE 10.15 Different ways of exerting a moment on a fluid: (a) rotary oscillation around axes of nonrevolution; (b) rotation around axes of nonrevolution; (c) by shear stresses.

equations describe the sound radiation of an oscillating moment, yielding

$$p(r, \varphi, \omega) = \left(\frac{1}{\pi c_0^2 r} \right) \omega^2 M \cos(\varphi) \sin^2(\psi) \quad \text{N/m}^2 \quad (10.55)$$

$$I(r, \varphi, \psi, \omega) = \left(\frac{1}{\rho_0 c_0^5} \right) \left(\frac{1}{\pi r} \right)^2 \omega^4 M^2 \cos^2(\varphi) \sin^4(\psi) \quad \text{N/m} \cdot \text{s} \quad (10.56)$$

$$W_{\text{rad}} = \left(\frac{4}{15\pi} \right) \left[\frac{M^2 k^4}{\rho_0 c_0} \right] = \left(\frac{4}{15\pi} \right) \left[\frac{M^2 \omega^4}{\rho_0 c_0^5} \right] \quad \text{N} \cdot \text{m/s} \quad (10.57)$$

The moment can be any combination of F and d provided that $kd \ll 1$.

As shown in Fig. 10.15, the moment M can be the result of the oscillatory rotation of solid bodies around axes of nonrotational symmetry, shear stresses acting on the fluid (as it is the main source of noise of high-speed jets), or any other acceleration motion that results in a increase of the rotary inertia of the body above that which would exist if the oscillatory rotation would be taking place in a vacuum. For a large number of body shapes this additional moment of rotary inertia M that acts on the fluid is given in Table 10.1 under heading M_x and for ellipsoids of different aspect ratios in Fig. 10.7.

Figure 10.15 shows various ways for the moment excitation of a fluid. Figure 10.15a depicts moment excitation by an oscillatory rotation of a body around a nonsymmetric axis, and Fig. 10.15b shows excitation by steady rotation along the same axes. Figure 10.15c shows moment excitation by shear forces, where $\vec{F} = \vec{\sigma} dS$ acts on a small fluid volume $dV = 2(dS)d$. The symbol $\vec{\sigma}$ is the fluctuating shear stress in newtons per square meter. This is the type of moment excitation found in the shear layer of high-speed jets and is treated in Chapter 15.

10.3 MEASURES TO REDUCE SOUND RADIATION

In this section we will discuss some of the measures noise control engineers and machine designers might employ to minimize the sound radiation of vibrating bodies. Most of these measures have been already discussed in a qualitative manner in Section 10.1. In this section we will show certain specific measures in

the form of conceptual sketches and give references where the reader can find more detailed design information.

Fish as Example for Minimized Added Mass. Through evolutionary development, fish have developed body shapes that minimize the reaction force on them when they accelerate their motion in the forward direction. If not in motion, most fish have their pectoral fins deployed perpendicular to their body. If they sense danger and want to dart away as fast as possible, the pectoral fins move rapidly backward (just like the oars of a paddleboat). For moving their pectoral fin in this direction they have a large added mass and can produce a large impulsive force on the water, which propels them forward. As the pectoral fins come near their body, the fins rotate 90° to align their body so that they contribute only a negligible amount to the friction coefficient. After this first impulsive burst, the pectoral fins are not deployed again until the fish want to use them to rapidly stop forward motion. With pectoral fins aligned the forward motion is propelled by the movement of the tail. At rest they deploy the pectoral fins again to be ready for the next dart.

To survive, predator fish must develop a body that minimizes the added mass for swimming in the forward direction so that they can generate a minimum of waterborne sound when they increase their speed (accelerate) when closing in on their prey. Otherwise the prey would sense the sound generated by their acceleration early enough to dart into a safe hiding space. In choosing body shapes that, for a given oscillatory velocity, radiate the least amount of sound in the direction of the body's motion, noise control engineers are well advised to choose the shape of the vibrating body to resemble the body of a fish.

As shown in Fig. 10.16, the most promising body shape would resemble that obtained by cutting two identical-size fish perpendicularly at the location of their largest cross-sectional area and fitting the two head-parts together so that the two mouths point in the opposite direction. For motion in a direction different

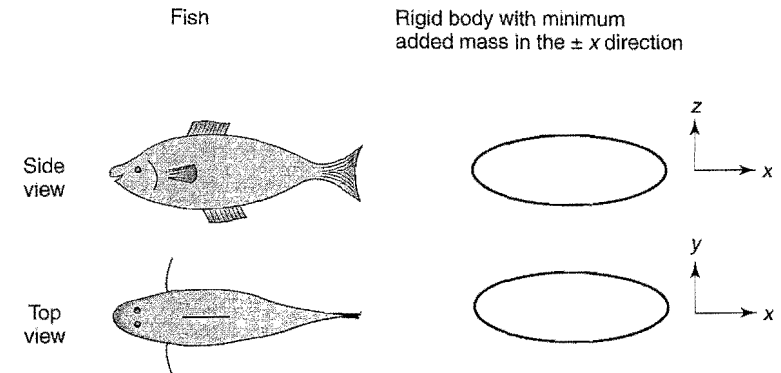


FIGURE 10.16 Learning from fish how to design bodies of minimum sound radiation owing to minimum added mass when oscillating in a specific direction.

from forward, especially in the direction perpendicular to it, the added mass of the fish is much larger, indicating the importance of the directional and shape dependence of the added mass of vibrating bodies of a geometry that is different from a sphere. The sphere has the distinction of having the same, large added mass for accelerating in any direction of motion. This large added mass is the reason that fish do not have a spherelike body (except some slow-swimming variety that inflate themselves to deceive their would-be predator). If fish would rotate instead of swim (mostly along straight paths), all of them would have a near-sphere-like body. On the other hand, if we wish to reduce the noise generated by the spinning of a rigid body, the sphere is the best choice. Surrounding a rotating nonspherical body with a spherical shell that rotates with the body will substantially reduce the noise radiation.

Reduction of Sound Radiation at Specific Frequencies. There is a class of noise sources, such as power transformers and turbine-generators, that radiate exclusively or predominantly tonal noise. Figures 10.17 and 10.18 show how the tonal noise produced by such equipment can be controlled by lining their radiating surface with Helmholtz resonator arrays tuned to the frequency (or frequencies) of their tonal noise output.

As shown in the lower sketch in Fig. 10.18, the noise radiation at the tuning frequency of the resonators is achieved by the low impedance of the resonator mouth in the vicinity of its resonance frequency. The fluid displaced by the solid part of the face of the resonator finds it easier to enter the resonator volume than to undergo compression. The smaller the compression, the lower is the radiated sound. Information on how to design Helmholtz resonators can be found in Chapter 8.

In the case of power transformers these tonal frequencies are 120, 240, 360, and 480 Hz in the United States and 100, 200, 300, and 400 Hz in Europe. Noise

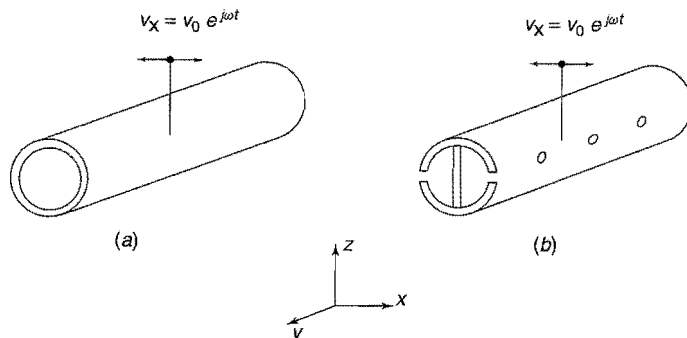


FIGURE 10.17 Frequency-selective reduction of sound generation owing to incorporation of Helmholtz resonators into the body: stiffened structural pipe beam vibrating perpendicular to its axes or rotating at constant rate as a spoke of a wheel.

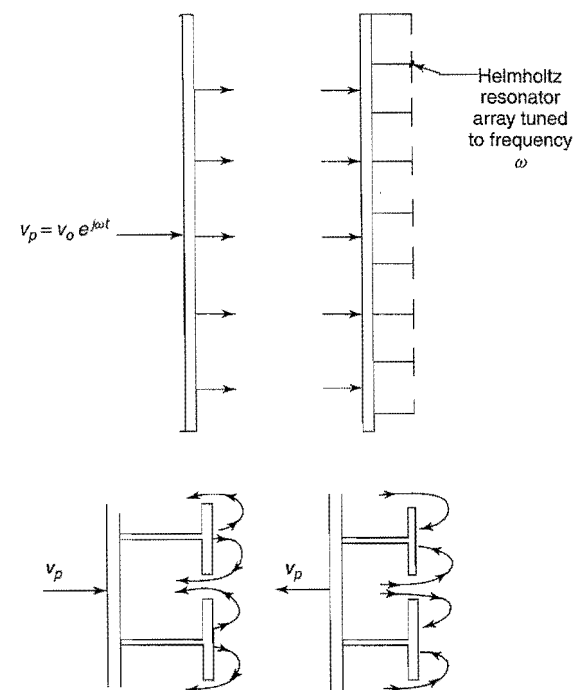


FIGURE 10.18 Frequency-selective reduction of sound radiation of a large flat surface, such as the outside wall of transformer tank, by lining it with a Helmholtz resonator array.

control measures for such noise sources and their advantages, disadvantages, and cost are compiled in reference 11.

Vacuum Bubbles. Figure 10.19 shows the concept of reducing the sound radiation of a transformer tank by lining the oil-side surface of the transformer tank with vacuum bubbles and how the presence of the vacuum bubbles, owing to their very large compliance, relieves the dynamic pressure on the tank wall and, consequently, reduces the sound radiation. As shown schematically in Fig. 10.20a, vacuum bubbles are twin curved thin shells with a small volume in between. When the cavity is evacuated, the shell nearly flattens out. In this evacuated final state they yield a compliance (inverse of the dynamic volume stiffness) that can be up to 50 times larger than the compliance of the air volume they replace and up to 1000 times that of the compliance of oil volume they replace. A unique feature of the vacuum bubbles is that there is vacuum in the cavity and there is no limit for the dynamic compliance, as there would be if the cavity were filled with a fluid. Below their resonance frequency, which is determined by the surface mass of the shell and its dynamic volume compliance, their volume compliance is independent of frequency. They work well even if the frequency approaches

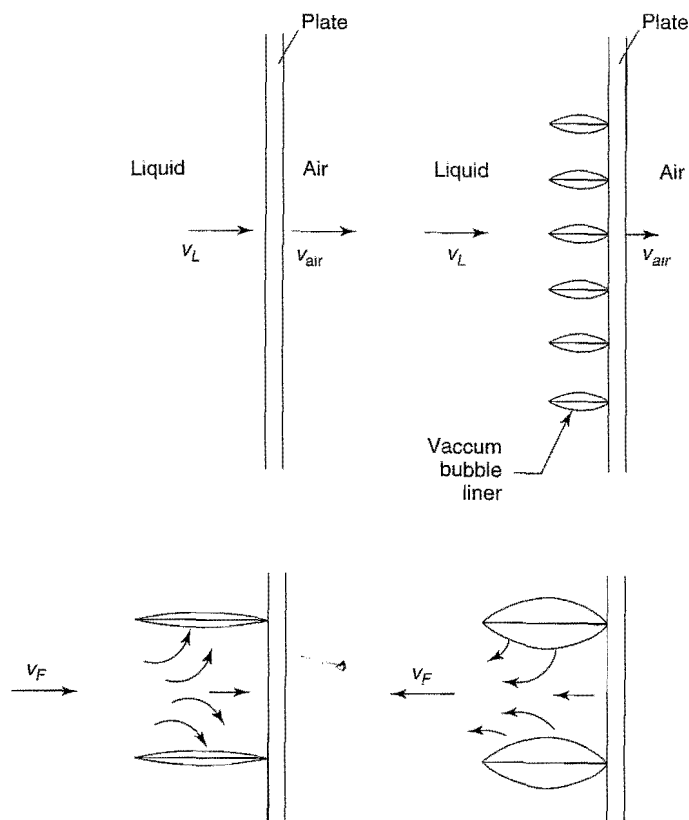


FIGURE 10.19 Reduction of the sound radiation from a transformer tank by lining the oil-side face of the tank wall with an array of vacuum bubbles of extremely high volume compliance. The vacuum bubbles can be oriented either perpendicular to the wall (as shown here) or parallel to it. Lower sketches indicate how the vacuum bubbles, which are much more compliant than either the oil or the tank wall, “soaks up” the momentum of the periodically approaching and receding oil. The velocity of the oil is due to the vibration of the transformer core driven by magnetostrictive forces.

zero. Information on how to design a vacuum bubbles for specific applications is given in references 12–14.

Figure 10.20 shows the geometry of a vacuum bubble (also called Scillator by its inventor Dr. Bschorr¹²) before and after evacuation of the cavity. As the name vacuum bubble implies, it is useful only in the evacuated state.

Vacuum bubbles are very sensitive to changes in static pressure (barometric pressure in air and depth-dependent pressure in liquids). Consequently, their most promising application would be in space stations, where the static air pressure can be controlled accurately.

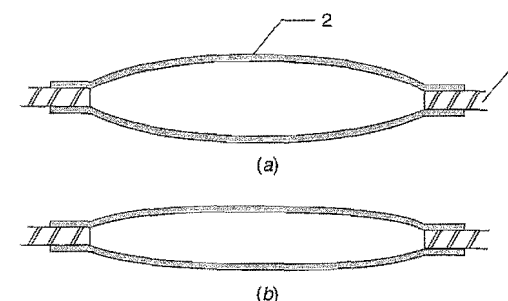


FIGURE 10.20 Conceptual sketch of a vacuum bubble. (a) Cross-sectional shape before evacuation of the cavity: 1, double-curved thin shell in the form of a top segment of a large-radius hollow sphere; 2, flat-plate strip base with punched holes. (b) Shape after evacuation of the cavity.

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CHAPTER 11

Interaction of Sound Waves with Solid Structures

ISTVÁN L. VÉR

Consultant in Acoustics, Noise, and Vibration Control
Stow, Massachusetts

The response of structures to dynamic forces or dynamic pressures is the subject of structural dynamics and acoustics. Structural dynamics is concerned predominantly with dynamic stresses severe enough to endanger structural integrity. Structural acoustics deals with low-level dynamic processes in structures resulting from excitation by forces, moments, and pressure fields. The primary interest in airborne and structure-borne noise problems is the prediction of

1. power input into the structure,
2. the response of the excited structure,
3. propagation of structure-borne sound to connecting structures, and
4. sound radiated by the vibrating structure.

The subject matter of this chapter is specific to noise control problems and thus is restricted to the audible frequency range and to air as the surrounding medium, though many of the concepts are directly applicable to liquid media or to higher or lower frequencies.

A typical noise control problem is illustrated in Fig. 11.1. A resiliently supported floor slab in the room to the left (source room) is excited by the periodic impacts of a tapping machine, while the microphone in the receiving room to the right registers the resulting noise. A part of the vibrational energy is dissipated in the floating slab, a part is radiated directly as sound into the source room, and the remainder is transmitted through the resilient layer into the building structure.

The radiated sound energy builds up a reverberant sound field in the source room that in turn excites the walls. The vibrations in the wall separating the two rooms, identified as path 1 in Fig. 11.1, radiate sound directly into the receiver room. The vibrations in the other partitions of the source room travel in the form of structure-borne sound to the six partitions of the receiving room and radiate

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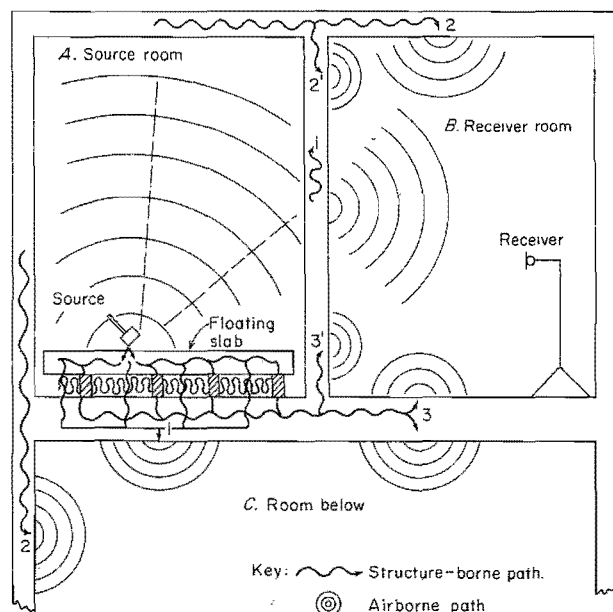


FIGURE 11.1 Sound transmission paths between an impact source in room A and a receiver in room B. Also shown are the paths to room C below.

sound into the receiver room. The structure-borne paths are identified by waves, 2, 2' and 3, 3'.

To reduce the noise level in the receiving room by a desired amount, the acoustical engineer must estimate the sound power transmitted from the source room to the receiver room through each path and then design appropriate measures to (1) reduce power input to the floating slab, (2) increase power dissipation in the slab, (3) increase vibration isolation, (4) reduce the structure-borne noise along its propagation path between source and receiver room, (5) reduce the sound radiated by the partitions of the source room, and (6) reduce the reverberation buildup of the sound in the receiver room by increasing sound absorption.

The aim of this chapter is to provide the information needed to solve such typical noise control engineering problems.

11.1 TYPES OF WAVE MOTION IN SOLIDS

Wave motion in solids can store energy in shear as well as in compression. The types of waves possible in solids include compressional waves, flexural waves, shear waves, torsional waves, and Rayleigh waves. Compressional waves are of practical importance in gases and liquids, which can store energy only in

compression. The different types of waves in solids result from different ways of stressing. For a wave to propagate in solids, liquids, and gases, the medium must be capable of storing energy alternatively in kinetic and potential form. Kinetic energy is stored in any part of a medium that has mass and is in motion, while potential energy is stored in parts that have undergone elastic deformation.

Solid materials are characterized by the following parameters:

Density	ρ_M	kg/m ³
Young's modulus	E	N/m ²
Poisson's ratio	ν	
Loss factor	η	

The shear modulus G is related to Young's modulus as

$$G = \frac{E}{2(1 + \nu)} \quad \text{N/m}^2 \quad (11.1)$$

The sketches on the left of Table 11.1 illustrate the deformation pattern typical for compressional, shear, torsional, and bending waves in bars and plates. Acoustically important parameters of solid materials are compiled in Table 11.2. Note that all tables in this chapter are placed at the end of the text. Thickness and weight per unit surface area of steel plates are listed in Table 11.10. Percentage open area of perforated metal is given in Table 11.11.

In dealing with structure-borne sound, we must distinguish between velocity and propagation speed. The term *velocity* refers to vibration velocity of the structure (i.e., the time derivative of the local displacement), which is linearly proportional to the excitation and will be designated by the letter v . As illustrated on the left of Table 11.1, the displacement and velocity are in the direction of wave propagation for longitudinal waves and perpendicular to it for shear and bending waves.

The term *speed* refers to the propagation speed of structure-borne sound, which is a characteristic property of the structure for each type of wave motion and is independent of the strength of excitation provided that the deformations are small enough to avoid nonlinearities. Propagation speeds will be designated by the letter c . We must distinguish between *phase speed* and *group speed* (or energy speed).

The phase speed c is defined in terms of (1) the wavelength λ , which is the distance in the propagation direction for which the phase of a sinusoidal wave changes 360°, and (2) the frequency f of the sinusoid. Consequently, $c = \lambda f$. Formulas to calculate phase speed of the various wave types are given on the right side of Table 11.1. Note that the phase speed for longitudinal, shear, and torsional waves is independent of frequency. Consequently, they are referred to as nondispersive waves. This means that the time history of an impulse, such as caused by a hammer blow striking axially on one end of a semi-infinite bar, as illustrated in Fig. 11.2a, will have the same shape regardless of where the

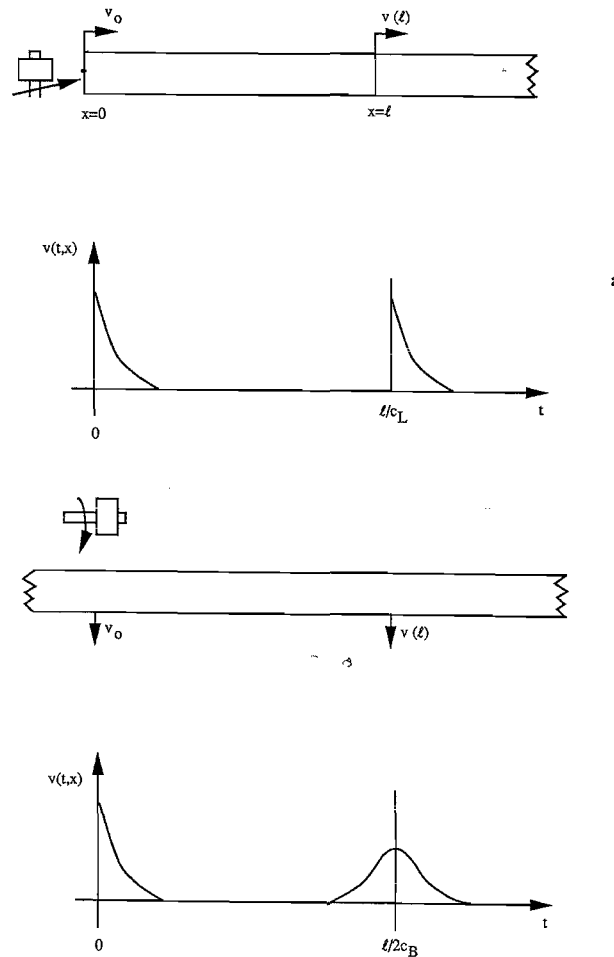


FIGURE 11.2 Time history of beam motion for impulsive excitation: (a) axial impact-generating nondispersive compressional waves; (b) normal impact-generating dispersive bending waves.

axial motion is sensed. Because the pulses sensed at different locations are time-delayed versions of each other, the propagation speed of the longitudinal wave can be determined experimentally as the ratio of the axial distance and the difference of arrival time. Since all frequency components travel at the same speed, the phase speed (which is defined only for a steady-state sinusoidal excitation) is identical with the speed of energy transport.

The phase speed of the bending wave can be easily measured by exciting the infinite beam with a steady-state sinusoidal force acting normal to the beam

axis and measuring the gradient of the phase $d\phi/dx$ along the beam. The phase velocity c_B is defined as

$$c_B = \frac{2\pi f}{|d\phi/dx|} \quad \text{m/s} \quad (11.2)$$

However, the situation is much more complicated in the case of a complex waveform such as illustrated in Fig. 11.2b, where an infinite beam is impacted normally and a bending-wave pulse that contains a wide band of frequencies is created. Because of their higher phase speed, high-frequency components speed ahead of the low-frequency component. Consequently, the width of the pulse increases as it travels along the beam. The speed of energy transport for the bending-wave pulse is not obvious. In case of light fluid loading (when the energy carried in the surrounding fluid is negligible), it is meaningful to define the energy transport speed as the ratio of transmitted power W and the energy density E'' (per unit length of beam). For a pulse whose spectrum peaks at frequency f the energy speed c_{BG} is twice the phase speed¹:

$$c_{BG}(f) = 2c_B(f) \quad \text{m/s} \quad (11.3)$$

Accordingly, the power transmitted by a plane bending wave in the direction of propagation per unit area of a structure normal to that direction is

$$W_S = c_{BG} E'' = 2c_B \rho_M v^2 \quad \text{W/m}^2 \quad (11.4)$$

where v is the rms value of the bending-wave velocity and ρ_M is the density of the material. According to Eq. (11.4), the bending-wave power in an infinite beam of cross section S is given by

$$W = 2c_B S \rho_M v^2 \quad \text{W} \quad (11.5)$$

and power transmitted by a plate of thickness h across a line length l aligned parallel to the wave front is

$$W = 2c_B h l \rho_M v^2 \quad \text{W} \quad (11.6)$$

11.2 MECHANICAL IMPEDANCE AND POWER INPUT

Except for the initial transient, the velocity at the excitation point is zero if a structure is subjected to a static force. Accordingly, a static force does not transmit power into a fixed structure. For dynamic excitation by a point force or moment [the moment $M = Fl_m$ is defined as a pair of forces (F) of equal magnitude but opposite direction acting simultaneously at a short distance ($\frac{1}{2}l_m$) on both sides of the excitation point], there is always a finite dynamic velocity or dynamic rotation at the excitation point. If the velocity at the excitation point

has a component that is in phase with the force, or the angular velocity has a component that is in phase with the moment, then power is fed continuously into the structure.

Mechanical Impedance

The mechanical impedance is a measure of how a structure "resists" outside forces or moments. A thorough understanding of mechanical impedances is a necessary requirement for understanding and for solving most noise control problems. The concept of mechanical impedance is best introduced by considering a harmonic force $F = \hat{F}e^{j\omega t}$ acting on an ideal lumped-parameter system such as a rigid unrestrained mass, a massless spring, or a dashpot. The velocity response is also a harmonic function of the form $v = \hat{v}e^{j\omega t}$. The force impedance Z_F is defined as

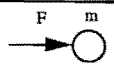
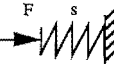
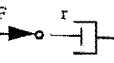
$$Z_F = \frac{\hat{F}}{\hat{v}} \quad \text{N} \cdot \text{s/m} \quad (11.7)$$

and the moment impedance Z_M as

$$Z_M = \frac{\hat{M}}{\hat{\theta}} \quad \text{N} \cdot \text{s/m} \quad (11.8)$$

where M is the exciting moment (torque) and θ is the angular velocity at the excitation point.

The point force impedance for these basic lumped elements are listed below:

Element	Z_F	
Mass	$j\omega m$	
Spring	$-js/\omega$	
Dashpot	r	

where m is the mass of a rigid body, s is the stiffness of the spring, and r is the resistance of the dashpot. Note that the direction of the force must go through the center of gravity of the mass so that the velocity response is free of rotation and is in the direction of the force. For the rigid mass and the massless spring the force impedance is imaginary, indicating that the force and velocity are in quadrature and the power input is zero.

Formulas for computing the point force and moment impedances of infinite structures are given in references 2-12 and in Tables 11.3 and 11.4, respectively. Both force and moment impedances are idealized concepts, defined for point

forces and point moments. Forces or moments can be considered applied at a point, provided that the surfaces over which they act are smaller than one-sixth of the structural wavelength. Naturally, the excited surface area must be large enough so that the force does not cause plastic deformation of the structure. There is proportionality between force and moment impedances in the form of

$$Z_M = l_M^2 Z_F \quad \text{N} \cdot \text{m/s} \quad (11.9)$$

where l_M is the effective length given in Table 11.5.

Power Input

For the dashpot the force impedance is real. The velocity is in phase with the force and the power that is fed continuously into the dashpot and dissipated in it is given by $\frac{1}{2}|\hat{F}|^2/r$. More generally, for a system with a complex force impedance Z_F , the input power is

$$W_{in} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} F(t)v(t) dt = \frac{1}{2}|\hat{F}| |\hat{v}| \cos \phi = \frac{1}{2}|\hat{F}|^2 \text{Re} \left\{ \frac{1}{Z_F} \right\} \quad \text{W} \quad (11.10)$$

where $\hat{F}$ and $\hat{v}$ are the peak amplitude of the exciting force and the velocity, ϕ is the relative phase between $F(t)$ and $v(t)$, and $\text{Re}\{\}$ refers to the real part of the bracketed quantity. Equation (11.10) is valid for any situation, not only for the dashpot.

Rigid masses, massless springs, and dashpots are useful abstractions that represent the dynamic properties of structural elements. At sufficiently low frequencies, the housing of a resiliently mounted pump behaves like a rigid mass, while the resilient rubber mount that supports it behaves like a spring. This lumped-parameter characterization of mechanical elements becomes invalid when, with increasing frequency, the structural wavelength becomes smaller than the largest dimensions of the structural element.

To deal with finite structures that are large compared with the structural wavelength, we need another useful abstraction. Regarding their input impedance—which governs power input—it is useful to consider such structures as infinite. The strategy for predicting the vibration response and sound radiation for these structures is to estimate the power input as if they were infinite and then predict the average vibration that such input power will cause in the actual finite structure. Because most structures are made of plates and beams, our strategy requires us to characterize the impedance of infinite plates and beams. The abstraction of infinite size is invoked so that no structural waves reflected from the boundaries come back to the excitation point. In this respect, structures can be considered infinite if no reflected waves that are coherent with the input come back to the excitation point because the boundaries are completely absorbing or because there is sufficient damping so that the response attributable to the reflected waves is small compared to that attributable to the local excitation.

In respect to point force excitation, infinite structures are characterized by their point force impedance $\tilde{Z}_{F\infty}$ or point input admittance (also called mobility) $\tilde{Y}_{F\infty}$, defined as

$$\tilde{Z}_{F\infty} = \frac{\tilde{F}_0}{\tilde{v}_0} \quad \text{N} \cdot \text{s/m} \quad (11.11)$$

$$\tilde{Y}_{F\infty} = \frac{1}{\tilde{Z}_{F\infty}} = \frac{\tilde{v}_0}{\tilde{F}_0} \quad \text{m/N} \cdot \text{s} \quad (11.12)$$

where $\tilde{F}_0$ is the applied point force and $\tilde{v}_0$ is the velocity response at the excitation point. The tilde above the symbols (which we will drop in further considerations) signifies that F_0 and v_0 are complex scalar quantities characterized by a magnitude and phase. Since $\tilde{v}_0$ usually has a component that is in phase with $\tilde{F}_0$ and one that is out of phase with $\tilde{F}_0$, the parameters $\tilde{Z}_F$ and $\tilde{Y}_F$ usually have both real and imaginary parts.

The power inputs to infinite structures excited by a point force or by a point velocity source (i.e., a vibration source of high internal impedance) are, respectively,

$$W_{\text{in}} = |\frac{1}{2}\hat{F}_0^2| \operatorname{Re} \left\{ \frac{1}{Z_{F\infty}} \right\} \quad \text{W} \quad (11.13a)$$

$$W'_{\text{in}} = G_{FF} \operatorname{Re} \left\{ \frac{1}{Z_{F\infty}} \right\} \quad \text{W/Hz} \quad (11.13b)$$

$$W_{\text{in}} = (\frac{1}{2}\hat{v}_0^2) \operatorname{Re}\{Z_{F\infty}\} \quad \text{W} \quad (11.14a)$$

$$W'_{\text{in}} = G_{VV} \operatorname{Re}\{Z_{F\infty}\} \quad \text{W/Hz} \quad (11.14b)$$

where $\hat{F}_0$ and $\hat{v}_0$ are peak amplitudes of the exciting sinusoidal force and velocity and G_{FF} and G_{VV} are the force and velocity spectral densities when the excitation is of broadband random nature. For excitation by a moment of peak amplitude $\hat{M}$ or by an enforced angular velocity of peak amplitude $\hat{\theta}$, the corresponding expressions are

$$W_{\text{in}} = |\frac{1}{2}\hat{M}_0^2| \operatorname{Re} \left\{ \frac{1}{Z_M} \right\} \quad \text{W} \quad (11.15a)$$

$$\frac{W'_{\text{in}}}{\text{Hz}} = G_{MM} \operatorname{Re} \left\{ \frac{1}{Z_M} \right\} \quad \text{W/Hz} \quad (11.15b)$$

$$W_{\text{in}} = |\frac{1}{2}\hat{\theta}| \operatorname{Re}\{Z_M\} \quad \text{W} \quad (11.16a)$$

$$W'_{\text{in}} = G_{\theta\theta} \operatorname{Re}\{Z_M\} \quad \text{W/Hz} \quad (11.16b)$$

Table 11.3 lists the point force impedance $Z_{F\infty}$ and Table 11.4 the moment impedance $Z_{M\infty}$ for semi-infinite and infinite structures. Table 11.5 lists the effective length l_M that connects the point force and moment impedances according

to Eq. (11.9). Information regarding power input to infinite structures for force, velocity, moment, and angular velocity excitation is compiled in Table 11.6. The most complete collection of impedance formulas, which includes beams, box beams, orthotropic and sandwich plates with elastic or honeycomb core, grills, homogeneous and rib-stiffened cylinders, spherical shells, plate edges, and plate intersections and transfer impedances of various kinds of vibration isolators, is presented in reference 12.

For approximate calculations or for cases where no formulas are given in Tables 11.3–11.6, the power input to infinite structures can be estimated according to Heckl^{10,11} as

$$W_{\text{in}} = \frac{\frac{1}{2}\hat{F}_0^2}{Z_{\text{eq}}} \quad \text{W} \quad (11.17)$$

$$W_{\text{in}} = (\frac{1}{2}\hat{v}_0^2)Z_{\text{eq}} \quad \text{W} \quad (11.18)$$

where Eq. (11.17) is used for localized force excitation (i.e., low-impedance vibration source) and Eq. (11.18) for localized velocity excitation (i.e., high-impedance vibration source) and Z_{eq} is estimated as

$$Z_{\text{eq}} = \omega\rho_M S \epsilon[\alpha\lambda] \quad \text{N} \cdot \text{s/m} \quad (\text{beam}) \quad (11.19a)$$

$$Z_{\text{eq}} = \omega\rho_M h \epsilon[\pi(\alpha\lambda)^2] \quad \text{N} \cdot \text{s/m} \quad (\text{plate}) \quad (11.19b)$$

$$Z_{\text{eq}} = \omega\rho_M \epsilon[\frac{4}{3}\pi(\alpha\lambda)^3] \quad \text{N} \cdot \text{s/m} \quad (\text{half space}) \quad (11.19c)$$

where ρ_M is the density of the material, S is the cross-sectional area of the beam, h is the plate thickness, λ is the wavelength of the motion excited most strongly (bending, shear, torsion, or compression), and ϵ is 1 when the structure is excited in the "middle" and 0.5 when it is excited at the end or at the edge (semi-infinite structure) and α is a number in the range of 0.16–0.6. If not known, it is customary to use $\alpha = 0.3$.

The equivalent impedance Z_{eq} in Eq. (11.19) has an instructive and easy-to-remember interpretation, namely that the magnitude of the point input impedance is roughly equal to the impedance of a lumped mass that lies within a sphere of radius $r \cong \frac{1}{3}\lambda$ centered at the excitation point. This portion of the structure is shown in the third column of Table 11.3. This principle can be extended to composite structures¹⁰ such as beam-stiffened plates where the combination of Eqs. (11.19a) and (11.19b) yields

$$Z_{\text{eq}} \cong \omega\rho_B S_B(\frac{1}{3}\lambda_{BB}) + \omega\rho_P h(\frac{1}{3}\lambda_{BP}) \quad \text{N} \cdot \text{s/m} \quad (11.20)$$

Another useful rule of thumb is that, regarding power input, excitation by a moment $\hat{M}$ and by an enforced angular velocity $\hat{\theta}$ can be represented by an equivalent point force $\hat{F}_{\text{eq}}$ or equivalent velocity $\hat{v}_{\text{eq}}$,¹¹

$$\hat{F}_{eq} \cong \frac{\hat{M}}{0.2\lambda_B} \quad \text{N} \quad (11.21)$$

$$\hat{v}_{eq} \cong \hat{\theta}(0.2\lambda_B) \quad \text{m/s} \quad (11.22)$$

If the excitation force extends over an area that is large compared with the bending wavelength, the power transmitted into the structure becomes substantially less than it would be for a concentrated force. For extended velocity source the power transmitted becomes significantly larger than it would be for a concentrated velocity excitation.

According to reference 5, the real part of the point force impedance, $\text{Re}\{Z_{F\infty}\}$, can be predicted according to Bode's theorem as

$$\text{Re}\{Z_{F\infty}\} \cong |Z_{eq}| \cos(\tfrac{1}{2}\epsilon\pi) \quad \text{N} \cdot \text{s/m} \quad (11.23)$$

where ϵ is defined here as an exponent of the Z_{eq} -versus-frequency curve (i.e., $Z_{eq} \sim \omega^\epsilon$), where $\epsilon = 0$ for homogeneous isotropic thick plates, $\epsilon = 0.5$ for beams in bending, and $\epsilon = \frac{2}{3}$ for cylinders below the ring frequency.

Parameters Influencing Power Input. When the excitation point is near a structural junction, part of the incident vibration wave is reflected. This reflected wave influences the velocity at the excitation point and thereby also the driving point impedance and consequently the power input. Fig. 11.3a shows the effect of the vicinity of a T-junction comprised of identical, anechoically terminated beams on the point force impedance $Z_2(x_0)$ as a function of the distance x_0 between the junction and the excitation point.¹³ As expected, the impedance increases rapidly with decreasing distance when $x_0 < \frac{1}{3}\lambda_B$. The junction has a considerable effect even for large x_0 , causing approximately a 2:1 fluctuation in the magnitude of the impedance. Figure 11.3b shows the measured distribution of the beam vibration velocity $|v(x)|$ along each of the three beam branches, indicating that the observed strong standing-wave pattern is limited to that part of beam 2 that lies between the junction and the excitation point.

The impedance formulas presented in Tables 11.3–11.6 are strictly valid only when the surrounding fluid medium has no appreciable effect on the response of the structure. This is generally true for air as the surrounding medium but not for water, which is 800 times more dense than air. The effect of fluid loading at low frequencies is equivalent¹⁴ to adding a virtual mass per unit area ρ'_s ,

$$\rho'_s = \frac{\rho_0 c_B(f)}{\omega} \cong \rho_0 [\tfrac{1}{6}\lambda_B(f)] \quad \text{kg/m}^2 \quad (11.24)$$

to the mass per unit area of the plate. In Eq. (11.24), ρ_0 is the density of the fluid and $c_B(f)$ and $\lambda_B(f)$ are the bending-wave speed and the wavelength of the free bending waves in the unloaded plate (see Table 11.1). Because ρ'_s is inversely proportional to $\omega^{1/2}$, fluid loading results in greater reduction of the response

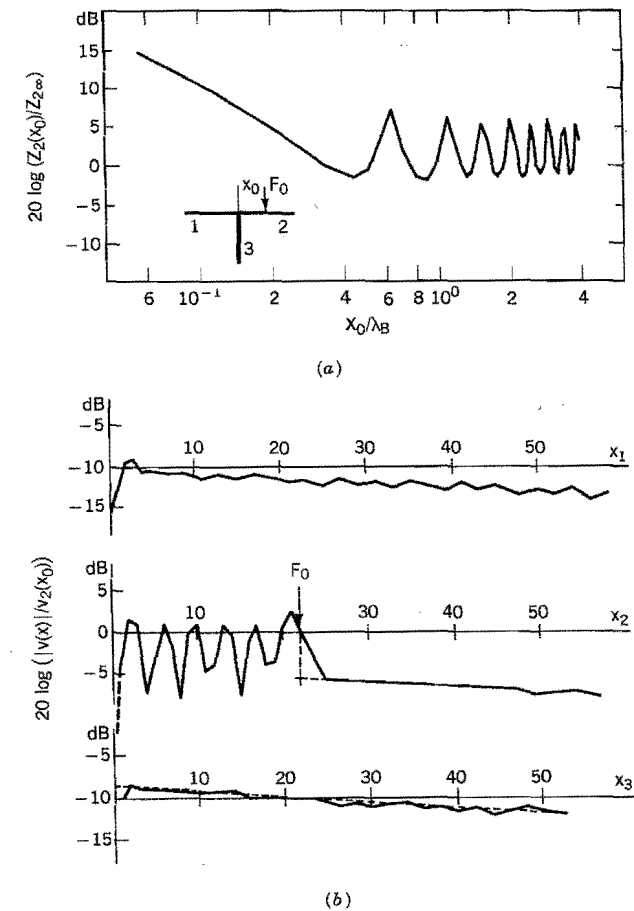


FIGURE 11.3 Effect of the vicinity of T-junction on the measured point force impedance and velocity response¹³: (a) normalized force impedance $Z_1(x_0)/Z_{1\infty}$ as a function of the normalized distance x_0/λ_B and x_i is the distance from the junction along beam i ; (b) distribution of normalized velocity, $v(x)/v_0$, along the three beam branches as numbered in (b).

at the excitation point at low than at high frequencies. Fluid-loading effects are treated in references 14–17.

11.3 POWER BALANCE AND RESPONSE OF FINITE STRUCTURES

For finite structures, the response at the excitation point, and accordingly also the input admittance, $Y = 1/Z$, depends on the contribution of the waves reflected from the boundaries or discontinuities. Consequently, Y varies with the location

of the excitation point and with frequency. However, both the space-averaged and frequency-averaged input admittances of the finite structure $\langle Y(x, f) \rangle_x$ and $\langle Y(x, f) \rangle_f$ equal the point input admittance of the equivalent infinite structure,¹⁸ namely,

$$\langle Y(x, f) \rangle_x = \langle Y(x, f) \rangle_f = Y_\infty \quad \text{m/N} \cdot \text{s} \quad (11.25)$$

Figures 11.4a and 11.4b show the typical variation of the real and imaginary parts of the point input admittance of a finite plate with location x and frequency f , respectively. This particular behavior has considerable practical importance:

1. The power introduced into a finite structure by a point force of random-noise character can be well approximated by the power that the same force would introduce into an equivalent infinite structure.
2. The power introduced into a finite structure by a large number of randomly spaced point forces can be approximated by the power the same forces would introduce into an equivalent infinite structure.

Resonant Modes and Modal Density

The peaks in Fig. 11.4a correspond to resonance frequencies of the finite structure, where waves reflected from the boundaries travel in closed paths such that they arrive at their starting point in phase. The spatial deformation pattern that corresponds to such a particular closed path is referred to as the *mode shape* and the frequency where it occurs as the *eigenfrequency* or *natural frequency* of the finite structure. The importance of such resonances lies in the high transverse velocity caused by the in-phase superposition of the multiple reflections that may result in increased sound radiation or fatigue.

Exact calculation of the natural frequencies is possible only for a few highly idealized structures. Fortunately, the modal density $n(f)$, which is defined as the average number of natural frequencies in a 1-Hz bandwidth, depends not too strongly on the boundary conditions. Accordingly, one can make a reliable statistical prediction of the modal density using the formulas obtained for the equivalent idealized system.

For example, the modal density of a thin, flat, homogeneous, isotropic plate of not too high aspect ratio two octaves above the first plate resonance is already well approximated by the modal density of a rectangular plate of equal surface area, which is given by¹

$$n(f) \approx \frac{\sqrt{12S}}{2c_L h} \quad \text{s} \quad (11.26)$$

where S is the area (one side) and h is the thickness. Note that $n(f)$ is independent of frequency and that it is large for large and thin plates.

The modal density of rooms of volume V one octave above the first room resonance is usually well approximated by the first term of the modal density of

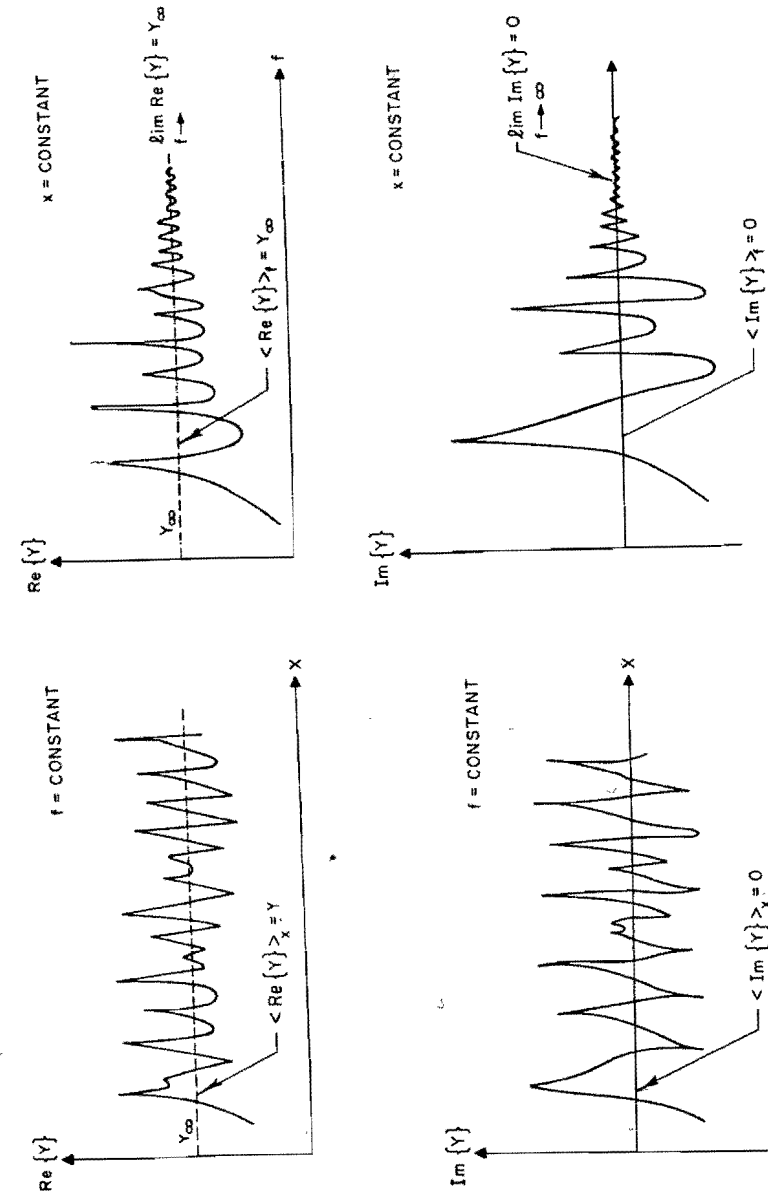


FIGURE 11.4 Typical fluctuations of the real part, $\text{Re}\{Y\}$, and imaginary part, $\text{Im}\{Y\}$, of the point input admittance of a finite plate as a function of (a) location x , (b) frequency f . $\langle \dots \rangle_x$ is spatial average at a fixed frequency; $\langle \dots \rangle_f$ is frequency average at a fixed location

a rectangular room with hard walls,

$$n(f) = \frac{4\pi V}{c_0^3} f^2 + \frac{\pi S}{c_0^2} + \frac{L}{8c_0} \quad \text{s} \quad (11.27)$$

where S is the total wall surface area and L is the longest dimension of the rectangular room. The first resonance frequency, modal densities, and mode shapes of a number of finite structures and the point input impedance (the inverse of the admittance) of the equivalent infinite structure are compiled in Table 11.7. More detailed information on mode shapes, natural frequencies, and modal densities are given in reference 19. The following important relationship exists between the modal density and the real part of the point force admittance of the equivalent infinite system¹:

$$\text{Re}\{Y_\infty\} = \frac{n(f)}{iM} \quad \text{m/N} \cdot \text{s} \quad i = \begin{cases} 4 & \text{for thin plate} \\ 2 & \text{for thin cylinder} \\ 1 & \text{for thin sphere} \end{cases} \quad (11.28)$$

where M is the total mass of the finite system.

Power Balance

The power balance given in Eq. (11.29),

$$W_{\text{in}} = W_d + W_{\text{tr}} + W_{\text{rad}} \quad \text{W} \quad (11.29)$$

states that, in steady state, the power introduced, W_{in} , equals the power dissipated in the structure, W_d , the power transmitted to connected structures, W_{tr} , and the power radiated as sound into the surrounding fluid, W_{rad} . If the excitation is by a point force of peak amplitude $\hat{F}_0$ and the structure is a homogeneous isotropic thin plate of thickness h , area S (one side), density ρ_M , with longitudinal wave speed c_L , immersed in a fluid of density ρ_0 , and speed of sound c_0 , Eq. (11.29) takes the form

$$\frac{1}{2} \hat{F}_0^2 \text{Re}\{Y_F\} = \langle v^2 \rangle S [\rho_s \omega \eta_d + \rho_s \omega \eta_{\text{tr}} + 2\rho_0 c_0 \sigma] \quad \text{W} \quad (11.30)$$

where $\rho_s = \rho_M h$ is the mass per unit area of the plate, $\omega = 2\pi f$ is the radian frequency, η_d and η_{tr} are the dissipative and transmissive loss factors, and σ is the sound radiation efficiency of the plate (see Section 11.6). Combining dissipative and transmissive losses into a single composite loss factor $\eta_c = \eta_d + \eta_{\text{tr}}$ and assuming $\text{Re}\{Y_F\} = Y_{F\infty} = 1/Z_{F\infty} = 1/(2.3\rho_s c_L t)$, as given in Table 11.3, and solving for the space-time averaged mean-square plate velocity $\langle v^2 \rangle$ yield

$$\langle v^2 \rangle = \frac{\hat{F}_0^2}{4.6\rho_s^2 c_L t \omega \eta_c S (1 + 2\rho_0 c_0 \sigma / \rho_s \omega \eta_c)} \quad \text{m}^2/\text{s}^2 \quad (11.31)$$

Example 11.1. Predict the space-time averaged velocity response $(\langle v^2 \rangle)^{1/2}$ of a 2-mm-thick, 1 × 2-m steel plate to a point force $\hat{F}_0 = 10$ N peak amplitude

at a frequency of 10 kHz when $\sigma = 1$ and $\eta_c = 0.01$. Compare $\langle v^2 \rangle$ with the mean-square velocity at the excitation point v_0^2 . The input parameters to be used in connection with Eq. (11.31) and Table 11.2 are

$$\begin{aligned} \hat{F}_0 &= 10 \text{ N} & \rho_s &= \rho_M t = 7700 \text{ kg/m}^3 \times 2 \times 10^{-3} \text{ m} = 15.4 \text{ kg/m}^2 \\ c_L &= 5050 \text{ m/s} & \omega &= 2\pi f = 2\pi \times 10^4 \text{ rad/s} & \eta_c &= 0.01 \\ S &= 2 \text{ m}^2 & \rho_0 &= 1.2 \text{ kg/m}^3 & c_0 &= 340 \text{ m/s} & \sigma &= 1 \end{aligned}$$

yielding

$$\begin{aligned} \langle v^2 \rangle &= \frac{10^2}{4.6 \times (15.4)^2 \times 5.05 \times 10^3 \times 2 \times 10^{-3} \times 2\pi \times 10^4 \times 10^{-2}} \\ &\quad \times \frac{1}{2(1 + 2 \times 1.2 \times 340 / 15.4 \times 2\pi \times 10^4 \times 2 \times 10^{-2})} \\ &= 6.7 \times 10^{-6} \text{ m/s}^2 \\ \sqrt{\langle v^2 \rangle} &= 2.6 \times 10^{-3} \text{ m/s} \\ v_0^2 &= \frac{1}{2} \hat{F}_0^2 Y_\infty = \frac{\hat{F}_0^2}{4.6\rho_s c_L t} = \frac{10^2}{4.6 \times 15.4 \times 5.03 \times 10^3 \times 2 \times 10^{-3}} \\ &= 0.14 \text{ m/s} \\ \frac{v_0^2}{\langle v^2 \rangle} &= \frac{(0.14)^2}{6.7} \times 10^6 = 2925 \quad \sqrt{\frac{\langle v^2 \rangle}{v_0^2}} = 54 \end{aligned}$$

11.4 REFLECTION AND TRANSMISSION OF SOUND AT PLANE INTERFACES

When a plane sound wave traveling in a homogeneous medium encounters a plane interface with another medium, it may be (1) totally reflected, (2) partially reflected, or (3) totally transmitted depending upon the angle of incidence, the propagation speed of sound, and the density of the materials on both sides of the interface. Interfaces of practical importance are between air and water, between air and solid materials, between air and porous materials (such as ground or sound-absorbing materials), and between layers of fibrous sound-absorbing material such as treated in Chapter 8.

The simplest case is when the wave front of the incident plane wave is parallel to the plane of the interface (i.e., the wave propagates normal to the interface), as shown schematically in Fig. 11.5. In this simple case the transmitted wave p_{tran} retains the propagation direction of the incident wave p_{inc} and the following

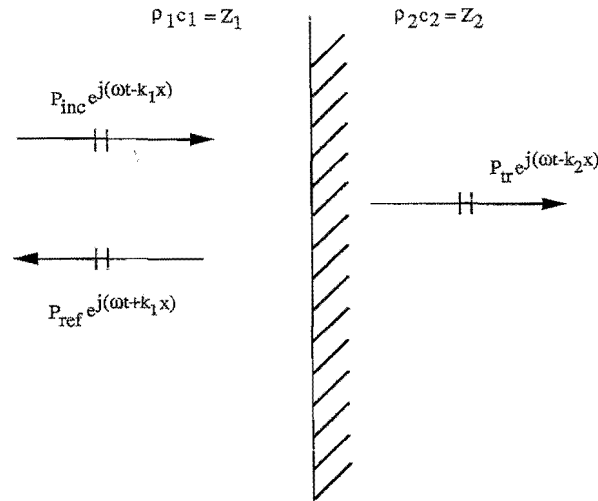


FIGURE 11.5 Reflection and transmission of a plane sound wave normally incident on the plane interface of an infinitely thick medium: p_{inc} , p_{ref} , p_{tr} , peak amplitudes of incident, reflected, and transmitted pressure; ρ , c , density and speed of sound of respective media.

relationships apply:

$$\frac{p_{tran}}{p_{inc}} = \frac{4R_e\{Z_2\}/R_e\{Z_1\}}{|Z_2/Z_1 + 1|^2} = \frac{2Z_2}{Z_2 + Z_1} \quad (11.32)$$

$$\frac{p_{ref}}{p_{inc}} = 1 - \frac{W_{tran}}{W_{ref}} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad (11.33)$$

and for the power transmission and reflections

$$\frac{W_{tran}}{W_{inc}} = 1 - \left| \frac{Z_2 - Z_1}{Z_2 + Z_1} \right|^2 \quad (11.34)$$

$$\frac{W_{ref}}{W_{inc}} = \left| \frac{Z_2 - Z_1}{Z_2 + Z_1} \right|^2 \quad (11.35)$$

where $Z_i = \rho_i c_i$ is the characteristic impedance of media i for plane waves and ρ_i and c_i are the density and sound speed of the medium. For an air-steel interface and for a water-steel interface where $Z_2/Z_1 = 3.9 \times 10^7/4.1 \times 10^2$ and $Z_2/Z_1 = 3.9 \times 10^7/1.5 \times 10^6$, respectively, Eqs. (11.32)–(11.35) yield

Interface	p_{tran}/p_{inc}	p_{ref}/p_{inc}	W_{tran}/W_{inc}	W_{ref}/W_{inc}
Air-steel	1.99998	0.99998	0.00004	0.99996
Water-steel	1.927	0.927	0.141	0.859

indicating that plane waves normally incident from air onto bulk solid materials transmit only an extremely small portion of the incident energy while those incident from liquids are able to transmit an important fraction of the incident sound energy.

If the plane wave arrives at the plane interface at an oblique angle ϕ_1 ($\phi_1 = 0$ is normal incidence), the angle of the reflected wave $\phi_r = \phi_i$ but the angle of the transmitted wave ϕ_2 depends on the ratio of the propagation speeds in the two materials according to Snell's law:

$$\frac{c_1}{c_2} = \frac{\sin \phi_1}{\sin \phi_2} \quad (11.36)$$

If the plane interface is between two semi-infinite fluids or between a fluid and a porous sound-absorbing material, only compressional waves are generated. However, at interfaces between a fluid and a solid the energy transmitted into the semi-infinite solid contains both compressional and shear waves. As illustrated in Fig. 11.6, the transmitted wave breaks toward the normal of the interface if $c_2 < c_1$ and away from the normal if $c_2 > c_1$.

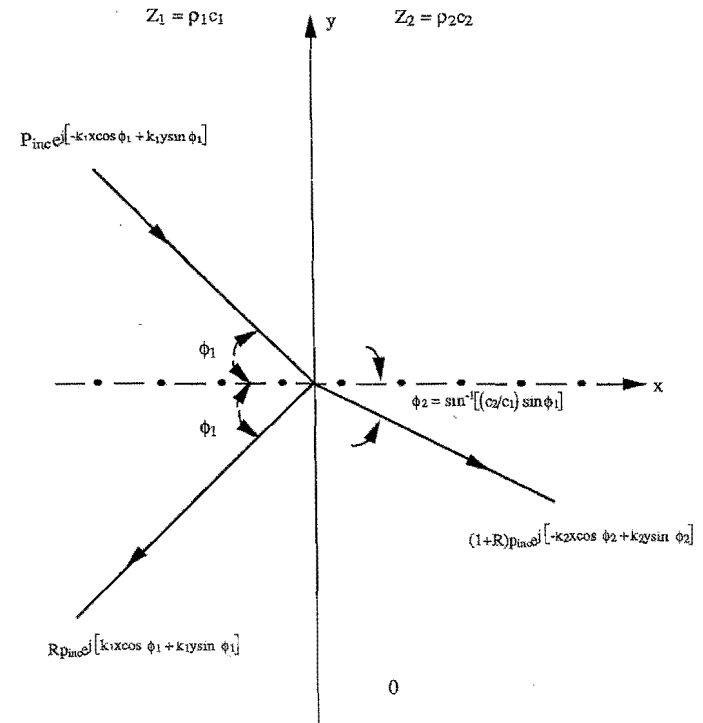


FIGURE 11.6 Reflection and transmission of an oblique-incidence plane sound wave at the plane interface of two semi-infinite thick medium, $k_1 = \omega/c_1$, $k_2 = \omega/c_2$.

If $c_2 < c_1$, there is always a transmitted wave even for grazing incidence ($\phi_1 = 90^\circ$), and for lossless media the power transmission reaches 100% at an oblique-incidence angle ϕ_1 when

$$\left(\frac{Z_1}{Z_2}\right)^2 = \frac{1 - (c_2/c_1)^2 \sin^2 \phi_1}{\cos^2 \phi_1} \quad (11.37)$$

If $c_2 > c_1$ sound transmission occurs only in a limited incidence angle range $0 < \phi_1 < \phi_{1L} = \sin^{-1}(c_1/c_2)$. For angles $\phi_1 > \phi_{1L}$ there is a total reflection and the sound wave penetrates into the second medium only in the form of a near field that exponentially decays with distance from the interface. The pressure reflection coefficient for oblique-incidence sound is given by

$$R(\phi_1) = \frac{p_{\text{ref}}(\phi_1)}{p_{\text{inc}}(\phi_1)} = \begin{cases} \frac{Z_2/\sqrt{1 - [(c_2/c_1) \sin \phi_1]^2} - Z_1/\cos \phi_1}{Z_2/\sqrt{1 - [(c_2/c_1) \sin \phi_1]^2} + Z_1/\cos \phi_1} & \text{for } c_2 < c_1 \text{ or } c_2 > c_1 \text{ and } \phi_1 < \phi_{1L} \\ 1 & \text{for } c_2 > c_1 \text{ and } \phi_1 > \phi_{1L} \end{cases} \quad (11.38)$$

The limiting angle for plane-wave sound transmission from air to steel is $\phi_{LC} = \sin^{-1}(c_0/c_s) = 3.8^\circ$ for compressional waves and $\phi_{LS} = \sin^{-1}(c_0/c_s) = 4.5^\circ$ for shear waves. For sound transmission from water to steel, the corresponding limiting angles are $\phi_{LC} = 13^\circ$ and $\phi_{LS} = 15^\circ$, respectively. For angles larger than these, there is total reflection and only exponentially decaying near fields exist in the solid.

11.5 POWER TRANSMISSION BETWEEN STRUCTURAL ELEMENTS

In the preceding sections, the power lost to a connected structure was considered only as an additional mechanism that increases the loss factor of the excited structure. In many practical problems, however, the power transmitted to a neighboring structure is the prime reason for a noise reduction program.

The power balance equation states that the power introduced into the directly excited structure is either dissipated in it or is transmitted to neighboring structures. Accordingly, if in a noise reduction problem the power is to be *confined* to the excited structure, the power *dissipated* in the structure must greatly exceed the power *transmitted* to the neighboring structures. This requires a *high loss factor* for the excited structure and a construction that *minimizes the power transmission* to neighboring structures. Methods to achieve high damping are the subject of Chapter 14.

Reduction of Power Transmission through a Change in Cross-Sectional Area

The simplest construction that causes a partial reflection of an incident compression or bending wave is a sudden change in cross-sectional area, as shown schematically in Fig. 11.7.

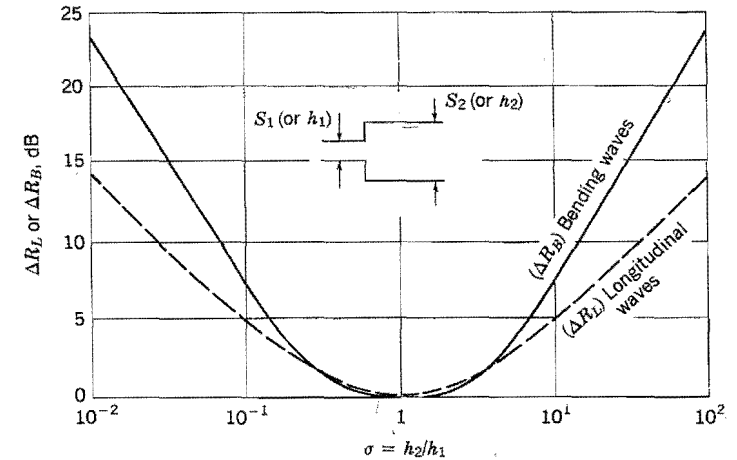


FIGURE 11.7 Attenuation at discontinuity in cross section as a function of thickness ratio. (After reference 1.)

Reflection Loss of Compression Waves. The reflection loss ΔR_L , defined as the logarithmic ratio of the incident to the transmitted power (for both sections of the same material), is calculated as¹

$$\Delta R_L = 20 \log \left[\frac{1}{2} (\sigma^{1/2} + \sigma^{-1/2}) \right] \text{ dB} \quad (11.39)$$

where $\sigma = S_2/S_1$ is ratio of the cross-sectional areas (see Fig. 11.7).

The reflection loss as a function of cross-sectional area ratio is plotted as the dashed line in Fig. 11.7. Since Eq. (11.39) is symmetrical for S_1 and S_2 , the reflection loss is independent of the direction of the incident wave. This equation is also valid for plates where $\sigma = h_2/h_1$ is the ratio of the thicknesses. Note that a 1:10 change in cross-sectional area yields only 4.8 dB reflection loss. To achieve 10 dB reflection loss, a 1:40 change in cross-sectional area would be necessary!

Reflection Loss of Bending Waves. The reflection loss for bending waves of perpendicular incidence at low frequencies is independent of frequency and is given by¹

$$\Delta R_B = 20 \log \frac{\frac{1}{2}(\sigma^2 + \sigma^{-2}) + (\sigma^{1/2} + \sigma^{-1/2}) + 1}{(\sigma^{5/4} + \sigma^{-5/4}) + (\sigma^{3/4} + \sigma^{-3/4})} \text{ dB} \quad (11.40)$$

The equation is also plotted in Fig. 11.7 (solid line).

We conclude from Fig. 11.7 that a change in cross-sectional area is not a practical way to achieve high reflection loss in load-bearing structures.

Reflection Loss of Free Bending Waves at an L-Junction

Structural elements that necessitate a change in the direction of a bending wave play an important role in structures. We consider here normally incident bending waves at a junction between two plates (or beams) at right angles: For low frequencies both the transmitted and reflected energy is predominantly in the form of bending waves. In this frequency range the reflection loss (logarithmic ratio of incident to transmitted power) for plates and beams of the same materials is given by¹

$$\Delta R_{BB} = 20 \log \left[\frac{\sigma^{5/4} + \sigma^{-5/4}}{\sqrt{2}} \right] \text{ dB} \quad (11.41)$$

This equation is plotted in Fig. 11.8. Because ΔR_{BB} is symmetric in σ , the reflection factor does not depend upon whether the original bending wave is incident from the thicker or from the thinner beam or plate. Note that the lowest reflection loss of 3 dB occurs for equal thicknesses ($\sigma = 1$). If the two plates or beams constituting the junction are of different material, replace the ratio $\sigma = h_2/h_1$ by

$$\sigma = \left(\frac{B_2 c_{B1}}{B_1 c_{B2}} \right)^{2/5} \quad (11.42)$$

where B and c_B are the bending stiffness and propagation speed of free bending waves, respectively (see Table 11.1). At higher frequencies the incident bending wave also excites longitudinal waves in the second structure.¹

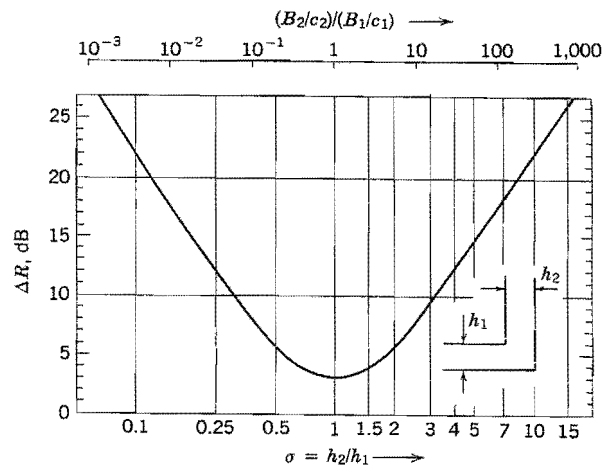


FIGURE 11.8 Attenuation of bending waves at corners (in absence of longitudinal wave interactions) as a function of thickness ratio. (After reference 1.)

Reflection Loss of Bending Waves through Cross Junctions and T-Junctions

Other structures that may provide a substantial reflection of an incident bending wave are the *cross junction* of walls shown schematically in Fig. 11.9 and the *T-junction* in Fig. 11.10. If a bending wave of perpendicular incidence reaches

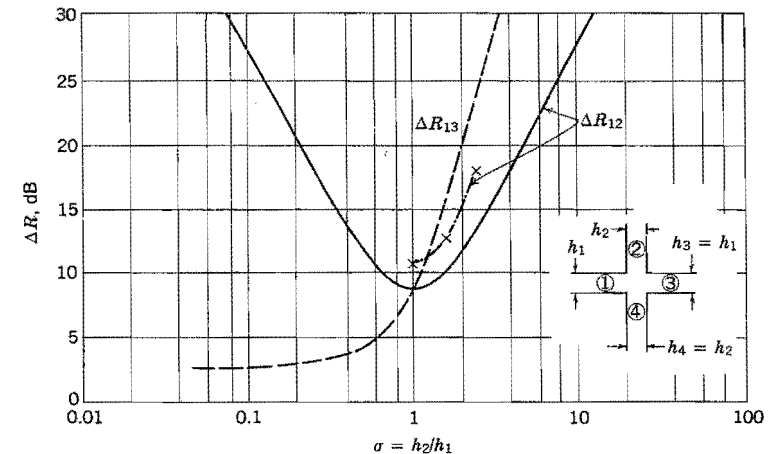


FIGURE 11.9 Attenuation of bending waves at plate intersections (in absence of longitudinal wave interactions) as a function of thickness ratio. (After reference 1.) (—) ΔR_{12} for random incidence computed by Kihlman.²⁰

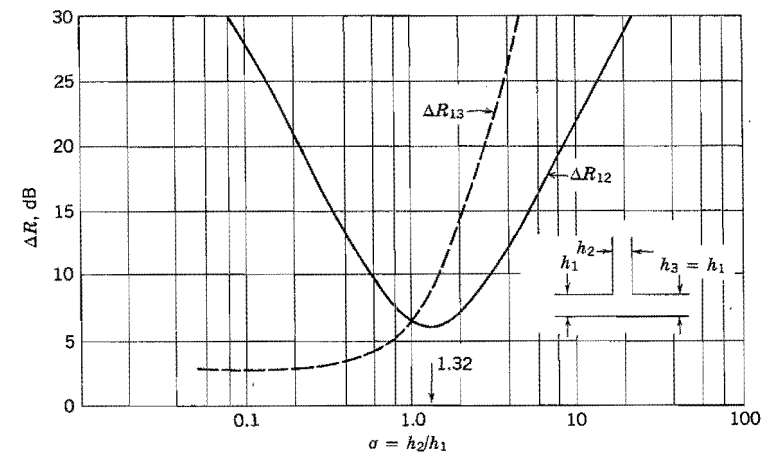


FIGURE 11.10 Attenuation of bending waves at plate intersections (in absence of longitudinal wave interactions) as a function of thickness ratio. (After reference 1.)

the cross junction from plate 1, it is partially reflected and partially transmitted to the other plates. The transmitted power splits into a number of different wave types, namely, bending waves in plate 3 and longitudinal and bending waves in plates 2 and 4. Because of the symmetry in the geometry, plates 2 and 4 will have the same excitation.¹

The reflection loss (defined as the logarithmic ratio of the power in the incident to that in the transmitted bending wave) is given as a function of the ratio of the plate thickness for plates or beams of the same material and is shown in Figs. 11.9 and 11.10. When the plates are made of different material, the ratio $\sigma = h_2/h_1$ is given by Eq. (11.42). The amplitudes of the bending waves transmitted without a change in direction are restrained by the perpendicular plate, and the reflection loss in this direction, ΔR_{13} , increases monotonically with increasing thickness of the restraining plate, h_2 . Since this plate effectively stops the vertical motion of the horizontal plate at the junction, even for very thin vertical walls, ΔR_{13} remains level at 3 dB, indicating that only the power carried by the bending moment can pass the junction. For those bending waves that change direction at the junction, the reflection loss becomes a minimum ($\Delta R_{12} = 9$ dB) at a thickness ratio $\sigma = h_2/h_1 = 1$ for the cross junction; corresponding numbers for the T-junction are 6.5 dB at a thickness ratio $\sigma = h_2/h_1 = 1.32$. The reflection loss then increases symmetrically for increasing or decreasing thickness ratio (h_2/h_1).

The transmission of free bending waves at cross junctions for random incidence has been computed and the reflection loss for a number of combinations of dense and lightweight concrete plates determined.²⁰ The results for ΔR_{12} are plotted in Fig. 11.9 (as x 's), which indicate that ΔR_{12} for random incidence is somewhat higher than that for normal incidence. It was also found that for random incidence ΔR_{12} is independent of frequency but ΔR_{13} decreases with increasing frequency.

Power Transmission from a Beam to a Plate

The structural parts of a modern building frequently include columns and structural floor slabs. Consequently, the power transmission from a beam to a plate, which models this situation, is of practical interest. Let us first consider the reflection loss for longitudinal and bending waves incident from the beam onto an infinite homogeneous plate.

Reflection Loss for Longitudinal Waves. When a longitudinal wave in the beam reaches the plate, its energy is partly reflected back up the beam and is partly transmitted to the plate in the form of a bending wave. The reflection loss (equal to the logarithmic ratio of the incident to the transmitted power) is given by²¹

$$\Delta R_L = -10 \log \left(1 - \left| \frac{Y_b - Y_p}{Y_b + Y_p} \right|^2 \right) \text{ dB} \quad (11.43)$$

where $Y_b = 1/Z_b =$ admittance of semi-infinite beam for longitudinal waves, $\text{m/N} \cdot \text{s}$

$Y_p =$ point admittance of infinite plate, $1/Z_p$, $\text{m/N} \cdot \text{s}$

Both Y_p and Y_b are real and frequency independent and can be found for infinite beams and plates from Table 11.3 by taking the reciprocal of the impedances, that is, $Y_b = 1/\rho_b c_{Lb} S_b$ and $Y_p = 1/2.3 \rho_p c_{Lp} h^2$.

Complete Power Transmission between Beam and Plate for Longitudinal Waves. Inspecting Eq. (11.43), we note that the reflection factor is zero (all the incident energy is transmitted to the plate) when $Y_b = Y_p$. Equating the values for Y_p and Y_p given above yields the requirements for complete power transmission from the beam to the plate:

$$S_b = 2.3 \frac{\rho_p c_{Lp} h^2}{\rho_b c_{Lb}} \text{ m}^2 \quad (11.44)$$

If the column and the slab are of the same material so that $\rho_p c_{Lp} = \rho_b c_{Lb}$, Eq. (11.44) simplifies to

$$S_b = 2.3 h^2 \text{ m}^2 \quad (11.45)$$

This equation says that for perfect power transfer from a beam of square cross section to a large plate, the cross dimension of the beam must be 1.52 times the thickness of the plate and for a beam of circular cross section the radius must be 0.86 times the plate thickness. Actually, this is well within the range of slab thicknesses-column cross section ratios commonly found in architectural structures. The reflection factor for different geometries of steel beam and plate connections has been measured.²¹ The results for a substantial mismatch (a) and for a near matching (b) are plotted in Fig. 11.11.

Reflection Loss for Bending Waves. When a beam carries a free bending wave, a part of the energy carried by the wave is transmitted to the plate by the effective bending moment and excites a radially spreading free bending wave in the plate. A part of the incident energy is reflected from the junction. Here the reflection loss is determined by the respective moment impedances^{1,21} of the plate and the beam:

$$\Delta R_b = -10 \log \frac{Y_b^M - Y_p^M}{Y_b^M + Y_p^M} \text{ dB} \quad (11.46)$$

where the moment admittances Y_b^M and Y_p^M are given by^{1,20}

$$Y_b^M = \frac{2}{1 + j} \frac{k_b^2}{\rho_b S_b c_{Bb}} \text{ m/N} \cdot \text{s} \quad (11.47)$$

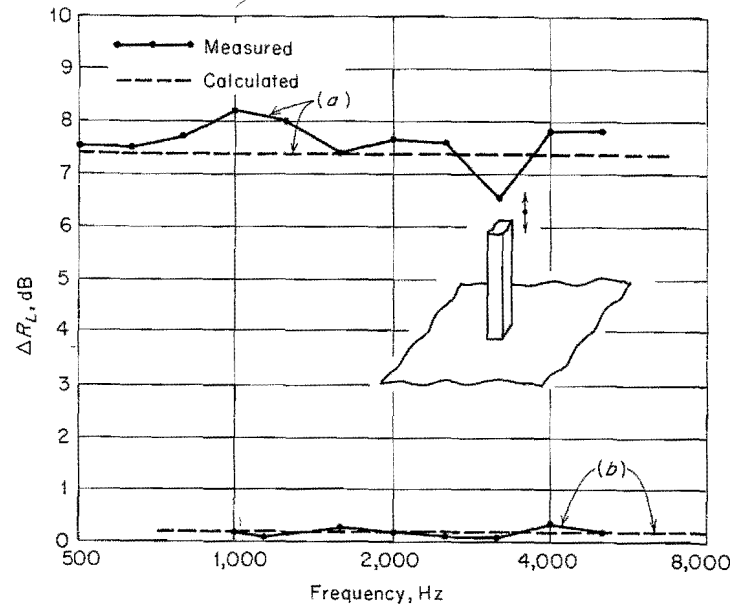


FIGURE 11.11 Reflection loss for longitudinal waves, ΔR_L , for a steel beam plate system: (a) plate thickness 2 mm, beam cross section 10×20 mm; (b) plate thickness 4 mm, beam cross section 5×10 mm. (After reference 21.)

and

$$Y_b^M = \frac{\omega}{16B_p} \left(1 + j \frac{4}{\pi} \ln \frac{1.1}{k_p a} \right) \text{ m/N} \cdot \text{s} \quad (11.48)$$

where k = wavenumber, m^{-1}

B_p = bending stiffness per unit width of plate, $\text{N} \cdot \text{m}$

a = effective distance of pair of point forces making up moment on plate, m; for rectangular and circular beam cross section $a_r = \frac{1}{3}d$ and $a_c = 0.59r$

d = side dimension of rectangular beam cross section (in direction of bending), m

r = radius of circular beam cross section, m

The reflection loss obtained²¹ for the bending-wave excitation of a steel rod of 1×2 cm cross-sectional area attached to a 0.2-cm-thick semi-infinite steel plate for two perpendicular directions of bending of the rod is plotted in Fig. 11.12.

Complete Power Transmission for Bending Waves. Since the moment impedances of the plate and beam are both frequency dependent, complete power transmission ($\Delta R_B = 0$) can occur only at a single frequency. The criteria for

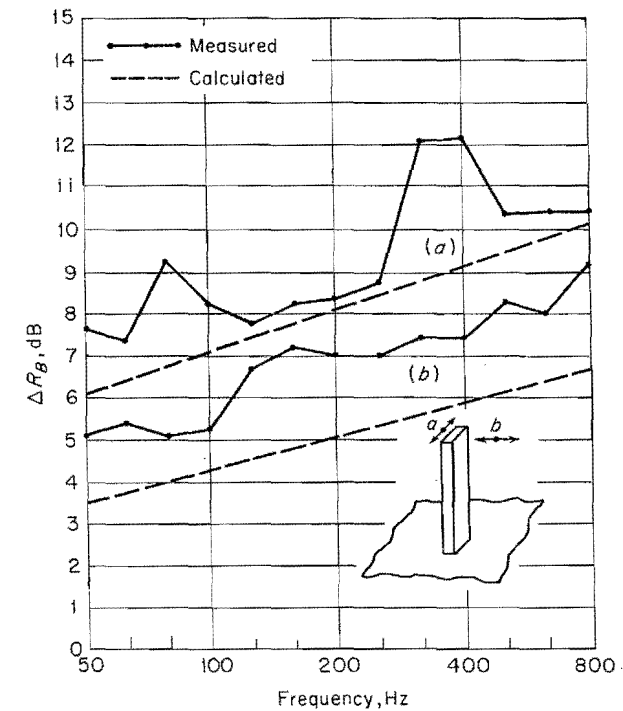


FIGURE 11.12 Reflection loss for bending waves, ΔR_B , for a steel beam plate system for two different directions of bending of the beam (a and b). Plate thickness 2 mm, beam cross section 10×20 mm. (After reference 21.)

perfect power transmission is achieved when both the real and imaginary parts of the moment impedances of the beam and plate are equal, which requires that

$$\lambda_b = 0.39 \frac{B_b}{B_p} \text{ m} \quad (11.49)$$

$$\lambda_p = 2.6a \text{ m} \quad (11.50)$$

where B_p = bending stiffness of beam, $\text{N} \cdot \text{m}^2$

λ_b = bending wavelength in beam, m

λ_p = bending wavelength in plate, m

Reduction of Power Transmission between Plates Separated by Thin Resilient Layer

In architectural structures it is customary to provide a so-called vibration break by inserting a thin layer of resilient material between structural elements. The

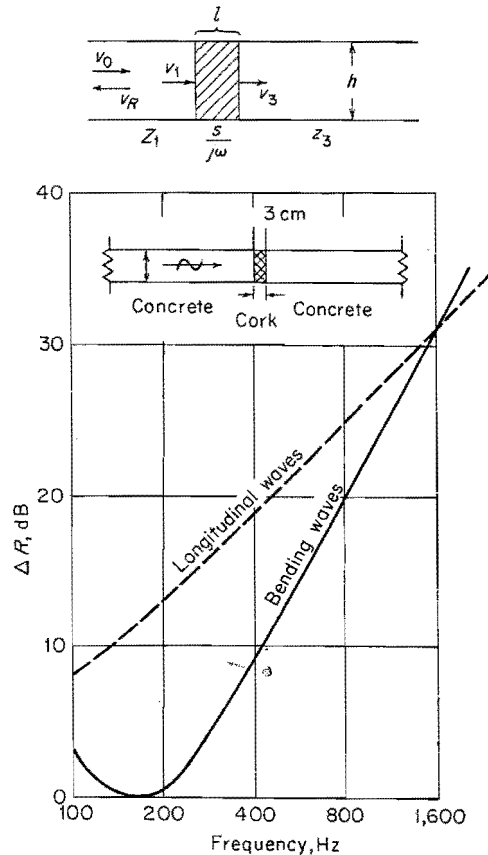


FIGURE 11.13 Attenuation due to an elastic interlayer of 3-cm-thick cork between 10-cm-thick slabs of concrete as a function of frequency. (After reference 1.)

geometry of such a vibration break is shown in Fig. 11.13. Often this construction serves also as an expansion joint.

Reflection Loss for Compression Waves. The reflection loss is given by¹

$$\Delta R_L = 10 \log \left[1 + \left(\frac{\omega Z_1}{2s_i} \right)^2 \right] \quad \text{dB} \quad (11.51)$$

where Z_1 = impedance of solid structure for compressional waves, $\text{N} \cdot \text{s/m}$
 s_i = stiffness of resilient layer in compression, N/m

Above a certain frequency ($\omega = 2s_i/Z_1$) the ΔR_L -versus-frequency curve increases with a 20-dB/decade slope with increasing frequency. Below this

frequency the resilient layer transmits the incident wave almost entirely. As an example,¹ the ΔR_L -versus-frequency curve for a 3-cm-thick layer of cork inserted between two 10-cm-thick concrete slabs (or columns) is shown in Fig. 11.13.

To achieve a high reflection loss, the resilient layer must be as soft as is permitted by the load-bearing requirements ($s_i/\omega \ll Z_1$). However, the stiffness of the layer cannot be reduced indefinitely by increasing the thickness. For frequencies where the thickness of the layer is comparable with the wavelength of compressional waves in the resilient material, the layer can no longer be considered as a simple spring characterized by its stiffness alone. The reflection loss in this frequency region is given by¹

$$\Delta R_L = 10 \log \left[\cos^2 k_i l + \frac{1}{4} \left(\frac{Z}{Z_i} + \frac{Z_i}{Z} \right)^2 \sin^2 k_i l \right] \quad \text{dB} \quad (11.52)$$

where k_i = wavenumber for compressional waves in resilient material, m^{-1}

Z = impedance of equivalent infinite structure for compression waves,
 $= Z_1 = Z_3$, $\text{N} \cdot \text{s/m}$

Z_i = impedance of equivalent infinite (length) resilient material for
 compression waves, $\text{N} \cdot \text{s/m}$

l = length of resilient layer, m

The impedances Z , Z_i and the wavenumber k_i are assumed real; thus Eq. (11.52) does not account for wave damping in the resilient material. As expected, $\Delta R_L = 0$ for $Z_i = Z$. The maximum of ΔR_L is reached when $k_i l = (2n + 1)(\frac{1}{2}\pi)$; here the reflection loss becomes

$$\Delta R_{L,\max} = 20 \log \frac{Z_i^2 + Z^2}{2ZZ_i} \quad \text{for } l = \frac{1}{2}(2n + 1)\lambda_i \quad (11.53)$$

For $k_i l = n\pi$ ($l = \frac{1}{2}n\lambda_i$), the denominator of Eq. (11.51) becomes unity independent of the magnitude Z , yielding

$$\Delta R_{L,\min} = 0 \quad \text{for } l = \frac{1}{2}n\lambda_i$$

Finally for $k_i l \ll 1$ and $Z_i \ll Z$, Eq. (11.52) simplifies to Eq. (11.51).

Reflection Loss for Bending Waves. The geometry in Fig. 11.13 suggests that for bending waves of perpendicular incidence the moments and forces acting on both sides of the junction must be equal. However, the transverse velocity and angular velocity on both sides of the junction are different because of shear and compressional deformation of the resilient layer. The elastic layer behaves quite differently for bending waves than it does for compressional waves.¹ The most striking difference is the *complete transmission* of the incident bending wave at a certain frequency and a *complete reflection* of it at another, higher frequency. Unfortunately, it turns out that the frequency of complete transmission

for architectural structures of interest usually occurs in the audio frequency range. As an example, Fig. 11.13 shows the reflection factor for bending waves as a function of frequency for a layer of 3-cm-thick cork between 10-cm-thick concrete slabs. The transmission is complete at a frequency of 170 Hz and then decreases with increasing frequency. The reflection loss for bending waves can be approximated by¹

$$\Delta R_B \cong 10 \log \left[1 + \frac{1}{4} \left(1 - \frac{E}{E_i} \frac{2\pi^3 l h^2}{\lambda_B^3} \right)^2 \right] \text{ dB} \quad (11.54)$$

where E = Young's modulus of structural material

E_i = Young's modulus of elastic material

l = length of elastic layer, m

h = thickness of structure, m

λ_B = wavelength of bending waves in structure, m

The bending wavelength in the structure for which the elastic layer provides a complete transmission of bending waves is given by

$$\lambda_{B, \text{trans}} = \pi \left(\frac{E}{E_i} h^2 l \right)^{1/3} \text{ m} \quad (11.55)$$

If one wishes to reduce the frequency of complete transmission, the ratio E/E_i and the length of the resilient layer must be large. However, the length of the resilient material should always be small compared with the wavelength of bending waves in the elastic layer to avoid resonances.

A complete reflection of the incident bending wave occurs when the bending wavelength in the plate equals π times the plate thickness, $\lambda_{Bs} = c_b/f_s = \pi h$. Consequently, the frequency where *complete reflection* occurs is *independent* of the dynamic properties and the length of the elastic layer and is given by the thickness and the dynamic properties of the plate or beam.

11.6 SOUND RADIATION

The sound radiation of small rigid bodies is treated in Chapter 10. This section deals exclusively with the sound radiation of thin flexible plates excited to vibration by point forces or by sound fields. Sound radiation of small rigid bodies is treated in Chapter 10.

Vibration of rigid and elastic structures forces the surrounding fluid or gas particles at the interface to oscillate with the same velocity as the vibrating structure and thus causes sound. The sound waves propagate in the form of compressional waves that travel with the speed of sound in the surrounding medium.

Infinite Rigid Piston

Conceptually the simplest sound-radiating structure is an infinite plane rigid piston. The motion of the piston forces the fluid particles to move along parallel lines that are perpendicular to the plane of the piston. There is no divergence that could lead to inertial reaction forces, such as those that can occur along the edges of a finite piston where the fluid can move to the side. Consequently, the reaction force per unit area (i.e., the sound pressure) is fully attributable to compression effects. This is the same situation as if the piston would be placed in a tube of rigid walls, as discussed already in Chapter 10. If the piston vibrates with velocity $\hat{v} \cos \omega t$, it generates a plane sound wave traveling perpendicular to the plane of the piston. The sound pressure as a function of distance is

$$p(x, t) = \hat{v} \rho_0 c_0 \cos(\omega t - k_0 x) \text{ N/m}^2 \quad (11.56)$$

and the radiated sound power per unit area is

$$W'_{\text{rad}} = 0.5 \hat{v}^2 \rho_0 c_0 = \langle v^2 \rangle_t \rho_0 c_0 \text{ W/m}^2 \quad (11.57)$$

where ρ_0 and c_0 are the density and speed of sound of the medium, $\omega = 2\pi f$ is the radian frequency, $k_0 = \omega/c_0 = 2\pi/\lambda_0$ is the wavenumber, $\lambda_0 = c_0/f$ is the wavelength of the radiated sound, and $\langle v^2 \rangle_t$ is the time-averaged mean-square velocity (i.e., $v = v_{\text{rms}}$).

Infinite Thin Plate in Bending

If a plane bending wave of velocity amplitude $\hat{v} = \sqrt{2}v$ and bending wave speed c_B travels on a thin plate in the positive x direction, the sound pressure as a function of x and perpendicular distance z is given by¹

$$\hat{p}(x, y) = \frac{j \hat{v} \rho_0 c_0 e^{j\omega t}}{\sqrt{(k_B/k_0)^2 - 1}} e^{-jk_B z} \exp(-z \sqrt{k_B^2 - k_0^2}) \text{ N/m}^2 \quad (11.58)$$

where $k_B = 2\pi f/c_B = 2\pi/\lambda_B$ and $k_0 = 2\pi f/c_0 = 2\pi/\lambda_0$ are the bending wavenumber in the plate and the wavenumber in the air, respectively. Inspecting Eq. (11.58), one finds that for $c_B(c_0 (k_B/k_0) 1)$ the sound pressure is 90° out of phase with the velocity at the interface so that no sound power is radiated by the plate. The sound pressure constitutes a near field that decays exponentially with increasing z . For $c_B > c_0$; $k_B/k_0 < 1$, Eq. (11.58) has the form

$$\hat{p}(x, y) = \frac{j \hat{v} \rho_0 c_0 e^{-j\omega t}}{\sqrt{1 - (k_B/k_0)^2}} e^{-jk_B x} \exp \left(-jk_0 z \sqrt{1 - \left(\frac{k_B}{k_0} \right)^2} \right) \text{ N/m}^2 \quad (11.59)$$

where the pressure and velocity are in phase at the interface ($z = 0$) and the sound power radiated by a unit area of the plate is

$$W'_{\text{rad}} = \begin{cases} \frac{0.5 \hat{v}^2 \rho_0 c_0}{\sqrt{1 - (\lambda_0/\lambda_B)^2}} & \text{W/m}^2 \text{ for } \lambda_B > \lambda_0 \\ 0 & \text{for } \lambda_B < \lambda_0 \end{cases} \quad (11.60)$$

which, with increasing frequency ($\lambda_0/\lambda_B \ll 1$), approaches that of an infinite rigid piston as given in Eq. (11.57).

Radiation Efficiency

It is customary to define the radiation efficiency of a vibrating body as

$$\sigma_{\text{rad}} = \frac{W_{\text{rad}}}{\langle v_n^2 \rangle \rho_0 c_0 S} \quad (11.61)$$

where $\langle v_n^2 \rangle$ is the normal component of the space-time average mean-square vibration velocity of the radiating surface of area S and W_{rad} is the radiated sound power. With this definition, Eqs. (11.57) and (11.60) yield for the piston

$$\sigma_{\text{rad}} = \begin{cases} 1 & \text{for infinite rigid piston} \\ 0 & \text{for } \lambda_B < \lambda_0 \text{ for infinite plate in bending} \\ [1 - (\lambda_0/\lambda_B)]^{-1/2} & \text{for } \lambda_B > \lambda_0 \text{ for infinite plate in bending} \\ (k_0 a)^2 / [1 + (k_0 a)^2] & \text{for pulsating sphere} \end{cases} \quad (11.62)$$

It is important to know that the radiation efficiency depends not only on the size and shape of the radiating body as compared with the wavelength but also on the manner the body is vibrating. If a sphere vibrates back and forth instead of pulsating, then the net volume displacement is zero and the radiation efficiency is²²

$$\sigma_{\text{rad}} = \frac{(k_0 a)^4}{4 + (k_0 a)^4} \quad (11.63)$$

Comparing Eqs. (11.62) and (11.63) reveals that at low frequencies a rigid body vibrating in translation radiates much less (approximately $\frac{1}{4}(k_0 a)^2$ times) sound power than a pulsating body of the same surface area. Radiation efficiencies of some typical structural elements are given in Table 11.8 (see references 22 and 23 for a more extensive collection). Radiation efficiencies for oscillating three-dimensional bodies of near-unity aspect ratio (such as a sphere or cube) are plotted in Fig. 11.14 and those of pipes of circular rods oscillating as rigid bodies in Fig. 11.15. Note that with increasing frequency, when the distance between the neighboring, out-of-phase moving parts of the vibrating body becomes larger than the wavelength of the radiated sound (i.e., $2\pi a > \lambda_0$ for oscillating bodies and $\lambda_B > \lambda_0$ for plates in bending), it becomes more difficult to push the air

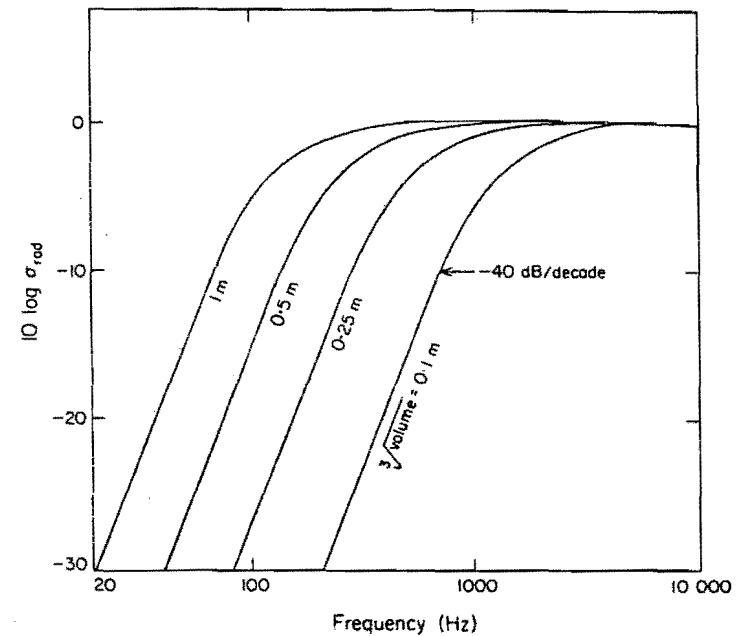


FIGURE 11.14 Radiation efficiency of oscillating three-dimensional bodies. (After reference 22.)

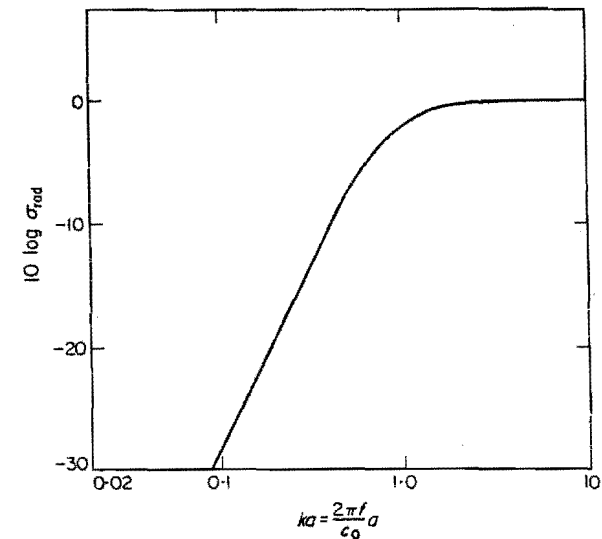


FIGURE 11.15 Radiation efficiency of oscillating rigid pipes and rods of radius a . (After reference 22.)

rapidly enough aside then to compress it, and the radiation efficiency approaches unity.

Point-Excited Infinite Thin Plate

When a very large homogeneous isotropic thin plate is excited by a point force of amplitude $\hat{F}_0$ or by an enforced local velocity of amplitude $\hat{v}_0$, free bending waves with a propagation speed of $c_B(f) \sim \sqrt{f}$ propagate radially from the excitation point. At low frequencies, where $c_B < c_0$, the free bending waves radiate no sound. Far from the excitation point, where the wave front approaches a straight line, the sound radiation that does occur is due to the in-phase vibration of the plate in the vicinity of the excitation point. The sound intensity radiation pattern has a $\cos^2 \phi$ dependence, where ϕ is the angle to the normal of the plate. The mechanical power input W_{in} , the radiated sound power W_{rad} , and the acoustical-mechanical conversion efficiency $\eta_{am} = W_{rad}/W_{in}$ are listed below:

Point Force Excitation

$$W_{in} = \frac{1}{2} \hat{F}_0^2 \frac{1}{2.3 \rho_s c_L h}$$

$$W_{rad}(f < f_c) = \hat{F}_0^2 \frac{\rho_0}{4\pi \rho_s^2 c_0}$$

$$\eta_{am}(f < f_c) = 0.37 \frac{\rho_0}{\rho_M} \frac{c_L}{c_0}$$

Point Velocity Excitation

$$(\frac{1}{2} \hat{v}_0^2) 2.3 \rho_s c_L h \quad (11.64)$$

$$\hat{v}_0^2 \frac{c_L^2 h^2 \rho_0}{2.38 c_0} \quad (11.65)$$

$$0.37 \frac{\rho_0}{\rho_M} \frac{c_L}{c_0} \quad (11.66)$$

where $\rho_s = \rho_M h$ is the mass per unit of the plate, ρ_M is the density and c_L the speed of longitudinal waves in the plate material, h is the plate thickness, and ρ_0 and c_0 are the density and speed of sound in the surrounding fluid.

Equations (11.64)–(11.66) contain the following, quite surprising, information:

1. W_{in} , W_{rad} , and η_{am} are independent of frequency and plate loss factor.
2. For point force excitation, the radiated sound power depends only on the mass per unit area of the plate ($W_{rad} \sim 1/\rho_s^2$) and not on stiffness.
3. For point velocity excitation, the radiated sound power depends only on stiffness ($W_{rad} \sim c_L^2 h^2$) and not on the plate material density.
4. The acoustical-mechanical conversion efficiency is the same for point force or point velocity excitation, and it is independent of plate thickness h and is a material constant [$\eta_{am} \sim (c_L/\rho_M)(\rho_0/c_0)$].

For noise control engineers, observations 2 and 3, embodied in Eq. (11.65), are very important. To minimize sound radiation from highly damped, point-excited, thin, platelike structures, the plate should have large mass per unit area for force excitation (e.g., by a low-impedance vibration source) and low stiffness (low E/ρ_M) for velocity excitation (by a high-impedance vibration source).

Equation (11.66) supplies the rationale for violin makers, who want to convert as large a portion of the mechanical power of the vibrating string as possible into radiated sound, to use wood ($\eta_{am} = 0.024$) and not steel ($\eta_{am} = 0.0023$) or lead ($\eta_{am} = 0.0004$) for the body of the violin.

Note also that the radiated sound power given in Eq. (11.65) usually represents the minimum achievable for a finite-size plate since it accounts only for the sound radiated from the vicinity of the excitation point.

Point-Excited Finite Plates

For finite-size plates, the sound power radiated has two components. The first component is radiated from the vicinity of the excitation point given in Eq. (11.65). The second component is radiated by the free bending waves as they interact with plate edges and discontinuities. The contribution of these two components to total radiated noise is represented by the first and second terms in Eqs. (11.67a) and (11.67b), where the first equation is valid for point force and the second for point velocity excitation:

$$W_{rad}^F \cong \hat{F}^2 \left[\frac{\rho_0}{4\pi \rho_s^2 c_0} + \frac{\rho_0 c_0 \sigma_{rad}}{4.6 \rho_s^2 c_L h \omega \eta_c} \right] W \quad (11.67a)$$

$$W_{rad}^v \cong \hat{v}^2 \left[\frac{\rho_0 c_L^2 h^2}{2.38 c_0} + 1.15 \left(\frac{c_L h}{\omega \eta_c} \right) \rho_0 c_0 \sigma_{rad} \right] W \quad (11.67b)$$

where η_c is the composite loss factor and σ_{rad} is the radiation efficiency of the plate for free bending waves. The second term in Eqs. (11.67) has been derived by the power balance (see Section 11.3) of the finite plate, assuming that the mechanical power input to the finite plate can be well approximated by the power input to the equivalent infinite plate.

Equations (11.67a) and (11.67b) can be used to assess the useful upper limit of the composite loss factor ($\eta_c^{\max}$) which, if exceeded, results only in added expense but no meaningful reduction of the radiated noise. This is done by equating the first and second terms in the square brackets and solving for η_c .

11.7 SOUND EXCITATION AND SOUND TRANSMISSION

The sound transmission process has the following three components: (1) description of the sound field at the source side, (2) prediction of the vibration response, and (3) sound radiation from the receiver side of the partition into the receiver room. Item 1 is treated in Chapter 7. Items 2 and 3 will be treated in this section. Regarding the appropriate way of analysis, partitions can be classified as either small or large compared with the acoustical wavelength.

Sound Transmission of Small Partitions

In the following analyses we define a partition small if its dimensions are small compared with the acoustical wavelength. Consequently, if the frequency is sufficiently low, even partitions of large size are considered "acoustically small."

The subject of sound transmission of acoustically small plates has been mostly ignored or treated only qualitatively in books on noise control engineering. The reason for this is that the traditional definition of sound transmission loss, which is based on the ratio of the incident and transmitted sound power, does not make sense in this low-frequency region where the sound power radiated into the receiver room is independent of the angle of incidence of the sound striking the source side of the partition. In this low-frequency region the partition will radiate the same sound power for grazing incidence (sound wave on the source side propagates parallel to the partition) than it does at normal incidence (where it propagates perpendicular to the plane of the partition).

It is instructive to start the investigation of sound transmission with a small, single, homogeneous, isotropic plate that is flash mounted in the test section between two reverberation rooms, as shown schematically in Fig. 11.16.

Let our investigation start at very low frequencies and proceed by gradually increasing the frequency of the incident sound. We will investigate the following cases:

1. The size of the panel, L , is small compared with the acoustical wavelength λ and the frequency of the incident sound wave, f , is small compared with the frequency of the first mechanical resonance of the plate, f_{M1} :

$$L \ll \lambda \quad \text{and} \quad f \ll f_{M1}$$

2. The size of the panel, L , is small compared with the acoustical wavelength λ and the frequency of the incident sound wave, f , matches the frequency of the first mechanical resonance of the plate, f_{M1} .

$$L \ll \lambda \quad \text{and} \quad f = f_{M1}$$

3. The size of the panel, L , is small compared with the acoustical wavelength λ and the frequency of the incident sound wave, f , is much larger than the frequency of the first mechanical resonance of the plate, f_{M1} .

$$L \ll \lambda \quad \text{and} \quad f \gg f_{M1}$$

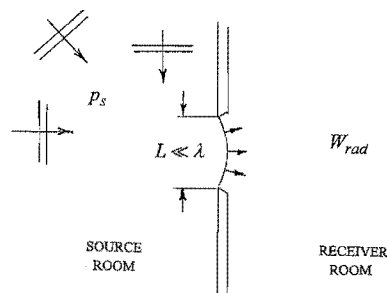


FIGURE 11.16 Sound Transmission of a small homogeneous isotropic partition.

Case 1 ($L \ll \lambda$ and $f \ll f_{M1}$). If the size of the partition is much smaller than the acoustical wavelength, the sound pressure, even at grazing incidence (i.e. the sound wave propagates parallel to the plate), is nearly constant across the source side of the plate. At any other angle of incidence, the sound pressure is even more evenly distributed across the plate than at grazing. At normal incidence, where the sound wave propagates perpendicular to the plane of the plate, the sound pressure is absolutely evenly distributed over the surface of the plate independent of the wavelength (for normal incidence the trace wavelength is infinitely large at all frequencies). In the case 1 frequency range the sound transmission of the plate is controlled by its volume compliance C_v defined as

$$C_v \equiv \frac{\Delta V}{p} \quad \text{m}^5/\text{N} \quad (11.68)$$

where C_v = volume compliance of plate, m^5/N

$p \approx$ sound pressure on source side of plate, N/m^2

As long as the dimensions of the plate are small compared with the acoustical wavelength, $p_s = 2p_{\text{inc}}$, where p_{inc} is the amplitude of the incident sound wave.

Considering now that the volume velocity $q = d\Delta V/dt = j\omega C_v p_s$ and using Eq. (10.49) from Chapter 10 the formula for the radiated sound power yields

$$W_{\text{rad}} = q^2 (\rho_0 c_0) \left(\frac{k^2}{2\pi} \right) = \frac{\omega^4 C_v^2 (\rho_0/c_0)}{2\pi} p_s^2 \quad \text{N} \cdot \text{m/s} \quad (11.69)$$

where ρ_0 = density of gas, kg/m^3

c_0 = speed of sound in gas, m/s .

Considering that our small plate radiates omnidirectionally into a half space, the sound pressure as a function of distance is

$$p(r) = \frac{p_s}{2\pi r} (\omega^2 C_v \rho_0) \quad \text{N}/\text{m}^2 \quad (11.70)$$

where r is the radial distance in meters from the center of the panel.

Formulas to predict the volume compliance C_v and resonance frequency f_{M1} for rectangular plates with simply supported and clamped edges are given in Chapter 12. Panels found in practical applications have a C_v that is larger than predicted for clamped edges and smaller than that predicted for simply supported edges. Also the resonance frequency of practical panels falls between those resonance frequencies predicted for clamped and simply supported edges.

Case 2 ($L \ll \lambda$ and $f \approx f_{M1}$). At the first resonance frequency of the panel, f_{M1} , the mass part of the panel impedance (which increases with increasing frequency) becomes equal in magnitude but opposite in phase to the stiffness part of the panel impedance, which is decreasing with increasing frequency. At and in the vicinity of the resonance frequency even a small sound pressure on the source side can cause a large volume velocity and the panel motion is limited only by energy

dissipation in the panel and by its radiation resistance. In the vicinity of f_{M1} the sound power radiated by the receiver side of the panel reaches a maximum. The dissipative losses in the panel cannot be predicted analytically. Consequently, the sound transmission cannot be predicted with satisfactory accuracy. In designing enclosures for equipment with a high-intensity tonal component at low frequencies the panels or subpanels should be chosen that their first resonance frequency should be substantially higher than the frequency of the low-frequency tonal component of the enclosed equipment. The first resonance frequency of platelike elements can be increased by curving them in two perpendicular directions to gain form stiffness.

Case 3 ($L \ll \lambda$ and $f \gg f_{M1}$). Above the first mechanical resonance frequency the velocity of the plate is controlled by the mass per unit area $\rho_s = \rho_M h$ ρ_M = density of plate material, kg/m³
 h = plate thickness, m

The volume velocity of the plate in this frequency region can be approximated by

$$q \approx p_s \frac{S}{j\omega\rho_s} \quad \text{m}^3/\text{s} \quad (11.71)$$

the radiated sound power by

$$W_{\text{rad}} \approx p_s^2 \frac{\omega^2 S^2 (\rho_0 c_0)}{2\pi} \quad \text{N} \cdot \text{m/s} \quad (11.72)$$

and the sound pressure at a distance r by

$$p(r) \approx p_s \frac{\omega^2 \rho_0^2 S}{2\pi r} \quad \text{N/m}^2 \quad (11.73)$$

where S is surface area (one side) of the plate in square meters.

More accurate prediction of the transmitted sound power can be obtained by determining the response of each of the volume-displacing modes of the plate responding to the spatially uniform sound pressure on the source side of the plate and determining the net volume velocity as the sum total of the volume velocities of all modes, as done in Chapter 6.

If the spatial distribution of the plate vibration $v(x, y)$ is known, the sound pressure in the receiver-side half space $p(x, y, z, \omega)$ can be computed as¹

$$p(x, y, z, \omega) = \frac{j\omega\rho_0}{2\pi} \int v(x, y, \omega) \frac{e^{-jkr}}{r} dS \quad \text{N/m}^2 \quad (11.74)$$

where r is the distance between the small surface element of the plate and the observation point. Equation (11.74) is valid only for a flat plate flash mounted in an infinite, flat, hard wall.

In the low-frequency region, where the plate dimensions are small compared with the acoustical wavelength ($\sqrt{S} \ll \lambda$), the transmitted sound power depends

only on the sound pressure on the source side of the plate p_s and does not depend on the angle of incidence or the sound power incident on the source side of the plate, W_{inc} . Consequently, the traditional definition of the sound transmission loss given in Eqs. (11.79) and (11.80) makes no sense in this frequency region.

Sound Transmission of Large Partitions

This section deals with the sound transmission loss of acoustically large partitions, where it is meaningful to characterize the excitation by the sound power incident on the source side of the partition and define the transmission coefficient τ and the sound transmission loss R as given in Eqs. (11.75) and (11.76), respectively.

The most common problem that noise control engineers have to deal with is the transmission of sound through solid partitions such as windows, walls, and floor slabs. The problem may be either prediction or design. The prediction problem is typically this: Given a noise source, a propagation path up to the partition, and the size and construction of the partition and the room acoustics parameters of the receiver room, predict the noise level in the receiver room. The design problem is typically stated as: Given a source, a propagation path, the room acoustics parameters of the receiver room, and a noise criterion (in the form of octave-band sound pressure levels), determine the construction of those partitions that would assure that the noise criteria are met with a sufficient margin of safety.

The transmission coefficient τ and sound transmission loss R , which characterize sound transmission through partitions, are defined as

$$\tau(\phi, \omega) = \frac{W_{\text{trans}}(\phi, \omega)}{W_{\text{inc}}(\phi, \omega)} \quad (11.75)$$

$$R(\phi, \omega) = 10 \log \frac{1}{\tau(\phi, \omega)} = 10 \log \frac{W_{\text{inc}}(\phi, \omega)}{W_{\text{trans}}(\phi, \omega)} \quad \text{dB} \quad (11.76)$$

where $W_{\text{inc}}(\phi, \omega)$ is the sound power incident at angle ϕ at frequency $\omega = 2\pi f$ on the source side and $W_{\text{trans}}(\phi, \omega)$ is the power transmitted (radiated by the receiver side).

Though knowledge of the sound transmission loss of a window or curtain wall for a particular angle of incidence may be desirable sometimes, it is a more common problem to characterize the transmission of sound between two adjacent rooms where the sound is incident on the separating partition from all angles with approximately equal probability. In such "random incident" sound fields, the sound intensity (energy incident on a unit area) I_{random} is related to the space-averaged mean-square sound pressure in the source room $\langle p^2 \rangle$ as

$$I_{\text{random}} = \frac{\langle p^2 \rangle}{4\rho_0 c_0} \quad \text{W/m}^2 \quad (11.77)$$

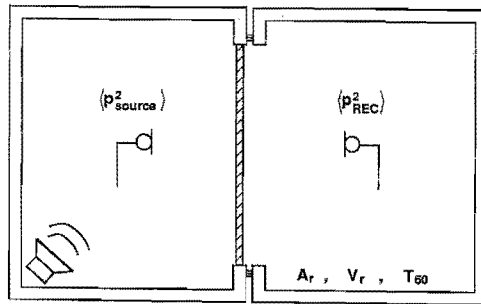


FIGURE 11.17 Experimental setup for laboratory measurements of the sound transmission loss of partitions (see text for definition of symbols).

Laboratory procedures ASTM E90-02²⁴ and ISO 140-1:1997,²⁵ adopted for measuring the sound transmission loss of partitions, are based on such a diffuse sound field obtained by utilizing large reverberation rooms (see Chapter 7) as source and receiver rooms. The measurement procedure depicted in Fig. 11.17 has three steps. The first step is to measure the sound power incident on the source-side face of the test partition of area S_w :

$$W_{inc} = I_{inc} S_w = \frac{\langle p_{source}^2 \rangle S_w}{4\rho_0 c_0} \quad \text{W} \quad (11.78)$$

by measuring the space-time average mean-square sound pressure level $\langle p_{source}^2 \rangle$ in the source room by spatially sampling the sound field. The second step is to measure the transmitted sound power W_{trans} from the power balance of the receiver room,

$$W_{trans} = \frac{\langle p_{rec}^2 \rangle A_r}{4\rho_0 c_0} \quad \text{W} \quad (11.79)$$

yielding the laboratory sound transmission loss

$$R_{lab} = 10 \log \frac{W_{inc}}{W_{trans}} = \langle L_p \rangle_s - \langle L_p \rangle_R + 10 \log \frac{S_w}{A_r} \quad \text{dB} \quad (11.80)$$

where A_r is the total absorption in the receiving room. The third step is to determine A_r from the known volume and the measured reverberation time T_{60} of the receiver room, as described in Chapter 7.

Once R_{lab} has been measured, it can be used for predicting the mean-square sound pressure in a particular receiver room acoustically characterized by its total absorption A_r through a partition of surface area S_w for an incident sound field of intensity I_{inc} as

$$\langle p_r^2 \rangle = \frac{\bar{\tau} I_{inc} S_w (4\rho_0 c_0)}{A_r} = \frac{\bar{\tau} S_w \langle p_{source}^2 \rangle}{A_r} \quad \text{N}^2/\text{m}^4 \quad (11.81a)$$

$$\langle \text{SPL}_R \rangle = \langle \text{SPL}_s \rangle - R_{lab} + 10 \log \frac{S_w}{A_r} \quad \text{dB} \quad (11.81b)$$

provided that the partition is not much smaller than tested and that edge conditions are not much different. Curves of measured random-incidence sound transmission loss versus frequency for standard windows, doors, and walls are available from manufacturers and should be used in design and prediction work. Measured sound transmission losses of some selected partitions are given by Bies and Hansen in their Table 8.1 (see the Bibliography).

The purpose of the discussion that follows is to identify the physical processes and the key parameters that control sound transmission through partitions and to provide analytical methods that further the development of an informed judgment needed for working with data obtained in laboratory measurements. Most importantly, however, this will focus on predicting sound transmission loss for situations that are different from those employed in the standardized laboratory measurements (i.e., for near-grazing incidence) and for the task of designing nonstandard partitions for unique applications.

Excitation of structures by an incident sound wave is significantly different from excitation by localized forces, moments, enforced velocities, or angular velocities. As discussed in the previous sections, the response of thin, platelike structures to localized excitation results in radially spreading free bending waves. The propagation speed of these waves, $c_B(f)$, is as unique a characteristic of the plate as is the period of a pendulum. The thinner the plate and the lower the frequency, the lower is the propagation speed.

If the structure is excited by an incident sound wave, forcing occurs simultaneously over the entire exposed surface of the plate. The incident sound enforces its spatial pattern on the plate, causing it to instantaneously conform to its trace. The trace “runs” along the plate with a speed that approaches the speed of sound in the source-side media when the sound runs nearly parallel to the plate (grazing incidence) and approaches infinity as the sound incidence approaches normal. This “sound-forced” response of the thin plate to sound excitation is referred to as the “forced wave.” In contrast to the free bending waves, the speed of the forced bending wave is independent of frequency, plate thickness, and mass per unit area (though the amplitude of the response depends on them). Because of their supersonic speed, forced waves radiate sound very efficiently at all frequencies (e.g., their radiation efficiency $\sigma_F \geq 1$), except for panels that are small compared with the acoustical wavelength.

Transmission of Normal-Incidence Plane Sound Waves through an Infinite Plate

It is advisable to introduce the complex process of sound transmission by considering first the least complicated case when a plane sound wave is normally incident on a uniform homogeneous, isotropic, flat plate of thickness h , as shown in Fig. 11.18. Because the pressure exerted on the plate is in phase over the

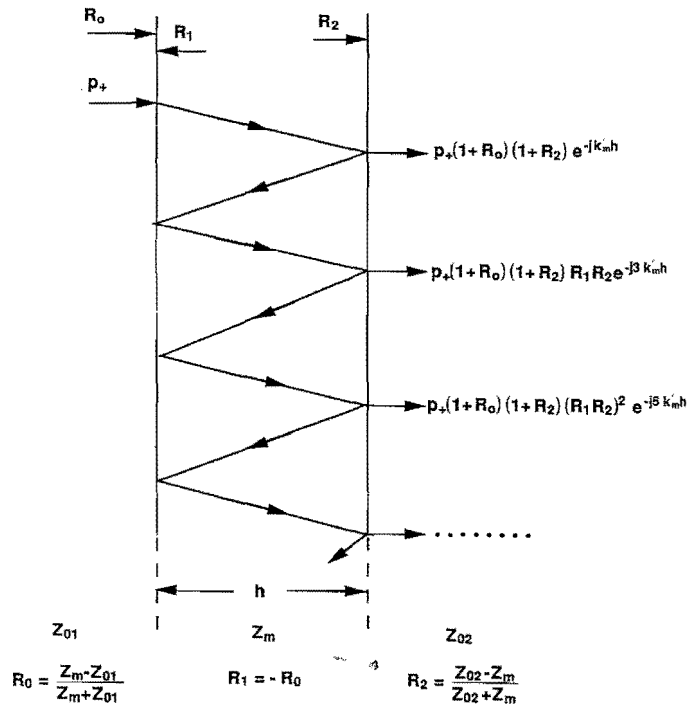


FIGURE 11.18 Transmission of a normally incident sound wave through a flat, homogeneous, isotropic plate. The transmitted pressure p_T is the sum of all the infinite components on the right. (See text for definition of symbols.)

entire surface of the plate, only compressional waves are excited. The sound wave of pressure amplitude p_+ propagates in a gas of characteristic plane-wave impedance $Z_{01} = \rho_{01}c_{01}$. It encounters the solid plate of characteristic impedance $Z_M = \rho c_L$ and on the transmitting side (right) will radiate sound into another gas of characteristic impedance $Z_{02} = \rho_{02}c_{02}$. The multiple reflection and transmission phenomena at the interfaces are governed by Eqs. (11.32) and (11.33). The amplitude of the transmitted sound pressure p_T in the receiver-side media is obtained by the summation of the transmitted components, as illustrated in Fig. 11.18, yielding²⁶

$$p_T = p_+ \frac{(1 + R_0)(1 + R_2)e^{-jk_m h}}{1 - R_1 R_2 e^{-j2k_m h}} \quad \text{N/m}^2 \quad (11.82)$$

where R_0 , R_1 , and R_2 are the reflection factors at the interfaces and directions indicated in Fig. 11.18 and h is the plate thickness. Propagation losses can be taken into account by a complex wavenumber $k'_m = k_m(1 + \frac{1}{2}j\eta)$, where $k_m = \omega/c_L$ is the wavenumber of compressional waves in the plate and η is the loss

factor. The normal-incidence sound transmission loss is defined as

$$R_N = 10 \log \frac{W_{\text{inc}}}{W_{\text{rad}}} = 10 \log \frac{p_+^2}{p_T^2} \quad \text{dB} \quad (11.83a)$$

Assuming that the same gas is on both sides of the plate, Eq. (11.82) yields

$$R_N = 10 \log \left[\cos^2 k'_m h + 0.25 \left(\frac{Z_0}{Z_m} + \frac{Z_m}{Z_0} \right)^2 \sin^2 k'_m h \right] \quad \text{dB} \quad (11.83b)$$

which for $|k'_m h| \ll 1$ yields the simple expression

$$R_N \cong 10 \log \left[1 + \left(\frac{\rho_s \omega}{2\rho_0 c_0} \right)^2 \right] \quad \text{dB} \quad (11.83c)$$

known as the normal-incidence mass law. In Eq. (11.83c), $\rho_s = \rho h$ is the mass per unit area of the plate and $\rho_0 c_0 = Z_0$ is the characteristic impedance of the gas, assumed the same on both sides.

Figure 11.19 shows the computed normal-incidence sound transmission loss of a 0.6-m- (2-ft-) thick dense concrete wall. At low frequencies where the plate thickness is less than one-sixth of the compressional wavelength ($f < c_L/6h$) the sound transmission loss follows the normal-incidence mass law of Eq. (11.83c), increasing by approximately 6 dB for each doubling of the frequency or the mass per unit area. At high frequencies compressional wave resonances in the plate

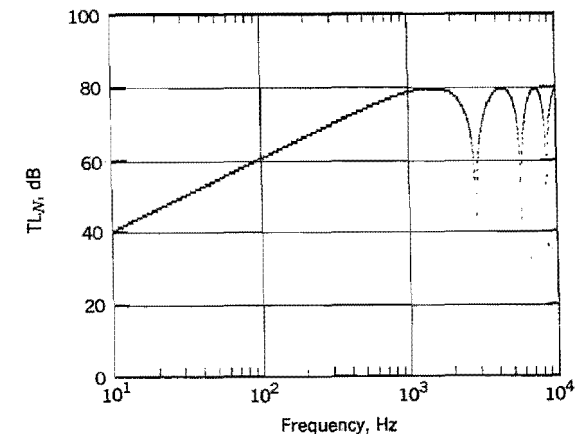


FIGURE 11.19 Normal-incidence sound transmission loss R_0 of a 0.6-m-thick dense concrete wall computed according to Eq. (11.83b) assuming $\eta = 0$.

occur and the R_N -versus-frequency curve exhibit strong minima at $f_n = nc_L/2h$, yielding

$$R_N^{\min} = 20 \log \left(1 + \frac{\pi}{4} \frac{Z_m}{Z_0} \eta n \right) \text{ dB} \quad (11.84a)$$

and the maximal achievable sound transmission loss can be approximated as

$$R_N^{\max} \cong 20 \log \frac{Z_m}{2Z_0} \text{ dB} \quad (11.84b)$$

Equation (11.84a) indicates complete transmission of the incident sound ($R_N^{\min} = 0$ dB) at compressional wave resonances if $\eta = 0$. However, Eq. (11.84a) indicates that even a small loss factor of $\eta = 0.001$ ensures that the normal-incidence transmission loss of a concrete partition does not dip below 24 dB at compressional wave resonances. Note, however, that no such dips are observed for random incidence because different incident angles correspond to different frequencies where such dips occur. As indicated in Eq. (11.84b), the maximum achievable normal-incidence sound transmission loss of homogeneous, isotropic single plates of any thickness is limited by the ratio of the characteristic impedances and is 80 dB for dense concrete, 94 dB for steel, and 68 dB for wood.

Transmission of Oblique-Incidence Plane Sound Waves through an Infinite Plate

The transmission of oblique-incidence sound through infinite plane plates can be formulated either in terms of shear and compressional waves, where the bending of the plate is considered as a superposition of these two wave types, or by utilizing the bending-wave equation of the plate. Both formulations are discussed below.

Combined Compressional and Shear Wave Formulation. The combined shear and compression wave formulation, which is valid also for thick plates, has been treated in Chapter 9 of the 1992 edition. It is too involved to present it here. The interested reader is referred to references 27 and 28 or to pages 248–288 of the 1992 edition of this book.

Separation Impedance Formulation. Sound transmission through plates can be conveniently characterized by the separation impedance Z_s defined as^{1,26}

$$Z_s = \frac{p_s - p_{\text{rec}}}{v_n} \text{ N} \cdot \text{s/m}^3 \quad (11.85)$$

where p_s is the complex amplitude of the sound pressure at the source-side face of the plate representing the sum of the incident and reflected pressures ($p_s = p_{\text{inc}} + p_{\text{refl}}$), p_{rec} is the complex amplitude of sound pressure on the receiver-side face, and v_n is the complex amplitude of normal velocity of the receiver-side face

of the plate. In the case of single panels, it is generally assumed that both faces of the panel vibrate in phase with the same velocity. The sound transmission coefficient of the plate τ and its sound transmission loss R are given by^{1,26}

$$\tau(\phi) = \frac{I_{\text{trans}}}{I_{\text{inc}}} = \left| 1 + \frac{Z_s \cos \phi}{2\rho_0 c_0} \right|^2 \quad (11.86)$$

$$R(\phi) = 10 \log \frac{1}{\tau(\phi)} = 10 \log \left(\left| 1 + \frac{Z_s}{2\rho_0 c_0 / \cos \phi} \right|^2 \right) \text{ dB} \quad (11.87)$$

where ϕ is the angle of sound incidence ($\phi = 0$ for normal incidence).

The formulation of the sound transmission in terms of the separation impedance of Eq. (11.87) lends itself exceptionally well to predicting the sound transmission loss of multilayer partitions where the constituent layers may be thin plates separated by air spaces with and without porous sound-absorbing material (such as double and triple walls), windows, and so on. The reason for this is that the separation impedance of such multilayer partitions can be expeditiously obtained by appropriately combining the separation impedances of the constituent layers.

The separation impedance for a thick isotropic plate is obtained by solving the bending-wave equation of the plate as formulated by Mindlin²⁹:

$$\begin{aligned} & \left(\nabla^2 - \frac{\rho_M}{G} \frac{\partial^2}{\partial t^2} \right) \left(B \nabla^2 - \frac{\rho_M h}{12} \frac{\partial^2}{\partial t^2} \right) \xi(x, y) + \rho_M h \frac{\partial^2}{\partial t^2} \xi(x, y) \\ & = \left(1 - \frac{B}{Gh} \nabla^2 + \frac{\rho_M h^2}{12G} \frac{\partial^2}{\partial t^2} \right) \Delta p(x, y, 0) \end{aligned} \quad (11.88)$$

where $\Delta p(x, y, 0)$ is the pressure differential across the plate, $\xi(x, y)$ is the plate displacement in the z direction normal to the plate surface, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian operator, ρ_M is the density, G is the shear modulus, B is the bending stiffness, and h is the thickness of the panel. The solution of Eq. (11.88) for a plane sound wave of incidence angle ϕ yields³⁰

$$Z_s = \frac{j \{ [\rho_M h + (\rho_M h^3/12 + \rho_M B/G)(\omega^2/c_0^2) \sin^2 \phi] \omega - [(B/c_0^4) \sin^4 \phi + \rho_M^2 h^3/(12G)] \omega^3 \}}{1 + B\omega^2 \sin^2 \phi/(Gc_0^2 h) - \rho_M h^2 \omega^2/(12G)} \text{ N} \cdot \text{s/m}^3 \quad (11.89)$$

Equation (11.89) can be approximated by

$$Z_s \cong Z_m + \left(\frac{1}{Z_B} + \frac{1}{Z_{\text{sh}}} \right)^{-1} \text{ N} \cdot \text{s/m}^3 \quad (11.90)$$

where $Z_m = j\omega\rho_M h$ is the mass impedance of the plate per unit area and $Z_B = -jB\omega^3 \sin^4 \phi/c_0^4$ and $Z_{\text{sh}} = -jGh\omega \sin^2 \phi/c_0^2$ are the bending-wave and shear wave impedances of the plate per unit area.

If $|Z_{sh}| \ll |Z_B|$, the plate responds predominantly in shear and $Z_s \cong Z_m + Z_{sh}$. In this case, Eq. (11.89) yields

$$R_{sh}(\phi) = 20 \log \left| 1 + \frac{j\omega\rho_M h[1 - (c_s/c_0)^2 \sin^2 \phi]}{2\rho_0 c_0 / \cos \phi} \right| \text{ dB} \quad (11.91)$$

Equation (11.91) indicates that trace coincidence between the incident sound wave and the free shear waves in the plate (which would lead to complete transmission) can be avoided at all incident angles provided that the panel is specifically designed to yield a low shear wave speed such that $c_s^2 = G/\rho < c_0^2 = P_0/\rho_0$. Unfortunately, this desirable condition cannot be met with homogeneous plates made of any construction-grade material. As we will discuss later, however, it is possible to satisfy the $c_s < c_0$ criterion with specially designed³¹ inhomogeneous sandwich panels without compromising the static strength of the panel and thereby preserving the mass law behavior described by

$$R_{sh}(\phi) \cong R_{mass}(\phi) = 20 \log \left(1 + \frac{\rho_s \omega \cos \phi}{2\rho_0 c_0} \right) \text{ dB} \quad (11.92)$$

which is representative of a limp panel of the same mass per unit area $\rho_s = \rho_M h$ as the shear panel.

Thin homogeneous panels are easier to bend than to shear so that $Z_B \ll Z_{sh}$. It follows that $Z_s \cong Z_m + Z_B$ and Eqs. (11.87) and (11.89) yield

$$R(\phi) = 10 \log \frac{1}{\tau(\phi)} = 10 \log \left| 1 + \frac{j\rho_s \omega [1 - (f/f_c)^2 \sin^4 \phi]}{2\rho_0 c_0 / \cos \phi} \right|^2 \text{ dB} \quad (11.93)$$

where f_c is the critical frequency where the speed of the free bending waves in the plate, $c_B(f_c)$, equals the speed of sound in the media. It is given by

$$f_c = \left(\frac{\omega_c}{2\pi} \right) = \left(\frac{c_0^2}{2\pi} \right) \left(\frac{\rho_s}{B} \right)^{1/2} \text{ Hz(s)}^{-1} \quad (11.94)$$

where ρ_s = mass per unit area, $\rho_M h$ kg/m²

h = plate thickness, m

B = bending stiffness of plate, $Eh^3/[12(1 - \nu^2)]$ N · m

Consequently, the product $f_c \rho_s$ is a constant for a specific plate material and liquid (usually air). It is given by

$$f_c \rho_s = \left(\frac{c_0^2}{2\pi} \right) \sqrt{12(1 - \nu^2)} \sqrt{\frac{\rho_M}{E}} \text{ Hz(s)}^{-1} \quad (11.95)$$

This product is listed in Table 11.2 for air as a surrounding medium. It is used to determine f_c as $f_c = \rho_s f_c / f_c$.

The coincidence frequency in air of homogeneous isotropic plates can be easily determined by cutting a narrow strip of length l in meters, supporting the two ends of the strip on a knife edge (to simulate simple supported edge conditions), and measuring the sag d in millimeters at the midpoint of the strip. The coincidence frequency is computed as

$$f_c = 1.65(10^2) \sqrt{\frac{d/\text{mm}}{l/\text{m}}} \text{ Hz(s)}^{-1} \quad (11.96)$$

Equation (11.93) indicates that above the critical frequency f_c given in Eq. (11.94), trace coincidence between the incident sound wave and the free bending waves in the plate occurs when $f = f_c / \sin^2 \phi$ (which is called the coincidence frequency) and would result in complete transmission if the plate had no internal damping.

It is customary to account for internal damping in Eq. (11.88) by introducing a complex Young's modulus $E' = E(1 + j\eta)$ that results in complex wavenumbers $k'_c = k_c(1 + \frac{1}{2}j\eta)$ and $k'_s = k_s(1 + \frac{1}{2}j\eta)$ and yields the following modified form of the sound transmission loss:

$$\begin{aligned} R(\phi) &= 10 \log \frac{1}{\tau(\phi)} \\ &= 10 \log \left\{ \left| 1 + \frac{\rho_s \omega}{2\rho_0 c_0 / \cos \phi} \left[\eta \left(\frac{f}{f_c} \right)^2 \sin^4 \phi + j \left[1 - \left(\frac{f}{f_c} \right)^2 \sin^4 \phi \right] \right] \right|^2 \right\} \text{ dB} \end{aligned} \quad (11.97)$$

Equation (11.97) indicates that in the vicinity of $f = f_c / \sin^2 \phi$ the curves of sound transmission loss versus frequency exhibit a minimum that is controlled by the damping.

Figure 11.20 shows the curve of sound transmission loss versus frequency for a 4.7-mm-($\frac{3}{16}$ -in.-) thick glass plate for normal ($\phi = 0^\circ$), $\phi = 45^\circ$, and near-grazing ($\phi = 85^\circ$) angles of incidence computed according to Eq. (11.97). Figure 11.20 illustrates the decrease in sound transmission loss that occurs with increasing angle of incidence (owing to the $\cos \phi$ term) and the trace-matching dip that occurs at $f = f_c / \sin^2 \phi$.

Transmission of Random-Incidence (Diffuse) Sound through an Infinite Plate

A plane wave impinging on the plate at one particular angle is not a typical problem. The sound field in a room is better modeled as a diffuse sound field, which is an ensemble of plane sound waves of the same average intensity traveling with equal probability in all directions. A region of unit area on the plate will be exposed, at any instant, to plane sound waves incident from all areas on

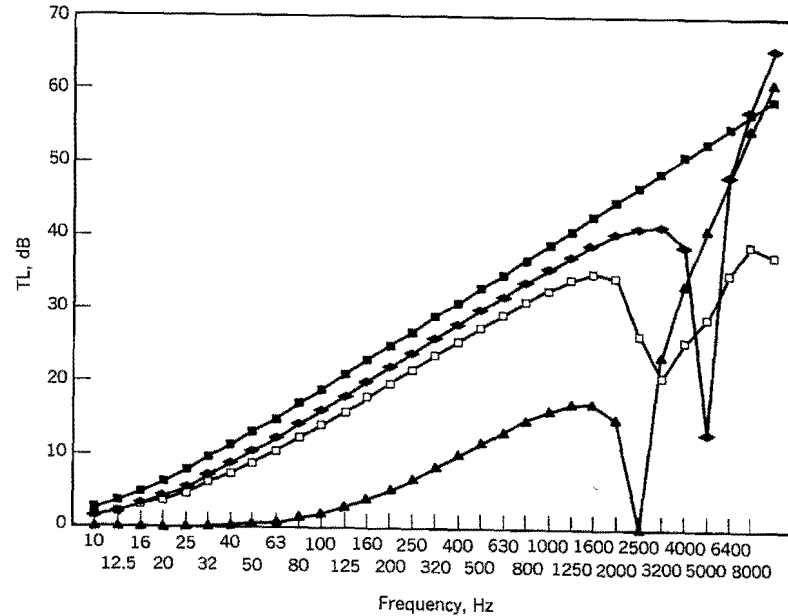


FIGURE 11.20 Computed TL-versus-frequency curve of a 3.7-mm- $(\frac{3}{16}$ -in.-) thick infinite glass plate for various angles of incidence: (■) normal ($\phi = 0^\circ$); (◆) $\phi = 45^\circ$; (▲) $\phi = 85^\circ$; (□) random.

a hemisphere whose center is the area on the plate. These waves are uncorrelated and have equal intensity. The sound intensity incident on the unit area of the plate from any particular angle will be the intensity of the plane wave at angle I_{inc} multiplied by the cosine of the angle of incidence. The total transmitted intensity is then

$$I_{trans} = \int_{\Omega} \tau(\phi) I_{inc} \cos \theta d\Omega \quad \text{W/m}^2 \quad (11.98)$$

The integration is over a hemisphere of solid angle Ω , where $d\Omega = \sin \phi d\phi d\theta$. Because I_{inc} is the same for all plane waves and τ is independent of the polar angle θ , an average transmission coefficient may be defined by

$$\bar{\tau} = \frac{\int_0^{\phi_{lim}} \tau(\phi) \cos \phi \sin \phi d\phi}{\int_0^{\phi_{lim}} \cos \phi \sin \phi d\phi} \quad (11.99)$$

where ϕ_{lim} is the limiting angle of incidence of the sound field. For random incidence, ϕ_{lim} is taken as $\frac{1}{2}\pi$, or 90° . The sound transmission coefficient $\tau(\phi)$ is that given in Eq. (11.97), and the random-incidence sound transmission loss is given by

$$R_{random} = 10 \log \frac{1}{\bar{\tau}} \quad \text{dB} \quad (11.100a)$$

At low frequencies ($f \ll f_c$) the random-incidence sound transmission loss (for $TL_N > 15$ dB) is found by averaging the argument of Eq. (11.97) over a range of ϕ from 0° to 90° to yield^{32,33}

$$R_{random} \cong R_0 - 10 \log(0.23 R_0) \quad \text{dB} \quad (11.100b)$$

which is commonly referred to as the random-incidence mass law.

It has become common practice to use the *field-incidence mass law*, which is defined (for $R_0 \geq 15$ dB) as³⁴

$$R_{field} \cong R_0 - 5 \quad \text{dB} \quad (11.101)$$

This result, which yields better agreement with measured data than Eq. (11.100b), approximates a diffuse incident sound field with a limiting angle ϕ_{lim} of about 78° in Eq. (11.99).³⁴

The mass law transmission losses R_0 , R_{field} , and R_{random} , valid for frequencies well below coincidence, are plotted versus $f\rho_s$ in Fig. 11.21.

Field-Incidence Sound Transmission for Thin Isotropic Plates, R_{field} . Equations (11.98) and (11.99) must be solved by numerical integration.

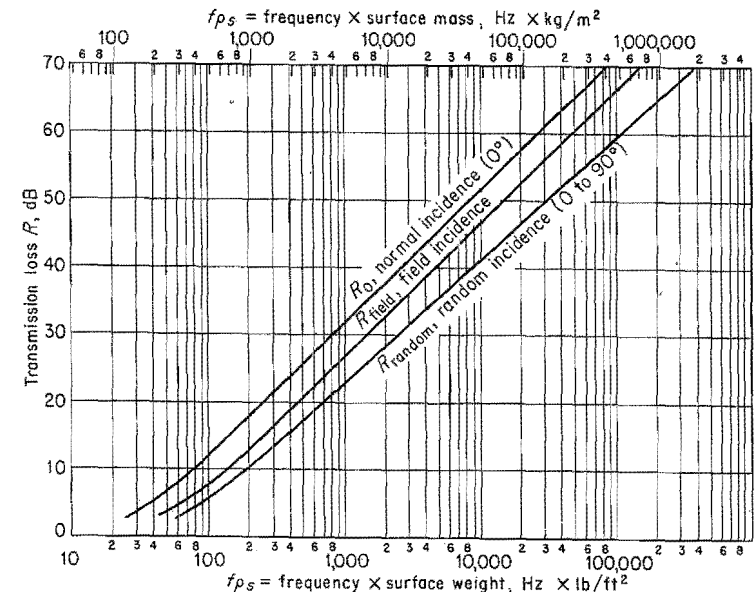


FIGURE 11.21 Theoretical sound transmission loss of large panels for frequencies well below coincidence ($f \leq 0.5 f_c$). Field incidence assumes a sound field that allows all angles of incidences up to 78° from normal.

The results of such an integration of the transmission coefficient between the angles 0° and 78° and application of Eq. (11.100a) to give the *field-incidence transmission loss* are presented in Fig. 11.22 for all values of f/f_c . The ordinate is the difference between the field-incidence transmission loss R_{field} and the normal-incidence mass law transmission loss at the critical frequency $R_0(f_c)$. The latter is easily determined from Eq. (11.83c) or from Fig. 11.21 when the mass per unit area and the critical frequency of the panel are known. Note that predicted transmission losses of less than about 15 dB for $f \ll f_c$ or less than 25 dB for $f \simeq f_c$ from Fig. 11.22 are not accurate.

Example 11.2. Calculate the normal-incidence mass law for an aluminum panel weighing 10 lb/ft^2 at a frequency of 500 Hz. Also determine the random-incidence and field-incidence mass laws. What is R_{field} at 2800 Hz when $\eta = 10^{-2}$?

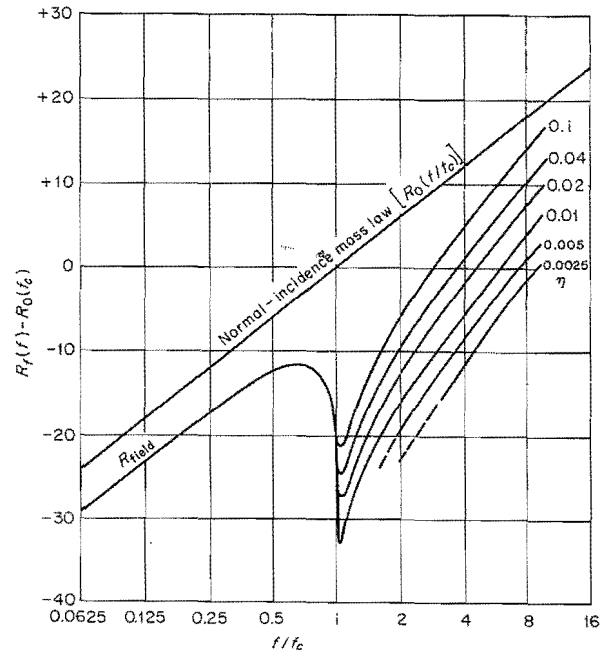


FIGURE 11.22 Field-incidence forced-wave transmission loss. The ordinate is the difference between the field-incidence transmission loss at the frequency f and the normal-incidence transmission loss at the critical frequency ($f/f_c = 1$). Note that for a predicted transmission loss of less than 15 dB or for the dashed areas on the figure, the transmission loss depends on both the surface weight and the loss factor, and the curves provide only a lower bound estimate to the actual transmission loss. Use of curve: (1) determine $\rho_s f_c$ from Table 11.2, (2) determine f_c , (3) determine $R_0(f_c)$ from Fig. 11.21 or Table 11.2, (4) read $R_f(f) - R_0(f_c)$ from Fig. 9.21 at the required η , and (5) $R_f(f) = [R_f(f) - R_0(f_c)] + R_0(f_c)$. Top curve is the normal-incidence mass law defined in Eq. (11.83c).

Solution The normal-incidence mass law is given by Eq. (11.83c) and the upper curve of Fig. 11.21. We have $f \rho_s = 500 \times 10 = 5000 \text{ Hz} \cdot \text{lb/ft}^2$. From Fig. 11.21, $R_0 = 45.5 \text{ dB}$. The random-incidence mass law is given by Eq. (11.100b) and the lower curve of Fig. 11.21; that is, $R_{\text{random}} = 35 \text{ dB}$. The field-incidence mass law is given by Eq. (11.101) and the middle curve of Fig. 11.21, that is, $R_{\text{field}} = 45.5 - 5 = 40.5 \text{ dB}$.

From Table 11.2, $\rho_s f_c = 7000 \text{ Hz} \cdot \text{lb/ft}^2$; $f_c = 7000/10 = 700 \text{ Hz}$ and $R_0(f_c) = 48 \text{ dB}$ from Fig. 11.21. Evaluating Fig. 11.22 at $f/f_c = 2800/700 = 4$ and $\eta = 0.01$, we get $R_f(f) - R_0(f_c) = -6 \text{ dB}$, yielding $R_r(f) = [R_r(f) - R_0(f_c)] + R_0(f_c) = -6 + 48 = 42 \text{ dB}$.

Sound Transmission for Orthotropic Plates. Sound transmission for orthotropic plates differs from that of isotropic plates because orthotropic plates have markedly different bending stiffnesses in the different principal directions. The difference in bending stiffness for plane plates may result from the anisotropy of the plate material (such as for wood caused by grain orientation) or from the construction of the plate such as corrugations, ribs, cuts, and so on. Consequently, the speed of free bending waves is different for these two directions and the orthotropic panel has two coincidence frequencies given by³⁵

$$f_{c1} = \frac{c_o^2}{2\pi} \sqrt{\frac{\rho_s}{B_x}} \text{ Hz} \quad (11.102a)$$

$$f_{c2} = \frac{c_o^2}{2\pi} \sqrt{\frac{\rho_s}{B_y}} \text{ Hz} \quad (11.102b)$$

where B_x is the bending stiffness for the stiffest direction and B_y the direction perpendicular to this. The random-incidence sound transmission loss of an orthotropic plate is predicted by³⁵

$$R_{\text{random}} \cong \begin{cases} 10 \log \left[\left(\frac{\rho_s \omega}{2\rho_0 c_0} \right)^2 \right] - 5 & \text{for } f \ll f_{c1} \\ 10 \log \left[\left(\frac{\rho_s \omega}{2\rho_0 c_0} \right)^2 \right] - 10 \log \left[\frac{1}{2\pi^3 \eta} \frac{f_{c1}}{f} \sqrt{\frac{f_{c1}}{f_{c2}}} \left(\ln \frac{4f}{f_{c1}} \right)^4 \right] & \text{for } f_{c1} < f < f_{c2} \\ 10 \log \left[\left(\frac{\rho_s \omega}{2\rho_0 c_0} \right)^2 \right] - 10 \log \frac{\pi f_{c2}}{2\eta f} & \text{for } f > f_{c2} \end{cases} \quad (11.103)$$

where η is the loss factor. For corrugated plates, as shown in Fig. 11.23, the bending stiffnesses can be approximated by

$$B_y = \frac{Eh^3}{12(1-\nu^2)} \text{ N} \cdot \text{m} \quad (11.104a)$$

$$B_x = B_y \left(\frac{s}{s'} \right) \quad \text{N} \cdot \text{m} \quad (11.104b)$$

where s and s' are defined in Fig. 11.23. Note that the increase in bending stiffness caused by corrugations, ribs, and stiffeners always results in a reduction of the sound transmission loss, while measures such as partial-depth saw cuts, which decrease bending stiffness, result in an increase of the sound transmission loss of plates.

Sound Transmission Loss for Inhomogeneous Plates. Sound transmission loss for inhomogeneous plates, such as appropriately designed sandwich panels, can be substantially higher than for homogeneous panels of the same mass per unit area, provided that such plates favor the propagation of the free shear waves (with frequency-independent propagation speed) rather than free bending waves for which the propagation speed increases with increasing frequency. However, they must be designed such that the shear wave speed remains below the speed of sound in air so that no trace coincidence occurs. Consequently, the sound transmission loss of such so-called shear wall panels closely approximates the field-incidence mass law. Information for designing such panels is given in reference 31. However, ordinary sandwich panels are very poor sound barriers because

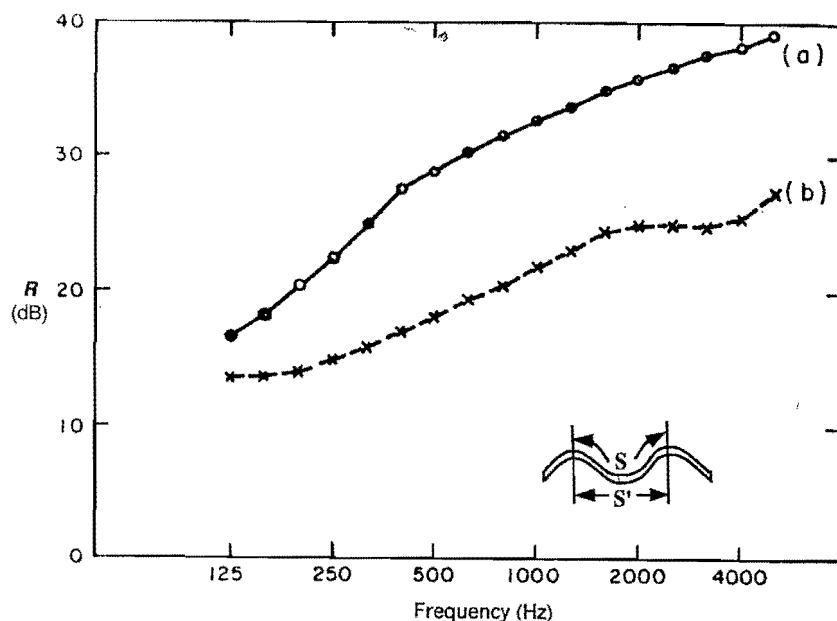


FIGURE 11.23 Measured sound transmission loss of steel plates (after Ref. 28): (a) plane plate, $\rho_s = 8 \text{ kg/m}^2$; (b) corrugated plate, $\rho_s = 11 \text{ kg/m}^2$; s , distance between corrugations along surface; s' , distance along straight line.

of their low mass and high bending stiffness that result in a coincidence frequency that usually falls in the middle of the audio frequency range. Dilation resonance, which occurs at the frequency where the combined stiffness impedance of the face plate and that of the enclosed air equals the mass impedance of the plate, also leads to further deterioration of the sound transmission loss of sandwich panels.

Sound Transmission through a Finite-Size Panel

For most architectural applications, where the first resonance frequency of typical platelike partitions is well below the frequency range of interest and the plate size is much larger than the acoustical wavelength, Eq. (11.97) or Fig. 11.22 (which are strictly valid only for infinitely large panels) can be used to predict the sound transmission loss of finite panels. In many industrial applications, the finite size of the panel must be taken into account.³⁶

In finite panels the sound-forced bending waves encounter the edges of the plate and generate free bending waves, such that the sum of the incident forced bending wave and the generated free bending wave satisfies the particular plate edge condition (e.g., zero displacement and angular displacement at a clamped edge). Consequently, the sound-forced bending waves continuously feed free bending wave energy into the finite panel and build up a reverberant, free bending wave field. The mean-square vibration velocity of this free bending wave field, $\langle v_{FR}^2 \rangle$ can be obtained using a power balance for the finite plate. The power introduced into the finite plate at the edges equals the power lost by the plate owing to viscous losses in the plate material, energy flow into connected structures, and sound radiation. The transmitted sound radiated by the finite panel is given by

$$W_{\text{rad}} = \langle v_{FO}^2 \rangle \rho_0 c_0 S \sigma_{FO} + \langle v_{FR}^2 \rangle \rho_0 c_0 S \sigma_{FR} \quad W \quad (11.105)$$

where $\langle v_{FO}^2 \rangle$ is the mean-square velocity of the sound-forced supersonic bending waves, $\sigma_{FO} \geq 1$ is the radiation efficiency of the forced waves, S is the surface area of the panel, and σ_{FR} is the radiation efficiency of the free bending waves. Since $\sigma_{FR} \ll 1$ below the critical frequency of the panel ($f \ll f_c$), it is frequently the case that the vibration response of the panel is controlled by the free bending waves (i.e., $\langle v_{FR}^2 \rangle \gg \langle v_{FO}^2 \rangle$) but the sound radiation is controlled by the less intense but more efficiently radiating forced waves.

The classical definition of sound transmission loss is $R = 10 \log(W_{\text{inc}}/W_{\text{trans}})$, where W_{inc} is the sound power incident at the source side and W_{trans} is that radiated from the receiver side of the panel. If the incident sound is a plane wave arriving at an incident angle ϕ ($\phi = 90^\circ$ for grazing incidence), then it is assumed that

$$W_{\text{inc}} = \frac{0.5 |\hat{p}_{\text{in}}|^2 S \cos \phi}{\rho_0 c_0} \quad W \quad (11.106)$$

This assumption leads to the dilemma that at grazing incidence ($\phi = 90^\circ$) no power is incident on the panel. It is common knowledge that grazing incidence sound excites the panel to forced vibrations, and the panel radiates sound into the receiver room when the forced bending waves in the panel and the sound wave at the receiver side, which run parallel to the panel, encounter the edges of the finite panel. This unresolved conceptual problem has been avoided³⁴ by limiting the incident angle range to 78° , in computing the field incidence sound transmission loss for the infinite panel according to Eq. (11.101), so as to yield reasonable agreement with laboratory measurements for panel sizes typically employed in such tests.

Obviously, it is not the incident sound power but the mean-square sound pressure on the source side that is forcing the panel. Since this quantity is proportional to the sound energy density in the source room $E_s = \langle p_s^2 \rangle / \rho_0 c_0^2$, it has been suggested^{37,38} that the sound transmission loss of a finite partition be defined as

$$R_E \equiv 10 \log \left(\frac{E_s S}{E_R A} \right) \text{ dB} \quad (11.107)$$

where $E_R = 4 W_{\text{trans}} / c_0 A$ is the energy density in the receiver room, S is the surface area of the panel (one side), and A is the total absorption in the receiver room. The transmitted sound power $W_{\text{trans}} = \frac{1}{4} c_0 A E_R$ is owing to the velocity of the plate. The forced response is dominated by the mass-controlled separation impedance $Z_s \simeq j\omega\rho_s$. The sound radiation of the sound-forced finite plate is controlled by its radiation impedance $Z_{\text{rad}} \simeq \text{Re}\{Z_{\text{rad}}\} = \rho_0 c_0 \sigma_F$. Consequently, the low-frequency sound transmission loss of the finite partition is predicted as³⁸

$$R_E \simeq R_0 - 3 - 10 \log \sigma_F \text{ dB} \quad (11.108)$$

where R_0 is the normal-incidence mass law sound transmission loss given in Eq. (11.83c) and σ_F is the forced-wave radiation efficiency of the finite panel given in Table 11.8. Note that σ_F depends on panel size as well as on incidence angle and can be smaller or larger than unity. This implies that the sound transmission loss of finite panels can be larger than the normal-incidence mass law even for grazing incidence if the size of the panel is small compared with the wavelength. When the panel size is much larger than the acoustical wavelength, σ_F approaches $1/\cos\phi$. For predicting the sound transmission loss of finite partitions over the entire low-frequency range ($f \ll f_c$), Eq. (11.108) should be used.

According to reference 38, the classical sound transmission formulas for infinite panels can be used to approximate the sound transmission loss of finite partitions by substituting $1/\sigma_F$ instead of $\cos\phi$ and $\sqrt{1 - 1/\sigma_F^2}$ instead of $\sin\phi$ in Eq. (11.97) and carrying out the integration in Eq. (11.99) from $\phi = 0$ to $\phi = 90^\circ$ to obtain an estimate of the random-incidence sound transmission loss that agrees well with laboratory measurements. There is no need to resort to limiting the incident angle range to 78° . The radiation efficiency for random-incidence (diffuse-field) sound-forced excitation is given in Table 11.8, and this expression

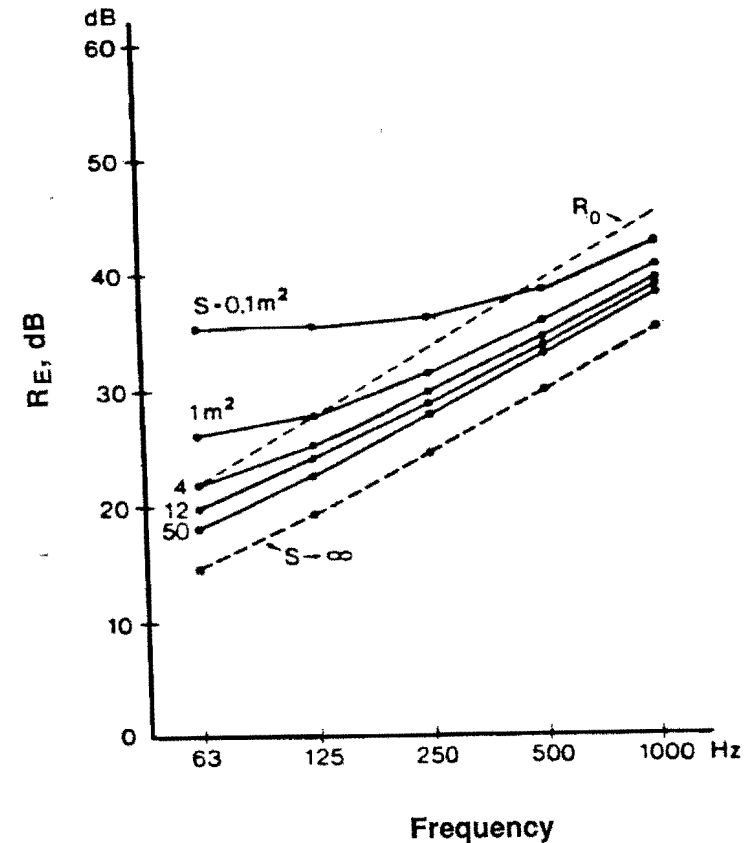


FIGURE 11.24 Random-incidence sound transmission loss of 25-kg/m² panels as a function of frequency with panel surface area S as a parameter, mass-controlled low-frequency region; computed according to Eq. (11.110). (After References 37 and 38.)

should be used in Eq. (11.108). As shown in Fig. 11.24, the random-incidence sound transmission loss of partitions of approximately 4 m² surface area, which are typically used in laboratory measurements, yield predicted values that are 5 dB below the normal-incidence mass law, giving theoretical justification for using the field-incidence mass law defined in Eq. (11.101) for partitions of this size. Note, however, that for small partitions Eq. (11.108) is more accurate. It should be pointed out that all of the sound transmission loss prediction formulas are valid only in the frequency region well above the first bending wave resonance of the panel where the forced response is mass controlled. For small, very stiff partitions the frequency range of interest may extend into the stiffness-controlled region below the frequency of the first bending wave resonance.* In this case,

*For prediction in this frequency region use Eqs. (11.70)–(11.73).

the volume compliance of the panel, as described in the beginning of this section in Chapter 12, should be used to predict sound transmission.

Empirical Method for Predicting Sound Transmission Loss of Single Partitions. An alternate technique useful in preliminary design is illustrated in Fig. 11.25. In essence, it considers the loss factor of the material to be determined completely by the material selection and substitutes a "plateau" or horizontal line for the peak and valley of the forced-wave analysis in the region of the critical frequency.^{34,39} Its use will be demonstrated in Example 11.3.

Example 11.3. Calculate the transmission loss of a $\frac{1}{8}$ -in.- (3-mm-) thick, 5×6.5 -ft (1.52×2 -m) aluminum panel by the alternate (plateau) method.

Solution From Table 11.2, the product of surface mass and critical frequency is $\rho_s f_c = 34,700 \text{ Hz} \cdot \text{kg/m}^2$. The $\frac{1}{8}$ -in.-thick aluminum plate has a surface density

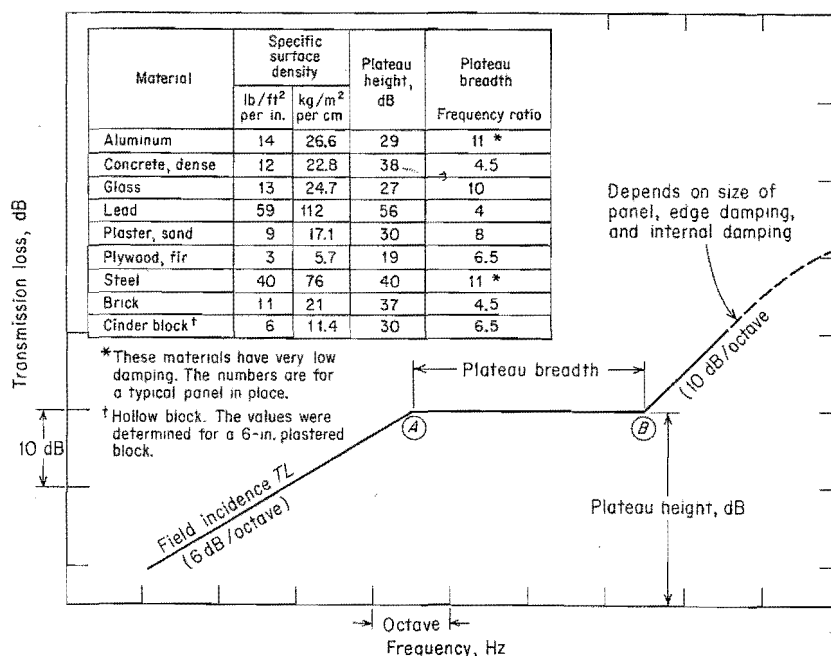


FIGURE 11.25 Approximate design chart for estimating the sound transmission loss of single panels. The chart assumes a reverberant sound field on the source side and approximates the behavior around the critical frequency with a horizontal line or plateau. The part of the curve to the left of A is determined from the field-incidence mass law curve (Fig. 11.21). The plateau height and length of the line from A to B are determined from the table. The part above B is an extrapolation. This chart is fairly accurate for large panels. Length and width of the panel should be at least 20 times the panel thickness.³⁴

of 8.5 kg/m^2 . From Fig. 11.21 we find that the normal-incidence transmission loss at the critical frequency is $R_0(f_c) = 48.5 \text{ dB}$ and that the field-incidence transmission loss at 1000 Hz is $R_{\text{field}}(8500 \text{ Hz} \cdot \text{kg/m}^2) = 31.5 \text{ dB}$.

The procedure by the plateau method is as follows^{34,39}:

1. Using semilog paper (with coordinates decibels versus log frequency), plot the field-incidence mass law transmission loss as a line with a 6-dB/octave slope through the point 31.5 dB at 1000 Hz.
2. From Fig. 11.25, the plateau height for aluminum is 29 dB. Plotting the plateau gives the intercept of the plateau with the field-incidence mass law curve at approximately 750 Hz.
3. From Fig. 11.25, the plateau width is a frequency ratio of 11. The upper frequency limit for the plateau is therefore $11 \times 750 \text{ Hz} = 8250 \text{ Hz}$.
4. From the point 29 dB, 8250 Hz, draw a line sloping upward at 10 dB/octave. This completes the plateau method estimate (see curve *b* in Fig. 11.34).

Sound Transmission through Double- and Multilayer Partitions

The highest sound transmission loss obtainable by a single partition is limited by the mass law. The way to "break this mass law barrier" is to use multilayer partitions such as double walls, where two solid panels are separated by an air space that usually contains fibrous sound-absorbing material and double windows where light transparency requirements do not allow the use of sound-absorbing materials.

The transmission of sound through a multilayer partition can be computed in a similar manner as the sound absorption coefficient of multilayer sound absorbers treated in Chapter 8. Figure 11.26 shows the situation where a plane sound wave of frequency $f = \omega/2\pi$ is incident at an angle ϕ on a panel that has N layers and $N + 1$ interfaces. The important boundary conditions are that the wavenumber component parallel to the panel surface $k_x = k \sin \phi$ must be the same in all of the layers and that the acoustical pressure and particle velocity at the interfaces of the layers must be continuous.⁴⁰⁻⁴²

The layers are characterized by their wave impedance formula, which relates the complex wave impedance at the input-side interface Z_I with that at the termination-side interface Z_T and by their pressure formula that relates the complex sound pressure at the input-side interface p_I to that at the terminal-side interface p_T .

The impedance and pressure formulas for an impervious orthotropic thin plate are given by⁴²

$$\begin{aligned} Z_I &= Z_T + Z_s \\ &= Z_T + j \left[\omega \rho_s - \frac{1}{\omega} (B_x k_x^4 + 2B_{xy} k_x^2 k_y^2 + B_y k_y^4) \right] \quad \text{N} \cdot \text{s/m}^3 \quad (11.109) \end{aligned}$$

$$p_I = p_T \frac{Z_I}{Z_T} \quad \text{N/m}^2 \quad (11.110)$$

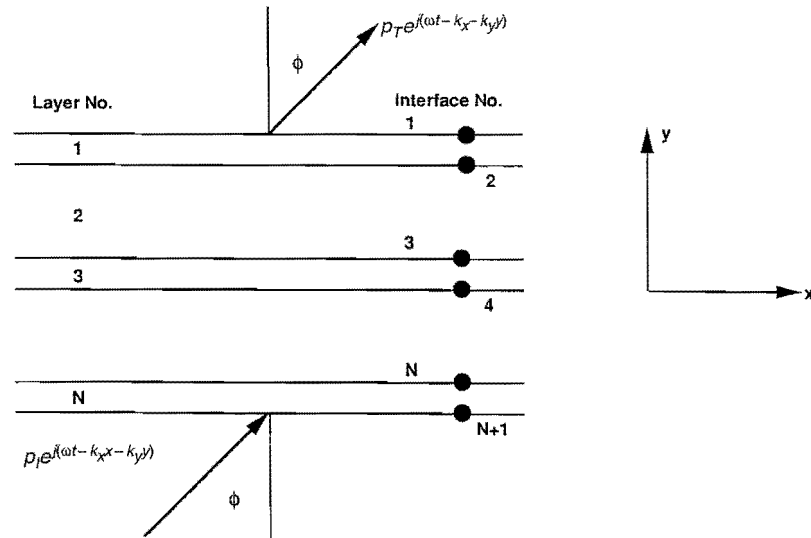


FIGURE 11.26 Transmission of a plane oblique incident sound wave through an infinite lateral dimension multilayer panel.

where ρ_s is the mass per unit area and B is the bending stiffness of the orthotropic plate.

For isotropic plates the second term in Eq. (11.109) reduces to $Z_s = Z_m + Z_B = j(\omega \rho_s - B k_x^4 / \omega)$. If the impervious layer is a composite of an isotropic homogeneous plate with a bonded damping material of thickness h_D , Young's modulus E_D , Poisson ratio ν_D , and damping loss factor η_D and the plate characteristics are thickness h_p , Young's modulus E_p , Poisson ratio ν_p , and loss factor η_p , then the complex bending stiffness $B = B_{\text{comp}}(1 + j\eta_{\text{comp}})$ is obtained^{1,42} from

$$B_{\text{comp}} = \frac{E_p h_p^3}{12(1 - \nu_p^2)} + \frac{E_D h_D (h_p + h_D)^2}{4(1 - \nu_D^2)} \quad \text{Nm} \quad (11.111)$$

$$\eta_{\text{comp}} = \frac{1}{4} B_{\text{comp}} (\eta_p E_p h_p + \eta_D E_D h_D) (h_p + h_D)^2 \quad (11.112)$$

For a porous sound-absorbing layer of thickness h the impedance and pressure formulas are given by⁴²

$$Z_I = Z_a \frac{k_a (1 + Z_a \Gamma_a / Z_T \Gamma_{ay}) e^{j\Gamma_{ay} h} + (1 - Z_a \Gamma_a / Z_T \Gamma_{ay}) e^{-j\Gamma_{ay} h}}{k_{ay} (1 + Z_a \Gamma_a / Z_T \Gamma_{ay}) e^{j\Gamma_{ay} h} - (1 - Z_a \Gamma_a / Z_T \Gamma_{ay}) e^{-j\Gamma_{ay} h}} \quad \text{N} \cdot \text{s/m} \quad (11.113)$$

$$p_T = \frac{p_I}{2} \left[\left(1 + \frac{Z_a \Gamma_a}{Z_I \Gamma_{ay}} \right) e^{-j\Gamma_{ay} h} + \left(1 - \frac{Z_a \Gamma_a}{Z_I \Gamma_{ay}} \right) e^{j\Gamma_{ay} h} \right] \quad \text{N/m}^2 \quad (11.114)$$

where $Z_a = \rho_0 c_0 Z_{an}$ is the complex characteristic acoustical impedance of the porous bulk material for plane waves and Z_{an} is its normalized value while $\Gamma_a = \Gamma_{an} k_0$ is the complex wavenumber of plane sound waves in the bulk porous material; $\Gamma_{ay}^2 = \Gamma_a^2 - k_x^2$. Formulas for computing Γ_{an} and Z_{an} on the basis of the flow resistivity of the porous material R_1 are given in Chapter 8 [see Eqs. (8.22) and (8.19)]. The simpler approximate formulas for Γ_a and Z_a given in reference 42 are less accurate and their use is not recommended.

In the special case of an air layer without porous sound-absorbing material, substitute $Z_a = \rho_0 c_0$ and $k_a = k_0$ into Eqs. (11.113) and (11.114).

The computation of the sound transmission loss of a layered partition proceeds as follows:

1. Set the termination impedance at the receiver-side interface (interface 1 in Fig. 11.26) to $Z_T = \rho_0 c_0 / \cos \phi$.
2. Apply the appropriate impedance formula for layer 1 and compute the input impedance at interface 2.
3. Using the impedance computed in step 2 as the termination impedance for layer 2, compute the input impedance at interface 3 and proceed down the chain of impedance calculations until the input impedance at the source-side interface (interface $N + 1$ in Fig. 11.26), Z_{N+1} , is obtained.
4. Compute the sound pressure at the source-side interface as the sum of the incident and reflected pressures $p_{N+1} = p_I [2\alpha / (\alpha + 1)] e^{j(\omega t - k_x x - k_y y)}$, where $\alpha = Z_{N+1} / (\rho_0 c_0 / \cos \phi)$.
5. Apply the appropriate pressure formulas in succession until the sound pressure at the receiver-side interface, p , is obtained.
6. Determine the transmission coefficient $\tau(\omega, \phi) = p_1^2 / p_I^2$ for all frequencies of interest.
7. Perform computational steps 1–6 for the incident angle range from $\phi = 0^\circ$ to $\phi = 90^\circ$ in one-third-degree increments.
8. Compute the random-incidence sound transmission coefficient for isotropic layers (see ref. 42 for orthotropic impervious layers) as

$$\tau_R(\omega) = \int_0^{\pi/2} \tau(\omega, \phi) \sin 2\phi d\phi$$

9. Compute the random-incidence sound transmission loss of the infinite layered partition as

$$R_{\text{random}}(\omega) = 10 \log \frac{1}{\tau_R(\omega)}$$

Figure 11.27 shows the random-incidence sound transmission loss of an infinite three-layer partition as well as that of its constituent layers, computed according to Eqs. (11.109)–(11.114). The partition consists of two 1-mm-thick steel plates and a 100-mm-thick air space that may or may not contain a fibrous sound-absorbing material. Figure 11.27 illustrates the benefit of using sound-absorbing

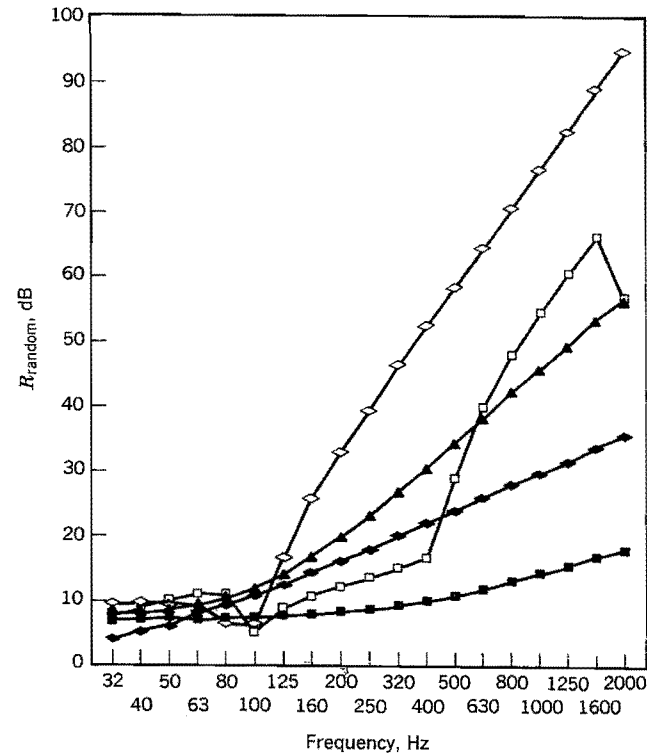


FIGURE 11.27 Computed random-incidence sound transmission loss of a double wall and its constituent layers: (■) 100-mm-thick fibrous absorber, flow resistivity $R_1 = 16,700 \text{ N/s} \cdot \text{m}^4$; (◆) 1-mm steel plate; (▲) 1-mm steel plate and 100-mm fibrous absorber. Double wall consisting of two 1-mm-thick steel plates: (□) 100-mm air space, no fibrous absorber; (◇) 100-mm air space with fibrous absorber.

layers. Note that below 500 Hz the double wall without sound-absorbing material in the air space provides substantially lower random-incidence sound transmission loss than a single 1-mm steel plate or the single plate combined with the sound-absorbing layer. The highest sound transmission loss is obtained when the air space is filled with the porous sound-absorbing material. In practical situations, leaks and structure-borne connections between the face plates at edges of the partition usually limit the maximally achievable sound transmission loss at high frequencies to a range of 40–70 dB. At low frequencies the finite size of the partition results in higher values than predicted for infinite partitions. The effect of the sound-absorbing material in the air space results in refraction of the oblique-incidence sound toward the normal, thereby reducing the dynamic stiffness of the air between the plates. The sound-absorbing material also prevents high sound energy buildup in the cavity. These result in a substantial

increase in sound transmission loss. The flow resistivity of the sound-absorbing material should be about $R_1 = 5000 \text{ N} \cdot \text{s/m}^4$.⁴³ Higher values of R_1 yield only diminishing returns.

The filling of the air space with a gas of 50% lower speed of sound than air (such as SF_6 or CO_2) has the same effect as the sound-absorbing material,^{44,45} as illustrated in Fig. 11.28. Using a light gas such as helium, which has three times higher speed of sound than air, also improves sound transmission loss to the same extent as a heavy-gas fill. In this case, the improvement is due to the higher speed of sound in the gas fill, which makes it easier to push the gas tangentially than to compress it. Double windows, which can be hermetically sealed and must be light transparent, are partitions where this beneficial effect can be exploited.

Empirical Method for Predicting Sound Transmission Loss of Double Partitions. Goesele⁴⁶ has proposed a simplified method to predict the sound transmission loss R of a double partition when the measured sound transmission losses of the two constituent single partitions R_I and R_{II} are available, there are no structure-borne connections, and the gap is filled with porous sound-absorbing material. The prediction is given as

$$R \cong R_I + R_{II} + 20 \log \left(\frac{4\pi f \rho_0 c_0}{s'} \right) \quad \text{dB} \quad (11.115)$$

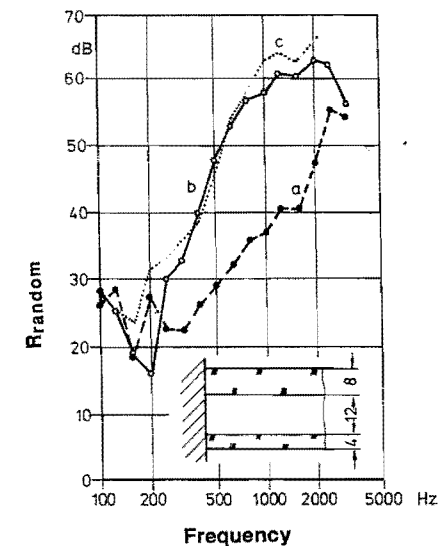


FIGURE 11.28 Improvement of the sound transmission loss of a double glass partition (no contact at the edges) owing to heavy gas (SF_6) fill of the gap: a, measured with air-filled gap; b, measured with SF_6 -filled gap; c, computed for mineral wool fill. (After Ref. 44.)

where

$$s' = \begin{cases} \frac{\rho_0 c_0^2}{d} & \text{for } f < f_d = \frac{c_0}{2\pi d} \quad \text{N/m}^3 \quad (11.116a) \\ 2\pi f \rho_0 c_0 & \text{for } f > f_d \quad \text{N/m}^3 \quad (11.116b) \end{cases}$$

is the dynamic stiffness per unit area of the gap and d is the gap thickness.

If no measured sound transmission loss data for the constituent single partitions are available, the sound transmission loss of the double wall made of two identical panels can be predicted on the basis of material properties as

$$R(f_R < f < f_c) \cong 20 \log \frac{\pi f \rho_{s1}}{\sqrt{2} \rho_0 c_0} + 40 \log \left(\frac{\sqrt{2} f}{f_R} \right) \quad \text{dB} \quad (11.117a)$$

$$R(f > f_R, f > f_c) \cong 40 \log \left[\frac{\pi f \rho_{s1}}{\rho_0 c_0} \sqrt{2\eta} \left(\frac{f}{f_c} \right)^{1/4} \right] + 20 \log \frac{4\pi f \rho_0 c_0}{s'} \quad \text{dB} \quad (11.117b)$$

where f_c is the critical frequency of the panels and f_R is the double-wall resonance frequency

$$f_R = \frac{1}{2\pi} \sqrt{\frac{2^3 \sqrt{2} s'}{\rho_{s1}}} \quad \text{Hz} \quad (11.118)$$

Figure 11.29 shows that Eq. (11.115) yields good agreement with measured data in the entire frequency region, while Eqs. (11.117a) and (11.117b) give good agreement only well below and well above the critical frequency but fail in the frequency region near the critical frequency. Prediction methods for the sound transmission loss of double walls with point and line bridges are given by reference 30 and by Bies and Hansen (see Bibliography).

Sound Transmission Loss of Ducts and Pipes

Pipes and ducts that carry high-intensity internal sound are excited into vibration and radiate sound to the outside. This sound transmission in the breakout direction (i.e., from inside to outside) is characterized by breakout sound transmission loss R_{io} , which is a measure of the rate at which sound energy from the interior of the duct radiated to the outside. When pipes and ducts traverse areas of high-intensity sound such as found in mechanical equipment rooms, the exterior sound field excites ductwall vibrations and the vibrating walls generate an internal sound field that can travel to distant quiet areas. This sound transmission from the outside to the inside direction is characterized by the breakin sound transmission loss R_{oi} , which is a measure of the rate at which sound energy from the exterior sound field enters the duct.

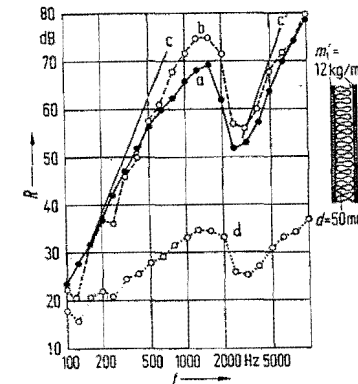


FIGURE 11.29 Sound transmission loss of a double partition consisting of two identical, 12.5-mm-thick gypsum boards separated by a 50-mm-thick gap filled with fibrous sound-absorbing material: *a*, measured values; *b*, predicted by Eq. (11.117); *c*, predicted by Eqs. (11.116a) and (11.116b); *d*, measured sound transmission loss of a single gypsum board wall. (After reference 46.)

The sound power level radiated by a duct or pipe of length l , L_w^{io} , is predicted as⁴⁷

$$L_w^{io}(l) = L_w^i(0) - R_{io} + 10 \log \left(\frac{Pl}{S} \right) + 10 \log(C) \quad \text{dB re } 10^{-12} \text{ W} \quad (11.119a)$$

where

$$C = \frac{1 - e^{-(\tau + \beta)l}}{(\tau + \beta)l} \quad (11.119b)$$

$$\tau = \frac{P}{S} \times 10^{-R_{io}/10} \quad (11.119c)$$

$$\beta = \frac{\Delta L_1}{4.34} \quad (11.119d)$$

where $L_w^i(0)$ is the sound power level in the duct at the source side, S is the cross-sectional area in square meters, P is the perimeter of the duct cross section in meters, and ΔL_1 is the sound attenuation in decibels per unit length inside the duct due to porous lining. Equation (11.119a) contains only measurable quantities and is used as a basis for the experimental evaluation of the breakout sound transmission loss R_{io} , by measuring the sound power $W_{io}(l)$ radiated by a test duct of length l into a reverberation room, the sound power in the duct at the source side, $W_i(0)$, and the sound attenuation inside the duct, ΔL_1 , and solving Eq. (11.119a) for R_{io} by iteration.

The sound power level of the sound propagating in one direction, $L_w^{io}(l)$ when a duct of length l traverses through a noisy area (and sound breaks into the duct is predicted by⁴⁷

$$L_w^{io}(l) = L_w^{inc} - R_{io} - 3 + C \quad \text{dB re } 10^{-12} \text{ W} \quad (11.120)$$

where L_w^{inc} is the sound power level of the sound incident on the duct of length l , R_{io} is the breakin sound transmission loss, and C is as defined in Eq. (11.119b). On the basis of reciprocity,⁴⁸ the following relationship exists between breakout and breakin sound transmission loss:

$$R_{oi} \cong R_{io} - 10 \log \left\{ 4\gamma \left[1 + 0.64 \frac{a}{b} \left(\frac{f_{cut}}{f} \right)^2 \right] \right\} \quad \text{dB} \quad (11.121)$$

where a and b are the larger and smaller sides of a rectangular duct cross section, f is the frequency, f_{cut} is the cutoff frequency of the duct, and γ is 1 below cutoff and 0.5 above cutoff. Empirical methods for predicting the breakout sound transmission loss of unlined, unlagged rectangular sheet metal ducts are given in references 48–50. Chapter 17 contains predicted values of octave-band breakout sound transmission loss versus frequency for rectangular sheet metal ducts of sizes most frequently used in low-velocity HVAC systems. Sound transmission loss predictions for round and flat-oval ducts are given elsewhere.^{49,50} Figure 11.30 shows the breakin sound transmission loss of an unlined, unlagged

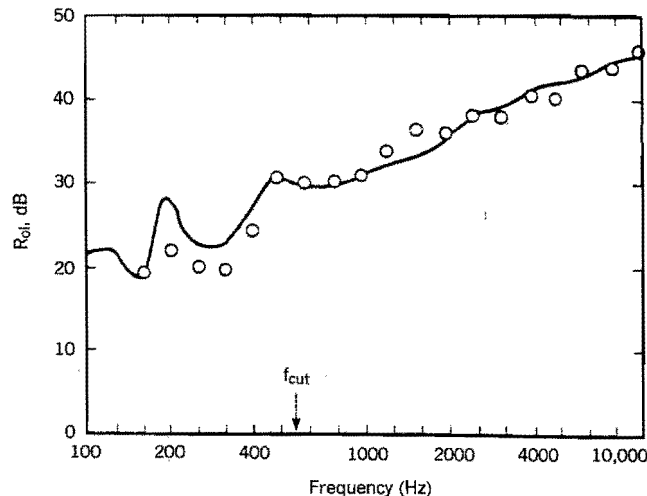


FIGURE 11.30 Breakin sound transmission loss R_{io} of a 0.3 m $\times$ 0.91 mm thick sheet metal duct (after Ref. 48): (—) measured; (○ ○ ○) predicted from measured R_{io} using Eq. (11.121).

sheet metal duct. The solid curve was obtained by directly measuring the breakin sound transmission loss while the open circles represent data points obtained by applying the reciprocity relationship embodied in Eq. (11.121) for the measured breakout sound transmission loss. The importance of Eq. (11.121) is that it makes it unnecessary to measure both R_{oi} and R_{io} separately because if one has been measured, the other can be predicted.

Sound Transmission through a Composite Partition

Partitions that separate adjacent rooms frequently consist of areas that have different sound transmission losses, such as a wall that contains a door with an uncovered key hole. If all parts of the composite partition are exposed to the same average sound intensity, I_{inc} , on the source side, then the sound power transmitted is

$$W_{trans} = I_{inc} \sum_{i=1}^n S_i \times 10^{-R_i/10} = I_{inc} \sum_{i=1}^n S_i \tau_i \quad \text{W} \quad (11.122a)$$

and the transmission loss of the composite partition is

$$R_{comp} = 10 \log \frac{W_{inc}}{W_{trans}} = -10 \log \sum_{i=1}^n \frac{S_i}{S_{tot}} \times 10^{-R_i/10} \quad \text{dB} \quad (11.122b)$$

where S_i is the surface area of each component, S_{tot} is the total area of all components, and R_i is the sound transmission loss of the i th component. The sound transmission loss of a small hole radius $a \ll \lambda_0$ in a thin plate of thickness h is well approximated by²⁶

$$R_{hole} \cong 20 \log \frac{h + 1.6a}{\sqrt{2}a} \quad \text{dB} \quad (11.123)$$

indicating that small holes in thin partitions ($h < a$) yield a frequency-independent sound transmission loss of $R_{hole} \cong 0$ dB. Note, however, that Eq. (11.123) is valid only for small round holes. Long narrow slits can have a “negative sound transmission loss.”⁵¹ The sound transmission loss of holes and slits can be increased substantially by sealing them with either porous sound-absorbing material or an elastomeric material or designing them as silencer joints. Prediction of the sound transmission loss of such acoustically sealed openings are given in references 52 and 53.

Flanking Sound Transmission

The sound transmission loss of partitions is measured in acoustical laboratories where sound transmission from the source room to the receiver room occurs only through the partition under test. However, if the same partition constitutes a part of a building, then sound can be transmitted through many paths, as shown schematically in Fig. 11.31. Path 1 represents the primary path, which is

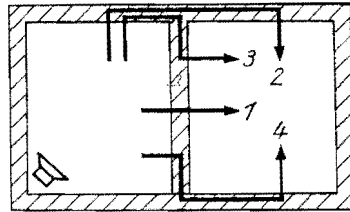


FIGURE 11.31 Sound transmission paths between two adjacent rooms in a typical building (after Reference 54): 1, primary path; 2–4, flanking paths.

characterized by sound transmission loss of the separating partition R_1 , usually available from laboratory measurements. The sound transmission loss for each of the $n = 4$ paths (1 the direct and 2, 3, and 4 the flanking paths) is defined as⁵⁴

$$R_n \equiv 10 \log \frac{W_1^{\text{inc}}}{W_{\text{trans}}^n} \quad \text{dB} \quad (11.124)$$

and the composite sound transmission loss, which combines sound transmission along each of the four paths, as

$$R_{\text{comp}} \equiv -10 \log \left(10^{-R_1/10} + \sum_{n=2}^4 \sum_{m=1}^4 10^{-R_{mn}/10} \right) \quad \text{dB} \quad (11.125)$$

where $m = 4$ represents the four sound-excited flanking partitions (i.e., the two sidewalls, floor, and ceiling of the receiver room) each of which transmit sound along each of the three flanking paths $n = 2, 3, 4$. Usually flanking path $n = 2$ contributes as much to the receiver room sound power as do the two other flanking paths $n = 3$ and $n = 4$ together. The contribution of the back wall of the source and receiver room is usually negligible. The process of flanking transmission along flanking paths $n = 2, 3, 4$ is as follows: (1) the sound field in the source room excites the flanking walls to vibration, (2) the vibration is transmitted through the wall junctions to the receiver room walls, and (3) the receiver room walls radiate sound power into the receiver room that adds to that transmitted by the separating partition through the direct path. If the source and receiver rooms have no common wall, the entire sound transmission takes place through flanking paths. For adjacent rooms with a common wall, the composite sound transmission loss given in Eq. (11.125) should be used to predict the sound pressure level in the receiver room. The component flanking transmission losses R_{mn} for homogeneous isotropic single-wall construction can be approximated by⁵⁴

$$R_{mn} = 10 \log \left[\frac{1}{4\pi\sqrt{12}} \left(\frac{\rho_M}{\rho_0} \right)^2 \frac{c_L h^3 \omega^2}{c_0^4} \eta_m \right] + \Delta L_{\text{junct}} + 10 \log \frac{S_1}{S_{\text{rad}}} - 10 \log(\sigma_m \sigma_{\text{rad}}) \quad \text{dB} \quad (11.126)$$

where ρ_M is density and c_L the longitudinal wave speed of the wall material, h is the thickness, η_m is the composite loss factor of the m th partition in the source room, and ΔL_{junct} is the attenuation of the structure-borne sound amplitude at the wall junction along the transmission path n . The symbol S_1 is the surface area of the separating partition and S_{rad} is the surface area of the partition in the receiver room involved in the transmission along path n . The symbols σ_m and σ_{rad} are the radiation efficiencies of the m th partition in the source room and the radiation efficiency of the partition in the receiver room that radiates sound owing to the sound-induced vibration of the m th partition in the source room transmitted through the n th path. A more accurate prediction of the effect of flanking paths on sound transmission loss can be made utilizing the statistical energy analysis method discussed in the next section.

11.8 STATISTICAL ENERGY ANALYSES

Statistical energy analysis (SEA) is a point of view in dealing with the vibration of complex resonant systems. It permits calculation of the energy flow between connected resonant systems, such as plates, beams, and so on, and between plates and the reverberant sound field in an enclosure.^{55–59}

System of Modal Groups

In respect to the energy E stored in a structure or in an acoustical volume, may be thought of as a *system of* resonant modes or resonators. First, let us consider the power flow between two groups of resonant modes of two coupled structures having their modal resonance frequencies within the same narrow frequency band $\Delta\omega$ (see Fig. 11.32).

We assume that each resonant mode of the first system (box 1 in Fig. 11.32) has the same energy. Also, assume that the coupling of the individual resonant modes of the first system with each resonance mode of the second system is approximately the same.

If we further assume that the waves carrying the energy in one system are uncorrelated with the waves carrying the energy gained through coupling to the

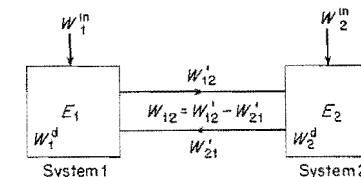


FIGURE 11.32 Block diagram illustrating power flow between two nondissipatively coupled systems.

other system, we can separate the power flow (each equation is written for a narrow frequency band $\Delta\omega$) as

$$W'_{12} = E_1 \omega \eta_{12} \quad W \quad (11.127)$$

$$W'_{21} = E_2 \omega \eta_{21} \quad W \quad (11.128)$$

where W'_{12} = power system 1 transmits to system 2, W

W'_{21} = power system 2 transmits to system 1, W

E_1 = total energy in system 1, m · kg/s

E_2 = total energy in system 2, m · kg/s

ω = center frequency of band, rad/s

η_{12} = coupling loss factor from system 1 to system 2, as defined in Eq. (11.127)

η_{21} = coupling loss factor from system 2 to system 1, as defined in Eq. (11.128)

The net power flow between the two systems is, accordingly,

$$W_{12} = W'_{12} - W'_{21} = E_1 \omega \eta_{12} - E_2 \omega \eta_{21} \quad W \quad (11.129)$$

Modal Energy E_m

Let us define modal energy as

$$E_m = \frac{E(\Delta\omega)}{n(\omega) \Delta\omega} \quad W \cdot s/Hz \quad (11.130)$$

where $E(\Delta\omega)$ = total energy in system in angular frequency band $\Delta\omega$

$n(\omega)$ = modal density, = number of modes in unit bandwidth ($\Delta\omega = 1$) centered on ω , the angular frequency

$\Delta\omega$ = bandwidth, rad/s

If the previously made assumptions about the equal distribution of energy in the modes and the same coupling loss factor are valid, then it may be shown that

$$\frac{\eta_{21}}{\eta_{12}} = \frac{n_1(\omega)}{n_2(\omega)} \quad (11.131)$$

where $n_1(\omega)$ = modal density of system 1 at frequency ω , s

$n_2(\omega)$ = modal density of system 2 at frequency ω , s

Equation (11.131) implies that for equal total energies in the two systems, $E_1 = E_2$, the system that has the *lower* modal density [lower $n(\omega)$] transfers more energy to the second system than is transferred from the second to the first system.

Combining Eqs. (11.129) and (11.131) yields*

$$W_{12} = \omega \eta_{12} n_1(\omega) [E_{m1} - E_{m2}] \Delta\omega \quad W \quad (11.132)$$

where W_{12} = net power flow between systems 1 and 2 in band $\Delta\omega$, centered at ω , W

E_{m1}, E_{m2} = modal energies for systems 1 and 2, respectively [see Eq. (11.130)], W · s/Hz

This equation is positive if the first term in the brackets is greater than the second.

The principle of the SEA method is given by Eq. (11.132), which is a simple algebraic equation with energy as the independent dynamic variable. It states that *the net power flow between two coupled systems in a narrow frequency band, centered at frequency ω , is proportional to the difference in the modal energies of the two systems at the same frequency. The flow is from the system with the higher modal energy to that with the lower modal energy.*

It may help to understand Eq. (11.132) if we use the thermodynamical analogy of heat transfer between two connected bodies of different temperature, where the heat flow is from the body of higher temperature to that of lower temperature and the net heat flow is proportional to the difference in temperature of the two bodies. Consequently, the modal energy E_m is analogous to temperature, and the net power flow W_{12} is analogous to heat flow. The case of equal modal energies of the two systems where the net power flow is zero is analogous to the equal temperature of the two bodies.

Equal Energy of Modes of Vibration

Equal energy of the modes within a group will usually exist if the wave field of the structure is diffuse. Also, since the frequency-adjacent resonance modes of a structure are coupled to each other by scattering and damping, there is always a tendency for the modal energy of resonant modes to equalize within a narrow frequency band even if the wave field is not diffuse.

Noncorrelation between Waves in the Two Systems

In sound transmission problems usually only one system is excited. The power W'_{12} transmitted to the nonexcited system builds up a semidiffuse vibration field in that system. Accordingly, the waves that carry the transmitted power W'_{21} back to the excited system are almost always sufficiently delayed and randomized in phase with respect to the waves carrying the incident power W'_{12} that there is little correlation between the two wave fields.

*Note that Eq. (11.131) is a necessary requirement if Eq. (11.132) is to obey the consistency relationship $W_{12} = -W_{21}$.

Realization of Equal Coupling Loss Factor

Equality of the coupling loss factors between individual modes within a group is a matter of grouping modes of similar nature. If the coupled system is a plate in a reverberant sound field, the *acoustically slow edge and corner modes* and *acoustically fast surface modes* are grouped separately.

Composite Structures

Composite structures generally consist of a number of elements such as plates, beams, stiffeners, and so on. We may divide a complex structure into its simpler member provided the wavelength of the structure-borne vibration is small compared with the characteristic dimensions of the elements. Where this is true, the modal density of a complex structure is approximately that of the sum of the modal densities of its elements. If the power input, the various coupling loss factors, and the power dissipated in each element are known, the power balance equations will yield the vibrational energy in the respective elements of the structure.

The dissipative loss factor for an element of a structure is obtained by separating that element from the rest of the structure and measuring its decay rate, as discussed in Chapter 14. The coupling loss factor can be determined experimentally from Eq. (11.132); however, the procedure is difficult. Theoretical solutions are available for the coupling loss factors of a few simple structural connections.⁵⁹ When the coupling loss factor between a sound field and a simple structure is desired, such a loss factor can be calculated from Eq. (11.131) if the radiation ratio of the structure is known, as shown in the next section.

Power Balance in a Two-Structure System

The power balance of the simple two-element system of Fig. 11.32 is given by the following two algebraic equations:

$$W_1^{\text{in}} = W_1^d + W_{12} \quad W \quad (11.133)$$

$$W_2^{\text{in}} = W_{12} + W_2^d \quad W \quad (11.134)$$

where W_1^{in} = input power to system 1, W
 W_1^d = power dissipated in system 1, W
 W_{12} = net power lost by system 1 through coupling* to system 2, = $W_{12}' - W_{21}'$, W
 W_2^{in} = input power to system 2, W
 W_2^d = power dissipated in system 2, W

*As in our previous analysis, we assume that the coupling is nondissipative.

The power dissipated in a system is related to the energy stored by that system, E_i , through the dissipative loss factor η_i , namely,

$$W_i^d = E_i \omega \eta_i \quad W \quad (11.135)$$

where E_i is energy stored in system i in newton-meters.

Assuming that the second system does not have direct power input ($W_2^{\text{in}} = 0$), the combination of Eqs. (11.130)–(11.132), (11.134), and (11.135) (with $i = 2$) yields the ratio of the energies stored in the two respective systems:

$$\frac{E_2}{E_1} = \frac{n_2}{n_1} \frac{\eta_{21}}{\eta_{21} + \eta_2} \quad (11.136)$$

If the coupling loss factor is very large compared to the loss factor in system 2, that is, if $\eta_{21} \gg \eta_2$, Eq. (11.136) yields the equality of the modal energies ($E_1/n_1 \Delta\omega = E_2/n_2 \Delta\omega$).

Diffuse Sound Field Driving a Freely Hung Panel

Let us now examine the special case of the excitation of a homogeneous panel (system 2) that hangs freely, exposed to the diffuse sound field of a reverberant room (system 1).

For this case, the total energies for each system are given by

$$E_1 = DV = \frac{\langle p^2 \rangle}{\rho_0 c_0^2} V \quad \text{N} \cdot \text{m} \quad (11.137)$$

$$E_2 = \langle v^2 \rangle \rho_s S \quad \text{N} \cdot \text{m} \quad (11.138)$$

where D = average energy density in reverberant room, N/m^2
 $\langle p^2 \rangle$ = mean-square sound pressure (space-time average), N^2/m^4
 V = room volume, m^3
 $\langle v^2 \rangle$ = mean-square plate vibration velocity (space-time average), m^2/s^2
 S = plate surface area (one side), m^2
 ρ_s = mass per unit area of panel, kg/m^2

To find the coupling loss factor η_{21} , we must first recognize that W_{21}' equals the power that the plate, having been excited into vibration, radiates back into the room. Thus

$$W_{\text{rad}} = W_{21}' = 2 \langle v^2 \rangle \rho c \sigma_{\text{rad}} S \equiv E_2 \omega \eta_{21} = \langle v^2 \rangle \rho_s S \omega \eta_{21} \quad W \quad (11.139)$$

where σ_{rad} = radiation ratio for plate, dimensionless
 W_{rad} = acoustical power radiated by both sides of plate, which accounts for factor 2, W

Solving for η_{21} yields

$$\eta_{21} = \frac{2\rho_0 c_0 \sigma_{\text{rad}}}{\rho_s \omega} \quad (11.140)$$

The modal density of the reverberant sound field in the room $n_1(\omega)$ and that of the thin homogeneous plate $n_2(\omega)$ are given in Table 11.7, namely,

$$n_1(\omega) \simeq \frac{\omega^2 V}{2\pi^2 c_0^3} \quad \text{s} \quad (11.141)$$

$$n_2(\omega) = \frac{\sqrt{12} S}{4\pi c_L h} \quad \text{s} \quad (11.142)$$

where c_L = propagation speed of longitudinal waves in plate material, m/s
 h = plate thickness, m

Inserting Eqs. (11.137)–(11.142) into Eq. (11.135) yields the desired relation between the sound pressure and plate velocity:

$$\langle v^2 \rangle = \langle p^2 \rangle \frac{\sqrt{12}\pi c_0^2}{2\rho_0 c_0 h c_L \rho_s \omega^2} \frac{1}{1 + \rho_s \omega \eta_2 / 2\rho_0 c_0 \sigma_{\text{rad}}} \quad \text{m}^2/\text{s}^2 \quad (11.143)$$

The mean-square (space–time average) acceleration of the panel is simply

$$\langle a^2 \rangle = \omega^2 \langle v^2 \rangle = \langle p^2 \rangle \frac{\sqrt{12}\pi c_0^2}{2\rho_0 c_0 h c_L \rho_s} \frac{1}{1 + \rho_s \omega \eta_2 / 2\rho_0 c_0 \sigma_{\text{rad}}} \quad \text{m}^2/\text{s}^4 \quad (11.144)$$

It can be shown that as long as the power dissipated in the plate is small compared with the sound power radiated by that plate ($\rho_s \omega \eta_2 \ll 2\rho_0 c_0 \sigma_{\text{rad}}$), the equality of the modal energies of the sound field and the plate yields the proper plate velocity and acceleration. Also, under this condition, the ratio of the mean-square plate acceleration to the mean-square sound pressure is independent of frequency.

In general, the plate response is always smaller than that calculated by the equality of the modal energies by the last factor on the right of Eq. (11.143) or (11.144), which equals the ratio of the power loss by acoustical radiation to the total power loss. In dealing with the excitation of structures by a sound field, the concept of equal modal energy often enables one to give a simple estimate for the upper bound of the structure's response.

Example 11.4. Calculate the rms velocity and acceleration of a 0.005-m- ($\frac{1}{10}$ -in.-) thick homogeneous aluminum panel resiliently suspended in a reverberant room. The space-averaged sound pressure level $\bar{L}_p = 100$ dB ($\sqrt{\langle p^2 \rangle} = 2$ N/m²) as measured in a one-third-octave band centered at a frequency $f = \omega/2\pi = 1000$ Hz. The appropriate constants of the panel and surrounding media are $\rho_s = 13.5$ kg/m²; $c_L = 5.2 \times 10^3$ m/s; $h = 5 \times 10^{-3}$ m; $\rho_0 = 1.2$ kg/m³; $c_0 = 344$ m/s; and $\eta_2 = 10^{-4}$.

Solution First, calculate the factor

$$\frac{\rho_s \omega \eta_2}{2\rho_0 c_0 \sigma_{\text{rad}}} = \frac{13.5 \times 2\pi \times 10^3 \times 10^{-4}}{2 \times 1.2 \times 344 \times \sigma_{\text{rad}}} = \frac{8.5}{820 \sigma_{\text{rad}}} \ll 1$$

According to the above inequality the mean-square acceleration of the panel given by Eq. (11.144) simplifies to

$$\langle a^2 \rangle \approx \langle p^2 \rangle \frac{\sqrt{12}\pi c_0^2}{2\rho_0 c_0 h c_L \rho_s} = 19 \quad \text{m}^2/\text{s}^4$$

or $a_{\text{rms}} = \sqrt{\langle a^2 \rangle} = 4.34$ m/s², an acceleration level of 113 dB re 10^{-5} m/s².

The mean-square velocity is

$$\langle v^2 \rangle = \frac{\langle a^2 \rangle}{\omega^2} = \frac{19}{4\pi^2 \times 10^6} = 4.76 \times 10^{-7} \quad \text{m}^2/\text{s}^2$$

or $v_{\text{rms}} = \sqrt{\langle v^2 \rangle} = 6.9 \times 10^{-4}$ m/s, a velocity level of 97 dB re 10^{-8} m/s.

Sound Transmission Loss of a Simple Homogeneous Structure by the SEA Method

The SEA method may be used to analyze the transmission of sound between two rooms coupled to each other by a single common, thin homogeneous wall.⁵⁸ (i.e., there are no flanking paths). System 1 is the ensemble of modes of the diffuse, reverberant sound field in the source room, resonant within the frequency band $\Delta\omega$. System 2 is an appropriately chosen group of vibration modes of the wall. System 3 is the ensemble of modes of the diffuse reverberant sound field in the receiving room, resonant within the frequency band $\Delta\omega$. A loudspeaker in the source room is the only source of power, and the power dissipated in each system is assumed to be large compared with the power lost to the other two systems through the coupling (see Fig. 11.33).

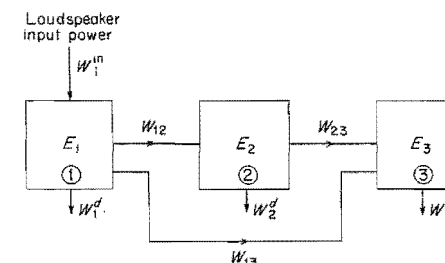


FIGURE 11.33 Block diagram illustrating the power flow in three-way coupled systems: W_{13} , transmission of sound by those modes whose resonance frequency lies outside the source band. The “nonresonant” modes are important below the critical frequency.

The procedure is as follows:

1. Relate W_1^{in} to the power lost by sound absorption in the room. This yields $\langle p_1^2 \rangle$, the space-averaged mean-square sound pressure in the source room.
2. Calculate $E_1 = \langle p_1^2 \rangle V_1 / \rho c^2$, where V_1 is the volume of the source room.
3. The reverberant sound power incident on the dividing wall of area S_2 is $W_{\text{inc}} = E_1 c S_2 / 4 V_1 = p_1^2 / 4 \rho c$.
4. From the power balance of the wall for resonant modes within the bandwidth, $\Delta\omega$ is next determined, that is, $W_{12} = W_2^d + W_{23}$, so that

$$W_{12} = \text{Eq. (11.135)} + 0.5 [\text{Eq. (11.139)}]$$

This sum equals Eq. (11.132). Because η_{12} of Eq. (11.32) is not well known, it is replaced by using Eq. (11.131) and the definition of loss factor η_{21} , which yields $\eta_{21} = \rho_0 c_0 \sigma_{\text{rad}} / \rho_s \omega$.

5. The vibrational energy of the wall is $E_2 = \langle v^2 \rangle \rho_s S_2$.
6. Combining steps 3, 4, and 5 yields the mean-square wall velocity $\langle v^2 \rangle$ as a function of mean-square source room pressure $\langle p_1^2 \rangle$.
7. The power radiated into the receiving room is

$$W_{23} = \rho_0 c_0 S_2 \sigma_{\text{rad}} \langle v^2 \rangle$$

8. Finally, the resonance transmission coefficient τ_r is found by dividing step 7 by step 3.
9. The *resonance transmission loss*, defined as $R_r = 10 \log(1/\tau_r)$, is computed from step 8 using Eq. (11.97) and assuming that $\rho_s \omega \eta_2 \gg 2 \rho_0 c_0 \sigma_{\text{rad}}$ to yield

$$R_r = 20 \log \left(\frac{\rho_s \omega}{2 \rho_0 c_0} \right) + 10 \log \left(\frac{f}{f_c} \frac{2}{\pi} \frac{\eta_2}{\sigma_{\text{rad}}^2} \right) \quad \text{dB} \quad (11.145)$$

The first term in Eq. (11.145) is approximately the normal-incidence mass law transmission loss R_0 , so that Eq. (11.145) becomes

$$R_r = R_0 + 10 \log \left(\frac{f}{f_c} \frac{2}{\pi} \frac{\eta_2}{\sigma_{\text{rad}}^2} \right) \quad \text{dB} \quad (11.146)$$

where f_c = critical frequency [see Eq. (11.94)], Hz
 η_2 = total loss factor of wall, dimensionless
 σ_{rad} = radiation efficiency for wall, dimensionless

Thus we have obtained the transmission loss between two rooms separated by a common wall using the SEA method.

Below the critical frequency and when the dimensions of the wall are large compared with the acoustical wavelength, the radiation factor σ_{rad} can be taken from Table 11.8.

It is important to note that if the sound transmission loss of an equivalent infinite wall is compared with the data measured and predicted by the SEA method, it is found that *above the critical frequency*, the transmission loss R for the infinite wall yields the same results as Eq. (11.146), which takes into account only the resonance transmission of a finite wall.

Below the critical frequency the sound transmission loss of a finite panel is more controlled by the contribution of those modes that have their resonance frequencies outside of the frequency band of the excitation signal than by those with resonance frequencies within that band. Since only the contributions of the latter are included in the previous SEA calculation, Eq. (11.146) usually overestimates the sound transmission loss of a finite panel below the critical frequency. Figure 11.34 shows that below the critical frequency for a $\frac{1}{8}$ -in.-thick aluminum panel the sound transmission loss of the resonant modes alone (curve *a*) is approximately 10 dB higher than that measured on the actual panel (curve *d*).

A *composite transmission factor* that approximately takes into account both the forced and resonance waves is closely approximated by

$$\frac{1}{\tau} = \frac{W_{\text{inc}}}{W_{\text{forced}} + W_{\text{res}}} = \frac{(\langle p^2 \rangle / 4 \rho c) S_2}{\langle p^2 \rangle (\pi \rho c S_2 / \rho_s^2 \omega^2) + \langle p^2 \rangle (\sqrt{12} \pi c^3 \rho \sigma_{\text{rad}}^2 S_2 / 2 \omega^3 \rho_s^2 c_L h \eta_2)} \quad (11.147)$$

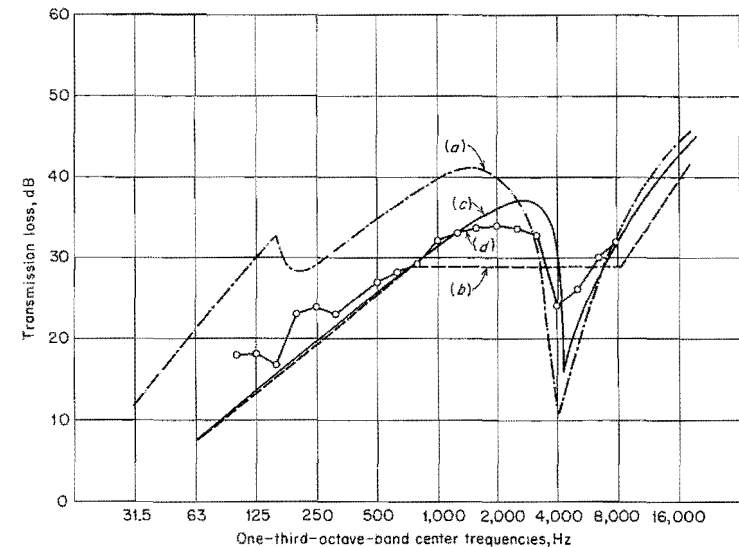


FIGURE 11.34 Comparison of experimental and theoretical transmission loss of a 5 ft $\times$ 6.5 ft $\times$ $\frac{1}{8}$ in. aluminum panel. The theoretical calculations are based on (a) resonance mode calculation, (b) plateau calculation, and (c) forced-wave calculation. Curve *d* shows the experimental results. (After Reference 58.)

At low frequencies where the first term in the denominator dominates, the transmission factor becomes

$$\frac{1}{\tau} \approx \frac{1}{\pi} \left(\frac{\rho_s \omega}{2\rho c} \right)^2 \quad (11.148)$$

and the sound transmission loss is [see Eq. (11.101)]

$$R = 10 \log \frac{1}{\tau} \approx R_0 - 5 = R_{\text{field}} \quad \text{dB} \quad (11.149)$$

At high frequencies where the second term in the denominator becomes dominant, the sound transmission loss is given by Eq. (11.146).

11.9 EQUIVALENCY BETWEEN THE EXCITATION OF A STRUCTURE SOUND FIELD AND POINT FORCE

In the mechanical equipment room of buildings, ships, and engine compartment of vehicles the boundaries (such as the walls, floor, and ceiling) are excited simultaneously by airborne noise and by dynamic forces. The airborne noise might emanate from the casing of machines and the dynamic forces might be those acting at the rigid or resilient attachment points of the machine to the floor or at the attachment points of pipes, conduits, or ducts (which are rigidly connected to the vibrating machine) to the wall or to the ceiling.

The noise control engineer is faced with the dilemma of predicting whether the airborne noise or the dynamic forces at the attachment points control the vibration response of the structure. The type of excitation that controls the vibration response of the structure will also control the airborne noise and vibration at distant noise- and vibration-sensitive receiver locations.

In his noise control engineering practice the author has been called upon to predict whether the ramble (low-frequency random noise) generated by the passage of subway trains in a nearby tunnel will be above or below the threshold of human hearing in a planned concert hall. In another project, he had to predict whether eye surgeons would be able to perform retina operations in a hospital located near another subway tunnel.

The dynamic forces acting in the tunnel floor owing to the wheel-rail interaction can be reduced substantially by mounting the track on a "floating slab" consisting of a thick concrete slab that is supported by resilient rubber mounts laid on the tunnel floor. However, the floating slab has no beneficial effect on reducing the airborne noise exposure of the walls and the roof of the tunnel. Actually, the floating slab results in an increase of the airborne noise, especially in the frequency range near its coincidence frequency where the propagation speed of bending waves in the floating slab coincides with the propagation speed of sound in air and the slab becomes a very efficient sound radiator. Consequently, the airborne sound excitation of the tunnel structure might be controlling the

low-frequency vibration response of the tunnel walls and the noise and vibration at a distant observer location.

To enable him to make a quantitative judgment on the relative importance of the airborne noise versus the force excitation of a platelike structure, the author has derived the relationship⁶⁰

$$F_{\text{eq}} = p \left(\frac{c_0}{f} \right) \sqrt{\frac{\sigma_{\text{rad}} S_f}{\pi}} = p \lambda \sqrt{\frac{\sigma_{\text{rad}} S_f}{\pi}} [2(10^{-5})] 10^{L_p/20} \quad \text{N} \quad (11.150)$$

where F_{eq} is the point force in newtons that generates the same free bending wave response on a partition of surface area S_f as a random-incidence sound field with a space-time average sound pressure p . The symbol σ_{rad} is the radiation efficiency of the partition. It is unity at frequencies above the coincidence frequency. The principle of reciprocity requires that σ_{rad} is also a measure of the degree of coupling of the sound waves to the vibration response of the structure. The symbol $\lambda = c_0/f$ is the acoustical wavelength in meters, c_0 is the speed of sound in air in meters per second, and f is the frequency in hertz (reciprocal seconds). The symbol L_p is the sound pressure level in decibels re 2×10^{-5} N/m².

Equation (11.150) can be written in a form that is easy to remember:

$$F_{\text{eq}}^2 = \left(\frac{4}{\pi} \right) \left(\frac{(pS)^2}{S/(\lambda/2)^2} \right) \approx \left(\frac{(pS)^2}{S/(\lambda/2)^2} \right) \quad \text{N}^2 \quad (11.151)$$

The numerator of Eq. (11.151) is a force squared (the product sound pressure and the area) and the denominator is the number of areas, each a half wavelength squared, that would fill the entire surface area S . The area $(\lambda/2)^2$ is where the sound pressure on the surface of the partition is in phase.

Equations (11.150) and (11.151) are extremely simple and universally useful because:

1. The force/sound pressure equivalency does not implicitly depend on the material properties of the partition such as the density, Young's modulus, loss factor, and geometry.
2. They do not depend on the density of the fluid:
3. They are also valid for partitions with fluid loading such as concrete slabs embedded in soil or steel plates with a liquid on the other side.

The reason for these unique properties of Eqs. (11.150) and (11.151) is that the vibration response to both sound and point force depends the same way on these properties. For example, a lightly damped structure will respond equally vigorously to sound and point force excitation, a plate made out of a material with high density will respond equally less vigorously to both sound and force excitation than one that is made of a less dense material, and so on. Equations (11.150) and (11.151) have been derived by assuming that the partition is large compared

with the bending wavelength and the room where the sound field is generated is large compared with the acoustical wavelength. Both the structure and the sound field respond in a multimodal fashion and their response is dominated by the resonant modes.

In noise control design the airborne noise strength of machines and equipment is given in the form of their sound power $W(f)$ or sound power level spectrum $L_w(f)$. The diffuse-field sound pressure in a room, resulting from the injected sound power, is given by

$$\frac{p^2}{4\rho_0 c_0} S_{\text{tot}} \bar{\alpha} = W_f \quad \text{N/m}^2 \quad (11.152)$$

Combining Eqs. (11.150) and (11.152) yields

$$F_{\text{eq}}(f) = W(f) \frac{\rho_0 c_0^3 \sigma_{\text{rad}}(f)}{f^2} \left(\frac{1}{\alpha} \right) \left(\frac{S_i}{S_{\text{tot}}} \right) \left(\frac{4}{\pi} \right) \quad \text{N}^2 \quad (11.153)$$

where $W(f)$ = sound power of machine, W

$F_{\text{eq}}(f)$ = equivalent point force acting on solid boundary that produces same resonant (i.e., free bending wave) vibration response of boundary as the sound field produced in room owing to sound power output of the machine, N/m²,

S_i = surface area (one side) of boundary directly excited by force, m²,

S_{tot} = total surface area of all boundaries of room, m²,

α = sound absorption coefficient of internal boundary surfaces

The validity of Eqs. (11.150), (11.151), and (11.153) have been briefly checked⁶¹ experimentally by exciting the boundaries of an underground fan room. First one wall was excited with a shaker and the point force $F(f)$ was measured with a force gauge built into the impedance head that connected the shaker to the fan room wall. Then, a sound field was generated in the fan room by a loudspeaker and the sound pressure level SPL(f) was measured with a calibrated microphone. For both types of excitation the force and the sound pressure level was generated at an identical series of pure-tone frequencies to maximize the signal-noise ratio. The response was measured by a geophone in the form of the ground vibration at a distant location.

Example 11.5. Predict whether a planned concert hall can be located on a site near an existing subway line without imposing speed limits on the subway train and without supporting the entire concert hall on resilient vibration isolation pads. The design goal is to keep the ramble noise in the concert hall below the threshold of human hearing, which is 35 dB in the 63-Hz center-frequency octave band.

The subway tunnel has a 10 ft × 10 ft cross section and the subway train is 100 ft long. The tunnel walls, roof, and floor are poured concrete of 0.4 m thickness. Analytical predictions carried out have indicated that mounting the

rails on a floating slab would reduce the total force acting on the tunnel floor to 200 N in the 63-Hz center-frequency octave band and that this dynamic force was expected to produce a sound pressure level of 30 dB in the concert hall. This is 5 dB lower than the human threshold of hearing. Sound-level measurements carried out in the tunnel during the passage of a subway train yielded a sound pressure level of 110 dB in the 63-Hz center-frequency octave band. Predict whether this airborne excitation will produce noise levels in the concert hall above the 35-dB threshold of hearing.

Solution The critical frequency f_c is predicted from Table 11.2. The density of concrete $\rho = 2300 \text{ kg/m}^3$, the mass per unit area $\rho_s = \rho h = 2300 \times 0.4 = 920 \text{ kg/m}^2$, $f_c \rho_s = 43,000$, and $f_c = 43,000/920 = 47 \text{ Hz}$; consequently, $\sigma_{\text{rad}}(f=63 \text{ Hz}) \simeq 1$. The surface area of the tunnel exposed to the high-level airborne noise excitation $S_{\text{tot}} = 10 \times 10 \times 100 = 10,000 \text{ ft}^2 = 929 \text{ m}^2$. With these values Eq. (11.150) yields

$$F_{\text{eq}} = \left(\frac{c_0}{f} \right) \sqrt{\frac{S_{\text{tot}}}{\pi}} 2(10^{-5}) 10^{\text{SPL}/20} = \left(\frac{340}{63} \right) \sqrt{\frac{929}{\pi}} 2(10^{-5}) 10^{110/20} = 586 \text{ N}$$

Consequently, the noise level in the concert hall that is attributable to the airborne sound excitation of the tunnel walls is predicted to be

$$\text{SPL}_{\text{Hall}}(f = 63 \text{ Hz}) = 30 + 20 \log \left(\frac{586}{200} \right) \simeq 39 \text{ dB}$$

This is 4 dB above the design goal, indicating that the design goal cannot be achieved without restricting the train speed or putting the concert hall on vibration isolators that would need to provide a high degree of isolation at 63 Hz.

11.10 RECIPROCITY AND SUPERPOSITION

The principles of reciprocity and superposition apply to linear systems with time-invariant parameters. Not only solid structures but fluid volumes at rest fall into this category. Consequently, reciprocity and superposition apply to systems that consist of solid structures surrounded by acoustical spaces and can be used to great advantage not only in structure-borne noise and airborne noise but also in structural acoustics, which deals with the interaction of sound waves with solid structures.

The principle of superposition, illustrated in Fig. 11.35, allows the use of the simplest excitation sources such as a point force source or a point-monopole sound source to explore the response to more complex excitation sources such as a moment acting on a structural element or an acoustical dipole radiating into an acoustical volume. The principle of reciprocity, which can be traced back to Lord Rayleigh,⁶² is illustrated in the upper three sketches in Fig. 11.36. It states that

$$F_2 v_2 = F_1 v_1 \quad W \quad (11.154a)$$

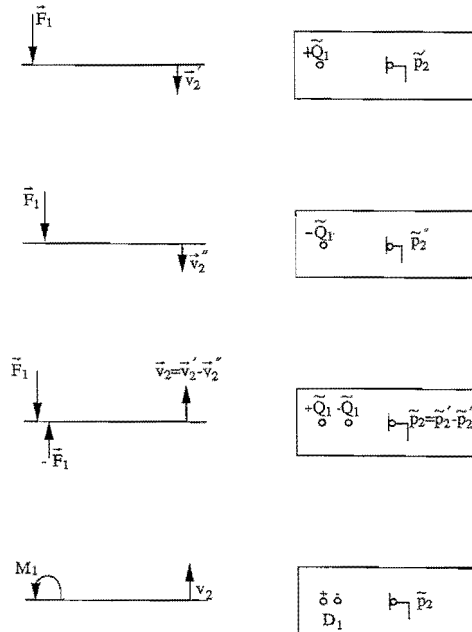


FIGURE 11.35 Use of superposition to predict the response of structures and acoustical spaces (v_2 and p_2) to complex excitation sources such as moments (M_1) and dipoles (D_1) on the basis of point force (F_1) and acoustical monopole (Q_1). Note that the structure and the acoustical space may be of arbitrary shape and the acoustical space may be unbound or bound by elastic or sound-absorbing boundaries and may contain an arbitrary number and size of rigid or elastic scatterers.

The symbol F_1 is (generalized) force when point 1 is source and point 2 is receiver and v_1 is (generalized) velocity when point 1 is receiver and point 2 is source. The vector product $F_1 v_1$ must yield the instantaneous power, or in complex notation $\text{Re}\{\frac{1}{2} F_1 v_1^*\} = \text{Re}\{\frac{1}{2} F_2 v_2^*\}$ must yield the time-averaged power. Note that F and v are vector quantities as signified in the figure by the arrow above the symbols. If v_1 and F_1 and v_2 and F_2 are measured or applied in the same direction, as illustrated by the sketch in the lower left side of Fig. 11.36, the vector notation can be exchanged for the less complicated scalar notation, where force and velocity are characterized by a magnitude and phase (i.e., $\tilde{F} = F e^{j\phi_f}$, $\tilde{v} = v e^{j\phi_v}$). In this case, the reciprocity takes the form of the equality of transfer functions

$$\frac{\tilde{v}_2}{\tilde{F}_1} = \frac{\tilde{v}_1}{\tilde{F}_2} \quad \text{m/N} \cdot \text{s} \quad (11.154b)$$

This should be kept in mind in our latter deliberations, where the special vector notation is not carried through. Since monopole strength $\tilde{Q}$ and sound pressure

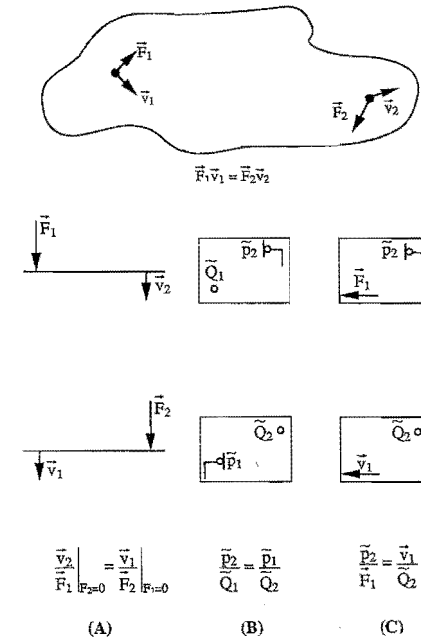


FIGURE 11.36 Principle of reciprocity as applied to a complex structure. Top sketch illustrates general principle. Lower sketches apply in special situations: (a) structure; (b) an acoustical space; (c) a structure coupled to an acoustical space. Symbols: F , point force vector; v , velocity response vector (measured by the same direction as F); Q , volume velocity of acoustical point source; p , sound pressure.

$\tilde{p}$ are scalar quantities (defined by their magnitude and phase), no directional restraints exist in the acoustical case illustrated in Fig. 11.36b.^{63,64} However, dipole and quadrupole sound sources (constructed from adjacent out-of-phase monopoles) have highly directional radiation characteristics and must be connected to directional quantities of the sound field such as pressure gradients dp/dx and d^2p/d^2x measured in the same direction relative to the orientation of the dipole or quadrupole sound source for the reciprocity to apply.

Table 11.9 contains useful reciprocity relationships applicable to higher order excitation sources and responses. These relationships automatically follow from the joint application of superposition and reciprocity to the appropriate combination of simple sources such as point forces and acoustical monopoles.

The principle of reciprocity can be used to considerable advantage in both experimental and analytical work. In experimental work it is difficult and cumbersome to excite complex structures by point forces and moments and to measure the sound pressure such excitation causes in the interior of a vehicle. It is almost always easier to obtain the sought transfer function between the acoustical pressure and the exciting force or moment by placing a small acoustical source of

known volume velocity at the microphone location and measuring the vibration response at the position and in the direction of the applied force and applying the reciprocity relationship illustrated in Fig. 11.36c.

The form of reciprocity illustrated in Fig. 11.36c is the most useful in structural acoustics. Its practical application is described in reference 64. In the direct experiment the force $\vec{F}_1$ is applied to the structure by a shaker (and its magnitude and phase are measured by a force gauge inserted between the shaker and the structure), and the magnitude and phase of the sound pressure p_2 are measured by a microphone. The phase of F_1 and p_2 are referenced to the voltage U applied to the shaker. In the reciprocal experiment—which is much easier to perform than the direct experiment—a small omnidirectional sound source (an enclosed loudspeaker whose diameter is smaller than one-quarter acoustical wavelength) with calibrated volume velocity response Q is placed at the former microphone location and the velocity response of the structure at the former excitation point v_2 is measured (in the same direction as the force was applied) by a small accelerometer. The phases of Q and v_2 are referenced to the voltage U applied to the loudspeaker sound source. The volume velocity calibration of the sound source (Q/U) is obtained by placing it in an anechoically terminated rigid tube, baffling it so it radiates only toward the anechoic termination, sweeping the loudspeaker voltage through the frequency range of interest, and measuring the transfer function (p/U), where U is the voltage applied to the loudspeaker and p is the sound pressure measured by a microphone located two tube diameters or further away from the source. The sought volume velocity calibration of the source is then computed as⁶⁵ $|Q/U| = |P/U|(S/\rho_0 c_0)$, where S is the cross-sectional area of the tube, ρ_0 is the density of air, and c_0 is the speed of sound in air. When phase information is important, the sound source can be calibrated in an anechoic chamber by measuring the sound pressure $p(r)$ at a large distance $r \gg \lambda_0$ away from the source and computing the volume velocity calibration $Q/U = [p(r)/U](4\pi r^2/\rho_0 c_0)e^{-j2\pi f r/c_0}$.

Figure 11.37 illustrates the application of reciprocity on a complex structural acoustical problem, namely, the prediction of the interior noise of an automobile to point force excitation of the shock tower. First, the shock tower was excited by a point force, and sound pressure generated at the driver's head position was measured to obtain the direct transfer function $\tilde{p}/F$ identified by the solid line. Next, the reciprocal experiment was carried out by placing a point sound source of known volume velocity $\tilde{Q}$ at the former location of the microphone and measuring the vibration velocity response of the shock tower. The transfer function $v/\tilde{Q}$ obtained this way is shown as the dotted curve in Fig. 11.37a. Referencing the phases of $\tilde{p}$ and F to the voltage applied to the shaker and that of v and $\tilde{Q}$ to the voltage applied to the loudspeaker source, not only the magnitude but also the phase of the reciprocal transfer functions $\tilde{p}/F$ and $v/\tilde{Q}$ can be retained so that the interior noise caused by many simultaneously acting forces and moments can be predicted. Figure 11.37b shows the unrolled phase of the transfer function pair.

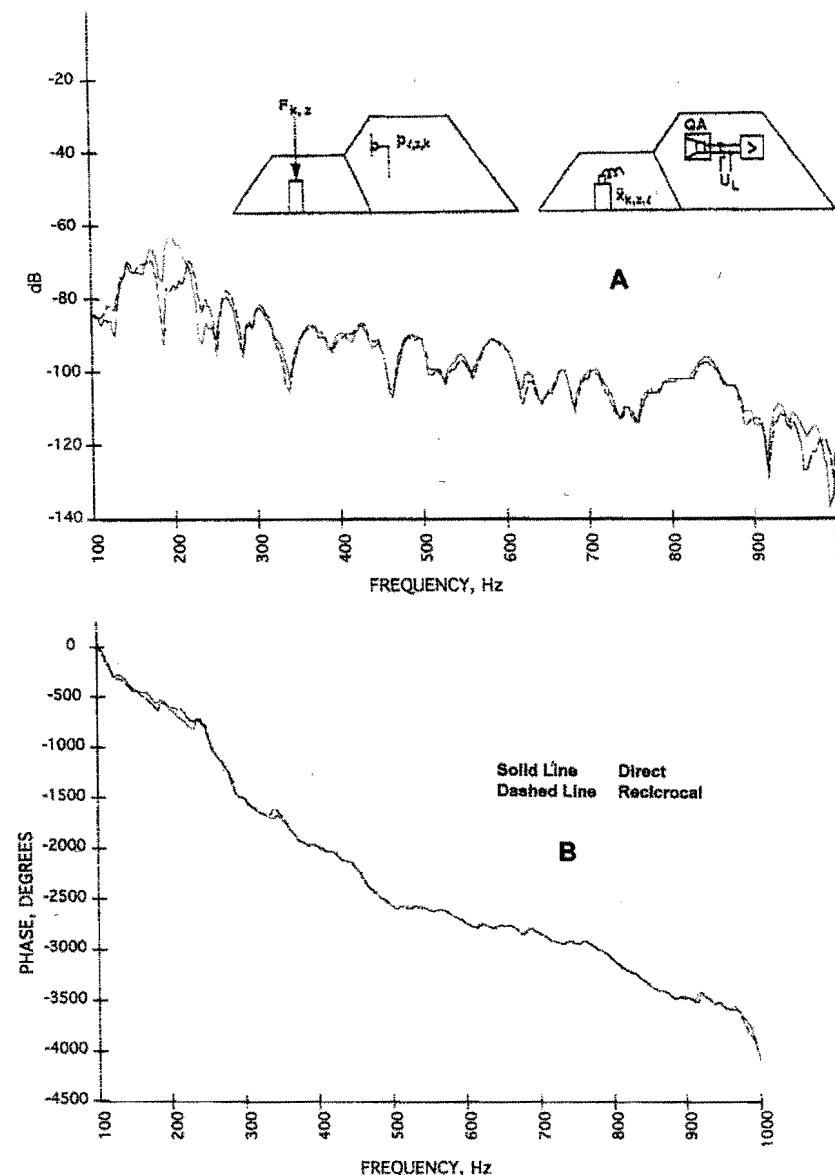


FIGURE 11.37 Pair of reciprocal transfer functions measured in a passenger vehicle: (a) direct, $\tilde{p}/F$; (b) reciprocal, $\tilde{v}/\tilde{Q}$.

Prediction of Noise Caused by Multiple Correlated Forces

The use of reciprocity and superposition in the case of multiple correlated force input is illustrated in Fig. 11.38. The problem is to predict the sound pressure p_R in the receiver room below after installation of a vibrating machine in the source room above. The building is constructed, and the machine manufacturer provides the magnitude, the direction, and the mutual phase ϕ_{12} of forces F_1 and F_2 the machine—when installed on soft springs—will impart to the floor. The prediction of p_R proceeds as illustrated in the lower part of Fig. 11.38 by measuring the reciprocal transfer functions $-v_1/\tilde{Q}_R = \tilde{p}_R/F_1$ and $-v_2/\tilde{Q}_R = \tilde{p}_R/F_2$ and utilizing the principle of superposition. The forces and velocities must be measured in the same direction. Performing the reciprocity prediction for a number of different loudspeaker positions in the receiver room, the spatial

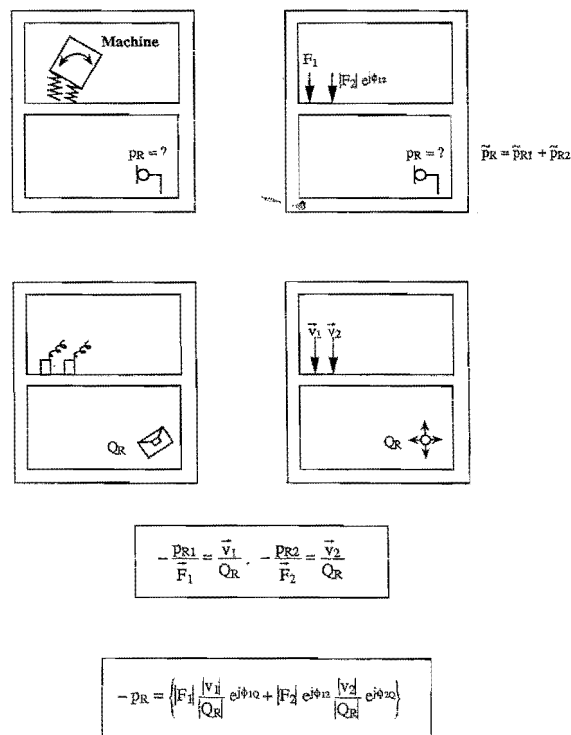


FIGURE 11.38 Use of reciprocity and superposition to predict the sound pressure in a room caused by two correlated forces $\tilde{F}_1$ and $\tilde{F}_2$ acting on the building structure. Top sketch represents actual situation and lower sketch the reciprocity prediction: ϕ_{1Q} and ϕ_{2Q} represent the phases of the transfer functions $\tilde{v}_1/\tilde{Q}_R$ and $\tilde{v}_2/\tilde{Q}_R$, respectively; each is conveniently referenced to the excitation voltage of the loudspeaker sound source.

variation of p_R can also be predicted. The methodology can be easily extended to more than two simultaneously acting, correlated forces.⁶⁶

The reciprocity is most useful in the early stages of design of aircraft and ground vehicles, well before a flightworthy version of the aircraft or a roadworthy version of the ground vehicle is available. Reciprocity helps the noise control engineer to find answers to some difficult questions:

1. What will be the contribution of the structure-borne noise at the engine firing rate to the noise in the passenger compartment?
2. Which engine mount transmits most of the structure-borne noise?
3. Which direction of vibration force is most critical?
4. Which mutual phasing of forces acting on the individual engine-mounting points is most critical?
5. Most importantly, what is the effect of changing the design of engine-mounting brackets on cabin noise?

All these questions can be answered without applying known forces in three orthogonal directions to each of the engine-mounting brackets.

Source Strength Identification by Reciprocity

When it is not feasible to measure directly the strength of noise and vibration sources during the operation of vehicles, equipment, and machinery, reciprocity can be used to obtain them indirectly. This is accomplished by measuring the noise or vibration during the operation of the equipment at a distant, accessible receiver location and—when the equipment is not operating—exciting it at these distant receiver locations and measuring the acoustical or structural response at the source location, which is now accessible. The principle is illustrated in Fig. 11.39. Common in the three problems shown in Fig. 11.39 is the knowledge of the location and nature of the excitation sources. Unknown are their magnitude and mutual phase, which must be determined by observation of response to these sources at distant locations and by reciprocity experiments as described below.

The upper left-hand side of Fig. 11.39 represents the case where the sound field in an enclosure is excited by two monopole sound sources of unknown strength and mutual phase $\tilde{Q}_1(?)$ and $\tilde{Q}_2(?)$ (e.g., the openings of the inlet pipe leading to two cylinders of a reciprocating compressor). The first experiment, illustrated in the upper sketch, is the measurement of the magnitude and mutual phase of the sound pressure $\tilde{p}_3$ and $\tilde{p}_4$ at accessible distant locations 3 and 4 obtained when both sources were operating simultaneously. The reciprocal experiments, illustrated in the two lower sketches, are performed when the sources are not operational by placing a monopole sound source of known volume velocity Q at the former microphone locations 3 and 4 and measuring the magnitude and phase of the sound pressure produced at the two former source locations $\tilde{p}_{13}$, $\tilde{p}_{23}$,

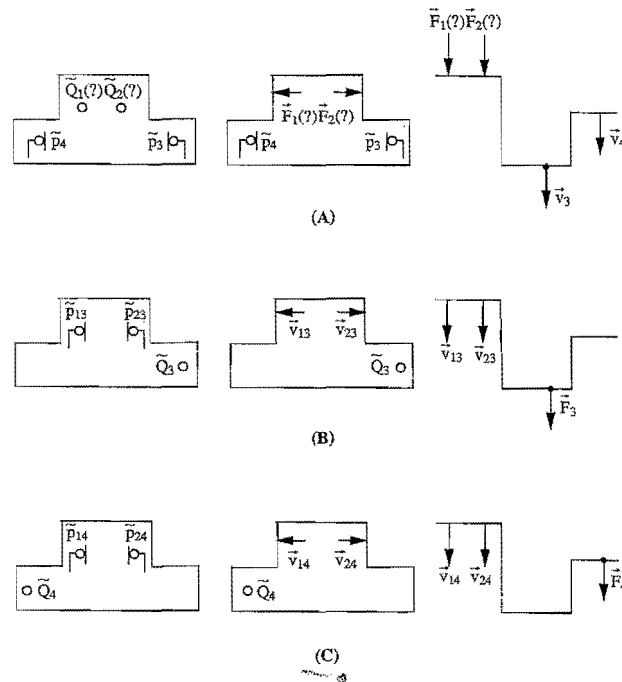


FIGURE 11.39 Source strength identification utilizing reciprocity and superposition: (a) response measurements at distant locations with sources active and (b, c) reciprocity measurements, sources inactive. (See text for explanation of symbols.)

$\tilde{p}_{14}$, and $\tilde{p}_{24}$. The phase of the sound pressure is referenced to the loudspeaker voltage.

The sketches in the middle column in Fig. 11.39 illustrate a case where the sound field in an enclosure is produced by two forces $\tilde{F}_1$ and $\tilde{F}_2$ of known location and direction but unknown magnitude and mutual phase, both acting simultaneously at the enclosure wall (e.g., forces caused by a vibration isolation-mounted reciprocating or rotating machine). In this case, the reciprocal experiment yields the vibration velocity responses at the former force application points $\tilde{v}_{13}$, $\tilde{v}_{23}$, $\tilde{v}_{14}$, and $\tilde{v}_{24}$, measured in the same direction as the force.

The situation shown in the sketch on the upper right-hand side in Fig. 11.39 illustrates the situation when the determination of the unknown magnitude and mutual phase of the two forces $\tilde{F}_1(?)$ and $\tilde{F}_2(?)$ must be diagnosed (e.g., forces transmitted to a structural floor by a vibration-isolated machine). In this case, the reciprocal experiment is carried out by exciting the building structure at the two distant observation points 3 and 4 by known forces $\tilde{F}_3$ and $\tilde{F}_4$ and measuring the velocity responses $\tilde{v}_{13}$, $\tilde{v}_{23}$, $\tilde{v}_{14}$, and $\tilde{v}_{24}$ at the former force excitation points 1 and 2. Utilizing the principle of reciprocity and superposition yields the following pairs of linear equations:

$$\tilde{p}_3 = \tilde{Q}_1(?) \left[\frac{\tilde{p}_{13}}{\tilde{Q}_3} \right] + \tilde{Q}_2(?) \left[\frac{\tilde{p}_{23}}{\tilde{Q}_3} \right] \quad \text{N/m}^2 \quad (11.155a)$$

$$\tilde{p}_4 = \tilde{Q}_1(?) \left[\frac{\tilde{p}_{14}}{\tilde{Q}_4} \right] + \tilde{Q}_2(?) \left[\frac{\tilde{p}_{24}}{\tilde{Q}_4} \right] \quad \text{N/m}^2 \quad (11.155b)$$

which can be solved for the two unknowns $\tilde{Q}_1(?)$ and $\tilde{Q}_2(?)$. Similarly, the set of equations for obtaining $\tilde{F}_1(?)$ and $\tilde{F}_2(?)$ in the situation illustrated in the center sketches in Fig. 11.39 are

$$\tilde{p}_3 = \tilde{F}_1(?) \left[\frac{\tilde{v}_{13}}{\tilde{Q}_3} \right] + \tilde{F}_2(?) \left[\frac{\tilde{v}_{23}}{\tilde{Q}_3} \right] \quad \text{N/m}^2 \quad (11.155c)$$

$$\tilde{p}_4 = \tilde{F}_1(?) \left[\frac{\tilde{v}_{14}}{\tilde{Q}_4} \right] + \tilde{F}_2(?) \left[\frac{\tilde{v}_{24}}{\tilde{Q}_4} \right] \quad \text{N/m}^2 \quad (11.155d)$$

and for that illustrated in the sketches on the right

$$\tilde{p}_3 = \tilde{F}_1(?) \left[\frac{\tilde{v}_{13}}{\tilde{F}_3} \right] + \tilde{F}_2(?) \left[\frac{\tilde{v}_{23}}{\tilde{F}_3} \right] \quad \text{N/m}^2 \quad (11.155e)$$

$$\tilde{p}_4 = \tilde{F}_1(?) \left[\frac{\tilde{v}_{14}}{\tilde{F}_4} \right] + \tilde{F}_2(?) \left[\frac{\tilde{v}_{24}}{\tilde{F}_4} \right] \quad \text{N/m}^2 \quad (11.155f)$$

In the case of n unknown excitation sources, the prediction equations represent an $n \times n$ matrix.

Extension of Reciprocity to Sound Excitation of Structures

As illustrated in Fig. 11.40, the reciprocity relationship can be extended for surface excitation of structures (e.g., by an incident sound wave). Consider first a small part of the surface of a cylindrical body (such as an aircraft fuselage) with surface area dA exposed to a local sound pressure of $\tilde{p}_1$ as illustrated in the upper left sketch in Fig. 11.40 resulting in a local force of $\tilde{F}_1 = \tilde{p}_1 dA$. For this force the reciprocity relationship shown in Fig. 11.36c yields $\Delta \tilde{p}_{R1} / \tilde{F}_1 = \Delta \tilde{v}_{1R} / \tilde{Q}_R$, which for $\tilde{F}_1 = \tilde{p}_1 dA$ and $\Delta \tilde{Q}_{1R} = \Delta \tilde{v}_{1R} dA$ becomes

$$\frac{\Delta \tilde{p}_R}{\tilde{p}_1} = \frac{\Delta \tilde{Q}_{1R}}{\tilde{Q}_R} \quad (11.156a)$$

When the structure is exposed to a complex sound field distribution $\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n$, as illustrated in the lower sketch, the resulting interior sound pressure at the receiver location, $\tilde{p}_R$, is given by

$$\tilde{p}_R = \sum_{i=1}^n \tilde{p}_i \left(\frac{\Delta \tilde{Q}_{iR}}{\tilde{Q}_R} \right) \quad \text{N/m}^2 \quad (11.156b)$$

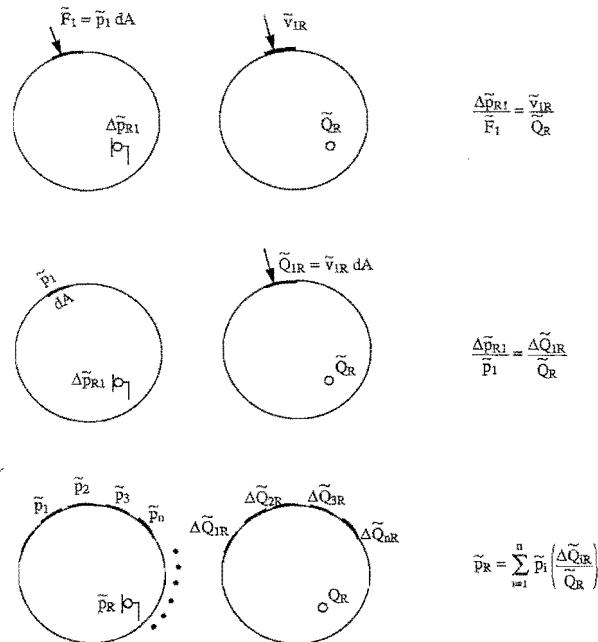


FIGURE 11.40 Extension of reciprocity to sound excitation of structures. Top: Reciprocity relationship for external pressure, $\tilde{p}_1$, acting on a small area (dA) of the structure and the resulting internal sound pressure, $\Delta\tilde{p}_{R1}$, and structural response v_{1R} produced by an internal point sound source of volume velocity Q_R located at the former receiver position. Middle: Substitution of volume velocity response $\Delta\tilde{Q}_{1R} = d\tilde{v}_{1R} dA$ into the reciprocity relationship. Bottom: Reciprocity relationship for external incident sound excitation $\tilde{p}_i$ and internal sound pressure $\tilde{p}_R$ and volume velocity responses of the structure $\Delta\tilde{Q}_{iR}$ produced by an internal point sound source of volume velocity $\tilde{Q}_R$.

where the transfer functions $\Delta\tilde{Q}_{iR}/\tilde{Q}_R$ represent the reciprocity calibration of the structure as a transducer. This extension of reciprocity by Fahy⁶⁷ has the advantage that the reciprocity calibration of the structure (in the form of discretized transfer functions $\Delta\tilde{Q}_{iR}/\tilde{Q}_R$) can be carried out with a capacitive transducer that directly measures the structure's local volume displacement $\Delta\tilde{Q}_{iR}/j\omega$. To obtain sufficient resolution, the side length of the square-shaped capacitive transducer used in measuring the volume displacement must not exceed one-eighth of the acoustical wavelength. Note that the bending wavelength in thin, plate-like structures is usually much smaller than the transducer size so that the capacitive transducer acts as a wavenumber filter, accounting only for those components of the vibration field that results in a net volume displacement. The high-wavenumber components, which result only in local near fields, are "averaged out." The additional advantage of the capacitive transducer is that it does not influence the vibration response of the structure.

Reciprocity can also be used in predicting the sound pressure attributable to the complex vibration pattern of a vibrating body in case where the radiated sound cannot be measured directly (e.g., other correlated vibration sources dominate the sound field). The reciprocity prediction proceeds in two steps. First the vibration pattern of the body is mapped during the operation of the equipment by measuring the vibration velocity $\tilde{v}_i$ at a large number of locations. The phase of the velocity responses is referenced to the velocity measured at a designated reference location. Next the machine is shut off and a point sound source of known volume velocity Q is placed at the receiver location where the sound pressure should be predicted and the sound pressure $\tilde{p}_i$ produced by the point sound source at the various locations along the stationary surface of the body is measured. The phase of the pressure responses is conveniently referenced to the voltage applied to the loudspeaker sound source. The sound pressure at the receiver location, $\tilde{p}_R$, attributable to the periodic vibration of body is predicted as

$$\tilde{p}_R = \sum_{i=1}^n \tilde{v}_i dA_i \frac{\tilde{p}_i}{Q} \cong \sum_{i=1}^n \Delta\tilde{Q}_i \frac{\tilde{p}_i}{Q} \quad \text{N/m}^2 \quad (11.156c)$$

where dA_i is the area and $\Delta\tilde{Q}_i = \tilde{v}_i dA_i$ is the volume velocity of the i th sample of the vibrating surface.

The solid line in Fig. 11.41 represents the directly measured sound pressure $\tilde{p}_R$ at a specific location in a room when a thin plate was excited by a shaker to a

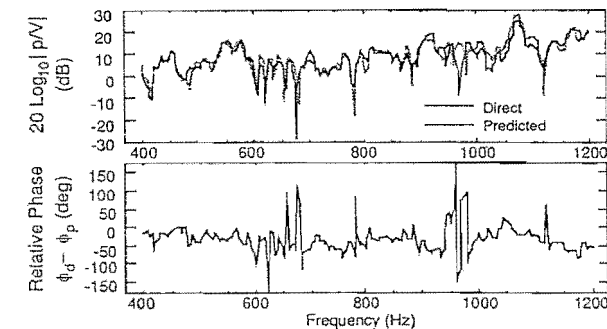


FIGURE 11.41 Sound pressure response at a specific location in a room produced by a thin plate excited by a shaker to complex vibration pattern. Solid line: Directly measured transfer function between sound pressure $\tilde{p}_R$ and excitation force F , phase is referenced to shaker voltage $\tilde{U}$. Dotted line: Transfer function predicted by measuring the magnitude and relative phase of the plate response, $\Delta\tilde{Q}_i = \tilde{v}_i dA$ at $i = 81$ position during shaker excitation and the magnitude and phase of the sound pressure $\tilde{p}_i$ at the surface of the stationary plate when a point sound source of volume velocity $\tilde{Q}_R$ is placed at the former receiver position and applying the reciprocity relationship $\tilde{p}_R = \sum_{i=1}^{81} \Delta\tilde{Q}_i (\tilde{p}_i / \tilde{Q}_R)$ (after Reference⁶⁷). Top curve, magnitude of pressure; lower curve, phase spectrum of directly measured minus phase spectrum of predicted pressure.

complex vibration pattern. The dotted curve represents the reciprocity prediction according to Eq. (11.156a) utilizing $n = 81$ sampling points on the plate.⁶⁷ The experimental results indicate the feasibility of using reciprocity predictions in engineering applications.

Reciprocity in Moving Media

Reciprocity requires that exchanging the function of the source and receiver should not result in any change in the sound propagation path. This is true only if the acoustical medium is at rest. As illustrated in Fig. 11.42, the propagation path between source and receiver remains the same if the exchange of source and receiver positions is accompanied by a reversal of the direction of the uniform mean flow. In this case, or in the case of low-Mach-number potential flow where the shear layer is small compared with the acoustical wavelength, the reciprocity also applies in moving media.⁶⁸ Reversal of potential flow is usually easy to accomplish in analytical calculations. However, in many experimental situations where the shear layer is not small compared with the acoustical wavelength, reciprocity does not apply.

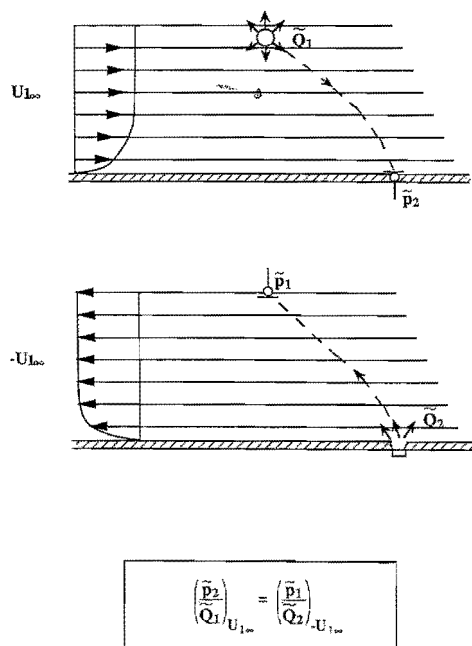


FIGURE 11.42 Reciprocity in moving media. Reversal of the function of source and receiver must be accompanied by reversing the flow direction. Streamlines must remain unchanged to assure that the propagation path between source and receiver remains the same.

Fundamental questions regarding reciprocity are dealt with in references 69–71. Analytical applications of reciprocity and superposition for predicting power input structures excited by complex airborne or structure-borne sources is presented in references 66, 72, and 73. Ship acoustics application of reciprocity are treated in references 74 and 75.

11.11 IMPACT NOISE

There are many practical cases where the excitation of a structure can be represented reasonably well by the periodic impact of a mass on its surface. Footfall in dwellings, punch presses, and forge hammers fall into this category. This section deals only with footfall noise in buildings. For the prediction and control of impact noise of machines and equipment, the reader is referred to a series of 10 papers^{76–85} that covers all aspects of impact noise of machinery.

Standard Tapping Machine

A standard tapping machine⁸⁶ is used to rate the impact noise isolation of floors in dwellings. This machine consists of five hammers equally spaced along a line, the distance between the two end hammers being about 40 cm. The hammers successively impact on the surface of the floor to be tested at a rate of 10 times per second. Each hammer has a mass of 0.5 kg and falls with a velocity equivalent to a free-drop height of 4 cm. The area of the striking surface of the hammer is approximately 7 cm²; the striking surface is rounded as though it were part of a spherical surface of 50 cm radius. The impact noise isolation capability of a floor is rated by placing the standard tapping machine on the floor to be tested and measuring the one-third-octave-band sound pressure level L'_p averaged in space in the room below.

$$L_n \equiv L_p - 10 \log \frac{A_0}{S\bar{\alpha}_{S,ab}} \quad \text{dB re } 2 \times 10^{-5} \text{ N/m}^2 \quad (11.157a)$$

where L_p = one-third-octave-band sound pressure level as measured, dB

$S\bar{\alpha}_{S,ab}$ = total absorption in receiving room (see Chapter 7), m²

A_0 = reference value of absorption, = 10 m²

The physical formulation of the problem of impact noise is that of the excitation of a plate by periodic force impulses. Such periodic forces can be presented by a Fourier series consisting of an infinite number of discrete-frequency components, each with amplitude F_n , given by

$$F_n = \frac{2}{T_r} \int_0^{T_r} F(t) \cos \frac{2\pi n}{T_r} t \, dt \quad \text{N} \quad (11.157b)$$

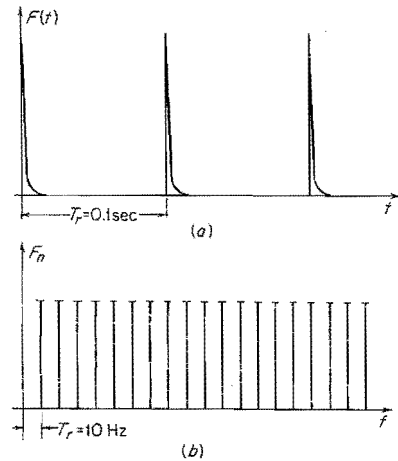


FIGURE 11.43 (a) Time function and (b) Fourier components of the force that a standard tapping machine exerts on a massive rigid floor.

where $T_r = 1/f_r = 0.1$ is the time interval between hammer strikes and $n = 1, 2, 3, \dots$. The curve in Fig. 11.43a shows the time function of the force $F(t)$, and that in Fig. 11.43b shows the amplitude of its Fourier components.

It is an experimental fact that when the hammer strikes a hard concrete slab, the duration of the force impulse is small compared even with the period of the highest frequency of interest in impact testing. For less stiff structures, like wooden floors, this assumption is not valid and the exact shape of $F(t)$ has to be determined and used in Eq. (11.157b). For a thick concrete slab the effective length of the force impulse is short enough so that $\cos[(2\pi n/T_r)t] \approx 1$, and all components have the same amplitude. Because the integral in Eq. (11.157b) is the momentum of a single hammer blow (assuming no rebound) equal to mv_0 (in kg · m/s), the amplitudes of the Fourier components of the force for a repetition frequency f_r are

$$F_n = 2f_r m v_0 \quad \text{N} \quad (11.158)$$

The velocity of the hammer at the instant of impact is

$$v_0 = \sqrt{2gh} \quad \text{m/s} \quad (11.159)$$

where h = falling height of hammer, m
 g = acceleration of gravity (9.8 m/s²)

Let us define a mean-square-force spectrum density S_{f_0} that when multiplied by the bandwidth will yield the value of the mean-square force in the same bandwidth,

$$S_{f_0} = \frac{1}{2} T_r F_n^2 = 4 f_r m^2 g h \quad \text{N}^2/\text{Hz} \quad (11.160)$$

For the standard tapping machine the numerical value of S_{f_0} is 4 N²/Hz. Accordingly, the mean-square force in an octave band $\Delta f_{\text{oct}} = f/\sqrt{2}$ is

$$F_{\text{rms}}^2(\text{oct}) = \frac{4}{\sqrt{2}} f \quad \text{N}^2 \quad (11.161)$$

The octave-band sound power level radiated by the impacted slab (which is assumed to be isotropic and homogeneous) into the room below is calculated by inserting Eq. (11.161) into Eq. (11.67a), which yields

$$L_w(\text{oct}) \approx 10 \log_{10} \left(\frac{\rho c \sigma_{\text{rad}}}{5.1 \rho_p^2 c_L \eta_p t^3} \right) + 120 \quad \text{dB re } 10^{-12} \text{ W} \quad (11.162)$$

where ρ = density of air, kg/m³

c = speed of sound in air, m/s

σ_{rad} = radiation factor of slab

ρ_p = density of slab material, kg/m³

c_L = propagation speed of longitudinal waves in slab material, m/s

η_p = composite loss factor of slab

t = thickness of slab, m

Note that the sound power level is independent of the center frequency of the octave, that doubling the slab thickness decreases the level of the noise radiated into the room below by 9 dB, and that the sound power level decreases with increasing loss factor.

Improvement of Impact Noise Isolation by an Elastic Surface Layer

Experience has shown that the impact noise level of even an 8–10-in.-thick dense concrete slab is too high to be acceptable. A further increase of thickness to reduce impact noise is not economical.

Impact noise may be reduced effectively by an elastic surface layer, much softer than the surface of the slab, applied to the structural slab. The resilient layer changes the shape of the force pulse and the amount of mechanical power introduced into the slab by the impacting hammer, as shown in Fig. 11.44.

We would expect, if the elastic layer is linear and nondissipative, that the velocity will be at its maximum v_0 at the instant of impact $t = 0$. It will then decrease to zero and the mass will rebound to nearly the same velocity (it is assumed the hammer is not permitted to bounce a second time) according to the function shown by curve *a* of Fig. 11.44. The force function is shown by curve *b*.

The improvement in impact noise isolation achieved by the addition of the soft surface layer is defined in terms of the logarithmic ratio⁸⁷

$$\Delta L_n = 20 \log \frac{F}{F'} = 20 \log \left(\frac{1 - n f_r / f_0}{\cos[(\pi/2)n(f_r/f_0)]} \right) \quad \text{dB} \quad (11.163)$$

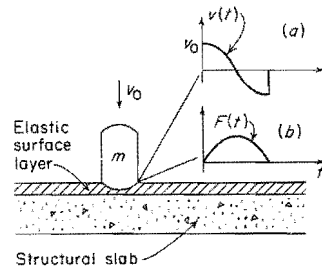


FIGURE 11.44 Velocity and force pulse of a single hammer blow on an elastic surface layer over a rigid slab: (a) velocity pulse; (b) force pulse.

where

$$n = 1, 2, 3, \dots \quad (11.164)$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{A_h}{m}} \sqrt{\frac{E}{h}} \quad \text{Hz} \quad (11.165)$$

F' , F = forces acting on slab with and without resilient surface layer, respectively, N

A_h = striking area of hammer, m^2

m = mass of hammer, kg

E = dynamic Young's modulus of elastic material, N/m

h = thickness of layer, m

The characteristic frequency f_0 of an elastic surface layer for the standard tapping machine is plotted in Fig. 11.45 as a function of E/h .

Equation (11.163), which assumes no damping, is plotted in Fig. 11.46 as a function of the normalized frequency f/f_0 . Below $f/f_0 = 1$, the improvement is zero. Above $f/f_0 = 1$ the improvement increases with an asymptotic slope of 40 dB/decade.

Figures 11.45 and 11.46 (use the 40-dB/decade asymptote) permit one to select an elastic surface layer to achieve a specified ΔL_n .

Example 11.6. The required improvement in impact noise isolation should be 20 dB at 300 Hz. Design a resilient covering for the concrete slab.

Solution From Fig. 11.46 we obtain $f/f_0 \approx 3$, which gives $f_0 = 100$ Hz. Entering Fig. 11.45 with this value of f_0 yields $E/h = 2.8 \times 10^8 \text{ N/m}^3$ (or $E/h \approx 1000 \text{ psi/in.}$). Any material having this ratio of Young's modulus to thickness will provide the required improvement. If we wish to select a 0.31-cm- ($\frac{1}{8}$ -in.-) thick layer, the dynamic modulus of the material should be $8.7 \times 10^5 \text{ N/m}^2$ (8000 psi). Since the dynamic modulus of most elastic materials is about twice the statically

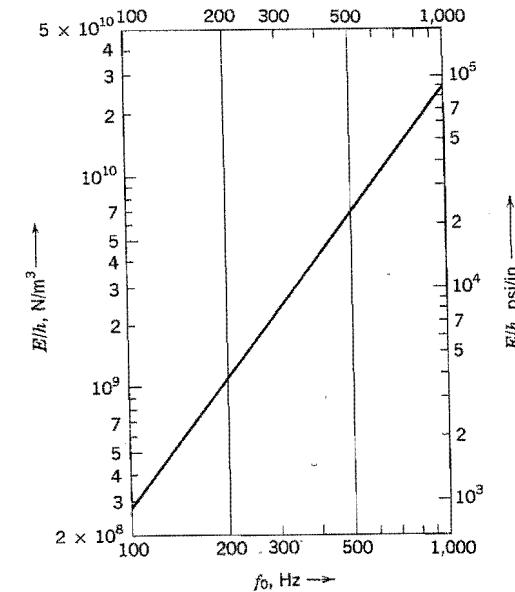


FIGURE 11.45 Chart for the selection of an elastic surface layer, where f_0 = characteristic frequency, E = Young's modulus, and h = thickness of layer.

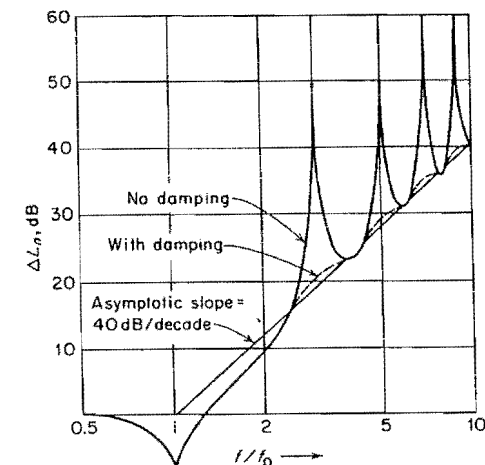


FIGURE 11.46 Improvement in impact noise isolation ΔL_n versus normalized frequency for a resilient surface layer (select f_0 to yield desired improvement).

measured Young's modulus,⁸⁸ a material with $E \leq 4.35 \times 10^5 \text{ N/m}^2$ (4000 psi) should be selected.

Frequently used materials for the elastic surface layer are rubberlike materials, vinyl-cork tile, or carpet. The impact isolation improvement curve (ΔL_n vs. frequency) has been measured and reported⁸⁹ for a large variety of elastic surface layer configurations.

The expected normalized impact sound level in the room below (see Fig. 11.1) for a composite floor (with a heavy structural slab) is that of the bare concrete structural floor minus the improvement caused by the elastic surface layer, that is,

$$L_{n,\text{comp}} = L_{n,\text{bare}} - \Delta L_n \quad (11.166)$$

where

$$L_{n,\text{bare}}(\text{oct}) = 116 + 10 \log \left(\frac{\rho c \sigma_{\text{rad}}}{5.1 \rho_p^2 c_L \eta_p t^3} \right) \text{ dB re } 2 \times 10^{-5} \text{ N/m}^2 \quad (11.167)$$

for homogeneous isotropic slabs.

Measured values of impact noise isolation of a large number of floor constructions and improvement of impact noise isolation by various surface layers are presented in reference 89.

Improvement through Floating Floors. It is often more practical to use a floating floor above a structural slab than a soft resilient surface layer. The advantages are that (1) both the impact noise isolation and the airborne sound transmission loss of the composite floor are improved and (2) the walking surface is hard. For analysis, floating floors can be categorized as either (1) locally reacting or (2) resonantly reacting, as defined below.

Locally Reacting Floating Floors. A locally reacting floor is one where the impact force of the hammer on the upper slab (slab 1) is transmitted to the structural slab (slab 2), primarily in the immediate vicinity of the excitation point, and where there is no spatially homogeneous reverberant vibration field on slab 1. In this case the bending waves in the floating slab are highly damped. If the Fourier amplitude of the force acting on plate 1 is given by Eq. (11.158), the reduction in transmitted sound level is⁸⁷

$$\Delta L_n = 20 \log \left[1 + \left(\frac{f}{f_0} \right)^2 \right] \approx 40 \log \frac{f}{f_0} \quad (11.168)$$

where

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{s'}{\rho_{s1}}} \quad (11.169)$$

and ρ_{s1} = mass per unit area of floating slab, kg/m^2

s' = dynamic stiffness per unit area of resilient layer between slab 1 and slab 2 including trapped air, N/m^3

Resonantly Reacting Floating Floors. If the floating slab is thick, rigid, and lightly damped, the impact force of the hammers excites a more-or-less spatially homogeneous reverberant bending wave field.

The improvement in impact noise isolation at higher frequencies, where the power dissipated in slab 1 exceeds the power transmitted to slab 2, can be approximated by^{87,90}

$$\Delta L_n \approx 10 \log \frac{2.3 \rho_{s1}^2 \omega^3 \eta_1 c_{L1} h_1}{n' s^2} \quad (11.170)$$

where h_1 = thickness of floating slab, m

c_{L1} = propagation speed of longitudinal waves in floating slab, m/s

ρ_{s1} = mass per unit area of floating slab, kg/m^2

η_1 = loss factor of floating slab

n' = number of resilient mounts per unit area of slab, m^2

s = stiffness of mount, N/m

Equation (11.170) indicates that, in contrast to the locally reacting case where ΔL_n increases at a rate of 40 dB for each decade increase in frequency, the increase is only 30 dB/decade if the loss factor of the floating slab η_1 is frequency independent. Another difference is the marked dependence of ΔL_n on this loss factor. The loss factor is determined both by the energy dissipated in the slab material itself and by the energy dissipated in the resilient mounts.

Figure 11.47 shows the improvement of a floating-floor system under impact by a standard tapping machine and high-heel shoes, respectively.⁹¹ The negative improvement in the vicinity of the resonance frequency f_0 can be observed.

Impact Noise Isolation versus Sound Transmission Loss

Whether a floor is excited by the hammers of a tapping machine or by an airborne sound field in the source room, it will in both cases radiate sound into the receiving room. There is a close relation between the airborne sound transmission loss R and the normalized impact noise level L_n for a given floor.

In the case of acoustical excitation, the sound power transmitted to the receiver room comprises the contribution of forced waves and of resonance waves. The forced waves usually dominate below the critical frequency of the slab and the resonance waves above. The sound power transmitted by exciting the slab by a standard tapping machine is made up of the contributions of the near-field component and of the reverberant component.

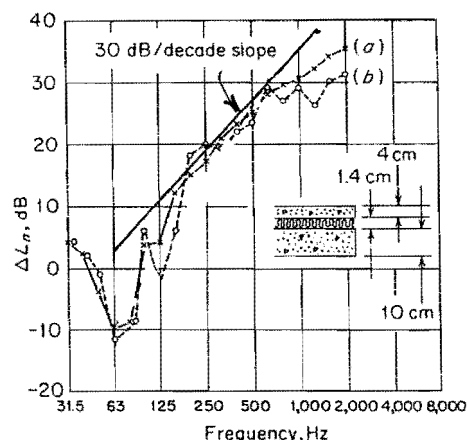


FIGURE 11.47 Improvement in impact noise isolation ΔL_n for a resonantly reacting floating floor for excitation by (a) a standard tapping machine and (b) high-heeled shoes. Note the negative ΔL_n in the vicinity of the resonance frequency. (After Ref. 91.)

The relation between sound transmission loss R and normalized impact noise level, assuming measurement in octave bands, is⁹²

$$L_n + R = 84 + 10 \log \left[\frac{S_{f0} f}{\sqrt{2}} \left(\frac{\rho / (2\pi \rho_s^2 c) + \rho c \sigma_{\text{rad}} / (2.3 \rho_s^2 c_L \omega \eta_p h)}{\pi \rho c / (\omega^2 \rho_s^2) + \pi \sqrt{12} c^3 \rho \sigma_{\text{rad}}^2 / (2 \rho_s^2 c_L \omega^3 \eta_p h)} \right) \right] \text{ dB} \quad (11.171)$$

where ρ_s = mass per unit area of slab, kg/m^2
 c_L = propagation speed of longitudinal waves in slab, m/s
 σ_{rad} = radiation factor of slab
 h = thickness of slab, m
 η_p = composite loss factor of slab
 S_{f0} = mean-square force spectrum density as given in Eq. (11.160), N^2/Hz

In the special case of a thick, lightly damped slab,

$$L_n + R = 43 + 30 \log f - 10 \log \sigma_{\text{rad}} - \Delta L_n \quad (11.172)$$

where ΔL_n represents the effect of the surface layer only. For a bare structural slab, by definition, $\Delta L_n = 0$.

Equation (11.172) states that the sum of the airborne sound transmission loss and the normalized impact noise level is independent of the physical characteristics of the structural slab above the critical frequency of the slab where $\sigma_{\text{rad}} \approx 1$.

Below the coincidence frequency where the forced waves control the airborne sound transmission loss but the impact noise isolation is still controlled by the resonant vibration of the impacted slab, Eq. (11.171) yields⁹²

$$R + L_n = 39.5 + 20 \log f - \Delta L_n - 10 \log \frac{\eta_p}{f_c \sigma_{\text{rad}}} \text{ dB} \quad (11.173)$$

where ΔL_n = effect of surface layer only (zero for structural slab), dB
 f_c = critical frequency of structural slab, Hz

In this case, the sum $R + L_n$ decibels depends on the physical characteristics of the slab and frequency. Figure 11.48 shows the measured sound transmission loss R and normalized impact sound level L_n as well as their sum for a typical floating floor. The measured and predicted values for the sum are in good agreement,

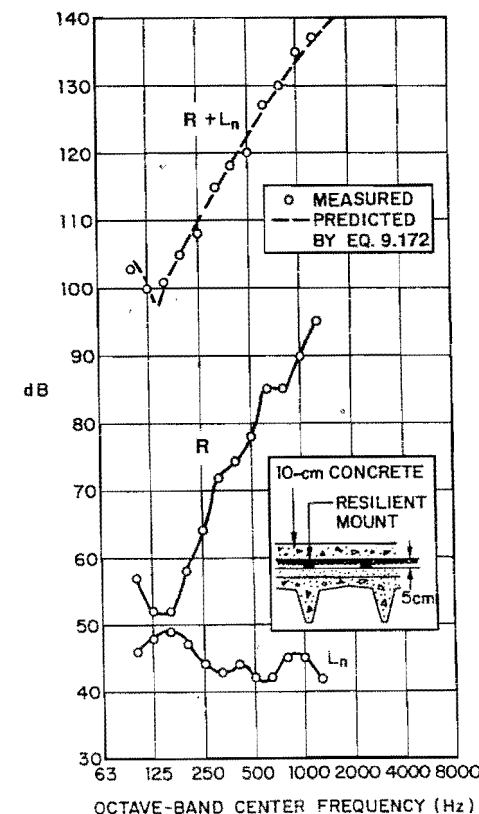


FIGURE 11.48 Measured sound transmission loss R and normalized impact sound level L_n and their sum ($R + L_n$) of a resonantly reacting floating floor assembly. Dotted curve: $R + L_n$ predicted by Eq. (11.172). (After reference 92.)

indicating that the precautionary measures taken to eliminate flanking have been successful.

In checking out the performance of floating floors in the field, it is advisable to measure both R and L_n . The discrepancy between the measured R and that calculated from Eq. (11.173) is a direct indication of flanking. By measuring the acceleration level on the wall surfaces in the source and receiving rooms during acoustical and impact excitation, the flanking paths can be immediately identified.

The measurement and rating of the impact noise isolation of floor assemblies is prescribed in ASTM E492-90 (1996), ASTM E989-89 (1999), and ASTM E1007-97.

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TABLE 11.1 Speeds of Sound in Solids, Deformation of Solids for Different Wave Types, and Formulas for Propagation Speed

	Longitudinal Waves Bar		$c_L = \sqrt{E/\rho_M}$
	Infinite plate		$c_L' = \sqrt{E/[\rho_M(1-\nu^2)]}$
	Infinite solid		$c_L'' = \sqrt{\frac{(E/\rho_M)(1-\nu)}{(1+\nu)(1-2\nu)}}$
	Shear Wave		$c_s = \sqrt{G/\rho_M} \quad G = E/(2+2\nu)$
	Torsional Wave Bar		$c_T = \sqrt{GK/\rho_M I}$ $= c_s \text{ for rods of circular cross section}$
	Bending Wave Plate		$c_B = \sqrt[4]{\omega^2 B/\rho_s}$

Young's modulus E , N/m^2 , relates the stress S (force per unit area) to the strain (change in length per unit length). Poisson's ratio ν is the ratio of the transverse expansion per unit length of a circular bar to its shortening per unit length, under a compressive stress, dimensionless. It equals about 0.3 for structural materials and nearly 0.5 for rubberlike materials. The density of the material is ρ_M , kg/m^3 . ρ_s is mass per unit area (kg/m^2) for plates and mass per unit length (kg/m) for bars, rods, or beams. The shear modulus G is the ratio of shearing stress to shearing strain, N/m^2 . I is the polar moment of inertia, m^4 . The torsional stiffness factor K relates a twist to the shearing strain produced, m^4 . The bending stiffness per unit width B equals $Eh^3/[12(1-\nu^2)]$ for a homogeneous plate, $\text{N}\cdot\text{m}$, where h is the thickness of the bar (or plate) in the direction of bending, m . For rectangular rods $B = Eh^3w/12$, where h is the cross-sectional dimension in the plane of bending and w that perpendicular to it (and width), m .

TABLE 11.2 Key Acoustical Parameters of Solid Materials

Material	Density ρ_M , kg/m^3	Young's Modulus E , N/m^2	Poisson Ratio ν	Speed of Sound c_L , m/s	Product of Surface Density and Critical Frequency $\rho_s f_c$		TL at Critical Frequency $R(f_c)$, dB	Internal Damping Factor for Bending at 1000 Hz, η^a	Acoustical-Mechanical Conversion Efficiency η_{am} , Eq. (11.66)
					$\text{Hz}\cdot\text{kg/m}^2$	$\text{Hz}\cdot\text{lb/ft}^2$			
Aluminum	2,700	7.16×10^{10}	0.34	5,150	34,700	7,000	48.5	10^{-4} – 10^{-2b}	2.5×10^{-3}
Copper	8,900	1.3×10^{11}	0.35	3,800	—	—	—	—	3.3×10^{-4}
Glass	2,500	6.76×10^{10}	— ^c	5,200	38,000	7,800	49.5	0.001–0.01 ^b	2.7×10^{-3}
Lead (chemical or tellurium)	11,000	1.58×10^{10}	0.43	1,200	605,000	124,000	73.5	0.015	1.4×10^{-4}
Plexiglas or Lucite	1,150	3.73×10^9	— ^c	1,800	35,400	7,250	49.0	0.002 ^b	2×10^{-3}
Steel	7,700	1.96×10^{11}	0.31	5,050	97,500	20,000	57.5	10^{-4} – 10^{-2b}	8.5×10^{-4}
Brick	1,900–2,300	— ^c	— ^c	— ^c	34,700–58,600	7,000–12,000	48.5 to 53	0.01	— ^c
Concrete, dense poured	2,300	2.61×10^{10}	— ^c	3,400	43,000	9,000	50.5	0.005–0.02	1.9×10^{-3}
Concrete (Clinker) slab, plastered on both sides 5 cm thick	1,500	—	—	—	48,800	10,000	51.5	0.005–0.02	— ^c
Masonry block	750	—	—	—	23,200	4,750	45.0	0.005–0.02	— ^c

(continued overleaf)

TABLE 11.2 (continued)

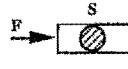
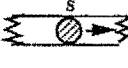
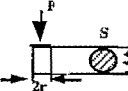
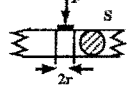
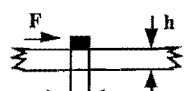
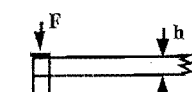
Material	Density ρ_M , kg/m ³	Young's Modulus E , N/m ²	Poisson Ratio ν	Speed of Sound c_L , m/s	Product of Surface Density and Critical Frequency $\rho_s f_c$		TL at Critical Frequency $R(f_c)$, dB	Internal Damping Factor for Bending at 1000 Hz, η^a	Acoustical-Mechanical Conversion Efficiency η_{am} , Eq. (11.66)
					Hz kg/m ²	Hz lb/ft ²			
Hollow cinder with 1.6 cm sand plaster, nominal thickness 15 cm (6 in.)	900	—	—	—	25,500	5,220	46.0	0.005–0.02	— ^c
Hollow dense concrete, nominal 15 cm (6 in.) thick	1100	—	—	—	23,000	4,720	45.0	0.007–0.02	— ^c
Hollow dense concrete, sand-filled voids, nominal 15 cm (6 in.) thick	1,700	— ^c	— ^c	— ^c	42,200	8,650	50.0	Varies with frequency	— ^c
Solid dense concrete, nominal 10 cm (4 in.) thick	1,700	— ^c	— ^c	— ^c	54,100	11,100	52.5	0.012	— ^c
Gypsum board 1.25–5 cm ($\frac{1}{2}$ –2 in.) thick	650	— ^c	— ^c	6,800	20,000	4,500	45.0	0.01–0.03	— ^c
Plaster, solid, on metal or gypsum lathe	1,700	— ^c	— ^c	— ^c	24,500	5,000	45.5	0.005–0.01	— ^c
Fir timber	550	— ^c	— ^c	3,800	4,880	1,000	31.5	0.04	9×10^{-3}
Plywood 0.6–3.12 cm ($\frac{1}{2}$ –2 in.) thick	600	— ^c	— ^c	— ^c	12,700	2,600	40	0.01–0.04	— ^c
Wood waste material bonded with plastic 23 kg/m ² (5 lb/ft ²)	750	— ^c	— ^c	— ^c	73,200	15,000	55.0	0.005–0.01	— ^c

^aThe range in values of η are based on limited data. The lower values are typical for material alone while the higher values are the maximum observed on panels in place.

^bThe loss factors for structures of these materials are sensitive to construction techniques and edge conditions.

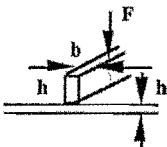
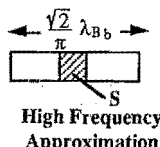
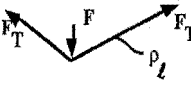
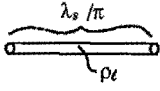
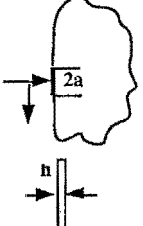
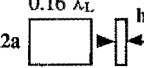
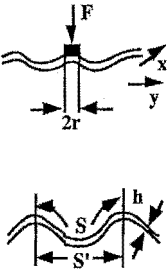
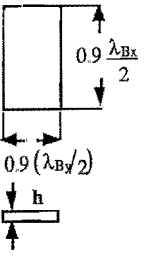
^cThe parameter either is not meaningful or is not available.

TABLE 11.3 Driving Force Point Impedance of Infinite Structures

Element	Picture	Equivalent Mass [see Eq. (11.19)]	Driving Point Force Impedance	Range of Validity	Auxiliary Expressions and Notes
Beam, in compression Semi-infinite Infinite		$\lambda_L/2 \pi$	$Z_F = \rho_M c_L S$	$S < (\lambda_s/4)^2$	$c_L = \sqrt{\frac{E}{\rho_M}}$
		λ_s/π	$Z_F = 2\rho_M c_L S$		$S = \text{cross-sectional area}$ $\lambda_s = \frac{1}{f} \sqrt{\frac{E}{2(1+\nu)\rho_M}}$
Beam in bending Semi-infinite Infinite		$\frac{\sqrt{2}}{2\pi} \lambda_B$	$Z_F = \frac{1}{2}(1+j)\rho_M S c_B$	$S < (\lambda_B/6)^2$ $2r > 9S/\lambda_s$	$r = \text{radius of contact area}$ $c_B = \left(\frac{EI\omega^2}{\rho_M S}\right)^{1/4}$ $\nu = \text{Poisson Ratio}$
		$\frac{\sqrt{2}}{\pi} \lambda_B$	$Z_F = 2(1+j)\rho_M S c_B$		$\lambda_B = c_B/f$ $I = \text{polar moment of inertia}$
Thin infinite plate Vertical force Horizontal force (in plane)		$\lambda_B/2$	$Z_F = 8\sqrt{B'\rho_M c_L h}$ $= 2.3\rho_M c_L h^2$	$h < \lambda_B/6$ $2r > 3h$ $\lambda_s \gg 2\pi r > 10h$	$B' = \left(\frac{Eh^3}{12(1-\nu^2)}\right)$ $\lambda_s = \frac{1}{f} \sqrt{\frac{E}{2(1+\nu)\rho_M}}$ $H = K + L$ $K = (1-\mu) \ln\left(\frac{\lambda_l}{\pi r}\right)$ $L = 2 \ln\left(\frac{\lambda_s}{\pi r}\right)$
		$\lambda_B/2$	$\frac{1}{Z_F} = \frac{\pi f}{4Gh}$ $\times \left(\frac{3-\mu}{2} + jH\right)$ $\text{Re}\{1/Z_F\} = [n_L(f) + n_s(f)]/8M$		
Thin semi-infinite plate		$\lambda_B/2$	$Z_F = 3.5\sqrt{B'\rho_M h}$ $\approx \rho_M c_L h^2$	$h < \lambda_B/6$ $2r > 3h$	$n_L(f) = \text{longitudinal modal density}$ $n_s(f) = \text{shear modal density}$ $M = \text{total mass}$ $\lambda_l = c_L/f$ $\lambda_s = c_s/f$ $\lambda_B = c_B/f$

(continued overleaf)

TABLE 11.3 (continued)

Element	Picture	Equivalent Mass [see Eq. (11.19)]	Driving Point Force Impedance	Range of Validity	Auxiliary Expressions and Notes
Beam-stiffened infinite thin plate			$Z_F = \frac{(1-j)k'}{4\rho'_s\omega}$ $Z_F \cong Z_{FB}$ $= 2(1+j)\rho_M S_b c_B$	$S_b < (\lambda_B/6)^2$ $2r > 9S_b/\lambda_B$	$\rho'_s = \rho_M S_B + 2\rho_M h/k_p$ $k' = \left(\frac{\rho'_s}{B}\right)^{1/4} \omega^{1/2} A$ $A = 1 - j \frac{\rho_M h}{2\rho'_s k_p}$ $k_p = \text{plate-bending wavenumber}$ $B = Eh^3 b/3$ $s = k_b/k_p$
Infinite string			$Z_F = 2\sqrt{\rho_l F_T}$		$c_s = \sqrt{F_T/\rho_l}$ $\rho_l = \text{mass per unit length}$ $F_T = \text{tension force}$
Semi-infinite plate, in-plane edge force			$\frac{1}{Z_F} = -\frac{j\omega^2}{\pi Eh} \log a$ $+ \frac{\omega}{4} \left(\frac{1}{D} + \frac{1}{S} \right)$		$D = \frac{Eh(1-\nu)}{(1-\nu)(1-2\nu)}$ $S = Gh$ $G = \text{shear modulus}$
Infinite corrugated plate			$Z_F = 8[(\rho_M h)^2 \times B_x B_y]^{1/4}$	$S \ll \lambda_B$	$B_y = \frac{Eh^3}{12(1-\nu^2)}$ $B_x \cong \left(\frac{1-\nu}{1+\nu}\right)^2 \frac{S}{S'} B_y$ $f_{c1} = \frac{c_0^2}{2\pi} \sqrt{\frac{\rho_M h}{B_x}}$ $f_{c2} = f_{c1} \sqrt{\frac{B_x}{B_y}}$

(continued overleaf)

TABLE 11.3 (continued)

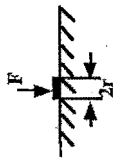
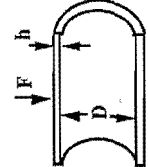
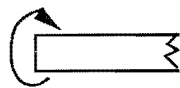
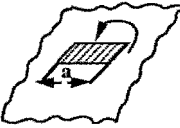
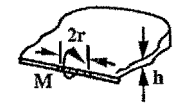
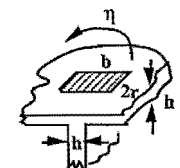
Element	Picture	Equivalent Mass [see Eq. (11.19)]	Driving Point Force Impedance	Range of Validity	Auxiliary Expressions and Notes
Elastic half space		—	$Z = \frac{-j0.64Gr}{f(1-\nu/2)} + 1.79r^2 \sqrt{\frac{4G\rho_M}{1-\nu}}$	$2r < \lambda_s/6$	ρ_M = density G = shear modulus ν = Poisson's ratio
Infinite cylindrical shell		—	$Z_\infty \approx 2.3\rho_M c_L h \frac{\sqrt{12}}{8\rho_M c_L h^2} \times \left(\frac{f}{f_R}\right)^{2/3}$ $\text{Re}\left(\frac{1}{Z_\infty}\right) = \frac{\sqrt{12}}{8\rho_M c_L h^2} \times \left(\frac{f}{f_R}\right)^{2/3}$	$f > 1.5 f_R$ $f < 0.7 f_R$	$f_R = c_L/\pi D$ ring frequency

TABLE 11.4 Moment Impedance Z_M of Infinite and Semi-Infinite Structures

Element	Picture	Driving Point Moment Impedance
Semi-infinite beam		
Free end		$\frac{(1-j)\rho_l c_B^3(f)}{8\pi^2 f^2}$
Pinned end		$\frac{(1-j)\rho_l c_B^3(f)}{4\pi^2 f^2}$
Infinite beam		$\frac{(1-j)\rho_l c_B^3(f)}{2\pi^2 f^2}$
Infinite homogeneous isotropic plate		$\frac{16\rho_M h c_L^2 k_B^{-2}(f)}{2\pi f [(1-j)1.27 \ln(k_B a/2.2)]}$
Semi-infinite homogeneous isotropic plate		$\frac{12\rho_M h c_L^2 k_B^{-2}(f)}{2\pi f [(1-j)3.35 \ln(kr/3.5)]}$
At joint of homogeneous isotropic plates		$\frac{\rho_M c_L^2 h^3}{75.4 f} \left\{ 16 \left[\frac{(1+j)1.27 \ln(kb/2.2)}{1 + (1.27 \ln kb/2.2)^2} \right] + 12 \left[\frac{(1+j)3.35 \ln(kr/3.5)}{1 + (3.35 \ln kr/3.5)^2} \right] \right\}$

Auxiliary expressions and notes:

 ρ_l = mass per unit length (kg/m) $c_B(f)$ = bending wave speed (m/s) in bending, $= \sqrt{2\pi f (EI/\rho_l)}^{1/4}$ E = Young's modulus (N/m²) I = area moment of inertia in bending (m⁴) f = frequency ρ_M = material density (kg/m³) $k_B(f) = 2\pi f/c_B(f)$ bending wavenumber h = plate thickness

TABLE 11.5 Effective Length l_M Connecting Force and Moment Impedance, $Z_M = l_M^2 Z_f$ for Infinite Beams and Plates

Element	Picture	Equivalent Force Pair	Effective Length l_M	Auxiliary Expressions
Beam in bending			$\frac{\lambda_B}{2\pi} \sqrt{j}$	$j = \sqrt{-1}$
Beam in torsion			$0.79i$	For hollow beams: $i = \sqrt{\frac{d_o^2 - d_i^2}{2}}$ d_o = outside diameter d_i = inside diameter
Plate, vertical moment			$\frac{\lambda_B / \sqrt{8\pi j}}{\sqrt{\ln(\lambda_B / \pi r)}}$	Bending waves: $3h < 2r \ll \lambda_B / \pi$
Plate, in-plane torsional moment			$\frac{r}{\sqrt{\ln(\lambda_s / \pi r)}}$	Shear waves: $3h < 2r \ll \lambda_s / \pi$ λ_s = shear wavelength

Source: After reference 6.

TABLE 11.6 Power Input to Structures

Element	Picture	Power Input to Infinite Element		Finite Elements		Auxiliary Expressions
		Force or Moment Excitation	Velocity or Angular Velocity Excitation	Onset of Infinite Behavior	W_{fin} / W_{inf}	
Beam in longitudinal wave motion; force or velocity excitation		$\frac{ \hat{F} ^2}{4\rho_M S c_L}$	$4 \hat{v} ^2 S \rho_M c_L$	$\omega > \frac{\pi c_L}{\eta l}$	$\frac{4}{\pi \eta}$	$c_L = \sqrt{E / \rho_M}$ l = length η = loss factor Q = torsion constant G = shear modulus J = mass moment of inertia per unit length $c_T = \sqrt{\frac{E / \rho_M}{G Q / J}}$ = torsional wave speed ρ_M = density E = Young's modulus I = second moment of inertia $\dot{\theta}$ = angular velocity $c_B = \sqrt{\omega \sqrt{\frac{E / I}{\rho_M S}}}$ = bending wave speed $B_p = \frac{h^3 E}{12(1 - \nu^2)}$ S = area
Beam in torsion moment or angular velocity excitation		$\frac{ \hat{M} ^2}{4G Q J}$	$4 \dot{\theta} ^2 \sqrt{G Q J}$	$\omega > \frac{\pi c_T}{\eta l}$	$\frac{4}{\pi \eta}$	
Beam in bending; force or velocity excitation		$\frac{ \hat{F} ^2}{8\rho_M S c_B(f)}$	$ \hat{v} ^2 S \rho_M c_B(f)$	$\omega > \frac{4\pi c_B(f)}{\eta l}$	$\frac{4\sqrt{2}}{\pi \eta}$	
Beam in bending; moment or angular velocity excitation		$\frac{ \hat{M} ^2 c_B(f)}{8EI}$	$\frac{ \dot{\theta} ^2 EI}{c_B(f)}$	$\omega > \frac{4\pi c_B(f)}{\eta l}$	$\frac{2\sqrt{2}}{\pi \eta}$	
Plate in bending; force or velocity excitation		$\frac{ \hat{F} ^2}{16\sqrt{B_p \rho_M h}} = \frac{ \hat{F} ^2}{46\rho h^2 c_L}$	$4\hat{v}^2 \sqrt{B_p \rho_M h} = 1.15\hat{v}^2 \rho_M h^2 c_L$	$\omega > \frac{8}{\eta l_1 l_2} \sqrt{\frac{B_p}{\rho_M h}}$	$\frac{32l_1 l_2}{\pi^2 \eta (l_1^2 + l_2^2)} \frac{\omega}{c_B}$	

(continued overleaf)

TABLE 11.6 (continued)

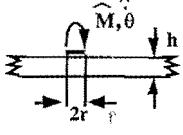
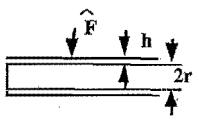
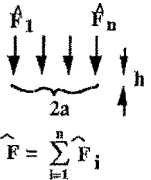
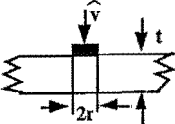
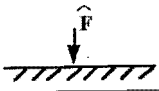
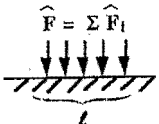
Element	Picture	Power Input to Infinite Element		Finite Elements		Auxiliary Expressions
		Force or Moment Excitation	Velocity or Angular Velocity Excitation	Onset of Infinite Behavior	W_{fin}/W_{inf}	
Plate in bending; moment or angular velocity excitation		$\sim \frac{\omega \hat{M} ^2}{16B_p}$ for $r > h$	$4 \hat{\theta} ^2 B_p$ $\omega \left\{ 1 + \left[\frac{4}{\pi} \ln \left(\frac{\omega r}{c_B} \right) - \frac{8}{\pi(1-\nu)} \left(\frac{h}{\pi r} \right)^2 \right]^2 \right\}$ $\omega > \frac{8}{\eta l_1 l_2} \sqrt{\frac{B_p}{\rho_M h}}$			
Thin-walled pipe in bending; force excitation		$\hat{F}^2 / (16\pi \rho_M r h \sqrt{c_L r \omega})$ for $f < 0.123 c_L h / r$ $\hat{F}^2 \sqrt{V/(2+V)} / (\omega \rho, 2\lambda^2 / \pi^2)$ for $f > 0.123 c_L h / r$				$V = \omega r / c_L$ $\lambda_p =$ bending wavelength in equivalent thickness plate $\rho_s = \rho_M h$
Plate in bending; multiple force excitation, equally spaced	 $\hat{F} = \sum_{i=1}^n \hat{F}_i$	$\hat{F}^2 [2J_1(z)/z]^2 / (16\sqrt{B_p \rho_M h})$				$Z = 2\pi a / \lambda_B$ $J_1 =$ Bessel function of order 1

TABLE 11.6 (continued)

Element	Picture	Power Input to Infinite Element		Finite Elements		Auxiliary Expressions
		Force or Moment Excitation	Velocity or Angular Velocity Excitation	Onset of Infinite Behavior	W_{finite}/W_{inf}	
Plate in bending; large area velocity excitation		$\hat{v}^2 \frac{\pi}{2} \rho_M c_B (r + 0.8\lambda_B) h$				
Elastic half space; single force		$\frac{48\hat{F}^2}{\omega \rho_M \pi \lambda_s^3}$				$\lambda_s = \sqrt{G/\rho_M/f}$ shear wavelength
Elastic half space equal multiple forces along a line, equally spaced	 $\hat{F} = \sum \hat{F}_i$	$\frac{16\hat{F}^2}{\omega \rho_M l \lambda_s^2}, l > \lambda_s/2$				

(continued overleaf)

TABLE 11.6 (continued)

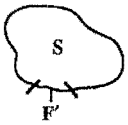
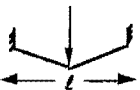
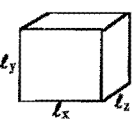
Element	Picture	Power Input to Infinite Element Nearby Monopole and Dipole Excitation	Auxiliary Expressions
Infinite plate excited by nearby:			
Monopole		$W_{\text{mon}} \cong \frac{Q^2 (\rho_o c_0)^2 \text{Re}\{Y_\infty\}}{(f_c/f) - 1} \times \exp\left(\frac{4\pi f}{c_0} y \sqrt{f_c/f - 1}\right), f < f_c$	Q = rms volume velocity Y_∞ = point force admittance
Lateral dipole		$y \ll \lambda_0/4$ light fluid loading $W_{\text{LD}} \cong 0.5 (k_B d)^2 W_{\text{mon}}, f < f_c$	k_B = free plate bending wavenumber f_c = critical coincidence frequency
Perpendicular dipole		$W_{\text{PD}} \cong W_{\text{mon}} (k_B d)^2 [1 - (f/f_c)]^2 \text{ (for } f < f_c \text{ and light fluid loading)}$	

TABLE 11.7 First Resonance Frequency, Mode Shape and Modal Density of Finite Structures

Element	Picture	Boundary Conditions ^a	First Resonance Frequency	Mode Shape $\phi(x, y, z)$	Modal Density ^b $n(\omega)$	Auxiliary Formulas
Beam in compression		f-f c-c	$c_L/2l$	$\cos(n\pi x/l)$ $\sin(n\pi x/l)$	$l/\pi c_L$	$\kappa = \sqrt{I/S}$ radius of gyration
Beam in bending		p-p f-f c-c c-f	$(\pi/2)(\kappa c_L/l^2)$ $(1/2\pi)(4.73/l)^2 \kappa c_L$ $(1/2\pi)(4.73/l)^2 \kappa c_L$ $(1/2\pi)(1.875/l)^2 \kappa c_L$	$\sqrt{2} \sin(k_n X)$ } See ref. 19	$\frac{l}{2\pi} \frac{1}{\sqrt{\omega \kappa c_L}}$	$k_n = \sqrt{2\pi f_n/c_L \kappa}$ a/b = aspect ratio h = plate thickness, m
Rectangular plate in bending		ffff ssss cccc	a/b 1 1.5 2.5 3.33 3.31 2.13 C_1 4.88 5.28 7.1 8.89 10.0 14.6 $f_1 = 10^3 C_1 (h/S) (c_L/c_{Lst})$	See ref. 19	$\frac{\sqrt{12} S}{4\pi c_L h}$	S = area, m ² $c_{Lst} = 5050$ m/s c_L = longitudinal wave speed

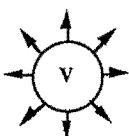
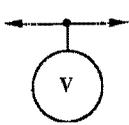
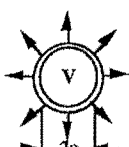
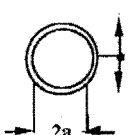
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TABLE 11.7 (continued)

Element	Picture	Boundary Conditions ^a	First Resonance Frequency	Mode Shape $\phi(x, y, z)$	Modal Density ^b $n(\omega)$	Auxiliary Formulas
Membrane			See ref. 18	See ref. 19	$\frac{S}{2\pi c_m}$	$F' =$ tension per unit length $\rho_s =$ mass per unit area $c_m = \sqrt{F'/\rho_s}$
String		c-c	$\pi c_s/l$	$\sin(n\pi x/l)$	$l/\pi c_s$	$c_s = \sqrt{F'/\rho_l}$ $F =$ tension force $\rho_l =$ mass per unit length
Rectangular air volume		Hard walls	$c_0/2l_{\max}$	$\cos\left(\frac{n_x\pi x}{l_x}\right)\cos\left(\frac{n_y\pi y}{l_y}\right)\cos\left(\frac{n_z\pi z}{l_z}\right)$ $n(\omega) = \frac{\omega^2 V}{2\pi^2 c_0^3} + \frac{S\omega}{4\pi c_0} + \frac{L}{16\pi c_0}$		$V = l_x l_y l_z$ $c_0 =$ speed of sound $n = 1, 2, 3, \dots$

^af = free; s = simply supported; c = clamped; p = pinned
^b $n(\omega) = n(f)/2\pi$, $S = 2(l_x l_y + l_x l_z + l_y l_z)$, $L = 4(l_x + l_y + l_z)$
Source: After reference 19.

TABLE 11.8 Radiation Efficiency of Vibrating Bodies

Body	Picture	σ_{rad}	Auxiliary Expressions
Small pulsating body		$\frac{(ka)^2}{1 + (ka)^2}$	$c_0 =$ speed of sound $k_0 = 2\pi f/c_0$ $a =$ source radius
Small oscillating rigid body		$\frac{(ka)^4}{4 + (ka)^4}$, see Fig. 11.14	
Pulsating pipe		$2/\pi k_0 a H_1(k_0 a) ^2$ for $(\pi/2)k_0 a \leq 2/\pi$	$H_1 =$ Hankel function, second kind, order 1 $k_0 = 2\pi f/c_0$
Oscillating pipe or rod		$2/\pi k_0 a H_1(k_0 a) ^2$, see Fig. 11.15	$H_1' =$ first derivative of H_1 in respect of its argument

(continued overleaf)

TABLE 11.8 (continued)

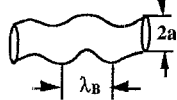
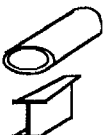
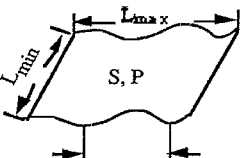
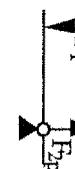
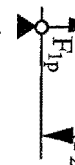
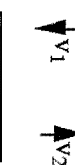
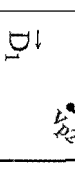
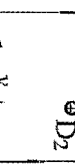
Body	Picture	σ_{rad}	Auxiliary Expressions
Circular pipes in bending		zero; $f < f_c$ $(k_0 a)^3 [1 - (f_c/f)]$; $f > f_c$; $k_0 a \ll 1$ 1; $f > f_c$; $k_0 a \gg 1.5$	$k_d^2 = k_0^2 - k_B^2$ $k_B = 2\pi/\lambda_B$ f_c = critical frequency where $k_B = k_0$
Rectangular and elliptic beams in bending		See ref. 22	
Infinite thin plate supporting free bending waves		0 for $f < f_c$ $1/[1 - (f_c/f)]^{1/2}$ for $f > f_c$	
Finite thin plate supporting free bending waves; plate surrounded by rigid baffle		$\frac{P c_0}{\pi S f_c} \sqrt{f/f_c} c_1$; $f < f_c$ $0.45(P/\lambda_c)^{1/2} (L_{\min}/L_{\max})^{1/4}$; $f = f_c$ $(1 - f_c/f)^{-1/2}$; $f > 1.3 f_c$ 1; $f \geq 1.3 f_c$	f_c = critical frequency See Eq. 11.94 $\lambda_c = c_0/f_c$ $S = L_{\max} L_{\min}$ = area (one side) $P = 2(L_{\max} + L_{\min})$ = perimeter $\beta = (f/f_c)^{1/2}$

TABLE 11.8 (continued)

Body	Picture	σ_{rad}	Auxiliary Expressions
		$g_1(\beta) = \begin{cases} ((4/\pi^4)[(1 - 2\beta^2)/\beta(1 - \beta^2)^{1/2}]) & ; f < 0.5 f_c \\ 0 & ; f > 0.5 f_c \end{cases}$ $g_2(\beta) = \left(\frac{1}{4\pi^2} \right) \frac{(1 - \beta^2) \ln[(1 + \beta)/(1 - \beta)] + 2\beta}{(1 - \beta^2)^{3/2}}; \quad C_1 = \begin{cases} 1 & \text{for simple supported edges} \\ \beta^2 \exp(10\lambda_c/P) & \text{for clamped edges} \end{cases}$ $\sigma_{\text{rad}} = \frac{P}{S} \frac{c_0}{\pi^2} \sqrt{\frac{f}{f_c^3}}$	
Thick finite plate supporting free bending waves		$0.45\sqrt{P/\lambda_0}$ for $f \leq f_b$ 1 for $f \gg f_b$	$f_b = f_c + \frac{5c_0}{P}$ P = perimeter
Infinite plate sound-forced waves		$\sigma_F = 1/\cos \phi$	ϕ = incidence angle, degrees
Finite square-plate oblique-incidence plane sound wave excitation	$\sigma_F = \min \left\{ \frac{A[(k_0/2)\sqrt{S}]}{1/\cos \phi} \text{ for } 0.1\lambda_0^2 < S < 0.4\lambda_0^2 \right.$ $\sigma_F = \min \left\{ \frac{[(0.5)^{(\phi/90)} \sqrt{k_0/2\sqrt{S}}]}{1/\cos \phi} \text{ for } S > 0.4\lambda_0^2 \right.$		$A = (0.5)(0.8)^{(\phi/90)}$ $\alpha = 1 - 0.34\phi/90$ $k_0 = 2\pi/\lambda_0 = 2\pi f/c_0$
Finite square-plate diffuse-sound-field excitation		$\sigma_F = 0.5[0.2 + \ln(k_0\sqrt{S})]$ for $k_0\sqrt{S} > 1$	

Source: After references 1, 23, and 28.

TABLE 11.9 Reciprocity Relationships for Higher Order Excitation Sources and Responses

	$\frac{\Theta_2}{M_1} = \frac{\Theta_1}{M_2}$	
	$\frac{v_2}{M_1} = \frac{\Theta_2}{F_2}$	
	$\frac{F_{2p}}{F_1} = \frac{F_{1p}}{F_2}$	
	$\frac{F_{2p}}{M_1} = \frac{\Theta_1}{v_2}$	
	$\frac{F_{2p}}{F_1} = \frac{v_1}{v_2}$	
	$\frac{v_{\theta 2}}{D_1} = \frac{v_{\theta 1}}{D_2}$	

Note: M = moment, v = velocity, v_p = particle velocity, Θ = angular velocity, F = force, F_p = force acting at pinned restraints, D = dipole sound source.

Source: After reference 63.

TABLE 11.10 USS Gauges and Weights of Steel Plates

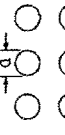
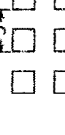
Gauge	Steel USS Gauge Rev.				Galvanized Steel USS Gauge				Stainless Chrome Alloy USS Gauge				Stainless Chrome Nickel USS Gauge		Monel USS Gauge			
	Thickness		Surface Weight		Thickness		Surface Weight		Thickness		Surface Weight		Surface Weight		Thickness		Surface Weight	
	in.	mm	lb/ft ²	kg/m ²	in.	mm	lb/ft ²	kg/m ²	in.	mm	lb/ft ²	kg/m ²	lb/ft ²	kg/m ²	in.	mm	lb/ft ²	kg/m ²
32	0.01	0.254			0.013	0.330	0.563	2.75	0.01	0.254	0.418	2.04	0.427	2.08				
31	0.011	0.279			0.014	0.356	0.594	2.90	0.0109	0.277	0.45	2.20	0.459	2.24				
30	0.012	0.305	0.5	2.44	0.0157	0.399	0.656	3.20	0.0125	0.318	0.515	2.51	0.525	2.56				
29	0.0135	0.343	0.563	2.75	0.0172	0.437	0.719	3.51	0.014	0.356	0.579	2.83	0.591	2.89				
28	0.0149	0.378	0.625	3.05	0.187	4.750	0.781	3.81	0.0156	0.396	0.643	3.14	0.656	3.20				
27	0.0164	0.417	0.688	3.36	0.0202	0.513	0.844	4.12	0.0171	0.434	0.708	3.46	0.721	3.52				
26	0.0179	0.455	0.75	3.66	0.0217	0.551	0.906	4.42	0.0187	0.475	0.772	3.77	0.787	3.84	0.0187	0.475	0.827	4.04
25	0.0209	0.531	0.875	4.27	0.0247	0.627	1.031	5.03	0.0218	0.554	0.901	4.40	0.918	4.48	0.0218	0.554	0.965	4.71
24	0.0239	0.607	1	4.88	0.0276	0.701	1.156	5.64	0.025	0.635	1.03	5.03	1.05	5.13	0.025	0.635	1.148	5.60
23	0.0269	0.683	1.125	5.49	0.0306	0.777	1.281	6.25	0.0281	0.714	1.158	5.65	1.181	5.77	0.0281	0.714	1.1286	5.51
22	0.0299	0.759	1.25	6.10	0.0336	0.853	1.406	6.86	0.0312	0.792	1.287	6.28	1.312	6.41	0.0312	0.792	1.424	6.95
21	0.0329	0.836	1.375	6.71	0.0366	0.930	1.531	7.47	0.0343	0.871	1.416	6.91	1.443	7.04	0.0343	0.871	1.562	7.63
20	0.0359	0.912	1.5	7.32	0.0396	1.006	1.656	8.08	0.0375	0.953	1.545	7.54	1.575	7.69	0.0375	0.953	1.7	8.30
19	0.0418	1.062	1.75	8.54	0.0456	1.158	1.906	9.31	0.0437	1.110	1.802	8.80	1.837	8.97	0.0437	1.110	1.975	9.64
18	0.0478	1.214	2	9.76	0.0516	1.311	2.156	10.53	0.05	1.270	2.06	10.06	2.1	10.25	0.05	1.270	2.297	11.21
17	0.0538	1.367	2.25	10.98	0.0575	1.461	2.406	11.75	0.0562	1.427	2.317	11.31	2.362	11.53	0.0562	1.427	2.572	12.56

(continued overleaf)

TABLE 11.10 (continued)

Gauge	Steel USS Gauge Rev.				Galvanized Steel USS Gauge				Stainless Chrome Alloy USS Gauge				Stainless Chrome Nickel USS Gauge		Monel USS Gauge			
	Thickness		Surface Weight		Thickness		Surface Weight		Thickness		Surface Weight		Surface Weight		Thickness		Surface Weight	
	in.	mm	lb/ft ²	kg/m ²	in.	mm	lb/ft ²	kg/m ²	in.	mm	lb/ft ²	kg/m ²	lb/ft ²	kg/m ²	in.	mm	lb/ft ²	kg/m ²
16	0.0598	1.519	2.5	12.21	0.0635	1.613	2.656	12.97	0.0625	1.588	2.575	12.57	2.625	12.82	0.0625	1.588	2.848	13.90
15	0.0673	1.709	2.812	13.73	0.071	1.803	2.969	14.49	0.0703	1.786	2.896	14.14	2.953	14.42	0.0703	1.786	3.216	15.70
14	0.0747	1.897	3.125	15.26	0.0785	1.994	3.281	16.02	0.0781	1.984	3.218	15.71	3.281	16.02	0.0781	1.984	3.583	17.49
13	0.0897	2.278	3.75	18.31	0.0934	2.372	3.906	19.07	0.0937	2.380	3.862	18.85	3.937	19.22	0.0937	2.380	4.272	20.86
12	0.1046	2.657	4.375	21.36	0.1084	2.753	4.531	22.12	0.1093	2.776	4.506	22.00	4.593	22.42	0.1093	2.776	5.007	24.44
11	0.1196	3.038	5	24.41	0.1233	3.132	5.156	25.17	0.125	3.175	5.15	25.14	5.25	25.63	0.125	3.175	5.742	28.03
10	0.1345	3.416	5.625	27.46	0.1382	3.510	5.781	28.22	0.1406	3.571	5.793	28.28	5.906	28.83	0.1406	3.571	6.431	31.40
9	0.1497	3.802	6.25	30.51	0.1532	3.891	6.406	31.27	0.1562	3.967	6.437	31.43	6.562	32.04	0.1562	3.967	7.166	34.98
8	0.1644	4.176	6.875	33.56	0.1681	4.270	7.031	34.33	0.1718	4.364	7.081	34.57	7.218	35.24	0.1718	4.364	7.855	38.35
7	0.1793	4.554	7.5	36.62					0.1875	4.763	7.59	37.05	7.752	37.85	0.1875	4.763	8.59	41.94

TABLE 11.11 Percentage Open Area of Perforated Plates

Diameter Divided by Centers or side of square divided by centers D/C or S/C	Round Holes, Standard Staggered		Round Holes, Standard Staggered		Square Holes, Straight Or	
						
.200	3.6	3.1	4.0	4.0	4.0	4.0
.225	4.6	4.0	5.1	5.1	5.1	5.1
.250	5.7	4.9	6.3	6.3	6.3	6.3
.275	6.9	5.9	7.6	7.6	7.6	7.6
.300	8.1	7.1	9.0	9.0	9.0	9.0
.325	9.6	8.3	10.6	10.6	10.6	10.6
.350	11.1	9.6	12.3	12.3	12.3	12.3
.375	12.8	11.0	14.1	14.1	14.1	14.1
.400	14.5	12.6	16.0	16.0	16.0	16.0
.425	16.4	14.2	18.1	18.1	18.1	18.1
.450	18.4	15.9	20.3	20.3	20.3	20.3
.475	20.5	17.7	22.6	22.6	22.6	22.6
.500	22.7	19.6	25.0	25.0	25.0	25.0
.525	25.0	21.6	27.6	27.6	27.6	27.6
.550	27.4	23.8	30.3	30.3	30.3	30.3
.575	30.0	26.0	33.1	33.1	33.1	33.1
.600	32.7	28.3	36.0	36.0	36.0	36.0
.625	35.4	30.7	39.1	39.1	39.1	39.1
.650	38.3	33.2	42.3	42.3	42.3	42.3
.675	41.3	35.8	45.6	45.6	45.6	45.6
.700	44.4	38.5	49.0	49.0	49.0	49.0
.725	47.7	41.3	52.6	52.6	52.6	52.6
.750	51.0	44.2	56.3	56.3	56.3	56.3
.775	54.4	47.2	60.0	60.0	60.0	60.0
.800	58.0	50.3	64.0	64.0	64.0	64.0
.825	61.7	53.5	68.0	68.0	68.0	68.0
.850	65.5	56.7	72.3	72.3	72.3	72.3
.875	69.5	60.1	76.6	76.6	76.6	76.6
.900	73.5	63.6	81.0	81.0	81.0	81.0
.925	77.6	67.2	85.6	85.6	85.6	85.6
.950	81.9	70.9	90.3	90.3	90.3	90.3

Enclosures, Cabins, and Wrappings

ISTVÁN L. VÉR

Consultant in Acoustics, Noise, and Vibration Control
Stow, Massachusetts

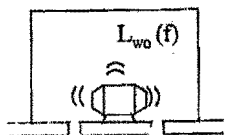
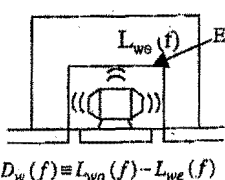
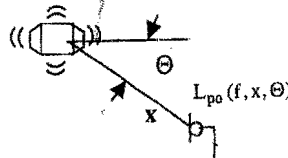
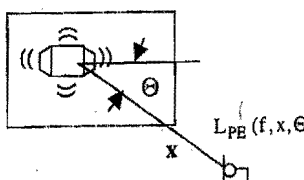
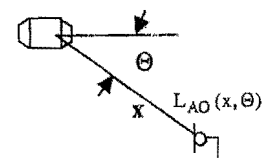
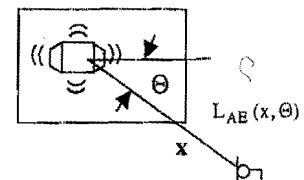
This chapter deals with the acoustical design of enclosures, cabins, and wrappings. Acoustical enclosures are structures that house a noise source (usually a machine) for protection of the environment from the noise emitted by the source. Cabins are fully enveloping structures specifically designed for protecting a human being from environmental noise. The human being protected by the cabin may be the operator of a noisy machine, supervisor of a manufacturing operation, or attendant of a road toll booth. Wrappings are acoustical structures which closely envelop the casing of machines, valves, and connected piping to provide a high degree of noise reduction at high frequencies, practically no reduction at low frequencies, and modest reduction in the frequency range in-between.

The recently issued international standard ISO 15667:2000E,¹ entitled "Guidelines for Noise Control by Enclosures and Cabins," defines a large number of acoustical performance ratings and specifies the measurement procedure of how to obtain them in the laboratory or in situ. These performance measures, which include single-number and spectral and directivity information, are listed in Table 12.1 for enclosures and in Table 12.2 for cabins. For purposes of easy comparison, these tables retain the somewhat cumbersome nomenclature used in the standard.

The technical content of ISO 15667:2000E is based, to a large extent, on the content of Chapter 13 of the 1992 edition of this book² and, consequently, the reader will find much of the information in this chapter. The standard contains excellent suggestions about the collection of input information, planning, and performance verification of acoustical enclosures and cabins.

Annex A provides a large number of sketches depicting (1) details for joining wall panels, (2) mounting the enclosure airtight to the floor, (3) seals around doors and observation windows, (4) vibration isolation of the enclosed machines, (5) pipe and shaft penetrations, (6) ventilation possibilities, and so on. Though the information contained in these sketches is useful, the author cautions the purchaser of acoustical enclosures to contract with an experienced provider of noise control hardware who has similar proven details and the necessary experience to

TABLE 12.1 Acoustical Performance Measures of Enclosures

Sound Power Insulation ^a D_W	Weighted Sound Power Insulation ^b D_{WW}	Sound Pressure Insulation ^c D_P	A-Weighted Sound Pressure Insulation ^d D_{PA}
 	$D_{WW}(f)$ is obtained from $D_W(f)$ according to ISO 717-1, resulting in a single-number value	 	 

L_{w0} = Sound power level of unenclosed source
 L_{we} = Sound power level of enclosed source according to ISO 11546-1 or ISO 110546-2
 $D_W(f) = R(f) + 10 \log[\alpha(f)]$
 $R(f)$ = sound transmission loss of enclosure wall according to ISO 140-3
 $\alpha(f)$ = average absorption coefficient of internal side of enclosure panels

$D_P(f, x, \theta) = L_{po}(f, x, \theta) - L_{pe}(f, x, \theta)$
 $L_{po}(f, x, \theta)$ = sound-pressure-level spectrum of unenclosed source at distance x , direction θ
 $L_{pe}(f, x, \theta)$ = sound-pressure-level spectrum of enclosed source at distance x , direction θ

$D_{PA}(X, \theta) = L_{AO}(X, \theta) - L_{AE}(X, \theta)$
 L_{AO} = dBA for unenclosed source
 L_{AE} = dBA for enclosed source

^a Contains no information about directivity. Limited usefulness if (1) source is highly directional and (2) source is much nearer to one wall than to others.

^b Used for rough comparison of different enclosures in cases where source spectrum is not known.

^c Best suited for detailed analyses of enclosure performance for different directions.

^d Single-number rating for particular machine of known sound power spectrum.

Note: According ISO 15667:2000.

TABLE 12.2 Acoustical Performance Measures of Cabins

Room	Cabin	Sound Pressure Insulation ^a D_P	A-Weighted Sound Pressure Insulation ^b D_{PA}	Apparent Sound Pressure Insulation ^c D'_P
		$D_P \equiv \langle L_{p_o}(f) \rangle - \langle L_{p_e}(f) \rangle$ $\langle L_{p_o}(f) \rangle$ = space-average sound pressure level in a room with semi-diffuse sound field $\langle L_{p_e}(f) \rangle$ = space-average sound pressure level inside the cabin	$D_{PA} \equiv L_{PA}(x) - L_{PAC}(x)$ $L_{PA}(x)$ = A-weighted sound pressure level in room at position x , without cabin $L_{PAC}(x)$ = A-weighted sound pressure level in cabin at the same position x	$D'_P \equiv L_{p_o}(f, x) - L_{p_e}(f, x)$ $L_{p_o}(f, x)$ = sound pressure level at frequency f in position x , arbitrary sound field distribution $L_{p_e}(f, x)$ = Sound pressure level at frequency f in position x inside cabin

^a $\langle L_{p_o}(f) \rangle$ assumed to be semidiffuse field^b $L_{PA}(x)$ is the actual A-weighted level in the Room.^c L_{p_o} is produced by an arbitrary sound pressure distribution

Note: According ISO 15667:2000

integrate them into the total design and successfully install them onsite. Giving the construction details included in Annex A to a local sheet metal shop without prior experience in producing noise control hardware may result in a low bid but it is very likely that the end result will be disappointing.

In Annex B the standard provides informative case studies where the non-expert reader can find typical ranges of acoustical performance of functional enclosures and cabins which have operable doors, inlet and discharge openings for ventilation, and so on.

The key difference between enclosures and wrapping is that, in the case of enclosures, the sound-absorbing layer is not in contact with the surface of the vibrating equipment, while in the case of wrappings, the porous sound-absorbing layer is in full surface contact with the vibrating body it surrounds. Because the porous sound-absorbing material of wrappings provides a full-surface, structure-borne connection between the vibrating equipment and the exterior layer, it must not only be a good sound absorber but also be highly resilient so as to prevent the transmission of vibration to the outer impervious layer where it can be radiated as sound. Sound-absorbing materials used in enclosures, where there is no contact between the porous material and the vibrating equipment, can have a fairly rigid skeleton. Wrappings are most frequently used to decrease the sound radiation of vibrating surfaces such as ducts and pipes and sometimes also to gain extra sound attenuation of acoustical enclosures. Since fibrous sound-absorbing materials, such as glass fiber and mineral wool, are good heat insulators, properly designed wrappings can provide both substantial acoustical and heat insulation. On the other hand, the heat-insulating properties of the enclosure may be detrimental and require that provision be made for auxiliary cooling of the interior of the enclosure to prevent the buildup of excessively high temperatures. Only the acoustical design aspects of enclosures and wrappings are treated in this chapter.

12.1 ACOUSTICAL ENCLOSURES

Depending on their size (compared with the acoustical and bending wavelength) acoustical enclosures can be termed either small or large. The enclosure is considered small* if both the bending wavelength is large compared with the largest wall panel dimensions and the acoustical wavelength is large compared with the largest interior dimension of the enclosure volume. In small acoustical enclosures, the interior volume has no acoustical resonances. If the largest dimension of the acoustical volume is $L_{\max} \leq \frac{1}{10}\lambda$, the sound pressure is evenly distributed within the volume. The enclosure is considered large if all of its interior dimensions are large compared with the acoustical wavelength and there are a large number of acoustical resonances in the interior volume in the frequency range of interest. Accordingly, even enclosures with large physical dimensions are acoustically small at very low frequencies while enclosures of small physical size are acoustically large at very high frequencies. In almost all acoustical enclosures the

*Small enclosures are also treated in Chapter 6.

enclosure walls already exhibit numerous structural resonances in the frequency range where the first acoustical resonance occurs.

If the enclosure has no mechanical connections to the enclosed equipment, it is termed *free standing*. If there are mechanical connections, then the enclosure is *equipment mounted*. Enclosures that very closely surround the enclosed equipment and the volume of the machine is comparable to the volume of the enclosure are called *close fitting*. Enclosures without acoustically significant openings are referred to as *sealed enclosures*, and those with significant acoustical leaks (intentional or unintentional) as *leaky acoustical enclosures*. Figure 12.1 shows various configurations of sealed acoustical enclosures. This chapter deals with the acoustical design of enclosures. Nonacoustical aspects—such as ventilation, safety, and economy—are treated in a handbook by Miller and Montone (see Bibliography). Construction details and advice for writing purchase specifications are given in a VDI guideline (also see Bibliography).

Insertion Loss as Acoustical Performance Measure

The insertion loss is the most appropriate descriptor for the acoustical performance of enclosures of all types. The operational definition of the insertion loss (IL) of an acoustical enclosure is illustrated in Fig. 12.2. For noise sources that will be positioned indoors, such as machinery in factory spaces, the sound-power-based insertion loss of the enclosure as indicated in Fig. 12.2a is the most meaningful. It is defined as

$$IL_w = 10 \log \left(\frac{W_0}{W_E} \right) = L_{w0} - L_{WE} \quad \text{dB} \quad (12.1)$$

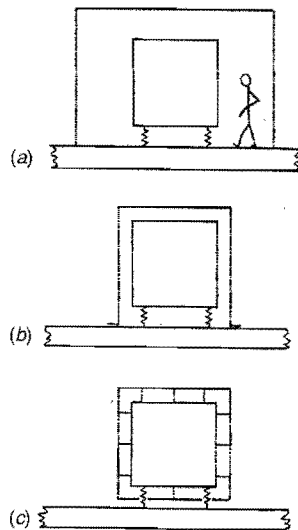


FIGURE 12.1 Enclosure types: (a) free standing, large; (b) free standing, close fitting; (c) equipment mounted, close fitting.

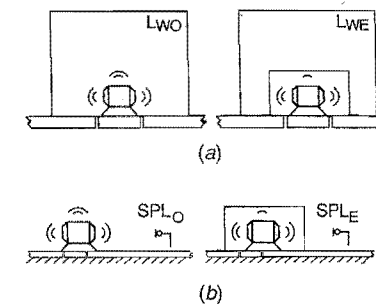


FIGURE 12.2 Operational definition of enclosure insertion loss IL: (a) power based; $IL_w \equiv L_{w0} - L_{WE}$, (b) sound pressure based; $IL_p \equiv SPL_O - SPL_E$.

where W_0 is the sound power radiated by the unenclosed source and L_{w0} the corresponding sound power level and W_E is the sound power radiated by the enclosed source and L_{WE} the corresponding sound power level. Both W_0 and W_E are measured in a reverberation room (see Chapter 4) or with the aid of a sound intensity meter.

For enclosures used with equipment deployed outdoors, a less precise but more easily implemented definition of insertion loss, the so-called pressure-based insertion loss, or IL_p , illustrated in Fig. 12.2b is most appropriate. It is defined as

$$IL_p = SPL_O - SPL_E \quad \text{dB} \quad (12.2)$$

where SPL_O is the average sound pressure level measured at a number of locations around the source without the enclosure and SPL_E is that measured with the source surrounded by the enclosure. The measurement positions may be chosen on a circle that is centered at the source location. The measurement distance should be at least three times the longest dimension of the enclosure. Equations (12.1) and (12.2) represent definitions readily implemented in the field or laboratory. Using these definitions one can readily determine whether a specific enclosure will or will not meet specific performance requirements stated in the form of a sound-power-level reduction, a sound-pressure-level reduction for a specified distance, or a sound-pressure-level reduction at a specified distance and direction. If the laboratory facilities are available, the sound-pressure-based insertion loss may also be measured in a large hemianechoic chamber.

Note that if the radiation patterns of both the unenclosed and enclosed source is omnidirectional, the two measures yield the same result ($IL_w = IL_p$).

Qualitative Description of Acoustical Performance

Figure 12.3 shows the typical shape of a curve of the acoustical insertion loss versus frequency of a free-standing, sealed acoustical enclosure. Region I is the small-enclosure region where neither the interior air volume nor the enclosure

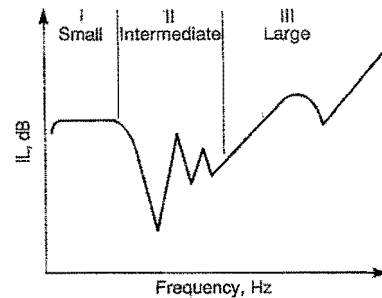


FIGURE 12.3 Typical curve of insertion loss versus frequency of a sealed, free-standing acoustical enclosure. Region I: panel stiffness controlled; damping and interior absorption are ineffective. Region II: resonance controlled; damping and interior absorption are effective. Region III: controlled by sound transmission loss; sound transmission loss and interior absorption are effective; usually limited by leaks.

wall panels exhibit any resonances. In this frequency region the insertion loss is frequency independent and is controlled by the ratio of the volume compliance (inverse of volume stiffness) of the enclosure walls and that of the enclosed air volume.

Region II in Fig. 12.3 is the intermediate region where the insertion loss is controlled by the resonant interaction of the enclosure structure with the enclosed acoustical volume. The region is characterized by a number of alternating maxima and minima in the insertion loss. Typically, the first and most important minima in the insertion loss occurs when the combined volume compliance of the interior air volume and the volume compliance of the wall panels matches the mass compliance of the wall panels. The insertion loss at this resonance frequency usually controls the low-frequency insertion loss of the enclosure and in some instances can become negative, signifying that the equipment with the enclosure may radiate more noise than without the enclosure. Additional minima of the insertion loss occur at acoustical resonances of the enclosure volume. Further minima in insertion loss occur when the frequencies of structural resonances of a wall panel and the frequencies of acoustical resonances of the enclosure volume coincide. It is imperative that enclosures designed for sound sources that radiate noise of predominantly tonal character (such as transformers, gears, reciprocating compressors and engines, etc.) should have no structural or acoustical resonances that correspond to the frequencies of the predominant components of the source noise.

Region III in Fig. 12.3 is the large-enclosure region where both the enclosure wall panels and the interior air volume exhibit a very large number of acoustical resonances. Here, statistical methods of room acoustics can be used to predict the sound field inside the enclosure (see Chapter 7) and the sound transmission through the enclosure walls (see Chapter 11). In this frequency region, the insertion loss is controlled by the interior sound absorption and by the sound

transmission loss (R) of the enclosure wall panels. The dip in the curve of insertion loss versus frequency in region III corresponds to the coincidence frequency of the wall panel (see Chapter 11), which for the most frequently used wall panels (1–2 mm steel or aluminum) falls above the frequency region of interest. The coincidence frequency may fall into the region of interest if the ratio of stiffness of the panel to its mass per unit area is high (e.g., a honeycomb panel).

The prediction of the insertion loss of small and large acoustical enclosures is treated in the following sections. Performance prediction in the intermediate frequency region requires a detailed finite-element analysis of the coupled mechanical acoustical system such as outlined in Chapter 6.

Small Sealed Acoustical Enclosures

For a small, sealed acoustical enclosure, where the sound pressure inside the cavity is evenly distributed, the insertion loss is given by^{4–5}

$$IL_{SM} = 20 \log \left(1 + \frac{C_v}{\sum_{i=1}^n C_{wi}} \right) \text{ dB} \quad (12.3)$$

where

$$C_v = \frac{V_0}{\rho c^2} \text{ m}^5/\text{N} \quad (12.4)$$

is the compliance of the gas volume inside the enclosure, V_0 is the volume of the gas in the enclosure volume, ρ is the density of the gas, c is the speed of sound of the gas, and C_{wi} is the volume compliance of the i th enclosure wall plate defined as

$$C_{wi} = \frac{\Delta V_{pi}}{p} \text{ m}^5/\text{N} \quad (12.5)$$

where ΔV_{pi} is the volume displacement of the i th enclosure wall plate in response to the uniform pressure p . It is assumed here that the enclosure is rectangular and is made of n separate, homogeneous, isotropic plates, each with its own volume compliance.

At frequencies below the first mechanical resonance of the isotropic enclosure wall panel, the volume compliance of a homogeneous, isotropic panel C_{pi} is given by³

$$C_{wi} = \frac{10^{-3} A_{wi}^3 F(\alpha)}{B_i} \text{ m}^5/\text{N} \quad (12.6)$$

where A_{wi} is the surface area of the i th wall panel and $F(\alpha)$ is given in Fig. 12.4 as a function of the aspect ratio $\alpha = a/b$ of the panel, where a is the longest and b is the smallest edge dimension of the wall panel. For homogeneous, isotropic wall panels, the bending stiffness per unit length of the panel is

$$B = \frac{Eh^3}{12(1-\nu^2)} \text{ N} \cdot \text{m} \quad (12.7)$$

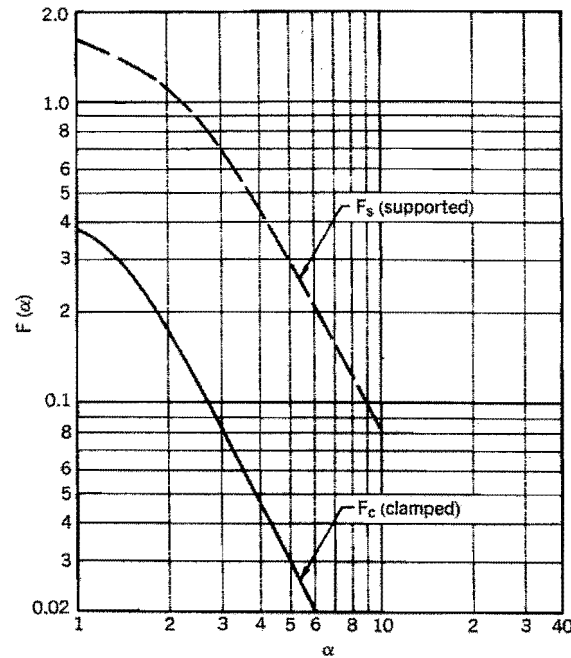


FIGURE 12.4 Plate volume compliance function $F(\alpha)$ plotted versus the aspect ratio $\alpha = a/b$ for homogeneous isotropic panels either with clamped or simply supported edges. (After Ref. 3).

where E is Young's modulus and h is the thickness and ν the Poisson ratio of the wall panel. For a rectangular enclosure combining Eqs. (12.3)–(12.7) yields

$$\text{IL}_{\text{SM}} = 20 \log \left[1 + \frac{V_0 E h^3}{12 \times 10^{-3} (1 - \nu^2) \rho c^2} \sum_{i=1}^6 \frac{1}{A_{wi}^3 F(\alpha_i)} \right] \text{ dB} \quad (12.8)$$

For the special case of a cubical enclosure with clamped walls of edge length a Eq. (12.8) yields

$$\text{IL}_s = 20 \log \left[1 + 41 \left(\frac{h}{a} \right)^3 \frac{E}{\rho c^2} \right] \text{ dB} \quad (12.9)$$

Equation (12.8) indicates that high insertion loss is achieved if the enclosure has small edge length, high aspect ratio, and large wall thickness, edge conditions are clamped, and the panels are made of a material of high Young's modulus. In short, for an enclosure with high insertion loss at low frequencies, the wall must be made as stiff as possible. According to Eq. (12.9), the low-frequency insertion

loss of a cubical steel enclosure of edge length $a = 300 \text{ mm}$, $E = 2 \times 10^{11} \text{ N/m}^2$, $\rho = 1.2 \text{ kg/m}^3$, and $c = 340 \text{ m/s}$ is for two different wall thicknesses h :

Wall Thickness h	Clamped		Simply Supported	
	IL_s	f_0	IL_s	f_0
3 mm	35.5 dB	296 Hz	24 dB	162 Hz
1.5 mm	18.5 dB	148 Hz	9 dB	81 Hz

Here f_0 is the resonance frequency of each of the identical homogeneous isotropic clamped panels. The insertion loss values listed in the table above are valid only at frequencies well below f_0 . Note in the table that much higher insertion loss is achieved with a clamped-edge condition than with simply supported edge conditions. In practice, clamped edges are almost impossible to achieve. Consequently, one should use the simply supported edge condition in the initial design to ensure that the designed performance will be achieved.

It is of considerable practical interest to know which material should be used for a small sealed cubical enclosure to yield the highest insertion loss at low frequencies for the same enclosure volume and same total weight. Considering that the total mass M of a cubical enclosure of edge length a is $M = 6\rho_M a^2 h$, Eq. (12.9) can be expressed as

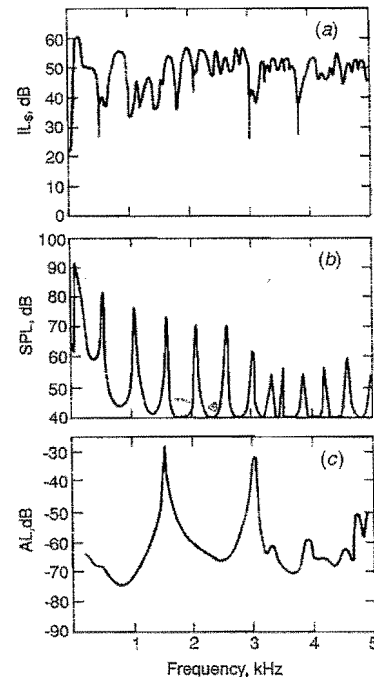
$$\text{IL}_s = 20 \log \left[1 + 0.19 \frac{M^3}{a^9 \rho c^2} \left(\frac{c_L}{\rho_M} \right)^2 \right] \text{ dB} \quad (12.10)$$

where $c_L = \sqrt{E/\rho_M}$ is the speed of longitudinal waves in the bulk enclosure material and ρ_M is the density of the enclosure material. Equation (12.10) indicates that for all materials giving the same enclosure mass M , the material with the highest c_L/ρ_M ratio yields the highest insertion loss. Table 12.3 lists the c_L/ρ_M ratio and the normalized low-frequency insertion loss ΔIL for frequently used materials, where ΔIL is defined as the low-frequency insertion loss of an enclosure made out of a specific material minus the insertion loss of an enclosure of the same volume and weight built of steel. Table 12.3 indicates that, in regard to low-frequency insertion loss, aluminum and glass are superior to steel and lead is the worst possible choice! Note, however, that this conclusion can be exactly opposite for a large enclosure if the coincidence frequency falls within the frequency range of interest.

Formstiff Small Enclosures. Because the insertion loss of small, sealed enclosures at very low frequencies ($L_{\text{max}} < \frac{1}{10} \lambda$) is controlled by the volume compliance of the enclosure walls, it is desirable to select constructions that provide the highest wall stiffness for the allowable enclosure weight. An enclosure consisting of a round, cylindrical body and two half-spherical end caps yields a very

TABLE 12.3 Difference in Low-Frequency Insertion Loss ΔIL and c_L/ρ_M Ratio for Different Construction Materials^a

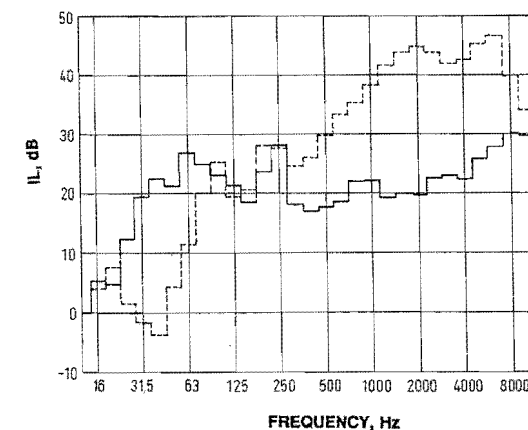
	Lead	Steel	Concrete	Plexiglass	Aluminum	Glass
c_L/ρ_M , m ⁴ /kg·s	0.11	0.65	1.5	1.6	1.9	2.1
ΔIL , dB	-31	0	+14	+15	+19	+20

^a ΔIL is for a cubical enclosure with identical sides.**FIGURE 12.5** Acoustical characteristics of a small, sealed, round cylindrical enclosure without interior absorption: (a) insertion loss IL; (b) resonant acoustical response of interior volume, SPL; (c) sound-induced vibration acceleration response of enclosure wall, AL.

stiff construction and much higher insertion loss than a rectangular enclosure of the same volume and weight. Figure 12.5 shows the acoustical characteristics of such a particular small, cylindrical enclosure with no internal sound-absorbing treatment. Curve A represents insertion loss versus frequency measured with an external source, indicating that insertion loss at most frequencies exceeds 55 dB. Curve B represents the sound pressure levels in the enclosed air volume when excited with an external source, indicating the presence of strong acoustical resonances at 550 Hz and above. Curve C represents the sound-induced vibration

acceleration of the cylindrical shell caused by an external sound source. Strong structural resonances are seen at 1.5, 3, and 3.75 kHz. Observing the three curves in Fig. 12.5, it is noted that the frequencies where the insertion loss is minimum coincide either with acoustical resonances of the interior volume or with structural resonances of the enclosure shell. The strong minima of the curve of insertion loss versus frequency in the vicinity of 1.5, 3, and 3.75 kHz are caused by coincidence of the structural resonance of the shell with acoustical resonances of the interior air volume.

Another way to achieve a low wall compliance for a given total weight is to make the enclosure walls out of a composite material consisting of a honeycomb core sandwiched between two lightweight plates. A special form of such high-stiffness, low-weight panel, described by Fuchs, Ackermann, and Frommhold,⁶ also provides high sound absorption at low frequencies in addition to the low volume compliance. In this case, the panel on its interior side has two double membranes. The first, thicker membrane, which is rigidly bonded to the honeycomb core, has round openings to make an inward-facing Helmholtz resonator out of each honeycomb cavity. A second, thin, membrane that is covering the first one is free to move over the resonator openings. The presence of this second, thin membrane lowers the resonance frequency of the Helmholtz resonators owing to the mass of the membrane covering the opening and increases the dissipation owing to air pumping. Such Helmholtz plates can be constructed from stainless steel, aluminum, or light-transparent plastic. They are fully sealed and can be hosed down for cleaning. Figure 12.6 shows the curve of insertion loss versus frequency of a cubical enclosure of 1 m edge length. The solid curve was obtained with enclosure walls made of the above-described Helmholtz plates of

**FIGURE 12.6** Measured insertion loss of two 1 × 1 × 1-m enclosures. (After Ref. 6.) Solid curve: Helmholtz plate walls, 10 cm thick, 8.5 kg/m², no sound-absorbing treatment. Dashed curve: Woodchip board walls, 1.3 cm thick, 5-cm-thick sound-absorbing layer, 10 kg/m².

10 cm thickness with a mass per unit area of 8.5 kg/m^2 without any additional sound-absorbing lining. The dotted curve was obtained with an enclosure of the same size, made of 1.3-cm-thick wood chip board with 5-cm sound-absorbing lining, yielding a mass per unit area of 10 kg/m^2 . The Helmholtz plate enclosure yields superior low-frequency performance and does not require extra sound-absorbing lining. However, the insertion loss of this enclosure at midfrequencies (above 250 Hz) is low because of the low coincidence frequency of the light and stiff panels.

Small Leaky Enclosures. Except for special constructions, such as hermetically sealed compressors, all enclosures are likely to be leaky and provide zero insertion loss as the frequency approaches zero. If an enclosure has a leak in the form of a round opening of radius a_L in a wall of thickness h , the leak represents a compliance $C_L = 1/j\omega Z_L$, where Z_L is the acoustical impedance of the leak given as

$$C_L = \frac{(\pi a_L^2)^2}{-\omega^2 \rho (h + \Delta h) \pi a_L^2 + j\omega R} \quad \text{m}^5/\text{N} \quad (12.11)$$

where h is the plate thickness, $\Delta h \approx 1.2a_L$ represents the end correction, and R is the real part of the impedance of the leak. The sound pressure inside the small, leaky enclosure owing to the operation of a source of volume velocity q_0 is

$$p_{\text{ins}}^{\text{leaky}} = \frac{q_0}{j\omega C_L + C_v + \sum_i C_{wi}} \quad \text{N/m}^2 \quad (12.12)$$

and the insertion loss of the small leaky enclosure is

$$\text{IL}_{\text{leaky}} = \text{IL}_L = 20 \log \frac{C_L + C_v + \sum_i C_{wi}}{C_L + \sum_i C_{wi}} \quad \text{dB} \quad (12.13)$$

where C_v and C_{wi} are given in Eqs. (12.4) and (12.5), respectively. Since C_L approaches infinity as the frequency approaches zero, the insertion loss of leaky enclosures approaches zero for any form of leak. The insertion loss becomes negative in the vicinity of the Helmholtz resonance frequency of the compliant leaky enclosure given by

$$f_{0L} = \frac{a_L c}{2} \sqrt{\frac{1}{\pi(h + \Delta h)(V_0 + \rho c \sum_i C_{wi})}} \quad \text{Hz} \quad (12.14)$$

As $C_L/\sum C_{wi}$ becomes small with increasing frequency, the insertion loss of the small, leaky enclosure, IL_L , approaches that of the sealed enclosure, IL_s . This behavior is illustrated in Fig. 12.7.

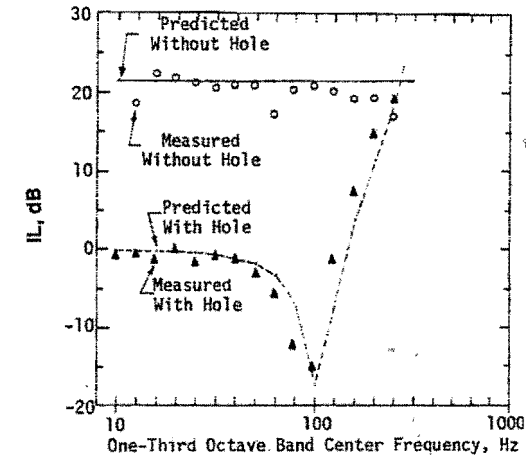


FIGURE 12.7 Measured and predicted sound-pressure-based insertion loss (outside-inside direction) of a $50 \times 150 \times 300$ -mm aluminum enclosure of 1.5-mm-thick wall with a single 9.4-mm-diameter hole (after Ref. 5): (—) predicted, sealed; (---) predicted, leaky; (○) measured, sealed; (▲) measured, leaky.

Close-Fitting, Sealed Acoustical Enclosures

A close-fitting acoustical enclosure is one in which a considerable portion of the enclosed volume is occupied by the equipment being quieted. Such enclosures are used in cases where the radiated noise must be reduced using a minimum of added volume. Vehicle engine enclosures, portable compressors, and transportable transformers fall into this category.

Free-standing, close-fitting enclosures, such as shown in Fig. 12.1b, have no mechanical connections to the vibrating source and the enclosure walls are excited to vibration only by the airborne path. Conversely, the walls of machine-mounted, close-fitting enclosures, such as depicted in Fig. 12.1c, are excited by both airborne and structure-borne paths.

The characteristic property of a close-fitting enclosure is that the air gap (the perpendicular distance between the vibrating surface of the machine and the enclosure wall) is small compared with the acoustical wavelength in most of the frequency range. The walls of the close-fitting enclosures are usually made of thin, flat sheet metal with a layer of sound-absorbing material such as glass fiber or acoustical foam on the interior face. The purpose of the sound-absorbing lining is to damp out the acoustical resonances in the air space.

Free-Standing, Close-Fitting Acoustical Enclosures. Free-standing, close-fitting enclosures, which have no structure-borne connection to the vibrating equipment, always achieve higher insertion loss than a similar machine-mounted enclosure can provide. If the enclosure is perfectly sealed, then its insertion loss

depends on (1) thickness, size, and material of the enclosure wall; (2) edge-joining conditions of the wall plates and their loss factors; (3) vibration pattern of the machine surface; (4) average thickness of the air gap between the wall and the machine; and (5) type of sound-absorbing material in the air gap.

Figure 12.8 shows the measured dependence of the power-based insertion loss of a model close-fitting enclosure consisting of 0.6×0.4 -m enclosure walls of thickness h and a variable spacing gap of thickness T . To obtain these data, an array of loudspeakers, which represented the vibrating surface of the enclosed machine, was phased to simulate various vibration patterns.⁸ Figure 12.8a shows

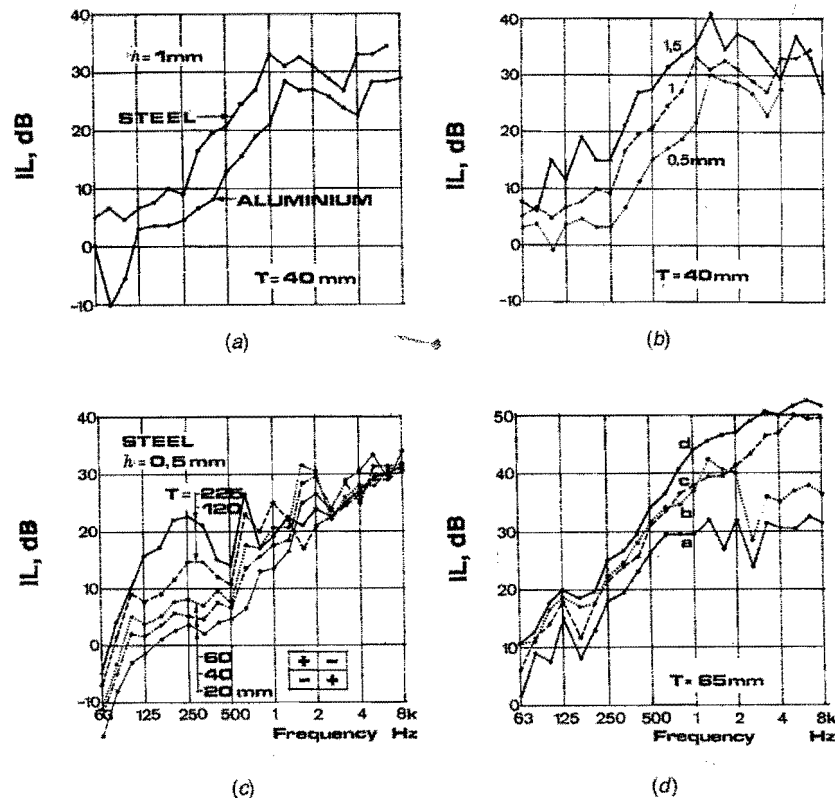


FIGURE 12.8 Effect of key parameters on the power-based insertion loss of a model close-fitting enclosure. Wall panel 0.6×0.4 . (After Ref. 8.) (a) Effect of the wall material; 4-cm air gap, in-phase excitation. (b) Effect of wall panel thickness, 4-cm air gap, in-phase excitation. (c) Effect of average air gap thickness; 0.5-mm steel plate, quadrupole excitation. (d) Effect of sound-absorbing material and structural damping; 1-mm steel plate, 6.5-cm air gap, in-phase excitation; a, no damping, no absorption in air space; b, with damping treatment; c, no damping, but absorption in air space; d, with damping and absorption.

the dependence of the measured insertion loss on frequency obtained for 1-mm-thick aluminum and steel wall plates, respectively. Because the steel and aluminum plates have practically the same dynamic behavior, the higher insertion loss of the steel wall panel is attributable to its higher mass. In the frequency range between 300 Hz and 1 kHz, the 9-dB average difference corresponds to the difference in density of the two materials. Figure 12.8b shows the effect of wall thickness. Here again the insertion loss increases with increasing wall thickness. Below 250 Hz, where the insertion loss is controlled by the stiffness of the air and the stiffness and damping of the wall panels, the IL_w changes little with increasing frequency. In the frequency range between 200 Hz and 1 kHz, where the IL_w is controlled by the volume stiffness of the air and by the mass per unit area of the wall, the IL_w increases with a slope of 40 dB/decade. Above 1 kHz, the insertion loss is limited by the acoustical resonances in the air space. Figure 12.8c shows that the insertion loss increases with increasing air gap thickness. Figure 12.8d shows the effect of sound-absorbing material in the air space and damping treatment of the enclosure wall on the achieved insertion loss. The sound-absorbing treatment in the air space prevents the acoustical resonances in the air space and results in a substantial increase of the insertion loss above 1 kHz. Structural damping helps to reduce the deleterious effect of the efficiently radiating plate resonance at 160 Hz. A combination of sound-absorbing treatment and structural damping results in a smooth, steeply increasing insertion loss with increasing frequency and provides a very high degree of acoustical performance.

One-Dimensional Model. According to Bryne, Fischer, and Fuchs,⁹ the insertion loss of free-standing, sealed, close-fitting acoustical enclosures can be predicted with reasonable accuracy with the simple one-dimensional model shown in Fig. 12.9. It is assumed that the machine wall (A) vibrates in phase (like a rigid piston) with velocity v_M and that the motion of the machine is not affected by the presence of the enclosure. The vibrating machine is surrounded by a

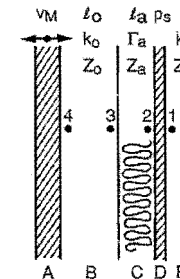


FIGURE 12.9 One-dimensional model for predicting the power-based insertion loss IL of free-standing, sealed close-fitting acoustical enclosures; A machine wall vibrating with velocity v_M ; B, air gap thickness l_0 ; C, sound-absorbing layer thickness l_a ; D, impervious enclosure wall of mass per unit area ρ_s ; E, free space into which sound is radiated. (After Ref. 9).

flat, limp, impervious enclosure wall (D) which has an internal sound-absorbing lining (C) of thickness l_a and there is an air gap (B) of thickness l_0 between the machine and the sound-absorbing layer. The sound pressure on the outside surface of the enclosure (point 1 in Fig. 12.9) is given by

$$p_1 = p_2 \left(1 - \frac{j\omega\rho_s}{Z_0} \right) \quad \text{N/m}^2 \quad (12.15)$$

where $Z_0 = \rho c$ is the characteristic impedance of air. The following equations are used to obtain p_1 as a function of the machine wall vibration velocity v_M :

$$p_3 = 0.5p_2 \left[\left(1 + \frac{Z_a}{Z_3} \right) e^{-j\Gamma_a l_a} + \left(1 - \frac{Z_a}{Z_3} \right) e^{j\Gamma_a l_a} \right] \quad \text{N/m}^2 \quad (12.16)$$

$$p_4 = v_M Z_4 = 0.5p_3 \left[\left(1 + \frac{Z_0}{Z_4} \right) e^{-jk_0 l_0} + \left(1 - \frac{Z_0}{Z_4} \right) e^{jk_0 l_0} \right] \quad \text{N/m}^2 \quad (12.17)$$

$$Z_2 = Z_0 + j\omega\rho_s \quad \text{N} \cdot \text{s/m}^3 \quad (12.18)$$

$$Z_3 = Z_a \frac{(1 + Z_a/Z_2)e^{j\Gamma_a l_a} + (1 - Z_a/Z_2)e^{-j\Gamma_a l_a}}{(1 + Z_a/Z_2)e^{j\Gamma_a l_a} - (1 - Z_a/Z_2)e^{-j\Gamma_a l_a}} \quad \text{N} \cdot \text{s/m}^3 \quad (12.19)$$

$$Z_4 = Z_0 \frac{(1 + Z_0/Z_3)e^{jk_0 l_0} + (1 - Z_0/Z_3)e^{-jk_0 l_0}}{(1 + Z_0/Z_3)e^{jk_0 l_0} - (1 - Z_0/Z_3)e^{-jk_0 l_0}} \quad \text{N} \cdot \text{s/m}^3 \quad (12.20)$$

where $k_0 = 2\pi f/c$ is the wavenumber, ρ the density, and c the speed of sound in air; Z_a is the complex characteristic impedance and Γ_a the complex propagation constant of the porous sound-absorbing material as given in Chapter 8.

The sound power radiated by the unenclosed machine wall is given by

$$W_0 = v_M^2 \rho c A \quad \text{W} \quad (12.21a)$$

and when it is enclosed with a free-standing, close-fitting enclosure

$$W_A = \frac{p_1^2}{\rho c} A \quad \text{W} \quad (12.21b)$$

yielding for the insertion loss

$$\text{IL}_F = 10 \log \frac{W_0}{W_A} = 20 \log \frac{v_M \rho c}{p_1^2} \quad \text{dB} \quad (12.22)$$

In deriving this equation, no account is taken of possible change in surface area or radiation efficiency between the enclosure surface and machine surface.

If the angle of sound incidence on the enclosure wall is known (i.e., a specific oblique angle or random), the power-based insertion loss of free-standing, close-fitting enclosures can be predicted on the basis of the two-dimensional model of sound transmission through layered media presented in Chapter 11.

Machine-Mounted, Close-Fitting Acoustical Enclosure. If the close-fitting enclosure is machine mounted, the rigid or resilient connections between the machine and the enclosure wall give rise to additional sound radiation of the enclosure wall. Assuming that the vibration velocity of the machine v_M is not affected by the connected enclosure, the additional sound power W_M radiated owing to the structural connections is

$$W_M = \sum_{i=1}^n F_i^2 \left(\frac{\rho}{\rho_s^2 c} + \frac{\rho c \sigma}{2.3 \rho_s^2 c_L h \omega \eta} \right) \quad \text{W} \quad (12.23)$$

where

$$F_i^2 = \frac{v_{Mi}^2}{|1/2.3 \rho_s^2 c_L h + j\omega/s|^2} \quad \text{N}^2 \quad (12.24)$$

In Eq. (12.24), F_i is the force transmitted by the i th attachment point, v_{Mi} is the vibration velocity of the machine at the i th attachment point, n is the number of point attachments between the machine and the homogeneous, isotropic enclosure wall, ρ_s is the mass per unit area of the enclosure wall, h is the wall thickness, c_L is the speed of longitudinal waves in the wall material, η is the loss factor, σ is the radiation efficiency, and s is the dynamic stiffness of the resilient mount connecting the enclosure wall to the machine ($s = \infty$ for rigid point connections). To minimize structure-borne transmission, it is advantageous to select attachment points at those locations on the machine that exhibit the lowest vibration. The insertion loss of the machine-mounted, close-fitting enclosure is

$$\text{IL}_{MM} = 10 \log_{10} \frac{W_0}{W_A + W_M} \quad \text{dB} \quad (12.25)$$

where W_0 and W_A are given in Eqs. (12.21a) and (12.21b).

Figure 12.10 shows a close-fitting enclosure investigated experimentally and analytically by Byrne, Fischer, and Fuchs⁹ in three configurations: (1) free standing, (2) rigidly machine mounted, and (3) resiliently machine mounted. Figure 12.11 shows the measured insertion loss obtained for each of the three configurations. The machine-mounted enclosure, which was supported from the machine at four points at each side, yield substantially lower insertion loss at high frequencies than the free-standing enclosure. Elastic mounting yields higher insertion loss than rigid mounting.

As shown by reference 9, the simple one-dimensional analytical model based on Eqs. (12.15)–(12.25) yields predictions that are in good agreement with measured data. The reason for this surprisingly good agreement is that the sound-absorbing treatment effectively prevents sound propagation (and the occurrence of acoustical resonances) in the plane parallel to the enclosure wall and also provides structural damping to the thin enclosure wall. Replacing the effective glass fiber sound-absorbing treatment with a thinner, less effective acoustical foam has resulted in substantial decrease in the insertion loss, indicating the crucial importance of an effective sound-absorbing treatment. Figure 12.12 shows

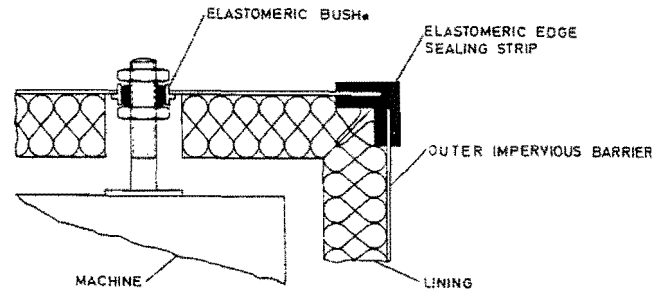


FIGURE 12.10 Machine-mounted, close-fitting enclosure; resilient mounting, resilient panel edge connections. (After Ref. 9).

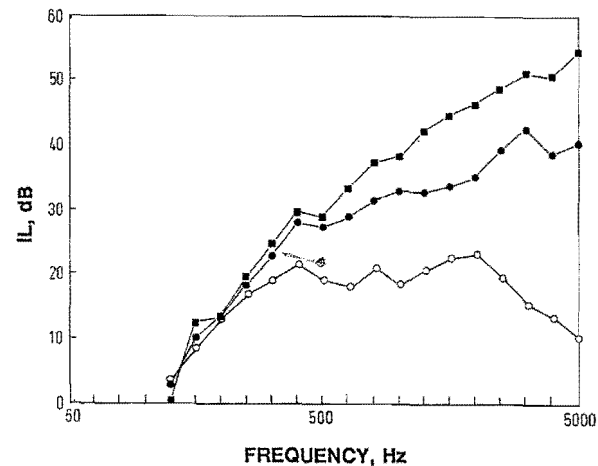


FIGURE 12.11 Insertion loss (IL_s) of a sealed, close-fitting enclosure made of 1-mm steel plates. (After Refs. 7 and 10.) Dimensions: $1 \times 0.6 \times 0.8$ m; machine, enclosure wall distance 50 mm; interior lining 50 mm thick; flow resistivity $R_1 = 2 \times 10^4 \text{ N} \cdot \text{s/m}^4$. (■) Free standing; (●) machine mounted, resilient mounting (see Fig. 12.10); (○) machine mounted, rigid mounting.

how the specific choice of edge connections of the enclosure plates affected the insertion loss of the machine-mounted, close-fitting enclosure. Elastically sealed edges, such as shown in Fig. 12.10, yield higher insertion loss than rigidly connected (welded) edges. The better performance obtained with elastically sealed edges is due to the reduced coupling between the in-plane motion of one plate (which radiates no sound) with the normal motion of the connected plate (that radiates sound efficiently). The elastic edge seal also increases the loss factor of the enclosure plates, thereby reducing their resonant vibration response.

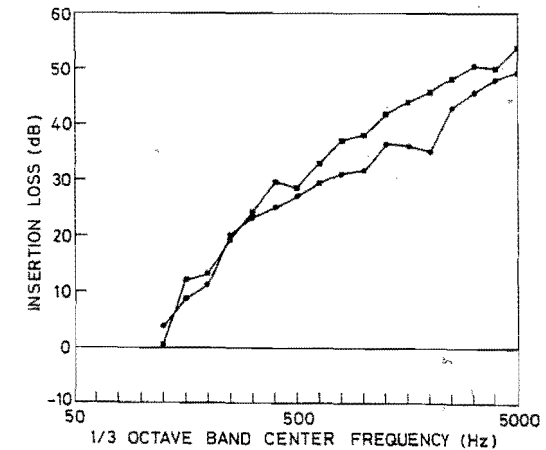


FIGURE 12.12 Effect of plate edge connection on the insertion loss of sealed, resiliently machine-mounted, close-fitting enclosure: (■) elastically sealed edge; (●) welded edge.

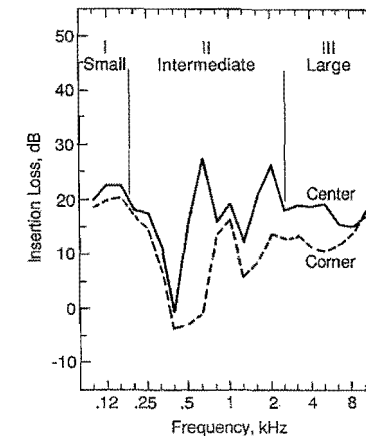


FIGURE 12.13 Effect of microphone position on the sound-pressure-based insertion loss measured in the outside-inside direction of a $50 \times 150 \times 300$ -mm (1.6-mm-thick) empty, plane, sealed, unlined aluminum acoustical enclosure. (After Ref. 11.) Solid line: center microphone location. Dashed line: corner microphone location.

Intermediate-Size Enclosures

The intermediate frequency region, designated as region II in Figs. 12.3 and 12.13, is defined as the frequency region where the enclosure walls, the enclosure air volume, or both exhibit resonances; but the resonances do not overlap so that statistical methods are not yet applicable.

Comparison of the solid and dashed curves in Fig. 12.13 show how the insertion loss, measured in the outside-inside direction (such as in a quiet control room in a noisy test facility), depends on the location of the microphone. Below 400 Hz, the sound pressure is uniformly distributed in the enclosure and the microphone in the corner measures the same sound pressure as that placed at the center. The first acoustical resonance occurs at 565 Hz, which results in a very low insertion loss at the enclosure corner in the 500- and 800-Hz center-frequency one-third-octave bands, as illustrated by the dashed curve, but in a high insertion loss in the same bands for the enclosure center location, as depicted by the solid curve. Fluctuations owing to structural resonances can be reduced by application of damping treatment to the panels (see Chapter 14) and those owing to acoustical resonances by internal sound-absorbing lining (see Chapter 8).

Because the insertion loss in this intermediate frequency range fluctuates widely with frequency and position, it is very difficult to make accurate analytical predictions of the insertion loss. Frequently, involved finite-element analysis (see Chapter 6), model-scale or full-scale experiments, or crude approximations must be employed. For crude approximations one connects the low- and high-frequency predictions to cover the intermediate range.

Enclosures without Internal Sound-Absorbing Treatment

There are cases where the danger of bacterial growth or the accumulation of combustible particles precludes the use of porous or fibrous sound-absorbing materials inside of acoustical enclosures. In such cases bare enclosures must be used. Simple analytical models yield zero insertion loss for such enclosures. Consequently, it is also of theoretical interest to treat this extreme case.

Acoustical Performance Prediction of Large, Bare Enclosures. Even if the enclosure is built from homogeneous, isotropic panels without any interior sound-absorbing treatment, the interior sound pressure will not be infinitely high and the insertion loss of the enclosure will not be zero.

The interior sound pressure does not go to infinity because the interior sound field loses power through (1) sound radiation of the walls by the forced bending waves, (2) sound radiation of the walls by the free bending waves, (3) energy dissipation in the panel by the free bending waves, and (4) the inevitable acoustical energy dissipation at the interior surfaces of the impervious enclosure panels owing to the acoustical shear layer and heat conduction losses.

The insertion loss of the enclosure will be finite because of the energy dissipation by processes 3 and 4 above.

The power balance of the interior sound field is given as

$$W_{\text{source}} = W_{\text{rad}}^{\text{forced}} + W_{\text{rad}}^{\text{free}} + W_d^{\text{free}} + W_d^{\text{shear}} \quad (12.26)$$

where W_{source} = sound power of enclosed source

$W_{\text{rad}}^{\text{forced}}$ = sound power radiated by forced bending waves

$W_{\text{rad}}^{\text{free}}$ = sound power radiated by free bending waves

W_d^{free} = sound power dissipated in plate by free bending waves

W_d^{shear} = sound power lost through shear and heat conduction at interior surface of impervious, solid walls

Unless the panel is very heavily damped, the power dissipated by the forced bending waves is small compared with that dissipated by the free bending waves and can be neglected.

Inserting into Eq. (12.26) the appropriate relationships from Section 11.8 and for α_{min} from reference 12 yields for the random-incidence sound absorption coefficient of the bare enclosure walls,

$$\bar{\alpha}_{\text{rand}} = \frac{W_{\text{loss}}^{\text{tot}}}{W_{\text{inc}}} = A + B + C + D \quad (12.27)$$

for the space-average mean-square sound pressure in the enclosure,

$$\bar{p}^2 = \frac{4\rho_0 c_0 W_{\text{source}}}{S_{\text{tot}}(A + B + C + D)} \quad \text{N}^2/\text{m}^4 \quad (12.28)$$

and for the insertion loss of the bare enclosure,

$$\text{IL}_{\text{bare}} = 10 \log \left(\frac{A + B + D}{A + BC} \right) \quad \text{dB} \quad (12.29)$$

where

$$A = \frac{1}{1 + (1/52)[\omega\rho_s/(\rho_0 c_0)]^2} \quad B = \frac{2\pi\sqrt{12}c_0^2\rho_0 c_0 \sigma_{\text{rad}}}{c_L \rho_M h^2 \omega^2}$$

$$C = \frac{\rho_0 c_0 \sigma_{\text{rad}}}{\rho_0 c_0 \sigma_{\text{rad}} + \rho_s \omega \eta} \quad D = \alpha_{\text{min}} = 0.72(10^{-4})\sqrt{\omega}$$

where S_{tot} = total interior surface area of enclosure, m^2 (assuming that all six partitions are made from the same panels)

ρ_s = mass per unit area of plate, $h\rho_M$, kg/m^2

ρ_M = density of plate material, kg/m^3

h = plate thickness, m

ω = radian frequency, $2\pi f$, Hz

ρ_0 = density of air, kg/m^3

c_0 = speed of sound, m/s

σ_{rad} = radiation efficiency of free bending waves on plate (see Chapter 11)

η_c = loss factor of plate (see Chapter 14)

Inspecting Eq. (12.29), note that for $\eta = 0$, $C = 1$, and $D \approx 0$, Eq. (12.29) yields $\text{IL} = 0$, as it should be. In the vicinity of the coincidence frequency (i.e.,

in the plateau region of the transmission loss), where $B \gg A$, the insertion loss of the bare enclosure approaches

$$IL \approx 10 \log \left(\frac{1}{C} \right) = 10 \log \left(\frac{\rho_0 c_0 \sigma_{\text{rad}} + \rho_s \omega \eta}{\rho_0 c_0 \sigma_{\text{rad}}} \right) \approx 10 \log \left(\frac{\rho_s \omega \eta}{\rho_0 c_0 \sigma_{\text{rad}}} \right) \quad (12.30)$$

Figure 12.14 compares the insertion loss predicted for an unlined, sealed enclosure [using Eq. (12.8) at low frequencies and Eq. (12.27) at high frequencies] with experimental data obtained by reference 11. There is a reasonably good agreement between the predicted and measured values at both low and high frequencies not only for this specific enclosure but also for a large variety of unlined sealed enclosures of different size and wall panel thickness, indicating that the prediction formulas embodied in Eqs. (12.8) and (12.27) yield reasonable estimates for engineering design. Figure 12.14 also indicates that without internal sound-absorbing treatment the insertion loss remains very modest even at high frequencies, highlighting the importance of sound absorption in enclosure design.

Large Acoustical Enclosures with Interior Sound-Absorbing Treatment

Acoustical enclosures are termed large at frequencies where both the enclosure wall panels and the enclosure volume exhibit a large number of resonant modes in a given frequency band and statistical methods for predicting the level of the interior sound field and the vibration response and sound radiation of the enclosure wall panels can be applied. Large acoustical enclosures used in industrial noise control have many paths for transmitting acoustical energy, as illustrated

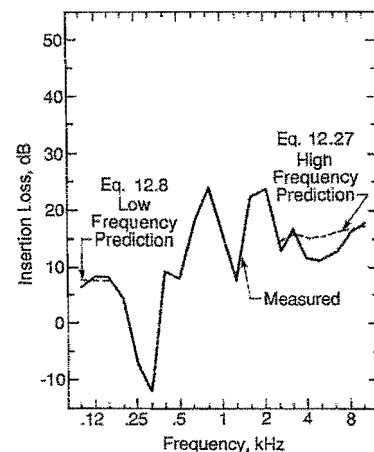


FIGURE 12.14 Comparison of measured and predicted pressure-based IL of a $50 \times 150 \times 300$ -mm (0.8-mm-thick) sealed, unlined aluminum enclosure, with source outside. (After Ref. 11).

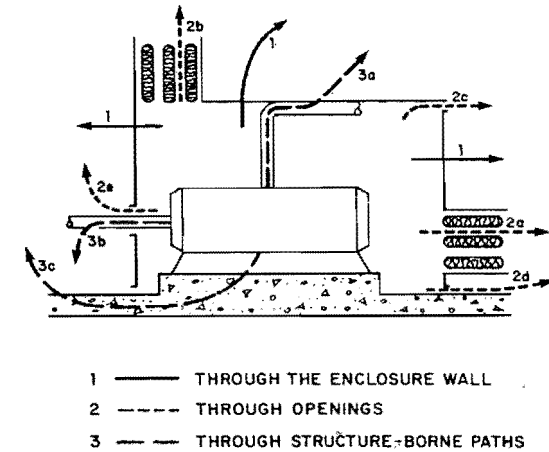


FIGURE 12.15 Paths of noise transmission from a typical acoustical enclosure.

schematically in Fig. 12.15. These paths can be grouped into three basic categories: (1) through the enclosure walls, (2) through openings, and (3) through structure-borne paths. The first group is characterized by the sound excitation of the enclosure wall by the interior sound field, resulting in sound radiation from the exterior wall surfaces. Sound transmission by this path is relatively well understood, and in most cases the sound power transmitted can be predicted with good engineering accuracy (see Chapter 11). The second group is characterized by sound energy escaping through openings in the enclosure wall such as air intake and exhaust ducts, gaps between panels, and gaps around the gasketing at the floor and doors. These are also fairly well understood¹³⁻¹⁵ but not so easily controlled. The third group is characterized by radiation of solid surfaces excited to vibration by dynamic forces, such as the enclosure walls when rigidly connected to the enclosed vibrating equipment, shafts and pipes that penetrate the enclosure wall, and the vibration of the uncovered portion of the floor. This path is difficult to predict without detailed information about the motion of the enclosed machine and its dynamic characteristics. Consequently, all possible efforts should be made to prevent any solid connections to the source. To obtain a balanced design, one must control each of these transmission paths and avoid overdesigning any one of them.

Analytical Model for Predicting Insertion Loss at High Frequencies.

A sound source enclosed in a large, acoustically lined enclosure will radiate approximately the same sound power as it would in the absence of the enclosure. To achieve a high power-based insertion loss, a high percentage of this radiated sound power must be dissipated (i.e., converted into heat) within the enclosure proper. This is accomplished by providing walls of high sound transmission loss

to contain the sound waves and by covering the interior with sound-absorbing treatment to convert the trapped acoustical energy into heat.

In analyzing the acoustical behavior of an enclosure at high frequencies, the first step is to predict the space-time average mean-square pressure of the diffuse sound field, $\langle p^2 \rangle$, within the enclosure proper. Once the interior sound field is predicted, one can determine the sound power escaping from the enclosure through the various paths. The mean-square value of the diffuse sound pressure in the interior of the enclosure is obtained by balancing (a) the sound power injected into the enclosure from all the noise sources with (b) the power loss through dissipation (in the sound-absorbing lining, in the air, and in the wall structure) and the sound transmission through the walls of the enclosure and through diverse openings. According to reference 4, a sound source of power output W_0 generates in the enclosure a reverberant field with a space-time average mean-square sound pressure $\langle p^2 \rangle$ given by

$$\langle p^2 \rangle = W_0 \frac{4\rho c}{\left\{ S_w \left[\alpha_w + \sum_i (S_{wi}/S_w) 10^{-R_{wi}/10} + D \right] + S_i \alpha_i + \sum_k S_{sk} + mV + \sum_j S_{Gj} 10^{-R_{Gj}/10} \right\}} N^2/m^4 \quad (12.31)$$

where

$$D = \left(\frac{4\pi\sqrt{12}\rho c^3\sigma}{c_L h \rho_s \omega^2} \right) \left(\frac{\rho_s \omega \eta}{\rho_s \omega \eta + 2\rho c \sigma} \right) \quad (12.32)$$

and S_w is the total interior wall surface, S_{wi} is the surface area of the i th wall, α_w is the average energy absorption coefficient of the walls, R_{wi} is the sound transmission loss of the i th wall, ρ_s is the mass per unit area of a typical wall panel, η is the loss factor of a typical wall panel in place, σ is the radiation efficiency of a typical panel, c_L is the propagation speed of longitudinal waves in the plate material, h is the wall panel thickness, $S_i \alpha_i$ is the total absorption in the interior in excess of the wall absorption (i.e., the machine body itself), S_{Gj} is the area of the j th leak or opening, R_{Gj} is the sound transmission loss of the j th leak or opening, S_{sk} is the face area of the k th silencer opening (assumed completely absorbing), m is the attenuation constant for air absorption, and V is the volume of the free interior space. The sound power incident on the unit surface area is given by

$$W_{inc} = \frac{\langle p^2 \rangle}{4\rho c} = \frac{W_0}{\{\dots\}} W \quad (12.33)$$

where $\{\dots\}$ represents the expression in the denominator in Eq. (12.31) and W_0 is the sound power output of the enclosed machine.

The terms in the denominator of Eq. (12.31) from left to right stand for (1) power dissipation by the wall absorption ($S_w \alpha_w$), (2) power loss through sound radiation of the enclosure walls ($\sum_i S_{wi} \times 10^{-R_{wi}/10}$), (3) power dissipation in the walls through viscous damping effects ($S_w D$), (4) power dissipation

by sound-absorbing surfaces in addition to the walls ($S_i \alpha_i$), (5) sound power loss to silencer terminals ($\sum_k S_{sk}$), (6) sound absorption in air (mV), and (7) sound transmission to the exterior through openings and gaps ($\sum_j S_{Gj} \times 10^{-R_{Gj}/10}$).

The relative importance of the various terms may differ widely for different enclosures and even for the same enclosure in the different frequency ranges. For example, for a small airtight enclosure without any sound-absorbing treatment, the power dissipation in the wall panels may be of primary importance, while it is usually negligible for enclosures with proper interior sound-absorbing treatment. Air absorption is important in large enclosures at frequencies above 1000 Hz. The sound power transmitted through the enclosure walls, W_{TW} , that through gaps and openings, W_{TG} , and that through silencers, W_{TS} , are given in Eqs. (12.34)–(12.36), respectively:

$$W_{TW} = \frac{W_0 (\sum_i S_{wi} \times 10^{-R_{wi}/10})}{\{\dots\}} W \quad (12.34)$$

$$W_{TG} = \frac{W_0 (\sum_j S_{Gj} \times 10^{-R_{Gj}/10})}{\{\dots\}} W \quad (12.35)$$

$$W_{TS} = \frac{W_0 (\sum_k S_{sk} \times 10^{-\Delta L_k/10})}{\{\dots\}} W \quad (12.36)$$

where ΔL_k is the sound attenuation through the silencer over opening k . The power-based insertion loss of the enclosure in the inside-outside direction is defined as

$$IL \equiv 10 \log_{10} \frac{W_0}{W_{TW} + W_{TG} + W_{TS} + W_{SB}} \text{ dB} \quad (12.37)$$

which takes the form

$$IL = 10 \log_{10} \frac{\{\dots\}}{\sum_i S_{wi} \times 10^{-R_{wi}/10} + \sum_k S_{sk} \times 10^{-\Delta L_k/10}} \times \frac{1}{\sum_j S_{Gj} \times 10^{-R_{Gj}/10} + W_{SB}(\{\dots\}/W_0)} \text{ dB} \quad (12.38)$$

where W_{SB} is the sound power transmitted through structure-borne paths considered separately according to Eqs. (12.23) and (12.24).

The denominator of Eq. (12.38) reveals that if full advantage is to be taken of the high transmission loss of the enclosure walls, the air paths through gaps, openings, and air intake and exhaust silencers and structure-borne paths must be controlled to a degree that their contribution to the sound radiation is small compared with the sound radiation of the walls. If these paths are well controlled and the dissipation is achieved by the sound-absorbing treatment of the interior wall surfaces and by the absorption in the interior, then for $R_{wi} = R_w$ the insertion loss of the enclosure is well approximated by

$$IL \cong 10 \log \left(1 + \frac{S_w \alpha_w + S_i \alpha_i}{S_w} \times 10^{R_w/10} \right) \text{ dB} \quad (12.39)$$

If we further assume that the second term in Eq. (12.39) is much larger than unity and call the total absorption in the interior of the enclosure $S_w\alpha_w + S_i\alpha_i = A$, Eq. (12.39) simplifies to

$$IL \cong R_w + 10 \log \frac{A}{S_w} \quad \text{dB} \quad (12.40)$$

This approximate formula would yield negative insertion loss for very small values of interior absorption. It has been widely promulgated in the acoustical literature without mention of its limited validity. Note that the insertion loss of a large enclosure can approach the sound transmission loss of the enclosure wall panels only if there are no leaks and α_w approaches unity.

Key Parameters Influencing the Insertion Loss of Acoustical Enclosures. The primary paths for the transmission and dissipation of sound in large acoustical enclosures are shown in the block diagram in Fig. 12.16. The key parameter affecting enclosure insertion loss are summarized in Table 12.4. Fisher and Veres¹⁰ and Kurtze and Mueller¹⁶ have carried out systematic experimental investigations on how (1) choice of wall panel parameters, (2) internal sound-absorbing lining, (3) leaks, and (4) vicinity of the enclosed machine to the enclosure walls and its vibration pattern influence the acoustical performance of enclosures. The model-machine sound source employed in reference 10 consisted of $1 \times 1.5 \times 2$ -m steel boxes of 1 mm wall thickness and were combined to yield a small ($3 \times 2 \times 1.5$ -m) and a large ($4 \times 2 \times 1.5$ -m) model machine. The

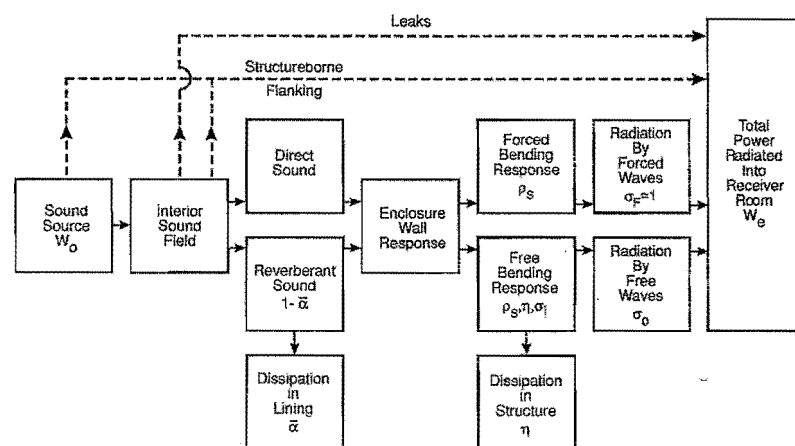


FIGURE 12.16 Block diagram of key components controlling the sound attenuation process in acoustical enclosures: W_0 , source sound power output; $\bar{\alpha}$, average sound absorption coefficient; ρ_s , mass per unit area; η , loss factor; σ_i , radiation efficiency of wall panels for radiation for inside direction; σ_0 , radiation efficiency of wall panels for radiation for outside direction; σ_F , 1 radiation efficiency of forced waves; W_e , total sound power radiated by the enclosure.

TABLE 12.4 Effect of Key Parameters on Sound Insertion Loss of Large Enclosures

Parameter	Symbol	Effect on Insertion Loss
Absorption coefficient of lining	$\bar{\alpha}$	Increases IL by reducing reverberant buildup
Distance between machine and enclosure wall	d	If d decreases beyond a certain limit, IL decreases at low frequencies (close-fitting enclosure behavior)
Thickness of wall panel	h	Increases IL
Density of wall panel material	ρ_M	Increases IL
Speed of longitudinal waves in panel material	$c_L = \sqrt{E/\rho_M}$	Decreases IL below critical frequency
Loss factor of wall panel	η	Increases IL, especially near and above critical frequency and at first panel resonance
Critical frequency of wall panel	$f_c = c^2/1.8c_L h$	The higher is f_c for a given mass per unit area, the higher is IL
Radiation efficiencies of wall panels	σ_i, σ_0	The higher is the radiation efficiency for inside direction, σ_i , and for outside direction, σ_0 , the smaller is IL
Stiffeners	—	Decrease IL by increasing σ
Leaks	—	Limit achievable IL, watch out for door gaskets and penetrations
Structure-borne flanking	—	Limit achievable IL; watch out for solid connections between vibrating machine and enclosure and for floor vibration

walls of the model machines could be excited by either an internal loudspeaker or an internal tapping machine to simulate both sound and structure-borne excitation. The investigated rectangular walk-in enclosure had $4.5 \times 2.5 \times 2$ m inside dimensions and was equipped with an operational personal-access door. The thickness and material of the wall panels as well as the thickness of the interior lining could be varied.

Wall Panel Parameters. Figure 12.17 shows the measured insertion loss versus frequency for three different values of the wall thickness h , indicating that above 1.5 mm, a further increase of the wall thickness brings only slight improvement at low frequencies and a slight deterioration at high frequencies near coincidence. Figure 12.17 also includes, as the dashed line, the field-incidence

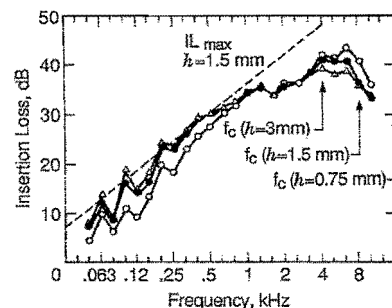


FIGURE 12.17 Curves of power-based insertion loss versus frequency measured (inside-outside direction) for a large walk-in enclosure for various thicknesses of steel plate, h . (After Ref. 10.) Sound-absorbing treatment, 70 mm thick, door sealed; f_c , coincidence frequency; $IL_{\max} = R_f$ represents the maximum achievable insertion loss. (Δ) $h = 3$ mm; ($\bullet$) $h = 1.5$ mm; ($\circ$) $h = 0.75$ mm.

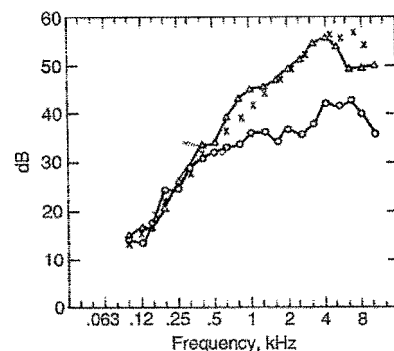


FIGURE 12.18 Comparison of measured enclosure insertion loss IL and panel sound transmission loss R (after Ref. 10): 1.5-mm-thick steel plate, 70-mm-thick sound-absorbing treatment; (Δ) R , measured; ($\times$) R , predicted according to Chapter 11; ($\circ$) IL measured.

sound transmission loss for a 1.5-mm-thick steel plate for comparison, indicating that up to 500 Hz, the insertion loss matches well the field-incidence sound transmission loss of the panel.

Figure 12.18 compares the measured sound transmission loss of a typical 2.1×1.2 -m wall panel and predicted sound transmission loss of an identical infinite-enclosure panel with the measured insertion loss obtained with an enclosure built with the same panels. The data indicate that the prediction formulas presented in Chapter 11 for infinite panels can be used to predict accurately the sound transmission loss of the finite panels of this specific size and that above 500 Hz the insertion loss is dominated by unintentional small leaks. According to reference 10, the sound-absorbing interior liner provides sufficient structural

damping of the wall panels so that additional damping treatment did not result in improved performance.

Stiffening the wall panels with exterior L-channels, which increase the radiation efficiency, resulted in slight deterioration of the acoustical performance. Using steel and wood chipboard of equal mass per unit area yielded different transmission loss but resulted in practically the same insertion loss because in the frequency range where the chipboard had its coincidence frequency the insertion loss was already controlled by leaks.

As a general rule, wall panels should be selected to have *high enough coincidence frequency* to be above the frequency region of interest and be *large enough* that their first structural resonance occurs below the frequency region of interest but heavy enough to yield a field-incidence mass law sound transmission loss that matches the insertion loss requirements. Steel plates 1.5 mm thick usually fulfill most of these requirements.

Stiff, lightweight panels (such as honeycombs) usually yield a critical frequency as low as 500 Hz and provide a very low sound transmission loss in this frequency region. Consequently, such panels should not be used in enclosure design unless the enclosure is required to provide high insertion loss at only very low frequencies and mid- and high-frequency performance is not required.

Effect of Sound-Absorbing Treatment. The proper choice of sound-absorbing treatment plays a more crucial role than the specific choice of wall material or wall thickness, as shown in Fig. 12.19. The sound-absorbing treatment helps to increase insertion loss by (1) reducing reverberant buildup in the enclosure at mid- and high frequencies, (2) increasing the transmission loss of the enclosure walls at high frequencies, and (3) covering up some of the unintentional leaks between adjacent panels and between panels and frames. Whenever feasible, the thickness of the interior sound-absorbing treatment should be chosen to yield a normal-incidence absorption coefficient $\alpha \geq 0.8$ in the frequency region of interest. As shown in Chapter 8, this can be achieved by a layer thickness $d \geq \frac{1}{10}\lambda$, where λ_L is the acoustical wavelength at the lower end of the frequency region, and by choosing a porous material of normalized flow resistance of $1.5 \leq R_1 d / \rho c < 3$.

Leaks. Leaks reduce the insertion loss of large enclosures just as they reduce it for small enclosures [see Eq. (12.13)]. The reduction in insertion loss due to leaks, ΔIL_L , is defined as

$$\Delta IL_L \equiv IL_s - IL_L \cong 10 \log(1 + \beta \times 10^{R_w/10}) \quad \text{dB} \quad (12.41)$$

where IL_s and IL_L are the insertion losses of the sealed and leaky enclosures, respectively, R_w is the sound transmission loss of the enclosure wall, and $\beta = (1/S_w) \sum_j S_{Gj} \times 10^{-R_{Gj}/10}$ is the leak ratio factor, S_{Gj} is the face area, and R_{Gj} is the sound transmission loss of the j th leak. The sound transmission loss of leaks can be positive or, in the case of longitudinal resonances in wide, rigid-walled gaps, also negative. For preliminary calculations it is customary to assume

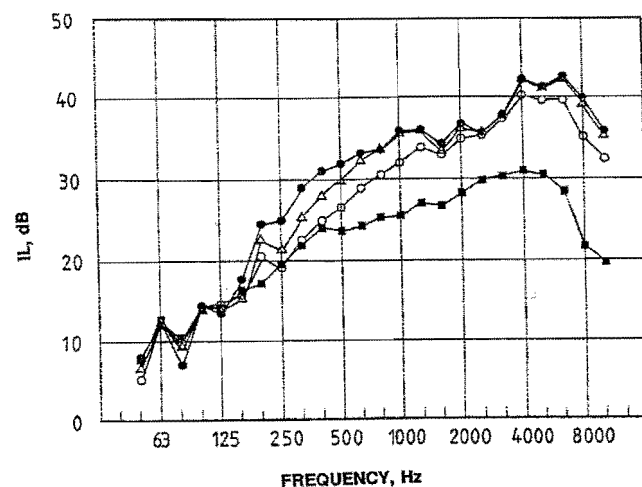


FIGURE 12.19 Measured insertion loss of the enclosure with 1.5-mm-thick steel plates for various thicknesses of the sound-absorbing material (after Ref. 10): (■) 0 mm (no absorption); (○) 20 mm; (△) 40 mm; (●) 70 mm.

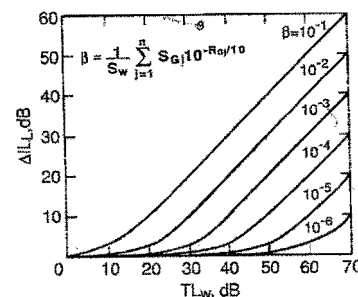


FIGURE 12.20 Decrease of enclosure insertion loss, ΔIL , as a function of the wall sound transmission loss R_w with the leak ratio factor β as parameter; S_w is the wall surface area and R_{Gj} and S_{Gj} are the sound transmission loss and face area of the j th leak, respectively.

$R_{Gj} = 0$. In this case, $\beta = \sum_j S_{Gj}/S_w$ is the ratio of the total face area of the leaks and gaps and the surface area (one side) of the enclosure walls.

Figure 12.20 is a graphical representation of Eq. (12.41) indicating that the higher is the sound transmission loss of the enclosure walls, the higher is the reduction in insertion loss caused by leaks. For example, for an enclosure assembled from wall panels that provide a R_w of 50 dB, the total area of all leaks must be less than 1/1000 percent ($\beta = 10^{-5}$) of the total enclosure surface area if the insertion loss is not to be decreased by more than 3 dB!

Machine Position. Experimental investigations by Fischer and Veres¹⁰ have shown that the insertion loss of an enclosure at low frequencies also depends on the specific position of the enclosed machine within the enclosure. At low frequencies, where the distance between the flat machine wall and the flat enclosure wall panel, d , becomes smaller than one-eighth of the acoustical wavelength ($d < \frac{1}{8}\lambda$), the observed decrease of the insertion loss (from its value obtained when the machine was positioned centrally so that all wall distances were large compared with $\frac{1}{8}\lambda$) was as high as 5 dB. Note that the machine-wall distance d includes the thickness of the internal lining. As a general rule, machines with omnidirectional sound radiation should be positioned centrally. It is especially important to avoid positioning the noisy side of machines very close to enclosure walls, doors, windows, and ventilation openings.

Machine Vibration Pattern. The insertion loss of an enclosure also depends on the specific vibration pattern of the machine, although that dependence appears to not be very strong. Experimental investigations¹⁰ have shown that variation of the insertion loss was about ± 2.5 dB when measured with a loudspeaker inside the model machine and with excitation of the model machine by an ISO tapping machine.

Flanking Transmission through the Floor. The potential insertion loss of an enclosure can be limited severely by flanking if the floor is directly exposed to vibration forces, the internal sound field of the enclosure, or both. Figure 12.21 shows typical enclosure installations ranging from the best to the worst with regard to flanking transmission via the floor. For most equipment it is imperative to provide vibration isolation, a structural break in the floor, or both to reduce the transmission of structure-borne excitation of the floor. Applicable prediction tools and isolation methods are described in Chapters 11 and 13. For the cases depicted in Figs. 12.21c,d, where the interior sound field directly impinges on the floor, the sound-induced vibration of the floor sets a limit to the achievable insertion loss of the enclosure. This limit can be estimated roughly from

$$IL_L \cong R_F + 10 \log \sigma_F \quad \text{dB} \quad (12.42)$$

where R_F is the sound transmission loss and σ_F is the radiation efficiency of the floor slab.

Relationship between Inside-Outside and Outside-Inside Transmission. Acoustical enclosures are most frequently used to surround a noisy equipment for reducing the noise exposure of a receiver located outside of the enclosure. Less frequently, the enclosure surrounds the receiver (i.e. it is a cabin) to reduce its noise exposure to a sound source located outside of the enclosure. The acoustical performance of the enclosure in the former case is given by its insertion loss in the inside-outside direction, IL_{io} , while the latter is given by its insertion loss in the outside-inside direction, IL_{oi} . All of

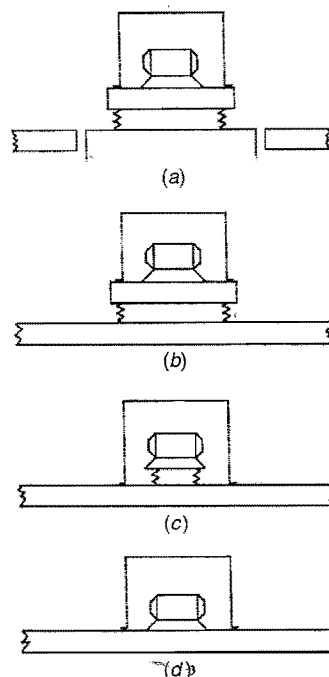


FIGURE 12.21 Rating of enclosure installations with regard to flanking transmission of the floor: (a) best; (b) good; (c) bad; (d) worst.

our considerations in this chapter have dealt with sound transmission from the inside to the outside. According to the principle of reciprocity (see Chapter 11), exchanging the position of the source and receiver does not affect the transfer function. Though it is strictly true only for point sound sources and receivers, it also can be applied to extended sound sources and averaged observer positions in an enclosure, yielding $IL_{io} = IL_{oi}$.

Example. To demonstrate the use of the previously provided design information, predict the insertion loss of the large, acoustically lined, walk-in enclosure investigated experimentally in reference 10. The enclosure is $4.5 \times 2.5 \times 2.0$ m high and is constructed of 1.5-mm-thick steel plates and has a 70-mm-thick interior sound-absorbing lining. The sound transmission loss of the wall panels, R_w , given in Fig. 12.18 by the triangular data points, and the sound absorption coefficient of the interior panel surfaces, $\bar{\alpha}$, taken from reference 10, as a curve that monotonically increases from 0.16 at 100 Hz and reaches a plateau of 0.9 at 600 Hz. The insertion loss is predicted by Eq. (12.38) using the following values: $S_{wi} = S_w = 39 \text{ m}^2$, $\sum_j S_{Gj} \times 10^{-R_{Gj}/10} = 10^4$, $S_{Gj} = 3.9 \times 10^{-3}$, assuming that leaks account for 1/100 percent of the wall surface area, that there are no silencer openings ($S_s = 0$) and no structure-borne connections ($W_{SB} = 0$), and

that air absorption (mV), the sound absorbed by the machine ($S_i \alpha_i$), and power lost due to dissipation in the panels ($S_w D$) and due to sound radiation ($10^{-R_w/10}$) are small compared with the power dissipated in the sound-absorbing treatment on the interior wall surfaces.

Then Eq. (12.38) simplifies to

$$IL_L \cong 10 \log \frac{S_w \bar{\alpha}}{\sum_i S_{wi} \times 10^{-R_{wi}/10} + \sum_j S_{Gj} \times 10^{-R_{Gj}/10}} \text{ dB} \quad (12.43)$$

The curve of insertion loss versus frequency predicted by Eq. (12.43) is represented by the open circles in Fig. 12.22, while the solid curve represents the measured data. The measured sound transmission loss of the wall panels, R_w , is the dashed line and represents the limiting insertion loss that could be achieved only if there would be no leaks and the random-incidence sound absorption coefficient $\bar{\alpha}_w$ would approach unity. Observing Fig. 12.22, note that the assumption of 1/100 percent leaks ($\beta = 10^{-4}$) yielded a prediction that matches reasonably well the measured data, which indicates that even such a very small percentage of leaks, which can be realized only if extreme care is exercised during erecting the enclosure and careful caulking of all leaks detected during the initial performance checkout and careful adjustment of door gaskets, can substantially decrease the potential insertion loss of an enclosure.

Partial Enclosures

When the work process of the machine or safety and maintenance requirements do not allow a full enclosure, a partial enclosure (defined as those with more than 10% open area) is used to reduce radiated noise. When a partial enclosure is far enough from the sound source not to cause an increase in source sound

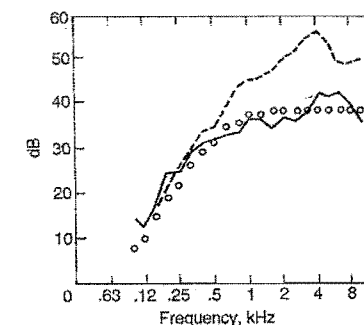


FIGURE 12.22 Typical acoustical performance of a leaky enclosure: (—) IL, measured by Ref. 10; (---) R_w , measured by Ref. 10; (O) IL, predicted by Eq. (12.43) for

$$\beta = \left(\sum_j \left(\frac{S_{Gj}}{S_w} \right) \times 10^{-R_{Gj}/10} \right) = 10^{-4}$$

power by hydrodynamic interactions of the near field of the source with edges of the opening and the walls of the partial enclosure are not rigidly connected to the vibrating machine to serve as a sounding board, a partial enclosure can provide a modest insertion loss of 5 dB or less. According to reference 17, the power-based insertion loss of partial enclosure can be estimated by

$$IL = 10 \log \left[1 + \alpha \left(\frac{\Omega_{\text{tot}}}{\Omega_{\text{open}}} - 1 \right) \right] \text{ dB} \quad (12.44)$$

where Ω_{tot} is the solid angle of sound radiation of the unenclosed source and $\Omega_{\text{open}} = S_{\text{open}}/r$ is the solid angle at which the enclosed source (located at a distance r) "sees" the opening of area S_{open} . Reference 17 contains construction details, curves of measured insertion loss versus frequency, and cost information of 54 different partial enclosures.

12.2 WRAPPINGS

Many machines and pipes need thermal insulation to provide protection of operating personnel or to prevent excessive heat loss. Typically, they need a minimum of 25-mm- (1-in.-) thick glass or ceramic fiber blanket to achieve proper protection. Since many hot equipment, such as turbines, boiler feed pumps, compressor valves, and pipelines, are also sources of intense noise, it is frequently feasible to achieve both heat and acoustical insulation by providing a (preferably) limp, impervious surface layer on top of the porous blanket. The impervious surface layer also provides protection for the porous blanket. Acoustically, the combination of the resilient blanket and the heavy, limp, impervious surface layer provide a spring-mass isolation system. At frequencies where the mass impedance of the surface layer exceeds the combined stiffness impedance of the skeleton of these flexible blankets and the entrapped air, the vibration amplitude of the surface layer becomes smaller than that of the machine surface and wrapping results in an insertion loss that monotonically increases with increasing frequency. The insertion loss of the wrapping is defined the same way as the insertion loss of a close-fitting enclosure and for flat wrappings can be predicted in a manner similar to that of the close-fitting enclosure.

The major difference between wrappings and close-fitting enclosures is that for wrappings the skeleton of the porous layer is in full surface contact with the vibrating surface of the machine, but for the close-fitting enclosures there is no such contact. Consequently, for wrappings the vibrating machine surface transmits a pressure to the impervious surface layer not only through sound propagation in the voids between the fibers but also through the skeleton of the porous material. At low frequencies where the thickness of the porous layer, L , is much smaller than the acoustical wavelength, the porous layer can be represented by a stiffness per unit area, S_{tot} , that according to Mechel¹⁸ can be estimated as

$$S_{\text{tot}} = S_M + S_L \left(1 - \frac{P}{A} \sqrt{\frac{\rho c^2}{L \pi f \gamma R_1 h'}} \right) \text{ N/m}^2 \quad (12.45)$$

where S_L is the stiffness of the trapped air,

$$S_L \cong \frac{\rho c^2}{\gamma L h'} \text{ N/m}^2 \quad (12.46)$$

γ the adiabatic exponent, h' the porosity, f the frequency, and R_1 the flow resistivity of the material, and S_M is given by

$$S_M = (2\pi f_0)^2 \rho_s \text{ N/m}^2 \quad (12.47)$$

In Eq. (12.47) the symbol f_0 represents the measured resonance frequency of a mass-spring system consisting of a small rectangular or square-shaped sample of the porous material covered with a metal plate of mass per unit area ρ_s . The resonance frequency is measured by putting the mass-spring system on top of a shaker table and performing a frequency sweep. The second term in Eq. (12.45) accounts for the air that escapes along the perimeter P of a test sample of surface area A , used in the experiment to determine the resonance frequency f_0 according to Eq. (12.47). Young's modulus of the porous material is then determined as $E = S_{\text{tot}}/L$, the speed of longitudinal waves in the porous layer as $c_L = \sqrt{E/\rho_M}$, where ρ_M is the density of the porous material, and the wavenumber as $k = 2\pi f/c_L$. Characteristics of some frequently used thermal insulation materials reported by Wood and Ungar¹⁹ are given below:

Material	Dynamic Stiffness per Unit Area, S_M (N/m ²)	Density, ρ_M (kg/m ³)
Erco-Mat ^a	1.3×10^7	138
Erco-Mat F ^a	2×10^6	104
Glass fiber ^b	4×10^4	12

^aNedled glass fiber insulation.

^bLow-density Owens Corning Fiberglas blanket.

Considering the porous heat insulating layer as a wave-bearing medium of density ρ and a complex Young's modulus $E' = E(1 + j\eta)$ and characterizing the impervious, limp surface layer by its mass per unit area ρ_s , Wood and Ungar¹⁹ derived the analytical formula

$$IL = 20 \log \left| \cos(kL) - \frac{\rho_s}{\rho_M L} (kL) \sin(kL) \right| \text{ dB} \quad (12.48)$$

where $k = \omega/\sqrt{E(1 + j\eta)/\rho_M}$ is the complex propagation constant of the porous layer, $\eta = 1/Q$ the loss factor, and $\rho_s \omega$ the mass impedance per unit area of the covering layer.

At very low frequencies, where $kL \ll 1$, Eq. (12.48) can be approximated by

$$IL_L = 20 \log \left[1 - \left(\frac{\omega}{\omega_n} \right)^2 \right] \text{ dB} \quad (12.49)$$

where $\omega_n = E/\rho_s L$ is the resonance frequency of a system consisting of a massless spring of stiffness E/L and a mass ρ_s (per unit area). According to Eq. (12.49), the cladding provides no insertion loss for $\omega \ll \omega_n$ and yields a negative insertion loss (amplifies the radiated sound) if $\omega \cong \omega_n$.

For frequencies above the resonance frequency ($\omega \gg \omega_n$) the blanket provides an attenuation that increases monotonically with increasing frequency. Noting that in Eq. (12.48) neither the cosine nor the sine term can exceed unity, an upper bound for the curve of insertion loss versus frequency at high frequencies is obtained by¹⁹

$$IL_L \leq 20 \log \left[1 + \frac{\rho_s}{\rho_L} (kL)^2 \right] \quad \text{dB} \quad (12.50)$$

Pipe Wrappings

Pipe wrappings consist of a resilient porous layer and an impervious jacket. The impervious jacket is usually sheet metal or loaded plastic. They achieve their acoustical performance in a similar manner as flat wrappings. However, there is an important difference. While flat wrappings do not increase the sound-radiating surface, when a wrapping is applied to a small-diameter pipe, the diameter of the impervious jacket can be substantially larger than that of the bare pipe. This increases both the radiating surface and the radiation efficiency. Accordingly, at low frequencies, below or slightly above the resonance frequency of the pipe wrapping, the insertion loss is negative. Positive insertion loss is usually achieved only above 200 Hz.

According to Michelsen, Fritz, and Sazenhofen,²⁰ the maximal achievable insertion loss of wrappings can be estimated by the empirical formula

$$IL_{\max} = \frac{40}{1 + 0.12/D} \log \frac{f}{2.2f_0} \quad \text{dB} \quad (12.51)$$

where

$$f_0 = 60/\sqrt{\rho_s L} \quad \text{Hz} \quad (12.52)$$

where L is the thickness of the porous resilient layer and D is the pipe diameter, both in meters, and ρ_s is the mass per unit area of the impervious jacket in kilograms per square meter.

Equation (12.51) is valid only if there are no structure-borne connections between the pipe and jacket and for frequencies $f \geq 2f_0$. Figure 12.23 shows curves of measured insertion loss versus frequency obtained in reference 20 for typical pipe wrapping consisting of a galvanized steel jacket, thickness 0.75–1 mm, and a porous resilient layer with the following parameters:

Thickness L	30, 60, 80, and 100 mm
Density	85–120 kg/m ³
Flow resistivity	$3 \times 10^4 \text{ N} \cdot \text{s/m}^4$
Dynamic Young's modulus	$2 \times 10^5 \text{ N/m}^2$

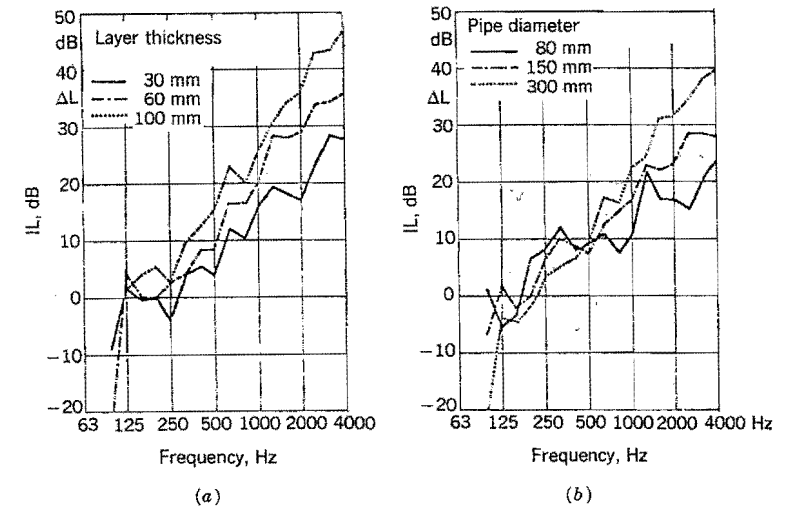


FIGURE 12.23 Curves of measured insertion loss versus frequency of pipe wrappings. (After Ref. 20.) (a) Effect of layer thickness. (b) Effect of pipe diameter.

Figure 12.23a shows the effect of layer thickness L on the insertion loss for a pipe of diameter $D = 300$ mm while Fig. 12.23b illustrates the effect of pipe diameter D for a constant layer thickness $L = 60$ mm. The insertion loss above 250 Hz increases with increasing thickness of the porous layer and with increasing diameter of the bare pipe. The standard deviation of the measured insertion loss around that predicted by Eq. (12.52) was 4 dB.

Note that spacers between the pipe and jacket result in insertion loss values that may be substantially lower than those predicted by Eq. (12.50) unless the spacers are less stiff dynamically than the porous layer.

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CHAPTER 13

Vibration Isolation

ERIC E. UNGAR AND JEFFREY A. ZAPFE

Acentech, Inc.
Cambridge, Massachusetts

13.1 USES OF ISOLATION

Vibration isolation refers to the use of comparatively resilient elements for the purpose of reducing the vibratory forces or motions that are transmitted from one structure or mechanical component to another. The resilient elements, which may be visualized as springs, are called vibration isolators. Vibration isolation is generally employed (1) to protect a sensitive item of equipment from vibrations of the structure on which it is supported or (2) to reduce the vibrations that are induced in a structure by a machine it supports. Vibration isolation may also be used to reduce the transmission of vibrations to structural components whose attendant sound radiation one wishes to control.

13.2 CLASSICAL MODEL

Mass-Spring-Dashpot System

Many aspects of vibration isolation can be understood from analysis of an ideal, linear, one-dimensional, purely translational mass-spring-dashpot system like that sketched in Fig. 13.1. The isolator is represented by the parallel combination of a massless spring of stiffness k (which produces a restoring force proportional to the displacement) and a massless damper with viscous damping coefficient c (which produces a force proportional to the velocity and opposing it). The rigid mass m , which here is taken to move only vertically and without rotation, corresponds either to an item to be protected (Fig. 13.1a) or to a machine frame on which a vibratory force acts (Fig. 13.1b).

Transmissibility

Although the mass-spring-dashpot diagrams of Figs. 13.1a and b are similar, they describe physically different situations. In Fig. 13.1a, the support S is

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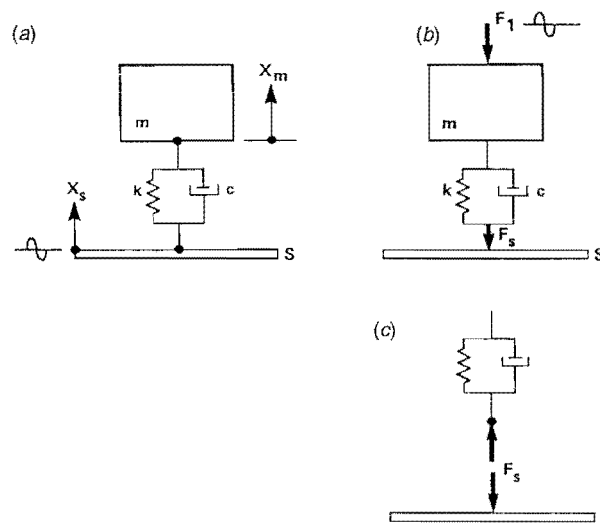


FIGURE 13.1 One-dimensional translational mass-spring-dashpot system: (a) excited by support motion; (b) excited by force acting on mass; (c) force F_s between isolator and support.

assumed to vibrate vertically with a prescribed amplitude X_s at a given frequency; the purpose of the isolator is to keep the displacement amplitude X_m of mass m acceptably small. In Fig. 13.1b, a force of a prescribed amplitude F_1 at a given frequency is assumed to act on mass m ; the purpose of the isolator is to keep the amplitude F_s of the force that acts on the support within acceptable limits, thereby also keeping the support's resulting motion adequately small.

For the situation represented by Fig. 13.1a, the ratio $T = X_m/X_s$ of the amplitude of the mass' displacement to that of the disturbing displacement of the support S is called the (motion) transmissibility. For the situation represented by Fig. 13.1b, the ratio $T_F = F_s/F_1$ of the amplitude of the force transmitted to the support S to that of the disturbing force is called the force transmissibility. In many practical instances corresponding to force excitation as represented by Fig. 13.1b, the support is so stiff or massive that its displacement may be taken to be zero. It turns out that the force transmissibility T_{F0} obtained with an immobile support (for the case of Fig. 13.1b) is given by the same expression as the motion transmissibility (for the case of Fig. 13.1a),* namely,^{1,2}

$$T = T_{F0} = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}} \quad (13.1)$$

*This equality holds not only for the simple system of Fig. 13.1, but also for any mathematically linear system.³ Because of this equality, the literature generally does not differentiate between the two types of transmissibility.

where $r = f/f_n$ is the ratio of the excitation frequency to the natural frequency of the mass-spring system and $\zeta = c/c_c$, which is called the *damping ratio*, is the ratio of the system's viscous damping coefficient c to its critical damping coefficient c_c .

The critical damping coefficient is given by $c_c = 2\sqrt{km} = 4\pi f_n m$. The natural frequency f_n of a spring-mass system obeys^{1,2}

$$2\pi f_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}} = \sqrt{\frac{g}{X_{st}}} \quad (13.2)$$

where g denotes the acceleration of gravity, W the weight associated with the mass m , and X_{st} the static deflection of the spring due to this weight. In customary units,

$$f_n(\text{Hz}) \approx \frac{15.76}{\sqrt{X_{st}(\text{mm})}} \approx \frac{3.13}{\sqrt{X_{st}(\text{in.})}} \quad (13.3)$$

The relation $X_{st} = k/W$ and Eq. (13.3) hold only if the spring is mathematically linear—that is, if the slope of the spring's force-deflection curve (which slope corresponds to the stiffness k) is constant. For small-amplitude vibrations about an equilibrium deflection one may take k in the first form of Eq. (13.2) as the slope dF/dx of the spring's force-deflection curve* at the spring's static deflection under an applied load W . However, for some practical isolators, notably those incorporating elastomeric materials, the effective dynamic stiffness may be considerably greater than that obtained from a quasi-static force-deflection curve.⁴

Figure 13.2 shows curves based on Eq. (13.1) for several values of the damping ratio. For small frequency ratios, $r \ll 1$, the transmissibility T is approximately equal to unity; the motion or force is transmitted essentially without attenuation or amplification. For values of r near 1.0, T becomes large (at $r = 1$ or $f = f_n$, $T = 1/2\zeta$); the system responds at resonance, resulting in amplification of the motion or force. All curves pass through unity at $r = \sqrt{2}$.

For $r > \sqrt{2}$, T is less than unity and decreases continually with increasing r . In this high-frequency range, which may be called the *isolation range*, the inertia of the mass plays the dominant role in limiting the mass excursion and thus in limiting the mass' response to support displacement or to forces acting on the mass. Thus, if one desires to achieve good isolation, which corresponds to small transmissibility, one needs to choose an isolator with the smallest possible k (i.e., with the largest practical static deflection X_{st}) in order to obtain the smallest f_n and the greatest value of $r = f/f_n$ for a given excitation frequency.

*Nonlinear spring elements for which $k = dF/dx$ is proportional to the applied load W in a given range are the basis for isolators which have the practical advantage of providing the same natural frequency for all loads in the given range.⁴

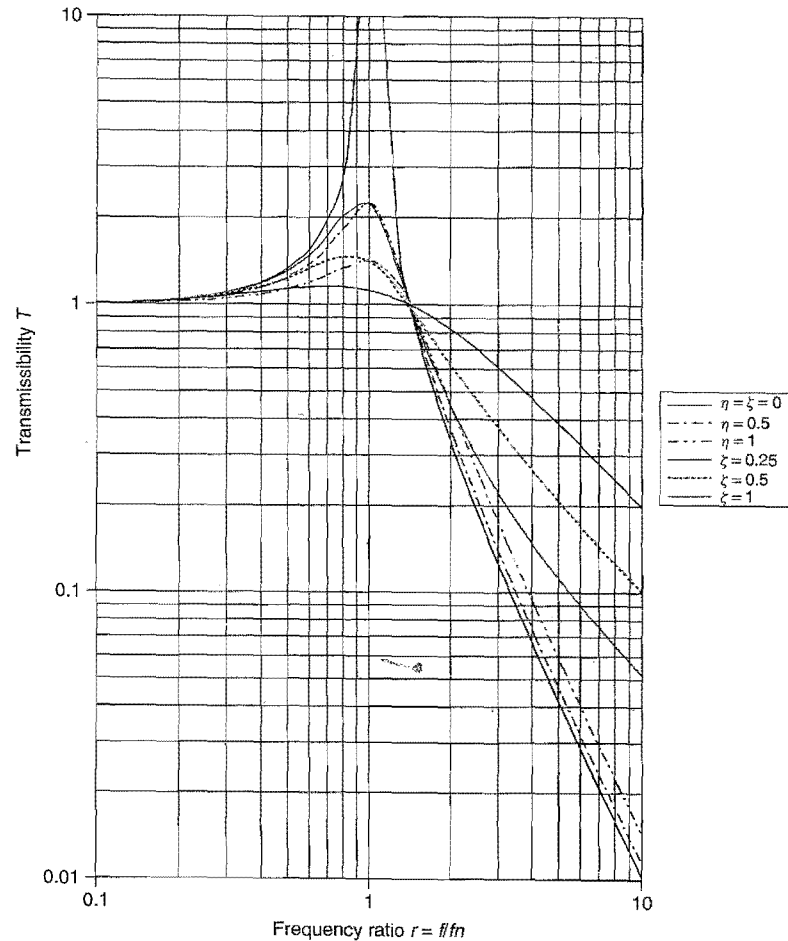


FIGURE 13.2 Transmissibility of mass-spring-damper system of Fig. 13.1 from Eqs. (13.1) and (13.4).

Isolation Efficiency

The performance of an isolation system is sometimes characterized by the isolation efficiency I , which is given by $I = 1 - T$. Whereas the transmissibility indicates the fraction of the disturbing motion or force that is transmitted, the isolation efficiency indicates the fraction by which the transmitted disturbance is less than the excitation. Isolation efficiency is often expressed in percent. For example, if the transmissibility is 0.0085, the isolation efficiency is 0.9915, or 99.15%, indicating that 99.15% of the disturbance does not “get through” the isolator.

Effect of Damping

In the isolation range, that is, for frequency ratios $r > \sqrt{2}$, increased viscous damping results in increased transmissibility, as evident from Fig. 13.2 or Eq. (13.1). Although this fact is mentioned in many texts, it is of little practical consequence for two reasons. First, in practice one rarely encounters systems with damping ratios that are greater than 0.1 unless high damping is designed into a system on purpose—and small values of the damping ratio, and certainly small changes in that damping ratio, have little effect on transmissibility. Second, Eq. (13.1) and Fig. 13.2 pertain only to viscously damped systems, in which a damper produces a retarding force that is proportional to the velocity. Although such systems have been studied most extensively (largely because they are relatively easy to analyze mathematically), the retarding forces in practical systems generally have other parameter dependences.

The effect of damping in practical isolators is generally better represented by *structural damping* than by viscous damping. In structural damping, the retarding force acts to oppose the motion, as in viscous damping, but is proportional to the displacement. For structurally damped systems,²

$$T = F_{F0} = \sqrt{\frac{1 + \eta^2}{(1 - r^2)^2 + \eta^2}} \quad (13.4)$$

where η denotes the *loss factor* of the system.*

If a structurally damped system has the same amplification at the natural frequency as a similar viscously damped system, then $\eta = 2\zeta$. The transmissibility of a structurally damped system in the isolation range increases much less rapidly with increasing damping than does that of a system with viscous damping, as evident from Fig. 13.2.

In practical isolation arrangements the damping typically is very small, that is, $\zeta < 0.1$. For such small amounts of damping, the transmissibility in the isolation range differs little from that for zero damping, so that one may approximate the transmissibility in that range by

$$T = T_{F0} = \frac{1}{|r^2 - 1|} \approx \left(\frac{f_n}{f}\right)^2 \quad (13.5)$$

where the rightmost expression applies for $r^2 = (f/f_n)^2 \gg 1$.

Effects of Inertia Bases

An isolated machine is often mounted on a massive support, generally called an *inertia base*, to increase the isolated mass. If the isolators are not changed as

*The loss factor η in Eq. (13.4) may be taken to vary with frequency as determined from experimental data. See Chapter 14.

the mass is increased, then the natural frequency of the system is reduced, the frequency ratio r corresponding to a given excitation frequency is increased, and the transmissibility is reduced; that is, isolation is improved.

However, practical considerations, such as the load-carrying capacities of isolators, usually dictate that the total isolator stiffness be increased* as the mass is increased, with the static deflection changing little. (In fact, conventional commercial isolators typically are specified in terms of the loads they can carry and the corresponding static deflection.) As evident from Eq. (13.3), the natural frequency remains unchanged if the static deflection remains unchanged, and addition of an inertia base then does not change the transmissibility.

Thus, inertia bases in practice typically do not improve isolation significantly. They are of some benefit, however: With a greater mass supported on stiffer springs, forces that act directly on the isolated mass produce smaller static and vibratory displacements of that mass.

Effect of Machine Speed

In a rotating or reciprocating machine, the dominant excitation forces generally are due to dynamic unbalance and occur at frequencies that correspond to the machine's rotational speed and/or multiples of that speed. Greater speed thus corresponds to increased excitation frequencies and larger values of r —and therefore to reduced transmissibility (or improved isolation). As evident from Eq. (13.1) or (13.5), the transmissibility²—the ratio of the transmitted to the excitation force—becomes smaller as the speed (and the excitation frequency) is increased; in the isolation range, the transmissibility is very nearly inversely proportional to the square of the excitation frequency. However, the excitation forces associated with unbalance vary as the square of speed (and frequency) and thus increase with speed about as much as the transmissibility decreases. The net result is that the magnitudes of the forces that are transmitted to the supporting structure are virtually unaffected by speed changes, although speed changes do affect the frequencies at which these forces occur.

Limitations of Classical Model

Although the linear single-degree-of-freedom model provides some useful insights into the behavior of isolation systems, it obviously does not account for many aspects of realistic installations. Clearly, real springs are not massless and may be nonlinear, and real machine frames and supporting structures are not rigid. Resiliently supported masses generally move not only vertically but also horizontally—and they also tend to rock.

In simple classical analyses, furthermore, the magnitude of the exciting force or motion is taken as constant and independent of the resulting response, whereas

*Note that the single spring and dashpot in the schematic diagrams of Fig. 13.1 represent the entire isolation system, which in reality may consist of many isolators. Addition of isolators amounts to an increase in the stiffness of the isolation system.

in actual situations the excitation often depends significantly on the response, as discussed in Section 13.4 under Loading of Sources.

13.3 ISOLATION OF THREE-DIMENSIONAL MASSES

General

Unlike the simple model shown in Fig. 13.1, where the mass can only move vertically without rotation and the system has only one natural frequency, an actual rigid three-dimensional mass has six degrees of freedom; it can translate in three coordinate directions and rotate about three axes. An elastically supported rigid mass thus has six natural frequencies. A nonrigid mass has many additional ones associated with its deformations. Obtaining effective isolation here requires that all of the natural frequencies fall considerably below the excitation frequencies of concern. Descriptions of the natural frequencies and responses of general isolated rigid masses are available^{3,5} but are so complex that they provide little practical insight and tend to be used only rarely for design purposes.

Coupling of Vertical Motion and Rocking

Figure 13.3 is a schematic diagram of a mass m supported on two isolators in a plane through its center of gravity, parallel to the plane of the paper (or on two rows of isolators extending in the direction perpendicular to the plane of the paper) with stiffnesses k_1 and k_2 located at distances a_1 and a_2 from the mass' center of gravity. One may visualize easily that a downward force applied at the center of gravity of the mass in general would produce not only a downward displacement of the center of gravity but also a rotation of the mass, the latter due to the moment resulting from the isolator forces. Similarly, purely vertical up-and-down motion of the support S would in general result in rocking of the mass, in addition to its vertical translation. Vertical and rocking motions here are said to be "coupled."

The natural frequency f_v at which pure vertical vibration would occur if the mass would not rock (i.e., the "uncoupled" vertical natural frequency) is given by

$$2\pi f_v = \sqrt{\frac{k_1 + k_2}{m}} \quad (13.6)$$

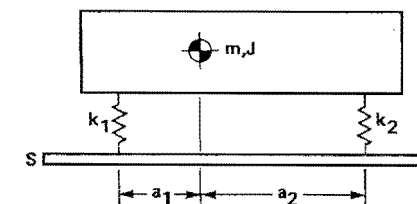


FIGURE 13.3 Mass m with moment of inertia J supported on two isolators.

The natural frequency f_r at which the mass would rock if its center of gravity would not move vertically (i.e., the uncoupled rocking frequency) is given by

$$2\pi f_r = \sqrt{\frac{k_1 a_1^2 + K_2 a_2^2}{J}} \quad (13.7)$$

where J denotes the mass polar moment of inertia about an axis through the mass' center of the gravity and perpendicular to the plane of the paper.

The rocking frequency f_r may be either smaller or larger than f_v . In the general case, the two natural frequencies of the system differ from both of these frequencies and correspond to motions that combine rotation and vertical translation (but without rotation out of the plane of the paper). These "coupled" motion frequencies always lie above and below the foregoing uncoupled motion frequencies; that is, one of the coupled motion frequencies lies below both f_v and f_r and the other lies above both f_v and f_r ; coupling in effect increases the spread between the natural frequencies.^{6,7}

The presence of coupled motions complicates the isolation problem because one needs to ensure that both of the coupled natural frequencies fall considerably below the excitation frequencies of concern. To eliminate this complication, one may select the locations and stiffnesses of the isolators so that the forces produced by them when the mass moves downward without rotation result in zero net moment about the center of gravity. This situation occurs if the isolators are linear and designed so that they have the same static deflection under the static loads to which they are subject. (Selection of isolators that have the same unloaded height as well as the same static deflection results also in keeping the isolated equipment level.) Then f_v and f_r of Eqs. (13.6) and (13.7) are the system's actual natural frequencies, and vertical forces at the center of gravity or vertical support motions produce no rocking.

Effect of Horizontal Stiffness of Isolators

In the foregoing discussion, horizontal translational motions and the effects of horizontal stiffnesses of the isolators were neglected. However, any real isolator that supports a vertical load also has a finite horizontal stiffness, and in some systems separate horizontally acting isolators may be present. Figure 13.4 is a schematic diagram of a mass on two vertical isolators (or on two rows of such isolators extending perpendicular to the plane of the paper) with stiffnesses k_1 and k_2 . The effects of the horizontal stiffnesses of these isolators or of separate horizontally acting isolators are represented by the horizontally acting spring elements with stiffnesses h_1 and h_2 , here assumed to act in the same plane at a distance b below the center of gravity of the mass.

The system of Fig. 13.4 has three degrees of freedom and therefore three natural frequencies. If the vertically acting isolators are selected and positioned so that they have the same static deflection, then (as discussed in the foregoing section) there is very little coupling between the vertical translational and the

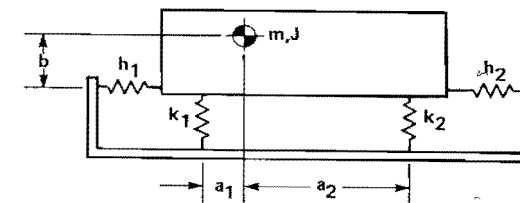


FIGURE 13.4 Mass m with moment of inertia J supported by two vertically and two collinear horizontally acting isolators.

rocking motions, and the natural frequency corresponding to vertical translation is given by Eq. (13.6). The other two natural frequencies, which are associated with combined rotation and horizontal translation, are given by the two values of f_H one may obtain from^{6,7}

$$\frac{f_H}{f_v} = (N \pm \sqrt{N^2 - SB})^{1/2} \quad (13.8)$$

where

$$N = \frac{1}{2} \left[S \left(1 + \frac{b^2}{r^2} \right) + B \right] \quad B = \frac{a_1^2 k_1 + a_2^2 k_2}{r^2 (k_1 + k_2)} \quad S = \frac{h_1 + h_2}{k_1 + k_2} \quad (13.8a)$$

Here, $r^2 = J/m$ represents the square of the radius of gyration of the mass about an axis through its center of gravity.

For a rectangular mass of uniform density, the center of gravity is in the geometric center and the radius of gyration r obeys $r^2 = \frac{1}{12}(H^2 + L^2)$, where H and L represent the lengths of the vertical and horizontal edges of the mass (in the plane of Fig. 13.4).

Nonrigidity of Mass or Support

If the mass of Fig. 13.1a is flexible instead of rigid and has a resonance at a certain frequency, then it will deflect (by deforming) considerably in response to excitation at that frequency, resulting in large transmissibility and thus in poor isolation. An analogous situation occurs if the system of Fig. 13.1b is excited at a resonance frequency of the support.

Figure 13.5 is a schematic diagram representing the vertical translational motion of a machine isolated from a nonrigid support, where the support is represented by a spring-mass system, corresponding, for example, to the static stiffness of a building's floor and to the effective mass that participates in the floor's vibration at its fundamental resonance. Isolator and floor damping have little effect on the off-resonance vibrations and may be neglected for the sake of simplicity. If f_M denotes the natural frequency of the isolated machine on a

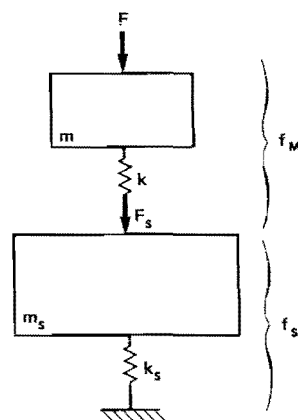


FIGURE 13.5 Schematic representation of isolated machine on nonrigid support structure. Machine is represented by mass m subject to oscillatory force F ; isolator is represented by spring k . Support structure is represented by effective mass m_s on spring k_s .

rigid support and f_s represents the natural frequency of the support without the machine in place, so that

$$2\pi f_M = \sqrt{\frac{k}{m}} \quad 2\pi f_s = \sqrt{\frac{k_s}{m_s}} \quad (13.9)$$

then the force transmissibility, that is, the ratio of the magnitude of the force F_s transmitted to the support to that of the excitation force F , may be written as

$$T_F = \frac{F_s}{F} = \left| \frac{1 - R^2}{(1 - R^2)(1 - R^2 G^2) - R^2/M} \right| \quad (13.10)$$

where $R = f/f_s$, $G = f_s/f_M$, and $M = m/m_s$ with f representing the excitation frequency.

Figure 13.6 shows a plot of the transmissibility calculated from Eq. (13.10) for the illustrative case where $M = \frac{1}{2}$, $G = 2$, together with a corresponding plot of the transmissibility that would be obtained with an immobile (infinitely rigid) support. For excitation frequencies that fall between the two resonance frequencies of the system with the nonrigid support, the transmissibility obtained with the nonrigid support is less than that with the rigid support; the reverse is true for excitation frequencies above the upper of the two resonance frequencies. For sufficiently high excitation frequencies, the difference between the two transmissibilities becomes negligible.

The two resonance frequencies f_c of the system with the nonrigid support may be found from

$$\left(\frac{f_c}{f_M}\right)^2 = P \pm \sqrt{P^2 - G^2} \quad P = \frac{1}{2}(1 + G^2 + M) \quad (13.11)$$

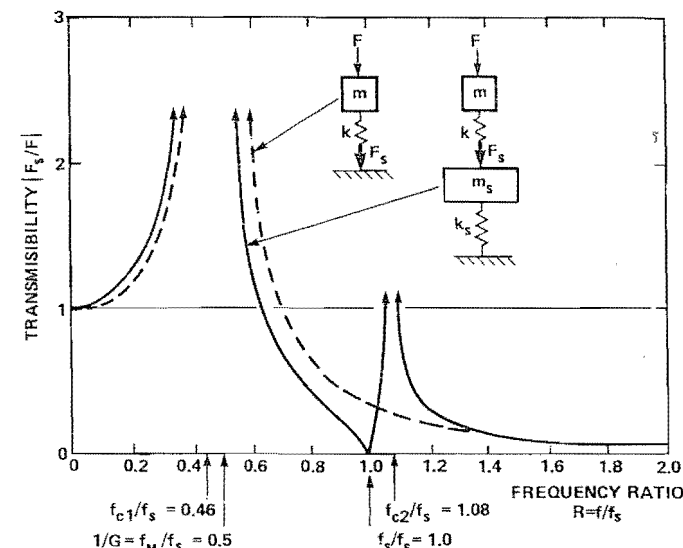


FIGURE 13.6 Comparison of transmissibilities obtained with rigid and nonrigid supports. Curves were obtained from Eqs. (13.1) and (13.10) for zero damping and for mass ratio $M = m/m_s = \frac{1}{2}$ and system frequency ratio $G = f_s/f_M = 2$. Resonance frequencies f_{c1} and f_{c2} of coupled system and f_M and f_s of uncoupled machine and support are indicated on the frequency scale.

with G and M defined as indicated after Eq. (13.10). The higher resonance frequency, obtained with the plus sign in Eq. (13.11), always lies above both f_M and f_s ; the lower resonance frequency, obtained with the minus sign, always lies below both f_M and f_s .

13.4 HIGH-FREQUENCY CONSIDERATIONS

Although the relations discussed in the foregoing sections usually suffice for analysis and design in cases where only relatively low frequencies are of concern (i.e., for frequencies that are generally below the resonance frequencies of the machine and support structures themselves), at higher frequencies there occur complications that may need to be taken into account. At high frequencies, the isolated item and the support structure cease to behave as rigid masses, isolators may exhibit internal resonances, and vibration sources tend to be affected by "loading."

Loading of Sources

Loading of a source refers to the reduction of the source's vibratory motion that results from a force that opposes this motion. For example, the vibrations of a

sheet metal enclosure tend to be reduced by the opposing forces produced when a hand is placed against it. Similarly, a factory floor that vibrates with a certain amplitude tends to vibrate less when a (nonvibrating) machine is bolted to it as the result of reaction forces produced by the machine.

In the latter example, if the machine is isolated from the floor, it is likely to produce smaller reaction forces on the floor than it would produce if it is bolted rigidly to the floor, and the floor may be expected to vibrate more if the machine is isolated than if it is rigidly fastened to the floor. Since greater floor vibrations then are transmitted to the machine, use of isolation will protect the machine less than anticipated from transmissibility considerations.

To be able to evaluate how much protection of the machine the isolator provides in this case, one needs a quantitative description of the source's response to loading. One may obtain such a description by measuring the motion produced by the source as it acts on several different masses or structures with different dynamic characteristics (impedances), which produce different (known or measurable) reaction forces. For prediction purposes, it often suffices to assume that the source's response to loading at any given frequency is "linear"; that is, the motion amplitude produced by the source decreases in proportion to the amplitude F_0 of the reaction force (which is equal to the amplitude of the force that the source generates). One may describe source motions at a given frequency equally well in terms of acceleration, displacement, or velocity, but use of velocity has become customary. For a linear source one may express the dependence of the source's velocity amplitude V_0 on the force as⁸

$$V_0 = V_{\text{free}} - M_s F_0 \quad (13.12)$$

Here V_{free} represents the velocity amplitude that the source generates if it is free of reaction forces, that is, if it produces zero force. (In general, the parameters V_{free} and M_s for a given source may be different for different frequencies.) The quantity M_s , which indicates how rapidly V_0 decreases with increasing F_0 , is known as the *source mobility* and may be found from $M_s = V_{\text{free}}/F_{\text{blocked}}$, where F_{blocked} denotes the force amplitude obtained if the source is "blocked" so that it has zero velocity V_0 .

One may readily verify that $M_s = 0$ corresponds to a *velocity source*, that is, to a source whose output velocity amplitude is constant, regardless of the magnitude of its output force F_0 . Similarly, infinite M_s corresponds to a *force source*, whose output force F_0 is constant and independent of its output velocity.⁸ (For example, a rotating unbalanced mass generates forces that are virtually independent of its support motions and thus acts essentially like a force source. On the other hand, a piston driven by a shaft with a large flywheel moves at essentially the same amplitude regardless of the force acting on it and thus behaves like a velocity source.)

Isolation Effectiveness^{3,8}

In the presence of significant source loading, one cannot evaluate the performance of an isolation system on the basis of transmissibility because in the definition

of transmissibility the magnitudes of the disturbances are prescribed and thus, in effect, are taken as constant. A measure of isolation performance that is useful in the presence of loading is the so-called *isolation effectiveness* E . Isolation effectiveness is defined as the ratio of the magnitude of the vibrational velocity of the item to be protected (called the "receiver") that results if the item is rigidly connected to the source to the magnitude of the receiver's velocity that is obtained if the isolator is inserted between the source and the receiver in place of the rigid connection. The definition of isolation effectiveness is analogous to that of insertion loss in airborne acoustics.

If the receiver velocity V_R is proportional to the force F_R that acts on the receiver, so that $V_R = M_R F_R$, where M_R is called receiver mobility,* then the isolation effectiveness may be expressed in terms of a ratio of forces acting on the receiver as well as of a ratio of receiver velocities, namely,

$$E = \frac{V_{Rr}}{V_{Ri}} = \frac{F_{Rr}}{F_{Ri}} \quad (13.13)$$

where the added subscript r refers to the case in which a rigid connection replaces the isolator and the subscript i refers to the situation where the isolator is present.

Whereas small transmissibility T corresponds to good isolation, it is large values of E that imply effective isolation. For this reason, the reciprocal of the effectiveness is sometimes used to characterize the performance of an isolation system. Although this reciprocal differs from the transmissibility T in the general situation where the source is affected by loading, in the special case of sources that are unaffected by loading (i.e., for sources that generate load-independent velocity or force amplitudes), $E = 1/T$.

Effectiveness of Massless Linear Isolator

An isolator may be considered as "massless" if it transmits whatever force is applied to it. (Equal and opposite forces must act on the two sides of a massless isolator if it is not to accelerate infinitely.) A "linear" isolator is one whose deflection is proportional to the applied force. The velocity difference across such an isolator at any frequency then is also proportional to the applied force, and the ratio of the magnitude of this velocity difference to that of the applied force is called the isolator's mobility M_I . Like the reciprocal of isolator stiffness, M_I is large for a soft isolator and zero for a rigid isolator.

For a massless linear isolator, the effectiveness obeys^{8,11}

$$E = \left| 1 + \frac{M_I}{M_S + M_R} \right| \quad (13.14)$$

*Mobilities of receivers at their attachment points may be estimated analytically for simple configurations (e.g., refs. 9 and 10) or may be measured (as functions of frequency). The velocities and forces usually are expressed in terms of complex numbers, or *phasors*, which indicate both the magnitudes and the relative phases of sinusoidally varying quantities. Mobilities then also are complex quantities in general.

At a resonance of the receiver, the receiver's vibratory velocity V_R resulting from a given force F_R is large, so that the receiver's mobility is large. In view of Eq. (13.14), the effectiveness of an isolator is small in the presence of such a resonance.

Equation (13.14) also indicates that the effectiveness is small if the source mobility M_S is large. For a force source, for which M_S is infinite, the effectiveness is equal to unity, implying that the receiver vibrates just as much with the isolator in place as it does if the isolator is replaced by a rigid connection. This initially somewhat surprising result is correct: After all, the force source generates the same force, regardless of the velocity or displacement it produces, and the isolator transmits all of this force, with a softer isolator merely leading to greater displacement of the source at its output point.

Consequences of Isolator Mass Effects

Isolator mass effects may be neglected—that is, an isolator may be considered as massless—as long as the frequencies under consideration are appreciably lower than the first internal or *standing-wave* resonance frequency of the isolator.* Such standing-wave resonances tend to reduce the isolator's effectiveness severely, as illustrated by Fig. 13.7. This figure shows the calculated transmissibility of a leaf spring modeled as a uniform cantilever beam. The upper left-hand corner of the plot may be recognized as the usual transmissibility curve (similar to Fig. 13.2) in the vicinity of the resonance frequency $f_n = (1/2\pi)\sqrt{k/m}$ obtained for a massless isolator. With increasing excitation frequency the transmissibility does not decrease monotonically, as it would for a massless spring (see curve for $m_{sp} = 0$); instead, there occur secondary peaks associated with standing-wave resonances of the beam. The frequency at which these peaks begin to occur increases as the ratio of the isolated mass m to the mass m_{sp} of the spring increases. Although in the figure only two peaks are shown for each mass ratio, there actually occurs a succession of peaks that become more closely spaced with increasing frequency. The magnitude of these peaks decreases with increasing damping.

To reduce the effects of standing-wave resonances, one thus needs to select an isolator with relatively high damping and a configuration for which the onset of standing-wave resonances occurs at comparatively high frequencies. This implies use of a material with high stiffness-to-weight ratio or, equivalently, with a high longitudinal wave velocity $\sqrt{E/\rho}$ (where E denotes the material's modulus of elasticity and ρ its density), and also use of a configuration with small overall dimensions.

*At lower frequencies, the only effect of the mass of the isolator is to reduce slightly the fundamental resonance frequency of the system. The modified resonance frequency may be calculated from the isolator stiffness and a mass consisting of the isolated mass plus a fraction of the mass of the isolator. If the isolator consists of a uniform spring or pad in compression or shear, the fraction is $\frac{1}{3}$; if the isolator consists of a uniform cantilever beam, the fraction is approximately 0.24.

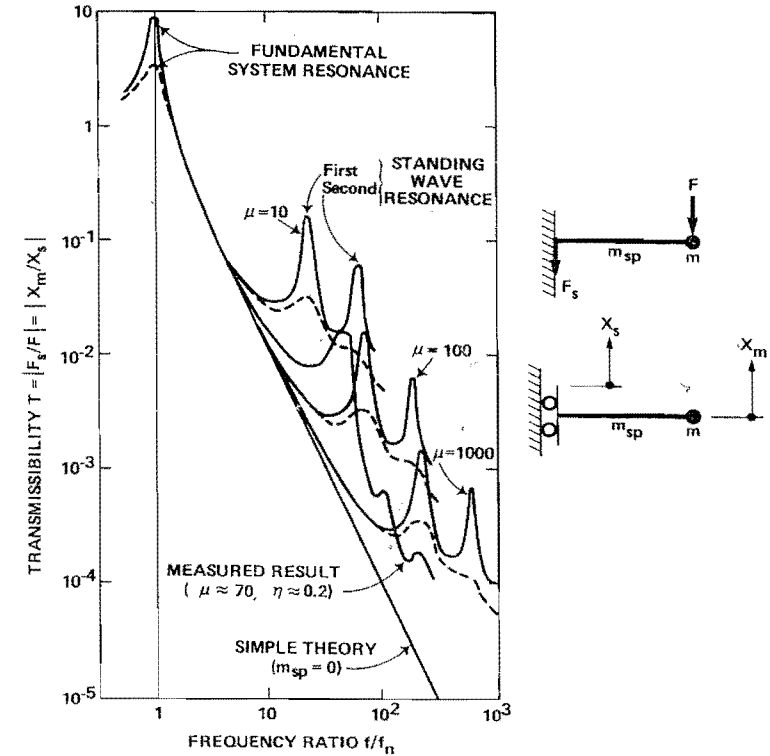


FIGURE 13.7 Effect of isolator mass and damping on transmissibility of uniform cantilever for three values of ratio $\mu = m/m_{sp}$ of isolated mass to mass of cantilever spring. (After Refs. 11 and 12.) Solid calculated lines are for loss factor $\eta = 0.1$; dashed lines for $\eta = 0.6$. (Measured result shown is from Ref. 13.) Frequency is normalized to f_n , the fundamental resonance frequency obtained with a massless spring.

The isolation effectiveness at any specified frequency of a system in which isolator mass effects are not negligible is given by⁸

$$E = \left| \frac{\alpha}{M_S + M_R} \right| \left| 1 + \frac{M_S}{M_{lsb}} + \frac{M_R}{M_{lrb}} \left(1 + \frac{M_S}{M_{lsf}} \right) \right| \quad (13.15)$$

$$\frac{1}{\alpha^2} = \frac{1}{M_{lrb}} \left(\frac{1}{M_{lsb}} - \frac{1}{M_{lsf}} \right)$$

where M_{lsb} denotes the isolator mobility (i.e., the velocity-to-force ratio) measured on the source side of the isolator if the receiver side of the isolator is "blocked" (i.e., prevented from moving), M_{lrb} denotes the mobility measured on the receiver side of the isolator if the source side is blocked, and M_{lsf} denotes the mobility measured on the source side if the receiver side is "free"

or unconstrained. One may readily verify that Eq. (13.15) reduces to Eq. (13.14) for massless isolators, for which $M_{lsb} = M_{lrb} = M_I$ and M_{lsf} is infinite.

13.5 TWO-STAGE ISOLATION

A force applied to one side of a massless isolator, as has been mentioned, must be balanced by an equal and opposite force at the other side of the isolator. This is not the case for an isolator that incorporates some mass, because the force applied to one side then is balanced by the sum of the inertia force and the force acting on the other side of the isolator. Thus, unlike for a massless isolator, the force transmitted by an isolator with mass can be less than the applied force.

One may realize such a force reduction benefit, even at low frequencies at which mass effects in isolators themselves are negligible, by adding a lumped mass “inside” an isolator. One may visualize this concept by considering a spring that is cut into two lengths with a rigid block of mass welded in place between the two parts, resulting in an isolator consisting of two lengths of spring with a mass between them. If a mass m is mounted atop this isolator, one obtains a system that may be represented by a diagram like that of Fig. 13.8a. Because this system consists of a cascade of two spring-mass systems, it is said to have two stages of isolation.

Transmissibility

The system of Fig. 13.8a has two natural frequencies f_b that may be found from

$$\left(\frac{f_b}{f_0}\right)^2 = Q \pm \sqrt{Q^2 - B^2} \quad Q = \frac{1}{2} \left(B^2 + 1 + \frac{k_2}{k_1} \right) \quad (13.16)$$

where

$$B = \frac{f_1}{f_0} \quad 2\pi f_1 = \sqrt{\frac{k_1 + k_2}{m_I}} \quad 2\pi f_0 = \frac{1}{\sqrt{m(1/k_1 + 1/k_2)}} \quad (13.16a)$$

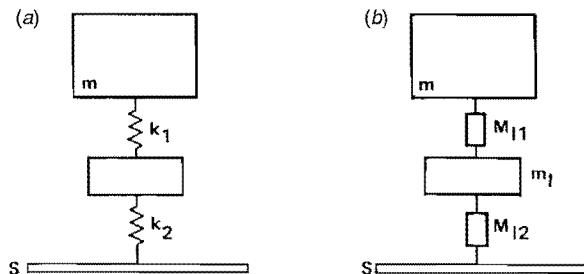


FIGURE 13.8 Two-stage isolation with intermediate mass m_I : (a) springs; (b) general isolation elements.

The frequency f_0 is the natural frequency of the system in the absence of any included mass m_I ; that is, it is the natural frequency of a conventional simple single-stage system. The frequency f_I is the natural frequency of mass m_I moving between the two springs, with mass m held completely immobile. The upper frequency f_b , which one obtains if one uses the plus sign before the square root, always is greater than both f_0 and f_I ; the lower frequency f_b , corresponding to the minus sign, always falls below both f_0 and f_I .

The transmissibility of a two-stage system like that of Fig. 13.8a obeys

$$\frac{1}{T} = \frac{1}{T_{F0}} = \frac{1}{B^2} \left(\frac{f}{f_0} \right)^4 - \left[1 + \frac{1 + k_2/k_1}{B^2} \right] \left(\frac{f}{f_0} \right)^2 + 1 \approx \left(\frac{f^2}{f_0 f_I} \right)^2 \quad (13.17)$$

where the last approximate expression applies for high frequencies, namely, for excitation frequencies that are much greater than both f_0 and f_I .

Figure 13.9 shows an illustrative plot of the transmissibility of an undamped two-stage system with $B = f_I/f_0 = 5$ and $k_2/k_1 = 1$, together with a plot of the transmissibility of an (undamped) single-stage system. The second natural frequency of the two-stage system (at $f/f_0 \approx 5.1$) is clearly evident, as is the subsequent rapid decrease in that system's transmissibility with increasing frequency. At high frequencies, that is, a little above the aforementioned second natural frequency, the transmissibility of the two-stage system may be seen to be smaller than that of a single-stage system with the same fundamental natural frequency. As evident from Eq. (13.17), the high-frequency transmissibility of a two-stage system varies inversely as the fourth power of the excitation frequency, whereas Eq. (13.5) indicates that the transmissibility of a single-stage system varies inversely as only the second power of the excitation frequency.

The advantage of a two-stage system over a single-stage system is that it results in greatly reduced transmissibility at high frequencies (i.e., above the higher of its two natural frequencies). It has the disadvantage that it introduces an additional transmissibility peak at a low frequency (i.e., at its second natural frequency). Thus, two-stage isolation is beneficial in general only if the aforementioned second natural frequency is somewhat lower than the lowest excitation frequency of concern. Thus, to reap the benefit of a two-stage system, one typically needs a relatively large intermediate mass m_I . Where several items need to be isolated, it often is advantageous to support these on a common massive platform (often called a “subbase” or “raft”), to isolate each item from the platform, and to isolate the platform from the structure that supports it. In this arrangement the platform serves as a relatively large intermediate mass for each of the isolated items, resulting in efficient two-stage isolation performance with comparatively small weight penalty.

Isolation Effectiveness

Although the foregoing results apply strictly only to isolators without damping, they also provide a reasonable approximation to the behavior of lightly damped systems, except near the natural frequencies. To account for high damping or more

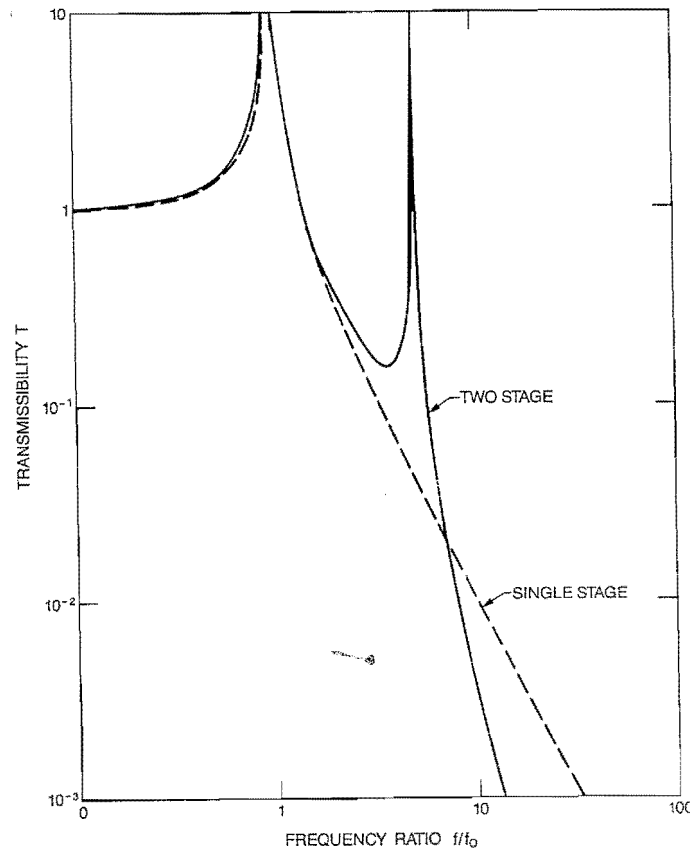


FIGURE 13.9 Transmissibility of two-stage system. Calculated from Eq. (13.17) for $k_2/k_1 = 1$ and $B = f_I/f_0 = 5$.

complicated linear isolator configurations (e.g., where each isolator is modeled by various series and parallel combinations of springs and dampers), it is convenient to represent each isolator by its mobility. A corresponding diagram appears in Fig. 13.8b. As has been discussed in Section 13.4, the source mobility M_S is a measure of a vibration source's susceptibility to loading effects, and the isolation effectiveness E is a measure of the isolation performance, which, unlike transmissibility, takes loading effects into account. If one takes S in Fig. 13.8b to represent a general linear source and replaces the mass m by a general linear receiver with mobility M_R , one obtains a general linear two-stage system whose effectiveness one may write as⁸

$$E = |E_1 + \Delta E| \quad E_1 = 1 + \frac{M_I}{M_S + M_R} \quad (13.18)$$

where

$$M_I = M_{I1} + M_{I2} \quad \Delta E = \frac{(M_{I1} + M_S)(M_{I2} + M_R)}{M_m(M_S + M_R)} \quad (13.18a)$$

One may recognize E_1 as corresponding to the effectiveness of a single-stage system [see Eq. (13.14)], that is, to a two-stage system with zero included mass, with M_I denoting the mobility of the two partial isolators in series. Thus, ΔE represents the effectiveness increase obtained by addition of the included mass m , whose mobility is M_m . Note that ΔE is inversely proportional to M_m , indicating that greater included masses generally result in greater effectiveness increases.

Optimization of Isolator Stiffness Distribution

Once one has selected the mobility M_I of the total isolator (or, equivalently, its compliance or stiffness), one needs to consider how to allocate this mobility among the components M_{I1} and M_{I2} . If one lets r_1 denote the fraction of the total mobility on the source side of the included mass, so that $M_{I1} = r_1 M_I$ and $M_{I2} = (1 - r_1) M_I$, it turns out that one may obtain the largest value of ΔE , namely,

$$\Delta E_{\max} = \frac{(M_I + M_S + M_R)^2}{4M_m(M_S + M_R)} \quad (13.19)$$

by making r_1 equal to its optimum value,*

$$r_{\text{opt}} = \frac{1}{2} \left(1 + \frac{M_R - M_S}{M_I} \right) \quad (13.20)$$

In view of Eq. (13.18a), in an efficient isolation system M_{I1} must be considerably greater than M_S , and also M_{I2} must be considerably greater than M_R . For such a system, one finds that $r_{\text{opt}} \approx \frac{1}{2}$ and that $\Delta E_{\max}$ may be approximated by replacing the expression in the parentheses of the numerator of Eq. (13.19) by M_I . Thus, if the total mobility of the isolator is sufficiently great, that is, if the total stiffness of the isolator is sufficiently small, one may generally obtain the greatest improvement $\Delta E_{\max}$ by allocating the same mobility or stiffness to the two isolator components.

It may be shown⁸ that placing a given mass "within" the isolator as described above, so as to obtain a two-stage system with two like isolator mobility components, results in greater effectiveness than placement of the mass directly at the receiver as long as $M_I \gg M_R$. This inequality usually is likely to be satisfied in practice, except at resonances of the receiver at which M_R is very large. Similarly, as long as $M_I \gg M_S$, placing the mass within the isolator results in greater

*This result applies strictly only if the various mobilities or mobility ratios are real quantities. It suffices for development of an intuitive understanding, although more complicated expressions apply in the general case where the mobilities are represented by complex quantities.

effectiveness than placement of the mass directly at the source. The foregoing inequality generally is likely to be satisfied in practice, except for sources that behave essentially like force sources and thus have very high mobility.

13.6 PRACTICAL ISOLATORS

A great many different isolators are available commercially in numerous sizes and load capacities, with various attachment means, and with many types of specialized features.⁴ Details concerning such isolators typically may be found in suppliers' catalogs.

Most commercial isolators incorporate metallic or elastomeric resilient elements. Metallic elements most often are in the form of coil springs but also occur in the form of flexural configurations such as leaf springs or conical Belleville washers. Coil springs are predominantly used in compression, usually because such tensile springs tend to involve configurations that give rise to stress concentrations and thus have lesser fatigue life. Coil spring isolator assemblies often involve parallel and/or series arrangements of springs in suitable housings, are designed to have the same stiffness in the lateral directions as in the axial direction, and may also incorporate friction devices (such as wire mesh inserts), snubbers to limit excursions due to large disturbances (such as earthquakes), and in-series elastomeric pads for enhanced damping and isolation at high frequencies. Spring systems in housings need to be installed with care to avoid binding between the housing elements and between these elements and the springs.

Some commercial metallic isolators employ pads or woven assemblies of wire mesh to provide both resilience and damping. Others use arrangements of coils or loops of wire rope not only to provide damping but also to serve as springs.

A great many commercially available isolators that employ elastomeric elements have these elements bonded or otherwise attached to support plates or sleeves that incorporate convenient means for fastening to other components. The isolators may be designed so that the elastomeric element is used in shear, torsion, compression, or a combination of these modes. There are also available a variety of elastomeric gaskets, grommets, sleeves, and washers, intended to be used with bolts or similar fasteners to provide both connection and isolation.

Elastomeric pads often are used as isolators by themselves, as are pads of other resilient materials, such as cork, felt, fiberglass, and metal mesh. Such pads often are convenient and relatively inexpensive; their areas can be selected to support the required loads, and their thicknesses can be chosen to provide the desired stiffness.

In the design and selection of pads of solid (in contrast to foamed) elastomeric materials, one needs to take into account that a pad's stiffness depends not only on its thickness and area but also on its shape and constraints. This behavior is due to the incompressibility of elastomeric materials, which essentially prevents a pad from changing its volume as it is compressed and thus in essence does not permit a pad to be compressed if it is confined so that its edges cannot bulge outward. A pad's freedom to compress may be characterized by its shape factor, defined

as the ratio of its loaded area to the total area of the edges that are free to bulge; the greater the shape factor, the greater is the pad's effective stiffness. However, a pad's freedom to deform while maintaining constant volume also is affected by how easily the loaded surfaces can slip relative to the adjacent surfaces; the more restricted this slippage, the greater the pad's effective stiffness. Some commercial isolation pads are furnished with top and bottom load-carrying surfaces bonded to metal or other stiff plates to eliminate the stiffness uncertainties due to unpredictable slippage.

To avoid the need for considering the shape factor in pad selection, many commercial isolation pad configurations have a multitude of cutouts (e.g., closely spaced arrays of holes) or ribs, which provide roughly constant amounts of bulging area per unit surface area. If ribbed or corrugated pads or pads with cutouts are used in stacks, plates of a stiff material (e.g., metal sheets) generally are used between pads to distribute the load on the load-bearing surfaces and to avoid having protrusions on one pad extending into openings on the adjacent pad.

So-called pneumatic, or air spring, isolators, which have found considerable use, obtain their resilience primarily from the compressibility of confined volumes of air. They may take the form of air-filled pillows of rubber or plastic, often with cylindrical or annular shapes, or they may consist essentially of piston-in-cylinder arrangements. Air springs can be designed to have small effective stiffnesses while supporting large loads and to have smaller heights than metal springs of equal stiffness. Practical air springs typically can provide fundamental resonance frequencies that may be as low as about 1 Hz. Some air spring configurations are laterally unstable under some load conditions and require the use of lateral restraints; some are available with considerable lateral stability.

Air springs of the piston-and-cylinder type can be provided with leveling controls, which automatically keep the isolated item's static position at a predetermined distance from a reference surface and (by use of several air springs and a suitable control system) at a predetermined inclination. The stiffness of a piston-type air spring is proportional to PA^2/V , where P denotes the air pressure, A the piston face area, and V the cylinder volume. The product $(P - P_0)A$, where P_0 represents the ambient atmospheric pressure, is equal to the static load carried by the spring. Lower stiffnesses may be obtained with a given area at a given air pressure by use of larger effective volumes; for this reason, some commercial air spring isolators are available with auxiliary tanks that communicate with the cylinder volume via piping. In some instances, a flow constriction in this piping is used to provide low-frequency damping. If the pressure in a piston-and-cylinder-type air spring is considerably greater than the atmospheric pressure, the pressure in the spring is nearly proportional to the load it supports. Because the spring's stiffness is proportional to this pressure, the natural frequency one obtains with such an air spring isolation system is essentially independent of the load, making air springs (like other constant-natural-frequency systems⁴) particularly useful for applications in which the loads are variable or uncertain.

Pendulum arrangements often are convenient means for obtaining horizontally acting isolation systems with low natural frequencies. One may calculate