

# Atomization and Sprays

Arthur H. Lefebvre

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# Atomization and Sprays

The science and technology of atomization have undergone considerable expansion, and atomization has now evolved into a major international, interdisciplinary field of research. It has become necessary for engineers to have a fuller understanding of basic atomization processes as well as characterization of sprays—needs that are met in this unparalleled hands-on reference source.

The text presents a scientific basis for understanding the physical processes of atomization and guidelines for atomizer design. Moreover, it demonstrates how measurement of particle size and velocity contribute to improved spray characterization.

Included in this unique coverage are such key areas as:

- Basic processes in atomization
- Disintegration of liquid jets and sheets
- Various types of atomizers
- Flow in atomizers
- Drop size distributions
- Various types of sprays
- Effects of liquid and gas properties on sprays
- Measuring spray characteristics
- Particle sizing
- Imaging

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## ATOMIZATION AND SPRAYS

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## GENERAL CONSIDERATIONS

### INTRODUCTION

The transformation of bulk liquid into sprays and other physical dispersions of small particles in a gaseous atmosphere is of importance in several industrial processes and has many other applications in agriculture, meteorology, and medicine. Numerous spray devices have been developed, and they are generally designated as atomizers or nozzles. The process of atomization is one in which a liquid jet or sheet is disintegrated by the kinetic energy of the liquid itself, or by exposure to high-velocity air or gas, or as a result of mechanical energy applied externally through a rotating or vibrating device. Because of the random nature of the atomization process the resultant spray is usually characterized by a wide spectrum of drop sizes.

Natural sprays include waterfall mists, rains, and ocean sprays. In the home, sprays are produced by shower heads, garden hoses, and hair sprays. They are commonly used in applying agricultural chemicals to crops, paint spraying, spray drying of wet solids, food processing, cooling of nuclear cores, gas-liquid mass transfer applications, dispersing liquid fuels for combustion, and many other applications.

Combustion of liquid fuels in diesel engines, spark ignition engines, gas turbines, rocket engines, and industrial furnaces is dependent on effective atomization to increase the specific surface area of the fuel and thereby achieve high rates of mixing and evaporation. In most combustion systems, reduction in mean fuel drop size leads to higher volumetric heat release rates, easier lightup, a wider burning range, and lower exhaust concentrations of pollutant emissions [1–3]. In other applications, however, such as crop spraying, small droplets must be avoided

because their settling velocity is low and, under certain meteorological conditions, they can drift too far downwind. Drop sizes are also important in spray drying and must be closely controlled to achieve the desired rates of heat and mass transfer.

During the past decade there has been a tremendous expansion of interest in the science and technology of atomization, which has now developed into a major international and interdisciplinary field of research. This growth of interest has been accompanied by large strides in the area of laser diagnostics for spray analysis and by a proliferation of mathematical models for spray combustion processes. It is becoming increasingly important for engineers to acquire a better understanding of the basic atomization process and to be fully conversant with the capabilities and limitations of all the relevant atomization devices. In particular, it is important to know which type of atomizer is best suited for any given application and how the performance of any given atomizer is affected by variations in liquid properties and operating conditions.

## ATOMIZATION

Sprays may be produced in various ways. There are several basic processes associated with all methods of atomization, such as the hydraulics of the flow within the atomizer, which governs the turbulence properties of the emerging liquid stream. The development of the jet or sheet and the growth of small disturbances, which eventually lead to disintegration into ligaments and then drops, are also of primary importance in determining the shape and penetration of the resulting spray as well as its detailed characteristics of number density, drop velocity, and drop size distributions as functions of time and space. All these characteristics are markedly affected by the internal geometry of the atomizer, the properties of the gaseous medium into which the liquid stream is discharged, and the physical properties of the liquid itself. Perhaps the simplest situation is the disintegration of a liquid jet issuing from a circular orifice, where the main velocity component lies in the axial direction and the jet is in laminar flow. In his classic study [4], Lord Rayleigh postulated the growth of small disturbances that eventually lead to breakup of the jet into drops having a diameter nearly twice that of the jet. A fully turbulent jet can break up without the application of any external force. Once the radial components of velocity are no longer confined by the orifice walls, they are restrained only by surface tension, and the jet disintegrates when the surface tension forces are overcome. The role of viscosity is to inhibit the growth of instabilities and generally delay the onset of disintegration. This causes atomization to occur farther downstream in regions of lower relative velocity; consequently, drop sizes are larger. In most cases, turbulence in the liquid, cavitation in the nozzle, and aerodynamic interaction with the surrounding air, which increases with air density, all contribute to atomization.

Many applications call for a conical or flat spray pattern to achieve the desired dispersion of drops for liquid-gas mixing. Conical sheets may be produced by

pressure-swirl nozzles in which a circular discharge orifice is preceded by a chamber in which tangential holes or slots are used to impart a swirling motion to the liquid as it leaves the nozzle. Flat sheets are generally produced either by forcing the liquid through a narrow annulus, as in fan spray nozzles, or by feeding it to the center of a rotating disk or cup. To expand the sheet against the contracting force of surface tension, a minimum sheet velocity is required and is produced by pressure in pressure-swirl and fan spray nozzles and by centrifugal force in rotary atomizers. Regardless of how the sheet is formed, its initial hydrodynamic instabilities are augmented by aerodynamic disturbances, so as the sheet expands away from the nozzle and its thickness declines, perforations are formed that expand toward each other and coalesce to form threads and ligaments. As these ligaments vary widely in diameter, when they collapse the drops formed also vary widely in diameter. Some of the larger drops created by this process disintegrate further into smaller droplets. Eventually a range of drop sizes is produced whose average diameter depends mainly on the initial thickness of the liquid sheet, its velocity relative to the surrounding gas, and the liquid properties of viscosity and surface tension.

A liquid sheet moving at high velocity can also disintegrate in the absence of perforations by a mechanism known as *wavy-sheet* disintegration, whereby the crests of the waves created by aerodynamic interaction with the surrounding gas are torn away in patches. Finally, at very high liquid velocities, corresponding to high injection pressures, sheet disintegration occurs close to the nozzle exit. However, although several modes of sheet disintegration have been identified, in all cases the final atomization process is one in which ligaments break up into drops according to the Rayleigh mechanism.

With prefilming airblast atomizers, a high relative velocity is achieved by exposing a slow-moving sheet of liquid to high-velocity air. Photographic evidence suggests that for low-viscosity liquids the basic mechanisms involved are essentially the same as those observed in pressure atomization, namely the production of drops from ligaments created by perforated-sheet and/or wavy-sheet disintegration.

A typical spray includes a wide range of drop sizes. Some knowledge of drop size distribution is helpful in evaluating process applications in sprays, especially in calculations of heat or mass transfer between the dispersed liquid and the surrounding gas. Unfortunately, no complete theory has yet been developed to describe the hydrodynamic and aerodynamic processes involved when jet and sheet disintegration occurs under normal atomizing conditions, so that only empirical correlations are available for predicting mean drop sizes and drop size distributions. Comparison of the distribution parameters in common use reveals that all of them have deficiencies of one kind or another. In one the maximum drop diameter is unlimited; in others the minimum possible diameter is zero or even negative. So far, no single parameter has emerged that has clear advantages over the others. For any given application the best distribution function is one that is easy to manipulate and provides the best fit to the experimental data.

The difficulties in specifying drop size distributions in sprays have led to

widespread use of various mean or median diameters. A median droplet diameter divides the spray into two equal parts by number, length, surface area, or volume [5]. Median diameters may be determined from cumulative distribution curves of the types shown in Fig. 3.6. In a typical spray the value of the median diameter, expressed in micrometers, will vary by a factor of about four depending on the median diameter selected for use. It is important therefore to decide which measure is the most suitable for a particular application. Some diameters are easier to visualize and comprehend, while others may appear in prediction equations that have been derived from theory or experiment. Some drop size measurement techniques yield a result in terms of one particular median diameter. In some cases a given median diameter is selected to emphasize some important characteristic, such as the total surface area in the spray. For liquid fuel-fired combustion systems and other applications involving heat and mass transfer to liquid drops, the Sauter mean diameter, which represents the ratio of the volume to the surface area of the spray, is often preferred. The mass median diameter, which is about 15 to 25% larger than the Sauter mean diameter, is also widely used. As Tate [5] has pointed out, the ratio of these two diameters is a measure of the spread of drop sizes in the spray.

## ATOMIZERS

Sprays may be produced in various ways. Essentially, all that is needed is a high relative velocity between the liquid to be atomized and the surrounding air or gas. Some atomizers accomplish this by discharging the liquid at high velocity into a relatively slow-moving stream of air or gas. Notable examples include the various forms of pressure atomizers and also rotary atomizers, which eject the liquid at high velocity from the periphery of a rotating cup or disk. An alternative approach is to expose the relatively slow-moving liquid to a high-velocity airstream. The latter method is generally known as *twin-fluid*, *air-assist*, or *airblast* atomization.

### Pressure Atomizers

When a liquid is discharged through a small aperture under high applied pressure, the pressure energy is converted into kinetic energy (velocity). For a typical hydrocarbon fuel, in the absence of frictional losses a nozzle pressure drop of 138 kPa (20 psi) produces an exit velocity of 18.6 m/s. As velocity increases as the square root of the pressure, at 689 kPa (100 psi) a velocity of 41.5 m/s is obtained, while 5.5 MPa (800 psi) produces 117 m/s.

**Plain orifice.** A simple circular orifice is used to inject a round jet of liquid into the surrounding air. Finest atomization is achieved with small orifices but, in practice, the difficulty of keeping liquids free from foreign particles usually limits the minimum orifice size to around 0.3 mm. Combustion applications for plain-

orifice atomizers include turbojet afterburners, ramjets, diesel engines, and rocket engines.

**Pressure-swirl (simplex).** A circular outlet orifice is preceded by a swirl chamber into which liquid flows through a number of tangential holes or slots. The swirling liquid creates a core of air or gas that extends from the discharge orifice to the rear of the swirl chamber. The liquid emerges from the discharge orifice as an annular sheet, which spreads radially outward to form a hollow conical spray. Included spray angles range from  $30^\circ$  to almost  $180^\circ$ , depending on the application. Atomization performance is generally good. Finest atomization occurs at high delivery pressures and wide spray angles.

For some applications a spray in the form of a solid cone is preferred. This can be achieved by using an axial jet or some other device to inject droplets into the center of the hollow conical spray pattern produced by the swirl chamber. These two modes of injection create a bimodal distribution of drop sizes, droplets at the center of the spray being generally larger than those near the edge.

**Square spray.** This is essentially a solid-cone nozzle, but the outlet orifice is specially shaped to distort the conical spray into a pattern that is roughly in the form of a square. Atomization quality is not as high as with conventional hollow-cone nozzles but, when used in multiple-nozzle combinations, a fairly uniform coverage of large areas can be achieved.

**Duplex.** A drawback of all types of pressure nozzles is that the liquid flow rate is proportional to the square root of the injection pressure differential. In practice, this limits the flow range of simplex nozzles to about 10:1. The duplex nozzle overcomes this limitation by feeding the swirl chamber through two sets of distributor slots, each having its own separate liquid supply. One set of slots is much smaller in cross-sectional area than the other. The small slots are termed *primary* and the large slots *secondary*. At low flow rates all the liquid to be atomized flows into the swirl chamber through the primary slots. As the flow rate increases, so does the injection pressure. At some predetermined pressure level a valve opens and admits liquid into the swirl chamber through the secondary slots.

Duplex nozzles allow good atomization to be achieved over a range of liquid flow rates of about 40:1 without the need to resort to excessively high delivery pressures. However, near the point where the secondary liquid is first admitted into the swirl chamber, there is a small range of flow rates over which atomization quality is poor. Moreover, the spray cone angle changes with flow rate, being widest at the lowest flow rate and becoming narrower as the flow rate is increased.

**Dual orifice.** This is similar to the duplex nozzle except that two separate swirl chambers are provided, one for the primary flow and the other for the secondary flow. The two swirl chambers are housed concentrically within a single nozzle body to form a "nozzle within a nozzle." At low flow rates all the liquid passes

through the inner primary nozzle. At high flow rates liquid continues to flow through the primary nozzle but most of the liquid is passed through the outer secondary nozzle, which is designed for a much larger flow rate. As with the duplex nozzle, there is a transition phase, just after the pressurizing valve opens, when the secondary spray draws its energy for atomization from the primary spray, so the overall atomization quality is relatively poor.

Dual-orifice nozzles offer more flexibility than duplex nozzles. For example, if desired it can be arranged for the primary and secondary sprays to merge just downstream of the nozzle to form a single spray. Alternatively, the primary and secondary nozzles can be designed to produce different spray angles, the former being optimized for low flow rates and the latter optimized for high flow rates.

**Spill return.** This is essentially a simplex nozzle, but with a return flow line at the rear or side of the swirl chamber and a valve to control the quantity of liquid removed from the swirl chamber and returned to supply. Very high turndown ratios are attainable with this design. Atomization quality is always good because the supply pressure is held constant at a high value, reductions in flow rate being accommodated by adjusting the valve in the spill return line. This construction provides a hollow-cone spray pattern, with some increase in spray angle as the flow is reduced.

**Fan spray.** Several different concepts are used to produce flat or fan-shaped sprays. The most popular type of nozzle is one in which the orifice is formed by the intersection of a V groove with a hemispheric cavity communicating with a cylindrical liquid inlet [5]. It produces a liquid sheet parallel to the major axis of the orifice, which disintegrates into a narrow elliptical spray.

An alternative method of producing a fan spray is by discharging the liquid through a plain circular hole onto a curved deflector plate. The deflector method produces a somewhat coarser spray pattern. Wide spray angles and high flow rates are attainable with this type of nozzle. Because the nozzle flow passages are relatively large, the problem of plugging is minimized.

A fan spray can also be produced by the collision of impinging jets. If two liquid jets are arranged to collide outside the nozzle, a flat liquid sheet is formed that is perpendicular to the plane of the jets. The atomization performance of this type of injector is relatively poor, and high stream velocities are necessary to approach the spray quality obtainable with other types of pressure nozzles. Extreme care must be taken to ensure that the jets are properly aligned. The main advantage of this method of atomization is the isolation of different liquids until they collide outside the nozzle.

### Rotary Atomizers

One widely used type of rotary atomizer comprises a high-speed rotating disk with means for introducing liquid at its center. The liquid flows radially outward across

the disk and is discharged at high velocity from its periphery. The disk may be smooth and flat or may have vanes or slots to guide the liquid to the periphery. At low flow rates, droplets form near the edge of the disk. At high flow rates, ligaments or sheets are generated at the edge and disintegrate into droplets. Small disks operating at high rotational speeds and low flow rates are capable of producing sprays in which drop sizes are fairly uniform. A 360° spray pattern is developed by rotating disks, which are usually installed in a cylindrical or conical chamber where an umbrellalike spray is created by downward gas currents [5].

Some rotary atomizers employ a cup instead of a disk. The cup is usually smaller in diameter and is shaped like an elongated bowl. In some designs the edge of the cup is serrated to encourage a more uniform drop size distribution in the spray. A flow of air around the periphery is sometimes used to shape the spray and to assist in transporting the droplets away from the atomizer. In contrast to pressure nozzles, rotary atomizers allow independent variation of flow rate and disk speed, thereby providing more flexibility in operation.

### Air-Assist Atomizers

In this type of nozzle the liquid is exposed to a stream of air or steam flowing at high velocity. In the *internal-mixing* configuration, gas and liquid mix within the nozzle before discharging through the outlet orifice. The liquid is sometimes supplied through tangential slots to encourage a conical discharge pattern. However, the maximum spray angle is limited to about 60°. The device tends to be energy inefficient, but it can produce a finer spray than simple pressure nozzles.

As its name suggests, in the *external-mixing* form of air-assist nozzle the high-velocity gas or steam impinges on the liquid at or outside the liquid discharge orifice. Its advantage over the internal-mixing type is that problems of back pressures are avoided because there is no internal communication between gas and liquid. However, it is less efficient than the internal-mixing concept, and higher gas flow rates are needed to achieve the same degree of atomization. Both types of nozzles can atomize high-viscosity liquids effectively.

### Airblast Atomizers

These devices function in a very similar manner to air-assist nozzles, and both types fall in the general category of twin-fluid atomizers. The main difference between air-assist and airblast atomizers is that the former use relatively small quantities of air or steam flowing at very high velocities (usually sonic), whereas the latter employ large amounts of air flowing at much lower velocities (<100 m/s). Airblast nozzles are thus ideally suited for atomizing liquid fuels in continuous-flow combustion systems, such as gas turbines, where air velocities of this magnitude are usually readily available. The most common form of airblast atomizer is one in which the liquid is first spread into a thin conical sheet and then exposed to high-velocity airstreams on both sides of the sheet. The atomi-

zation performance of this *prefilming* type of airblast nozzle is superior to that of the alternative *plain-jet* airblast nozzle, in which the liquid is injected into the airstream in the form of one or more discrete jets.

### Other Types

Most practical atomizers are of the pressure, rotary, or twin-fluid type. However, many other forms of atomizers have been developed that are useful in special applications.

**Electrostatic.** A liquid jet or film is exposed to an intense electrical pressure that tends to expand its area. This expansion is opposed by surface tension forces. If the electrical pressure predominates, droplets are formed. Droplet size is a function of the electrical pressure, the liquid flow rate, and the physical and electrical properties of the liquid. The low liquid flow rates associated with electrostatic atomizers have tended to limit their practical applications to electrostatic painting and nonimpact printing.

**Ultrasonic.** The liquid to be atomized is fed through or over a transducer and horn, which vibrates at ultrasonic frequencies to produce the short wavelengths necessary for fine atomization. The system requires a high-frequency electrical input, two piezoelectric transducers, and a stepped horn. The concept is well suited for applications that require very fine atomization and a low spray velocity. At present, an important application of ultrasonic atomizers (nebulizers) is for medical inhalation therapy, where very fine sprays and the absence of gas to effect atomization are important attributes.

**Sonic (whistle).** Gas is accelerated within the device to sonic velocity and impinges on a plate or annular cavity (resonance chamber). The sound waves produced are reflected into the path of the incoming liquid [6]. The frequency of the sound waves is around 20 kHz, and this serves to disintegrate the liquid into small droplets ranging downward in size from 50  $\mu\text{m}$ . The sonic and pneumatic effects are difficult to isolate from each other. Efforts have been made to design nozzles that operate above the audible frequency limit to reduce the nuisance of noise [7]. However, in some applications the attendant sound field may benefit the process (e.g., combustion) for which the resultant spray is required.

**Windmill.** Many aerial applications of pesticides require a narrow spectrum of drop sizes. Conventional rotary disk atomizers can provide such a spectrum but only when operating in the ligament mode at low flow rates. By making radial cuts at the periphery of a disk and twisting the tips of the segments, the disk can be converted into a windmill that will rotate rapidly when inserted into an airflow at aircraft flight speed. According to Spillmann and Sanderson [8], the disk windmill constitutes an ideal rotary atomizer for the aerial application of pesticides. It

provides a narrow spectrum of drop sizes in the range most suitable for herbicides, at relatively high flow rates.

**Vibrating capillary.** This type of droplet generator was first used to study the collision and coalescence of small water droplets. It consists of a hypodermic needle vibrating at its resonant frequency and can produce uniform streams of drops down to 30  $\mu\text{m}$  in diameter. The size and frequency with which the droplets can be produced depend on the flow rate of the liquid through the needle, the needle diameter, the resonant frequency, and the amplitude of oscillation of the needle tip.

**Flashing liquid jets.** An orifice downstream of which a high-pressure liquid flash vaporizes to shatter the liquid into small droplets can produce a fairly regular spray pattern. Flashing dissolved gas systems have been studied by Brown and York [9], Sher and Elata [10], and Marek and Cooper [11]. Solomon et al. [12] have shown that flashing even small quantities of dissolved gas (mole fractions less than 15%) can effect a significant improvement in atomization. However, these beneficial effects cannot be realized unless they are promoted by fitting an expansion chamber just upstream of the discharge orifice. The need for this expansion chamber stems from the low bubble growth rate for dissolved gas systems. This low bubble growth rate appears to pose a fundamental limitation to the practical application of flashing injection by means of dissolved gas systems.

**Effervescent atomization.** This method of atomization overcomes the basic problems associated with flashing dissolved gas systems. No attempt is made to dissolve any air or gas in the liquid. Instead, the gas is injected at low velocity into the flowing liquid stream at some point upstream of the discharge orifice. The pressure differential between the atomizing gas and the liquid into which it is injected is only a few centimeters of water and is only what is needed to prevent the liquid from flowing back up the gas line. Studies by Lefebvre et al. [13] have shown that good atomization can be achieved at much lower liquid injection pressures than the values normally associated with pressure atomization.

Schematic diagrams illustrating the principal design features of the most important of the atomizers described above are shown in Fig. 1.1. The relative merits of these and other atomizers are listed in Table 1.1.

## FACTORS INFLUENCING ATOMIZATION

The performance of any given type of atomizer depends on its size and geometry and on the physical properties of the dispersed phase (i.e., the liquid being atomized) and the continuous phase (i.e., the gaseous medium into which the droplets are discharged).

For plain-orifice pressure nozzles and plain-jet airblast atomizers, the dimen-

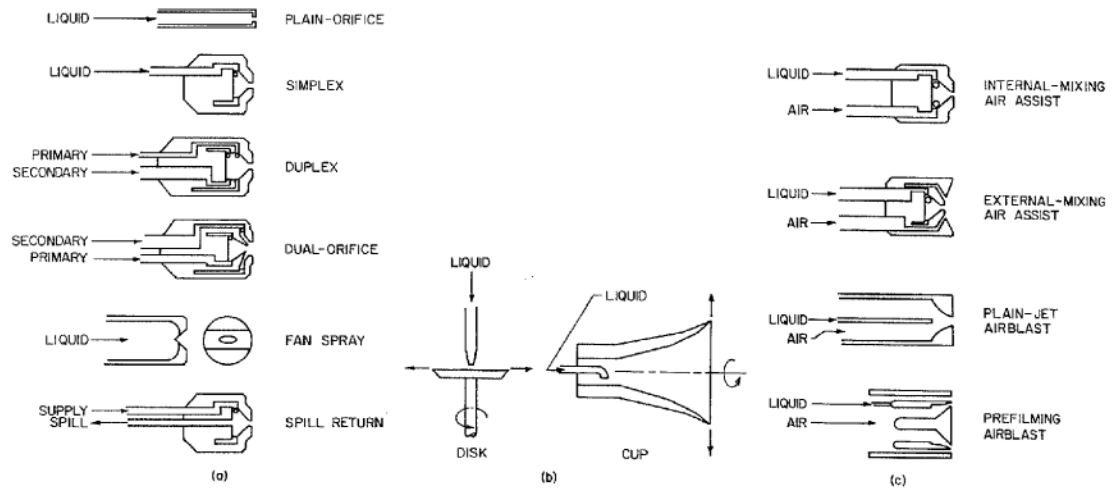


Figure 1.1 (a) Pressure atomizers. (b) Rotary atomizers. (c) Twin-fluid atomizers.

sion most important for atomization is the diameter of the final discharge orifice. For pressure-swirl, rotary, and prefilming airblast atomizers, the critical dimension is the thickness of the liquid sheet as it leaves the atomizer. Theory predicts, and experiment confirms, that mean drop size is roughly proportional to the square root of the liquid jet diameter or sheet thickness. Thus, provided the other key parameters that affect atomization are maintained constant, an increase in atomizer scale (size) will impair atomization.

### Liquid Properties

The flow and spray characteristics of most atomizers are strongly influenced by the liquid properties of density, viscosity, and surface tension. In theory, the mass flow rate through a pressure nozzle varies with the square root of liquid density. However, as Tate [5] has pointed out, in practice it is seldom possible to change the density without affecting some other liquid property, so this relationship must be interpreted cautiously. The significance of density for atomization performance is diminished by the fact that most liquids exhibit only minor differences in this property. Moreover, the modest amount of available data on the effect of liquid density on mean drop size suggests that its influence is quite small.

One way of defining a spray is in terms of the increase in liquid surface area resulting from atomization. The surface area before breakup is simply that of the liquid cylinder as it emerges from the nozzle. After atomization, the area is the sum of the surface areas of all the individual droplets. This multiplication factor provides a direct indication of the level of atomization achieved and is useful in applications that emphasize surface phenomena such as evaporation and absorption. Surface tension is important in atomization because it represents the force that resists the formation of new surface area. The minimum energy required for atomization is equal to the surface tension multiplied by the increase in liquid surface area. Whenever atomization occurs under conditions where surface tension forces are important, the Weber number, which is the ratio of the inertial force to the surface tension force, is a useful dimensionless parameter for correlating drop size data. Commonly encountered surface tensions range from  $0.073 \text{ kg/s}^2$  for water to  $0.027 \text{ kg/s}^2$  for petroleum products. For most pure liquids in contact with air, the surface tension decreases with an increase in temperature and is independent of the age of the surface [14].

In many respects, viscosity is the most important liquid property. Although in an absolute sense its influence on atomization is no greater than that of surface tension, its importance stems from the fact that it affects not only the drop size distributions in the spray but also the nozzle flow rate and spray pattern. An increase in viscosity lowers the Reynolds number and also hinders the development of any natural instabilities in the jet or sheet. The combined effect is to delay disintegration and increase the size of the drops in the spray.

The effect of viscosity on flow within the nozzle is complex. In hollow-cone nozzles a modest increase in viscosity can actually increase the flow rate. It does this by thickening the liquid film in the discharge orifice, thereby raising the ef-

Table 1.1 Relative merits of various types of atomizers

| Type              | Description   | Advantages                                                                                                                                                        | Drawbacks                                                                                                                          | Applications                                                                                     |
|-------------------|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|
| Pressure atomizer | Plain orifice | 1. Simple, cheap<br>2. Rugged                                                                                                                                     | 1. Narrow spray angle<br>2. Solid spray cone                                                                                       | Diesel engines, jet engine afterburners, ramjets                                                 |
|                   | Simplex       | 1. Simple, cheap<br>2. Wide spray angle (up to 180°)                                                                                                              | 1. Needs high supply pressures<br>2. Cone angle varies with pressure differential and ambient gas density                          | Gas turbines and industrial furnaces                                                             |
|                   | Duplex        | Same as simplex, plus good atomization over a very wide range of liquid flow rates                                                                                | Spray angle narrows as liquid flow rate is increased                                                                               | Gas turbine combustors                                                                           |
|                   | Dual orifice  | 1. Good atomization<br>2. Turndown ratio as high as 50:1<br>3. Relatively constant spray angle                                                                    | 1. Atomization poor in transition range<br>2. Complexity in design<br>3. Susceptibility of small passages to blockage              | Wide range of aircraft and industrial gas turbines                                               |
|                   | Spill return  | 1. Simple construction<br>2. Good atomization over entire flow range<br>3. Very large turndown ratio<br>4. Large holes and flow passages obviate risk of blockage | 1. Spray angle varies with flow rates<br>2. Power requirements higher than with other pressure nozzles except at maximum discharge | Various types of combustor<br>Has good potential for slurries and fuels of low thermal stability |
|                   | Fan spray     | 1. Good atomization<br>2. Narrow elliptical pattern sometimes advantageous                                                                                        | Needs high supply pressures                                                                                                        | High-pressure coating operations<br>Annular combustors                                           |
| Rotary            | Spinning disk | 1. Nearly uniform atomization possible with small disks rotating at high speeds<br>2. Independent control of atomization quality and flow rate                    | Produces a 360° spray pattern                                                                                                      | Spray drying<br>Crop spraying                                                                    |
|                   | Rotary cup    | Capable of handling slurries                                                                                                                                      | May require air blast around periphery                                                                                             | Spray drying<br>Spray cooling                                                                    |

|               |                 |                                                                                                    |                                                                                                                                      |                                                                                |
|---------------|-----------------|----------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| Air assist    | Internal mixing | 1. Good atomization<br>2. Large passages prevent clogging<br>3. Can atomize high-viscosity liquids | 1. Liquid can back up in air line<br>2. Requires auxiliary metering device<br>3. Needs external source of high-pressure air or steam | Industrial furnaces<br>Industrial gas turbines                                 |
|               | External mixing | Same as internal mixing, plus construction prevents backing up of liquid into the air line         | 1. Needs external source of air or steam<br>2. Does not permit high liquid/air ratios                                                | Same as internal mixing                                                        |
| Airblast      | Plain jet       | 1. Good atomization<br>2. Simple, cheap                                                            | 1. Narrow spray angle<br>2. Atomizing performance inferior to prefilming airblast                                                    | Industrial gas turbines                                                        |
|               | Prefilming      | 1. Good atomization especially at high ambient air pressures<br>2. Wide spray angle                | Atomization poor at low air velocities                                                                                               | Wide range of industrial and aircraft gas turbines                             |
| Ultrasonic    |                 | 1. Very fine atomization<br>2. Low spray velocity                                                  | Cannot handle high flow rates                                                                                                        | Medical sprays<br>Humidification<br>Spray drying<br>Acid etching<br>Combustion |
| Electrostatic |                 | Very fine atomization                                                                              | Cannot handle high flow rates                                                                                                        | Paint spraying<br>Printing                                                     |

fective flow area. At high viscosities, however, the flow rate usually diminishes with increasing viscosity. With pressure-swirl nozzles an increase in viscosity generally produces a narrower spray angle. At very high viscosities the normal conical spray may collapse into a straight stream of relatively large ligaments and drops. An increase in liquid viscosity invariably has an adverse effect on atomization quality, because when viscous losses are large, less energy is available for atomization and a coarser spray results. In airblast atomizers, liquid velocities are usually much lower than in pressure nozzles. In consequence, the drop sizes produced by airblast nozzles tend to be less sensitive to variations in liquid viscosity.

Table 1.2 lists the relevant physical properties of some of the liquids used in spray applications. The viscosity of these liquids ranges from 0.001 kg/m s for water to 0.5 kg/m s for heavy fuel oil. The viscosity of liquids generally decreases with an increase in temperature. It is customary to heat up many of the heavier fuel oils, partly to reduce pumping power requirements but also to improve atomization.

Some fluids, for example, slurries of liquids and solid powders, are characterized by a nonlinear relationship between shear stress and shear strain rate. For such liquids, which are called non-Newtonian, it is necessary to specify the shear rate with the viscosity. The apparent reduction in viscosity with increasing shear rate highlights the need to minimize pressure losses in the supply lines and nozzles. This reduction is also desirable because the viscosities of non-Newtonian fluids have less effect on atomization if a high shear rate is produced in the liquid film formed by the nozzle [14]. Very little secondary atomization will occur once the drops are formed, due to the increase in apparent viscosity at the lower shear rate.

The influence of temperature on viscosity and surface tension is illustrated for some widely used hydrocarbon fuels in Figs. 8.31 and 1.2, respectively.

### Ambient Conditions

The ambient gas into which sprays are injected can vary widely in pressure and temperature. This is especially true of liquid fuel-fired combustion systems. In diesel engines, critical and supercritical pressure and temperature conditions are encountered. In gas turbine combustors, fuel sprays are injected into highly turbulent, swirling recirculating streams of reacting gases. In industrial furnaces, the fuel is sprayed into high-temperature flames of recirculating combustion products. With pressure-swirl atomizers the spray angle decreases markedly with increase in ambient gas density until a minimum angle is reached beyond which any further increase in ambient density has no effect on spray angle. Ambient gas density also has a strong influence on the mean drop sizes produced by pressure-swirl atomizers. If the ambient pressure is raised continuously above the normal atmospheric value, the mean drop size increases initially until a maximum value is reached and then slowly declines. The reasons for this unusual relationship between ambient pressure and mean drop size are discussed in Chapter 6.

The spray patterns generated by pressure-swirl atomizers are also affected by

**Table 1.2 Properties of liquids**

| Liquid               | Temperature<br>(K) | Viscosity<br>(kg/m s) | Density<br>(kg/m <sup>3</sup> ) | Surface tension<br>(kg/s <sup>2</sup> ) |
|----------------------|--------------------|-----------------------|---------------------------------|-----------------------------------------|
| Acetone              | 273                | 0.000400              |                                 | 0.0261                                  |
|                      | 293                | 0.00032               | 792                             | 0.0237                                  |
|                      | 300                | 0.000300              |                                 |                                         |
|                      | 303                | 0.000295              |                                 |                                         |
|                      | 313                | 0.00028               |                                 | 0.02116                                 |
| Ammonia              | 284                |                       |                                 | 0.0234                                  |
|                      | 300                | 0.00013               |                                 |                                         |
|                      | 307                |                       |                                 | 0.0181                                  |
| Aniline              | 283                | 0.0065                |                                 | 0.0441                                  |
|                      | 288                | 0.0053                | 1000                            | 0.040                                   |
|                      | 323                | 0.00185               |                                 | 0.0394                                  |
|                      | 373                | 0.00085               |                                 |                                         |
| Benzene              | 273                | 0.00091               | 899                             |                                         |
|                      | 283                | 0.00076               |                                 | 0.0302                                  |
|                      | 293                | 0.00065               | 880                             | 0.0290                                  |
|                      | 303                | 0.00056               |                                 | 0.0276                                  |
|                      | 323                | 0.00044               |                                 |                                         |
|                      | 353                | 0.00039               |                                 |                                         |
| Butane               | 300                | 0.00016               |                                 | 0.0116                                  |
| Carbon tetrachloride | 273                | 0.00133               |                                 |                                         |
|                      | 293                | 0.00097               | 799                             | 0.0270                                  |
|                      | 323                | 0.00065               |                                 |                                         |
|                      | 373                | 0.00038               |                                 | 0.0176                                  |
| Castor oil           | 283                | 2.42                  |                                 |                                         |
|                      | 288                |                       | 969                             |                                         |
|                      | 293                | 0.986                 |                                 |                                         |
|                      | 303                | 0.451                 |                                 |                                         |
|                      | 313                | 0.231                 |                                 |                                         |
|                      | 373                | 0.0169                |                                 |                                         |
| Chloroform           | 273                | 0.0007                |                                 |                                         |
|                      | 293                | 0.00058               | 1489                            | 0.02714                                 |
|                      | 303                | 0.00051               |                                 |                                         |
| Cottonseed oil       | 289                |                       | 926                             |                                         |
|                      | 293                | 0.0070                |                                 |                                         |
| Creosote             | 313                | 0.0070                |                                 |                                         |
| <i>n</i> -Decane     | 293                | 0.00092               |                                 |                                         |
|                      | 300                |                       |                                 | 0.0233                                  |
|                      | 300                | 0.000035              |                                 | 0.0007                                  |
| Ethyl alcohol        | 273                | 0.00177               |                                 | 0.02405                                 |
|                      | 293                | 0.0012                | 791                             | 0.02275                                 |
|                      | 303                | 0.0010                |                                 | 0.02189                                 |
|                      | 323                | 0.0007                |                                 |                                         |
|                      | 343                | 0.00050               |                                 |                                         |
| Ethylene glycol      | 293                | 0.01990               |                                 |                                         |
|                      | 313                | 0.00913               |                                 |                                         |
|                      | 333                | 0.00495               |                                 |                                         |
|                      | 353                | 0.00302               |                                 |                                         |
|                      | 373                | 0.00199               |                                 |                                         |
|                      | 288                | 0.172                 | 930                             | 0.0250                                  |
| Fuel oil (light)     | 313                | 0.047                 | 916                             | 0.0230                                  |
|                      | 356                | 0.0083                | 880                             | 0.0210                                  |

**Table 1.2 Properties of liquids (Continued)**

| Liquid              | Temperature<br>(K) | Viscosity<br>(kg/m s) | Density<br>(kg/m <sup>3</sup> ) | Surface tension<br>(kg/s <sup>2</sup> ) |
|---------------------|--------------------|-----------------------|---------------------------------|-----------------------------------------|
| Fuel oil (medium)   | 313                | 0.215                 | 936                             | 0.0230                                  |
|                     | 378                | 0.0134                | 897                             | 0.0200                                  |
| Fuel oil (heavy)    | 313                | 0.567                 | 970                             | 0.0230                                  |
|                     | 366                | 0.037                 | 920                             | 0.0210                                  |
|                     | 400                | 0.015                 | 900                             | 0.0200                                  |
| Gas oil             | 288                | 0.0060                | 850                             | 0.0240                                  |
|                     | 313                | 0.0033                | 863                             | 0.0230                                  |
| Glycerin            | 273                | 12.1                  | 1260                            | 0.0630                                  |
|                     | 288                | 2.33                  |                                 |                                         |
|                     | 293                | 0.622                 |                                 | 0.0630                                  |
| Heptane             | 273                | 0.00052               |                                 |                                         |
|                     | 300                | 0.00038               |                                 | 0.0194                                  |
|                     | 313                | 0.00034               |                                 |                                         |
|                     | 343                | 0.00026               |                                 | 0.0194                                  |
| Hexane              | 273                | 0.00040               |                                 |                                         |
|                     | 293                | 0.00033               |                                 | 0.0184                                  |
|                     | 300                | 0.00029               |                                 | 0.0176                                  |
|                     | 323                | 0.00025               |                                 |                                         |
| Hydrazine           | 274                | 0.00129               |                                 |                                         |
|                     | 293                | 0.00097               |                                 |                                         |
|                     | 298                |                       |                                 | 0.0915                                  |
| Kerosine            | 293                | 0.0016                | 800                             | 0.0260                                  |
| Linseed oil         | 288                |                       | 942.0                           |                                         |
|                     | 303                | 0.0331                |                                 |                                         |
|                     | 363                | 0.0071                |                                 |                                         |
| Machine oil (light) | 288.6              | 0.114                 |                                 |                                         |
|                     | 310.8              | 0.0342                |                                 |                                         |
|                     | 373                | 0.0049                |                                 |                                         |
| Machine oil (heavy) | 288.6              | 0.661                 |                                 |                                         |
|                     | 310.8              | 0.127                 |                                 |                                         |
| Mercury             | 273                | 0.0017                | 13600                           |                                         |
|                     | 293                | 0.00153               | 13550                           | 0.480                                   |
|                     | 313                | 0.00045               |                                 |                                         |
| Methyl alcohol      | 273                | 0.00082               | 810                             | 0.0245                                  |
|                     | 293                | 0.00060               |                                 | 0.0226                                  |
|                     | 300                | 0.00053               |                                 | 0.0221                                  |
|                     | 323                | 0.00040               |                                 |                                         |
| Naphthalene         | 353                | 0.00097               |                                 |                                         |
|                     | 373                | 0.00078               |                                 |                                         |
|                     | 400                |                       |                                 | 0.0288                                  |
| Nonane              | 300                |                       |                                 | 0.0223                                  |
| <i>n</i> -Octane    | 293                | 0.00054               |                                 | 0.0218                                  |
|                     | 300                | 0.0005                |                                 | 0.0210                                  |
| Olive oil           | 283                | 0.138                 |                                 |                                         |
|                     | 288                |                       | 918                             |                                         |
|                     | 291                |                       |                                 | 0.0331                                  |
|                     | 293                | 0.0840                |                                 |                                         |
|                     | 343                | 0.0124                |                                 |                                         |
| Pentane             | 273                | 0.00029               |                                 |                                         |
|                     | 300                | 0.00022               |                                 | 0.0153                                  |
| Propane             | 300                | 0.000098              |                                 | 0.0064                                  |

**Table 1.2 Properties of liquids (Continued)**

| Liquid     | Temperature (K) | Viscosity (kg/m s) | Density (kg/m <sup>3</sup> ) | Surface tension (kg/s <sup>2</sup> ) |
|------------|-----------------|--------------------|------------------------------|--------------------------------------|
| Toluene    | 273             | 0.00077            | 870                          | 0.0277                               |
|            | 293             | 0.00059            |                              | 0.0285                               |
|            | 303             | 0.00053            |                              | 0.0274                               |
|            | 343             | 0.00035            |                              |                                      |
| Turpentine | 273             | 0.00225            | 870                          |                                      |
|            | 283             | 0.00178            |                              | 0.0270                               |
|            | 303             | 0.00127            |                              |                                      |
|            | 343             | 0.000728           |                              |                                      |
| Water      | 291             |                    |                              | 0.073                                |
|            | 300             | 0.00085            |                              | 0.0717                               |

the liquid injection pressure differential  $\Delta P_L$ . The ejector action of the high-velocity spray generates air currents, which cause the spray angle to contract. This effect is aggravated by the increase in spray velocity that accompanies an increase in  $\Delta P_L$ . Thus, although increasing  $\Delta P_L$  has no effect on the spray angle immediately downstream of the nozzle, it causes appreciable contraction of the spray pattern farther downstream.

With plain-orifice atomizers an increase in ambient gas density leads to a wider spray angle. This is because the increase in aerodynamic drag on the drop-

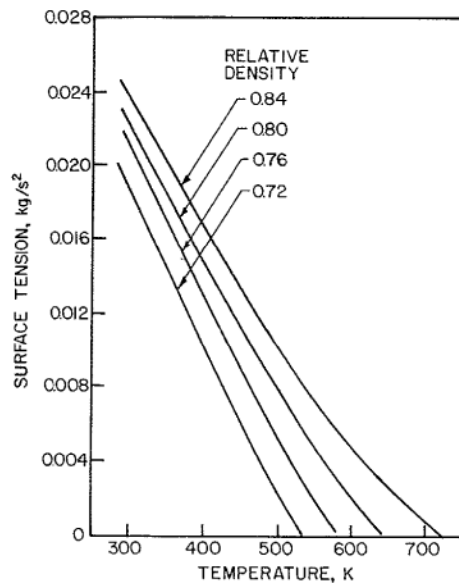


**Figure 1.2** Surface tension-temperature relationship for hydrocarbon fuels of varying relative densities.

lets, created by an increase in gas density, tends to produce a greater deceleration in the axial direction than in the radial direction.

The spray patterns produced by airblast atomizers tend to be fairly insensitive to variations in ambient gas density. All but the largest drops in the spray tend to follow the streamlines of the airflow pattern generated at the nozzle exit by the various swirlers and shaped passages within the nozzle. This airflow pattern generally remains fixed and independent of air density, apart from second-order Reynolds number and Mach number effects. However, if the "natural" cone angle of the spray, i.e., the spray angle with no air flowing, is markedly different from that of the air, then the change in aerodynamic drag forces produced by a change in air density will affect the resulting spray pattern. In general, an increase in air density will cause the spray pattern to adhere more closely to the streamlines of the atomizing air.

## SPRAY CHARACTERISTICS

The importance of drop size for the many combustion and industrial processes that utilize liquid sprays is attested to by the proliferation of techniques available for drop size measurement. No single technique is completely satisfactory but each technique has its own advantages and drawbacks, depending on the application. Direct methods include those in which individual drops are collected on slides for subsequent measurement and counting or in which droplets are frozen and sized as solid particles. With the impaction method the drops are sorted on the basis of inertial differences. Depending on its size, a droplet may impact or fail to impact on a solid surface or may follow a different trajectory. This allows all the drops in a spray to be sorted into different size categories.

High-speed photography is often used to provide instantaneous images of the drops in a spray, which are recorded for subsequent counting or analysis. The method has the important advantage of being nonintrusive, but it cannot give the temporal distribution of drop sizes as produced by the atomizer. This is because the rate of deceleration of the drops downstream of the atomizer is higher for the smaller droplets, so photographs taken in this region indicate a preponderance of small drops. The spatial distribution of drop sizes given by instantaneous photography can be converted into the true temporal distribution by multiplying the number of droplets in any given size range by the average velocity for that size range.

In recent years considerable advances have been made in the development of laser diagnostic techniques for measuring particle size and velocity in sprays. High-speed pulsed microphotography, cinematography, and holography are being used to study drop size distributions and spray structure. Much of the tedium normally involved in detailed studies of drop size distributions in various regions of the spray can now be alleviated using automatic image analysis, in which droplet images are enlarged, counted, and sorted by electronic scanning devices and analyzed by microprocessors [15].

A most convenient and effective method for assessing and comparing sprays is Fraunhofer diffraction particle sizing, which uses a line-of-sight measurement through the spray. Various models of this instrument are commercially available for applications to both continuous and intermittent sprays. Recent developments include new computer software for providing a rapid printout of size distributions for on-line analysis.

Laser Doppler anemometry and interferometry, interfaced with signal processing and high-speed data acquisition systems, are now being used in simultaneous measurements of drop size, drop velocity, and local drop drag coefficients.

Summaries of the methods used in spray analysis prior to 1970 have been prepared by Putnam et al. [16] and Tate and Olsen [17]. These and more recent developments are described in some detail in Chapter 9.

**Table 1.3 Spray applications**

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|                                                                                                             |
|-------------------------------------------------------------------------------------------------------------|
| Production or processing                                                                                    |
| Spray drying (dairy products, coffee and tea, starch pharmaceuticals, soaps and detergents, pigments, etc.) |
| Spray cooling                                                                                               |
| Spray reactions (absorption, roasting, etc.)                                                                |
| Atomized suspension technique (effluents, waste liquors, etc.)                                              |
| Powdered metals                                                                                             |
| Treatment                                                                                                   |
| Evaporation and aeration                                                                                    |
| Cooling (spray ponds, towers, reactors, etc.)                                                               |
| Humidification and misting                                                                                  |
| Air and gas washing and scrubbing                                                                           |
| Industrial washing and cleaning                                                                             |
| Coating                                                                                                     |
| Surface treatment                                                                                           |
| Spray painting (pneumatic, airless, and electrostatic)                                                      |
| Flame spraying                                                                                              |
| Insulation, fibers, and undercoating materials                                                              |
| Multicomponent resins (urethanes, epoxies, polyesters, etc.)                                                |
| Particle coating and encapsulation                                                                          |
| Combustion                                                                                                  |
| Oil burners (furnaces and heaters, industrial and marine boilers)                                           |
| Diesel fuel injection                                                                                       |
| Gas turbines (aircraft, marine, automotive, etc.)                                                           |
| Rocket fuel injection                                                                                       |
| Miscellaneous                                                                                               |
| Medicinal sprays                                                                                            |
| Dispersion of chemical agents                                                                               |
| Agricultural spraying (insecticides, herbicides, fertilizer solutions, etc.)                                |
| Foam and fog suppression                                                                                    |
| Printing                                                                                                    |
| Acid etching                                                                                                |

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Source: Tate [5].

## APPLICATIONS

A compilation by Tate [5] of some of the most important applications is contained in Table 1.3. As the cost, complexity, atomizing performance, and energy consumption vary widely between different types of atomizers, it is important to select the best atomizer for any given application. The following factors enter into the proper selection: properties of the liquid to be atomized, e.g., density, viscosity, surface tension, and temperature; ambient gas properties, such as pressure, temperature, and flow pattern; particle sizes and percent solids in suspensions, slurries, and pastes; maximum flow rate; range of flow rates (turndown ratio); required mean drop size and drop size distribution; liquid or gas pressures available for nozzles, or power required for rotary atomizers; conditions that may contribute to wear and corrosion; size and shape of vessel, enclosure, or combustor containing spray; economics of spray operation taking into account initial cost, operating expenses, and depreciation; and safety considerations [5].

## GLOSSARY

Some of the terms frequently used in descriptions of atomizers and sprays are defined below. These definitions are necessarily brief, and no attempt has been made to include all qualifying considerations.

*Air-assist nozzle.* Nozzle in which high-velocity air or steam is used to enhance pressure atomization at low liquid flow rates.

*Airblast atomizer.* Atomizer in which a liquid jet or sheet is exposed to air flowing at high velocity.

*Air core.* Cylindrical void space within the rotating liquid in a simplex swirl chamber.

*Arithmetic mean diameter.* Linear mean diameter of drops in spray.

*Atomization.* Process whereby a volume of liquid is disintegrated into a multiplicity of small drops.

*Beam steering.* Refraction of a laser beam due to density gradients in the continuous phase.

*Breakup length.* Length of continuous portion of jet measured from nozzle exit to point where breakup occurs.

*Cavitation.* Formation of bubbles by gas or vapor released in flow regions of low static pressure; affects discharge coefficient and jet breakup.

*Cavitation number.* Ratio of pressure differential to downstream pressure; indicator of propensity for cavitation.

*Combined spray.* Spray produced when both stages flow simultaneously in a dual-orifice or piloted-airblast nozzle.

*Continuous phase.* Medium, usually gaseous, in which atomization occurs.

*Critical flow rate.* Liquid flow rate corresponding to the transition from one mode of atomization to another.

- Critical Weber number.* Value of Weber number above which a single drop will split into two or more drops.
- Cumulative distribution.* Plot of percentage by number, surface area, or volume of drops whose diameter is less than a given drop diameter.
- Discharge coefficient.* Ratio of actual flow rate to theoretical flow rate.
- Discharge orifice.* Final orifice through which liquid is discharged into the ambient gas.
- Dispersed phase.* Liquid to be atomized.
- Dispersion.* Ratio of the volume of a spray to the volume of the liquid contained within it.
- Drooling.* Sluggish dripping of liquid from a nozzle while spraying, usually caused by impingement of the spray on some surface other than the orifice from which the liquid is discharging [18].
- Drop coalescence.* Collision of two drops to form a single drop.
- Droplet size.* Diameter of a spherical droplet, usually expressed in micrometers.
- Droplet uniformity index.* Indication of the range of drop sizes in a spray relative to the median diameter.
- Drop saturation.* Droplet population exceeding the capability of the sizing instrument or method.
- Dual-orifice atomizer.* Atomizer consisting of two simplex nozzles fitted concentrically one inside the other.
- Duplex nozzle.* Nozzle featuring a swirl chamber with two sets of tangential swirl ports, one set being the primary ports for low flows and the other the larger secondary ports for handling high flow rates.
- Effective evaporation constant.* Value of evaporation constant that includes heat-up period and convective effects.
- Electrostatic atomizer.* Atomizer in which electrical pressure is used to overcome surface tension forces and achieve atomization.
- Equivalent spray angle.* Angle formed by drawing two straight lines from the nozzle discharge orifice through the center of the liquid mass in the left and right lobes of the spray.
- Evaporation constant.* Indication of the rate of change of drop surface area during steady-state evaporation.
- External mixing nozzle.* Air-assist atomizer in which high-velocity gas impinges on a liquid at or outside the final orifice.
- Extinction.* Percentage of light removed from original direction; indicator of the extent to which measurements of mean drop size are affected by multiple scattering. Also termed *obscuration*.
- Fan spray.* Spray in the shape of a sector of a circle of about 75° angle; elliptical in cross section.
- Film thickness.* Thickness of annular liquid sheet as it discharges from the atomizer.
- Flat spray.* Same as fan spray.
- Flow number.* Effective flow area of a nozzle, usually expressed as the ratio of mass or volumetric flow rate to the square root of injection pressure differential.

*Flow rate.* Amount of liquid discharged during a given period of time; normally identified with all factors that affect flow rate, such as pressure differential and liquid density.

*Frequency distribution curve.* Plot of liquid volume per size class.

*Heat transfer number.* Indicator of rate of evaporation due to heat transfer to droplet from surrounding gas.

*Heat-up period.* Initial phase of droplet evaporation prior to attainment of steady-state conditions.

*Hollow-cone spray.* Spray in which most of the droplets are concentrated at the outer edge of a conical spray pattern.

*Impingement.* Collision of two round liquid jets or collision of a jet of liquid with a stationary deflector.

*Impinging jet atomizer.* Atomizer in which two liquid jets collide outside the nozzle to produce a liquid sheet perpendicular to the plane of the jets.

*Internal mixing nozzle.* Air-assist atomizer in which gas and liquid mix within the nozzle before discharging through the outlet orifice.

*Mass transfer number.* Indicator of rate of evaporation due to mass transfer.

*Mass (volume) median diameter.* Diameter of a drop below or above which 50% of the total mass (volume) of drops lies.

*Mean drop size.* A given spray is replaced by a fictitious one in which all the drops have the same diameter while retaining certain characteristics of the original spray.

*Monodisperse spray.* Spray containing drops of uniform size.

*Multiple scattering.* When spray number density is high some drops obscure part of signal generated by others, leading to biased diffraction patterns.

*Normal distribution.* Distribution of drop sizes based on the random occurrence of a given drop size.

*Obscuration.* Percentage of light removed from original direction; indicator of the extent to which measurements of mean drop size are affected by multiple scattering. Also known as *extinction*.

*Ohnesorge number.* Dimensionless group obtained by dividing the square root of Weber number by Reynolds number, which eliminates velocity from both; indicator of jet or sheet stability.

*Patternation.* A measure of the uniformity of the circumferential distribution of liquid in a conical spray. The term *radial patternation* is also used to describe the radial distribution of liquid within a conical spray.

*Patternator.* Two types are available: one designed to measure the radial liquid distribution in a conical spray, the other to measure the uniformity of the circumferential liquid distribution.

*Plain-orifice atomizer.* Atomizer in which liquid is ejected at high velocity through a small round hole; the best known example is a diesel injector.

*Polydisperse spray.* Spray containing drops of different sizes.

*Prefilmer.* Solid surface on which a thin continuous liquid film is formed.

*Pressure atomizer.* Single-fluid atomizer in which the conversion of pressure into kinetic energy results in a high relative velocity between the liquid and the surrounding gas.

*Relative density.* Ratio of the mass of a given volume of liquid to the mass of an equal volume of water; the temperature of both liquids must be stated. For example, relative density = 0.81 at 289/277 K indicates that the mass of the liquid was measured at 289 K and divided by the mass of an equal volume of water at 277 K. Formerly called specific gravity.

*Relative span factor.* Indicator of the range of drop sizes relative to the mass median diameter.

*Reynolds number.* Dimensionless ratio of inertial force to viscous force.

*Rosin-Rammler distribution.* Drop size distribution described in terms of two parameters, one of which provides a measure of the spread of drop sizes.

*Rotary atomizer.* Atomizer in which liquid is discharged from the edge of a rotating disk, cup, or slotted wheel.

*Sauter mean diameter.* Diameter of a droplet whose surface-to-volume ratio is equal to that of the entire spray.

*Shroud air.* A flow of air over the atomizer face to prevent deposition of carbon; also used to modify spray characteristics at low flow rates.

*Simplex nozzle.* Nozzle that employs a single swirl chamber to produce a well-atomized spray of wide cone angle.

*Skewness.* The axis of the nozzle spray cone is not colinear with the central axis of the nozzle; the maximum departure is stated in degrees [18].

*Slinger system.* Rotary atomizer employed in some small gas turbines.

*Solid-cone spray.* Spray in which the droplets are fairly uniformly distributed throughout a conical spray volume.

*Spatial sampling.* Measurement of drops contained within a volume under conditions such that contents of volume do not change during any single measurement.

*Spill-return nozzle.* Basically a simplex atomizer with provision for liquid to be removed from the swirl chamber and returned to supply; provides good atomization even at lowest flow rates.

*Spitting.* Large, irregular drops of liquid intermittently produced by otherwise uniformly fine spray; sometimes caused by internal flow leaks within the nozzle [18].

*Spray angle.* Angle formed by two straight lines drawn from the discharge orifice to cut the spray contours at a specific distance from the atomizer face.

*Spray axis.* Intersection of two planes of symmetry of the spray [5]; for symmetrical sprays the spray axis coincides with the centerline of the angle.

*Stability curve.* Graph showing relationship between jet velocity and breakup length.

*Streak.* Very narrow sector of the spray with more or less than the average concentration of droplets.

*Surface tension.* Property that resists expansion of liquid surface area. Surface tension forces must be overcome by aerodynamic, centrifugal, or pressure forces to achieve atomization.

*Swirl chamber.* Conical or cylindrical cavity having tangential inlets that impart a swirling motion to the liquid.

*Temporal sampling.* Measurement of drops that pass through a fixed area during a specific time interval.

- Transition range.* Small flow range of a dual-stage nozzle over which atomization quality is relatively poor; occurs when the pressurizing valve first opens to allow secondary liquid flow into the nozzle.
- Turndown ratio.* Ratio of maximum rated liquid flow to minimum rated liquid flow.
- Twin-fluid atomizer.* Generic term encompassing all nozzle types in which atomization is achieved using high-velocity air, gas, or steam.
- Ultrasonic atomizer.* Atomizer in which a vibrating surface is used to cause a liquid film to become unstable and disintegrate into drops.
- Upper critical point.* Point on stability curve where breakup changes from varicose to sinuous.
- Varicose.* Term used to describe appearance of liquid jet during breakup without interaction with air.
- Velocity coefficient.* Ratio of actual discharge velocity to the theoretical velocity corresponding to the total pressure differential across the nozzle.
- Viscosity.* Liquid property that has a marked effect on atomization quality and spray angle and also affects pumping power requirements; very dependent on liquid temperature.
- Visibility technique.* Drop sizing interferometry used in conjunction with laser Doppler anemometry to measure both drop size and velocity.
- Weber number.* Dimensionless ratio of momentum force to surface tension force.
- Web bulb temperature.* Droplet surface temperature during steady-state evaporation.
- Whistle atomizer.* Atomizer in which sound waves are used to shatter a liquid jet into droplets.
- Wide-range nozzle.* Nozzle designed to provide good atomization over a wide range of liquid flow rates; best known example is dual-orifice nozzle.
- Windmill atomizer.* Rotary atomizer used for aerial application of pesticides; unique feature is use of wind forces to provide rotary motion.

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## BASIC PROCESSES IN ATOMIZATION

### INTRODUCTION

The atomization process is essentially one in which bulk liquid is converted into small drops. Basically, it can be considered as a disruption of the consolidating influence of surface tension by the action of internal and external forces. In the absence of such disruptive forces, surface tension tends to pull the liquid into the form of a sphere, since this has the minimum surface energy. Liquid viscosity exerts a stabilizing influence by opposing any change in system geometry. On the other hand, aerodynamic forces acting on the liquid surface may promote the disruption process by applying an external distorting force to the bulk liquid. Breakup occurs when the magnitude of the disruptive force just exceeds the consolidating surface tension force.

Many of the larger drops produced in the initial disintegration process are unstable and undergo further disruption into smaller drops. Thus, the final range of drop sizes produced in a spray depends not only on the drop sizes produced in primary atomization but also on the extent to which these drops are further disintegrated during secondary atomization.

This chapter is devoted primarily to a review of the various mechanisms that have been proposed to account for the manner in which a jet or sheet emerging from an atomizer is broken down into drops. The process of jet disintegration is of primary importance for the design of plain-orifice atomizers of the types used extensively in diesel and rocket engines. The mechanisms of sheet disintegration

have direct relevance to the design of rotary atomizers for spray drying and to the performance of pressure-swirl and prefilming airblast atomizers, which are widely used in oil burners and gas turbine combustors. In view of their importance in the overall atomization process, the various mechanisms of liquid particle breakup are considered first.

## STATIC DROP FORMATION

Before embarking on a discussion of practical atomization processes, it is worthwhile to consider briefly the most elementary form of atomization, which is the quasi-static case of the hanging or pendant drop. This is exemplified by the slow discharge of a liquid from the end of a burette or a dripping faucet. When the gravity force on the liquid exceeds the attaching surface tension force, the liquid is pulled away from its attachment and a drop forms. The mass of the drop formed is determined by equating the gravitational and surface tension forces on the drop. For the slow emission of liquid from a thin circular tube of diameter  $d_o$ , the mass of the drop formed is

$$m_D = \frac{\pi d_o \sigma}{g} \quad (2.1)$$

The size of the spherical drop corresponding to this mass is given by

$$D = \left( \frac{6d_o \sigma}{\rho_L g} \right)^{1/3} \quad (2.2)$$

For a sharp-edged opening 1 mm in diameter, Eq. (2.2) would predict drop sizes of 3.6 and 2.6 mm for water and kerosine, respectively. If the hole size were reduced to 10  $\mu\text{m}$  in diameter, the predicted drop sizes for the two liquids would be 784 and 560  $\mu\text{m}$ , respectively.

For the breakaway of a drop from a flat horizontal wetted surface, which involves a more complex balance of gravitational and surface tension forces, Tamada and Shibaki [1] derived the following expression for drop size:

$$D = 3.3 \left( \frac{\sigma}{\rho_L g} \right)^{0.5} \quad (2.3)$$

This equation predicts that drops formed slowly by the action of gravity on a liquid film would have diameters of 9 and 5 mm for water and kerosine, respectively. Thus, the formation of drops by a dripping mechanism inevitably entails large drops and low flow rates. Although common in nature, this mechanism is clearly ineffective for practical applications, most of which call for high liquid flow rates and fine atomization. Gravitational forces are significant only in the formation of large drops and become negligibly small for the range of drop sizes of practical interest, which normally lie between 1 and 300  $\mu\text{m}$ .

## BREAKUP OF DROPS

When atomization occurs as a result of interaction between a liquid and the surrounding air, the overall atomization process involves several interacting mechanisms, among which is the splitting up of the larger drops during the final stages of disintegration. It is of interest, therefore, to examine the various ways in which a single drop of liquid can break up under the action of aerodynamic forces.

A rigorous mathematical solution for the breakup of a drop would demand an exact knowledge of the distribution of aerodynamic pressures on the drop. However, as soon as the drop is deformed by these pressures, the pressure distribution around it also changes and either a state of equilibrium between the external aerodynamic forces and the internal forces due to surface tension and viscosity is attained or further deformation follows, leading to possible breakup of the drop.

The influence of variations in the distribution of air pressure around a drop was examined by Klüsener [2]. Under equilibrium conditions the internal pressure at any point on the drop surface,  $p_I$ , is just sufficient to balance the external aerodynamic pressure  $p_A$  and the surface tension pressure  $p_\sigma$  so that

$$p_I = p_A + p_\sigma = \text{constant} \quad (2.4)$$

Note that for a spherical drop

$$p_\sigma = \frac{4\sigma}{D} \quad (2.5)$$

Clearly, a drop can remain stable as long as a change in air pressure at any point on its surface can be compensated by a corresponding change in  $p_\sigma$  such that  $p_I$  remains constant. However, if  $p_A$  is large compared with  $p_\sigma$ , then any appreciable change in  $p_A$  cannot be compensated by a corresponding change in  $p_\sigma$  to maintain  $p_I$  constant. In this situation, the external pressure  $p_A$  may deform the drop to an extent that leads to further reduction in  $p_\sigma$  and finally to disruption of the drop into smaller drops [3]. For these smaller drops, the higher value of  $p_\sigma$  as indicated by Eq. (2.5) may be large enough to accommodate the variations in  $p_A$ . If not, further subdivision may occur until  $p_\sigma$  is large enough to maintain a constant value of  $p_I$  at all points on the drop surface. When this stage is reached the drop is stable and no further breakup can occur.

These considerations lead to the concept of a critical drop size. For drops slightly larger than the critical size, the breakup time increases for decreasing size, until the stable drop has an infinite breakup time [4]. The influence of liquid viscosity, by opposing deformation of the drop, is to increase the breakup time. If the aerodynamic force on the drop declines during the breakup time, no breakup may occur, although the initial aerodynamic force was large enough to produce breakup.

### Drop Breakup in Flowing Air

The investigation of drop breakup in flowing air has a long history, dating back to the beginning of this century. Large free-falling drops in still air and smaller drops in a steady stream of air were first considered by Lenard [5] and Hochschwender [6]. Since then, this breakup process has been studied extensively both theoretically and experimentally [7–30].

High-speed photographs have revealed that globules or drops of liquid can split up under the action of aerodynamic forces in a number of ways, depending on the flow pattern around them. Hinze [7] identified the following three basic types of deformation, as illustrated in Fig. 2.1:

1. The drop is flattened to form an oblate ellipsoid (lenticular deformation). Subsequent deformation depends on the magnitude of the internal forces causing the deformation. It is conjectured that the ellipsoid is converted into a torus, which becomes stretched and disintegrates into small drops.
2. The initial drop becomes elongated to form a long cylindrical thread or ligament, which breaks up into small drops (cigar-shaped deformation).
3. Local deformations on the drop surface create bulges and protuberances, which eventually detach themselves from the parent drop to form smaller drops.

According to Hinze [7], deformation of type 1 occurs if the drop is subjected to aerodynamic pressures or viscous stresses produced by parallel and rotating flows. Plane hyperbolic and Couette flows are needed to produce type 2 deformation, while type 3 (described as “bulgy” deformation) occurs in irregular flow patterns. Thus, the preference for any particular type of deformation and breakup depends partly on the physical properties of the gas and liquid phases, namely their densities, viscosities, and interfacial tension, and also on the flow pattern around the drop.

In general, the breakup of a drop in a flowing stream is controlled by the dynamic pressure, surface tension, and viscous forces. For liquids of low viscosity, the deformation of a drop is determined primarily by the ratio of the aerodynamic forces, represented by  $0.5\rho_A U_R^2$ , and the surface tension forces, which are related to  $\sigma/D$ . Forming a dimensionless group from these two opposing forces yields the Weber number,  $\rho_A U_R^2 D / \sigma$ . The higher the Weber number, the larger

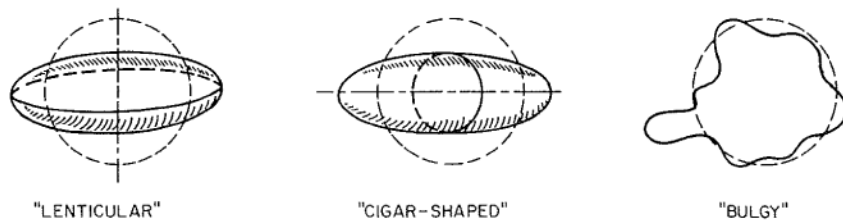


Figure 2.1 Basic types of globule deformation (Hinze [7]).

are the deforming external pressure forces compared with the reforming surface tension forces.

For any given liquid, the initial condition for breakup is achieved when the aerodynamic drag is just equal to the surface tension force, i.e.,

$$C_D \frac{\pi D^2}{4} 0.5 \rho_A U_R^2 = \pi D \sigma \quad (2.6)$$

Rearranging these terms provides the dimensionless group

$$\left( \frac{\rho_A U_R^2 D}{\sigma} \right)_{\text{crit}} = \frac{8}{C_D} \quad (2.7)$$

where the subscript crit denotes that a critical condition has been reached. As the first term in Eq. (2.7) is the Weber number, the equation may be written as

$$\text{We}_{\text{crit}} = \frac{8}{C_D} \quad (2.8)$$

For a relative velocity  $U_R$  the maximum stable drop size is obtained from Eq. (2.7) as

$$D_{\text{max}} = \frac{8\sigma}{C_D \rho_A U_R^2} \quad (2.9)$$

and the critical relative velocity at which the drop will disrupt is given by

$$U_{R,\text{crit}} = \left( \frac{8\sigma}{C_D \rho_A D} \right)^{0.5} \quad (2.10)$$

Many different experimental techniques have been used to investigate the breakup of large drops. They include (1) free fall from towers and stairwells, (2) suspension of drops in vertical wind tunnels with air velocity adjusted to keep the drops stationary, and (3) use of shock tubes to create supersonic velocities. The knowledge gained from these studies has direct application to the analysis of sprays.

Lane [18] has shown experimentally, and Hinze [7] has confirmed theoretically, that the mode of drop disintegration depends on whether the drop is subjected to steady acceleration or is suddenly exposed to a high-velocity gas stream. With steady acceleration the drop becomes increasingly flattened, and at a critical relative velocity it is blown out into the form of a hollow bag attached to a roughly circular rim, as illustrated in Fig. 2.2. On disintegration the bag produces a shower of very fine drops, while the rim, which contains at least 70% of the mass of the original drop, breaks up into larger drops.

A drop suddenly exposed to a fast airstream disintegrates in an entirely different manner. Instead of being blown out into a thin hollow bag anchored to a rim, the drop is deformed in the opposite direction and presents a convex surface to the flow of air. The edges of the saucer shape are drawn out into a thin sheet and then into fine filaments, which break into drops.

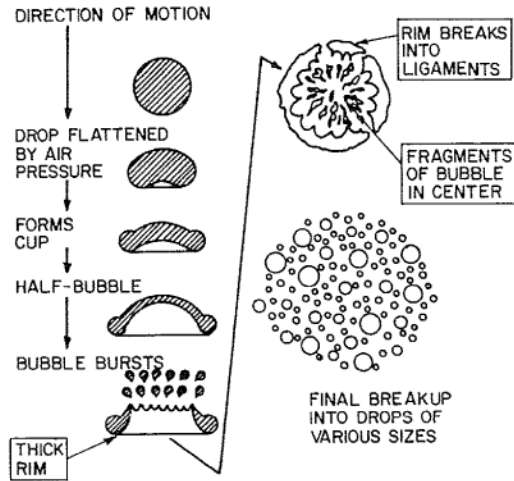


Figure 2.2 Breakup of a spherical drop by interaction with ambient air (Lane [18], Simmons [31]).

For the first mode of breakup, Lane [18] has established that at normal atmospheric conditions there is a limiting value of the relative velocity  $U_{R_{crit}}$  below which breakup does not occur. For liquids whose surface tensions lie in the range 0.028 to 0.475 kg/s<sup>2</sup>, the relationship is given by

$$U_{R_{crit}} \propto \left( \frac{\sigma}{D} \right)^{0.5} \quad (2.11)$$

This equation should be compared with Eq. (2.10). For water, Eq. (2.11) becomes

$$U_{R_{crit}} = \frac{784}{\sqrt{D}} \quad (2.12)$$

where  $U_R$  is in meters per second and  $D$  in micrometers.

From the data of Merrington and Richardson [9], Hinze [7] estimated  $We_{crit}$  to be 22 for a free-falling drop. For low-viscosity liquid drops suddenly exposed to a high-velocity airstream, the critical value of the Weber number was estimated to be 13.

Taylor [10] has provided an explanation for the observation that critical Weber numbers for drops subjected to a steady stream are almost twice as high as those for drops suddenly exposed to an airflow at constant speed. According to Taylor, the reason for this may be seen by considering the drop as a vibrating system and comparing the suddenly applied force  $F_T$  and the gradually increasing force  $F_S$  required to produce the same extension. The maximum distortion the drop can sustain without breaking occurs when  $F_T = 0.5F_S$ .

Hinze's value of 13 for the critical Weber number may be compared with

$8/C_D$  from Eq. (2.8), 10.6 from the data of Lane [18], 10.3 for mercury drops in air obtained by Haas [19], and 7.2 to 16.8 (with an average of about 13.0) for water, methyl alcohol, and a low-viscosity silicone oil obtained by Hanson et al. [20].

To account for the influence of liquid viscosity on drop breakup, Hinze [7] used a viscosity group defined as

$$Z = \frac{\sqrt{We}}{Re} \quad (2.13)$$

This dimensionless group represents the ratio of an internal viscosity force to an interfacial surface tension force [13]. Some idea of the significance of the group may be gained by examining these forces [21]. The interfacial tension force per unit area is represented by  $\sigma/D$ , while the friction force per unit area,  $\tau$ , is given by the product of liquid viscosity and the velocity gradient within the drop, i.e.,  $\mu_L \delta U / \delta x$ . If the drop is oscillating at its natural frequency of vibration,  $\omega$ , then  $\tau$  is of the order  $\mu_L \omega$ . The frequency is related to the properties of the drop by

$$\omega^2 = \frac{2\sigma n(n+1)(n-1)(n+2)}{\pi^2 D^3 [\rho_L(n+1) + \rho_A n]} \quad (2.14)$$

where  $n$  depends on the mode of vibration. The most important mode is the first, for which  $n = 2$ . Since the density of the liquid is usually much higher than that of the surrounding air, for  $n = 2$  Eq. (2.14) becomes

$$\tau = \mu_L \omega \approx \frac{4\mu_L}{\pi D} \left( \frac{\sigma}{D\rho_L} \right)^{0.5} \quad (2.15)$$

The ratio of the friction force to the surface tension force is thus

$$\frac{\tau}{\sigma D} = \frac{4}{\pi} \frac{\mu_L}{(\rho_L \sigma D)^{0.5}} = \left( \frac{4}{\pi} \right) Oh \quad (2.16)$$

This equation illustrates the relevance of the Ohnesorge number  $Oh$  to the atomization process. However, as Sleicher [21] has pointed out, before the drops break up they are likely to become distorted to an extent far beyond the linear region of validity of Eq. (2.15). Therefore, the viscosity forces represented in this group may not be as important as those that resist the stretching of the drop by the flow field.

According to Hinze [7], the effect of viscosity on the critical Weber number can be expressed by a relation of the form

$$We_{crit} = \dot{We}_{crit} [1 + f(Oh)] \quad (2.17)$$

where  $\dot{We}_{crit}$  is the critical Weber number for zero viscosity.

This form is considered acceptable because  $Oh$  goes to zero as  $\mu_L$  goes to zero. Thus, when  $\mu_L$  is so small that it offers no resistance to drop deformation,

the initial value of the Weber number, as given by Eq. (2.17), reverts to the value for zero viscosity.

The data of Hanson et al. [20] provide qualitative support for Eq. (2.17) but do not agree in detail. Brodkey [22] has proposed the following empirical relationship:

$$\text{We}_{\text{crit}} = \dot{\text{We}}_{\text{crit}} + 14\text{Oh} \quad (2.18)$$

which is claimed to be accurate within 20%.

### Drop Breakup in Turbulent Flow Fields

The foregoing discussion is based on the assumption of a high relative velocity between the drop and the surrounding gas. In many practical situations, a high relative velocity may not exist or may be difficult to determine. In such cases it is perhaps more logical to assume that the dynamic pressure forces of the turbulent motion determine the size of the largest drops. Kolmogorov [23] and Hinze [7] took this view and further assumed that since the breakup was to be considered local, the principles of isotropic turbulence would be valid. The kinetic energy of a turbulent fluctuation increases with increasing wavelength. Thus, velocity differences having a wavelength equal to  $2D$  will produce a higher dynamic pressure than those due to fluctuations of shorter wavelength. If these fluctuations are assumed to be responsible for the breakup of drops, the critical Weber number becomes

$$\text{We}_{\text{crit}} = \frac{\rho_A \bar{u}^2 D_{\text{max}}}{\sigma} \quad (2.19)$$

where  $\bar{u}^2$  is the average value across the whole flow field of the squares of velocity differences over a distance equal to  $D_{\text{max}}$  [7]. For isotropic turbulence, the main contributions to the kinetic energy are made by fluctuations where the Kolmogorov energy distribution law is valid. For this region, where the turbulence pattern is determined solely by the energy input per unit mass per unit time,  $E$ , it can be shown that

$$\bar{u}^2 = C_1(ED)^{2/3} \quad (2.20)$$

where, according to Batchelor [24],  $C_1 = 2$ .

For low-viscosity liquids ( $\text{Oh} \ll 1$ ) Eq. (2.19) may be written as

$$\text{We}_{\text{crit}} = \frac{2\rho_A}{\sigma} E^{2/3} D_{\text{max}}^{5/3} \quad (2.21)$$

and

$$D_{\text{max}} = C \left( \frac{\sigma}{\rho_A} \right)^{3/5} E^{-2/5} \quad (2.22)$$

where  $C$  is a constant to be determined experimentally. This simple result can also be derived directly from dimensional analysis on the assumption that only  $\sigma$ ,  $\rho_A$ , and  $E$  determine the size of the largest drops.

Hinze [7] used Clay's [25] experimental data to calculate the value of the constant  $C$  in Eq. (2.22). Clay's apparatus consisted of two coaxial cylinders, of which the inner one rotated. The space between the cylinders was filled with two immiscible fluids, one of which formed discrete drops. Clay determined drop size distributions as a function of energy input and from these calculated the value of  $D_{95}$ , the value of  $D$  for which 95% of the total dispersed liquid is contained in drops of smaller diameter. By assuming  $D_{\max} = D_{95}$ , Hinze used Clay's data to derive a value for  $C$  of 0.725 (see Fig. 2.3). Equation (2.22) thus becomes

$$D_{\max} = 0.725 \left( \frac{\sigma}{\rho_A} \right)^{0.6} E^{-0.4} \quad (2.23)$$

Substitution of this relation for  $D_{\max}$  into Eq. (2.21) gives

$$We_{\text{crit}} = 1.18 \quad (2.24)$$

Experimental data on breakup in an isotropic field are nonexistent, so direct verification of Eqs. (2.23) and (2.24) is not possible. Sleicher [21] has shown that these equations are not valid for pipe flow. In a pipe system, breakup occurs as a result of a balance between surface forces, velocity fluctuations, pressure fluctuations, and the steep velocity gradients.

The work of Kolmogorov [23] and Hinze [7] on the splitting of drops in turbulent flows has been modified by Sevik and Park [26]. They suggest that resonance can cause drop breakup in turbulent flow fields when the characteristic turbulence frequency matches the lowest or natural frequency mode of an entrained fluid particle. Since damping is very small, such drops will deform very violently if the existing frequency corresponds to one of their resonant frequencies. By setting a characteristic frequency of the turbulence equal to such a res-

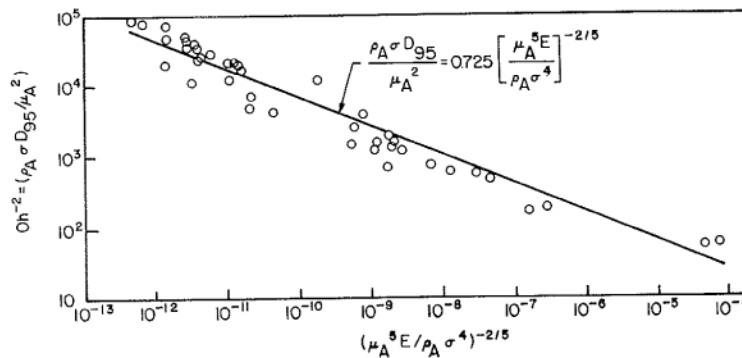


Figure 2.3 Maximum drop size as a function of energy input (Hinze [7]).

onant frequency, Sevik and Park were able to predict theoretically the critical Weber numbers corresponding to both Clay's drop-splitting experiments and their own bubble-splitting experiments. For drops it was found that

$$We_{crit} = 1.04 \quad (2.25)$$

as compared with Hinze's value of 1.18.

### Drop Breakup in Viscous Flow Fields

This mechanism of breakup applies to fluid globules surrounded by viscous fluid where there is a strong velocity gradient in the vicinity of the globules. In this situation the Reynolds numbers that are characteristic of the flow field may be so small that dynamic forces are no longer important, and breakup is controlled entirely by viscous and surface tension forces. If the viscous forces are large enough, the interfacial tension forces are unable to prevent the globule from splitting.

The first basic experiments on the splitting of drops under the action of viscous and surface tension forces were made by Taylor [10] in 1934. His test apparatus was designed to generate carefully controlled flow patterns, and several liquids with different viscosities were used. Among the observations made by Taylor, many of which were subsequently explained by Tomotika [27], are the following:

1. Under the action of viscous shear, a drop elongates into the shape of a prolate ellipsoid.
2. The deformation is governed by a dimensionless group,  $\mu_c SD/\sigma$ , where  $\mu_c$  is the viscosity of the continuous phase and  $S$  is the maximum velocity gradient in the external flow field.
3. Breakup of the drop occurs at a critical value of the Weber number that depends on the continuous flow field.

The experimental data obtained by Meister and Scheele [28] demonstrate the effects of the viscosities of both the dispersed and continuous phases, as well as their viscosity ratio, on atomization. At this point it is appropriate to make a distinction between the *dispersed* and *continuous* phases. The dispersed phase refers to the liquid to be atomized, whereas the continuous phase represents the medium, usually gaseous, in which the atomization occurs. By injecting heptane into various concentrations of aqueous glycerine solutions, Meister and Scheele showed that variation of continuous-phase viscosity has no appreciable effect on the atomization process. The viscosity of the dispersed phase, however, delays breakup at higher velocities, thereby impeding atomization.

The analysis of Tomotika [27], as reviewed by Miesse [29], suggests that the perturbation wavelength has a minimum value (for maximum growth rate) when the viscosity ratio  $\mu_L/\mu_A$  is almost unity. Either decreasing or increasing the value of this ratio reduces the tendency for drop breakup. The same conclusion was reached by Hinze [7], who showed from Taylor's experimental data that a min-

imum value for  $We_{crit}$  of less than 1 is obtained for viscosity ratios between 1 and 5.

Taylor's theory has been modified by several workers over the years. For example, Rumscheidt and Mason [30] proposed that breakup occurs when

$$We_{crit} = \frac{1 + (\mu_L/\mu_A)}{1 + (19/16)(\mu_L/\mu_A)} \quad (2.26)$$

which varies only between 1 and 0.84 as  $\mu_L/\mu_A$  varies from zero to infinity.

Sevik and Park [26] pointed out that the Taylor mechanism of globule deformation applies only if both the undeformed and elongated drops are small compared with local regions of viscous flow. When the Reynolds number of the external flow field is large, as it is in most practical applications, the spatial dimensions of such local regions are very small compared with the drop sizes. Under these circumstances, the determining factor is the dynamic pressure caused by the velocity changes over distances of the order of the globule diameter. It is also clear from Taylor's experiments that it is very difficult to atomize liquids that show a high viscosity ratio. This explains why, in practice, dispersion by pure viscous flow is restricted to emulsification processes.

## DISINTEGRATION OF LIQUID JETS

When a liquid jet emerges from a nozzle as a continuous body of cylindrical form, the competition set up on the surface of the jet between the cohesive and disruptive forces gives rise to oscillations and perturbations. Under favorable conditions the oscillations are amplified and the liquid body disintegrates into drops. This process is sometimes referred to as *primary* atomization. If the drops so formed exceed the critical size, they further disintegrate into drops of smaller size, a process known as *secondary* atomization.

The phenomenon of jet disintegration has been subjected to theoretical and experimental investigation for more than 100 years. A comprehensive review of jet flow was made by Krzywoblocki [32], and other reviews of relevance to liquid jet flow are also available [29, 33–38]. The following discussion is restricted to liquid jets injected into a gaseous atmosphere. It applies to any combination of liquid (the dispersed phase) and gas (the continuous phase), but for convenience the surrounding gas is usually referred to as air.

The properties of jets of most interest are the continuous length (which provides a measure of the growth rate of the disturbance) and the drop size (which is a measure of the wave number of the most unstable disturbance). Also of interest is the manner in which the jet is disrupted.

The earliest investigations into jet flow phenomena appear to have been carried out by Bidone [39] and Savart [40]. Bidone's research was concerned with the geometric forms of jets produced by nozzles of noncircular cross section, while Savart supplied the first quantitative data related to jet disintegration. His

results showed that if the jet diameter is kept constant, the length of the continuous part of the jet is directly proportional to jet velocity. He also observed that, for constant jet velocity, the length of a jet is directly proportional to its diameter.

Plateau [41] is credited with the first theoretical investigation into jet instability. He showed that a cylindrical column of liquid is unstable if its length exceeds its perimeter; otherwise, two drops would be formed whose total surface area would be less than that of the original cylinder. Plateau's work helped to explain the results of Savart and provided the basis for Rayleigh's more widely known theory of jet stability.

In an early mathematical analysis, Rayleigh [11] employed the method of small disturbances to predict the conditions necessary to cause the collapse of a liquid jet issuing at low velocity, for example, a low-speed water jet in air. This comprehensive analysis has been reviewed and summarized by McCarthy and Molloy [42].

Rayleigh compared the surface energy (directly proportional to the product of surface area and surface tension) of the disturbed configuration with that of the undisturbed column. He then calculated the potential energy of the disturbed configuration (relative to the equilibrium value) as

$$E_s = \frac{\pi\sigma}{2d} (\gamma^2 + n^2 - 1)b_n^2 \quad (2.27)$$

where  $E_s$  = potential surface energy  
 $d$  = jet diameter  
 $b_n$  = constant in Fourier series expansion  
 $\gamma$  = dimensionless wave number =  $2\pi/\lambda$   
 $\lambda$  = wavelength of disturbance  
 $n$  = any positive integer (including zero)

For nonsymmetrical disturbances,  $n \gg 1$  and  $E_s$  is always positive, indicating that the system is always stable to this class of disturbance. When  $n = 0$  and  $\gamma < 1$ , which is the case for symmetrical disturbances, Eq. (2.27) shows that  $E_s$  is negative and the system is unstable to this class of disturbance. Hence, a liquid jet that is affected by surface tension forces only will become unstable to any axisymmetrical disturbance whose wavelength satisfies the inequality

$$\lambda > \pi d$$

which corresponds to

$$\gamma < 1$$

The conclusion to be drawn from Rayleigh's analysis of the breakup of non-viscous liquid jets under laminar flow conditions is that all disturbances on a jet with wavelengths greater than its circumference will grow. Furthermore, his results show that one class of disturbance will grow fastest and eventually control the breakup. Although actual liquid jets are viscous, turbulent, and subjected to

surrounding air influences, the conclusions of Rayleigh have found general acceptance in later theories as valid first approximations.

Rayleigh's contribution to the mathematical treatment of the breakup of jets stemmed from his recognition that jet breakup is a dynamic problem and that the rate of collapse is important. By assuming that  $b_n$  in Eq. (2.27) is proportional to  $\exp(qt)$ , where  $q$  is the exponential growth rate of disturbance, Rayleigh showed that the exponential growth rate of the fastest-growing disturbance is given by

$$q_{\max} = 0.97 \left( \frac{\sigma}{\rho_L d^3} \right)^{0.5} \quad (2.28)$$

and  $\lambda_{\text{opt}}$ , corresponding to  $q_{\max}$ , is

$$\lambda_{\text{opt}} = 4.51d \quad (2.29)$$

After breakup, the cylinder of length  $4.51d$  becomes a spherical drop, so that

$$4.51d \times \frac{\pi}{4} d^2 = (\pi/6) D^3$$

and hence

$$D = 1.89d \quad (2.30)$$

Thus for the Rayleigh mechanism of breakup the average drop size is nearly twice the diameter of the undisturbed jet.

Figure 2.4 shows an idealization of Rayleigh breakup for a falling liquid jet and breakup as it actually occurs. It may be noted in Fig. 2.4a that this mode of drop breakup is quite regular; the liquid jet first starts to run down and eventually

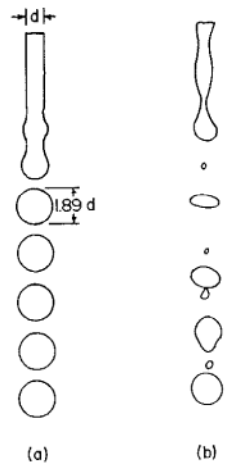


**Figure 2.4** Comparison of (a) idealized jet breakup with (b) actual breakup as indicated by high-speed photographs.

collapses to form drops of uniform size and spacing. Figure 2.4b, which is based on high-speed photographs, shows the appearance of the jet in this region. The formation and growth of axisymmetric disturbances are evident, as is the characteristic dumbbell shape just prior to breakup. The jet pinches off at the ends of the connecting cylinder, which in turn becomes unstable and breaks down into drops. The drops formed from this cylinder usually coalesce, so the end result is a pattern of large drops with small single "satellite" drops between them [35].

Tyler [43] later measured the frequency of formation of drops as a jet disintegrated and related it to the wavelength of the disturbance. By assuming that the volume of the spherical drops formed by disintegration of the jet is equal to the volume of a cylinder with diameter equal to that of the undisturbed jet and with wavelength equal to the most rapidly growing disturbance, Tyler obtained the following results:

$$\frac{D}{d} = \left(1.5 \frac{\lambda}{d}\right)^{1/3} \quad (2.31)$$

$$\lambda_{\text{opt}} = 4.69d \quad (2.32)$$

$$D = 1.92d \quad (2.33)$$

In view of the close agreement between his experimental results and the predictions of Rayleigh's mathematical analysis [see Eqs. (2.29) and (2.30)], Tyler concluded that cylindrical jets do break up under the conditions required for maximum instability, as predicted by Rayleigh's theory.

A more general theory for disintegration at low jet velocities was developed by Weber [44], who extended Rayleigh's analysis to include viscous liquids. He assumed that any disturbance causes rotationally symmetrical oscillations of the jet, as illustrated in Fig. 2.5a. If the wavelength of the initial disturbance is less

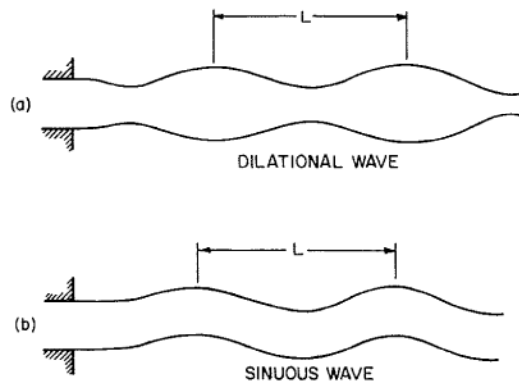


**Figure 2.5** (a) Jet with rotationally symmetric disturbance. (b) Jet disturbance causing wave formation.

than  $\lambda_{\min}$ , the surface forces tend to damp out the disturbance. If  $\lambda$  is greater than  $\lambda_{\min}$ , the surface tension forces tend to increase the disturbance, which eventually leads to disintegration of the jet. There is, however, one particular wavelength,  $\lambda_{\text{opt}}$ , that is most favorable for drop formation. For nonviscous liquids

$$\lambda_{\min} = \pi d \quad (2.34)$$

$$\lambda_{\text{opt}} = \sqrt{2}\pi d = 4.44d \quad (2.35)$$

For viscous liquids

$$\lambda_{\min} = \pi d \quad (2.36)$$

$$\lambda_{\text{opt}} = \sqrt{2}\pi d \left( 1 + \frac{3\mu_L}{\sqrt{\rho_L \sigma d}} \right)^{0.5} \quad (2.37)$$

Thus, for nonviscous liquids the value of  $\lambda/d$  required to produce maximum instability is 4.44, which is close to the value of 4.51 predicted by Rayleigh for this case. It is of interest to note that the minimum wavelength is the same for both viscous and nonviscous liquids, but the optimum wavelength is greater for viscous liquids.

Weber next examined the effect of air resistance on the disintegration of jets into drops. He found that air friction shortens both the minimum wavelength and the optimum wavelength for drop formation. Recalling that for nonviscous liquids and zero air velocity

$$\lambda_{\min} = 3.14d \quad (2.34)$$

$$\lambda_{\text{opt}} = 4.44d \quad (2.35)$$

at 15 m/s relative air velocity

$$\lambda_{\min} = 2.2d \quad (2.38)$$

$$\lambda_{\text{opt}} = 2.8d \quad (2.39)$$

Thus, the effect of relative air velocity is to reduce the optimum wavelength for jet breakup.

Weber also considered the case where the air motion induces wave formation and showed that this can occur only at relative air velocities above a certain minimum value. For glycerin the minimum velocity for wave formation is about 20 m/s. At this velocity the theoretical breakup distance is infinite. Increasing the velocity reduces the breakup distance; for example, at 25, 30, and 35 m/s, the corresponding wavelengths are  $3.9d$ ,  $3.0d$ , and  $1.3d$ , respectively.

Haenlein [12] presented experimental evidence in support of Weber's theoretical analysis. Using nozzles with a length-to-bore ratio of 10 and liquids of various viscosities and surface tensions, he showed that for liquids of high viscosity (0.85 kg/m s) the ratio of wavelength to jet diameter producing maximum instability could range from 30 to 40, in contrast to the value of 4.5 predicted by the Rayleigh theory for nonviscous jets.

Haenlein identified four distinct regimes of breakup in the disintegration of a liquid jet.

1. Drop formation without the influence of air. This is the mechanism studied by Rayleigh. The term "varicose" is sometimes used to describe the appearance of the jet in this regime. Radially symmetric waves, as illustrated in Fig. 2.6a, are formed by the interaction of primary disturbances in the liquid and surface tension forces. This regime is characterized by a linear relationship between the length of the jet prior to breakup and the jet velocity. Weber calculated the breakup time to be proportional to  $d_0^{1.5}$  for nonviscous jets and proportional to  $d_0$  for viscous jets.
2. Drop formation with air influence (Fig. 2.6b). As the jet velocity is increased, the aerodynamic forces of the surrounding air are no longer negligible and tend to accentuate the waves formed under regime 1.
3. Drop formation due to waviness of the jet (Fig. 2.6c). This regime is associated with increasing effectiveness of aerodynamic forces and lessened relative influence of surface tension. The term "sinuous" has been used to describe the jet in this regime.
4. Complete disintegration of the jet, i.e., atomization. The liquid is broken up at the nozzle in a chaotic and irregular manner.

Although these four separate regimes can be clearly identified, there is no sharp demarcation between them. From a practical viewpoint it is unfortunate that regime 4, which is the normal operating condition for the plain-orifice atomizer, is not easily described.

Perhaps the most commonly quoted criteria for classifying jet disintegration

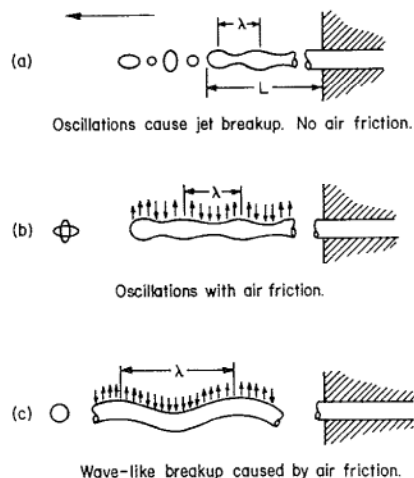


Figure 2.6 Mechanisms of drop formation (Haenlein [12]).

are those proposed by Ohnesorge [13]. From photographic records of jet disintegration, Ohnesorge classified the data according to the relative importance of gravitational, inertial, surface tension, and viscous forces. He used dimensionless analysis with good effect to show that the breakup mechanism of a jet could be expressed in three stages, each stage characterized by the magnitudes of the Reynolds number and a dimensionless number  $Z$ , which is obtained as

$$Z = \left( \frac{U_L^2 \rho_L d_o}{\sigma} \right)^{0.5} \left( \frac{U_L d_o \rho_L}{\mu_L} \right)^{-1} = \frac{\mu_L}{(\rho_L \sigma d_o)^{0.5}} \quad (2.40)$$

This group is sometimes referred to as the stability number, the viscosity group [7], or the Ohnesorge number (Oh).

Ohnesorge showed that the various mechanisms of jet breakup could be divided into three regions on a graph of Ohnesorge number versus Reynolds number according to the rapidity of drop formation:

1. At low Reynolds numbers, the jet disintegrates into large drops of fairly uniform size. This is the Rayleigh mechanism of breakup.
2. At intermediate Reynolds numbers, the breakup of the jet is by jet oscillations with respect to the jet axis. The magnitude of these oscillations increases with air resistance until complete disintegration of the jet occurs. A wide range of drop sizes is produced.
3. At high Reynolds numbers, atomization is complete within a short distance from the discharge orifice.

Ohnesorge's chart is shown in Fig. 2.7. Since for a given liquid and orifice size the Oh number is constant, a variation of Reynolds number on the chart follows a horizontal line. Thus, at low Reynolds numbers (region I) the jet structure is predominantly varicose and the mode of breakup follows the Rayleigh mechanism. With increase in Reynolds number the mode moves into region II, where the jet oscillates about its axis, having a twisted or sinuous appearance (see Fig. 2.5b). Passing through this narrow band, region III is reached, in which atomization occurs at the orifice from which the jet emerges.

For region III, Castleman [8] proposed a mechanism for jet disruption based on the observations of Sauter [14] and Scheubel [15]. According to Castleman, the most important factor in the process of jet disintegration is the effect of the relative motion between the outer jet layer and the air; this, combined with air friction, causes irregularities in the previously smooth liquid surface and the production of unstable ligaments. As the relative air speed increases, the size of the ligaments decreases and their life becomes shorter; upon their collapse, much smaller drops are formed, in accordance with Rayleigh's theory.

Miesse [29] found that his experimental data would fall in the appropriate area in Fig. 2.7 only if the boundary line between regions II and III was translated to the right, as indicated by the dashed line in Fig. 2.7. The equation of the modified boundary line was determined by Miesse as

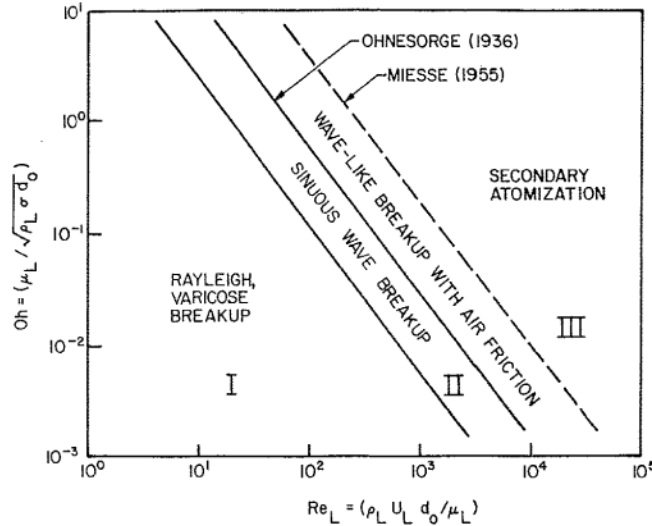


Figure 2.7 Classification of modes of disintegration (Ohnesorge [13]).

$$Oh = 100Re^{-0.92} \quad (2.41)$$

or

$$We^{0.5} = 150Oh^{-0.087} \quad (2.42)$$

In a more recent study, Reitz [45] attempted to resolve some of the uncertainties surrounding the Ohnesorge chart. His analysis was based on interpretation of the data on diesel sprays obtained by himself and other workers, including Giffen and Muraszew [3] and Haenlein [12]. According to Reitz [45], the following four regimes of breakup are encountered as the liquid injection velocity is progressively increased.

1. Rayleigh jet breakup. This is caused by the growth of axisymmetric oscillations of the jet surface, induced by surface tension. Drop diameters exceed the jet diameter.
2. First wind-induced breakup. The surface tension effect is now augmented by the relative velocity between the jet and the ambient gas, which produces a static pressure distribution across the jet, thereby accelerating the breakup process. As in regime 1, breakup occurs many jet diameters downstream of the nozzle. Drop diameters are about the same as the jet diameter.
3. Second wind-induced breakup. Drops are produced by the unstable growth of short-wavelength surface waves on the jet surface caused by the relative motion of the jet and the ambient gas. This wave growth is opposed by surface tension. Breakup occurs several diameters downstream of the nozzle exit. Average drop diameters are much less than the jet diameter.

4. Atomization. The jet disrupts completely at the nozzle exit. Average drop diameters are much less than the jet diameter.

These four regimes are shown in Fig. 2.8 and described in Table 2.1.

### Influence of Jet Velocity Profile

It is probable that some of the peculiarities and anomalies associated with experimental work on jet stability arise from differences in the velocity profile and turbulence properties of the jet as it emerges from the nozzle. Schweitzer [33] provided an excellent qualitative description of hydraulic turbulence in jets and orifices. The jet emerges from the nozzle in either a laminar or turbulent state. When the liquid particles flow in streams parallel to the axis of the tube, the flow is laminar. When the paths of the liquid particles cross each other in a random manner, the particles having various transverse velocity components, the flow is turbulent. Laminar flow is promoted by the absence of any disturbances to the flow, a rounded entrance to the tube, and high liquid viscosity, whereas turbulent flow is promoted by high flow velocity, large tube size, surface roughness, rapid changes in tube cross-sectional area, and protuberances projecting into the flowing stream. Irrespective of the length of the tube, an initially laminar flow will remain laminar if its Reynolds number is lower than the critical Reynolds number. Conversely, an originally turbulent flow will remain turbulent if its Reynolds number remains above the critical value.

In a smooth tube with no flow disturbances, an initially laminar flow can

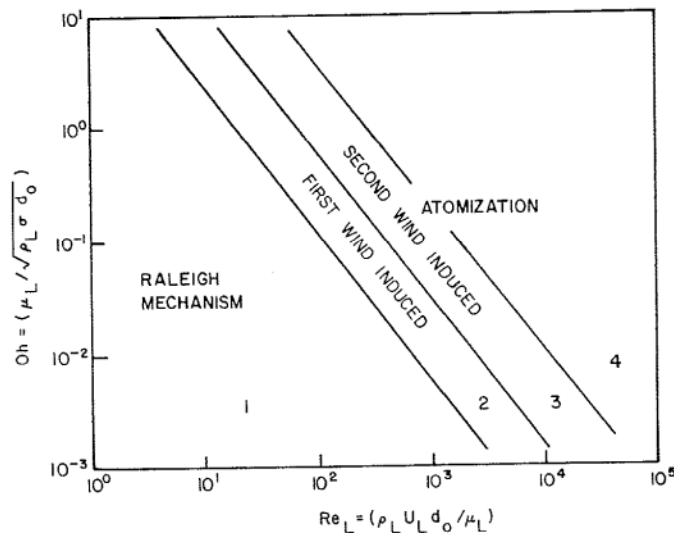


Figure 2.8 Classification of modes of disintegration (Reitz [45]).

**Table 2.1 Classification of jet breakup regimes**

| Regime | Description                 | Predominant drop formation mechanism                                       | Criteria for transition to next regime     |
|--------|-----------------------------|----------------------------------------------------------------------------|--------------------------------------------|
| 1      | Rayleigh breakup            | Surface tension force                                                      | $We_A > 0.4$<br>$We_A > 1.2 + 3.4Oh^{0.9}$ |
| 2      | First wind-induced breakup  | Surface tension force; dynamic pressure of ambient air                     | —                                          |
| 3      | Second wind-induced breakup | Dynamic pressure of ambient air opposed by surface tension force initially | $We_A > 40.3$<br>$We_A > 13$               |
| 4      | Atomization                 | Unknown                                                                    | —                                          |

Source: Reitz [45].

remain laminar up to Reynolds numbers much higher than the critical value, but if the Reynolds number exceeds the critical value, only a small disturbance to the flow is required to initiate a transition to turbulent flow.

As Schweitzer [33] pointed out, there is a general tendency to associate turbulent flow with a Reynolds number higher than critical, although in reality the flow is sometimes laminar at Reynolds numbers higher than critical and may be turbulent or semiturbulent at Reynolds numbers below the critical value. A semi-turbulent flow comprises a turbulent core and a laminar envelope, as illustrated in Fig. 2.9.

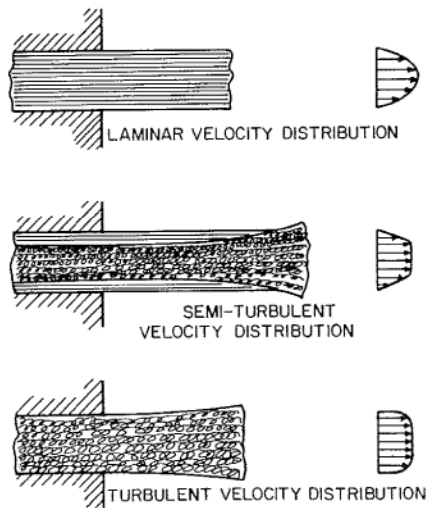


**Figure 2.9** Various velocity distributions in jets as suggested by Giffen and Muraszew [3].

The critical Reynolds number may be defined as the value below which, in a long straight cylindrical tube, any disturbances in the flow will damp out. Above the critical Reynolds number, disturbances in the flow never damp out, no matter how long the tube is [33]. The critical Reynolds number so defined was found by Schiller [46] to be around 2320. In attempts to standardize the velocity profile in the emerging jet, many workers have used long tubes as nozzles to ensure that the jet initially possesses either a fully developed laminar (parabolic) profile or a fully developed turbulent velocity profile.

Spray nozzles are usually of compact size, with short orifices to minimize pressure losses. The state of turbulence in the flow at the nozzle exit is determined by the flow state upstream of the orifice and by disturbances generated in the approach passages and within the orifice itself. For a given nozzle all these factors are constant, so the nature of the flow—laminar, turbulent, or semiturbulent—is dictated by the Reynolds number.

The strong influence of the velocity profile of the emerging jet on its subsequent disintegration has been described by Schwietzer [33], McCarthy and Molloy [42], and Sterling [35, 47]. With laminar flow, the velocity distribution in the jet immediately downstream of the orifice varies in a parabolic manner, rising from zero at the outer surface to a maximum at the jet axis. If the jet is injected into quiescent or slow-moving air, there is no appreciable velocity difference between the outer skin of the jet and the adjacent air. Consequently the necessary conditions for atomization by air friction forces do not exist. However, after a certain distance, the combined effects of air friction and surface tension forces create surface irregularities that ultimately lead to disintegration of the jet.

If the flow at the orifice is fully turbulent, the radial velocity component soon leads to disruption of the surface film followed by general disintegration of the jet. It should be noted that when the issuing jet is fully turbulent, no aerodynamic forces are needed for breakup. Even when injected into a vacuum, the jet will disintegrate solely under the influence of its own turbulence.

If the jet is semiturbulent, as illustrated in Fig. 2.9, the annulus of laminar flow surrounding the turbulent core tends to prevent the liquid particles in the core from reaching and disrupting the jet surface. At the same time, the influence of air friction is minimal, due to the very low relative velocity between the jet surface and the surrounding air. Thus, jet disintegration does not occur close to the orifice exit. However, farther downstream, the faster turbulent core outpaces its protective laminar layer and then disintegrates in the normal manner of a turbulent jet. Alternatively, and to some extent simultaneously, a redistribution of energy takes place between the laminar and turbulent components of the total flow, which produces a flattening of the jet velocity profile. This process brings to the jet surface liquid particles with radial components of velocity. These particles disrupt the jet surface so that ultimately it disintegrates into drops.

The change in velocity profile (usually termed velocity profile "relaxation") that occurs downstream of the nozzle exit can have an important influence on the stability of the jet and on its subsequent breakup into drops [42]. As soon as a jet leaves a nozzle, i.e., as soon as the physical constraint of the nozzle wall is

removed, the process of velocity profile relaxation occurs by a mechanism of momentum transfer between transverse layers within the jet. Thus, in addition to the normal jet-destabilizing forces discussed above, there exists an additional disruptive mechanism arising from the internal motions associated with profile relaxation.

It is well known that the kinetic energy per unit mass of flowing gas or liquid is very dependent on the fluid velocity profile. For example, in pipe flow with a fully developed parabolic velocity profile (i.e., laminar flow) the kinetic energy per unit mass of fluid is exactly twice what it would be for a flat velocity profile (plug flow) with the same average velocity. Following McCarthy and Molloy [42], a quantity  $\epsilon$  can be defined such that

$$\epsilon = \frac{\int_0^A U_r^3 dA}{U^3 A} \quad (2.43)$$

where  $U_r$  is the local fluid velocity and  $U$  the average fluid velocity over area  $A$ . For the three different flow situations we have

1. For plug flow,  $\epsilon = 1$ .
2. For fully developed turbulent flow,  $\epsilon = 1.1$  to  $1.2$ .
3. For fully developed laminar flow,  $\epsilon = 2.0$ .

Thus, for flow in a pipe, the quantity  $\epsilon$  represents the ratio of the kinetic energy to the equivalent kinetic energy under plug flow conditions.

When a fully developed laminar jet emerges from a nozzle, its parabolic profile relaxes into a flat profile at the same average velocity. This process is accompanied by a reduction in  $\epsilon$  from 2 to 1 that involves a considerable redistribution of energy within the jet, leading to the creation of forces that can be quite violent, causing the jet to burst. The phenomenon of "bursting breakup" was first observed and explained by Eisenklam and Hooper [48] and was also noted by Rupe [49]. Obviously, jets with fully developed turbulent profiles on exit ( $\epsilon = 1.1$  to  $1.2$ ) are only slightly susceptible to profile relaxation effects.

For high-velocity jets it is now generally believed that the action of the surrounding air or gas is the primary cause of atomization, although jet turbulence is a contributing factor because it ruffles the surface of the jet, making it more susceptible to aerodynamic effects. The different roles played by liquid turbulence and air friction allow the distinction to be made between *primary* and *secondary* atomization. Primary atomization is related to jet breakup by the action of internal forces, such as turbulence, inertial effects, or those arising from velocity profile relaxation and surface tension. Secondary atomization always involves the action of aerodynamic forces in addition to those present in primary atomization [42]. Aerodynamic forces promote atomization by acting directly on the surface of the jet and by splitting the drops formed in primary atomization into smaller droplets.

### Stability Curve

Many investigators have characterized jet behavior by determining experimentally the relationship between jet velocity and breakup length. The latter is defined as the length of the continuous portion of the jet, measured from the nozzle to the breakup point where drop formation occurs.

**Laminar flow region.** The general shape of the length-velocity curve is shown in Fig. 2.10. The initial dashed portion of this curve below A corresponds to drip flow. Point A denotes the lower critical velocity at which the drip flow changes to jet flow. From A to B the breakup length  $L$  increases linearly with velocity. This portion of the curve corresponds to disintegration of the jet due to surface forces as studied by Rayleigh and Weber. Weber [44] has shown that a small axisymmetric disturbance  $\delta_0$  will grow at an exponential rate  $q_{\max}$  until  $\delta$  acquires a value equal to the jet radius. If  $t_b$  is the time required for breakup, it is assumed that

$$r_0 = \delta_0 \exp(q_{\max} t_b) \quad (2.44)$$

Hence

$$t_b = \frac{\ln(d/2\delta_0)}{q_{\max}} \quad (2.45)$$

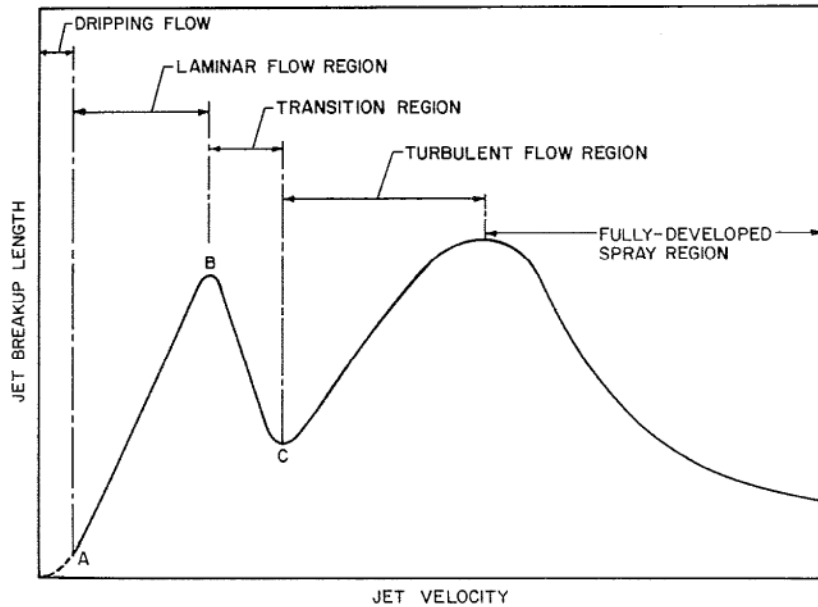


Figure 2.10 Jet stability curve indicating change of breakup length with jet velocity.

As

$$t_b = \frac{L}{U}$$

$$L = \frac{U}{q_{\max}} \ln \left( \frac{d}{2\delta_0} \right) \quad (2.46)$$

From Eq. (2.28) we have

$$q_{\max} = 0.97 \left( \frac{\sigma}{\rho_L d^3} \right)^{0.5} \quad (2.28)$$

Substituting for  $q_{\max}$  from Eq. (2.28) into Eq. (2.46) gives

$$\frac{L}{d} = 1.03 U \left( \ln \frac{d}{2\delta_0} \right) \left( \frac{\rho_L d}{\sigma} \right)^{0.5} \quad (2.47)$$

or

$$L = 1.03 d \text{We}^{0.5} \ln(d/2\delta_0) \quad (2.48)$$

Equation (2.48) is thus a prediction of the breakup length of a liquid jet subjected only to inertial and surface tension forces.

For the breakup length of a viscous jet in the absence of air friction, Weber's analysis yields an expression of the form

$$L = U \left( \ln \frac{d}{2\delta_0} \right) \left[ \left( \frac{\rho_L d^3}{\sigma} \right)^{0.5} + \frac{3\mu_L d}{\sigma} \right] \quad (2.49)$$

which may be rewritten as

$$L = d \text{We}^{0.5} (1 + 3\text{Oh}) \ln \left( \frac{d}{2\delta_0} \right) \quad (2.50)$$

Thus from Eqs. (2.48) and (2.50) we see that breakup length is proportional to  $d^{1.5}$  for nonviscous liquids and proportional to  $d$  for viscous liquids.

Unfortunately, the initial disturbance ( $d/2\delta_0$ ) cannot be determined a priori. Its value will depend on the particular experimental conditions of nozzle geometry and liquid flow rate. For glycol and glycerol/water solutions, Grant and Middleman [50] determined an average value of 13.4 for  $\ln(d/2\delta_0)$  and also provided the following more general correlation:

$$\ln \left( \frac{d}{2\delta_0} \right) = 7.68 - 2.66\text{Oh} \quad (2.51)$$

Weber's analytical results for jet breakup length have not, in general, been supported by subsequent experimental evidence. In fact, this lack of agreement has caused some investigators to discount Weber's theory entirely. It was left to Sterling and Sleicher [47] to show that reported discrepancies between theory and

experiment could be due to relaxation of the velocity profile in the jet, as discussed earlier. They also showed that, in the absence of velocity profile relaxation, Weber's theory overestimates the influence of aerodynamic forces. They then modified Weber's theory to account for the viscosity of the ambient gas. Details of their analysis are beyond the scope of this work, but the significant improvement in data prediction afforded by their approach is illustrated in Figs. 2.11 and 2.12.

Mahoney and Sterling [51] later extended the results of Sterling and Sleicher [47] to obtain a universal equation for the length of laminar Newtonian jets. Their equation is strictly valid only for jets with an initially uniform velocity profile, but it appears to be satisfactory also for jets with an initial parabolic profile when the Oh number (based on jet properties) is large. It is essentially the same as Eq. (2.50) but has an extra term that is a function of Oh and We.

$$L = d We^{0.5} (1 + 3Oh) \ln\left(\frac{d}{2\delta_0}\right) / f(Oh, We) \quad (2.52)$$

For details of the expressions used in estimating  $f(Oh, We)$  in terms of the jet velocity and liquid properties, reference should be made to the original paper [51].

The success of Eq. (2.52) in predicting breakup length is illustrated in Fig. 2.13. This figure shows a comparison of measured breakup lengths obtained by

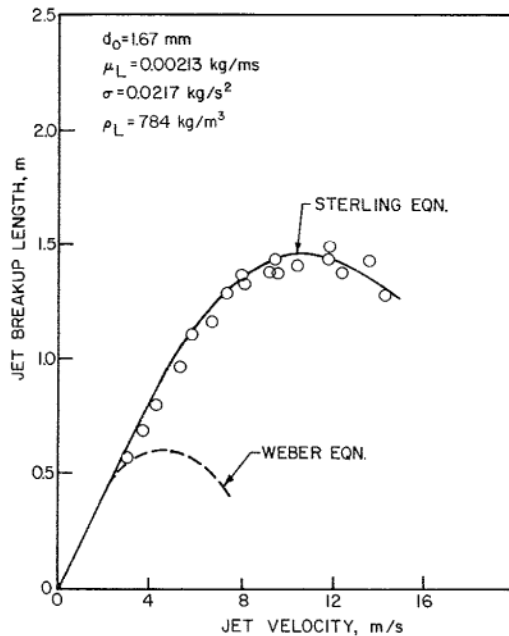


Figure 2.11 Comparison of experimental data with predictions of Weber [44] and Sterling [35].

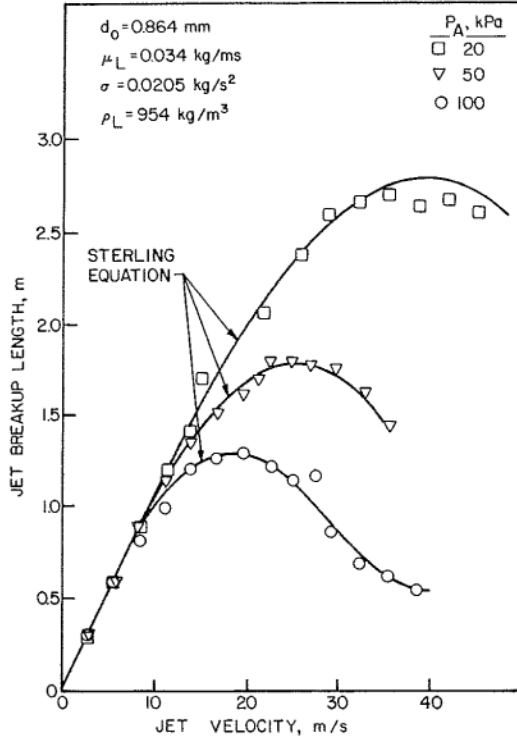


Figure 2.12 Comparison of experimental data with predicted values (*Sterling and Sleicher [47]*).

Phinney and Humphries [52] for sharp-edged orifices with jet lengths predicted from Eq. (2.52). The agreement is clearly very satisfactory.

A further empirical correlation of data for the region AB in Fig. 2.10 is the following, due to Grant and Middleman [50].

$$L = 19.5d We^{0.5}(1 + 3Oh)^{0.85} \quad (2.53)$$

**Upper critical point.** According to Haenlein [12], point B on the stability curve corresponds to the change in the breakup mechanism from varicose to sinuous (see Fig. 2.5b). For jets with fully developed parabolic velocity profiles, Grant and Middleman [50] developed the following empirical correlation for the upper critical point:

$$Re_{crit} = 3.25Oh^{-0.28} \quad (2.54)$$

According to Weber, the upper critical point marks the sudden impact of air resistance on jet stability and is solely dependent on the relative velocity between the jet surface and the surrounding gaseous medium. Other investigators have attributed this critical point to the onset of turbulence in the jet. It seems probable

that both mechanisms operate together in promoting more rapid jet disintegration, but which is dominant has yet to be resolved.

Van de Sande and Smith [53] have proposed the following expression for the critical Reynolds number at which the jet changes from laminar to turbulent flow:

$$\text{Re}_{\text{crit}} = 12,000 \left( \frac{1}{d} \right)^{-0.3} \quad (2.55)$$

This equation is intended to apply to the transition region, shown as BC in Fig. 2.10, where the changeover from a laminar to a turbulent jet occurs.

**Turbulent jets.** A turbulent jet is defined as one in which the flow is turbulent at the nozzle exit. Turbulent jets tend to have a “milky” surface. This is caused by light scattering from ruffles on the surface produced by random fluctuations of the velocity components, as opposed to the clear glassy appearance of laminar jets.

The notion that atomization is due to aerodynamic interaction between the liquid and the gas leading to unstable wave growth on the liquid jet surface, as postulated by Castleman [16], has not found universal acceptance. It is agreed that aerodynamically induced wave growth needs time to develop and, therefore, an undisturbed length should be observed at the nozzle exit. However, Reitz and Bracco [54] have pointed out that the most unstable wavelength and the undis-

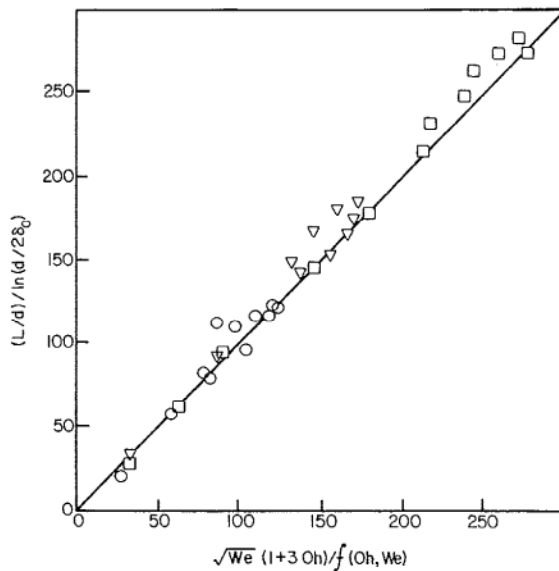


**Figure 2.13** Comparison of measured jet length to jet length predicted by Eq. (2.52) (Mahoney and Sterling [51]).

turbed length could be much smaller than the jet diameter and therefore difficult to observe experimentally. This is borne out by the remarkably clear and detailed photograph obtained for a water jet by Taylor and Hoyt [55], shown in Fig. 2.14, which provides strong support for Castleman's hypothesis.

Alternative mechanisms for the disintegration of turbulent jets have been proposed by various workers. For example, DeJuhasz [56] has suggested that the jet breakup process starts within the nozzle itself and is strongly influenced by turbulence. As discussed earlier, Schweitzer [33] took the view that the radial velocity components generated in turbulent pipe flow could produce immediate disruption of the jet at the nozzle exit. Bergwerk [57] hypothesized that liquid cavitation phenomena inside the nozzle could create large-amplitude pressure disturbances in the flow, leading to atomization. Sadek [58] also considered that cavitation bubbles could influence the atomization process. Other workers, including Eisenklam and Hooper [48] and Rupe [49], have suggested that jet breakup is due to velocity profile relaxation, which also accounts for the greater stability of turbulent jets. From their review of experimental work on turbulent jets and their own experimental observations, Reitz and Bracco [54] concluded that no single mechanism is responsible for jet breakup in all cases and often a combination of factors is involved.

Regardless of the detailed mechanisms of the atomization process, for turbulent jet flow with strong interaction between the liquid and the ambient gaseous medium, the jet breakup length increases with increase in velocity, as indicated by the region on the stability curve beyond point C in Fig. 2.10. Empirical correlations for breakup data in this region have been developed by several workers, including Vitman [59], Lienhard and Day [60], Phinney [36], and Lafrance [61]. For turbulent jet emerging from long smooth tubes, Grant and Middleman [50] suggested the following empirical relationship:

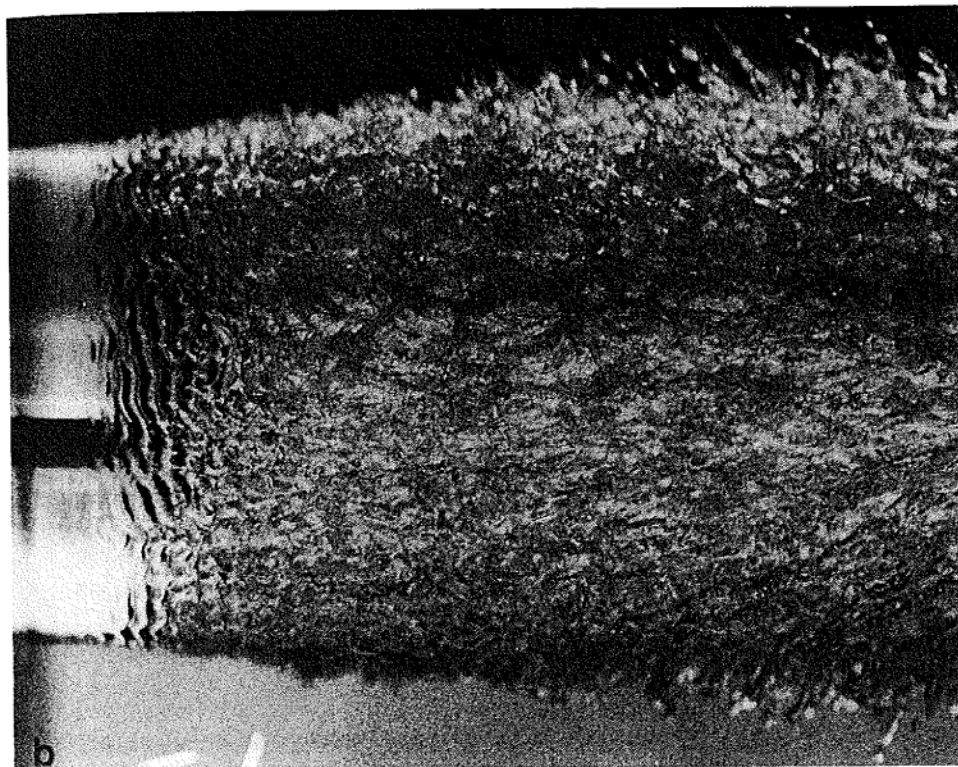
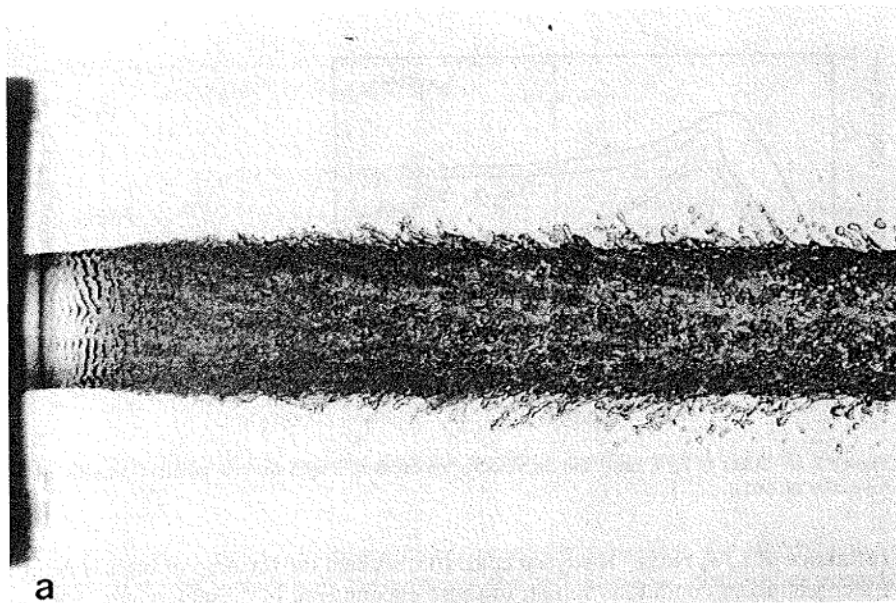
$$L = 8.51d_o We^{0.32} \quad (2.56)$$

while Baron's [62] correlation of Miesse's [29] water data gives

$$L = 538d_o We^{0.5} Re^{-0.625} \quad (2.57)$$

It is uncertain what happens to the shape of the stability curve if the jet velocity is increased indefinitely. In their review of experimental data in this region, McCarthy and Molloy [42] note that Tanasawa and Toyoda [63] assert that  $L/d_o$  increases continually with increasing velocity, while Yoshizawa et al. [64] claim that  $L/d_o$  decreases with increasing velocity. According to Hiroyasu et al. [65] and Arai et al. [66], the breakup length increases with jet velocity up to a maximum value, beyond which any further increase in velocity causes breakup length to decline, as illustrated in Fig. 2.10. Some of the results on which these conclusions are based are shown in Figs. 2.15 to 2.19, from which the data points have been removed for clarity.

**Figure 2.14** (a) High-speed photograph of water jet showing surface wave instabilities and spray detachment; (b) enlarged view of water jet just downstream of nozzle exit (Taylor and Hoyt [55]).



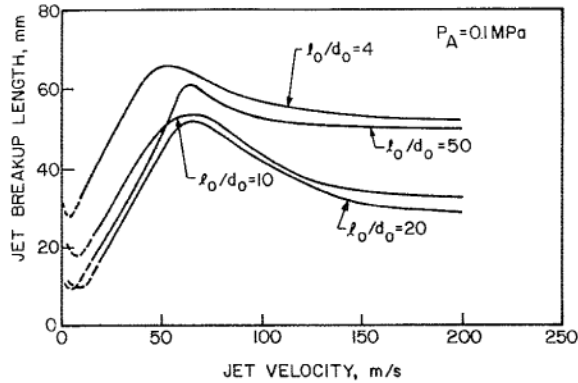


**Figure 2.15** Effect of  $l_0/d_0$  ratio and jet velocity on breakup length for low ambient pressure (Hiroyasu et al. [65]).

**Influence of  $l_0/d_0$  ratio.** Hiroyasu et al. [65] studied the breakup of high-velocity water jets under conditions similar to those encountered in diesel engines. Figure 2.15 shows the measured values of breakup length obtained for several nozzle  $l_0/d_0$  ratios at jet velocities up to 200 m/s when injecting into air at normal atmospheric pressure. This figure shows breakup length increasing with injection velocity up to a maximum at around 60 m/s, beyond which any further increase in jet velocity causes breakup length to decline. The influence of  $l_0/d_0$  on breakup length shows no clear trend. In the range of jet velocities of most practical interest, i.e.,  $>50$  m/s, it is of interest to note that for  $l_0/d_0$  ratios of 10 and 20, breakup length may be increased by either reducing the  $l_0/d_0$  ratio to 4 or increasing it to 50. Similar data on the effect of  $l_0/d_0$  on  $L$ , obtained at an ambient air pressure of 3 MPa (30 atm) are shown in Fig. 2.16. At this high pressure the influence of

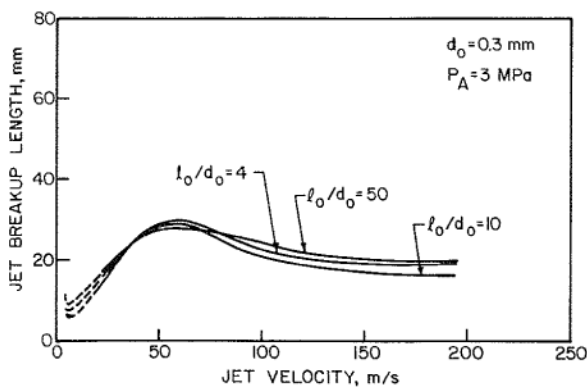


**Figure 2.16** Effect of  $l_0/d_0$  ratio and jet velocity on breakup length for high ambient pressure (Hiroyasu et al. [65]).

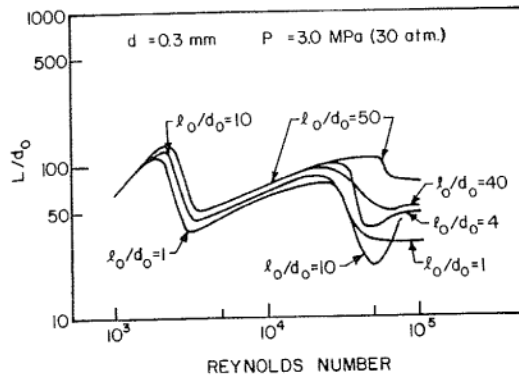


Figure 2.17 Effect of  $l_o/d_o$  ratio and Reynolds number on breakup length (Arai et al. [66]).

$l_o/d_o$  is clearly much less pronounced, presumably because the aerodynamic effects on the jet surface now far outweigh the hydrodynamic instabilities generated within the liquid upstream of the nozzle exit.

Arai et al. [66] carried out a similar series of tests on the influence of  $l_o/d_o$  on breakup length for high ambient air pressures (3 MPa). Diesel-type nozzles were used with water as the test fluid. Their results are shown in Fig. 2.17. For Reynolds numbers higher than 30,000,  $L$  is seen to diminish with increase in  $Re$ , pass through a minimum, and then reach an almost constant value when  $Re$  exceeds 50,000. According to Arai et al. [66], when the nozzle  $l_o/d_o$  ratio is much higher than 10, the strong turbulence created by separated flow at the nozzle entrance is reduced and the velocity profile of the internal flow changes to that of fully developed turbulent flow. Thus the breakup length of the  $l_o/d_o = 50$  nozzle is longer than that of the  $l_o/d_o = 10$  nozzle in the spray flow region. However, for small  $l_o/d_o$  ratios, the turbulence in the flow through the nozzle is not fully developed. This causes the breakup length to increase with  $l_o/d_o$  in the range from 1 to 4 in the spray flow region.

**Influence of ambient pressure.** In diesel engines, fuel is injected from plain-orifice atomizers into air at pressures of 3 MPa and higher. Some perception of the strong influence of ambient gas pressure on breakup length may be gained by comparison of Figs. 2.15 and 2.16. It is clear from these figures that an increase in pressure causes the breakup length to diminish and at the same time reduces the influence of  $l_o/d_o$  on breakup length, as discussed above.

The effect of ambient pressure on breakup length is shown more directly in Fig. 2.18. The curves drawn in this figure indicate a strong effect of pressure in the range 0.1 to 3 MPa (1 to 30 atm). Within this range, a 30-fold increase in gas pressure produces a 3-fold reduction in breakup length. Increase in ambient gas pressure from 3 to 4 MPa appears to have little effect on breakup length. For all pressures, breakup length increases with injection velocity up to around 60

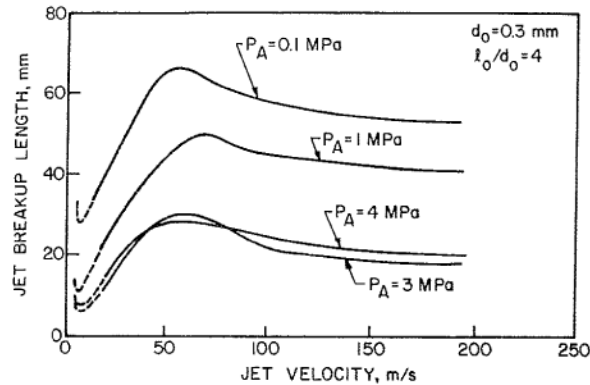


Figure 2.18 Influence of ambient pressure on breakup length (Hiroyasu *et al.* [65]).

m/s, beyond which any further increase in velocity causes breakup length to decline.

The results obtained by Arai *et al.* [66] on the influence of pressure on breakup length are shown in Fig. 2.19. In the laminar and transition flow regions the effect of ambient pressure is quite small, but in the turbulent flow region its influence is more pronounced. The results shown in Fig. 2.19 for the fully developed spray region are in broad agreement with those of Hiroyasu *et al.* [65]; both sets of data indicate a marked reduction in breakup length with increase in gas pressure. However, the results of Arai *et al.* generally conform more closely to the stability curve drawn in Fig. 2.10.

An interesting feature of Fig. 2.19 is that for a pressure of 0.4 MPa (4 atm) there is a range of Reynolds numbers over which the breakup length has two values. Starting from a Reynolds number below this region, increase in Reynolds number causes the breakup length to assume the smaller value. However, if the

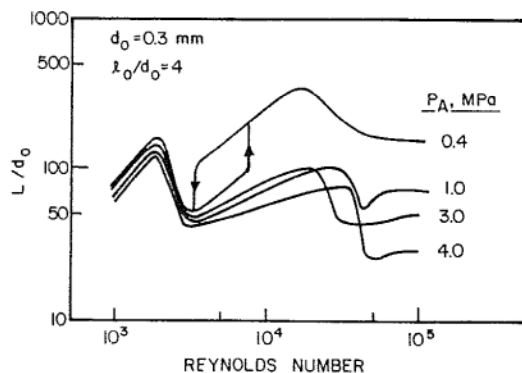


Figure 2.19 Influence of ambient pressure on breakup length (Arai *et al.* [66]).

Reynolds number is falling as it approaches and passes through this region, higher values of breakup length are obtained. Arai et al. attribute this phenomenon to the effects of flow separation and reattachment within the nozzle discharge orifice.

**Influence of transverse airflow.** Some atomizing devices produce jets of liquid that are disintegrated by exposure to a cross-flowing airstream. Kitamura and Takahashi [67] have studied this type of atomization. The breakup length was measured from film negatives exposed by a high-speed flash. The distance along the jet axis, the shape of which is a curved line, was used as the breakup length. Their results for water are shown in Fig. 2.20. Similar results were obtained for ethanol and aqueous glycerol solutions. They demonstrate that an increase in air velocity increases the aerodynamic destabilizing effects at the jet surface, thereby accelerating jet breakup. High-speed photographs reveal that the liquid jet is disintegrated by liquid disturbances at low air velocities, as illustrated in Fig. 2.6b, and by sinuous wave formation at high jet velocities, of the type shown in Fig. 2.6c.

## DISINTEGRATION OF LIQUID SHEETS

Many atomizers do not form jets of liquid, but rather form flat or conical sheets. Impingement of two liquid streams can produce a flat sheet. Conical sheets can be obtained if a liquid flowing in a pipe is deflected through an annular orifice, the form of the sheet being governed by the angle of deflection. Conical sheets are also generated in pressure-swirl and prefilming airblast nozzles, where the

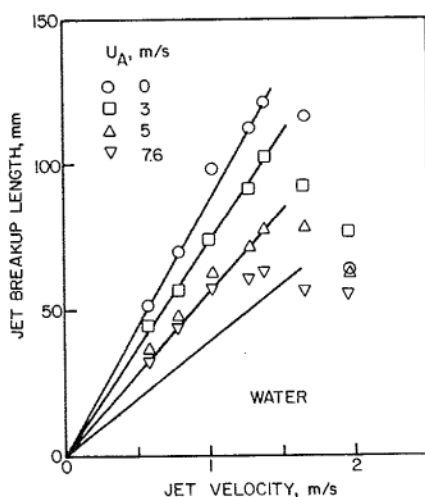


Figure 2.20 Effect of transverse airflow on breakup length (Kitamura and Takahashi [67]).

liquid issues from an orifice with a tangential velocity component resulting from its passage through one or more tangential or helical slots. Another widely used method of producing a flat circular sheet is by feeding the liquid to the center of a rotating disk or cup.

When a sheet of liquid emerges from a nozzle, its subsequent development is influenced mainly by its initial velocity and the physical properties of the liquid and the ambient gas. To expand the sheet against the contracting surface tension force, a minimum sheet velocity is required, which is provided by pressure, aerodynamic drag, or centrifugal forces, depending on whether the atomizer is pressure-swirl, prefilming-airblast, or rotary, respectively. Increasing the initial velocity expands and lengthens the sheet until a leading edge is formed where equilibrium exists between surface tension and inertial forces.

Fraser and Eisenklam [68] defined three modes of sheet disintegration, described as *rim*, *wave*, and *perforated-sheet* disintegration. In the rim mode, forces created by surface tension cause the free edge of a liquid sheet to contract into a thick rim, which then breaks up by a mechanism corresponding to the disintegration of a free jet. When this occurs, the resulting drops continue to move in the original flow direction, but they remain attached to the receding surface by thin threads that also rapidly break up into rows of drops. This mode of disintegration is most prominent where the viscosity and surface tension of the liquid are both high. It tends to produce large drops, together with numerous small satellite droplets.

In perforated-sheet disintegration, holes appear in the sheet and are delineated by rims formed from the liquid that was initially included inside. These holes grow rapidly in size until the rims of adjacent holes coalesce to produce ligaments of irregular shape that finally break up into drops of varying size.

Disintegration can also occur in the absence of perforations through the generation of a wave motion on the sheet whereby areas of the sheet, corresponding to a half or full wavelength of the oscillation, are torn away before the leading edge is reached. These areas rapidly contract under the action of surface tension, but they may suffer disintegration by air action or liquid turbulence before a regular network of threads can be formed. This type of wavy-sheet disintegration is evident in the photograph shown in Fig. 2.21.

As Fraser [69] has pointed out, the orderliness of the disintegration process and the uniformity of production of threads have a large influence on the drop size distribution. Perforations occurring in the sheet at the same distance from the orifice have a similar history, and thus thread diameters tend to be uniform and drop sizes fairly constant in perforated-sheet disintegration. However, wavy-sheet disintegration is highly irregular, and consequently drop sizes are much more varied.

Atomizers that discharge the liquid in the form of a sheet are usually capable of exhibiting all three modes of sheet disintegration. Sometimes two different modes occur simultaneously, and their relative importance can greatly influence both the mean drop size and the drop size distribution. Dombrowski, Eisenklam, Fraser, and co-workers [68–74] made numerous studies on the mechanisms of

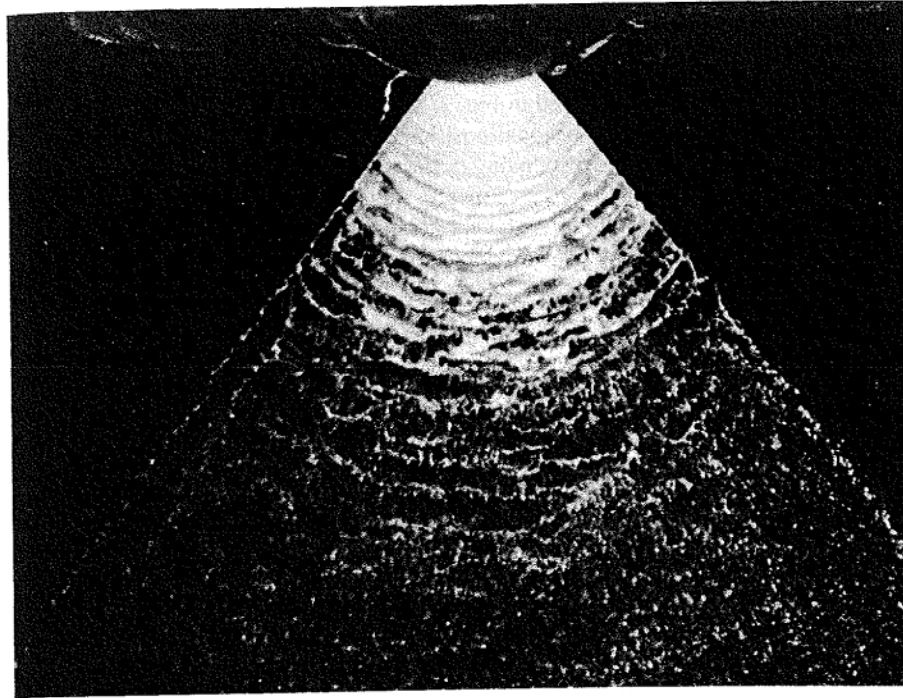


**Figure 2.21** Photograph illustrating atomization by wavy-sheet disintegration (*courtesy of Ransburg Gema, Inc.*).

sheet disintegration. In the early 1950s Dombrowski and Fraser provided useful insight into the manner of liquid sheet breakup, using an improved photographic technique and a specially designed source of lighting combining high intensity with very short duration. They established that ligaments are caused principally by perforations in the liquid sheet. If the holes are caused by air friction, the ligaments break up very rapidly. However, if the holes are created by other means, such as turbulence in the nozzle, the ligaments are broken more slowly. From a large number of tests on a wide variety of liquids, Dombrowski and Fraser [72] concluded that (1) liquid sheets with high surface tension and viscosity are most resistant to disruption and (2) the effect of liquid density on sheet disintegration is negligibly small.

### Flat Sheets

The mechanism for the disintegration of flat liquid sheets has been studied theoretically and experimentally by York et al. [17], who concluded that instability

and wave formation at the interface between the continuous and discontinuous phases are the major factors in the breakup of a sheet of liquid into drops. They considered the system of forces acting on the slightly disturbed surface of a liquid sheet moving in air, as illustrated in Fig. 2.22. Surface tension forces try to return the protuberance back to its original position, but the air experiences a local decrease in static pressure (corresponding to the local increase in velocity) that tends to expand the protuberance farther outward. This corresponds to the normal pattern of wind-induced instability, where surface tension forces oppose any movement of the interface from its initial plane and attempt to restore equilibrium, while the aerodynamic forces increase any deviation from the interface and thereby promote instability.

At the boundary between a liquid and the ambient air the balance of forces may be expressed by the equation

$$p_L - p_A = -\sigma \frac{d^2 h}{dx^2} \quad (2.58)$$

where  $h$  is the displacement of liquid from the equilibrium position and  $x$  is distance along the liquid sheet.

To formulate the problem, York et al. [17] considered a two-dimensional infinite sheet of liquid of finite thickness with air on both sides. By neglecting viscous effects and assuming irrotational flow, the velocities can be obtained from a velocity potential. Using calculated velocities and Bernoulli's equation, the pressures may then be estimated and the displacements  $h$  determined. As in the case of a liquid jet, an exponential increase in wave amplitude occurs under certain conditions. The amplitude increase is given by

$$h_t = A \exp(\beta t) \quad (2.59)$$

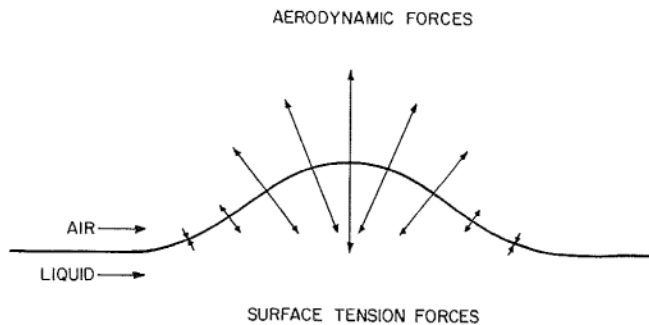


Figure 2.22 System of forces acting on the distributed interface of a liquid sheet moving in air.

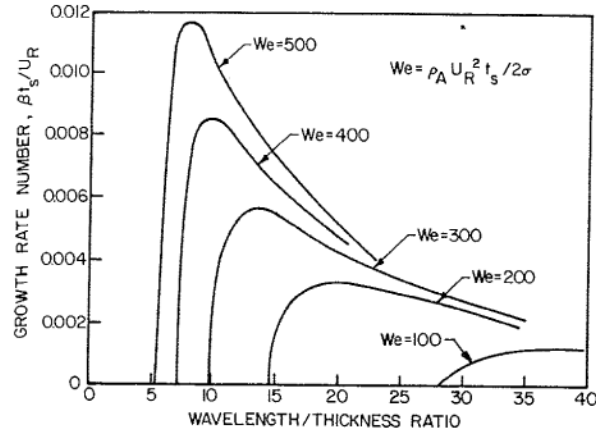


Figure 2.23 Effect of Weber number on wave growth for air and water (York *et al.* [17]).

where  $h_t$  is the amplitude at time  $t$ ,  $A$  the amplitude of the initial disturbance, and  $\beta$  a number that determines the growth rate of the disturbance. If the amplitude of the initial disturbance,  $A$ , is known, then the time for disintegration can be calculated from the relation

$$t = \beta^{-1} \ln \left( \frac{t_s}{2A} \right) \quad (2.60)$$

where  $t_s$  is the thickness of the liquid sheet. A graph based on the analysis of York *et al.* is plotted in Fig. 2.23 and illustrates the effect of Weber number on growth rate for a liquid sheet in air at atmospheric pressure.

Figure 2.23 shows that the growth rate has a clearly defined maximum for a given Weber number, especially at high Weber numbers. A disturbance of that wavelength will dominate the interface and rapidly disintegrate the sheet. Figure 2.23 also indicates a fairly precise lower limit for  $\lambda/t_s$  below which the interface is stable. This demonstrates that short wavelengths of disturbance on thick sheets are stable unless wind velocities or Weber numbers are very high.

If the wavelength  $\lambda^*$  corresponding to the maximum growth rate for a given Weber number ( $U_R^2 t_s \rho_A / \sigma$ ) is used to calculate the corresponding Weber number ( $U_R^2 \lambda^* \rho_A / \sigma$ ), the two Weber numbers can be plotted on a single graph to show the effect of varying the sheet thickness alone. This is done in Fig. 2.24, which demonstrates that large variations in sheet thickness create only small changes in  $\lambda^*$ , especially at low density ratios. For example, for water and air at normal atmospheric pressure, a hundredfold variation in sheet thickness changes the wavelength  $\lambda^*$  only 10%.

Using photographs of the flat liquid sheet, Hagerty and Shea [75] obtained experimental values of the growth rate factor ( $\beta$ ) for sinuous waves and compared them with the theoretically predicted curve calculated from the equation

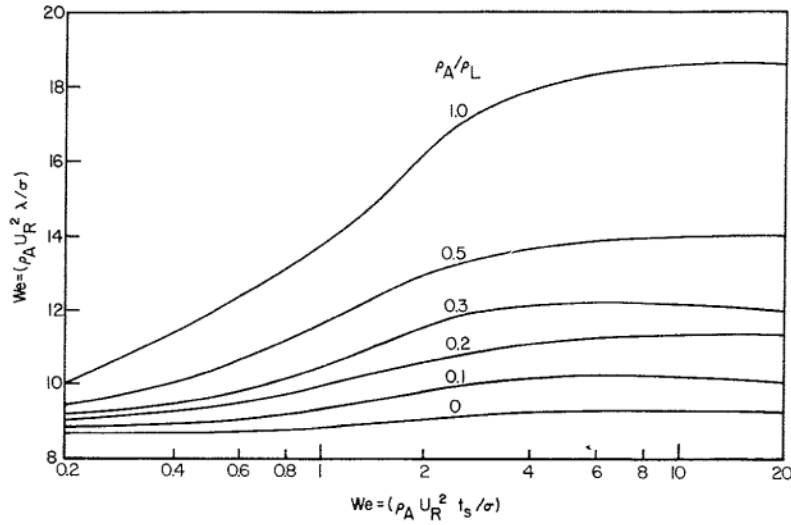


**Figure 2.24** Effect of sheet thickness on wavelength corresponding to maximum growth rate (York *et al.* [17]).

$$\beta = \left[ \frac{n^2 U_R^2 (\rho_A / \rho_L) - n^3 \sigma / \rho_L}{\tanh n(t_s/2)} \right]^{0.5} \quad (2.61)$$

where  $n$  is the wave number of the disturbing wave ( $=\omega/U$ ),  $U_R$  the relative velocity between liquid and air, and  $\omega$  the wave frequency. The results of this comparison are shown in Fig. 2.25.

Their analysis also showed that the lowest stable frequency is given by

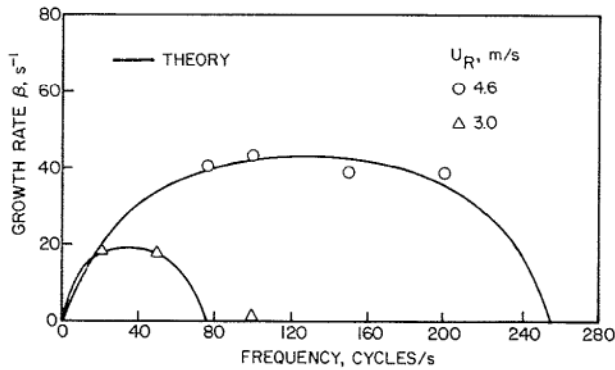


**Figure 2.25** Comparison of predicted growth rate with experimental data (Hagerty and Shea [75]).

$$f_{\min} = \frac{\omega}{2\pi} = \frac{\rho_A U_R^3}{2\pi\sigma} = \frac{U_R We}{2\pi t_s} \quad (2.62)$$

where  $We = \rho_A U_R^2 t_s / \sigma$ .

Frequency, velocity, and wavelength are related by

$$f = \frac{U}{\lambda} \quad (2.63)$$

Hence, from Eqs. (2.62) and (2.63), the minimum wavelength for an unstable system is obtained as

$$\lambda_{\min} = \frac{2\pi\sigma}{\rho_A U_R^2} \quad (2.64)$$

From an analysis of the unstable oscillations of a liquid sheet moving in still air, Squire [76] derived the following expression for minimum wavelength:

$$\lambda_{\min} = \frac{2\pi t_s \rho_L}{\rho_A (We - 1)} \quad (2.65)$$

where  $We = \rho_L U_R^2 t_s / \sigma$ . Since usually  $We \gg 1$ ,

$$\lambda_{\min} = \frac{2\pi\sigma}{\rho_A U_R^2}$$

which is identical to Eq. (2.64) as derived by Hagerty and Shea [75].

The optimum wavelength for sheet disintegration,  $\lambda_{\text{opt}}$ , is the one that has the maximum growth rate  $\beta_{\text{max}}$ . For  $We \gg 1$ , Squire obtained

$$\lambda_{\text{opt}} = \frac{4\pi\sigma}{\rho_A U_R^2} \quad (2.66)$$

and

$$\beta_{\text{max}} = \frac{\rho_A U_R^2}{\sigma (\rho_L t_s)^{0.5}} \quad (2.67)$$

**Influence of surrounding gaseous medium.** According to Dombrowski and Johns [71], the instability of thin liquid sheets resulting from interaction with the surrounding gaseous medium gives rise to rapidly growing surface waves. Disintegration occurs when the wave amplitude reaches a critical value; fragments of the sheet are torn off and rapidly contract into unstable ligaments under the action of surface tension, and drops are produced as the ligaments break down according to theories of varicose instability (see Fig. 2.21).

Fraser et al. [73] investigated the mechanism of disintegration of liquid sheets impinged on by high-velocity airstreams. They used a special system in which flat circular liquid sheets were produced from a spinning cup, while the atomizing airstream was admitted through an annular gap located axially symmetrically to

the cup. Photographic examination showed circumferential waves initiating at the position of impact of the airstream in the sheet, and the liquid sheet was observed to break down into drops through the formation of unstable ligaments.

Rizk and Lefebvre [77] studied the influence of initial liquid film thickness on spray characteristics. They used two specially designed airblast atomizers that were constructed to produce a flat liquid sheet across the centerline of a two-dimensional air duct, with the liquid sheet exposed on both sides to high-velocity air. From analysis of the processes involved and correlations of the experimental data, it was found that high values of liquid viscosity and liquid flow rate result in thicker films. It was also observed that thinner liquid films break down into smaller drops according to the relationship  $SMD \propto t^{0.4}$ . This is an interesting result since, if other parameters are held constant, liquid film thickness is directly proportional to nozzle size, which implies that SMD should be proportional to (atomizer linear scale)<sup>0.4</sup>. This, in fact, is precisely the result obtained by El-Shanawany and Lefebvre [78] in their study of the effect of nozzle size on SMD.

Previous workers had noted a similar relationship. For example, the analyses of York et al. [17], Hagerty and Shea [75], and Dombrowski and Johns [71] all suggest that mean drop diameter is roughly proportional to the square root of the film thickness. In addition, the photographic studies of film disintegration carried out by Fraser et al. [73] show that, for sheets breaking down through the formation of unstable ligaments, the ligament diameter depends mainly on the sheet thickness.

Rizk and Lefebvre [77] also examined the mechanism of sheet disruption and drop formation using very high speed flash photography of 0.2  $\mu$ s duration. Some typical photographs are shown in Figs. 2.26 and 2.27. Figure 2.26 was obtained with water at an air velocity of 55 m/s. It shows clearly the type of atomization process postulated by Dombrowski and Johns [71], in which the liquid/air interaction produces waves that become unstable and disintegrate into fragments. These fragments then contract into ligaments, which in turn break down into drops. With increase in air velocity, the liquid sheet disintegrates earlier and ligaments are formed nearer the lip. These ligaments tend to be thinner and shorter and disintegrate into smaller drops. With liquids of high viscosity, the wavy-surface mechanism is no longer present. Instead, the liquid is drawn out from the atomizing lip in the form of long ligaments. When atomization occurs, it does so well downstream of the atomizing lip in regions of relatively low velocity. In consequence, drop sizes tend to be higher. This type of atomization, as illustrated in Fig. 2.27 for a liquid of viscosity 0.017 kg/m s and an air velocity of 91 m/s, conforms closely to the ligament theory of Castelman [8].

The fact that thicker liquid sheets produce thicker ligaments, which disintegrate into larger drops, highlights the importance of spreading the liquid into a very thin sheet to achieve the finest atomization. Rizk and Lefebvre [77] found that the thickness of the sheet depends on both air and liquid properties. High values of liquid viscosity and/or liquid flow rate result in thicker films, while variations in surface tension appear to have no effect on the thickness of the sheet. However, for liquids of low surface tension, the sheet disintegrates more readily under the action of the airflow and the resulting ligaments are shorter.

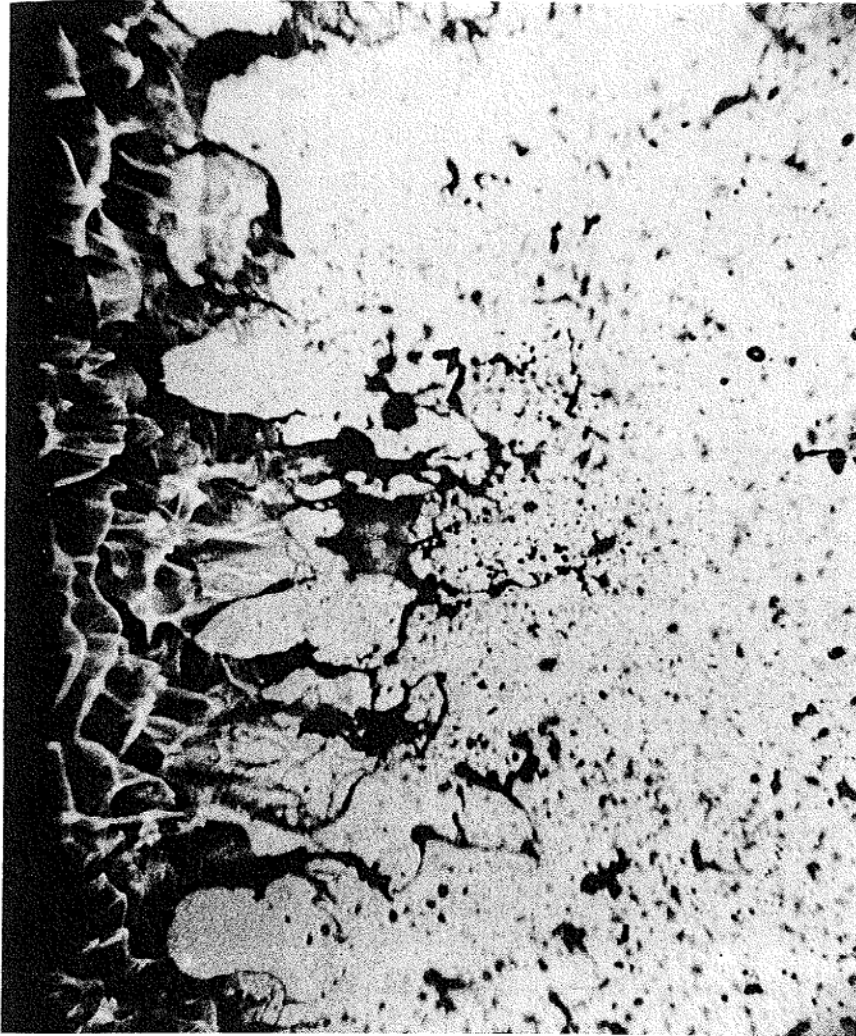


**Figure 2.26** Mechanism of atomization for a low-viscosity liquid; airflow is from left to right (*Rizk and Lefebvre [77]*).

**Breakup length.** Arai and Hashimoto [79] studied the disintegration of liquid sheets injected into a coflowing airstream. Breakup lengths were determined by averaging values obtained from many photographs. For a constant liquid sheet thickness, Fig. 2.28 shows that breakup length decreases with increase in the relative velocity between the air and the liquid. Also indicated in the figure is that breakup length increases as the liquid sheet velocity increases or as the liquid viscosity decreases. An empirical equation for breakup length was derived as

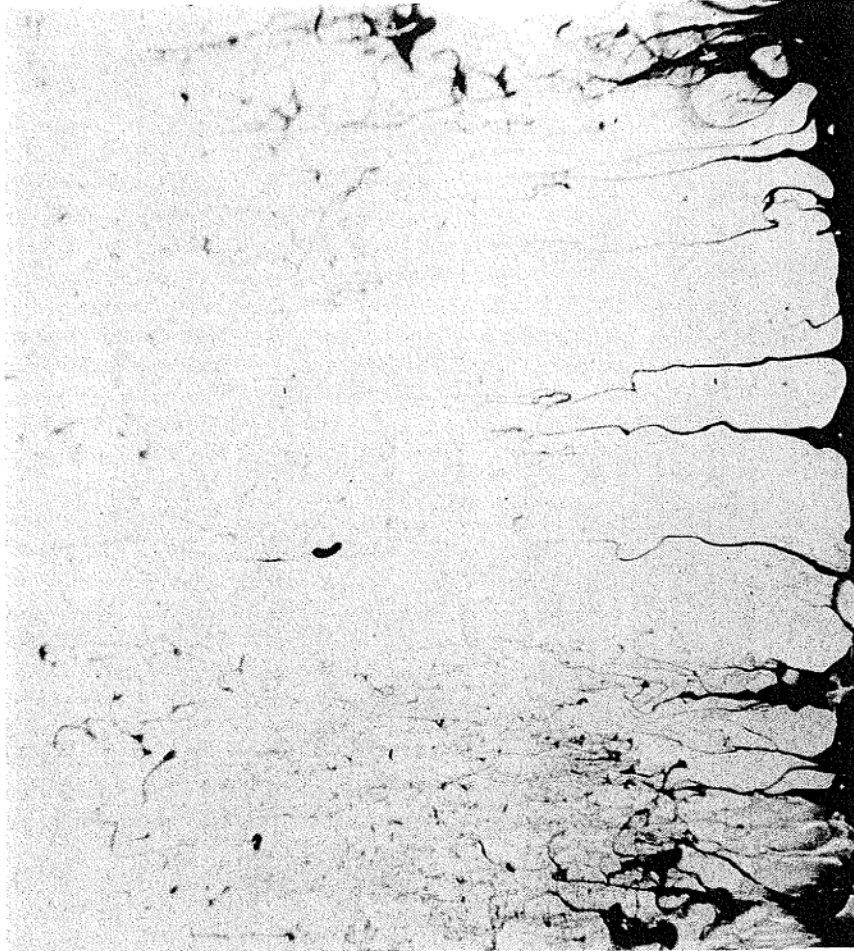


**Figure 2.27** Mechanism of atomization for a high-viscosity liquid; airflow is from left to right (*Rizk and Lefebvre [77]*).

$$L = 0.123 t_s^{0.5} We^{-0.5} Re^{0.6} \quad (2.68)$$

where  $We = t_s \rho_A U_R^2 / 2\sigma$  and  $Re = t_s U_L \rho_L / \mu_L$ .

### Conical Sheets

The theory of attenuating conical sheets in hollow-cone, pressure-swirl nozzles has yet to be developed, but there is evidence that the radius of curvature has a destabilizing effect on the fluctuations, so that conical sheets tend to be shorter than flat sheets [80].

By making certain approximations in their analysis, York et al. [17] were able to make a rough estimate of the size of the drops produced by a pressure-swirl nozzle. Waves form near the nozzle, and those with the wavelength for maximum growth cause periodic thickening of the liquid sheet in a direction normal to the flow. Rings break off from the conical sheet, and the liquid volume contained in the rings can be estimated as the volume of a ribbon cut out of the sheet with a thickness equal to that of the sheet at the breakup distance and a width equal to one wavelength. These cylindrical ligaments then disintegrate into drops according to the Rayleigh mechanism. The resulting mean drop diameter is estimated as

$$D = 2.13(t_s \lambda^*)^{0.5} \quad (2.69)$$

where  $t_s$  is the sheet thickness and  $\lambda^*$  is the wavelength for maximum growth rate, as estimated from Fig. 2.24.

York et al. [17] found that the infinite flat sheet model used in their theoretical analysis was not approximated closely by the conical spray in their experiments, so the agreement between theory and experiment was only qualitative. Hagerty and Shea [75] overcame this problem by selecting for study a system that produced a flat sheet of liquid which could be subjected to waves of any desired frequency. The plane sheet was 15 cm wide and 1.6 mm thick, and its velocity could be varied continuously up to 7.6 m/s. Their analysis included both sinuous and dilation waves, as illustrated in Fig. 2.29. Sinuous waves are produced when the two surfaces of the sheet oscillate in phase, whereas dilation waves are the result of out-of-phase motion of the two surfaces. According to Fraser et al. [70], dilation waves in a sheet may be neglected since their degree of instability is always less than that of sinuous ones.

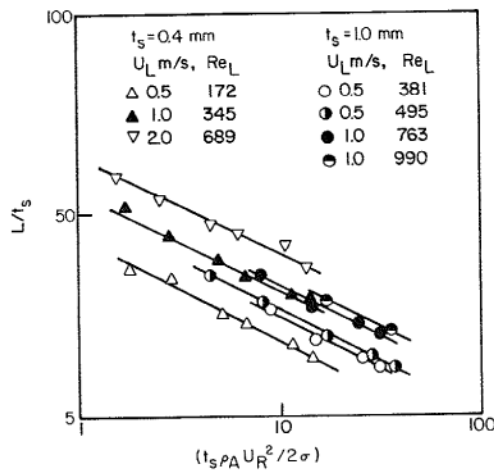


Figure 2.28 Breakup length of a flat liquid sheet in a coflowing airstream (Arai and Hashimoto [79]).

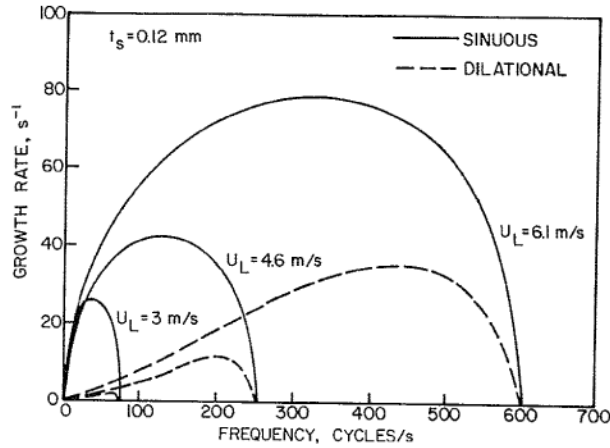


Figure 2.29 Growth rate versus frequency for sinuous and dilational waves (Hagerty and Shea [75]).

### Fan Sheets

Fraser et al. [70] extended the theories of Hagerty and Shea [75] and Squire [76] to derive an expression for the drop sizes produced by the breakup of a low-viscosity, fan spray sheet. Their model assumes that the most rapidly growing ( $\beta_{\max}$ ) wave is detached at the leading edge in the form of a ribbon a half-wave-length wide ( $\lambda_{\text{opt}}/2$ ). This ribbon immediately contracts into a filament of diameter  $D_L$ , which subsequently disintegrates into drops of equal diameter. The mechanism is represented schematically in Fig. 2.30.

By equating the ribbon and ligament volumes, the ligament diameter is obtained as

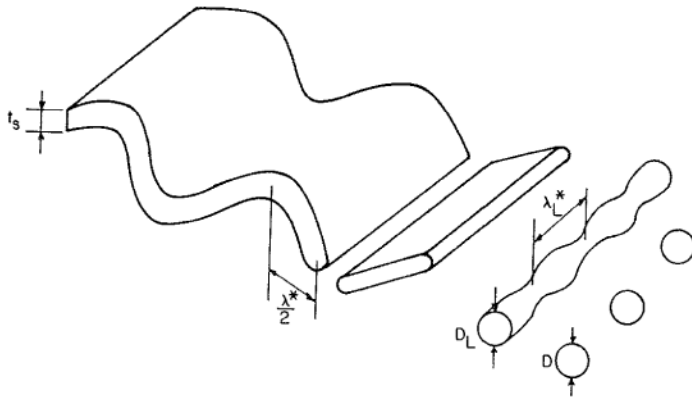


Figure 2.30 Successive stages in the idealized breakup of a wavy sheet (Fraser et al. [70]).

$$D_L = \left( \frac{2}{\pi} \lambda_{\text{opt}} t_s \right)^{0.5} \quad (2.70)$$

In accordance with Rayleigh's analysis [11], the collapse of a ligament produces drops of diameter

$$D = 1.89 D_L \quad (2.30)$$

and hence

$$D = \text{const}(\lambda_{\text{opt}} t_s)^{0.5} \quad (2.71)$$

The sheet thickness at breakup is derived as

$$t_s = \left( \frac{1}{2H^2} \right)^{1/3} \left( \frac{k^2 \rho_A^2 U_R^2}{\rho_L \sigma} \right)^{1/3} \quad (2.72)$$

where  $k$  = spray nozzle parameter,  $H = \ln(h^0/h_0)$ ,  $h^0$  = amplitude at breakup, and  $h_0$  = initial amplitude at orifice.

From tests carried out with varying density and liquid velocity, Fraser et al. [70] determined that  $H$  remains sensibly constant. Their final expression for mean drop diameter is obtained by combining Eqs. (2.66), (2.71), and (2.72) to yield

$$D \propto \left( \frac{\rho_L}{\rho_A} \right)^{1/6} \left( \frac{k\sigma}{\rho_L U_R^2} \right)^{1/3} \quad (2.73)$$

For a given pressure-swirl or fan spray nozzle,  $k$  is constant and  $U_R$  is equal to the sheet velocity  $U_L$ , which is related to the nozzle pressure differential by

$$\Delta P_L = 0.5 \rho_L U_L^2 \quad (2.74)$$

Thus, for a given nozzle

$$D \propto \left( \frac{k\sigma \rho_L^{0.5}}{\Delta P_L \rho_A^{0.5}} \right)^{1/3} \quad (2.75)$$

The dimensions of  $k$  in Eq. (2.73) are meters squared, and from the description of  $k$  provided by Fraser et al. it appears to be proportional to the nozzle flow number. Substituting  $k = FN$  in Eq. (2.73) gives

$$D \propto \left( \frac{\rho_L}{\rho_A} \right)^{1/6} \left( \frac{FN\sigma}{\rho_L U_L^2} \right)^{1/3} \quad (2.76)$$

By substituting in the above equation

$$FN = \frac{\dot{m}_L}{\sqrt{\rho_L \Delta P_L}} \quad (2.77)$$

it becomes

$$D \propto \left( \frac{\dot{m}_L \sigma}{\rho_A^{0.5} \Delta P_L^{1.5}} \right)^{1/3} \quad (2.78)$$

This equation shows how mean drop size is influenced by the nozzle operating conditions of injection pressure differential and liquid flow rate. Equation (2.78) also indicates that drop sizes decrease with increase in ambient air density, a result that was confirmed by Fraser et al.'s measurements of the drop sizes produced by the disintegration of water sheets at subatmospheric pressure. Subsequent research by other workers has generally supported their conclusion that an increase in ambient air pressure leads to a diminution of mean drop size.

Equations (2.75)–(2.78) apply solely to flow situations where the liquid viscosity is negligibly small. Dombrowski and Johns [71] and Hasson and Mizrahi [81] examined the more realistic case where the liquid has finite viscosity and the thickness of the sheet diminishes as it moves away from the orifice. The latter workers were able to correlate their measurements of Sauter mean diameter for a fan-spray nozzle by using Eq. (2.75), which was developed theoretically for inviscid flow, empirically correlated for viscosity,

$$\text{SMD} = 0.071 \left( \frac{t_s x \sigma \mu_L^{0.5}}{\rho_L^{0.5} U_L^2} \right)^{1/3} \text{ cm} \quad (2.79)$$

where  $x$  is the distance downstream from the nozzle and cgs units are employed.

This equation provided a satisfactory correlation of the experimental data over the range of viscosities from 0.003 to 0.025 kg/m s.

## SUMMARY

The theoretical work described in this chapter has made a valuable contribution to our understanding of the fundamental mechanisms involved in atomization, but it has not culminated in quantitative descriptions that can be used to design and predict the performance of practical atomizers. That is why most of the experimental data have been accumulated in the form of empirical and semiempirical equations of the type presented in Chapter 6. Nevertheless, from the theoretical, experimental, and photographic evidence now available, certain general conclusions can be drawn regarding the key factors that govern jet and sheet disintegration and the manner and extent of their influence on the drop sizes produced in the spray. It is now clear that important spray characteristics, such as mean drop size and drop size distribution, are dependent on a large number of variables such as nozzle geometry, the physical properties of the liquid being atomized, and the physical properties, turbulence characteristics, and flow conditions in the surrounding gas.

Theoretical and experimental studies on jet and sheet disintegration have both demonstrated the need to group these variables into nondimensional parameters to clarify their effects on the atomization process. As in most flow systems, the Reynolds grouping, which represents the ratio of momentum forces to viscous drag forces, usefully describes the flow state of the emerging jet in terms of both velocity profile and the magnitude of the radial velocity components that promote

jet disintegration. The flow characteristics of most relevance to atomization are velocity, velocity profile, and turbulence properties, all of which can contribute effectively to jet or sheet disintegration, especially under conditions where aerodynamic influences are relatively small.

In reality, aerodynamic forces are seldom small, and the Weber number, which represents the ratio of the disruptive aerodynamic forces to the restoring surface tension forces, becomes very significant. In fact, for low-viscosity liquids subjected to ambient gas at high relative velocities, the Weber grouping becomes dominant.

Another important dimensionless grouping is the Ohnesorge number. This group contains only the properties of the globules formed in primary atomization before they split up into smaller drops during secondary atomization. It is sometimes called a stability group because it provides an indication of the resistance of a globule to further disintegration, but it is also called a viscosity group because it accounts for the effect of liquid viscosity on the globule. Other dimensionless groups are obtained as the density ratios of the gas and liquid phases and various geometric ratios of the spray generating system.

The fundamental principle of the disintegration of a liquid consists of increasing its surface area, usually in the form of a cylindrical rod or sheet, until it becomes unstable and disintegrates into drops. If the issuing jet is in laminar flow, a vibration in the jet or an external disturbance causes disintegration as predicted by the Rayleigh breakup mechanism. If the jet is fully turbulent, it can break up without the application of external aerodynamic forces. Jet or sheet disintegration is promoted by increases in relative velocity and is impaired by increases in liquid viscosity. Regardless of all other effects, the breakup process is always accelerated by air resistance, which increases with air density. Where the air resistance is high, as in most practical applications, the mutually opposing aerodynamic and surface tension forces acting on the liquid surface give rise to oscillations and perturbations. Under favorable conditions these oscillations are amplified and the liquid body breaks up. Jets break up into drops, while sheets disintegrate into ligaments and then into drops. If the drops so formed exceed the stable maximum limit, they further disintegrate into smaller drops until all the drops are below the critical size.

The random nature of the atomization process, as summarized above, means that most sprays are characterized by a wide range of drop sizes. The methods used to describe and quantify drop size distributions form the subject of the next chapter.

## NOMENCLATURE

|       |                  |
|-------|------------------|
| $A$   | area, $m^2$      |
| $C_D$ | drag coefficient |
| $D$   | drop diameter, m |

|              |                                                                 |
|--------------|-----------------------------------------------------------------|
| $D_L$        | ligament diameter, m                                            |
| $d$          | jet diameter, m                                                 |
| $d_o$        | orifice diameter, m                                             |
| $E$          | rate of energy input per unit mass                              |
| $E_s$        | potential surface energy                                        |
| $f$          | frequency                                                       |
| $g$          | acceleration due to gravity                                     |
| $h$          | displacement (amplitude) of liquid from equilibrium position, m |
| $h_0$        | amplitude at orifice, m                                         |
| $h^0$        | amplitude at breakup, m                                         |
| $h_t$        | amplitude at time $t$ , m                                       |
| $L$          | breakup length, m                                               |
| $l$          | length of jet, m                                                |
| $l_o$        | orifice length, m                                               |
| $m_D$        | mass of drop, kg                                                |
| $\dot{m}$    | mass flow rate, kg/s                                            |
| $n$          | order of mode of vibration, or wave number of disturbing wave   |
| Oh           | Ohnesorge number                                                |
| $\Delta P_L$ | nozzle liquid pressure differential, Pa                         |
| $p_i$        | internal pressure, Pa                                           |
| $p_\sigma$   | surface tension pressure, Pa                                    |
| Re           | Reynolds number                                                 |
| $r$          | jet radius, m                                                   |
| $S$          | maximum velocity gradient in external flow field                |
| $t_b$        | breakup time, s                                                 |
| $t_s$        | sheet thickness, m                                              |
| $U$          | axial velocity, m/s                                             |
| $\bar{u}$    | rms value of fluctuating velocity component, m/s                |
| $x$          | distance downstream of nozzle, m                                |
| We           | Weber number                                                    |
| $\alpha$     | dimensionless wave number ( $=2\pi/\lambda$ )                   |
| $\beta$      | number that determines growth rate of disturbance               |
| $\delta$     | amplitude of disturbance, m                                     |
| $\lambda$    | wavelength of disturbance, m                                    |
| $\lambda^*$  | wavelength for maximum growth rate, m                           |
| $\mu$        | dynamic viscosity, kg/m s                                       |
| $\rho$       | density, kg/m <sup>3</sup>                                      |
| $\sigma$     | surface tension, kg/s <sup>2</sup>                              |
| $\omega$     | natural frequency of vibration                                  |

## Subscripts

|     |                |
|-----|----------------|
| $A$ | air            |
| $L$ | liquid         |
| $R$ | relative value |
| $0$ | initial value  |

|      |                |
|------|----------------|
| crit | critical value |
| max  | maximum value  |
| min  | minimum value  |
| opt  | optimum value  |

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## DROP SIZE DISTRIBUTIONS OF SPRAYS

### INTRODUCTION

A spray is generally considered as a system of drops immersed in a gaseous continuous phase. Examples of natural sprays include rain, drizzle, fog, and waterfall mists. Figure 3.1 indicates the range of drop sizes as they occur in certain natural phenomena and also as commonly produced by atomizers.

Most practical atomizers generate drops in the size range from a few micrometers up to around 500  $\mu\text{m}$ . Owing to the heterogeneous nature of the atomization process, the threads and ligaments formed by the various mechanisms of jet and sheet disintegration vary widely in diameter, and the resulting main drops and satellite drops vary in size correspondingly. Practical nozzles do not, therefore, produce sprays of uniform drop size at any given operating condition; instead, the spray can be regarded as a spectrum of drop sizes distributed about some arbitrarily defined mean value. Only under certain special conditions (such as are obtained, for example, with a rotary cup atomizer operating within a limited range of liquid flow rates and rotational speeds) can a fairly homogeneous spray be produced. Thus, in addition to mean drop size, another parameter of importance in the definition of a spray is the distribution of drop sizes it contains.

### GRAPHICAL REPRESENTATION OF DROP SIZE DISTRIBUTIONS

An instructive picture of drop size distribution may be obtained by plotting a histogram of drop size, each ordinate representing the number of drops whose

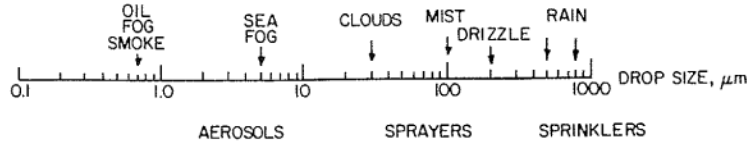


Figure 3.1 Spectrum of drop sizes [1].

dimensions fall between the limits  $D - \Delta D/2$  and  $D + \Delta D/2$ . A typical histogram of this type is shown in Fig. 3.2, in which  $\Delta D = 17 \mu\text{m}$ . If, instead of plotting the number of drops, the volume of the spray corresponding to a range of drop sizes between  $D - \Delta D/2$  and  $D + \Delta D/2$  is plotted as a histogram of drop size, the resulting distribution is skewed to the right, as shown in Fig. 3.3, due to the weighting effect of the larger drops.

As  $\Delta D$  is made smaller, the histogram assumes the form of a frequency curve that may be regarded as a characteristic of the spray, provided it is based on sufficiently large samples. Such a curve, shown in Fig. 3.4, is usually referred to as a *frequency distribution curve*. The ordinate values may be expressed in several alternative ways: as the number of drops with a given diameter, the relative number or fraction of the total, or the fraction of the total number per size class. If the ordinate is expressed in the last manner, the area under the frequency distribution curve must be equal to 1.0.

It is evident that incremental frequency plots may be constructed directly from drop size distribution data by plotting  $\Delta N_i / N \Delta D_i$  versus  $D$  or  $\Delta Q_i / Q \Delta D_i$  versus  $D$ , where  $\Delta N_i$  is the number increment within  $\Delta D_i$  and  $\Delta Q_i$  is the volume increment within  $\Delta D_i$ . The  $\Delta Q_i$  can be obtained as

$$\Delta Q_i = \Delta N_i \left( \frac{\pi}{6} \right) [0.5(D_{i1} + D_{i2})]^3 \quad (3.1)$$

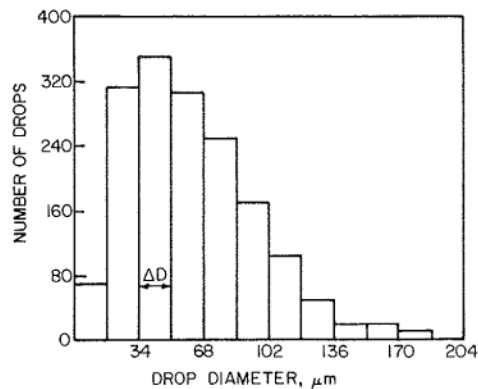


Figure 3.2 Typical drop size histogram.

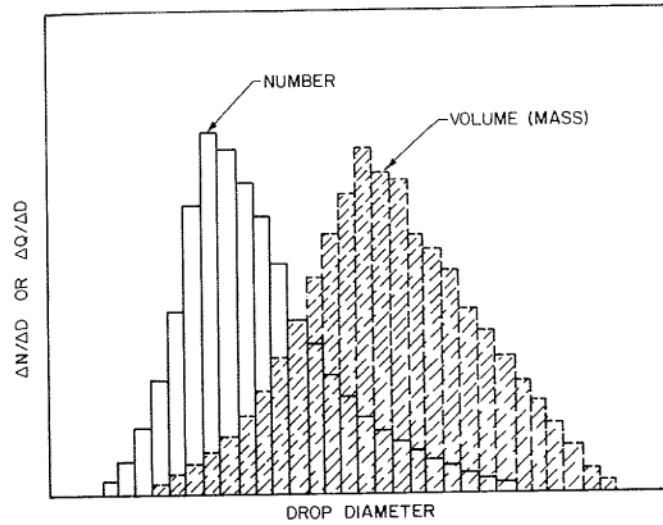


Figure 3.3 Drop size histograms based on number and volume.

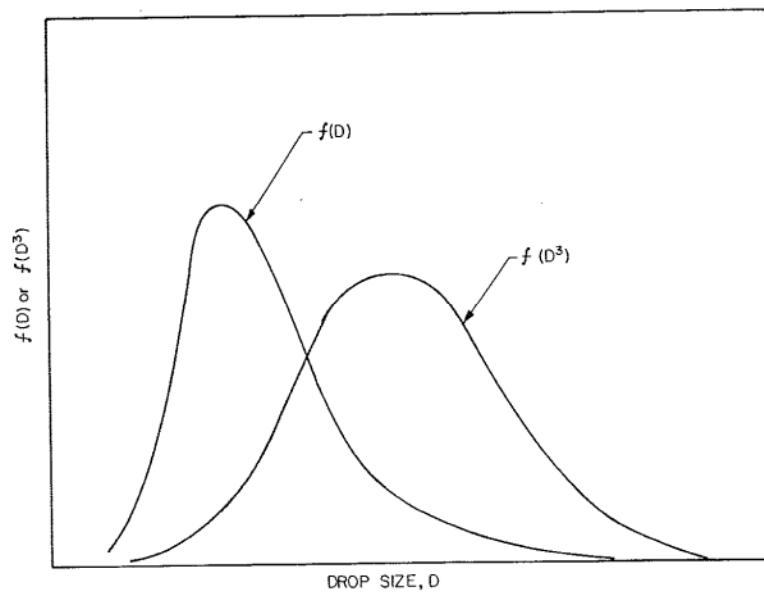


Figure 3.4 Drop size frequency distribution curves based on number and volume.

where  $D_{i_1}$  and  $D_{i_2}$  are the upper and lower boundaries in the  $\Delta D_i$  ( $i$ th) drop size class.

If the surface area or volume of the drops in the spray is plotted versus diameter, the distribution curve is again skewed to the right, as shown in Fig. 3.4, due to the weighting effect of the larger diameters. Figure 3.5 illustrates the use of this type of curve to demonstrate the effect on drop size distribution of a change in operating conditions, in this case an increase in air velocity through an airblast atomizer.

In addition to representing the drop size distribution with a frequency plot, it is also informative to use a *cumulative distribution* representation. This is essentially a plot of the integral of the frequency curve, and it may represent the percentage of the total number of drops in the spray below a given size or the percentage of the total surface area or volume of a spray contained in drops below a given size. Cumulative distribution curves plotted on arithmetic coordinates have the general shape shown in Fig. 3.6; the ordinate may be the percentage of drops by number, surface area, or volume whose diameter is less than a given drop diameter. Figure 3.7 shows cumulative distributions corresponding to the frequency distribution curves of Fig. 3.5.

## MATHEMATICAL DISTRIBUTION FUNCTIONS

Because the graphical representation of drop size distribution is laborious and not easily related to experimental results, many workers have attempted to replace it with mathematical expressions whose parameters can be obtained from a limited number of drop size measurements. Suitable mathematical expressions would have the following attributes [3]:

1. Provide a satisfactory fit to the drop size data.
2. Allow extrapolation to drop sizes outside the range of measured values.
3. Permit easy calculation of mean and representative drop diameters and other parameters of interest.
4. Provide a means of consolidating large amounts of data.
5. Ideally, furnish some insight into the basic mechanisms involved in atomization.

In the absence of any fundamental mechanism or model on which to build a theory of drop size distributions, a number of functions have been proposed, based on either probability or purely empirical considerations, that allow the mathematical representation of measured drop size distributions. Those in general use include normal, log-normal, Nukiyama-Tanasawa, Rosin-Rammler, and upper-limit distributions. As the basic mechanisms involved in atomization are not clearly understood and no single distribution function can represent all drop size data, it is usually necessary to test several distribution functions to find the best fit to any given set of experimental data.

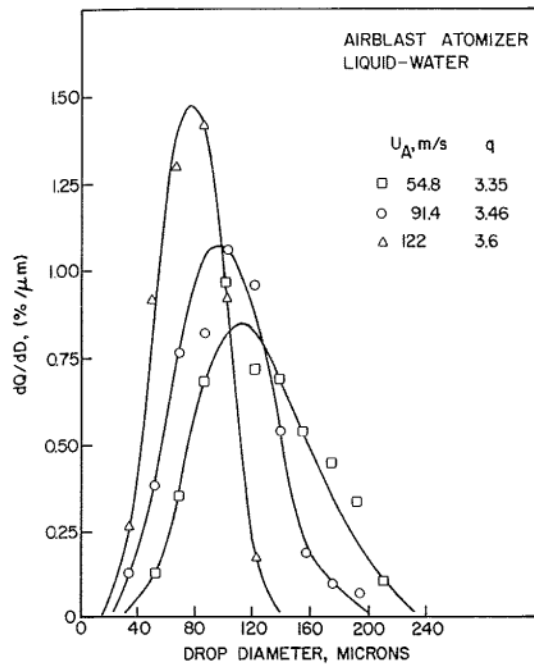


Figure 3.5 Graphs illustrating the effect of atomizing air velocity on drop size distribution [2].

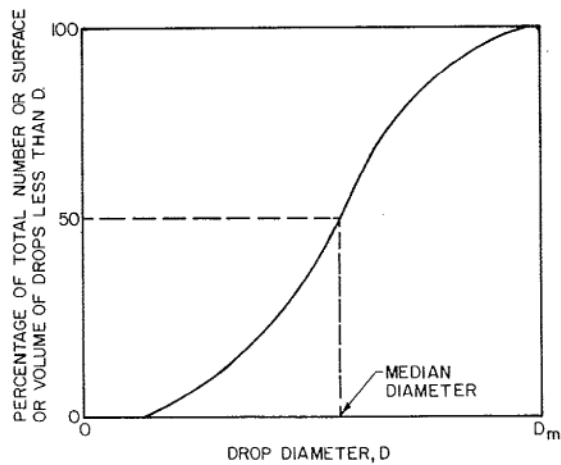


Figure 3.6 Typical shape of cumulative drop size distribution curve.

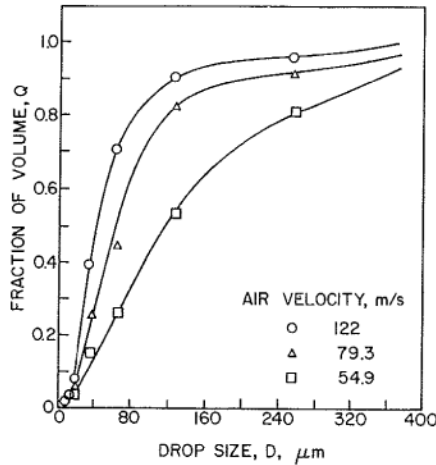


Figure 3.7 Effect of atomizing air velocity on cumulative volume distribution.

### Normal Distribution

This distribution function is based on the random occurrence of a given drop size. It is comparatively simple to use, but its application is limited to processes that are random in nature and where no specific bias is present. It is usually expressed in terms of a number distribution function  $f(D)$  that gives the number of particles of a given diameter  $D$ . We have

$$\frac{dN}{dD} = f(D) = \frac{1}{\sqrt{2\pi}s_n} \exp\left[-\frac{1}{2s_n^2}(D - \bar{D})^2\right] \quad (3.2)$$

where  $s_n$  is a measure of the deviation of values of  $D$  from a mean value  $\bar{D}$ . The term  $s_n$  is usually referred to as the *standard deviation*, and  $s_n^2$  is the variance, as defined in standard textbooks on statistics. A plot of the distribution function is shown in Fig. 3.8. It is usually described as the *standard normal curve*. The area under the curve from  $-\infty$  to  $+\infty$  is equal to 1, and the areas on either side of the  $y$  axis are equal.

The integral of the standard normal curve is the cumulative standard number distribution function  $F(D)$ . By substituting into Eq. (3.2) the expression

$$t = \frac{D - \bar{D}}{s_n} \quad (3.3)$$

and noting that  $\bar{D} = 0$  and  $s_n = 1$  for the standard normal curve,  $F(D)$  can be derived as

$$F(D) = \left(\frac{1}{\sqrt{2\pi}}\right) \int_{-\infty}^D \exp -(t^2/2) dt \quad (3.4)$$

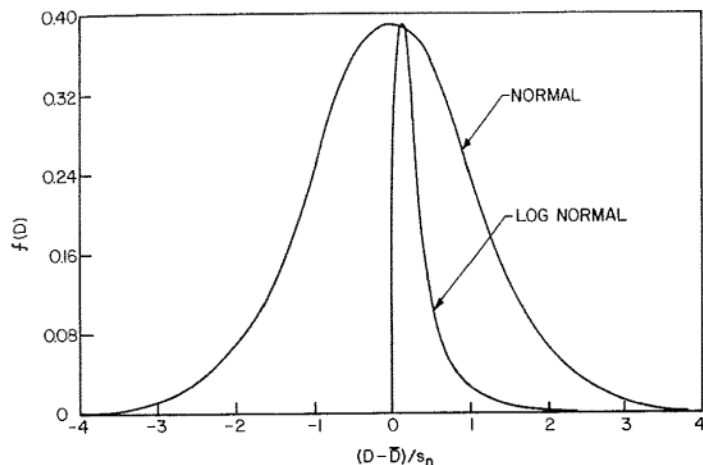


Figure 3.8 Normal and log-normal distributions.

Tabulated values of this integral may be found in most mathematical handbooks. Equation (3.4) indicates that if the data conform to a normal distribution, they will lie on a straight line when plotted on arithmetic-probability graph paper. This type of graph can be used in conjunction with Eq. (3.4) to define the mean diameter and the standard deviation.

### Log-normal Distribution

Many particle size distributions that occur in nature have been found to follow the *Gaussian* or *normal* distribution law if the logarithm of the particle diameter is used as the variable. With this modification Eq. (3.2) becomes

$$\frac{dN}{dD} = f(D) = \frac{1}{\sqrt{2\pi D s_g}} \exp \left[ -\frac{1}{2s_g^2} (\ln D - \ln \bar{D}_{ng})^2 \right] \quad (3.5)$$

where  $\bar{D}_{ng}$  is the number geometric mean drop size and  $s_g$  is the geometric standard deviation. The terms  $s_g$  and  $\bar{D}_{ng}$  have the same significance on a log-probability graph as  $s_n$  and  $\bar{D}$  do on an arithmetic-probability graph.

From inspection of Eq. (3.5) it is evident that when drop size data fit this type of function, the logarithm of the diameter is distributed normally. Thus letting  $y = \ln(\bar{D}/\bar{D}_{ng})$  reduces Eq. (3.5) to the normal distribution form, Eq. (3.2). Equation (3.5) is also plotted in Fig. 3.8 to compare it with the normal distribution function. Log-normal functions can also be written for surface and volume distributions as

$$\text{Surface distribution: } f(D^2) = \frac{1}{\sqrt{2\pi D s_g}} \exp \left[ -\frac{1}{2s_g^2} (\ln D - \ln \bar{D}_{sg})^2 \right] \quad (3.6)$$

where  $\bar{D}_{sg}$  is the geometric surface mean diameter

$$\text{Volume distribution: } f(D^3) = \frac{1}{\sqrt{2\pi D s_g}} \exp - \left[ \frac{1}{2s_g^2} (\ln D - \ln \bar{D}_{vg})^2 \right] \quad (3.7)$$

where  $\bar{D}_{vg}$  is the geometric mass or volume mean diameter.

The relationships between the various mean diameters can be expressed in terms of the number geometric mean diameter  $\bar{D}_{ng}$ . For example,

$$\text{Surface: } \ln \bar{D}_{sg} = \ln \bar{D}_{ng} + 2s_g^2 \quad (3.8)$$

$$\text{Volume: } \ln \bar{D}_{vg} = \ln \bar{D}_{ng} + 3s_g^2 \quad (3.9)$$

$$\text{Volume/surface (SMD): } \ln \bar{D}_{vsg} = \ln \bar{D}_{ng} + 2.5s_g^2 \quad (3.10)$$

## EMPIRICAL DISTRIBUTION FUNCTIONS

Several empirical relationships have been proposed to characterize the distribution of drop sizes in a spray. None of these is universally better than any other, and the extent to which any particular function matches any given set of data depends largely on the mechanism of disintegration involved. Some of the functions most frequently used in the analysis and correlation of drop size data are given below.

### Nukiyama and Tanasawa

One relatively simple mathematical function that adequately describes the actual distribution is that due to Nukiyama and Tanasawa [4]:

$$\frac{dN}{dD} = aD^p \exp -(bD)^q \quad (3.11)$$

This expression contains four independent constants, namely  $a$ ,  $b$ ,  $p$ , and  $q$ . Most of the commonly used size distribution functions represent either simplifications or modifications of this function. One example is the Nukiyama-Tanasawa [4] equation in which  $p = 2$ :

$$\frac{dN}{dD} = aD^2 \exp -(bD)^q \quad (3.12)$$

Dividing this equation through by  $D^2$  and taking logarithms of both sides gives

$$\ln \left( \frac{1}{D^2} \frac{dN}{dD} \right) = \ln a - bD^q \quad (3.13)$$

For any given set of data a value of  $q$  may be assumed and a graph plotted of  $\ln(D^{-2} dN/dD)$  against  $D^q$ . If the assumed value of  $q$  is correct, this plot will yield a straight line from which values of  $a$  and  $b$  may be determined.

### Rosin-Rammler

At present the most widely used expression for drop size distribution is one that was originally developed for powders by Rosin and Rammler [5]. It may be expressed in the form

$$1 - Q = \exp -(D/X)^q \quad (3.14)$$

where  $Q$  is the fraction of the total volume contained in drops of diameter less than  $D$ , and  $X$  and  $q$  are constants. Thus, by applying the Rosin-Rammler relationship to sprays, it is possible to describe the drop size distribution in terms of the two parameters  $X$  and  $q$ . The exponent  $q$  provides a measure of the spread of drop sizes. The higher the value of  $q$ , the more uniform is the spray. If  $q$  is infinite, the drops in the spray are all the same size. For most sprays the value of  $q$  lies between 1.5 and 4. However, for rotary atomizers  $q$  can be as high as 7.

Although it assumes an infinite range of drop sizes, the Rosin-Rammler expression has the virtue of simplicity. Moreover, it permits data to be extrapolated into the range of very fine drops, where measurements are most difficult and least accurate.

A typical Rosin-Rammler plot is shown in Fig. 3.9. The value of  $q$  is obtained as the slope of the line, while  $X$ , which is a representative diameter of some kind, is given by the value of  $D$  for which  $1 - Q = \exp -1$ . Solution of this equation yields the result that  $Q = 0.632$ ; that is,  $X$  is the drop diameter such that 63.2% of the total liquid volume is in drops of smaller diameter.

### Modified Rosin-Rammler

From analysis of a considerable body of drop size data obtained with pressure-swirl nozzles, Rizk and Lefebvre [6] found that although the Rosin-Rammler

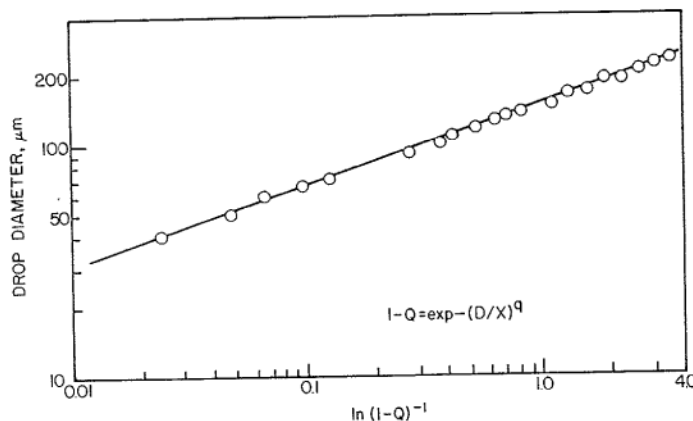


Figure 3.9 Typical Rosin-Rammler plot.

expression provides an adequate data fit over most of the drop size range, there is occasionally a significant deviation from the experimental data for the larger drop sizes. By rewriting the Rosin-Rammler equation in the form

$$1 - Q = \exp - \left( \frac{\ln D}{\ln X} \right)^q \quad (3.15)$$

the volume distribution equation is given by

$$\frac{dQ}{dD} = q \frac{(\ln D)^{q-1}}{D(\ln X)^q} \exp - \left( \frac{\ln D}{\ln X} \right)^q \quad (3.16)$$

which gives a much better fit to the drop size data, as illustrated in Figs. 3.10 and 3.11 from reference [7]. However, many more comparative assessments must be performed before the modified version can be claimed superior to the original Rosin-Rammler formula.

### Upper-Limit Function

Mugele and Evans [8] analyzed the various functions used to represent drop size distribution data by computing mean diameters for the experimental data and comparing them with means calculated from the distribution functions given above. The empirical constants used in these functions were determined from the experimental distributions. As a result of their analysis, Mugele and Evans proposed a so-called *upper-limit function* as being the best way of representing the drop size distributions of sprays. This is a modified form of the log-probability equation that is based on the normal distribution function. The volume distribution equation is given by

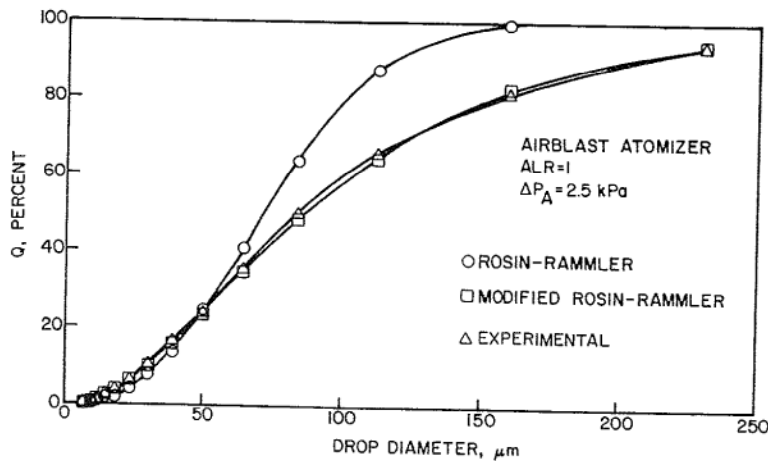


Figure 3.10 Comparison of Rosin-Rammler and modified Rosin-Rammler distributions (Rizk [7]).

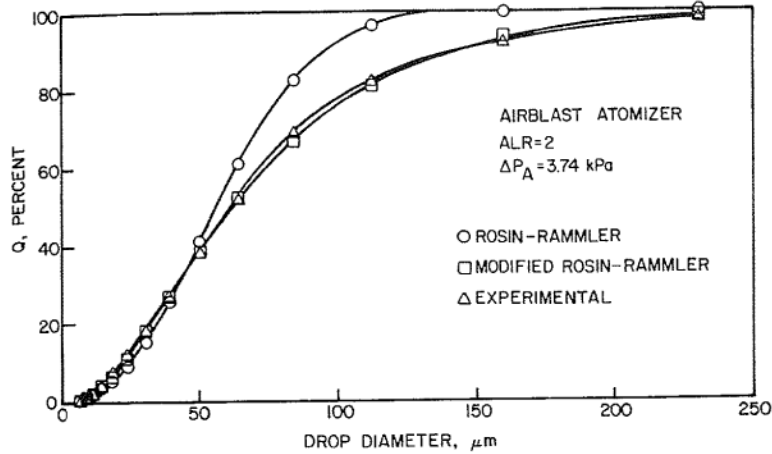


Figure 3.11 Comparison of Rosin-Rammler and modified Rosin-Rammler distributions (Rizk [7]).

$$\frac{dQ}{dy} = \delta \exp \frac{-\delta^2 y^2}{\sqrt{\pi}} \quad (3.17)$$

where

$$y = \ln \frac{aD}{D_m - D} \quad (3.18)$$

As  $y$  goes from  $-\infty$  to  $+\infty$ ,  $D$  goes from  $D_0$  (minimum drop size) to  $D_m$  (maximum drop size), while  $\delta$  is related to the standard deviation of  $y$  and, hence, of  $D$ ;  $a$  is a dimensionless constant. The Sauter mean diameter is given by

$$\text{SMD} = \frac{D_m}{1 + a \exp(1/4\delta^2)} \quad (3.19)$$

It follows that a reduction in  $\delta$  implies a more uniform distribution.

The upper-limit distribution function assumes a realistic spray of finite minimum and maximum drop sizes, but it involves difficult integration that requires the use of log-probability paper. The value of  $D_m$  must be assumed, and usually many trials are needed to find the most suitable value.

Mugele and Evans [8] have summarized the various statistical formulas for computing mean diameters and variances. The principal conclusion appears to be that the drop size distribution should be given in each case by the best empirical representation obtainable. Until atomization mechanisms can be suitably related to one or more distribution functions, there seems to be no theoretical justification for a belief (or expectation) that one function is generally superior to another for representing drop size distribution. Probably the best reasons for selecting a given distribution function would be (1) mathematical simplicity, (2) ease of manipu-

lation in computations, and (3) consistency with the physical phenomena involved.

For further information on empirical equations for drop size distribution, reference should be made to references [1, 3, 8, 9, 10].

## MEAN DIAMETERS

In many calculations of mass transfer and flow processes it is convenient to work only with mean or average diameters instead of the complete drop size distribution. The concept of mean diameter has been generalized and its notation standardized by Mugele and Evans [8]. One of the most common mean diameters is  $D_{10}$ , where

$$D_{10} = \frac{\int_{D_0}^{D_m} D(dN/dD) dD}{\int_{D_0}^{D_m} (dN/dD) dD} \quad (3.20)$$

Other mean diameters of interest include the

$$\text{Surface mean: } D_{20} = \left[ \frac{\int_{D_0}^{D_m} D^2(dN/dD) dD}{\int_{D_0}^{D_m} (dN/dD) dD} \right]^{1/2} \quad (3.21)$$

$$\text{Volume mean: } D_{30} = \left[ \frac{\int_{D_0}^{D_m} D^3(dN/dD) dD}{\int_{D_0}^{D_m} (dN/dD) dD} \right]^{1/3} \quad (3.22)$$

In general, we have [8]

$$(D_{ab})^{a-b} = \frac{\int_{D_0}^{D_m} D^a(dN/dD) dD}{\int_{D_0}^{D_m} D^b(dN/dD) dD} \quad (3.23)$$

where  $a$  and  $b$  may take on any values corresponding to the effect investigated, and the sum  $a + b$  is called the order of the mean diameter.

Equation (3.23) may also be written as

$$D_{ab} = \left[ \frac{\sum N_i D_i^a}{\sum N_i D_i^b} \right]^{1/(a-b)} \quad (3.24)$$

where  $i$  denotes the size range considered,  $N_i$  is the number of drops in size range  $i$ , and  $D_i$  is the middle diameter of size range  $i$ . Thus, for example,  $D_{10}$  is the linear average value of all the drops in the spray;  $D_{30}$  is the diameter of a drop whose volume, if multiplied by the number of drops, equals the total volume of the sample; and  $D_{32}$  (SMD) is the diameter of the drop whose ratio of volume to surface area is the same as that of the entire spray. These and other important mean diameters are listed in Table 3.1, along with their fields of application as suggested by Mugele and Evans [8].

## REPRESENTATIVE DIAMETERS

For most engineering purposes the distribution of drop sizes in a spray may be represented concisely as a function of two parameters (as in the Rosin-Rammler expression, for example), one of which is a representative diameter and the other a measure of the range of drop sizes. In some instances it may be advantageous to introduce another term, such as a parameter to express minimum drop size, but basically there must be at least two parameters to describe the drop size distribution.

**Table 3.1 Mean diameters and their applications**

| $a$ | $b$ | $a + b$<br>(order) | Symbol   | Name of mean<br>diameter  | Expression                                                 | Application                            |
|-----|-----|--------------------|----------|---------------------------|------------------------------------------------------------|----------------------------------------|
| 1   | 0   | 1                  | $D_{10}$ | Length                    | $\frac{\sum N_i D_i}{\sum N_i}$                            | Comparisons                            |
| 2   | 0   | 2                  | $D_{20}$ | Surface area              | $\left( \frac{\sum N_i D_i^2}{\sum N_i} \right)^{1/2}$     | Surface area<br>controlling            |
| 3   | 0   | 3                  | $D_{30}$ | Volume                    | $\left( \frac{\sum N_i D_i^3}{\sum N_i} \right)^{1/3}$     | Volume controlling,<br>e.g., hydrology |
| 2   | 1   | 3                  | $D_{21}$ | Surface area-length       | $\frac{\sum N_i D_i^2}{\sum N_i D_i}$                      | Absorption                             |
| 3   | 1   | 4                  | $D_{31}$ | Volume-length             | $\left( \frac{\sum N_i D_i^3}{\sum N_i D_i} \right)^{1/2}$ | Evaporation,<br>molecular diffusion    |
| 3   | 2   | 5                  | $D_{32}$ | Sauter (SMD)              | $\frac{\sum N_i D_i^3}{\sum N_i D_i^2}$                    | Mass transfer,<br>reaction             |
| 4   | 3   | 7                  | $D_{43}$ | De Brouckere or<br>Herdan | $\frac{\sum N_i D_i^4}{\sum N_i D_i^3}$                    | Combustion<br>equilibrium              |

There are many possible choices of representative diameter, each of which could play a role in defining the distribution function. The various possibilities include the following:

$D_{0.1}$  = drop diameter such that 10% of total liquid volume is in drops of smaller diameter.

$D_{0.5}$  = drop diameter such that 50% of total liquid volume is in drops of smaller diameter. This is the mass median diameter (MMD).

$D_{0.632}$  = drop diameter such that 63.2% of total liquid volume is in drops of smaller diameter. This is  $X$  in Eq. (3.14).

$D_{0.9}$  = drop diameter such that 90% of total liquid volume is in drops of smaller diameter.

$D_{0.999}$  = drop diameter such that 99.9% of total liquid volume is in drops of smaller diameter.

$D_{\text{peak}}$  = value of  $D$  corresponding to peak of drop size frequency distribution curve.

The locations of various representative diameters on a drop size frequency curve (assuming a Rosin-Rammler distribution) are shown in Fig. 3.12.

Chin and co-workers [11, 12] have argued in support of the Rosin-Rammler distribution function because it is simple to use and can readily be obtained using a standard drop size analyzer. A further useful advantage is that all the represen-

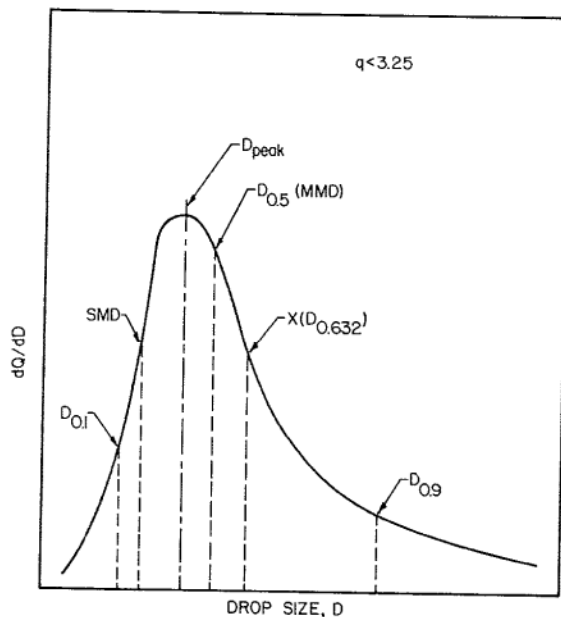


Figure 3.12 Locations of various representative diameters (Chin and Lefebvre [11]).

tative diameters in the spray are uniquely related to each other via the distribution parameter  $q$ . For example, we have [12]

$$\frac{\text{MMD}}{\text{SMD}} = (0.693)^{1/q} \Gamma\left(1 - \frac{1}{q}\right) \quad (3.25)$$

where  $\Gamma$  denotes the gamma function.

This equation demonstrates that the ratio MMD/SMD is not constant, as sometimes asserted, but is a unique function of  $q$ .

The definition of  $D_{\text{peak}}$  is evident from inspection of Fig. 3.12. Clearly, at  $D_{\text{peak}}$ ,  $d^2Q/dD^2 = 0$ , so differentiation of Eq. (3.14) gives

$$\frac{d^2Q}{dD^2} = q(q-1) \frac{D^{q-2}}{X^q} \exp\left(-\left(\frac{D}{X}\right)^q\right) - \left(q \frac{D^{q-1}}{X^q}\right)^2 \exp\left(-\left(\frac{D}{X}\right)^q\right) = 0 \quad (3.26)$$

Hence,

$$\frac{D_{\text{peak}}}{X} = \left(1 - \frac{1}{q}\right)^{1/q} \quad (3.27)$$

From Eq. (3.14) we have

$$\frac{D_{0.1}}{X} = (0.1054)^{1/q} \quad (3.28)$$

$$\frac{D_{0.9}}{X} = (2.3025)^{1/q} \quad (3.29)$$

Also,

$$\frac{\text{MMD}}{X} = (0.693)^{1/q} \quad (3.30)$$

and

$$\frac{\text{SMD}}{X} = \left[\Gamma\left(1 - \frac{1}{q}\right)\right]^{-1} \quad (3.31)$$

Other useful relationships are

$$\frac{D_{0.1}}{\text{MMD}} = (0.152)^{1/q} \quad (3.32)$$

$$\frac{D_{0.9}}{\text{MMD}} = (3.32)^{1/q} \quad (3.33)$$

$$\frac{D_{0.999}}{\text{MMD}} = (9.968)^{1/q} \quad (3.34)$$

It is of interest to examine where MMD and SMD lie on the distribution curve in relation to the peak diameter. From Eqs. (3.27) and (3.30) we have

$$\frac{D_{\text{peak}}}{\text{MMD}} = \frac{(1 - 1/q)^{1/q}}{(0.6931)^{1/q}} = \left(1.4428 - \frac{1.4428}{q}\right)^{1/q} \quad (3.35)$$

which shows that  $\text{MMD} = D_{\text{peak}}$  when  $q = 3.2584$ . Referring to Fig. 3.12, this means that MMD will appear on the left-hand side or the right-hand side of  $D_{\text{peak}}$  depending on whether  $q$  is greater or less than 3.2584, respectively.

From Eqs. (3.27) and (3.31) we have

$$\frac{D_{\text{peak}}}{\text{SMD}} = \left(1 - \frac{1}{q}\right)^{1/q} \Gamma\left(1 - \frac{1}{q}\right) \quad (3.36)$$

From this equation it is apparent that  $D_{\text{peak}}$  will always be larger than SMD, so SMD must always lie on the left-hand side of  $D_{\text{peak}}$ .

Several representative diameters are defined in Table 3.2, and the ratios of different diameters are shown in Table 3.3 and Figs. 3.13 to 3.15. Figures 3.16 and 3.17 show, respectively, the effects on the drop size frequency distribution curves and the cumulative drop size distribution curves of changes in SMD and  $q$ . These figures were calculated using Eq. (3.38).

From inspection of Table 3.3 it is evident that

1. The ratio of MMD/SMD is always greater than one. For  $q \geq 3$  it changes only little, as illustrated in Fig. 3.15.
2. For a spray having  $q = 3$ ,  $D_{0.9}$  is only 50% higher than  $D_{0.5}$ , but for  $q \leq 1.7$ ,  $D_{0.9}$  is more than double  $D_{0.5}$ .
3. For many sprays  $q$  lies between 2 and 2.8, and SMD is between 80 and 84% of the peak diameter.

From Eq. (3.25) and Eqs. (3.27) to (3.36) the general conclusion is that when the Rosin-Rammler expression is used, the ratio of any two representative diameters is always a unique function of  $q$ . In Eq. (3.14), instead of using  $X$ , we can use any representative diameter and it will give the same distribution. For example, Eq. (3.14) can be rewritten as

**Table 3.2 Some representative diameters**

| Symbol            | Name                       | Value for Rosin-Rammler distribution  | Position on $Q$ versus $D$ plot                |
|-------------------|----------------------------|---------------------------------------|------------------------------------------------|
| $D_{0.1}$         | Peak diameter              | $X(0.1054)^{1/q}$                     | $Q = 10\%$                                     |
| $D_{\text{peak}}$ |                            | $X\left(1 - \frac{1}{q}\right)^{1/q}$ | Peak point on $\frac{dQ}{dD}$ versus $D$ curve |
| $D_{0.5}$         | Mass median diameter (MMD) | $X(0.693)^{1/q}$                      | $Q = 50\%$                                     |
| $D_{0.632}$       | Characteristic diameter    | $X$                                   | $Q = 63.2\%$                                   |
| $D_{0.9}$         | Maximum diameter           | $X(2.3025)^{1/q}$                     | $Q = 90\%$                                     |
| $D_{0.999}$       |                            | $X(6.9077)^{1/q}$                     | $Q = 99.9\%$                                   |

**Table 3.3 Relationship between Rosin-Rammler distribution parameter  $q$  and other spray parameters**

| $q$ | $\frac{D_{0.9}}{D_{0.5}}$ | $\frac{D_{peak}}{D_{0.5}}$ | $\frac{D_{peak}}{SMD}$ | $\frac{D_{0.5}}{SMD}$ | $Q_{HSDM} (\%)$ | $\Gamma\left(1 - \frac{1}{q}\right)$ | $\frac{D_{peak}}{X}$ | $\frac{D_{0.9}}{X}$ | $\frac{D_{0.1}}{X}$ | $\frac{D_{0.5}}{X}$ | $SMD/X$ | $\frac{\Delta = D_{0.9} - D_{0.1}}{D_{0.5}}$ | $\frac{\Delta_R = D_{0.999} - D_{0.5}}{D_{0.5}}$ |
|-----|---------------------------|----------------------------|------------------------|-----------------------|-----------------|--------------------------------------|----------------------|---------------------|---------------------|---------------------|---------|----------------------------------------------|--------------------------------------------------|
| 1.2 | 2.71952                   | 0.30494                    | 1.2506                 | 4.1013                | 11.965          | 5.5673                               | 0.22467              | 2.00376             | 0.15331             | 0.73681             | 0.17965 | 2.51143                                      | 5.7939                                           |
| 1.4 | 2.35134                   | 0.53100                    | 1.2870                 | 2.4238                | 18.18           | 3.1496                               | 0.40868              | 1.81437             | 0.20041             | 0.76967             | 0.31755 | 2.09695                                      | 4.1671                                           |
| 1.6 | 2.11772                   | 0.68120                    | 1.2841                 | 1.8851                | 22.23           | 2.3707                               | 0.54171              | 1.68417             | 0.24500             | 0.79527             | 0.42187 | 1.80966                                      | 3.2081                                           |
| 1.8 | 1.94832                   | 0.78125                    | 1.2701                 | 1.6257                | 25.10           | 1.9930                               | 0.63730              | 1.58939             | 0.28645             | 0.81577             | 0.50180 | 1.59719                                      | 2.5871                                           |
| 2.0 | 1.82262                   | 0.84935                    | 1.2534                 | 1.4757                | 27.26           | 1.7727                               | 0.70711              | 1.51743             | 0.32459             | 0.83255             | 0.56418 | 1.43275                                      | 2.1570                                           |
| 2.2 | 1.72582                   | 0.89683                    | 1.2367                 | 1.3790                | 28.95           | 1.6291                               | 0.75918              | 1.46098             | 0.35955             | 0.84654             | 0.61388 | 1.30110                                      | 1.8437                                           |
| 2.4 | 1.64910                   | 0.93068                    | 1.2212                 | 1.3122                | 30.31           | 1.5288                               | 0.79885              | 1.41554             | 0.39155             | 0.85837             | 0.65415 | 1.19295                                      | 1.6065                                           |
| 2.6 | 1.58685                   | 0.95529                    | 1.2071                 | 1.2636                | 31.48           | 1.4550                               | 0.82967              | 1.37820             | 0.42083             | 0.86852             | 0.68788 | 1.1023                                       | 1.4213                                           |
| 2.8 | 1.53536                   | 0.97348                    | 1.1943                 | 1.2269                | 32.36           | 1.3956                               | 0.85402              | 1.34698             | 0.44767             | 0.87731             | 0.71506 | 1.02507                                      | 1.2731                                           |
| 3.0 | 1.49210                   | 0.98712                    | 1.1830                 | 1.1984                | 33.15           | 1.3542                               | 0.87358              | 1.32050             | 0.47231             | 0.88500             | 0.73848 | 0.95841                                      | 1.1520                                           |
| 3.2 | 1.45524                   | 0.99747                    | 1.1726                 | 1.1756                | 33.84           | 1.3183                               | 0.88950              | 1.29775             | 0.49498             | 0.89178             | 0.75857 | 0.90019                                      | 1.0514                                           |
| 3.4 | 1.42348                   | 1.00539                    | 1.1633                 | 1.1571                | 34.43           | 1.2889                               | 0.90263              | 1.27801             | 0.51588             | 0.89781             | 0.77591 | 0.84888                                      | 0.9665                                           |
| 3.6 | 1.39582                   | 1.0115                     | 1.1549                 | 1.1418                | 34.95           | 1.2642                               | 0.91357              | 1.26071             | 0.5352              | 0.90320             | 0.79103 | 0.80327                                      | 0.8940                                           |
| 3.8 | 1.37154                   | 1.01624                    | 1.1473                 | 1.1290                | 35.40           | 1.2434                               | 0.92278              | 1.24543             | 0.55311             | 0.90805             | 0.80430 | 0.76242                                      | 0.8314                                           |
| 4.0 | 1.35004                   | 1.01992                    | 1.1403                 | 1.118                 | 35.83           | 1.2253                               | 0.93060              | 1.23184             | 0.56973             | 0.91244             | 0.81613 | 0.72564                                      | 0.7768                                           |

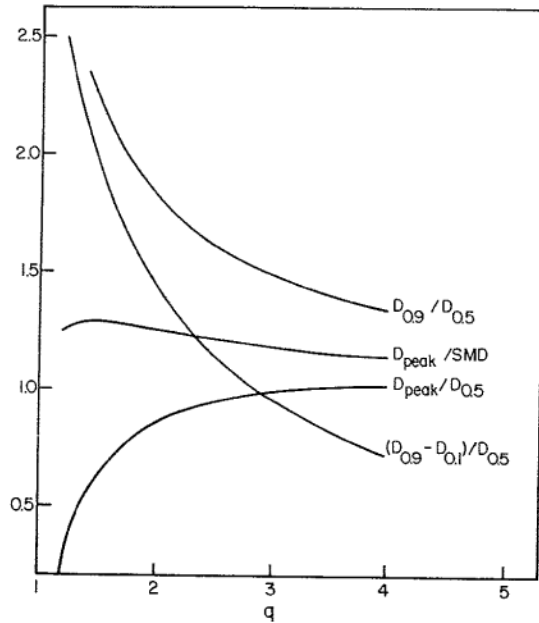


**Figure 3.13** Relationship between Rosin-Rammler distribution parameter  $q$  and various spray characteristics (Chin and Lefebvre [11]).

$$Q = 1 - \exp - \left[ 0.693 \left( \frac{D}{\text{MMD}} \right)^q \right] \quad (3.37)$$

or

$$Q = 1 - \exp - \left[ \Gamma \left( 1 - \frac{1}{q} \right)^{-q} \left( \frac{D}{\text{SMD}} \right)^q \right] \quad (3.38)$$

Although Eq. (3.14) is simpler than Eq. (3.38), the latter is strongly recommended because it shows clearly both the fineness and the spread of drop sizes in the spray.

From Table 3.3 it can be seen that, for a constant SMD of 50  $\mu\text{m}$ , changing  $q$  from 2 to 3 will produce the following changes in MMD and  $X$ :

| $q$                            | 2     | 2.2   | 2.4   | 2.6   | 2.8   | 3.0   |
|--------------------------------|-------|-------|-------|-------|-------|-------|
| $D_{0.5}$ (MMD), $\mu\text{m}$ | 73.78 | 68.95 | 65.61 | 63.18 | 61.35 | 59.90 |
| $X$ , $\mu\text{m}$            | 88.62 | 81.45 | 76.45 | 72.69 | 69.93 | 67.70 |

Some drop size analyzers give a direct estimate of  $D_{0.9}$ ,  $D_{0.1}$ ,  $D_{0.5}$ , SMD, etc., but others can only estimate  $X$  and  $q$ . The above equations, along with Table 3.3 and Figs. 3.13–3.15, can then be used to obtain SMD,  $D_{0.5}$ ,  $D_{0.9}$ , and  $D_{0.1}$ .

It is of interest to note in Table 3.3 that as  $q$  increases (toward a more uniform spray), the volume fraction  $Q$  at which  $D = \text{SMD}$  also increases. This means that, relatively speaking, SMD is increasing. This somewhat curious trend occurs because increase in  $q$  causes all the drops in the spray to shift closer to  $D_{0.5}$ , which eliminates many of the smallest drops. For many combustion systems, where very small drops are needed to provide high initial fuel evaporation rates for rapid ignition, a more uniform distribution of drop sizes in the spray may not always be desirable.

It is clearly important to distinguish between the concept of a representative diameter and a diameter that provides an indication of atomization quality. Any of the diameters listed in Table 3.2 can be used as a representative diameter to describe drop size distribution, but only the SMD can properly indicate the fineness of the spray from a combustion viewpoint. Even if the mass median diameter ( $D_{0.5}$ ) is decreasing, it is still not certain whether the spray is finer, because if  $q$  also changes it is possible that SMD will either increase or remain sensibly constant. For the same reason, other representative diameters such as  $D_{\text{peak}}$ ,  $X$ , and  $D_{0.9}$  cannot indicate the fineness of the spray. It is strongly recommended, therefore, that for combustion applications the Sauter mean diameter be used to describe atomization quality, since the use of any representative diameter could lead to erroneous conclusions about the fineness of the spray.

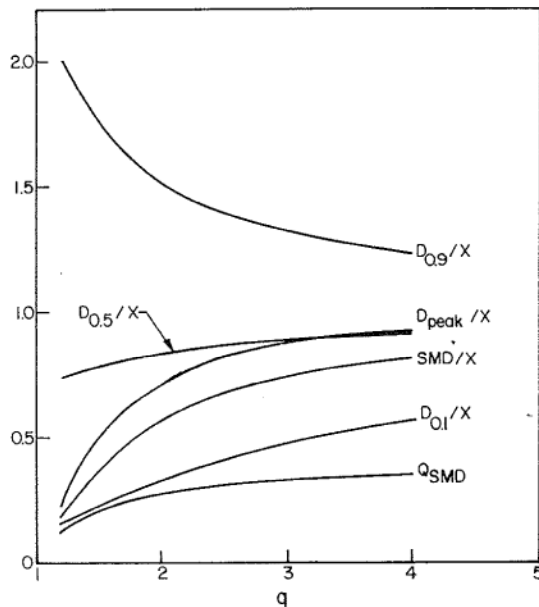


**Figure 3.14** Relationship between Rosin-Rammler distribution parameter  $q$  and various spray characteristics (Chin and Lefebvre [11]).