

# NOISE AND VIBRATION CONTROL ENGINEERING

PRINCIPLES AND APPLICATIONS

SECOND EDITION

ISTVÁN L. VÉR  
LEO L. BERANEK

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**SECOND EDITION**

Edited by

**István L. Vér  
and  
Leo L. Beranek**



**WILEY**

**JOHN WILEY & SONS, INC.**

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Published by John Wiley & Sons, Inc., Hoboken, New Jersey.  
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**Library of Congress Cataloging-in-Publication Data:**

Noise and vibration control engineering : principles and applications / edited  
by István L. Ver.—2nd ed.

p. cm.

Includes bibliographical references and index.

ISBN-13 978-0-471-44942-3

ISBN-10 0-471-44942-3

1. Noise control. 2. Vibration. 3. Soundproofing. I. Vér, I. L. (István  
L.), 1934—

TD892.N6512 2005

620.2'3—dc22

2005001372

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

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## PREFACE

The aim of this edition continues to be the presentation of the latest information on the most frequently encountered noise and vibration problems. We have endeavored to introduce new chapters and to update those chapters where the field has advanced. New or fully rewritten chapters are Sound Generation, Noise Control in Heating, Ventilating, and Air Conditioning Systems, Active Control of Noise and Vibration, Sound-Absorbing Materials and Sound Absorbers, Outdoor Sound Propagation, Criteria for Noise in Buildings and Communities, and Acoustical Standards for Noise and Vibration Control. Substantial new information has been added to Passive Silencers. All other chapters have been reviewed for timeliness.

Worldwide, there has been increased interest in noise and vibration control. Much of this interest has been generated by the expanding activities in the countries of the European Union and the Far East. Workshops on the latest developments in global noise policy are being held annually—the latest in Prague (Czech Republic) in 2004 and in Rio de Janeiro (Brazil) in 2005. There are signs of expanded interest in noise policy in the United States. Consumers are demanding quiet to a greater degree, the best example being the improved quiet interiors of automobiles. Other consumer products with greater noise control are already following or are likely to follow. Manufacturers must be alert to the increased competitiveness of imported products.

We are particularly indebted to John Bradley, Richard D. Godfrey, Colin H. Hanson, William W. Lang, George C. Mailing, Jr., Howie Noble, Robert Preuss, and Paul Schomer, who rendered help and criticism during the preparation of this book, and to the many individuals and organizations who gave us permission to use copyrighted technical information. We are also indebted to Acentech, Inc., BBN Technologies, Inc., Mueller-BBM Corporation and Harris Miller, Miller & Hanson, Inc., for encouraging members of their senior technical staff to contribute chapters and for assistance in many ways. It gives us special pleasure to acknowledge the help of Kathy Coleman and Jane Schultz, who assisted in the preparation of many chapters, and to Robert L. Argentieri, Fred Bernardi, and Robert H. Hilbert our editors at John Wiley & Sons, for their effective help and guidance in the production of this book.

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January 2005

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## CHAPTER 1

# Basic Acoustical Quantities: Levels and Decibels

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## 1.1 BASIC QUANTITIES OF SOUND WAVES

### Sound Waves and Noise

In the broadest sense, a *sound wave* is any disturbance that is propagated in an elastic medium, which may be a gas, a liquid, or a solid. Ultrasonic, sonic, and infrasonic waves are included in this definition. Most of this text deals with sonic waves, those sound waves that can be perceived by the hearing sense of a human being. *Noise* is defined as any perceived sound that is objectionable to a human being. The concepts basic to this chapter can be found in references 1–7. Portions are further expanded in Chapter 2.

### Sound Pressure

A person who is not deaf perceives as sound any vibration of the eardrum in the audible frequency range that results from an incremental variation in air pressure at the ear. A variation in pressure above and below atmospheric pressure is called *sound pressure*, in units of pascals (Pa).<sup>\*</sup> A young person with normal hearing can perceive sound in the frequency range of roughly 15 Hz (hertz) to 16,000 Hz, defined as the normal audible frequency range.

Because the hearing mechanism responds to sound pressure, it is one of two quantities that is usually measured in engineering acoustics. The normal ear is most sensitive at frequencies between 3000 and 6000 Hz, and a young person can detect pressures as low as about 20  $\mu\text{Pa}$ , which, when compared to the normal atmospheric pressure ( $101.3 \times 10^3 \text{ Pa}$ ) around which it varies, is a fractional variation of  $2 \times 10^{-10}$ .

<sup>\*</sup>One pascal (Pa) = 1 newton/meter squared ( $\text{N/m}^2$ ) = 10 dynes/ $\text{cm}^2$

## Pure Tone

A pure tone is a sound wave that can be represented by the equation,

$$p(t) = p_0 \sin(2\pi f)t \quad (1.1)$$

where  $p(t)$  is the instantaneous, incremental, sound pressure (above and below atmospheric pressure),  $p_0$  is the maximum amplitude of the instantaneous sound pressure, and  $f$  is the frequency, that is, the number of cycles per second, expressed in hertz. The time  $t$  is in seconds.

## Period

A full cycle occurs when  $t$  varies from zero to  $1/f$ . The  $1/f$  quantity is known as the period  $T$ . For example, the period  $T$  of a 500-Hz wave is 0.002 sec.

## Root-Mean-Square Amplitude

If we wish to determine the mean value of a full cycle of the sine wave of Eq. (1.1.) (or any number of full cycles), it will be zero because the positive part equals the negative part. Thus, the mean value is not a useful measure. We must look for a measure that permits the effects of the rarefactions to be added to (rather than subtracted from) the effects of the compressions.

One such measure is the root-mean-square (rms) sound pressure  $p_{rms}$ . It is obtained, first, by squaring the value of the sound pressure disturbance  $p(t)$  at each instant of time. Next the squared values are added and averaged over one or more periods. The rms sound pressure is the square root of this time average. The rms value is also called the *effective value*. Thus

$$p_{rms}^2 = \frac{1}{2} p_0^2 \quad (1.2)$$

or

$$p_{rms} = 0.707 p_0 \quad (1.3)$$

In the case of nonperiodic sound pressures, the integration interval should be long enough to make the rms value obtained essentially independent of small changes in the length of the interval.

## Sound Spectra

A sound wave may be comprised of a pure tone (single frequency, e.g., 1000 Hz), a combination of single frequencies harmonically related, or a combination of single frequencies not harmonically related, either finite or infinite in number. A combination of a finite number of tones is said to have a *line spectrum*. A combination of an infinite (large) number of tones has a *continuous spectrum*. A continuous-spectrum noise for which the amplitudes versus time occur with a

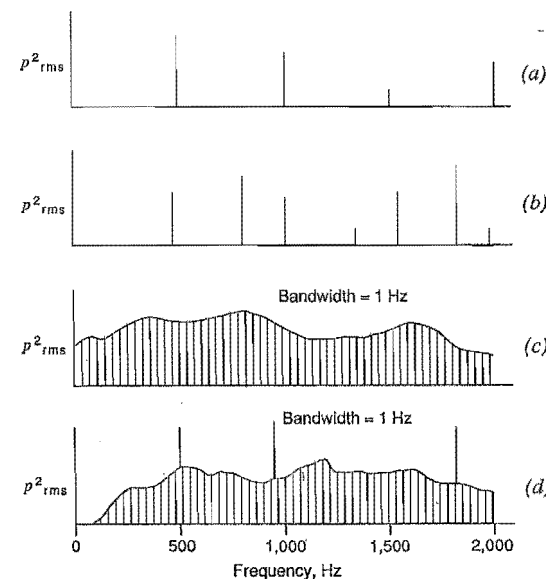


FIGURE 1.1 Mean-square sound pressure spectra: (a) harmonically related line spectrum; (b) inharmonically related line spectrum; (c) continuous power spectral density spectrum; (d) combination line and continuous power spectral density spectrum (complex spectrum).

normal (Gaussian) distribution is called *random noise*. Three of these types of noise are shown by the frequency spectra in Figs. 1.1a–c. A combination of a line and a continuous spectrum, called a complex spectrum, is shown in Fig. 1.1d.

Regardless of which type of sound wave is considered, when propagating at normal sound pressure amplitudes (to avoid nonlinearity) in air over reasonably short distances (so that sound attenuation in the air itself, which becomes significant at frequencies above 1000 Hz, can be neglected), the waveform is unchanged. Thus, a violin heard at a distance of 30 m sounds the same as at 5 m, although it is less loud.

## Sound Intensity

The second quantity commonly measured in engineering acoustics is *sound intensity*, defined as the continuous flow of power carried by a sound wave through an incrementally small area at a point in space. The units are watts per square meter ( $\text{W/m}^2$ ). This quantity is important for two reasons. First, at a point in free space, it is related to the total power radiated into the air by a sound source and, second, it bears at that point a fixed relation to the sound pressure.

Sound intensity at a point is directional (a vector) in the sense that the position of the plane of the incrementally small area can vary from being perpendicular

to the direction in which the wave is traveling to being parallel to that direction. It has its maximum value,  $I_{\max}$ , when its plane is perpendicular to the direction of travel. When parallel, the sound intensity is zero. In between, the component of  $I_{\max}$  varies as the cosine of the angle formed by the direction of travel and a line perpendicular to the incremental area.

Another equation, which we shall develop in the next chapter, relates sound pressure to sound intensity. In an environment in which there are no reflecting surfaces, the sound pressure at any point in any type of freely traveling (plane, cylindrical, spherical, etc.) wave is related to the maximum intensity  $I_{\max}$  by

$$p_{\text{rms}}^2 = I_{\max} \cdot \rho c \quad \text{Pa}^2 \quad (1.4)$$

where  $p_{\text{rms}}$  = rms sound pressure, Pa (N/m<sup>2</sup>)

$\rho$  = density of air, kg/m<sup>3</sup>

$c$  = speed of sound in air, m/s [see Eq. (1.7)]

$N$  = force, N

### Sound Power

A sound source radiates a measurable amount of power into the surrounding air, called *sound power*, in watts. If the source is nondirectional, it is said to be a *spherical sound source* (see Fig. 1.2). For such a sound source the measured (maximum) sound intensities at all points on an imaginary spherical surface centered on the acoustic center of the source are equal. Mathematically,

$$W_s = (4\pi r^2) I_s(r) \quad \text{W} \quad (1.5)$$

where  $I_s(r)$  = maximum sound intensity at radius  $r$  at surface of an imaginary sphere surrounding source, W/m<sup>2</sup>

$W_s$  = total sound power radiated by source in watts, W (N · m/s)

$r$  = distance from acoustical center of source to surface of imaginary sphere, m

A similar statement can be made about a line source; that is, the maximum sound intensities at all points on an imaginary cylindrical surface around a *cylindrical sound source*,  $I_c(r)$ , are equal:

$$W_c = (2\pi r l) I_c(r) \quad \text{W} \quad (1.6)$$

where  $W_c$  = total sound power radiated by cylinder of length  $l$ , W

$r$  = distance from acoustical centerline of cylindrical source to imaginary cylindrical surface surrounding source

### Inverse Square Law

With a spherical source, the radiated sound wave is spherical and the total power radiated in all directions is  $W$ . The sound intensity  $I(r)$  must decrease with

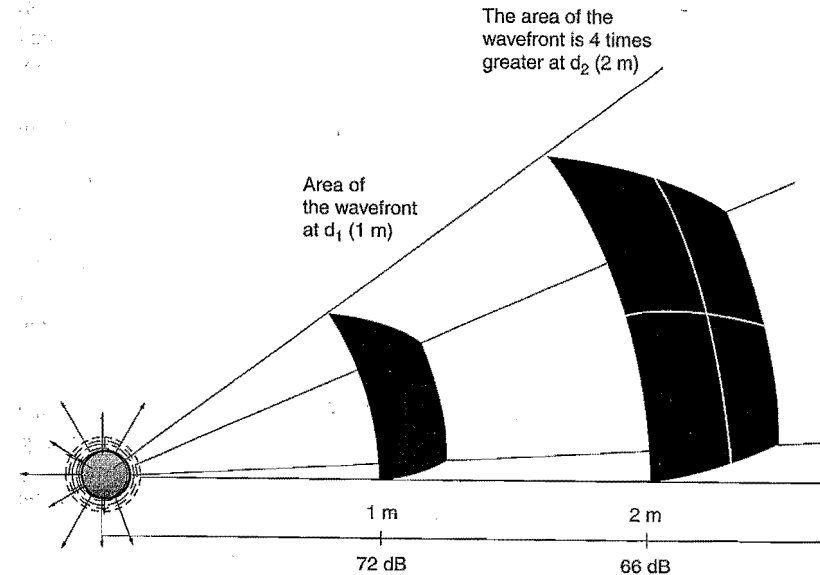


**FIGURE 1.2** Generation of a one-dimensional spherical wave. A balloonlike surface pulsates uniformly about some equilibrium position and produces a sound wave that propagates radially away from the balloon at the speed of sound. (Courtesy of Ref. 8.)

distance in proportion to the sound pressure squared, that is  $W/4\pi r^2$  [Eq. (1.5)]. Hence the name inverse square law. This is illustrated by the diagram in Fig. 1.2, where at a distance of 1 m the area of the wave front shown is  $a$  square meters and at 2 m it becomes  $4a$  square meters. To preserve sound energy, the same amount of power flows through the larger area; thus Fig. 1.2 diagrammatically shows the intensity decreasing by a factor of 4, or, as shown later, 6 dB, from 72 to 66 dB.

### Particle Velocity

Consider that the surface of the spherical source of Fig. 1.2 is expanding and contracting sinusoidally. During the first half of its sinusoidal motion, it pushes the air particles near its surface outward. Because air is elastic and compresses, the pressure near the surface will increase. This increased pressure overcomes the inertia of the air particles a short distance away and they move outward. That outward movement causes the pressure to build up at this new distance and, in turn, pushes more removed particles outward. This outward movement of the disturbance takes place at the speed of sound.

In the next half of the sinusoid, the sinusoidally vibrating spherical source reverses direction, creating a drop of pressure near its surface that pulls nearby

air particles toward it. This reverse disturbance also propagates outward with the speed of sound. Thus, at any one point in space, there will be sinusoidal to-and-fro movement of the particles, called the *particle velocity*. Also at any point, there will be sinusoidal rise and fall of sound pressure.

From the basic equations governing the propagation of sound, as is shown in the next chapter, we can say the following:

1. In a *plane wave* (approximated at a large distance from a point source) propagated in free space (no reflecting surfaces) the sound pressure and the particle velocity reach their maximum and minimum values at the same instant and are said to be in phase.
2. In such a wave the particles move back and forth along the line in which the wave is traveling. In reference to the spherical radiation discussed above, this means that the particle velocity is always perpendicular to the imaginary spherical surface (the wave front) in space. This type of wave is called a longitudinal, or compressional, wave. By contrast, a transverse wave is illustrated by a surface wave in water where the particle velocity is perpendicular to the water surface while the wave propagates in a direction parallel to the surface.

### Speed of Sound

A sound wave travels outward at a rate dependent on the elasticity and density of the air. Mathematically, the *speed of sound* in air is calculated as

$$c = \sqrt{\frac{1.4P_s}{\rho}} \quad \text{m/s} \quad (1.7)$$

where  $P_s$  = atmospheric (ambient) pressure, Pa  
 $\rho$  = density of air, kg/m<sup>3</sup>

For all practical purposes, the speed of sound is dependent only on the absolute temperature of the air. The equations for the speed of sound are

$$c = 20.05\sqrt{T} \quad \text{m/s} \quad (1.8)$$

$$c = 49.03\sqrt{R} \quad \text{ft/s} \quad (1.9)$$

where  $T$  = absolute temperature of air in degrees Kelvin, equal to 273.2 plus the temperature in degrees Celsius

$R$  = absolute temperature in degrees Rankine, equal to 459.7 plus the temperature in degrees Fahrenheit

For temperatures near 20°C (68°F), the speed of sound is

$$c = 331.5 + 0.58^\circ\text{C} \quad \text{m/s} \quad (1.10)$$

$$c = 1054 + 1.07^\circ\text{F} \quad \text{ft/s} \quad (1.11)$$

### Wavelength

Wavelength is defined as the distance the pure-tone wave travels during a full period. It is denoted by the Greek letter  $\lambda$  and is equal to the speed of sound divided by the frequency of the pure tone:

$$\lambda = cT = \frac{c}{f} \quad \text{m} \quad (1.12)$$

### Sound Energy Density

In standing-wave situations, such as sound waves in closed, rigid-wall tubes, rooms containing little sound-absorbing material, or reverberation chambers, the quantity desired is not sound intensity, but rather the *sound energy density*, namely, the energy (kinetic and potential) stored in a small volume of air in the room owing to the presence of the standing-wave field. The relation between the *space-averaged sound energy density*  $D$  and the *space-averaged squared sound pressure* is

$$D = \frac{p_{av}^2}{\rho c^2} = \frac{P_{av}^2}{1.4P_s} \quad \text{W} \cdot \text{s/m}^3 \quad (\text{J/m}^3 \text{ or simply N/m}^2) \quad (1.13)$$

where  $p_{av}^2$  = space average of mean-square sound pressure in a space, determined from data obtained by moving a microphone along a tube or around a room or from samples at various points, Pa<sup>2</sup>  
 $P_s$  = atmospheric pressure, Pa; under normal atmospheric conditions, at sea level,  $P_s = 1.013 \times 10^5$  Pa

## 1.2 SOUND SPECTRA

In the previous section we described sound waves with line and continuous spectra. Here we shall discuss how to quantify such spectra.

### Continuous Spectra

As stated before, a continuous spectrum can be represented by a large number of pure tones between two frequency limits, whether those limits are apart 1 Hz or thousands of hertz (see Fig. 1.3). Because the hearing system extends over a large frequency range and is not equally sensitive to all frequencies, it is customary to measure a continuous-spectrum sound in a series of contiguous frequency bands using a sound analyzer.

Customary bandwidths are one-third octave and one octave (see Fig. 1.4 and Table 1.1). The rms value of such a filtered sound pressure is called the one-third-octave-band or the octave-band sound pressure, respectively. If the filter bandwidth is 1 Hz, a plot of the filtered *mean-square* pressure of a *continuous-spectrum*

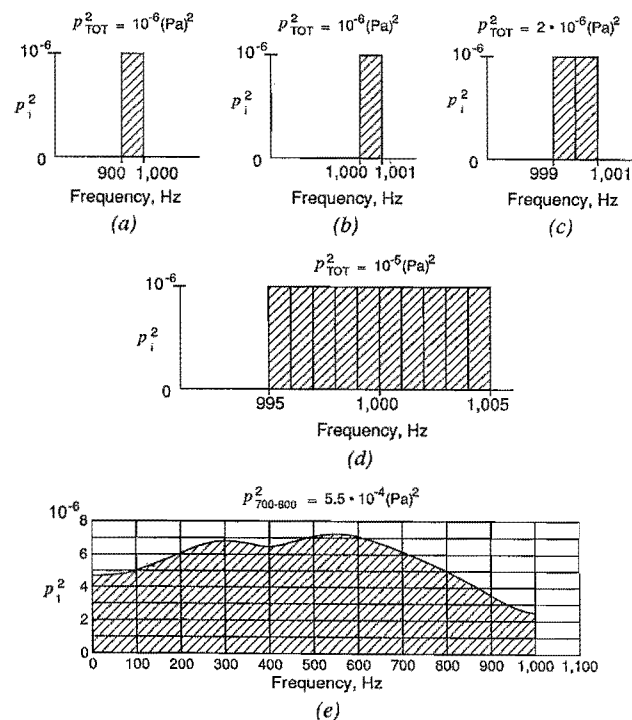


FIGURE 1.3 Power spectral density spectra showing the linear growth of total mean-square sound pressure when the bandwidth of noise is increased. In each case,  $p_i^2$  is the mean-square sound pressure in a band 1 Hz wide.

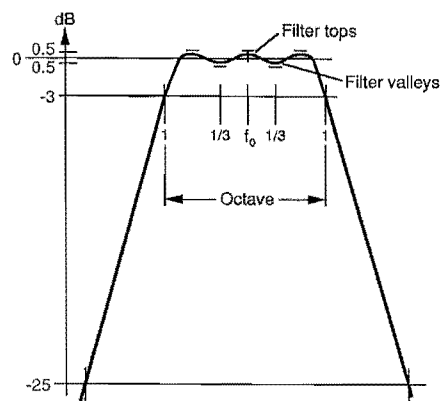


FIGURE 1.4 Frequency response of one manufacturer's octave-band filter. The 3-dB down points for the octave band and for a one-third-octave-band filter are indicated.

TABLE 1.1 Center and Approximate Cutoff Frequencies (Hz) for Standard Set of Contiguous-Octave and One-Third-Octave Bands Covering Audio Frequency Range<sup>a</sup>

| Band | Octave           |        |                  | One-Third Octave |        |                  |
|------|------------------|--------|------------------|------------------|--------|------------------|
|      | Lower Band Limit | Center | Upper Band Limit | Lower Band Limit | Center | Upper Band Limit |
| 12   |                  |        |                  |                  |        |                  |
| 13   | 11               | 16     | 22               | 14.1             | 16     | 17.8             |
| 14   |                  |        |                  | 17.8             | 20     | 22.4             |
| 15   |                  |        |                  | 22.4             | 25     | 28.2             |
| 16   | 22               | 31.5   | 44               | 28.2             | 31.5   | 35.5             |
| 17   |                  |        |                  | 35.5             | 40     | 44.7             |
| 18   |                  |        |                  | 44.7             | 50     | 56.2             |
| 19   | 44               | 63     | 88               | 56.2             | 63     | 70.8             |
| 20   |                  |        |                  | 70.8             | 80     | 89.1             |
| 21   |                  |        |                  | 89.1             | 100    | 112              |
| 22   | 88               | 125    | 177              | 112              | 125    | 141              |
| 23   |                  |        |                  | 141              | 160    | 178              |
| 24   |                  |        |                  | 178              | 200    | 224              |
| 25   | 177              | 250    | 355              | 224              | 250    | 282              |
| 26   |                  |        |                  | 282              | 315    | 355              |
| 27   |                  |        |                  | 355              | 400    | 447              |
| 28   | 355              | 500    | 710              | 447              | 500    | 562              |
| 29   |                  |        |                  | 562              | 630    | 708              |
| 30   |                  |        |                  | 708              | 800    | 891              |
| 31   | 710              | 1,000  | 1,420            | 891              | 1,000  | 1,122            |
| 32   |                  |        |                  | 1,122            | 1,250  | 1,413            |
| 33   |                  |        |                  | 1,413            | 1,600  | 1,778            |
| 34   | 1,420            | 2,000  | 2,840            | 1,778            | 2,000  | 2,239            |
| 35   |                  |        |                  | 2,239            | 2,500  | 2,818            |
| 36   |                  |        |                  | 2,818            | 3,150  | 3,548            |
| 37   | 2,840            | 4,000  | 5,680            | 3,548            | 4,000  | 4,467            |
| 38   |                  |        |                  | 4,467            | 5,000  | 5,623            |
| 39   |                  |        |                  | 5,623            | 6,300  | 7,079            |
| 40   | 5,680            | 8,000  | 11,360           | 7,079            | 8,000  | 8,913            |
| 41   |                  |        |                  | 8,913            | 10,000 | 11,220           |
| 42   |                  |        |                  | 11,220           | 12,500 | 14,130           |
| 43   | 11,360           | 16,000 | 22,720           | 14,130           | 16,000 | 17,780           |
|      |                  |        |                  | 17,780           | 20,000 | 22,390           |

<sup>a</sup>From Ref. 6.

sound versus frequency is called the *power spectral (or spectrum) density spectrum*. Narrow bandwidths are commonly used in analyses of machinery noise and vibration, but the term "spectral density" has no meaning in cases where pure tones are being measured.

The mean-square (or rms) sound pressure can be determined for each of the contiguous frequency bands and the result plotted as a function of frequency (Fig. 1.3e).

**Bandwidth Conversion.** It is frequently necessary to convert sounds measured with one set of bandwidths to a different set of bandwidths or to reduce both sets of measurements to a third set of bandwidths. Let us imagine that we have a machine that at a point in space produces a mean-square sound pressure of  $p_1^2 = 10^{-6} \text{ Pa}^2$  in a 1-Hz bandwidth between 999 and 1000 Hz (Fig. 1.3a). Now imagine that we have a second machine the same distance away that radiates the same power but is confined to a bandwidth between 1000 and 1001 Hz (Fig. 1.3b). The total spectrum now becomes that shown in Fig. 1.3c and the total mean-square pressure is twice that in either band. Similarly, 10 machines would produce 10 times the mean-square sound pressure of any one (Fig. 1.3d).

In other words, if the power spectral density spectrum in a frequency band of width  $\Delta f$  is flat (the mean-square sound pressures in all the 1-Hz-wide bands,  $p_1^2$ , within the band are equal), the total mean-square sound pressure for the band is given by

$$p_{\text{tot}}^2 = p_1^2 \frac{\Delta f}{\Delta f_0} \text{ Pa}^2 \quad (1.14)$$

where  $\Delta f_0 = 1 \text{ Hz}$ .

As an example, assume that we wish to convert the power spectral density spectrum of Fig. 1.3e, which is a plot of  $p_1^2(f)$ , the mean-square sound pressure in 1-Hz bands, to a spectrum for which the mean-square sound pressure in 100-Hz bands,  $p_{\text{tot}}^2$ , is plotted versus frequency. Let us consider only the 700–800-Hz band. Because  $p_1^2(f)$  is not equal throughout this band, we could painstakingly determine and add together the actual  $p_1^2$ 's or, as is more usual, simply take the average value for the  $p_1^2$ 's in that band and multiply by the bandwidth. Thus, for each 100-Hz band, the total mean-square sound pressure is given by Eq. (1.14), where  $p_1^2$  is the average 1-Hz band quantity throughout the band. For the 700–800-Hz band, the average  $p_1^2$  is  $5.5 \times 10^{-6}$  and the total is  $5.5 \times 10^{-6} \times 100 \text{ Hz} = 5.5 \times 10^{-4} \text{ Pa}^2$ .

If mean-square sound pressure levels have been measured in a specific set of bandwidths such as one-third-octave bands, it is possible to present accurately the data in a set of wider bandwidths such as octave bands by simply adding together the mean-square sound pressures for the component bands. Obviously, it is not possible to reconstruct a narrower bandwidth spectrum accurately (e.g., one-third-octave bands) from a wider bandwidth spectrum (e.g., octave bands.) However, it is sometimes necessary to make such a conversion in order to compare sets of data measured differently. Then the implicit assumption has to be made that the narrower band spectrum is continuous and monotonic within the larger band. In either direction, the conversion factor for each band is

$$p_B^2 = p_A^2 \frac{\Delta f_B}{\Delta f_A} \quad (1.15)$$

where  $p_A^2$  is the measured mean-square sound pressure in a bandwidth  $\Delta f_A$  and  $p_B^2$  is the desired mean-square sound pressure in the desired bandwidth  $\Delta f_B$ .

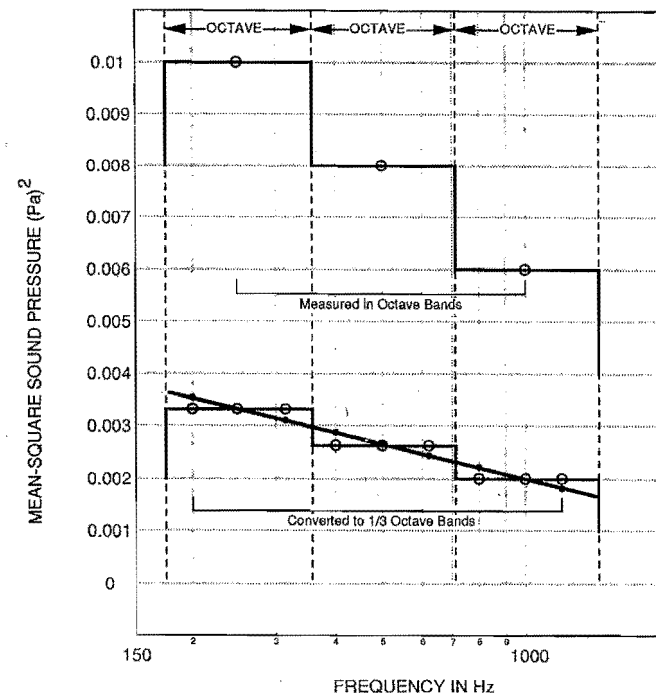


**FIGURE 1.5** Conversion of octave-band mean-square sound pressures into third-octave-band mean-square sound pressures. Such conversion should be made only where necessary under the assumption that the third-octave-band mean-square pressure levels decrease monotonically with band midfrequency. The sloping solid curve is the assumed-correct converted spectrum.

As an example, assume that we have measured a sound with a continuous spectrum using a one-octave-band filterset and have plotted the intensity for the contiguous bands versus the midfrequency of each band (see the upper three circles in Fig. 1.5). Assume that we wish to convert to an approximate one-third-octave-band spectrum to make comparisons with other data possible. Not knowing how the mean-square sound pressure varies throughout each octave band, we assume it to be continuous and monotonic and apply Eq. (1.15). In this example, approximately,  $\Delta f_B = \frac{1}{3} \Delta f_A$  for all the bands (see Table 1.1), and the mean-square sound pressure in each one-third-octave band with the same midfrequency will be one-third that in the corresponding one-octave band. The data would be plotted versus the midfrequency of each one-third-octave band (see Table 1.1 and Fig. 1.5). Then the straight, sloping line is added under the assumption that the true one-third-octave spectrum is monotonic.

### Complex Spectra

The mean-square sound pressure resulting from the combination of two or more pure tones of different amplitudes  $p_1, p_2, p_3$  and different frequencies  $f_1, f_2, f_3$  is given by

$$p_{\text{rms}}^2(\text{total}) = p_1^2 + p_2^2 + p_3^2 + \dots \quad (1.16)$$

The mean-square sound pressure of two pure tones of the same frequency but different amplitudes and phases is found from

$$p_{\text{rms}}^2(\text{total}) = p_1^2 + p_2^2 + 2p_1p_2 \cos(\theta_1 - \theta_2) \quad (1.17)$$

where the phase angle of each wave is represented by  $\theta_1$  or  $\theta_2$ .

Comparison of Eqs. (1.16) and (1.17) reveals the importance of phase when combining two sine waves of the same frequency. If the phase difference  $\theta_1 - \theta_2$  is zero, the two waves are in phase and the combination is at its maximum value. If  $\theta_1 - \theta_2 = 180^\circ$ , the third term becomes  $-2p_1p_2$  and the sum is at its minimum value. If the two waves are equal in amplitude, the minimum value is zero.

If one wishes to find the mean-square sound pressure of a number of waves all of which have different frequencies except, say, two, these two are added together according to Eq. (1.17) to obtain a mean-square pressure for them. Then this mean-square pressure and the mean-square pressures of the remainder of the components are summed according to Eq. (1.16).

### 1.3 LEVELS<sup>6</sup>

Because of the wide range of sound pressures to which the ear responds (a ratio of  $10^5$  or more for a normal person), sound pressure is an inconvenient quantity to use in graphs and tables. This is also true for the other acoustical quantities listed above. Early in the history of the telephone it was decided to adopt logarithmic scales for representing acoustical quantities and the voltages encountered in associated electrical equipment.

As a result of that decision, sound powers, intensities, pressures, velocities, energy densities, and voltages from electroacoustic transducers are commonly stated in terms of the logarithm of the ratio of the measured quantity to an appropriate reference quantity. Because the sound pressure at the threshold of hearing at 1000 Hz is about 20  $\mu\text{Pa}$ , this was chosen as the fundamental reference quantity around which the other acoustical references have been chosen.

Whenever the magnitude of an acoustical quantity is given in this logarithmic form, it is said to be a *level in decibels (dB) above or below a zero reference level* that is determined by a *reference quantity*. The argument of the logarithm is always a ratio and, hence, is dimensionless. The level for a very large ratio, for example the power produced by a very powerful sound source, might be given with the unit *bel*, which equals 10 dB.

### Power and Intensity Levels

**Sound Power Level.** Sound power level is defined as

$$L_W = 10 \log_{10} \frac{W}{W_0} \text{ dB re } W_0 \quad (1.18)$$

and conversely

$$W = W_0 \text{ antilog}_{10} \frac{L_W}{10} = W_0 \times 10^{L_W/10} \text{ W} \quad (1.19)$$

where  $W$  = sound power, W (watts)

$W_0$  = reference sound power, standardized at  $10^{-12}$  W

As seen in Table 1.2, a *ratio* of 10 in the power  $W$  corresponds to a *level difference* of 10 dB regardless of the reference power  $W_0$ . Similarly, a ratio of 100 corresponds to a level difference of 20 dB. Power ratios of less than 1 are allowable: They simply lead to negative levels. For example (see Table 1.2), a power ratio of 0.1 corresponds to a level difference of  $-10$  dB.

Column 4 of Table 1.2 gives sound power levels relative to the standard reference power level  $W_0 = 10^{-12}$  W in watts.

Some sound power ratios and the corresponding sound-power-level differences are given in Table 1.3. We note from the last line that the sound power level for the *product of two ratios* is equal to the *sum of the levels* for the two ratios. For example, determine  $L_W$  for the quantity  $2 \times 4$ . From Table 1.3,

TABLE 1.2 Sound Powers and Sound Power Levels

| Radiated Sound Power $W$ , watts |                                 | Sound Power Level $L_W$ , dB |                                     |
|----------------------------------|---------------------------------|------------------------------|-------------------------------------|
| Usual Notation                   | Equivalent Exponential Notation | Relative to 1 W              | Relative to $10^{-12}$ W (standard) |
| 100,000                          | $10^5$                          | 50                           | 170                                 |
| 10,000                           | $10^4$                          | 40                           | 160                                 |
| 1,000                            | $10^3$                          | 30                           | 150                                 |
| 100                              | $10^2$                          | 20                           | 140                                 |
| 10                               | $10^1$                          | 10                           | 130                                 |
| 1                                | 1                               | 0                            | 120                                 |
| 0.1                              | $10^{-1}$                       | -10                          | 110                                 |
| 0.01                             | $10^{-2}$                       | -20                          | 100                                 |
| 0.001                            | $10^{-3}$                       | -30                          | 90                                  |
| 0.000,1                          | $10^{-4}$                       | -40                          | 80                                  |
| 0.000,01                         | $10^{-5}$                       | -50                          | 70                                  |
| 0.000,001                        | $10^{-6}$                       | -60                          | 60                                  |
| 0.000,000,1                      | $10^{-7}$                       | -70                          | 50                                  |
| 0.000,000,01                     | $10^{-8}$                       | -80                          | 40                                  |
| 0.000,000,001                    | $10^{-9}$                       | -90                          | 30                                  |

**TABLE 1.3** Selected Sound Power Ratios and Corresponding Power-Level Differences

| Sound Power Ratio<br>$W/W_0, R$ | Sound-Power-Level Difference <sup>a</sup><br>$10 \log W/W_0, L_W \text{ (dB)}$ |
|---------------------------------|--------------------------------------------------------------------------------|
| 1000                            | 30                                                                             |
| 100                             | 20                                                                             |
| 10                              | 10                                                                             |
| 9                               | 9.5                                                                            |
| 8                               | 9.0                                                                            |
| 7                               | 8.5                                                                            |
| 6                               | 7.8                                                                            |
| 5                               | 7.0                                                                            |
| 4                               | 6.0                                                                            |
| 3                               | 4.8                                                                            |
| 2                               | 3.0                                                                            |
| 1                               | 0.0                                                                            |
| 0.9                             | -0.5                                                                           |
| 0.8                             | -1.0                                                                           |
| 0.7                             | -1.5                                                                           |
| 0.6                             | -2.2                                                                           |
| 0.5                             | -3.0                                                                           |
| 0.4                             | -4.0                                                                           |
| 0.3                             | -5.2                                                                           |
| 0.2                             | -7.0                                                                           |
| 0.1                             | -10                                                                            |
| 0.01                            | -20                                                                            |
| 0.001                           | -30                                                                            |
| $R_1 \times R_2$                | $L_{W_1} + L_{W_2}$                                                            |

<sup>a</sup>To the nearest 0.1 dB.

$L_W = 3.0 + 6.0 = 9.0$  dB, which is the sound power level for the ratio 8. Similarly,  $L_W$  for a ratio of 8000 equals the sum of the levels for 8 and 1000, that is,  $L_W = 9 + 30 = 39$  dB.

**Sound Intensity Level.** Sound intensity level, in decibels, is defined as

$$\text{Intensity level} = L_I = 10 \log \frac{I}{I_{\text{ref}}} \quad \text{dB re } I_{\text{ref}} \quad (1.20)$$

where  $I$  = sound intensity whose level is being specified,  $\text{W/m}^2$   
 $I_{\text{ref}}$  = reference intensity standardized as  $10^{-12} \text{ W/m}^2$

Sound power levels should not be confused with intensity levels (or with sound pressure levels, which are defined next), which also are expressed in decibels. Sound power is a measure of the *total* acoustical power radiated by a source in watts. Sound intensity and sound pressure specify the acoustical "disturbance"

produced at a point removed from the source. For example, their levels depend on the distance from the source, losses in the intervening air path, and room effects (if indoors). A helpful analogy is to imagine that sound power level is related to the total rate of heat production of a furnace, while either of the other two levels is analogous to the temperature produced at a given point in a dwelling.

### Sound Pressure Level

Almost all microphones used today respond to sound pressure, and in the public mind, the word *decibel* is commonly associated with sound pressure level or A-weighted sound pressure level (see Table 1.4). Strictly speaking, sound pressure level is analogous to intensity level, because, in calculating it, pressure is first squared, which makes it proportional to intensity (power per unit area):

$$\begin{aligned} \text{Sound pressure level} = L_p &= 10 \log \left[ \frac{p(t)}{p_{\text{ref}}} \right]^2 \\ &= 20 \log \frac{p(t)}{p_{\text{ref}}} \quad \text{dB re } p_{\text{ref}} \end{aligned} \quad (1.21)$$

where  $p_{\text{ref}}$  = reference sound pressure, standardized at  $2 \times 10^{-5} \text{ N/m}^2$  (20  $\mu\text{Pa}$ ) for airborne sound; for other media, references may be 0.1  $\text{N/m}^2$  (1  $\text{dyn/cm}^2$ ) or 1  $\mu\text{N/m}^2$  (1  $\mu\text{Pa}$ )

$p(t)$  = instantaneous sound pressure, Pa

Note that  $L_p$  re 20  $\mu\text{Pa}$  is 94 dB greater than  $L_p$  re 1 Pa.

As we shall show shortly,  $p(t)^2$  is only proportional to sound intensity if its mean-square value is taken. Thus, in Eq. (1.21),  $p(t)$  would be replaced by  $p_{\text{rms}}$ .

The relations among sound pressure levels (re 20  $\mu\text{Pa}$ ) for pressures in the meter-kilogram-second (mks), centimeter-gram-second (cgs), and English systems of units are shown by the four nomograms of Fig. 1.6.

## 1.4 DEFINITIONS OF OTHER COMMONLY USED LEVELS AND QUANTITIES IN ACOUSTICS

Analogous to sound pressure level given in Eq. (1.21), *A-weighted sound pressure level*  $L_A$  is given by

$$L_A = 10 \log \left[ \frac{p_A(t)}{p_{\text{ref}}} \right]^2 \quad \text{dB} \quad (1.22)$$

where  $p_A(t)$  is the instantaneous sound pressure measured using the standard frequency-weighting A (see Table 1.4).

TABLE 1.4 A and C Electrical Weighting Networks for Sound-Level Meter<sup>a</sup>

| Frequency, Hz | A-Weighting Relative Response, dB | C-Weighting Relative Response, dB |
|---------------|-----------------------------------|-----------------------------------|
| 10            | -70.4                             | -14.3                             |
| 12.5          | -63.4                             | -11.2                             |
| 16            | -56.7                             | -8.5                              |
| 20            | -50.5                             | -6.2                              |
| 25            | -44.7                             | -4.4                              |
| 31.5          | -39.4                             | -3.0                              |
| 40            | -34.6                             | -2.0                              |
| 50            | -30.2                             | -1.3                              |
| 63            | -26.2                             | -0.8                              |
| 80            | -22.5                             | -0.5                              |
| 100           | -19.1                             | -0.3                              |
| 125           | -16.1                             | -0.2                              |
| 160           | -13.4                             | -0.1                              |
| 200           | -10.9                             | 0                                 |
| 250           | -8.6                              | 0                                 |
| 315           | -6.6                              | 0                                 |
| 400           | -4.8                              | 0                                 |
| 500           | -3.2                              | 0                                 |
| 630           | -1.9                              | 0                                 |
| 800           | -0.8                              | 0                                 |
| 1,000         | 0                                 | 0                                 |
| 1,250         | +0.6                              | 0                                 |
| 1,600         | +1.0                              | -0.1                              |
| 2,000         | +1.2                              | -0.2                              |
| 2,500         | +1.3                              | -0.3                              |
| 3,150         | +1.2                              | -0.5                              |
| 4,000         | +1.0                              | -0.8                              |
| 5,000         | +0.5                              | -1.3                              |
| 6,300         | -0.1                              | -2.0                              |
| 8,000         | -1.1                              | -3.0                              |
| 10,000        | -2.5                              | -4.4                              |
| 12,500        | -4.3                              | -6.2                              |
| 16,000        | -6.6                              | -8.5                              |
| 20,000        | -9.3                              | -11.2                             |

<sup>a</sup>These numbers assume a flat, diffuse-field (random-incidence) response for the sound-level meter and microphone.

Average sound level  $L_{av,T}$  is given by

$$L_{av,T} = 10 \log \frac{(1/T) \int_0^T p^2(t) dt}{p_{ref}^2} \text{ dB} \quad (1.23)$$

where  $T$  is the (long) time over which the averaging takes place.

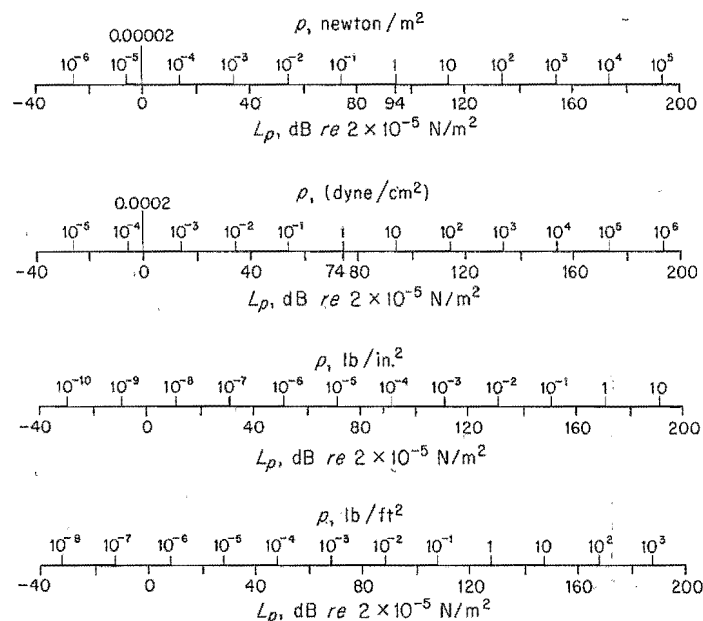


FIGURE 1.6 Charts relating  $L_p$  (dB re 20  $\mu$ Pa) to  $p$  in  $\text{N/m}^2$  (Pa),  $\text{dyn/cm}^2$ ,  $\text{lb/in.}^2$ , and  $\text{lb/ft}^2$ . For example, 1.0 Pa = 94 dB re 20  $\mu$ Pa.

Average A-weighted sound level  $L_{A,T}$  (also called  $L_{eq}$ , equivalent continuous A-weighted noise level) is given by

$$L_{A,T} = L_{eq} = 10 \log \frac{(1/T) \int_0^T p_A^2(t) dt}{p_{ref}^2} \text{ dB} \quad (1.24)$$

The time  $T$  must be specified. In noise evaluations, its length is usually one to several hours, or 8 h (working day), or 24 h (full day).

Day-night sound (noise) level  $L_{dn}$  is given by

$$L_{dn} = 10 \log \frac{1}{24} \left[ \frac{\int_{07:00}^{22:00} p_A^2(t) dt}{p_{ref}^2} + \frac{\int_{22:00}^{07:00} 10 p_A^2(t) dt}{p_{ref}^2} \right] \text{ dB} \quad (1.25)$$

where the first term covers the "daytime" hours from 07:00 to 22:00 and the second term covers the nighttime hours from 22:00 to 07:00. Here, the nighttime noise levels are considered to be 10 dB greater than they actually measure. The A-weighted sound pressure  $p_A$  is sampled frequently during measurement.

A-weighted sound exposure  $E_{A,T}$  is given by

$$E_{A,T} = \int_{t_1}^{t_2} p_A^2(t) dt \quad \text{Pa}^2 \cdot \text{s} \quad (1.26)$$

This equation is not a level. The term  $E_{A,T}$  is proportional to the energy flow (intensity times time) in a sound wave in the time period  $T$ . The period  $T$  starts and stops at  $t_1$  and  $t_2$ , respectively.

A-weighted noise exposure level  $L_{E_{A,T}}$  is given by

$$L_{E_{A,T}} = 10 \log \left( \frac{E_{A,T}}{E_0} \right) \quad \text{dB} \quad (1.27)$$

where  $E_0$  is a reference quantity, standardized at  $(20 \mu\text{Pa})^2 \cdot \text{s} = (4 \times 10^{-10} \text{ Pa})^2 \cdot \text{s}$ . However, the International Organization for Standardization standard ISO 1999:1990-01-5, on occupational noise level, uses  $E_0 = (1.15 \times 10^{-5} \text{ Pa})^2 \cdot \text{s}$ , because, for an 8-h day,  $L_{E_{A,T}}$ , with that reference, equals the average A-weighted sound pressure level  $L_{A,T}$ . The two reference quantities yield levels that differ by 44.6 dB. For a single impulse, the time period  $T$  is of no consequence provided  $T$  is longer than the impulse length and the background noise is low.

*Hearing threshold for setting "zero" at each frequency on a pure-tone audiometer* is the standardized, average, pure-tone threshold of hearing for a population of young persons with no otological irregularities. The standardized threshold sound pressure levels at the frequencies 250, 500, 1000, 2000, 3000, 4000, 6000, and 8000 Hz are, respectively, 24.5, 11.0, 6.5, 8.5, 7.5, 9.0, 8.0, and 9.5 dB measured under an earphone. An audiometer is used to determine the difference at these frequencies between the threshold values of a person (the lowest sound pressure level of a pure tone the person can detect consistently) and the standardized threshold values. Measurements are sometimes also made at 125 and 1500 Hz.<sup>7</sup>

*Hearing impairment (hearing loss)* is the number of decibels that the permanent hearing threshold of an individual at each measured frequency is above the zero setting on an audiometer, in other words, a change for the worse of the person's threshold of hearing compared to the normal for young persons.

*Hearing threshold levels associated with age* are the standardized pure-tone thresholds of hearing associated solely with age. They were determined from tests made on the hearing of persons in a certain age group in a population with no otological irregularities and no appreciable exposure to noise during their lives.

*Hearing threshold levels associated with age and noise* are the standardized pure-tone thresholds determined from tests made on the hearing of individuals who had histories of higher than normal noise exposure during their lives. The average noise levels and years of exposure were determined by questioning and measurement of the exposure levels.

*Noise-induced permanent threshold shift (NIPTS)* is the shift in the hearing threshold level caused solely by exposure to noise.

## 1.5 REFERENCE QUANTITIES USED IN NOISE AND VIBRATION

### American National Standard

The American National Standards Institute has issued a standard (ANSI S1.8-1989, Reaffirmed 2001) on "Reference Quantities for Acoustical Levels." This standard is a revision of ANSI S1.8-1969. The authors of this book have been surveyed for their opinions on preferred reference quantities. Table 1.5 is a combination of the standard references and of references preferred by the authors. The two references are clearly distinguished. All quantities are stated in terms of the International System of units (SI) and in British units.

### Relations among Sound Power Levels, Intensity Levels, and Sound Pressure Levels

As a practical matter, the reference quantities for sound power, intensity, and sound pressure (in air) have been chosen so that their corresponding levels are interrelated in a convenient way under certain circumstances.

The threshold of hearing at 1000 Hz for a young listener with acute hearing, measured under laboratory conditions, was determined some years ago as a sound pressure of  $2 \times 10^{-5}$  Pa. This value was then selected as the reference pressure for sound pressure level.

Intensity at a point is related to sound pressure at that point in a free field by Eq. (1.14). A combination of Eqs. (1.4), (1.20), and (1.21) yields the sound intensity level

$$\begin{aligned} L_I &= 10 \log \frac{I}{I_{\text{ref}}} = 10 \log \frac{p^2}{\rho c I_{\text{ref}}} \\ &= 10 \log \frac{p^2}{p_{\text{ref}}^2} + 10 \log \frac{p_{\text{ref}}^2}{\rho c I_{\text{ref}}} \\ L_I &= L_p - 10 \log K \quad \text{dB re } 10^{-12} \text{ W/m}^2 \end{aligned} \quad (1.28)$$

where  $K = \text{const} = I_{\text{ref}} \rho c / p_{\text{ref}}^2$ , which is dependent upon ambient pressure and temperature; quantity  $10 \log K$  may be found from Fig. 1.7, or,

$$K = \rho c / 400$$

The quantity  $10 \log K$  will equal zero, that is,  $K = 1$ , when

$$\rho c = \frac{p_{\text{ref}}^2}{I_{\text{ref}}} = \frac{4 \times 10^{-10}}{10^{-12}} = 400 \text{ mks rays} \quad (1.29)$$

We may also rearrange Eq. (1.28) to give the sound pressure level

$$L_p = L_I + 10 \log K \quad \text{dB re } 2 \times 10^{-5} \text{ Pa} \quad (1.30)$$

**TABLE 1.5 Reference Quantities for Acoustical Levels from American National Standard ANSI S1.8-1989 (Reaffirmed 2001) and As Preferred by Authors**

| Name                                    | Definition                        | Preferred Reference Quantities                                                              |                                                     |
|-----------------------------------------|-----------------------------------|---------------------------------------------------------------------------------------------|-----------------------------------------------------|
|                                         |                                   | SI                                                                                          | British                                             |
| Sound pressure level (gases)            | $L_p = 20 \log_{10}(p/p_0)$ dB    | $p_0 = 20 \mu\text{Pa} = 2 \times 10^{-5} \text{ N/m}^2$                                    | $2.90 \times 10^{-9} \text{ lb/in}^2$               |
| Sound pressure level (other than gases) | $L_p = 20 \log_{10}(p/p_0)$ dB    | $p_0 = 1 \mu\text{Pa} = 10^{-6} \text{ N/m}^2$                                              | $1.45 \times 10^{-10} \text{ lb/in}^2$              |
| Sound power level                       | $L_W = 10 \log_{10}(W/W_0)$ dB    | $W_0 = 1 \text{ pW} = 10^{-12} \text{ N} \cdot \text{m/s}$                                  | $8.85 \times 10^{-12} \text{ in} \cdot \text{lb/s}$ |
|                                         | $L_W = \log_{10}(W/W_0)$ bel      | $W_0 = 1 \text{ pW} = 10^{-12} \text{ N} \cdot \text{m/s}$                                  | $8.85 \times 10^{-12} \text{ in} \cdot \text{lb/s}$ |
| Sound intensity level                   | $L_I = 10 \log_{10}(I/I_0)$ dB    | $I_0 = 1 \text{ pW/m}^2 = 10^{-12} \text{ N/m} \cdot \text{s}$                              | $5.71 \times 10^{-15} \text{ lb/in} \cdot \text{s}$ |
| Vibratory force level                   | $L_{F0} = 20 \log_{10}(F/F_0)$ dB | $F_0 = 1 \mu\text{N} = 10^{-6} \text{ N}$                                                   | $2.25 \times 10^{-7} \text{ lb}$                    |
| Frequency level                         | $N = \log_{10}(f/f_0)$ dB         | $f_0 = 1 \text{ Hz}$                                                                        | $1.00 \text{ Hz}$                                   |
| Sound exposure level                    | $L_E = 10 \log_{10}(E/E_0)$ dB    | $E_0 = (20 \mu\text{Pa})^2 \cdot \text{s} = (2 \times 10^{-5} \text{ Pa})^2 \cdot \text{s}$ | $8.41 \times 10^{-18} \text{ lb}^2/\text{in}^4$     |

The quantities listed below are not officially part of ANSI S1.8. They either are listed there for information or are included here as the authors' choice.

|                                                   |                                |                                                          |                                                   |
|---------------------------------------------------|--------------------------------|----------------------------------------------------------|---------------------------------------------------|
| Sound energy level given in ISO 1683:1983         | $L_e = 10 \log_{10}(e/e_0)$ dB | $e_0 = 1 \text{ pJ} = 10^{-12} \text{ N} \cdot \text{m}$ | $8.85 \times 10^{-12} \text{ lb} \cdot \text{in}$ |
| Sound energy density level given in ISO 1683:1983 | $L_D = 10 \log_{10}(D/D_0)$ dB | $D_0 = 1 \text{ pJ/m}^3 = 10^{-12} \text{ N/m}^2$        | $1.45 \times 10^{-16} \text{ lb/in}^2$            |
| Vibration acceleration level                      | $L_a = 20 \log_{10}(a/a_0)$ dB | $a_0 = 10 \mu\text{m/s}^2 = 10^{-5} \text{ m/s}^2$       | $3.94 \times 10^{-4} \text{ in./s}^2$             |
| Vibration acceleration level in ISO 1683:1983     | $L_a = 20 \log_{10}(a/a_0)$ dB | $a_0 = 1 \mu\text{m/s}^2 = 10^{-6} \text{ m/s}^2$        | $3.94 \times 10^{-5} \text{ in./s}^2$             |
| Vibration velocity level                          | $L_v = 20 \log_{10}(v/v_0)$ dB | $v_0 = 10 \text{ nm/s} = 10^{-8} \text{ m/s}$            | $3.94 \times 10^{-7} \text{ in./s}$               |
| Vibration velocity level in ISO 1683:1983         | $L_v = 20 \log_{10}(v/v_0)$ dB | $v_0 = 1 \text{ nm/s} = 10^{-9} \text{ m/s}$             | $3.94 \times 10^{-8} \text{ in./s}$               |
| Vibration displacement level                      | $L_d = 20 \log_{10}(d/d_0)$ dB | $d_0 = 10 \text{ pm} = 10^{-11} \text{ m}$               | $3.94 \times 10^{-10} \text{ in}$                 |

Notes: Decimal multiples and submultiples of SI units are formed as follows:  $10^{-1}$  = deci (d),  $10^{-2}$  = centi (c),  $10^{-3}$  = milli (m),  $10^{-6}$  = micro ( $\mu$ ),  $10^{-9}$  = nano (n), and  $10^{-12}$  = pico (p). Also J = joule =  $\text{W} \cdot \text{s}$  (N · m), N = newton, and Pa = pascal =  $1 \text{ N/m}^2$ . Note that  $1 \text{ lb} = 4.448 \text{ N}$ .

Although some international standards differ, in this text, to avoid confusion between power and pressure, we have chosen to use  $W$  instead of  $P$  for power; and to avoid confusion between energy density and voltage, we have chosen  $D$  instead of  $E$  for energy density. The symbol lb means pound force.

In recent international standardization  $\log_{10}$  is written lg and  $20 \log_{10}(a/b) = 10 \lg(a^2/b^2)$ , i.e., "20" is never used.

**TABLE 1.6 Ambient Pressures and Temperatures for Which  $\rho c(\text{Air}) = 400 \text{ mks rays}$**

| $P_s$ , Pa          | Ambient Pressure           |                              | Ambient Temperature $T$ |                  |
|---------------------|----------------------------|------------------------------|-------------------------|------------------|
|                     | m of Hg, $0^\circ\text{C}$ | in. of Hg, $0^\circ\text{C}$ | $^\circ\text{C}$        | $^\circ\text{F}$ |
| $0.7 \times 10^5$   | 0.525                      | 20.68                        | -124.3                  | -192             |
| $0.8 \times 10^5$   | 0.600                      | 23.63                        | -78.7                   | -110             |
| $0.9 \times 10^5$   | 0.675                      | 26.58                        | -27.0                   | -17              |
| $1.0 \times 10^5$   | 0.750                      | 29.54                        | +30.7                   | +87              |
| $1.013 \times 10^5$ | 0.760                      | 29.9                         | 38.9                    | 102              |
| $1.1 \times 10^5$   | 0.825                      | 32.5                         | 94.5                    | 202              |
| $1.2 \times 10^5$   | 0.900                      | 35.4                         | 164.4                   | 328              |
| $1.3 \times 10^5$   | 0.975                      | 38.4                         | 240.4                   | 465              |
| $1.4 \times 10^5$   | 1.050                      | 41.3                         | 322.4                   | 613              |

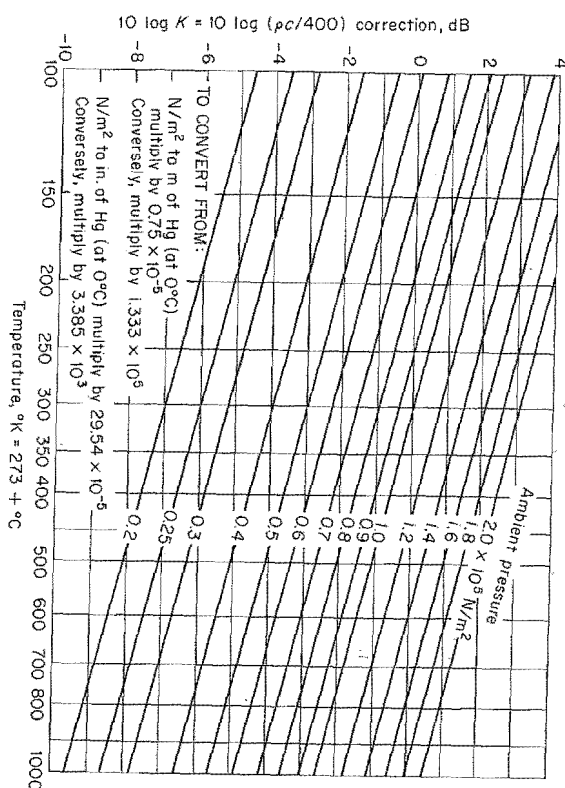


**FIGURE 1.7** Chart determining the value of  $10 \log(\rho c/400) = 10 \log K$  as a function of ambient temperature and ambient pressure. Values for which  $\rho c = 400$  are also given in Table 1.6.

In Table 1.6, we show a range of ambient pressures and temperatures for which  $\rho c = 400 \text{ mks rays}$ . We see that for average atmospheric pressure, namely,  $1.013 \times 10^5 \text{ Pa}$ , the temperature must equal  $38.9^\circ\text{C}$  ( $102^\circ\text{F}$ ) for  $\rho c = 400 \text{ mks rays}$ . However, if  $T = 22^\circ\text{C}$  and  $P_s = 1.013 \times 10^5 \text{ Pa}$ ,  $\rho c \approx 412$ . This yields

a value of  $10 \log(\rho c/400) = 10 \log 1.03 = 0.13$  dB, an amount that is usually not significant in acoustics.

Thus, for most noise measurements, we neglect  $10 \log K$  and in a free progressive wave let

$$L_p \approx L_I \quad (1.31)$$

Otherwise, the value of  $10 \log K$  is determined from Fig. 1.7 and used in Eq. (1.28) or (1.30).

Under the condition that the *intensity is uniform over an area  $S$* , the sound power and the intensity are related by  $W = IS$ . Hence, the sound power level is related to the intensity level as follows:

$$10 \log \frac{W}{10^{-12}} = 10 \log \frac{I}{10^{-12}} + 10 \log \frac{S}{S_0} \quad (1.32)$$

$$L_W = L_I + 10 \log S \quad \text{dB re } 10^{-12} \text{ W}$$

where  $S$  = area of surface,  $\text{m}^2$   
 $S_0 = 1 \text{ m}^2$

Obviously, only if the area  $S = 1.0 \text{ m}^2$  will  $L_W = L_I$ . Also, observe that the relation of Eq. (1.32) is not dependent on temperature or pressure.

## 1.6 DETERMINATION OF OVERALL LEVELS FROM BAND LEVELS

It is necessary often to convert sound pressure levels measured in a series of contiguous bands into a single-band level encompassing the same frequency range. The level in the all-inclusive band is called the *overall level*  $L(\text{OA})$  given by

$$L_p(\text{OA}) = 20 \log \sum_{i=1}^n 10^{L_{pi}/20} \quad \text{dB} \quad (1.33)$$

$$L_p(\text{OA}) = 10 \log \sum_{i=1}^n 10^{L_{pi}/10} \quad \text{dB} \quad (1.34)$$

The conversion can also be accomplished with the aid of Fig. 1.8. Assume that the contiguous band levels are given by the eight numbers across the top of Fig. 1.9. The frequency limits of the bands are not important to the method of calculation as long as the bands are contiguous and cover the frequency range of the overall band. To combine these eight levels into an overall level, start with any two, say, the seventh and eighth bands. From Fig. 1.8 we see that whenever the difference between two band levels,  $L_1 - L_2$ , is zero, the combined level is 3 dB higher. If the difference is 2 dB (the sixth band level minus the new level of 73 dB), the sum is 2.1 dB greater than the larger (75 + 2.1 dB). This procedure is followed until the overall band level is obtained, here, 102.1 dB.

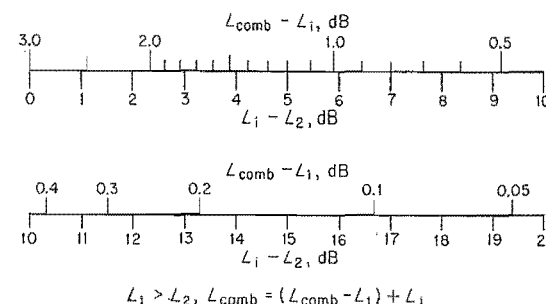


**FIGURE 1.8** Nanogram for combining two sound levels  $L_1$  and  $L_2$  (dB). Levels may be power levels, sound pressure levels, or intensity levels. Example:  $L_1 = 88$  dB,  $L_2 = 85$  dB,  $L_1 - L_2 = 3$  dB. Solution:  $L_{\text{comb}} = 88 + 1.8 = 89.8$  dB.

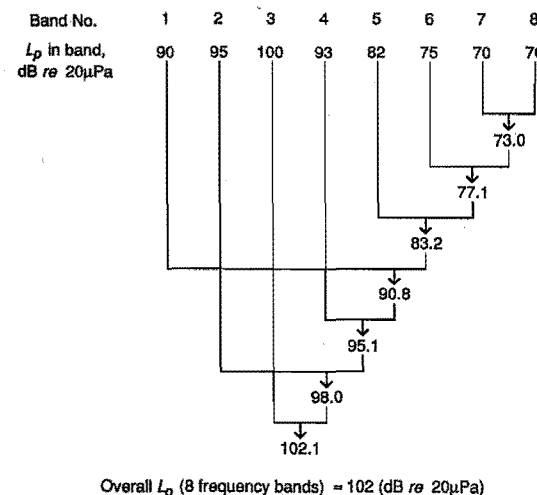


**FIGURE 1.9** Determination of an overall sound pressure level from levels in frequency bands (see also Fig. 1.10).

If we set  $L_{\text{comb}} - L_1 = A$ , then  $A$  is the number to be added to  $L_1$  (the larger) to get  $L_{\text{comb}}$ .

It is instructive to combine the bands in a different way, as is shown in Fig. 1.10. The first four bands are combined first; then the second four bands are combined. The levels of the two wider band levels are then combined. It is seen that the overall level is determined by the first four bands alone. This example points up the fact that characterization of a noise by its overall level may be completely inadequate for some noise control purposes because it may ignore a large

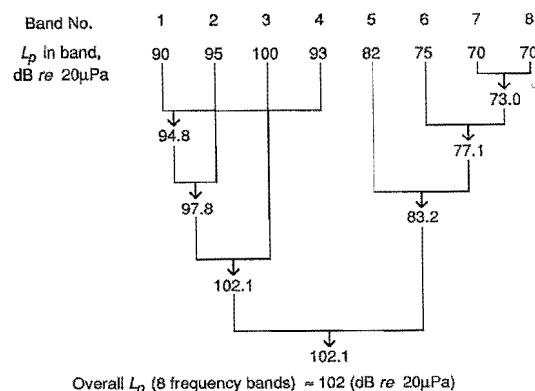


FIGURE 1.10 Alternate determination of an overall sound pressure level from levels in frequency bands (see also Fig. 1.9).

portion of the frequency spectrum. If the data of Fig. 1.10 represented a genuine noise control situation, the 102.1 overall level might be meaningless for some applications. For example, the sound pressure levels in the four highest bands might be the cause of annoyance or interference with speech communication, as is discussed in Chapter 19.

Finally, it should be remembered that in almost all noise control problems, it makes no sense to deal with small fractions of decibels. Rarely does one need a precision of 0.2 dB in measurements, and quite often it is adequate to quote levels to the nearest decibel.

## REFERENCES

1. L. L. Beranek, *Acoustics*, Acoustical Society of America, Melville, NY, 1986.
2. M. Moeser, *Engineering Acoustics*, Springer, New York, 2004.
3. D. A. Bies and C. H. Hansen, *Engineering Noise Control*, 2nd ed., E&FN Spon, New York, 2002.
4. P. M. Morse and K. U. Ingard, *Theoretical Acoustics*, Princeton University Press, Princeton, NJ, 1987.
5. L. E. Kinsler, A. R. Frey, A. B. Coppens, J. B. Sanders, and L. Kinsler, *Fundamentals of Acoustics*, 4th ed., Wiley, New York, 1999.
6. L. L. Beranek, *Noise and Vibration Control*, rev. ed., Institute of Noise Control Engineering, Poughkeepsie, NY, 1988.
7. L. L. Beranek, *Acoustical Measurements*, rev. ed., Acoustical Society of America, Melville, NY, 1988.
8. Charles M. Salter Associates, *Acoustics*, William Stout, San Francisco, 1998.

## CHAPTER 2

# Waves and Impedances

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## 2.1 THE WAVE EQUATION<sup>1</sup>

Sound waves must obey the laws of physics. For gases these include Newton's second law of motion, the gas law, and the law of conservation of mass. Combined, these equations produce the wave equation that governs the behavior of sound waves regardless of the surroundings in which they occur.

### Equation of Motion

The equation of motion (also called the force equation) is obtained by the application of Newton's second law to a small volume of gas in a homogeneous medium. Imagine the small volume of gas to be enclosed in a packet with weightless flexible sides and assume that there is negligible drag (friction) between particles inside and outside the packet. Suppose that this small volume exists in a part of the medium where the sound pressure  $p$  [actually  $p(t)$ ] increases at the space rate of

$$\text{grad } p = \mathbf{i} \frac{\partial p}{\partial x} + \mathbf{j} \frac{\partial p}{\partial y} + \mathbf{k} \frac{\partial p}{\partial z} \quad (2.1)$$

where  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  are unit vectors in the  $x$ ,  $y$ , and  $z$  directions, respectively. Obviously,  $\text{grad } p$  is a vector quantity.

The difference between the forces acting on the sides of the packet is a force  $\mathbf{f}$  equal to the rate at which the force changes with distance times the incremental dimensions of the box:

$$\mathbf{f} = - \left[ \mathbf{i} \left( \frac{\partial p}{\partial x} \Delta x \right) \Delta y \Delta z + \mathbf{j} \left( \frac{\partial p}{\partial y} \Delta y \right) \Delta x \Delta z + \mathbf{k} \left( \frac{\partial p}{\partial z} \Delta z \right) \Delta x \Delta y \right] \quad (2.2)$$

Note that a positive gradient causes the packet to accelerate in the negative direction.

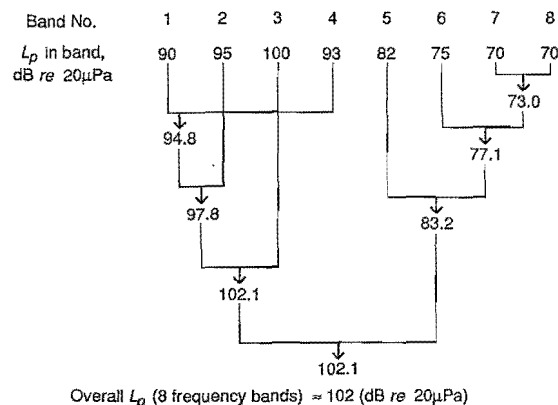


**FIGURE 1.10** Alternate determination of an overall sound pressure level from levels in frequency bands (see also Fig. 1.9).

portion of the frequency spectrum. If the data of Fig. 1.10 represented a genuine noise control situation, the 102.1 overall level might be meaningless for some applications. For example, the sound pressure levels in the four highest bands might be the cause of annoyance or interference with speech communication, as is discussed in Chapter 19.

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5. L. E. Kinsler, A. R. Frey, A. B. Coppens, J. B. Sanders, and L. Kinsler, *Fundamentals of Acoustics*, 4th ed., Wiley, New York, 1999.
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Note that a positive gradient causes the packet to accelerate in the negative direction.

Division of both sides of the equation by  $\Delta x \Delta y \Delta z = V$  gives the force per unit volume acting to accelerate the box,

$$\frac{\mathbf{f}}{V} = -\text{grad } p \quad (2.3)$$

By Newton's law, the force per unit volume of Eq. (2.3) equals the time derivative of the momentum per unit volume of the box. Because the box is a deformable packet, the mass inside is constant. Hence,

$$\frac{\mathbf{f}}{V} = -\text{grad } p = \frac{M}{V} \frac{D\mathbf{q}}{Dt} = \rho' \frac{D\mathbf{q}}{Dt} \quad (2.4)$$

where  $\mathbf{q}$  is the average vector velocity of the gas in the packet,  $\rho'$  is the average density of the gas in the packet, and  $M = \rho'V$  is the total mass of the gas in the packet.

The partial derivative  $D/Dt$  is not a simple one but represents the total rate of the change of velocity of the particular bit of gas in the packet regardless of its position that is, because its position changes when a sound wave hits it:

$$\frac{D\mathbf{q}}{Dt} = \frac{\partial \mathbf{q}}{\partial t} + q_x \frac{\partial \mathbf{q}}{\partial x} + q_y \frac{\partial \mathbf{q}}{\partial y} + q_z \frac{\partial \mathbf{q}}{\partial z} \quad (2.5)$$

where  $q_x$ ,  $q_y$ , and  $q_z$  are the components of the vector particle velocity  $\mathbf{q}$ .

If  $\mathbf{q}$  is small enough, the rate of change of momentum of the particles in the box can be approximated by the rate of change of momentum at a fixed point  $D\mathbf{q}/Dt \doteq \partial \mathbf{q}/\partial t$ , and the instantaneous density  $\rho'$  can be approximated by the average density  $\rho$ . Then

$$-\text{grad } p = \rho \frac{\partial \mathbf{q}}{\partial t} \quad (2.6)$$

### Gas Law

At audible frequencies, the wavelength of a sound wave is long compared to the spacing between air molecules, so that expansions and contractions at two different parts of the medium occur so rapidly that there is no time for heat exchange between points of differing instantaneous pressures. Hence, the compressions and expansions are adiabatic. From elementary thermodynamics,

$$PV^\gamma = \text{const} \quad (2.7)$$

where  $\gamma$  for air, hydrogen, oxygen, and nitrogen equals 1.4. If we let  $P = P_s + p$  and  $V = V_s + \tau$ , where  $P_s$  and  $V_s$  are the undisturbed pressure and volume of the packet, we get, for small values of incremental pressure  $p$  and incremental volume  $\tau$ ,

$$\frac{p}{P_s} = \frac{\gamma \tau}{V_s} \quad (2.8)$$

The time derivative of Eq. (2.8) yields

$$\frac{1}{P_s} \frac{\partial p}{\partial t} = -\frac{\gamma}{V_s} \frac{\partial \tau}{\partial t} \quad (2.9)$$

### Continuity Equation

The continuity equation is a statement that the mass of the gas in the deformable packet is constant. Thus the change in the incremental volume  $\tau$  depends only on the divergence of the vector displacement  $\xi$ :

$$\tau = V_s \text{div } \xi \quad (2.10)$$

or

$$\frac{\partial \tau}{\partial t} = V_s \text{div } \mathbf{q} \quad (2.11)$$

where  $\mathbf{q}$  is the instantaneous (vector) particle velocity.

### Wave Equation in Rectangular Coordinates

The three-dimensional wave equation is given by combining Eqs. (2.6), (2.9), and (2.11) and setting

$$c^2 = \frac{\gamma P_s}{\rho} \quad (2.12)$$

which yields

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad (2.13)$$

where

$$\nabla^2 p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \quad (2.14)$$

The one-dimensional wave equation is simply

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad (2.15)$$

We could also have eliminated  $p$  in the combination of the three equations and retained  $\mathbf{q}$ , in which case we would have had

$$\nabla^2 \mathbf{q} = \frac{1}{c^2} \frac{\partial^2 \mathbf{q}}{\partial t^2} \quad (2.16)$$

## 2.2 SOLUTIONS TO THE ONE-DIMENSIONAL WAVE EQUATION

### General Solution

The general solution to Eq. (2.15) is the sum of two terms,

$$p(x, t) = f_1\left(t - \frac{x}{c}\right) + f_2\left(t + \frac{x}{c}\right) \quad \text{Pa} \quad (2.17)$$

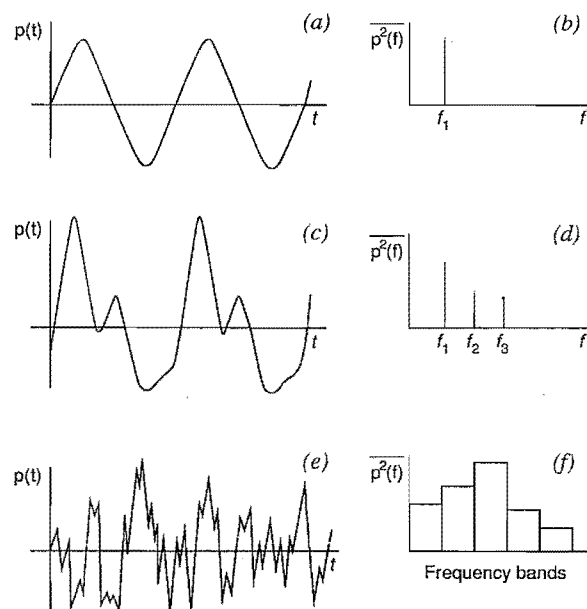


**FIGURE 2.1** (a, c, e) Forms of time function  $f_1$  or  $f_2$  of Eq. (2.17). Corresponding spectra are shown on the right. (b, d) Line spectra. (f) Complex spectrum (very large number of tones in each band).

where  $f_1$  and  $f_2$  are arbitrary functions. As we shall illustrate shortly, the first term represents an outgoing wave and the second term a backward-traveling wave. The functions  $f_1$  and  $f_2$  represent the shapes of the two sound waves being propagated. Examples of typical time histories and spectra of  $p(t)$  at a fixed location are given in Fig. 2.1. We also recognize  $c$  as the speed of sound in air.

### Outwardly Traveling Plane Wave

An apparatus for producing an outward-traveling plane wave is shown in Fig. 2.2. A piston at the left moving sinusoidally generates a sound wave that travels outward in the positive  $x$  direction and becomes absorbed in the anechoic termination so that no reflected wave exists. Equation (2.17) becomes

$$p(x, t) = f_1\left(t - \frac{x}{c}\right) = P_R \cos k(x - ct) \quad \text{Pa} \quad (2.18)$$

where  $P_R$  is the peak amplitude of the sound pressure.

Let us choose the space and the time origins, as shown by the left-hand sine wave in Fig. 2.2, so that  $P_R$  has its maximum value at  $x = 0$  and  $t = 0$ . After a time  $t_1$ , the wave will have traveled a distance  $x_1 = ct_1$ . Similarly for  $x_2 = 2x_1 = 2ct_1$ .

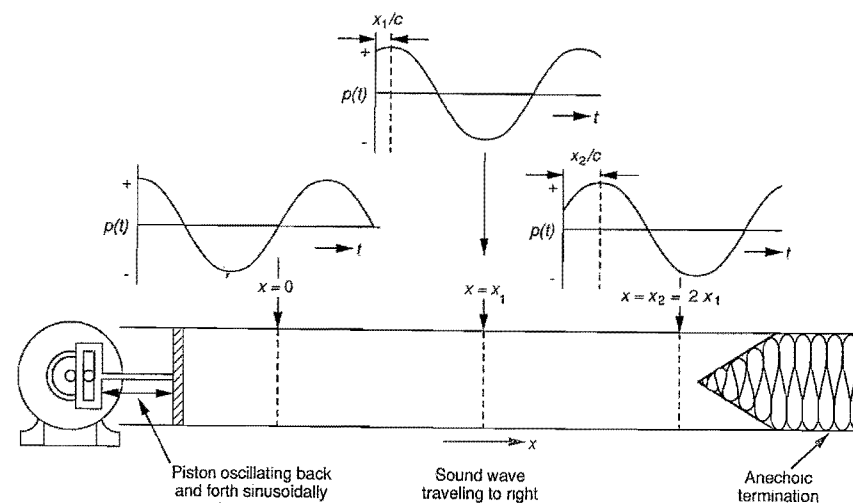


**FIGURE 2.2** Apparatus for producing a plane forward-traveling sound wave. A plane wave generated by the piston at the left travels to the right and is absorbed by the anechoic termination. The three waves at the top give the variation in sound pressure with time at the three points indicated,  $x = 0$ ,  $x = x_1$ , and  $x = x_2 = 2x_1$ .

Figure 2.3 shows a set of four spatial timeshots taken at  $t = 0, \frac{1}{4}T, \frac{1}{2}T, \frac{3}{4}T$ , where  $T$  is the time period of the piston machine. Each shows the sound pressure over a spatial extent of one wavelength,  $\lambda = c/f = cT$ . The 20 vertical lines along each snapshot enable one to observe the spatial variation of sound pressure at a given point at the four different times.

Snapshot (a) represents the pressure-versus-distance relation for times  $t = 0, T, 2T, 3T, \dots, nT$ . The maximum value  $+P_R$  exists at  $x = 0$ . Because the wave is periodic in space, the maximum value must also occur at  $x = \lambda, 2\lambda, 3\lambda, \dots$

Snapshot (b) shows the sound pressure a quarter of a period,  $\frac{1}{4}T$ , later, that is, the wave in (a) has moved to the right a distance equal to  $\frac{1}{4}\lambda$  to become the wave in (b). Similarly for (c) and (d). To convince yourself that the wave is traveling to the right, allow your eyes to jump successively from (a) to (b) to (c) to (d) and note that the peak,  $+P_R$ , moves successively to the right.

**Wavenumber.** The cosine function of Eq. (2.18) repeats its value every time the argument increases  $2\pi$  radians ( $360^\circ$ ). From the definition of wavelength,  $\lambda = c/f = cT$ , we can write this periodicity condition as

$$\cos[k(x + \lambda - ct)] = \cos[k(x - ct) + 2\pi] \quad (2.19)$$

so that  $k\lambda = 2\pi$  and  $k = 2\pi/\lambda$  radians per meter. We see that the meaning of the parameter  $k$ , called the *wavenumber*, is a kind of “spatial frequency.”

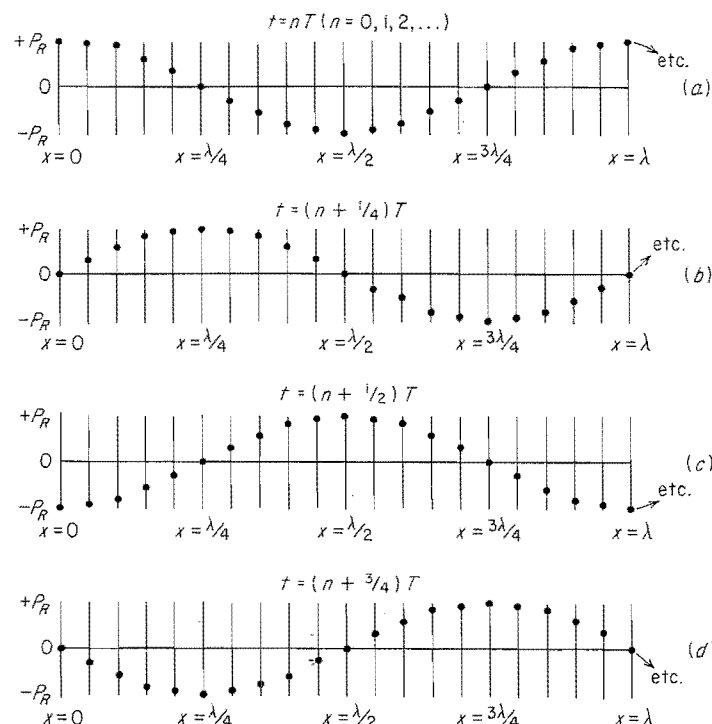


**FIGURE 2.3** Graphs showing sound pressure in a plane free-progressive wave traveling from left to right at 20 equally spaced axial locations at four instants of time  $t$ . The wave is produced by a source at the left and travels to the right with the speed  $c$ . The length of time it takes a wave to travel a distance equal to a wavelength is called the period  $T$ . Forward-traveling wave:  $p(x, t) = P_R \cos k(x - ct)$ ;  $k = 2\pi/\lambda = 2\pi/(cT) = 2\pi f/c = \omega/c$ .

The argument of the cosine in Eq. (2.18) may be written in any one of the following ways:

$$\begin{aligned} k(x - ct) &= \frac{2\pi}{\lambda}(x - ct) = 2\pi f \left( \frac{x}{c} - t \right) = 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \\ &= \frac{2\pi x}{\lambda} - 2\pi ft = kx - \omega t \end{aligned}$$

From Eq. (2.19) we can write the equations for the snapshots of Fig. 2.3 as

$$\begin{aligned} \text{(a)} \quad t &= nT & p &= P_R \cos \frac{2\pi x}{\lambda} \\ \text{(b)} \quad t &= \left(n + \frac{1}{4}\right)T & p &= P_R \cos \left( \frac{2\pi x}{\lambda} - \frac{\pi}{2} \right) \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad t &= \left(n + \frac{1}{2}\right)T & p &= P_R \cos \left( \frac{2\pi x}{\lambda} - \pi \right) \\ \text{(d)} \quad t &= \left(n + \frac{3}{4}\right)T & p &= P_R \cos \left( \frac{2\pi x}{\lambda} - \frac{3\pi}{2} \right) \end{aligned}$$

where  $n = 0, 1, 2, 3, \dots$

### Root-Mean-Square Sound Pressure

As discussed in Chapter 1, a measure of the strength of a sound wave is needed that avoids the problem of directly averaging over a cycle. Such an average is zero. The measure that has been standardized is the rms sound pressure  $p_{\text{rms}}$ . Its magnitude is 0.707 times the peak value  $P_R$  [Eq. (2.18)]. The rms value is also called the *effective value*.

### Particle Velocity

From Eq. (2.6) we may derive a relation between sound pressure  $p$  and particle velocity  $u$ , where  $u$  is the component of  $\mathbf{q}$  in the  $x$  direction. In one dimension,

$$-\frac{\partial p}{\partial x} = \rho \frac{\partial u}{\partial t} \quad (2.20)$$

Substitution of Eq. (2.18) into Eq. (2.20) gives

$$-u = \frac{1}{\rho} \int \frac{\partial p}{\partial x} dt = \frac{-P_R}{\rho c} \cos k(x - ct) \quad (2.20a)$$

or

$$u = \frac{p}{\rho c} \quad (2.21)$$

where  $\rho$  = time-averaged density of air, = 1.18 kg/m<sup>3</sup> for normal room temperature  $T = 22^\circ\text{C}$  (71.6°F) and atmospheric pressure  $P_s = 0.751$  m (29.6 in.) Hg  
 $c$  = speed of sound [see Eqs. (1.5)–(1.8)], which at normal temperatures of 22°C equals 344 m/s (1129 ft/s)  
 $\rho c$  = 406 mks rayls (N · s/m<sup>3</sup>) at normal room temperature and pressure; at other  $T$ 's and  $P_s$ 's,  $\rho c$  is found from Fig. 1.7

### Intensity

A freely traveling progressive sound wave transmits energy. We define this energy transfer as the *intensity*  $I$ , the energy that flows through a unit area in unit time. The units are watts per square meter (N/m · s). It has its maximum value,  $I_{\text{max}}$ , when the plane of the unit area is perpendicular to the direction in which the wave

is traveling. Intensity, analogous to electrical power, equals the time average of the product of sound pressure and particle velocity,

$$I_{\max} = \overline{p \cdot u} \quad \text{W/m}^2 \quad (\text{N/m} \cdot \text{s}) \quad (2.22)$$

For the wave of Figs. 2.2 and 2.3,

$$I_{\max} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \frac{P_R^2}{\rho c} \cos^2 k(x - ct) dt \quad (2.23)$$

where  $I(\theta) = I_{\max} \cos \theta$ ,  $\theta$  being the angle between the direction of travel of the wave and a line perpendicular to the plane of the unit area through which the flow of sound power is being determined.

Let  $T = \infty$  be a time long enough that  $I_{\max}$  has reached its asymptotic value within experimental error.

Because the time average of the cosine is zero,

$$I_{\max} = \frac{P_R^2}{2\rho c} = \frac{p_{\text{rms}}^2}{\rho c} \quad \text{W/m}^2 \quad (2.24)$$

where  $p_{\text{rms}}$  is the square root of the mean (time) square value of  $p(t)$ , as can be demonstrated by finding  $I\rho c$  from Eq. (2.24) and  $W$  is watts.

### Backward-Traveling Plane Wave

A backward-traveling plane wave may be produced by interchanging the source and termination of Fig. 2.2. The wave now travels in the  $-x$  direction and is described by

$$p(x, t) = P_L \cos k(x + ct) \quad \text{Pa} \quad (2.25)$$

Comparison of Eqs. (2.18) and (2.25) show that, if the two variables  $x$  and  $ct$  are separated by a negative sign, the wave travels in the positive direction, and if the two variables are separated by a positive sign, the direction reverses.

The four "snapshots" of Fig. 2.4 illustrate the backward-traveling wave. Allowing your eyes to jump from (a) to (b) to (c) to (d) and following the movement of  $+P_L$  from right to left convinces one that this is true.

### One-Dimensional Spherical Wave

**Sound Pressure.** The equation for the sound pressure associated with a free-progressive, spherically traveling sound wave, produced as shown in Fig. 1.2 in Chapter 1, is

$$p(r, t) = \frac{A}{r} \cos k(r - ct) \quad \text{Pa} \quad (2.26)$$

where  $A$  is an amplitude factor with dimension newtons per meter. Because the sign between  $r$  and  $ct$  is negative, the wave is traveling outward in the positive

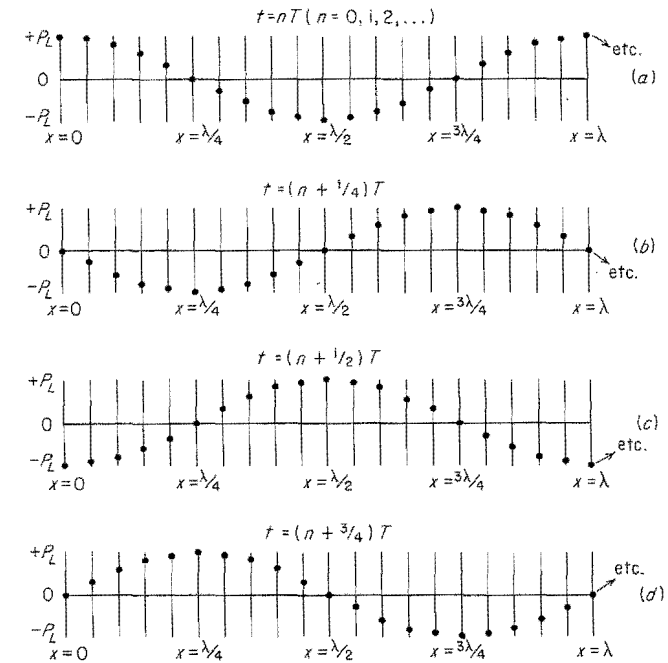


**FIGURE 2.4** Graphs showing sound pressure in a plane free-progressive wave traveling from right to left at 20 equally spaced axial locations at four instants of time  $t$ . The wave is produced by a source at the right (or by being reflected from a boundary at the right) and travels to the left with a speed  $c$ . The period  $T$  is defined as for Fig. 2.3. Backward-traveling wave:  $p(x, t) = P_L \cos[k(x + ct)]$ ;  $k = 2\pi/\lambda = 2\pi/(cT) = 2\pi f/c = \omega/c$ .

$r$  direction. In a spherical wave, the pressure amplitude is inversely proportional to the radial distance  $r$ .

**Particle Velocity.** The particle velocity for a spherical wave, using Eq. (2.20a) with  $r$  substituted for  $x$ , is

$$\begin{aligned} u(r, t) &= -\frac{1}{\rho} \int \left[ \frac{A}{r} k \sin k(r - ct) + \frac{A}{r^2} \cos k(r - ct) \right] dt \\ &= -\frac{1}{\rho} \left[ \frac{-kA}{kcr} \cos k(r - ct) - \frac{A}{r^2 kc} \sin k(r - ct) \right] \end{aligned}$$

or

$$u(r, t) = \frac{A}{\rho cr} \cos k(r - ct) \left[ 1 + \frac{1}{kr} \tan k(r - ct) \right] \quad (2.27)$$

For large values of  $kr$ ,

$$u(r, t) \doteq \frac{p(r, t)}{\rho c} \quad k^2 r^2 \gg 1 \quad (2.28)$$

For very small values of  $kr$ ,  $k^2r^2 \ll 1$ ,

$$\begin{aligned} u(r, t) &\doteq \frac{A}{k\rho cr^2} \sin k(r - ct) \\ &\doteq \frac{p(r, t)}{\rho ckr} \angle 90^\circ \text{ re phase of } p(r, t) \end{aligned} \quad (2.29)$$

Equations (2.27) and (2.29) show that as one approaches the center of a spherical source, the sound pressure and particle velocity become progressively more out of phase, approaching  $90^\circ$  in the limit.

**Intensity.** For a freely traveling spherical wave, Eq. (2.26) states that for *all values of  $r$* , the sound pressure varies as  $1/r$ . Because  $u = p/\rho c$  [Eq. (2.28)] for the case  $k^2r^2 \gg 1$ , we can write for all values of  $r$  that

$$I_{\max} \text{ at radius } r = \overline{p \cdot u} = \frac{p_{\text{rms}}^2 \text{ at } r}{\rho c} \quad \text{W/m}^2 \quad (2.30)$$

### 2.3 SOUND POWER OUTPUT OF ELEMENTARY RADIATORS

#### Monopole (Radiating Sphere)

The total power  $W_M$  radiated from a simple source, a pulsating sphere, called a monopole, is given in Table 2.1, where

where  $\hat{Q}_s$  = source strength,  $= 4\pi a^2 \hat{v}_r$ ,  $\text{m}^3/\text{s}$

$\hat{v}_r$  = peak value of velocity of sinusoidally pulsating surface,  $\text{m/s}$

$a$  = radius of pulsating sphere,  $\text{m}$

$k = 2\pi f/c$ ,  $\text{m}^{-1}$

$\rho c$  = characteristic impedance of gas, 406 mks rayls ( $\text{N} \cdot \text{s}/\text{m}^3$ ) for air at normal room temperature and atmospheric pressure

#### Dipole (Two Closely Spaced Monopoles)<sup>2</sup>

By definition, two monopoles constitute a dipole when  $(kd)^2 \ll 1$  and when they vibrate  $180^\circ$  out of phase. A dipole has a figure-eight radiation pattern, with minimum radiation in the direction perpendicular to a line connecting the two monopoles. The total sound power,  $W_D$ , radiated is given in the second row of Table 2.1, where  $\hat{Q}_s$ ,  $\hat{v}_r$ ,  $a$ ,  $k$ , and  $\rho c$  are as given above and

where  $d$  = separation between monopoles,  $\text{m}$

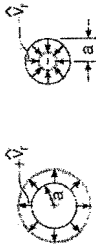
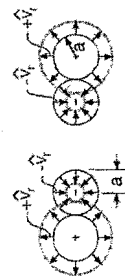
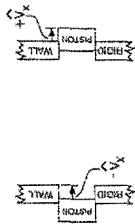
#### Oscillating Sphere<sup>3,4</sup>

An oscillating sphere is defined as a rigid sphere moving axially, back and forth, around its rest position. The total power,  $W_{OS}$ , radiated is given in the third row of Table 2.1,  $\rho c$  and  $k$  are as defined for a monopole, and

where  $\hat{v}_x$  = peak back-and-forth velocity,  $\text{m/s}$

$a$  = radius of oscillating sphere,  $\text{m}$

TABLE 2.1 Sound Power Output of Elementary Radiators

| Source Type        | Source Behavior $180^\circ$ Phase Difference                                          | Sound Power Output, W                                                                                                      | Auxiliary Expressions                                       |
|--------------------|---------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|
| Monopole           |  | $W_M = \frac{\rho c k^2}{8\pi(1 + k^2 a^2)} \hat{Q}_s^2$                                                                   | $\hat{Q}_s = 4\pi a^2 \hat{v}_r$                            |
| Dipole             |   | $W_D = \frac{\rho c k^4 d^2}{12\pi} \hat{Q}_s^2$                                                                           | $d$ = distance between monopoles                            |
| Oscillating sphere |  | $W_{OS} = \frac{2\pi \rho c k^4 a^6}{3(4 + k^4 a^4)} \hat{v}^2$                                                            | $\hat{v}_x$ = peak sinusoidal vibration velocity            |
| Baffled piston     |  | $W_{BP} = \frac{\rho c k^2}{4\pi} \hat{Q}^2 \quad ka \ll 1$<br>$W_{BP} = \frac{\rho c}{2\pi a^2} \hat{Q}^2 \quad ka \gg 1$ | $\hat{Q} = \pi a^2 \hat{v}_x$ where $\hat{v}_x$ is as above |

### Baffled Piston<sup>5</sup>

An axially vibrating diaphragm in an infinite plate is called a baffled piston. The total power radiated to one side of the rigid wall,  $W_{BP}$ , is given in the fourth row of Table 2.1, both for the case of a piston whose radius is small compared to a wavelength,  $ka \ll 1$ , and vice versa,  $ka \gg 1$ , where

where  $\hat{v}_x$  = peak axial velocity of piston, m/s  
 $a$  = radius of piston, m

The other quantities are as defined above.

## 2.4 INTERFERENCE AND RESONANCE

The sound pressure and incremental density in a sound wave are generally very small in comparison with the equilibrium values on which they are superposed. This is certainly true for speech and music waves. As a result, it is possible in such acoustical situations to determine the effect of two sound waves in the same space by simple linear addition of the effects of each sound wave separately. This is a statement of the principle of *superposition*.

In the previous section we presented a series of spatial snapshots for plane sound waves traveling to the right (Fig. 2.3) and to the left (Fig. 2.4). According to the principle of superposition, the effect of the sum of these two waves will be the sum of their effects, which we can see graphically by adding Figs. 2.3 and 2.4. The result is shown in Fig. 2.5, where we have set the amplitude of the forward-traveling wave  $P_R$  equal to the amplitude of the backward-traveling wave  $P_L$ .

The interference of the two waves has produced a surprising change. No longer does the sound pressure at one place occur to the right or to the left of that place at the next instant. The wave no longer travels; it is a standing wave. We see that at each point in space the sound pressure varies sinusoidally with time, except at the points  $x = \frac{1}{4}\lambda$  and  $\frac{3}{4}\lambda$ , where the pressure is always zero. The maximum value of the pressure variation at different points is different, being greatest at  $x = 0$ ,  $x = \frac{1}{2}\lambda$ , and  $x = \lambda$ . The sound pressures at the points between the points  $x = \frac{1}{4}\lambda$  and  $x = \frac{3}{4}\lambda$  always vary together, that is, increase or decrease in phase. At the same times the sound pressures for the points to the left of  $x = \frac{1}{4}\lambda$  and to the right of  $x = \frac{3}{4}\lambda$  decrease or increase together (in phase). Thus all pressures are in time phase in the standing wave, but there is a space difference of phase of  $180^\circ$  between the sound pressures at the points at  $x = 0$  and  $x = \frac{1}{2}\lambda$ .

Remembering that  $P_R = P_L = P$ , that is, that the amplitude of the wave traveling to the right is equal to the amplitude of the wave traveling to the left, we find that the sum of the two waves is

$$\begin{aligned} p(x, t) &= P \cos[k(x - ct)] + P \cos[k(x + ct)] \\ &= 2P(\cos kx)(\cos 2\pi ft) \quad \text{Pa} \end{aligned} \quad (2.31)$$

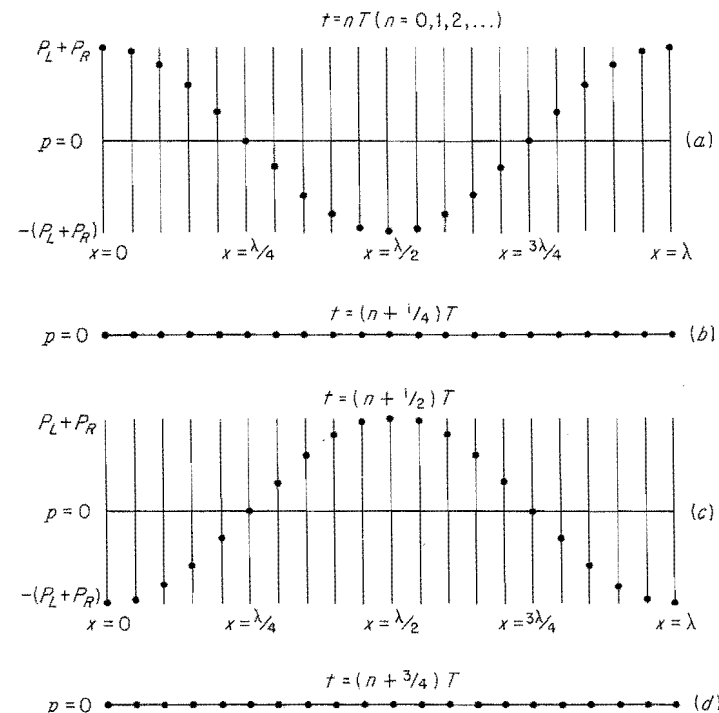


FIGURE 2.5 Graphs giving sound pressure in a plane standing wave at 20 places in space at four instants of time  $t$ . The wave is produced by two sources equal in strength at the right and left of the graphs (or the right source is a perfectly reflecting boundary that sends back a wave of equal amplitude to that produced by the source on the left). Standing wave:  $p(x, t) = 2P(\cos kx)(\cos 2\pi ft)$ , where  $k$  is the wavenumber.

From Eqs. (2.18), (2.20), and (2.31) we see very clearly the differences between a standing and a traveling wave. In a traveling wave distance  $x$  and time  $t$  occur as a sum or difference in the argument of the cosine. Hence, for the traveling wave, by adjusting both time and distance (according to the speed of sound) in the argument of the cosine, we can always keep the argument and thus the magnitude of the cosine the same. In Eq. (2.31) distance and time no longer appear together in the argument of a single cosine. So the same sound pressure cannot occur at an adjacent point in the space at a later time.

Standing waves will exist in any regular enclosure. In a rectangular room, for example, three classes of standing waves may exist (see Chapter 6). One class includes all waves that are perpendicular to one pair of opposing walls, that is, that travel at grazing incidence to two pairs of walls, the  $(n_x, 0, 0)$ ,  $(0, n_y, 0)$ ,  $(0, 0, n_z)$  modes of vibration. A second class travels at grazing incidence to only one pair of walls, the  $(n_x, n_y, 0)$ ,  $(n_x, 0, n_z)$ ,  $(0, n_y, n_z)$  modes of vibration. A third class involves all walls at oblique angles of incidence, the  $(n_x, n_y, n_z)$  modes of

vibration. Each free-standing wave in an acoustical space is called a normal mode of vibration, or simply a resonance. The frequencies at which resonant standing waves can exist are related to the separation between the reflecting surfaces. For example, the lowest frequency for a resonant standing wave in a one-dimensional system consisting of two rigid parallel walls is given by

$$f = \frac{c}{2d} \text{ Hz} \quad (2.32)$$

where  $f$  = lowest frequency for resonant standing wave, Hz  
 $c$  = speed of sound, m/s  
 $d$  = distance separating two reflecting surfaces, m

Resonant standing waves can also exist at every integral multiple of this frequency. That is to say,

$$f = \frac{nc}{2d} \text{ Hz} \quad (2.33)$$

where  $n$  is an integer 1, 2, 3, ....

## 2.5 IMPEDANCE AND ADMITTANCE

Reference to Eqs. (2.18) and (2.21) reveals that the magnitudes of sound pressure and particle velocity are directly proportional to each other. Also, in the special case of plane-wave sound propagation, the time dependence of sound pressure is exactly the same as the time dependence of particle velocity, and at any point in the wave there is no phase difference between the two quantities. Thus, in a plane sound wave the ratio of sound pressure to particle velocity at all instants of time is a constant equal to  $\rho c$ .

In general, however, for linear (small-signal) acoustical phenomena in the steady state, there is a difference in the time functions of sound pressure and particle velocity, leading to a phase difference of one relative to the other. Thus, at any point the particle velocity may lead or lag the sound pressure. In many situations, both the ratio of the magnitudes and the relative phase may be functions of frequency.

In several of the chapters that follow, it is convenient in acoustical design to avoid separate consideration of steady-state sound pressure and steady-state particle velocity (or other quantities that may be derived from them, such as force and volume velocity) and instead to deal with either one and with their complex ratio, as defined below.

### Complex Notation

The foundations for the designation of steady-state signals with the same frequency but different phases by complex notation are expressed in the identities

$$|A| \cos(\omega t + \theta_1) \equiv \text{Re } \bar{A} e^{j\omega t} \quad (2.34)$$

where  $|A|$   $\equiv$  amplitude of cosine function

$\theta_1 \equiv$  phase shift at time  $t = 0$

Re  $\equiv$  "real part of"

$j \equiv \sqrt{-1}$

and

$$\bar{A} \equiv A_{\text{Re}} + jA_{\text{Im}} = |A| e^{j\theta_1} \quad (2.35)$$

$$|A| \equiv \sqrt{A_{\text{Re}}^2 + A_{\text{Im}}^2} \quad (2.36)$$

$$\theta_1 \equiv \tan^{-1} \left( \frac{A_{\text{Im}}}{A_{\text{Re}}} \right) \quad (2.37)$$

We note also that

$$e^{j\theta} \equiv \cos \theta + j \sin \theta \quad (2.38)$$

so that

$$A_{\text{Re}} \equiv |A| \cos \theta \quad (2.39)$$

$$A_{\text{Im}} \equiv |A| \sin \theta \quad (2.40)$$

These equations say that a cosinusoidal time-varying function, given by the left-hand side of Eq. (2.34), can be represented by the real-axis projection of a vector of magnitude  $|A|$  given by Eq. (2.36), rotating at a rate  $\omega$  radians per second. The angle  $\theta_1$  is the angle of the vector (in radians) relative to the positive real axis at the instant of time  $t = 0$ .

We might therefore express a time-varying steady-state sound pressure or force by

$$\bar{A} e^{j\omega t} = |A| e^{j\omega t} e^{j\theta_1} \quad (2.41)$$

Also, a time-varying steady-state velocity or volume velocity might be expressed by

$$\bar{B} e^{j\omega t} = |B| e^{j\omega t} e^{j\theta_2} \quad (2.42)$$

### Definitions of Complex Impedance

**Complex Impedance  $Z$ .** In general, complex impedance is defined as

$$\bar{Z} \equiv \frac{\bar{A}}{\bar{B}} = \frac{|A| e^{j(\omega t + \theta_1)}}{|B| e^{j(\omega t + \theta_2)}} = \frac{|A| e^{j\theta_1}}{|B| e^{j\theta_2}} = |Z| e^{j\theta} \quad (2.43)$$

and

$$\bar{Z} \equiv R + jX = \sqrt{R^2 + X^2} e^{j\theta} = |Z| e^{j\theta} \quad (2.44)$$

where  $\bar{Z}$  = complex impedance as given above

$A$  = steady-state pressure or force

- $B$  = steady-state velocity or volume velocity  
 $|Z|$  = magnitude of complex impedance  
 $\theta$  = phase angle between the time functions  $A$  and  $B$ ,  $= \theta_1 - \theta_2$   
 $R, X$  = real and imaginary parts, respectively, of complex impedance  $\bar{Z}$   
 $R$  = resistance,  $\text{Re } \bar{Z}$   
 $X$  = reactance,  $\text{Im } \bar{Z}$

Often  $|A|$  and  $|B|$  are taken to be the rms values of the phenomena they represent, although, if so taken, a factor of  $\sqrt{2}$  must be added to both sides of Eq. (2.34) to make them correct in a physical sense. Whether amplitudes or rms values are used makes no difference in the impedance ratio.

Complex impedances are of several types, according to the quantities involved in the ratios. Types common in acoustics are given below.

**Acoustical Impedance  $Z_A$ .** The acoustical impedance at a given surface  $S$  is defined as the complex ratio of (a) sound pressure averaged over the surface to (b) volume velocity through it. The volume velocity  $U = uS$ . The surface may be either a hypothetical surface in an acoustical medium or the moving surface of a mechanical device. The unit is  $\text{N} \cdot \text{s}/\text{m}^5$ , also called the mks acoustical ohm. That is,

$$Z_A = \frac{P}{U} \quad \text{N} \cdot \text{s}/\text{m}^5 \quad (\text{mks acoustical ohms}) \quad (2.45)$$

**Specific Acoustical Impedance  $Z_s$ .** The specific acoustical impedance is the complex ratio of the sound pressure at a point of an acoustical medium or mechanical device to the particle velocity at that point. The unit is  $\text{N} \cdot \text{s}/\text{m}^3$ , also called the mks rayl. That is,

$$Z_s = \frac{P}{u} \quad \text{N} \cdot \text{s}/\text{m}^3 \quad (\text{mks rayls}) \quad (2.46)$$

**Mechanical Impedance  $Z_M$ .** The mechanical impedance is the complex ratio of the force acting on a specific area of an acoustical medium or mechanical device to the resulting linear velocity through or of that area, respectively. The unit is the  $\text{N} \cdot \text{s}/\text{m}$ , also called the mks mechanical ohm. That is,

$$Z_M = \frac{f}{u} \quad \text{N} \cdot \text{s}/\text{m} \quad (\text{mks mechanical ohms}) \quad (2.47)$$

**Characteristic Resistance  $\rho c$ .** The characteristic resistance is the ratio of the sound pressure at a given point to the particle velocity at that point in a free, plane, progressive sound wave. It is equal to the product of the density of the medium and the speed of sound in the medium ( $\rho c$ ). It is analogous to the characteristic impedance of an infinitely long, dissipationless transmission line. The unit is the  $\text{N} \cdot \text{s}/\text{m}^3$ , also called the mks rayl. In the solution of problems in this book we shall assume for air that  $\rho c = 406$  mks rayls, which is valid for a temperature of  $22^\circ\text{C}$  ( $71.6^\circ\text{F}$ ) and a barometric pressure of 0.751 m (29.6 in.) Hg.

**Normal Specific Acoustical Impedance  $Z_{sn}$ .** At the boundary between air and a denser medium (such as a porous acoustical material) we find a further definition necessary, as follows: When an alternating sound pressure  $p$  is produced at the surface of an acoustical material, an alternating velocity  $u$  of the air particles is produced through the surface. The to-and-fro motions of the air particles may be at any angle relative to the surface. The angle depends both on the angle of incidence of the sound wave and on the nature of the acoustical material. For example, if the material is porous and has very low density, the particle velocity at the surface is nearly in the same direction as that in which the wave is propagating. By contrast, if the surface were a large number of small-diameter tubes packed side by side and oriented perpendicular to the surface, the particle velocity would necessarily be only perpendicular to the surface. In general, the direction of the particle velocity at the surface has both a normal (perpendicular) component and a tangential component.

The normal specific acoustical impedance (sometimes called the unit-area acoustical impedance) is defined as the complex ratio of the sound pressure  $p$  to the normal component of the particle velocity  $u_n$  at a plane, in this example at the surface of the acoustical material. Thus

$$Z_{sn} = \frac{P}{u_n} \quad \text{N} \cdot \text{s}/\text{m}^3 \quad (\text{mks rayls}) \quad (2.48)$$

### Definition of Complex Admittance

Complex admittance is the reciprocal of complex impedance. In all ways, it is handled by the same set of rules as given by Eqs. (2.34)–(2.44). Thus, the complex admittance corresponding to the complex impedance of Eq. (2.43) is

$$\bar{Y} = \frac{\bar{B}}{\bar{A}} = \frac{|B| e^{j\theta_2}}{|A| e^{j\theta_1}} = |Y| e^{j\phi} \quad (2.49)$$

$$\text{where } |Y| = 1/|Z| \\ \phi = -\theta$$

The choice between impedance and admittance is sometimes made according to whether  $|A|$  or  $|B|$  is held constant during a measurement. Thus, if  $|B|$  is held constant,  $|Z|$  is directly proportional to  $|A|$  and is used. If  $|A|$  is held constant,  $|Y|$  is directly proportional to  $|B|$  and is used.

### REFERENCES

1. W. J. Cunningham, "Application of Vector Analysis to the Wave Equation," *J. Acoust. Soc. Am.*, **22**, 61 (1950); R. V. L. Hartley, "Note on 'Application of Vector Analysis to the Wave Equation,'" *J. Acoust. Soc. Am.*, **22**, 511 (1950).
2. H. Kuttruff, *Room Acoustics*, Halstead, New York, 2000.

3. M. Moeser, *Engineering Acoustics*, Springer-Verlag, New York, 2004.
4. L. Cremer, *Vorlesungen über Technische Akustik*, Springer-Verlag, New York, 1971, Section 2.5.2.
5. L. L. Beranek, *Acoustics*, Acoustical Society of America, Melville, NY, 1986, Sections 4.3 and 7.5.

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**CHAPTER 3**

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## Data Analysis

ALLAN G. PIERSOL

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For studies related to noise and vibration control engineering, the analysis of measured acoustical noise and/or vibration data may be accomplished with a number of goals in mind. The most important of these goals can be divided into four broad categories: (a) an assessment of the severity of an environment, (b) the identification of system response properties, (c) the identification of sources, and (d) the identification of transmission paths. The first goal is commonly accomplished using one-third-octave-band-level calculations, or perhaps frequency-weighted overall level measurements, as described in Chapter 1. The other goals often require more advanced data analysis procedures. Following a brief discussion of the general types of acoustical and vibration data of common interest, the most important data analysis procedures for accomplishing goals (b)–(d) are outlined, and important applications of the results from such analyses are summarized. Because of the broad and intricate nature of the subject, heavy use of references is employed to cover details.

### 3.1 TYPES OF DATA SIGNALS

Acoustical and vibration data are commonly acquired in the form of analog time history signals produced by appropriate transducers (details of acoustical and vibration transducers and signal conditioning equipment are available from the data acquisition documents listed in the Bibliography and the literature published by acoustical and vibration measurement system manufacturers). The signals are generally produced with the units of volts but can be calibrated into appropriate engineering units (g, m/s, Pa, etc.) as required. From a data analysis viewpoint, it is convenient to divide these time history signals into two broad categories, each with two subcategories, as follows:

1. Deterministic data signals: (a) steady-state signals; (b) transient signals.
2. Random data signals: (a) stationary signals; (b) nonstationary signals.

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1. Deterministic data signals: (a) steady-state signals; (b) transient signals.
2. Random data signals: (a) stationary signals; (b) nonstationary signals.

### Deterministic Data

Deterministic data signals are those for which it is theoretically feasible to determine a mathematical equation that would predict future time history values of the signal (within reasonable experimental error), based upon a knowledge of the applicable physics or past observations of the signal. The most common type of deterministic signal, called a periodic signal, has a time history  $x(t)$  that exactly repeats itself after a constant time interval  $T_p$ , called the period of the signal; that is,

$$x(t) = x(t \pm T_p) \quad (3.1)$$

The most common sources of periodic acoustical and vibration signals are constant-speed rotating machines, including propellers and fans. Ideally, such signals would have only one dominant frequency, allowing them to be represented by a simple sine wave. However, it is more likely that the periodic source will produce a complex signal that must be described, using a Fourier series representation,<sup>1</sup> by a collection of harmonically related sine waves, as illustrated in Fig. 3.1. In any case, periodic signals are called steady state because their average properties (mean value, mean-square value, and spectrum) do not vary with time.

There are steady-state acoustical and vibration signals that are not rigorously periodic, for example, the data produced by a collection of independent (unsynchronized) periodic sources, such as the propellers on a multiengine propeller airplane. Such nonperiodic steady-state signals are referred to as almost periodic. Most nonperiodic deterministic signals, however, are also not steady state; that is, their average properties change with time. An important type of time-varying

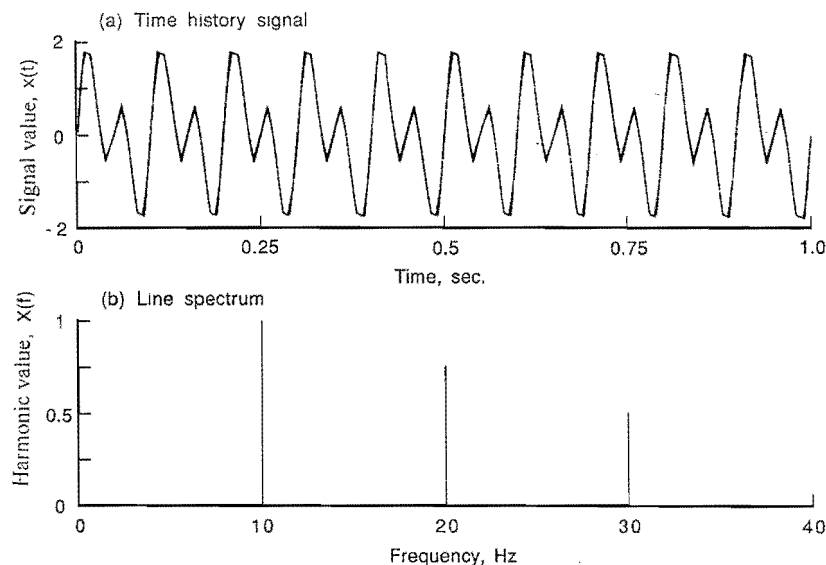


FIGURE 3.1 Time history and line spectrum for periodic signal.

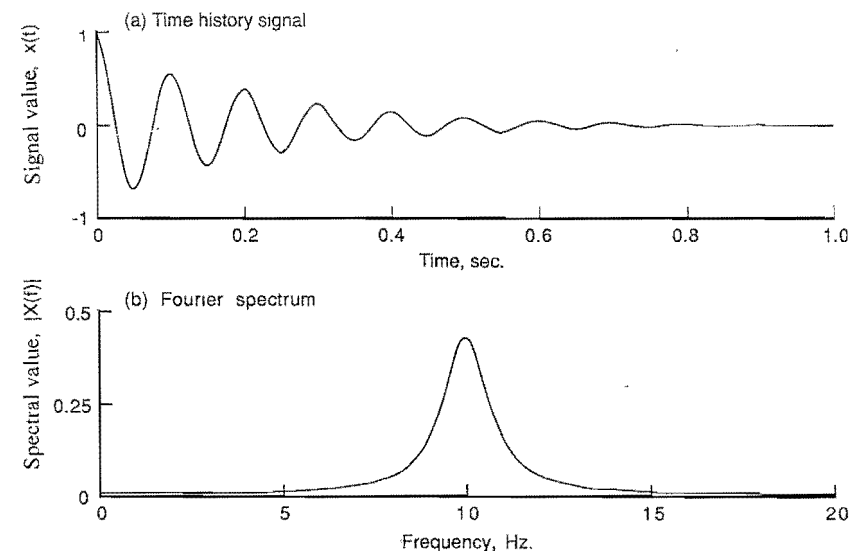


FIGURE 3.2 Time history and Fourier spectrum for deterministic transient signal.

signal is one that begins and ends within a reasonable measurement time interval. Such signals are called transient signals. Examples of deterministic transient signals include well-controlled impacts, sonic booms, and aircraft landing loads. Such data can be described, using a Fourier integral representation,<sup>1</sup> by a continuous spectrum, as illustrated for an exponentially decaying oscillation in Fig. 3.2.

### Random Data

Random acoustical and vibration data signals may be broadly defined as all signals that are not deterministic, that is, where it is not theoretically feasible to predict future time history values based upon a knowledge of the applicable physics or past observations. In some cases, the border between deterministic and random signals may be blurred. For example, the pressure field produced by a high-speed fan at its blade passage rate with a uniform inflow would be deterministic, but turbulence in the inflow would introduce a random property to the pressure signal. In other cases, the data will be more fully random in character (sometimes called "strongly mixed"). Examples include the pressure fields generated by fluid dynamic boundary layers (flow noise), the loads produced by atmospheric turbulence, and the acoustical noise caused by the exhaust gas mixing from an air blower. These sources of acoustical and vibration signals cover a wide frequency range and have totally haphazard time histories, as illustrated in Fig. 3.3.

When the mechanisms producing random acoustical or vibration data are time invariant, the average properties of the resulting signals will also be time invariant. Such random data are said to be stationary. Unlike steady-state deterministic data, stationary random data must be described by a continuous spectrum.

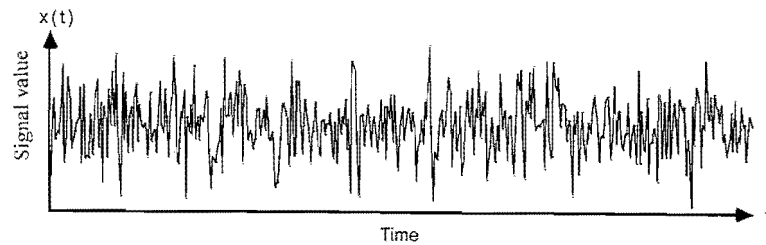


FIGURE 3.3 Time history for stationary random signal.

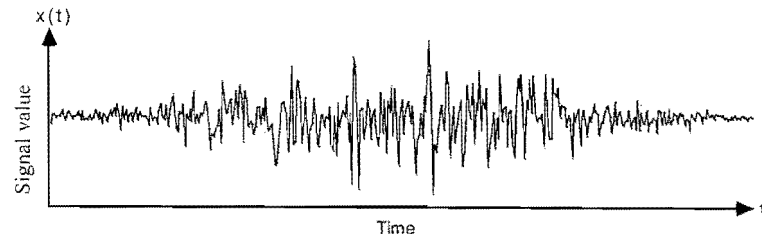


FIGURE 3.4 Time history for nonstationary random signal.

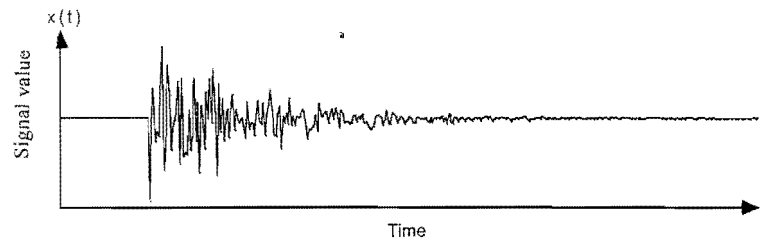


FIGURE 3.5 Time history for stochastic transient signal.

Furthermore, because of the probabilistic character of the data, the measurement of the spectrum and all other signal properties of interest will involve statistical sampling errors that do not occur in the analysis of deterministic signals. These statistical errors will be summarized later.

If the average properties of random signals vary with translations in time, the signals are said to be nonstationary. An illustration of nonstationary data is shown in Fig. 3.4. Although there is a well-developed theoretical methodology for the analysis of arbitrary nonstationary signals,<sup>1</sup> the analysis procedures often require more data than are commonly available and further involve extensive and complex computer calculations. An exception is a special class of nonstationary signals, called stochastic transients, that begin and end within a reasonable measurement time interval. Common sources of stochastic transients are hard impact loads and pyrotechnic devices. An illustration of a stochastic transient signal is shown in Fig. 3.5. Such data can be analyzed by procedures similar to those used

to describe deterministic transients discussed earlier, except now there will be a statistical sampling error problem that must be addressed.

### 3.2 MEAN AND MEAN-SQUARE VALUES

The most rudimentary measures of any steady-state or stationary data signal are the mean value and the mean-square (ms) value (or variance), which provide single-valued descriptions of the central tendency and dispersion of the signal. Mean and ms values of signals can be measured using either digital computations<sup>1</sup> or analog instruments.<sup>2</sup> Hence, appropriate algorithms are presented for both digital and analog analysis procedures.

#### Mean Values

For acoustical and vibration signals, the mean value is commonly zero, simply because most transducers used for acoustical pressure and vibration acceleration measurements do not sense static values. However, if a transducer is used that senses static values and the central tendency of the signal is of interest, the mean value for a time history measurement,  $x(t)$ , of duration  $T_r$  is computed by

$$m_x = \frac{1}{T_r} \int_0^{T_r} x(t) dt \quad (3.2a)$$

For digital data with a sampling interval of  $\Delta t$ ,  $x(t) = x(n \Delta t)$ ,  $n = 1, 2, \dots, N$ , and the mean value is computed by

$$m_x = \frac{1}{N} \sum_{n=1}^N x(n \Delta t) \quad (3.2b)$$

The mean value  $m_x$  in Eq. (3.2) has units of volts and is essentially the quantity computed by a direct current (DC) voltmeter. If the data signal is periodic, the calculation in Eq. (3.2) will be accurate as long as  $T_r$  (or  $N \Delta t$ ) is an integer multiple of the period  $T_p$ . For random data, the calculation will involve a statistical sampling error that is a function of  $T_r$  (or  $N$ ) and the spectral characteristics of the signal,<sup>1</sup> to be discussed later.

#### Mean-Square Values

The ms value of a steady-state or stationary acoustical or vibration signal  $x(t)$  [or a digitized signal  $x(n \Delta t)$ ] is defined by

$$w_x = \frac{1}{T_r} \int_0^{T_r} x^2(t) dt = \frac{1}{N} \sum_{n=1}^N x^2(n \Delta t) \quad (3.3)$$

The ms value  $w_x$  in Eq. (3.3) has the units of volts squared, which is proportional to power or power per unit area, and hence  $w_x$  is often referred to as the overall

"power" or "intensity" of the acoustical or vibration signal. If the mean value of the signal is zero, the ms value in Eq. (3.3) is equal to the variance of  $x(t)$ , defined as

$$s_x^2 = \frac{1}{T_r} \int_0^{T_r} [x(t) - \mu_x]^2 dt = \frac{1}{N} \sum_{n=1}^N [x(n \Delta t) - \mu_x]^2 \quad (3.4)$$

where  $\mu_x$  is the true mean value of the signal ( $m_x$  as  $T_r \rightarrow \infty$ ). The positive square roots of the quantities defined in Eqs. (3.3) and (3.4),  $w_x^{1/2}$  and  $s_x$ , are called the rms value and the standard deviation, respectively, of the signal  $x(t)$ . Again,  $w_x^{1/2} = s_x$  if the mean value of the signal is zero, as will be assumed henceforth. The value  $s_x$  in Eq. (3.4), or  $w_x^{1/2}$  in Eq. (3.3) when  $\mu_x = 0$ , is essentially the quantity measured by a true rms voltmeter (not to be confused with an alternating current (AC) voltmeter that employs a linear rectifier calibrated to measure the correct rms value for a sine wave and reads about 1 dB low when measuring random noise). As for mean-value calculations, the ms value calculation in Eq. (3.3) will be precise for periodic data if  $T_r$  is an integer multiple of the period  $T_p$ . For random data, however, there will be a statistical sampling error that is a function of  $T_r$  (or  $N$ ) and the spectral characteristics of the signal,<sup>1</sup> to be discussed later.

### Weighted Averages

The averaging operation indicated in Eqs. (3.2)–(3.4) is a simple linear sum of prior values over a specific time interval. This type of average, referred to as an unweighted or linear average, is the natural way one would average any set of discrete data values and is the simplest way for a digital computer to calculate an average value. However, some acoustical and vibration data analyses are still performed using analog instruments, where a relatively expensive operational amplifier is required to accomplish an unweighted average. Hence, the averaging operation in analog instruments is commonly weighted so that it can be accomplished using inexpensive passive circuit elements.<sup>2</sup> The most common averaging circuit used by analog instruments is a simple low-pass filter consisting of a series resistor and shunt capacitor (commonly called an RC filter) that produces an exponentially weighted average. For a ms value estimate (assuming the mean value is zero), the exponentially weighted average estimate is given by

$$w_x(t) = \frac{1}{K} \int_0^t x^2(\tau) \exp\left(-\frac{t-\tau}{K}\right) d\tau \quad (3.5)$$

where  $K = RC$  is the product of the numerical values of the resistance in ohms and capacitance in farads used in the averaging circuit. The term  $K$  has units of time (seconds) and is referred to as the *time constant* of the averaging circuit. An exponentially weighted averaging circuit provides a continuous average value estimate versus time, which is based on all past values of the signal. It follows

that after starting the average calculation, a period of time must elapse before the indicated average value is accurate. As a rule of thumb, when averaging a steady-state or stationary signal, at least four time constants ( $t > 4K$ ) must elapse to obtain an average value estimate with an error of less than 2%. The error in question is a bias error. For the analysis of random data signals, there will also be a statistical sampling error, to be discussed later.

### Running Averages

When the acoustical or vibration data of interest have average properties that are time varying (nonstationary data), "running" time averages are often used to describe the data. For the case of a ms value estimate, this could be accomplished by executing Eq. (3.3) repeatedly over short, contiguous time segments of duration  $T \ll T_r$ . Exponentially weighted averaging is particularly convenient for the computation of running averages because it produces a continuous average value estimate versus time. However, a near-continuous estimate can also be generated using an unweighted average (as is more desirable for digital data analysis) by simply recomputing an average value every data-sampling interval  $\Delta t$  rather than only at the end of the averaging time  $T$ . An illustration of a running average for the rms value of typical nonstationary acoustical data measured near an airport during an aircraft flyover is shown in Fig. 3.6 (the measurement in this illustration is a frequency-weighted rms value called *perceived noise level*).

The basic requirement in the computation of a running average is to select an averaging time  $T$  (or averaging time constant  $K$ ) that is short enough not to smooth out the time variations of the data property being measured but long enough to suppress statistical sampling errors in the average value estimate at any time (assuming the data are at least partially random in character). Analytical procedures for selecting the averaging time  $T$  that provides an optimum compromise between the smoothing and statistical sampling errors have been formulated,<sup>1</sup> but trial-and-error procedures coupled with experience will usually provide adequate results. Also, many of the simpler acoustical and vibration

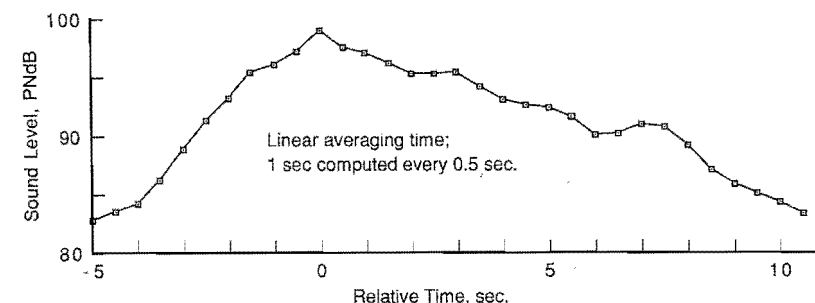


FIGURE 3.6 Running average for weighted rms value of aircraft flyover noise. (Courtesy of Acoustic Analysis Associates, Inc., Canoga Park, CA.)

measurement instruments used in the field have fixed averaging times ("fast" and "slow" averaging circuits) built into the instrument.<sup>2</sup>

### Statistical Sampling Errors

As mentioned earlier, there are no fundamental errors (other than instrument and calibration errors) associated with the calculation of mean and ms values of periodic data signals, assuming the averaging time is an integer multiple of the period of the signal. For random data signals, however, there will be a statistical sampling error due to the fact that the record duration and averaging time can never be long enough to cover all unique signal values. It is convenient to describe this statistical sampling error in terms of a normalized standard deviation of the resulting estimate, called the normalized random error (also called the coefficient of variation). The normalized random error of an estimate,  $\hat{\theta}$ , for a signal property  $\theta$  is defined as

$$\epsilon_r[\hat{\theta}] = \frac{\sigma[\hat{\theta}]}{\hat{\theta}} \quad (3.6)$$

where  $\sigma[\cdot]$  denotes the standard deviation as defined in Eq. (3.4) with  $T_r \rightarrow \infty$  and the hat denotes an estimate. The interpretation of the normalized random error is as follows. If a signal property  $\theta$  is repeatedly estimated with a normalized random error of, say, 0.1, then about two-thirds of the estimates,  $\hat{\theta}$ , will be within  $\pm 10\%$  of the true value of  $\theta$ .

The normalized random errors of the mean, ms, and rms value estimates are summarized in Table 3.1.<sup>1</sup> In these error equations,  $B_s$  is a measure of the spectral bandwidth of the data signal, defined as<sup>1</sup>

$$B_s = \frac{w^2}{\int_0^\infty G^2(f) df} \quad (3.7)$$

where  $w$  is the ms value and  $G(f)$  is the auto (power) spectrum of the data signal, to be addressed later. The error formulas in Table 3.1 are approximations that are valid for  $\epsilon_r \leq 0.20$ . Note in Table 3.1 that the random error for mean-value estimates is dependent on the true mean ( $\mu_x$ ) and standard deviation ( $\sigma_x$ ) of the signal as well as the bandwidth ( $B_s$ ) and averaging time ( $T$  or  $K$ ). However, the

TABLE 3.1 Normalized Random Errors for Mean, ms, and rms Value Estimates

| Signal Property        | Normalized Random Error, $\epsilon_r$    |                                              |
|------------------------|------------------------------------------|----------------------------------------------|
|                        | Linear Averaging with Averaging Time $T$ | RC Weighted Averaging with Time Constant $K$ |
| Mean value, $m_x$      | $\sigma_x / [\mu_x (2B_s T)^{1/2}]$      | $\sigma_x / [\mu_x (4B_s K)^{1/2}]$          |
| ms value, $w_x$        | $1 / (B_s T)^{1/2}$                      | $1 / (2B_s K)^{1/2}$                         |
| rms value, $w_x^{1/2}$ | $1 / (4B_s T)^{1/2}$                     | $1 / (8B_s K)^{1/2}$                         |

random error for ms and rms value estimates is a function only of the bandwidth and averaging time.

### Synchronous Averaging

Periodic vibration and acoustical data signals produced by rotating machinery (including propellers and fans) are sometimes contaminated by additive, extraneous noise such that the measured signal is  $x(t) = p(t) + n(t)$ , where  $p(t)$  is the periodic signal of interest and  $n(t)$  is the noise. In such cases, the signal-to-noise ratio of the periodic signal can be strongly enhanced by the procedure of synchronous averaging,<sup>3</sup> where the data record is divided into a collection of segments  $x_i(t)$ ,  $i = 1, 2, \dots, q$ , each starting at exactly the same phase angle during a period of  $p(t)$ . The collection of segments can then be ensemble averaged to extract  $p(t)$  from the extraneous noise as well as other periodic components that are not harmonically related to  $p(t)$  as follows:

$$p(t) \approx \frac{1}{q} \sum_{i=1}^q x_i(t) \quad (3.8)$$

Synchronous averaging is illustrated in Fig. 3.7 for the pressure field generated in the plane of the propeller on the sidewall of a propeller airplane powered by a reciprocating engine. With 1000 ensemble averages (accomplished in less than 2 min), the procedure extracts the propeller pressure signal cleanly from the engine and boundary layer turbulence noise.

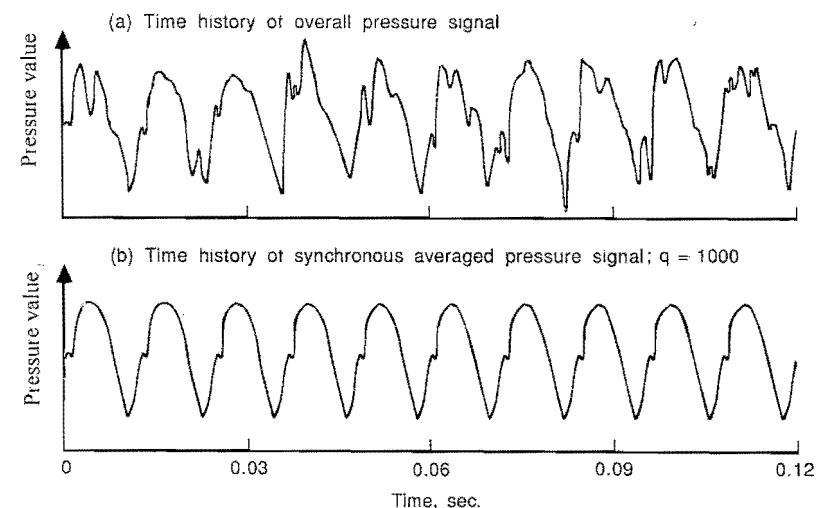


FIGURE 3.7 Original and synchronous averaged time histories for pressure measurement on sidewall of propeller airplane.

The primary requirement for synchronous averaging is the ability to initiate new records at a desired instant during a period of  $p(t)$ . This is most effectively accomplished using a trigger signal that is a noise-free indicator of the phase during each period of  $p(t)$ . For rotating machines, a noise-free trigger signal is commonly obtained using either an optical detector or a magnetic pulse generator on the rotating element of the machine. The time base accuracy of the trigger signal determines the accuracy of the magnitude of the resulting synchronous averaged signal; that is, time base errors in the trigger signal cause a reduction in the indicated signal amplitude with increasing frequency. The signal-to-noise ratio enhancement for the synchronous averaged signal is given in decibels by  $10 \log_{10} q$ , where  $q$  is the number of segments used in the ensemble-averaging operation<sup>3</sup>.

### 3.3 SPECTRAL FUNCTIONS

The ms value of a digital signal constitutes an overall measure of the "power" or "intensity" represented by the measured quantity, but much more useful information is provided by a frequency decomposition of the signal values. As noted in Chapter 1, the computation of rms values in one-third-octave bands is widely used for the frequency analysis of acoustical data and sometimes for vibration data as well. However, the more advanced signal-processing techniques needed for system, source, and path identification problems require the computation of frequency spectra with a much finer resolution than one-third-octave bands. Furthermore, if the data are random in character, a frequency analysis in terms of power quantities per unit frequency (hertz) greatly facilitates the desired evaluations of the data signals.

Prior to 1965, most analysis of acoustical and vibration data signals, including the calculation of frequency spectra, was accomplished by analog instruments. Highly resolved frequency spectra were generally computed using narrow-band width analog filters, often employing mechanical elements such as resonant crystals or magnetostrictive devices.<sup>2</sup> Of course, narrow-bandwidth frequency spectra could also be computed, even at that time, on a digital computer using Fourier transform software, but the computations were time consuming and expensive because of the large number of data values needed to represent wide-bandwidth acoustical and vibration signals. In 1965, this situation changed dramatically with the introduction of an algorithm for the fast computation of Fourier series coefficients<sup>4</sup> that reduced the required computer calculations by several orders of magnitude. Various versions of this algorithm have since come into wide use and are generally referred to as fast Fourier transform (FFT) algorithms. The vast majority of all current narrow-bandwidth spectral analysis of acoustical and vibration data signals are performed using FFT algorithms.

#### The FFT Algorithm

Using lowercase letters for functions of time and uppercase letters for functions of frequency, the Fourier transform of a time history signal,  $x(t)$ , which is measured

over the time interval  $0 \leq t \leq T$ , is defined for all frequencies (both positive and negative) by

$$X(f, T) = \int_0^T x(t) e^{-j2\pi f t} dt \quad (3.9a)$$

In terms of a digital time series of  $N$  data values where  $x(t) = x(n \Delta t)$ ,  $n = 0, 1, \dots, N-1$  (starting the indexing at  $n = 0$  is helpful in maintaining consistent relationships between time and frequency functions), the Fourier transform may be written as

$$X(f, T) = X(k \Delta f, N) = \Delta t \sum_{n=0}^{N-1} x(n \Delta t) \exp(-j2\pi f n \Delta t) \quad (3.9b)$$

where the spectral components are generally complex valued and are defined only at  $N$  discrete frequencies given by

$$f_k = k \Delta f = \frac{k}{N \Delta t} \quad k = 0, 1, 2, \dots, N-1 \quad (3.9c)$$

The finite Fourier transform defined in Eq. (3.9), when divided by  $T$  (or  $N \Delta t$ ), essentially yields the conventional Fourier series coefficients for a periodic function under the assumption that the time history record  $x(t)$  is one period (or an integer multiple of one period) of the periodic function being analyzed. The Fourier components are unique only out to  $k = \frac{1}{2}N$ , that is, out to the frequency  $f_k = 1/(2 \Delta t)$ , commonly called the Nyquist frequency,  $f_N$ , of the digitized signal. The Nyquist frequency is that frequency for which there are only two sample values per cycle and, hence, aliasing will initiate.<sup>1</sup> Comparing the digital result in Eq. (3.9b) with the analog formulation in Eq. (3.9a), the first  $\frac{1}{2}N + 1$  Fourier coefficients, from  $k = 0$  to  $k = \frac{1}{2}N$ , define the spectral components at nonnegative frequencies, while the last  $\frac{1}{2}N - 1$  Fourier coefficients, from  $k = \frac{1}{2}N + 1$  to  $k = N - 1$ , essentially define the spectral components at negative frequencies.

The details of the various FFT algorithms are fully documented in the literature (see refs. 1 and 4 and the signal analysis documents listed in the Bibliography). It is necessary here only to note a few basic characteristics of the most common algorithm used for acoustical and vibration work, which is commonly referred to as the *Cooley-Tukey algorithm* in recognition of the authors of the 1965 paper<sup>4</sup> that initiated the wide use of such algorithms:

1. It is convenient to restrict the number of data values for each FFT to a power of 2, that is,  $N = 2^p$ , where values of  $p = 8$  to  $p = 12$  are commonly used.
2. The fundamental frequency resolution of the Fourier components will be  $\Delta f = 1/(N \Delta t)$ .
3. The Nyquist frequency where aliasing will initiate, denoted by  $f_N$ , occurs at the  $k = \frac{1}{2}N$  Fourier component, that is,  $f_N = 1/(2 \Delta t)$ .

4. The first  $\frac{1}{2}N + 1$  Fourier components up to the Nyquist frequency are related to the last  $\frac{1}{2}N - 1$  components above the Nyquist frequency by  $X(k) = X^*(N - k)$ ,  $k = 0, 1, 2, \dots, N - 1$ , where the asterisk denotes complex conjugate.
5. The Fourier components defined only for positive frequencies (called a one-sided spectrum) are given by  $X(0)$ ,  $X(N/2)$ , and  $2X(k)$ ;  $k = 1, 2, \dots, N/2 - 1$ .

### Line and Fourier Spectral Functions

For deterministic signals that are periodic, a frequency decomposition or spectrum of the signal is directly obtained by computing the Fourier series coefficients of the signal over at least one period of the signal using an FFT algorithm. Assuming the mean value equals zero, the one-sided spectra of Fourier components for a periodic signal,  $p(t)$ , are given by

$$P(f) = \frac{2X(f, T)}{T} \quad f > 0 \text{ or } k = 1, 2, \dots, \frac{N}{2} - 1 \quad (3.10)$$

where  $X(f, T)$  is as defined in Eq. (3.9). The Fourier component magnitudes,  $|P(f)|$ , are usually plotted versus frequency in the form of a discrete-frequency spectrum (often called a *line spectrum*), as illustrated earlier in Fig. 3.1b. Of course, each Fourier component is a complex number that defines a phase as well as a magnitude (the phases for the Fourier components in Fig. 3.1 are all zero). However, the phase information is generally retained only in those applications where there may be a need to reconstruct the signal time history or determine peak values.

**Aliasing.** Because of the aliasing problem inherent in digital spectral analysis,<sup>1</sup> it is important to assure there are no spectral components in the data signal being analyzed above the Nyquist frequency [ $f_N = 1/(2 \Delta t)$ ] for the analysis. This can be guaranteed only when the analog signal is low-pass filtered prior to the digitization to remove any spectral components that may exist in the signal above  $f_N$ . The low-pass filters employed to accomplish this task are commonly referred to as *antialiasing* filters and should always be used for all spectral analysis.

**Leakage Errors and Tapering.** Ideally, in the analysis of periodic data, the spectral computations should be performed over an exact integer multiple of one period to avoid truncation errors in the Fourier series calculation. In practice, however, the computations are often terminated at a time that is convenient for the FFT algorithm and not related to the exact period of the data. The resulting truncation error leads to a phenomenon called side-lobe leakage<sup>1,5</sup> that can severely distort the desired results. To suppress this leakage, it is common to taper the measured time history signal in a manner that forces the values at the start and finish of the measurement to be zero, so as to eliminate the discontinuity between the beginning and ending data values. Numerous tapering functions

(often called “windows”) have been proposed over the years,<sup>5</sup> but one of the earliest and still most widely used is the *cosine-squared* taper (commonly called the *Hanning* window) given by

$$u_h(t) = 1 - \cos^2 \frac{\pi t}{T} \quad 0 \leq t \leq T \quad (3.11)$$

The FFT is then performed on the signal,  $y(t) = x(t)u_h(t)$ , rather than directly on the original measured signal,  $x(t)$ . Leakage suppression can also be accomplished by equivalent operations in the frequency domain.

For deterministic signals that are not periodic and further have a well-defined beginning and end (transients), a spectrum of the signal is directly obtained by computing the Fourier transform of the signal over the entire duration of the signal, again using an FFT algorithm (to obtain a one-sided spectrum, the actual computation is  $2X(k \Delta f)$ ,  $k = 1, 2, \dots, N/2 - 1$ ). The Fourier spectrum magnitude is plotted as a continuous function of frequency, as illustrated previously in Fig. 3.2b. Similar to the spectra for periodic signals, there is also a phase function associated with Fourier spectra, but it is generally retained only if the signal time history is to be reconstructed or peak values are of interest. As long as the FFT computation is performed over a measurement duration that covers the entire duration of the transient event, there is no side-lobe leakage problem in the analysis.

As a final point on transient signal analysis, it should be mentioned that transient data signals, particularly those produced by short-duration mechanical shocks, are often analyzed by a technique called the *shock response spectrum*,<sup>6</sup> which essentially defines the peak response of a hypothetical collection of single-degree-of-freedom mechanical systems to the transient input. The shock response spectrum can be a valuable tool for assessing the damaging potential of mechanical shock loads on equipment but is not particularly useful for noise and vibration reduction applications.

### Auto (Power) Spectral Density Functions

The autospectral density function (also called the “power” spectral density function) provides a convenient and consistent measure of the frequency composition of random data signals. The autospectrum, denoted by  $G_{xx}(f)$ , is most easily visualized as the ms value of the signal passed through a narrow-bandpass filter divided by the filter bandwidth, as illustrated in Fig. 3.8. In equation form,

$$\hat{G}_{xx}(f) = \frac{1}{T \Delta f} \int_0^T x^2(f, \Delta f, t) dt \quad (3.12)$$

where  $x(f, \Delta f, t)$  denotes the signal passed by the narrow-bandpass filter with a center frequency  $f$  and a bandwidth  $\Delta f$ . To obtain the exact autospectral density function, the operations in Fig. 3.8 would theoretically be carried out in the limit as  $T \rightarrow \infty$  and  $\Delta f \rightarrow 0$  such that  $T \Delta f \rightarrow \infty$ . It is clear from Fig. 3.8 and

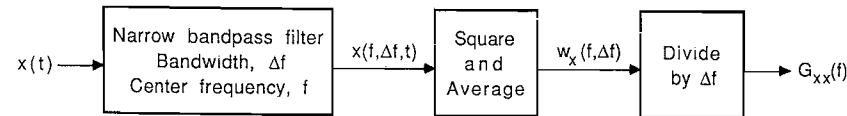


FIGURE 3.8 Autospectral density function measurement by analog filtering operations.

Eq. (3.12) that the units of the autospectral density function are volts squared per hertz.

The operations shown in Fig. 3.8 represent the way autospectra were computed by analog instruments<sup>2</sup> prior to the introduction of FFT algorithms and the transition to digital data analysis procedures. Today, with the ready availability of FFT hardware and software, the autospectral density function (at positive frequencies only) is estimated directly by<sup>1</sup>

$$G_{xx}(f) = \frac{2}{n_d T} \sum_{i=1}^{n_d} |X_i(f, T)|^2 \quad f > 0 \quad (3.13)$$

where  $X_i(f, T)$  is the FFT of  $x(t)$  computed over the  $i$ th data segment of duration  $T$ , as defined in Eq. (3.9), and  $n_d$  is the number of disjoint (statistically independent) data segments used in the calculation. To obtain the exact autospectral density function, the operations in Eq. (3.13) would theoretically be carried out in the limit as  $T \rightarrow \infty$  and  $n_d \rightarrow \infty$ . As will be seen later, the number of averages  $n_d$  determines the random error in the estimate, while the segment duration  $T$  for each FFT computation determines the resolution and, hence, a potential bias error in the estimate. The collection of disjoint data segments needed to estimate a statistically reliable autospectrum is usually created by dividing the total available measurement duration  $T_r$  into a sequence of contiguous segments of duration  $T$ , as illustrated in Fig. 3.9. It follows that  $n_d = T_r/T = \Delta f T_r$ , often referred to as the *BT product* of the estimate. It can be shown<sup>1</sup> that Eq. (3.13) is equal to the result in Eq. (3.12) when the appropriate limits are imposed, that is, when  $T \rightarrow \infty$  and  $\Delta f \rightarrow 0$  in Eq. (3.12) and  $T \rightarrow \infty$  and  $n_d \rightarrow \infty$  in Eq. (3.13). Note that the autospectral density function is always a real number (there is no phase information associated with autospectra).

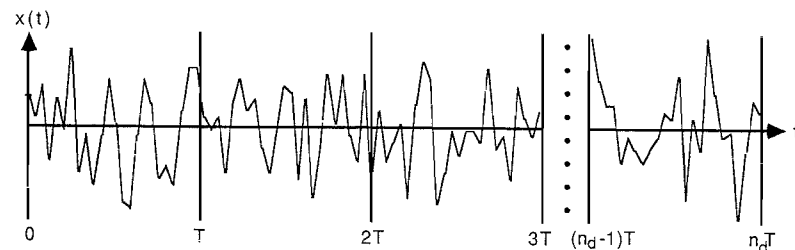


FIGURE 3.9 Subdivision of measured time history into  $n_d$  contiguous segments.

A number of grooming operations are commonly employed to enhance the quality of autospectral density estimates. A few of the more important ones are as follows (see the noted references for details).

**Antialiasing Filters.** As for periodic data analysis, to avoid aliasing, it is important that the random signal being analyzed have no spectral values above the Nyquist frequency,  $f_N = 1/(2 \Delta t)$ . Hence, the analog signal must always be low-pass filtered prior to digitization to suppress any spectral content that may exist above  $f_N$ .<sup>1</sup>

**Tapering Windows.** Since random data signals essentially have an infinite period, there will always be a truncation error associated with the selected segment duration  $T$ . Hence, tapering operations (windows) are commonly used in random signal analysis to suppress the side-lobe leakage problem. Of the numerous available tapering functions,<sup>5</sup> the cosine-squared (Hanning) window defined in Eq. (3.11) is the most widely used.

**Overlapped Processing.** Although tapering operations on segments of the measured signal are desirable to suppress leakage, they also increase the bandwidth of the effective spectral window associated with the analysis.<sup>5</sup> If it is desired to maintain the same spectral window bandwidth with tapering that would have been achieved without tapering, the segment duration  $T$  for the analysis must be increased. However, assuming the total duration of the measurement  $T_r$  is fixed, this will reduce the number of disjoint averages  $n_d$  and increase the random error of the spectral estimates. This increase in random error can be counteracted by computing the spectrum with overlapped segments, rather than contiguous segments.<sup>7,8</sup> A 50% overlap is commonly used in such cases.

**Zoom Transforms.** As discussed earlier, FFT algorithms are usually implemented with a fixed number of data points. Hence once a desired upper frequency limit for an analysis (the Nyquist frequency  $f_N$ ) has been chosen, the resolution of the analysis,  $\Delta f$ , as defined in Eq. (3.9c), is also fixed. Situations often arise when the desired upper frequency limit and frequency resolution are not compatible with the number of data points used by the FFT computation. In these cases, a finer resolution for a given value of  $f_N$  can be achieved using computation techniques referred to as *zoom transform* procedures.<sup>1</sup> The most common zoom transform techniques employ a complex demodulation calculation that essentially segments the frequency range of the signal into contiguous bands that are then analyzed separately.

**Cross-Spectral Density Functions.** The solution of acoustical and vibration control problems involving random processes is often facilitated by the identification of a linear dependence (correlation) between two measurements at different locations. The basic parameter that defines the linear dependence between two measured random signals,  $x(t)$  and  $y(t)$ , as a function of frequency

is the cross-spectral density function, which is estimated (at positive frequencies only) by<sup>1</sup>

$$G_{xy}(f) = \frac{2}{n_d T} \sum_{i=1}^{n_d} X_i^*(f, T) Y_i(f, T) \quad f > 0 \quad (3.14a)$$

where  $X_i(f, T)$  and  $Y_i(f, T)$  are the FFTs of  $x(t)$  and  $y(t)$ , respectively, computed over the  $i$ th simultaneous data segments of duration  $T$ ,  $n_d$  is the number of disjoint records used in the calculation, and the asterisk denotes complex conjugate. As for autospectra, the exact cross-spectral density function would be obtained in the limit as  $T \rightarrow \infty$  and  $n_d \rightarrow \infty$ . All of the computational considerations and grooming procedures discussed for autospectra apply to cross-spectra as well. Unlike the autospectrum, however, the cross-spectrum is generally a complex number that includes both magnitude and phase information and, hence, may be denoted in complex polar notation as

$$G_{xy}(f) = |G_{xy}(f)| e^{j\theta_{xy}(f)} \quad (3.14b)$$

### Coherence Functions

For many applications, it is more convenient to work with a normalized version of the cross-spectral density function, called the coherence function (sometimes called coherency squared), which is defined as<sup>1</sup>

$$\gamma_{xy}^2(f) = \frac{|G_{xy}(f)|^2}{G_{xx}(f)G_{yy}(f)} \quad (3.15)$$

The coherence function is a real-valued quantity bounded by zero and unity, that is,

$$0 \leq \gamma_{xy}^2(f) \leq 1 \quad (3.16)$$

where a value of zero means there is no linear dependence and a value of unity means there is a perfect linear dependence between the signals  $x(t)$  and  $y(t)$  at the frequency  $f$ . A coherence value that is less than unity at one or more frequencies is usually indicative of one of the following situations<sup>9</sup>:

1. Extraneous noise is present in the measurements.
2. The frequency resolution of the spectral estimates is too wide.
3. The system relating  $y(t)$  to  $x(t)$  has time-dependent parameters.
4. The system relating  $y(t)$  to  $x(t)$  is not linear.
5. The output  $y(t)$  is due to other inputs besides  $x(t)$ .

By carefully designing an experiment to minimize the first four possible reasons for a low coherence, the fifth reason provides the basis for a powerful procedure to identify acoustical noise and/or vibration sources. Specifically, if it

is known that a constant-parameter linear system exists between a source and a receiver location, the source signal is measured with an adequate signal-to-noise ratio, and the spectra of the source and receiver signals are estimated with an adequate frequency resolution, then the coherence function defines the fractional portion of the receiver signal autospectral density that is due to the measured source signal. This is the basis for the coherent output power relationship, which is discussed and illustrated in Section 3.5.

### Statistical Sampling Errors

There are no statistical sampling errors associated with the calculation of spectra for periodic signals, assuming the averaging time is an integer multiple of the period of the signal. The same is true of the calculation of Fourier spectra for deterministic transient signals, assuming the averaging time is longer than the transient. The calculation of spectral density quantities for random signals, however, will involve a random sampling error, as discussed previously in Section 3.2. First-order approximations for these random errors in autospectra, cross-spectra, and coherence function estimates are summarized in Table 3.2.<sup>1</sup> The random errors are presented in terms of the normalized random error (coefficient of variation) defined in Eq. (3.6), except for estimates of the cross-spectrum phase where the random error is given in terms of the standard deviation of the estimated phase angle in radians.

Beyond the random errors, there is also a bias error problem in the estimation of spectral density functions that occurs at peaks and valleys in the estimates. This bias error is caused by the finite-resolution bandwidth used for the calculations. For auto- and cross-spectral density magnitude estimates, the bias error is approximated in normalized terms by<sup>1,9</sup>

$$\epsilon_b[\hat{G}(f)] = \frac{b[\hat{G}(f)]}{G(f)} = -\frac{1}{3} \left( \frac{\Delta f}{B_r} \right)^2 \quad (3.17)$$

where  $b[\cdot]$  denotes the bias error incurred by estimating  $G(f)$  by its biased value  $\hat{G}(f)$ ,  $\Delta f$  is the frequency resolution of the analysis, and  $B_r$  is the half-power-point bandwidth of a spectral peak in either  $G_{xx}(f)$  or  $|G_{xy}(f)|$  at that frequency. There is no general bias error equation for coherence function estimates, but error relationships have been formulated for special cases.<sup>10</sup>

TABLE 3.2 Normalized Random Errors for Autospectra, Cross-Spectra, and Coherence Function Estimates

| Signal Property                                 | Normalized Random Error $\epsilon_r$ or Standard Deviation $\sigma_r$     |
|-------------------------------------------------|---------------------------------------------------------------------------|
| Autospectral density function, $G_{xx}(f)$      | $\epsilon_r = 1/n_d^{1/2}$                                                |
| Cross-spectral density magnitude, $ G_{xy}(f) $ | $\epsilon_r = 1/[n_d \gamma_{xy}^2(f)]^{1/2}$                             |
| Cross-spectral density phase, $\theta_{xy}(f)$  | $\sigma_r = [1 - \gamma_{xy}^2(f)]^{1/2} / [2n_d \gamma_{xy}^2(f)]^{1/2}$ |
| Coherence function, $\gamma_{xy}^2(f)$          | $\epsilon_r = [1 - \gamma_{xy}^2(f)] / [0.5n_d \gamma_{xy}^2(f)]^{1/2}$   |

### 3.4 CORRELATION FUNCTIONS

Certain noise and vibration control problems that involve relatively wide bandwidth random data signals are best addressed using time domain signal-processing procedures, as opposed to the frequency domain spectral analysis techniques discussed in the previous section. The basic calculation of interest is the correlation function between two random data signals,  $x(t)$  and  $y(t)$ , which is estimated by

$$R_{xy}(\tau) = \frac{1}{T - \tau} \int_0^{T-\tau} x(t)y(t + \tau) dt \quad (3.18a)$$

where  $\tau$  is a time delay. In digital notation,

$$R_{xy}(r \Delta t) = \frac{1}{N - r} \sum_{n=1}^{N-r} x[n \Delta t]y[(n + r) \Delta t] \quad (3.18b)$$

where  $r$  is a lag number corresponding to a time delay of  $r \Delta t$ . The general quantity estimated in Eq. (3.18) is called the cross-correlation function between the signals  $x(t)$  and  $y(t)$ . For the special case where  $x(t) = y(t)$ ,

$$R_{xx}(\tau) = \frac{1}{T - \tau} \int_0^{T-\tau} x(t)x(t + \tau) dt \quad (3.19)$$

is called the autocorrelation function of  $x(t)$ . Note that for  $\tau = 0$ , the autocorrelation function is simply  $w_x$ , the ms value of the signal. In both Eqs. (3.18) and (3.19), the estimated quantities will become exact in the limit as the averaging time  $T \rightarrow \infty$ . For finite values of  $T$ , there will be a random sampling error in the estimates, to be discussed later.

The correlation function is related to the spectral density function through a Fourier transform,<sup>1</sup>

$$G_{xy}(f) = 2 \int_{-\infty}^{\infty} R_{xy}(\tau) e^{-j2\pi f \tau} d\tau \quad (3.20)$$

Equation (3.20), often called the *Wiener-Khinchine relationship*, is the basis for computing correlation functions in practice. Specifically, the spectral density function is first computed by the FFT procedures outlined in Section 3.3. An inverse Fourier transform of the spectral density function is then computed to obtain the correlation function. Due to the remarkable efficiency of the FFT algorithm, this approach requires substantially fewer calculations than that needed to compute Eq. (3.18b) directly. However, due to the *circular effects* associated with the FFT algorithm, a number of special operations are needed to obtain correct results, as detailed in reference 1.

#### Correlation Coefficient Function

For many applications, it is more convenient to work with the normalized cross-correlation function between  $x(t)$  and  $y(t)$ , called the correlation coefficient

function, which is given by (assuming the mean value is zero)

$$\rho^2(\tau) = \frac{R_{xy}^2(\tau)}{R_{xx}(0)R_{yy}(0)} = \frac{R_{xy}^2(\tau)}{w_x w_y} \quad (3.21)$$

The correlation coefficient function (sometimes called the squared correlation coefficient function) is similar to the coherence function, defined in Section 3.3, in that it is a real-valued quantity bounded by zero and unity, that is,

$$0 \leq \rho_{xy}^2(\tau) \leq 1 \quad (3.22)$$

where a value of zero means there is no linear dependence and a value of unity means there is a perfect linear dependence<sup>1</sup> between  $x(t)$  and  $y(t)$  at the time displacement  $\tau$ . Hence, the correlation coefficient function is interpreted much like the frequency domain coherence function discussed in Section 3.3, except the correlation coefficient function applies to the entire frequency range of the two signals while the coherence function applies to specific frequencies. Also, from Eq. (3.14b), time delay information in the correlation coefficient function is related to the phase information in the cross-spectral density function by

$$\theta(f) = 2\pi f \tau \quad (3.23)$$

Hence, the phase of the cross-spectrum can be valuable for extracting time delay information when the time delay is a function of frequency.<sup>11</sup>

#### Statistical Sampling Errors

When applied to random data signals, the computation of correlation functions will involve a statistical sampling error. In terms of a normalized random error defined in Eq. (3.6), the error in a cross-correlation estimate can be approximated by<sup>1</sup>

$$\epsilon_r[\hat{R}_{xy}(\tau)] = \left( \frac{1 + 1/\rho_{xy}^2(\tau)}{2B_s T_r} \right)^{1/2} \quad (3.24)$$

where  $\hat{R}_{xx}(\tau)$  is an estimate of  $R_{xx}(\tau)$ ,  $T_r$  is the total measurement duration over which the computations are performed, and  $B_s$  is the smallest statistical bandwidth for the two data signals, as defined in Eq. (3.7).

### 3.5 DATA ANALYSIS APPLICATIONS

The applications for signal analysis in noise and vibration studies are extensive and can become quite elaborate.<sup>9</sup> However, as mentioned in the introduction to this chapter, there are three specific application areas of special interest for noise and vibration control problems: (a) the identification of system response properties, (b) the identification of excitation sources, and (c) the identification of transmission paths.

### Identification of System Response Properties

The control of noise and vibration is often facilitated by the determination of gain factors between excitation sources and receiver location responses. The fundamental measurement of interest here is the frequency response function (sometimes called the transfer function) between the two points of interest. Given an excitation source signal  $x(t)$  and a simultaneously measured response signal  $y(t)$ , the frequency response function between the source and receiver signal is given by<sup>1</sup>

$$H_{xy}(f) = \frac{G_{xy}(f)}{G_{xx}(f)} \quad (3.25a)$$

where the auto- and cross-spectral density functions are as defined in Eqs. (3.13) and (3.14), respectively. The frequency response function is generally a complex-valued quantity that is more conveniently expressed in complex polar notation as

$$H_{xy}(f) = |H_{xy}(f)| e^{i\phi_{xy}(f)} \quad (3.25b)$$

where the magnitude function  $|H_{xy}(f)|$  is the gain factor and the argument  $\phi_{xy}(f)$  is the phase factor between  $x(t)$  and  $y(t)$ . In the more advanced applications, such as normal-mode analysis,<sup>12</sup> both the gain and phase factor are needed. In many elementary applications, however, only the gain factor may be of interest.

The normalized random error in frequency response magnitude (gain factor) estimates is approximated by<sup>1</sup>

$$\epsilon_r[|\hat{H}_{xy}(f)|] \approx \frac{[1 - \gamma_{xy}^2(f)]^{1/2}}{[2n_d \gamma_{xy}^2(f)]^{1/2}} \quad (3.26)$$

where  $\hat{H}_{xy}(f)$  is an estimate of  $H_{xy}(f)$ ,  $\gamma_{xy}^2(f)$  is the coherence function between the source and receiver signals, and  $n_d$  is the number of disjoint averages used to compute the autospectra and cross-spectra from which the gain factor is calculated. The random error in frequency response phase estimates is the same as given for the phase of cross-spectral density estimates in Table 3.2.

Like coherence function estimates, the random error in a gain factor estimate approaches zero as the coherence function approaches unity, even for a small number of averages in the spectral density estimates. Hence, if the coherence function is large, the gain factor can be estimated with greater accuracy than the spectral density estimates used in its computation.

There are several sources of bias errors in gain factor estimates,<sup>1,9</sup> but the most significant is due to the frequency resolution bias error in the spectral density functions used to compute the gain factor, as given by Eq. (3.17). As a rule of thumb, if there are at least four spectral components between the half-power points of peaks in the spectral data, that is, if  $\epsilon_b[G(f)] < 0.02$  in Eq. (3.17), then the bias error in the gain factor estimate should be negligible.

As an illustration of the application of gain factor estimates, consider the experiment illustrated in Fig. 3.10, involving two vibration measurements made on a

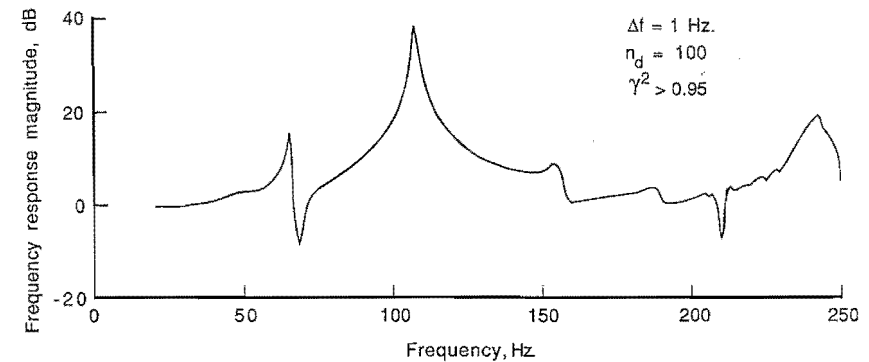


FIGURE 3.10 Gain factor estimate for component in space vehicle payload.

simulated spacecraft payload during a vibration test. One of the measurements is near the mounting point of the payload, and the other is on a critical payload element where vibration may adversely affect the payload performance. The gain factor clearly reveals a frequency region (around 110 Hz) where vibration at the mounting point is greatly magnified at the critical element of concern, due to a strong normal-mode response (resonance) of the payload at this frequency. It follows that efforts to reduce the vibration should be concentrated in this frequency region.

### Identification of Periodic Excitation Sources

The identification of periodic acoustical and vibration excitations can usually be accomplished by a straightforward narrow-bandwidth spectral analysis plus a knowledge of the rpm of all rotating machinery producing the acoustical noise and/or vibration. This is illustrated in Fig. 3.11, which shows the spectrum for the vibration on the floor of a microelectronics manufacturing facility with extensive

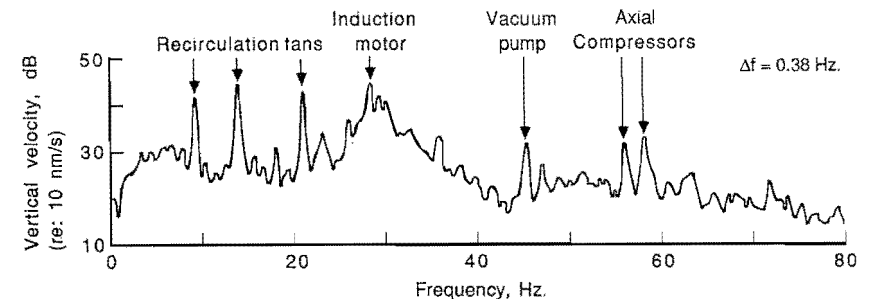


FIGURE 3.11 Fourier spectrum of floor vibration in microelectronics facility. (Courtesy of BBN Laboratories, Inc., Canoga Park, CA.)

air conditioning equipment. It is seen that the more intense spectral peaks in the vibration data can be directly identified with specific rotating machines simply through a knowledge of the rotational frequencies. Of course, this approach cannot separate the vibration contributions of two rotating machines that operate at exactly the same frequency. However, as long as there is some difference in the rotational speed between two machines, say,  $\Delta g$  hertz, the contributions of the machines can theoretically be separated by a spectral analysis with a frequency resolution  $\Delta f$  that is less than  $\Delta g$ . In practice, it is desirable to have

$$\Delta f = \frac{1}{T} < \frac{\Delta g}{3} \quad (3.27)$$

where  $T$  is the segment duration for the spectral computation. Since the signals are periodic or almost periodic, there is no random error associated with the resulting spectral estimates.

### Identification of Random Excitation Sources

When two or more sources of an acoustical and/or vibration environment are random in character and further cover essentially the same frequency range, the identification of the contributions of the individual sources is more difficult. If possible, the contribution of each source of excitation should be identified by turning off all but one of the sources so that the effects of each source can be measured individually. If this is not possible, then the principle of coherent output power might be applied. Specifically, measure the acoustical or vibration signal produced at the source location for each of a collection of  $q$  suspected sources, denoted by  $x_i(t)$ ,  $i = 1, 2, \dots, q$ . For each individual source signal, simultaneously measure the receiver signal  $y(t)$  and compute the coherence function between the  $i$ th source and receiver signals using Eq. (3.15). Under proper conditions, the autospectrum of the receiver signal due solely to the contribution of the  $i$ th source signal is given by

$$G_{y;i}(f) = \gamma_{iy}^2(f) G_{yy}(f) \quad (3.28)$$

where  $G_{y;i}(f)$  reads "the portion of the autospectrum of the receiver signal that is due only to the source signal  $x_i(t)$ ." Equation (3.28) is called the coherent output power relationship. If used properly, it can be a powerful tool for separating the contributions of various possible sources of random excitation in acoustical noise and vibration problems. The primary requirements for the proper application of the coherent output power relationship are as follows<sup>9</sup>:

1. The candidate sources of excitation (or the responses in the immediate vicinity of the sources) must be measured accurately and with negligible measurement noise. However, since the coherence function is dimensionless, any type of transducer (pressure, velocity, acceleration, or displacement) can be used for the source measurements as long as it generates a signal that has a linear relationship with the excitation phenomenon.

2. The candidate sources must be statistically independent, and there must be no interference (crosstalk) in the measurement of any one source due to energy propagating from the other sources; that is, the coherence functions among the measured source signals must all be zero.
3. There must be no significant feedback or nonlinear effects between the candidate source and receiver signals.

The normalized random error for coherent output power measurements is approximated by<sup>1</sup>

$$\epsilon_r[\hat{G}_{y;i}(f)] \approx \frac{[2 - \gamma_{iy}^2(f)]^{1/2}}{[n_d \gamma_{iy}^2(f)]^{1/2}} \quad (3.29)$$

where  $\hat{G}_{y;i}(f)$  is an estimate of  $G_{y;i}(f)$ ,  $\gamma_{iy}^2(f)$  is the coherence function between the  $i$ th source signal and the receiver signal, and  $n_d$  is the number of disjoint averages used to compute the autospectra and cross-spectra from which the coherent output is calculated. It is clear from Eq. (3.29) that the number of disjoint records (and hence the total measurement duration,  $T_r = n_d T$ ) required for an accurate coherent output power calculation will be substantial when  $\gamma_{iy}^2(f) \ll 1$ , as will commonly occur if there are numerous independent sources contributing to  $y(t)$ .

A serious bias error can occur in coherent output power calculations due to time delays between the source and receiver signals that may arise when there is a substantial distance between the source and receiver measurement positions<sup>1,9,10,13</sup> or the measurements are made in a reverberant environment<sup>9,14</sup>. Since the two measurements are usually recorded and analyzed on a common time base, time delays between the source and receiver signals will cause a portion of the received signal to be uncorrelated with the source signal. These time-delay-induced bias errors can be suppressed by the use of precomputation delays or the selection of an appropriately long block duration  $T$  in the data analysis, as detailed in references 9, 13, and 14.

To illustrate the coherent output power calculation, consider the experiment outlined in Fig. 3.12, where a panel section excited by a broadband random vibration source radiates acoustical noise to a receiver microphone. The autospectrum of the radiated noise,  $y(t)$ , as seen by the receiver microphone with no other

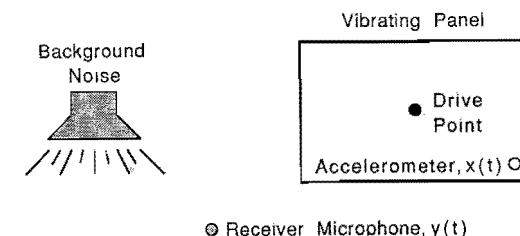


FIGURE 3.12 Acoustical source identification experiment with radiating panel in background noise.

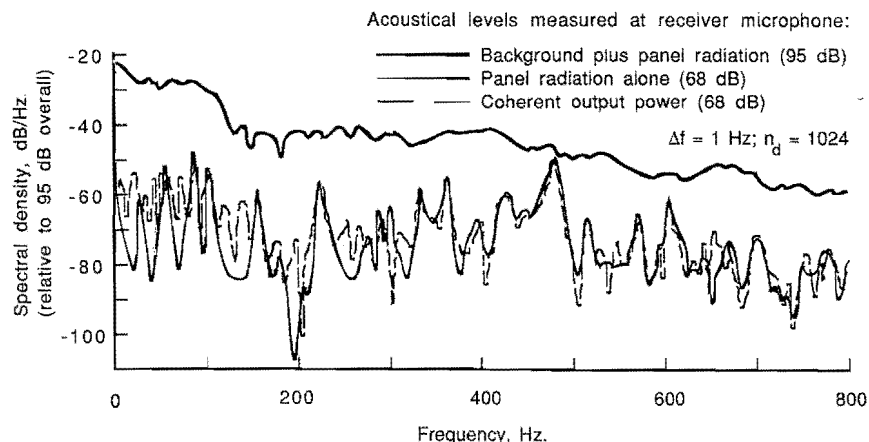


**FIGURE 3.13** Overall and coherent output power spectra for radiating panel in background noise. (From Ref. 9 with the permission of the authors.)

noise sources present, is shown by the thin solid line in Fig. 3.13. Statistically independent background noise is now introduced by a speaker to produce acoustical energy with an overall value that is about 500 times more intense (27 dB higher) than the panel-radiated noise. The autospectrum of the total received microphone signal due to both the panel radiation plus the background noise is also shown in Fig. 3.13, by the heavy solid line. Finally, the coherent output power between the microphone and an accelerometer mounted on the panel is computed with the background noise present. This result is shown by the dashed line in Fig. 3.13. It is seen that the coherent output power calculation extracts the autospectrum of the radiated panel noise, as measured by an accelerometer, from the intense background noise with reasonable accuracy at most frequencies. The reason the procedure works in this example is that the radiated noise from the panel has a linear relationship with the panel motion measured by the accelerometer. Furthermore, since the panel is driven from only one point, the vibration response at any one point on the panel is representative of the vibration at all points.

### Identification of Propagation Paths

Another analysis of great importance in acoustical noise and vibration control problems is the identification of the physical path or paths by which energy from a source of excitation travels to a receiver location. For those cases involving broadband random energy that propagates in a nondispersive manner (with a frequency-independent propagation speed), such as airborne noise, the identification of propagation paths can often be accomplished by a cross-correlation analysis. Specifically, assume a source signal  $x(t)$  propagates in a nondispersive manner through  $r$  paths to produce a receiver signal  $y(t)$ . For simplicity, further

assume the propagation paths have uniform (frequency-independent) gain factors denoted by  $H_i$ ,  $i = 1, 2, \dots, r$ . It follows that

$$y(t) = H_1 x(t - \tau_1) + H_2 x(t - \tau_2) + \dots + H_r x(t - \tau_r) \quad (3.30)$$

where  $\tau_i$ ,  $i = 1, 2, \dots, r$ , are the propagation times through each of the paths. Then, from Eqs. (3.18) and (3.19),

$$R_{xy}(\tau) = H_1 R_{xx}(\tau - \tau_1) + H_2 R_{xx}(\tau - \tau_2) + \dots + H_r R_{xx}(\tau - \tau_r) \quad (3.31)$$

In words, the cross-correlation function between the source and receiver signals will be a series of superimposed autocorrelation functions, each associated with a nondispersive propagation path and centered on a time delay equal to the propagation time along that path. Referring to Eq. (3.20), if the source signal has a wide bandwidth, these autocorrelation peaks will decay rapidly when  $\tau$  deviates from  $\tau_i$  and will be sharply defined in the cross-correlation estimate, as illustrated in Fig. 3.14. Noting that the propagation time for each path is the ratio of distance to propagation speed, the physical path associated with each correlation peak can usually be identified from a knowledge of the length of the path and the propagation speed of the nondispersive waves in the medium forming the path. Finally, the portion of the receiver signal ms value that propagated through a specific path is proportional to the square of the magnitude of the correlation peak associated with that path. The normalized random error associated with the estimate of  $R_{xy}(\tau)$  in Eq. (3.31) is given by Eq. (3.24).

For a cross-correlation analysis to be effective in identifying different nondispersive propagation paths, several requirements must be met, as follows.

1. The source  $x(t)$  must be a broadband random signal.
2. The propagation paths must have reasonably uniform gain factors.
3. The propagation time through each path must be different from all other paths.

As a rule of thumb,<sup>9</sup> the difference in the propagation times through any pair of paths must be  $\Delta t > 1/B_s$ , where  $B_s$  is the spectral bandwidth of the receiver



**FIGURE 3.14** Cross-correlation function between source and receiver signals with two nondispersive propagation paths.

signal, as defined in Eq. (3.7). As the spectral bandwidth becomes small, the peaks in the cross-correlation function spread, and the ability to identify peaks representing individual propagation paths diminishes. There are a number of other signal-processing operations that will enhance the ability to detect individual propagation paths between two signals when the bandwidth is not wide.<sup>9</sup> Also, the use of envelope functions generated by Hilbert transforms can further enhance such detections.<sup>1</sup> Nevertheless, in the limiting narrow-band case where the source signal is a sine wave (or any periodic function), the identification of individual propagation paths cannot be achieved by any signal-processing procedure no matter how large a difference there is between the propagation times through the various paths.

To illustrate this application of cross-correlation analysis to a propagation path identification problem, consider the experiment shown in Fig. 3.15, which involves a speaker that produces acoustical noise with a bandwidth of approximately 8 kHz. Two microphones are used to measure the noise, one located in front of the speaker and another located 0.68 m from the speaker. There is a wall behind the receiver microphone that causes a back reflection producing a second path between the source and receiver microphones with a length of 1.7 m. The computed cross-correlation function between the source and receiver signals is shown in Fig. 3.16. It is seen that two maxima appear in the cross-correlation estimate at 2 and 5 ms. Noting that the speed of sound in air at room

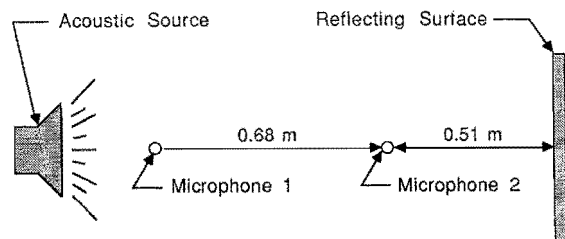


FIGURE 3.15 Acoustical propagation path experiment with back reflection.

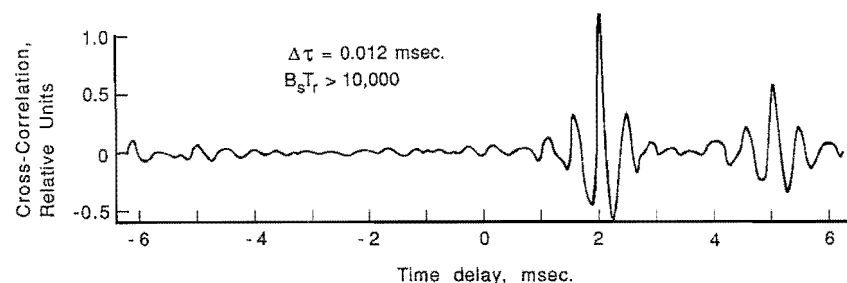


FIGURE 3.16 Cross-correlation function between two microphone signals with back reflection. (From Ref. 9 with the permission of the authors.)

temperature is about 340 m/s, these two peaks clearly identify the direct path and the back reflection. The magnitude of the correlation peak corresponding to the back reflection is only about 40% of the magnitude of peak corresponding to the direct path, as would be expected due to spherical spreading loss; that is, the reflected path is 2.5 times longer than the direct path and, hence, should be about 8 dB lower in level.

The cross-correlation analysis procedure often works well in multipath acoustical problems where the propagation is in the form of longitudinal waves that are nondispersive. In structural vibration problems, there may also be some nondispersive longitudinal wave propagation, but most vibratory energy in structures propagates in the form of flexural waves, which are dispersive,<sup>15</sup> that is, the propagation velocity is a function of frequency. Also, flexural waves strongly reflect and/or scatter at locations where there are changes in either the material properties or the geometry of the structural path. These facts greatly complicate the detection of individual propagation paths in multipath structural vibration problems. Nevertheless, meaningful results can sometimes be obtained through the judicious use of bandwidth-limited cross-correlation analyses<sup>1,9</sup> if the structural paths are reasonably homogeneous.

## REFERENCES

1. J. S. Bendat and A. G. Piersol, *RANDOM DATA: Analysis and Measurement Procedures*, 3rd ed., Wiley, New York, 2000.
2. L. L. Beranek, *Acoustical Measurements*, rev. ed., Acoustical Society of America, Woodbury, NY, 1988.
3. H. Himelblau et al., *Handbook for Dynamic Data Acquisition and Analysis*, IEST-RP-DTE012.1, Institute of Environmental Sciences and Technology, Rolling Meadows, IL, 1994.
4. J. W. Cooley and J. W. Tukey, "An Algorithm for the Machine Calculation of Complex Fourier Series," *Math. Comput.*, **19**, 297-301 (1965).
5. F. J. Harris, "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform," *Proc. IEEE*, **66**(1), 51-83, (1978).
6. S. Rubin, "Concepts in Shock Data Analysis," in C. M. Harris and A. G. Piersol (Eds.), *Harris' Shock and Vibration Handbook*, 5th ed., McGraw-Hill, New York, 2002.
7. P. D. Welch, "The Use of Fast Fourier Transforms for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms," *IEEE Trans. Audio Electroacoust.*, **AU-15**(2), 70-73 (1967).
8. A. H. Nuttall, "Spectral Estimates by Means of Overlapped Fast Fourier Transformed Processing of Windowed Data," NUSC TR-4169, Naval Underwater Systems Center, New London, CT, October 1971.
9. J. S. Bendat and A. G. Piersol, *Engineering Applications of Correlation and Spectral Analysis*, 2nd ed. Wiley, New York, 1993.
10. H. Schmidt, "Resolution Bias Errors in Spectral Density, Frequency Response and Coherence Function Estimates," *J. Sound Vib.*, **101**(3), 347-427 (1985).

11. A. G. Piersol, "Time Delay Estimation Using Phase Data," *IEEE Trans. Acoust. Speech Signal Proc.*, **ASSP-29**(3), 471–477 (1981).
12. R. J. Allemang and D. L. Brown, "Experimental Modal Analysis," in C. M. Harris and A. G. Piersol (Eds.), *Harris' Shock and Vibration Handbook*, 5th ed., McGraw-Hill, New York, 2002.
13. M. W. Trethewey and H. A. Evensen, "Time-Delay Bias Errors in Estimating Frequency Response and Coherence Functions from Windowed Samples of Continuous and Transient Signals," *J. Sound Vib.*, **97**(4), 531–540 (1984).
14. K. Verhulst and J. W. Verheij, "Coherence Measurements in Multi-Delay Systems," *J. Sound Vib.*, **62**(3), 460–463 (1979).
15. L. Cremer, M. Heckl, and E. E. Ungar, *Structure-Borne Sound*, 2nd ed., Springer-Verlag, New York, 1988.

## BIBLIOGRAPHY

- Brigham, E. O., *The Fast Fourier Transform and Its Applications*, Prentice-Hall, Englewood Cliffs, NJ, 1988.
- Crocker, M. J., *Handbook of Acoustics*, Wiley, New York, 1998.
- Doebelin, E. O., *Measurement Systems: Application and Design*, 5th ed., McGraw-Hill, New York, 2004.
- Harris, C. M., *Handbook for Acoustic Measurements and Noise Control*, 3rd ed., McGraw-Hill, New York, 1991.
- Harris, C. M., and A. G. Piersol, *Harris' Shock and Vibration Handbook*, 5th ed., McGraw-Hill, New York, 2002.
- Hassall, E. R., and K. Zaven, *Acoustic Noise Measurements*, 5th ed., Bruel & Kjaer, Naerum, Denmark, 1989.
- Mitra, S. K., and J. F. Kaiser, *Handbook for Digital Signal Processing*, Wiley, New York, 1993.
- Newland, D. E., *Random Vibrations, Spectral and Wavelet Analysis*, 3rd ed., Wiley, New York, 1993.
- Oppenheim, A. V., and R. W. Schaffer, *Discrete-Time Signal Processing*, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1999.
- Smith, S. W., *The Scientist and Engineer's Guide to Digital Signal Processing*, 2nd ed., California Technical Publishing, San Diego, CA, 2002. (Online: <http://www.dspguide.com/pdfbook.htm>)
- Wirsching, P. H., T. L. Paez, and H. Ortiz, *Random Vibrations, Theory and Practice*, Wiley, New York, 1995.
- Wright, C. P., *Applied Measurement Engineering*, Prentice-Hall, Englewood Cliffs, NJ, 1995.

## CHAPTER 4

# Determination of Sound Power Levels and Directivity of Noise Sources

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## 4.1 INTRODUCTION

Noise control can be considered as a system problem, the system containing three major parts: the source, the path, and the receiver.<sup>1</sup>

In any noise control problem, sound energy from the source or sources travels over a multiplicity of paths, both in solid structures and in air, to reach the receiver—an individual, a group of people, a microphone or other instrument, or a structure that is affected by the noise. Three action words are associated with the source–path–receiver model: *emission*, *transmission*, and *immission*. Sound energy that is emitted by a noise source is transmitted to a receiver where it is immitted. Transmission is treated in Chapters 5–7, 10, and 11. Immission is treated in Chapter 19.

Sound pressure level is the physical quantity usually used to describe a sound field quantitatively because the ear responds to sound pressure. A sound-level meter may be used to readily measure the sound pressure level at the location in the sound field occupied by the receiver. Therefore, the preferred descriptor of immission is the sound pressure level in decibels. However, the sound pressure

11. A. G. Piersol, "Time Delay Estimation Using Phase Data," *IEEE Trans. Acoust. Speech Signal Proc.*, **ASSP-29**(3), 471–477 (1981).
12. R. J. Allemang and D. L. Brown, "Experimental Modal Analysis," in C. M. Harris and A. G. Piersol (Eds.), *Harris' Shock and Vibration Handbook*, 5th ed., McGraw-Hill, New York, 2002.
13. M. W. Trethewey and H. A. Evensen, "Time-Delay Bias Errors in Estimating Frequency Response and Coherence Functions from Windowed Samples of Continuous and Transient Signals," *J. Sound Vib.*, **97**(4), 531–540 (1984).
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## BIBLIOGRAPHY

- Brigham, E. O., *The Fast Fourier Transform and Its Applications*, Prentice-Hall, Englewood Cliffs, NJ, 1988.
- Crocker, M. J., *Handbook of Acoustics*, Wiley, New York, 1998.
- Doebelin, E. O., *Measurement Systems: Application and Design*, 5th ed., McGraw-Hill, New York, 2004.
- Harris, C. M., *Handbook for Acoustic Measurements and Noise Control*, 3rd ed., McGraw-Hill, New York, 1991.
- Harris, C. M., and A. G. Piersol, *Harris' Shock and Vibration Handbook*, 5th ed., McGraw-Hill, New York, 2002.
- Hassall, E. R., and K. Zaven, *Acoustic Noise Measurements*, 5th ed., Bruel & Kjaer, Naerum, Denmark, 1989.
- Mitra, S. K., and J. F. Kaiser, *Handbook for Digital Signal Processing*, Wiley, New York, 1993.
- Newland, D. E., *Random Vibrations, Spectral and Wavelet Analysis*, 3rd ed., Wiley, New York, 1993.
- Oppenheim, A. V., and R. W. Schaffer, *Discrete-Time Signal Processing*, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1999.
- Smith, S. W., *The Scientist and Engineer's Guide to Digital Signal Processing*, 2nd ed., California Technical Publishing, San Diego, CA, 2002. (Online: <http://www.dspguide.com/pdfbook.htm>)
- Wirsching, P. H., T. L. Paez, and H. Ortiz, *Random Vibrations, Theory and Practice*, Wiley, New York, 1995.
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Sound pressure level is the physical quantity usually used to describe a sound field quantitatively because the ear responds to sound pressure. A sound-level meter may be used to readily measure the sound pressure level at the location in the sound field occupied by the receiver. Therefore, the preferred descriptor of immission is the sound pressure level in decibels. However, the sound pressure

level by itself is not a satisfactory quantity to describe the strength of a noise source (emission) because the sound pressure level varies with distance from the source and with the acoustical environment in which the source operates.

Two quantities are needed to describe the strength of a noise source, its *sound power level* and its *directivity*. The sound power level is a measure of the total sound power radiated by the source in all directions and is usually stated as a function of frequency, for example, in one-third-octave bands. The sound power level is then the preferred descriptor for the emission of sound energy by noise sources. The sound power level is usually expressed in decibels.\*

The directivity of a source is a measure of the variation in its sound radiation with direction. Directivity is usually stated as a function of angular position around the acoustical center of the source and also as a function of frequency. Some sources radiate sound energy nearly uniformly in all directions. These are called nondirectional sources (see Fig. 4.1). Generally, such sources are small in size compared to the wavelength of the sound radiated. Most practical sources are somewhat directional (see Fig. 4.2); that is, they radiate more sound in some directions than in others. Measures of source directivity are presented in Section 4.12.

From the sound power level and directivity, it is possible to calculate the sound pressure levels produced by the source in the acoustical environment in which it operates. This is not an easy task, however, because the resulting sound

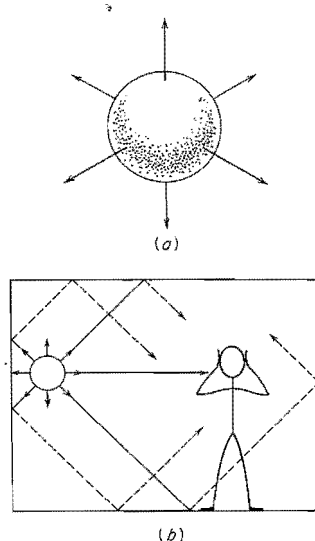


**FIGURE 4.1** Sound source that radiates uniformly in all directions: (a) sound source in free space; (b) same source in enclosure showing reflections from interior surfaces. Solid lines show the direct sound; dashed lines show the reflected (reverberant) sound.

\*Some industries have adopted the use of the bel, where 1 bel = 10 dB, as the unit of sound power level to distinguish clearly between sound pressure level in decibels and sound power level in bels.

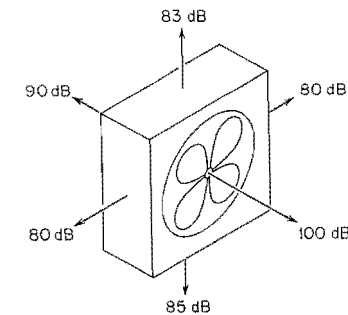


**FIGURE 4.2** Sound source with directive radiation into free space. This behavior is typical of equipment noise.

pressure levels at points in the room depend not only on the characteristics of the source but also on the characteristics of the room itself. The sound energy from the source is reflected, absorbed, and scattered by the room boundaries, and some energy is transmitted through these boundaries to adjacent spaces. Thus, the same source could produce quite different sound pressure levels in different rooms or environments. This underscores the difference between *emission* and *immission* and illustrates why sound pressure level is not a good descriptor for noise emission from a source. Sound fields outdoors and in rooms are discussed in detail in Chapters 5–7.

A source may set a nearby surface into vibration if it is rigidly attached to that surface, causing more sound power to be radiated than if the source were vibration isolated. Both the operating and mounting conditions of the source therefore influence the amount of sound power radiated as well as the directivity of the source. Nonetheless, the sound power level alone is useful for comparing the noise radiated by machines of the same type and size as well as by machines of different types and sizes; determining whether a machine complies with a specified upper limit of noise emission; planning in order to determine the amount of transmission loss or noise control required; and engineering work to assist in developing quiet machinery and equipment.

## 4.2 SOUND POWER LEVELS OF SOURCES

Even though the sound power produced by a noisy machine is only a very small fraction of the total mechanical power that the machine produces, the range of sound powers produced by sources of practical interest is enormous—from less than a microwatt to megawatts. Shaw<sup>2</sup> has estimated the radiation ratio of a wide variety of noise sources.

A single-number descriptor for noise source emission is obtained when the sound power as a function of frequency is weighted using the A-frequency weighting curve. The result is the A-weighted sound power (Table 1.4). The

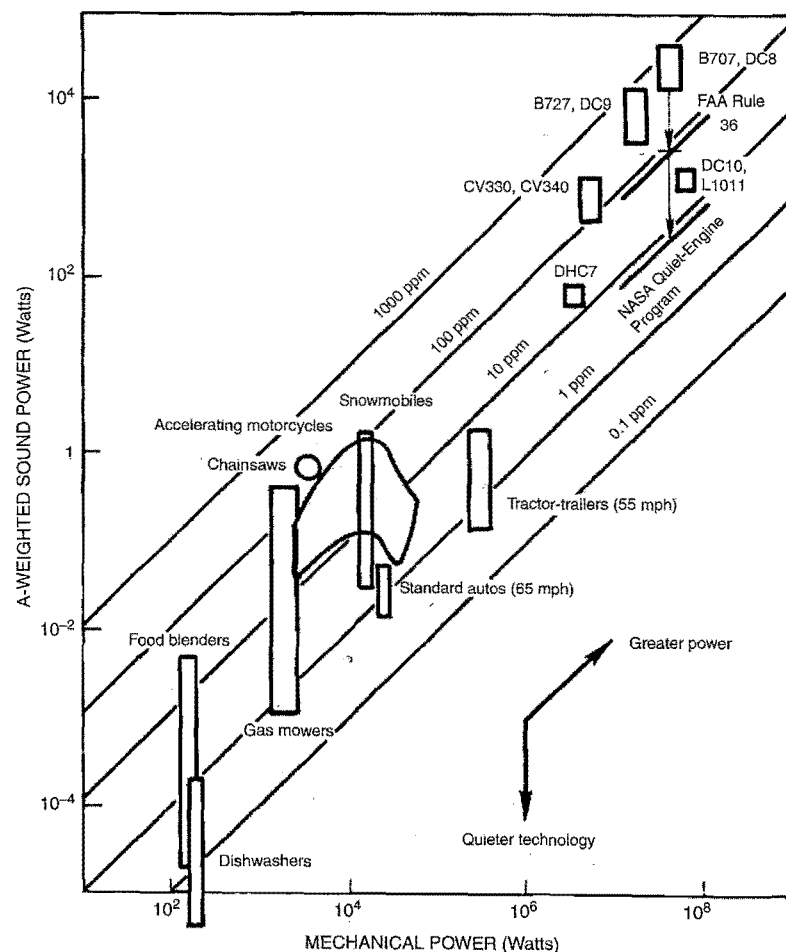


**FIGURE 4.3** Estimated values of A-weighted sound power versus mechanical power for various machines. The diagonal lines are lines of constant mechanoacoustical efficiency (sound power per mechanical power) in parts per million. The line labeled FAA Rule 36 approximates 1975 noise levels for new aircraft designs, while the goal of a research program in progress at the time the figure was published is labeled NASA Quiet-Engine Program.

radiation ratio is the ratio of the A-weighted sound power level of the source and the mechanical power.

Estimates of the conversion ratio are shown in Fig. 4.3. The diagonal lines in the figure give conversion ratios ranging from  $10^{-3}$  to  $10^{-7}$ .

The sound powers of sources of practical interest cover a range of more than 12 orders of magnitude. Hence, it is convenient to express a sound power on a

logarithmic scale using an internationally agreed-upon sound power,  $10^{-12}$  W (watts), as the reference for the logarithm (see Chapter 1). The A-weighted sound power level\* in decibels is defined as

$$L_{WA} = 10 \log \left( \frac{W_A}{W_0} \right) \quad (4.1)$$

where  $W_A$  = A-weighted sound power

$L_{WA}$  = A-weighted sound power level, dB

$W_0$  = reference sound power, internationally agreed upon as  $10^{-12}$  W

Consequently

$$L_{WA} = 10 \log W_A + 120 \text{ dB re } 10^{-12} \text{ W} \quad (4.2)$$

The A-weighted sound power level ( $L_{WA}$ ) is usually expressed in decibels but may be expressed in bels. Since there is an order-of-magnitude difference between sound power levels in bels (emission) and sound pressure levels in decibels (immission), the ambiguity of expressing both emission and immission values in decibels is avoided. Use of this convention is particularly important in dealing with the public. In many countries, people who are unfamiliar with the technical details of acoustics are unable to distinguish between different quantities that are expressed in decibels. Nonetheless, noise control engineers and other practitioners in the field usually find it convenient to express sound power levels in decibels.

**Example 4.1.** A sound source radiates an A-weighted sound power of 3 W. Find the A-weighted sound power level in decibels.

**Solution**

$$\begin{aligned} L_{WA} &= 10 \log 3 + 120 \\ &= 4.8 + 120 = 124.8 \text{ dB re } 10^{-12} \text{ W} \end{aligned}$$

A level is a dimensionless quantity, and it is imperative that the reference be stated to avoid confusion.

### 4.3 RADIATION FIELD OF A SOUND SOURCE

#### Near Field, Far Field, and Reverberant Field

Since the sound power emitted by a source must be determined by measurement of a field quantity such as sound pressure or sound intensity (the sound energy

\*In the past, the A-weighted sound power level was sometimes designated by the term *noise power emission level*, abbreviated NPEL. This usage is no longer common.

flowing through a unit area in a unit time), it is important to understand the radiation field of a sound source when it is placed in various acoustical environments. The character of the radiation field of a typical noise source usually varies with distance from the source. In the vicinity of the source, the particle velocity is not necessarily in the direction of propagation of the sound wave, and an appreciable tangential velocity component may exist at any point. This is the near field. It is characterized by appreciable variations of the sound pressure with distance from the source along a given radius, even when the source is in a free unbounded space commonly referred to as a free field. Moreover, in the near field, the sound intensity is not simply related to the mean-square value of the sound pressure.

The distance from the source to which the near field extends is dependent on the frequency, on a characteristic source dimension, and on the phases of the radiating parts of the surface of the source. The characteristic dimension may vary with frequency and angular orientation. It is difficult, therefore, to establish limits for the near field of an arbitrary source with any degree of accuracy. It is often necessary to explore the sound field experimentally.

In the far field, the sound pressure level decreases by 6 dB for each doubling of the distance from the source, provided that either the source is in free space (no boundaries to reflect the sound) or the reverberant field has not yet been reached (see Fig. 4.4). In this free-field part of the far field, the particle velocity is primarily in the direction of propagation of the sound wave.

If the source is radiating inside an enclosure, fluctuations of sound pressure with position are observed in the reverberant part of the far field, that is, in the region

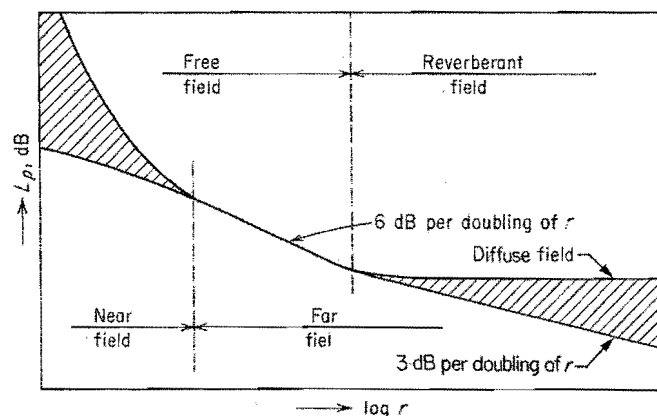


**FIGURE 4.4** Variation of sound pressure level in an enclosure along a radius  $r$  from a typical noise source. The free field, reverberant field, near field, and far field are shown. The free far field indicates the region where the sound pressure level  $L_p$  decreases at the rate of 6 dB for each doubling of distance from the acoustical center of the source, although this region is often very short. In the reverberant far field, the sound pressure level in a highly reverberant room is constant. The lower edge of the shaded region is typical of sound fields in furnished rooms of dwellings and offices.

where the waves reflected from the boundaries of the enclosure are superimposed upon the direct field from the source (see Fig. 4.4). In a highly reverberant room in the region where the direct sound pressure from the source is considerably smaller than the contribution from the reflected sound, the sound pressure level reaches a value essentially independent of the distance from the source. This region may approximate an ideal *diffuse field* in which reflected energy propagates equally in all directions and the sound energy density is uniform.

As discussed in more detail in Chapter 7, in furnished rooms in dwellings and offices where the sound field is neither a free field nor a diffuse field, the sound pressure level decreases by about 3 dB for each doubling of distance from the source.

#### 4.4 SOUND INTENSITY, SOUND POWER, AND SOUND PRESSURE

##### Sound Intensity and Sound Power

The sound energy (in joules,  $1 \text{ J} = 1 \text{ N} \cdot \text{m}$ ) or the sound power (in joules per second or watts) radiated into free space by a source spreads out over a larger and larger area as the sound wave travels outward from the source. As a result, the sound intensity (in watts per square meter) and the sound pressure (in pascals) at a point in the sound field decreases as the distance from the source increases. Sound intensity, defined in Section 2.2 as the amount of sound energy flowing through a unit area per unit time, is a vector quantity  $\mathbf{I}$  having magnitude  $I = |\mathbf{I}|$  and direction  $\hat{\mathbf{r}}$  pointing in the direction of sound energy propagation. If a closed surface that surrounds the source is selected, the sound power radiated by the source can be calculated from the following integral:

$$W = \int_S \mathbf{I} \cdot d\mathbf{S} \quad (4.3)$$

where  $W$  = sound power, W

$\mathbf{I}$  = time-average sound intensity vector,  $\text{W/m}^2$

$d\mathbf{S}$  = infinitesimal element of surface area (a vector oriented normal to surface)

$S$  = area of closed surface that surrounds source

Expanding the dot product in Eq. (4.3) allows the surface integral to be written in terms of scalar quantities as follows:

$$\begin{aligned} W &= \int_S \mathbf{I} \cdot d\mathbf{S} = \int_S (|\mathbf{I}|\hat{\mathbf{r}}) \cdot (|d\mathbf{S}|\hat{\mathbf{n}}) \\ &= \int_S (I dS) \times (\hat{\mathbf{r}} \cdot \hat{\mathbf{n}}) = \int_S I dS \cos \theta = \int_S I_n dS \end{aligned} \quad (4.4)$$

where  $I_n = I \cos \theta$  is the component of sound intensity *normal* to the surface at the location of  $d\mathbf{S}$ ;  $dS$  is the magnitude of the elemental surface area vector;

$\hat{r}$  is the unit vector in the direction of sound propagation;  $\hat{n}$  is the unit vector normal to the surface; and  $\theta$  is the angle between  $\hat{r}$  and  $\hat{n}$ . It should be noted that the maximum value of the intensity is equal to its magnitude,  $I_{\max} = I$ , and is attained in the direction  $\hat{r}$ ; in general, components of the intensity vector, such as  $I_n$  (or components in the Cartesian directions  $\hat{x}$ ,  $\hat{y}$ , or  $\hat{z}$ ) are less than  $I_{\max}$ .

The integral may be carried out over a spherical or hemispherical surface that surrounds the source. Other regular surfaces, such as a parallelepiped or a cylinder, are also used in practice, and, in principle, any closed surface can be used. If the source is nondirectional and the integration is carried out over a spherical surface having a radius  $r$  and centered on the source, sound intensity and sound power are related by

$$I(\text{at } r) = I_n(\text{at } r) = \frac{W}{S} = \frac{W}{4\pi r^2} \quad \text{W/m}^2 \quad (4.5)$$

where  $I$  = magnitude of intensity on the surface (at radius  $r$ )

$I_n$  = normal component of intensity on the surface (at radius  $r$ )

$W$  = sound power, W

$S$  = area of spherical surface,  $= 4\pi r^2$ ,  $\text{m}^2$

$r$  = radius of sphere, m

In general, a source is directional, and the sound intensity is not the same at all points on the surface. Consequently, an approximation must be made to evaluate the integral of Eq. (4.4). It is customary to divide the measurement surface into a number of subsegments each having an area  $S_i$  and to approximate the normal component of the sound intensity on each surface subsegment. The sound power of the source may then be calculated by a summation over all of the surface subsegments:

$$W = \sum_i I_{n_i} S_i \quad \text{W} \quad (4.6)$$

where  $I_{n_i}$  = normal component of sound intensity averaged over  $i$ th segment of area,  $\text{W/m}^2$

$S_i$  =  $i$ th segment of area,  $\text{m}^2$

$i$  = number of segments

Equation (4.6) may be expressed logarithmically as

$$L_W = 10 \log \sum_i S_i \times 10^{L_{I_i}/10} \quad (4.7)$$

where  $L_W$  = sound power level, dB re  $10^{-12}$  W

$S_i$  = area of the  $i$ th segment,  $\text{m}^2$

$L_{I_i}$  = normal sound intensity level averaged over the  $i$ th area segment, dB re  $10^{-12}$   $\text{W/m}^2$  (where the subscript  $n$  has been omitted for simplicity)

When each of the subareas  $S_i$  on the measurement surface has the same area,  $S_\varepsilon$ , Eq. (4.7) reduces to

$$W = \sum_{i=1}^N I_{n_i} S_i = S_\varepsilon \sum_{i=1}^N I_{n_i} = \left( \frac{1}{N} \sum_{i=1}^N I_{n_i} \right) \times (N \times S_\varepsilon) = \bar{I}_n \times S \quad \text{W} \quad (4.8)$$

where  $\bar{I}_n = (1/N) \sum_{i=1}^N I_{n_i}$ , average normal sound intensity over the measurement surface,  $\text{W/m}^2$

$S$  = total area of the measurement surface,  $\text{m}^2$

Equation (4.8) may be expressed logarithmically as

$$L_W = 10 \log \frac{\bar{I}_n}{I_0} + 10 \log \frac{S}{S_0} \quad (4.9)$$

$$L_W = L_I + 10 \log \frac{S}{S_0} \quad (4.10)$$

where  $L_W$  = sound power level, dB re  $10^{-12}$  W

$L_I$  = normal sound intensity level, dB re  $10^{-12}$   $\text{W/m}^2$

$S$  = area of measurement surface,  $\text{m}^2$

$S_0 = 1 \text{ m}^2$

$I_0$  = reference sound intensity, internationally agreed upon as  $10^{-12}$   $\text{W/m}^2$

Equation (4.10) is usually used to determine the sound power level of a source from the sound intensity level except when the source is highly directional. For directional sources, the subareas of the measurement surface may be selected to be unequal and Eq. (4.7) should be used.

### Free-Field Approximation for Sound Intensity

From Eq. (2.24) or (2.30), for the far field of a source radiating into free space, the magnitude of the intensity at a point a distance  $r$  from the source is

$$I(\text{at } r) = \frac{p_{\text{rms}}^2(\text{at } r)}{\rho c} \quad \text{W/m}^2 \quad (4.11)$$

where  $\rho c$  = characteristic resistance of air (see Section 2.5),  $\text{N} \cdot \text{s/m}^3$ , equal to about 406 mks rayls at normal room conditions

$p_{\text{rms}}$  = rms sound pressure at  $r$ ,  $\text{N/m}^2$

Strictly speaking, this relationship is only correct in the far field of a source radiating into free space. Good approximations to free-space, or "free-field," conditions can be achieved in properly designed anechoic or hemi-anechoic rooms, or outdoors. Hence, Eq. (4.11) is approximately correct in the far field of a

source over a reflecting plane provided that the space above the reflecting plane remains essentially a free field at distance  $r$  (see Section 4.3). Even if the free field is not perfect and a small fraction of the sound is reflected from the walls and ceiling of the room, an "environmental correction" may be introduced to allow valid measurements to be taken in the room. Environmental corrections are introduced in Section 4.9. The relations below are widely used in standards that determine the sound power of a source from a measurement of sound pressure level.

If a closed measurement surface is placed around a source so that all points on the surface are in the far field and the intensity vector is assumed to be essentially normal to the surface so that  $I = I_n$  at all points on the surface, then Eqs. (4.7) and (4.11) can be combined to yield

$$W = \frac{1}{\rho c} \sum_i p_i^2 S_i \quad (4.12)$$

where  $p_i$  = average rms sound pressure over area segment  $S_i$ ,  $N/m^2$

We may express Eq. (4.12) logarithmically as

$$L_W = 10 \log \sum_i S_i \times 10^{L_{pi}/10} - 10 \log D \quad (4.13)$$

where  $L_W$  = sound power level, dB re  $10^{-12}$  W  
 $L_{pi}$  = sound pressure level over the  $i$ th area segment, dB re  $2 \times 10^{-5}$  N/m<sup>2</sup>  
 $S_i$  = area of  $i$ th segment, m<sup>2</sup>  
 $10^{L_{pi}/10} = p_i^2 / p_{ref}^2$   
 $p_{ref} = 2 \times 10^{-5}$  N/m<sup>2</sup>  
 $D = \rho c W_0 / p_{ref}^2 = \rho c / 400$

and values for  $10 \log D$  may be found from Fig. 1.7; at normal temperatures and pressures, the  $10 \log D$  term is negligible.

Note that in Eq. (4.13), if the A-weighted sound power level in bels were being computed, the constant 10 in front of the logarithm would disappear, and the result (with  $D = 1$ ) would be

$$L_{WA} = \log \sum_i S_i \times 10^{L_{pAi}/10} \quad (4.14)$$

When each subsegment  $S_i$  has an equal area and  $10 \log D = 0$ , in analogy with Eqs. (4.8)–(4.10), Eq. (4.13) can be expressed as

$$L_W = \langle L_p \rangle_S + 10 \log \left( \frac{S}{S_0} \right) \quad (4.15)$$

where  $L_W$  = sound power level, dB re  $10^{-12}$  W

$\langle L_p \rangle_S$  = sound pressure level averaged on a mean-square basis over the measurement surface (surface sound pressure level, dB re  $2 \times 10^{-5}$  N/m<sup>2</sup>)

$S$  = area of the measurement surface, m<sup>2</sup>

$S_0 = 1$  m<sup>2</sup>

Equation (4.15) is usually used to determine the sound power level of a source from the sound pressure level except when the source is highly directional. For directional sources, the subareas of the measurement surface may be selected to be unequal; Eq. (4.13) should be used (usually with  $10 \log D = 0$ ).

Hence, the sound power level of a source can be computed from sound pressure level measurements made in a free field. Equations (4.13) and (4.15) are widely used in standardized methods for determination of sound power levels in a free field or in a free field over a reflecting plane.

### Sound Power Determination in a Diffuse Field

The sound power level of a source can also be computed from sound-pressure-level measurements made in an enclosure with a diffuse sound field because in such a field the sound energy density is constant; it is directly related to the mean-square sound pressure and, therefore, to the sound power radiated by the source. The sound pressure level in the reverberant room builds up until the total sound power absorbed by the walls of the room is equal to the sound power generated by the source. The sound power is determined by measuring the mean-square sound pressure in the reverberant field. This value is either compared with the mean-square pressure of a source of known sound power output (*comparison method*) or calculated directly from the mean-square pressure produced by the source and a knowledge of the sound-absorptive properties of the reverberant room (*direct method*). Depending on the method used, either Eq. (4.16) or Eq. (4.18) is used to determine the sound power level of the source in a diffuse field.

Diffuse sound fields can be obtained in laboratory reverberation rooms. Sufficiently close engineering approximations to diffuse-field conditions can be obtained in rooms that are fairly reverberant and irregularly shaped. When these environments are not available or when it is not possible to move the noise source under test, other techniques valid for in situ determination of sound power level may be used and are described later in this chapter.

All procedures described in this chapter apply to the determination of sound power levels in octave or one-third-octave bands. The techniques are independent of bandwidth. The A-weighted sound power level is obtained by summing (on a mean-square basis) the octave-band or one-third-octave-band data after applying the appropriate A-weighting corrections. A-weighting values are listed in Table 1.4.

Nonsteady and impulsive noises are difficult to measure under reverberant-field conditions. Measurements on such noise sources should be made either under free-field conditions or using one of the techniques described in Chapter 18.

#### 4.5 MEASUREMENT ENVIRONMENTS

Three different types of laboratory environments in which noise sources are measured are found in modern laboratories: anechoic rooms (free field), hemi-anechoic rooms (free field over a reflecting plane), and reverberation rooms (diffuse field). In an anechoic room, all of the boundaries are highly absorbent, and the free-field region extends very nearly to the boundaries of the room. Because the "floor" itself is absorptive, anechoic rooms usually require a suspended wire grid or other mechanism to support the sound source, test personnel, and measurement instruments. A hemi-anechoic room has a hard, reflective floor, but all other boundaries are highly absorbent. Both anechoic and hemi-anechoic environments are used to determine the sound power level of a source, but the hemi-anechoic room is clearly more practical for testing large, heavy sources. The sound power level is derived from Eq. (4.7) if the sound intensity on a surface surrounding the source is measured directly or from Eq. (4.13) if the sound pressure level is measured on a surface surrounding the source and located in the far field.

In a reverberation room, where all boundaries are acoustically hard and reflective, the reverberant field extends throughout the volume of the room except for a small region in the vicinity of the source. The sound power level of a source may be determined from an estimate of the average sound pressure level in the diffuse-field region of the room coupled with a knowledge of the absorptive properties of the boundaries.

The sound pressure field in an ordinary room such as an office or laboratory space that has not been designed for acoustical measurements is neither a free field nor a diffuse field. Here the relationship between the sound intensity and the mean-square pressure is more complicated. Instead of measuring the mean-square pressure, it is usually more advantageous to use a sound intensity analyzer that measures the sound intensity directly (see Section 4.10). By sampling the sound intensity at defined locations in the vicinity of the source, the sound power level of the source can be determined. Equation (4.10) is used to determine the sound power levels from the sound intensity levels. If the subareas of the measurement surface are unequal, Eq. (4.7) may be used.

#### 4.6 INTERNATIONAL STANDARDS FOR DETERMINATION OF SOUND POWER USING SOUND PRESSURE

##### ISO Standards

The International Organization for Standardization (ISO) has published a series of international standards, the ISO 3740 series,<sup>3-13</sup> which describes several methods for determining the sound power levels of noise sources. Table 4.1 summarizes the applicability of each of the basic standards of the ISO 3740 series. The most important factor in selecting an appropriate noise measurement method is the ultimate use of the sound-power-level data that are to be obtained.

TABLE 4.1 Description of the ISO 3740 Series of Standards for Determination of Sound Power

| ISO Standard                           | Test Environment                                  | Criteria for Suitability of Test Environment                                                           | Volume of Source                                            | Character of Noise                                    | Limitation on Background Noise              | Sound Power Levels Obtainable                      | Optional Information Available                                                                               |
|----------------------------------------|---------------------------------------------------|--------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------|---------------------------------------------|----------------------------------------------------|--------------------------------------------------------------------------------------------------------------|
| ISO 3741<br>Precision<br>(Grade 1)     | Reverberation room meeting specified requirements | Room volume and reverberation time to be qualified                                                     | Preferably less than 2% of room volume                      | Steady, broadband, narrow-band, or discrete frequency | $\Delta L \geq 10$ dB,<br>$K_1 \leq 0.5$ dB | A weighted and in one-third-octave or octave bands | Other frequency-weighted sound power levels                                                                  |
| ISO 3743-1<br>Engineering<br>(Grade 2) | Hard-walled room                                  | Volume $\geq 40$ m <sup>3</sup> ,<br>$\alpha \leq 0.20$                                                | Preferably less than 1% of room volume                      | Any, but no isolated bursts                           | $\Delta L \geq 6$ dB,<br>$K_1 \leq 1.3$ dB  | A weighted and in octave bands                     | Other frequency-weighted sound power levels                                                                  |
| ISO 3743-2<br>Engineering<br>(Grade 2) | Special reverberation room                        | $70 \text{ m}^3 \leq V \leq 300 \text{ m}^3$ , $0.5 \text{ s} \leq T_{\text{room}} \leq 1.0 \text{ s}$ | Preferably less than 1% of room volume                      | Any, but no isolated bursts                           | $\Delta L \geq 4$ dB,<br>$K_1 \leq 2.0$ dB  | A weighted and in octave bands                     | Other frequency-weighted sound power levels                                                                  |
| ISO 3744<br>Engineering<br>(Grade 2)   | Essentially free field over a reflecting plane    | $K_2 \leq 2$ dB                                                                                        | No restrictions; limited only by available test environment | Any                                                   | $\Delta L \geq 6$ dB,<br>$K_1 \leq 1.3$ dB  | A weighted and in one-third-octave or octave bands | Source directivity; SPL as a function of time; single-event SPL; other frequency-weighted sound power levels |

(continued overleaf)

TABLE 4.1 (continued)

| ISO Standard                                  | Test Environment                                                                    | Criteria for Suitability of Test Environment                                        | Volume of Source                                                         | Character of Noise                                    | Limitation on Background Noise              | Sound Power Levels Obtainable                      | Optional Information Available              |
|-----------------------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|--------------------------------------------------------------------------|-------------------------------------------------------|---------------------------------------------|----------------------------------------------------|---------------------------------------------|
| ISO 3745 Precision (Grade 1)                  | Anechoic or hemi-anechoic room                                                      | Specified requirements; measurement surface must lie wholly inside qualified region | Characteristic dimension not greater than 1/2 measurement surface radius | Any                                                   | $\Delta L \geq 10$ dB,<br>$K_1 \leq 0.5$ dB | A weighted and in one-third-octave or octave bands | Same as above plus sound energy level       |
| ISO 3746 Survey (Grade 3)                     | No special test environment                                                         | $K_2 \leq 7$ dB                                                                     | No restrictions; limited only by available test environment              | Any                                                   | $\Delta L \geq 3$ dB,<br>$K_1 \leq 3$ dB    | A weighted                                         | Sound pressure levels as a function of time |
| ISO 3747 Engineering or Survey (Grade 2 or 3) | Essentially reverberant field in situ, subject to stated qualification requirements | Specified requirements                                                              | No restrictions; limited only by available test environment              | Steady, broadband, narrow-band, or discrete frequency | $\Delta L \geq 6$ dB,<br>$K_1 \leq 1.3$ dB  | A weighted from octave bands                       | Sound pressure levels as a function of time |

Notes:  $\Delta L$  is the difference between the sound pressure levels of the source-plus-background noise and the background noise alone;  $K_1$  is the correction for background noise, defined in the associated standard;  $V$  is the test room volume;  $\alpha$  is the sound absorption coefficient;  $T_{60}$  is the nominal reverberation time for the test room, defined in the associated standard;  $K_2$  is the environmental correction, defined in the associated standard; SPL is an abbreviation for "sound pressure level."

The principal uses of sound-power-level data include the development of quieter machines and equipment, compliance testing of products in production, acoustical comparisons of several products that may be of the same or different type and size, and preparing acoustical noise declarations for the general public.

In making a decision on the appropriate measurement method to be used, several factors should be considered: (a) the size of the noise source, (b) the movability of the noise source, (c) the test environments available for the measurements, (d) the character of the noise emitted by the noise source, and (e) the grade (classification) of accuracy required for the measurements. The methods described in this chapter are consistent with those of the ISO 3740 series. A set of standards with the same objectives is available from the ANSI or the Acoustical Society of America (ASA).

## 4.7 DETERMINATION OF SOUND POWER IN A DIFFUSE FIELD

### Characteristics of Reverberation Rooms

Measurements of the sound power level of a device or machine may be performed in a laboratory reverberation room (see Section 7.8). The determination of the sound power level of a noise source in such a room is based on the premise that the measurements are performed entirely in the diffuse (reverberant) sound field (see Fig. 4.4). In the reverberant field, the average sound pressure level is essentially uniform, although there are fluctuations from point to point, and it is related to the sound power radiated by the source [see (Eq. 4.18)]. Information in regard to the directivity of the source cannot be obtained in a diffuse sound field.

The minimum volume of the room depends on how low in frequency valid measurements are to be taken. For example, if measurements in the 100-Hz one-third-octave band are desired, the minimum volume is recommended to be 200 m<sup>3</sup> in ISO 3741. On the other hand, if measurements are only needed down to the 200-Hz band, 70 m<sup>3</sup> will be adequate. Maximum room volume is constrained by the adverse effects of air absorption and the ability to make valid high-frequency measurements; a volume less than 300 m<sup>3</sup> is generally recommended. The equipment being tested should have a volume no greater than 2% of the room volume. The absorption coefficient of the surface that is closest to the equipment being evaluated (usually the floor) should not exceed 0.06, and the remaining surfaces of the room should be highly reflective, such that the reverberation time (in seconds) is greater than the ratio of volume  $V$  (m<sup>3</sup>) to surface area  $S$  (m<sup>2</sup>):  $T_{60} > V/S$ . In addition to these requirements on room volume and absorption, ISO 3741 includes requirements for background noise levels; temperature, humidity, and atmospheric pressure; instrumentation and calibration; installation and operation of the source under test; minimum distance between microphone and source; and performance requirements for the reference sound source, if used.

If the room is to be used to measure equipment that has discrete-frequency components in its noise emissions, it is often necessary to use additional microphone positions and additional source positions, and ISO 3741 includes

detailed procedures to determine whether or not such positions are required. These procedures must be followed for each source tested, or, alternatively, the reverberation room itself can be "qualified" for the measurement of discrete-frequency components (see below). The need for additional source positions may be reduced by adding low-frequency sound absorption to the room or through the use of rotating diffusers.

### Room Qualification

The ISO 3741 standard contains two detailed procedures for the qualification of reverberation rooms for the determination of sound power levels, one for qualifying the room for the measurement of broadband sound (Annex E) and one for qualifying the room for the measurement of discrete-frequency components (Annex A). For sources of broadband sound, the room is qualified by using a reference sound source placed at different positions in the room and determining the standard deviation of the measured space-averaged sound pressure levels in the room for each position. A minimum of six positions of the source are required, each with specified constraints on the distance between them, the distance from walls, and the distance from the microphone. The room is qualified according to ISO 3741 for the measurement of broadband sound if the standard deviation does not exceed the values given in Table 4.2.

The qualification of reverberation rooms for the measurement of noise that contains discrete-frequency components in the spectrum is complicated and time consuming. However, this only has to be performed once, and the benefit can be great. If the chamber is qualified for the measurement of discrete tones, there is no longer a need to perform initial tests to determine the number of microphone and source positions for each source under investigation.

Essentially, a "calibrated" loudspeaker is placed in the reverberation room and is driven by a series of discrete-frequency tones in each one-third-octave band. For example, there are 22 frequencies spaced 1 H apart in the 100-Hz one-third-octave band and 23 frequencies spaced 5 H apart in the 500-Hz octave band. The average sound pressure level in the room is determined at each frequency and corrected for the loudspeaker response (previously determined in a hemi-anechoic

**TABLE 4.2 Qualification Requirements for Reverberation Room Used for Measurement of Broadband Noise Sources**

| Octave-Band Center Frequencies, Hz | One-Third-Octave-Band Center Frequencies, Hz | Maximum Allowable Standard Deviation, dB |
|------------------------------------|----------------------------------------------|------------------------------------------|
| 125                                | 100–160                                      | 1.5                                      |
| 250, 500                           | 200–630                                      | 1.0                                      |
| 1000, 2000                         | 800–2500                                     | 0.5                                      |
| 4000, 8000                         | 3150–10,000                                  | 1.0                                      |

Source: ISO 3741: 1999.

Note: Annex A of ISO 3741 contains detailed procedures for the discrete-frequency qualification.

**TABLE 4.3 Qualification Requirements for Reverberation Room Used for Measurement of Narrowband Noise Sources**

| Octave Band Center Frequency, Hz | One-Third-Octave-Band Center Frequency, Hz | Maximum Allowable Standard Deviation, dB |
|----------------------------------|--------------------------------------------|------------------------------------------|
| 125                              | 100–160                                    | 3.0                                      |
| 250                              | 200–315                                    | 2.0                                      |
| 500                              | 400–630                                    | 1.5                                      |
| 1000, 2000                       | 800–2500                                   | 1.0                                      |

room). The room is qualified according to ISO 3741 if the standard deviation of the level in each one-third-octave band does not exceed the values given in Table 4.3.

### Experimental Setup

An array of fixed microphone positions or a single microphone that traverses a path (often circular) in the reverberant room may be used to determine the average sound pressure level in the reverberant field. The number of fixed microphone positions,  $N_M$ , required depends on the results of an initial series of sound-pressure-level measurements using six positions. If the standard deviation of these initial measurements,  $s_M$ , is less than or equal to 1.5, then the original six microphone positions will suffice (i.e., the noise is essentially broadband). If  $s_M > 1.5$ , it is assumed that the source emits discrete tones and a larger number of microphone positions is usually required to obtain an adequate sampling of the sound field. In this case,  $N_M$  could range anywhere from 6 to 30 depending on the frequency and the magnitude of  $s_M$ . When a traversing microphone is used, the path length  $l$  must be at least  $(\lambda/2)N_M$ , where  $\lambda$  is the wavelength of sound at the lowest midband frequency of interest. The microphone path or array should be positioned in the room so that no microphone position is within a minimum distance  $d_{\min}$  of the equipment being evaluated. The distance  $d_{\min}$  is determined differently in the comparison method and the direct method for determination of sound power. These requirements are discussed below.

If the noise source under test is typically associated with a hard floor, wall, edge, or corner, it should be placed in a corresponding position in the reverberation room. Otherwise, it should be placed no closer than 1.5 m from any wall of the room. The source should not be placed near the geometric center of the room since in that location many of the resonant modes of the room would not be excited. For rectangular reverberation rooms, the source should be placed asymmetrically relative to the boundaries. ISO 3741 gives further information for special source locations and installation conditions.

Near the boundaries of the room and close to other reflecting surfaces such as stationary or rotating diffusers, the sound field will depart from the ideal state

of diffusion. ISO 3741 prescribes the following conditions on the microphone positions used during the measurements:

When using fixed microphone positions:

1. No position shall be closer than 1.0 m from any surface of or within the room.
2. No position shall be closer than  $d_{\min}$  from the source (defined below).
3. The distance between microphone positions shall be at least  $\lambda/2$ , where  $\lambda$  is the wavelength of sound at the lowest midband frequency of interest.

When using one or more continuous microphone traverses:

1. No point on the traverse shall be closer than 1.0 m from any surface of the room.
2. No point on the traverse shall be closer than 0.5 m from any surface of a rotating diffuser.
3. No position shall be closer than  $d_{\min}$  from the source (defined below).
4. The microphone traverse should not lie in any plane within  $10^\circ$  of a room surface.
5. The length of the traverse shall be at least  $l \geq 3\lambda$ , where  $\lambda$  is the wavelength of sound at the lowest midband frequency of interest.
6. If multiple traverses are used, the minimum distance between their paths shall be at least  $\lambda/2$ .

### Comparison Method

The procedure for determining the sound power level of a noise source by the comparison method<sup>6</sup> requires the use of a reference sound source (see Fig. 4.5 and ref. 13) of known sound power output. Using microphone positions that meet the above requirements, the procedure is essentially as follows:

1. With the equipment being evaluated at a suitable location in the room, determine, in each frequency band, the average sound pressure level (on a mean-square basis) in the reverberant field using the microphone array or traverse described above.
2. Replace the source under test with the reference sound source and repeat the measurement to obtain the average level for the reference sound source.

The sound power level of the source under test,  $L_W$ , for a given frequency band is calculated as

$$L_W = L_{Wr} + (\langle L_p \rangle - \langle L_p \rangle_r) \quad (4.16)$$

where  $L_W$  = one-third-octave-band sound power level for source being evaluated, dB re  $10^{-12}$  W

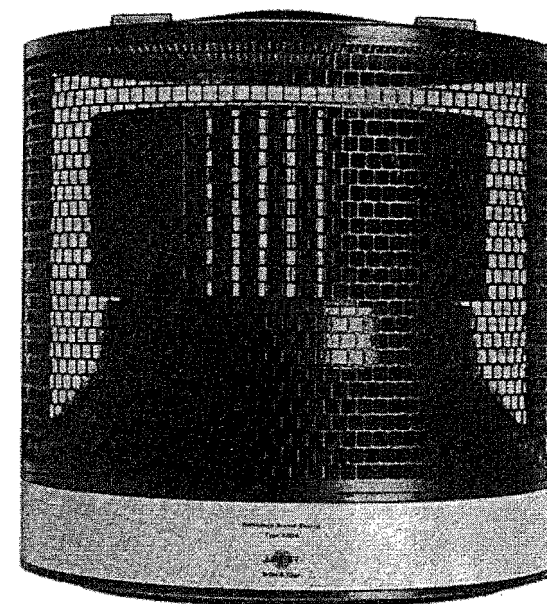


FIGURE 4.5 Reference sound source, Brüel and Kjær Type 4204. (Courtesy of Brüel and Kjær, Inc.)

$\langle L_p \rangle$  = space-averaged one-third-octave-band sound pressure level of source being evaluated, dB re  $2 \times 10^{-5}$  N/m<sup>2</sup>

$L_{Wr}$  = calibrated one-third-octave-band sound power level of reference source, dB re  $10^{-12}$  W

$\langle L_p \rangle_r$  = space-averaged one-third-octave-band sound pressure level of reference sound source, dB re  $2 \times 10^{-5}$  N/m<sup>2</sup>

To make certain that the reverberant sound field predominates in the determination of the average sound pressure level, the minimum distance  $d_{\min}$  between the microphone(s) and the equipment being evaluated should be at least

$$d_{\min} = 0.4 \times 10^{(L_{Wr} - L_{pr})/20} \quad (4.17)$$

where  $L_{Wr}$  and  $L_{pr}$  are as defined for Eq. (4.16). [Note that although Eq. (4.17) states the actual requirement, ISO 3741 also recommends that the constant be 0.8 instead of 0.4 to ensure that the microphone is in the reverberant field.]

### Direct Method

The direct method does not use a reference sound source. Instead, this method requires that the sound-absorptive properties of the room be determined by

measuring the reverberation time in the room for each frequency band. Measurement of  $T_{60}$  is described in ISO 3741.

With this method, the space-averaged sound pressure level for each frequency band of the source being evaluated is determined as described above for the comparison method. The sound power level of the source is found from<sup>4</sup>

$$L_W = \bar{L}_p + \left\{ 10 \log \frac{A}{A_0} + 4.34 \frac{A}{S} + 10 \log \left( 1 + \frac{S \times c}{8 \times V \times f} \right) - 25 \log \left[ \frac{427}{400} \sqrt{\frac{273}{273 + \theta}} \times \frac{B}{B_0} \right] - 6 \right\} \text{ dB re } 10^{-12} \text{ W} \quad (4.18)$$

where  $L_W$  = band sound power level of sound source under test

$\bar{L}_p$  = band space-averaged sound pressure level of sound source under test, dB re  $2 \times 10^{-5}$  N/m<sup>2</sup>

$A$  = equivalent absorption area of the room, =  $(55.26/c)(V/T_{\text{rev}})$ , m<sup>2</sup>

$V$  = room volume, m<sup>3</sup>

$T_{\text{rev}}$  = reverberation time for particular band

$A_0$  = reference absorption area, 1 m<sup>2</sup>

$S$  = total surface area of room, m<sup>2</sup>

$V$  = room volume, m<sup>3</sup>

$f$  = midband frequency of measurement, Hz

$c$  = speed of sound at temperature  $\theta$ , =  $20.05\sqrt{273 + \theta}$ , m/s

$\theta$  = temperature, °C

$B$  = atmospheric pressure, Pa

$B_0$  =  $1.013 \times 10^5$  Pa

$V_0$  = 1 m<sup>3</sup>

$T_0$  = 1 s

To make certain that the reverberant sound field predominates in the determination of the average sound pressure level, the minimum distance  $d_{\text{min}}$  between the microphone(s) and the equipment being evaluated should be at least<sup>4</sup>

$$d_{\text{min}} = 0.08 \sqrt{\frac{V/V_0}{T/T_0}} \text{ m} \quad (4.19)$$

where  $V$  and  $T_{\text{rev}}$  are as defined above. [Note that although Eq. (4.19) states the actual requirement, ISO 3741 also recommends that the constant be 0.16 instead of 0.08 to ensure that the microphone is in the reverberant field.]

**Example 4.2.** Assume a room at temperature 21.4°C with a volume of 200 m<sup>3</sup>, a surface area  $S = 210$  m<sup>2</sup>, and a reverberation time at 100 Hz of 3 s. The space-averaged sound pressure level  $\langle L_p \rangle$  in the diffuse field with a given machine

operating is 100 dB. Find the sound power level for this machine. Assume a discrete-frequency spectrum and an atmospheric pressure of 1000 mbars.

**Solution** Use Eq. (4.18) to determine the sound power. The wavelength corresponding to 100 Hz is 3.44 m:

$$c = 20.05\sqrt{273 + 21.4} = 344 \text{ m/s}$$

$$A = \frac{55.26}{344} \left( \frac{200}{3} \right) = 10.7 \text{ m}^2$$

The sound power level is

$$L_W = 100 + \left\{ 10 \log \frac{10.7}{1} + 4.34 \frac{10.7}{210} + 10 \log \left( 1 + \frac{210 \times 344}{8 \times 200 \times 100} \right) - 25 \log \left[ \frac{427}{400} \sqrt{\frac{273}{273 + 21.4}} \right] - 6 \right\}$$

$$= 100 + (10.3 + 0.22 + 1.62 - 0.3 - 6)$$

$$= 106.4 \text{ dB re } 10^{-12} \text{ W (100 Hz mean frequency)}$$

#### 4.8 DETERMINATION OF SOUND POWER IN A FREE FIELD USING SOUND PRESSURE MEASUREMENTS

Determination of the sound power level produced by a device or a machine may be performed in a laboratory anechoic or hemi-anechoic room. Alternatively, a hemi-anechoic environment may be provided at an open-air site above a paved area, distant from reflecting surfaces such as buildings, and with a low background noise level. This environment approximates a large room with sound-absorptive treatment on ceilings and walls with the equipment under test mounted on the hard, reflecting floor. Detailed information on anechoic and hemi-anechoic rooms is presented in Section 7.9.

The determination of the sound power level radiated in an anechoic or a hemi-anechoic environment is based on the premise that the reverberant field is negligible at the positions of measurement for the frequency range of interest. Thus, the total radiated sound power may be obtained by a spatial integration, over a hypothetical surface that surrounds the source, of the component of sound intensity normal to the surface of the source [see Eq. (4.3)]. When all points of the measurement surface are in the far field of the source, the magnitude of the intensity can be assumed to be equal to  $p^2/\rho c$  [see Eq. (4.11)]. With the additional assumption that the magnitude of the intensity is equal to the normal component of sound intensity (i.e., the direction of sound propagation is essentially normal to the surface at the measurement points), the relationships given in Eqs. (4.12) and

(4.13) can be used to determine the sound power level of a source from simple measurements of sound pressure level. This is the basis for two key international standards for the determination of sound power levels of noise sources, ISO 3744<sup>8</sup> and ISO 3745,<sup>9</sup> discussed below. Essentially, a measurement surface is chosen and microphone positions are defined over this surface. Sound-pressure-level measurements are taken at each microphone position for each frequency band, and from these, the sound power levels are computed.

### Selection of a Measurement Surface

The international standards discussed below allow a variety of measurement surfaces to be used; some are discussed here. In selecting the shape of the measurement surface to be used for a particular source, an attempt should be made to choose one where the direction of sound propagation is approximately normal to the surface at the various measurement points. For example, for small sources that approximate a point source, the selection of a spherical or hemispherical surface may be the most appropriate—the direction of sound propagation will be essentially normal to this surface. For machines in a hemi-anechoic environment that are large and in the shape of a box, the parallelepiped measurement surface may be preferable. For “tall” machines in a hemi-anechoic environment having a height much greater than the length and depth, a cylindrical measurement surface<sup>14,15</sup> may be the most appropriate.

### Measurement in Hemi-Anechoic Space

The sound power determination in a hemi-anechoic space may be performed according to ISO 3744<sup>8</sup> for engineering-grade accuracy or according to ISO 3745<sup>9</sup> for precision-grade accuracy. ISO 3744 is strictly for hemi-anechoic environments, while ISO 3745 includes requirements for both hemi-anechoic and fully anechoic environments. These standards specify requirements for the measurement surfaces and locations of microphones, procedures for measuring the sound pressure levels and applying certain corrections, and the method for computing the sound power levels from the surface-average sound pressure levels. In addition, they provide detailed information and requirements on criteria for the adequacy of the test environment and background noise, calibration of instrumentation, installation and operation of the source, and information to be reported. Several annexes in each standard include information on measurement uncertainty and the qualification of the test rooms.

In terms of allowable measurement surfaces, ISO 3744 currently specifies the hemisphere and the parallelepiped,\* while ISO 3745 explicitly defines only the hemisphere and the sphere. However, ISO 3745 includes a clause for “other microphone arrangements” and cites as an example papers describing the cylindrical measurement surface.<sup>14,15</sup> If a hemisphere is used to determine the location

\*At the time of this writing, a revision to ISO 3744 was under consideration, which included the cylindrical measurement surface as well.

of the microphone positions, it has its center on the reflecting plane beneath the acoustical center of the sound source. To ensure that the measurements are carried out in the far field, the radius of the hemisphere for ISO 3744 measurements should be equal to at least two “characteristic” source dimensions and generally not less than 1.0 m. For ISO 3745, the requirements are slightly more stringent with the radius being at least twice the *largest* source dimension or three times the distance of the acoustical center of the source from the reflecting plane (whichever is larger) and at least  $\lambda/4$  of the lowest frequency of interest. No microphone position can lie outside of the region qualified for measurements (the free-field region), and this requirement generally prevents microphones from being located too close to the walls. Outdoors, atmospheric effects are likely to influence the measurements if the radius of the test hemisphere is much greater than about 15 m, even in favorable weather.

Figure 4.6 gives an array of 20 microphone positions that may be used for measurements according to ISO 3744<sup>8</sup> for sources that emit predominantly broadband sound. The concern when measuring in a hemi-anechoic room is that reflections of sound from the hard floor may cause far-field interference at the microphone positions. This becomes more problematic when the noise emissions contain discrete tones; a small change in the position of the microphone may result in large variations in measured sound pressure level in those bands containing the tones. Therefore, for sources that emit discrete tones, ISO 3744 specifies a different microphone array, one having a greater distribution in the vertical direction. For precision measurements made in a hemi-anechoic environment according to ISO 3745,<sup>9</sup> a microphone array of at least 20 positions, each with a different vertical height, is required. The coordinates of this array are given in Table 4.4. Procedures are specified in both standards for determining whether or not the number of microphones is sufficient and for defining additional positions, if required. Note that the simultaneous deployment of a very large number of microphones may result in a situation where the support structures of these microphones become effective scatterers of the direct sound at high frequencies and can introduce substantial errors. Scanning the sound field with a single microphone can eliminate this error.

A parallelepiped array of microphone positions is frequently used for the determination of sound power levels for box-shaped machines and equipment. Detailed requirements for the selection of microphone positions and the criteria for the sufficiency of the number of microphones are given in ISO 3744.<sup>8</sup> A minimum of nine positions is required, but this number rapidly increases as the size of the source under test increases. The basic nine-position parallelepiped arrangement is illustrated in Fig. 4.7. As can be seen, this array is limited in its sampling in the vertical direction and so should be used with caution if the source is directional or emits discrete tones.

Another convenient measurement surface standardized in at least one industry test code<sup>16</sup> is the cylindrical microphone array illustrated in Fig. 4.8. The use of this array facilitates the measurement of tall-aspect-ratio sound sources such as data-processing equipment installed in racks. Since the array is usually

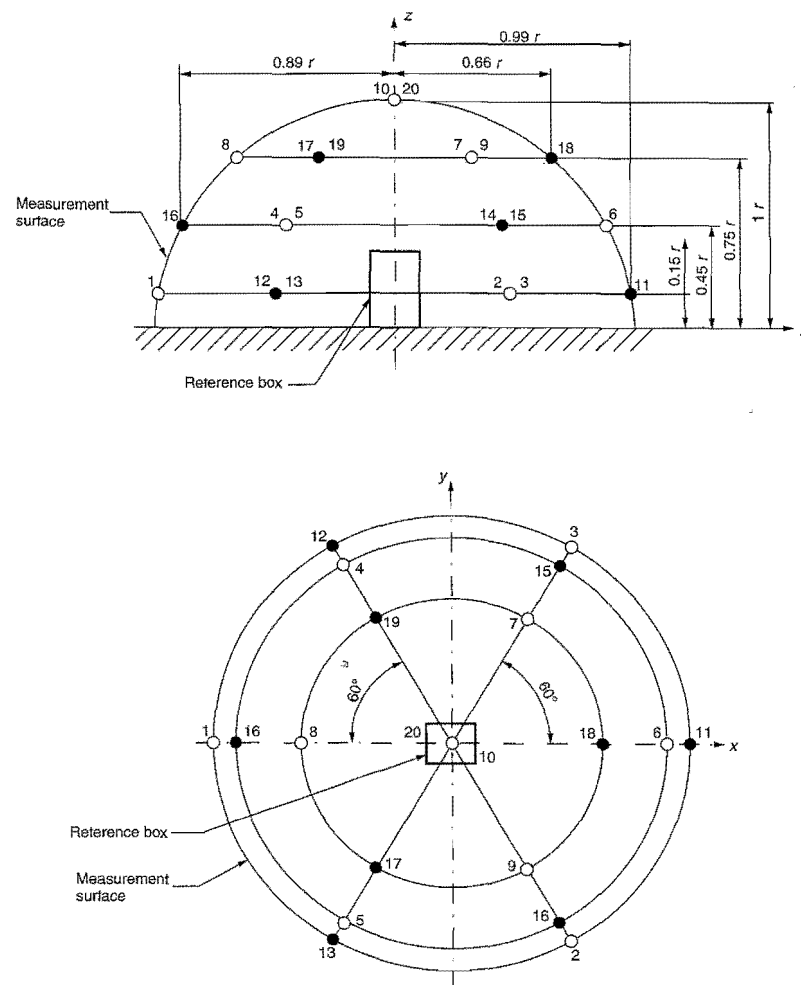


**FIGURE 4.6** Microphone positions for a hemispherical measurement surface according to ISO CD 3744 (N1497). (Courtesy of the International Organization for Standardization, Geneva, Switzerland.)

implemented using continuously traversing microphones and a sufficient number of vertical heights, the accuracy is generally improved over that of the parallelepiped.<sup>14,15</sup>

The general procedure for determining the sound power level of the source according to either ISO 3744 or ISO 3745 can be summarized as follows:

1. The test room is set up and checked out and all environmental conditions are recorded.

**TABLE 4.4** Twenty Microphone Positions on Surface of Hemisphere As Defined in ISO 3745

| Position Number | $x/r$ | $y/r$ | $z/r$ |
|-----------------|-------|-------|-------|
| 1               | -1.00 | 0     | 0.025 |
| 2               | 0.50  | -0.86 | 0.075 |
| 3               | 0.50  | 0.86  | 0.125 |
| 4               | -0.49 | 0.85  | 0.175 |
| 5               | -0.49 | -0.84 | 0.225 |
| 6               | 0.96  | 0     | 0.275 |
| 7               | 0.47  | 0.82  | 0.325 |
| 8               | -0.93 | 0     | 0.375 |
| 9               | 0.45  | -0.78 | 0.425 |
| 10              | 0.88  | 0     | 0.475 |
| 11              | -0.43 | 0.74  | 0.525 |
| 12              | -0.41 | -0.71 | 0.575 |
| 13              | 0.39  | -0.68 | 0.625 |
| 14              | 0.37  | 0.64  | 0.675 |
| 15              | -0.69 | 0     | 0.725 |
| 16              | -0.32 | -0.55 | 0.775 |
| 17              | 0.57  | 0     | 0.825 |
| 18              | -0.24 | 0.42  | 0.875 |
| 19              | -0.38 | 0     | 0.925 |
| 20              | 0.11  | -0.19 | 0.975 |

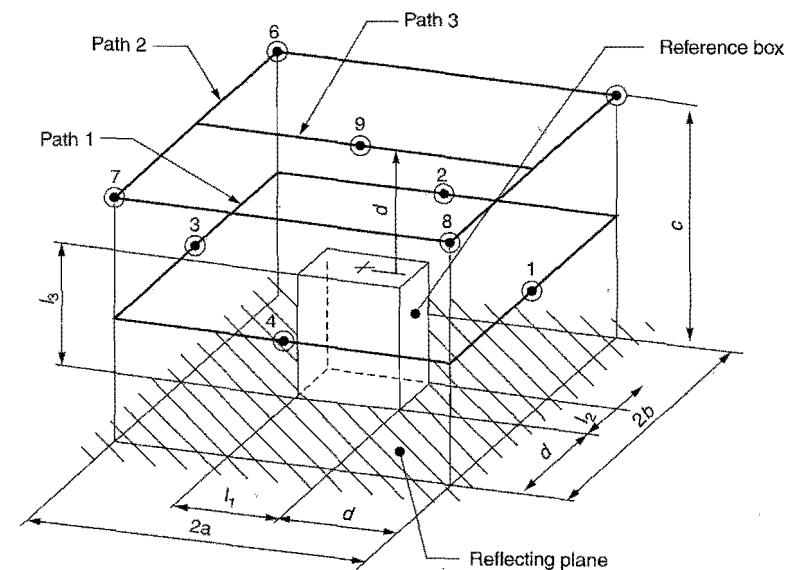


**FIGURE 4.7** Microphone positions for a rectangular measurement surface according to ISO CD 3744 (N1497). (Courtesy of the International Organization for Standardization, Geneva, Switzerland.)

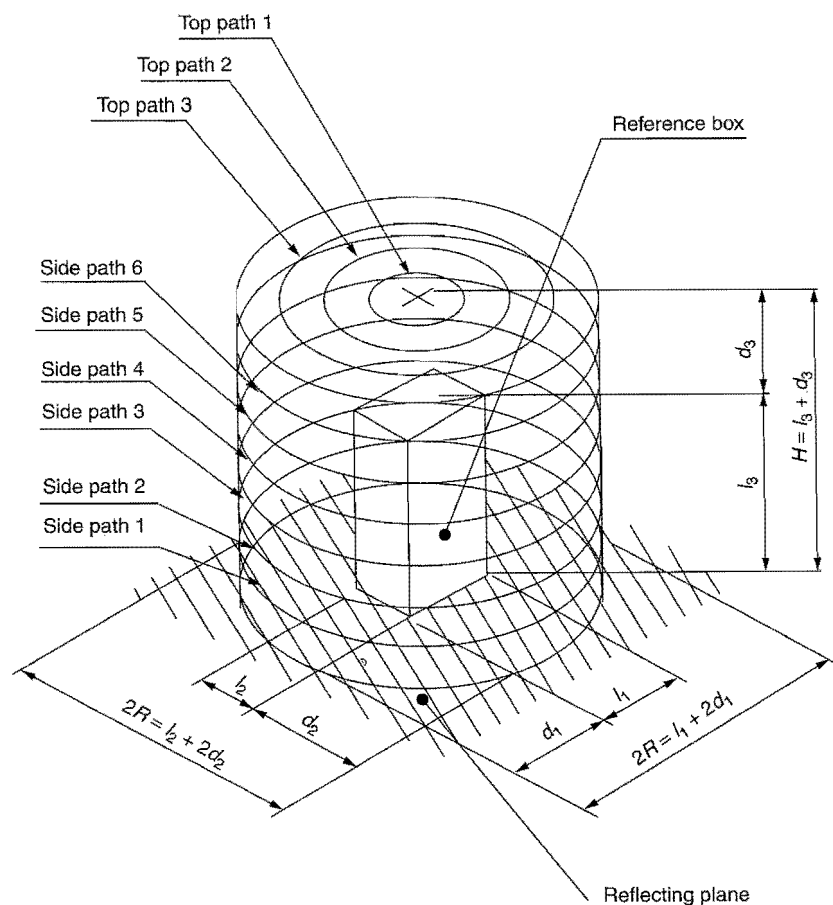


**FIGURE 4.8** Microphone positions for a cylindrical measurement surface according to ISO CD 3744 (N1497). (From ISO CD 9997: 1999 Amended, Revision of Annex B (N1479). Courtesy of the International Organization for Standardization, Geneva, Switzerland.)

2. The microphones and instrumentation are calibrated.
3. The sound source is installed.
4. The measurement surface is selected and the microphones are set up in their proper positions.
5. The sound source under test is operated under specified conditions and sound pressure levels are measured at each microphone position for each frequency band of interest.
6. The sound source is turned off and the background noise levels are measured.

7. The sound pressure data are corrected for background noise and for environmental conditions (see below), if necessary.
8. The surface average band sound pressure levels are calculated.
9. The band sound power levels are calculated from the latter, taking into account corrections for meteorological conditions.

Equation (4.15) is used to determine the sound power level of the source when the subareas of the measurement surface associated with each microphone position are all equal. For highly directional sources, the standards require to increase the number of subareas of the measurement surface in the region of high directivity and to measure the sound pressure levels associated with each subarea. The sound power level can then be determined by using Eq. (4.13) for unequal subareas. The correction for background noise, denoted  $K_1$ , is specified in terms of the difference between the sound pressure levels measured with and without the source running,  $\Delta L_i$ , as

$$K_1 = -10 \log(1 - 10^{-0.1\Delta L_i}) \text{ dB} \quad (4.20)$$

### Measurement in Anechoic Space

In certain instances, the sound power level of a noise source must be determined in a totally free field, without the influence of a reflecting plane beneath the source. The reflecting plane not only causes constructive and destructive interference patterns in the far field—especially pronounced when the noise emissions contain discrete frequency components—but also may affect the radiated sound power of the source itself, especially at low frequencies.<sup>17</sup> Thus for critical measurements of tonal sources, measurements of noise sources that in normal use are not mounted over a hard surface, measurements of directivity, and other specialized measurements, a fully anechoic test environment may be desirable. In this case, the space-averaged mean-square sound pressure level is generally determined over a spherical measurement surface, and measurements are made according to ISO 3745.<sup>9</sup> Most of the procedures and requirements discussed above for hemi-anechoic measurements apply here also, with the exception of the measurement surfaces.

An array of at least 20 microphone positions is specified in ISO 3745, defined over the spherical surface by the Cartesian coordinates given in Table 4.5. If requirements on the sufficiency of the number of microphones given in ISO 3745 are not met, then the number of positions must be doubled, to 40. If the sound source is highly directional, the number of subareas in the regions of high directivity should be increased, as mentioned above. When measurements are made in an anechoic space according to ISO 3745, the sound power level of each frequency band is computed according to either Eq. (4.15) for equal subareas or Eq. (4.13) for unequal subareas. For the precision measurements of ISO 3745, however, two constants are included (instead of the single constant  $D$ ), one to

correct for meteorological conditions and one to "normalize" the measurements to specified "reference" meteorological conditions.

#### 4.9 ENVIRONMENTAL CORRECTIONS AND DETERMINATION OF SOUND POWER LEVEL

In the previous sections, the only corrections applied for the determination of the average sound pressure level on the measurement surface were for the presence of background noise. However, if reflections from the room surfaces affect the measured surface sound pressure levels in any of the frequency bands of interest, a second correction may be required to account for these reflections. This so-called environmental correction, denoted  $K_2$ , is computed for both A-weighted values and individual frequency bands and is subtracted directly from the measured sound pressure level (after corrections for background noise are applied). The ISO 3744 standard<sup>8</sup> generally limits the environmental correction to a maximum of 2 dB. The correction may be determined in several ways:

1. By comparing the calibrated sound power level of a reference source,  $L_{Wr}$ , with the measured sound power level of the same source in the room,  $L_W$ . The environmental correction is then computed as  $K_2 = L_W - L_{Wr}$ .

TABLE 4.5 Twenty Microphone Positions on Sphere As Defined in ISO 3745

| Position Number | $x/r$ | $y/r$ | $z/r$ |
|-----------------|-------|-------|-------|
| 1               | -1.00 | 0     | 0.05  |
| 2               | 0.49  | -0.86 | 0.15  |
| 3               | 0.48  | 0.84  | 0.25  |
| 4               | -0.47 | 0.81  | 0.35  |
| 5               | -0.45 | -0.77 | 0.45  |
| 6               | 0.84  | 0     | 0.55  |
| 7               | 0.38  | 0.66  | 0.65  |
| 8               | -0.66 | 0     | 0.75  |
| 9               | 0.26  | -0.46 | 0.85  |
| 10              | 0.31  | 0     | 0.95  |
| 11              | 1.00  | 0     | -0.05 |
| 12              | -0.49 | 0.86  | -0.15 |
| 13              | -0.48 | -0.84 | -0.25 |
| 14              | 0.47  | -0.81 | -0.35 |
| 15              | 0.45  | 0.77  | -0.45 |
| 16              | -0.84 | 0     | -0.55 |
| 17              | -0.38 | -0.66 | -0.65 |
| 18              | 0.66  | 0     | -0.75 |
| 19              | -0.26 | 0.46  | -0.85 |
| 20              | -0.31 | 0     | -0.95 |

2. By first determining the "equivalent sound absorption area" of the room,  $A$ , and computing the environmental correction as  $K_2 = 10 \log[1 + 4(S/A)]$  decibels, where  $S$  is the area of the measurement surface. There are two methods currently specified in ISO 3744 for determining the value of  $A$ : a method using the measured reverberation time in the room and the so-called two-surface method. There is also a third method under consideration for standardization, a direct method using a calibrated reference sound source. The latest version of ISO 3744 should be consulted for details of using these methods.
3. By using the "approximate" method for determining  $A$ , which is then used only for determining an A-weighted value of the environmental correction,  $K_{2A} = 10 \log[1 + 4(S/A)]$ . Here the mean sound absorption coefficient  $\alpha$  is estimated from Table 4.6.<sup>10</sup> The equivalent sound absorption area  $A$  is then calculated from  $A = \alpha S_V$ , where  $S_V$  is the total area of the boundary surfaces of the test room (walls, floor, ceiling) in square meters.

Reference should also be made to ISO 3746<sup>10</sup>, a survey-grade standard for the determination of A-weighted sound power in rooms where the environmental correction may range up to 7 dB. This standard may be useful for taking measurements in situ, that is, when the source cannot be moved into a laboratory environment. The uncertainty of the A-weighted sound power level determined according to ISO 3746 is greater than that obtained when ISO 3744 or ISO 3745 is used. The average A-weighted sound pressure level is determined either on the parallelepiped measurement surface of Fig. 4.9 or the hemispherical measurement surface of Fig. 4.10, each shown with the minimum number of microphones required. Table 4.7 gives the coordinates for the four key microphone positions

TABLE 4.6 Approximate Values of Mean Sound Absorption Coefficient  $\alpha$

| Mean Sound Absorption Coefficient $\alpha$ | Description of Room                                                                                                                                                    |
|--------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0.05                                       | Nearly empty room with smooth hard walls made of concrete, brick, plaster, or tile                                                                                     |
| 0.1                                        | Partly empty room; room with smooth walls                                                                                                                              |
| 0.15                                       | Room with furniture; rectangular machinery room; rectangular industrial room                                                                                           |
| 0.2                                        | Irregularly shaped room with furniture; irregularly shaped machinery room or industrial room                                                                           |
| 0.25                                       | Room with upholstered furniture; machinery or industrial room with a small amount of sound-absorbing material on ceiling or walls (e.g., partially absorptive ceiling) |
| 0.35                                       | Room with sound-absorbing materials on both ceiling and walls                                                                                                          |
| 0.5                                        | Room with large amounts of sound-absorbing materials on ceiling and walls                                                                                              |

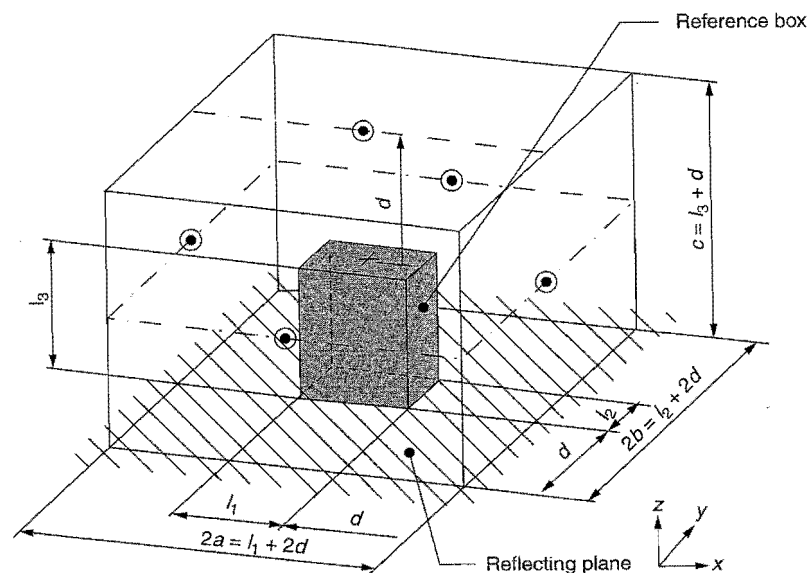


**FIGURE 4.9** Microphone positions for a rectangular measurement surface according to ISO 3746. (Courtesy of the International Organization for Standardization, Geneva, Switzerland.)

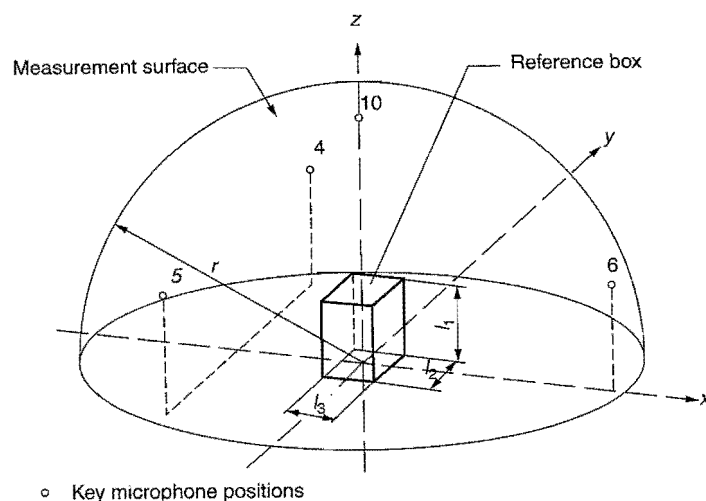


**FIGURE 4.10** Microphone positions for a hemispherical measurement surface according to ISO 3746. (Courtesy of the International Organization for Standardization, Geneva, Switzerland.)

**TABLE 4.7** Microphone Positions on Hemisphere for Survey Measurements According to ISO 3746

| Microphone Position | Microphone Positions<br>(All Heights, $z = 0.6R$ ) |       |       |
|---------------------|----------------------------------------------------|-------|-------|
|                     | $x/r$                                              | $y/r$ | $z/r$ |
| 4                   | -0.45                                              | 0.77  | 0.45  |
| 5                   | -0.45                                              | -0.77 | 0.45  |
| 6                   | 0.89                                               | 0     | 0.45  |
| 10                  | 0                                                  | 0     | 1.0   |
| 14                  | 0.45                                               | -0.77 | 0.45  |
| 15                  | 0.45                                               | 0.77  | 0.45  |
| 16                  | -0.89                                              | 0     | 0.45  |
| 20                  | 0                                                  | 0     | 1.0   |

Note: The key microphone positions are 4, 5, 6, and 10. Additional microphone positions are 14, 15, 16, and 20.

shown in Fig. 4.10 for the hemisphere, along with the four additional positions that might be required under certain circumstances.

The measured sound pressure levels are first corrected for background noise according to Eq. (4.20), but for ISO 3746 only the A-weighted value,  $K_{1A}$ , is needed. The environmental correction,  $K_{2A}$ , is determined using methods similar to those mentioned above for ISO 3744, including the approximate method. The value of  $K_{2A}$  should not exceed 7 dB (or, the ratio  $S/A$  should be less than or equal to 1). The A-weighted sound power level is finally calculated according to Eq. (4.15).

#### 4.10 DETERMINATION OF SOUND POWER USING SOUND INTENSITY

The fundamental procedures for the determination of sound power from sound intensity are formulated in Section 4.4. The sound intensity is measured over a selected surface enclosing the source. In principle, the integral over any surface totally enclosing the source of the scalar product (dot product) of the sound intensity vector and the associated elemental area vector provides a measure of the sound power radiated directly into the air by all sources located within the enclosing surface.

The precision of sound power determination based on sound intensity when the value of the intensity is calculated from measurements of sound pressure in a free field strongly depends on a number of factors. These include source type, measurement area related to source size, and presence of standing waves. The relationship between sound intensity and sound pressure in Eq. (4.11) is valid only in the far field of the free waves radiated by a source. Experience with the measurements on real sources has revealed that errors as large as 10 dB can

occur if the selection of the measurement surface is unsuitable or if standing waves affect the measured sound pressure.

In recent years, methods and techniques for direct measurement of sound intensity have been developed. This direct measurement has made it possible to determine the sound power of a variety of source configurations and to make measurements in environments which are not suitable for use in sound power determinations based on measurement of sound pressure. These include sound power determination from measurements in the near field of large sources on test stands, elimination of the adverse effects of standing waves when measuring in enclosures, partial sound power determination of parts of the source with the entire source operating, and the analysis of the source behavior by finding the areas of sound radiation and absorption.

Other acoustical quantities are, by definition, based on sound power. These quantities include sound transmission coefficient, sound absorption coefficient, and radiation efficiency. Either the direct or indirect method described here can be used to determine the sound power for these purposes.

Sound intensity is defined as the sound power that propagates perpendicularly through a unit area. The sound power radiated by a source can be calculated using Eq. (4.5). Sound intensity can be calculated from the product of the sound pressure and particle velocity at a given field point. Because both of these quantities are functions of time, the sound intensity calculated from this time-dependent product is called instantaneous intensity. However, in noise control, it is more practical to work with time-averaged quantities so that acoustic intensity for periodic sound of frequency  $f$  and period  $T = 1/f$  is defined at point  $r$  by

$$\mathbf{I}(r) = \frac{1}{T} \int_0^T p(r, t) \mathbf{u}(r, t) dt \quad (4.21)$$

where  $\mathbf{u}$  is the particle velocity and  $p$  is the sound pressure. The sound intensity  $\mathbf{I}$  is a vector describing the time-averaged flow of power per unit area in watts per square meter normal to the intensity direction. This is typically applicable to the tonal components of the noise spectrum. The random components of the noise spectra usually consist of stationary noise and the length of the averaging time  $T$  is, in principle, related to the required measurement precision. In most situations, the sound power is determined in octave bands or one-third-octave bands, and the averaging time depends on the filter response—as specified in the instrumentation standards.

To determine the sound intensity, both the pressure and the particle velocity have to be measured. While good pressure microphones are available, precision measurement microphones for the particle velocity are not available. Recently, prototypes of velocity microphones that can measure all three components of the velocity vector by means of a hot-wire technique have been developed.<sup>18</sup> However, the standards for sound power measurements are based on a two-microphone (pressure microphones) technique for the velocity determination. This is based on Euler's equation, which links the particle velocity with the sound pressure

gradient. This equation has the form

$$\mathbf{u} = - \int_0^t \frac{1}{\rho} \nabla p dt \quad (4.22)$$

where  $\rho$  is the density of the medium,  $\nabla p$  is the pressure gradient, and  $t$  is the integration time, which, as before, depends on the time function defining the noise. When measuring sound power, we are usually interested in the intensity vector component that is normal to the measurement surface. In this case, the gradient of  $p$  is replaced by  $\partial p / \partial x$ , where  $x$  is the direction of the normal to the measurement surface. The measurement of this gradient is approximated by  $\partial p / \partial x \cong \Delta p / \Delta x \cong (p_1 - p_2) / \Delta x$ . The pressure difference  $p_1 - p_2$  is measured using two microphones spaced a distance  $\Delta x$  apart, which must be much smaller than the wavelength of sound. Figure 4.11 shows a two-microphone probe used to measure the pressure gradient. The magnitude of the measured sound intensity vector component  $\hat{I}_x$  in the direction  $x$  is determined from

$$\hat{I}_x = - \frac{1}{2\rho \Delta x} E \left\{ [p_2(t) + p_1(t)] \int_0^t [p_2(t) - p_1(t)] dt \right\} \quad (4.23)$$

where  $E$  is the expected value representing the time averaging and the pressures are on the two microphone diaphragms spaced a distance  $\Delta x$  apart. Figure 4.12 shows a block diagram of an intensity measurement instrument which calculates the intensity component  $\hat{I}_x$  using Eq. (4.23).

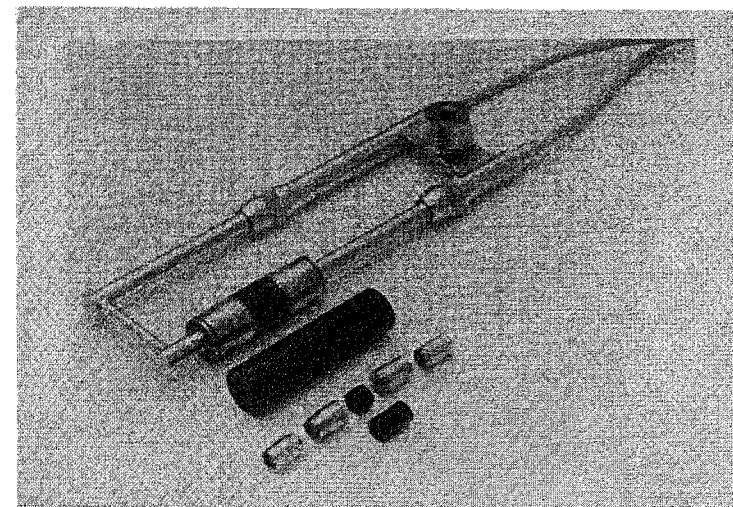


FIGURE 4.11 Sound intensity probe showing two  $\frac{1}{2}$ -in.-diameter microphones separated precisely by a 1.2-cm spacer. Just beneath is a 5-cm spacer. Below are two  $\frac{1}{4}$ -in. microphones separated by a 0.6-cm spacer. (Courtesy of Brüel and Kjær, Inc.)

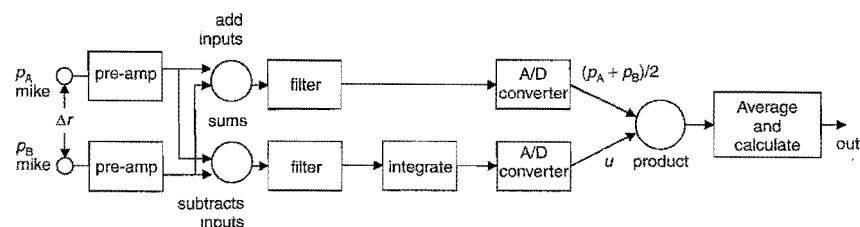


**FIGURE 4.12** Block diagram of an analog/digital sound intensity analyzer using octave-band filters. Alternatively, the preamplifier outputs could be suitably amplified and converted to digital format. Then all further processing would be done digitally.

The instrument is also provided with filters with selectable bandwidths to be able to measure the sound power in frequency bands and also to calculate the total power with A-frequency weighting, as required by some standards. Because sound intensity is often used to locate the areas of sound energy radiation or absorption from the surface of a noise source, it is also very practical to measure the sound intensity in very narrow bands. This is conveniently accomplished by using a dual-channel FFT analyzer. The sound intensity measurement is based on a Fourier transform of Eq. (4.23):

$$\hat{I}_x(\omega) = \frac{1}{\omega \rho \Delta x} \text{Im}[S_{p_1 p_2}(\omega)] \quad (4.24)$$

where  $\text{Im}[S_{p_1 p_2}(\omega)]$  is the imaginary part of the cross-spectra of the output from the two microphones as measured by the FFT analyzer. This measurement is particularly suitable to analyze the sound power radiation from sources with pronounced line spectra. All sound sources which operate with mechanical periodicity such as rotating engines, fans, vehicles, and similar equipment usually have strong line spectra. The output of the FFT analyzer is usually connected to a computer, which executes the desired postprocessing. The most common calculations are the energy in the lines, in octave or one-third-octave bands, and the total energy, either linear or with A-frequency weighting, as required by some standards. To determine the energy in a band, at least 10 lines of the FFT analysis are generally needed.

The precision of intensity measurements depends on many factors, which can be summarized into two groups: the precision of the instrumentation and the precision of the sampling of the radiated intensity and subsequent calculation of the total radiated power.

There are two standards that define instrumentation requirements for intensity measurements: International Electrotechnical Commission (IEC) 1043<sup>19</sup> and ANSI 1.9-1996.<sup>20</sup> In principle, the intensity measured by an intensity meter should be the same as the intensity in a plane wave measured by a pressure microphone and calculated from the equation  $I = p^2/\rho c$ . An intensity meter consists

of an intensity probe and a processor. The standards define allowable tolerances for both the probe and the processor for class 1 and 2 intensity instruments.

The probe shown in Fig. 4.11 consists of two pressure microphones separated by a spacer of a length selected by the user. The sound pressure is calculated from the arithmetic average of the microphone pressures. As shown in Eq. (4.23), the particle velocity is approximated by the pressure gradient, which depends on the difference of the microphone pressures. The selected microphone distance must be small enough to avoid a bias error at high frequencies. Figure 4.13 shows the bias error as a function of frequency and microphone distance. This type of bias error cannot be corrected.

Another bias error can occur at low frequencies if the microphones are too close together and not sufficiently phase matched. Figure 4.14 shows the bias error for a 0.3° microphone phase mismatch. The lower the frequency, the greater is the error. Increasing the microphone distance will decrease this error, but a large distance will cause, as mentioned above, a finite distance bias error. Fortunately, modern processors and signal processing can compensate for the phase mismatch error. Essential details are provided in the standards. Due to the existence of these two different bias errors, when the frequency range of the measured noise is large, the measurement usually needs to be repeated using two different microphone separations.

Another important quantity defining the useful frequency range of the instrument is the dynamic capability  $L_d$ , defined as

$$L_d = \theta_{pIR} - K \quad (4.25)$$

where  $\theta_{pIR}$  is the pressure minus residual intensity index and  $K = 7$  for a 1-dB allowable measurement error. The importance and usefulness of  $L_d$  are shown and

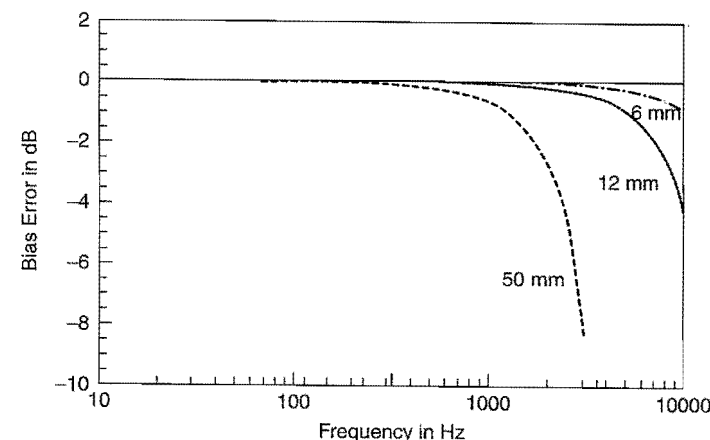


**FIGURE 4.13** Intensity probe bias error as a function of frequency with different microphone distances as a parameter.

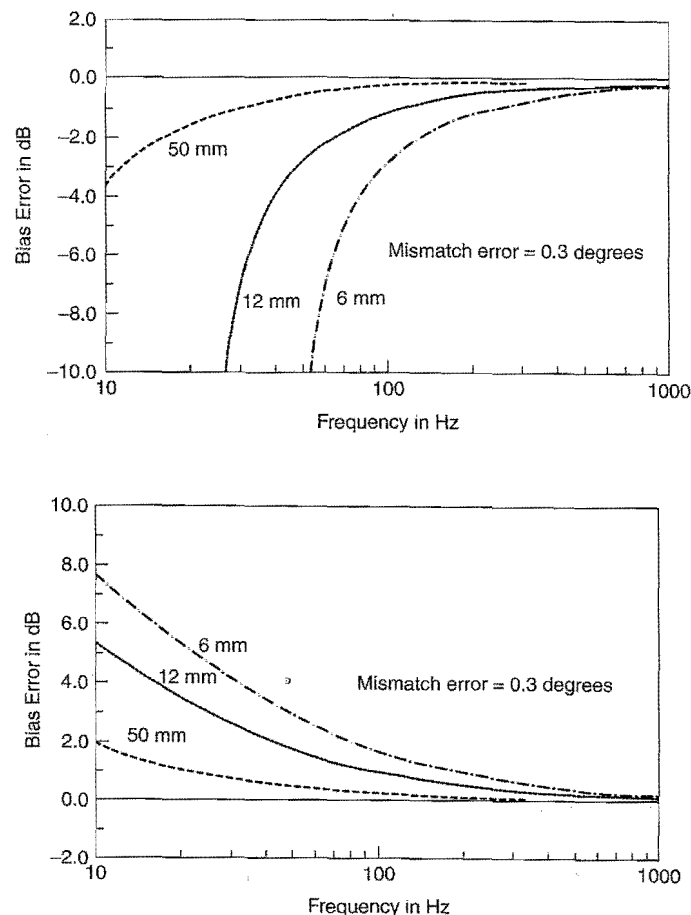


FIGURE 4.14 Intensity probe relative bias error as a function of frequency for different microphone distances as a parameter and mismatch error of  $0.3^\circ$  (plus and minus).

explained in Fig. 4.15. The pressure-residual intensity index  $\theta_{pIR}$  can be obtained by placing both microphones in a small cavity in which the sound field is excited by an external source. In this way, both microphones are exposed to an identical sound pressure level  $L_p$  and the residual intensity level  $L_{IR}$  is measured using the intensity meter (usually a FFT analyzer). Ideally, the measured residual intensity should be zero, but due to the microphone mismatch, noise, and measurement instrumentation channel mismatch and other factors,  $L_{IR}$  is finite (see Fig. 4.15).

Both  $L_p$  and  $L_I$  are measured for an actual noise source and the pressure minus intensity index  $L_{pI} = L_p - L_I$  is calculated. The intersection of the dynamic capability curve with  $L_I$  determines the lowest usable frequency. A phase mismatch will increase  $L_{IR}$  and shift the lowest usable frequency higher.

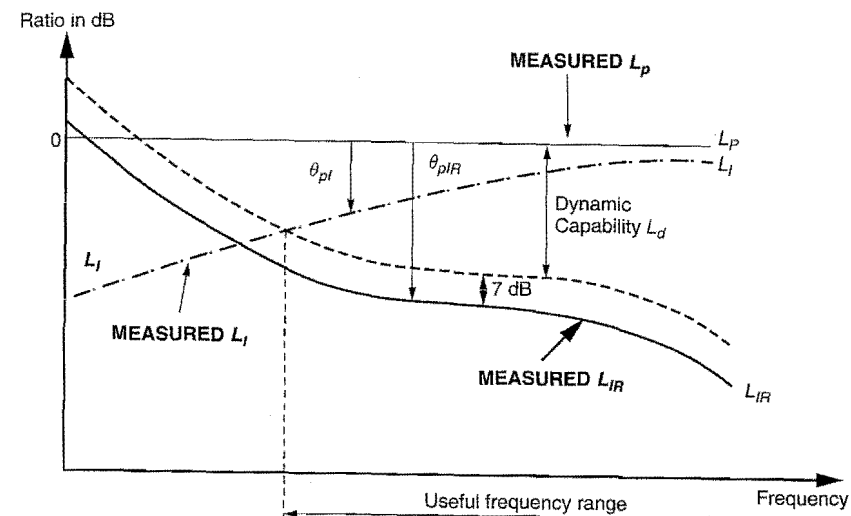


FIGURE 4.15 Dynamic capability of a sound intensity measurement system for 1-dB error as obtained from the pressure-intensity index. The values of  $L_p$  and  $L_I$  measured with the microphone in a small cavity are used to determine the residual pressure minus intensity index  $\theta_{pIR} = L_p - L_{IR}$ . The values of  $L_p$  and  $L_I$  on an actual noise source are used to determine the pressure minus intensity index,  $\theta_{pI} = L_p - L_I$ .

The pressure minus intensity index  $L_{pI} = L_p - L_I$  obtained from the measurement of the actual noise source is an important quantity that characterizes at the measurement point the combination of the source properties and the sound field properties, particularly the effects of the wall reflections. The standards for the sound power measurements and other literature<sup>21</sup> provide essential details.

A typical machine (e.g., an engine) usually operates under conditions different from those under which it would be tested in a reverberant or anechoic room. Moreover, the radiation impedance "seen" by the source may be different from that in a controlled acoustical environment. Hence, its radiated power may be somewhat dependent on how it is mounted and on the proximity of surrounding surfaces.

The intensity technique permits, in most situations, measurement of the sound power of a source of any size operating in its natural environment or on a test stand. In many situations, the sound power radiated from parts of a source can be measured while the whole source is operating. In addition, the intensity technique has become an analytical tool to determine the sound power radiation from different regions of the surface of the source so that the areas of major power radiation can be found. These tasks require an extensive general knowledge of the characteristics of the sound fields, near and far, from the source, wave interference, reflected wave fields, and diffuse sound fields. The details of these important subjects can be found in the literature.<sup>21,22</sup>

Standards which use the direct measurement of sound intensity have been prepared by the ISO and are based on extensive research as well as practical experience gained from measurements. Part 1 of ISO 9614<sup>23</sup> is based on the sampling of sound intensity at discrete points. The intensity probe is moved from point to point and held at each point for a time sufficiently long to make the temporal averaging error small. Part 2 of the same standard uses a scanning technique. The intensity probe is moved over a prescribed path with a sufficiently low speed to satisfy the temporal averaging criterion. The measurement surface is subdivided into smaller sampling areas to secure the required uniformity in the motion of the hand-held probe. Part 3 of the same standard defines the conditions for using the scanning technique for more exact measurements than in Part 2. In practice, the intensity probe is held perpendicular to the measurement surface so that the scalar product converts into an algebraic product.

One of the principal advantages of using the intensity method in field situations is that the results are, in principle, independent of sources outside the measurement surface. The sound energy of external sources propagates through the measurement surface without contributing to the measured power of the sources within the surface—provided that no sound energy due to external sources is absorbed within the surface. Similarly, sound waves reflected from room boundaries or standing waves, unless they are too strong, do not affect the results of a sound power measurement. The standards are applicable to stationary sources located in a nonmoving medium. Because of instrumentation limitations, the frequency range is generally limited to the one-third-octave bands from 50 Hz to 6.3 kHz. A-frequency weighted data are calculated from one-third-octave-band data in this frequency range or from octave-band levels in the frequency range 63 Hz–4 kHz. The correction factors are given in Table 1.4.

Determination of the measurement uncertainty is an important component of the measurements. All standards define the methods and procedures for its determination. Before starting the measurement, the acoustical environment has to be examined for extraneous intensity, wind, gas flow, vibrations, and temperature. The next step consists of the calibration and field check of the instrumentation as specified in IEC 1043<sup>19</sup> or ANSI 1.9–1996.<sup>20</sup> The selection of the measurement surface is important. This is usually performed in two steps. First, an initial surface is selected and an initial measurement performed. The results are tested using a set of indicators that define the characteristics of both the pressure and intensity fields on the measurement surface.<sup>23,24</sup> If the values of these “field indicators” as specified in the standard are not satisfactory, the steps above have to be modified and the measurement repeated. Important field indicators are defined below.

The initial measurement surface is usually selected following the shape of the source at a distance greater than 0.5 m, unless that position is over an area that radiates an insignificant proportion of the sound power of the source under test. The selection of the number of measurement points depends on the shape, segments, and size of the measurement surface. A minimum of 10 points must be selected (greater number obviously leads to better precision, particularly at

higher frequencies). If the source is large, one point per square meter of the measurement surface is usually selected, provided that the total number is not less than 50. If extraneous sound penetrates into the measurement area, the number of measurement points must be increased.

After both the sound pressure and the sound intensity are measured at all points, the results are tested by the field indicators in all frequency bands. Depending on the outcome from these indicators, the number and distribution of the measurement points and the distance of the measurement surface from the source may have to be changed. The standards provide tables and flow charts that define the actions to be taken.

The purpose of the indicators is to ensure a sufficient precision of the measurement. The statistical distributions of both the sound pressure and the sound intensity over the measurement surface depend on the source shape and its environment, primarily standing waves caused by sound reflections. Therefore, a general formula for the measurement error does not exist and the error must be determined experimentally. Figure 4.16 shows the flow diagram for the implementation of ISO 9614-1.

The field indicators as defined in ISO 9614 are as follows:  $F_1$ , temporal variability of the sound field;  $F_2$ , surface pressure–intensity;  $F_3$ , negative partial power; and  $F_4$ , field nonuniformity. The field indicators require measuring both the sound pressure and intensity. If the criteria for the indicators are not satisfied, the measurement arrangements must be modified. This concerns mainly the change of distance from the measurement surface and increase of the number of measurement points. The flow diagram indicates the actions to be taken. Table 4.8 provides detailed information on these actions.

Indicator  $F_1$  checks the stationarity of the intensity field from several short time-average estimates of the sound intensity at one point of the measurement surface. Its value should be less than 0.6. This should assure that the source operation is steady and the environmental effects are not time variable.

Indicator  $F_2$  is calculated from pressure square averages over the measurement area, converted into a pressure level  $L_p$ . Similarly, the intensity level  $L_I$  is determined from the arithmetic average of the sound intensities in individual points, all taken with positive sign, irrespective of the power flow out or the measurement surface at a particular point. Indicator  $F_2$  must be smaller than the dynamic capability indicator  $L_d$  as defined by Eq. (4.25) and shown in Fig. 4.15 in order to keep the error caused by the instrumentation less than 1 dB for  $K = 7$ . This indicator is particularly important at low frequencies, as it is apparent in Fig. 4.15.

Indicator  $F_3$  is similar to indicator  $F_2$  except that the intensity level  $L_I$  is determined from intensity values with respect to its sign, which means that  $I$  is negative at the points where the sound power flows into the measurement surface. This can be caused by extraneous sources or strong reflections due to source environment. Thus  $F_3 - F_2$ , which is supposed to be less than 3 dB, is linked to the ratio of the sound power radiated out of the measurement surface to the sound power entering the measurement surface. Because both bias and

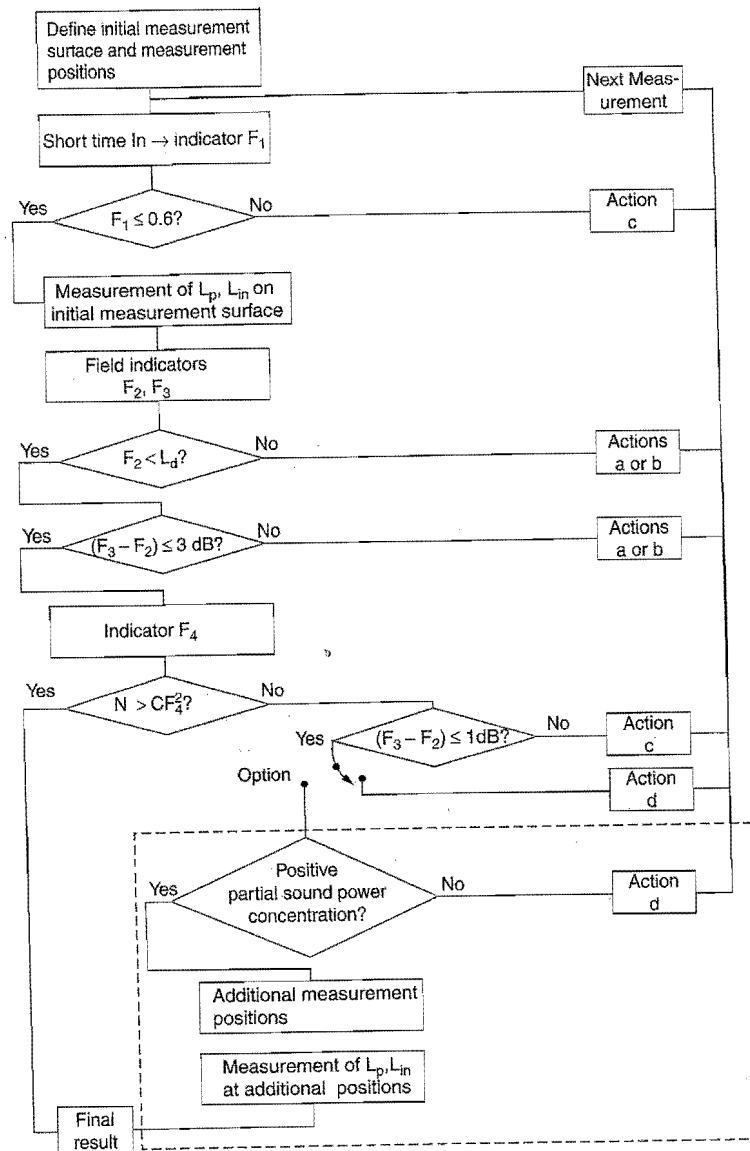


**FIGURE 4.16** Flow diagram for the implementation of ISO 9614. The action codes defined in the center column of this figure are defined in Table 4.8. Calculation of the required number of measurement points requires a factor,  $C$ , which is defined in Table 4.9. The path enclosed in dashed lines represents an optimal procedure designed to minimize the number of additional measurement positions required on the initial measurement surface. (Courtesy of the International Organization for Standardization, Geneva, Switzerland.)

**TABLE 4.8** Actions to Be Taken to Increase Grade of Accuracy of Determination

| Criterion                                                                                                                                               | Action Code <sup>a</sup> | Action                                                                                                                                                                                                                                                                                                              |
|---------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $F_1 > 0.6$                                                                                                                                             | e                        | Take action to reduce the temporal variability of extraneous intensity, or measure during periods of less variability, or increase the measurement period at each position (if appropriate)                                                                                                                         |
| $F_2 > L_d$ or<br>$(F_3 - F_2) > 3 \text{ dB}$                                                                                                          | a                        | In the presence of significant extraneous noise and/or strong reverberation, reduce the average distance of the measurement surface from the source to a minimum average value of 0.25 m; in the absence of significant extraneous noise and/or strong reverberation, increase the average measured distance to 1 m |
|                                                                                                                                                         | or                       |                                                                                                                                                                                                                                                                                                                     |
|                                                                                                                                                         | b                        | Shield measurement surface from extraneous noise sources or take action to reduce sound reflections toward the source                                                                                                                                                                                               |
| Criterion 2 not satisfied and<br>$1 \text{ dB} \leq (F_3 - F_2) \leq 3 \text{ dB}$                                                                      | c                        | Increase the density of measurement positions uniformly to satisfy criterion 2                                                                                                                                                                                                                                      |
| Criterion 2 not satisfied and<br>$(F_3 - F_2) \leq 1 \text{ dB}$ , and the<br>procedure of ISO 9614<br>Section 8.3.2 either fails or<br>is not selected | d                        | Increase average distance of measurement surface from source using the same number of measurement positions or increase the number of measurement positions on the same surface                                                                                                                                     |

<sup>a</sup>See Fig. 4.16.

random errors depend on  $F_3 - F_2$ , the 3-dB criterion satisfies the requirement to keep the bias errors low.

Indicator  $F_4$  is the spatial variance of sound intensity measured at discrete points normalized to the average. The signs of intensity values are considered. This indicator reflects the variability of the power flow over the measurement surface. The higher is  $F_4$ , the more measurement points are needed. The number of the measurement points  $N$  is given by  $N > C \times F_4^2$ , where  $C$  is a factor which depends on frequency and required precision grade, as shown in Table 4.9.

Part 2 of ISO 9614 defines the measurement of both the sound pressure and intensity by scanning. The measurement area is subdivided into usually plane segments over which the probe is moved ("scanned") perpendicularly to the surface so that spatial averages of the measured quantities are obtained. The recommended moving pattern and speed are specified in the standard. Experimental evidence indicates that the results using the scanning technique are generally more precise than using point measurements.

TABLE 4.9 Values for Factor C

| Octave Band<br>Center Frequencies, Hz | One-Third-Octave-<br>Band Center<br>Frequencies, Hz | C                      |                          |                     |
|---------------------------------------|-----------------------------------------------------|------------------------|--------------------------|---------------------|
|                                       |                                                     | Precision<br>(Grade 1) | Engineering<br>(Grade 2) | Survey<br>(Grade 3) |
| 63–125                                | 50–160                                              | 19                     | 11                       |                     |
| 250–500                               | 200–630                                             | 29                     | 19                       |                     |
| 1000–4000                             | 800–5000                                            | 57                     | 29                       |                     |
|                                       | 6300                                                | 19                     | 14                       |                     |
| A weighted <sup>a</sup>               |                                                     |                        |                          | 8                   |

<sup>a</sup>63 Hz–4 kHz or 50 Hz–6.3 kHz.

Part 3 of ISO 9614 is also based on the scanning technique. The requirements for the measurements and the tolerances to be satisfied are more strict than in Part 2.

In addition to the cited international standards, the ANSI has developed a standard, ANSI S12.12-1992,<sup>25</sup> which mirrors, in its fundamental concept and measurement procedures, the ISO standards. This standard contains a greater number of field indicators and provides more details on the measurement procedures. The selection of any of these standards depends on the product to be measured, the purpose of the sound power determination, and commercial criteria.

ECMA International has also issued a standard for determining sound power from sound intensity using a scanning technique.<sup>26</sup> The standard is intended for use with computer and business equipment.

After substantial experience with the determination of sound power via sound intensity has been achieved, it is expected that these standards will be revised.

#### 4.11 SOUND POWER DETERMINATION IN A DUCT

The most common application of in-duct measurements is to determine the sound power radiated by air-moving devices. The sound power level of a source in a duct can be determined according to ISO 5136<sup>27</sup> from sound-pressure-level measurements, provided that the sound field in the duct is essentially a plane progressive wave, using the equation

$$L_W = L_p + 10 \log \frac{S}{S_0} \quad (4.26)$$

where  $L_W$  = level of total sound power traveling down duct, dB re  $10^{-12}$  W  
 $L_p$  = sound pressure level measured just off centerline of duct, dB re  
 $2 \times 10^{-5}$  N/m<sup>2</sup>  
 $S$  = cross-sectional area of duct, m<sup>2</sup>  
 $S_0 = 1 \text{ m}^2$

The above relation assumes not only a nonreflecting termination for the end of the duct opposite the source but also a uniform sound intensity across the duct. At frequencies near and above the first cross resonance of the duct, the latter assumption is no longer satisfied. Also, when following the measurement procedures of ISO 5136, several correction factors are incorporated into Eq. (4.26) to account for microphone response and atmospheric conditions.

Equation (4.26) can still be used provided  $L_p$  is replaced by a suitable space average  $\langle L_p \rangle$  obtained by averaging the mean-square sound pressures obtained at selected radial and circumferential positions in the duct or by using a traversing circumferential microphone. The number of measurement positions across the cross section used to determine  $\langle L_p \rangle$  will depend on the accuracy desired and the frequency. (See ref. 27, Section 6.2.)

In practical situations, reflections occur at the open end of the duct, especially at low frequencies. The effect of branches and bends must be considered.<sup>28</sup> When there is flow in the duct, it is also necessary to surround the microphone by a suitable windscreen (see Chapter 14). This is necessary to reduce turbulent pressure fluctuations at the microphone, which can cause an error in the measured sound pressure level.

#### 4.12 DETERMINATION OF SOURCE DIRECTIVITY<sup>29,30</sup>

Most sources of sound of practical interest are directional to some degree. If one measures the sound pressure level in a given frequency band a fixed distance away from the source, different levels will generally be found for different directions. A plot of these levels in polar fashion at the angles for which they were obtained is called the *directivity pattern* of the source. A directivity pattern forms a three-dimensional surface, a hypothetical example of which is sketched in Fig. 4.17. The particular pattern shown exhibits rotational symmetry about the direction of maximum radiation, which is typical of many noise sources. At low frequencies, many sources of noise are nondirectional—or nearly so. As the frequency increases, directivity also increases. The directivity pattern is usually determined in the far (free) field (see Fig. 4.4). In the absence of obstacles and reflecting surfaces other than those associated with the source itself,  $L_p$  decreases at the rate of 6 dB per doubling of distance.

##### Directivity Factor

A numerical measurement of the directivity of a sound source is the directivity factor  $Q$ , a dimensionless quantity. To understand the meaning of the directivity factor, we must first compare Figs. 4.17 and 4.18. We see in Fig. 4.18 the directivity pattern of a nondirectional source. It is a sphere with a radius equal in length to  $L_{pS}$ , the sound pressure level in decibels measured at distance  $r$  from a source radiating a total sound power  $W$ . The sources of Figs. 4.17 and 4.18 both radiate the same total sound power  $W$ , but because the source of Fig. 4.17

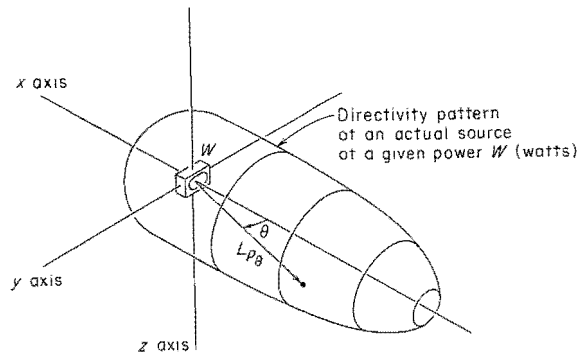


**FIGURE 4.17** Directivity pattern of a noise source radiating sound power  $W$  into free space. A particular sound pressure level  $L_p$  is shown as the length of a vector terminating on the surface of the directivity pattern at angle  $\theta$ . Sound pressure levels were measured at various angles  $\theta$  and at a fixed distance  $r$  from the actual source in free space.

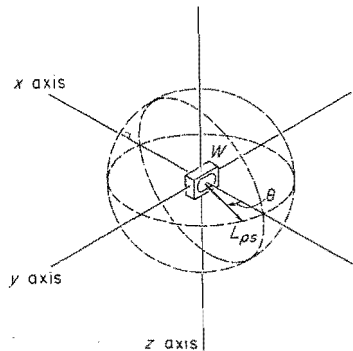


**FIGURE 4.18** Spherical directivity pattern of a nondirectional source radiating acoustic power  $W$  into free space. At all angles  $\theta$  and distance  $r$ , the sound pressure level equals  $L_{pS}$ , where  $L_{pS} = 10 \log(10^{12} \times W/4\pi r^2)$ .

is directional, it radiates more sound than that of Fig. 4.18 in some directions and less in others.

To derive a directivity factor  $Q$ , we must assume that the directivity pattern does not change shape regardless of the radius  $r$  at which it is measured. For example, if  $L_p$  at a particular angle is 3 dB greater than at a second angle, the 3-dB difference should be the same whether  $r$  is 1, 2, 10, or 100 m. This can only be determined in the far field of a source located in anechoic space.

The directivity factor  $Q_\theta$  is defined as the ratio of (1) the mean-square sound pressure  $p_\theta^2$  [(N/m<sup>2</sup>)<sup>2</sup>] at angle  $\theta$  and distance  $r$  from an actual source radiating  $W$  watts to (2) the mean-square sound pressure  $p_s^2$  at the same distance from a nondirectional source radiating the same acoustic power  $W$ . Alternatively,  $Q_\theta$

is defined as the ratio of the intensity in the direction of propagation ( $W/m^2$ ) at angle  $\theta$  and distance  $r$  from an actual source to the intensity at the same distance from a nondirectional source, both sources radiating the same sound power  $W$ . Thus

$$Q_\theta = \frac{p_\theta^2}{p_s^2} = \frac{I_\theta}{I_s} = \frac{10^{L_{p\theta}/10}}{10^{L_{pS}/10}} \quad (\text{dimensionless}) \quad (4.27)$$

or

$$Q_\theta = 10^{(L_{p\theta} - L_{pS})/10} \quad (4.28)$$

where  $L_{p\theta}$  = sound pressure level measured a distance  $r$  and an angle  $\theta$  from a source radiating power  $W$  into an anechoic space (see Fig. 4.17)

$L_{pS}$  = sound pressure level measured at a distance  $r$  from a nondirectional source of power  $W$  radiating into anechoic space (see Fig. 4.18)

Note that  $Q_\theta$  is for the angle  $\theta$  at which  $L_{p\theta}$  was measured and that  $L_{pS}$  and  $L_{p\theta}$  are for the same distance  $r$ .

### Directivity Index

The directivity index (DI) is simply defined as

$$DI_\theta = 10 \log Q_\theta \quad \text{dB} \quad (4.29)$$

or

$$DI_\theta = L_{p\theta} - L_{pS} \quad (4.30)$$

Obviously, a nondirectional source radiating into spherical space has  $Q_\theta = 1$  and  $DI = 0$  at all angles  $\theta$ .

### Relations between $L_{p\theta}$ , Directivity Factor, and Directivity Index

The sound pressure level for the nondirectional source of Fig. 4.18 is

$$L_{pS} = 10 \log \frac{p^2 \text{ at distance } r}{4 \times 10^{-10}} \quad \text{dB} \quad (4.31)$$

From Eqs. (4.5) and (4.11) and taking the quantity  $D = \rho c W_0 / p_{\text{ref}}^2$  to be small,  $L_{pS}$  is given by (see Fig. 4.18)

$$L_{pS} = 10 \log \frac{W \times 10^{12}}{4\pi r^2} \quad \text{dB} \quad (4.32)$$

From Eqs. (4.28) and (4.32), we find that

$$L_{p\theta} = 10 \log \frac{WQ_\theta \times 10^{12}}{4\pi r^2} \quad (4.33)$$

where  $W$  is in watts and  $r$  is in meters. In logarithmic form

$$L_{p\theta} = L_W + DI_\theta - 20 \log r - 11 \text{ dB} \quad (4.34)$$

where  $r$  is the distance from the acoustical center of the source in meters.

### Determination of Directivity Index in Spherical Space

The directivity index  $DI_\theta$  of a sound source in free space at angle  $\theta$  and for a given frequency band is computed from

$$DI_\theta = L_{p\theta} - \langle L_p \rangle_S \quad (4.35)$$

where  $L_{p\theta}$  = sound pressure level measured at distance  $r$  and angle  $\theta$  from source, dB

$\langle L_p \rangle_S$  = sound pressure level averaged over test sphere of radius  $r$  (and area  $4\pi r^2$ ) centered on and surrounding source

### Determination of Directivity Index in Hemispherical Space

The directivity index  $DI_\theta$  of a sound source on a rigid plane at angle  $\theta$  and for a given frequency band is computed from

$$DI_\theta = L_{p\theta} - \langle L_p \rangle_H + 3 \text{ dB} \quad (4.36)$$

where  $L_{p\theta}$  = sound pressure level measured distance  $r$  and angle  $\theta$  from source, dB

$\langle L_p \rangle_H$  = sound pressure level of space-averaged mean-square pressure averaged over a test hemisphere of radius  $r$  (and area  $2\pi r^2$ ) centered on and surrounding source

The 3 dB in this equation is added to  $\langle L_p \rangle_H$  because the measurement was made over a hemisphere instead of a full sphere, as defined in Eq. (4.37). The reason for this is that the intensity at radius  $r$  is twice as large if a source radiates into a hemisphere as compared to a sphere. That is, if a nondirectional source were to radiate uniformly into hemispherical space,  $DI_\theta = DI = 3 \text{ dB}$ .

### Determination of Directivity Index in Quarter-Spherical Space

Some pieces of equipment are normally associated with more than one reflecting surface, for example, an air conditioner standing on the floor against a wall. The power level of noise sources of this type may be measured with those surfaces in place. This is done best in a test room with anechoic walls but with one hard wall forming an "edge" with the hard floor. The general considerations of the

preceding paragraphs apply here as well. One determines the sound pressure level averaged over the quarter-sphere,  $\langle L_p \rangle_H$ , and also determines  $L_{p\theta}$  as before. The directivity index is given by

$$DI_\theta = L_{p\theta} - \langle L_p \rangle_Q + 6 \text{ dB} \quad (4.37)$$

### REFERENCES

1. R. H. Bolt and K. U. Ingard, "System Considerations in Noise-Control Problems," in C. M. Harris (Ed.), *Handbook of Noise Control*, 1st ed., McGraw-Hill, New York, 1957, Chapter 22.
2. E. A. G. Shaw, "Noise Pollution—What Can Be Done?" *Phys. Today*, 28(1), 46 (1975).
3. ISO 3740, "Acoustics—Determination of Sound Power Levels of Noise Sources—Guidelines for the Use of Basic Standards," International Organization for Standardization, Geneva, Switzerland, 2000.
4. ISO 3741, "Acoustics—Determination of Sound Power Levels of Noise Sources Using Sound Pressure—Precision Methods for Reverberation Rooms," International Organization for Standardization, Geneva, Switzerland, 1999.
5. ISO 3741, "Acoustics—Determination of Sound Power Levels of Noise Sources Using Sound Pressure—Precision Methods for Reverberation Rooms," International Organization for Standardization, Geneva, Switzerland, 1999. Correction 1:2001.
6. ISO 3743-1, "Acoustics—Determination of Sound Power Levels of Noise Sources—Engineering Methods for Small, Movable Sources in Reverberant Fields—Part 1: Comparison Method for Hard-Walled Test Rooms," International Organization for Standardization, Geneva, Switzerland, 1994.
7. ISO 3743-2, "Acoustics—Determination of Sound Power Levels of Noise Sources Using Sound Pressure—Engineering Methods for Small, Movable Sources in Reverberant Fields—Part 2: Methods for Special Reverberation Test Rooms," International Organization for Standardization, Geneva, Switzerland, 1994.
8. ISO 3744, "Acoustics—Determination of Sound Power Levels of Noise Sources Using Sound Pressure—Engineering Method in an Essentially Free Field over a Reflecting Plane," International Organization for Standardization, Geneva, Switzerland, 1994.
9. ISO 3745, "Acoustics—Determination of Sound Power Levels of Noise Sources Using Sound Pressure—Precision Methods for Anechoic and Hemi-Anechoic Rooms," International Organization for Standardization, Geneva, Switzerland, 2003.
10. ISO 3746, "Acoustics—Determination of Sound Power Levels of Noise Sources Using Sound Pressure—Survey Method Using an Enveloping Measurement Surface over a Reflecting Plane," International Organization for Standardization, Geneva, Switzerland, 1995.
11. ISO 3746, "Acoustics—Determination of Sound Power Levels of Noise Sources Using Sound Pressure—Survey Method Using an Enveloping Measurement Surface over a Reflecting Plane," International Organization for Standardization, Geneva, Switzerland, 1995. Correction 1:1995.
12. ISO 3747, "Acoustics—Determination of Sound Power Levels of Noise Sources Using Sound Pressure—Comparison Method in Situ," International Organization for Standardization, Geneva, Switzerland, 2000.

13. ISO 6926, "Acoustics—Requirements for the Performance and Calibration of Reference Sound Sources Used for the Determination of Sound Power Levels," International Organization for Standardization, Geneva, Switzerland, 1999.
14. M. A. Nobile, B. Donald, and J. A. Shaw, "The Cylindrical Microphone Array: A Proposal for Use in International Standards for Sound Power Measurements," Proc. NOISE-CON 2000 (CD-ROM), paper 1pNSc2, 2000.
15. M. A. Nobile, J. A. Shaw, and R. A. Boyes, "The Cylindrical Microphone Array for the Measurement of Sound Power Level: Number and Arrangement of Microphones," Proc. INTER-NOISE 2002 (CD-ROM), paper N318, 2002.
16. ISO 7779, "Acoustics—Measurement of Airborne Noise Emitted by Information Technology and Telecommunications Equipment (Second Edition)," International Organization for Standardization, Geneva, Switzerland, 1999.
17. R. V. Waterhouse, "Output of a Sound Source in a Reverberation Chamber and Other Reflecting Environments," *J. Acoust. Soc. Am.*, **30**, 4–13 (1958).
18. Hans-Elias de Bree, "The Microflown Report," Amsterdam, The Netherlands, [www.Microflown.com](http://www.Microflown.com), April 2001.
19. IEC 1043, "Instruments for the Measurement of Sound Intensity—Measurement with Pairs of Pressure Sensing Microphones," International Electrotechnical Commission, Geneva, Switzerland, 1993.
20. ANSI S1.9, "Instruments for the Measurement of Sound Intensity," Acoustical Society of America, Melville, NY, 1996.
21. F. J. Fahy, *Sound Intensity*, 2nd ed., E&FN SPON, Chapman & Hall, London, 1995.
22. J. Adin Man III and J. Tichy, "Near-Field Identification of Vibration Sources, Resonant Cavities, and Diffraction Using Acoustic Intensity Measurements," *J. Acoust. Soc. Am.*, **90**, 720–729 (1991).
23. ISO 9614, "Determination of Sound Power Levels of Noise Sources Using Sound Intensity Part 1: Measurement at Discrete Points"—1993; "Part 2: Measurement by Scanning"—1994; "Part 3: Precision Method for Measurement by Scanning"—2000, International Organization for Standardization, Geneva, Switzerland.
24. F. Jacobsen, "Sound Field Indicators: Useful Tools," *Noise Control Eng. J.*, **35**, 37–46 (1990).
25. ANSI S12.12, "Engineering Method for the Determination of Sound Power Levels of Noise Sources Using Sound Intensity," Acoustical Society of America, Melville, NY, 1992.
26. ECMA-160: "Determination of Sound Power Levels of Computer and Business Equipment using Sound Intensity Measurements; Scanning Method in Controlled Rooms," 2nd edition, ECMA International, Geneva, Switzerland, December 1992. A free download is available from [www.ecma-international.org](http://www.ecma-international.org).
27. ISO 5136, "Acoustics—Determination of Sound Power Radiated into a Duct by Fans and Other Air Moving Devices—In-Duct Method," International Organization for Standardization, Geneva, Switzerland, 2003.
28. P. K. Baade, "Effects of Acoustic Loading on Axial Flow Fan Noise Generation," *Noise Control Eng. J.*, **8**(1), 5–15 (1977).
29. L. L. Beranek, *Acoustical Measurements*, Acoustical Society of America, Woodbury, NY, 1988.
30. L. L. Beranek, *Acoustics*, Acoustical Society of America, Woodbury, NY, 1986.

## CHAPTER 5

# Outdoor Sound Propagation

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## 5.1 INTRODUCTION

This chapter deals with the description and prediction of sound due to sources in an outdoor environment. Specifically, it deals with the propagation path from source to receiver. Propagation is affected by geometrical spreading, ground effects including reflection and refraction due to temperature and wind speed vertical gradients, attenuation from intervening barriers, general reflections and reverberation, atmospheric absorption, and attenuation from intervening vegetation.

## 5.2 GENERAL DISCUSSION

The atmosphere is in constant motion due to wind and sun at amplitudes that are large compared to the amplitudes of sound—particle velocity. This constant motion results in considerable distortion of sound waves and considerable variability of propagation conditions. Ever since the careful observations and first scientific modeling of outdoor sound propagation by O. Reynolds, Lord Rayleigh, and Lord Kelvin in the nineteenth century,<sup>1</sup> numerous experimental and mathematical studies have provided detailed understanding of the effects of mechanical and thermal turbulence, humidity (including fog), boundary conditions at the ground surface, and obstacles such as trees, walls, and buildings in the propagation path.

Unfortunately, much of the vast amount of information published on outdoor sound propagation in scientific papers is not relevant to the practical control of noise from recreational and industrial facilities or from road, rail, and air

13. ISO 6926, "Acoustics—Requirements for the Performance and Calibration of Reference Sound Sources Used for the Determination of Sound Power Levels," International Organization for Standardization, Geneva, Switzerland, 1999.
14. M. A. Nobile, B. Donald, and J. A. Shaw, "The Cylindrical Microphone Array: A Proposal for Use in International Standards for Sound Power Measurements," Proc. NOISE-CON 2000 (CD-ROM), paper 1pNSc2, 2000.
15. M. A. Nobile, J. A. Shaw, and R. A. Boyes, "The Cylindrical Microphone Array for the Measurement of Sound Power Level: Number and Arrangement of Microphones," Proc. INTER-NOISE 2002 (CD-ROM), paper N318, 2002.
16. ISO 7779, "Acoustics—Measurement of Airborne Noise Emitted by Information Technology and Telecommunications Equipment (Second Edition)," International Organization for Standardization, Geneva, Switzerland, 1999.
17. R. V. Waterhouse, "Output of a Sound Source in a Reverberation Chamber and Other Reflecting Environments," *J. Acoust. Soc. Am.*, **30**, 4–13 (1958).
18. Hans-Elias de Bree, "The Microflown Report," Amsterdam, The Netherlands, www.Microflown.com, April 2001.
19. IEC 1043, "Instruments for the Measurement of Sound Intensity—Measurement with Pairs of Pressure Sensing Microphones," International Electrotechnical Commission, Geneva, Switzerland, 1993.
20. ANSI S1.9, "Instruments for the Measurement of Sound Intensity," Acoustical Society of America, Melville, NY, 1996.
21. F. J. Fahy, *Sound Intensity*, 2nd ed., E&FN SPON, Chapman & Hall, London, 1995.
22. J. Adin Man III and J. Tichy, "Near Field Identification of Vibration Sources, Resonant Cavities, and Diffraction Using Acoustic Intensity Measurements," *J. Acoust. Soc. Am.*, **90**, 720–729 (1991).
23. ISO 9614, "Determination of Sound Power Levels of Noise Sources Using Sound Intensity Part 1: Measurement at Discrete Points"—1993; "Part 2: Measurement by Scanning"—1994; "Part 3: Precision Method for Measurement by Scanning"—2000, International Organization for Standardization, Geneva, Switzerland.
24. F. Jacobsen, "Sound Field Indicators: Useful Tools," *Noise Control Eng. J.*, **35**, 37–46 (1990).
25. ANSI S12.12, "Engineering Method for the Determination of Sound Power Levels of Noise Sources Using Sound Intensity," Acoustical Society of America, Melville, NY, 1992.
26. ECMA-160: "Determination of Sound Power Levels of Computer and Business Equipment using Sound Intensity Measurements; Scanning Method in Controlled Rooms," 2nd edition, ECMA International, Geneva, Switzerland, December 1992. A free download is available from www.ecma-international.org.
27. ISO 5136, "Acoustics—Determination of Sound Power Radiated into a Duct by Fans and Other Air Moving Devices—In-Duct Method," International Organization for Standardization, Geneva, Switzerland, 2003.
28. P. K. Baade, "Effects of Acoustic Loading on Axial Flow Fan Noise Generation," *Noise Control Eng. J.*, **8**(1), 5–15 (1977).
29. L. L. Beranek, *Acoustical Measurements*, Acoustical Society of America, Woodbury, NY, 1988.
30. L. L. Beranek, *Acoustics*, Acoustical Society of America, Woodbury, NY, 1986.

## CHAPTER 5

# Outdoor Sound Propagation

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Unfortunately, much of the vast amount of information published on outdoor sound propagation in scientific papers is not relevant to the practical control of noise from recreational and industrial facilities or from road, rail, and air

traffic. Relevance is determined by the rating criteria required by governmental agencies for assessment of community environments, such as described in the ISO 1996 series and various governmental regulations. Relevant criteria most often are given in the form of two acoustical descriptors: the equivalent continuous and the average maximum A-weighted sound pressure level,  $L_{eq}$  (as defined in Chapter 2) and  $L_{A,max}$  (e.g., during a single-vehicle pass-by).

For that purpose, mainly average favorable conditions for sound propagation (plus the frequency of such conditions) need to be considered. Unfavorable conditions, which result in a large and uncertain range of low sound pressure levels, play a minor role in Europe.

For example, when favorable conditions occur during 50% of the time and unfavorable conditions result in at least 5-dB lower levels, then the equivalent continuous level

$$L_{eq} = 10 \lg \left( \frac{50}{100} 10^{L_{fav}/10} + \frac{50}{100} 10^{L_{unfav}/10} \right) \text{ dB}$$

$$< L_{fav} - 3 \text{ dB} + 10 \lg(1 + 10^{-5/10}) \text{ dB} = L_{fav} - 1.8 \text{ dB} \quad (5.1)$$

$$> L_{fav} - 3 \text{ dB}$$

lies some 2–3 dB below the average maximum level  $L_{fav}$  that occurs under favorable propagation conditions. It never lies outside that range, independent of the actual distribution of levels  $L_{unfav}$  for unfavorable propagation conditions.

The techniques in this chapter concentrate on such average favorable conditions for sound propagation to receiver positions about 4 m above the ground (first story above the ground story or higher). Additional techniques will be needed in governmental jurisdictions that require assessment of somewhat lower levels received 1.5 m above the ground—as is common within the United States and Canada.

In general, the level of outdoor sound decays with increasing distance between source and receiver. This geometrical divergence is most important for attenuation near the source, while meteorological conditions dominate attenuation further away.

Barriers, buildings, and hills that interrupt direct propagation from source to receiver are most important for excess attenuation. Of less importance are the effects of atmospheric absorption, porous ground (with receivers above 3–5 m), trees, single reflections from buildings, and reverberation in forests, valleys, and street canyons. Even though such effects are well understood, their computation requires input that is often not available in engineering practice—such as the spatial distribution of relative humidity or the effective flow resistivity of the ground between source and receiver. For planning purposes, engineering estimates or conventions have to be employed, rather than detailed models, to account for such effects.

In a homogeneous atmosphere at rest, geometrical divergence of sound from point sources is described by rigorous solutions of the wave equation in spherical

coordinates. In these solutions, sound travels on a straight course in all directions at the same speed.

In reality, however, sound paths are not straight, because sound speed varies with temperature and wind velocity, mainly as a function of height above the ground. Underwater sound propagation is similarly dependent on height, due to the variation of salt content with height. To account for refraction of sound under water, various specialized mathematical models have been developed: ray theory, the spectral method or fast-field program (FFP), the normal mode, and the parabolic equation (PE).<sup>2</sup> Except for the normal-mode model, which is based on essentially two-dimensional fields between the bottom and the surface of an ocean, all these models have been proposed for airborne sound as well.

In addition, for airborne sound propagation the particular effects of ground impedance can be computed with a boundary element model (BEM)—although that model is limited to a nonrefracting (homogeneous) atmosphere. Also in the acoustical literature is a Meteo-BEM (presently limited to linear sound speed profiles) and a generalized-terrain PE. The latter is limited to axis-symmetric cases, just like the PE, but is also applicable to terrain profiles with moderate slope.<sup>3</sup>

In spite of these advanced models, the much-simpler ray theory has now gained engineering importance, exclusively. In the past, ray theory was dominant because of excessive computation-time requirements for FFP- and PE-like models. At present, the comparison of results obtained with FFP (within the limits of reliable atmospheric input data involving the Monin-Obukhov boundary layer theory) has shown no particular advantage to FFP over ray theory. Instead, the two models show surprising consistency for downwind conditions up to a distance of 2000 m.<sup>4,5</sup>

At a reception point outdoors, only the A-weighted overall sound pressure level is considered in most practical cases. Further calculation of sound transmission into buildings, which requires information about the spectral distribution, is limited to special problems of building acoustics. Consequently, older engineering estimates neglect the frequency dependence of propagation losses. Instead, they assume a single frequency band that represents the attenuation of A-weighted overall sound pressure from sources with typical spectra. Due to increased computer capabilities, advanced engineering models now include calculations in frequency bands. Except for distant propagation to receiver heights of 1.5 m, full-octave bands are sufficient—both for the precision obtainable and the requirement for traceable details. One-third-octave bands are needed for special cases only.

Regulations that allow various calculation procedures with frequency bands of different widths have a great disadvantage. The simpler procedure is not always on the safe side, and so users quickly learn to choose the procedure that yields “better” results, from their point of view. To avoid ambiguous results of this type, one should follow well-established governmental regulations and conventions—such as ISO 9613-2<sup>6</sup> in Europe and the regulations of funding authorities in the United States and Canada.

### 5.3 SOURCES

#### Point Sources at Rest

Historically, acousticians distinguished between point, line, and area sources. Modern computer capabilities allow for the exclusive use of large numbers of point sources to approximate lines and areas. Such approximation is officially considered satisfactory in Europe, but not in the United States, where federal regulations generally require use of line source algorithms and computer programs. Such line source programs eliminate the possibility that gaps in noise barriers, for example, will be ignored by computations when the approximating point sources are not sufficiently dense along the roadway.

European computer programs automatically break up extended sources into sufficiently small elements, which can be described as point sources. For this purpose, it is generally sufficient that

- the largest dimension of a source element be less than half the distance between source and receiver,
- the sound power be about equally distributed over the source element, and
- about the same propagation conditions exist from all points on the source element to the receiver.

The last requirement pertains to effects of the ground and obstacles in the propagation path. If they are relevant, the permissible difference in height above the ground between two source elements is typically less than 0.3 m. Whether a particular computer program follows these rules needs to be carefully determined by test cases.

Point sources are described by

- the location  $(x, y, z)$  of the center,
- the sound power level  $L_W$  in frequency bands (relative to 1 pW), and
- the directivity index  $D_i$  in frequency bands (in one or two dimensions).

Preferred frequency bands are octave bands with center frequencies from 63 to 8000 Hz. Lower frequencies may be important—for example, in the vicinity of jet engine test cells. They need special consideration. Higher frequencies are subject to strong and variable atmospheric attenuation. They can be neglected outdoors.

When available, one should use measured sound spectra and directivity of specific noise sources. If these are not available but the A-weighted overall sound power level is known, Table 5.1 contains an estimate of the octave-band spectra applicable to a surprising number of sound sources—such as roadway, railway and aircraft traffic, rifle fire, muffled diesel engines, and many industrial noise sources. For comparatively large and slow sources or for substantial vibration damping, the spectrum may be shifted to lower frequencies by one octave. On the other hand, comparatively small and fast-moving sources with little vibration

TABLE 5.1 Typical Unweighted and A-Weighted Octave-Band Source Spectra Relative to A-Weighted Overall Sound Power

| Octave-Band Center frequency, Hz       | 63  | 125 | 250 | 500 | 1000 | 2000 | 4000 | 8000 |
|----------------------------------------|-----|-----|-----|-----|------|------|------|------|
| Unweighted: $L_{W,oct} - L_{WA}$ , dB  | -2  | 1   | -1  | -3  | -5   | -8   | -12  | -23  |
| A weighted: $L_{WA,oct} - L_{WA}$ , dB | -28 | -15 | -10 | -6  | -5   | -7   | -11  | -24  |

damping may have a spectrum that is shifted to higher frequencies by one octave. When the frequency band around 500 Hz is taken as an equivalent for the attenuation of A-weighted overall sound, this simplifying assumption is relative to one octave band below the average maximum of A-weighted octave-band noise.

The directivity index is normalized so that the average value of  $10^{D_i/10}$  in all directions is unity. In many cases of rotational symmetry, the directivity index is sufficiently described in one dimension. Examples are smoke stacks and gunfire. For receivers on level ground, the directivity index is often described in the horizontal plane only. A practical example is the directivity of sound from aircraft engines during ground run-up tests. For turbo-jet aircraft on the ground, a typical dependence of directivity on angle and frequency is plotted in Fig. 5.1. Highest values occur in the direction  $\phi = 120^\circ$  from the forward direction and in the 1000-Hz frequency band.

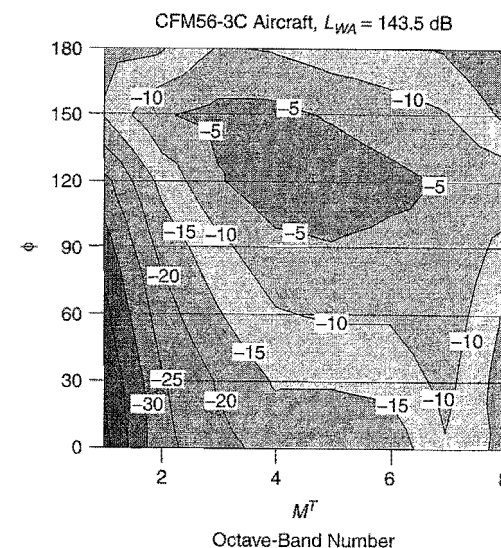


FIGURE 5.1 Combination of directivity and octave-band weighting for a turbojet engine with high bypass ratio at takeoff power setting. Octave-band number 2 is centered at 125 Hz and number 8 at 8000 Hz. Data are for a CFM-56-3C aircraft engine with  $L_{WA} = 143.5$  dB.

The basic relation between the sound pressure level  $L_p$  at a distance  $d$  from a source and the sound power level  $L_W$  of the source is given by

$$L_p(d, \phi) = L_W + D_I(\phi) + D_\Omega - A(d, \phi) \quad (5.2)$$

where  $A(d, \phi)$  = transfer function due to effect of all elements in propagation path; attenuating or enhancing effects are discussed in Section 5.4

$D_I(\phi)$  = directivity index

$D_\Omega$  = index that accounts for sound propagation into solid angles less than  $4\pi$  steradians

When the sound power output of a source is determined from outdoor measurements of the sound pressure level  $L_p(d, \phi)$  in various directions  $\phi$  at a certain distance  $d$  from the source, it is important to apply the same transfer function  $A(d, \phi)$  from the sound power level to the sound pressure level as is applied in the opposite direction.

The description of sound emission in terms of the sound power level generally includes the assumption that the source radiates into free space (solid angle  $\Omega = 4\pi$ ). Instead, when the source is located on the ground, and therefore the solid angle of radiation is  $2\pi$ , two equivalent assumptions can account for the ground: (1) an additional incoherent image source beneath the ground, with the same sound power, or (2) a correction of the sound power level  $L_W$  by  $D_\Omega = 3$  dB in Eq. (5.2). In general, when the source is located at a height  $h_S$  and the receiver at a height  $h_R$  above reflecting ground, the correcting index is

$$D_\Omega = 10 \lg \left[ 1 + \frac{d^2 + (h_S - h_R)^2}{d^2 + (h_S + h_R)^2} \right] \text{ dB} \quad (5.3)$$

from geometrical consideration of incoherent image sources.<sup>6</sup> Furthermore, in some cases the ground effect  $A_{gr}$  from source to receiver, which is part of  $A(d)$  in Eq. (5.2), must be taken into account.

When the source is located close to a wall or in a corner formed by two walls, sound pressure at some distance would include reflections from these walls as well. Similar to Eq. (5.3), the effect of these reflections is a level difference of about  $D_\Omega = 6$  dB for a wall and about 9 dB for a corner. Alternatively—and necessarily for sound-absorbing walls—image sources may be considered separately.

### Moving Sources

For traffic noise, it is common practice to consider the motion of sources along straight lines. Compared to continuous sources at rest, moving sources yield

- variable sound pressure levels at a stationary receiver,
- variable pitch of tonal components at a stationary receiver (Doppler effect), and
- different radiation due to different acoustical loading of the surrounding air.

The last of these effects is well described by theory, but in practice either included in the overall description of sound emission or neglected at low velocities.

The variation in pitch, from a higher pitch during vehicle approach to a lower pitch after passage, is determined by the factor  $(1 + M)/(1 - M)$ , in which  $M = V/c$  is the Mach number resulting from the vehicle velocity  $V$  and the sound velocity  $c$ . For road traffic, this multiplicative factor roughly corresponds to one-third of an octave, but for high-speed maglev trains, it may reach a full octave. In addition, for maglev trains, the sinusoidal excitation of the track, which is due to the groove-passage frequency of magnets over the long-stator, results in a broadband maximum noise at receivers close to the track—when radiated frequencies shift upward from some parts of the track and downward from other parts (see Fig. 5.2). Pure tones from the vibration signal of the magnets are not audible during train passage.

For moving vehicles of any type, the variation in sound pressure level during approach and pass-by is generally not taken into account, except for its energy mean and, in some cases, its maximum value. An integrating sound-level meter is used to determine the total sound energy and then to report the pass-by's sound exposure level (SEL), which is the level of the integrated sound energy referred to a time interval of 1 s and is also called the single-event level. Since the direction of sound approach to a receiver from a long, straight track is equally distributed horizontally over  $180^\circ$ , Eq. (5.2) can be used without its directivity index to convert the SEL into a sound power level. Spreading this sound power over the distance the source travels in 1 h results in a sound power per unit length of track—the strength of a line source for one vehicle per hour. For further calculations, the line source is broken up into straight-line segments in the United States and Canada or into point sources in Europe.

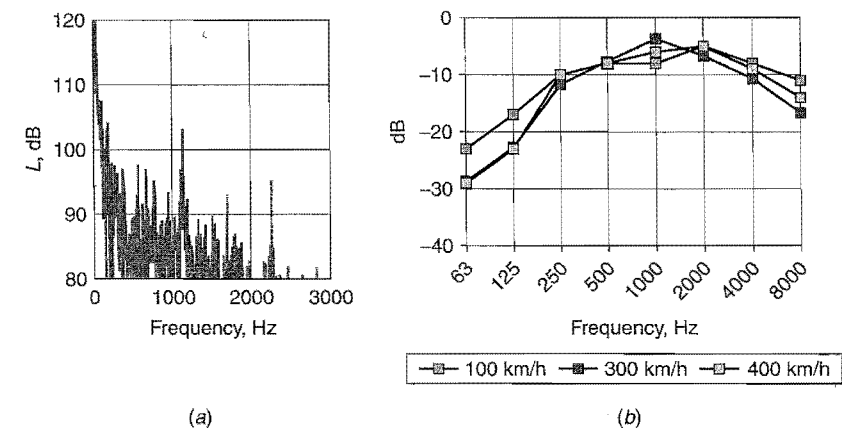


FIGURE 5.2 (a) Narrow-band spectrum of sound pressure level measured close to the magnets of a maglev train at 355 km/h. (b) A-weighted octave-band spectrum relative to A-weighted overall sound pressure level at some distance from a maglev train.

Although this procedure is normally applied to individual road and rail vehicles, the emission from the homogeneous section of a long train is usually evaluated differently. In such cases, the time integral of the squared sound pressure may be limited, without significant energy loss, to just when the center part of the homogeneous section passes by. This integral describes the sound emission from the homogeneous section in the same way as the SEL describes it for the entire train. Consequently, the same procedure can be used here (distribute the sound power over the distance the source travels in one hour) to determine the sound power per unit length of track for the train section selected. Sound power contributions from rolling noise of different vehicles are then added together. Contributions from sources at different heights need to be considered as separate line sources.

#### 5.4 ELEMENTS IN PROPAGATION PATH

##### Overview

The transfer function of sound from a source to a receiver is determined by the sum of all attenuations along a particular ray path and by the contribution from all paths of direct and reflected sound. Attenuations account for spherical spreading  $A_{\text{div}}$ , ground effect  $A_{\text{gr}}$ , diffraction by barriers  $A_{\text{bar}}$ , partial reflections  $A_{\text{refl}}$ , atmospheric absorption  $A_{\text{att}}$ , and miscellaneous others  $A_{\text{misc}}$ . They are described in the following.

##### Sound Propagation in Homogeneous Free Space over Ground

In the geometrical, high-frequency approximate solution of the Helmholtz equation for sound pressure  $p$ ,

$$\Delta p + \left(\frac{2\pi f}{c}\right)^2 p = -\delta^2(\mathbf{r} - \mathbf{r}_S) \quad (5.4)$$

propagation of sound with frequency  $f$  from a point source at  $\mathbf{r} = \mathbf{r}_S$  is described by ray theory. Rapid variations in phase are distinguished from slow variations in amplitude due to geometrical spreading and energy loss mechanisms. The ray trajectories are perpendicular to the surface of constant phase, which forms the wave front. The direction of average energy flux follows that of the trajectories. The amplitude of the field at any point can be obtained from the density of rays.

At short ranges, it is reasonable to assume straight rays. In free space, spherical spreading over a distance  $d$  results in an attenuation of

$$A_{\text{div}} = 10 \lg \frac{4\pi d^2}{d_0^2} \text{ dB} \quad (5.5)$$

where  $d_0 = 1$  m. An observer above partially reflecting ground receives not only the direct ray but also a ground reflection, as sketched in Fig. 5.3. For simplicity,

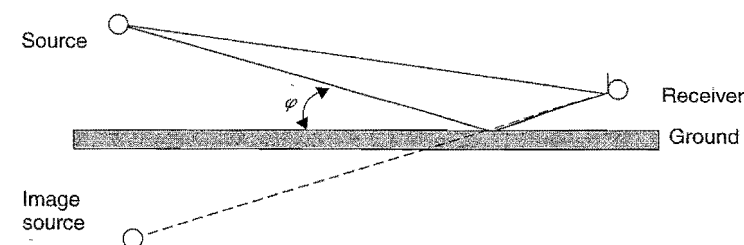


FIGURE 5.3 Ground reflection of a straight ray.

the plane-wave reflection coefficient

$$R_p = \frac{Z_s \sin \varphi - Z_0}{Z_s \sin \varphi + Z_0} \quad (5.6)$$

of a locally reacting surface of the ground, with impedance  $Z_s$ , may be used to determine the magnitude/phase effect of the reflection. The characteristic wave impedance of air,  $Z_0 = \rho_0 c_0$ , is generally very small compared to the magnitude of the ground impedance  $Z_s$ . Therefore, the reflection coefficient is usually about 1, except for very small angles  $\varphi$ —that is, when  $|Z_s| \sin \varphi \ll Z_0$  and the reflection coefficient is about  $-1$ . Under the latter condition, the direct ray destructively interferes with the ground reflection and causes relatively low sound pressure close to the ground. In essence, the source and image source form a dipole, causing little radiation parallel to the ground. When the source is much closer to the ground than the receiver—e.g., for road traffic noise—the ground effect near the source can be attributed to a vertical radiation characteristic of the source. According to measurements over grassland near the shoulder of highways, the directivity index  $D_I(\phi)$  of A-weighted road traffic noise drops from 0 dB at an elevation angle of  $15^\circ$  to about  $-5$  dB at  $0^\circ$ . Generalized per reciprocity for arbitrary source and receiver heights, this relation converts to a reduction in overall A-weighted sound level<sup>6</sup>:

$$A_{\text{gr},D} = \left[ 4.8 - \frac{h_{\text{av}}}{d} \left( 34 + \frac{600 \text{ m}}{d} \right) \right] \text{ dB} > 0, \quad (5.7)$$

where  $d$  is the source–receiver distance and

$$h_{\text{av}} = \frac{1}{2}(h_S + h_R) \quad (5.8)$$

is the average height of the sound ray from a source at height  $h_S$  to a receiver at height  $h_R$ .

Within a receiver distance of less than 200 m, this description is quite consistent with calculations of the Ontario noise regulation<sup>7</sup> from the equation

$$A_{\text{gr},O} = 10 \lg \frac{r}{15 \text{ m}} \text{ dB} > 0 \quad (5.9)$$

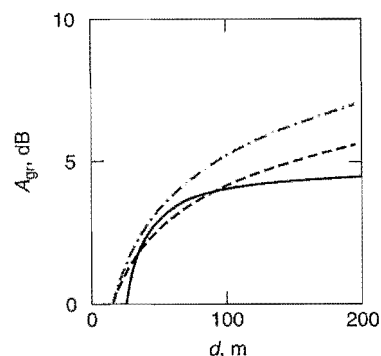


**FIGURE 5.4** Excess attenuation  $A_{gr}$  over grassland as a function of distance  $d$  for an average height of 2 m: (—) according to Eq. (5.7); (---) according to Eq. (5.9) with  $G = 0.5$ ; (- - -) according to Eqs. (5.9) and (5.10).

with

$$0 \leq G \equiv 0.75 \left( 1 - \frac{h_{av}}{12.5 \text{ m}} \right) \leq 0.66 \quad (5.10)$$

for an average height of 2 m (or a receiver height of 4 m for a source close to the ground), as shown in Fig. 5.4.

More precise models have been developed to account for the curvature of wave fronts using a spherical reflection factor and possibly for an extended reaction of soft ground (e.g., fresh-fallen snow).<sup>8</sup> Such models explain a pronounced interference dip of sound pressure for sources and receivers within 2 m above the ground. This dip typically occurs in octave bands centered at 250 and 500 Hz. A dominant parameter of these more precise models is the effective flow resistivity of the ground. United States and Canadian agencies generally require the application of such models to properly account for this dip to receivers 1.5 m above the ground (the required receiver height). Note that this dip is caused by interference, not energy absorption at the ground.

However, near-complete destructive interference of two rays requires about equal amplitude and opposite phases. Diffuse rather than specular ground reflection, plus phase distortion due to thermal and wind turbulence in the propagation path, reduces or inhibits such near-complete interference, especially at large distances. Predicted ground attenuation  $A_{gr}$  of more than 20 dB for average lawn surface is unrealistic.

From an engineering point of view (especially in Europe), this interference effect is of little use. Some sound-level reduction may be experienced on a terrace surrounded by a lawn. But neighbors to industrial plants cannot be protected from sound impinging on their bedroom windows (4 m or more above the ground) by interfering reflections from a substantially lower ground. In Europe, planning of new roadways or commercial activities does not depend on the ground conditions of adjacent property, as it does in the United States and Canada.

### Refraction of Sound in Inhomogeneous Atmosphere

Very important for outdoor sound propagation is the inhomogeneity of the atmosphere. The normal state of air is one of "convective equilibrium," in which the sound velocity  $c$  varies with temperature and wind velocity as a function of height above the ground. Higher temperature close to the ground results in higher sound velocity there and therefore in curvature of sound rays toward the sky. A component of wind in the direction of sound propagation causes refraction of sound rays toward the ground, because wind speed always increases with height. Linear wind and temperature profiles result in rays following a catenary curve that can be approximated by a circular arc with radius

$$R = \frac{1}{a \cos \phi} \quad (5.11)$$

as already described by Lord Rayleigh,<sup>1</sup> where  $a$  denotes the sound speed gradient due to vertical temperature and wind gradients. In the Nordic countries of Europe, measurements of wind and temperature are specified at heights of 0.5 and 10 m above level ground, expressly to determine  $a$  from the relation<sup>9</sup>

$$a = \frac{10^{-3}}{3.2 \text{ m}} \left( \frac{0.6 \Delta T}{1^\circ \text{C}} + \frac{\Delta u}{1 \text{ m/s}} \right) \quad (5.12)$$

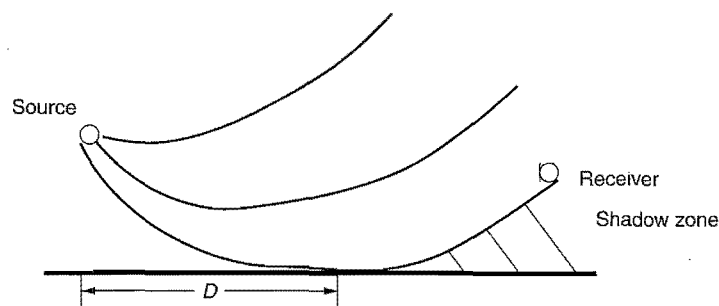
where  $\Delta T = T(10 \text{ m}) - T(0.5 \text{ m})$  is the difference in temperature and  $\Delta u = u(10 \text{ m}) - u(0.5 \text{ m})$  is the difference in wind speed component at those two heights.

For negative values of  $a$ , causing upward refraction, there is a limiting arc between source and receiver at a distance

$$D = \sqrt{\frac{2}{|a|}} \left( \sqrt{h_S} + \sqrt{h_R} \right) \quad (5.13)$$

that just grazes the ground (see Fig. 5.5). Beyond this distance, sound can reach the receiver not along a ray but only by diffraction. The limiting arc determines the boundary of an acoustical shadow zone. For source and receiver height  $h_S = 0 \text{ m}$  and  $h_R = 4 \text{ m}$ , respectively, and a moderate gradient  $a = -10^{-4} \text{ m}^{-1}$ , this distance  $D = 282 \text{ m}$ . Stronger gradients reduce the distance, but typically not below 100 m. Consequently, meteorological conditions are often neglected for sound propagation over less than 100 m. To ensure reliable measurements of industrial noise at a minimum height of 4 m, the Nordic countries specify a minimum value  $a = -10^{-4} \text{ m}^{-1}$  at distances from 50 to 200 m and  $a > -10^{-4} \text{ m}^{-1}$  at larger distances.

Most noticeable is the case of temperature inversion ( $a > 0$ ), which happens on calm days during dawn and dusk when dampness covers the ground and sunshine is restricted to the upper layers of the atmosphere. With temperature inversion, traffic noise is trapped in the lowest layer by downward refraction

FIGURE 5.5 Sound rays and shadow formation for negative values of  $a$ .

and repeated reflections from the ground. As a result, sound can be heard over very large distances, due to divergence in only two (rather than three) dimensions—that is, no divergence upward. Even stronger effects can be observed for sound propagation over lakes and wide rivers.

Opposite effects occur from sound propagation over hot spots, as is well known from the poor quality of communication over an open fire. Ground surfaces warmed up by the sun or by industrial facilities may cause upward refraction and scattering of sound out of straight rays, both effects resulting in excess attenuation.

The combined effects of ground and micrometeorology have been studied in detail for the development of a heuristic model for outdoor sound propagation.<sup>10</sup> This model accounts for

- ray theory for the fastest path in air from source to receiver;
- one or more ground reflections during sound propagation downwind or for temperature inversion conditions, the number depending on the magnitude of the positive sound speed gradient;
- diffraction of sound into the shadow zone for a negative sound speed gradient;
- spherical reflection coefficient for refracted sound rays incident on the ground, depending on the effective flow resistivity of the ground; and
- reduced coherence of the various contributions to the sound pressure at the receiver, due to different travel times of these components and turbulence of the atmosphere.

Data calculated from this model have been compared to data measured over grassland at relatively low receiver positions with the following results:

- High excess attenuations calculated for negative gradients  $a$  at distances of more than 400 m have not been observed. Up to 1600 m, the average attenuation values were limited to approximately 20 dB in the frequency band centered at 630 Hz.

- For positive gradients  $a$ , average measured excess attenuations were limited to a few decibels and did not increase significantly with distance from 400 to 1600 m in the frequency bands below 200 Hz and above 630 Hz. Calculated values of excess attenuation came out higher.
- A pronounced excess attenuation in the frequency bands from 200 to 1000 Hz was both calculated and measured for low wind and a small positive gradient  $a$ .

A much simpler model for the ground effect on sound propagation outdoors has been adopted in ISO 9613-2<sup>6</sup>:

- Downwind (or a positive gradient  $a$ ) is considered exclusively.
- The ground is either hard ( $G = 0$ ) or porous ( $G = 1$ ). If only a fraction of the ground is porous,  $G$  takes on a value between 0 and 1, the value being the fraction of the region that is porous.
- Three regions for ground reflection are distinguished: (1) the source region stretching  $30h_S$  (maximum) toward the receiver, (2) the receiver region stretching  $30h_R$  (maximum) toward the source, and (3) if the source and receiver regions do not overlap, a middle region in between.
- Similar to the angular parameter involved in Eq. (5.7), a parameter

$$q = \left(1 - 60 \frac{h_{\text{eff}}}{d}\right) \text{ dB} > 0 \quad (5.14)$$

is introduced to account for reflections in the middle region by a ground attenuation equal to  $-3q$  in the octave band centered at 63 Hz and equal to  $-3q(1 - G_m)$  in all the higher octave bands up to 8 kHz center frequency, where  $G_m$  is the value of  $G$  in the middle region. Together with the assumption that sound is radiated into a half-space with solid angle  $2\pi$ , there is a 6-dB increase in sound pressure level due to correlated ground reflection at 63 Hz, independent of ground porosity, and at higher frequencies for hard ground. Over porous ground, a 3-dB increase at higher frequencies is due to uncorrelated reflections.

- For the source and the receiver region,  $q = 0.5$  accounts for the splitting of regions and  $G_m$  is replaced by  $G_S$  or  $G_R$ , where the indices  $S$  and  $R$  refer to the source and receiver regions, respectively. In addition, the following attenuations are taken into account for both regions in four octave bands:

| Midband Frequency,<br>Hz | Attenuation Contribution,<br>dB                                                        |
|--------------------------|----------------------------------------------------------------------------------------|
| 125                      | $3Ge^{-(h-5m)/2.9m} (1 - e^{-d/50m})$<br>$+ 5.7Ge^{-(h/3.3m)^2} (1 - e^{-(d/600m)^2})$ |
| 250                      | $8.6Ge^{-(h/3.3m)^2} (1 - e^{-d/50m})$                                                 |
| 500                      | $14Ge^{-(h/1.5m)^2} (1 - e^{-d/50m})$                                                  |
| 1000                     | $5Ge^{-(h/1.05m)^2} (1 - e^{-d/50m})$                                                  |

(5.15)

Largest attenuation occurs over porous ground in the octave band with center frequency 500 Hz. Consequently, the simplifying assumption that this frequency band is representative for the attenuation of A-weighted overall sound may not be conservative. Instead, it could actually underestimate the sound pressure level at the receiver.

The attenuation linearly increases with distance  $d$  up to about  $d = 50$  m and then approaches a limit, except for the low-frequency band around 125 Hz, where a further extending ground wave is assumed.

Equations (5.15) show an exponential decay of attenuation with the square of receiver height. For the octave band at 500 Hz, the normalizing height in that band's equation is 1.5 m, roughly two wavelengths of sound. At 1000 Hz, this normalizing height is 1.05 m. The ratio of height to wavelength is similar in the relevant octave bands around 250 and 1000 Hz, indicating that the model is based on physical considerations together with experimental findings.

Despite essential simplifications compared to the heuristic model, the procedure described in ISO 9613-2 for octave-band calculations of ground attenuation is often replaced in practice by the procedure of Eq. (5.7) for A-weighted overall sound, mainly because ground factors  $G$  are often ambiguous.

The application of Eq. (5.15) to road and rail traffic noise with low source height is questionable. Ground effects near the source are already included in measured data. Uneven surfaces due to ditches for drainage are not covered by the model. Ground effects near the receiver can be excluded by an appropriate minimum height. Consequently, it has been decided that German guidelines for prediction of rail traffic noise, presently under revision, shall not account for frequency-dependent ground effects.

In contrast, U.S. requirements for road traffic computations consist of a more detailed computer model based on acoustical theory—the U.S. Federal Highway Administration (FHWA) Traffic Noise Model (TNM).<sup>11</sup> Version 2.5 of this model has been compared with more than 100 h of measured sound levels ( $L_{eq}$ ) at 17 highway sites, most with flat terrain, with and without noise barriers. Measurements ranged to 400 m from the roadway during daytime hours under varying wind conditions. Under these conditions, the average difference between results of computations and measurements was within approximately 1 dB.<sup>12</sup>

## Barriers

Any large and dense object that causes an acoustical shadow zone in the path of a sound ray is considered as a sound barrier. Such objects include walls, roofs, buildings, or the ground itself if it forms an edge or a hill. Sound penetrates into such shadow zones by diffraction and thereby suffers barrier attenuation  $D_z$ . The amount of attenuation is primarily determined by the Fresnel number

$$N = \frac{2z}{\lambda} \quad (5.16)$$

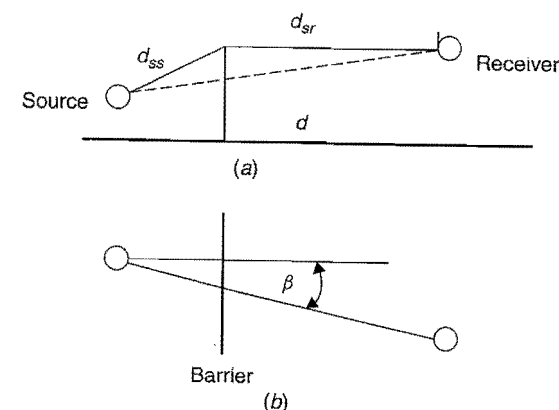


FIGURE 5.6 Barrier between source and receiver: (a) cross sectional view; (b) plan view.

where  $z$  is the increase in path length (the extension of a hypothetical rubber band between source and receiver) caused by one or more of the barrier edges and  $\lambda = c/f$  is the wavelength of sound at frequency  $f$ . Note that  $z$  decreases with increasing angle of sound incidence  $\beta$  (see Fig. 5.6).

For  $z = 0$  there is already a small barrier attenuation of about 5 dB plus an interference pattern at the edge of the shadow zone. Details of this interference pattern require a substantial computational effort that is not needed for engineering purposes. Therefore, preferred are simple approximations that are derived from extrapolations of the diffraction model and the ray-tracing model. When both models provide the same result for the line of sight over a hill or over undulating ground, the calculation of barrier insertion loss should be based on a limiting ( $z = 0$ ) ground attenuation of about 5 dB independent of frequency, as described by Eq. (5.7). This consideration leads to the barrier insertion loss:

$$A_{\text{bar}} = \begin{cases} D_z - A_{\text{gr}} > 0 & \text{for diffraction over top edge of barrier} \\ D_z > 0 & \text{for diffraction around vertical edge of barrier} \end{cases} \quad (5.17)$$

where  $D_z$  is defined in Eq. (5.18).

Inclined barrier edges are considered to be top edges whenever the angle of inclination against the ground is less than  $45^\circ$ . Otherwise they are considered to be vertical edges.

Note in Eq. (5.17) that an intervening barrier causes the loss of all ground effects. At a height of 4 m or more over porous ground, this approximation is relatively accurate. Close to hard ground, however, it is not correct. Pressure doubling due to hard ground, which results in negative values of  $A_{\text{gr}}$ , persists even with intervening noise barriers and cannot result in larger values of  $A_{\text{bar}}$  than for porous ground.

The calculation of barrier attenuation is based on Huygens' model for wave fields, which takes all insonified points in a plane perpendicular to the ray from

source to receiver as new sound sources. Points on a radius around the ray radiate with the same phase, varying between positive and negative values with increasing radius. Intervals between "phase zeros" in the hypothetical plane are called Fresnel zones. In free space, the contributions from neighboring Fresnel zones cancel each other, except for the center zone with the Fresnel radius  $\sqrt{\lambda d_{so}}$ , where  $d_{so}$  is the distance from the source to the plane. This center zone is the only uncanceled portion of the secondary wave front of Huygens' model and therefore is responsible for the entire sound field at the receiver. When this center zone is blocked by a barrier, the direct ray disappears.

In addition, a barrier partially blocks outer Fresnel zones and thereby inhibits the complete cancellation of their contributions. In the limit of many blocked Fresnel zones both above and on either side of the direct ray from the source to the receiver by a sufficiently high and long barrier, the intensity of the uncanceled portion decays with  $1/N$ ; that is, the barrier attenuation increases with increasing path length difference over the top edge and with frequency.

Refined calculations of barrier attenuation include sound reflection at the barrier surface. When either the source or the receiver is close to a reflecting barrier surface, then the image source behind the barrier contributes almost as much to the sound field as does the actual source. This effect reduces the barrier attenuation but is often negligibly small.

Ground reflections can be important. When either the source or the receiver is high above the ground, ground-reflected contributions are relatively small due to relatively large path length differences  $\approx z$ . But such high sources and receivers are generally not the case. Consequently, barrier attenuation over ground is computed 3 dB lower than the attenuation by a diffracting half-plane.

For outdoor sound propagation, the coherence of sound at different points in the sound field is reduced by wind and temperature fluctuations. This sets a limit for interference, depending upon the Fresnel radius and Fresnel number. Furthermore, one has to account for the curvature of rays due to gradients of wind and temperature. In the downwind direction, this curvature reduces the effective barrier height and the diffraction angle and, consequently, reduces the barrier attenuation—depending on the distance between the source, barrier, and receiver.

Theoretical and heuristic models have been developed to account for many or all of the effects just described. For prediction purposes, they suffer from the need for extremely detailed or unavailable input. In contrast, ISO 9613-2 requires less input—perhaps the upper limit of detail that can typically be obtained. In that standard, barrier attenuation is approximated by

$$D_z = 10 \lg (3 + 2NC_2C_3K_{\text{met}}) \text{ dB} < 20 \text{ dB for } C_3 = 1 (<25 \text{ dB for } C_3 > 1) \quad (5.18)$$

where  $C_2 = 20$  and includes the effects of (1) sound rays from the source via the barrier edge to the receiver, (2) sound rays from the image source in the ground, and (3) sound rays to the image receiver in the ground (if ground reflections are taken into account separately in special cases, then  $C_2 = 40$ );  $C_3 = 1$  for

diffraction at a single edge but can increase to 3 for diffraction at two or more edges in series, according to

$$C_3 = \frac{1 + (5 \lambda/e)^2}{1/3 + (5 \lambda/e)^2} \quad (5.19)$$

with  $e$  as the path length from the first to the last diffracting edge; and  $K_{\text{met}}$  is a meteorological correction factor which accounts for down-wind ray curvature and is calculated from

$$K_{\text{met}} = \exp \left( -\frac{1}{2000\text{m}} \sqrt{\frac{d_{ss}d_{sr}d}{2z}} \right) \quad (5.20)$$

where  $d_{ss}$  is the path length from the source to the first diffracting edge,  $d_{sr}$  is the path length from the last diffracting edge to the receiver, and  $d$  is the path length without the barrier.

All of Eqs. (5.17)–(5.20) are conventions that are based on first-order approximations of theoretical relations, plus field experience, for sound propagation in a refracting atmosphere with curvature parameter  $a > 0$ . They can be applied for barriers of infinite or finite length on the ground, with one to three perpendicular diffracting edges. The barrier length must be large enough to meet the requirements for reflectors (see below). For example, this is true for typical railway barriers of 2 m height above the railhead. At a distance of 25 m and a height of 3.5 m above the railhead, the A-weighted sound pressure level of freight cars is reduced by about 11 dB.

Calculations of ISO 9613-2 do not include the case of barriers aloft, like shelter roofs at gas stations. If diffraction at a single edge is dominant for the received sound, the calculation should still be applicable, however. Contributions from more than one edge may be assumed as incoherent for sufficiently large barriers. Minimum dimensions must meet the requirements for reflectors.

### Reflectors and Reverberation

Any large object—a wall, roof, building, or road sign, for example—that has plane surfaces in the path of a sound ray and that blocks an area with Fresnel radius at least  $\sqrt{\lambda d_{so}}$  is considered to be a reflector. More specifically according to ISO 9613-2, the projection of the minimum dimension of the object in the direction of sound incidence shall meet the requirement

$$l_{\min} \cos \beta > \sqrt{\frac{2\lambda}{1/d_{so} + 1/d_{or}}} \quad (5.21)$$

where  $\beta$  is the angle of sound incidence on the reflecting plane and  $d_{so}$  and  $d_{or}$  are the path lengths from the reflection point to the source and to the receiver, respectively. Planes approximate nonplanar surfaces. Specular reflections are considered exclusively in terms of an image source (see Fig. 5.7).

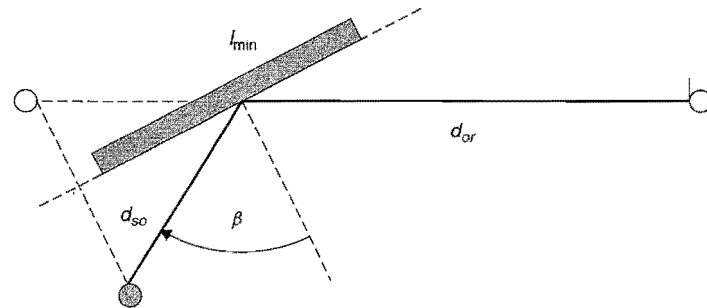


FIGURE 5.7 Specular reflection from a plane with minimum dimension  $l_{\min}$ .

A reflection coefficient\*  $R < 1$  of the reflector reduces the sound power level of the image source by  $-10 \lg(R)$  dB. Diffuse reflections from rough building facades, which result in scattering of sound toward the sky, are generally taken into account with a reflection loss of 1 or 2 dB. The directivity of a source needs to be considered in the direction of a receiver image.

### Atmospheric Absorption

The attenuation due to atmospheric absorption

$$A_{\text{atm}} = \alpha d \quad (5.22)$$

is theoretically well understood and fully described in ISO 9613-1.<sup>13</sup> For prediction purposes, one has to select the appropriate conditions of relative humidity, temperature, and ambient pressure. When low temperature and high humidity are assumed, the attenuation is relatively low around 500 Hz. In contrast, in the frequency bands at and above 4000 Hz, attenuation is high enough to reduce contributions to the A-weighted overall level to negligible values at distances of a few hundred meters. On the other hand,  $A_{\text{atm}}$  itself is mostly negligible in the frequency bands at and below 125 Hz. Table 5.2 contains selected values.

TABLE 5.2 Atmospheric Attenuation Coefficient  $\alpha$  for Octave Bands of Noise at Selected Nominal Midband Frequencies<sup>6</sup>

| Temperature,<br>°C | Relative<br>Humidity, % | Atmospheric Attenuation Coefficient $\alpha$ , dB/km |           |           |           |            |            |            |            |
|--------------------|-------------------------|------------------------------------------------------|-----------|-----------|-----------|------------|------------|------------|------------|
|                    |                         | 63<br>Hz                                             | 125<br>Hz | 250<br>Hz | 500<br>Hz | 1000<br>Hz | 2000<br>Hz | 4000<br>Hz | 8000<br>Hz |
| 10                 | 70                      | 0.1                                                  | 0.4       | 1.0       | 1.9       | 3.7        | 9.7        | 33         | 117        |
| 20                 | 70                      | 0.1                                                  | 0.3       | 1.1       | 2.8       | 5.0        | 9.0        | 23         | 77         |
| 15                 | 50                      | 0.1                                                  | 0.5       | 1.2       | 2.2       | 4.2        | 10.8       | 36         | 129        |
| 15                 | 80                      | 0.1                                                  | 0.3       | 1.1       | 2.4       | 4.1        | 8.3        | 24         | 83         |

\*ISO 9613-2 uses  $\rho$  instead of  $R$ .

### Effects of Ground Cover and Trees

The consideration of ground cover and trees is often excluded from engineering prediction of excess attenuation in the propagation path of sound. The reason is twofold. The effect is small and unreliable. The situation is not only subject to seasonal variations but may change with different land use.

However, much money can be wasted building roadway noise barriers longer than truly needed if tree attenuation is ignored. This happens when barrier lengths are needlessly extended to reduce sound coming from far up and down the roadway—needlessly because trees adequately reduce that flanking sound by themselves. Note that sound arriving perpendicularly from the roadway might typically pass through only 50 m of trees, whereas sound arriving from far up and down the roadway passes through 500–1000 m of trees—for significant attenuation.

## 5.5 INTERACTION OF ELEMENTS AND CONTRIBUTIONS FROM VARIOUS PATHS

### Standard Regulations for Consideration of Interactions

The attenuations discussed for individual elements in the propagation path of a sound ray are added to yield the total attenuation:

$$A = A_{\text{div}} + A_{\text{gr}} + A_{\text{bar}} + A_{\text{atm}} \quad (5.23)$$

where  $A_{\text{div}}$  = attenuation due to spherical spreading, predicted according to Eq. (5.5)

$A_{\text{gr}}$  = attenuation due to ground effects, predicted according to Eqs. (5.7)–(5.10) and (5.14)–(5.15)

$A_{\text{bar}}$  = attenuation due to diffraction at barrier edge, predicted according to Eqs. (5.16)–(5.20)

$A_{\text{atm}}$  = attenuation due to atmospheric absorption, predicted according to Eq. (5.22)

The only interaction taken into account is that of barriers and ground, as described by Eq. (5.17). Reflections are assumed to result in incoherent contributions at the receiver.

### Interaction of Barriers and Ground at Various Distances

Barriers on both sides of a roadway or railway provide at least two contributions to the total sound field at a receiver outside of the barriers—one by diffraction over the barrier at the receiver side and the second by reflection from the barrier on the other side—which may also suffer diffraction over the barrier on the receiver side (see Fig. 5.8). The second contribution is lower due to higher geometrical attenuation  $A_{\text{div}}$  but possibly higher due to lower barrier attenuation

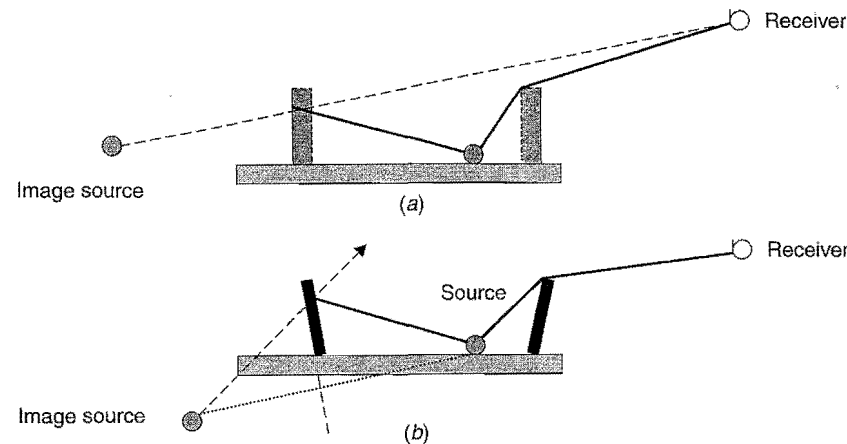


**FIGURE 5.8** Sound rays from traffic lane shielded on both sides by (a) absorptive barriers and (b) inclined barriers.

$A_{\text{bar}}$ . It generally needs to be reduced by absorptive or inclined barrier surfaces, which reduce the reflection toward the receiver.

Frequently discussed is the question of whether ground attenuation should be completely neglected for sound diffracted over the top of a barrier, as in Eq. (5.17). The reasoning to account for some ground attenuation in the barrier shadow zone is based on the assumption of an equivalent source at the diffracting edge. However, this assumption is not consistent with diffraction theory. There is no ray going out from the edge suffering attenuation just from divergence, but the diffracted field penetrates deeper and deeper into the shadow zone until it reaches the receiver. Since the ground effect on the receiver side is mostly determined by interference with a ground reflection close to the receiver, one has to consider two differently diffracted contributions, which are differently attenuated. Thus, the chance for considerable interference is reduced, particularly at high receiver positions. Of course, such an argument is valid only for meteorological conditions that provide no shadow formation. For greater accuracy at receiver positions near the ground, this interference is not neglected in the United States and Canada.

#### Barriers Close to Trees, with Gaps and Slots

The attenuation performance of barriers can be reduced by various influences. Past concern centered on trees or bushes that exceed the height of a nearby barrier. Experiments have shown that high-frequency components of noise are scattered by leaves into the barrier shadow zone.<sup>14</sup> However, this effect seems to be small compared to the better acceptability of "green" barriers.

Calculations with Eq. (5.18) assume that diffracted sound dominates in the barrier shadow zone. For this to be true, sound passing through the barrier must be negligible (down 5 dB or more) compared to the diffracted sound, which holds for a barrier relatively free of holes in its face and also free of a continuous gap between it and the ground. More sound energy gets through such holes/gaps than geometrically incident on the openings. Also required is a sufficiently large transmission loss through the surface material of the barrier. A mass per unit surface area (excluding framing) of  $20 \text{ kg/m}^2$  is usually sufficient to provide a transmission loss of 25 dB or more at 500 Hz. Such weight is easily achievable with thickness and materials needed for mechanical stability of the barrier.

#### Procedures Applied by Computer Software

Engineering computer software for noise mapping is based on geometrical data provided by geographical information systems. The ground is often described by plane triangles; roads, railways, and barriers by sections of straight lines; and individual sources by points. Houses are modeled by rectangles. The handling of numerous data for larger areas requires a high degree of sophistication.

A sound ray may experience several reflections and may be diffracted on the path from source to receiver. Because of computer speed, current computer software can deal with many reflections with reasonable computation times. However, current software is not necessarily precise, due to approximate modeling of reflectors and reflection losses. Normally only three reflections are taken into account. Former consideration of reverberation in street canyons is excluded from current computations.

An essential part of the software is its validation by test cases<sup>15</sup>. This task does not aim for physical correctness but for consistency with the referenced calculation schemes. For this purpose, the program must be run in the same mode for test cases as for real applications—for example, the setting of software switches for accelerated computation.

## 5.6 ENGINEERING APPROACHES TO ACCOUNT FOR METEOROLOGY

### Correction Terms $C_{\text{met}}$ and $C_0$

The transfer function of Eq. (5.2), as specified by the attenuation in Eq. (5.23), leads from source properties to the equivalent continuous A-weighted sound pressure level  $L_{AT}(\text{DW})$  for meteorological conditions that are favorable to sound propagation. This may be appropriate to meet specific restrictive requirements. A slightly better balance between interests of industry and neighborhood is the long-term A-weighted sound pressure level  $L_{AT}(\text{LT})$ , where the time interval  $T$  is several months or a year. Such a period will normally include a variety of sound propagation conditions, both favorable and unfavorable.

According to ISO 9613-2, a value  $L_{AT}(LT)$  may be obtained from the value of  $L_{AT}(DW)$  by subtraction of a meteorological correction term,

$$C_{met} = \begin{cases} 0 & \text{if } d \leq 10(h_S + h_R) \\ C_0 \left( 1 - 10 \frac{h_S + h_R}{d} \right) & \text{if } d > 10(h_S + h_R) \end{cases} \quad (5.24)$$

where  $C_0$  (in decibels) depends on local meteorological statistics for wind speed and directions plus temperature gradients. Similar to Eq. (5.7), this subtracted term depends on the ratio of the effective height of the sound ray and distance, so that Fig. 5.4 gives a qualitative impression, except for distance. The correction term  $C_{met}$  is applied at somewhat larger distances. Experience indicates that values of  $C_0$  are limited in practice between 0 and approximately 5 dB, with values in excess of 2 dB being exceptional.

### Local Weather Statistics

Among the various procedures proposed for the calculation of  $C_0$  from weather statistics is the following:

$$C_0(\alpha, g) = -10 \lg \left( \sum_{i=0}^{I-1} Q \frac{W_i(\theta_i)}{2} [1 + g - (1 - g) \cos(\theta_{rec} - \theta_i)] + 1 - Q \right) \text{ dB} \quad (5.25)$$

where  $\theta_{rec}$  = angle between North and the source-receiver line  
 $g$  = parameter between 0.01 and 0.1 that accounts for attenuation in upwind direction  
 $\theta_i$  = angle between North and  $i$ th wind direction  
 $W_i$  = probability for  $i$ th direction of wind  
 $I$  = number of wind directions  
 $1 - Q$  = probability for no wind (calm)

Values of  $g = 0.1$  and  $g = 0.01$  correspond to a maximum upwind attenuation of 10 and 20 dB, respectively. Crosswind attenuation is less than 3 dB. Such values are effective for single point sources and wind always from the same direction. For extended sources (e.g., traffic lines or larger industrial areas), the appropriate averaging procedure typically results in values slightly smaller than 2 dB.

## 5.7 UNCERTAINTIES

### General

The uncertainty of sound pressure levels calculated for a receiver position results from the uncertainty  $\Delta L_w$  of the source level, determined, among other matters, by uncertain assumptions about the operating conditions of a sound source, the number of vehicles on a road, the roughness of rail heads, and the uncertainty

$\Delta A$  of the propagation losses. In the following, the uncertainty  $\Delta A$  is discussed exclusively. However, note that uncertainties about source levels cannot always be neglected.

### Elements

Uncertainty about spreading losses is generally small. Except for air traffic, distances between source and receiver can be determined very precisely. Effects of road traffic distribution on several parallel lanes are generally small if the most inner and outer lanes are modeled separately.

Uncertainty is significantly affected by effects of ground and meteorology. Very pronounced uncertainty has been obtained by Schomer<sup>16</sup> from long-term measurements with a loudspeaker centered at a height of 0.6 m above grassland and receivers at 1.2 m at distances up to 800 m. The standard deviation describing high portions of received octave-band levels is shown in Fig. 5.9.

At low frequencies up to 125 Hz and also at high frequencies above 2000 Hz, the standard deviation increases approximately continuously with distance and frequency, as one would expect. Anomalies can be seen in the intervening frequency bands. These anomalies can be attributed to variable ground interference. The low measurement height of 1.2 m may exaggerate the effect, though this receiver height is important to computations in the United States and Canada.

For special barrier-top design, a considerable uncertainty about barrier attenuation is found in the comparison of standard calculations with results calculated or measured in laboratories. However, in field tests such designs have hardly ever shown level differences of more than 1 dB at a distance of more than 25 m from a roadside or railway barrier. Source or receiver positions close to a barrier,

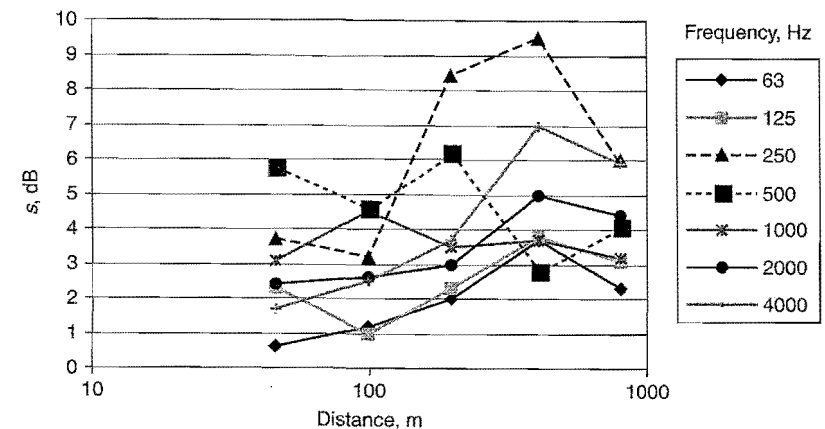


FIGURE 5.9 Standard deviation  $s$  of octave-band sound pressure levels for downwind condition at various distances from a loudspeaker centered at a height of 0.6 m and receiver height 1.2 m above grassland.

where reflections from the barrier surface or deep shadow zones play a role, are rarely met in practice, and therefore the laboratory effects are not observed in the field.

There is substantial uncertainty about the validity of the factor  $C_3$  in Eq. (5.19). In many laboratory cases, multiple diffraction has shown an increase in barrier attenuation by more than 5 dB. But these cases are often related to barrier attenuations in excess of 20 dB. In practice, they depend significantly upon specific propagation paths from source to receiver.

Significant uncertainty exists for the meteorological correction factor of Eq. (5.20). It is matched to numerous data obtained for downwind situations at distances up to 1000 m over porous ground. Estimates for this uncertainty are about 2 dB, independent of distance, since the magnitude of barrier attenuation decreases with distance.

Uncertainty due to the roughness and absorption of reflecting surfaces is approximately 1 dB. In addition, limiting the number of reflections to three is sometimes not sufficient to let even a single computed sound ray enter through a passageway into a courtyard. It is obvious from such considerations that ray theory and specular reflections are limited to relatively open areas. Otherwise, standard deviations of more than 5 dB may occur. It is this consideration that partially argues for the U.S. and Canadian retention of line segment sources instead of point source approximations.

Uncertainty about atmospheric absorption may be calculated from the uncertainties of relative humidity, temperature, and ambient pressure. At low frequencies, that uncertainty is very small. At high frequencies, it is of little concern for engineering purposes. At frequencies around 500 Hz, the standard deviation is usually small compared to that of ground effects.

### Overall A-Weighted Sound Pressure Level

The uncertainty for sound attenuation along the path of a ray in any frequency band increases with the square root of the sum of variances due to geometry, ground effects, barriers, reflectors, and atmospheric absorption. When several rays contribute to the sound pressure level at the receiver, the variances must be weighted according to their energy and the sum is divided by the total energy. This summation of uncorrelated partial uncertainties results in a reduction of the total uncertainty.

The uncertainty for the A-weighted overall sound pressure level may be calculated from the variances in octave bands, weighted according to their energy. When a number of  $n$  frequency bands equally contribute to the overall level, the uncertainty is reduced by a factor  $1/\sqrt{n}$ .

Approximately consistent with estimated accuracies in ISO 9613-2 is the standard deviation reported by Schomer<sup>16</sup> for the A-weighted overall sound pressure level received from a source of pink noise.<sup>14</sup> At distances from 100 to 800 m and an average height above grassland of 0.9 m, that standard deviation was about 3 dB. At an average height above 5 m, ISO 9613-2 estimates an accuracy

of 1 dB. Advanced procedures for calculation of outdoor noise levels aim for an accuracy with a standard deviation of 1 dB up to a distance of 1000 m over flat terrain and of 2.5 dB over hilly terrain.<sup>3</sup>

### REFERENCES

1. Lord Rayleigh, *Theory of Sound*, 2nd ed., Dover, New York, 1877, reissued 1945, Vol. 2, p. 128 ff.
2. W. A. Kuperman, "Propagation of Sound in the Ocean," in M. J. Crocker (Ed.), *Encyclopedia of Acoustics*, Wiley, New York, 1997, Chapter 36.
3. F. de Roo and I. M. Noordhoek, "Harmonoise WP2—Reference Sound Propagation Model," Proc. DAGA 03, Aachen, Germany, pp. 354–355.
4. K. Attenborough et al., "Benchmark Cases for Outdoor Sound Propagation," *J. Acoust. Soc. Am.*, **97**(1), 173–191 (1995).
5. R. Matuschek and V. Mellert, "Vergleich von technischen Prognoseprogrammen für die Schallimmission mit physikalischen Berechnungen der Schallausbreitung im Freien" ("Comparison of Technical Prediction Programs for Sound Immission with Physical Calculations of Sound Propagation Outdoors"), Proc. DAGA 03, Aachen, Germany, pp. 428–429.
6. ISO 9613-2, "Acoustics—Attenuation of Sound During Propagation Outdoors—Part 2: General Method of Calculation," International Organization for Standardization, Geneva, Switzerland, 1996.
7. H. Gidamy, C. T. Blaney, C. Chiu, J. E. Coulter, M. Delint, L. G. Kende, A. D. Lightstone, J. D. Quirt, and V. Schroter, "ORNAMENT: Ontario Road Noise Analysis Method for Environment and Transportation," Environment Ontario, Noise Assessment and Systems Support Unit, Advisory Committee on Road Traffic Noise, 1988.
8. L. C. Sutherland and G. A. Daigle, "Atmospheric Sound Propagation," in M. J. Crocker (Ed.), *Encyclopedia of Acoustics*, Wiley, New York, 1997, Chapter 32.
9. J. Kragh, "A New Meteo-Window for Measuring Environmental Noise from Industry," Technical Report LI 359/93, Danish Acoustical Institute, Lyngby, Denmark, 1993.
10. A. L'Espérance, P. Herzog, G. A. Daigle, and J. R. Nicolas, "Heuristic Model for Outdoor Sound Propagation Based on an Extension of the Geometrical Ray Theory in the Case of a Linear Sound Speed Profile," *Appl. Acoust.* **37**, 111–139, (1992).
11. G. S. Anderson, C. S. Y. Lee, G. G. Fleming, and C. W. Menge, *FHWA Traffic Noise Model, Version 1.0: User's Guide*, Report FHWA-PD-96-009 and DOT-VNTSC-FHWA-98-1, U.S. Department of Transportation, Federal Highway Administration, Washington, DC, January 1998.
12. J. L. Rochat and G. G. Fleming, *Validation of FHWA's Traffic Noise Model (TNM): Phase 1, Addendum*, Addendum to Report No. FHWA-EP-02-031 and DOT-VNTSC-FHWA-02-01 U.S. Department of Transportation, Federal Highway Administration, Washington, DC (in publication).
13. ISO 9613-1, "Acoustics—Attenuation of Sound During Propagation Outdoors—Part 1: Calculation of the Absorption of Sound by the Atmosphere," International Organization for Standardization, Geneva, Switzerland, 1993.
14. T. van Renterghem and D. Botteldooren, "Effect of a Row of Trees Behind Noise Barriers in Wind," *Acta Acustica united with Acustica*, **88**, 869–878 (2002).

15. DIN 45687, "Acoustics—Software Products for the Calculation of the Sound Propagation Outdoors—Quality Requirements and Test Conditions" (draft), Deutsches Institut für Normung, Berlin, Germany, 2004.
16. P. D. Schomer, "A Statistical Description of Ground-to-Ground Sound Propagation," *Noise Control Eng. J.*, 51(2), 69–89 (2003).

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**CHAPTER 6**

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## Sound in Small Enclosures

DONALD J. NEFSKE AND SHUNG H. SUNG

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Warren, Michigan

### 6.1 INTRODUCTION

Only in an anechoic room may sound waves travel outwardly in any direction without encountering reflecting surfaces. In practice, one must deal with every shape and size of enclosure containing an infinite variety of sound-diffusing, sound-reflecting, and sound-absorbing objects and surfaces. However, in small rooms of fairly regular shape, with smooth walls, the sound is not statistically diffuse and will depend on the acoustical modal response in the room. Typical examples where this is the case are the passenger compartments of transportation vehicles, ductwork and small rooms in buildings, enclosures used to enhance the response of audio equipment, and enclosures designed for sound isolation. Because of the need to understand the modal nature of sound in such enclosures, this chapter will describe the governing equations, the modal theory, and its application for determining and controlling the interior acoustical response. While the modal theory for rectangular enclosures is described in most standard textbooks on acoustics,<sup>1–5</sup> the extended approach for irregular geometries and the use of advanced numerical techniques are more amenable to practical application and are also described here.

### 6.2 SOUND IN A VERY SMALL ENCLOSURE

Before considering the more general case, it will be instructive to consider the sound pressure response in a very small enclosure. Frequently, in noise control problems, a noise source is enclosed in a very small box to prevent it from radiating noise to the exterior, as conceptually represented in Fig. 6.1a. When the noise source has a frequency low enough so that the wavelength of the

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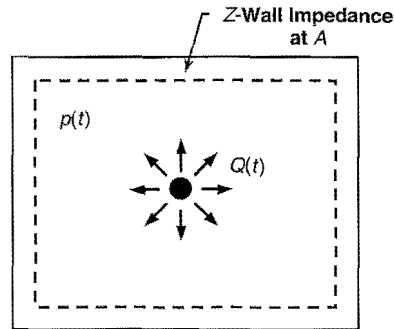
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## 6.1 INTRODUCTION

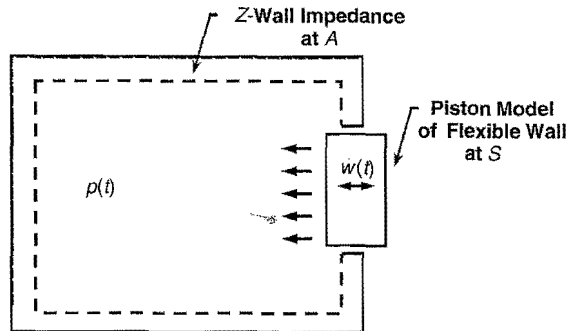
Only in an anechoic room may sound waves travel outwardly in any direction without encountering reflecting surfaces. In practice, one must deal with every shape and size of enclosure containing an infinite variety of sound-diffusing, sound-reflecting, and sound-absorbing objects and surfaces. However, in small rooms of fairly regular shape, with smooth walls, the sound is not statistically diffuse and will depend on the acoustical modal response in the room. Typical examples where this is the case are the passenger compartments of transportation vehicles, ductwork and small rooms in buildings, enclosures used to enhance the response of audio equipment, and enclosures designed for sound isolation. Because of the need to understand the modal nature of sound in such enclosures, this chapter will describe the governing equations, the modal theory, and its application for determining and controlling the interior acoustical response. While the modal theory for rectangular enclosures is described in most standard textbooks on acoustics,<sup>1–5</sup> the extended approach for irregular geometries and the use of advanced numerical techniques are more amenable to practical application and are also described here.

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(a) Interior Noise Source



(b) Vibrating Flexible Wall

**FIGURE 6.1** Sound in small enclosure with impedance boundaries generated by (a) interior noise source and (b) piston model of flexible-wall vibration. Symbols:  $p(t)$ , sound pressure;  $Q(t)$ , volume velocity;  $Z$ , wall impedance;  $A$ , impedance wall area;  $\dot{w}(t)$ , piston velocity;  $S$ , piston area.

sound is long compared to the largest distance of the box, the sound pressure produced by the source will be uniform throughout the entire cavity. A uniform sound pressure field will also be created in an enclosure when the walls of the enclosure are forced to vibrate at low frequency by the pressures of an external sound source, as conceptually represented by the vibrating piston in Fig. 6.1b. Frequently, the walls of the enclosure will be damped by applying panel-damping treatment or absorption material over the walls.

The uniform sound pressure  $p(t)$  in such an enclosure will satisfy an extended form of Eq. (2.9) as  $(1/P)dp/dt + (\gamma/V)d\tau/dt = (\gamma/V)Q(t)$ , where  $Q(t)$  is the volume velocity of the sound source ( $\text{m}^3/\text{s}$ ),  $V$  is the enclosure volume,  $P$  is the ambient air pressure, and  $\tau(t)$  is the volume variation. At the wall area  $A$  ( $\text{m}^2$ )

in Fig. 6.1a or b, the specific acoustical impedance is  $Z = p/u$  and the particle velocity is  $u(t)$ , so  $d\tau/dt = Au = Ap/Z$ . Since  $c^2 = \gamma P/\rho$  by Eq. (2.12), the uniform sound pressure in the enclosure satisfies

$$\frac{dp}{dt} + 2\delta p = \left(\frac{\rho c^2}{V}\right) Q(t) \quad (6.1)$$

where  $\delta = \rho c^2 A/(2VZ)$ . For steady-state excitation,  $Q(t) = \hat{Q} \cos(\omega t + \phi)$ , which is the volume velocity of the source operating at the forcing frequency  $f = \omega/2\pi$  (hertz) with the volume velocity amplitude  $\hat{Q}$  and phase  $\phi$ . The steady-state sound pressure in the enclosure is then  $p(t) = p_0 \cos(\omega t + \theta_0)$ , with the sound pressure amplitude and phase given by

$$p_0 = \frac{\rho c^2 \hat{Q}}{V[(\omega + 2\delta_r)^2 + (2\delta_i)^2]^{1/2}} \quad \text{N/m}^2 \quad (6.2)$$

$$\theta_0 = \phi - \tan^{-1} \frac{\omega + 2\delta_r}{2\delta_i} \quad \text{rad}$$

Here  $\delta = \delta_r + i\delta_i$  is a complex damping factor ( $i = \sqrt{-1}$ ) that accounts for the acoustical impedance  $Z = R + iX$  of the wall area  $A$ , where

$$\delta_r = \frac{cA}{2V} \text{Re} \frac{\rho c}{Z} \quad \delta_i = \frac{cA}{2V} \text{Im} \frac{\rho c}{Z} \quad \text{s}^{-1} \quad (6.3)$$

For the interior noise source in Fig. 6.1a,  $Q(t)$  is taken as positive for outward volume flow, while for the flexible-wall vibration in Fig. 6.1b,  $Q(t) = -S\dot{w}(t)$  is an equivalent volume velocity with outward piston velocity  $\dot{w}(t)$  taken as positive.

From Eq. (6.2), we see that the amplitude  $p_0$  of the sound pressure in the enclosure depends, not only on the noise source amplitude  $\hat{Q}$  and forcing frequency  $f = \omega/2\pi$ , but also on the enclosure volume  $V$  and total wall impedance in  $\delta$ . For a rigid-wall enclosure,  $|Z| \rightarrow \infty$ , so that  $\delta_r, \delta_i = 0$ , and

$$p_0 = \frac{\rho c^2 \hat{Q}}{\omega V} \quad \text{N/m}^2 \quad (6.4)$$

$$\theta_0 = \phi - \frac{1}{2}\pi \quad \text{rad}$$

In this case, the sound pressure depends only on the amplitude and frequency of the volume velocity of the source and on the enclosure volume. The sound pressure lags the volume velocity by exactly  $90^\circ$ , indicating that the source does not radiate any acoustical power.

**Example 6.1.** The piston of Fig. 6.1b has an area of  $1 \text{ cm}^2$  and is driven harmonically with a peak-to-peak displacement of  $4 \text{ mm}$  at a frequency of  $100 \text{ Hz}$ .

The volume of the cavity is  $0.0125 \text{ m}^3$  and it has rigid walls. What is the sound pressure level in the cavity?

**Solution** For harmonic displacement excitation  $w = \hat{w} \sin \omega t$ , and we obtain  $\dot{w} = (\omega \hat{w}) \cos \omega t = \hat{\dot{w}} \cos \omega t$ , so that  $\hat{\dot{w}} = \omega \hat{w}$ . Therefore,  $\hat{Q} = S \hat{\dot{w}} = S \omega \hat{w}$  and from Eq. (6.4),

$$p_0 = \frac{\rho c^2 S \hat{w}}{V} = \rho c^2 \frac{\Delta \hat{V}}{V} \quad \text{N/m}^2 \quad (6.5)$$

where  $\Delta \hat{V} = S \hat{w}$  is the volume change of the enclosure due to the piston displacement. Inserting the appropriate numerical values, we have  $\Delta \hat{V} = 10^{-4} \times 2 \times 10^{-3} = 2 \times 10^{-7} \text{ m}^3$ , so that

$$p_0 = 1.21 \times 343^2 \times \frac{2 \times 10^{-7}}{1.25 \times 10^{-2}} = 2.28 \text{ N/m}^2$$

The sound pressure level is then

$$L_p = 20 \log_{10} \frac{p_{\text{rms}}}{p_{\text{ref}}} = 20 \log_{10} \frac{0.707 \times 2.28}{2 \times 10^{-5}} = 98 \text{ dB}$$

**Example 6.2.** A noise source is enclosed in a very small box, as in Fig. 6.1a, that has flexible but very stiff walls. Determine the formula for the interior sound pressure.

**Solution** For a mass-spring-dashpot model of the box walls, the acoustical impedance is  $Z = A^{-1}[C + i\omega(M - K/\omega^2)]$ , where  $A$  is the wall surface area and  $C, M, K$  are overall values of wall damping, mass, and stiffness. For very stiff walls,  $Z \approx -iK/A\omega$  so that, from Eq. (6.3),

$$\delta_r = 0 \quad \delta_i = \frac{\omega \rho c^2 A^2 / V}{2 K} = \frac{\omega K_{\text{air}}}{2 K} \text{ s}^{-1}$$

where  $K_{\text{air}} = \rho c^2 A^2 / V$  (N/m) is the *stiffness* of the air and  $K$  (N/m) is the wall stiffness. Substituting  $\delta_r$  and  $\delta_i$  into Eq. (6.2) then gives

$$p_0 = \frac{1}{1 + K_{\text{air}}/K} \frac{\rho c^2 \hat{Q}}{\omega V} \quad \text{N/m}^2 \quad (6.6)$$

$$\theta_0 = \phi - \frac{1}{2}\pi \quad \text{rad}$$

Note that the sound pressure in an enclosure with compliant walls is lower than the sound pressure in an equivalent rigid-wall enclosure given by Eq. (6.4), and the addition of damping to the walls can be shown to produce a similar effect.

**Example 6.3.** A *Helmholtz resonator* is a rigid-wall enclosure with a small aperture of cross-sectional area  $S$  that connects the enclosure to a column of air of length  $L$  that oscillates as the piston in Fig. 6.1b. Determine the natural frequency.

**Solution** The mass of the column of air is  $M = \rho SL$  and the force on it is  $pS$ . Hence we must have  $M\ddot{w} = pS$ , where  $\dot{w}$  is the velocity of the air column. Since  $\hat{\dot{w}} = \omega \hat{w}$ , we obtain  $\hat{Q} = S \hat{\dot{w}} = S^2 p_0 / \omega M$ . Substituting this for  $\hat{Q}$  in Eq. (6.4) gives  $(1 - \rho c^2 S^2 / \omega^2 M V) p_0 = 0$  or  $(1 - K_{\text{air}} / \omega^2 M) p_0 = 0$ , where  $K_{\text{air}} = \rho c^2 S^2 / V$  is the air stiffness. The natural frequency is then  $\omega_0 = \sqrt{K_{\text{air}} / M} = c \sqrt{S / LV}$ .

### 6.3 GOVERNING EQUATIONS FOR ACOUSTICAL MODAL RESPONSE

For larger enclosures or for higher frequencies, the sound pressure field in the enclosure is no longer uniform but depends on the acoustical modal response in the enclosure. More importantly, the sound pressure can be amplified considerably near discrete frequencies corresponding to the acoustical cavity resonances. This modal nature of sound in an enclosure results from the superposition of sound waves that propagate according to the well-known *acoustical wave equation* [Eq. (2.13)],

$$\nabla^2 p - \frac{\ddot{p}}{c^2} = 0 \quad \text{N/m}^4 \quad (6.7)$$

where  $\ddot{p}$  denotes the second partial derivative with respect to time  $t$ .

Noise sources interior to an enclosed cavity can be included as forcing terms in the wave equation. For the example of a monopole source (e.g., a loudspeaker in a cabinet) as in Fig. 6.1a, the time-varying mass flow rate is  $\dot{m}(x, y, z, t) = \rho \dot{Q}(x, y, z, t)$  (kg/s), so that

$$\nabla^2 p - \frac{\ddot{p}}{c^2} = -\frac{\rho \dot{Q}}{V} \quad \text{N/m}^4 \quad (6.8)$$

where  $\dot{Q} = \partial Q / \partial t$ . Other interior sources can be represented as combinations of monopole sources or else can be included directly in the wave equation in a similar manner. For simple harmonic motion,  $p(x, y, z, t) = \text{Re}[\hat{p}(x, y, z) \exp(i\omega t)]$  and  $\dot{Q}(x, y, z, t) = \text{Re}[\hat{\dot{Q}}(x, y, z) \exp(i\omega t)]$ , and we obtain the inhomogeneous *Helmholtz equation* for the steady-state sound pressure response,

$$\nabla^2 \hat{p} + \left(\frac{\omega}{c}\right)^2 \hat{p} = -\frac{i\omega \rho \hat{\dot{Q}}}{V} \quad \text{N/m}^4 \quad (6.9)$$

where  $f = \omega / 2\pi$  (Hz) is the forcing frequency of the vibration and  $\lambda = c / f$  (m) is the wavelength of the sound produced by the source.

The boundary conditions for  $p$  determine the reflection, absorption, and transmission of the sound waves at the enclosure's surfaces and are derived from fluid

mechanical considerations.<sup>6</sup> For small-amplitude motions, a momentum balance at the boundary requires that the air particle velocity  $u$  normal to the boundary surface be related to  $p$  through

$$\frac{1}{\rho} \frac{\partial p}{\partial n} = -\dot{u} \quad \text{m/s}^2 \quad (6.10)$$

where  $\partial/\partial n$  is the outward surface-normal derivative. For an impervious wall surface,  $u$  is the normal-velocity component of the air into the pores of the surface. Table 6.1 lists the boundary conditions for different wall surfaces and air interfaces that are characterized by their acoustical impedance  $Z$  and surface-normal vibration velocity  $\dot{w}$ . In what follows, we will investigate the steady-state, random, and transient sound pressure response in enclosures with flexible, absorbent boundaries where  $Z$  and  $\dot{w}$  are defined as in Table 6.1.

#### 6.4 NATURAL FREQUENCIES AND MODE SHAPES

Acoustical resonances that result in high sound pressure occur at discrete natural frequencies in an enclosed cavity. The acoustical resonances are found from the free-vibration solution of the wave equation by substituting  $p_n = p_{n0} \Psi_n(x, y, z) \exp(i\omega_n t)$ , which gives

$$\nabla^2 \Psi_n + \left(\frac{\omega_n}{c}\right)^2 \Psi_n = 0 \quad \text{m}^{-2} \quad (6.11)$$

with free-vibration boundary conditions for the walls. The nondimensional pressure distributions  $\Psi_n(x, y, z)$  for  $n = 0, 1, 2, \dots$  are the *mode shapes*, with  $f_n = \omega_n/2\pi$  (Hz) being the corresponding *natural frequencies*. For absorbent boundaries, the mode shapes and natural frequencies are complex, and the acoustical modes are damped. However, for rigid boundaries ( $|Z| \rightarrow \infty$ ) or for fully reflective open boundaries ( $|Z| = 0$ ), one obtains real, undamped modes, or *standing waves*, that depend only on the geometrical shape of the cavity.

Table 6.2 gives formulas for the natural frequencies and mode shapes of the undamped acoustical modes of a few regular-shape enclosures with rigid walls as well as those of a tube with fully reflective and open boundaries. (A more complete list can be found in ref. 7.) One can show that the acoustical modes are orthogonal over the cavity volume such that

$$\int_V \Psi_m \Psi_n dV = \begin{cases} 0 & m \neq n \\ V_n & m = n \end{cases} \quad \text{m}^3 \quad (6.12)$$

and constitute *normal modes* of the enclosure. In Table 6.2, the number of indices used to identify each mode is based on the dimensionality of the enclosure, whereas we have used a single index to identify a mode, so the equivalence for a three-dimensional enclosure would be  $f_n \equiv f_{ijk}$  and  $\Psi_n \equiv \Psi_{ijk}$ .

TABLE 6.1 Acoustical Boundary Conditions

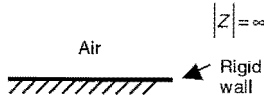
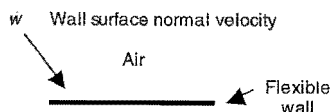
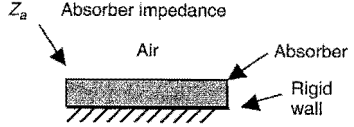
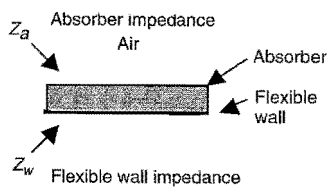
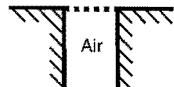
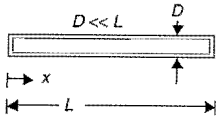
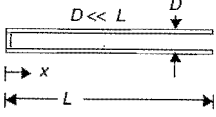
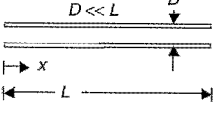
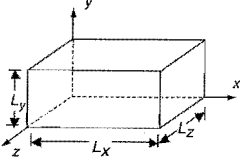
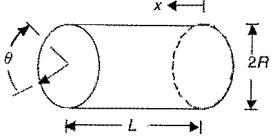
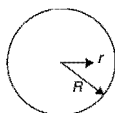
| Type                         | Boundary Condition                                                                                                                                                                                                                                                                                           | Air Particle Velocity                                              |
|------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------|
| 1. Rigid wall                |  $\frac{\partial p}{\partial n} = 0$                                                                                                                                                                                      | $u = 0$                                                            |
| 2. Flexible wall             |  $\frac{1}{\rho} \frac{\partial p}{\partial n} = -\ddot{w}$                                                                                                                                                               | $u = \dot{w}$                                                      |
| 3. Absorber on rigid wall    |  $\frac{1}{\rho} \frac{\partial p}{\partial n} = -\frac{1}{Z_a} \frac{\partial p}{\partial t}$                                                                                                                            | $u = \frac{p}{Z_a}$                                                |
| 4. Absorber on flexible wall |  $\begin{aligned} \frac{1}{\rho} \frac{\partial p}{\partial n} &= -\frac{1}{Z_a} \frac{\partial p}{\partial t} - \ddot{w} \\ &= -\left(\frac{1}{Z_a} + \frac{1}{Z_w}\right) \frac{\partial p}{\partial t} \end{aligned}$ | $u = \frac{p}{Z_a} + \dot{w}$<br>$= \frac{p}{Z_a} + \frac{p}{Z_w}$ |
| 5. Open pressure release     |  $p = 0$                                                                                                                                                                                                                | Determine from analysis                                            |
| 6. Open plane wave           |  $\frac{1}{\rho} \frac{\partial p}{\partial n} = -\frac{1}{Z_{\text{air}}} \frac{\partial p}{\partial t}$                                                                                                               | $u = \frac{p}{Z_{\text{air}}}$                                     |

TABLE 6.2 Acoustical Modes and Natural Frequencies<sup>a</sup>

| Description                                 | Figure                                                                             | Natural Frequency, $f_{ijk}$ (Hz)                                                                                               | Mode Shape, $\Psi_{ijk}$                                                                                                                                              |
|---------------------------------------------|------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. Slender tube both ends closed            |   | $\frac{ic}{2L}$<br>$D \ll \lambda$ where $\lambda = c/f$                                                                        | $\cos \frac{i\pi x}{L}$ $i = 0, 1, 2, \dots$                                                                                                                          |
| 2. Slender tube one end closed one end open |   | $\frac{ic}{4L}$<br>$D \ll \lambda$ where $\lambda = c/f$                                                                        | $\cos \frac{i\pi x}{L}$ $i = 1, 3, 5, \dots$                                                                                                                          |
| 3. Slender tube both ends open              |   | $\frac{ic}{2L}$<br>$D \ll \lambda$ where $\lambda = c/f$                                                                        | $\sin \frac{i\pi x}{L}$ $i = 1, 2, 3, \dots$                                                                                                                          |
| 4. Closed rectangular volume                |   | $\frac{c}{2} \left( \frac{i^2}{L_x^2} + \frac{j^2}{L_y^2} + \frac{k^2}{L_z^2} \right)^{1/2}$                                    | $\cos \frac{i\pi x}{L_x} \cos \frac{j\pi y}{L_y} \cos \frac{k\pi z}{L_z}$<br>$i, j, k = 0, 1, 2, \dots$                                                               |
| 5. Closed cylindrical volume                |  | $\frac{c}{2\pi} \left( \frac{\lambda_{jk}^2}{R^2} + \frac{i^2\pi^2}{L^2} \right)^{1/2}$<br>$\lambda_{jk}$ from Table 6.2a below | $J_j \left( \lambda_{jk} \frac{r}{R} \right) \cos \frac{i\pi x}{L} \begin{cases} \sin j\theta \\ \text{or} \\ \cos j\theta \end{cases}$<br>$i, j, k = 0, 1, 2, \dots$ |

## Modes Symmetric about Center

6. Closed spherical volume



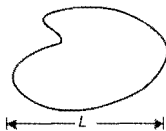
$$\frac{\lambda_i c}{2\pi R}$$

 $\lambda_i$  from Table 6.2b below

$$\frac{R}{\lambda_i r} \sin \frac{\lambda_i r}{R}$$

$$i = 0, 1, 2, \dots$$

7. Arbitrary closed volume



$L$ —Maximum linear dimension  
fundamental natural frequency  
(approximate):

$$\frac{c}{2L}$$

Finite-element analysis  
Boundary element analysis

Table 6.2a

| $\lambda_{jk}$                                                               | $j$     |         |         |         |         |         |         |
|------------------------------------------------------------------------------|---------|---------|---------|---------|---------|---------|---------|
| $k$                                                                          | 0       | 1       | 2       | 3       | 4       | 5       | 6       |
| 0                                                                            | 0       | 1.8412  | 3.0542  | 4.2012  | 5.3176  | 6.4156  | 7.5013  |
| 1                                                                            | 3.8317  | 5.3314  | 6.7061  | 8.0152  | 9.2824  | 10.5199 | 11.7349 |
| 2                                                                            | 7.0156  | 8.5363  | 9.9695  | 11.3459 | 12.6819 | 13.9872 | 15.2682 |
| 3                                                                            | 10.1730 | 11.7060 | 13.1704 | 14.5859 | 15.9641 | 17.3128 | 18.6374 |
| $\lambda_{j=0,k} = \pi(k + 1/4)$ for $k \geq 3$ ( $J'_j(\lambda_{jk}) = 0$ ) |         |         |         |         |         |         |         |

Table 6.2b

| $i$                                                                        | 0 | 1      | 2      | 3       | 4       |
|----------------------------------------------------------------------------|---|--------|--------|---------|---------|
| $\lambda_i$                                                                | 0 | 4.4934 | 7.7253 | 10.9041 | 14.0662 |
| $\lambda_i = \pi(i + 1/2)$ for $i \geq 4$ ( $\tan \lambda_i = \lambda_i$ ) |   |        |        |         |         |

<sup>a</sup>From Ref. 6.

It is noteworthy that the first natural frequency of a cavity that is completely enclosed by rigid walls is zero,  $f_0 = 0$ , and is a *uniform pressure mode* (sometimes called a *Helmholtz mode*) with  $\Psi_0 = 1$  and

$$V_0 = \int_V \Psi_0^2 dV = V \quad \text{m}^3 \quad (6.13)$$

where  $V$  is the volume of the enclosure. Also, the fundamental frequency of the first spatially varying mode in an enclosed cavity is approximately  $f_1 = c/2L$  (Hz), where  $L$  is the maximum linear dimension of the cavity.

For a cavity with an open boundary, the  $n = 0$  mode is absent because a (nonzero) uniform pressure cannot be sustained. Also, the natural frequencies and mode shapes in Table 6.2 for a tube with an open boundary are approximate because the boundary condition ( $p \approx 0$ ) does not fully model the exact physics of the acoustical behavior at an open boundary. The accuracy of the formulas in Table 6.2 for an open tube increases with increasing slenderness of the tube. In general, the dimension of the open boundary must be small compared with the acoustical wavelength so that the sound is fully reflected from the open boundary.

**Example 6.4.** A closed rectangular cavity has dimensions  $0.41 \times 0.51 \times 0.61$  m. From Table 6.2, frame 4, the formulas for the normal-mode frequencies and mode shapes are

$$f_{ijk} = \frac{c}{2} \sqrt{\left(\frac{i}{L_x}\right)^2 + \left(\frac{j}{L_y}\right)^2 + \left(\frac{k}{L_z}\right)^2} \quad \text{Hz} \quad (6.14)$$

$$\Psi_{ijk} = \cos \frac{i\pi x}{L_x} \cos \frac{j\pi y}{L_y} \cos \frac{k\pi z}{L_z}$$

Let  $L_x = 0.61$  m,  $L_y = 0.51$  m, and  $L_z = 0.41$  m. (a) Find the natural frequencies of the  $i = 2, j = 0, k = 0$ ; the  $i = 1, j = 1, k = 0$ ; and the  $i = 2, j = 1, k = 0$  normal modes of vibration. (b) Plot the sound pressure distribution for these three normal modes of vibration. The speed of sound, adjusted for temperature, is 347.3 m/s.

### Solution

(a) From Eq. (6.13) we have

$$f_{2,0,0} = \frac{347.3}{2} \sqrt{\left(\frac{2}{0.61}\right)^2} = 569.3 \text{ Hz}$$

$$f_{1,1,0} = \frac{347.3}{2} \sqrt{\left(\frac{1}{0.61}\right)^2 + \left(\frac{1}{0.51}\right)^2} = 443.8 \text{ Hz}$$

$$f_{2,1,0} = \frac{347.3}{2} \sqrt{\left(\frac{2}{0.61}\right)^2 + \left(\frac{1}{0.51}\right)^2} = 663.4 \text{ Hz}$$

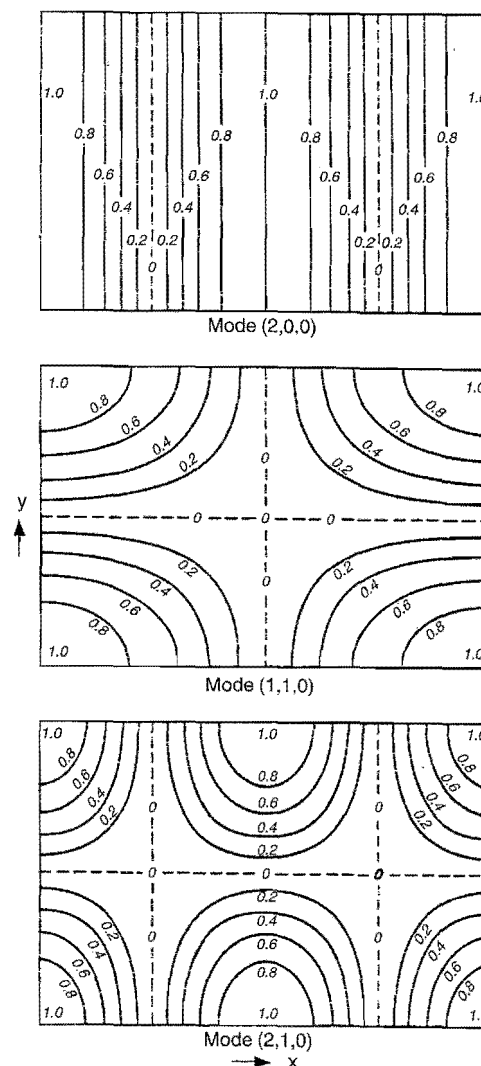


**FIGURE 6.2** Sound pressure contour plots of a section through a rectangular enclosure for three different modes of vibration. The numbers on the plots indicate the relative sound pressure amplitude.

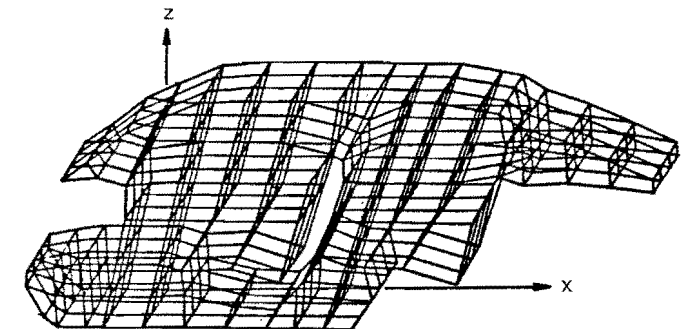
(b) The pressure distributions  $|\Psi_{ijk}|$  of the modes are shown in Fig. 6.2, where the normalization is such that the maximum pressure at the corners is  $|\Psi_{ijk}| = 1$ . The minimum sound pressure occurs at the nodal surfaces for which  $|\Psi_{ijk}| = 0$ . Since  $k = 0$  for all three modes, the pressure distributions are uniform in the  $z$  direction.

### 6.5 NUMERICAL METHODS FOR ACOUSTICAL ANALYSIS

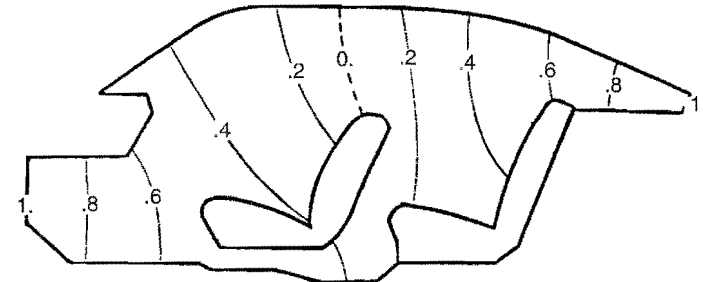
For simple geometries and boundary conditions, the acoustical modes can be expressed analytically as in Table 6.2, but for more complicated geometries and boundary conditions numerical approaches such as the finite-element method or the boundary-element method are required.<sup>8</sup> The finite-element method is based upon the representation of the interior of the enclosure by a finite number of interconnected *volume* elements. The boundary-element method is based upon the representation of only the boundary surface of the enclosure by a finite number of interconnected *planar* elements. While both the finite-element method and the boundary-element method were originally developed for structural analysis, they are applicable to numerous nonstructural problems, and the acoustical cavity problem falls into this category. The major advantage of the boundary-element method is the reduction in modeling effort that is required because only the surface of the enclosure needs to be meshed. On the other hand, the finite-element method generally results in banded, frequency-independent, real matrices that require significantly less computational time when solving for the acoustical response.

To illustrate a numerical acoustical modal analysis of a complex-shaped enclosure, Fig. 6.3 shows the application of the acoustical finite-element method to an automobile passenger compartment. The acoustical finite-element model in Fig. 6.3a was developed from linear hexahedral and pentahedral elements, where only a half-model is required because of symmetry.<sup>9</sup> The model can be solved to obtain the acoustical modes by implementing a structural finite-element code and using the *structural-acoustical analogy*<sup>10</sup> outlined in Table 6.3. By specifying the structural material properties in terms of the acoustical material properties as shown in Table 6.3, the structural equations of motion reduce to the acoustical wave equation and the structural boundary conditions reduce to the required acoustical boundary conditions. The accuracy of the modal solution will be proportional to  $(n/N)^2$  when using linear elements, where  $n$  is the mode number and  $N$  is the number of elements in a particular direction. Generally, accuracy to within 10% in frequency can be expected for the first four modes in a particular direction when there are about 10 linear elements in that direction.

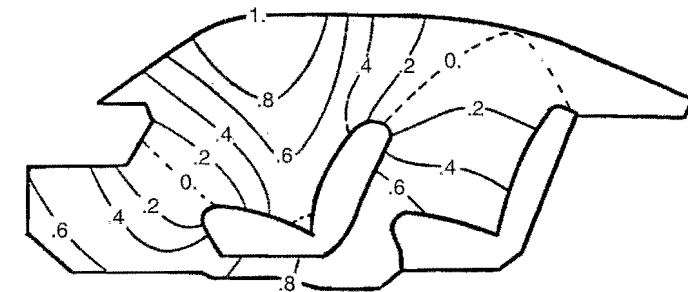
The first acoustical mode predicted by the passenger compartment finite-element model in Fig. 6.3a is shown in Fig. 6.3b. It is a longitudinal mode with the nodal surface passing vertically through the passenger compartment. The nodal surface on which  $|\Psi_n| = 0$  and the antinodal surfaces on which  $|\Psi_n|$  is a local maximum are of practical importance because they indicate low- and high-noise regions, respectively, which result from excitation of the mode. For the example in Fig. 6.3b, excitation of the mode may go unnoticed by a front-seat occupant because of the proximity of the nodal surface, but it may be heard by a rear-seat occupant. The situation is just the opposite for the second mode illustrated in Fig. 6.3c. Because of the irregular geometrical shape of the compartment, these modes could not be predicted by simple formulas like those presented in Table 6.2. However, the numerical approach provides a method to predict the



(a) Acoustic Finite Element Model



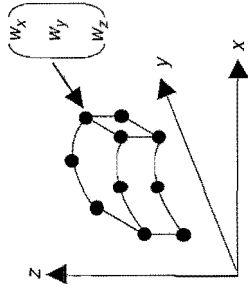
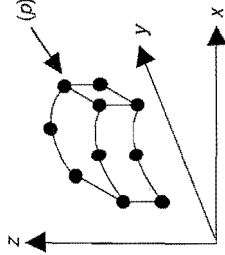
(b) First Resonant Mode at 73 Hz



(c) Second Resonant Mode at 130 Hz

**FIGURE 6.3** Acoustical finite-element analysis of automobile passenger compartment to determine the acoustical mode shapes and natural frequencies. The mode shapes become increasingly more complex as the natural frequency increases and as the complexity of the enclosure geometry increases.

TABLE 6.3 Structural-Acoustical Analogy

| Description              | Structure                                                                                                                                                                                                                                                                                                             | Analogy                                                                                                                                                                                                                   | Acoustical Cavity                                                                                                                                                                                                                                                              |
|--------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Finite element           |                                                                                                                                                                                                                                    | $w_k = p$<br>$N_i^T = N_i$                                                                                                                                                                                                |                                                                                                                                                                                               |
| Constitutive equations   | $w_k = \sum_{i=1}^N (W_k)_i N_i^T \quad k = x, y, z$ $\begin{pmatrix} \sigma_{xx} \\ \tau_{xy} \\ \tau_{xz} \end{pmatrix} = \begin{pmatrix} G_{11} & G_{14} & G_{16} \\ G_{14} & G_{44} & G_{46} \\ G_{16} & G_{46} & G_{66} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \gamma_{xy} \\ \gamma_{xz} \end{pmatrix}$ | $\downarrow$ $\begin{pmatrix} \partial w_x \\ \partial w_y \\ \partial w_z \end{pmatrix} = \begin{pmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{pmatrix}$ | $p = \sum_{i=1}^N p_i N_i$ $\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} -1/\rho & 0 & 0 \\ 0 & -1/\rho & 0 \\ 0 & 0 & -1/\rho \end{pmatrix} \begin{pmatrix} \partial p / \partial x \\ \partial p / \partial y \\ \partial p / \partial z \end{pmatrix}$ |
| Equilibrium equations    | $\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = -\rho_s \ddot{w}_x$                                                                                                                                                                        | $\left\{ \begin{array}{l} \text{Material properties} \\ G_{11} = 1/\rho \\ G_{44} = 1/\rho \\ G_{66} = 1/\rho \\ G_{14} = 0 \\ G_{16} = 0 \\ G_{46} = 0 \\ \rho_s = 1/\rho c^2 \end{array} \right.$                       | $\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = -\frac{1}{\rho c^2} \ddot{p}$                                                                                                                                           |
| Finite-element equations | $([K] - \omega^2 [M])\{w\} = \{0\}$ $K_{ij} = \int_V (\nabla N_i)^T [G] (\nabla N_j) dV$ $M_{ij} = \int_V \rho_s N_i N_j dV$                                                                                                                                                                                          |                                                                                                                                                                                                                           | $([K] - \omega^2 [M])\{p\} = \{0\}$ $K_{ij} = \int_V \frac{1}{\rho} (\nabla N_i)^T (\nabla N_j) dV$ $M_{ij} = \int_V \frac{1}{\rho c^2} N_i N_j dV$                                                                                                                            |

acoustical modes, which can then be used to predict the forced sound pressure response by the methods of the following sections.

## 6.6 FORCED SOUND PRESSURE RESPONSE IN ENCLOSURE

Often the sound pressure response in an enclosure for known sound sources is of interest. The enclosure may be complicated in shape, and it may have walls that are both flexible and absorbent as well as multiple interior surfaces. If the undamped modes  $\Psi_n(x, y, z)$  and natural frequencies  $f_n = \omega_n/2\pi$  for  $n = 0, 1, 2, \dots$  are known, the forced acoustical response in such an enclosure can be directly expressed using the *modal analysis* technique as the *normal-mode expansion*,<sup>11,12</sup>

$$p(x, y, z, t) = \sum_n P_n(t) \Psi_n(x, y, z) \quad \text{N/m}^2 \quad (6.15)$$

where the  $P_n$  are time-varying coefficients that must be determined to satisfy the acoustical wave equation, the boundary conditions, and the initial conditions.

For flexible and absorbent boundaries described by the boundary conditions in Table 6.1, the modal coefficients  $P_n(t)$  are determined from

$$\ddot{P}_n + 2\delta_n \dot{P}_n + \omega_n^2 P_n = \frac{\rho c^2}{V_n} F_n(t) \quad \text{N/m}^2 \cdot \text{s}^{-2} \quad (6.16)$$

where  $V_n$  is given by Eq. (6.12) and  $F_n(t)$  is the *modal force*,

$$F_n(t) = \int_V \frac{\dot{Q}}{V} \Psi_n dV - \int_S \dot{w} \Psi_n dS \quad \text{m}^3/\text{s}^2 \quad (6.17)$$

$\dot{Q}$  is the volume velocity of interior noise sources, and  $\dot{w}$  is the vibration velocity of the wall panels at the boundary surface  $S$ . The integrations are carried out over the enclosure volume  $V$  and the wall panel surface  $S$ , respectively. In Eq. (6.16),  $\delta_n$  is the *modal damping constant* or *modal damping decrement*, which as defined here is complex and given by

$$\delta_n = \delta_n^r + i\delta_n^i = \frac{\rho c^2}{2V_n} \int_A \frac{\Psi_n^2}{Z} dA \quad \text{s}^{-1} \quad (6.18)$$

where  $Z = R + iX$  is the complex wall impedance and the integration is carried out over the impedance wall area  $A$ . Table 6.1 determines the particular interpretation of  $\dot{w}$  and  $Z$  in Eqs. (6.17) and (6.18) for different boundaries. The undamped modes  $\Psi_n$  are assumed to satisfy the rigid-wall boundary condition ( $\partial \Psi_n / \partial n = 0$ ) on both the wall panel surface  $S$  and the impedance boundary  $A$ .

From Eq. (6.18), the real and imaginary parts of  $\delta_n$  can be expressed as [cf. Eq. (6.3)]

$$\delta_n^r = \frac{cA_n}{2V_n} \text{Re} \frac{\rho c}{Z_n} \quad \delta_n^i = \frac{cA_n}{2V_n} \text{Im} \frac{\rho c}{Z_n} \quad \text{s}^{-1} \quad (6.19)$$

where  $Z_n$  is an average impedance for the  $n$ th mode and

$$\frac{\rho c}{Z_n} = \frac{1}{A_n} \int_A \frac{\rho c}{Z} \Psi_n^2 dA \quad \text{where} \quad A_n = \int_A \Psi_n^2 dA \quad (6.20)$$

When the wall impedance  $Z$  does not vary over the wall surface area  $A$ , Eq. (6.20) results in  $\rho c/Z_n \equiv \rho c/Z$ , as expected. Recalling the real and imaginary interpretation of the acoustical impedance in Eq. (6.19), we observe that  $\delta_n^r$  relates to the wall resistivity ( $\sim$  damping) and  $\delta_n^i$  relates to the wall reactance ( $\sim$  flexibility). In the above formulation, we have assumed that "light damping" exists such that  $|\delta_n^r| \ll \omega_n$  and  $|\delta_n^i| \ll \omega_n$ . For more "heavily damped" enclosures that often occur in practice, the modal equations (6.16) are coupled through the boundary impedance and the wall flexibility, and they must be solved simultaneously. A complete discussion of the procedure is given in references 12 and 13.

**Example 6.5.** The acoustical modes  $\Psi_n$  and natural frequencies  $\omega_n$  are given by the finite-element analysis in Table 6.3. Determine the normal-mode expansion for the forced sound pressure response.

**Solution** From the finite-element analysis, the acoustical modes are given as the  $M \times N$  matrix

$$\{\Psi_1 \Psi_2 \cdots \Psi_n \cdots \Psi_N\} = \begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1n} & \cdots & \psi_{1N} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{2n} & \cdots & \psi_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \psi_{m1} & \psi_{m2} & \cdots & \psi_{mn} & \cdots & \psi_{mN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \psi_{M1} & \psi_{M2} & \cdots & \psi_{Mn} & \cdots & \psi_{MN} \end{bmatrix} \quad (6.21)$$

where each row  $m$  corresponds to the response at a particular grid point of the finite-element model and each column  $n$  corresponds to the response of a particular mode. In matrix form, the normal-mode expansion in Eq. (6.15) for the forced sound pressure response then becomes

$$\begin{bmatrix} p_1(t) \\ p_2(t) \\ \vdots \\ p_m(t) \\ \vdots \\ p_M(t) \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1n} & \cdots & \psi_{1N} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{2n} & \cdots & \psi_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \psi_{m1} & \psi_{m2} & \cdots & \psi_{mn} & \cdots & \psi_{mN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \psi_{M1} & \psi_{M2} & \cdots & \psi_{Mn} & \cdots & \psi_{MN} \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ \vdots \\ P_n(t) \\ \vdots \\ P_N(t) \end{bmatrix} \quad (6.22)$$

Each modal coefficient  $P_n(t)$  is determined from Eq. (6.16) with  $\omega_n$  known and  $V_n = \{\Psi_n\}^T [M] \{\Psi_n\}$ , where superscript T indicates the transpose and  $[M]$  is

developed in the finite-element analysis as in Table 6.3. The modal forcing in Eq. (6.16) is

$$\{F_n(t)\} = [\Psi]^T (\{\dot{Q}\} - [S]\{\ddot{w}\}) \quad (6.23)$$

where  $\{\dot{Q}\}$  and  $\{\ddot{w}\}$  are vectors of the grid point volume acceleration and wall panel acceleration and  $[S]$  is a matrix of surface areas associated with each wall panel grid. Similarly, the damping constant in Eq. (6.16) is

$$\{\delta_n\} = \frac{\rho c^2}{2V_n} [\Psi]^T \left[ \frac{A}{Z} \right] [\Psi] \quad (6.24)$$

where  $[A/Z]$  is a matrix of grid point impedance values weighted by the surface area  $A_m$  associated with each grid point  $m$ . Most structural finite-element computer codes, when adapted for acoustical normal-mode analysis according to Table 6.3, also have capabilities for modal frequency response and modal transient response to give the forced sound pressure response in the form of Eq. (6.22).

## 6.7 STEADY-STATE SOUND PRESSURE RESPONSE

The steady-state sound pressure in an enclosure is often of interest where the noise emanates from a point source, as in Fig. 6.1a. For a point source located at  $(x_0, y_0, z_0)$  and having a steady-state volume velocity  $\hat{Q} \cos(\omega t + \phi)$ , the mass flow rate from the source can be mathematically represented using the Dirac delta function as  $\dot{m}(x, y, z, t) = \rho \hat{Q} \delta(x - x_0) \delta(y - y_0) \delta(z - z_0) \cos(\omega t + \phi)$ . At a particular forcing frequency  $f = \omega/2\pi$ , the sound pressure response in the enclosure can then be obtained from Eqs. (6.15)–(6.17) by using the *frequency response function* technique,

$$p(x, y, z, t) = \sum_n p_n(x, y, z, \omega) \cos(\omega t + \theta_n) \quad \text{N/m}^2 \quad (6.25)$$

where

$$p_n(x, y, z, \omega) = \frac{\rho c^2 \omega \hat{Q} \Psi_n(x, y, z) \Psi_n(x_0, y_0, z_0)}{V_n [(\omega^2 - \omega_n^2 + 2\delta_n^i \omega)^2 + (2\delta_n^r \omega)^2]^{1/2}} \quad \text{N/m}^2 \quad (6.26)$$

and

$$\theta_n(\omega) = \phi - \tan^{-1} \frac{\omega^2 - \omega_n^2 + 2\delta_n^i \omega}{2\delta_n^r \omega} \quad \text{rad} \quad (6.27)$$

The amplitude of the modal sound pressure is  $|p_n|$ . When  $\hat{Q} = 1$ ,  $|p_n|$  is the amplitude of the modal frequency response function.

Equation (6.25) shows that the steady-state sound pressure at one point in the enclosure can be considered as the superposition of numerous components of the

same frequency  $\omega$  but with different amplitudes  $|p_n|$  and phase angles  $\theta_n$ . If the source location  $(x_0, y_0, z_0)$  or the observer location  $(x, y, z)$  in Eq. (6.26) is on the nodal surface of a particular mode  $n$ , then the minimum participation of that mode will be observed in the response. It is also noteworthy that the solution is symmetric in the coordinates  $(x_0, y_0, z_0)$  of the sound source and the point of observation  $(x, y, z)$ . If we put the sound source at  $(x, y, z)$ , we observe at point  $(x_0, y_0, z_0)$  the same sound pressure as we did at  $(x, y, z)$  when the source was at  $(x_0, y_0, z_0)$ . This is the famous reciprocity theorem (see Chapter 10) that can sometimes be applied with advantage to measurements in room acoustics.

When only the uniform-pressure mode ( $\omega_0 = 0$ ) is excited, Eqs. (6.25)–(6.27) reduce to  $p = p_0 \cos(\omega t + \theta_0)$ , where

$$p_0(\omega) = \frac{\rho c^2 \hat{Q}}{V[(\omega + 2\delta_0^i)^2 + (2\delta_0^r)^2]^{1/2}} \quad \text{N/m}^2 \quad (6.28)$$

$$\theta_0(\omega) = \phi - \tan^{-1} \frac{\omega + 2\delta_0^i}{2\delta_0^r} \quad \text{rad}$$

which are equivalent to Eqs. (6.2). The uniform-pressure mode applies to the case of sound sources operating at very low frequencies, well below the first ( $n = 1$ ) resonance of the enclosure. This requirement is generally met if  $L < \lambda/10$ , where  $L$  is the maximum linear dimension of the cavity and  $\lambda = c/f$  is the wavelength of sound generated by the source operating at the forcing frequency  $f = \omega/2\pi$ .

**Example 6.6.** For the rectangular cavity of Example 6.4, determine the steady-state sound pressure response in the cavity for loudspeaker excitation when (a) the enclosure walls are rigid and there is no damping, (b) one wall is covered uniformly with absorption material of known impedance  $Z$ , and (c) the damping is expressed in terms of the *critical damping ratio*  $\zeta_n$ . Assume the loudspeaker acts as a simple monopole source.

### Solution

- (a) For an undamped ( $\delta_n^r = 0$ ) and rigid-wall ( $\delta_n^i = 0$ ) enclosure, we obtain, from Eqs. (6.25)–(6.27),

$$p(x, y, z, t) = \rho c^2 \hat{Q} \sum_{i=0}^I \sum_{j=0}^J \sum_{k=0}^K \frac{\omega \Psi_{ijk}(x, y, z) \Psi_{ijk}(x_0, y_0, z_0)}{V_{ijk}[\omega^2 - \omega_{ijk}^2]} \times \cos\left(\omega t + \phi - \frac{\pi}{2}\right) \quad \text{N/m}^2 \quad (6.29)$$

where  $\Psi_{ijk}$  and  $f_{ijk} = \omega_{ijk}/2\pi$  are given in Eq. (6.14),  $V_{ijk}$  is determined from Eq. (6.12), and  $I, J, K$  are the number of modes we include to

obtain a converged solution. For a closed rectangular enclosure,

$$\frac{V_{ijk}}{V} = \varepsilon_i \varepsilon_j \varepsilon_k \quad \text{where} \quad \varepsilon_n = \begin{cases} 1 & \text{for } n = 0 \\ \frac{1}{2} & \text{for } n \geq 1 \end{cases} \quad (6.30)$$

with  $V = L_x L_y L_z$ . Note from Eq. (6.14) that for every mode of vibration the sound pressure is a maximum at the corners of the rectangular enclosure. Also, for every mode of vibration for which one of the indexes  $i, j$ , or  $k$  is *odd*, the sound pressure is zero at the *center* of the enclosure; hence at the geometrical center of the enclosure only one-eighth of the modes of vibration produce a finite sound pressure. Extending this further, at the center of any one wall, the modes for which two of the indexes ( $i, j, k$ ) are odd will have zero pressure, so that only one-fourth of them will participate. Finally, at the center of one edge of the enclosure, the modes for which one index is odd will have zero pressure, so that only one-half of them participate there.

- (b) Assume the uniform absorption material is on the  $z = 0$  wall. Then, from Eqs. (6.19) and (6.20),

$$\delta_{ijk}^r = \frac{c A_{ijk}}{2 V_{ijk}} \operatorname{Re} \frac{\rho c}{Z_{ijk}} = \frac{c}{2 L_z \varepsilon_k} \operatorname{Re} \frac{\rho c}{Z} \quad \text{s}^{-1}$$

$$\delta_{ijk}^i = \frac{c A_{ijk}}{2 V_{ijk}} \operatorname{Im} \frac{\rho c}{Z_{ijk}} = \frac{c}{2 L_z \varepsilon_k} \operatorname{Im} \frac{\rho c}{Z} \quad \text{s}^{-1} \quad (6.31)$$

where  $\rho c/Z_{ijk} = \rho c/Z$  and  $A_{ijk} = \varepsilon_i \varepsilon_j A_z$  with  $A_z = L_x L_y$ . Substituting Eq. (6.31) into Eqs. (6.25)–(6.27), one can evaluate the series solution for the sound pressure when  $Z$  is known.

- (c) From the theory of vibration (Chapter 13), the critical damping ratio  $\zeta_n$  is related to the damping constant through  $\delta_n = \zeta_n \omega_n$ . In complex form,  $\zeta_n = \zeta_n^r + i \zeta_n^i$ , so that  $\delta_n^r = \zeta_n^r \omega_n$  and  $\delta_n^i = \zeta_n^i \omega_n$ , which can be substituted into Eqs. (6.25)–(6.27) to evaluate the sound pressure for given  $\zeta_n$ .

Figure 6.4 shows the predicted versus measured sound pressure level in a rectangular enclosure for volume-velocity (loudspeaker) excitation. The response was predicted by using Eqs. (6.25)–(6.27) as developed in Example 6.6. Figure 6.4a is the response in the undamped, rigid-wall enclosure, where the resonance peaks resulting from the excitation of the rigid-wall cavity modes are identified. Theoretically, the response at these resonances in an undamped enclosure should be infinite. In practice, however, damping is present even in an enclosure with very rigid walls since viscous and thermal losses occur in the air and at the boundaries. Modal damping provides a convenient method of accounting for these losses; and it is easily included in the solution as described in Example 6.6(c). Since the losses are primarily resistive, a real value of modal damping is used. Figure 6.4b shows the sound pressure response when sound-absorptive material of known impedance covers the bottom wall of the enclosure. The predicted response is obtained using the complex “damping” formulas given

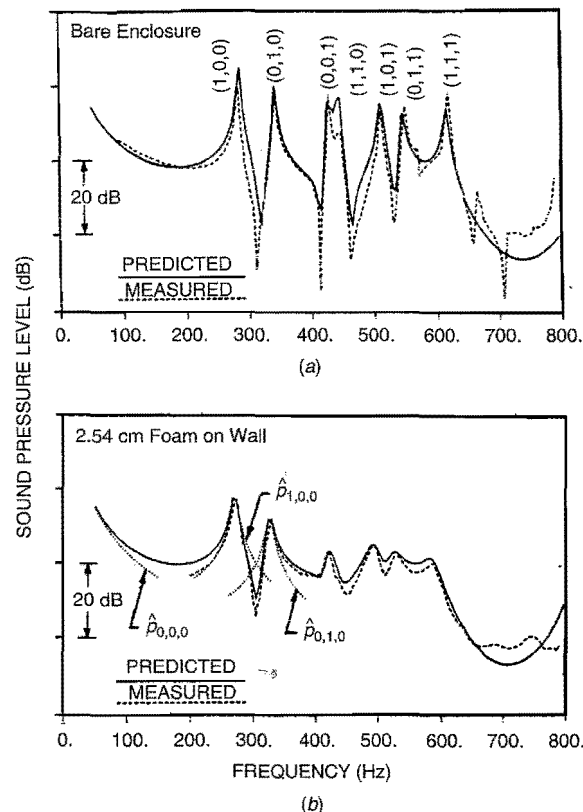


**FIGURE 6.4** Comparisons of predicted versus measured of sound-pressure-level curves for a constant volume-velocity source in a rectangular enclosure of dimensions  $0.61 \times 0.51 \times 0.41$  m. The microphone was at (0.51 m, 0.10 m, 0.30 m) and the source was at (0.0 m, 0.10 m, 0.10 m). (a) Bare enclosure with 0.5% modal damping. (b) Acoustical foam 2.54 cm thick with measured impedance  $\rho c/Z = (0.12 + 0.47i)(f/800)$  for  $0 \leq f < 800$  Hz covering the  $0.61 \times 0.51$ -m wall.

in Eq. (6.31) and by using the modes based on the reduced height of the enclosure due to the absorption material thickness. The resonant responses of the  $|p_{0,0,0}|$ ,  $|p_{1,0,0}|$ , and  $|p_{0,1,0}|$  terms in the series in Eq. (6.25) are also shown. The attenuation and frequency shift of the resonant peaks are evident by comparing Figs. 6.4a and 6.4b, and these are due to the resistive and reactive nature of the absorption material.

## 6.8 ENCLOSURE DRIVEN AT RESONANCE

If a point source of volume velocity  $Q$  is driven by a steady-state generator of frequency  $\omega$  and if that frequency equals a normal-mode frequency  $\omega_n$ , then the

modal sound pressure from Eq. (6.26) is

$$p_n(x, y, z, \omega_n) = \frac{\rho c^2 \hat{Q} \Psi_n(x, y, z) \Psi_n(x_0, y_0, z_0)}{V_n [(2\delta_n^i)^2 + (2\delta_n^r)^2]^{1/2}} \quad \text{N/m}^2 \quad (6.32)$$

From this we conclude that  $|p_n| \rightarrow \infty$  when both  $\delta_n^r = 0$  and  $\delta_n^i = 0$ , so that  $\omega_n$  is the resonance frequency when we have an enclosure with undamped and rigid walls.

For an enclosure with reactive walls ( $\delta_n^i \neq 0$ ,  $\delta_n^r = 0$ ), the frequency dependence in the denominator of Eq. (6.26) can be approximated by

$$\omega^2 - \omega_n^2 + 2\delta_n^i \omega \approx \omega^2 - \omega_n^2 \left(1 - \frac{2\delta_n^i}{\omega_n}\right) = \omega^2 - \omega_{n0}^2 \quad (6.33)$$

where

$$\omega_{n0} = \omega_n \sqrt{1 - \frac{2\delta_n^i}{\omega_n}} \quad \text{rad/s} \quad (6.34)$$

Since now  $|p_n| \rightarrow \infty$  when  $\omega \rightarrow \omega_{n0}$ , we see that the new resonance frequency of an enclosure with undamped, reactive walls is  $\omega_{n0}$ . The resonance frequency shifts from the rigid-wall resonance value  $\omega_n$  due to the reactance  $\delta_n^i$  of the walls, and  $\omega_{n0}$  can be either higher or lower than the rigid-wall frequency  $\omega_n$ , depending on whether  $\delta_n^i < 0$  or  $\delta_n^i > 0$ , respectively. From Eq. (6.27), the phase angle between the modal pressure and the volume velocity source at resonance (i.e., when  $\omega = \omega_{n0}$ ) is either  $0^\circ$  or  $180^\circ$ , and this provides a means of experimentally identifying the resonance frequency  $\omega_{n0}$  and the enclosure reactance  $\delta_n^i$ .

The enclosure dissipation  $\delta_n^r$  controls the amplitude of the resonant peak when  $\omega = \omega_{n0}$  since

$$p_n(x, y, z, \omega_{n0}) = \frac{\rho c^2 \hat{Q} \Psi_n(x, y, z) \Psi_n(x_0, y_0, z_0)}{2\delta_n^r V_n} \quad \text{N/m}^2 \quad (6.35)$$

The dissipation also shifts the resonance frequency an additional amount. The actual resonance frequency  $\omega_{\text{res}}$  at which  $|p_n|$  becomes a maximum can be shown to be

$$\omega_{\text{res}} = \sqrt{\omega_{n0}^2 - 2(\delta_n^r)^2} = \sqrt{\omega_n^2 - 2\delta_n^i \omega_n - 2(\delta_n^r)^2} \quad \text{rad/s} \quad (6.36)$$

The dissipative effect ( $\delta_n^r$ ) is always to reduce the resonance frequency, but this effect is of second order and generally less important than the reactive effect ( $\delta_n^i$ ), which is of first order, unless  $\delta_n^r$  is large. In general, therefore, to first order, the resonance frequency of an enclosure is  $\omega_{\text{res}} = \omega_{n0}$  with  $\omega_{n0}$  given by Eq. (6.34). The half-power bandwidth of the resonant peak is the width of the resonance curve at 3 dB below the peak power (6 dB below the peak pressure) and is

$$\Delta\omega_{\text{res}} = 2\delta_n^r \quad \text{rad/s} \quad (6.37)$$

This provides a means of experimentally determining the enclosure dissipation  $\delta_n^r$ .

**Example 6.7.** Determine the resonance frequency of the first mode in Fig. 6.4b. The measured wall impedance is  $\rho c/Z = (0.12 + 0.47i)(f/800)$ .

**Solution** For the (1, 0, 0) mode, we have  $f_{1,0,0} = 284.8$  Hz from Eq. (6.14). For this frequency,  $\rho c/Z = (0.12 + 0.47i)(284.8/800) = 0.043 + 0.167i$ . Noting that the thickness of the absorption material is 2.54 cm, we have  $L_z = 0.41 - 0.0254 = 0.3846$  m, and from Eq. (6.31),

$$\delta_{1,0,0}^r = \frac{347.3}{2 \times 0.3846 \times 1} \times 0.043 = 19.6$$

$$\delta_{1,0,0}^i = \frac{347.3}{2 \times 0.3846 \times 1} \times 0.167 = 76.2$$

Substituting these into Eq. (6.36) gives the resonance frequency

$$\begin{aligned} f_{\text{res}} &= \frac{\omega_{\text{res}}}{2\pi} = f_n \sqrt{1 - \frac{2\delta_n^i}{\omega_n} - 2\left(\frac{\delta_n^r}{\omega_n}\right)^2} \\ &= 284.8(1 - 0.085 - 0.00024)^{1/2} = 272.4 \text{ Hz} \end{aligned}$$

Actually, the calculated frequency should be slightly greater than this because the impedance is frequency dependent and should be evaluated at  $f_{\text{res}}$ . An iterative calculation gives  $f_{\text{res}} = 273$  Hz, while the measured resonance frequency in Fig. 6.4b is 274 Hz.

## 6.9 FLEXIBLE-WALL EFFECT ON SOUND PRESSURE

A flexible wall may exhibit structural resonances and affect the sound pressure in two ways: (a) by acting as a noise source when exterior structural or pressure loads excite vibrations of the wall and (b) by acting as a boundary impedance that can alter the cavity sound pressure. Both of these effects may occur simultaneously in a room with flexible walls, in which case a coupled *structural-acoustical analysis* is required to predict the sound pressure response. We shall first consider each of the two effects of a flexible wall separately and then consider the coupled structural-acoustical response.

### Flexible Wall as Noise Source

When the vibration velocity  $\dot{w}(x, y, z, t)$  of a flexible wall is known, the boundary condition in Table 6.1, frame 2 or 4, can be applied to Eqs. (6.16) and (6.17). The forcing is through the equivalent modal volume-velocity of the wall vibration,

$$Q_n = \hat{Q}_n \cos(\omega t + \phi_n) = - \int_S \dot{w} \Psi_n dS \quad \text{m}^3/\text{s} \quad (6.38)$$

where the vibration velocity of the wall  $\dot{w} = \hat{w} \cos(\omega t + \phi_w)$  is assumed to have a known amplitude  $\hat{w}(x, y, z)$  and phase  $\phi_w(x, y, z)$ . The steady-state sound pressure response is then obtained as in Eqs. (6.26) and (6.27),

$$p_n(x, y, z, \omega) = \frac{\rho c^2 \omega \hat{Q}_n \Psi_n(x, y, z)}{V_n[(\omega^2 - \omega_n^2 + 2\delta_n^i \omega)^2 + (2\delta_n^r \omega)^2]^{1/2}} \quad \text{N/m}^2 \quad (6.39)$$

$$\theta_n(\omega) = \phi - \tan^{-1} \frac{\omega^2 - \omega_n^2 + 2\delta_n^i \omega}{2\delta_n^r \omega} \quad \text{rad} \quad (6.40)$$

where  $\delta_n^r, \delta_n^i$  relate to the layer of absorption material of acoustical impedance  $Z_a$  covering the flexible wall.

Figure 6.5a shows the calculation of the forced acoustical response in the automobile passenger compartment using the above method with measured panel acceleration amplitude and phase data to represent the wall vibration and with measured wall impedance data to represent the absorption materials. The solution is expressed in matrix form as in Eq. (6.22) with the acoustical modes obtained from a finite-element analysis (Fig. 6.3). Figure 6.5a shows that a large spatial variation in sound pressure level occurs in the passenger compartment because of the modal nature of the acoustical response. Figure 6.5b shows the sound pressure computed separately for the vibration of each individual wall panel. The individual sound pressures can be combined, by considering their magnitude and phase, to obtain the resultant sound pressure, which is shown in Fig. 6.5b. This provides a method to identify the major noise sources and the extent to which they must be controlled to yield a specified noise reduction. For example, Fig. 6.5b illustrates the large amplitude of the sound pressure due to the back-window vibration. Also note that the elimination of the roof vibration would increase the noise at the driver's ear.

### Flexible Wall as Reactive Impedance

Wall panel vibration can generally be expressed in the normal-mode expansion form

$$w = \sum_m W_m(t) \Phi_m(x, y, z) \quad \text{m} \quad (6.41)$$

where  $\Phi_m$  are the structural vibration mode shapes and  $W_m$  are the modal amplitudes. If we consider the coupling between the wall panel and the cavity, for the  $m$ th structural mode and the  $n$ th acoustical mode, the modal acoustical impedance of the wall can be expressed as

$$Z_{mn} = (S_{mn})^{-1} \left[ C_m + i\omega \left( M_m - \frac{K_m}{\omega^2} \right) \right] \quad \text{N/m}^2 \cdot \text{s}^{-1} \quad (6.42)$$

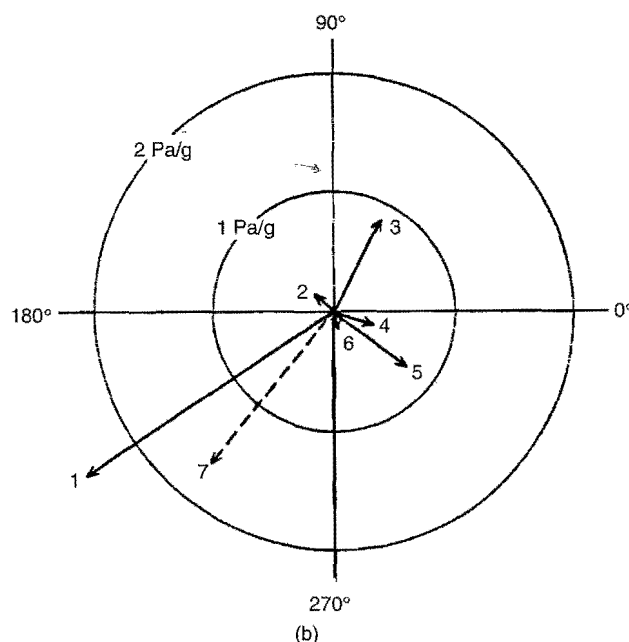
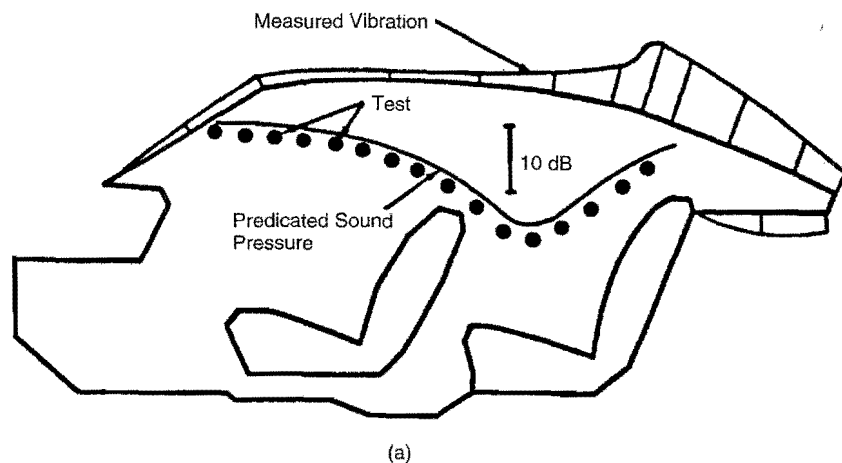


**FIGURE 6.5** (a) Comparison of measured versus predicted sound-pressure-level spatial variation in automobile passenger compartment for 40-Hz structural excitation. (b) Polar amplitude-phase diagram indicating panel contributions to resultant sound pressure at driver's ear (1, back window; 2, rear floor; 3, roof; 4, windshield; 5, rear shelf; 6, front floor; 7, resultant).

where  $M_m$ ,  $C_m$ , and  $K_m$  are the modal mass, damping, and stiffness of the  $m$ th wall mode, and

$$S_{mn} = \int_S \Phi_m \Psi_n dS \quad \text{m}^2 \quad (6.43)$$

where  $S_{mn}$  is the *structural-acoustical coupling*. [Note that when  $S_{mn} = 0$ , there is no coupling between the structural mode  $m$  and the acoustical mode  $n$ , and Eq. (6.42) does not apply. In practice, when  $S_{mn}$  is sufficiently small, the coupling can be neglected to simplify the solution.]

For an undamped enclosure, the structural-acoustical coupling modifies Eq. (6.26) to

$$p_n(x, y, z, \omega) = \frac{\rho c^2 \omega \hat{Q} \Psi_n(x, y, z) \Psi_n(x_0, y_0, z_0)}{V_n |\omega^2 - \omega_{n0}^2|} \left| \frac{\omega^2 - \Omega_m^2}{\omega^2 - \Omega_{m0}^2} \right| \quad \text{N/m}^2 \quad (6.44)$$

where  $\Omega_m = \sqrt{K_m/M_m}$  (rad/s) is the natural frequency of the  $m$ th wall mode and  $\omega_{n0}^2$  and  $\Omega_{m0}^2$  are solutions of

$$(\omega^2 - \omega_n^2)(\omega^2 - \Omega_m^2) - D^2 \omega^2 = 0 \quad (6.45)$$

where  $D = \sqrt{K_{\text{air}}/M_m}$  with  $K_{\text{air}} = \rho c^2 S_{mn}^2 / V_n$  (N/m) being the *stiffness* of the  $n$ th acoustical mode relative to the  $m$ th structural mode.

Equation (6.44) shows that the flexible wall introduces an additional resonance at  $\omega = \Omega_{m0}$ , which will appear as a resonance peak in the sound pressure response and corresponds to the wall resonance. It can be shown from Eq. (6.45) that, with wall modes at frequencies below the rigid-wall cavity resonance ( $\Omega_m < \omega_n$ , the case of *mass-controlled* boundaries), the cavity resonance is raised ( $\omega_{n0} > \omega_n$ ) and the wall resonance is lowered ( $\Omega_{m0} < \Omega_m$ ). With wall modes at frequencies above the rigid-wall cavity resonance ( $\Omega_m > \omega_n$ , the case of *stiffness-controlled* boundaries), the cavity resonance is lowered ( $\omega_{n0} < \omega_n$ ) and the wall resonance is raised ( $\Omega_{m0} > \Omega_m$ ). In addition, the wall acts as a vibration absorber when  $\omega = \Omega_m$ , so that the modal sound pressure  $p_n = 0$ . However, the total sound pressure  $p$  may not be zero because of the participation of other acoustical modes. A Helmholtz resonator used as a vibration absorber as described in Chapter 8 has impedance  $Z_R$  and performs in a similar manner.

### Coupled Structural-Acoustical Response

Above we have considered the coupling of one acoustical mode  $n$  with one structural mode  $m$ , which is the case when a single coupling coefficient  $S_{mn}$  dominates for the mode pair  $(m, n)$ . In practice, however, each acoustical mode  $n$  may couple with several structural modes, and the general case can be treated

by substituting Eq. (6.41) into the second integral of Eq. (6.17) to obtain, from Eq. (6.16),

$$\ddot{P}_n + 2\delta_n \dot{P}_n + \omega_n^2 P_n + \frac{\rho c^2}{V_n} \sum_m S_{mn} \ddot{W}_m = \frac{\rho c^2}{V_n} F_n(t) \quad \text{N/m}^2 \cdot \text{s}^2 \quad (6.46)$$

where the summation is carried out over all coupled structural modes  $m$ . Similarly, each structural mode  $m$  may in general couple with several acoustical modes, and the corresponding equation for the structural modal amplitude  $W_m(t)$  in Eq. (6.41) is

$$\ddot{W}_m + 2\Delta_m \dot{W}_m + \Omega_m^2 W_m - \frac{1}{M_m} \sum_n S_{mn} P_n = \frac{R_m(t)}{M_m} \quad \text{m/s}^2 \quad (6.47)$$

where the summation is carried out over all coupled acoustical modes  $n$ . In this latter equation for the structure,  $\Delta_m$  is the modal damping constant,  $\Omega_m$  is the natural frequency,  $M_m$  is the modal mass, and  $R_m(t)$  is the modal force. The coupling between the acoustical and structural systems makes it necessary to solve Eqs. (6.46) and (6.47) simultaneously for all coupled  $P_n$  and  $W_m$ . A complete derivation of the equations and discussion of the procedure can be found in reference 12, and its finite-element implementation is described in references 14 and 15.

To illustrate a typical coupled structural-acoustical response, Fig. 6.6 shows an application to a small metallic box used to isolate instruments from an external noise field (cf. Fig. 6.1b). The analysis of such enclosures to predict their noise attenuation is discussed in detail in Chapter 11. The finite-element method can be used to model the box wall panels and the acoustical cavity (Figs. 6.6b, c) in order to compute the uncoupled structural and acoustical modes. Equations (6.46) and (6.47) are then solved for the coupled frequency response, where the exterior noise field in Fig. 6.6a is specified as an oscillating external pressure  $\hat{p}_E \cos \omega t$  applied uniformly to the wall panels. The noise attenuation inside the enclosure is characterized by its *insertion loss*, defined in Chapter 11 as  $20 \log_{10}(\hat{p}_I/\hat{p}_E)$ , where  $\hat{p}_I$  is the interior sound pressure. The predicted versus measured insertion loss is shown in Fig. 6.6d. The structural-acoustical coupling effect is particularly evident in the rms panel vibration shown in Fig. 6.6e, which compares the coupled versus the uncoupled (i.e., in vacuo) panel response and illustrates the significant effect that the acoustical cavity modes have on the structural vibration response.

## 6.10 RANDOM SOUND PRESSURE RESPONSE

For steady-state, deterministic sound source excitation, the sound pressure response can be obtained by the frequency response function technique of Eqs. (6.25)–(6.27). However, for sound source excitation that is nondeterministic, it is generally necessary to utilize a random-analysis technique

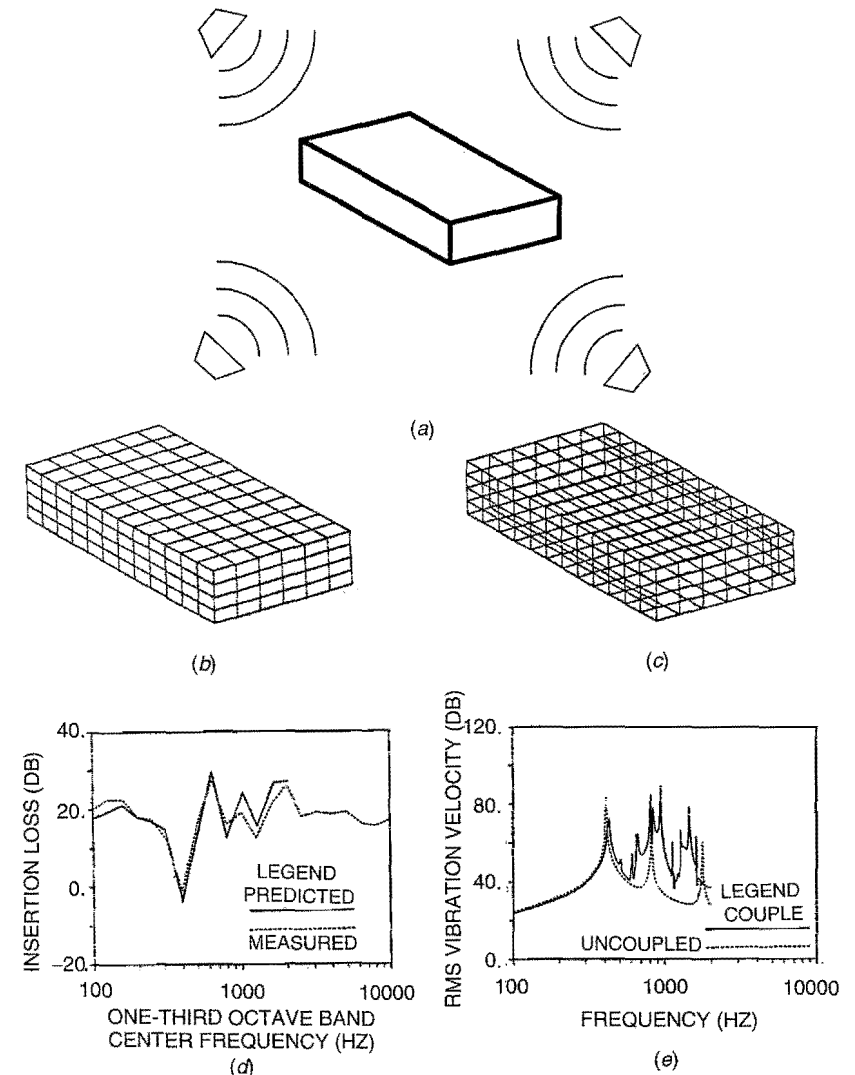


**FIGURE 6.6** Structural-acoustical analysis to predict insertion loss of a  $30 \times 15 \times 5$ -cm unlined aluminum box with 0.16-cm-thick walls: (a) box in uniform noise environment; (b) structural finite-element model of box wall panels; (c) acoustical finite-element model of box cavity; (d) predicted versus measured insertion loss at box center; (e) coupled versus uncoupled surface-averaged vibration of  $30 \times 15$ -cm wall panel. [Measured data in (d) from Chapter 11].

to obtain the sound pressure response. For example, irregular pressure fluctuations generated by turbulence can result in complicated sound sources that are more readily measured or defined as random excitations than as deterministic excitations. Also, interactions with irregular surfaces such as roads, runways, and pathways are more readily represented as random loads than as deterministic loads. Under certain conditions (stationarity) the average frequency content of these excitations may be represented by a spectral density function (Chapter 3). Then by using the spectral density representation of the input and the frequency response function technique to determine the sound pressure output, conventional random-analysis theory may be utilized to obtain the sound pressure spectral density response.

From random-analysis theory, nondeterministic stationary processes can be characterized by their auto- (power) spectral density function and their cross-spectral density function.<sup>16</sup> The power spectral density (PSD) function of a physical variable such as sound pressure  $p(t)$  is defined as a real-magnitude function  $S_p(\omega)$ , where

$$S_p(\omega) = \lim_{T \rightarrow \infty} \frac{2}{T} \left| \int_0^T e^{-i\omega t} p(t) dt \right|^2 \quad (\text{N/m}^2)^2/\text{Hz} \quad (6.48)$$

Similarly, the PSD function  $S_Q(\omega)$  for a volume-velocity point source  $Q(t)$  is

$$S_Q(\omega) = \lim_{T \rightarrow \infty} \frac{2}{T} \left| \int_0^T e^{-i\omega t} Q(t) dt \right|^2 \quad (\text{m}^3/\text{s})^2/\text{Hz} \quad (6.49)$$

The PSD functions of the sound pressure response and sound source excitation are related via the frequency response function as

$$S_p(x, y, z, \omega) = \sum_n p_n^2(x, y, z, \omega) S_Q(\omega) \quad (\text{N/m}^2)^2/\text{Hz} \quad (6.50)$$

where  $S_p(x, y, z, \omega)$  is the sound pressure PSD response and  $p_n(x, y, z, \omega)$  is the modal frequency response function in Eq. (6.26) for unit sound source excitation  $\hat{Q} = 1$ . Equation (6.50) is the random-response equivalent of Eq. (6.25) for the steady-state response. However, for random response, only the amplitude of the modal frequency response function is required and not the phase information in  $\theta_n(\omega)$  in Eq. (6.27).

Another useful result from random-analysis theory is that, if several sound sources  $Q_1(t), Q_2(t), \dots, Q_a(t), \dots$  are statistically independent, then the cross-correlation between any pair of sources is zero, and the total sound pressure PSD response is equal to the sum of the PSD responses due to the individual sources. That is,

$$\begin{aligned} S_p(x, y, z, \omega) &= \sum_a S_{p_a}(x, y, z, \omega) \\ &= \sum_a \sum_n p_{an}^2(x, y, z, \omega) S_{Q_a}(\omega) \quad (\text{N/m}^2)^2/\text{Hz} \end{aligned} \quad (6.51)$$

Finally, when two sound sources  $Q_a(t), Q_b(t)$  are statistically correlated, the degree of correlation is related by a cross-spectral density function  $S_{Q_a Q_b}(\omega)$  given by

$$S_{Q_a Q_b}(\omega) = \lim_{T \rightarrow \infty} \frac{2}{T} \left[ \left( \int_0^T e^{-i\omega t} Q_a(t) dt \right)^* \left( \int_0^T e^{-i\omega t} Q_b(t) dt \right) \right] \quad (\text{m}^3/\text{s})^2/\text{Hz} \quad (6.52)$$

where the asterisk denotes complex conjugate. Unlike the PSD function, the cross-spectral density function is generally a complex number that includes both amplitude and phase information. For correlated sound sources, the spectral density of the sound pressure response is then a complex number given by

$$\begin{aligned} S_p(x, y, z, \omega) &= \sum_a \sum_b \sum_m \sum_n (p_{am}(x, y, z, \omega) e^{i\theta_m(\omega)}) (p_{bn}(x, y, z, \omega) e^{i\theta_n(\omega)})^* \\ &\quad \times S_{Q_a Q_b}(\omega) \quad (\text{N/m}^2)^2/\text{Hz} \end{aligned} \quad (6.53)$$

where the phase relationships  $\theta_m(\omega), \theta_n(\omega)$  are from Eq. (6.27) with  $\phi = 0$ . In the case where the sound sources are uncorrelated, Eq. (6.53) reduces to the previous real-magnitude sound pressure PSD functions in Eqs. (6.50) and (6.51).

**Example 6.8.** Determine the sound pressure PSD response in the very small enclosure in Fig. 6.1a with rigid walls when (a) there is a single sound source  $Q_a$  in the enclosure with PSD amplitude  $S_{Q_a} = S(\omega)$ ; (b) there are two uncorrelated interior sound sources  $Q_a, Q_b$  in the enclosure with PSD amplitudes  $S_{Q_a} = S(\omega), S_{Q_b} = S(\omega)$ ; and (c) there are two interior sound sources with  $S_{Q_a} = S(\omega), S_{Q_b} = S(\omega)$  but perfectly correlated so that  $S_{Q_a Q_b} = S(\omega)$ .

#### Solution

- (a) For a rigid-wall enclosure with a single sound source, substituting Eq. (6.4) into Eq. (6.50) gives

$$S_p(x, y, z, \omega) = p_0^2(x, y, z, \omega) S_{Q_a} = \left( \frac{\rho c^2}{\omega V} \right)^2 S_{Q_a} = \left( \frac{\rho c^2}{\omega V} \right)^2 S(\omega)$$

- (b) For two uncorrelated sound sources in the enclosure, Eq. (6.51) gives

$$S_p(x, y, z, \omega) = \left( \frac{\rho c^2}{\omega V} \right)^2 (S_{Q_a} + S_{Q_b}) = 2 \left( \frac{\rho c^2}{\omega V} \right)^2 S(\omega)$$

- (c) For two correlated sound sources in the enclosure, Eq. (6.54) gives

$$S_p(x, y, z, \omega) = \left( \frac{\rho c^2}{\omega V} \right)^2 (S_{Q_a} + S_{Q_b} + 2S_{Q_a Q_b}) = 4 \left( \frac{\rho c^2}{\omega V} \right)^2 S(\omega)$$

Thus, for uncorrelated sound sources, doubling the number of the sound sources in the enclosure *doubles* the sound pressure PSD response (3 dB increase), as in (b). On the other hand, for perfectly correlated sound sources, doubling the number of the sound sources *quadruples* the sound pressure PSD response (6 dB increase), as in (c).

The random-analysis formulas in Eqs. (6.50)–(6.53) are also applicable to structural analysis, by replacing the acoustical frequency response functions and sound source excitations with the corresponding structural frequency response functions and structural excitations.<sup>16</sup> Also, the coupled structural–acoustical analysis provided by Eqs. (6.46) and (6.47) can be applied to predict the sound pressure PSD response for random excitations.<sup>17</sup> As an example, Fig. 6.7 illustrates the predicted interior sound pressure PSD response in an automobile passenger compartment for the vehicle traveling at constant speed  $V$  over a randomly rough road. In this example, random road excitation occurs at the tire patch (Fig. 6.7a) and the sound pressure response in the passenger compartment results from the transmitted vibration to the vehicle body panels. Equations (6.46) and (6.47) can be used to obtain the sound pressure frequency response functions by employing the vehicle finite-element models in Figs. 6.7b,c. Equation (6.51) is then used to predict the sound pressure PSD response by applying the road profile PSD function in Fig. 6.7d as excitation at each tire patch at the vehicle speed  $V$ . Figure 6.7e shows the predicted sound pressure PSD response in the passenger compartment versus the 95% confidence band based on the measured responses in nominally identical vehicles. The relative participation of the sound sources and body panels to the sound pressure PSD response can also be identified by the methods in Chapter 3 and reference 17.

### 6.11 TRANSIENT SOUND PRESSURE RESPONSE

When the source of sound in a room is turned off, the sound dies out, or decays, at a rate that depends on the dissipation, or damping, in the room. The acoustical response that exists in the room can be found from the transient solution of Eq. (6.16), which is

$$P_n(t) = \frac{\rho c^2}{V \omega_{nD}} \int_0^t F_n(\tau) e^{-\delta_n^r(t-\tau)} \sin[\omega_{nD}(t-\tau)] d\tau \quad \text{N/m}^2 \quad (6.54)$$

where  $\omega_{nD} = \sqrt{\omega_n^2 - (\delta_n^r)^2 + (\delta_n^i)^2}$  is the “damped” modal frequency of the acoustical response in the enclosure. Equation (6.54) is the particular solution that satisfies zero initial conditions. For general initial conditions, the following free-vibration response must be added to the above equation:

$$P_n(t) = e^{-\delta_n^r t} \left[ \frac{\dot{P}_n(0) + P_n(0) \delta_n^r}{\omega_{nD}} \sin \omega_{nD} t + P_n(0) \cos \omega_{nD} t \right] \quad \text{N/m}^2 \quad (6.55)$$

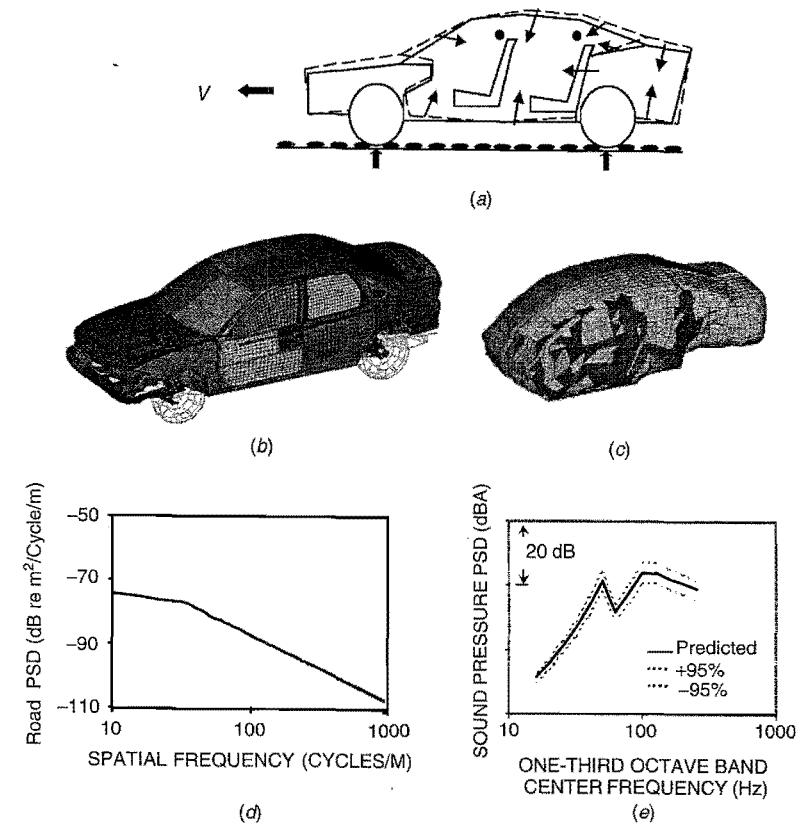


**FIGURE 6.7** Structural–acoustical analysis to predict sound pressure PSD response in vehicle traveling at constant speed  $V$  on randomly rough road: (a) interior road noise generation; (b) structural finite-element model of vehicle; (c) acoustical finite-element model of passenger-trunk compartment; (d) road profile power spectral density; (e) predicted A-weighted sound pressure PSD response at front seat occupant ear location versus 95% confidence interval.

Equation (6.55) can be used to determine the decay of the modes after the sound source is turned off at  $t = 0$ , when the initial conditions  $P_n(0)$  and  $\dot{P}_n(0)$  are known. For light damping, each mode of vibration behaves independently of the others, and the total process of sound decay is the summation in Eq. (6.15) of the sound pressures associated with all of the individual modes of vibration that fall within the frequency band of interest. The long-time sound decay may be different than the short-time sound decay because of the different damping factors and initial conditions for the modes.

The *reverberation time*  $T$  is defined as the time in seconds required for the level of sound to drop by 60 dB or for the pressure to drop to  $\frac{1}{1000}$  of its initial

value (see Chapter 7). One can similarly define a *modal* reverberation time  $T_n$  as that for which the sound pressure decays in that mode by 60 dB or  $\frac{1}{1000}$  of its initial value. Since the reverberation time is that associated with the decaying part ( $-\delta_n^r t$ ) of the solution in Eq. (6.55),

$$T_n = \frac{6.91}{\delta_n^r} = \frac{13.82 V_n}{c A_n \operatorname{Re}(\rho c / Z_n)} \quad \text{s} \quad (6.56)$$

where we have substituted for  $\delta_n^r$  from Eq. (6.19). With  $c = 343$  m/s ( $20^\circ\text{C}$ ) and expressed in terms of the random-incidence absorption coefficient,  $\alpha_n = 8 \operatorname{Re}(\rho c / Z_n)$ , with  $V$  in cubic meters and  $A$  in square meters, one obtains the modal reverberation time ( $n > 0$ )

$$T_n = 0.322 \frac{V_n}{\alpha_n A_n} \quad \text{s} \quad (6.57)$$

which can be evaluated for a given  $Z$  by using the formulas in Eqs. (6.20) and (6.12). The uniform-pressure mode ( $n = 0$ ) must be treated separately. The general transient solution of Eq. (6.16) when  $n = 0$  reduces to

$$P_0(t) = P_0(0)e^{-2\delta_0^r t} + \frac{\rho c^2}{V} \int_0^t \int_0^\tau F_0(\sigma) d\sigma e^{-2\delta_0^r(t-\tau)} d\tau \quad \text{N/m}^2 \quad (6.58)$$

and the reverberation time for the decay of a uniform sound pressure is  $T_0 = 6.91/2\delta_0^r = 6.91V/cA \operatorname{Re}(\rho c/Z)$  so that, for  $V$  in cubic meters and  $A$  in square meters,

$$T_0 = 0.161 \frac{V}{\alpha A} \quad \text{s} \quad (6.59)$$

In this case, the formula for the reverberation time of a uniform sound pressure is identical to the formula of Sabine for the reverberation time of a diffuse sound field.

If the pressure-time history is dominated by a single mode, then the reverberation time is equal to the appropriate modal reverberation time. If the sound source is turned off at  $t = 0$ , the magnitude of the sound pressure associated with a particular mode at a response location ( $x, y, z$ ) is given by Eq. (6.55). The decay of the response from its maximum amplitude [ $\dot{P}_n(0) = 0$ ] can then be written as

$$p_n(x, y, z, t) = P_n(0)\Psi_n(x, y, z)e^{-\delta_n^r t} \cos(\omega_n t + \theta_n) \quad \text{N/m}^2 \quad (6.60)$$

where  $\theta_n = \tan^{-1}(\delta_n^r/\omega_n D)$  is the modal phase. If we take the rms time average of  $\cos(\omega_n t + \theta_n)$  and the rms spatial average of  $\Psi_n(x, y, z)$  and we designate the resultant as  $\bar{p}_n(t)$ , then Eq. (6.60) indicates that on a plot of  $\log \bar{p}_n$  versus

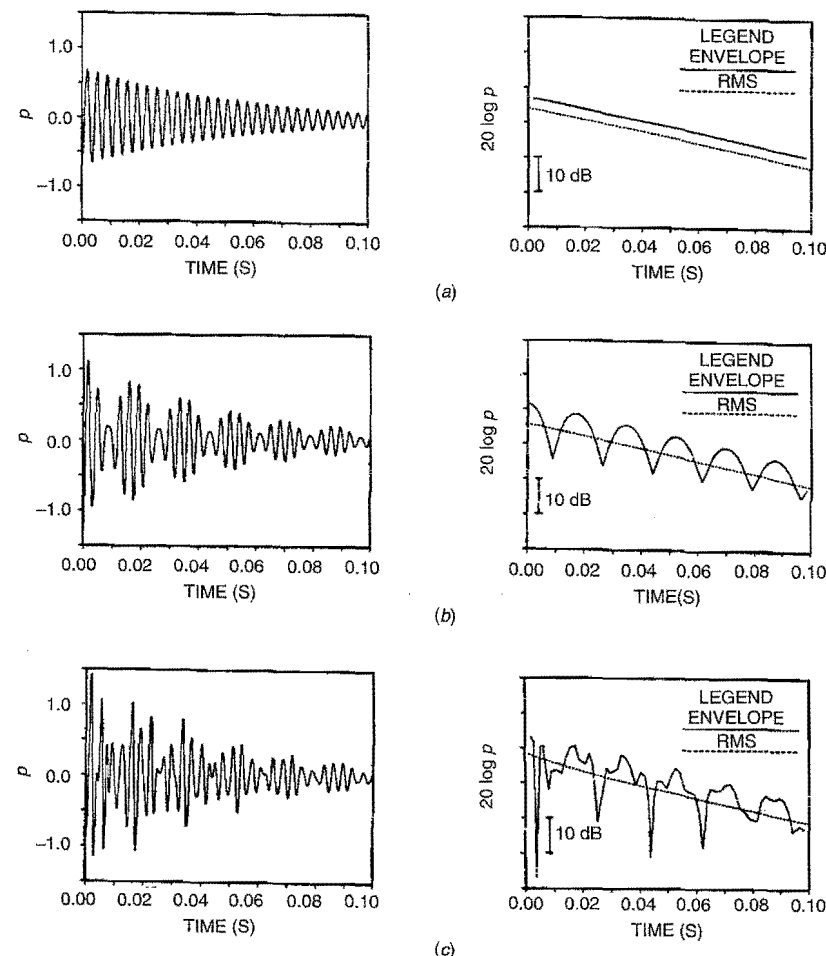


**FIGURE 6.8** Sound pressure decay curves for rectangular enclosure of Fig. 6.6: (a) for the (1, 0, 0) mode of vibration; (b) for the (1, 0, 0) and (0, 1, 0) modes together; (c) for all modes up to 800 Hz. The graphs on the left show the course of the instantaneous sound pressure at the microphone location, and those on the right show the curve of the envelope of the left graphs and the rms pressure of Eq. (6.61) plotted on a log  $p$ -versus- $t$  coordinate system.

time, as in Fig. 6.8a, both the envelope of the sound pressure and the rms sound pressure decay linearly with time and have the constant reverberation time  $T_n$ .

When many normal modes of vibration (each with its own amplitude, phase, resonance frequency, and damping constant) decay simultaneously, the total rms

(spatial and temporal) sound pressure is obtained from Eq. (6.15) as

$$\bar{p}(t) = \sqrt{\bar{p}_I^2 + \bar{p}_{I+1}^2 + \cdots + \bar{p}_{I+N}^2} \quad \text{N/m}^2 \quad (6.61)$$

where  $I$  is the first mode in the frequency band considered and  $I + N$  is the last mode in the decay. In this case, the decay envelope is generally not linear as above, even when the modal reverberation times do not vary greatly from one mode to another so that all modes in the frequency band have a similar damping constant. The decay envelope is irregular, as in Figs. 6.8*b,c*, because the modes of vibration have different frequencies and beat with each other during the decay. However, the rms decay from Eq. (6.61) will be nearly linear if the modal reverberation times are similar (Fig. 6.8*b*) or if the long-time decay is governed by a least-damped mode, which may occur when the modal reverberation times or initial conditions differ (Fig. 6.8*c*).

This brief description of sound decay in small rooms is applicable to rooms of any size and shape, and it can be extended to include the effects of structural-acoustical coupling. The fact that sound fields in large enclosures involve too many modes for the calculations to be practical does not mean that there are not distinct normal modes of vibration, each with its own natural frequency and damping constant. Alternate and more practical methods for handling large enclosures involving large numbers of modes are discussed in the next chapter.

## REFERENCES

1. P. M. Morse and K. U. Ingard, *Theoretical Acoustics*, Princeton University Press, Princeton, NJ, 1968.
2. L. L. Beranek (Ed.), *Noise and Vibration Control*, rev. ed., Institute of Noise Control Engineers, Poughkeepsie, NY, 1988.
3. H. Kuttruff, *Room Acoustics*, Applied Science, London, 1979.
4. L. E. Kinsler, A. R. Frey, A. B. Coppins, and J. V. Saunders, *Fundamentals of Acoustics*, 4th ed., Wiley, New York, 1999.
5. L. L. Beranek and I. L. Ver, *Noise and Vibration Control Engineering*, Wiley, New York, 1992.
6. E. H. Dowell, C. F. Chao, and D. A. Bliss, "Absorption Material Mounted on a Moving Wall-Fluid/Wall Boundary Condition," *J. Acoust. Soc. Am.*, **70**(1), 244-245 (1981).
7. R. D. Blevins, *Formulas for Natural Frequency and Mode Shape*, Krieger, Malabar, FL, 1995.
8. M. J. Crocker (Ed.), *Handbook of Noise and Vibration Control*, Chapter 6: "Numerical Acoustical Modeling (Finite Element Modeling)" and Chapter 7: "Numerical Acoustical Modeling (Boundary Element Modeling)," Wiley, New York, 2006.
9. M. M. Kamal and J. A. Wolf, Jr. (Eds.), *Modern Automotive Structural Analysis*, Van Nostrand Reinhold, New York, 1982.
10. G. C. Everstine, "Structural Analogies for Scalar Field Problems," *Int. J. Numer. Methods Eng.*, **17**, 471-476 (1981).

11. P. M. Morse and R. H. Bolt, "Sound Waves in Rooms," *Rev. Modern Phys.*, **16**(2), 69-150 (1944).
12. E. H. Dowell, G. F. Gorman III, and D. A. Smith, "Acoustoelasticity: General Theory, Acoustic Natural Modes and Forced Response to Sinusoidal Excitation, Including Comparisons with Experiment," *J. Sound Vib.*, **52**(4), 519-542 (1977).
13. E. H. Dowell, "Reverberation Time, Absorption, and Impedance," *J. Acoust. Soc. Am.*, **64**(1), 181-191 (1978).
14. D. J. Nefske, J. A. Wolf, Jr., and L. J. Howell, "Structural-Acoustic Finite Element Analysis of the Automobile Passenger Compartment: A Review of Current Practice," *J. Sound Vib.*, **80**(2), 247-266 (1982).
15. S. H. Sung and D. J. Nefske, "A Coupled Structural-Acoustic Finite Element Model for Vehicle Interior Noise Analysis," *ASME J. Vib. Acoust. Stress Reliab. Design*, **106**, 314-318 (1984).
16. D. E. Newland, *An Introduction to Random Vibration, Spectral and Wavelet Analysis*, Wiley, New York, 1994.
17. S. H. Sung and D. J. Nefske, "Component Mode Synthesis of a Vehicle Structural-Acoustic System Model," *Am. Inst. Aero. Astro. J.*, **24**(6), 1021-1026 (1986).

## Sound in Rooms

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Acoustical problems experienced by room users include difficult verbal communication in classrooms and conference rooms, impaired student learning and teacher voice problems in classrooms, hearing loss in industrial workshops, and inadequate speech privacy in open-plan offices. Room sound fields comprise both "signals" (useful sounds, such as speech) and detrimental "noise". To modify room sound fields and improve acoustical conditions, the acoustician must understand the relationship between the room user and activity, the sound sources, the room and its contents, and the characteristics of room sound fields. Models for predicting room sound fields are of primary importance. They allow the acoustical conditions to be optimized during design and permit sound control measures to be evaluated for cost effectiveness in new or existing rooms. In this chapter, we take an energy-based approach, ignoring phase, interference, and modal effects discussed in Chapter 6. This is justified, except perhaps at low frequencies, since we are interested in rooms which are large compared with the sound wavelength, which are of complex shape and may not be empty. Further, we are interested in wide-band (total A-weighted, octave- or third-octave band) results. In this chapter, the sound field is quantified by the time-averaged, mean-square sound pressure  $p^2$  in  $(\text{N/m}^2)^2$  or the associated sound pressure level  $L_p$  in decibels ( $L_p = 10 \log_{10}[p^2/p_0^2]$ ,  $p_0 = 2 \times 10^{-5} \text{ N/m}^2$ ). In the case of multiple sources or sound reflections, the total energy is the sum of the individual energy contributions:  $p_{\text{tot}}^2 = \sum p_i^2$ ,  $L_{p,\text{tot}} = 10 \log_{10}(\sum 10^{L_{pi}/10})$ .

Sound sources radiate energy over some frequency range and, at each frequency, with a certain intensity. The rate of emission of energy is described by the sound power  $W$  in watts or the associated sound power level  $L_w$  in decibels ( $L_w = 10 \log_{10}(W/W_0)$ ,  $W_0 = 10^{-12} \text{ W}$ ), usually measured in octave- or third-octave