

INDUCTION MOTORS, RECIPROCATING COMPRESSORS AND VARIABLE FREQUENCY DRIVES

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Abstract:

The use of variable frequency drives (VFD) is becoming increasingly popular with induction motor driven reciprocating gas compressors. Reciprocating compressors, being rich sources of torque pulsation frequencies, too often create motor component reliability issues. The problem is compounded when variable speed is introduced because of the difficulty in avoiding resonance. This paper reviews key torsional vibration concepts and common motor failure modes. An overview of field testing methods is presented. Also, the unique concerns with variable frequency drives are discussed.

Introduction

Torsional vibration problems are commonly experienced with motor driven reciprocating compressors. This paper is intended to provide the user with sufficient knowledge to effectively handle new motor and drive purchases as well as field issues.

A reciprocating compressor is one of the most arduous applications in the motor industry. General vibration levels are extremely high. Numerous strong torsional excitations exist creating torsional vibration related component failures. No forewarning of failure is provided from external vibration, as even extreme torsional vibration will not manifest in this manner. This problem, which has persisted with reciprocating machinery since the turn of the century, has been and still is the catalyst for numerous engineering advances. References such as [1] attest to early torsional system problems and their significant engineering efforts.

The traditional method for good system design has been to perform sufficient analysis to ensure all resonances are avoided and thereby provide trouble free operation. The current trend to variable frequency drives has resulted in a need to re-think this traditional approach of "avoidance" in order to provide a system which can operate over the largest possible speed range.

Torsional Vibration Concepts

Crank Effort Torque

The translation of piston gas forces to equivalent crankshaft torque defines the non-resonant motor torque demand. The harmonic content of the compressor will depend upon the running speed and the compressor's staging. Physically, staging is the interconnection of some cylinder outlets with the inlet of others. This is done to provide the necessary compressor volume and pressure performance. Single, double and triple staged units are commonly used. The different compressor stages will have different piston sizes and the resulting pressure and inertia forces will vary. The physics of this creates the rich harmonic content of reciprocating compressors. **Figure 1** shows a plot of crank effort torque versus rotation angle. This can be broken down by frequency analysis into its harmonic content as shown in **Table 1**. Crank effort torque would be equivalent to motor shaft torque if the shafting system were completely rigid. However, the torsional flexibility inherent in any shaft system can result in the amplification of crank effort torque. This creates the need for further detailed analysis.

TABLE I
Crank Effort Torque Harmonic Content

Harmonic Order	Amplitude Lbf-IN	Percentage
1	15918	13.98
2	57877	50.83
3	2002	1.76
4	10461	9.19
5	16189	14.22
6	802	0.70
7	4159	3.65
8	5314	4.67
9	1827	1.60
10	1201	1.05
11	552	0.48
12	1826	1.60

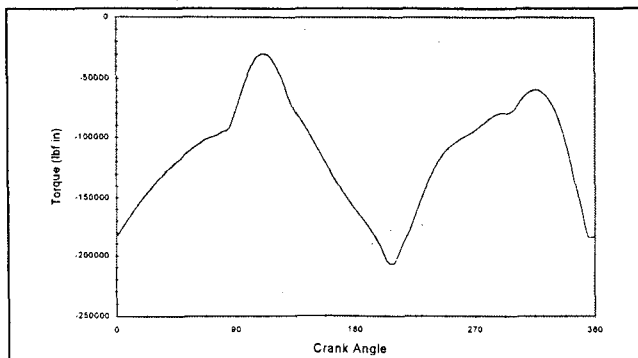


FIGURE 1. Crank Effort Torque

Undamped Resonance Analysis

The Undamped Resonance Analysis is the basic calculation which is done to determine the system's torsional natural frequencies. The complete shaft system is modeled as a series of lumped masses and springs.

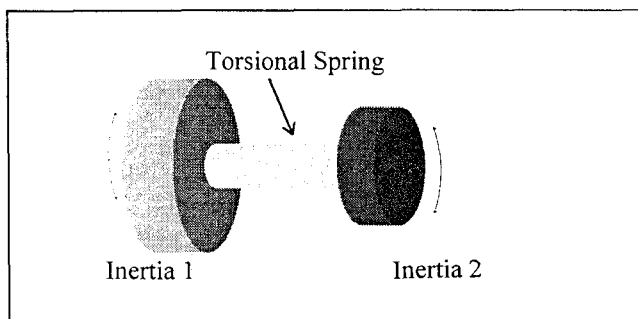


FIGURE 2. Two Mass Lumped Model

For example, the simple '2 mass - one spring' system shown in Figure 2 will yield a single torsional frequency when calculated per its fundamental equation.

$$\omega_n = \sqrt{K_{12}(I_1 + I_2) / I_1 I_2} \quad (1)$$

where

ω_n = fundamental natural frequency

K_{12} = shaft torsional stiffness, unit torque per radian

I_1, I_2 = torsional inertia, WK^2 , mass times moment arm squared

A typical shaft system is shown in Figure 3. It consists of the motor shaft, coupling, flywheel and compressor crankshaft. The lumped mass model for this system is shown below the shaft system in the same figure. The motor is modeled as a single inertia and shaft spring constant. The crank throws, flywheel and coupling are also modeled as individual lumped inertias. Rules are available to help in obtaining an adequate idealization of these system components. This can be supplemented with finite element models to determine effective stiffnesses if necessary.

The output of this analysis is a number of torsional natural frequencies (one less than the number of inertias) and the system mode shapes. The calculated frequencies are best viewed on a Campbell Diagram, as shown in Figure 4, where operating speed is plotted versus the natural frequencies. Harmonic lines (orders) are drawn to show potential excitation forces. The intersection of a harmonic line with the natural frequency line indicates a condition of resonance (also known as the interference point). The speed at which the frequencies are coincident can be called a *critical speed*.

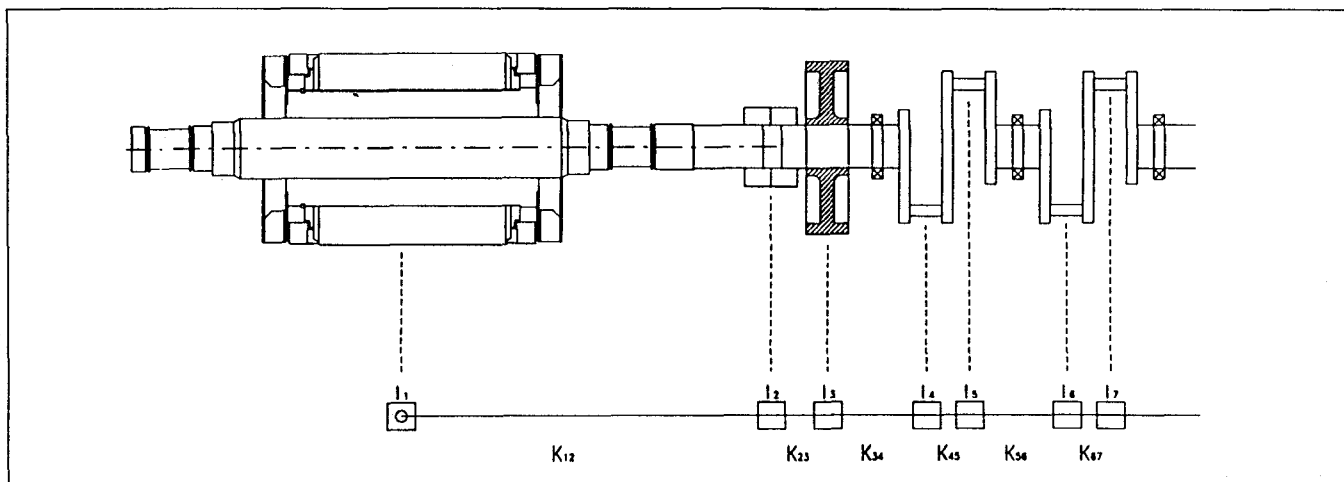


FIGURE 3. Shaft System

Mode shape plots show the system deflection pattern and are helpful in anticipating the effect of tuning changes. What Undamped Resonance Analysis does not do, is predict system torque levels.

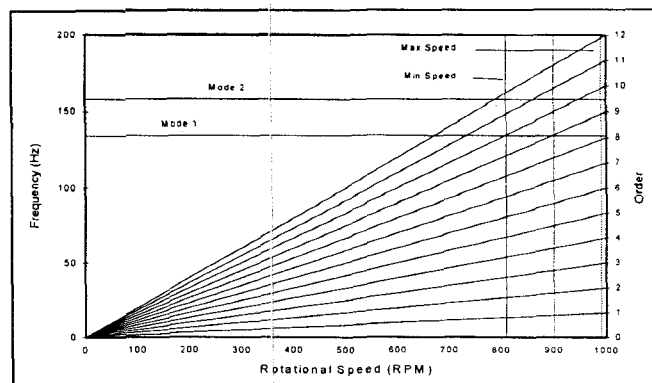


FIGURE 4. Campbell Diagram

Forced Response Study

A Forced Response Study is the method used to predict shaft torque. It is a specialized analysis requiring a proven software package and an experienced user. The compressor load cases need to be accurately defined. It is quite common that partial load compressor operation creates the highest torque pulsation levels and therefore these cases need thorough analysis. A lumped mass model similar to the undamped analysis model is again used. Quite often it will be broken down into more masses and springs in order to provide more output data. The analysis then analytically accounts for the excitation forces (crank effort torque) and the sources of system damping.

System Tuning

Once we have completed the torsional analysis we may wish to modify the system to obtain a better overall response. This can be done by changes to: Torsional Stiffness, Component Inertia, Damping, Excitation Force and Excitation Frequency. The latter two items, are out of this paper's scope.

Torsional Stiffness: When shaft stiffness changes, so do the torsional resonant frequencies. This is a practical approach to system tuning during the design phase either through changes to shaft diameter or overall length. The selection of bearing type will influence the overall length as antifriction bearings are generally more compact than sleeve. As well, diameters will be restricted by available bearing sizes and possibly the need for a self cooling bearing arrangement.

The coupling torsional stiffness is also variable within a manufacturers product range by selecting alternate sizes and flexible member type / thickness. More significant changes are usually possible by switching to a design that utilizes a different flexibility concept or material.

Inertia: As previously stated, inertia is one of the essential variables that determines the system's torsional natural frequencies. Minor frequency shifts for tuning purposes are commonly done with the addition of donut flywheels on the compressor crankshaft, weighted cross heads and sometimes coupling size. Within the motor, the inertia is generally set by the electrical design. Significant inertia changes are possible by changing frame size, but capital equipment budgets usually prevent this. Historically, a large percentage of compressors have incorporated significant added inertia in the form of a drive end flywheel. The purpose of this can be two fold. First is to reduce the crank effort torque pulsations and the associated current pulsations by the increased momentum and second is to lower the torsional natural frequencies. The decision to utilize a flywheel should be based upon benefits proven by the Torsional System Analysis not by personal preference. Major inertia changes will also shift the mode shapes which can be helpful in optimizing the performance of a damper.

System Damping: Increased system damping can greatly decrease the peak torsional response when running close to resonance. This is commonly done with a soft coupling, which has damping components, usually rubber, to absorb the torsional vibration. These couplings will also change the torsional natural frequencies due to their reduced torsional stiffness. Soft couplings, although widely available, have fallen out of favour due to higher maintenance. The damping components, generally rubber, tend to degrade and wear out from the heat generated in absorbing the torsional energy.

Viscous torsional dampers are also available. These have been used for years on engine driven reciprocating compressors and may start finding application with VFD installations that run on resonance.

Current Pulsation Analysis

This analysis, which is normally performed by the motor manufacturer, calculates the supply current pulsations and motor air gap torque. NEMA and other industry standards, like API, limit the amount of current variation permissible. This helps insure the quality of the electrical supply for other applications and users. Normal inputs include electrical motor parameters, system inertia and crank effort torques. It does not normally include any system resonance effects. This simplification can result in discrepancies between predictions and actual measurements due to the system's modal behavior. For example, on many 1500 to 3000 HP installations, torsional vibration is predominately at the fundamental mode. The

shaft system deflection is such that minimal motion takes place at the motor rotor. When this system is in resonance very little load variation is seen at the motor air gap and hence current pulsations remain at modest levels. Actual field tests have recorded shaft extension torque pulsations of 150 percent while the current pulsation level was below 5 percent. If on the other hand, motor torsional vibration is at a significant level compared to peak system values then current pulsation can become an indicator of the system's vibration. In conclusion, the torsional system behavior must be known before any assumptions about current pulsation significance can be made.

Torsional Failure Modes

Fatigue Failure

Components that fail from torsional vibration are usually failing due to metal fatigue. Metal fatigue is defined as failure at relatively low stress levels caused by repeated load cycling. Stress levels as low as 2 percent of the single application load limit can cause failure if factors, such as surface finish, are extremely poor.

The relationship between alternating stress levels and the number of cycles to failure is shown with a SN diagram. See **Figure 5**. Note that as the stress level is reduced the cycles to failure increase. For most metals, a point is reached where life becomes essentially infinite. This is called the endurance limit and is labeled in the above figure. The endurance limit for steel is normally between 2 & 10 million cycles. For aluminum the endurance limit is not reached until between 1×10^8 and 1×10^9 cycles.

The endurance limit for steel is typically one half its ultimate tensile strength when tested with a carefully prepared, flaw free, laboratory sample. The endurance limit of actual machine elements will be considerably less than this test sample. This difference is accounted for with the following equation, [2], [3].

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \quad (2)$$

where S_e = endurance limit of component
 S'_e = endurance limit of test sample
 k_a = surface factor
 k_b = size factor
 k_c = reliability factor
 k_d = temperature factor (not normally used with motors)
 k_e = stress concentration factor
 k_f = miscellaneous-effects factor(not normally used with motors)

These factors account for a majority of the phenomena affecting fatigue. A brief description of k_a , k_b , k_c and k_e are contained in the following subsection. As k_d and k_f are not normally applied to motors they are not discussed.

Surface Finish: Fatigue cracks usually start on the surface of a component, at microscopic irregularities. The surface quality, as defined by the manufacturing process, will make significant differences to the available fatigue strength. This effect will also vary based on the component's tensile strength. For example, a 4140 heat treated shaft with a tensile strength of 120,000 psi will have a 25 percent fatigue strength improvement, when the finish is upgraded from machined to ground. Another 10 percent improvement is possible when going to a polished finish.

Size Factor: The larger the rough material size the higher the probability for inherent defects from the steel making process. This is accounted with the following formula, [2]

$$k_b = (d / 7.6)^{-0.068} \quad (3)$$

where d = shaft diameter in millimeters

Reliability Factor: The published endurance limit test data usually represents mean life values. Generally there is a large scatter to be found in fatigue and is commonly accounted for with a reliability factor [3]. In recent years there has been a trend to account for this effect at the safety factor evaluation phase.

Stress Concentrations: Where the geometry of a component changes higher than nominal stresses result. This is known as a stress concentration. The sharpness of this change will directly influence the peak values that result. **Figure 6** details this effect for round shafts in torsion.

Mean Stress Effects: The fatigue life of a component is also influenced by the level of mean stress. Mean stress is defined as the stresses resulting from the nominal (non-alternating) load. This effect, as shown in **Figure 7**, is most often predicted by the Modified Goodman Criterion although the Gerber and Soderberg methods are favored by some industries. The Modified Goodman method in equation form is as follows:

$$S_a = S_{ut} S_e \sigma_a' / (S_e \sigma_m' + S_{ut} \sigma_a') \quad (4)$$

where S_a = allowable alternating stress
 S_{ut} = ultimate tensile strength
 S_e = endurance limit
 σ_a' = actual alternating stress
 σ_m' = actual mean stress

There is also a corresponding life increase when local compressive stress is added. This is commonly done on stress concentrations by Shot Peening [4], [5].

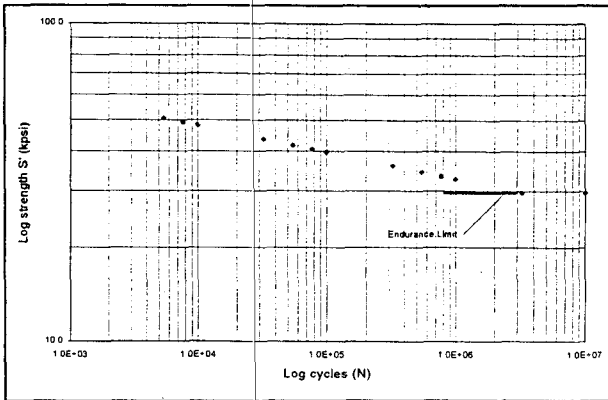


FIGURE 5. SN Diagram

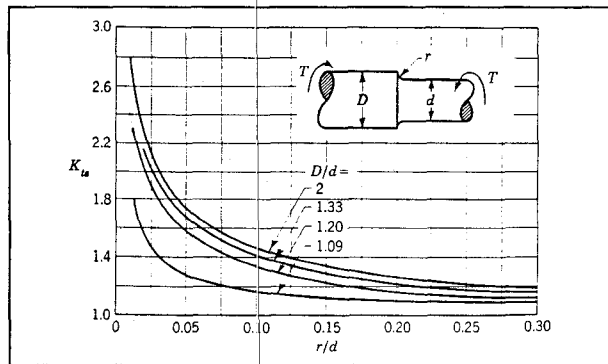


FIGURE 6. Torsional Stress Concentration

Component Life Requirements

For a typical motor / compressor system the torsional vibration occurs between the third and sixth harmonic of running speed. This generates millions of cycles per day and thus the rotor system must be designed for infinite life. Conversely, transient torsional stress occurring from start-up and shut down would normally be designed for a finite life, such as API's 5000 cycle requirement. Note that transient torques are not usually a problem with VFD due to their smooth starting characteristics.

Motor Failure Modes

Shaft Extension: The common shaft extension designs with their key attributes are shown in Table II.

TABLE II
Shaft Extension Attributes

Type of Design	Attributes
keyed, straight fit	low cost, coupling requires light shrink fit to prevent fretting, lowest fatigue performance
keyless, straight fit	lowest initial shaft cost, requires well defined shrink fit, may present testing and disassembly problems, improved fatigue performance
keyless, taper fit	high cost, simplifies removal for maintenance, improved fatigue performance
integral hub	highest cost, usually only available on forgings, may affect internal component design due to hub OD assembly restriction, best fatigue performance due to large fillet radii at shaft transition

On standard keyed shaft extensions, the use of sled runner keyways provide a reduction in the stress concentration compared to the end milled type. [6]. The full benefit of large keyway corner radii and good surface finish should be utilized. In order to maintain surface finish, the user must be careful that maintenance activities on the coupling do not gouge or score the shaft. Always use sufficient heating during coupling removal so that puller forces are minimized.

Shaft Steps: This is a very common failure location within the industry. If the fillet radii at these steps are too sharp or surface finish too rough then fatigue cracking can occur. A complete Shaft Fatigue Analysis will allow the motor manufacturer to optimize the diameters, radii, shaft material and surface finishes.

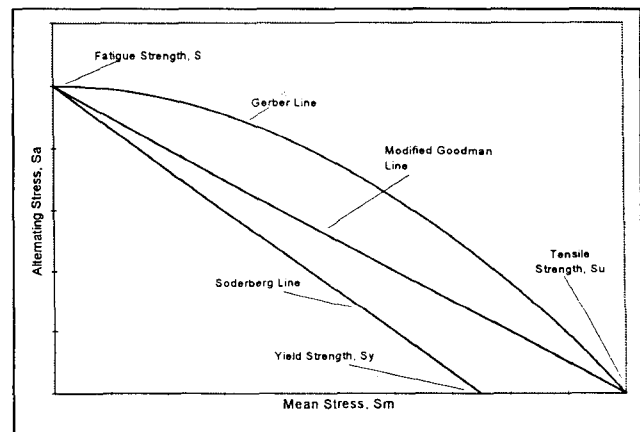


Figure 7 - Goodman Diagram

Spider Bars: In the 1000 to 5000 horsepower size range, welded spider bars are a very common motor design practice. See **Figure 8**. The welds, which join the low carbon steel spider bars to the alloy steel shafting, such as AISI 4140, are very critical and highly stressed. High levels of pre-heat and a good post welding heat treatment are necessary to control heat affected zone hardening. Process variables must be well controlled to prevent hydrogen cracking. Attention to the geometry between bar and shaft is necessary to control the stress concentration. The peak stress level will be greatly reduced if all spider bars carry an equal share of the torsional load. Also note that one piece forgings are available but costly.

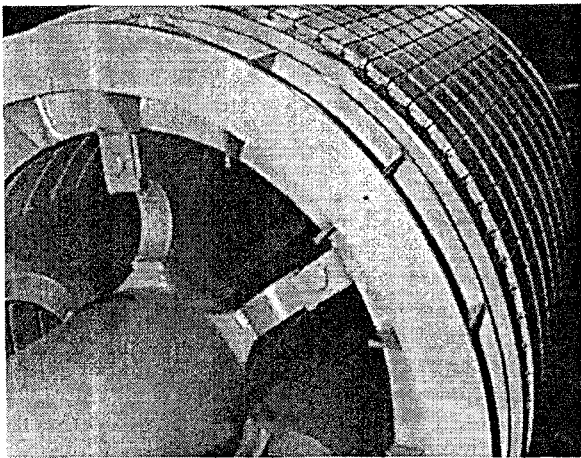


Figure 8. View of Rotor Showing Spider Bars

Rotor Lamination to Shaft Interface: The rotor lamination to shaft interface must be designed to transmit the peak torque without relative motion. The presence of any relative motion will result in fretting wear which will escalate causing rotor end fingers or duct fingers to come loose. Also amplification of the torque pulsations can occur from the impact forces of loose items hitting their adjoining components. This situation will lead to an accelerated failure of the torque carrying spider bars and lamination assembly. A dependable prevention method is to have sufficient shrink fit between the laminations and the shaft / spider bars. This approach provides equal load sharing between spider bars and reduces fatigue stress levels as a result of the compressive shrink fit forces. Improved fatigue performance results.

Laminations: Sufficient lamination tooth thickness must be available to withstand the inertial loading of the rotor bars. Likewise the rotor bars should be a precision fit in their slots to prevent vibration. Slot bridge lamination tip failures are not commonly associated with Reciprocating Compressor applications since torsional vibration does not directly increase their stress level.

Direct Mounted Cooling Fans: Historically, cooling fans have been a major cause of motor and generator failures. This is primarily due to unacceptable stress concentrations or operation on resonance. Like any component subjected to fatigue, stress levels must be minimized, stress concentrations reduced and surface finish improved where possible. Fans should be tuned above the first 10 harmonics to prevent resonance and close attention paid to all torsional modes of vibration. With TEFV enclosures, the opportunity exists to completely remove these fans and eliminate the potential problem without penalizing motor thermal performance.

Field Testing

Prior to commissioning, the need for a torsional field test must be reviewed. Is the reliability of the analysis deemed to be sufficient for the application? Questions such as the following should be asked:

- Has the required communication and actions taken place between all parties?
- Is the mass-elastic data accurate?
- Is the torsional analysis software field proven?
- Was a Forced Response Analysis Performed?
- Is the compressor still going to operate per the analysis conditions?
- Was a thorough sample of VFD operating conditions investigated?
- Were VFD effects considered?
- Is the cost of a field test justified in terms of improved field reliability?

If sufficient doubt exists then a field test may be prudent.

Torsional testing of an installed system is now commonly performed. Numerous testing methods and systems exist. The following is a brief overview.

Telemetry

This method is a well proven, commercially available test procedure. Set-up involves the installation of strain gages on the shaft surface and a rotating plus stationary antenna system. Data is transmitted from the strain gages using FM radio frequency transmission. An actual test set-up is shown in **Figure 9**. Telemetry provides a direct measurement of shaft torque which is independent of any analytical models. The test setup can be field load calibrated to further improve accuracy. Strain gages can be installed on a variety of internal components, such as fans and spider bars. Caution must be exercised in the strain gage placement to avoid stress concentrations which can distort the readings.

Accelerometers

This method involves the installation of an angular accelerometer assembly, usually on one of the system's free shaft ends. It provides reliable data to determine the

resonant frequencies. To estimate system torque levels, the acceleration data must be utilized in conjunction with a torsional system model.

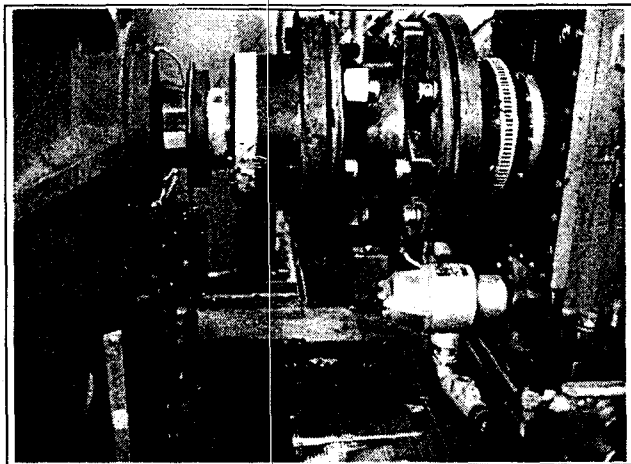


Figure 9. Telemetry Test

Displacement Methods

Numerous methods are available which involve the measurement of shaft surface displacement. These include:

- Eddy Current Probes and Toothed Wheels
- Strobe Light and Degree Scale
- Doppler Effect Laser Systems

The laser system, which is relatively new, offers good accessibility for tight axial locations. The other methods are essentially outdated.

VFD Considerations For Reciprocating Compressors

When considering drive generated torque harmonics, of primary concern are the drive generated current harmonics. These are generally proportional in relation, however harmonics will vary between drive types and motor designs. The primary torque pulsations generated by harmonic currents will exist at 6X, 12X and 18X the drive output frequency. With a 2 pole motor, this multiple of output frequency coincides with multiples of running speed and equals multiples of 3X the number of poles. With each additional pole set, the current/torque generated pulsation as a multiple of running speed also increases in multiples of 3X the number of poles. The level of torque pulsations will depend on harmonic current levels in the drive output current, the generation of harmonic currents in the rotor and their interaction with airgap flux. For a system with a base speed of 900rpm (pulsations at 24X,

48X, 72X rpm) and a first natural frequency of 70Hz, the drive torque pulsations clear interference after 11.7hz output or 19.4% of base speed. Although the levels of pulsations originating from harmonic currents are a fraction of the load torque harmonics, a harmonic rich environment such as a reciprocating compressor may have coincident torque pulsations from different sources.

Within motor designs, changes in leakage inductance and effective rotor resistance will change harmonic torque levels even with a constant current harmonic content at the motor input. Due to the relatively large stator impedance compared to rotor impedance, especially at higher frequencies, most of the harmonic currents will pass to the rotor. Higher leakage inductance levels will lower the amount of harmonic current which passes through the rotor circuit. Leakage inductance and magnetizing current in an induction motor increase with the number of poles for a given shape (constant length/radius of rotor) [7]. From reference [8] the amount of harmonic current absorbed by the motor is inversely proportional to the output frequency and motor leakage inductance. Increased motor losses due to harmonic current heating can impose limits on the torque/speed capabilities on motors without separate cooling.

Voltage Source Inverter (VSI) drives have excellent performance over a wide speed range and usually have higher switching frequencies in the Pulse Width Modulated (PWM) patterns which they use in creating output voltage waveforms. Harmonic elimination alternatives are good with higher switching frequencies. Given a number of pulses per half wave in a PWM pattern, choosing firing angles specifically can eliminate certain output harmonics. When lower order harmonics are eliminated it is usually accompanied by increased levels of higher order voltage harmonics. Higher dv/dt resulting from faster switching can also increase insulation stress. Inverter duty motors usually compensate for this with increased insulation on the first few turns of stator windings. Multi-level VSI drives provide a lower step change in the PWM pattern and lessen the stress on insulation.

Current Source Inverter drives have a wide power range, good performance over a wide speed range with PWM control and a large torque range limited primarily by motor breakdown Torque. Harmonic elimination can be achieved at lower switching frequencies with good sinusoidal likeness in motor waveforms which limits motor losses. This is the most commonly used drive on existing motors. With a Current Source Inverter PWM Drive the higher frequency harmonics are absorbed by the capacitor between inverter and motor. An electrical natural frequency results from the LC motor/capacitor circuit that can amplify current harmonics near this frequency in the output waveform. As in VSI drives, lower torque harmonics result from a higher leakage reactance. With the use of harmonic elimination, total distortion on current is usually less than 5% and torque pulsations less than 1%.

Drive regulator responses can also play a role in the lower frequency torsional resonances. In most compressor applications, speed regulation is well within the capability of the drive. There is usually margin for adjustment on the drive regulator circuits to ensure no torsionals are excited by speed or current regulator oscillations. Being aware of the regulator bandwidths and the frequency of compressor oscillations is important when monitoring vibrations. A VSI drive may experience regulator interference at higher frequencies than a CSI drive. A multipulse inverter and motor is another approach that will reduce torque harmonics due to cancellation of current harmonics. Avoiding drive output frequencies that cause interference is another option if the process allows it.

Compressor torque oscillations and system resonances can sometimes produce a ripple on the motor generated voltage which can affect drive response and even distribution system electrical harmonics, should the ripple frequency coincide with natural frequencies on the electrical distribution system. On a CSI drive the size of the DC link will determine the level of interference between the input system with rectifier and the output system with inverter.

Design for VFD Operation

The majority of system analysis has in the past been based on the principal of resonant frequency avoidance. This may not be possible when operating over a broad speed range. The system analysis must therefore be concerned with the calculation of the peak torque pulsations over that full speed and full compressor operating conditions. The use of an additional source of damping may be necessary if peak pulsation levels cannot be safely handled. It is also possible to reduce peak levels by optimizing the compressor design. If frequency avoidance within the operating speed range is being considered, then the minimum speed range necessary to meet the design torque pulsation level must be calculated. Hence, for any system utilizing a VFD, a Forced Response Analysis is a must.

There has been a large volume of technical papers presented on the electrical motor requirements for VFD applications. Far less has been written about the mechanical requirements. For reciprocating compressor applications there are no all encompassing motor design standards. There is a need for a thorough analysis over the full operating range. Close attention must be paid to rotor stress concentrations and potential component looseness. If the system is designed without additional damping, then it should be expected that it will be pushing the limits of any reciprocating compressor motor design.

Motor Design Standards

The following is a summary of *Motor Design Standard* recommendations:

- Define the torque pulsation level for design
- Define the analytical method and safety factors for fatigue calculations
- Use large shaft step fillet radii confirmed with the fatigue analysis
- Never use end milled keyways
- Consider upgraded shaft extension designs
- Ensure laminations have an adequate shrink fit to the shaft
- Consider using TEFV enclosures to eliminate internal fans
- Resonance test all internal and external fans

Summary

There are numerous road blocks to adequate system design. Generally, they fall into three main categories; Misconceptions, Inadequate Analysis & Testing, and Communication.

Misconceptions

In typical applications, over 1000 horsepower, the oscillating motor shaft torque is greater than the mean value. This fact and other technicalities already mentioned are often not realized due to the common misconceptions in the Industry such as:

1. current pulsations directly relate to torque pulsations
2. crank effort torques are the same as shaft torques
3. undamped torsional natural frequency calculations are sufficient analysis
4. radial vibration relates to torsional performance

Inadequate Analysis & Testing

Torsional system analysis needs to accurately predict system behavior. This requires a Forced Response Analysis which has been proven by field testing. The motor, like reciprocating compressors, needs a fatigue analysis. A subsequent check of analysis accuracy by means of a commissioning field test is often justified considering capital outlay and downtime costs.

Communications

The necessary system design is an extremely interactive process which involves contributions from the: User, Packager, Engineering Consultants, Coupling Manufacturer, Compressor Company, Motor Company, and Drive Company. The need to coordinate and formalize the information flow should not be underestimated. Many times the necessary work is being done individually but the lack of co-ordination leads to field problems.

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