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Complex Conjugate

The complex conjugate of a complex number $z \equiv a + b i$ is defined to be

$$\bar{z} \equiv a - b i. \quad (1)$$

The conjugate matrix of a matrix $\mathbf{A} = (a_{ij})$ is the matrix obtained by replacing each element a_{ij} with its complex conjugate, $\bar{\mathbf{A}} = (\bar{a}_{ij})$ (Arfken 1985, p. 210).

The complex conjugate is implemented in the Wolfram Language as `Conjugate[z]`.

Note that there are several notations in common use for the complex conjugate. Applied physics and engineering texts tend to prefer z^* , while most modern math and theoretical physics texts favor $\bar{z}$. Unfortunately, the notation z^* is also commonly used to denote adjoint operators matrices. Because of these mutually contradictory conventions, care is needed when consulting the literature. In this work, $\bar{z}$ is used to denote the complex conjugate.

Common notational conventions for complex conjugate are summarized in the table below.

notation	references
$\bar{z}$	This work; Abramowitz and Stegun (1972, p. 16), Anton (2000, p. 528), Harris and Stocker (1998, p. 21), Golub and Van Loan (1996, p. 15), Kaplan (1981, p. 28), Kaplan (1992, p. 572), Krantz (1999, p. 2), Kreyszig (1988, p. 568), Roman (1987, p. 534), Strang (1988, p. 220), Strang (1993)

z^*

Arfken (1985, p. 356), Bekefi and Barrett (1987, p. 616), Press et al. (1989, p. 397), Harris and Stocker (1998, p. 21), Hecht (1998, p. 18), Herkommer (1999, p. 262)

In linear algebra, it is common to apply both the complex conjugate and transpose to the same matrix. The matrix obtained from a given matrix $\mathbf{A}$ by this combined operation is commonly called the conjugate transpose $\mathbf{A}^H$ of $\mathbf{A}$. However, the terms adjoint matrix, adjugate matrix, Hermitian conjugate, and Hermitian adjoint are also used, as are the notations $\mathbf{A}^\dagger$ and $\mathbf{A}^*$. In this work, $\mathbf{A}^H$ is used to denote the conjugate transpose matrix and $a^\dagger$ is used to denote the adjoint operator.

By definition, the complex conjugate satisfies

$$\overline{\overline{z}} = z. \quad (2)$$

The complex conjugate is distributive under complex addition,

$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}, \quad (3)$$

since

$$\overline{(a_1 + i b_1) + (a_2 + i b_2)} = \overline{(a_1 + a_2) + i (b_1 + b_2)} \quad (4)$$

$$= (a_1 + a_2) - i (b_1 + b_2) \quad (5)$$

$$= (a_1 - i b_1) + (a_2 - i b_2) \quad (6)$$

$$= \overline{a_1 + i b_1} + \overline{a_2 + i b_2}, \quad (7)$$

and distributive over complex multiplication,

$$\overline{z_1 z_2} = \overline{z_1} \overline{z_2}, \quad (8)$$

since

$$\overline{(a_1 + i b_1) (a_2 + i b_2)} = \overline{(a_1 a_2 - b_1 b_2) + i (a_1 b_2 + a_2 b_1)} \quad (9)$$

$$= (a_1 a_2 - b_1 b_2) - i (a_1 b_2 + a_2 b_1) \quad (10)$$

$$= (a_1 - i b_1) (a_2 - i b_2) \quad (11)$$

$$= \overline{a_1 + i b_1} \overline{a_2 + i b_2}. \quad (12)$$

SEE ALSO

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