Hopfield Neural Network Implementation of the Optimal CDMA Multiuser Detector

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Abstract— **We investigate the application of Hopfield Neural Networks (HNN) to the problem of multiuser detection in Spread Spectrum/CDMA communication systems. It is shown that the NP-complete problem of minimizing the objective function of the Optimal Multiuser Detector (OMD) can be translated into minimizing an HNN "energy" function, thus allowing to take advantage of the ability of HNNs to perform very fast gradient descent algorithms in analog hardware and produce in real-time suboptimal solutions to hard combinatorial optimization problems. The performance of the proposed HNN receiver is evaluated via computer simulations and compared to that of other suboptimal schemes as well as to that of the OMD for both the synchronous and the asynchronous CDMA transmission cases. It is shown that the HNN detector exhibits a number of attractive properties and that it provides in fact a powerful generalization of a well-known and extensively studied suboptimal scheme, namely the Multistage Detector (MSD).**

Keywords— **Hopfield Neural Networks, Digital Communications, CDMA Multi-user Detectors.**

I. Introduction - Background

In recent years there has been a considerable interest in using highly parallel implementations of analog VLSI neural networks to solve a variety of problems in digital communications while meeting the increasing demand for very high speed efficient data transmission over physical communication channels [1,2,3,4,5,6,7].

The optimum centralized demodulation of the information sent simultaneously by several users through a shared Gaussian multiple access channel is a very important problem arising in multipoint-to-point digital communication networks (e.g. radio networks, local-area networks, uplink satellite channels, uplink cellular communications). In the *Code Division Multiple Access* (CDMA) technique, each transmitter modulates a different signature signal waveform, which is known to the receiver. At the receiver, the incoming signal is the sum of the signals transmitted by each individual user.

To demodulate the received signal we need to suppress the inherent channel noise, often modeled as an additive Gaussian process, and the Multiple Access Interference (MAI). If the waveforms assigned to each of the users are orthogonal and the transmitted signals are antipodal ($\{-1, +1\}$) then the *conventional detector* (CD) can recover the information by first passing the received signal through a bank of filters matched to the users' signature waveforms, and then deciding on the information bits based on the sign of the output. One of the major limitations of the conventional detector is that it is not *near-far* resistant, i.e. its performance degrades severely when powers of the transmitting users are dissimilar.

Verdu [8], has shown that optimum, near-far resistant multiuser demodulation can be achieved via the maximization of an integer quadratic objective function. In [9], Verdu showed that in both the *synchronous* and the *asynchronous* transmission cases, optimum multi-user demodulation is an *NP-complete* problem, and therefore, research efforts have concentrated on the development of suboptimal receivers, that exhibit good near-far resistance properties,

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have low computational complexity, and achieve Bit-Error-Rate (BER) performance that is comparable to that of the optimal receiver.

Among the many suboptimal multiuser detectors proposed in the literature we mention the *decorrelating detector* developed by Lupas and Verdu [10], that is linear in nature and complexity and achieves near-optimal performance, assuming that the users' signals form a linearly independent set and the spreading codes of all users are known. Another suboptimal detector is the *multistage detector* (MSD) developed by Varanasi and Aazhang [11], that relies on improving each stage's estimate by subtracting the estimate of the MAI obtained by the previous stage. Recently, Aazhang et. al. [1], and U. Mitra and H. V. Poor [12], have proposed feedforward neural network based multi-user detectors. While their performance is shown to be very good for a very small number of synchronous or asynchronous users, their hardware complexity (number of neurons and training time) appears to be *exponential* in the number of users, as conjectured in [1]. Furthermore it is only empirically possible to determine the number of neurons in the hidden layer as the number of users increases.

In this paper we elaborate on an alternative multiuser detector first introduced in [13] that is based on the ability of the *Hopfield Neural Networks* (HNN) [14] to give very fast suboptimal solutions to hard combinatorial optimization problems. The remaining on this paper is organized as follows: In Section II we present the optimal CDMA multiuser detection problem formulation. In Section III we provide briefly some background on HNNs; then in Section IV we show how the objective function of the OMD problem can be translated into an HNN energy function and prove that the proposed HNN receiver is a powerful generalization of the multistage detector. In Section V we compare the performance of the HNN to that of the MSD for the two-user synchronous transmission case and identify *Regions of Failure* of both receivers as a function of the near-far ratio and the cross-correlation of the user signature waveforms. In Section VI the performance of the proposed HNN detector is evaluated via simulations and compared to that of other proposed detectors for the synchronous transmission case. In Section VII we present an investigation on the behavior of the OMD's objective function both analytically and experimentally and formulate conditions that can lead to a significant reduction of the search space over which the optimization needs to be performed. In Section VIII, simulation results for the case of asynchronous CDMA transmission with a small number of users are presented, and finally Section IX contains a summary of the main results and a discussion of our current research directions.

II. Problem Statement - Multiuser Detection Schemes

Assume that K active transmitters share the same Gaussian channel at a given time instance. A signature waveform $s_k(t)$, time limited in the interval $t \in [0,\ T]$, is assigned to each transmitter. Let us denote the i^{th} information bit of the k^{th} user as $b_k^{(i)} \in \{+1,\ -1\}$. In a general CDMA system, the signal at a receiver is the superposition of K transmitted signals and additive channel noise:

$$r(t) = \sum_{i=-P}^{P} \sum_{k=1}^{K} b_k^{(i)} s_k(t - iT - \tau_k) + n(t), \quad t \in \mathbf{R} \qquad (1)$$

In (1) $\tau_k \in [0,\ T)$ are the relative time delays between the users and $2P+1$ is the packet size. In case that the stations cooperate to maintain synchronism, it holds that $\tau_k = 0, \quad k = 1, \ldots, K$. As we mentioned in the previous section, the Conventional Detector (CD) consists of a bank of filters matched to the signature waveforms of each user, and a simple thresholding device that produces an estimate $\hat{b}_k^{(i)}$ for the i^{th} information bit of the k^{th} user based on the sign of the i^{th} output of the k^{th} matched filter:

$$\begin{aligned} y_k^{(i)} &= \int_{iT-\tau_k}^{(i+1)T-\tau_k} r(t) s_k(t - iT - \tau_k) dt \\ \mathbf{b}_{CD}^{(i)} &= sign(\mathbf{y}^{(i)}) \end{aligned} \qquad (2)$$

where $\mathbf{y}^{(i)} = [y_0^{(i)} \quad y_1^{(i)} \quad \ldots y_{K-1}^{(i)}]^T$. The structure of the conventional detector is depicted in Figure 1.

On the other hand, the *Optimal Multiuser Detector* (OMD) produces an estimate for the information vector transmitted at the discrete time instant i, based on the maximization of the logarithm of the likelihood function. In the *synchronous* case it holds [9] that:

$$\mathbf{b}_{OMD}^{(i)} = \arg \max_{\mathbf{b} \in \{+1,\,-1\}^K} \{2\mathbf{y}^{(i)^T}\mathbf{b} - \mathbf{b}^T\mathbf{H}\mathbf{b}\} \qquad (3)$$

where $\mathbf{H} \in \mathbf{R}^{K \times K}$ is the (symmetric) matrix of signal cross-correlations:

$$h_{kl} = \int_0^T s_k(t)s_l(t)dt \qquad (4)$$

Because of the exponential growth of the computational complexity of the OMD with the number of active users, suboptimal detection schemes have been proposed. One such detection scheme with a number of attractive properties including low computational complexity, is the *Multistage Detector* (MSD) proposed by Varanasi and Aazhang [11]. The MSD consists of a collection of stages $m = 1, 2, \ldots$ each producing an estimate $\mathbf{b}_{MSD}^{(i)}(m)$ as follows:

$$\mathbf{b}_{MSD}^{(i)}(m+1) = sign\left(\mathbf{y}^{(i)} - (\mathbf{H} - \mathbf{E})\mathbf{b}_{MSD}^{(i)}(m)\right) \qquad (5)$$

where $\mathbf{E}$ is a diagonal matrix with elements $e_{ii} = \int_0^T s_i^2 dt$ (signal energies). The output of the first stage ($m = 1$) is initialized to the estimate of the conventional detector. As we will show in Section IV, an MSD with an infinite number of stages may converges to a *local minimum* of the OMD objective function. The MSD has been well studied and analyzed in the literature and it is shown to exhibit in many cases a BER performance very close to that of the OMD while it remains insensitive to near-far problems.

In the *asynchronous* transmission case, the optimum receiver problem can again be expressed in the form of (3) by defining the matrices $\mathbf{H}(j) \in \mathbf{R}^{K \times K}$, $j = -1,\ 0,\ 1$ as,

$$h_{kl}(j) = \int_{-\infty}^{\infty} s_k(t - \tau_k)s_l(t + jT - \tau_l)dt \qquad (6)$$

and the matrix $\tilde{\mathbf{H}} \in \mathbf{R}^{(2P+1)K \times (2P+1)K}$ as,

$$\tilde{\mathbf{H}} = \begin{bmatrix} \mathbf{H}(0) & \mathbf{H}(-1) & 0 & \ldots & 0 \\ \mathbf{H}(1) & \mathbf{H}(0) & \mathbf{H}(-1) & & \vdots \\ 0 & \mathbf{H}(1) & \mathbf{H}(0) & \ddots & 0 \\ \vdots & & \ddots & \ddots & \mathbf{H}(-1) \\ 0 & \ldots & 0 & \mathbf{H}(1) & \mathbf{H}(0) \end{bmatrix} \qquad (7)$$

Then, the optimum receiver for the asynchronous case, can be formulated as a larger size combinatorial optimization problem of the same form as in the synchronous case:

$$\tilde{\mathbf{b}}_{OMD}^{(m)} = \arg \max_{\tilde{\mathbf{b}} \in \{+1,\,-1\}^{(2P+1)K}} \{2\tilde{\mathbf{y}}^{(m)^T}\tilde{\mathbf{b}} - \tilde{\mathbf{b}}^T\tilde{\mathbf{H}}\tilde{\mathbf{b}}\} \qquad (8)$$

where $\tilde{\mathbf{y}}^{(m)^T}$ is the row vector consisting of the sampled outputs of the matched filter-bank corresponding to the m^{th} data packet. Note that if the packet length is relatively large (e.g. $2P + 1 = 511$ bits), even for small to moderate number of users the computational effort required for the solution of (8) becomes prohibitively large for real-time implementations due to the NP-complete nature of the optimization problem.

III. Hopfield Neural Networks

Hopfield Neural Networks (HNN) [15, 16], are single layer networks with output feedback consisting of simple processors (neurons) that can collectively provide good solutions to difficult optimization problems. An HNN with 4 neurons is depicted in Figure 2. A connection between two processors in established through a conductance T_{ij} which transforms the voltage outputs of amplifier j to a current input for amplifier i. Externally supplied bias currents I_i are also present in every processor j.

Each neuron i, receives a weighted sum of the activations of other neurons in the network, and updates its activation according to the rule:

$$V_i = g(U_i) = g\left(\sum_{j \neq i} T_{ij} V_j + I_i\right) \qquad (9)$$

The function $g(U_i)$ can be either a binary or antipodal thresholding function for the case of the McCulloch-Pitts

neurons

$$V_i = g(U_i) = sign(U_i) \tag{10}$$

or any monotonically increasing nonlinear function. One example of such a nonlinear function often used in simulations is the *sigmoid* function, defined by:

$$V_i = g(U_i) = sigm(\alpha U_i) = \frac{1 - e^{-\alpha U_i}}{1 + e^{-\alpha U_i}} \tag{11}$$

where α is a positive constant that controls the slope of the nonlinearity. In particular, when $\alpha \rightarrow \infty$, then $g(U_i) \rightarrow sign(U_i)$.

It has been shown [14], that in the case of symmetric connections ($T_{ij} = T_{ji}$), the equations of motion for the activation of the neurons of a HNN *always* lead to convergence to a *stable state*, in which the output voltages of all the amplifiers remain constant. Also, when the diagonal elements (T_{ii}) are zero and the width of the amplifier gain curve is narrow, (i.e. the nonlinear activation function $g(\cdot)$ approaches the antipodal thresholding function), the stable states of a network of N neuron units are the *local* minima of the quantity (*energy function*):

$$E = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} T_{ij}V_iV_j - \sum_{i=1}^{N} V_iI_i \tag{12}$$

The *equations of motion* for the i^{th} neuron may be described in terms of the energy function (12) as follows:

$$\frac{dU_i}{dt} = -\frac{\partial E}{\partial V_i} - \frac{U_i}{\tau} = -\frac{U_i}{\tau} + \sum_{i \neq j} T_{ij}V_j + I_i \tag{13}$$

where $\tau = RC$ is the *time constant* of the RC circuit connected to neuron i. With the exception of pathological cases, when the matrix $\mathbf{T}$ is negative or positive definite, networks with vanishing diagonal elements have minima *only* at the *corners* of the N-dimensional hypercube $[-1 \quad +1]^N$ [14]. Hopfield Neural Networks have been employed extensively to solve a variety of difficult combinatorial optimization problems [17,18].

IV. The HNN Detector - Relation to the Multistage Detector

It is apparent from Eqn. (3) that the OMD objective function is very similar to a HNN energy function. The cross-correlation matrix $\mathbf{H}$ is symmetric since

$$h_{kl} = \int_0^T s_k(t)s_l(t)dt = \int_0^T s_l(t)s_k(t)dt = h_{lk}$$

Moreover, Eqn. (3) can be rewritten as:

$$\begin{aligned}
&\mathbf{b}_{OMD}^{(i)} = \arg\min_{\mathbf{b}\in\{+1,\,-1\}^K}\{-\mathbf{y}^{(i)^T}\mathbf{b} + \tfrac{1}{2}\mathbf{b}^T\mathbf{H}\mathbf{b}\} = \\
&= \arg\min_{\mathbf{b}\in\{+1,\,-1\}^K}\{-\mathbf{y}^{(i)^T}\mathbf{b} + \tfrac{1}{2}\mathbf{b}^T(\mathbf{H}-\mathbf{E})\mathbf{b} + \\
&\tfrac{1}{2}\mathbf{b}^T\mathbf{E}\mathbf{b}\} = \\
&\arg\min_{\mathbf{b}\in\{+1,\,-1\}^K}\{-\mathbf{y}^{(i)^T}\mathbf{b} + \tfrac{1}{2}\mathbf{b}^T(\mathbf{H}-\mathbf{E})\mathbf{b}\}
\end{aligned} \tag{14}$$

since $\mathbf{b}^T\mathbf{E}\mathbf{b}$ is always a positive number. The matrix $-(\mathbf{H}-\mathbf{E})$ is symmetric, and has zero diagonal elements since

$$h_{ii} = \int_0^T s_i^2\,dt = e_{ii}$$

Therefore, the OMD objective function can be directly translated into the energy function of a Hopfield Neural Network (Eqn. (12)) with weight matrix $\mathbf{T} = -(\mathbf{H}-\mathbf{E})$ and biases $\mathbf{I} = \mathbf{y}^{(i)}$.

The structure of the HNN multiuser receiver is shown in Figure 3. The sampled outputs of the matched filters are converted to currents and fed as biases into the HNN section. For each sampling instance i, the initial state (OP-AMP output) is initialized to zero such that no feedback takes place and the initial output of each amplifier k becomes equal to $g(y_k^{(i)})$ where $g(\cdot)$ is the nonlinearity implemented by the neurons. After the initialization, the network is let to converge to a final stable state, and at the end of a fixed predetermined time period (in the order of a few RC constants), the detectors estimate is being computed as the sign of the network's outputs. Note that for the HNN receiver, the initial state of the HNN coincides with the estimate of the conventional detector. As in the case of the MSD, this scheme assumes that the energies of the active users vary relatively slowly and can be estimated via other techniques, such that the HNN weights can be preset according to the energies of the users and the known values of the cross-correlations of their signature waveforms.

The obvious advantage of the HNN detector over other proposed suboptimal detection schemes lies in its small convergence time. The main part of the receiver can be implemented by relatively simple analog VLSI hardware with convergence times in the order of a few nanoseconds, while other detectors have to be implemented using digital microprocessors or ASICs that, not only are much slower than the analog implementation but also consume much more power (which becomes especially important in applications such as hand-held and mobile wireless communications [2]).

However, as it will be demonstrated shortly, speed and low power consumption are not the only advantages of the HNN detector. We will show that the well-known and studied Multistage Detector (MSD) is in fact a special case of a discrete-time approximation of the proposed HNN detector. In particular, under certain conditions, the HNN detector corresponds to an infinite number of stages MSD, thus sharing the same near-far resistance and near-optimal performance characteristics that the MSD is known to exhibit. Moreover, the additional flexibility that the HNN detector allows for, in terms of controlling the RC constants of the analog circuits and the slopes of the nonlinearities, facilitates the design of HNN receivers capable of delivering near-optimal performance in regions determined by the relative energies and cross-correlation values of the user signature waveforms.

Consider the discrete-time approximation of Eqn. (13) that describes the equations of motion of the i^{th} neuron of the HNN (set $dt = 1$):

$$\Delta U_i = -\frac{U_i}{\tau} + \sum_{i \neq j} T_{ij} V_j + I_i \tag{15}$$

If the activation function of the neurons is the $sign(\cdot)$ function, the dynamics of the i^{th} neuron at the discrete-time instant $t = m+1$, are described by the following equation:

$$\begin{aligned} &V_i(m+1) = g(U_i(m+1)) = sign(U_i(m) + \Delta U_i(m)) = \\ &= sign(U_i(m) - \tfrac{U_i(m)}{\tau} + \textstyle\sum_{i \neq j} T_{ij} V_j(m) + I_i) \end{aligned} \tag{16}$$

By setting $\tau = 1$ and substituting in Eqn. (16) for the values of $\mathbf{T}$ and $\mathbf{I}$ for the proposed HNN detector, (16) becomes:

$$V_i(m+1) = sign\left(y_i - \sum_{i \neq j} h_{ij} V_j(m) \right)$$

which can be rewritten in matrix form as:

$$\mathbf{V}(m+1) = sign(\mathbf{y} - (\mathbf{H} - \mathbf{E})\mathbf{V}(m)) \tag{17}$$

Now by comparing Eqn. (17) and Eqn. (5) we see that for the special case in which the RC constant of the HNN circuit is equal to $\tau = 1$ and $g(\cdot) = sign(\cdot)$, the estimate of the $(m+1)^{th}$ stage of a MSD coincides with the output of a discrete-time approximation of this HNN at time instant $t = m+1$. Moreover, since the update of the estimate of each MSD stage is being performed synchronously, an infinite number of stages MSD is essentially equivalent to a discrete HNN operating in synchronous (fully parallel) updating mode [18] which may lead to limit cycles of length 2, as it was proved by Bruck [19]. An analog HNN with continuous updating, or an asynchronously updated discrete HNN, does not suffer from this limit cycles problem and thus in principle it can outperform even an infinite number of stages MSD.

In the next section we will experimentally demonstrate that for the two synchronous users case, the HNN detector can be designed to outperform the MSD detector, and that its performance can be improved even more by adapting the value of the RC constant and slope of the nonlinearity of the neuron units.

V. Comparison of HNN and MSD Detectors

In this section we compare the performance of the HNN detector to a 2-stage MSD detector. Consider a $K = 2$ synchronous users, noiseless case. Assume that at a given time instant the symbols transmitted by the two users are $\mathbf{b}_{sent} = [b_0 \;\; b_1]^T$, and that the energy of the second user remains constant while the energy of the first user is varying.

The cross correlation matrix can be expressed as:

$$\mathbf{H} = \mathbf{E}^{1/2}\mathbf{H}_{norm}\mathbf{E}^{1/2} = \\ = \begin{bmatrix} e^{1/2} & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & h \\ h & 1 \end{bmatrix} \cdot \begin{bmatrix} e^{1/2} & 0 \\ 0 & 1 \end{bmatrix} \quad (18)$$

where, $h \in (-1, 1)$ is the normalized cross-correlation of the users' signature waveforms and $e \in \mathbf{R}$ is the near-far-ratio which coincides with the energy of the first user.

In the noiseless case, the sampled outputs of the matched filter-bank can be expressed as:

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{b}_{sent} \quad (19)$$

By evaluating the outputs of a multiuser detector for various values of h and e, and comparing them with the originally transmitted symbols $\mathbf{b}_{sent}$, we can estimate the *Region Of Failure* (ROF) for the particular detector. In Figure 4, we compare the CD, 2-stage MSD, OMD, and a *randomly initialized* HNN detector by plotting their ROFs (dark regions) and assuming that the originally transmitted information vector is $\mathbf{b}_{send} = [b_0 \quad b_1]^T = [-1 \quad -1]^T$. In each of the plots, the vertical axis corresponds to the decimal logarithm of the near-far ratio ($log_{10}\ e$), and the horizontal to the value of the cross-correlation h. For each point in the ROF, $\mathbf{b}_{detected} \neq \mathbf{b}_{sent}$.

As we can see from Figure 4, for heavily correlated signature waveforms, the CD exhibits the worst performance producing the correct estimate only when the energies of the users are close to each other. The 2-stage MSD detector performs well when one of the users is much stronger than the other (since the strong user can be estimated more accurately), but fails in a region where the energy of one user is larger than that of the other but not enough to allow its correct estimation. For the noiseless case and for the range of h and e shown in Figure 4, the OMD never fails to produce the correct estimate for the transmitted symbols. The randomly initialized HNN exhibits a much larger ROF than the 2 stage MSD. In particular, since a randomly initialized HNN is guaranteed to converge only to a local minimum of the OMD's objective function, its ROF corresponds to objective functions with either two local minima or a unique global minimum that is however different from the originally transmitted symbols.

However, as we have shown in the previous section, an HNN detector initialized with the outputs of the conventional detector (matched filter bank) $\mathbf{y}$ corresponds to, under certain conditions, an infinite-number-of-stages long MSD detector. The particular conditions, are that the neurons' nonlinearity is very steep (α is large in Eqn. (11)) such that it approximates the $sign(\cdot)$ function, and that the $\tau = RC$ constant of the circuit is equal to one. In Figure 5 we plot the ROFs for such an HNN detector with $\tau = RC = 1.0$ and $\alpha = 100$ and we compare it to the ROF of the 2-stage MSD. As we can see from Figure 5, the ROF of the HNN detector roughly coincides with that of the 2-stage MSD. From Figure 6, where we plot again the ROFs of the HNN (with $\alpha = 5.0$ and $\tau = RC = 1.0$) and the MSD, we can see that the ROF of the HNN can be made smaller by making the nonlinearity of the OP-AMPs smoother. Finally, in Figure 7 we plot the ROF of the HNN for $\alpha = 5.0$ and several values of the RC constant. It is evident from Figure 7, that by increasing slightly the value of RC, the ROF of the HNN can be made much smaller than that of the MSD and can approach that of the OMD.

In Figure 8, we compare the CD, 2-stage MSD, HNN and OMD detectors by evaluating their error probabilities in a 2 users' synchronous CDMA case. The decimal logarithm of the Bit-Error-Rates (BER) are plotted versus the value of the near-far-ratio (e_2/e_1 in decibel) for $h = 0.7$. The Signal-to-Noise-Ratio (SNR) of user 1 is fixed at 8dB, and the BER of user 1 is being evaluated (number of incorrectly detected bits/total number of bits transmitted). As we can see from Figure 8, the CD exhibits the worst BER performance of all the other detectors. The 2-stage MSD detector exhibits worst performance when the energies of the two users are not very dissimilar as expected. The HNN's and OMD's performance are practically indis-

tinguishable except for the case of equal energy users in which the OMD performed slightly better than the HNN. When the energies of the users become very dissimilar, the 2-stage MSD, the HNN and the OMD achieve comparable performance.

VI. Synchronous CDMA Simulations Results

In our simulations it is assumed that a number of synchronous users K is transmitting over a Gaussian channel. The Direct-Sequence Spread-Spectrum Binary PSK (DSSP-BPSK) signaling system [20] is used. In DSSP the code waveforms are given by:

$$s_k(t) = A_k \alpha_k(t) \cos(\omega_c t + \theta_k), \quad k = 1, \ldots, K \tag{20}$$

where A_k is the signal amplitude, ω_c is the carrier frequency and $\theta_k \in [0,\ 2\pi)$ is the phase angle. A spreading sequence $\alpha^{(k)} = [\alpha_0^{(k)}, \alpha_1^{(k)}, \ldots \alpha_{L-1}^{(k)}]$, $\alpha_j \in \{+1,\ -1\}$ is assigned to each one of the K users, and the time limited code waveform $\alpha_k(t)$ becomes:

$$\alpha_k(t) = \sum_{j=0}^{L-1} \alpha_j^{(k)} \Pi(t - jT_c) \tag{21}$$

where $\Pi(t)$ is the unit rectangular pulse of duration $T_c = 2\pi/\omega_c$, and $T = LT_c$. In DSSP-BPSK if $\omega_c T$ is an integer multiple of 2π much greater than 1, then the elements of the $\mathbf{H}(i)$ matrices can expressed as:

$$h_{kl}(0) = \begin{cases} (E_k E_l)^{1/2} \hat{R}_{kl}(\tau_k - \tau_l) \cos(\phi_{kl}), & \text{if } l \le k \\ (E_k E_l)^{1/2} \hat{R}_{lk}(\tau_l - \tau_k) \cos(\phi_{kl}), & \text{if } l > k \end{cases} \tag{22}$$

$$h_{kl}(1) = \begin{cases} 0, & \text{if } l \le k \\ (E_k E_l)^{1/2} \hat{R}_{kl}(T + \tau_l - \tau_k) \cos(\phi_{kl}), & \text{if } l > k \end{cases} \tag{23}$$

where $\hat{R}_{kl}(\tau) = T^{-1} \int_\tau^T \alpha_k(t-\tau)\alpha_l(t)dt$, for $0 \le \tau \le T$, is the partial continuous-time correlation function, $E_k = TA_k^2/2$ is the k^{th} user's energy, and $\phi_{kl} = |(\tau_k - \tau_l)\omega_c + \theta_l - \theta_k|$.

In the *synchronous* case, $\tau_k = 0, \quad k = 1, \ldots, K$ and therefore:

$$\begin{aligned} h_{kl}(0) &= \tfrac{1}{L}(E_k E_l)^{1/2} \left(\textstyle\sum_{j=0}^{L-1} \alpha_j^{(k)} \alpha_j^{(l)} \right) \cos(\theta_k - \theta_l) \\ h_{kl}(1) &= 0 \end{aligned} \tag{24}$$

Example 1 *(BER vs. SNR, K=3 users , spreading factor L=7)*

A set of three synchronous users transmit employing the Gold sequences of length $L = 7$ as their spreading sequences [21]. The power of the first user is 2 dB stronger than that of each of the other users. The *cumulative* BER (defined as the ratio of the total number of incorrect bits detected at the multiuser receiver by the total number of bits transmitted by the $K = 3$ users) achieved by the CD, MSD2, MSD3, HNN and OMD is evaluated versus the value of the SNR of the weak users. The HNN detector had RC constant $\tau = 5.5$ and $\alpha = 5.0$. Since the energies of the users are very similar, the MSD detectors exhibit poor performance as it is shown in Figure 9.

Example 2 *(BER vs. near-far ratio, K=5 users, spreading factor L=7)*

In this example we compare the CD, 2- and 3-stage MSD, HNN and OMD detectors for $K = 5$ active synchronous users. The spreading factor is equal to $L = 7$ and the spreading sequences assigned to the users are derived from Gold sequences [21]. The SNR of users 2 through 5 is kept fixed at 8 dB, while the energy of the first user is varying. The decimal logarithm of the cumulative BER is being plotted versus the value of the near-far-ratio (E_1/E_i in dB). The HNN detector had parameters $RC = 2.5$ and $\alpha = 5.0$. As we can see from Figure 10, the HNN 5-user detector performs substantially better than the 2- and 3-stage MSD detectors.

VII. The behavior of the HNN objective function

Since the HNN detector is only guaranteed to converge to a *local* minimum of the OMD objective function, if the number of such local minima is large and the initialization

of the HNN random, the probability of converging to a solution different than the solution of the OMD increases, and so does the BER. Therefore, the performance of the HNN detector is expected to depend on the *average* number of local minima of the objective function in Eqn. (3). If this number grows too fast with the size of the optimization problem, then poor performance may be observed in the detection of a very large number of users transmitting large data packets, as usually the case in practical asynchronous CDMA transmission.

To experimentally estimate the rate of growth of the number of local minima of the objective function (3) we computed, using exhaustive search, all its local minima for the number of synchronous users K ranging from two to ten. The results shown in Figure 11 correspond to 1,000 test cases per value of K, each test being performed with different random channel noise realization and poorly designed spreading codes. For each test case, $K-1$ users have the same energy per bit and the K^{th} user has 10 times the energy of the other users (maximum near-far ratio = 10). Furthermore the weak users' SNR was set to 10dB. As we can see from Figure 11 the growth of the average number of local minima is approximately proportional to $K^{\frac{1}{2}}$ for large K.

Under certain conditions, the objective function of the HNN exhibits a *unique* local minimum that coincides therefore with the solution of the OMD (global minimum). Consider the objective function of Eqn. (3):

$$f(\mathbf{H}, \mathbf{y}, [b_0 \;\; b_1 \;\; b_2 \;\;]^T) = -\mathbf{y}^T\mathbf{b} + \frac{1}{2}\mathbf{b}^T\mathbf{H}\mathbf{b} \qquad (25)$$

Assume that $\mathbf{b}^*$ is a *local minimum* of (25) Then, $f(\mathbf{H}, \mathbf{y}, \mathbf{b}^*)$ has to be smaller than the value of $f(\cdot)$ evaluated at every neighboring vertex of the 3-dimensional hypercube $[-1, +1]^3$. This translates to the conditions:

$$\begin{aligned}
f(\mathbf{H}, \mathbf{y}, [+b_0^* \;\; +b_1^* \;\; +b_2^* \;\;]^T) &< f(\mathbf{H}, \mathbf{y}, [-b_0^* \;\; +b_1^* \;\; +b_2^* \;\;]^T) \\
f(\mathbf{H}, \mathbf{y}, [+b_0^* \;\; +b_1^* \;\; +b_2^* \;\;]^T) &< f(\mathbf{H}, \mathbf{y}, [+b_0^* \;\; -b_1^* \;\; +b_2^* \;\;]^T) \\
f(\mathbf{H}, \mathbf{y}, [+b_0^* \;\; +b_1^* \;\; +b_2^* \;\;]^T) &< f(\mathbf{H}, \mathbf{y}, [+b_0^* \;\; +b_1^* \;\; -b_2^* \;\;]^T)
\end{aligned} \qquad (26)$$

or equivalently,

$$\begin{aligned}
b_0^*(y_0 - b_1^* h_{10} - b_2^* h_{20}) &> 0 \\
b_1^*(y_1 - b_0^* h_{01} - b_2^* h_{21}) &> 0 \\
b_2^*(y_2 - b_0^* h_{02} - b_1^* h_{12}) &> 0
\end{aligned} \qquad (27)$$

Assume now that for some user (e.g. $k = 1$) it holds that:

$$|y_k| > \sum_{j=0,\;\; j \neq k}^{K} |h_{jk}| \qquad (28)$$

Then, $sign(y_1 - b_0^* h_{01} - b_2^* h_{21}) = sign(y_1)$ for any choice of b_0^*, b_2^*, and therefore, to satisfy (27), b_1^* has also to be equal to $sign(y_1)$. Since the OMD's estimate is the global minimum of (25), if (28) holds for $k = 1$ then it has to be true that $b_1^{OMD} = sign(y_1)$.

Thus, whenever (28) is true, for some user(s) k, the OMD solution coincides with that output(s) of the conventional detector and consequently inspection of conditions (28) can give a-priori information about the number of local minima of the HNN and/or their location. This fact can be used to better initialize the HNN detector and even to reduce the size of the HNN that is needed to solve the original OMD problem. We have followed this approach in [4, 22] where we formulated a *hybrid* signal pre-processing/neural post-processing approach for solving demodulation problems with more than 10 asynchronous CDMA users and packets of length larger or equal to 31 (optimization problems with at least 310 parameters). If *all* conditions (28) are satisfied (for every k), a *unique* local minimum that coincides with the output of the CD is guaranteed.

VIII. Asynchronous CDMA Simulation Results

In the asynchronous CDMA transmission case, the size of the optimization problem that needs to be solved, in order to detect the information sent by all the users to the central receiver, increases drastically. If K users transmit packets of length $2P+1$ the corresponding Hopfield Neural Network receiver will have $K \cdot (2P + 1)$ neuron units. However, due to the sparsity of the cross-correlation matrix $\tilde{\mathbf{H}}$, the number of interconnections required for the HNN is only

$(3(2P+1)-2)\cdot K^2$ i.e. it is still in $O(K^2)$ as for the synchronous transmission case.

When the length of the transmitted packet size is relatively small and the number of active asynchronous users is small to moderate, an extended version of the HNN detector proposed for the synchronous case can be employed. Computer simulations for $K=3$ asynchronous users transmitting packets of length $2P+1=31$ have been performed with promising results. Due to the large size of the optimal detector, its digital computer (using MATLAB) simulation takes prohibitively long time and therefore comparisons are reported here only with respect to the conventional detector. HNN detectors with $g(\cdot)=sign(\cdot)$ and $RC=1.0$ were used.

Example 1 *(BER vs. SNR, K=3 asynchronous users, spreading factor L = 127)*

In our first experiment, $K=3$ asynchronous users employ the optimized Gold sequences [21] of length $L=127$. The energy of one of the users is 10 times larger than the energy of each one of the other two users, such that the maximum near-far-ratio is equal to 10, and the packet length has been chosen to be equal to 31 bits. The cumulative BER has been calculated by simulating both the conventional and the proposed HNN detector for 10^6 transmitted sets of symbols for each value of the SNR, randomly drawn from a uniform distribution.

Figure 12 shows that the HNN detector has a much better performance than the conventional detector. During the simulations, the values of the delays (τ_1,τ_2,τ_3) (see Eqn. (1)) have been changed randomly every 500 symbols, so that the BER curves in Figure 12 represent the performance of the detectors averaged over all possible delays.

Example 2 *(BER vs. SNR, K=3 asynchronous users, spreading factor L = 4 with conditions of worst case interference)*

In Figure 13 we evaluated the HNN detector for the example reported in [23], where the spreading codes and the values of relative delays have been chosen such as to result in conditions of worst case interference. In particular, $K=3$ asynchronous users employ spreading codes: $a^{(1)}=(1,1,1,1)$, $a^{(2)}=(1,1,-1,-1)$ and $a^{(3)}=(-1,1,-1,1)$, and their relative delays are $\tau_1=0.0$, $\tau_2=0.5T$ and $\tau_3=0.75T$. The powers of all the users are the same.

As we can see in Figure 13, the HNN detector achieves acceptable performance, whereas the the conventional detector's performance is very poor.

IX. Conclusions-Further Research Directions

We have investigated how the well known ability of Hopfield Neural Networks to provide fast suboptimal solutions to hard combinatorial optimization problems can be exploited to efficiently implement the CDMA Optimal Multiuser Detector. The proposed HNN detector was proven to be a powerful generalization of the Multistage Detector that can be implemented using analog VLSI circuits. The proposed HNN receiver has been evaluated via extensive simulations and found to: outperform the conventional detector by orders of magnitude, exceed the performance of the MSD, and approach the performance of the OMD.

The widespread demand for CDMA in a variety of communication systems ranging from satellite to cellular and personal communications, in conjunction with the low hardware complexity and high convergence speed, makes the proposed HNN a very attractive post-processing stage that could improve the performance of any other proposed suboptimal centralized multiuser detection scheme.

The proposed HNN-based detector has a hardware complexity (number of neurons) that is *linear* in the number of users K and does not require training. Since it can be implemented directly in analog VLSI hardware, its computational cost per symbol is *constant* irrespectively of the number of users. On the contrary practical multi-layer per-

ceptron based CDMA multi-user receivers may require an exponential (in K) number of neurons, and consequently extensive training. Furthermore it is not clear how one can find the optimal feed-forward network structure as the number of active users changes, and also whether the number of symbols needed to adequately train such large size receivers will render them useless in practice. Therefore we believe that overall HNNs are more suitable as neural network based multi-user CDMA receiver structures, and we expect them to find many applications in high speed communications.

Although in the asynchronous case the size of the optimization problem that needs to be solved is considerably larger, the proposed HNN detector performs quite well as the simulations suggest. This is probably due to the sparsity of the user signatures correlation matrix. However, the size of the HNN that would be required for the implementation of a practical receiver might become a problem when the number of the asynchronous users and/or the packet length becomes very large (in the order of hundreds of users). We are currently working on developing methods to reduce the size of the quadratic optimization problem that the HNN will have to solve, so that available analog HNN IC implementations can be used to build practical receivers. The number of local minima that the OMD objective function exhibits depends on the additive channel noise, the relative powers and delays of the users and the spreading codes assigned to them. A number of simple conditions can be derived to either guarantee a unique local minimum or to reduce the search space over which the optimization is being performed and consequently the size and the complexity of the HNN required to implement the detector [4, 22].

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Elias Manolakos is a member of the IEEE Signal Processing Society's Technical Committees on VLSI Signal Processing and Neural Networks. He is the Program Chair of the 1995 IEEE International Workshop on Neural Networks for Signal Processing and has participated in the organizing and program committees for several IEEE Conferences. He has authored more than 30 papers in refereed archive Journals and Conference Proceedings. For more information on his research activities you may use `mosaic` and navigate to the home page `http://www.cdsp.neu.edu/info/manolakos.html`.

Figure Captions

All Figure captions are given here. Figures start on the next page and are in the sequence listed below.

Fig. 1 The Conventional multi-user CDMA Detector.

Fig. 2 A simple Hopfield Neural Network. Small boxes model synaptic conductances.

Fig. 3 Structure of the HNN multiuser CDMA detector.

Fig. 4 Regions of Failure (dark regions) for CD, 2-stage MSD, OMD, and randomly initialized HNN: Vertical axis = log_{10}(near-far-ratio); Horizontal axis = normalized signature waveforms cross-correlation.

Fig. 5 Regions of Failure of the HNN detector with $\alpha = 100$, $\mathbf{U}(0) = \mathbf{y}$ and $\tau = RC = 1$. The ROF of the HNN almost coincides with that of the 2-stage MSD with the same initialization.

Fig. 6 Regions of Failure of the HNN detector with $\alpha = 5.0$, $\mathbf{U}(0) = \mathbf{y}$ and $\tau = RC = 1.0$. The ROF of the HNN becomes smaller than that of the 2-stage MSD with the same initialization, as the non-linearity becomes smoother.

Fig. 7 HNN detector with $g(x) = sigm(5.0 \cdot x)$, and $\mathbf{U}(0) = \mathbf{y}$: Regions of Failure versus the value of the $\tau = RC$ constant for fixed $\alpha = 5.0$. The ROF of the 2-stages MSD is included for graphical reference purposes. As the RC constant increases, the HNN's ROF approaches that of the OMD.

Fig. 8 Error probability comparison of the CD, 2-stage MSD, HNN and OMD detectors for $h = 0.7$ and Signal-to-Noise Ratio of user 1 fixed at 8 dB. HNN parameters: $\alpha = 5.0$, $RC = 2.5$.

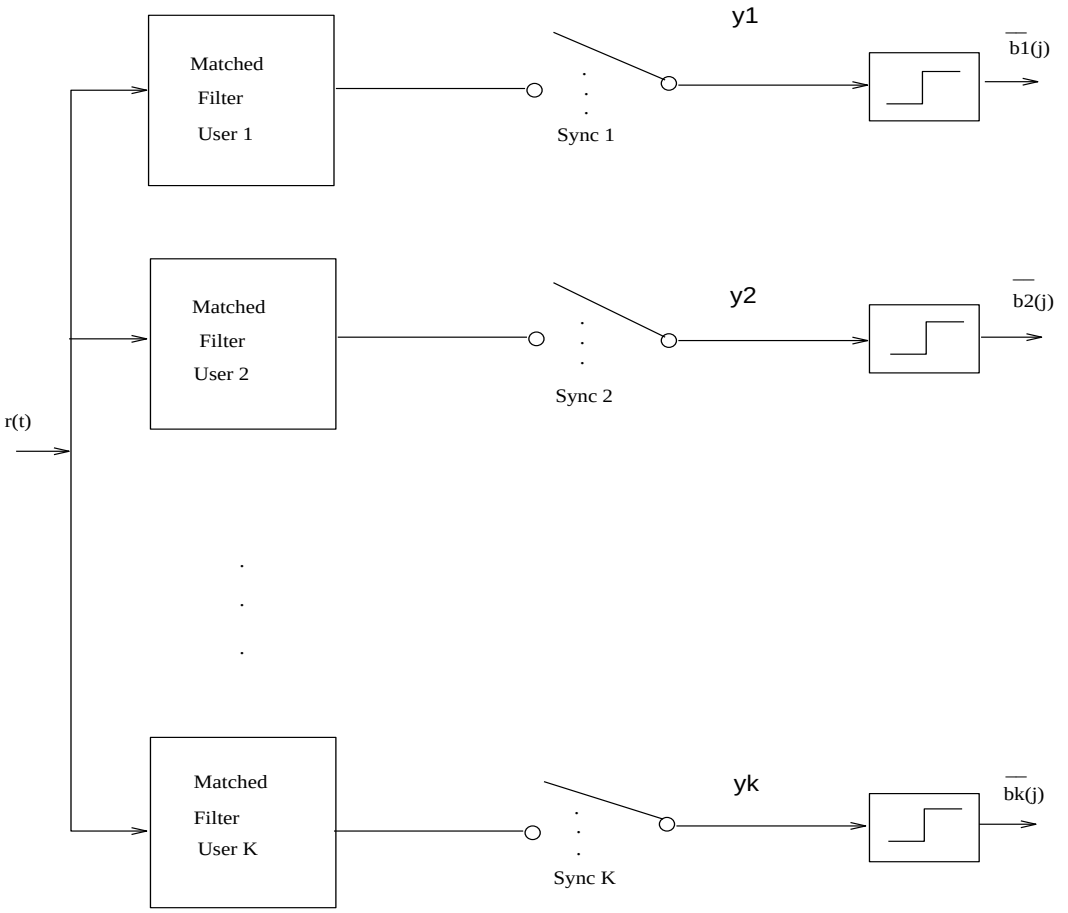
Fig. 9 Error probability comparison of the CD, 2-stage and 3-stage MSD, HNN and OMD detectors for 3 synchronous users versus the Signal-to-Noise Ratio of users 2-3. Maximum near-far-ratio is 2 dB. HNN parameters: $\alpha = 5.0$, $\tau = RC = 5.5$.

Fig. 10 Error probability comparison of the CD, 2-stage and 3-stage MSD, HNN and OMD detectors for $K = 5$ synchronous users and Signal-to-Noise Ratio of users 2-5 fixed at 8 dB. HNN parameters: $\alpha = 5.0$, $\tau = RC = 2.5$.

Fig. 11 Experimental evaluation of growth of the number of local minima of the OMD's objective function as the number of synchronous CDMA users increases. Curve (1) corresponds to the maximum and curve (2) to the average number of local minima.

Fig. 12 Performance achieved by conventional and HNN detectors with $K = 3$ asynchronous CDMA users, $L = 127$ (Gold sequences) and near-far ratio=10.

Fig. 13 Performance achieved by conventional and HNN detector with $K = 3$ asynchronous CDMA users, $L = 4$ under conditions of worst-case interference.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Filter
User 1
Matched
Filter
User 2
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y1
y2
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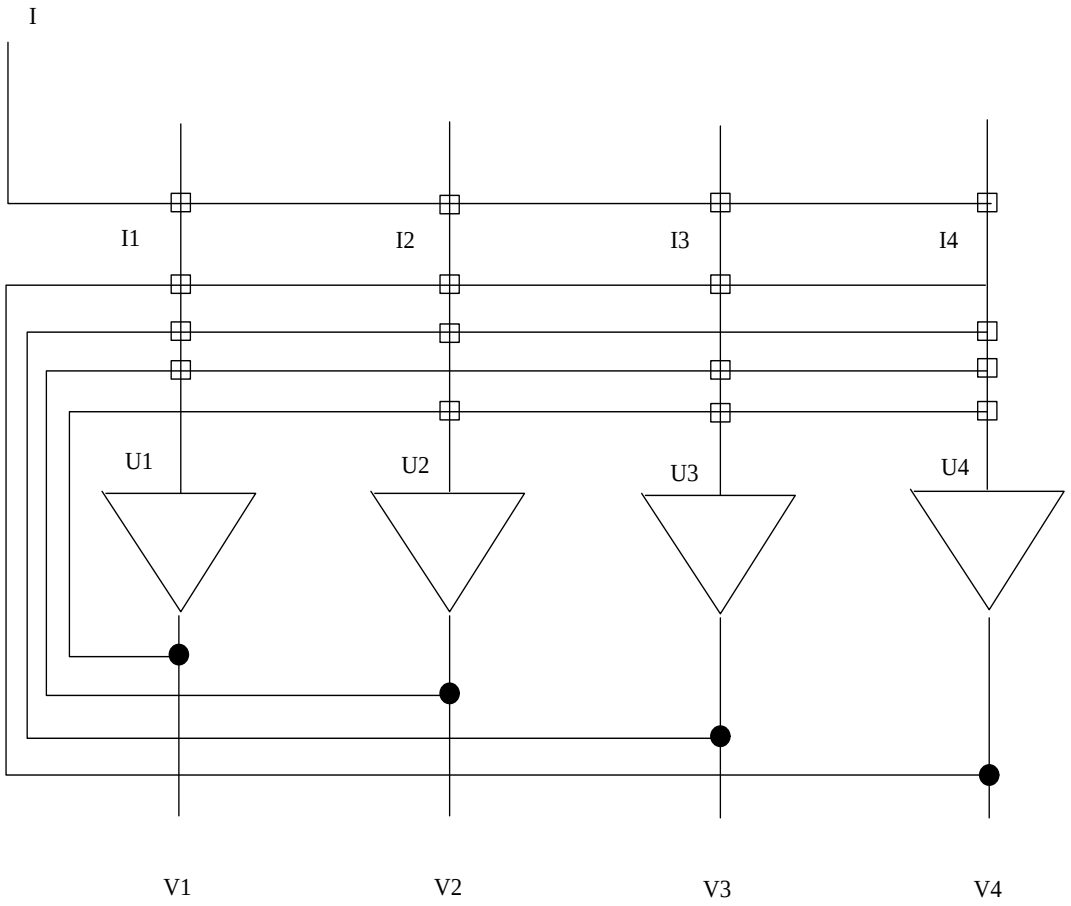
b2(j)
bk(j)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I2
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V1
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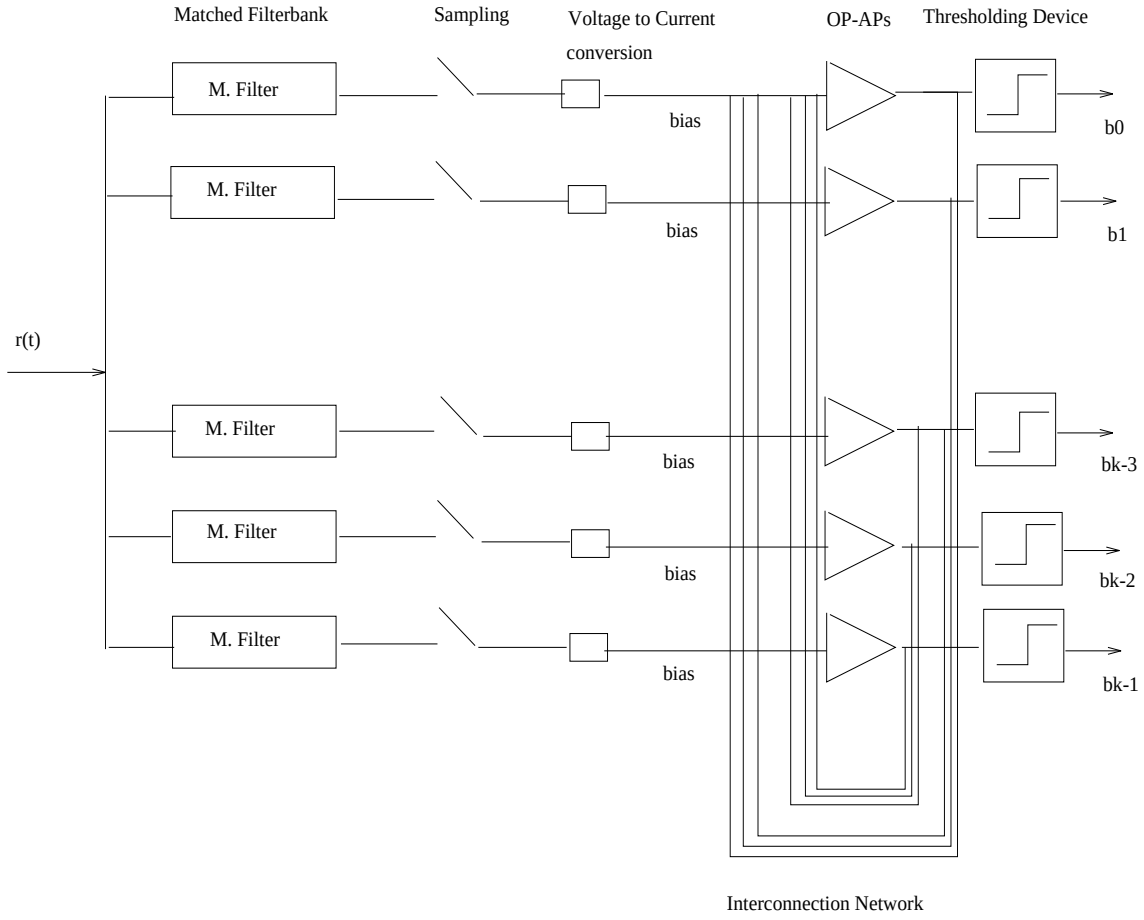
V3
V4


Matched Filterbank
Sampling
Voltage to Current conversion
OP-APs
Thresholding Device
M. Filter
M. Filter
M. Filter
M. Filter
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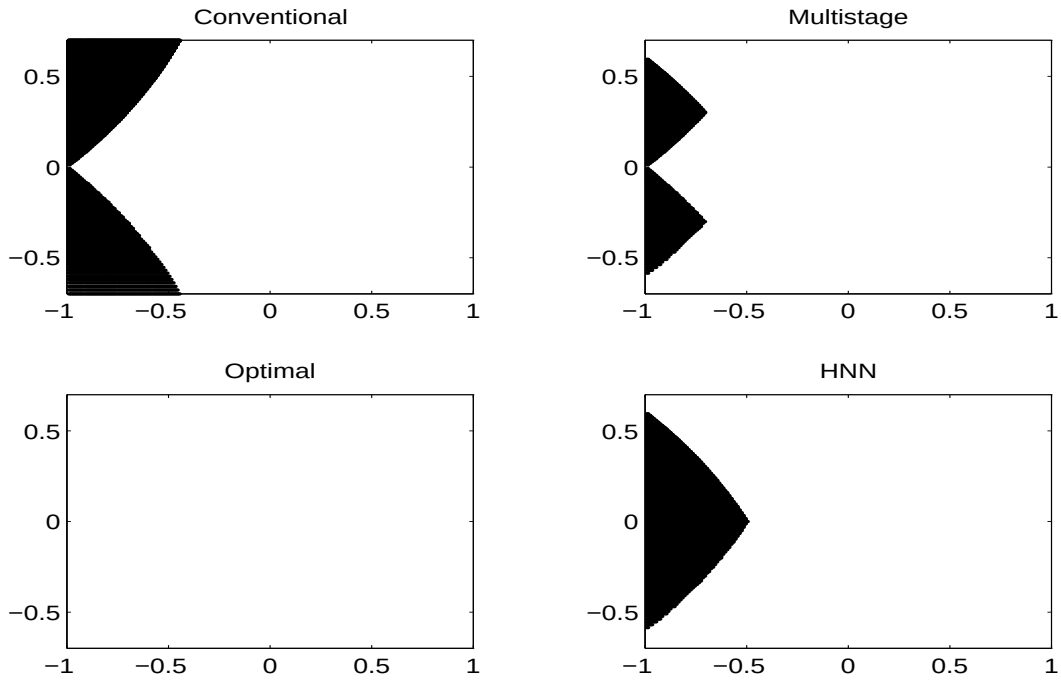
bias
bias
bias
bias
b0
b1
bk-3
bk-2
bk-1
Interconnection Network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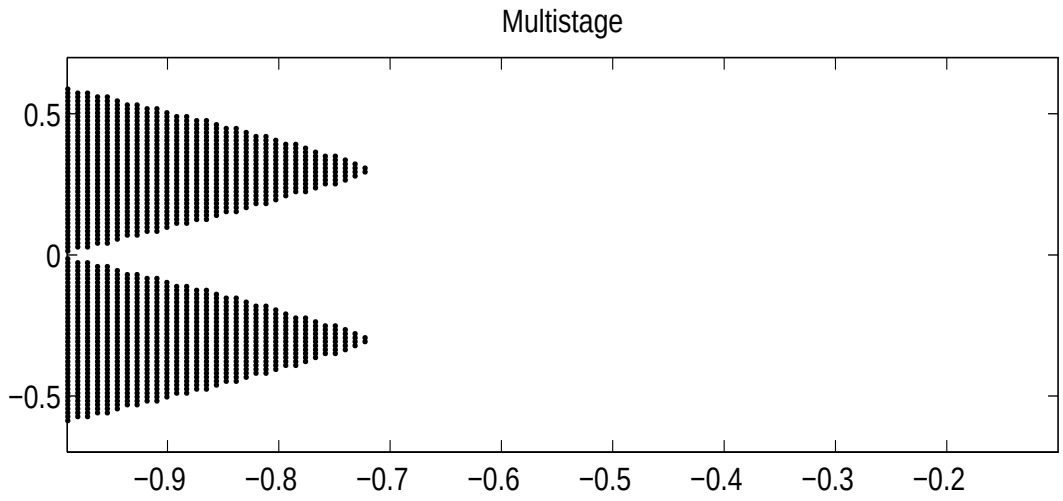
−1
−0.5
0
0.5
1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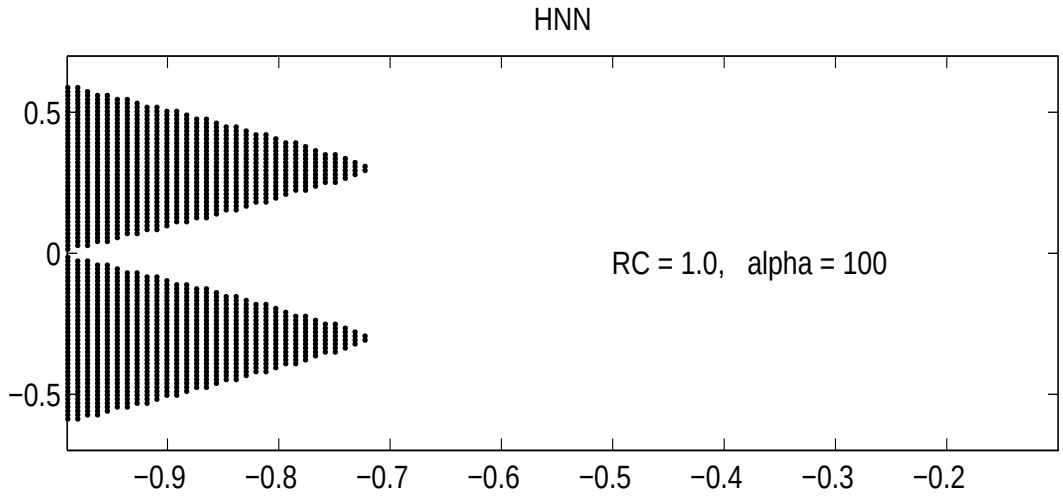
−0.2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−0.5
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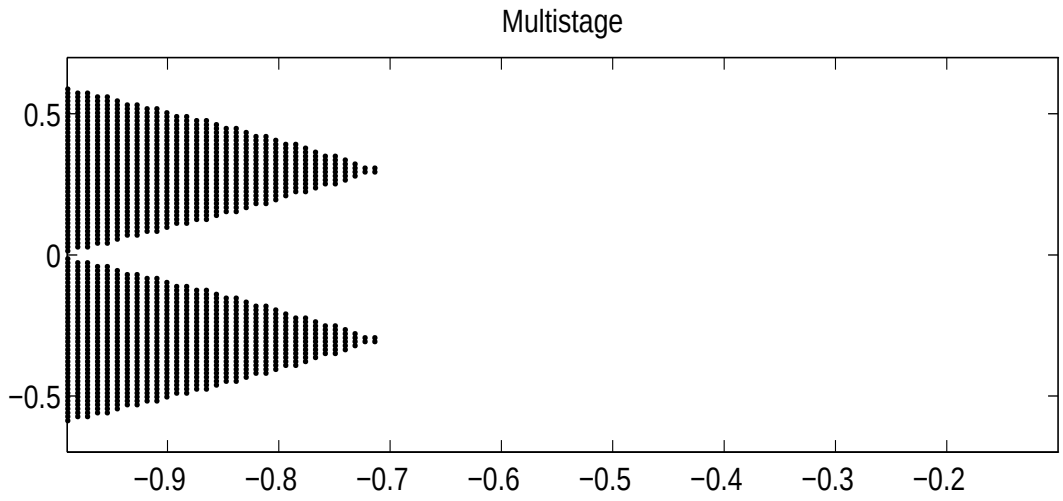
−0.4
−0.3
−0.2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0
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−0.9
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−0.3
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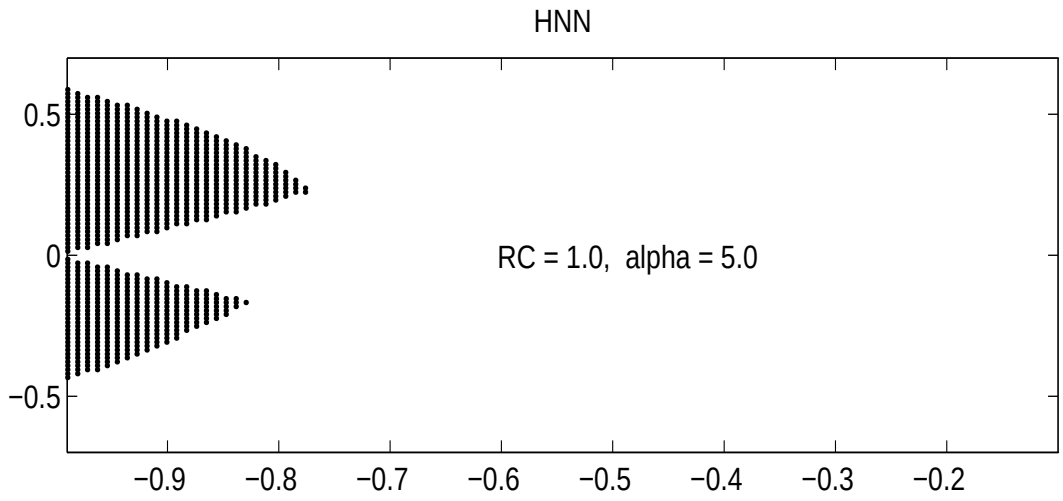
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0
−0.5
RC = 1.0, alpha = 5.0
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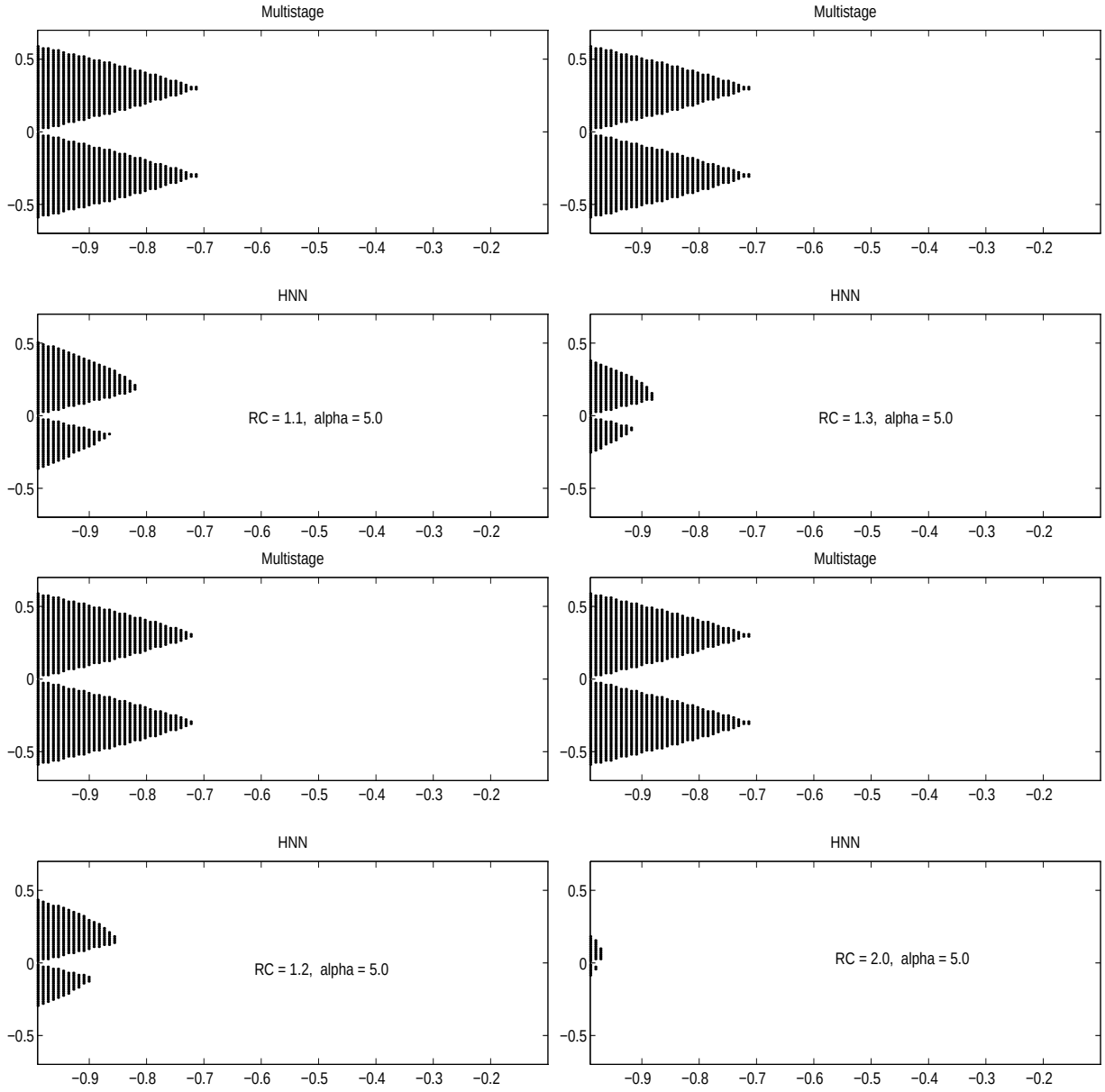
−0.3
−0.2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Multistage
HNN
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HNN
RC = 1.3, alpha = 5.0
Multistage
Multistage
HNN
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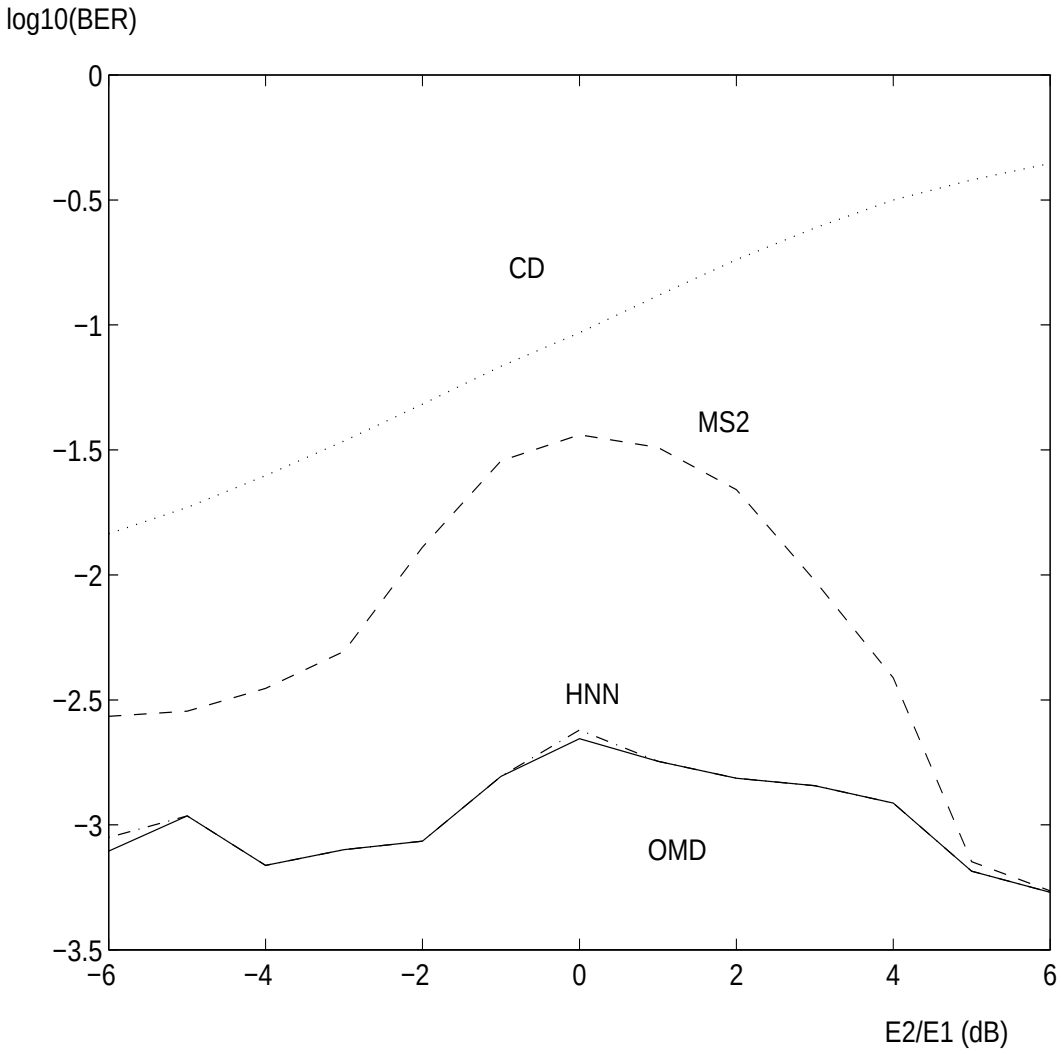
HNN
RC = 2.0, alpha = 5.0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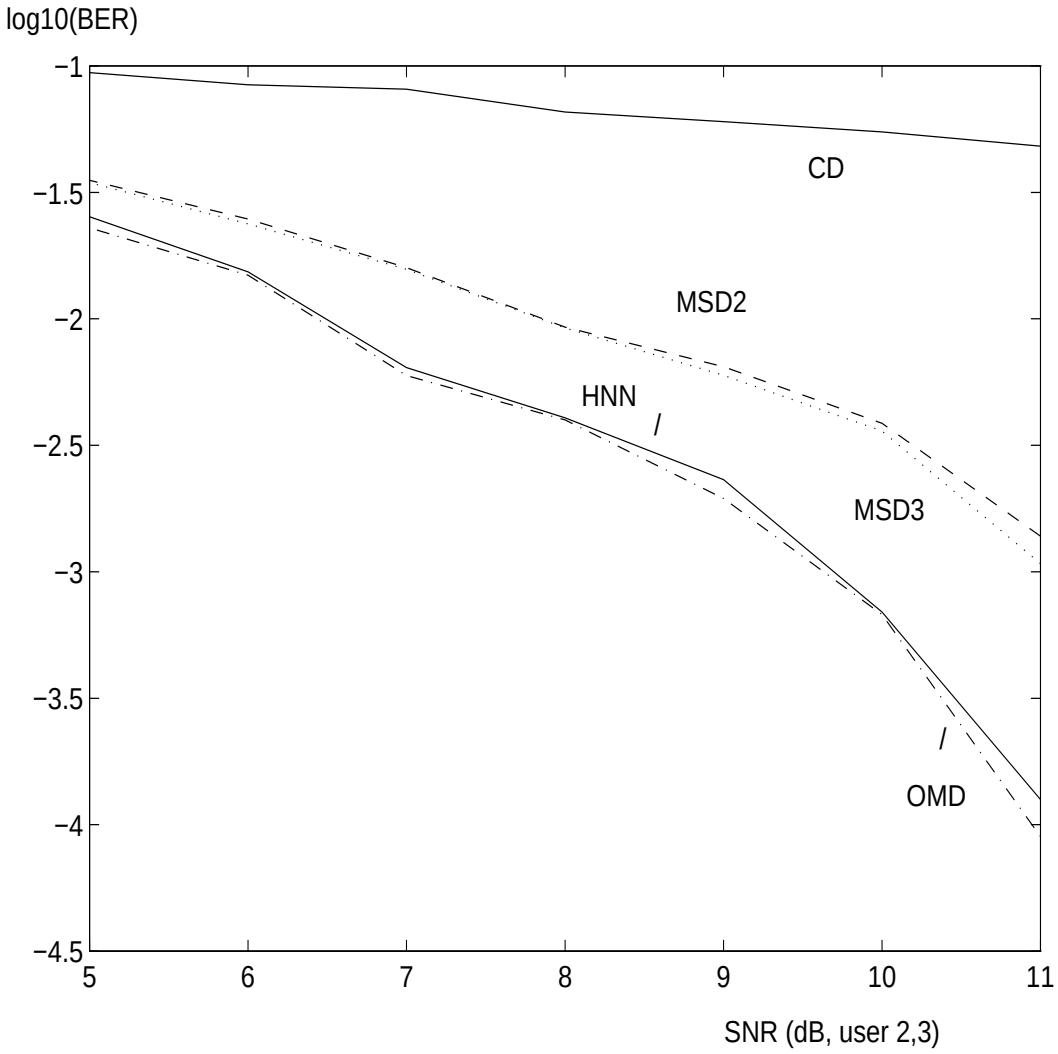
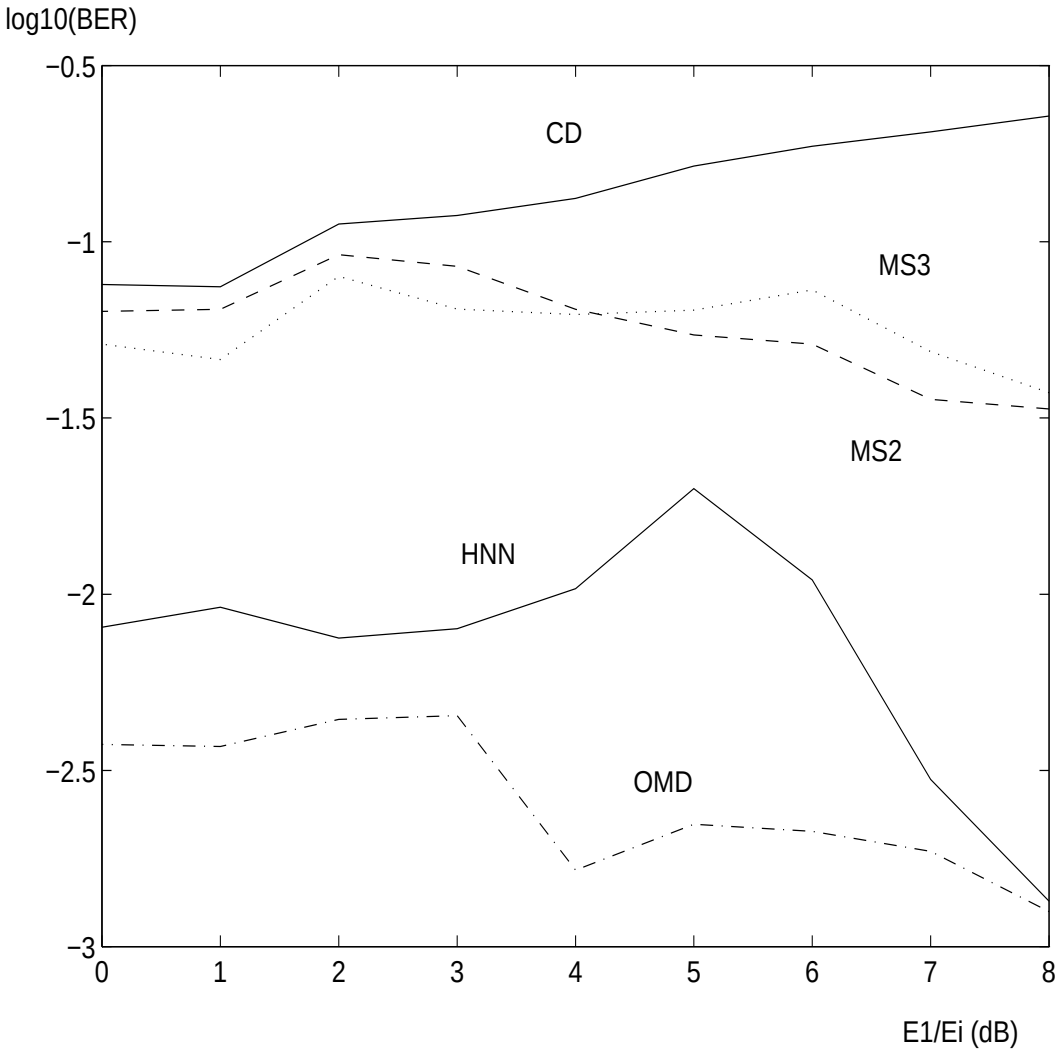
−0.5
−1
−1.5
−2
−2.5
−3
−3.5
−6
−4
−2
0
2
4
6
E2/E1 (dB)
CD
MS2
HNN
OMD


log10(BER)
−1
−1.5
−2
−2.5
−3
−3.5
−4
−4.5
5
6
7
8
9
10
11
CD
MSD2
HNN
MSD3
OMD
SNR (dB, user 2,3)


log10(BER)
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−2.5
−3
0
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2
3
4
5
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MS2
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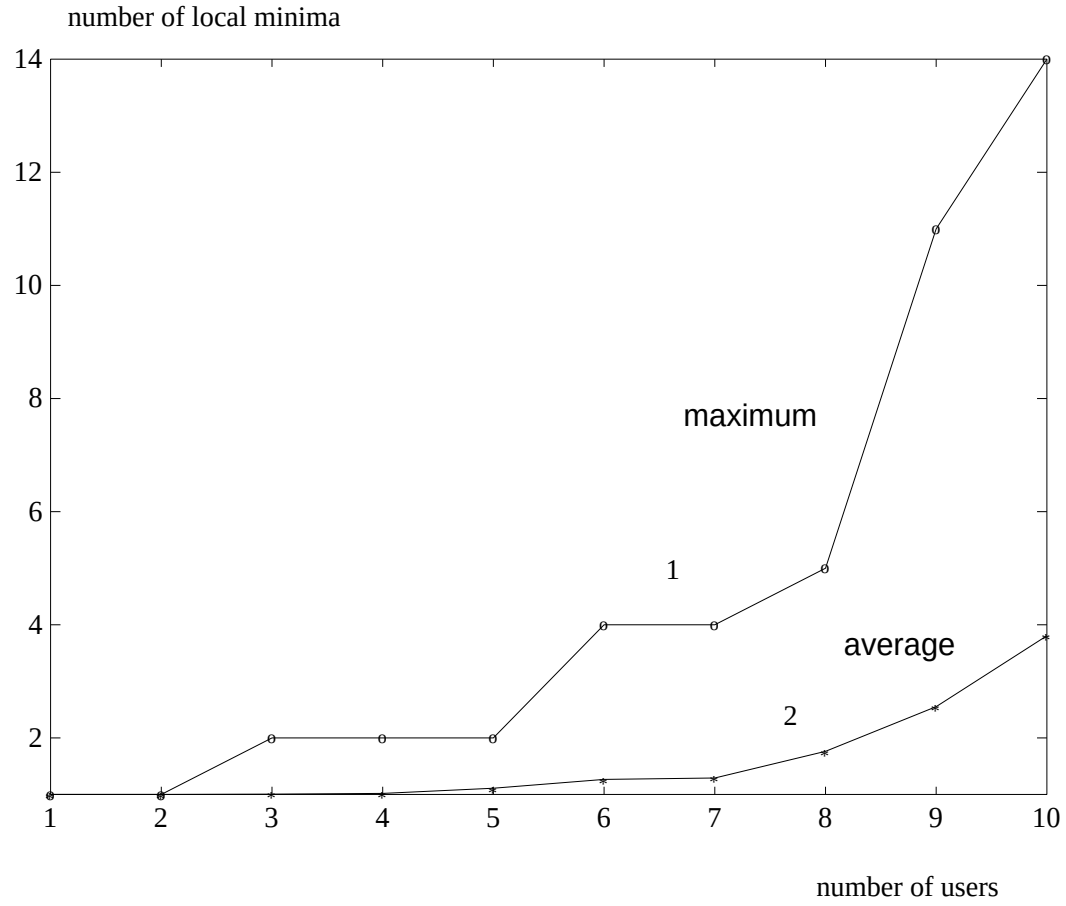
OMD
E1/Ei (dB)


number of local minima
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12
10
8
6
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4
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6
7
8
9
10
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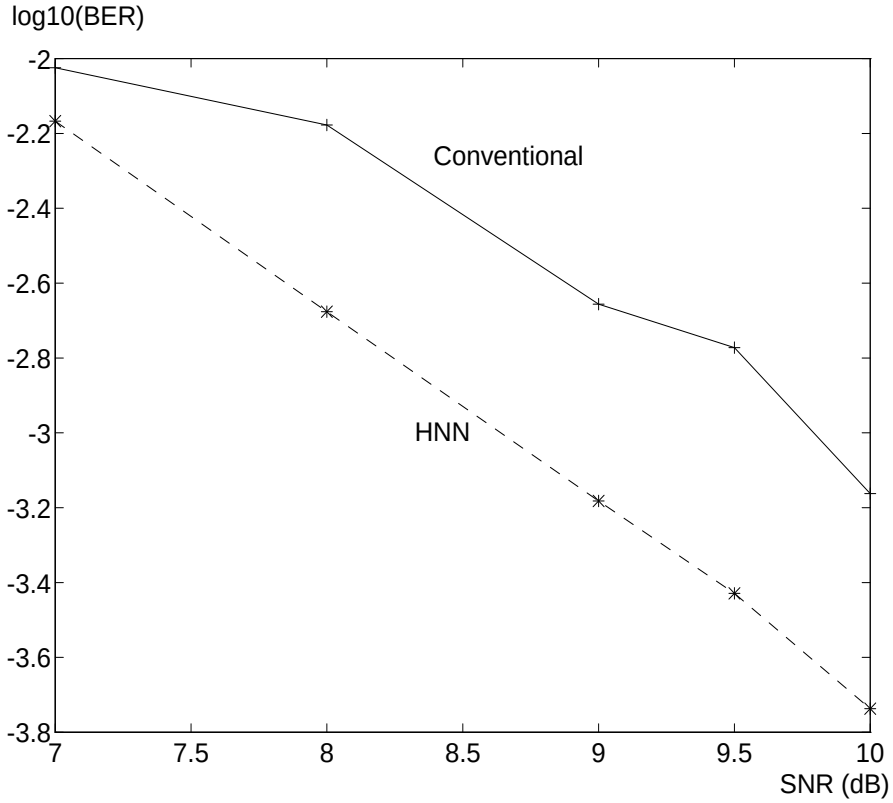
1
average
2
number of users


log10(BER)
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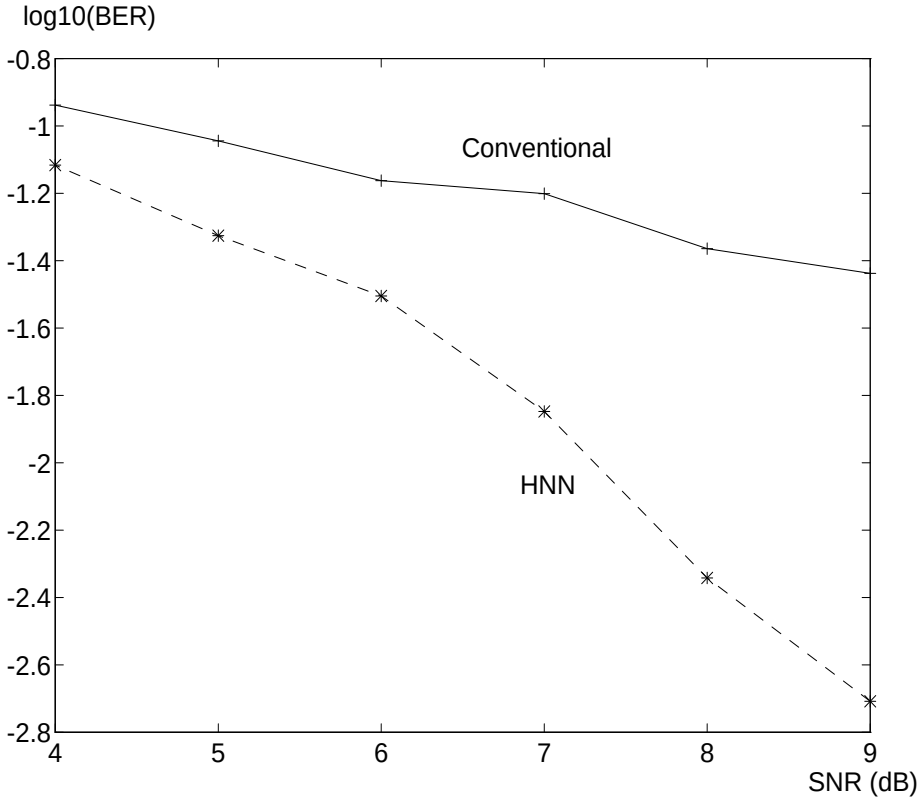
-2.2
-2.4
-2.6
-2.8
-3
-3.2
-3.4
-3.6
-3.8
Conventional
HNN
7
7.5
8
8.5
9
9.5
10
SNR (dB)


log10(BER)
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-1.2
-1.4
-1.6
-1.8
-2
-2.2
-2.4
-2.6
-2.8
4
5
6
7
8
9
SNR (dB)
Conventional
HNN