

A Low-Complexity Iterative Multiuser Receiver for Turbo-Coded DS-CDMA Systems

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Abstract—Optimal joint multiuser detection and decoding for direct-sequence code-division multiple-access (DS-CDMA) systems with forward error correction normally requires prohibitively high computational complexity. A suboptimal solution with low complexity is therefore appealing for use in practical applications. In this paper, we propose a low-complexity iterative multiuser receiver for turbo-coded DS-CDMA systems. The proposed approach consists of a modified decorrelating decision-feedback detector (MDDFD) and K single-user turbo decoders, where K is the number of users in the DS-CDMA system. The MDDFD is derived on the basis of maximizing a likelihood probability and has a feature that it can use the reliability information from the turbo decoders' output. In addition, the MDDFD can deliver interference-cancelled soft outputs to the turbo decoders where the calculation of transition metrics is modified appropriately. Both performance analysis and computer simulation results have indicated that the reliability information from the turbo decoders' output can enhance the multiuser detection capability of the MDDFD. Computer simulations have also shown that the proposed iterative multiuser receiver outperforms the conventional DDFD-based multiuser receiver in terms of the bit-error probability.

Index Terms—Code-division multiple access, forward error correction, iterative methods, multiuser detection, turbo codes.

I. INTRODUCTION

DIRECT-SEQUENCE code-division multiple access (DS-CDMA) has become very popular in wireless communications [1]. In a DS-CDMA system, a number of users simultaneously transmit information over a common channel using preassigned spreading codes (or signature waveforms). By selecting mutually orthogonal spreading codes for all users, multiple-access interference-free single-user performance could be achieved. However, the orthogonality condition is usually not satisfied perfectly for spreading codes used in practice, and thus multiple-access interference (MAI) arises; the MAI problem is particularly severe when another user's spreading code is highly correlated with the desired user's spreading code or when another user is much stronger than the desired user in power. The use of a set of conventional single-user detectors (i.e., a bank of

single-user matched filters followed by quantizers) seems to be a simple solution to this problem, but the corresponding performance is unsatisfactory when MAI is high [2]. As compared to the conventional detection approach, the optimal multiuser detector [3] has much better performance; however, its computational complexity is exponentially increased with the number of users, making it unsuitable for practical applications. To balance this tradeoff, some suboptimal multiuser detectors that vary in performance and complexity have been proposed in the literature, including the linear decorrelating detectors [4], the multistage detectors [5], and the decorrelating decision-feedback detectors [6]. An overview of various multiuser detection approaches was provided in [7].

Forward error correction (FEC) coding techniques are normally employed in communication systems to improve the transmission performance. In particular, it was emphasized by Viterbi [8] that spread spectrum CDMA systems can exhibit their full potential when they are combined with FEC. A number of FEC codes have been considered for CDMA systems, such as convolutional codes [9] and trellis codes [10], [11]. In [12], Li and Imai considered the newly appeared powerful turbo codes [13] for spread spectrum systems and indicated that the system capacity can be greatly increased.

In order to obtain better performance, several efforts have been made for optimal joint multiuser detection and FEC decoding in DS-CDMA systems. One major problem associated with such an optimal approach is that the computational complexity is usually prohibitively high. For example, the maximum-likelihood sequence estimator for convolutionally coded DS-CDMA systems proposed by Giallorenzi *et al.* [14] is an optimal solution of this kind, where the computational complexity is exponentially increased with the number of users in the system and the number of states in each user's encoder. In contrast, a suboptimal solution with lower complexity to joint multiuser detection and FEC decoding seems to be more attractive for use in practical applications. In [15], Reed *et al.* described a suboptimal approach that is actually an iterative multiuser receiver structure consisting of a multiuser likelihood metric calculator and K single-user soft-input/soft-output (SISO) decoders, where K is the number of users in the DS-CDMA system. As shown in [15], such an iterative receiver can achieve near-single-user performance, but it still involves very high computational complexity for calculating the likelihood metrics.

In this paper, we propose a low-complexity iterative multiuser receiver for turbo-coded DS-CDMA systems. The proposed approach consists of a modified decorrelating decision-feedback detector (MDDFD) and K single-user turbo decoders.

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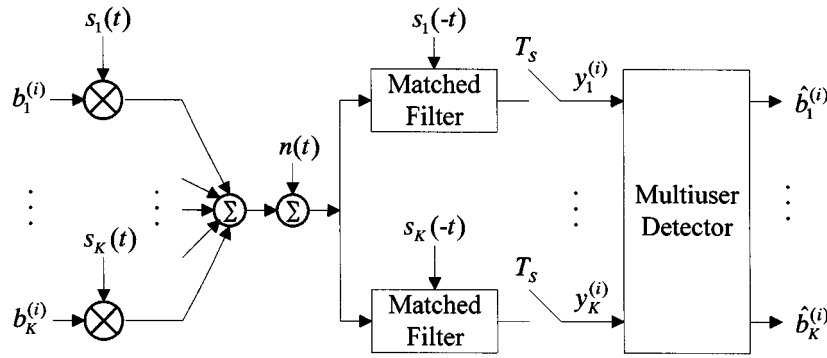


Fig. 1. The DS-CDMA system model.

The MDDFD is derived based on maximizing a likelihood probability and has a feature that it can process the reliability information from the turbo-decoders' output to generate interference-cancelled soft outputs for use in the turbo decoders. For the proposed approach, the calculation of transition metrics in the turbo decoders is also appropriately modified.

The rest of this paper is organized as follows. In Section II, we describe the DS-CDMA system model used in the development of the proposed multiuser receiver. In Section III, we review the conventional decorrelating decision-feedback detector and derive the MDDFD, where a performance analysis is conducted to explain the behavior of the MDDFD. Section IV presents an iterative multiuser receiver structure for turbo-coded DS-CDMA systems, and Section V demonstrates some performance comparisons based on computer simulation results. Finally, conclusions are given in Section VI.

II. SYSTEM MODEL

In the following, we consider a synchronous DS-CDMA system, where the receiver can recover timing and carrier phases of all users perfectly. Fig. 1 shows the corresponding K -user bit-synchronous DS-CDMA system model. Each user transmits a block of N bits and those bits for user k are represented by $\{b_k^{(1)}, b_k^{(2)}, \dots, b_k^{(N)}\}$, where $b_k^{(i)} \in \{+1, -1\}$ is the transmitted bit of user k during the i th bit period and $k \in \{1, 2, \dots, K\}$. With these notations, the K users' bits during the i th bit period can be expressed as a vector $\mathbf{b}^{(i)} = [b_1^{(i)} \ b_2^{(i)} \ \dots \ b_K^{(i)}]^T$, and all the K users' bits can be expressed as a $K \times N$ matrix $\mathbf{b} = [\mathbf{b}^{(1)} \ \mathbf{b}^{(2)} \ \dots \ \mathbf{b}^{(N)}]$.

Assume that the k th user is assigned with a signature waveform $s_k(t)$ of symbol period T_s and the communication channel is with an additive white Gaussian noise (AWGN) process $n(t)$. Then the input signal to the receiver is

$$r(\mathbf{b}, t) = \sum_{i=1}^N \sum_{k=1}^K b_k^{(i)} \sqrt{w_k} s_k(t - iT_s) + n(t) \quad (1)$$

where w_k is the received energy of the k th user. After the received signal $r(\mathbf{b}, t)$ has been processed by a bank of K matched filters, each matched to a signature waveform, a sequence of N output vectors $\mathbf{y}^{(i)} = [y_1^{(i)} \ y_2^{(i)} \ \dots \ y_K^{(i)}]^T$, $i = 1, 2, \dots, N$, are produced, where $y_k^{(i)}$ is the sampled output of the k th matched filter for the i th bit period. It was shown in [3] that $\mathbf{y}^{(i)}$ is a sufficient statistic for maximum-likelihood detection of vector $\mathbf{b}^{(i)}$

of the K users' bits in bit period i . Also, the relation between $\mathbf{y}^{(i)}$, $\mathbf{b}^{(i)}$, and the noise samples can be formulated as follows:

$$\mathbf{y}^{(i)} = \mathbf{R} \mathbf{w} \mathbf{b}^{(i)} + \mathbf{z}^{(i)} \quad (2)$$

where $\mathbf{R}$ is the $K \times K$ signature-waveform cross-correlation matrix, $\mathbf{z}^{(i)}$ is a K -dimensional colored Gaussian noise vector, and $\mathbf{w} = \text{diag} [\sqrt{w_1} \ \sqrt{w_2} \ \dots \ \sqrt{w_K}]$ is a diagonal matrix.

III. DECORRELATING DECISION-FEEDBACK DETECTORS

A decorrelating decision feedback detector (DDFD) can eliminate all interference from other users (i.e., MAI) in the DS-CDMA system provided that the feedback decisions are correct [6], where the decisions for users are made in the nonincreasing order of received users' energies, i.e., $w_1 \geq w_2 \geq \dots \geq w_K$. In practical systems, the users' energies need to be estimated and sorted appropriately. For convenience, we will not deal with such problems and assume that the users' energies are known and in the nonincreasing order at the receiver.

A. The Conventional DDFD

The structure of the conventional DDFD is briefly explained as follows. Consider the discrete-time system model described by (2). Since the cross-correlation matrix $\mathbf{R}$ is positive definite, it can be factored as $\mathbf{R} = \mathbf{F}^T \mathbf{F}$, where $\mathbf{F}$ is a lower triangular matrix obtained from the Cholesky decomposition algorithm [16]. By applying a filter with response $(\mathbf{F}^T)^{-1}$ to the K matched filters' output vector $\mathbf{y}^{(i)}$ in (2), we obtain a decision statistic $\tilde{\mathbf{y}}^{(i)}$ with whitened noise components, i.e.,

$$\tilde{\mathbf{y}}^{(i)} = \mathbf{F} \mathbf{w} \mathbf{b}^{(i)} + \tilde{\mathbf{z}}^{(i)} \quad (3)$$

where $\tilde{\mathbf{z}}^{(i)}$ is a K -dimensional white Gaussian noise vector with covariance matrix $\sigma^2 \mathbf{I}$ and the k th component of $\tilde{\mathbf{y}}^{(i)}$ is

$$\tilde{y}_k^{(i)} = f_{k,k} \sqrt{w_k} b_k^{(i)} + \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} b_j^{(i)} + \tilde{z}_k^{(i)}. \quad (4)$$

The decision for the first (strongest) user is simply made according to hard decision on data $\tilde{y}_1^{(i)}$, i.e.,

$$\hat{b}_1^{(i)} = \text{sign}(\tilde{y}_1^{(i)}). \quad (5)$$

For the k th user ($k \geq 2$), MAI from the $k-1$ stronger users can be estimated as $\sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \hat{b}_j^{(i)}$, and the corresponding decision can be made by

$$\hat{b}_k^{(i)} = \text{sign} \left(\hat{y}_k^{(i)} - \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \hat{b}_j^{(i)} \right). \quad (6)$$

For the conventional DDFD, detection of each user's bit in the i th bit period relies on the decision statistic $\hat{y}^{(i)}$ and the decisions made for all stronger interfering users. In particular, the detector for the strongest user in this system does not involve feedback from the other users' decisions and is equivalent to the linear decorrelating detector [4]. On the other hand, the detector for the weakest user utilizes feedback from all the other users' decisions and can achieve the single-user performance when the feedback decisions are correct. Although this decision-feedback approach is intuitive and simple, it must be appropriately modified if the reliability information is taken into account for improving multiuser detection performance.

B. The Modified DDFD

The modified DDFD (MDDFD) is derived as follows. The decision $\hat{b}_k^{(i)}$ for user k is made by searching for $b_k^{(i)} \in \{+1, -1\}$ that maximizes the joint probability $p(\hat{y}_k^{(i)}, b_k^{(i)} | \hat{b}_1^{(i)}, \dots, \hat{b}_{k-1}^{(i)})$, where $\{\hat{b}_1^{(i)}, \dots, \hat{b}_{k-1}^{(i)}\}$ denotes the estimated bits of the $k-1$ stronger users for bit period i . That is,

$$\hat{b}_k^{(i)} = \arg \left\{ \max_{b_k^{(i)} \in \{+1, -1\}} p(\hat{y}_k^{(i)}, b_k^{(i)} | \hat{b}_1^{(i)}, \dots, \hat{b}_{k-1}^{(i)}) \right\}. \quad (7)$$

By applying Bayes' rule, one obtains

$$\begin{aligned} p(\hat{y}_k^{(i)}, b_k^{(i)} | \hat{b}_1^{(i)}, \dots, \hat{b}_{k-1}^{(i)}) \\ = p(\hat{y}_k^{(i)} | \hat{b}_1^{(i)}, \dots, \hat{b}_{k-1}^{(i)}, b_k^{(i)}) \cdot p(b_k^{(i)} | \hat{b}_1^{(i)}, \dots, \hat{b}_{k-1}^{(i)}) \\ = p(\hat{y}_k^{(i)} | \hat{b}_1^{(i)}, \dots, \hat{b}_{k-1}^{(i)}, b_k^{(i)}) \cdot p(b_k^{(i)}). \end{aligned} \quad (8)$$

The second equality follows from the assumption that the K users' bits are statistically independent.

Since the maximization problem in (7) is actually a binary hypothesis-testing problem, we can equivalently solve it by using the following rule:

The decision would favor $\hat{b}_k^{(i)} = +1$, if

$$\begin{aligned} p(\hat{y}_k^{(i)} | \hat{b}_1^{(i)}, \dots, \hat{b}_{k-1}^{(i)}, b_k^{(i)} = +1) \cdot p(b_k^{(i)} = +1) \\ \geq p(\hat{y}_k^{(i)} | \hat{b}_1^{(i)}, \dots, \hat{b}_{k-1}^{(i)}, b_k^{(i)} = -1) \cdot p(b_k^{(i)} = -1); \end{aligned} \quad (9)$$

otherwise, $\hat{b}_k^{(i)} = -1$.

Furthermore, from (4) and with the assumption of $\hat{b}_j(i) = b_j(i)$, $j = 1, \dots, k-1$, it is straightforward to write the following Gaussian distribution:

$$\begin{aligned} p(\hat{y}_k^{(i)} | \hat{b}_1^{(i)}, \dots, \hat{b}_{k-1}^{(i)}, b_k^{(i)}) \\ = C_{N_0} \cdot \exp \left\{ -\frac{1}{2\sigma^2} \left(\hat{y}_k^{(i)} - \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \hat{b}_j^{(i)} - f_{k,k} \sqrt{w_k} \cdot b_k^{(i)} \right)^2 \right\} \end{aligned} \quad (10)$$

where the constant C_{N_0} normalizes the probability mass. By substituting (10) into (9) and with appropriate mathematical manipulation, we obtain

$$\begin{aligned} \exp \left\{ \frac{1}{\sigma^2} \left(\hat{y}_k^{(i)} - \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \hat{b}_j^{(i)} \right) \cdot (f_{k,k} \sqrt{w_k}) \right\} \\ \cdot p(b_k^{(i)} = +1) \geq \exp \left\{ \frac{1}{\sigma^2} \left(\hat{y}_k^{(i)} - \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \hat{b}_j^{(i)} \right) \right. \\ \left. \cdot (-f_{k,k} \sqrt{w_k}) \right\} \cdot p(b_k^{(i)} = -1). \end{aligned} \quad (11)$$

Rearranging this inequality and taking logarithm on its both sides yields

$$L_{a,k}^{(i)} \geq -\frac{2}{\sigma^2} \left(\hat{y}_k^{(i)} - \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \hat{b}_j^{(i)} \right) \cdot (f_{k,k} \sqrt{w_k}) \quad (12)$$

where $L_{a,k}^{(i)} = \log\{p(b_k^{(i)} = +1)/p(b_k^{(i)} = -1)\}$ is the *a priori* information for bit $b_k^{(i)}$. Based on (12), we can derive a decision rule for the MDDFD as follows:

$$\hat{b}_k^{(i)} = \begin{cases} +1, & \text{if } A_k^{(i)} \geq 0 \\ -1, & \text{if } A_k^{(i)} < 0 \end{cases} \quad (13)$$

where

$$A_k^{(i)} = \left(\hat{y}_k^{(i)} - \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \hat{b}_j^{(i)} \right) \cdot (f_{k,k} \sqrt{w_k}) + \frac{\sigma^2}{2} \cdot L_{a,k}^{(i)}.$$

The output $\zeta_k^{(i)}$ of the MDDFD is defined as

$$\begin{aligned} \zeta_k^{(i)} = \hat{y}_k^{(i)} - \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \hat{b}_j^{(i)} \\ = f_{k,k} \sqrt{w_k} \cdot b_k^{(i)} + \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \Delta b_j^{(i)} + \tilde{z}_k^{(i)} \end{aligned} \quad (14)$$

where $\Delta b_j^{(i)} = b_j^{(i)} - \hat{b}_j^{(i)}$. The summation term $\sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \Delta b_j^{(i)}$ represents the residual MAI, which is not equal to zero when incorrect decision feedback occurs in the MDDFD. If the MDDFD completely eliminates MAI, i.e., $\Delta b_j^{(i)} = 0$ for $j = 1, \dots, k-1$, $\zeta_k^{(i)}$ can be treated as a Gaussian random variable with mean $f_{k,k} \sqrt{w_k} \cdot b_k^{(i)}$ and variance σ^2 .

The structure of the MDDFD is shown in Fig. 2, where $g_{k,j} = f_{k,j} \sqrt{w_j}$. The energy sorter performs users' energies estimation and arranges the received energies in the nonincreasing order. The triangular matrix filter $(\mathbf{F}^T)^{-1}$ is used to obtain the whitened decision statistic $\tilde{\mathbf{y}}^{(i)}$ as defined in (3). The MDDFD output $\zeta_k^{(i)}$ is computed according to (14). It can be seen in Fig. 2 that the decision for the k th user's bit involves the *a priori* information $L_{a,k}^{(i)}$ of the corresponding bit $b_k^{(i)}$. With the aid of the *a priori* information, it can be expected that the MDDFD will outperform the conventional DDFD in terms of the interference cancellation capability.

C. Performance Analysis of the MDDFD

The bit-error probability at the MDDFD output is evaluated as follows. We assume that the bit-error probability is independent

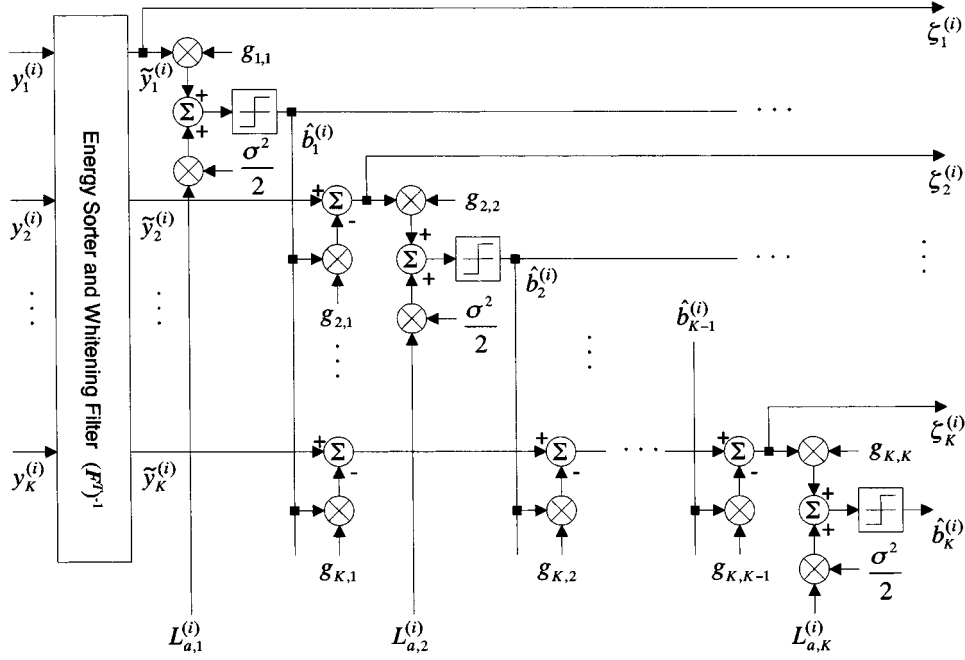


Fig. 2. The structure of the MDDFD.

of bit positions, thereby removing the bit index i in the following performance analysis. The bit-error probability of the k th user is

$$P_k = P_k^- \cdot p(b_k = -1) + P_k^+ \cdot p(b_k = +1) \quad (15)$$

where

$$\begin{aligned} P_k^- &= p(\hat{b}_k = +1 | b_k = -1) \\ &= \int_{-\infty}^{\infty} p(A_k \geq 0 | L_{a,k}, b_k = -1) \\ &\quad \cdot p(L_{a,k} | b_k = -1) \cdot dL_{a,k} \end{aligned} \quad (16)$$

$$\begin{aligned} P_k^+ &= p(\hat{b}_k = -1 | b_k = +1) \\ &= \int_{-\infty}^{\infty} p(A_k < 0 | L_{a,k}, b_k = +1) \\ &\quad \cdot p(L_{a,k} | b_k = +1) \cdot dL_{a,k} \end{aligned} \quad (17)$$

with

$$\begin{aligned} A_k &= \left(\tilde{y}_k - \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \hat{b}_j \right) \cdot (f_{k,k} \sqrt{w_k}) \\ &\quad + \frac{\sigma^2}{2} \cdot L_{a,k}. \end{aligned} \quad (18)$$

Note that, in (18), $\hat{b}_j$'s are assumed to give the correct decisions for b_j 's, i.e., $b_j = \hat{b}_j$ for $j = 1, \dots, k-1$. To take into account the decision error probability of the $k-1$ stronger users, an approach similar to that described in [6] is employed. First, by using the expression for $\tilde{y}_k$ in (4), we have

$$\begin{aligned} \tilde{y}_k &= \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \hat{b}_j \\ &= f_{k,k} \sqrt{w_k} \cdot b_k + \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \Delta b_j + \tilde{z}_k \end{aligned} \quad (19)$$

where $\Delta b_j = b_j - \hat{b}_j$ represents the decision error pattern for user j . We can find the bit-error probability of user k by averaging the conditional error probability given a particular error pattern for users $1, \dots, k-1$ over all possible error patterns, i.e., $(\Delta b_1, \Delta b_2, \dots, \Delta b_{k-1}) \in \{+2, 0, -2\}^{k-1}$. Then (16) becomes

$$\begin{aligned} P_k^- &= \sum_{(\Delta b_1, \dots, \Delta b_{k-1})} \int_{-\infty}^{\infty} p\left(\frac{\tilde{z}_k}{\sigma} \geq B^-\right) \cdot p(L_{a,k} | b_k = -1) \\ &\quad \cdot dL_{a,k} \cdot p(\Delta b_1, \dots, \Delta b_{k-1}) \\ &= \sum_{(\Delta b_1, \dots, \Delta b_{k-1})} \int_{-\infty}^{\infty} Q(B^-) \cdot p(L_{a,k} | b_k = -1) \\ &\quad \cdot dL_{a,k} \cdot p(\Delta b_1, \dots, \Delta b_{k-1}) \end{aligned} \quad (20)$$

where $Q(x) = \int_x^{\infty} (1/\sqrt{2\pi}) e^{-v^2/2} dv$ and

$$\begin{aligned} B^- &= \frac{f_{k,k} \sqrt{w_k}}{\sigma} - \frac{\sigma \cdot L_{a,k}}{2f_{k,k} \sqrt{w_k}} - \frac{1}{\sigma} \\ &\quad \cdot \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \Delta b_j \end{aligned} \quad (21)$$

The same procedure can be applied to (17) to obtain

$$\begin{aligned} P_k^+ &= \sum_{(\Delta b_1, \dots, \Delta b_{k-1})} \int_{-\infty}^{\infty} Q(B^+) \cdot p(L_{a,k} | b_k = +1) \\ &\quad \cdot dL_{a,k} \cdot p(\Delta b_1, \dots, \Delta b_{k-1}) \end{aligned} \quad (22)$$

where

$$\begin{aligned} B^+ &= \frac{f_{k,k} \sqrt{w_k}}{\sigma} + \frac{\sigma \cdot L_{a,k}}{2f_{k,k} \sqrt{w_k}} - \frac{1}{\sigma} \cdot \sum_{j=1}^{k-1} f_{k,j} \sqrt{w_j} \cdot \Delta b_j. \end{aligned} \quad (23)$$

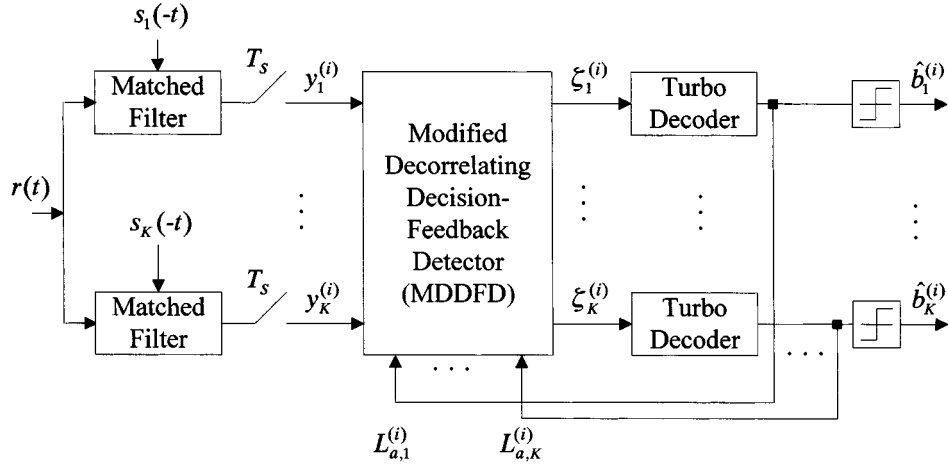


Fig. 3. The proposed iterative multiuser receiver for turbo-coded DS-CDMA systems.

TABLE I
COMPUTATIONAL COMPLEXITY COMPARISON OF TWO ITERATIVE MULTIUSER RECEIVERS

Method	Procedures	Complexity (FLOP's Per Information Bit)
The Proposed Receiver	Whitening Filtering	$K^2/(RK)$
	MDDFD (1 Iteration)	$(3K^2/2 + 3K/2 + 1)/(RK)$
The Receiver Presented by Reed <i>et al.</i> [15]	Likelihood Calculation	$2^K(7K^3/6 + 4K^2 + 6K + 4)/(RK)$
	Metric Generation (1 Iteration)	$2^K(K-1)/(RK)$

With (15), (20), and (22), the bit-error probability for the k th user can be calculated if the distributions $p(L_{a,k}|b_k = -1)$ and $p(L_{a,k}|b_k = +1)$ are known. If we further assume that the binary data of users are equally probable, i.e., $p(b_i = +1) = p(b_i = -1) = 1/2$, and $p(L_{a,k}|b_k = -1) = p(-L_{a,k}|b_k = +1)$, the bit-error probability is simply $P_k = P_k^- = P_k^+$. Note that, for the distributions $P(L_{a,k}|b_k)$, $L_{a,k}$ is the real metric computed at the decoder, not the theoretical expression in (12). Moreover, if the *a priori* information $L_{a,k}$ gives the same sign as b_k , it can be seen from (20) to (23) that the term $\sigma \cdot L_{a,k}/(2f_{k,k}\sqrt{w_k})$ contributes to increase the argument in $Q(x)$, thereby reducing the error probability.

IV. AN ITERATIVE MULTIUSER RECEIVER

Based on the derived MDDFD, we further propose an iterative multiuser receiver structure for turbo-coded DS-CDMA system, as shown in Fig. 3. This iterative multiuser receiver consists of the MDDFD and K single-user turbo decoders. In the first multiuser detection iteration, the *a priori* information of users' bits is not available for the MDDFD in making decisions, i.e., $L_{a,k}^{(i)} = 0$ for $k = 1, \dots, K$ and $i = 1, \dots, N$; therefore, its performance is simply the same as that of the DDFD. The MDDFD delivers interference-cancelled (if all decisions are correct) soft outputs $\xi_k^{(i)}$'s to the input of the turbo decoders. After several turbo decoder iterations, the log-likelihood ratios of data bits at the turbo decoders' output are fed back to the input of the MDDFD as the *a priori* information $L_{a,k}^{(i)}$ for the next multiuser detection iteration.

Owing to the structure of the MDDFD, we have to modify the transition metrics calculation in the turbo decoders. Assume that

the BCJR algorithm [17] is used for decoding a turbo code consisting of two 2^v -state recursive systematic convolutional (RSC) codes linked by an interleaver. To begin with, we brief the transition metrics calculation in the original BCJR algorithm. Since the two component decoders used for turbo decoding are identical, it is sufficient to deal with one of them. Consider a transition from state m' to state m with input bit $d^{(j)} = \ell$ (a systematic bit) at the first RSC encoder that generates a redundant bit $Y_{\ell,m'}$, where $\ell, Y_{\ell,m'} \in \{0, 1\}$ and $m, m' \in \{0, 1, \dots, 2^v - 1\}$. Let $x^{(j)}$ and $y^{(j)}$ represent the observed data for the systematic bit and the redundant bit, respectively, at the turbo decoder in the case of an AWGN channel. Then the transition metrics have the following form:

$$\begin{aligned} \gamma^{(j)}(\ell, m', m) = & \exp \left\{ \frac{x^{(j)} \cdot (2\ell - 1)}{\sigma^2} \right\} \\ & \cdot \exp \left\{ \frac{y^{(j)} \cdot (2Y_{\ell,m'} - 1)}{\sigma^2} \right\} \\ & \cdot \frac{\exp\{\ell \cdot \tilde{L}_a^{(j)}\}}{1 + \exp\{\tilde{L}_a^{(j)}\}} \end{aligned} \quad (24)$$

where $x^{(j)}$ is Gaussian distributed with mean $2\ell - 1$ and variance σ^2 , $y^{(j)}$ is Gaussian distributed with mean $2Y_{\ell,m'} - 1$ and variance σ^2 , and $\tilde{L}_a^{(j)}$ is the *a priori* information of the systematic bit.

In the following, we describe how to calculate the transition metrics in the proposed multiuser receiver. Without loss of generality, the turbo decoder for user k is considered, where the corresponding block of N bits transmitted consists of the systematic (uncoded) bits and the redundant (coded) bits. Soft out-

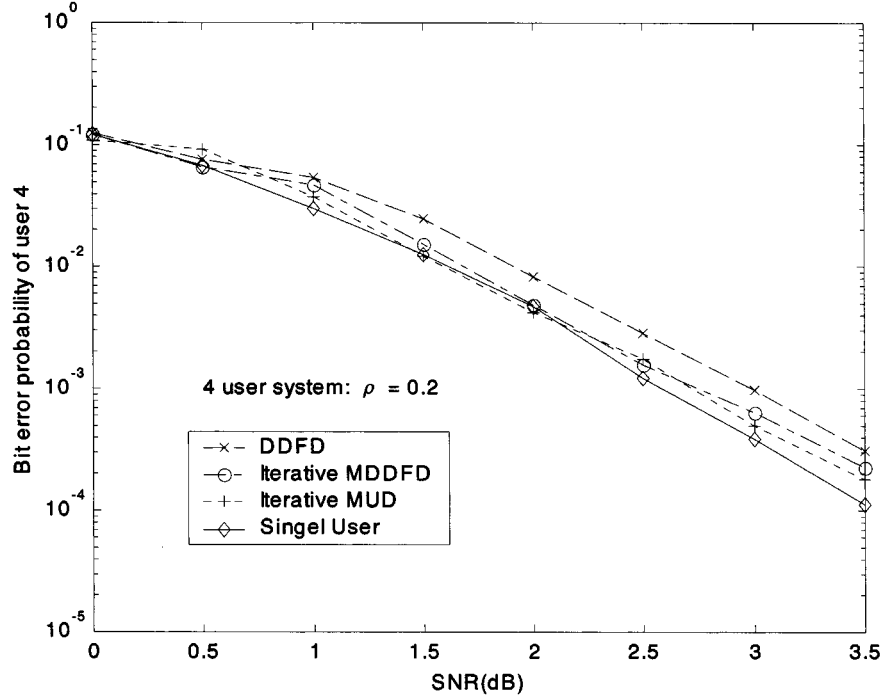


Fig. 4. Performance comparison of multiuser receivers with $\rho = 0.2$.

puts $\zeta_k^{(i)}$, $i = 1, \dots, N$, from the MDDFD can be demultiplexed according to the puncturing rule used in the turbo encoder to obtain $\zeta_{x,k}^{(j)}$, $\zeta_{y1,k}^{(j)}$, and $\zeta_{y2,k}^{(j)}$, $j = 1, \dots, N_t$, where $N_t = N \cdot R$ is the block size of the turbo code used and R is the code rate. For the above notations, $\zeta_{x,k}^{(j)}$ corresponds to the systematic bits, and $\zeta_{y1,k}^{(j)}$ and $\zeta_{y2,k}^{(j)}$ are related to the redundant bits generated from the first and second RSC encoders, respectively. From the discussion immediately following (14), we assume that $\zeta_{x,k}^{(j)}$, $\zeta_{y1,k}^{(j)}$, and $\zeta_{y2,k}^{(j)}$ are Gaussian random variables with mean $\pm f_{k,k} \sqrt{w_k}$ and variance σ^2 . Then the transition metrics calculation in the BCJR algorithm for the first component decoder can be modified as follows:

$$\begin{aligned} \gamma^{(j)}(\ell, m', m) = & \exp \left\{ \frac{\zeta_{x,k}^{(j)} \cdot f_{k,k} \sqrt{w_k} \cdot (2\ell - 1)}{\sigma^2} \right\} \\ & \cdot \exp \left\{ \frac{\zeta_{y1,k}^{(j)} \cdot f_{k,k} \sqrt{w_k} \cdot (2Y_{\ell,m'} - 1)}{\sigma^2} \right\} \\ & \cdot \frac{\exp\{\ell \cdot \tilde{L}_{a,k}^{(j)}\}}{1 + \exp\{\tilde{L}_{a,k}^{(j)}\}} \end{aligned} \quad (25)$$

where $\tilde{L}_{a,k}^{(j)}$ is the *a priori* information of a systematic bit for user k . Note that this *a priori* information is set to zero initially and updated appropriately during the turbo decoding process. Furthermore, since the MDDFD requires the *a priori* information for both the systematic bits and the redundant bits, the turbo decoders are modified to produce the log-likelihood ratios for these bits. The corresponding modification can be found in [18].

The iterative manner in the proposed multiuser receiver is similar to that in the multiuser receiver developed by Reed *et al.* [15]. However, the latter requires relatively high computational complexity. We have analyzed the computational com-

plexity of the proposed receiver in the terms of the number of floating-point operations (FLOPs) required per information bit. Table I shows the complexity of the proposed MDDFD-based receiver, where K is the number of users and R is the code rate of the turbo code in use. For the purpose of comparison, the computational complexity of Reed *et al.*'s receiver [15] is also included. It can be seen from Table I that the complexity of Reed *et al.*'s receiver grows exponentially with K while the proposed receiver has complexity linear in K .

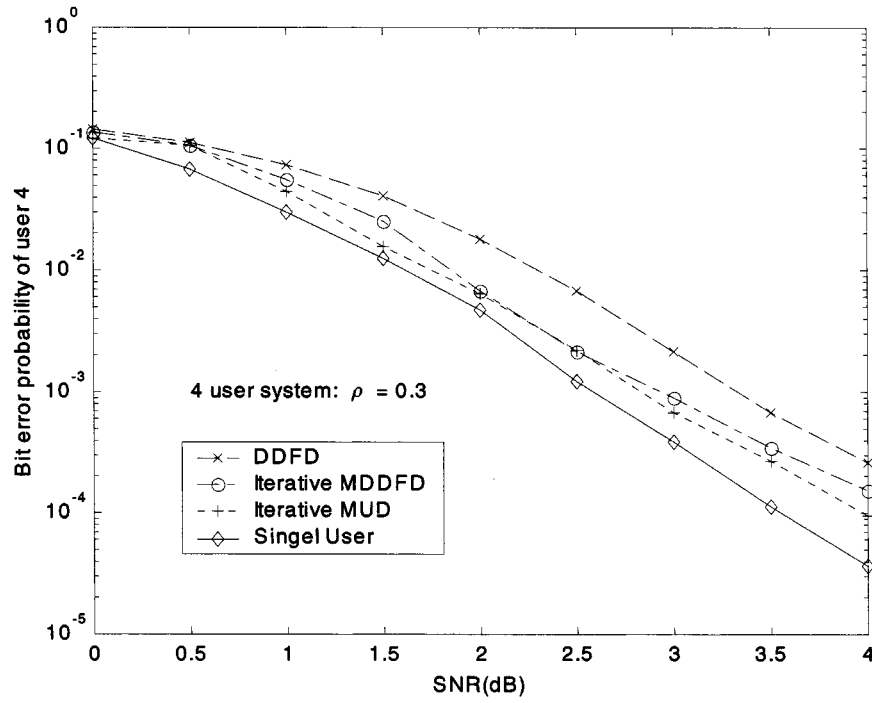
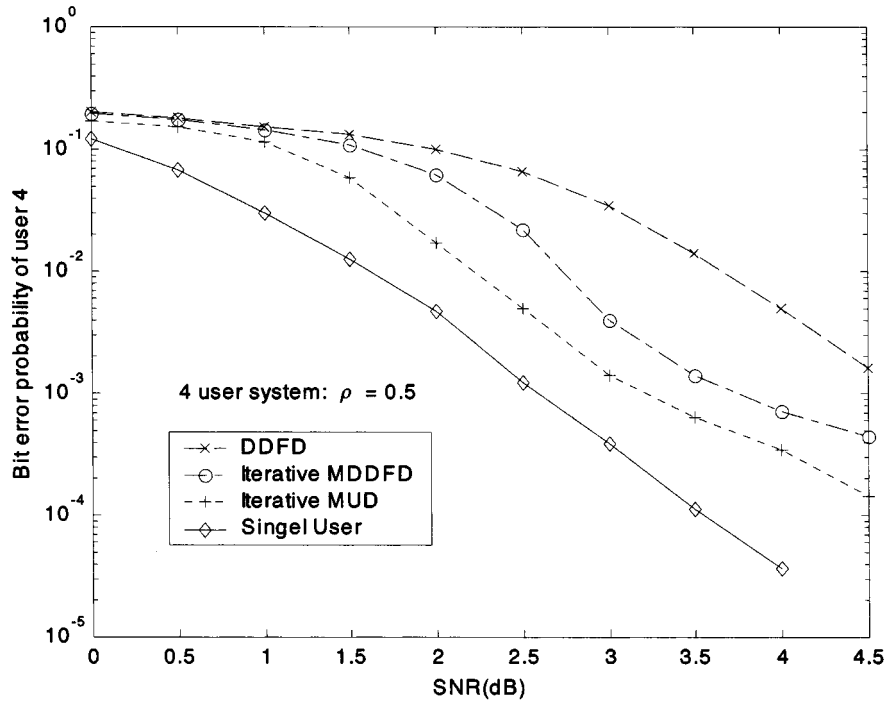
V. SIMULATION RESULTS

The bit-error probability performance of the proposed iterative multiuser receiver was investigated through computer simulations. A rate $R = 1/2$ turbo code with two 4-state ($v = 2$; octal generator 7, 5) RSC codes and a helical block interleaver of length $N_t = 210$ bits were used for simulations. The BCJR algorithm with appropriate transition metrics modifications presented in the previous section was employed for turbo decoding. For making comparisons of system performance under various cross-correlations among spreading codes, all simulations were performed on the K -symmetric channel [19] that is characterized by a cross-correlation matrix given by

$$R = \begin{bmatrix} 1 & \rho & \dots & \rho \\ \rho & 1 & \dots & \rho \\ \rho & \rho & 1 & \dots \\ \dots & \dots & \rho & 1 \end{bmatrix} \quad (26)$$

where ρ is referred to as the cross-correlation parameter.

In the simulations, we considered a 4-user ($K = 4$) DS-CDMA system, where all users were assumed to have equal powers. Figs. 4–7 show the bit-error probability performance of the multiuser receivers for various cross-correlation parameters. In these figures, the bit-error probability of user 4

Fig. 5. Performance comparison of multiuser receivers with $\rho = 0.3$.Fig. 6. Performance comparison of multiuser receivers with $\rho = 0.5$.

is illustrated. The curve labeled “DDFD” represents the performance of the conventional DDFD followed by turbo decoding with nine iterations. The performance of the proposed iterative multiuser receiver is shown by the curve labeled “Iterative MDDFD.” We performed three multiuser detection iterations, each followed by three turbo decoder iterations. For comparison, the iterative multiuser receiver developed by Reed *et al.* [15] was also simulated in a similar iterative manner and its performance is plotted as “Iterative MUD.” For each of these

receivers, the total number of turbo decoder iterations is equal to nine.

For a low cross-correlation parameter of $\rho = 0.2$, “Iterative MDDFD” and “Iterative MUD” are close to the single-user performance. For a moderate cross-correlation parameter of $\rho = 0.3$, “Iterative MDDFD” is still close to “Iterative MUD” and is approximately 0.4 dB away from the single-user performance at the bit-error probability of 10^{-3} . In high cross-correlation cases, “Iterative MDDFD” is inferior to “Iterative

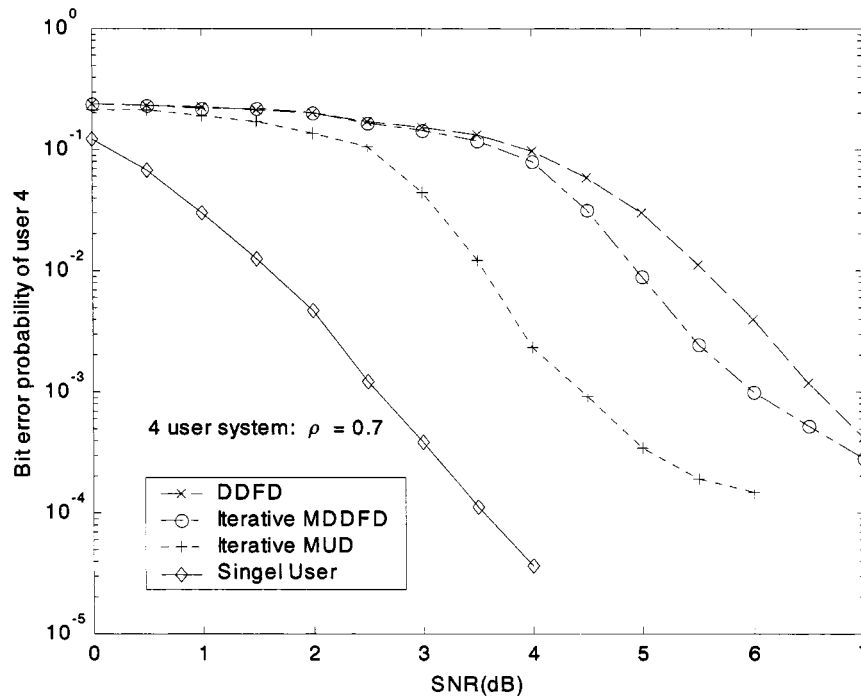


Fig. 7. Performance comparison of multiuser receivers with $\rho = 0.7$.

MUD,” where the performance loss is about 0.5 dB for $\rho = 0.5$ and about 1.0 dB for $\rho = 0.7$ at the bit-error probability of 10^{-3} . Note that the proposed iterative multiuser receiver always outperforms the conventional DDFD without iterative multiuser detection.

VI. CONCLUSION

In this paper, we have proposed a low-complexity iterative multiuser receiver for turbo-coded DS-CDMA systems. The proposed iterative multiuser receiver consists of a modified DDFD and K single-user turbo decoders, where K is the number of users in the system. The modified DDFD is derived based on maximizing a likelihood probability and has a feature that it can use the reliability information from the turbo decoders’ output. In addition, it can deliver interference-cancelled soft outputs to the turbo decoders if all feedback decisions are correct. For the proposed approach, the calculation of the transition metrics in the turbo decoders is also appropriately modified.

Computer simulations have shown that the performance of the proposed iterative multiuser receiver is close to the single-user performance when the cross-correlations among spreading codes are low. Although the proposed receiver has some extent of performance loss as compared to the single-user case, it always provides a performance improvement over the conventional DDFD-based receiver without iterative multiuser detection.

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