

Kymogram detection and kymogram-correlated image reconstruction from subsecond spiral computed tomography scans of the heart

Marc Kachelrieß,^{a)} Dirk-Alexander Sennst, Wolfgang Maxlmoser, and Willi A. Kalender
Institute of Medical Physics, University of Erlangen-Nürnberg, Germany

(Received 11 October 2001; accepted for publication 18 April 2002; published 20 June 2002)

Subsecond single-slice, multi-slice or cone-beam spiral computed tomography (SSCT, MSCT, CBCT) offer great potential for improving heart imaging. Together with the newly developed phase-correlated cardiac reconstruction algorithms 180°MCD and 180°MCI [Med. Phys. **27**, 1881–1902 (2000)] or related algorithms provided by the CT manufacturers, high image quality can be achieved. These algorithms require information about the cardiac motion, i.e., typically the simultaneously recorded electrocardiogram (ECG), to synchronize the reconstruction with the cardiac motion. Neither data acquired without ECG information (standard patients) nor acquisitions with corrupted ECG information can be handled adequately. We developed a method to extract the appropriate information about cardiac motion directly from the measured raw data (projection data). The so-called kymogram function is a measure of the cardiac motion as a function of time t or as a function of the projection angle α . In contrast to the ECG which is a global measure of the heart's electric excitation, the kymogram is a local measure of the heart motion at the z -position $z(\alpha)$ at projection angle α . The patient's local heart rate as well as the necessary synchronization information to be used with phase-correlated algorithms can be extracted from the kymogram by using a series of signal processing steps. The kymogram information is shown to be adequate to substitute the ECG information. Computer simulations with simulated ECG and patient measurements with simultaneously acquired ECG were carried out for a multislice scanner providing $M=4$ slices to evaluate these new approaches. Both the ECG function and the kymogram function were used for reconstruction. Both were highly correlated regarding the periodicity information used for reconstruction. In 21 out of 25 consecutive cases the kymogram approach was equivalent to the ECG-correlated reconstruction; only minor differences in image quality between both methods were observed. For one patient the synchronization information detected by the ECG monitor turned out to be wrong; here, the kymogram constituted the only approach that provided useful reconstructions. Patient studies with 12 and 16 slices indicate the usefulness of our approach for cone-beam CT scans. Kymogram-correlated reconstructions also appear to have the potential to improve imaging of pericardial lung areas in general. © 2002 American Association of Physicists in Medicine. [DOI: 10.1118/1.1487861]

Key words: Computed tomography (CT), multi-slice spiral CT (MSCT), cone-beam spiral CT (CBCT), heart, ECG, kymogram

I. INTRODUCTION

Noninvasive imaging of the heart is an important issue because coronary artery disease is one of the leading causes of death in western civilizations. Since most of the cardiac imaging techniques available today [cardiac ultrasound, fluoroscopy, conventional computed tomography (CT), spiral CT, electron-beam CT and magnetic resonance tomography] provide insufficient information or are not readily available¹ dedicated reconstruction algorithms using the simultaneously acquired patient electrocardiogram (ECG) as additional information for reconstruction have been developed in recent years.^{2–5} These algorithmic developments together with vast improvements in CT technology, namely the reduced rotation time down to 0.42 s per revolution and the introduction of multi-slice and cone-beam spiral CT (MSCT and CBCT) systems, allowed for the recent breakthrough of cardiac CT imaging.

For phase-correlated image reconstruction the user typically specifies a reconstruction phase c_R relative to the R

peaks of the ECG. For example, 70% of $R-R$ is a typical selection of the reconstruction phase. Alternatively, absolute timing specifications may be used. Due to the synchronous recording of the ECG signal, the cardiac ECG phase $c_E(\alpha)$ is known for each view angle α . This information allows to use only those raw data for reconstruction that are as close as possible to c_R . In other words, the phase-correlated reconstruction algorithms use data windows that satisfy $c_E(\alpha) \approx c_R$ for reconstruction.

Two classes of phase-correlated algorithms are in use today: Partial scan approaches that use 180° data intervals and cardio interpolation approaches that combine data of adjacent heart cycles and rotations to improve temporal resolution. Advantages of the interpolation algorithms over the partial scan algorithms have been demonstrated clearly.^{2–4,6} These findings are also reflected by current investigations of other groups and by the manufacturers who recently started to switch from half scan or partial scan to “multisector” or “pulse-rate adaptive” reconstruction algorithms.^{7,8}

In this paper, we propose an alternative measure of cardiac motion that may serve as a fall-back solution to substitute the use of the cardiac ECG phase $c_E(\alpha)$ or that may be applied in situations where no ECG is available or where using ECG acquisition is inconvenient.

The motion detection shall be computationally efficient in order not to increase the total reconstruction time. A raw data-based approach appears to be most promising. The function to be extracted from the raw data is called the kymogram; the synchronization information derived therefrom is the cardiac kymogram phase and is denoted by $c_K(\alpha)$. (We have chosen the term kymogram to give credit to conventional kymogram methods that are available since the mid-19th century and that use an instrument to graphically record variations or undulations, as of the heart and blood vessels but also of the vocal chord. The name kymogram is derived from kymo, the water nymph of greek mythology. Further information can be found at www.kymogram.com)

We derive the kymogram function using a raw data-based COM (center of mass) tracking, followed by a few signal processing steps.⁹ This paper specifies the algorithms and demonstrates the performance of the kymogram detection in a simulation study, in one cadaver and in a patient study including 25 individuals consecutively scheduled for coronary CT angiography. Comparisons to ECG-correlated cardiac imaging and to standard (not phase-correlated) reconstructions are given.

II. MATERIALS AND METHODS

Throughout this paper we use the notations and definitions stated in the list below. The variables α (fan-beam projection angle), ϑ (parallel beam projection angle), t (time of a projection), and z (z position of a projection) are related as

$$\frac{\alpha}{2\pi} = \frac{\vartheta}{2\pi} = \frac{t}{t_{\text{rot}}} = \frac{z}{d}.$$

We will make use of this one-to-one correspondence to switch between these variables, if convenient.

*	convolution symbol
$\delta(\cdot)$	Dirac's delta function
$\Pi(\cdot)$	rectangle function with support $[-\frac{1}{2}, \frac{1}{2}]$ and area 1
$\Pi_a^*(\cdot)$	$(1/ a)\Pi(\cdot/a)$
$\text{sinc}(\cdot)$	sinc-function, $\text{sinc}(x) = \sin(\pi x)/\pi x$
α	projection angle, $\alpha \in [0, 2\pi)$ for a sequence (360°) scan or z-interpolated data, $\alpha \in \mathbb{R}$ for a spiral scan
β	angle within fan, $\beta \in [-\frac{1}{2}\Phi, \frac{1}{2}\Phi]$
c_R	cardiac reconstruction phase (the center phase of the data used for reconstruction)
$c_\chi(t)$	cardiac phase as a function of time, $c_\chi(t) \in [0, 1)$. $\chi = E$ for ECG, $\chi = K$ for kymogram, and $\chi = S$ for simulated functions

$C_\chi(t)$	absolute cardiac phase given as $C_\chi(t) = n + c_\chi(t)$ where n denotes the number of heart beats that have already occurred
Δc_χ	constant phase offset to be added to the kymogram phase (modulo 1) to best agree with $c_\chi(t)$
d	table feed distance per 360° rotation
Φ	fan angle, in our case $\Phi = 52^\circ$
f_H	patient's heart rate, typically $40 \text{ min}^{-1} \leq f_H \leq 120 \text{ min}^{-1}$
$L_\chi(t)$	phase lag function, $L_\chi(t) = C_\chi(t) - C_K(t)$
M	number of simultaneously scanned slices
p	pitch, $p = d/MS$
$p(\alpha, \beta)$	fan-beam projection data (neglecting the z-component of the detector)
$p(\vartheta, \xi)$	parallel-beam projection data (neglecting the z-component of the detector)
S	nominal slice thickness
t_{rot}	time for a 360° rotation
$x_c(\cdot), y_c(\cdot)$	center-of-mass coordinates as a function of the view angle ϑ , of time t , or of z-position z (we have the equivalence $\vartheta/2\pi = t/t_{\text{rot}} = z/d$)
$x_{uc}(\cdot), y_{uc}(\cdot)$	coordinates of the unbiased center of mass
$x'_{uc}(\cdot), y'_{uc}(\cdot)$	coordinates of the unbiased center of mass given in the local inertia coordinate system
$x_{buc}(\cdot), y_{buc}(\cdot)$	coordinates of the bandpass filtered, unbiased center of mass
$x'_{buc}(\cdot), y'_{buc}(\cdot)$	coordinates of the bandpass filtered, unbiased center of mass given in the local inertia coordinate system
z axis	axis of rotation
z_R	arbitrary selectable reconstruction position
(C/W)	notation used for the window setting, C is the window's center in HU, W its width in HU

The freely selectable parameter c_R is used to select the desired cardiac phase; c_R determines the relative center of the time window with respect to each R - R interval of the ECG or relative to the kymogram period. Arithmetics using the cardiac phase c are meant to be modulo 1 to take into account its periodicity.

Patient and cadaver measurements were performed on a SOMATOM Volume Zoom MSCT scanner (Siemens Medical Solutions, Forchheim, Germany). The scan protocol used for 25 cardiac patients is the MSCT coronary angiography protocol $t_{\text{rot}} = 0.5 \text{ s}$, $M \times S = 4 \times 1 \text{ mm}$, $d = 1.5 \text{ mm}$ (i.e., $p = 0.375$) which is established today as a standard scan protocol for thin-section cardiac CTA (CT angiography) with 4-slice CT systems.^{5,6,10,11}

We also performed standard thorax scans with 16-slices and coronary angiography scans with 12-slices on a SOMATOM Sensation 16 CBCT scanner (Siemens Medical Solu-

tions, Forchheim, Germany). The standard thorax protocol was $t_{\text{rot}} = 0.5$ s, $M \times S = 16 \times 0.75$ mm, $d = 6.0$ mm (i.e., $p = 0.5$) and covers a range of 360 mm in 30 s. The cone-beam coronary angiography protocol we used was $t_{\text{rot}} = 0.42$ s, $M \times S = 12 \times 0.75$ mm, $d = 3.0$ mm (i.e., $p = 0.333$) and covers 160 mm in 22 s. Our Sensation 16 is a work-in-progress installation without ECG option and hence we cannot quantitatively assess the kymogram results for those patients scanned on the cone-beam CT.

Further, spiral cone-beam data for a 4-slice and a 12-slice scanner of the cardiac motion phantom (see Ref. 3 for a description thereof) were simulated using the dedicated simulation software ImpactSIM (VAMP GmbH, Möhrendorf, Germany).

The kymogram detection algorithm is implemented as the Kymo taskcard of the Syngo Explorer (VAMP GmbH, Möhrendorf, Germany). The algorithm generates an ECG compatible kymogram file from the raw data which is stored in the DICOM database of Syngo™ (Siemens Medical Solutions, Erlangen, Germany). The file is then used to synchronize the phase-correlated reconstruction algorithms.

Reconstructions are performed with the cardio interpolation approach 180°MCI (Multi-slice cardio interpolation) and ASSR CI (advanced single-slice rebinning cardio interpolation) for 4-slice and 16-slice data, respectively.^{3,12} The reconstruction algorithms are implemented on a standard PC using the dedicated image reconstruction and image evaluation software ImpactIR (VAMP GmbH, Möhrendorf, Germany).

III. KYMOGRAM DETECTION

A. Center of mass tracking

The first step in the kymogram detection approach which we present here is to compute an estimate of the center of mass of the slice currently scanned. Our method is stimulated by and is a generalization of the algorithm published in Ref. 13. We make use of the well-known fact that the COM of the object projects for each parallel view angle ϑ onto the COM of the parallel projection. To establish this fact, we denote the parallel projection through an object $\mu(x, y)$ in two dimensions as

$$p(\vartheta, \xi) = \int dx dy \mu(x, y) \delta(x \cos \vartheta + y \sin \vartheta - \xi).$$

Now, one can immediately validate the following properties which hold $\forall \vartheta$:

$$\int dx dy \mu(x, y) = \int d\xi p(\vartheta, \xi) \quad (\text{conservation of mass}), \quad (1a)$$

$$\begin{pmatrix} \cos \vartheta \\ \sin \vartheta \end{pmatrix} \cdot \int dx dy \begin{pmatrix} x \\ y \end{pmatrix} \mu(x, y) = \int d\xi \xi p(\vartheta, \xi) \quad (\text{conservation of first moment}). \quad (1b)$$

These relations state that the COM of the object function projects onto the COM of the corresponding parallel projection.

Therefore, it seems promising to use the measured projection data to compute the projection COM $\xi_c(\vartheta)$ for each parallel view angle ϑ and to estimate the object COM therefrom. For moving objects, the object COM will also vary in time—and thus as a function of ϑ —and hence the function $\xi_c(\vartheta)$ should allow us to draw conclusions on the object motion provided that a small range of ϑ , say an interval of length $2\pi f$ with f denoting the fraction of a full rotation contributing to the estimation procedure, suffices to perform the estimation.

The scan geometry is a fan-beam geometry with a single-slice, multi-slice or cone-beam spiral trajectory, but we are in need of 2D parallel data to use the relations derived above. Hence, we must convert the measured data accordingly. Fortunately, it suffices to regard the central slice only, and one may neglect the fact that the table translates by a distance of $d(f + \Phi/\pi)$ while acquiring the data needed for one COM estimation. Experiments showed that neither taking into account more slices nor compensating for the table movement by performing a z-interpolation between adjacent detector slices improve the outcome of our study. This can be explained by the signal processing steps that follow the COM detection. They remove all biased dependencies along the z direction (see below) and the algorithm becomes insensitive to axial variations. Due to these reasons, the following considerations neglect the table movement and all but the central detector slice.

To perform the conversion between fan-beam and parallel-beam geometry we denote the measured projection data as $p(\alpha, \beta)$ with $\alpha \in \mathbb{R}$ being the rotation angle of the gantry and $\beta \in \frac{1}{2}[-\Phi, \Phi]$ being the angle within the fan. We need the center of mass of each projection in parallel coordinates as a function of ϑ , i.e., $\xi_c(\vartheta)$. Due to the known relations $\xi = -R_F \sin \beta$, where R_F denotes the distance of the focal spot to the isocenter, and $\alpha = \vartheta - \beta$ we can change the integration variable ξ to β by using the Jacobian $d\xi = -R_F \cos \beta d\beta$ and get

$$\begin{aligned} \xi_c(\vartheta) &= \frac{\int d\xi \xi p(\vartheta, \xi)}{\int d\xi p(\vartheta, \xi)} \\ &= - \frac{\int d\beta R_F^2 \sin \beta \cos \beta p(\vartheta - \beta, \beta)}{\int d\beta R_F \cos \beta p(\vartheta - \beta, \beta)} \\ &= - \frac{1}{2} R_F \frac{\int d\beta \sin(2\beta) p(\vartheta - \beta, \beta)}{\int d\beta \cos \beta p(\vartheta - \beta, \beta)}. \end{aligned}$$

As mentioned, no correction for the shift in z direction is made. Further, one notes that the data contributing to $\xi_c(\vartheta)$ stems from a gantry angle range of size Φ and therefore contains temporal contributions of size $t_{\text{rot}}\Phi/2\pi$ which is far

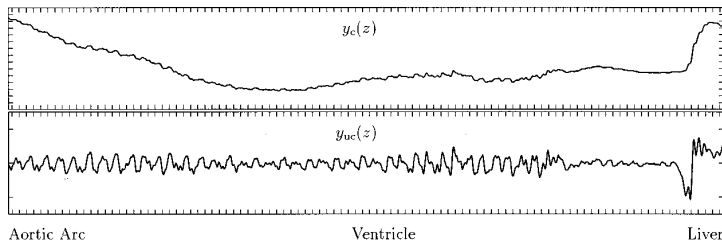


FIG. 1. Biased and unbiased y coordinate of the estimated COM as a function of the z position. The tick mark spacing is 1 mm in z and y direction.

below the time of one cardiac cycle. Since the heart, as the moving part of the object, occupies only a small fraction of the complete field of measurement (which corresponds to the fan angle Φ) the temporal contributions of the heart to the computation of ξ_c are even less and can be neglected.

Denoting the image center of mass as x_c and y_c we have shown that

$$x_c \cos \vartheta + y_c \sin \vartheta - \xi_c(\vartheta) = 0$$

should hold $\forall \vartheta$. This can be solved in a least square manner by minimizing

$$\frac{1}{2\pi f} \int_{\vartheta-f\pi}^{\vartheta+f\pi} d\vartheta (x_c \cos \vartheta + y_c \sin \vartheta - \xi_c(\vartheta, m))^2$$

in the range $[\vartheta - f\pi, \vartheta + f\pi]$ with $f > 0$. In the following we will formulate the integral as a convolution with $\Pi_{2\pi f}^*(\vartheta)$, for convenience. The minimization (derivative with respect to x_c and y_c) yields the system of equations

$$\begin{pmatrix} 1+C & S \\ S & 1-C \end{pmatrix} \begin{pmatrix} x_c \\ y_c \end{pmatrix} = \begin{pmatrix} X_C \\ X_S \end{pmatrix}$$

with the solution

$$\begin{pmatrix} x_c \\ y_c \end{pmatrix} = \frac{1}{1-C^2-S^2} \begin{pmatrix} 1-C & -S \\ -S & 1+C \end{pmatrix} \begin{pmatrix} X_C \\ X_S \end{pmatrix},$$

where we use the abbreviations

$$C = \cos(2\vartheta) * \Pi_{2\pi f}^*(\vartheta) = \text{sinc } 2f \cos 2\vartheta,$$

$$S = \sin(2\vartheta) * \Pi_{2\pi f}^*(\vartheta) = \text{sinc } 2f \sin 2\vartheta,$$

$$X_C = 2\xi_c(\vartheta) \cos(\vartheta) * \Pi_{2\pi f}^*(\vartheta),$$

$$X_S = 2\xi_c(\vartheta) \sin(\vartheta) * \Pi_{2\pi f}^*(\vartheta).$$

It should be noted that the denominator, which is the determinant of the coefficient matrix, yields $1-C^2-S^2=1-\text{sinc}^2 2f$ which is always positive and thus indicates the regularity of the coefficient matrix. The fraction f should not be chosen too small to avoid numerical instabilities. Values f between 10% and 50% turn out to be sufficient to yield accurate results. The overall results do not depend on the exact choice of f ; we use $f=10\%$ in the following.

B. Further signal processing

The COM signal we have computed in the preceding section needs to undergo a few signal processing steps until the synchronization peaks of the cardiac motion can be detected.

For convenience, we will now switch from the independent variable ϑ to the variable $t = t_{\text{rot}} \vartheta / 2\pi$ which denotes the time or to the variable $z = d \vartheta / 2\pi$ which denotes the z position. Functions of ϑ are now overloaded and we also note t or z as the function's argument. Thereby, we emphasize which parameter is of relevance. For example, when smoothing the function in the z direction we use z and when performing a temporal filtering we use t while being aware that there is a one-to-one correspondence between ϑ , t , and z .

Further, if the x component is the only component noted it is implicitly assumed that the y component and other motion components are treated equivalently.

1. Bias correction

The COM functions $x_c(t)$ and $y_c(t)$ are biased due to (slow) variations in the patient cross section introduced by the continuous translation of the patient table during the spi-

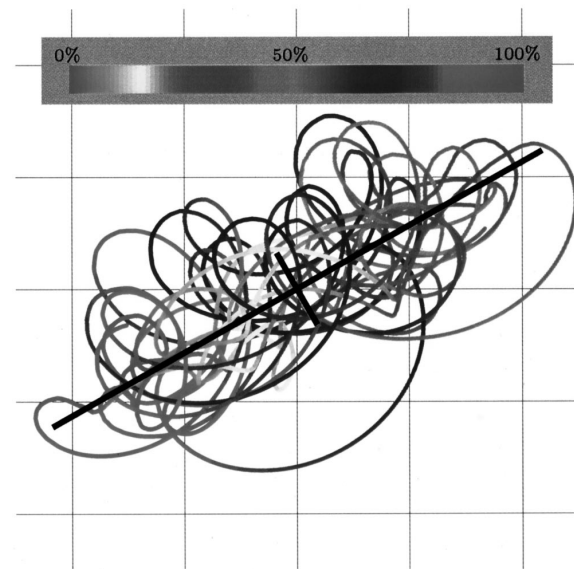


FIG. 2. Parametric plot (x_{uc} vs y_{uc} as a function of t) of the unbiased COM for a complete scan (many heart cycles). The grid spacing is 0.1 mm. The corresponding cardiac ECG phase $c_E(t)$ has been used to set the hue of the plot. The legend shows the hue to phase mapping. Regions of similar hue are spatially relatively close. The two global principal axes are shown as well. Their length was chosen proportional to Θ'_{xx} and Θ'_{yy} , respectively. For this patient we have $\Theta'_{xx}=6.8\Theta'_{yy}$ and $\phi=29.5^\circ$.

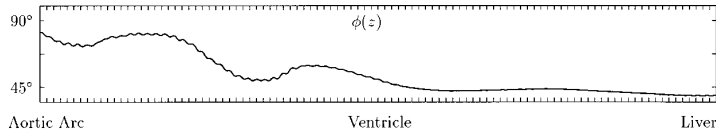


FIG. 3. The function $\phi(z)$ for the complete scan. Ticks are spaced at 1 mm in the z direction and 22.5° in the ϕ direction, respectively.

ral scan. To remove the bias we subtract the running mean of the COM using a rectangular filter of width Δt . Thus, the unbiased version of the signal is given as

$$x_{uc}(t) = x_c(t) - x_c(t) * \Pi_{\Delta t}^*(t).$$

A value of $\Delta t = 1$ s is large enough to avoid averaging out components of the cardiac motion. The value is motivated by the fact that cardiac motion usually is of the order $f_H \geq 60 \text{ min}^{-1}$. Anyhow, frequencies below this are suppressed smoothly by the bias procedure and heart rates down to about 30 bpm can be detected as well.

To illustrate the necessity of bias correction Fig. 1 shows the functions $y_c(z)$ and $y_{uc}(z)$ for a typical patient. The unbiased signal is of low amplitude (submillimeter); its variations are mainly due to cardiac motion.

2. Principal heart axes

The two available signals $x_{uc}(z)$ and $y_{uc}(z)$ represent an estimate of the in-plane cardiac motion as a function of time or of z position. We now seek to reduce the information to a one-dimensional signal from which we can easily detect synchronization peaks.

Assume that cardiac motion takes place in the y direction only. Then, the signal in x direction would be low and probably worthless. A parametric plot of $(x_{uc}(z), y_{uc}(z))$ as a function of z for a typical patient indicates that there is a preferred axis of in-plane motion (Fig. 2). Projecting the COM functions onto this principle axis of motion should consequently yield a useful COM function even if the motion takes place along one axis only.

We determine the principal axis of motion (and its orthogonal counterpart) by taking the inertia tensor

$$\Theta(t) = \begin{pmatrix} \Theta_{xx} & \Theta_{xy} \\ \Theta_{xy} & \Theta_{yy} \end{pmatrix}(t)$$

$$\Theta_{xx}(t) = \Pi_{\Delta t}^*(t) * x_{uc}^2(t),$$

with the moments of inertia defined as

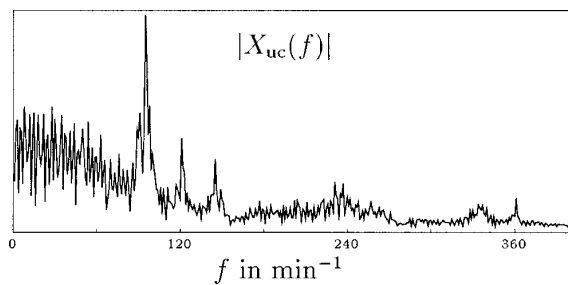


FIG. 4. Spectrum of $x'_{uc}(t)$ scaled to arbitrary units for frequencies up to 400 min^{-1} . Patient heart rate is 97 min^{-1} which obviously coincides with the dominating peak.

$$\Theta_{xy}(t) = \Pi_{\Delta t}^*(t) * x_{uc}(t) y_{uc}(t),$$

$$\Theta_{yy}(t) = \Pi_{\Delta t}^*(t) * y_{uc}^2(t).$$

Then, we perform a principal axes transformation such that the transformed moment of inertia is diagonal,

$$\Theta'(t) = \begin{pmatrix} \Theta'_{xx} & 0 \\ 0 & \Theta'_{yy} \end{pmatrix}(t).$$

The width Δt of the contributing running interval is chosen as $\Delta t = 10$ s to cover at least about 10 heart beats (assuming a heart rate of at least 60 beats per minute).

The diagonal values of Θ' are the eigenvalues of Θ . Without loss of generality we demand $\Theta'_{xx} \geq \Theta'_{yy}$. We denote the corresponding rotation angle of the underlying orthogonal transformation as $\phi(t)$ and we use the transformation matrix to express the COM coordinates in the new eigensystem

$$\begin{pmatrix} x'_{uc}(t) \\ y'_{uc}(t) \end{pmatrix} = \begin{pmatrix} \cos \phi(t) & \sin \phi(t) \\ -\sin \phi(t) & \cos \phi(t) \end{pmatrix} \begin{pmatrix} x_{uc}(t) \\ y_{uc}(t) \end{pmatrix}.$$

Here, $x'_{uc}(t)$ is expected to be the most pronounced signal since it corresponds to the main inertia axis (largest eigenvalue).

The advantage of computing the new coordinate system adaptively, i.e., over a range Δt , is that the algorithm can adopt to changing (e.g., slowly rotating) principal axes.

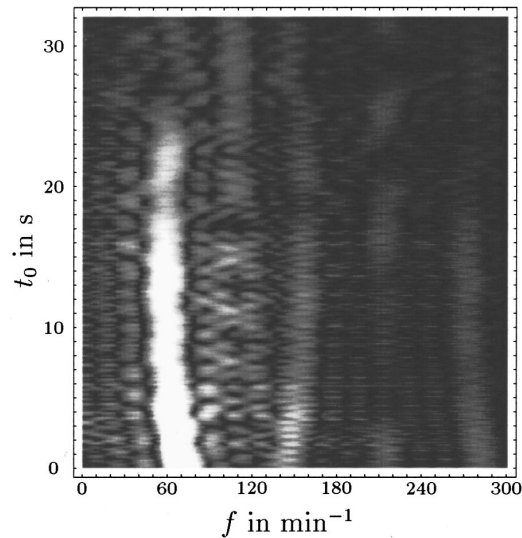


FIG. 5. The windowed Fourier transform $X_{uc}(f, t_0)$ for frequencies up to 300 min^{-1} . The patient's heart rate lies between 60 min^{-1} and 70 min^{-1} . For further discussion of this specific patient see Sec. V C.

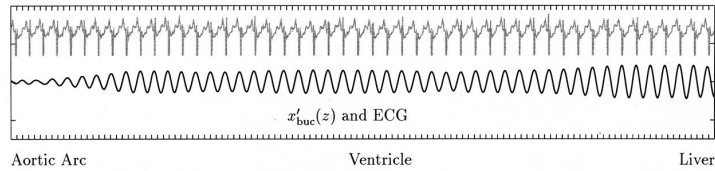


FIG. 6. Kymogram $x'_{buc}(t)$ and the ECG signal as a function of z position/time. Tick spacing is 1 mm in the z direction and in the x' direction; the ECG signal is scaled to arbitrary units.

Since this range constitutes the average of at least 10 cardiac cycles, the determination of the heart axis is insensitive to extra systoles or arrhythmic behavior.

For illustration purposes the principal axes shown in Fig. 2 are the global ($\Delta t = \infty$) axes and do not reflect the adaptive behavior of the algorithm. The function $\phi(t)$ is shown in Fig. 3 for another patient.

3. Bandpass filtering

The quality of the unbiased COM signal proved to be insufficient to directly detect periodicity information. The spectrum of $x_{uc}(t)$ and $y_{uc}(t)$ —a patient example is given in Fig. 4—shows dominating peaks at the heart frequency f_H and at the scanner's rotation frequency $1/t_{rot}$.

Therefore, a bandpass filter must be centered about f_H to remove the undesired parts of the spectrum. Since the heart frequency may change as a function of time, e.g., from 60 min^{-1} at scan start to 100 min^{-1} at scan end, a simple bandpass filter will not suffice. We rather perform a windowed Fourier transform assuming a time window $w(t)$ with $\int dt w^2(t) = 1$:

$$X_{uc}(f, t_0) = \int dt x_{uc}(t) w(t - t_0) e^{-2\pi i f t}.$$

The window function used is

$$w(t) = \frac{2}{\sqrt{3}\Delta t} \cos^2\left(\frac{\pi}{2} \frac{t}{\Delta t}\right) \Pi\left(\frac{1}{2} \frac{t}{\Delta t}\right).$$

An appropriate width, that allows to detect frequencies down to 20 min^{-1} is $\Delta t = 3 \text{ s}$. Hence, all clinically relevant cases ($f_H > 40 \text{ min}^{-1}$) can be handled.

From the modulus of these functions we detect the maximum. The frequencies taken into account lie between 40 min^{-1} and $1/t_{rot}$. The frequency where this maximum occurs is denoted as $f_H(t_0)$ and bandpass filtering is performed by multiplying the windowed Fourier transform with a smooth function H that is centered about $f_H(t_0)$.

$$X_{buc}(f, t_0) = X_{uc}(f, t_0) H(f - f_H(t_0)).$$

Currently, we use

$$H(f) = \cos^2\left(\frac{\pi}{2} \frac{f}{\Delta f_H}\right) \Pi\left(\frac{1}{2} \frac{f}{\Delta f_H}\right),$$

where Δf_H denotes the full width at half maximum of the bandpass filter function. A value of $\Delta f_H = 20 \text{ min}^{-1}$ turned out to be sufficient. We have tested the method with varying values of Δf_H and found the method to be stable for values between 10 and 30 min^{-1} (which is also indicated by the width of the peak in Fig. 4). An example of a windowed Fourier transform for a typical patient is given in Fig. 5.

The temporal domain signal is then reconstructed by simple inversion:

$$x_{buc}(t) = \iint dt_0 df X_{buc}(f, t_0) w(t - t_0) e^{2\pi i f t}.$$

The kymogram function $x'_{buc}(t)$ and the patient's ECG signal are shown in Fig. 6; the close correlation of the periodicity information of these two signals is apparent.

4. Peak detection

Finally, the synchronization peaks are detected from $x'_{buc}(t)$ by simply detecting the ascending zero crossings of the first derivative. If we denote the location of the peaks as t_n we construct the cardiac kymogram phase as

$$c_K(t) = \frac{t - t_n}{t_{n+1} - t_n} + \Delta c_\chi \bmod 1$$

$$\text{with } n \text{ such that } t_n \leq t < t_{n+1}.$$

This definition is equivalent to the definition of the cardiac ECG phase $c_E(t)$ which quantifies the location of t relative to the previous and next R peak. The constant offset Δc_χ reflects the remaining degree of freedom in choosing the kymogram phase. If we do not intend to compare our results to an ECG function, we set this offset to zero. For ECG comparisons, however, we choose its value to minimize the phase-lag between ECG and kymogram (see below). Doing so, we can, for example, reconstruct ECG and kymogram correlated at a reconstruction phase of $c_R = 70\%$ while ensuring maximum consistency between the two reconstructed

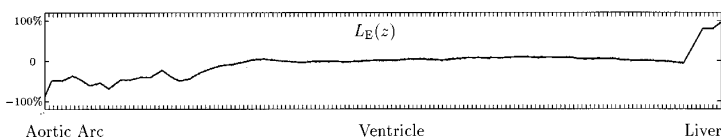
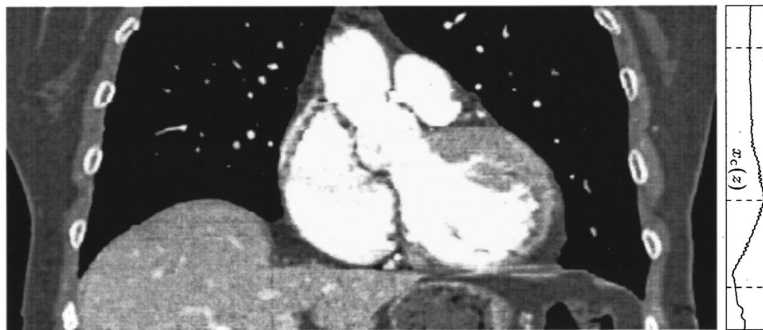


FIG. 7. An example ECG lag function $L_E(z)$ corresponding to the ECG and kymogram shown in Fig. 6.



a) Sagittal MPR



b) Coronal MPR



c) Transaxial slices (in cranio-caudal order)

FIG. 8. MPRs and transaxial slices of the complete patient thorax exemplify the behavior of the COM curves. (a) Sagittal section and $y_c(z)$. (b) Coronal section and $x_c(z)$. The global behavior of the COM coordinates correlates well with the anatomy shown. (c) Transaxial slices for illustration purposes. Their z positions correspond to the dashed lines of the x_c and y_c plots. The images are reconstructed kymogram-correlated.

volumes. Without the offset correction it could happen that kymogram images at, say, 70% are best comparable to ECG images at 43%.

IV. TEST METHODS

A. Comparing $c_K(t)$ and $c_\chi(t)$

We define the monotonically increasing (absolute) phase function

$$C_\chi(t) = n + c_\chi(t) \quad \text{with } n \text{ such that } t_n \leq t < t_{n+1}.$$

The integer part of C_χ counts the number of heart beats that have occurred and the fractional part gives the relative position between two synchronization points. Obviously, C_χ is strict monotonically increasing in t and has an inverse which we denote as C_χ^{-1} . Now, we can regard the phase lag function

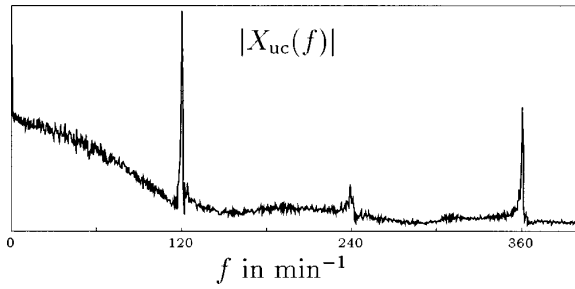


FIG. 9. Spectrum of $x'_{uc}(t)$ as a function of frequency f for the cadaver scan. The peaks at $1/t_{rot}$ and multiples thereof are clearly visible. The ordinate is scaled to arbitrary units.

$$L_X(t) = C_X(t) - C_K(t)$$

which should be constant for an ideal correlation between the ECG and the kymogram approach. The ECG lag for a typical patient is given in Fig. 7. Please note the plateau in the scan center which indicates a good correlation between ECG and kymogram for the plateau region. The plateau's height is within 5% distance to 0% due to the offset correction Δc_X . Since the heart is not reached fully at scan start and since the scan has left the heart at scan end there are deviations from the 0% optimum at the left and right side of the plot.

B. Simulated motion functions

For test purposes, we simulated phantom data of our cardiac motion phantom using various motion functions. It turned out that the computation of $\xi_c(\vartheta)$ based on these simulated raw data yields exactly the same result as when directly prescribing

$$\xi_c(\vartheta) = x_{sim}(\vartheta) \cos \vartheta + y_{sim}(\vartheta) \sin \vartheta \quad (2)$$

with $\vartheta = 2\pi t/t_{rot}$ and the COM coordinates $x_{sim}(\vartheta)$ and $y_{sim}(\vartheta)$. Since our intent is to evaluate the kymogram detection algorithm we will therefore not concentrate on the simulated raw data but on the directly generated function ξ_c and its evaluation.

Thus, we simulated a number of COM-functions $x_{sim}(t)$ and $y_{sim}(t)$ and used (2) to compute the projected COM. To complete our simulation, we had to additionally define the sync peaks t_n of the simulated motion. This defines the simulated phase $c_S(t)$ and the absolute phase $C_S(t)$.

The simulated input signal $\xi_c(\vartheta)$ was then used for the described signal processing steps that detect the kymogram peaks. The detected peaks are compared to the simulated sync peaks by computing the phase lag function $L_S(t)$ for simulated motion.

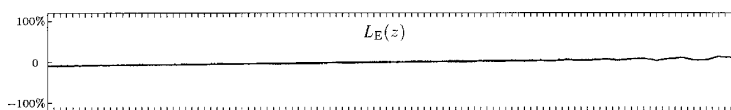


FIG. 10. The chirp's lag function $L_S(z)$. The frequency of motion increases linearly with the z position from 40 min^{-1} to 110 min^{-1} . The tick spacing in z direction is 1 mm.

1. Sine

As a simple test function we used a sine function in x direction defined in $0 \leq t \leq 40 \text{ s}$:

$$x_{sim}(t) = 1.0 \text{ mm} \times \sin(2\pi t \times f_H),$$

$$y_{sim}(t) = 0,$$

the heart rates were chosen as $f_H = 60 \text{ min}^{-1}$, 70 min^{-1} , 80 min^{-1} , and 100 min^{-1} . We define the sine's sync peaks as the ascending zeroes of the motion's x component:

$$t_n = n/f_H \quad \text{with } 0 \leq n \leq f_H \times 40 \text{ s.}$$

2. Chirp

As a more complicated test function we define a chirp signal in the x direction. The increasing heart rate helps to evaluate the performance of our kymogram algorithms with patients of slowly varying heart rate.

For $0 \leq t \leq T = 40 \text{ s}$ we define

$$x_{sim}(t) = 5.0 \text{ mm} \quad \text{constant offset}$$

$$+ 0.1 \text{ mm} \times \sin(2\pi t/(20t_{rot})) \quad \text{low frequency bias}$$

$$+ 1.0 \text{ mm} \times \sin(2\pi t(at+b)) \quad \text{actual signal}$$

$$+ 1.0 \text{ mm} \times \sin(2\pi t/t_{rot}) \quad \text{high frequency drop in}$$

$$+ 1.0 \text{ mm} \times \sin(4\pi t/t_{rot}) \quad \text{first harmonic of drop in}$$

$$y_{sim}(t) = 0$$

to compute $\xi_c(\vartheta) = x_{sim}(\vartheta) \cos \vartheta + y_{sim}(\vartheta) \sin \vartheta$. The actual signal is a chirp starting with $f_H(0) = b = 40 \text{ min}^{-1}$ and ending with $f_H(T) = 2aT + b = 110 \text{ min}^{-1}$ (a and b have been chosen accordingly). Our aim is to observe the behavior of the kymogram detection during the frequency sweep from 40 min^{-1} to 110 min^{-1} . The drop in at $1/t_{rot} = 120 \text{ min}^{-1}$ and its harmonic mimic the resonance behavior of the scanner.

The function's sync peaks t_n are defined as the ascending zeroes of the actual signal

$$t_n(at_n + b) = n.$$

V. RESULTS

The following results are based on detecting synchronization information from x'_{buc} which has turned out, as expected, to be the most pronounced signal.

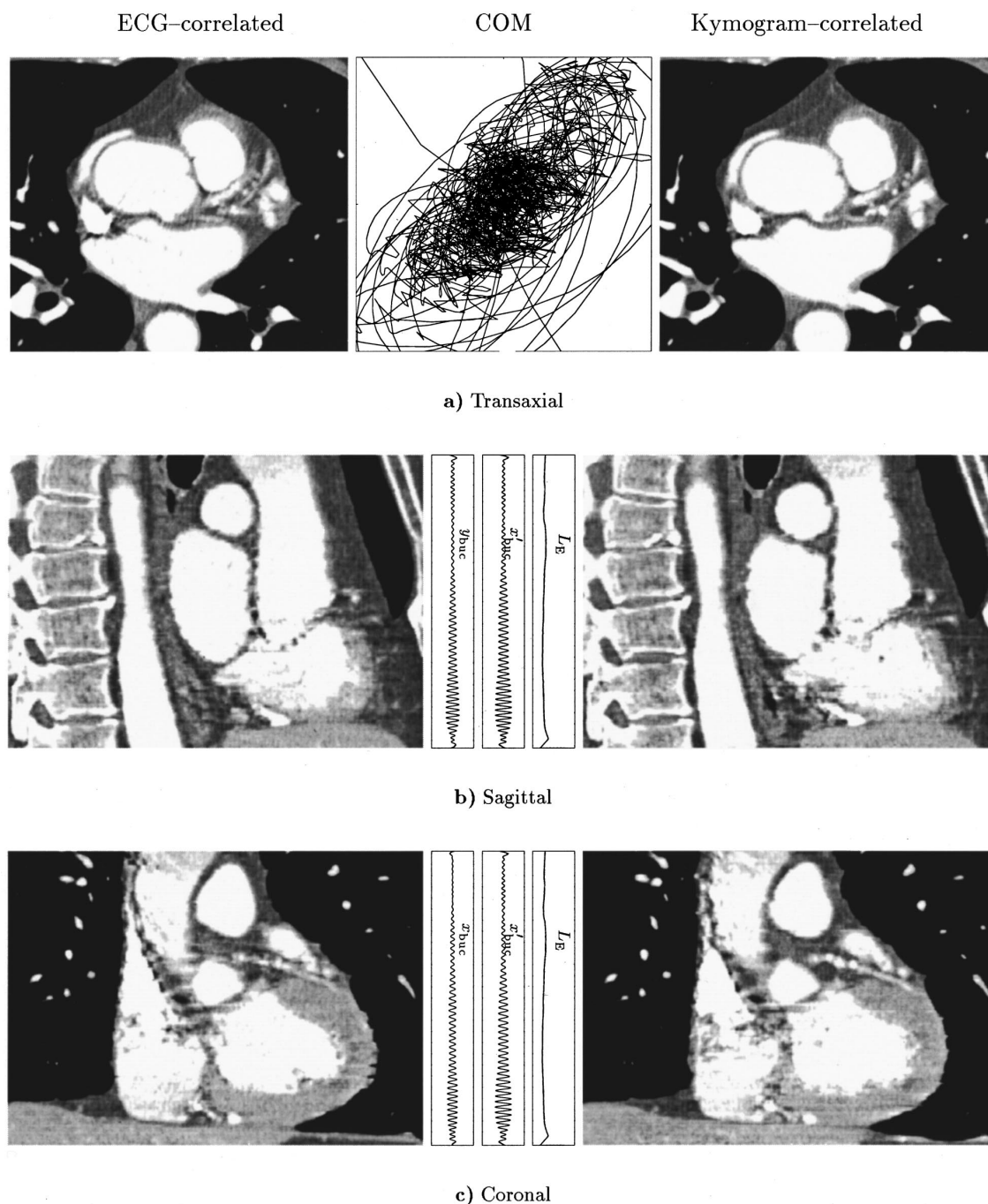


FIG. 11. Transaxial and MPR images of a cardiac patient reconstructed with 180° MCI. The parametric COM plot ranges from -0.75 mm to 0.75 mm for the unbiased x and y direction. The functions shown with the MPRs are aligned to the MPRs in z direction. The x and y dimensions are scaled to fit the available space. The lag function $L_E(z)$ and the primed x coordinate $x'_{buc}(z)$ are identical for both MPR sections; they are repeated to show the correlation to the anatomy. The patient's mean heart rate is $f_H = 65 \text{ min}^{-1}$ and the reconstruction phase was chosen as $c_R = 50\%$.

A. General results

To demonstrate the good correlation between the detected COM functions x_c and y_c as a function of the z position we

have prepared Fig. 8. It shows a sagittal and a coronal multiplanar reformation (MPR) of the complete thorax cross section (and not only of the heart) together with the corresponding COM estimation. In addition, three transaxial slices are



a) Standard reconstruction



b) ECG-based reconstruction



c) Kymogram-based reconstruction

FIG. 12. Comparison of a standard reconstruction with the phase-correlated ECG based and kymogram based reconstructions. Transaxial, sagittal, and coronal slices are shown. The patient's mean heart rate is $f_H = 52 \text{ min}^{-1}$ and the reconstruction phase was chosen as $c_R = 50\%$.

shown. The COM's slow variations, i.e., the bias that is to be removed, corresponds to the patient's anatomy. The shift at the lower end of the coronal plot, for example, is induced by the transition from the heart to the liver which introduces a shift of x_c towards the patient's right side (appearing left in the image).

To determine the source of the 120 min^{-1} resonance frequency that was visible in the spectral plots of Sec. III B 3 and that appears for all patient data, we have scanned a

cadaver. As can be seen from Fig. 9 the resonance frequency appears even for motionless objects. Tests with simulated monochromatic raw data (pure line integrals) using the same scanner geometry showed no such resonance behavior. Consequently, nonlinear effects such as scattering and beam hardening have to be considered as the cause of this artifact. These nonlinearities violate the assumptions of (1) and therefore introduce an artificial modulation into the COM curve. Due to this intrinsic resonance, we cannot detect cardiac mo-

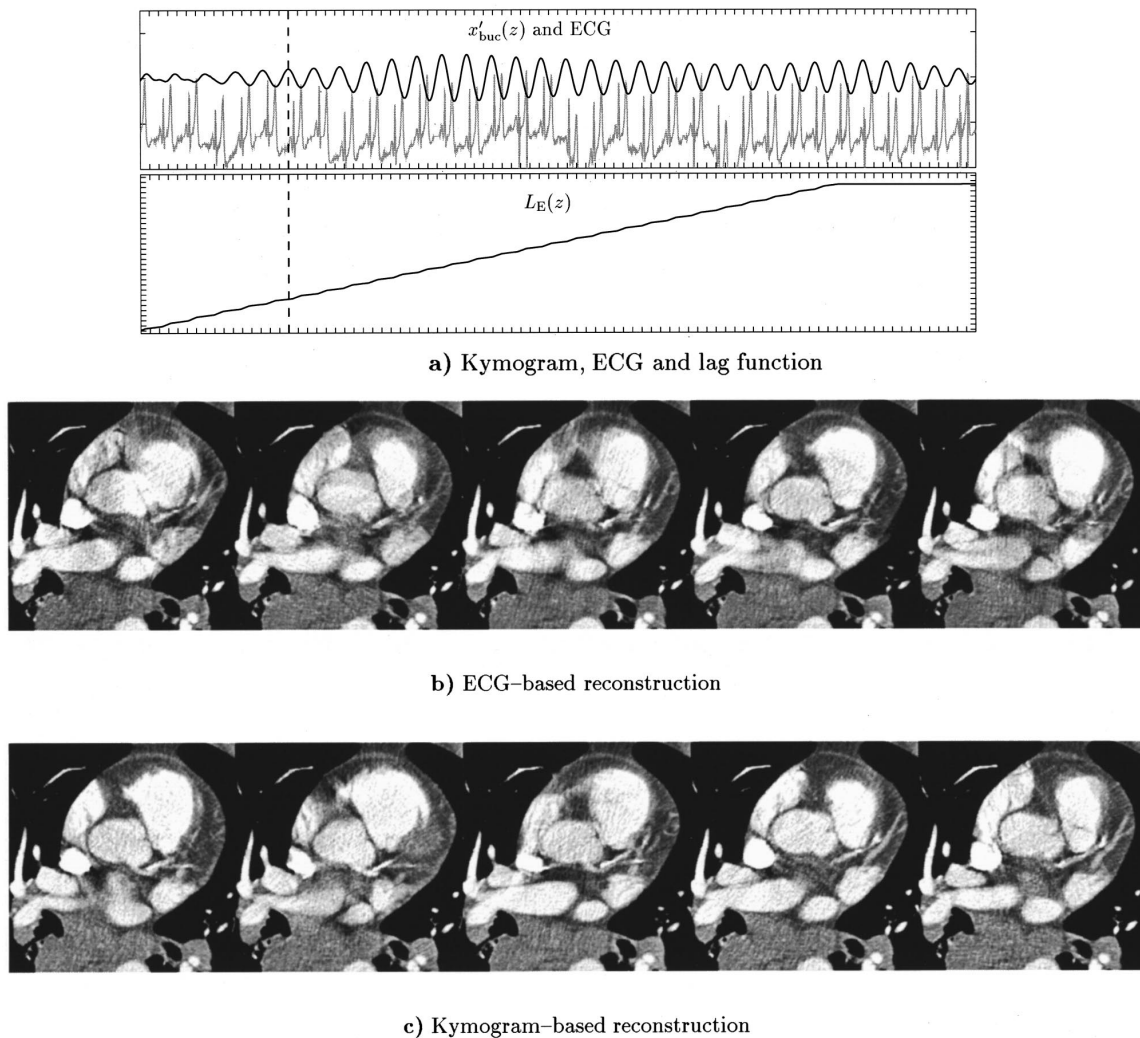


FIG. 13. For this patient, the *T* wave was misinterpreted as an *R* peak by the ECG monitor. The lag function therefore shows a strong increase, except for the last fifth of the scan where the ECG monitor started to correctly interpret the ECG signal. Tick spacing is 1 mm in the *z* direction, 1 mm for x'_{buc} and 100% for the lag function, respectively. The ECG is scaled to arbitrary units. The vertical dashed line indicates the slice position of the transaxial images. The reconstructions are performed in cardiac ECG- and kymogram-phase steps of 20%. The ECG-correlated images are blurred and show overlaid structures since different cardiac phases are mixed into each image. The patient's mean heart rate, as detected by the kymogram algorithm, is $f_H = 68 \text{ min}^{-1}$ and the reconstruction phases were chosen in steps of 20%.

tion at $f_H = 1/t_{\text{rot}}$ and within a 10% window around this frequency. This disadvantage will become irrelevant with future scanners that rotate faster.

B. Simulation study

The resulting lag functions for the simulated sine motions show no lag between the detected and the simulated peaks, i.e., $L_S(z) \equiv 0$. This is a first indication that the new method is free of systematic errors.

The chirp function, which is much more demanding, yields almost optimal results for the complete frequency range. This is indicated by the chirp's lag function in Fig. 10. The lag values are not perfectly zero. They rather increase

almost linearly from -5% to $+5\%$. This can be well accepted regarding the facts that (a) these small deviations are not oscillating but change slowly over the scan and (b) the range of heart rates covered with this experiment within a single scan is far from clinical reality.

C. Patient results

To evaluate our approach we have studied data of 25 routine patients scheduled for coronary CT angiography. Phase-correlated reconstructions were performed using the ECG-based and the kymogram-based approach for comparison. In addition, standard reconstructions which are not phase-correlated have been carried out.

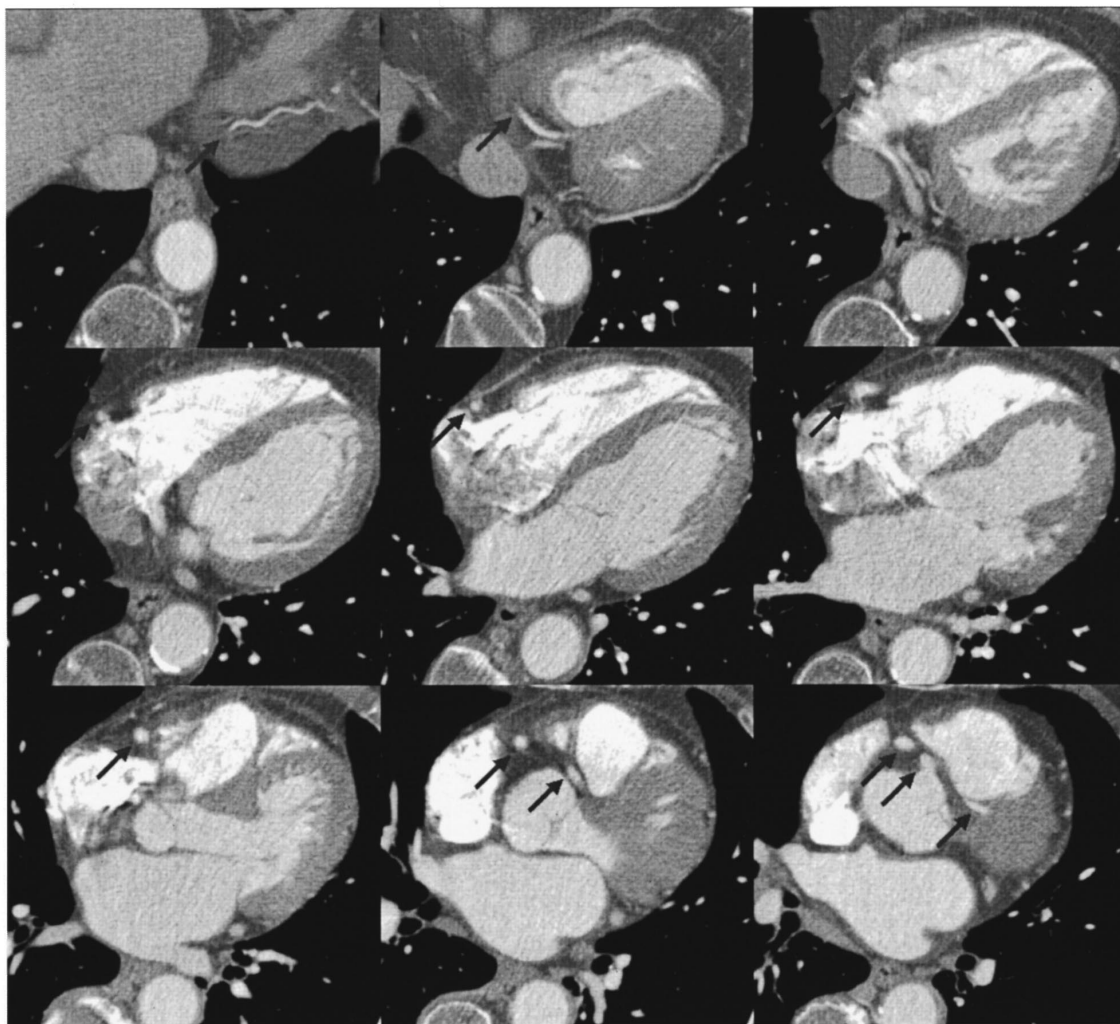


FIG. 14. Kymogram-based ASSR CI reconstruction of 12-slice cardiac CT data. The images are spaced by 9 mm in caudo-cranial order from top left to bottom right. Arrows point at the right coronary artery which can be depicted clearly and at full length.

Figure 11 shows the ECG-correlated and the kymogram-correlated reconstructions together with the COM plots and the kymogram functions. Image quality of both approaches appears to be the same although some discontinuities in the MPR displays differ. The kymogram functions x_{buc} , y_{buc} , and x'_{buc} nicely show the correlation of the cardiac anatomy and the amplitude of motion: the aortic motion alone yields a lower amplitude of the COM motion than the motion of the ventricles. The lag function is almost horizontal and indicates a good correlation between ECG and kymogram.

Another patient is shown in Fig. 12 which compares standard, ECG-correlated and kymogram-correlated reconstructions. Both the ECG-correlated and the kymogram-correlated reconstruction are, not surprising, clearly superior to the standard approach although substantial motion artifacts still remain for certain heart beats. Apart from slightly fewer discontinuities in the kymogram reconstruction the two phase-

correlated reconstructions yield comparable image quality. Please note, the kymogram-based reconstruction and the standard reconstruction are both based on the same prerequisites: A standard CT scan without additional ECG acquisition. The improvement in image quality, however, is significant.

For a few patient cases, the ECG signal may be defective or the ECG monitor's *R*-peak detection algorithms may fail. In the case presented in Fig. 13, the *R*-peak detection did not only detect the *R* waves but also the *T* waves—the *T* wave typically appears less pronounced than the *R* wave—of the patient. Therefore, the patient's heart rate appeared to be twice as high. Since the ECG file was not corrupt, this error remained unnoticed during ECG-correlated reconstruction. The kymogram, in contrast, detected the cardiac motion correctly. Results are shown in Fig. 13. The phase lag is strongly increasing since the ECG signal reports a twofold increase of

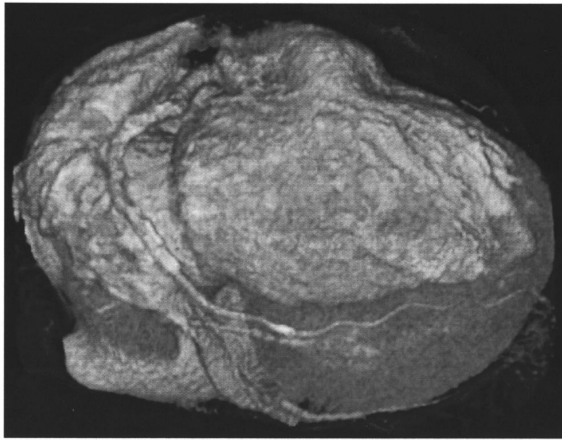


FIG. 15. Kymogram-based ASSR CI reconstruction of 12-slice cardiac CT data viewed from anterior/caudal using volume rendering. The coronary arteries are nicely depicted at full length.

the cardiac phase as compared to the kymogram signal. Close to the scan end the ECG monitor started to detect correctly and, consequently, the lag function turns into a horizontal line. Figures 13(b) and 13(c) show the ECG correlated and the kymogram-correlated reconstructions of a single slice for the phases (ECG and kymogram) 10%, 30%, 50%, 70%, and 90%. Whereas the kymogram-correlated reconstruction depicts the heart in a full cycle of motion, the ECG-correlated images appear to be mixed and, even more drastically, the artifact content is increased due to overlapping structures. This can be explained by the fact that the reconstruction algorithm, based on the false ECG interpretation, combines data segments corresponding to different (true) heart phases. Figure 5 shows the windowed Fourier transform of that patient; the true heart rate between 60 min^{-1} and 70 min^{-1} can be clearly seen from this density plot.

As a further demonstration Fig. 14 gives an impression of the kymogram performance on 12-slice cone-beam CT with a rotation time of 0.42 s. Unfortunately, we have no ECG available with our CBCT scanner and cannot compare the image quality of kymogram-correlated CBCT to ECG-correlated CBCT. Nevertheless, the high quality of the transaxial reconstruction and the high quality of the volume rendering in Fig. 15 indicate a good correlation of the kymogram and the cardiac motion also for cone-beam CT data.

D. Statistical evaluation

To quantify the kymogram algorithm in comparison to the ECG signal we performed a statistical evaluation of the lag function for 25 consecutive cardiac patients. First, the z positions that did not cover the heart were excluded from the data. Then, the mean and the standard deviation of the lag function's values were computed. The mean values are equivalent to the offset Δc_E introduced in Sec. III B 4. The

TABLE I. Patient mean heart rate, mean lag offset Δc_E and lag function variation σ_{L_E} in relation to the gold standard ECG. The last two lines show the mean and standard deviation over the respective columns. (Patient 7 was excluded because of disturbed ECG data.)

Patient	f_H (ECG)	f_H (Kymo)	Δc_E	σ_{L_E}
1	75 min^{-1}	75 min^{-1}	5%	3.05%
2	64 min^{-1}	64 min^{-1}	1%	1.38%
3	69 min^{-1}	69 min^{-1}	55%	5.45%
4	89 min^{-1}	87 min^{-1}	4%	7.51%
5	75 min^{-1}	75 min^{-1}	97%	1.46%
6	70 min^{-1}	70 min^{-1}	1%	3.19%
7	...	68 min^{-1}	...	...
8	70 min^{-1}	69 min^{-1}	3%	4.40%
9	54 min^{-1}	54 min^{-1}	41%	2.20%
10	101 min^{-1}	101 min^{-1}	53%	24.5 %
11	87 min^{-1}	87 min^{-1}	15%	3.08%
12	79 min^{-1}	79 min^{-1}	30%	2.75%
13	61 min^{-1}	61 min^{-1}	38%	4.84%
14	64 min^{-1}	64 min^{-1}	93%	4.65%
15	66 min^{-1}	66 min^{-1}	49%	1.25%
16	69 min^{-1}	69 min^{-1}	2%	2.52%
17	67 min^{-1}	67 min^{-1}	97%	1.45%
18	75 min^{-1}	74 min^{-1}	97%	2.18%
19	65 min^{-1}	65 min^{-1}	94%	4.65%
20	52 min^{-1}	52 min^{-1}	29%	2.72%
21	84 min^{-1}	84 min^{-1}	2%	2.11%
22	70 min^{-1}	71 min^{-1}	47%	3.75%
23	78 min^{-1}	78 min^{-1}	2%	2.13%
24	61 min^{-1}	61 min^{-1}	39%	4.15%
25	97 min^{-1}	97 min^{-1}	2%	8.81%
mean:	73 min^{-1}	73 min^{-1}	...	4.53%
sigma:	13 min^{-1}	13 min^{-1}	...	4.85%

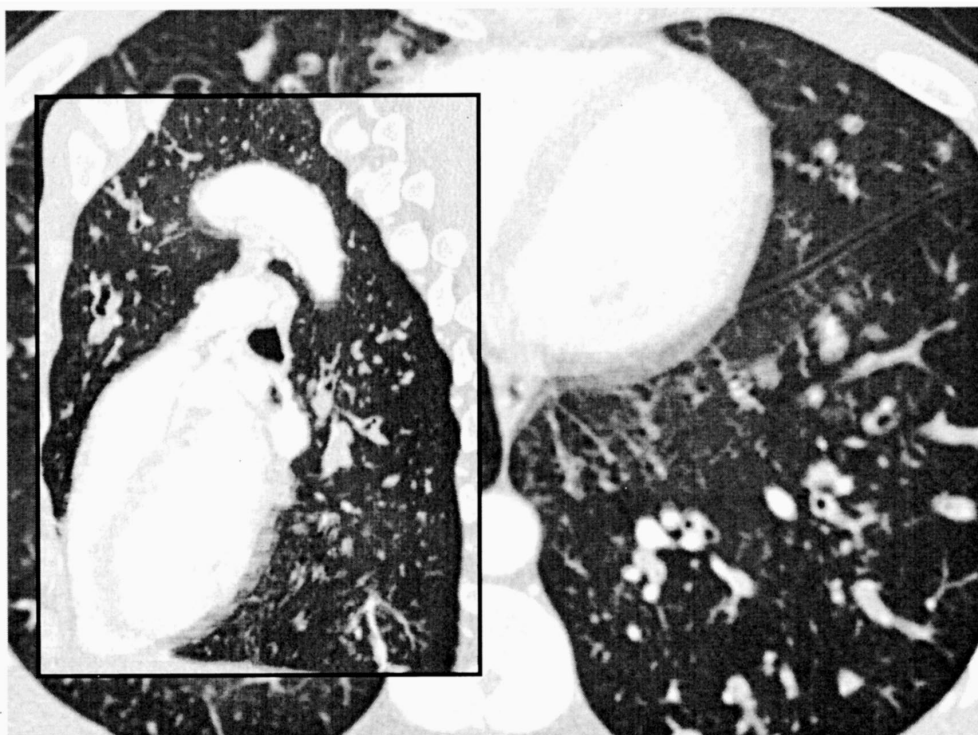
standard deviation σ_{L_E} is a measure of the kymogram quality if one assumes the ECG to be the gold standard for cardiac motion synchronization. Results of our analysis are given in Table I. Please note that only 4-slice data enter this table since we have no ECG available for the cone-beam scans.

The table shows a good match between the ECG and the kymogram. The deviation of the lag function is below 5% in 84% of all cases. This is also indicated by the good agreement in detected heart rate. As patients 10 and 25 indicate, the kymogram tends to match the ECG less closely for higher heart rates.

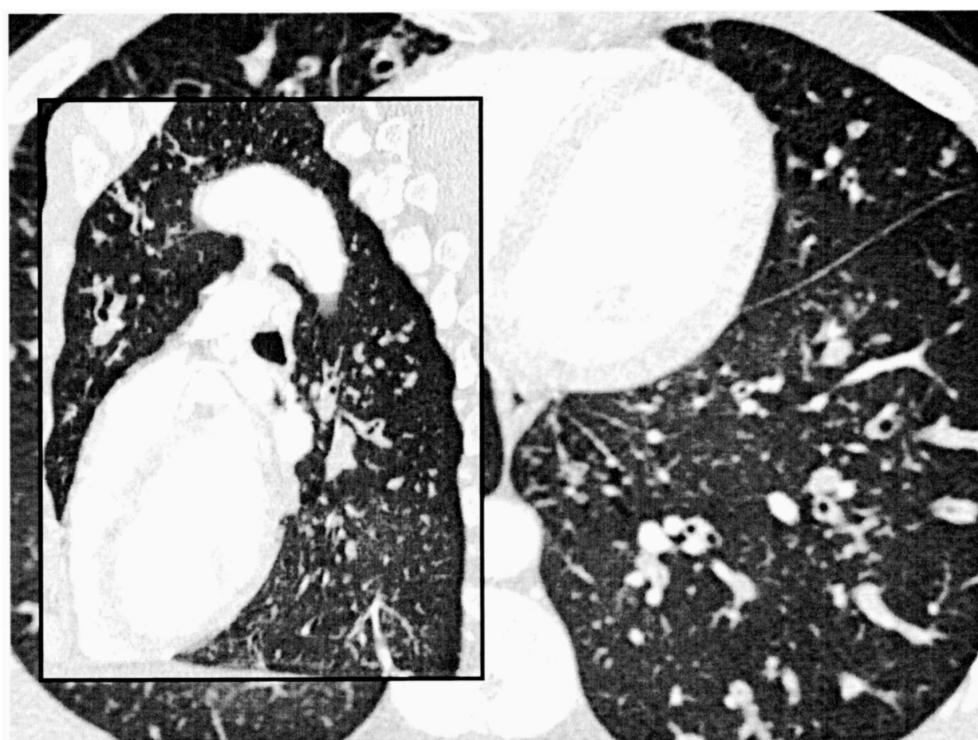
VI. DISCUSSION AND CONCLUSIONS

The kymogram approach as presented in this paper involves a number of ad hoc steps, such as data windowing and filtering, that are not strictly derived from theory. However, experience has shown that these steps are necessary. This fact and the overall results obtained justify our procedure.

In general, it has turned out that information nearly equivalent to the ECG's phase information can be obtained from the raw data at the level of the heart; slight deteriorations for heart rates that approach the scanner's resonance frequency have been observed. Regarding that the improvements in image quality obtained with the kymogram compared to standard reconstructions are equivalent to the im-



a) ASSR Std reconstruction



b) Kymogram-correlated ASSR CI reconstruction

FIG. 16. Improvements that can be obtained for a standard thorax scan when using kymogram-correlated reconstruction.

provements obtained by reconstructing ECG-correlated, which were the basis for modern cardiac CT,² is highly impressive and should stimulate further research on motion-correlated reconstruction.

Although the studies have been presented mainly for $t_{\text{rot}} = 0.5$ s, there is no loss of generality since our results scale according to the rotation time of the scanner. Assume a scanner with t'_{rot} , then the results must be simply scaled according to $f'_H = f_H t_{\text{rot}} / t'_{\text{rot}}$. Consequently, CT scanners with shorter rotation times allow us to detect cardiac motion for heart rates above 120 min^{-1} as well. This is the case with our CBCT scanner that shows the resonance at 143 min^{-1} .

For most cases, comparable image quality has been achieved with both phase-correlated approaches. There may be cases (that have not occurred in our study) where the kymogram approach fails due to severe arrhythmia of the patient which would violate the assumption of slowly varying heart rate. But there are also cases where the kymogram is superior to the ECG. In particular, the case of misinterpreted *T* waves which happened in one of 25 consecutive cardiac patients yielded superior images with the kymogram approach and where the kymogram gave the hint for detecting the ECG monitor's failure.

To summarize, the novel approach of motion detection appears to be a viable alternative to ECG-based reconstruction. It is possible to use the kymogram to substitute the ECG information but it also seems of high value to have both approaches available since potential errors (that may occur for the ECG but also for the kymogram) may be compensated thus improving the overall image quality. A potential disadvantage is that the kymogram cannot be used for prospective triggering or dose modulation. Prospective methods require the prediction of cardiac motion at least 180° in advance and the kymogram algorithm consists of signal processing steps that involve more than 720° of data. Hence, the kymogram detection is not as localized as a prospective operation would require.

An important additional point is that the kymogram technique may be useful for thoracic and lung imaging since it allows for motion compensation not only in the heart but also in pericardial lung regions which often suffer from blurring of diagnostically important details. To fortify this suggestion we have used *standard* thorax data with 0.5 s rotation and with a pitch of 0.5 acquired with a 16-slice CT scanner to perform both a standard and a kymogram-based ASSR CI reconstruction. The significant improvements obtained with the new method are demonstrated in Fig. 16. A pitch value lower than one is the only precondition that must be satisfied

for this lung imaging technique. Due to the high volume coverage of modern cone-beam scanners this means no restriction to clinical routine.

ACKNOWLEDGMENTS

This work was supported by grant AZ 262/98 by "Bayerische Forschungsförderung, D-80333 München, Germany." We thank Dr. Stephan Achenbach, Dr. Ulrich Baum, Dr. Michael Lell, and Dr. Dieter Ropers who carried out the patient studies for a very efficient and pleasant cooperation.

^aElectronic mail: marc.kachelriess@imp.uni-erlangen.de

¹W. Stanford, B. H. Thompson, and R. M. Weiss, "Coronary artery calcification: Clinical significance and current methods of detection," *Am. J. Roentgenol.* **161**, 1139–1146 (1993).

²M. Kachelrieß and Willi A. Kalender, "Electrocardiogram-correlated image reconstruction from subsecond spiral computed tomography scans of the heart," *Med. Phys.* **25**, 2417–2431 (1998).

³M. Kachelrieß, S. Ulzheimer, and W. A. Kalender, "ECG-correlated image reconstruction from subsecond multi-slice spiral CT scans of the heart," *Med. Phys.* **27**, 1881–1902 (2000).

⁴M. Kachelrieß, S. Ulzheimer, and W. A. Kalender, "ECG-correlated imaging of the heart with subsecond multislice CT," *IEEE Trans. Med. Imaging* **19**, 888–901 (2000).

⁵B. Ohnesorge, T. Flohr, C. Becker, A. Kopp, U. Schöpf, U. Baum, A. Knez, K. Klingenberg-Regn, and M. Reiser, "Cardiac imaging by means of electrocardiographically gated multislice spiral CT: initial experience," *Radiology* **217**, 564–571 (2000).

⁶S. Ulzheimer, Ph.D. thesis, Friedrich–Alexander–Universität Erlangen–Nürnberg, 2001.

⁷T. Pan and S. Yun, "Cardiac CT with variable gantry speeds and multi-sector reconstruction," *Radiology* **217**, 438 (2000).

⁸T. Flohr, B. Ohnesorge, C. Becker, A. Kopp, S. Halliburton, and A. Knez, "A reconstruction concept for ECG-gated multi-slice spiral CT of the heart with pulse-rate adaptive optimization of spatial and temporal resolution," *Radiology* **217**, 438 (2000).

⁹M. Kachelrieß and W. Kalender, "Computertomograph mit objektbezogener Bewegungsartefaktreduktion und Extraktion der Objektbewegungsinformation (Kymogramm)," European Patent Office, 1999, Patent Specification EP 1 061 474 A1.

¹⁰S. Achenbach, U. Baum, S. Ulzheimer, M. Kachelrieß, D. Ropers, T. Giesler, W. Bautz, W. G. Daniel, W. A. Kalender, and W. Moshage, "Visualization of the coronary artery lumen by contrast-enhanced multi-detector spiral CT with retrospective ECG gating," *Circulation* **102**, II–398 (2000).

¹¹S. Achenbach, S. Ulzheimer, U. Baum, M. Kachelrieß, D. Ropers, T. Giesler, W. Bautz, W. G. Daniel, W. A. Kalender, and W. Moshage, "Noninvasive coronary angiography by retrospectively ECG-gated multislice spiral CT," *Circulation* **102**, 2823–2828 (2000).

¹²M. Kachelrieß, T. Fuchs, R. Lapp, D. Alexander Sennst, S. Schaller, and W. Kalender, "Image to volume weighting generalized ASSR for arbitrary pitch 3D and phase-correlated 4D spiral cone-beam CT reconstruction," *Proceeding of the 6th International Meeting on Fully 3D Image Reconstruction*, pp. 179–182, 2001.

¹³S. G. Azevedo, D. J. Schneberk, P. J. Fitch, and H. E. Martz, "Calculation of the rotational centers in computed tomography sinograms," *IEEE Trans. Nucl. Sci.* **37**, 1525–1540 (1990).