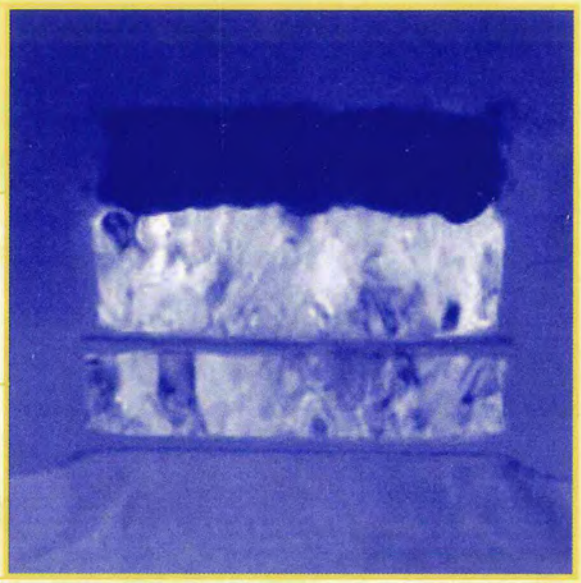
SEMICONDUCTOR DEVICES

Physics and Technology



2nd Edition

S.M. Sze

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2ND EDITION

Semiconductor Devices

Physics and Technology

S. M. SZE
UMC Chair Professor
National Chiao Tung University
National Nano Device Laboratories
Hsinchu, Taiwan



JOHN WILEY & SONS, INC.

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Cover Photography: A transmission-electron micrograph of a floating-gate nonvolatile semiconductor memory with a magnification of 100,000 times. (Photography courtesy of George T. T. Sheng.) For a discussion of the device, see Chapters 1, 6, and 14.

This book was typeset in *New Caledonia* by *Argosy Publishing* and printed and bound by *R. R. Donnelley and Sons, Inc. (Willard)*. The cover was printed by *The Lehigh Press*.

The paper in this book was manufactured by a mill whose forest management programs include sustained yield harvesting of its timberlands. Sustained yield harvesting principles ensure that the number of trees cut each year does not exceed the amount of new growth.

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Library of Congress Cataloging in Publication Data:
Sze, S. M., 1936-
Semiconductor devices, physics and technology/S.M. Sze.—2nd ed.
p. cm.
Includes bibliographical references and index.
ISBN 0-471-33372-7 (cloth: alk. paper)
1. Semiconductors. I. Title.
TK7871.85 .S9883 2001

621.3815'2—dc21

2001026003

ISBN 0-471-33372-7

Printed in the United States of America

10 9 8 7 6 5 4

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In Memory of My Mentors

Dr. L. J. Chu *Academia Sinica*

Dr. R. M. Ryder *Bell Laboratories*

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Preface

The book is an introduction to the physical principles of modern semiconductor devices and their advanced fabrication technology. It is intended as a textbook for undergraduate students in applied physics, electrical and electronics engineering, and materials science. It can also serve as a reference for practicing engineers and scientists who need an update on device and technology developments.

WHAT'S NEW IN THE SECOND EDITION

- 50% of the material has been revised or updated. We have added many sections that are of contemporary interest such as flash memory, Pentium chips, copper metallization, and eximer-laser lithography. On the other hand, we have omitted or reduced sections of less important topics to maintain the overall book length.
- We have also made substantial changes in updating the pedagogy. We have adopted a two-color format for all illustrations to enhance their presentation; and all important equations are boxed.
- All device and material parameters have been updated or corrected . For example, the intrinsic carrier concentration in silicon at 300K is 9.65×10^9 cm^{-3}, replacing the old value of 1.45×10^{10} cm^{-3}. This single change has an impact on at least 30% of the problem solutions.
- To improve the development of each subject, sections that contain graduate-level mathematics or physical concepts have been omitted or moved to the Appendixes, at the back of the book.

TOPICAL COVERAGE

- Chapter 1 gives a brief historical review of major semiconductor devices and key technology developments. The text is then organized into three parts.
- Part I, Chapters 2–3, describes the basic properties of semiconductors and their conduction processes, with special emphasis on the two most important semiconductors: silicon (Si) and gallium arsenide (GaAs). The concepts in Part I will be used throughout this book. These concepts requires a background knowledge of modern physics and college calculus.
- Part II, Chapters 4–9, discusses the physics and characteristics of all major semiconductor devices. We begin with the p–n junction which is the key building block of most semiconductor devices. We proceed to bipolar and field-effect devices and then cover microwave, quantum-effect, hot-electron, and photonic devices.
- Part III, Chapters 10–14, deals with processing technology from crystal growth to impurity doping. We present the theoretical and practical aspects of the major steps in device fabrication with an emphasis on integrated devices.

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KEY FEATURES

Each chapter includes the following features:

- The chapter starts with an overview of the topical contents. A list of learning goals is also provided.
- The second edition has tripled the worked-out examples that apply basic concepts to specific problems.
- A chapter summary appears at the end of each chapter to summarize the important concepts and to help the student review the content before tackling the homework problems that follow.
- The book includes about 250 homework problems, over 50% of them new to the second edition. Answers to odd-numbered problems, which have numerical solutions are provided in Appendix L at the back of the book.

COURSE DESIGN OPTIONS

The second edition can provide greater flexibility in course design. The book contains enough material for a full-year sequence in device physics and processing technology. Assuming three lectures per week, a two-semester sequence can cover Chapters 1–7 in the first semester, leaving Chapters 8–14 for the second semester. For a three-quarter sequence, the logical break points are Chapters 1–5, Chapters 6-9, and Chapters 10–14.

A two-quarter sequence can cover Chapters 1–5 in the first quarter. The instructor has several options for the second quarter. For example, covering Chapters 6, 11, 12, 13, and 14 produces a strong emphasis on the MOSFET and its related process technologies, while covering Chapters 6–9 emphasizes all major devices. For a one-quarter course on semiconductor device processing, the instructor can cover Section 1.2 and Chapters 10–14.

A one-semester course on basic semiconductor physics and devices can cover Chapters 1–7. A one-semester course on microwave and photonic devices can cover Chapters 1–4, 7–9. If the students already have some familiarity with semiconductor fundamentals, a one-semester course on Submicron MOSFET: Physics and Technology can cover Chapters 1, 6, 10–14. Of course, there are many other course design options depending on the teaching schedule and the instructor's choice of topics.

TEXTBOOK SUPPLEMENTS

- Instructor's Manual. A complete set of detailed solutions to all the end-of-chapter problems has been prepared. These solutions are available free to all adopting faculty.
- The figures used in the text are available, in electronic format, to instructors from the publisher. Instructors can find out more information at the publisher's website at: http: //www.wiley.com/college/sze

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Acknowledgments

Many people have assisted me in revising this book. I would first like to express my deep appreciation to my colleagues at the National Nano Device Laboratories for their contributions in improving the content of the text, in suggesting state-of-the-art illustrations, and in providing homework problems and solutions: Dr. S. F. Hu on Chapter 2, Dr. W. F. Wu on Chapter 3, Dr. S. H. Chan on Chapter 4, Dr. T. B. Chiou on Chapter 5, Dr. H. C. Lin on Chapter 6, Dr. J. S. Tsang on Chapter 7, Dr. G. W. Huang on Chapter 8, Dr. J. D. Guo on Chapter 9, Dr. S. C. Wu on Chapter 10, Dr. T. C. Chang on Chapter 11, Drs. M. C. Liaw and M. C. Chiang on Chapter 12, Dr. F. H. Ko on Chapter 13, and Dr. T. S. Chao on Chapter 14.

I have benefited significantly from suggestions made by the reviewers: Profs. C. Y. Chang, C. H. Chen, T. Y. Huang, B. Y. Tsui, and T. J. Yang of the National Chiao Tung University, Profs. Y. C. Chen and M. K. Lee of the National Sun Yet-sen University, Prof. C. S. Lai of the Chang Gung University, Prof. W. Y. Liang of the University of Cambridge, Dr. K. K. Ng of Bell Laboratories, Lucent Technologies, Prof. W. J. Tseng of the National Cheng Kung University, Prof. T. C. Wei of the Chung Yuan University, Prof. Y. S. G. Wu of the National Tsing Hwa University, Dr. C. C. Yang of the National Nano Device Laboratories, Prof. W. L. Yang of Feng Chia University, and Dr. A. Yen of the Taiwan Semiconductor Manufacturing Company.

I am further indebted to Mr. N. Erdos for technical editing of the manuscript, Ms. Iris Lin for typing the many revisions of the draft and the final manuscript, and Ms. Y. G. Yang of the Semiconductor Laboratory, the National Chiao Tung University who furnished the hundreds of technical illustrations used in the book. In each case where an illustration was used from another published source, I have received permission from the copyright holder. Even through all illustrations were then adopted and redrawn, I appreciate being granted these permissions. I wish to thank Mr. George T. T. Sheng of the Macronix International Company for providing the flash-memory transmission electron micrography, which is shown in the cover design. I wish also to thank Mr. A. Mutlu, Mr. S. Short, and Ms. R. Steward of Intel Corporation for providing the photographs of the first microprocessor (Intel 4004) and its latest version (Pentium 4).

At John Wiley and Sons, I wish to thank Mr. G. Telecki and Mr. W. Zobrist who encouraged me to undertake the project. I wish also to acknowledge the Spring Foundation of the National Chiao Tung University for the financial support. I would especially like to thank the United Microelectronics Corporation (UMC), Taiwan, ROC, for the UMC Chair Professorship grant that provided the environment to work on this book.

Finally, I am grateful to my wife Therese for her continued support and assistance in this and many previous book projects. I also like to thank my son Raymond (Doctor of Medicine) and my daughter-in-law Karen (Doctor of Medicine), and my daughter Julia (Certified Financial Analyst) and my son-in-law Bob (President, Cameron Global Investment, LLC), who have helped me in their capacities as my medical advisors and financial advisors, respectively.

S. M. Sze
Hsinchu, Taiwan
March 2001

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CHAPTER 1

Introduction

- 1.1 SEMICONDUCTOR DEVICES
- 1.2 SEMICONDUCTOR TECHNOLOGY
- SUMMARY

As an undergraduate in applied physics, electrical engineering, electronics engineering, or materials science, you might ask why you need to study semiconductor devices. The reason is that semiconductor devices are the foundation of the electronic industry, which is the largest industry in the world with global sales over one trillion dollars since 1998. A basic knowledge of semiconductor devices is essential to the understanding of advanced courses in electronics. This knowledge will also enable you to contribute to the Information Age, which is based on electronic technology.

Specifically, we cover the following topics:

- Four building blocks of semiconductor devices.
- Eighteen important semiconductor devices and their roles in electronic applications.
- Twenty important semiconductor technologies and their roles in device processing.
- Technology trends toward high-density, high-speed, low-power consumption, and nonvolatility.

1.1 SEMICONDUCTOR DEVICES

Figure 1 shows the sales volume of the semiconductor-device–based electronic industry in the past 20 years and projects sales to the year 2010. Also shown are the gross world product (GWP) and the sales volumes of automobile, steel, and semiconductor industries.[1,2] We note that the electronic industry has surpassed the automobile industry in 1998. If the current trends continue, in year 2010 the sales volume of the electronic industry will reach three trillion dollars and will constitute about 10% of GWP. The semiconductor industry, which is a subset of the electronic industry, will grow at an even higher rate to surpass the steel industry in the early twenty-first century and to constitute 25% of the electronic industry in 2010.

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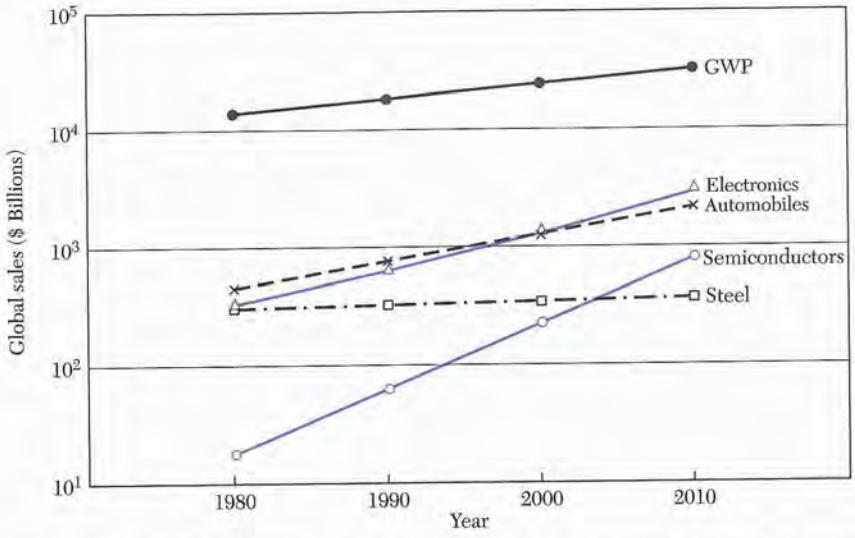


Fig. 1 Gross world product (GWP) and sales volumes of the electronics, automobile, semiconductor, and steel industries from 1980 to 2000 and projected to 2010.[1,2]

1.1.1 Device Building Blocks

Semiconductor devices have been studied for over 125 years.[3] To date, we have about 60 major devices, with over 100 device variations related to them.[4] However, all these devices can be constructed from a small number of device building blocks.

Figure 2*a* is the metal-semiconductor interface, which is an intimate contact between a metal and a semiconductor. This building block was the first semiconductor device ever studied (in the year 1874). This interface can be used as a rectifying contact, that is, the device allows electrical current to flow easily only in one direction, or as an ohmic contact, which can pass current in either direction with a negligibly small voltage drop. We can use this interface to form many useful devices. For example, by using a rectifying contact as the *gate** and two ohmic contacts as the *source* and *drain*, we can form a *MESFET* (metal-semiconductor field-effect transistor), an important microwave device.

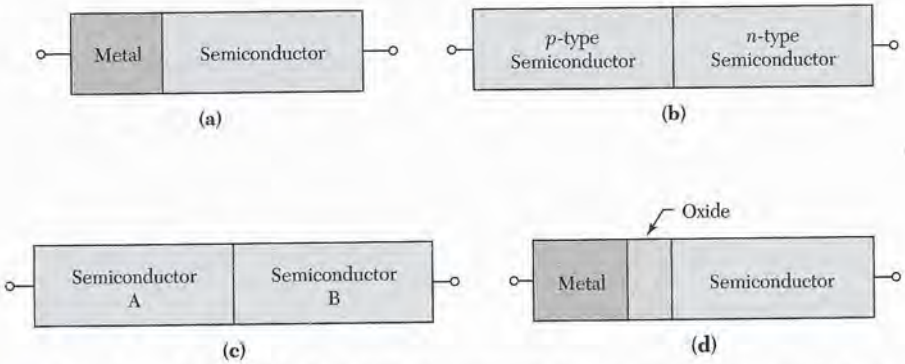


Fig. 2 Basic device building blocks. (*a*) Metal-semiconductor interface; (*b*) *p*–*n* junction; (*c*) heterojunction interface; and (*d*) metal-oxide-semiconductor structure.

* The italicized terms in this paragraph and in subsequent paragraphs are defined and explained in Part II of the book.

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The second building block is the *p–n* junction (Fig. 2*b*), which is formed between a *p- type* (with positively charged carriers) and an *n-type* (with negatively charged carriers) semiconductors. The *p–n* junction is a key building block for most semiconductor devices, and *p–n* junction theory serves as the foundation of the physics of semiconductor devices. By combining two *p–n* junctions, that is, by adding another *p*-type semiconductor, we form the *p–n–p bipolar transistor*, which was invented in 1947 and had an unprecedented impact on the electronic industry. If we combine three *p–n* junctions to form a *p–n–p–n* structure, it is a switching device called a *thyristor*.

The third building block (Fig. 2*c*) is the heterojunction interface, that is, an interface formed between two dissimilar semiconductors. For example, we can use *gallium arsenide* (GaAs) and *aluminum arsenide* (AlAs) to form a heterojunction. Heterojunctions are the key components for high-speed and photonic devices.

Figure 2*d* shows the metal-oxide-semiconductor (MOS) structure. The structure can be considered a combination of a metal-oxide interface and an oxide-semiconductor interface. By using the MOS structure as the gate and two *p–n* junctions as the source and drain, we can form a *MOSFET* (MOS field-effect transistor). The MOSFET is the most important device for advanced *integrated circuits*, which contains tens of thousands of devices per integrated circuit chip.

1.1.2 Major Semiconductor Devices

Some major semiconductor devices are listed in Table 1 in chronological order; those with a superscript *b* are two-terminal devices, otherwise they are three-terminal or four-terminal devices.[3] The earliest systematic study of semiconductor devices (metal-semiconductor contacts) is generally attributed to Braun,[5] who in 1874 discovered that the resistance of contacts between metals and metal sulfides (e.g., copper pyrite) depended on the magnitude and polarity of the applied voltage. The electroluminescence phenomenon (for the *light-emitting diode*) was discovered by Round[6] in 1907. He observed the generation of yellowish light from a crystal of carborundom when he applied a potential of 10 V between two points on the crystals.

TABLE 1 Major Semiconductor Devices

Year	Semiconductor Device[a]	Author(s)/Inventor(s)	Ref.
1874	Metal-semiconductor contact[b]	Braun	5
1907	Light emitting diode[b]	Round	6
1947	Bipolar transistor	Bardeen, Brattain, and Shockley	7
1949	*p–n* junction[b]	Shockley	8
1952	Thyristor	Ebers	9
1954	Solar cell[b]	Chapin, Fuller, and Pearson	10
1957	Heterojunction bipolar transistor	Kroemer	11
1958	Tunnel diode[b]	Esaki	12
1960	MOSFET	Kahng and Atalla	13
1962	Laser[b]	Hall et al	15
1963	Heterostructure laser[b]	Kroemer, Alferov and Kazarinov	16,17
1963	Transferred-electron diode[b]	Gunn	18
1965	IMPATT diode[b]	Johnston, DeLoach, and Cohen	19

(continued)

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TABLE 1 *(continued)*

Year	Semiconductor Device[a]	Author(s)/Inventor(s)	Ref.
1966	MESFET	Mead	20
1967	Nonvolatile semiconductor memory	Kahng and Sze	21
1970	Charge-coupled device	Boyle and Smith	23
1974	Resonant tunneling diode[b]	Chang, Esaki, and Tsu	24
1980	MODFET	Mimura et al.	25
1994	Room-temperature single-electron memory cell	Yano et al.	22
2001	20 nm MOSFET	Chau	14

[a]MOSFET, metal-oxide-semiconductor field-effect transistor; MESFET, metal-semiconductor field-effect transistor; MODFET, modulation-doped field-effect transistor.

[b]Denotes a two-terminal device, otherwise it is a three- or four-terminal device.

In 1947, the point-contact transistor was invented by Bardeen and Brattain.[7] This was followed by Shockley's[8] classic paper on *p–n* junction and bipolar transistor in 1949. Figure 3 shows the first transistor. The two point contacts at the bottom of the triangular quartz crystal were made from two stripes of gold foil separated by about 50 μm (1μm = 10^{-4} cm) and pressed onto a semiconductor surface. The semiconductor used was germanium. With one gold contact forward biased, that is, positive voltage with respect to the third terminal, and the other reverse biased, the *transistor action* was observed, that is, the input signal was amplified. The bipolar transistor is a key semiconductor device and has ushered in the modern electronic era.

In 1952, Ebers[9] developed the basic model for the thyristor, which is an extremely versatile switching device. The *solar cell* was developed by Chapin, et al.[10] in 1954 using a silicon *p–n* junction. The solar cell is a major candidate for obtaining energy from the sun because it can convert sunlight directly to electricity and is environmentally benign. In 1957, Kroemer[11] proposed the heterojunction bipolar transistor to improve the transistor performance; this device is potentially one of the fastest semiconductor devices. In 1958, Esaki[12] observed negative resistance characteristics in a heavily doped *p–n* junction, which led to the discovery of the *tunnel diode*. The tunnel diode and its associated tunneling phenomenon are important for ohmic contacts and carrier transport through thin layers.

The most important device for advanced integrated circuits is the MOSFET, which was reported by Kahng and Atalla[13] in 1960. Figure 4 shows the first device using a thermally oxidized silicon substrate. The device has a gate length of 20 μm and a gate oxide thickness of 100 nm (1 nm = 10^{-7} cm). The two keyholes are the source and drain contacts, and the top elongated area is the aluminum gate evaporated through a metal mask. Although present-day MOSFETs have been scaled down to the deep-submicron regime, the choice of silicon and thermally grown silicon dioxide used in the first MOSFET remains the most important combination of materials. The MOSFET and its related integrated circuits now constitute about 90% of the semiconductor device market. An ultrasmall MOSFET with a channel length of 20 nm has been demonstrated recently.[14] This device can serve as the basis for the most advanced integrated circuit chips containing over one trillion ($>10^{12}$) devices.

In 1962, Hall et al.[15] first achieved lasing in semiconductors. In 1963, Kroemer[16] and Alferov and Kazarinov[17] proposed the *heterostructure laser*. These proposals laid the foundation for modern laser diodes, which can be operated continuously at room temperature. Laser diodes are the key components for a wide range of applications, including

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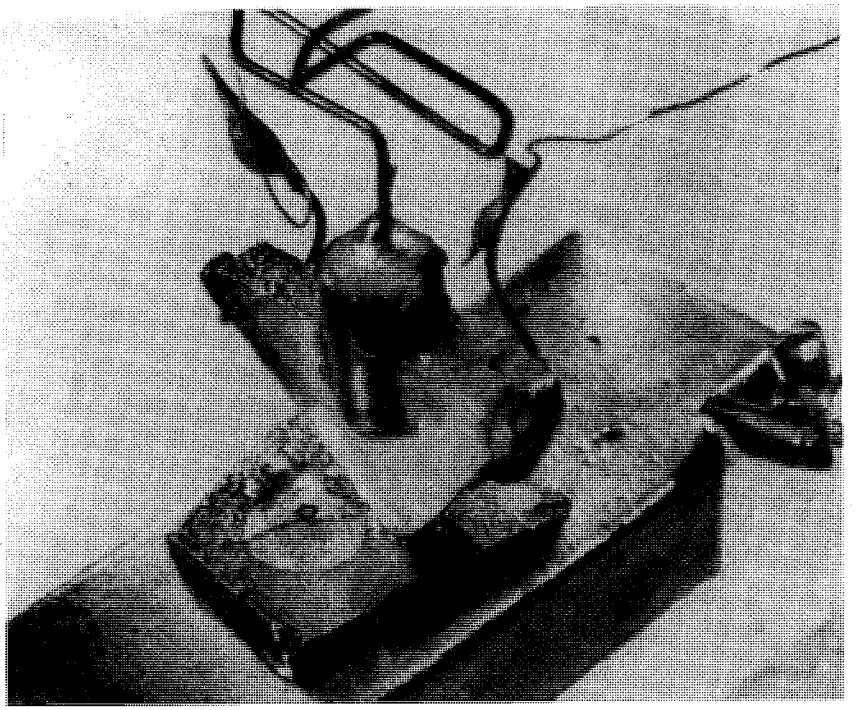


Fig. 3 The first transistor.[7] (Photograph courtesy of Bell Laboratories.)



Fig. 4 The first metal-oxide-semiconductor field-effect transistor.[13] (Photograph courtesy of Bell Laboratories.)

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digital video disk, optical-fiber communication, laser printing, and atmospheric–pollution monitoring.

Three important microwave devices were invented or realized in the next 3 years. The first device is the *transferred-electron diode* (TED; also called Gunn diode) by Gunn[18] in 1963. The TED is used extensively in such millimeter-wave applications as detection systems, remote controls, and microwave test instruments. The second device is the *IMPATT diode*; its operation was first observed by Johnston et al.[19] in 1965. IMPATT diodes can generate the highest continuous wave (CW) power at millimeter-wave frequencies of all semiconductor devices. They are used in radar systems and alarm systems. The third device is the MESFET, invented by Mead[20] in 1966. It is a key device for monolithic microwave integrated circuits (MMIC).

An important semiconductor memory device was invented by Kahng and Sze[21] in 1967. This is the *nonvolatile semiconductor memory* (NVSM), which can retain its stored information when the power supply is switched off. A schematic diagram of the first NVSM is shown in Fig.5*a*. Although it is similar to a conventional MOSFET, the major difference is the addition of the *floating gate*, in which semipermanent charge storage is possible. Because of its attributes of nonvolatility, high device density, low-power consumption, and electrical rewritability (e.g., the stored charge can be removed by applying voltage to the control gate), NVSM has become the dominant memory for portable electronic systems such as the cellular phone, notebook computer, digital camera, and smart card.

A limiting case of the floating-gate nonvolatile memory is the *single-electron memory cell* (SEMC) shown in Fig.5*b*. By reducing the length of the floating gate to ultra-

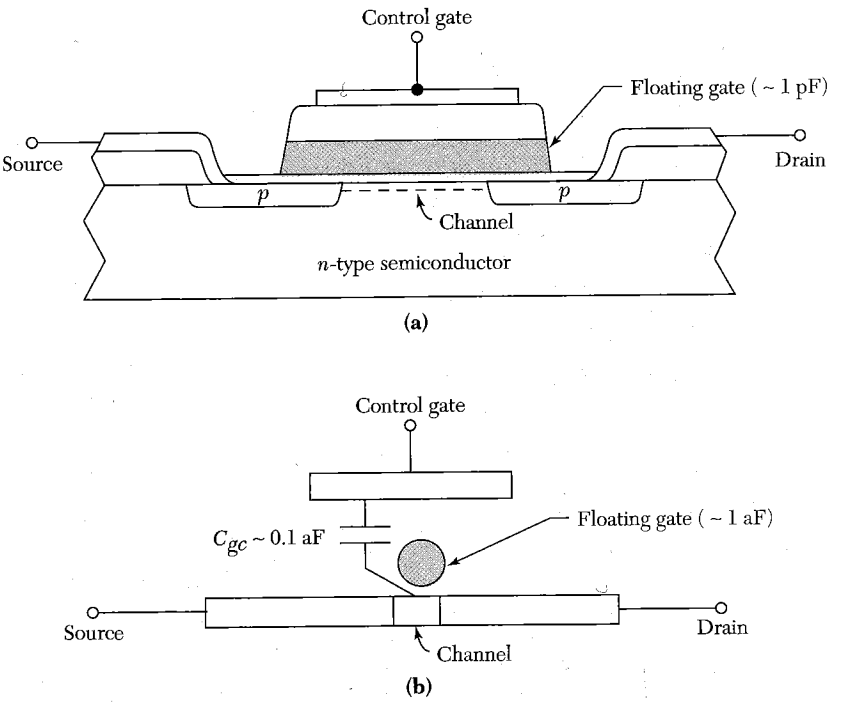


Fig. 5 (*a*) A schematic diagram of the first nonvolatile semiconductor memory (NVSM) with a floating gate.[21] (*b*) A limiting case of the floating-gate NVSM—the single-electron memory cell.[22]

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small dimensions (e.g., 10 nm), we obtain the SEMC. At this dimension, when an electron moves into the floating gate, the potential of the gate will be altered so that it will prevent the entrance of another electron. The SEMC is an ultimate floating-gate memory cell, since we need only one electron for information storage. The operation of a SEMC at room temperature was first demonstrated by Yano et al.[22] in 1994. The SEMC can serve as the basis for the most advanced semiconductor memories that can contain over one trillion bits.

The *charge-coupled device* (CCD) was invented by Boyle and Smith[23] in 1970. CCD is used extensively in video cameras and in optical sensing applications. The resonant tunneling diode (RTD) was first studied by Chang et al.[24] in 1974. RTD is the basis for most quantum-effect devices, which offer extremely high density, ultrahigh speed, and enhanced functionality because it permits a greatly reduced number of devices to perform a given circuit function. In 1980, Minura et al.[25] developed the *MODFET* (modulation-doped field-effect transistor). With the proper selection of heterojunction materials, the MODFET is expected to be the fastest field-effect transistor.

Since the invention of the bipolar transistor in 1947, the number and variety of semiconductor devices have increased tremendously as advanced technology, new materials, and broadened comprehension have been applied to the creation of new devices. In Part II of the book, we consider all the devices listed in Table 1. It is hoped that this book can serve as a basis for understanding other devices not included here and perhaps not even conceived of at the present time.

1.2 SEMICONDUCTOR TECHNOLOGY

1.2.1 Key Semiconductor Technologies

Many important semiconductor technologies have been derived from processes invented centuries ago. For example, the lithography process was invented in 1798; in this first process, the pattern, or image, was transferred from a stone plate (litho).[26] In this section, we consider the milestones of technologies that were applied for the first time to semiconductor processing or developed specifically for semiconductor-device fabrication.

Some key semiconductor technologies are listed in Table 2 in chronological order. In 1918, Czochralski[27] developed a liquid-solid monocomponent growth technique. The Czochralski growth is the process used to grow most of the crystals from which silicon wafers are produced. Another growth technique was developed by Bridgman[28] in 1925. The Bridgman technique has been used extensively for the growth of gallium arsenide and related compound semiconductor crystals. Although the semiconductor properties of silicon have been widely studied since early 1940, the study of semiconductor compounds was neglected for a long time. In 1952, Welker[29] noted that gallium arsenide and its related III–V compounds were semiconductors. He was able to predict their characteristics and to prove them experimentally. The technology and devices of these compounds have since been actively studied.

The diffusion of impurity atoms in semiconductors is important for device processing. The basic diffusion theory was considered by Fick[30] in 1855. The idea of using diffusion techniques to alter the type of conductivity in silicon was disclosed in a patent in 1952 by Pfann.[31] In 1957, the ancient lithography process was applied to semiconductor-device fabrication by Andrus.[32] He used photosensitive etch-resistant polymers (photoresist) for pattern transfer. Lithography is a key technology for the semiconductor industry. The continued growth of the industry has been the direct result of improved lithographic technology. Lithography is also a significant economic factor, currently representing over 35% of the integrated-circuit manufacturing cost.

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TABLE 2 Key Semiconductor Technologies

Year	Technology[a]	Author(s)/Inventor(s)	Ref.
1918	Czochralski crystal growth	Czochralski	27
1925	Bridgman crystal growth	Bridgman	28
1952	III-V compounds	Welker	29
1952	Diffusion	Pfann	31
1957	Lithographic photoresist	Andrus	32
1957	Oxide masking	Frosch and Derrick	33
1957	Epitaxial CVD growth	Sheftal, Kokorish, and Krasilov	34
1958	Ion implantation	Shockley	35
1959	Hybrid integrated circuit	Kilby	36
1959	Monolithic integrated circuit	Noyce	37
1960	Planar process	Hoerni	38
1963	CMOS	Wanlass and Sah	39
1967	DRAM	Dennard	40
1969	Polysilicon self-aligned gate	Kerwin, Klein, and Sarace	41
1969	MOCVD	Manasevit and Simpson	42
1971	Dry etching	Irving, Lemons, and Bobos	43
1971	Molecular beam epitaxy	Cho	44
1971	Microprocessor (4004)	Hoff et al.	45
1982	Trench isolation	Rung, Momose, and Nagakubo	46
1989	Chemical mechanical polishing	Davari et al.	47
1993	Copper interconnect	Paraszczak et al.	48

[a] CVD, chemical vapor deposition; CMOS, complementary metal-oxide-semiconductor field-effect transistor; DRAM, dynamic random access memory; MOCVD, metalorganic CVD.

The oxide masking method was developed by Frosch and Derrick[33] in 1957. They found that an oxide layer can prevent most impurity atoms from diffusing through it. In the same year, the epitaxial growth process based on chemical vapor deposition technique was developed by Sheftal et al.[34] Epitaxy, derived from the Greek word epi, meaning on, and taxis, meaning arrangement, describes a technique of crystal growth to form a thin layer of semiconductor materials on the surface of a crystal that has a lattice structure identical to that of the crystal. This method is important for the improvement of device performance and the creation of novel device structures.

In 1958, Shockley[35] proposed the method of using ion implantation to dope the semiconductors. Ion implantation has the capability of precisely controlling the number of implanted dopant atoms. Diffusion and ion implantation can complement each other for impurity doping. For example, diffusion can be used for high-temperature, deep-junction processes, whereas ion implantation can be used for lower-temperature, shallow-junction processes.

In 1959, a rudimentary integrated circuit (IC) was made by Kilby.[36] It contained one bipolar transistor, three resistors, and one capacitor, all made in germanium and connected by wire bonding—a hybrid circuit. Also in 1959, Noyce[37] proposed the monolithic IC by fabricating all devices in a single semiconductor substrate (monolith means single stone) and connecting the devices by aluminum metallization. Figure 6 shows the first monolithic IC of a flip-flop circuit containing six devices. The aluminum interconnection lines were obtained by etching evaporated aluminum layer over the entire oxide surface using the lithographic technique. These inventions laid the foundation for the rapid growth of the microelectronics industry.

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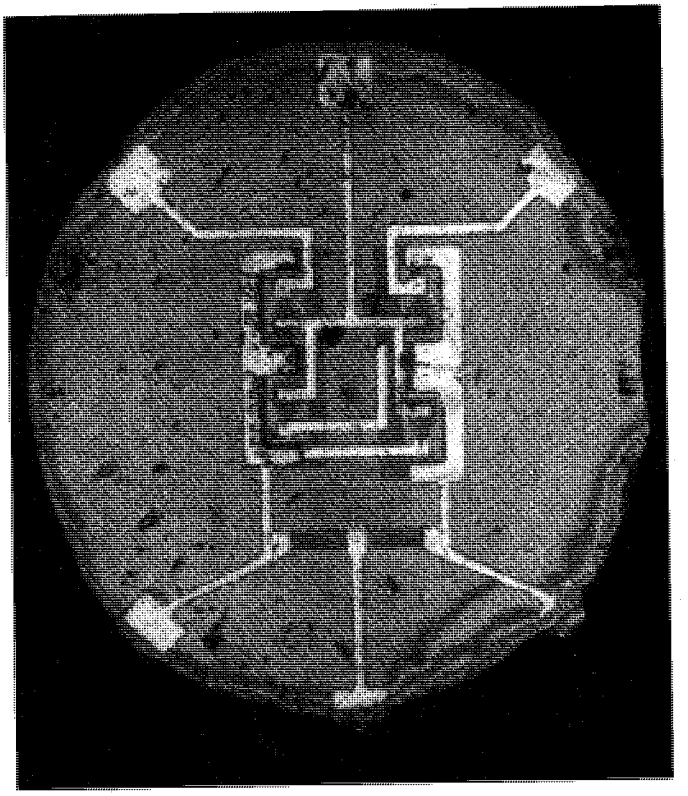


Fig. 6 The first monolithic integrated circuit.[37] (Photograph courtesy of Dr. G. Moore.)

The "planar" process was developed by Hoerni[38] in 1960. In this process, an oxide layer is formed on a semiconductor surface. With the help of a lithography process, portions of the oxide can be removed and windows cut in the oxide. Impurity atoms will diffuse only through the exposed semiconductor surface, and p–n junctions will form in the oxide window areas.

As the complexity of the IC increased, we have moved from *NMOS* (n-channel MOSFET) to *CMOS* (complementary MOSFET) technology, which employs both NMOS and *PMOS* (p-channel MOSFET) to form the logic elements. The CMOS concept was proposed by Wanlass and Sah [39] in 1963. The advantage of CMOS technology is that logic elements draw significant current only during the transition from one state to another (e.g., from 0 to 1) and draw very little current between transitions, allowing power consumption to be minimized. CMOS technology is the dominant technology for advanced ICs.

In 1967, an important two-element circuit, the dynamic random access memory (DRAM), was invented by Dennard.[40] The memory cell contains one MOSFET and one charge-storage capacitor. The MOSFET serves as a switch to charge or discharge the capacitor. Although DRAM is volatile and consumes relatively high power, we expect that DRAM will continue to be the first choice among various semiconductor memories for nonportable electronic systems in the foreseeable future.

To improve the device performance, the polysilicon self-aligned gate process was proposed by Kerwin et al.[41] in 1969. This process not only improved device reliability, it also reduced parasitic capacitances. Also in 1969, the metalorganic chemical vapor deposition

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(MOCVD) method was developed by Manasevit and Simpson.[42] This is a very important epitaxial growth technique for compound semiconductors such as GaAs.

As the device dimensions were reduced, a dry etching technique was developed to replace wet chemical etching for high-fidelity pattern transfer. This technique was initiated by Irving et al.[43] in 1971 using a $CF_4 - O_2$ gas mixture to etch silicon wafers. Another important technique developed in the same year is molecular beam epitaxy by Cho.[44] This technique has the advantage of near-perfect vertical control of composition and doping down to atomic dimensions. It is responsible for the creation of numerous photonic devices and quantum-effect devices.

In 1971, the first microprocessor was made by Hoff et al.[45] They put the entire central processing unit (CPU) of a simple computer on one chip. It was a four-bit microprocessor (Intel 4004), shown in Fig. 7, with a chip size of 3 mm × 4 mm, and it contained 2300 MOSFETs. It was fabricated by a *p*-channel, polysilicon gate process using an 8 μm design rule. This microprocessor performed as well as those in $300,000 IBM computers of the early 1960s—each of which needed a CPU the size of a large desk. This was a major breakthrough for the semiconductor industry. Currently, microprocessors constitute the largest segment of the industry.

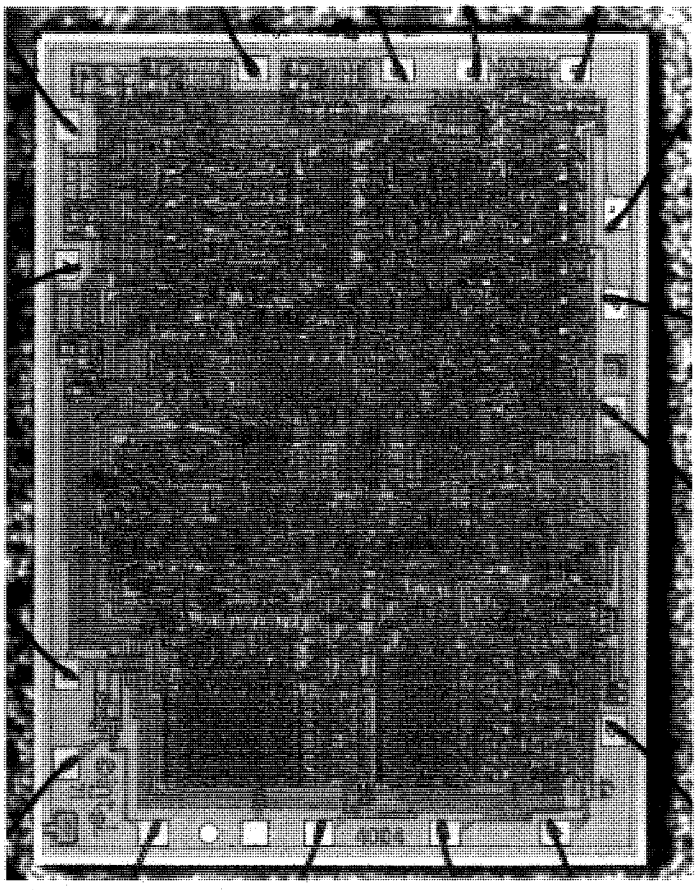


Fig. 7 The first microprocessor.[45] (Photograph courtesy of Intel Corp.)

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Since early 1980, many new technologies have been developed to meet the requirements of ever-shrinking minimum feature lengths. We consider three key technologies: trench isolation, chemical-mechanical polishing, and the copper interconnect. The trench isolation technology was introduced by Rung et al.[46] in 1982 to isolate CMOS devices. This approach eventually replaced all other isolation methods. In 1989, the chemical-mechanical polishing method was developed by Davari et al.[47] for global planarization of the interlayer dielectrics. This is a key process for multilevel metallization. At submicron dimensions, a widely known failure mechanism is electromigration, which is the transport of metal ions through a conductor due to the passage of an electrical current. Although aluminum has been used since the early 1960s as the interconnect material, it suffers from electromigration at high electrical current. The copper interconnect was introduced in 1993 by Paraszczak et al.[48] to replace aluminum for minimum feature lengths approaching 100 nm. In Part III of this book, we consider all the technologies listed in Table 2.

1.2.2 Technology Trends

Since the beginning of the microelectronics era, the smallest line width or the minimum feature length of an integrated circuit has been reduced at a rate of about 13% per year.[49] At that rate, the minimum feature length will shrink to about 50 nm in the year 2010. Device miniaturization results in reduced unit cost per circuit function. For example, the cost per bit of memory chips has halved every 2 years for successive generations of DRAMs. As device dimension decreases, the intrinsic switching time also decreases. The device speed has improved by four orders of magnitude since 1959. Higher speeds lead to expanded IC functional throughput rates. In the future, digital ICs will be able to perform date processing and numerical computation at terabit-per-second rates. As the device becomes smaller, it consumes less power. Therefore, device miniaturization also reduces the energy used for each switching operation. The energy dissipated per logic gate has decreased by over one million times since 1959.

Figure 8 shows the exponential increase of the actual DRAM density versus the year of first production from 1978 to 2000. The density increases by a factor of 2 every 18 months. If the trends continue, we expect that DRAM density will increase to 8 Gb in the year 2005 and to 64 Gb around the year 2012. Figure 9 shows the exponential increase of the microprocessor computational power. The computational power also increases by a factor of 2 every 18 months. Currently, a Pentium-based personal computer has the same computational power as that of a supercomputer, CRAY 1, of the late 1960s; yet, it is three orders of magnitude smaller. If the trends continue, we will reach 100 GIP (billion instructions per second) in the year 2010.

Figure 10 illustrates the growth curves for different technology drivers.[50] At the beginning of the modern electronic era (1950–1970), the bipolar transistor was the technology driver. From 1970 to 1990, the DRAM and the microprocessor based on MOS devices were the technology drivers because of the rapid growth of personal computers and advanced electronic systems. Since 1990, nonvolatile semiconductor memory has been the technology driver, mainly because of the rapid growth of portable electronic systems.

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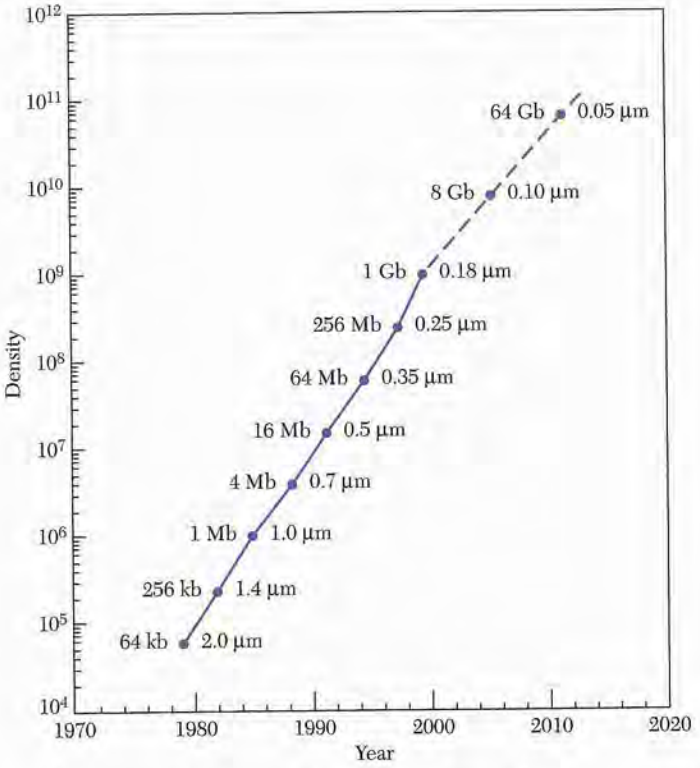


Fig. 8 Exponential increase of dynamic random access memory density versus year based on the Semiconductor Industry Association (SIA) roadmap.[49]

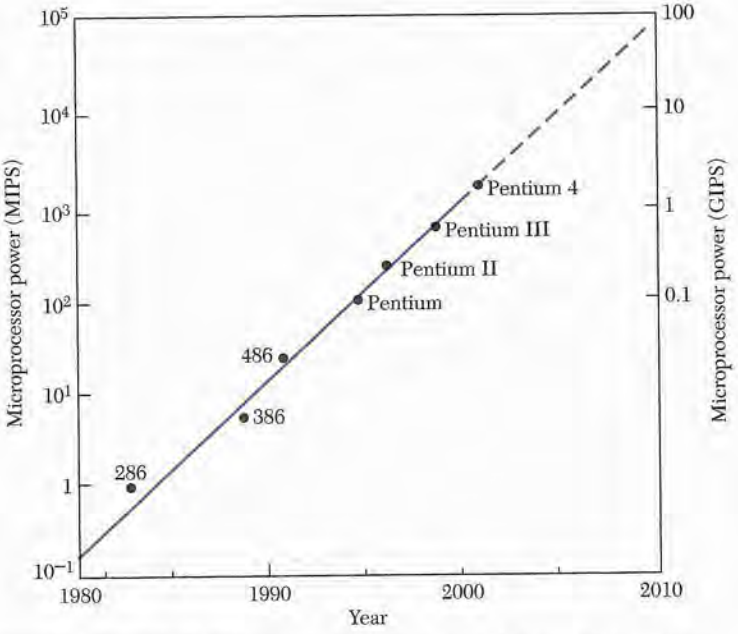


Fig. 9 Exponential increase of microprocessor computational power versus year.

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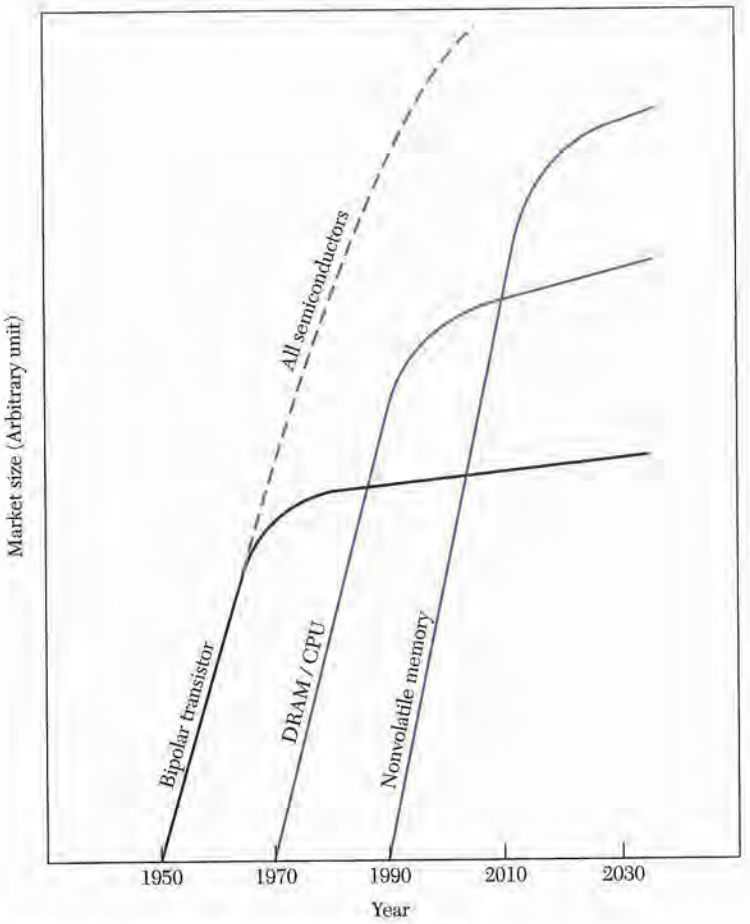


Fig. 10 Growth curves for different technology drivers.[50]

SUMMARY

Although the *semiconductor-device* field is a relatively new area of study,* it has enormous impact on our society and the global economy. This is because semiconductor devices serve as the foundation of the largest industry in the world—the electronic industry.

In this introductory chapter, we have presented a historical review of major semiconductor devices from the first study of metal-semiconductor contact in 1874 to the fabrication of an ultrasmall 20-nm MOSFET in 2001. Of particular importance are the invention of the bipolar transistor in 1947, which ushered in the modern electronic era; the development of the MOSFET in 1960, which is the most important device for integrated circuits; and the invention of the nonvolalite semiconductor memory in 1967, which has been the technology driver of the electronic industry since 1990.

* Semiconductor devices and materials have been studied since the early nineteenth century. However, many traditional devices and materials have been studied for a much longer time. For example, steel was first studied in 1200 BC (over 3000 years ago).

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We have also described key semiconductor technologies. The origins of many technologies can be traced back to the late eighteenth and early nineteenth centuries. Of particular importance are the development of the lithographic photoresist in 1957, which established the basic pattern-transfer process for semiconductor devices; the invention of the integrated circuits in 1959, which was seminal to the rapid growth of the microelectronic industry; and the developments of the DRAM in 1967 and the microprocessor in 1971, which constitute the two largest segments of the semiconductor industry.

We have a vast literature on semiconductor-device physics and technology.[51] To date, more than 300,000 papers have been published in this field, and the grand total may reach one million papers in the year 2012. In this book, each chapter deals with a major device or a key technology. Each is presented in a clear and coherent fashion without heavy reliance on the original literature. However, we have selected a few important papers at the end of each chapter for reference and for further reading.

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CHAPTER 2

Energy Bands and Carrier Concentration in Thermal Equilibrium

- 2.1 SEMICONDUCTOR MATERIALS
- 2.2 BASIC CRYSTAL STRUCTURE
- 2.3 BASIC CRYSTAL GROWTH TECHNIQUE
- 2.4 VALENCE BONDS
- 2.5 ENERGY BANDS
- 2.6 INTRINSIC CARRIER CONCENTRATION
- 2.7 DONORS AND ACCEPTORS
- SUMMARY

In this chapter, we consider some basic properties of semiconductors. We begin with a discussion of crystal structure, which is the arrangement of atoms in a semiconductor. This is followed by a brief description of the crystal growth technique. We then present the concepts of valence bonds and energy bands, which relate to conduction in semiconductors. Finally, we discuss the concept of carrier concentration in thermal equilibrium. These concepts are used throughout this book.

Specifically, we cover the following topics:

- Element and compound semiconductors and their basic properties.
- The diamond structure and its related crystal planes.
- The bandgap and its impact on electrical conductivity.
- The intrinsic carrier concentration and its dependence on temperature.
- The Fermi level and its dependence on carrier concentration.

2.1 SEMICONDUCTOR MATERIALS

Solid-state materials can be grouped into three classes—insulators, semiconductors, and conductors. Figure 1 shows the range of electrical conductivities σ (and the corresponding resistivities $\rho = 1/\sigma$)° associated with some important materials in each of the three classes. Insulators such as fused quartz and glass have very low conductivities, on the order of

° A list of symbols is given in Appendix A.

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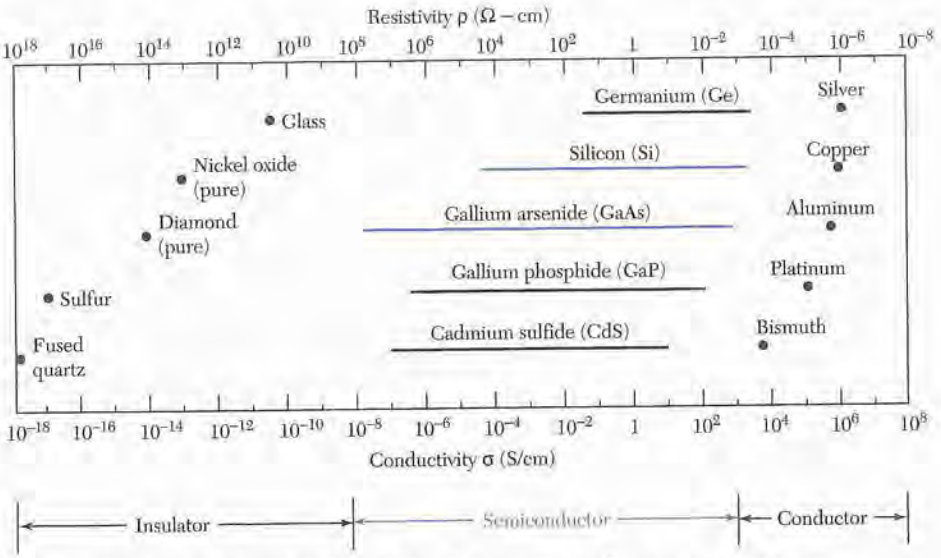


Fig. 1 Typical range of conductivities for insulators, semiconductors, and conductors.

$10^{-18} - 10^{-8}$ S/cm; and conductors such as aluminum and silver have high conductivities, typically from 10^4 to 10^6 S/cm.* Semiconductors have conductivities between those of insulators and those of conductors. The conductivity of a semiconductor is generally sensitive to temperature, illumination, magnetic field, and minute amounts of impurity atoms (typically, about 1 μg to 1 g of impurity atoms in 1 kg of semiconductor materials). This sensitivity in conductivity makes the semiconductor one of the most important materials for electronic applications.

2.1.1 Element Semiconductors

The study of semiconductor materials began in the early nineteenth century.[1] Over the years many semiconductors have been investigated. Table 1 shows a portion of the periodic table related to semiconductors. The element semiconductors, those composed of single

TABLE 1 Portion of the Periodic Table Related to Semiconductors

Period	Column II	III	IV	V	VI
2		B Boron	C Carbon	N Nitrogen	O Oxygen
3	Mg Magnesium	Al Aluminum	Si Silicon	P Phosphorus	S Sulfur
4	Zn Zinc	Ga Gallium	Ge Germanium	As Arsenic	Se Selenium
5	Cd Cadmium	In Indium	Sn Tin	Sb Antimony	Te Tellurium
6	Hg Mercury		Pb Lead		

* The international system of units is presented in Appendix B.

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species of atoms, such as silicon (Si) and germanium (Ge), can be found in Column IV. In the early 1950s, germanium was the major semiconductor material. Since the early 1960s silicon has become a practical substitute and has now virtually supplanted germanium as a material for semiconductor fabrication. The main reasons we now use silicon are that silicon devices exhibit better properties at room temperature, and high-quality silicon dioxide can be grown thermally. There is also an economic consideration. Device-grade silicon costs much less than any other semiconductor material. Silicon in the form of silica and silicates comprises 25% of the Earth's crust, and silicon is second only to oxygen in abundance. Currently, silicon is one of the most studied elements in the periodic table; and silicon technology is by far the most advanced among all semiconductor technologies.

2.1.2 Compound Semiconductors

In recent years a number of compound semiconductors have found applications for various devices. The important compound semiconductors as well as the two element semiconductors are listed[2] in Table 2. A binary compound semiconductor is a combination of two elements from the periodic table. For example, gallium arsenide (GaAs) is a III-V compound that is a combination of gallium (Ga) from Column III and arsenic (As) from Column V.

In addition to binary compounds, ternary compounds and quaternary compounds are made for special applications. The alloy semiconductor $Al_xGa_{1-x}As$, which has Al and Ga from Column III and As from Column V is an example of a tenary compound, whereas quaternary compounds of the form $A_xB_{1-x}C_yD_{1-y}$ can be obtained from combination of many binary and ternary compound semiconductors. For example, GaP, InP, InAs, plus GaAs can be combined to yield the alloy semiconductor $Ga_xIn_{1-x}As_yP_{1-y}$. Compared with the element semiconductors, the preparation of compound semiconductors in single-crystal form usually involves much more complex processes.

Many of the compound semiconductors have electrical and optical properties that are different from silicon. These semiconductors, especially GaAs, are used mainly for high-speed electronic and photonic applications. Although we do not know as much about the technology of compound semiconductors as we do about that of silicon, advances in silicon technology have also helped progress in compound semiconductor technology. In this book we are concerned mainly with device physics and processing technology of silicon and gallium arsenide.

2.2 BASIC CRYSTAL STRUCTURE

The semiconductor materials we will be studying are single crystals, that is, the atoms are arranged in a three-dimensional periodic fashion. The periodic arrangement of atoms in a crystal is called a *lattice*. In a crystal, an atom never strays far from a single, fixed position. The thermal vibrations associated with the atom are centered about this position. For a given semiconductor, there is a *unit cell* that is representative of the entire lattice; by repeating the unit cell throughout the crystal, one can generate the entire lattice.

2.2.1 Unit Cell

A generalized primitive, three-dimensional unit cell is shown in Fig. 2. The relationship between this cell and the lattice is characterized by three vectors $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$, which need

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TABLE 2 Semiconductor Materials[2]

General Classification	Semiconductor Symbol	Semiconductor Name
Element	Si	Silicon
	Ge	Germanium
Binary compound		
IV-IV	SiC	Silicon carbide
III-V	AlP	Aluminum phosphide
	AlAs	Aluminum arsenide
	AlSb	Aluminum antimonide
	GaN	Gallium nitride
	GaP	Gallium phosphide
	GaAs	Gallium areside
	GaSb	Gallium antimonide
	InP	Indium phosphide
	InAs	Indium arsenide
	InSb	Indium antimonide
II-VI	ZnO	Zinc oxide
	ZnS	Zinc sulfide
	ZnSe	Zinc selenide
	ZnTe	Zinc telluride
	CdS	Cadmium sulfide
	CdSe	Cadmium selenide
	CdTe	Cadmium telluride
	HgS	Mercury sulfide
IV-VI	PbS	Lead sulfide
	PbSe	Lead selenide
	PbTe	Lead telluride
Ternary compound	$Al_xGa_{1-x}As$	Aluminum gallium arsenide
	$Al_xIn_{1-x}As$	Aluminum indium arsenide
	$GaAs_{1-x}P_x$	Gallium arsenic phosphide
	$Ga_xIn_{1-x}As$	Gallium indium arsenide
	$Ga_xIn_{1-x}P$	Gallium indium phosphide
Quaternary compound	$Al_xGa_{1-x}As_ySb_{1-y}$	Aluminum gallium arsenic antimonide
	$Ga_xIn_{1-x}As_{1-y}P_y$	Gallium indium arsenic phosphide

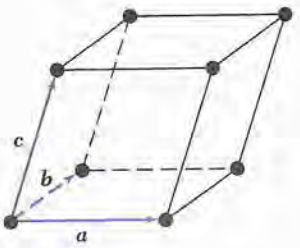


Fig. 2 A generalized primitive unit cell.

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not be perpendicular to each other and which may or may not be equal in length. Every equivalent lattice point in the three-dimensional crystal can be found using the set

$$\boldsymbol{R} = m\boldsymbol{a} + n\boldsymbol{b} + p\boldsymbol{c} \tag{1}$$

where m, n, and p are integers.

Figure 3 shows some basic cubic-crystal unit cells. Figure 3*a* shows a simple cubic (sc) crystal; it has an atom at each corner of the cubic lattice, and each atom has six equidistant nearest-neighbor atoms. The dimension a is called the lattice constant. In the periodic table, only polonium is crystallized in the simple cubic lattice. Figure 3*b* is a body-centered cubic (bcc) crystal where, in addition to the eight corner atoms, an atom is located at the center of the cube. In a bcc lattice, each atom has eight nearest-neighbor atoms. Crystals exhibiting bcc lattices include those of sodium and tungsten. Figure 3*c* shows the face-centered cubic (fcc) crystal that has one atom at each of the six cubic faces in addition to the eight corner atoms. In this case, each atom has 12 nearest-neighbor atoms. A large number of elements exhibit the fcc lattice form, including aluminum, copper, gold, and platinum.

EXAMPLE 1

If we pack hard spheres in a bcc lattice such that the atom in the center just touches the atoms at the corners of the cube, find the fraction of the bcc unit cell volume filled with hard spheres.

SOLUTION Each corner sphere in a bcc unit cell is shared with eight neighboring cells; thus, each unit cell contains one-eighth of a sphere at each of the eight corners for a total of one sphere. In addition, each unit cell contains one central sphere. We have the following:

Spheres (atoms) per unit cell = (1/8) × 8 (corner) + 1 (center) = 2;

Nearest-neighbor distance (along the diagonal AE in Fig. 3*b*) = $a\sqrt{3}/2$;

Radius of each sphere = $a\sqrt{3}/4$;

Volume of each sphere = $4\pi/3 \times (a\sqrt{3}/4)^3 = \pi a^3\sqrt{3}/16$; and

Maximum fraction of unit cell filled = Number of spheres × volume of each sphere/total volume of each unit cell = $2\pi a^3\sqrt{3}/16\,a^3 = \pi\sqrt{3}/8 \approx 0.68$.

Therefore, about 68% of the bcc unit cell volume is filled with hard spheres, and about 32% of the volume is empty.

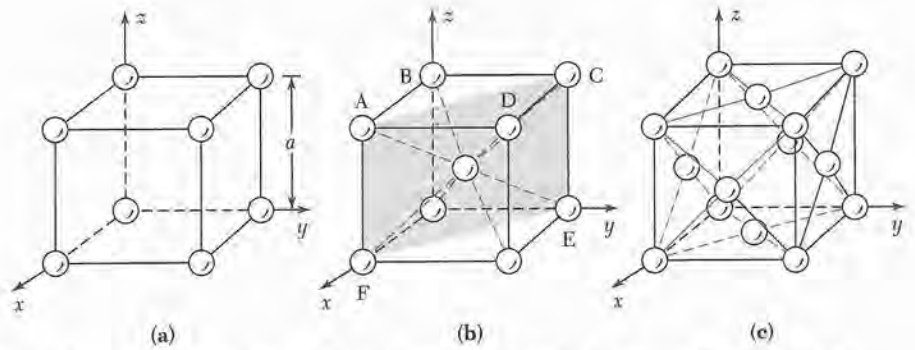


Fig. 3 Three cubic-crystal unit cells. (*a*) Simple cubic. (*b*) Body-centered cubic. (*c*) Face-centered cubic.

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2.2.2 The Diamond Structure

The element semiconductors, silicon and germanium, have a diamond lattice structure as shown in Fig. 4*a*. This structure also belongs to the fcc crystal family and can be seen as two interpenetrating fcc sublattices with one sublattice displaced from the other by one-quarter of the distance along the body diagonal of the cube (i.e., a displacement of $a\sqrt{3}/4$). Although chemically identical, the two sets of atoms belonging to the two sublattices are different from the point view of the crystal structure. It can be seen in Fig. 4*a* that if a corner atom has one nearest neighbor in the body diagonal direction, then it has no nearest neighbor in the reverse direction. Consequently, it requires two such atoms in the unit cell. Alternatively, a unit cell of a diamond lattice consists of a tetrahedron in which each atom is surrounded by four equidistant nearest neighbors that lie at the corners (refer to the spheres connected by darkened bars in Fig. 4*a*).

Most of the III-V compound semiconductors (e.g., GaAs) have a *zincblende lattice*, shown in Fig. 4*b*, which is identical to a diamond lattice except that one fcc sublattice has Column III atoms (Ga) and the other has Column V atoms (As). Appendix F at the end of the book gives a summary of the lattice constants and other properties of important element and binary compound semiconductors.

EXAMPLE 2

At 300 K the lattice constant for silicon is 5.43 Å. Calculate the number of silicon atoms per cubic centimeter and the density of silicon at room temperature.

SOLUTION There are eight atoms per unit cell. Therefore,

$8/a^3 = 8/(5.43 \times 10^{-8})^3 = 5 \times 10^{22}$ atoms/cm^3; and

Density = no. of atoms/cm^3 × atomic weight/Avogadro constant = 5×10^{22} (atoms/cm^3) × 28.09 (g/mol)/6.02×10^{23} (atoms/mol) = 2.33 g/cm^3.

2.2.3 Crystal Planes and Miller Indices

In Fig. 3*b* we note that there are four atoms in the *ABCD* plane and five atoms in the *ACEF* plane (four atoms from the corners and one from the center) and that the atomic spacing is different for the two planes. Therefore, the crystal properties along different planes are different, and the electrical and other device characteristics can be dependent

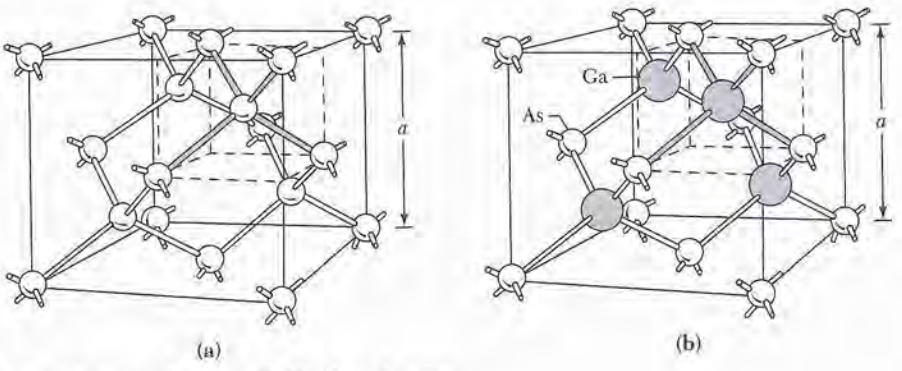


Fig. 4 (*a*) Diamond lattice. (*b*) Zincblende lattice.

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on the crystal orientation. A convenient method of defining the various planes in a crystal is to use *Miller indices*.[3] These indices are obtained using the following steps:

1. Find the intercepts of the plane on the three Cartesian coordinates in terms of the lattice constant.
2. Take the reciprocals of these numbers and reduce them to the smallest three integers having the same ratio.
3. Enclose the result in parentheses (hkl) as the Miller indices for a single plane.

EXAMPLE 3

As shown in Fig. 5, the plane has intercepts at a, $3a$, and $2a$ along the three coordinates. Taking the reciprocals of these intercepts, we get 1, ⅓, and ½. The smallest three integers having the same ratio are 6, 2, and 3 (obtained by multiplying each fraction by 6). Thus, the plane is referred to as a (623)-plane. ◀

Figure 6 shows the Miller indices of important planes in a cubic crystal.[c] Some other conventions are the following:

1. $(\bar{h}kl)$: For a plane that intercepts the x-axis on the negative side of the origin, such as $(\bar{1}00)$.
2. $\{hkl\}$: For planes of equivalent symmetry, such as $\{100\}$ for (100), (010), (001), $(\bar{1}00)$, $(0\bar{1}0)$, and $(00\bar{1})$ in cubic symmetry.
3. $[hkl]$: For a crystal direction, such as $[100]$ for the x-axis. By definition, the $[100]$-direction is perpendicular to (100)-plane, and the $[111]$-direction is perpendicular to the (111)-plane.
4. $\langle hkl \rangle$: For a full set of equivalent directions, such as $\langle 100 \rangle$ for $[100]$, $[010]$, $[001]$, $[\bar{1}00]$, $[0\bar{1}0]$, and $[00\bar{1}]$.

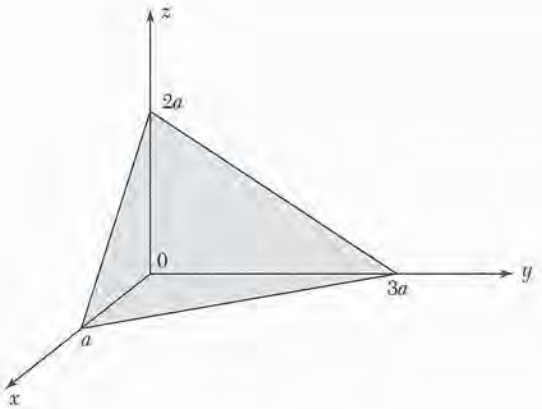


Fig. 5 A (623)-crystal plane.

[c] In Chapter 6, we show that the $\langle 100 \rangle$ orientation is preferred for silicon metal-oxide-semiconductor field-effect transistors (MOSFETs).

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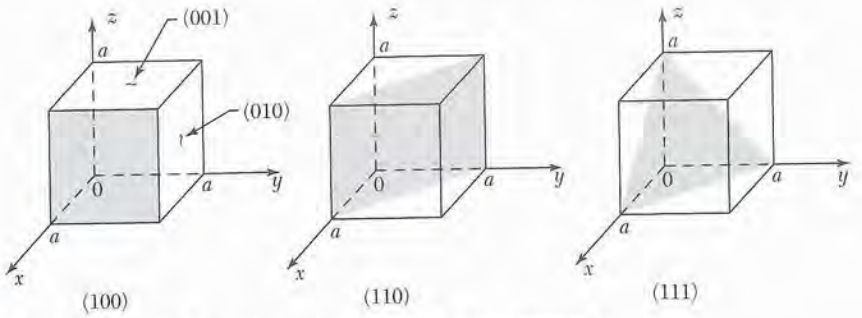


Fig. 6 Miller indices of some important planes in a cubic crystal.

2.3 BASIC CRYSTAL GROWTH TECHNIQUE

In this section, we consider briefly how we grow semiconductor crystals, specifically the silicon crystal because 95% of the semiconductor materials used by the electronic industry is silicon.

The starting material for silicon is a relatively pure form of sand (SiO_2) called quartzite. It is mixed with various forms of carbon and allowed to react to form silicon (98% pure):

$$SiC + SiO_2 \rightarrow Si\ (solid) + SiO\ (gas) + CO\ (gas). \tag{2}$$

The silicon product is reacted with hydrogen chloride to form trichlorosilane,

$$Si\ (solid) + 3\ HCl\ (gas) \rightarrow SiHCl_3(gas) + H_2(gas). \tag{3}$$

The trichlorosilane is decomposed using an electric current in a chamber with a controlled ambient, producing rods of ultrapure polycrystallice silicon (i.e., silicon material that contains many single-crystal regions with different size and orientation with respect to one another),

$$SiHCl_3\ (gas) + H_2(gas) \rightarrow Si\ (solid) + 3\ HCl\ (gas). \tag{4}$$

The polycrystalline silicon is now ready for the crystal-growing process. Figure 7 shows many pieces of polycrystalline silicon in a silica (SiO_2) crucible.



Fig. 7 Photograph of polycrystalline silicon in a silica crucible.

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The most common crystal growth method is the Czochralski technique. Figure 8 shows a schematic drawing of Czochralski crystal puller. The crucible containing the polycrystalline silicon is heated either by radio-frequency induction or by a thermal resistance method to the melting point of silicon (1412°C). The crucible rotates during the growth to prevent the formation of local hot or cold regions.

The atmosphere around the crystal-growing apparatus or crystal puller is controlled to prevent contamination of the molten silicon. Argon is often used as the ambient gas. When the temperature of the silicon has stabilized, a piece of silicon with a suitable orientation (e.g., ⟨111⟩), which is called the seed crystal, is lowered to the melt and is the starting point for the subsequent growth of a much larger crystal. As the bottom of the seed crystal begins to melt in the molten silicon, the downward motion of the rod holding the seed is reversed. As the seed crystal is slowly withdrawn from the melt (Fig. 9), the molten silicon adhering to the crystal freezes or solidifies, using the crystal structure of the seed crystal as a template. The seed crystal is, therefore, used to initiate the growth of the ingot with the correct crystal orientation. The rod continues its upward movement, forming an even larger crystal. The crystal growth is terminated when the silicon in the crucible is depleted. By carefully controlling the temperature of the crucible and the rotation speeds of the crucible and the rod, precise control of the diameter of the crystal can be maintained. Figure 10 shows a 200 mm diameter silicon crystal ingot. The desired impurity concentration is obtained by adding impurities to the melt in the form of heavily doped silicon prior to crystal growth. A more detailed discussion on the crystal growth of silicon as well as other semiconductors can be founded in Chapter 10.

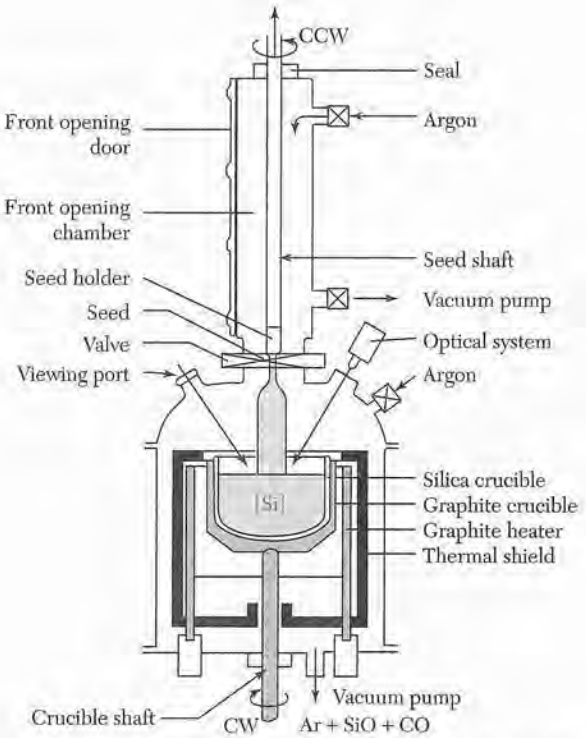


Fig. 8 Simplified schematic drawing of the Czochralski puller. Clockwise (CW), counterclockwise (CCW).

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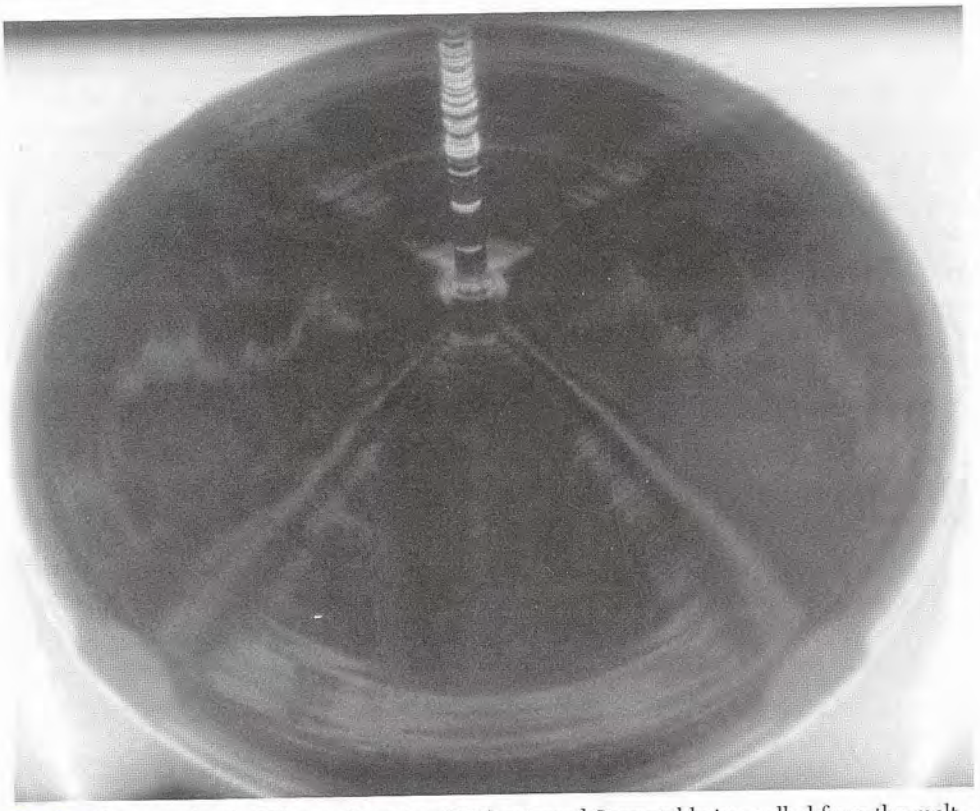


Fig. 9 Photograph of a 200 mm diameter, (100)-oriented Si crystal being pulled from the melt. (Photograph courtesy of Taisil Electronic Materials Corp., Taiwan.)

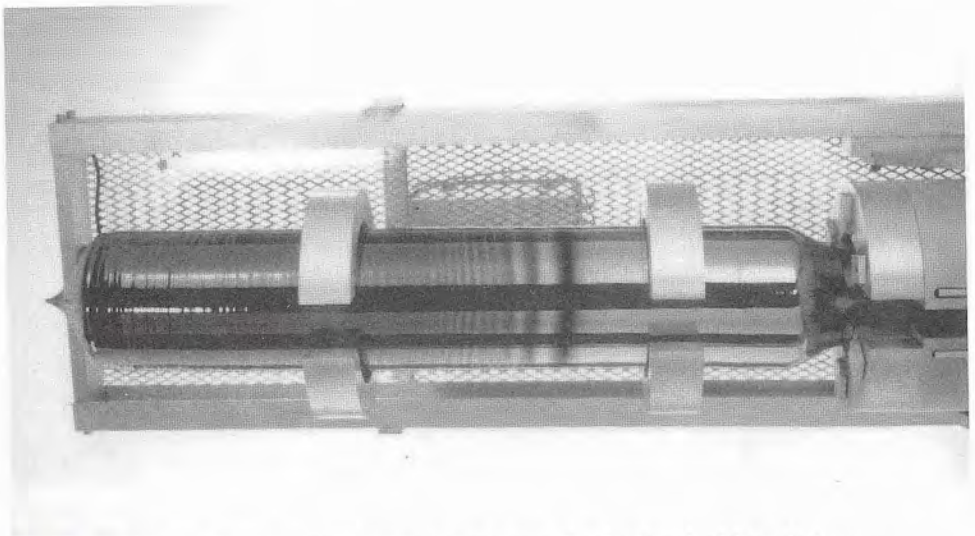


Fig. 10 A 200 mm diameter silicon crystal ingot grown by the Czochralski technique.

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2.4 VALENCE BONDS

As discussed in Section 2.2, each atom in a diamond lattice is surrounded by four nearest neighbors. Figure 11*a* shows the tetrahedron bonds of a diamond lattice. A simplified two-dimensional bonding diagram for the tetrahedron is shown in Fig. 11*b*. Each atom has four electrons in the outer orbit, and each atom shares theses valence electrons with its four neighbors. This sharing of electrons is known as *covalent bonding*; each *electron pair* constitutes a covalent bond. Covalent bonding occurs between atoms of the same element or between atoms of different elements that have similar outer-shell electron configurations. Each electron spends an equal amount of time with each nucleus. However, both electrons spend most of their time between the two nuclei. The force of attraction for the electrons by both nuclei holds the two atoms together.

Gallium arsenide crystallizes in a zincblende lattice, which also has tetrahedron bonds. The major bonding force in GaAs is also due to the covalent bond. However, gallium arsenide has a small ionic contribution that is an electrostatic attractive force between each Ga^+ ion and its four neighboring As^- ions, or between each As^- ion and its four neighboring Ga^+ ions. Electronically, this means that the paired bonding electrons spend slightly more time in the As atom than in the Ga atom.

At low temperatures, the electrons are bound in their respective tetrahedron lattice; consequently, they are not available for conduction. At higher temperatures, thermal vibrations may break the covalent bonds. When a bond is broken or partially broken, a free electron results that can participate in current conduction. Figure 12*a* shows the situation when a valence electron in silicon becomes a free electron. An electron deficiency is left in the covalent bond. This deficiency may be filled by one of the neighboring electrons, which results in a shift of the deficiency location, as from location *A* to location *B* in Fig. 12*b*. We may therefore consider this deficiency as a particle similar to an electron. This fictitious particle is called a *hole*. It carries a positive charge and moves, under the influence of an applied electric field, in the direction opposite to that of an electron. Therefore, both the electron and the hole contribute to the total electric current. The concept of a hole is analogous to that of a bubble in a liquid. Although it is actually the liquid that moves, it is much easier to talk about the motion of the bubble in the opposite direction.

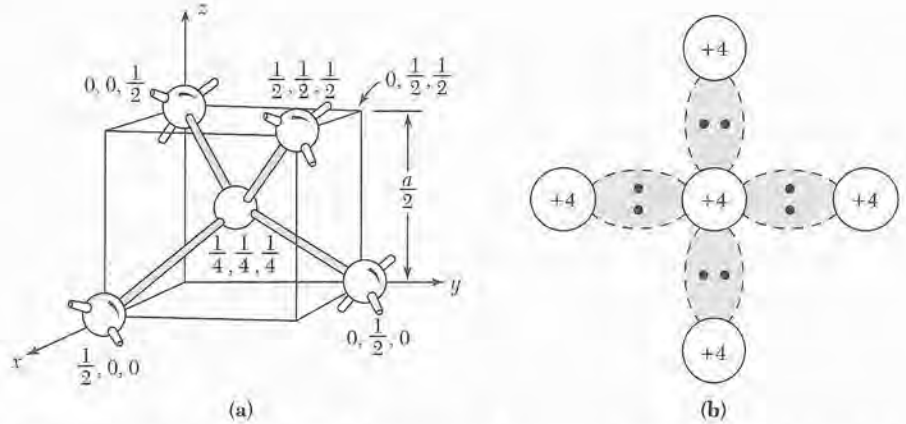


Fig. 11 (*a*) A tetrahedron bond. (*b*) Schematic two-dimensional representation of a tetrahedron bond.

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2.5 ENERGY BANDS

2.5.1 Energy Levels of Isolated Atoms

For an isolated atom, the electrons can have discrete energy levels. For example, the energy levels for an isolated hydrogen atom are given by the Bohr model[4]:

$$E_H = -m_0 q^4 / 8\varepsilon_0^2 h^2 n^2 = -13.6/n^2 \text{ eV}, \tag{5}$$

where m_0 is the free-electron mass, q is the electronic charge, ε_0 is the free-space permittivity, h is the Planck constant, and n is a positive integer called the principal quantum number. The quantity eV (electron volt) is an energy unit corresponding to the energy gained by an electron when its potential is increased by one volt. It is equal to the product of q (1.6×10^{-19} coulomb) and one volt, or 1.6×10^{-19} J. The discrete energies are –13.6 eV for the ground state energy level ($n = 1$), – 3.4 eV for the first excited-state energy level ($n = 2$), etc. Detailed studies reveal that for higher principle quantum numbers ($n \geq 2$), energy levels are split according to their angular momentum quantum number ($\ell = 0, 1, 2, \ldots, n - 1$).

We now consider two identical atoms. When they are far apart, the allowed energy levels for a given principal quantum number (e.g., $n = 1$) consist of one doubly degenerate level, that is, both atoms have exactly the same energy. When they are brought closer, the doubly degenerate energy levels will spilt into two levels by the interaction between the atoms. As N isolated atoms are brought together to form a solid, the orbits of the outer electrons of different atoms overlap and interact with each other. This interaction, including those forces of attraction and repulsion between atoms, causes a shift in the energy levels, as in the case of two interacting atoms. However, instead of two levels, N separate but closely spaced levels are formed. When N is large, the result is an essentially continuous band of energy. This band of N levels can extend over a few eV depending on the inter-atomic spacing for the crystal. Figure 13 shows the effect, where the parameter a represents the equilibrium interatomic distance of the crystal.

The actual band splitting in a semiconductor is much more complicated. Figure 14 shows an isolated silicon atom that has 14 electrons. Of the 14 electrons, 10 occupy deeplying energy levels whose orbital radius is much smaller than the interatomic separation in the crystal. The four remaining valence electrons are relatively weakly bound and can be involved in chemical reactions. Therefore, we only need to consider the outer shell

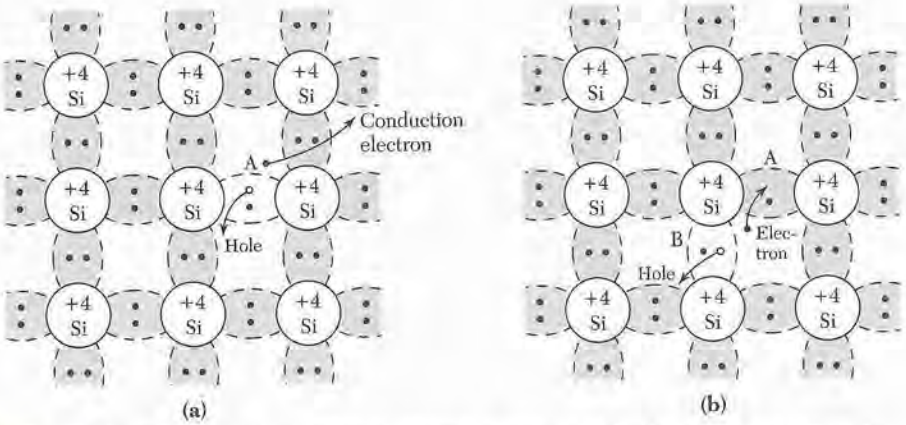


Fig. 12 The basic bond representation of intrinsic silicon. (*a*) A broken bond at position A, resulting in a conduction electron and a *hole*. (*b*) A broken bond at position B.

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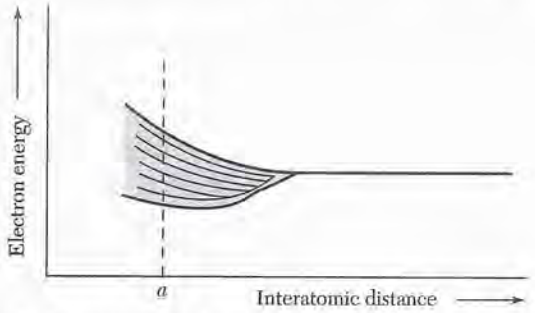


Fig. 13 The splitting of a degenerate state into a band of allowed energies.

(the $n = 3$ level) for the valence electrons, since the two inner shells are completely full and tightly bound to the nucleus. The 3s subshell (i.e., for $n = 3$ and $\ell = 0$) has two allowed quantum states per atom. This subshell will contain two valence electrons at $T = 0$ K. The 3p subshell (i.e., $n = 3$, and $\ell = 1$) has six allowed quantum states per atom. This subshell will contain the remaining two valence electrons of an individual silicon atom.

Figure 15 is a schematic diagram of the formation of a silicon crystal from N isolated silicon atoms. As the interatomic distance decreases, the 3s and 3p subshell of the N silicon atoms will interact and overlap. At the equilibrium interatomic distance, the bands will again split, with four quantum states per atom in the lower band and four quantum states per atom in the upper band. At a temperature of absolute zero, electrons occupy the lowest energy states, so that all states in the lower band (*the valence band*) will be full and all states in the upper band (*the conduction band*) will be empty. The bottom of the conduction band is called E_C, and the top of the valence band is called E_V. The *bandgap energy* E_g between the bottom of the conduction band and the top of the valence band ($E_C - E_V$) is the width of the forbidden energy gap, as shown on the far left of Fig. 15. Physically, E_g is the energy required to break a bond in the semiconductor to free an electron to the conduction band and leave a hole in the valence band.

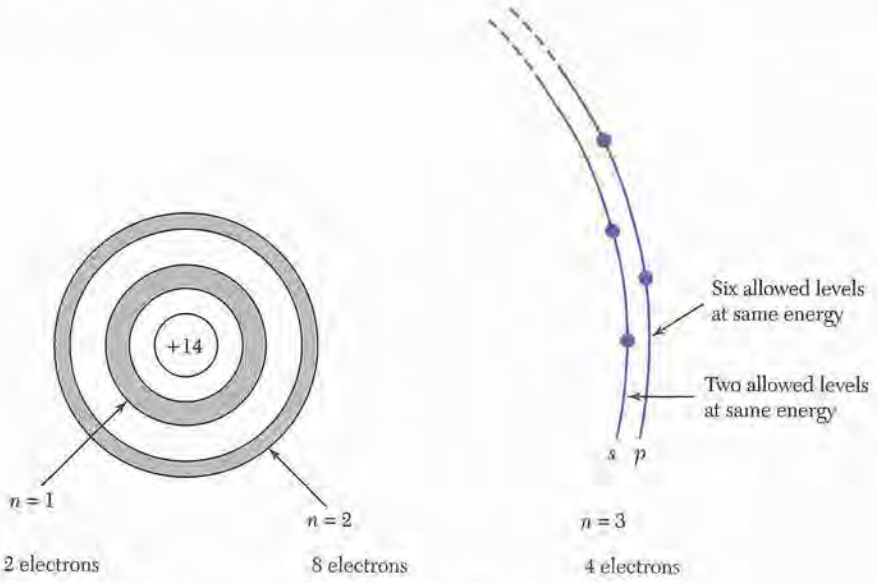


Fig. 14 Schematic representation of an isolated silicon atom.

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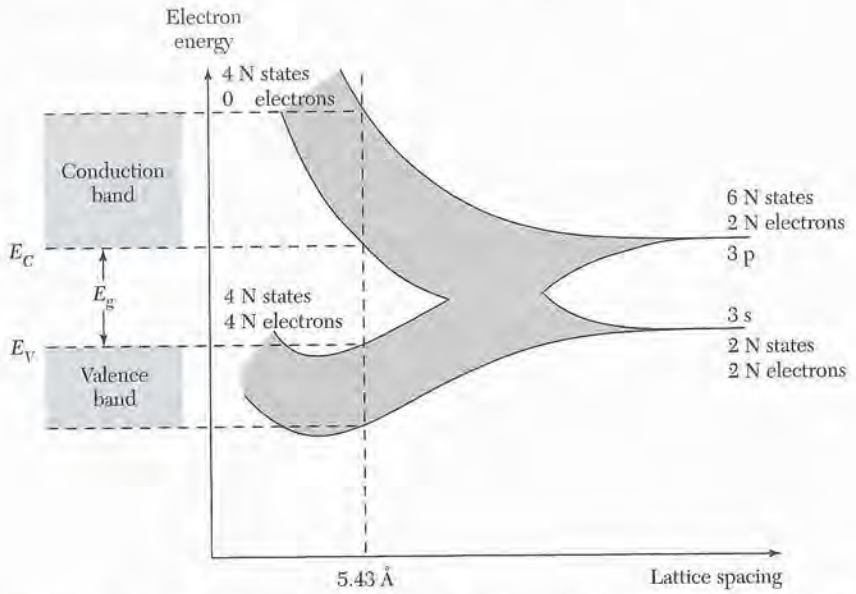


Fig. 15 Formation of energy bands as a diamond lattice crystal is formed by bringing isolated silicon atoms together.

2.5.2 The Energy-Momentum Diagram

The energy E of a free electron is given by

$$E = \frac{p^2}{2m_0}, \tag{6}$$

where p is the momentum and m_0 is the free-electron mass. If we plot E vs. p, we obtain a parabola as shown in Fig. 16. In a semiconductor crystal, an electron in the conduction band is similar to a free electron in that it is relatively free to move about in the crystal. However, because of the periodic potential of the nuclei, Eq. 6 can no longer be valid. However, it turns out that we can still use Eq. 6 if we replace the free-electron mass in Eq. 6 by an effective mass m_n (the subscript n refers to the negative charge on an electron), that is,

$$E = \frac{p^2}{2m_n}. \tag{7}$$

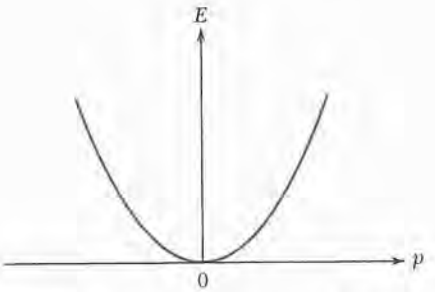


Fig. 16 The parabolic energy (E) vs. momentum (p) curve for a free electron.

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The electron effective mass depends on the properties of the semiconductor. If we have an energy-momentum relationship described by Eq.7, we can obtain the effective mass from the second derivative of E with respect to p:

$$m_n \equiv \left(\frac{d^2E}{dp^2} \right)^{-1}. \tag{8}$$

Therefore, the narrower the parabola, corresponding to a larger second derivative, the smaller the effective mass. A similar expression can be written for holes (with effective mass m_p where the subscript p refers to the positive charge on a hole). The effective-mass concept is very useful because it enables us to treat electrons and holes essentially as classical charged particles.

Figure 17 shows a simplified energy-momentum relationship of a special semiconductor with an electron effective mass of $m_n = 0.25\, m_0$ in the conduction band (the upper parabola) and a hole effective mass of $m_p = m_0$ in the valence band (the lower parabola). Note that the electron energy is measured upward and the hole energy is measured downward. The spacing at $p = 0$ between these two parabolas is the bandgap E_g, shown previously in Fig. 15.

The actual energy-momentum relationships (also called energy-band diagram) for silicon and gallium arsenide are much more complex. They are shown in Fig. 18 for two crystal directions. We note that the general features in Fig. 18 are similar to those in Fig. 17. First of all, there is a bandgap E_g between the bottom of the conduction band and the top of the valence band. Second, near the minimum of the conduction band or the maximum of the valence band, the E-p curves are essentially parabolic. For silicon, Fig 18*a*, the maximum in the valence band occurs at $p = 0$, but the minimum in the conduction band occurs along the [100] direction at $p = p_c$. Therefore, in silicon, when an electron makes a transition from the maximum point in the valence band to the minimum point in the conduction, it requires not only an energy change ($\geq E_g$) but also some momentum change ($\geq p_c$).

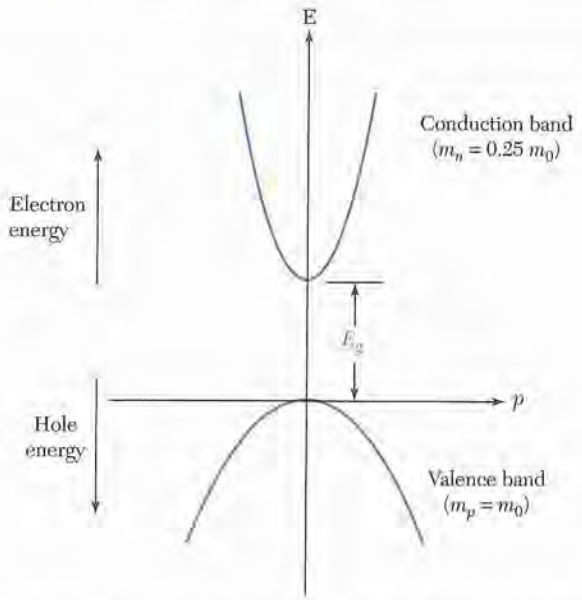


Fig. 17 A schematic energy-momentum diagram for a special semiconductor with $m_n = 0.25\, m_0$ and $m_p = m_0$.

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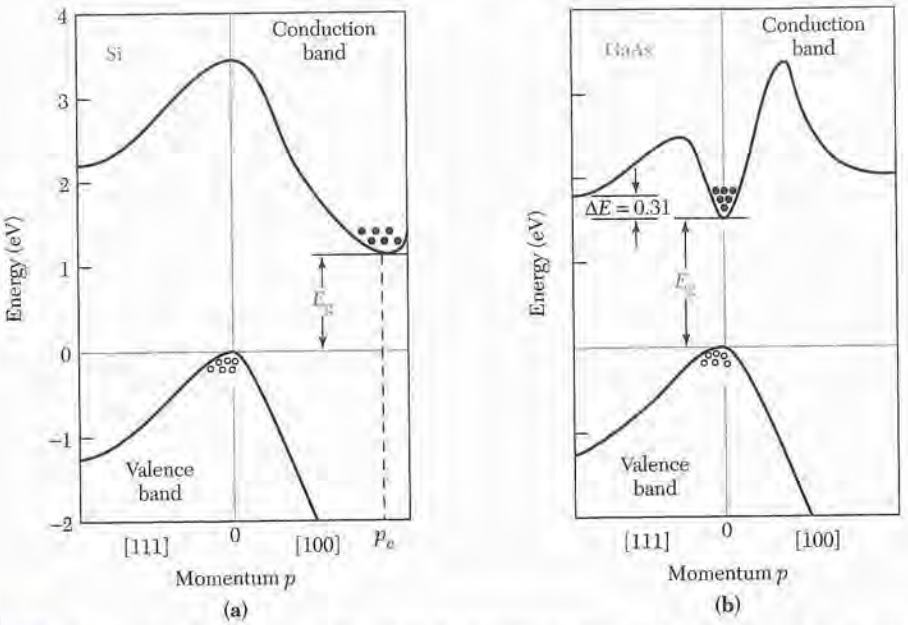


Fig. 18 Energy band structures of Si and GaAs. Circles (∘) indicate holes in the valence bands and dots (•) indicate electrons in the conduction bands.

For gallium arsenide, Fig. 18*b*, the maximum in the valence band and the minimum in the conduction band occur at the same momentum ($p = 0$). Thus, an electron making a transition from the valence band to the conduction band can do so without a change in momentum.

Gallium arsenide is called a *direct semiconductor*, because it does not require a change in momentum for an electron transition from the valence band to the conduction band. Silicon is called an *indirect semiconductor,* because a change of momentum is required in a transition. This difference between direct and indirect band structures is very important for light-emitting diodes and semiconductor lasers. These devices require direct semiconductors for efficient generation of photons (refer to Chapter 9).

We can obtain the effective mass from Fig. 18 using Eq. 8. For example, for gallium arsenide with a very narrow conduction–band parabola, the electron effective mass is $0.063\ m_0$, where for silicon, with a wider conduction–band parabola, the electron effective mass is $0.19\ m_0$.

2.5.3 Conduction in Metals, Semiconductors, and Insulators

The enormous variation in electrical conductivity of metals, semiconductors, and insulators as shown in Fig. 1 may be explained qualitatively in terms of their energy bands. We shall see that the electron occupation of the highest band or of the highest two bands determines the conductivity of a solid. Figure 19 shows the energy band diagrams of three classes of solids—metals, semiconductors, and insulators.

Metals

The characteristics of a metal (also called a conductor) include a very low value of resistivity, and the conduction band either is partially filled (such as Cu) or overlaps the valence band (such as Zn or Pb) so that there is no bandgap, as shown in Fig. 19*a*. As a conse-

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quence, the uppermost electrons in the partially filled band or electrons at the top of the valence band can move to the next-higher available energy level when they gain kinetic energy (e.g., from an applied electric field). Electrons are free to move with only a small applied field in a metal because there are many unoccupied states close to the occupied energy states. Therefore, current conduction can readily occur in conductors.

Insulators

In an insulator such as silicon dioxide (SiO_2), the valence electrons form strong bonds between neighboring atoms. These bonds are difficult to break, and consequently there are no free electrons to participate in current conduction at or near room temperature. As shown in the energy band diagram Fig. 19*c*, insulators are characterized by a large bandgap. Note that electrons occupy all energy levels in the valence band and all energy levels in the conduction band are empty. Thermal energy* or the energy of an applied electric field is insufficient to raise the uppermost electron in the valence band to the conduction band. Thus, although an insulator has many vacant states in the conduction band that can accept electrons, so few electrons actually occupy conduction band states that the overall contribution to electrical conductivity is very small, resulting in a very high resistivity. Therefore, silicon dioxide is an insulator; it can not conduct current.

Semiconductors

Now, consider a material that has a much smaller energy gap, on the order of 1 eV (Fig. 19*b*). Such materials are called semiconductors. At $T = 0$ K, all electrons are in the valence band, and there are no electrons in the conduction band. Thus, semiconductors are poor conductors at low temperatures. At room temperature and under normal atmosphere, values of E_g are 1.12 eV for Si and 1.42 eV for GaAs. The thermal energy kT at room temperature is a good fraction of E_g, and an appreciable numbers of electrons are thermally excited from the valence band to the conduction band. Since there are many empty states in the conduction band, a small applied potential can easily move these electrons, resulting in a moderate current.

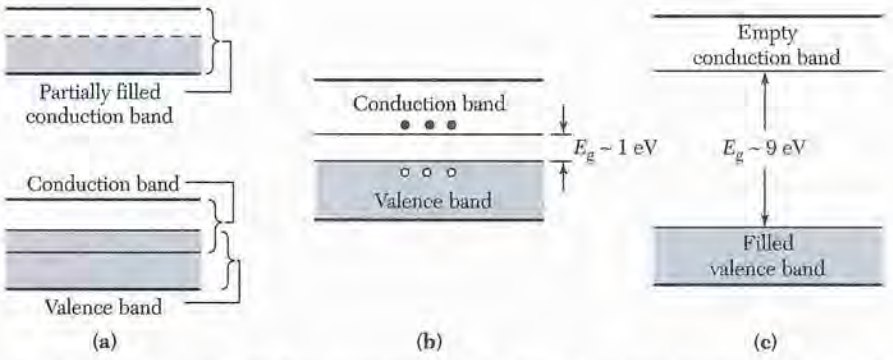


Fig. 19 Schematic energy band representations of (*a*) a conductor with two possibilities (either the partially filled conduction band shown at the upper portion or the overlapping bands shown at the lower portion), (*b*) a semiconductor, and (*c*) an insulator.

* The thermal energy is of the order of kT. At room temperature, kT is 0.026 eV, which is much smaller than the bandgap of an insulator.

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2.6 INTRINSIC CARRIER CONCENTRATION

We now derive the carrier concentration in thermal equilibrium condition, that is, the steady-state condition at a given temperature without any external excitations such as light, pressure, or electric field. At a given temperature, continuous thermal agitation results in the excitation of electrons from the valence band to the conduction band and leaves an equal number of holes in the valence band. An *intrinsic semiconductor* is one that contains relatively small amounts of impurities compared with the thermally generated electrons and holes.

To obtain the electron density (i.e., the number of electrons per unit volume) in an intrinsic semiconductor, we first evaluate the electron density in an incremental energy range dE. This density $n(E)$ is given by the product of the density of states $N(E)$, that is, the density of allowed energy states (including electron spin) per energy range per unit volume,* and by the probability of occupying that energy range $F(E)$. Thus, the electron density in the conduction band is given by integrating $N(E)\,F(E)\,dE$ from the bottom of the conduction band (E_C initially taken to be $E = 0$ for simplicity) to the top of the conduction band E_{top}:

$$n = \int_0^{E_{top}} n(E)dE = \int_0^{E_{top}} N(E)F(E)dE, \tag{9}$$

where n is in cm^{-3}, and $N(E)$ is in $(cm^3\text{–}eV)^{-1}$.

The probability that an electron occupies an electronic state with energy E is given by the Fermi–Dirac distribution function, which is also called the Fermi distribution function

$$\boxed{F(E) = \frac{1}{1 + e^{(E - E_F)/kT}},} \tag{10}$$

where k is the Boltzmann constant, T is the absolute temperature in degrees Kelvin, and E_F is the energy of the Fermi level. The Fermi energy is the energy at which the probability of occupation by an electron is exactly one-half. The Fermi distribution is illustrated in Fig. 20 for different temperatures. Note that $F(E)$ is symmetrical around the Fermi energy E_F. For energies that are $3kT$ above or below the Fermi energy, the exponential term in Eq. 10 becomes larger than 20 or smaller than 0.05, respectively. The Fermi distribution function can thus be approximated by simpler expressions:

$$F(E) \cong e^{-(E - E_F)/kT} \quad \text{for} \quad (E - E_F) > 3kT, \tag{11a}$$

and

$$F(E) \cong 1 - e^{-(E - E_F)/kT} \quad \text{for} \quad (E - E_F) < 3kT. \tag{11b}$$

Equation 11*b* can be regarded as the probability that a hole occupies a state located at energy E.

Figure 21 shows schematically from left to right the band diagram, the density of states $N(E)$, which varies as $\sqrt{E}$ for a given electron effective mass, the Fermi distribution function, and the carrier concentrations for an intrinsic semiconductor. The carrier concentration can be obtained graphically from Fig. 21 using Eq. 9; that is, the product of $N(E)$ in Fig. 21*b* and $F(E)$ in Fig. 21*c* gives the $n(E)$-versus-E curve (upper curve) in Fig. 21*d*. The upper shaded area in Fig. 21*d* corresponds to the electron density.

* The density of states $N(E)$ is derived in Appendix H.

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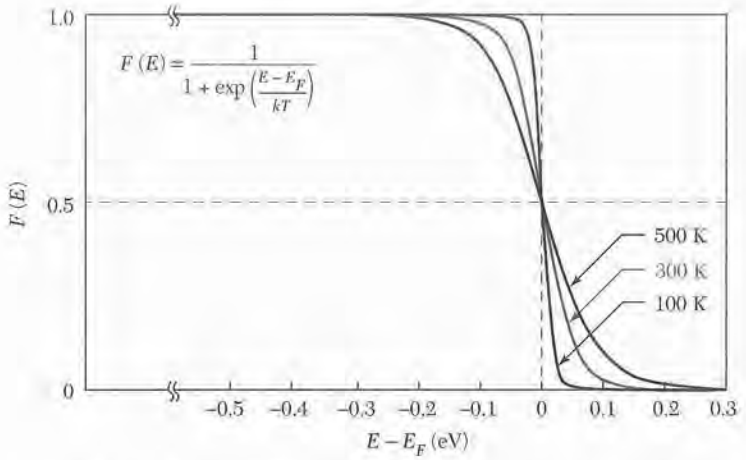


Fig. 20 Fermi distribution function $F(E)$ versus $(E - E_F)$ for various temperatures.

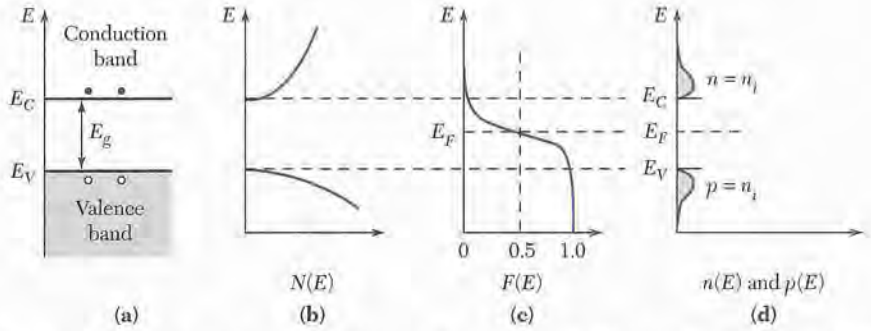


Fig. 21 Instrinsic semiconductor. (*a*) Schematic band diagram. (*b*) Density of states. (*c*) Fermi distribution function. (*d*) Carrier concentration.

There are a large number of allowed states in the conduction band. However, for an intrinsic semiconductor there will not be many electrons in the conduction band. Therefore, the probability of an electron occupying one of these states is small. There also are a large number of allowed states in the valence band. By contrast, most of these are occupied by electrons. Thus, the probability of an electron occupying one of these states in the valence band is nearly unity. There will be only a few unoccupied electron states, that is, holes, in the valence band. As can be seen, the Fermi level is located near the middle of the bandgap (i.e., E_F is many kT below E_C). Substituting the last equation in Appendix H and Eq.11*a* into Eq. 9 yields*

$$n = \frac{2}{\sqrt{\pi}} N_C (kT)^{-3/2} \int_0^{\infty} E^{1/2} \exp\left[-(E - E_F)/kT\right] dE, \tag{12}$$

where

$$N_C \equiv 12(2\pi m_n kT/h^2)^{3/2} \text{ for Si} \tag{13a}$$

$$\equiv 2(2\pi m_n kT/h^2)^{3/2} \text{ for GaAs.} \tag{13b}$$

* We have taken E_{top} to be ∞, because $F(E)$ becomes very small when $(E - E_C) >> kT$.

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If we let $x \equiv E/kT$, Eq. 12 becomes

$$n = \frac{2}{\sqrt{\pi}} N_C \exp(E_F / kT) \int_0^\infty x^{1/2} e^{-x} dx. \tag{14}$$

The integral in Eq. 14 is of the standard form and equals $\sqrt{\pi}/2$. Therefore, Eq. 14 becomes

$$n = N_C \exp(E_F/kT). \tag{15}$$

If we refer to the bottom of the conduction band as E_C instead of $E = 0$, we obtain for the electron density in the conduction band

$$\boxed{n = N_C \exp[-(E_C - E_F)/kT],} \tag{16}$$

where N_C defined in Eq.13 is the *effective density of states* in the conduction band. At room temperature (300 K), N_C is 2.86×10^{19} cm^{-3} for silicon and 4.7×10^{17} cm^{-3} for gallium arsenide.

Similarly, we can obtain the hole density p in the valence band:

$$\boxed{p = N_V \exp[-(E_F - E_V)/kT],} \tag{17}$$

and

$$N_V \equiv 2(2\pi m_p kT/h^2)^{3/2}, \tag{18}$$

where N_V is the *effective density of states in the valence band* for both Si and GaAs. At room temperature, N_V is 2.66×10^{19} cm^{-3} for silicon and 7.0×10^{18} cm^{-3} for gallium arsenide.

For an intrinsic semiconductor, the number of electrons per unit volume in the conduction band is equal to the number of holes per unit volume in the valence band, that is, $n = p = n_i$ where n_i is the *intrinsic carrier density*. This relationship of electrons and holes is depicted in Fig. 21*d*. Note that the shaded area in the conduction band is the same as that in the valence band.

The Fermi level for an intrinsic semiconductor is obtained by equating Eq.16 and Eq. 17:

$$E_F = E_i = (E_C + E_V)/2 + (kT/2) \ln(N_V/N_C). \tag{19}$$

At room temperature, the second term is much smaller than the bandgap. Hence, the intrinsic Fermi level E_i of an intrinsic semiconductor generally lies very close to the middle of the bandgap.

The intrinsic carrier density is obtained from Eqs. 16, 17, and 19:

$$\boxed{np = n_i^2,} \tag{20}$$

$$\boxed{n_i^2 = N_C N_V \exp(-E_g/kT),} \tag{21}$$

and

$$\boxed{n_i = \sqrt{N_C N_V} \exp(-E_g/2kT),} \tag{22}$$

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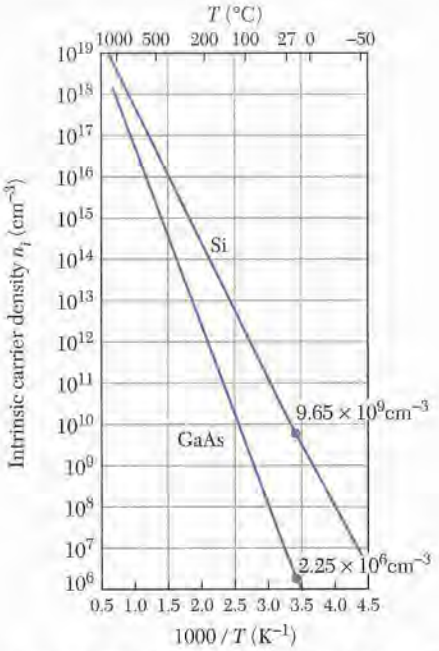


Fig. 22 Intrinsic carrier densities in Si and GaAs as a function of the reciprocal of temperature.[5-7]

where $E_g \equiv E_C - E_V$. Figure 22 shows the temperature dependence of n_i for silicon and gallium arsenide.[5] At room temperature (300 K), n_i is 9.65×10^9 cm^{-3} for silicon[6] and 2.25×10^6 cm^{-3} for gallium arsenide.[7] As expected, the larger the bandgap, the smaller the intrinsic carrier density.

2.7 DONORS AND ACCEPTORS

When a semiconductor is doped with impurities, the semiconductor becomes *extrinsic* and impurity energy levels are introduced. Figure 23*a* shows schematically that a silicon atom is replaced (or substituted) by an arsenic atom with five valence electrons. The arsenic atom forms covalent bonds with its four neighboring silicon atoms. The fifth electron has a relatively small binding energy to its host arsenic atom and can be "ionized" to become a conduction electron at a moderate temperature. We say that this electron has been "donated" to the conduction band. The arsenic atom is called a *donor* and the silicon becomes *n*-type because of the addition of the negative charge carrier. Similarly, Fig. 23*b* shows that when a boron atom with three valence electrons substitutes for a silicon atom, an additional electron is "accepted" to form four covalent bonds around the boron, and a positively charged "hole" is created in the valence band. This is a *p*-type semiconductor, and the boron is an *acceptor*.

We can estimate the *ionization energy* for the donor E_D by replacing m_0 with the electron effective mass m_n and taking into account the semiconductor permittivity ε_s in the hydrogen atom model, Eq. 5:

$$E_D = \left(\frac{\varepsilon_0}{\varepsilon_S}\right)^2 \left(\frac{m_n}{m_0}\right) E_H. \tag{23}$$

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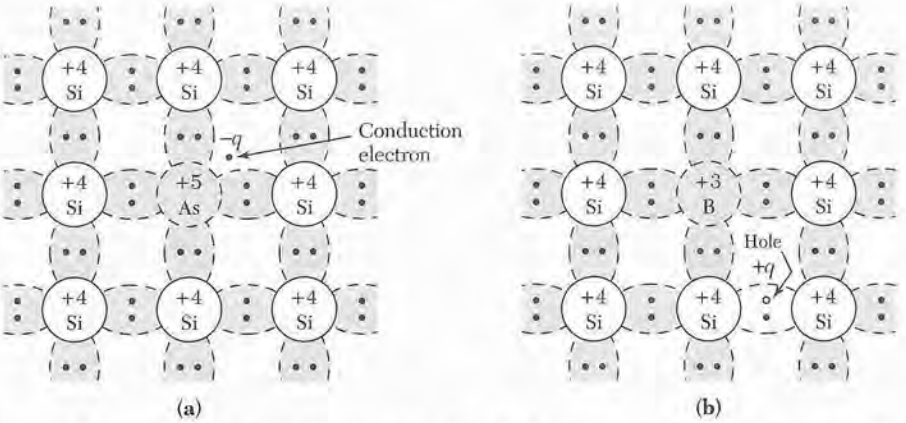


Fig. 23 Schematic bond pictures for (*a*) *n*-type Si with donor (arsenic) and (*b*) *p*-type Si with acceptor (boron).

The ionization energy for donors, measured from the conduction band edge, as calculated from Eq. 23 is 0.025 eV for silicon and 0.007 eV for gallium arsenide. The hydrogen atom calculation for the ionization level of acceptors is similar to that for donors. We consider the unfilled valence band as a filled band plus a hole in the central force field of a negative charged acceptor. The calculated ionization energy, measured from the valence band edge, is 0.05 eV for both silicon and gallium arsenide.

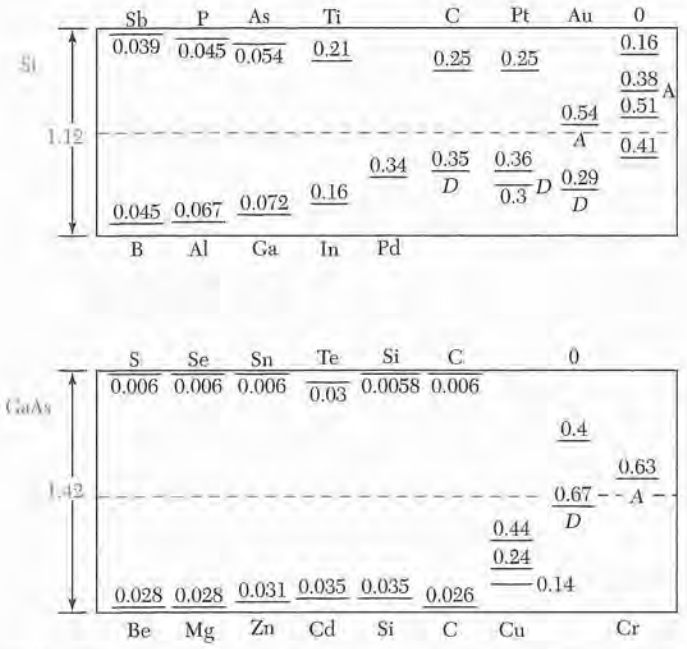


Fig. 24 Measured ionization energies (in eV) for various impurities in Si and GaAs. The levels below the gap center are measured from the top of the valence band and are acceptor levels unless indicated by *D* for donor level. The levels above the gap center are measured from the bottom of the conduction band and are donor levels unless indicated by *A* for acceptor level.[8]

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This simple hydrogen atom model cannot account for the details of the ionization energy, particularly for the deep impurity levels in semiconductors (i.e., with ionization energies $\geq 3\,kT$). However, the calculated values do predict the correct order of magnitude of the true ionization energies for shallow impurity levels. Figure 24 shows the measured ionization energies for various impurities in silicon and gallium arsenide.[8] Note that it is possible for a single atom to have many levels; for example, oxygen in silicon has two donor levels and two acceptor levels in the forbidden energy gap.

2.7.1 Nondegenerate Semiconductor

In our previous discussion, we have assumed that the electron or hole concentration is much lower than the effective density of states in the conduction band or the valence band, respectively. In other words, the Fermi level E_F is at least $3kT$ above E_V or $3kT$ below E_C. For such cases, the semiconductor is referred to as a *nondegenerate* semiconductor.

For shallow donors in silicon and gallium arsenide, there usually is enough thermal energy to supply the energy E_D to ionize all donor impurities at room temperature and thus provide the same number of electrons in the conduction band. This condition is called complete ionization. Under a complete ionization condition, we can write the electron density as

$$n = N_D, \tag{24}$$

where N_D is the donor concentration. Figure 25*a* illustrates complete ionization where the donor level E_D is measured with respect to the bottom of the conduction band and equal concentrations of electrons (which are mobile) and donor ions (which are immobile) are shown. From Eqs. 16 and 24, we obtain the Fermi level in terms of the effective density of states N_C and the donor concentration N_D:

$$E_C - E_F = kT \ln (N_C/N_D). \tag{25}$$

Similarly, for shallow acceptors as shown in Fig. 25*b*, if there is complete ionization, the concentration of holes is

$$p = N_A, \tag{26}$$

Where N_A is the acceptor concentration. We can obtain the corresponding Fermi level from Eqs. 17 and 26:

$$E_F - E_V = kT \ln (N_V/N_A). \tag{27}$$

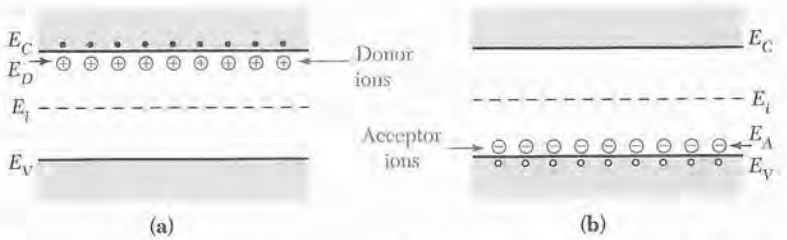


Fig. 25 Schematic energy band representation of extrinsic semiconductors with (*a*) donor ions and (*b*) acceptor ions.

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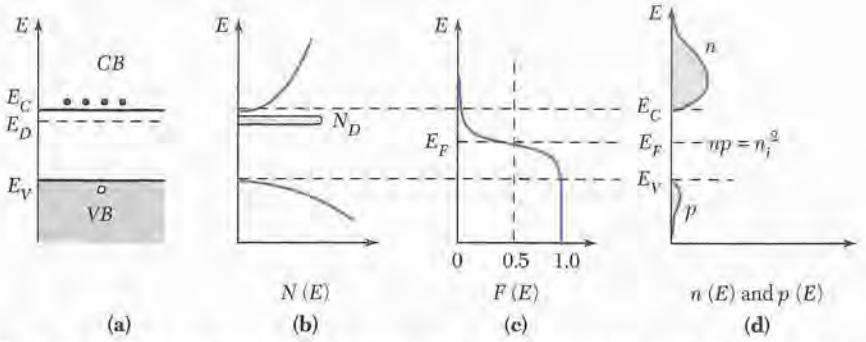


Fig. 26 n-Type semiconductor. (*a*) Schematic band diagram. (*b*) Density of states. (*c*) Fermi distribution function. (*d*) Carrier concentration. Note that $np = n_i^2$.

From Eq. 25 we can see that the higher the donor concentration, the smaller the energy difference $(E_C - E_F)$, that is, the Fermi level will move closer to the bottom of the conduction band. Similarly, for higher acceptor concentration, the Fermi level will move closer to the top of the valence band. Figure 26 illustrates the procedure for obtaining the carrier concentration. This figure is similar to that shown in Fig. 21. However, the Fermi level is closer to the bottom of the conduction band, and the electron concentration (upper shaded area) is much larger than the hole concentration (lower shaded area).

It is useful to express electron and hole densities in terms of the intrinsic carrier concentration n_i and the intrinsic Fermi level E_i since E_i is frequently used as a reference level when discussing extrinsic semiconductors. From Eq.16 we obtain

$$n = N_C \exp[-(E_C - E_F)/kT],$$
$$= N_C \exp[-(E_C - E_i)/kT] \exp[(E_F - E_i)/kT],$$

or

$$\boxed{n = n_i \exp[(E_F - E_i)/kT],} \tag{28}$$

and similarly,

$$\boxed{p = n_i \exp[(E_i - E_F)/kT].} \tag{29}$$

Note that the product of n and p from Eqs. 28 and 29 equals n_i^2. This result is identical to that for the intrinsic case, Eq. 20. Equation 20 is called the *mass action law*, which is valid for both intrinsic and extrinsic semiconductors under thermal equilibrium conduction. In an extrinsic semiconductor, the Fermi level moves toward either the bottom of the conduction band (n-type) or the top of the valence band (p-type). Either n- or p-type carriers will then dominate, but the product of the two types of carriers will remain constant at a given temperature.

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EXAMPLE 4

A silicon ingot is doped with 10^{16} arsenic atoms/cm^3. Find the carrier concentrations and the Fermi level at room temperature (300 K).

SOLUTION At 300 K, we can assume complete ionization of impurity atoms. We have

$$n \approx N_D = 10^{16}\,\text{cm}^{-3}.$$

From Eq. 20, $$p \approx n_i^2/N_D = (9.65 \times 10^9)^2/10^{16} = 9.3 \times 10^3\,\text{cm}^{-3}.$$

The Fermi level measured from the bottom of the conduction band is given by Eq. 25:

$$E_C - E_F = kT \ln(N_C/N_D)$$
$$= 0.0259 \ln(2.86 \times 10^{19}/10^{16}) = 0.205\ \text{eV}.$$

The Fermi level measured from the intrinsic Fermi level is given by Eq. 28:

$$E_F - E_i \approx kT \ln(N_D/n_i)$$
$$= 0.0259 \ln(10^{16}/9.65 \times 10^9) = 0.358\ \text{eV}.$$

These results are shown graphically in Fig. 27. ◀

If both donor and acceptor impurities are present simultaneously, the impurity that is present in a greater concentration determines the type of conductivity in the semiconductor. The Fermi level must adjust itself to preserve charge neutrality, that is, the total negative charges (electrons and ionized acceptors) must equal the total positive charges (holes and ionized donors). Under complete ionization condition, we have

$$\boxed{n + N_A = p + N_D.} \tag{30}$$

Solving Eqs. 20 and 30 yields the equilibrium electron and hole concentrations in an n-type semiconductor:

$$\boxed{n = \tfrac{1}{2}\left[N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2}\right],} \tag{31}$$

$$\boxed{p_n = n_i^2/n_n.} \tag{32}$$

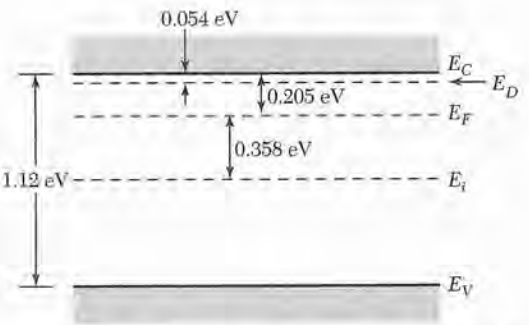


Fig. 27 Band diagram showing Fermi level E_F and intrinsic Fermi level E_i.

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The subscript n refers to the n-type semiconductor. Because the electron is the dominant carrier, it is called the *majority carrier*. The hole in the n-type semiconductor is called the *minority carrier*. Similarly, we obtain the concentration of holes (majority carrier) and electrons (minority carrier) in a p-type semiconductor:

$$p_p = \tfrac{1}{2}\left[N_A - N_D + \sqrt{(N_D - N_A)^2 + 4n_i^2} \right], \tag{33}$$

$$n_p = n_i^2 / p_p \tag{34}$$

The subscript p refers to the p-type semiconductor.

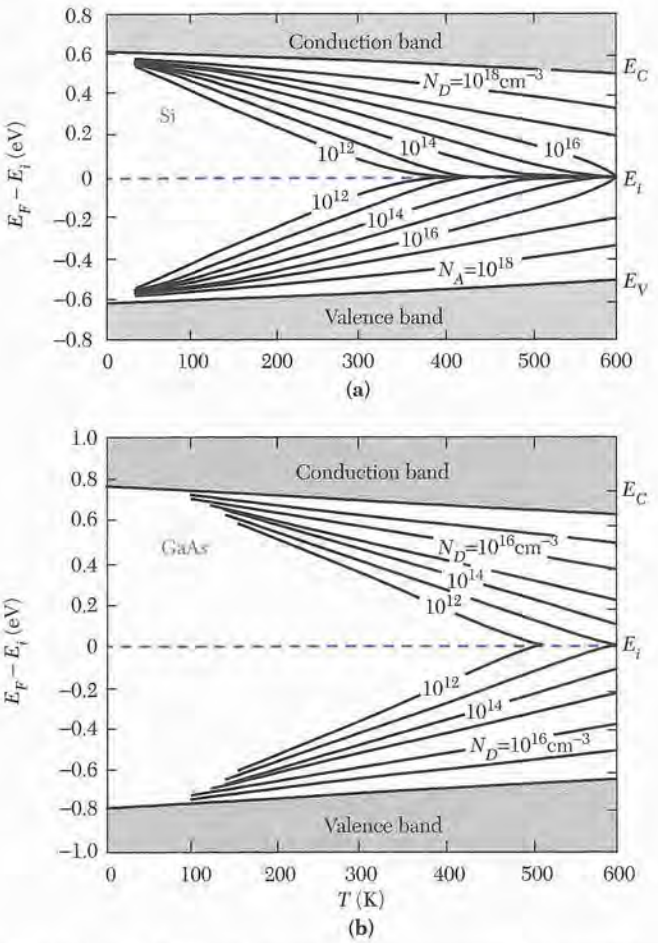


Fig. 28 Fermi level for Si and GaAs as a function of temperature and impurity concentration. The dependence of the bandgap on temperature is shown.[9]

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Generally, the magnitude of the net impurity concentration $|N_D - N_A|$ is larger than the intrinsic carrier concentration n_i; therefore, the above relationships can be simplified to

$$n_n \approx N_D - N_A \quad \text{if} \quad N_D > N_A, \tag{35}$$

$$p_p \approx N_A - N_D \quad \text{if} \quad N_A > N_D. \tag{36}$$

From Eqs. 31 to 34 together with Eqs. 16 and 17, we can calculate the position of the Fermi level as a function of temperature for a given acceptor or donor concentration. Figure 28 shows a plot of these calculations for silicon[9] and gallium arsenide. We have incorporated in the figure the variation of the bandgap with temperature (see Problem 8). Note that as the temperature increases, the Fermi level approaches the intrinsic level, that is, the semiconductor becomes intrinsic.

Figure 29 shows electron density in Si as a function of temperature for a donor concentration of $N_D = 10^{15}$ cm^{-3}. At low temperatures, the thermal energy in the crystal is not sufficient to ionize all the donor impurities present. Some electrons are "frozen" at the donor level and the electron density is less than the donor concentration. As the temperature is increased, the condition of complete ionization is reached, (i.e., $n_n = N_D$). As the temperature is further increased, the electron concentration remains essentially the same over a wide temperature range. This is the extrinsic region. However, as the temperature is increased even further, we reach a point where the intrinsic carrier concentration becomes comparable to the donor concentration. Beyond this point, the semiconductor becomes intrinsic. The temperature at which the semiconductor becomes intrinsic depends on the impurity concentrations and the bandgap value and can be obtained from Fig. 22 by setting the impurity concentration equal to n_i.

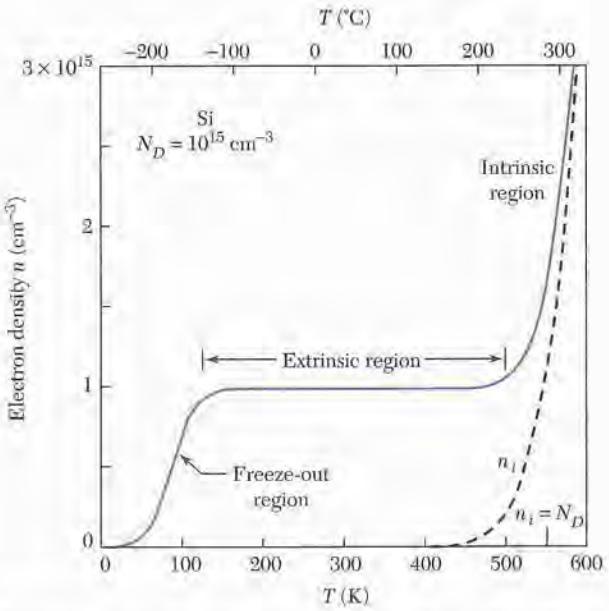


Fig. 29 Electron density as a function of temperature for a Si sample with a donor concentration of 10^{15} cm^{-3}.

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2.7.2 Degenerate Semiconductor

When the doping concentration becomes equal or larger than the corresponding effective density of states, we can no longer use the approximation of Eq. 11 and the electron density, Eq. 9, has to be integrated numerically. For very heavily doped n-type or p-type semiconductor, E_F will be above E_C or below E_V. The semiconductor is referred to as a *degenerate* semiconductor.

An important aspect of high doping is the bandgap narrowing effect, that is, high impurity concentration causes a reduction of the bandgap. The bandgap reduction ΔE_g for silicon at room temperature is given by

$$\Delta E_g = 22\left(\frac{N}{10^{18}}\right)^{1/2} \text{ meV}, \tag{37}$$

where the doping is in cm^{-3}. For example, for $N_D \leq 10^{18}\ \text{cm}^{-3}$, $\Delta E_g \leq 0.022$ eV, which is less than 2% of the original bandgap. However, for $N_D \geq N_C = 2.86 \times 10^{19}\text{cm}^{-3}$, $\Delta E_g \geq 0.12$ eV, which is a significant fraction of E_g.

SUMMARY

At the beginning of the chapter we listed a few important semiconductor materials. The properties of semiconductors are determined to a large extent by the crystal structure. We have defined the Miller indices to describe the crystal surfaces and crystal orientations and briefly described how semiconductor crystals are grown. More detailed discussions can be found in Chapter 10.

The bonding of atoms and the electron energy-momentum relationship in a semiconductor were considered in connection with the electrical properties. The energy band diagram can be used to understand why some materials are good conductors of electric current whereas others are poor conductors. We have also shown that changing the temperature or the amount of impurities can drastically vary the conductivity of a semiconductor.

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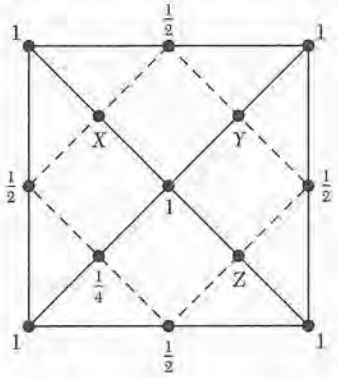
PROBLEMS (* DENOTES DIFFICULT PROBLEMS)

FOR SECTION 2.2 BASIC CRYSTAL STRUCTURE

1. (a) What is the distance between nearest neighbors in silicon?

   (b) Find the number of atoms per square centimeter in silicon in the (100), (110), and (111) planes.

2. If we project the atoms in a diamond lattice onto the bottom surface with the heights of the atoms in unit of the lattice constant shown in the figure below. Find the heights of the three atoms (X, Y, Z) on the figure.



3. Find the maximum fraction of the unit cell volume, which can be filled by identical hard spheres in the simple cubic, face-centered cubic, and diamond lattices.

*4. Calculate the tetrahedral bond angle, the angle between any pair of the four bonds in a diamond lattice. (Hint: represent the four bonds as vectors of equal length. What must be the sum of the four vectors equal? Take components of this vector equation along the direction of one of these vectors.)

5. If a plane has intercepts at $2a$, $3a$, and $4a$ along the three Cartesian coordinates, where a is the lattice constant, find the Miller indices of the planes.

6. (a) Calculate the density of GaAs (the lattice constant of GaAs is 5.65 Å, and the atomic weights of Ga and As are 69.72 and 74.92 g/mol, respectively).

   (b) A gallium arsenide sample is doped with tin. If the tin displaces gallium atoms in the crystal lattice, are donors or acceptors formed? Why? Is the semiconductor n- or p-type?

FOR SECTION 2.3 BASIC CRYSTAL GROWTH TECHNIQUE

7. (a) Does silicon or silicon dioxide have a higher melting point? Why?

   (b) Why is a seed crystal used for crystal growth?

   (c) Why is the crystal orientation of a wafer important?

   (d) What two variables are used to control the diameter of the silicon rod?

FOR SECTION 2.5 ENERGY BANDS

8. The variation of silicon and GaAs bandgaps with temperature can be expressed as $E_g(T) = E_g(0) - \alpha T^2/(T + \beta)$, where $E_g(0) = 1.17$ eV, $\alpha = 4.73 \times 10^{-4}$ eV/K, and $\beta = 636$ K for

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silicon; and $E_g(0) = 1.519$ eV, $\alpha = 5.405 \times 10^{-4}$ eV/ K, and $\beta = 204$ K for GaAs. Find the bandgaps of Si and GaAs at 100 K and 600 K.

FOR SECTION 2.6 INTRINSIC CARRIER CONCENTRATION

*9. Derive Eq. 17. (Hint: In the valence band, the probability of occupancy of a state by a hole is $[1 - F(E)]$.)

10. At room temperature (300 K) the effective density of states in the valence band is 2.66×10^{19} cm^{-3} for silicon and 7×10^{18} cm^{-3} for gallium arsenide. Find the corresponding effective masses of holes. Compare these masses with the free-electron mass.

11. Calculate the location of E_i in silicon at liquid nitrogen temperature (77 K), at room temperature (300 K), and at 100°C (let $m_p = 1.0\ m_0$ and $m_n = 0.19\ m_0$). Is it reasonable to assume that E_i is in the center of the forbidden gap?

12. Find the kinetic energy of electrons in the conduction band of a nondegenerate n-type semiconductor at 300 K.

13. (a) For a free electron with a velocity of 10^7 cm/s, what is its de Broglie wavelength.

(b) In GaAs, the effective mass of electrons in the conduction band is $0.063\ m_0$. If they have the same velocity, find the corresponding de Broglie wavelength.

14. The intrinsic temperature of a semiconductor is the temperatures at which the intrinsic carrier concentration equals the impurity concentration. Find the intrinsic temperature for a silicon sample doped with 10^{15} phosphorus atoms/cm^3.

FOR SECTION 2.7 DONORS AND ACCEPTORS

15. A silicon sample at $T = 300$ K contains an acceptor impurity concentration of $N_A = 10^{16}$ cm^{-3}. Determine the concentration of donor impurity atoms that must be added so that the silicon is n-type and the Fermi energy is 0.20 eV below the conduction band edge.

16. Draw a simple flat energy band diagram for silicon doped with 10^{16} arsenic atoms/cm^3 at 77 K, 300 K, and 600 K. Show the Fermi level and use the intrinsic Fermi level as the energy reference.

17. Find the electron and hole concentrations and Fermi level in silicon at 300 K (a) for 1×10^{15} boron atoms/cm^3 and (b) for 3×10^{16} boron atoms/cm^3 and 2.9×10^{16} arsenic atoms/cm^3.

18. A Si sample is doped with 10^{17} As atoms/cm^3. What is the equilibrium hole concentration p_0 at 300 K? Where is E_F relative to E_i?

19. Calculate the Fermi level of silicon doped with 10^{15}, 10^{17}, and 10^{19} phosphorus atoms/cm^3 at room temperature, assuming complete ionization. From the calculated Fermi level, check if the assumption of complete ionization is justified for each doping. Assume that the ionized donors is given by $n = N_D[1 - F(E_D)] = \dfrac{N_D}{1 + e^{(E_F - E_D)/kT}}$.

20. For an n-type silicon sample with 10^{16} cm^{-3} phosphorous donor impurities and a donor level at $E_D = 0.045$ eV, find the ratio of the neutral donor density to the ionized donor density at 77 K where the Fermi level is 0.0459 below the bottom of the conduction band. The expression for ionized donors is given in Prob. 19.

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CHAPTER 3

Carrier Transport Phenomena

- 3.1 CARRIER DRIFT
- 3.2 CARRIER DIFFUSION
- 3.3 GENERATION AND RECOMBINATION PROCESSES
- 3.4 CONTINUITY EQUATION
- 3.5 THERMIONIC EMISSION PROCESS
- 3.6 TUNNELING PROCESS
- 3.7 HIGH-FIELD EFFECTS
- SUMMARY

In this chapter, we investigate various transport phenomena in semiconductor devices. The transport processes include drift, diffusion, recombination, generation, thermionic emission, tunneling, and impact ionization. We consider the motion of charge carriers (electrons and holes) in semiconductors under the influence of an electric field and a carrier concentration gradient. We also discuss the concept of the nonequilibrium condition where the carrier concentration product pn is different from its equilibrium value n_i^2. Returning to an equilibrium condition through the generation-recombination processes is considered next. We then derive the basic governing equations for semiconductor device operation, which includes the current density equation and the continuity equation. This is followed by a discussion of thermionic emission and tunneling process. The chapter closes with a brief discussion of high-field effects, which include velocity saturation and impact ionization.

Specifically, we cover the following topics:

- The current density equation and its drift and diffusion components.
- The continuity equation and its generation and recombination components.
- Other transport phenomena, including the thermionic emission, tunneling, transferred-electron effect, and impact ionization.
- Methods to measure key semiconductor parameters such as resistivity, mobility, majority-carrier concentration, and minority-carrier lifetime.

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3.1 CARRIER DRIFT

3.1.1 Mobility

Consider an n-type semiconductor sample with uniform donor concentration in thermal equilibrium. As discussed in Chapter 2, the conduction electrons in the semiconductor conduction band are essentially free particles, since they are not associated with any particular lattice or donor site. The influence of crystal lattices is incorporated in the effective mass of conduction electrons, which differs somewhat from the mass of free electrons. Under thermal equilibrium, the average thermal energy of a conduction electron can be obtained from the theorem for equipartition of energy, $1/2\,kT$ units of energy per degree of freedom, where k is Boltzmann's constant and T is the absolute temperature. The electrons in a semiconductor have three degrees of freedom; they can move about in a three-dimensional space. Therefore, the kinetic energy of the electrons is given by

$$\frac{1}{2} m_n v_{th}^2 = \frac{3}{2} kT, \tag{1}$$

where m_n is the effective mass of electrons and v_{th} is the average thermal velocity. At room temperature (300 K) the thermal velocity of electrons in Eq. 1 is about 10^7 cm/s for silicon and gallium arsenide.

The electrons in the semiconductor are therefore moving rapidly in all directions. The thermal motion of an individual electron may be visualized as a succession of random scattering from collisions with lattice atoms, impurity atoms, and other scattering centers, as illustrated in Fig. 1*a*. The random motion of electrons leads to a zero net displacement of an electron over a sufficiently long period of time. The average distance between collisions is called the *mean free path*, and the average time between collisions is called the *mean free time* τ_c. For a typical value of 10^{-5} cm for the mean free path, τ_c is about 1 ps (ie., $10^{-5}/v_{th} \cong 10^{-12}$ s).

When a small electric field $\mathscr{E}$ is applied to the semiconductor sample, each electron will experience a force $-q\mathscr{E}$ from the field and will be accelerated along the field (in the opposite direction to the field) during the time between collisions. Therefore, an additional velocity component will be superimposed upon the thermal motion of electrons. This additional component is called the *drift velocity*. The combined displacement of an electron due to the random thermal motion and the drift component is illustrated in Fig. 1*b*. Note that there is a net displacement of the electron in the direction opposite to the applied field.

We can obtain the drift velocity v_n by equating the momentum (force × time) applied to an electron during the free flight between collisions to the momentum gained by the electron in the same period. The equality is valid because in a steady state all momen-

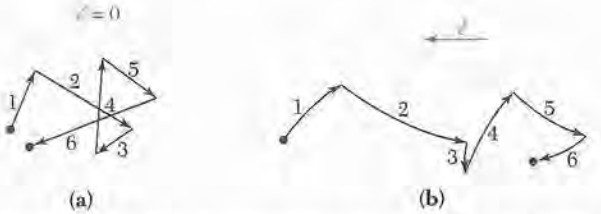


Fig. 1 Schematic path of an electron in a semiconductor. (*a*) Random thermal motion. (*b*) Combined motion due to random thermal motion and an applied electric field.

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tum gained between collisions is lost to the lattice in the collision. The momentum applied to an electron is given by $-q\mathscr{E}\tau_c$, and the momentum gained is $m_n v_n$. We have

$$-q\mathscr{E}\tau_c = m_n v_n \tag{2}$$

or

$$v_n = -\left(\frac{q\tau_c}{m_n}\right)\mathscr{E} \tag{2a}$$

Equation 2*a* states that the electron drift velocity is proportional to the applied electric field. The proportionality factor depends on the mean free time and the effective mass. The proportionality factor is called the *electron mobility* μ_n in units of $cm^2/V\text{-}s$, or

$$\mu_n \equiv \frac{q\tau_c}{m_n} \tag{3}$$

Thus,

$$\boxed{v_n = -\mu_n \mathscr{E}.} \tag{4}$$

Mobility is an important parameter for carrier transport because it describes how strongly the motion of an electron is influenced by an applied electric field. A similar expression can be written for holes in the valence band:

$$\boxed{v_p = \mu_p \mathscr{E},} \tag{5}$$

where v_p is the hole drift velocity and μ_p is the hole mobility. The negative sign is removed in Eq. 5 because holes drift in the same direction as the electric field.

In Eq. 3 the mobility is related directly to the mean free time between collisions, which in turn is determined by the various scattering mechanisms. The two most important mechanisms are lattice scattering and impurity scattering. Lattice scattering results from thermal vibrations of the lattice atoms at any temperature above absolute zero. These vibrations disturb the lattice periodic potential and allow energy to be transferred between the carriers and the lattice. Since lattice vibration increases with increasing temperature, lattice scattering becomes dominant at high temperatures; hence the mobility decreases with increasing temperature. Theoretical analysis[1] shows that the mobility due to lattice scattering μ_L will decrease in proportion to $T^{-3/2}$.

Impurity scattering results when a charge carrier travels past an ionized dopant impurity (donor or acceptor). The charge carrier path will be deflected owing to Coulomb force interaction. The probability of impurity scattering depends on the total concentration of ionized impurities, that is, the sum of the concentration of negatively and positively charged ions. However, unlike lattice scattering, impurity scattering becomes less significant at higher temperatures. At higher temperatures, the carriers move faster; they remain near the impurity atom for a shorter time and are therefore less effectively scattered. The mobility due to impurity scattering μ_I can theoretically be shown to vary as $T^{3/2}/N_T$, where N_T is the total impurity concentration.[2]

The probability of a collision taking place in unit time, $1/\tau_c$, is the sum of the probabilities of collisions due to the various scattering mechanisms:

$$\frac{1}{\tau_c} = \frac{1}{\tau_{c,\,\text{lattice}}} + \frac{1}{\tau_{c,\,\text{impurity}}}, \tag{6}$$

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or

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I}. \tag{6a}$$

Figure 2 shows the measured electron mobility as a function of temperature for silicon with five different donor concentrations.[3] The inset shows the theoretical temperature dependence of mobility due to both lattice and impurity scatterings. For lightly doped samples (e.g., the sample with doping of 10^{14} cm^{-3}), the lattice scattering dominates, and the mobility decreases as the temperature increases. For heavily doped samples, the effect of impurity scattering is most pronounced at low temperatures. The mobility increases as the temperature increases, as can be seen for the sample with doping of 10^{19} cm^{-3}. For a given temperature, the mobility decreases with increasing impurity concentration because of enhanced impurity scatterings.

Figure 3 shows the measured mobilities in silicon and gallium arsenide as a function of impurity concentration at room temperature.[3] Mobility reaches a maximum value at low impurity concentrations; this corresponds to the lattice-scattering limitation. Both electron and hole mobilities decrease with increasing impurity concentration and eventually approach a minimum value at high concentrations. Note also that the mobility of electrons is greater than that of holes. Greater electron mobility is due mainly to the smaller effective mass of electrons.

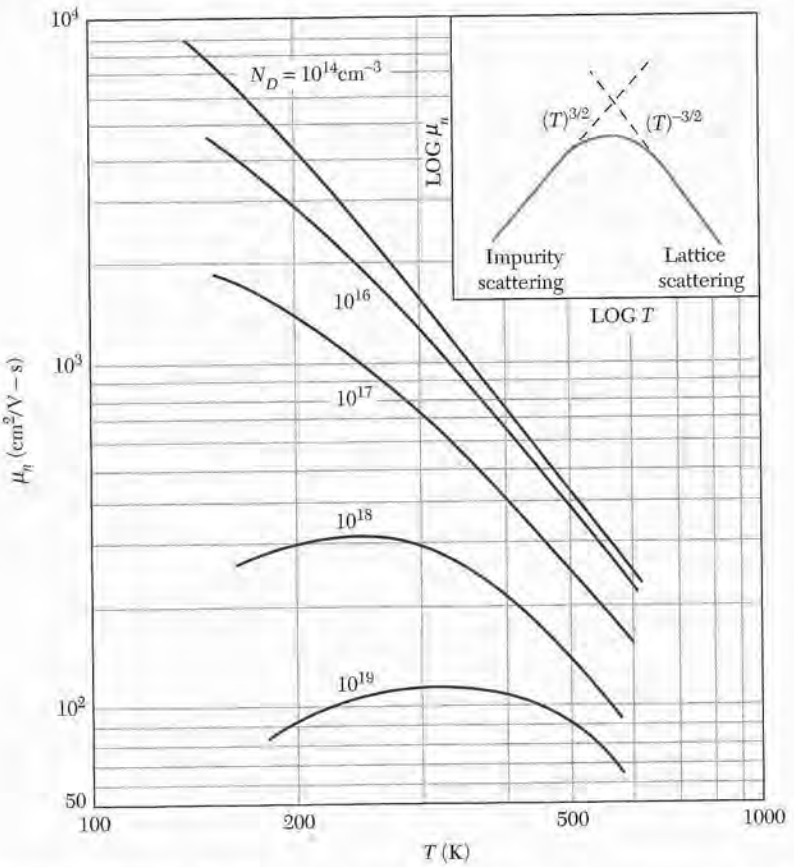


Fig. 2 Electron mobility in silicon versus temperature for various donor concentrations. Insert shows the theoretical temperature dependence of electron mobility.[3]

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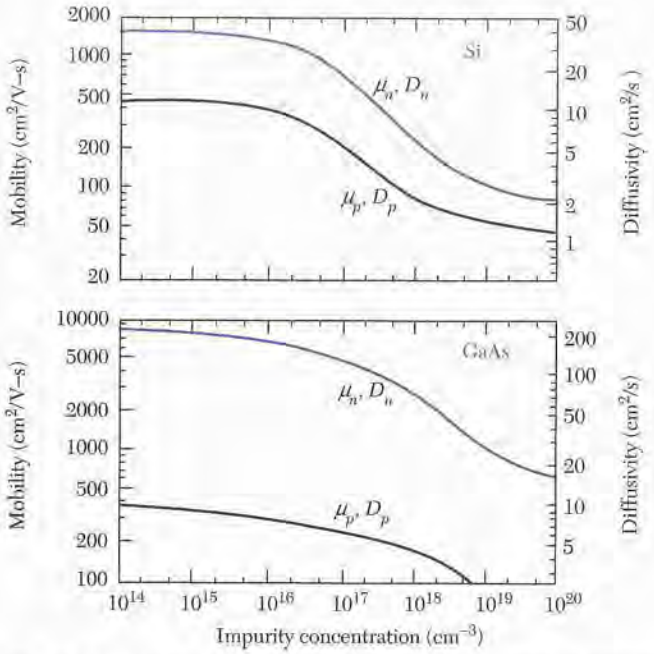


Fig. 3 Mobilities and diffusivities in Si and GaAs at 300 K as a function of impurity concentration.[3]

EXAMPLE 1

Calculate the mean free time of an electron having a mobility of 1000 cm²/V-s at 300 K; also calculate the mean free path. Assume $m_n = 0.26\, m_0$ in these calculations.

SOLUTION From Eq. 3, the mean free time is given by

$$\tau_c = \frac{m_n \mu_n}{q} = \frac{(0.26 \times 0.91 \times 10^{-30}\ \text{kg}) \times (1000 \times 10^{-4}\ \text{m}^2/\text{V-s})}{1.6 \times 10^{-19}\ \text{C}}$$
$$= 1.48 \times 10^{-13}\ \text{s} = 0.148\ \text{ps}.$$

The mean free path is given by

$$l = v_{th}\tau_c = (10^7\ \text{cm/s})(1.48 \times 10^{-13}\text{s}) = 1.48 \times 10^{-6}\ \text{cm}$$
$$= 14.8\ \text{nm}.$$

3.1.2 Resistivity

We now consider conduction in a homogeneous semiconductor material. Figure 4*a* shows an *n*-type semiconductor and its band diagram at thermal equilibrium. Figure 4*b* shows the corresponding band diagram when a biasing voltage is applied to the right-hand terminal. We assume that the contacts at the left-hand and right-hand terminals are ohmic, that is, there is negligible voltage drop at each of the contacts. The behavior of ohmic contacts are considered in Chapter 7. As mentioned previously, when an electric field $\mathscr{E}$

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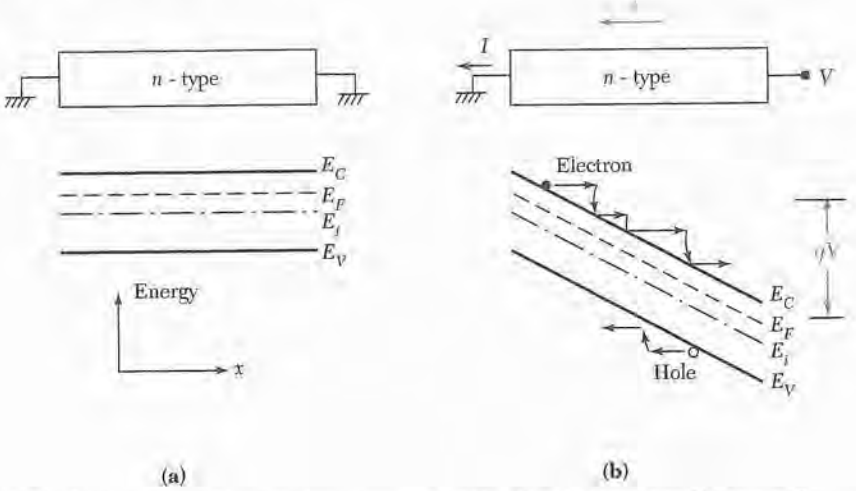


Fig. 4 Conduction process in an n-type semiconductor (a) at thermal equilibrium and (b) under a biasing condition.

is applied to a semiconductor, each electron will experience a force $-q\mathscr{E}$ from the field. The force is equal to the negative gradient of potential energy; that is,

$$-q\mathscr{E} = -(\text{gradient of electron potential energy}) = -\frac{dE_C}{dx}. \tag{7}$$

Recall that in Chapter 2 the bottom of the conduction band E_C corresponds to the potential energy of an electron. Since we are interested in the gradient of the potential energy, we can use any part of the band diagram that is parallel to E_C (e.g., E_F, E_i, or E_V, as shown in Fig. 4b). It is convenient to use the intrinsic Fermi level E_i because we shall use E_i when we consider p–n junctions in Chapter 4. Therefore, from Eq. 7 we have

$$\mathscr{E} = \frac{1}{q}\frac{dE_C}{dx} = \frac{1}{q}\frac{dE_i}{dx}. \tag{8}$$

We can define a related quantity ψ as the *electrostatic potential* whose negative gradient equals the electric field:

$$\mathscr{E} \equiv -\frac{d\psi}{dx}. \tag{9}$$

Comparison of Eqs. 8 and 9 gives

$$\psi = -\frac{E_i}{q}, \tag{10}$$

which provides a relationship between the electrostatic potential and the potential energy of an electron. For a homogeneous semiconductor shown in Fig. 4b, the potential energy and E_i decrease linearly with distance; thus, the electric field is a constant in the negative x-direction. Its magnitude equals the applied voltage divided by the sample length.

The electrons in the conduction band move to the right side as shown in Fig. 4b. The kinetic energy corresponds to the distance from the band edge (i.e., E_C for electrons). When an electron undergoes a collision, it loses some or all of its kinetic energy to the lattice and drops toward its thermal equilibrium position. After the electron has lost some

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or all its kinetic energy, it will again begin to move toward the right, and the same process will be repeated many times. Conduction by holes can be visualized in a similar manner but in the opposite direction.

The transport of carriers under the influence of an applied electric field produces a current called the *drift current*. Consider a semiconductor sample shown in Fig. 5, which has a cross-sectional area A, a length L, and a carrier concentration of n electrons/cm^3. Suppose we now apply an electric field $\mathscr{E}$ to the sample. The electron current density J_n flowing in the sample can be found by summing the product of the charge $(-q)$ on each electron times the electron's velocity over all electrons per unit volume n:

$$J_n = \frac{I_n}{A} = \sum_{i=1}^{n}(-qv_i) = -qnv_n = qn\mu_n\mathscr{E}, \tag{11}$$

where I_n is the electron current. We have employed Eq. 4 for the relationship between v_n and $\mathscr{E}$.

A similar argument applies to holes. By taking the charge on the hole to be positive, we have

$$J_p = qpv_p = qp\mu_p\mathscr{E}. \tag{12}$$

The total current flowing in the semiconductor sample due to the applied field $\mathscr{E}$ can be written as the sum of the electron and hole current components:

$$J = J_n + J_p = (qn\mu_n + qp\mu_p)\mathscr{E}. \tag{13}$$

The quantity in parentheses is known as *conductivity*:

$$\sigma = q(n\mu_n + p\mu_p). \tag{14}$$

The electron and hole contributions to conductivity are simply additive.

The corresponding resistivity of the semiconductor, which is the reciprocal of σ, is given by

$$\boxed{\rho \equiv \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}} \tag{15}$$

Generally, in extrinsic semiconductors, only one of the components in Eq. 13 or 14 is significant because of the many orders-of-magnitude difference between the two carrier densities. Therefore, Eq. 15 reduces to

$$\rho = \frac{1}{qn\mu_n} \tag{15a}$$

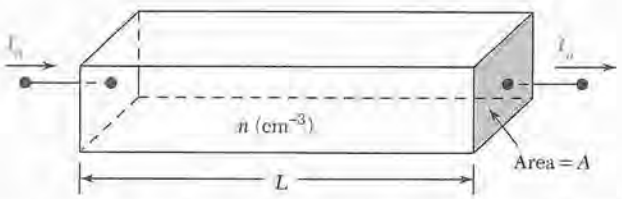


Fig. 5 Current conduction in a uniformly doped semiconductor bar with length L and cross-sectional area A.

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for an n-type semiconductor (since $n >> p$), and to

$$\rho = \frac{1}{qp\mu_p} \tag{15b}$$

for a p-type semiconductor (since $p >> n$).

The most common method for measuring resistivity is the four-point probe method shown in Fig. 6. The probes are equally spaced. A small current I from a constant-current source is passed through the outer two probes and a voltage V is measured between the inner two probes. For a thin semiconductor sample with thickness W that is much smaller than the sample diameter d, the resistivity is given by

$$\rho = \frac{V}{I} \cdot W \cdot CF \qquad \Omega\text{-cm}, \tag{16}$$

where CF is a well-documented "correction factor." The correction factor depends on the ratio of d/s, where s is the probe spacing. When $d/s > 20$, the correction factor approaches 4.54.

Figure 7 shows the measured room-temperature resistivity as a function of the impurity concentration for silicon and gallium arsenide.[3] At this temperature and for low impurity concentrations, all donor (e.g., P and As in Si) or acceptor (e.g., B in Si) impurities that have shallow energy levels will be ionized. Under these conditions, the carrier concentration is equal to the impurity concentration. From these curves we can obtain the impurity concentration of a semiconductor if the resistivity is known, or vice versa.

EXAMPLE 2

Find the room-temperature resistivity of an n-type silicon doped with 10^{16} phosphorus atoms/cm^3.

SOLUTION At room temperature we assume that all donors are ionized; thus,

$$n \approx N_D = 10^{16}\ \text{cm}^{-3}.$$

From Fig. 7 we find $\rho \cong 0.5\ \Omega$–cm. We can also calculate the resistivity from Eq. 15*a*:

$$\rho = \frac{1}{qn\mu_n} = \frac{1}{1.6 \times 10^{-19} \times 10^{16} \times 1300} = 0.48 \qquad \Omega\text{-cm}.$$

The mobility μ_n is obtained from Fig. 3.

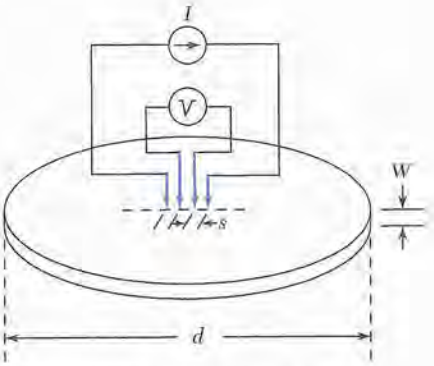


Fig. 6 Measurement of resistivity using a four-point probe.[3]

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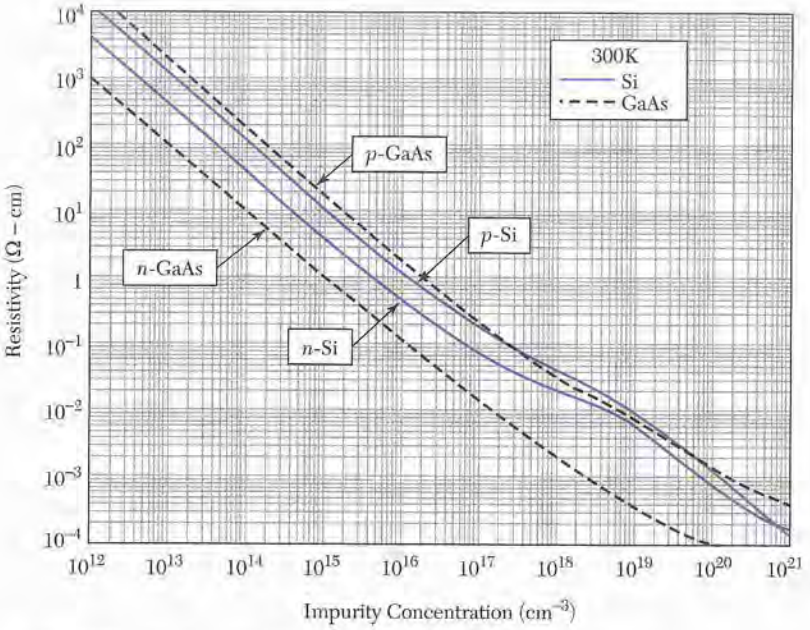


Fig. 7 Resistivity versus impurity concentration[3] for Si and GaAs.

3.1.3 The Hall Effect

The carrier concentration in a semiconductor may be different from the impurity concentration, because the ionized impurity density depends on the temperature and the impurity energy level. To measure the carrier concentration directly, the most commonly used method is the Hall effect. Hall measurement is also one of the most convincing methods to show the existence of holes as charge carriers, because the measurement can give directly the carrier type. Figure 8 shows an electric field applied along the x-axis and a magnetic field applied along the z-axis. Consider a p-type semiconductor sample. The Lorentz force $q\mathbf{v} \times \mathbf{B}$ ($= qv_xB_z$) due to the magnetic field will exert an average upward force on the holes flowing in the x-direction. The upward directed current causes an

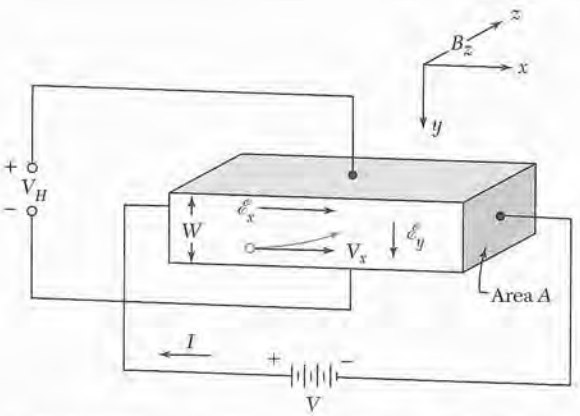


Fig. 8 Basic setup to measure carrier concentration using the Hall effect.

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accumulation of holes at the top of the sample that gives rise to a downward-directed electric field $\mathscr{E}_y$. Since there is no net current flow along the y-direction in the steady state, the electric field along the y-axis exactly balances the Lorentz force; that is,

$$q\mathscr{E}_y = qv_x B_z, \tag{17}$$

or

$$\mathscr{E}_y = v_x B_z. \tag{18}$$

Once the electric field $\mathscr{E}_y$ becomes equal to $v_x B_z$, no net force along the y-direction is experienced by the holes as they drift in the x-direction.

The establishment of the electric field is known as the *Hall effect*. The electric field in Eq. 18 is called the *Hall field*, and the terminal voltage $V_H = \mathscr{E}_y W$ (Fig. 8) is called the *Hall voltage*. Using Eq. 12 for the hole drift velocity, the Hall field $\mathscr{E}_y$ in Eq. 18 becomes

$$\mathscr{E}_y = \left(\frac{J_p}{qp}\right) B_z = R_H J_p B_z, \tag{19}$$

where

$$R_H \equiv \frac{1}{qp}. \tag{20}$$

The Hall field $\mathscr{E}_y$ is proportional to the product of the current density and the magnetic field. The proportionality constant R_H is the *Hall coefficient*. A similar result can be obtained for an n-type semiconductor, except that the Hall coefficient is negative:

$$R_H = -\frac{1}{qn}. \tag{21}$$

A measurement of the Hall voltage for a known current and magnetic field yields

$$p = \frac{1}{qR_H} = \frac{J_p B_z}{q\mathscr{E}_y} = \frac{(I/A)B_z}{q(V_H/W)} = \frac{IB_z W}{qV_H A}, \tag{22}$$

where all the quantities in the right-hand side of the equation can be measured. Thus, the carrier concentration and carrier type can be obtained directly from the Hall measurement.

EXAMPLE 3

A sample of Si is doped with 10^{16} phosphorus atoms/cm^3. Find the Hall voltage in a sample with $W = 500\ \mu\text{m}$, $A = 2.5 \times 10^{-3}$ cm^2, $I = 1$ mA, and $B_z = 10^{-4}$ Wb/cm^2.

SOLUTION The Hall coefficient is

$$R_H = -\frac{1}{qn} = -\frac{1}{1.6\times10^{-19}\times10^{16}} = -625\ \text{cm}^3/\text{C}.$$

The Hall voltage is

$$\begin{aligned} V_H = \mathscr{E}_y W &= \left(R_H \frac{I}{A} B_z\right) W \\ &= \left(-625 \cdot \frac{10^{-3}}{2.5\times10^{-3}} \cdot 10^{-4}\right) 500\times10^{-4} \\ &= -1.25\ \text{mV}. \end{aligned}$$

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3.2 CARRIER DIFFUSION

3.2.1 Diffusion Process

In the preceding section, we considered the drift current, that is, the transport of carriers when an electric field is applied. Another important current component can exist if there is a spatial variation of carrier concentration in the semiconductor material. The carriers tend to move from a region of high concentration to a region of low concentration. This current component is called *diffusion current*.

To understand the diffusion process, let us assume an electron density that varies in the x-direction, as shown in Fig. 9. The semiconductor is at uniform temperature, so that the average thermal energy of electrons does not vary with x, only the density $n(x)$ varies. Consider the number of electrons crossing the plane at $x = 0$ per unit time and per unit area. Because of finite temperature, the electrons have random thermal motions with a thermal velocity v_{th} and a mean free path l (note that $l = v_{th}\tau_c$, where τ_c is the mean free time.) The electrons at $x = -l$, one mean free path away on the left side, have equal chances of moving left or right; and in a mean free time τ_c, one half of them will move across the plane $x = 0$. The average rate of electron flow per unit area F_1 of electrons crossing plane $x = 0$ from the left is then

$$F_1 = \frac{\frac{1}{2}n(-l) \cdot l}{\tau_c} = \frac{1}{2}n(-l) \cdot v_{th}. \tag{23}$$

Similarly, the average rate of electron flow per unit area F_2 of electrons at $x = l$ crossing plane $x = 0$ from the right is

$$F_2 = \frac{1}{2}n(l) \cdot v_{th}. \tag{24}$$

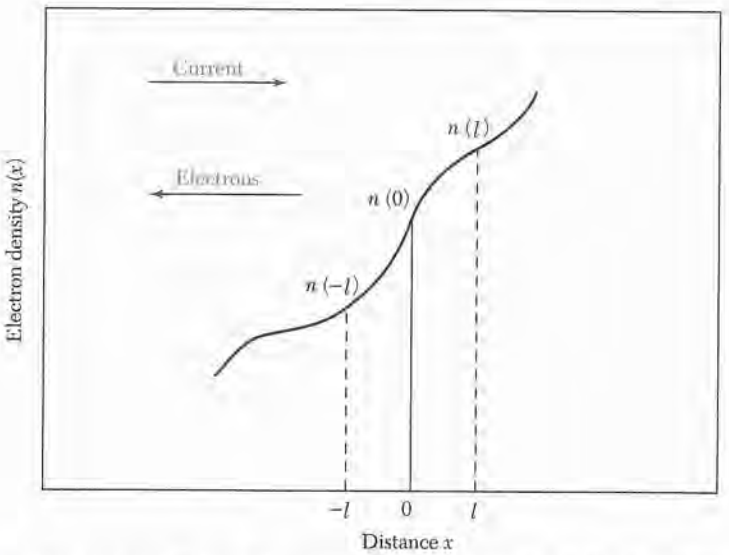


Fig. 9 Electron concentration versus distance; l is the mean free path. The directions of electron and current flows are indicated by arrows.

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The net rate of carrier flow from left to right is

$$F = F_1 - F_2 = \frac{1}{2} v_{th}[n(-l) - n(l)]. \tag{25}$$

Approximating the densities at $x = \pm l$ by the first two terms of a Taylor series expansion, we obtain

$$F = \frac{1}{2} v_{th} \left\{ \left[n(0) - l\frac{dn}{dx} \right] - \left[n(0) + l\frac{dn}{dx} \right] \right\}$$
$$= -v_{th} l \frac{dn}{dx} \equiv -D_n \frac{dn}{dx}, \tag{26}$$

where $D_n \equiv v_{th} l$ is called the *diffusion coefficient* also called the *diffusivity*. Because each electron carriers a charge $-q$, the carrier flow gives rise to a current

$$J_n = -qF = qD_n \frac{dn}{dx}. \tag{27}$$

The diffusion current is proportional to the spatial derivative of the electron density. Diffusion current results from the random thermal motion of carriers in a concentration gradient. For an electron density that increases with x, the gradient is positive, and the electrons will diffuse toward the negative x-direction. The current is positive and flows in the direction opposite to that of the electrons as indicated in Fig. 9.

EXAMPLE 4

Assume that, in an n-type semiconductor at $T = 300$ K, the electron concentration varies linearly from 1×10^{18} to 7×10^{17} cm^{-3} over a distance of 0.1 cm. Calculate the diffusion current density if the electron diffusion coefficient is $D_n = 22.5$ cm^2/s.

SOLUTION The diffusion current density is given by

$$J_{n,\,\text{diff}} = qD_n \frac{dn}{dx} \approx qD_n \frac{\Delta n}{\Delta x}$$
$$= (1.6 \times 10^{-19})(22.5)\left(\frac{1 \times 10^{18} - 7 \times 10^{17}}{0.1} \right) = 10.8 \quad \text{A/cm}^2.$$

3.2.2 Einstein Relation

Equation 27 can be written in a more useful form using the theorem for the equipartition of energy for this one-dimensional case. We can write

$$\frac{1}{2} m_n v_{th}^{\,2} = \frac{1}{2} kT. \tag{28}$$

From Eqs. 3, 26, and 28 and using the relationship $l = v_{th}\tau_c$, we obtain

$$D_n = v_{th} l = v_{th}(v_{th}\tau_c) = v_{th}^2 \left(\frac{\mu_n m_n}{q} \right) = \left(\frac{kT}{m_n} \right) \left(\frac{\mu_n m_n}{q} \right), \tag{29}$$

or

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$$D_n = \left(\frac{kT}{q}\right)\mu_n. \tag{30}$$

Equation 30 is known as the *Einstein relation*. It relates the two important constants (diffusivity and mobility) that characterize carrier transport by diffusion and by drift in a semiconductor. The Einstein relation also applies between D_p and μ_p. Values of diffusivities for silicon and gallium arsenide are shown in Fig. 3.

EXAMPLE 5

Minority carriers (holes) are injected into a homogeneous n-type semiconductor sample at one point. An electric field of 50 V/cm is applied across the sample, and the field moves these minority carriers a distance of 1 cm in 100 μs. Find the drift velocity and the diffusivity of the minority carriers.

SOLUTION

$$v_p = \frac{1 \text{ cm}}{100 \times 10^{-6} \text{ s}} = 10^4 \quad \text{cm/s};$$

$$\mu_p = \frac{v_p}{\mathscr{E}} = \frac{10^4}{50} = 200 \quad \text{cm}^2/\text{V-s};$$

$$D_p = \frac{kT}{q}\mu_p = 0.0259 \times 200 = 5.18 \quad \text{cm}^2/\text{s}.$$

3.2.3 Current Density Equations

When an electric field is present in addition to a concentration gradient, both drift current and diffusion current will flow. The total current density at any point is the sum of the drift and diffusion components:

$$J_n = q\mu_n n\mathscr{E} + qD_n\frac{dn}{dx}. \tag{31}$$

where $\mathscr{E}$ is the electric field in the x-direction.

A similar expression can be obtained for the hole current:

$$J_p = q\mu_p p\mathscr{E} - qD_p\frac{dp}{dx}. \tag{32}$$

We use the negative sign in Eq. 32 because for a positive hole gradient the holes will diffuse in the negative x-direction. This diffusion results in a hole current that also flows in the negative x-direction.

The total conduction current density is given by the sum of Eqs. 31 and 32:

$$J_{cond} = J_n + J_p. \tag{33}$$

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The three expressions (Eqs. 31–33) constitute the current density equations. These equations are important for analyzing device operations under low electric fields. However, at sufficiently high electric fields the terms $\mu_n \mathscr{E}$ and $\mu_p \mathscr{E}$ should be replaced by the saturation velocity v_s discussed in Section 3.7.

3.3 GENERATION AND RECOMBINATION PROCESSES

In thermal equilibrium the relationship $pn = n_i^2$ is valid. If excess carriers are introduced to a semiconductor so that $pn > n_i^2$, we have a *nonequilibrium situation*. The process of introducing excess carriers is called *carrier injection*. Most semiconductor devices operate by the creation of charge carriers in excess of the thermal equilibrium values. We can introduce excess carriers by optical excitation or forward-biasing a p–n junction (discussed in Chapter 4).

Whenever the thermal-equilibrium condition is disturbed (i.e., $pn \neq n_i^2$), processes exist to restore the system to equilibrium (i.e., $pn = n_i^2$). In the case of the injection of excess carriers, the mechanism that restores equilibrium is recombination of the injected minority carriers with the majority carriers. Depending on the nature of the recombination process, the released energy that results from the recombination process can be emitted as a photon or dissipated as heat to the lattice. When a photon is emitted, the process is called radiative recombination; otherwise, it is called nonradiative recombination.

Recombination phenomena can be classified as direct and indirect processes. Direct recombination, also called band-to-band recombination, usually dominates in direct-bandgap semiconductors, such as gallium arsenide, whereas indirect recombination via bandgap recombination centers dominates in indirect bandgap semiconductors, such as silicon.

3.3.1 Direct Recombination

Consider a direct-bandgap semiconductor in thermal equilibrium. The continuous thermal vibration of lattice atoms causes some bonds between neighboring atoms to be broken. When a bond is broken, an electron-hole pair is generated. In terms of the band diagram, the thermal energy enables a valence electron to make an upward transition to the conduction band, leaving a hole in the valence band. This process is called carrier generation and is represented by the generation rate G_{th} (number of electron-hole pairs generated per cm^3 per second) in Fig. 10*a*. When an electron makes a transition downward from the conduction band to the valence band, an electron-hole pair is annihilated. This reverse process is called recombination; it is represented by the recombination rate R_{th} in Fig. 10*a*. Under thermal equilibrium conditions, the generation rate G_{th} must equal

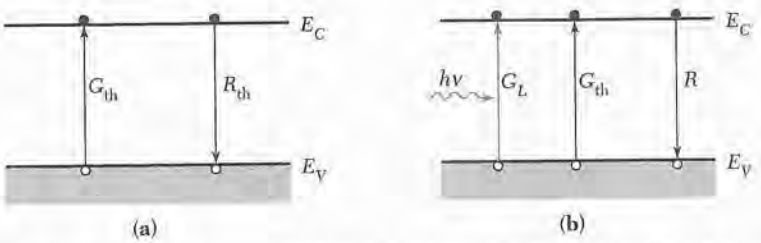


Fig. 10 Direct generation and recombination of electron-hole pairs: (*a*) at thermal equilibrium and (*b*) under illumination.

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the recombination rate R_{th}, so that the carrier concentrations remain constant and the condition $pn = n_i^2$ is maintained.

When excess carriers are introduced to a direct-bandgap semiconductor, the probability is high that electrons and holes will recombine directly, because the bottom of the conduction band and the top of the valence band are lined up and no additional momentum is required for the transition across the bandgap. The rate of the direct recombination R is expected to be proportional to the number of electrons available in the conduction band and the number of holes available in the valence band; that is,

$$R = \beta np, \tag{34}$$

where β is the proportionality constant. As discussed previously, in thermal equilibrium the recombination rate must be balanced by the generation rate. Therefore, for an n-type semiconductor, we have

$$G_{th} = R_{th} = \beta n_{no} p_{no}. \tag{35}$$

In this notation for carrier concentrations the first subscript refers to the type of the semiconductor. The subscript o indicates an equilibrium quantity. The n_{no} and p_{no} represent electron and hole densities, respectively, in an n-type semiconductor at thermal equilibrium. When we shine a light on the semiconductor to produce electron-hole pairs at a rate G_L (Fig. 10*b*), the carrier concentrations are above their equilibrium values. The recombination and generation rate become

$$R = \beta n_n p_n = \beta(n_{no} + \Delta n)(p_{no} + \Delta p), \tag{36}$$

$$G = G_L + G_{th}, \tag{37}$$

where Δn and Δp are the excess carrier concentrations, given by

$$\Delta n = n_n - n_{no} \tag{38a}$$

$$\Delta p = p_n - p_{no}, \tag{38b}$$

and $\Delta n = \Delta p$ to maintain overall charge neutrality.

The net rate of change of hole concentration is given by

$$\frac{dp_n}{dt} = G - R = G_L + G_{th} - R. \tag{39}$$

In steady state, $dp_n / dt = 0$. From Eq. 39 we have

$$G_L = R - G_{th} \equiv U, \tag{40}$$

where U is the net recombination rate. Substituting Eqs. 35 and 36 into Eq. 40 yields

$$U = \beta(n_{no} + p_{no} + \Delta p)\Delta p. \tag{41}$$

For low-level injection Δp, $p_{no} << n_{no}$, Eq. 41 is simplified to

$$U \equiv \beta n_{no} \Delta p = \frac{p_n - p_{no}}{\frac{1}{\beta n_{no}}}, \tag{42}$$

Therefore, the net recombination rate is proportional to excess minority carrier concentration. Obviously, $U = 0$ in thermal equilibrium. The proportionality constant $1/\beta n_{no}$ is called the *lifetime* τ_p of the excess minority carriers, or

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$$U \equiv \frac{p_n - p_{no}}{\tau_p}, \tag{43}$$

where

$$\tau_p \equiv \frac{1}{\beta n_{no}}. \tag{44}$$

The physical meaning of lifetime can best be illustrated by the transient response of a device after the sudden removal of the light source. Consider an n-type sample, as shown in Fig. 11*a*, that is illuminated with light and in which the electron-hole pairs are generated uniformly throughout the sample with a generation rate G_L. The time-dependent expression is given by Eq. 39. In steady state, from Eqs. 40 and 43

$$G_L = U = \frac{p_n - p_{no}}{\tau_p} \tag{45}$$

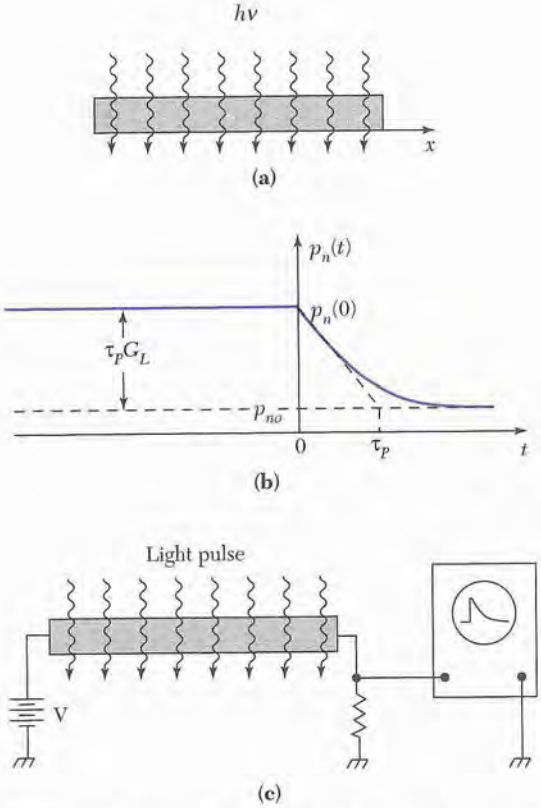


Fig. 11 Decay of photoexcited carriers. (*a*) n-type sample under constant illumination. (*b*) Decay of minority carriers (holes) with time. (*c*) Schematic setup to measure minority carrier lifetime.

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or

$$p_n = p_{no} + \tau_p G_L. \tag{45a}$$

If at an arbitrary time, say $t = 0$, the light is suddenly turned off, the boundary conditions are $p_n(t = 0) = p_{no} + \tau_p G_L$, as given by Eq. 45*a*, and $p_n(t \to \infty) = p_{no}$. The time-dependent expression of Eq. 39 becomes

$$\frac{dp_n}{dt} = G_{th} - R = -U = -\frac{p_n - p_{no}}{\tau_p} \tag{46}$$

and the solution is

$$p_n(t) = p_{no} + \tau_p G_L \exp\left(-t/\tau_p\right). \tag{47}$$

Figure 11*b* shows the variation of p_n with time. The minority carriers recombine with majority carriers and decay exponentially with a time constant τ_p, which corresponds to the lifetime defined in Eq. 44.

This case illustrates the main idea of measuring the carrier lifetime using the photoconductivity method. Figure 11*c* shows a schematic setup. The excess carriers, generated uniformly throughout the sample by the light pulse, cause a momentary increase in the conductivity. The increase in conductivity manifests itself by a drop in voltage across the sample when a constant current is passed through it. The decay of the conductivity can be observed on an oscilloscope and is a measure of the lifetime of the excess minority carriers.

EXAMPLE 6

A Si sample with $n_{no} = 10^{14}$ cm^{-3} is illuminated with light and 10^{13} electron-hole pairs/cm^3 are created every microsecond. If $\tau_n = \tau_p = 2$ μs, find the change in the minority carrier concentration.

SOLUTION Before illumination

$$p_{no} = n_i^2 / n_{no} = (9.65 \times 10^9)^2 / 10^{14} \approx 9.31 \times 10^5 \text{ cm}^{-3}.$$

After illumination

$$p_n = p_{no} + \tau_p G_L = 9.31 \times 10^5 + 2 \times 10^{-6} \times \frac{10^{13}}{1 \times 10^{-6}} \approx 2 \times 10^{13} \text{ cm}^{-3}.$$

3.3.2 Indirect Recombination

For indirect-bandgap semiconductors, such as silicon, a direct recombination process is very unlikely, because the electrons at the bottom of the conduction band have nonzero momentum with respect to the holes at the top of the valence band (see Chapter 2). A direct transition that conserves both energy and momentum is not possible without a simultaneous lattice interaction. Therefore the dominant recombination process in such semiconductors is indirect transition via localized energy states in the forbidden energy gap.[4] These states act as stepping stones between the conduction band and the valence band.

Figure 12 shows various transitions that occur in the recombination process through intermediate-level states (also called recombination centers). We illustrate the charging state of the center before and after each of the four basic transitions takes place. The arrows in the figure designate the transition of the electron in a particular process. The illustration is for the case of a recombination center with a single energy level that is neutral

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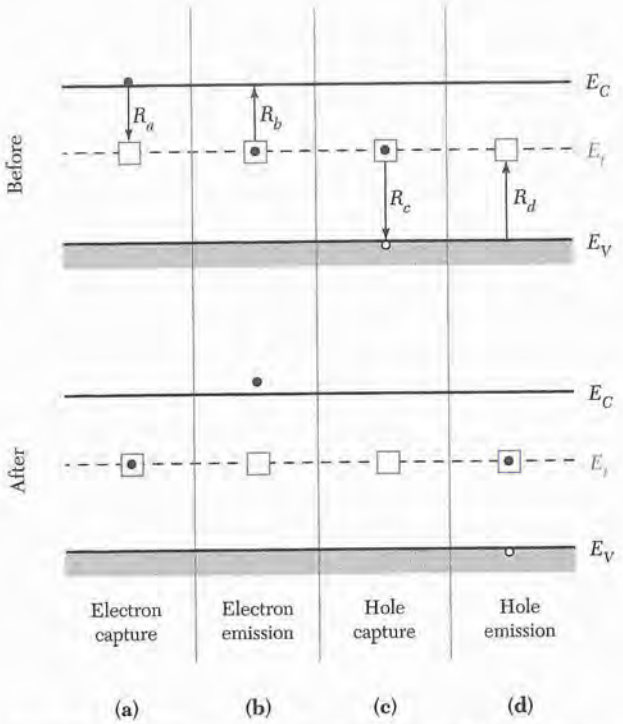


Fig. 12 Indirect generation-recombination processes at thermal equilibrium.

when not occupied by an electron or negative when it is occupied. In indirect recombination, the derivation of the recombination rate is more complicated. The detailed derivation is given in Appendix I, and the recombination rate is given by [4]

$$U = \frac{v_{th}\sigma_n\sigma_p N_t\left(p_n n_n - n_i^2\right)}{\sigma_p[p_n + n_i e^{(E_t - E_t)/kT}] + \sigma_n[n_n + n_i e^{(E_t - E_t)/kT}]}, \tag{48}$$

where v_{th} is the thermal velocity of carriers given in Eq. 1, N_t is the concentration of the recombination center in the semiconductor, and σ_n is the capture cross section of electrons. The quantity σ_n describes the effectiveness of the center to capture an electron and is a measure of how close the electron has to come to the center to be captured. σ_p is the capture cross section of holes.

We can simplify the general expression for the dependence of U on E_t by assuming equal electron and hole capture cross sections, that is, $\sigma_n = \sigma_p = \sigma_o$. Equation 48 then becomes

$$U = v_{th}\sigma_o N_t \frac{\left(p_n n_n - n_i^2\right)}{p_n + n_n + 2n_i \cosh\left(\dfrac{E_t - E_i}{kT}\right)}. \tag{49}$$

Under a low-injection condition in an n-type semiconductor so that $n_n >> p_n$, the recombination rate can be written as

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$$U \approx v_{th}\sigma_o N_t \frac{p_n - p_{no}}{1 + \left(\frac{2n_i}{n_{no}}\right)\cosh\left(\frac{E_t - E_i}{kT}\right)} = \frac{p_n - p_{no}}{\tau_p}. \tag{50}$$

The recombination rate for indirect recombination is given by the same expression as Eq. 43; however, τ_p depends on the locations of the recombination centers.

3.3.3 Surface Recombination

Figure 13 shows schematically the bonds at a semiconductor surface.[5] Because of the abrupt discontinuity of the lattice structure at the surface, a large number of localized energy states or generation-recombination centers may be introduced at the surface region. These energy states, called *surface states*, may greatly enhance the recombination rate at the surface region. The kinetics of surface recombination are similar to those considered before for bulk centers. The total number of carriers recombining at the surface *per unit area* and unit time can be expressed in a form analogous to Eq. 48. For a low-injection condition, and for the limiting case where electron concentration at the surface is essentially equal to the bulk majority carrier concentration, the total number of carriers recombining at the surface per unit area and unit time can be simplified to

$$U_s \cong v_{th}\sigma_p N_{st}(p_s - p_{no}), \tag{51}$$

where p_s denotes the hole concentrations at the surface, and N_{st} is the recombination center density per unit area in the surface region. Since the product $v_{th}\sigma_p N_{st}$ has its dimension in centimeters per second, it is called the *low-injection surface recombination velocity* S_{lr}:

$$S_{lr} \equiv v_{th}\sigma_p N_{st}. \tag{52}$$

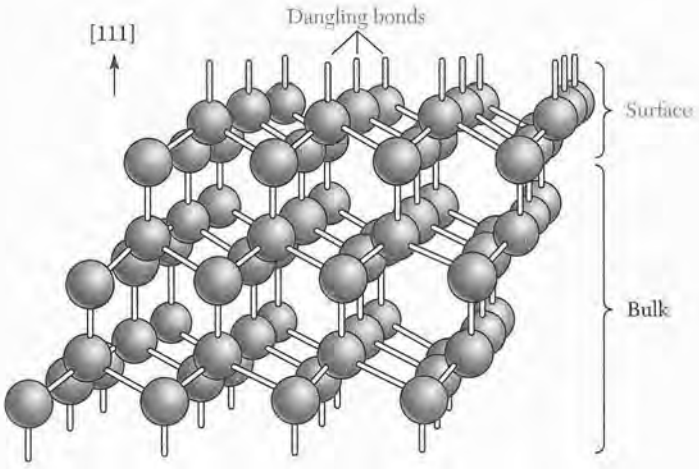


Fig. 13 Schematic diagram of bonds at a clean semiconductor surface. The bonds are anisotropic and differ from those in the bulk.[5]

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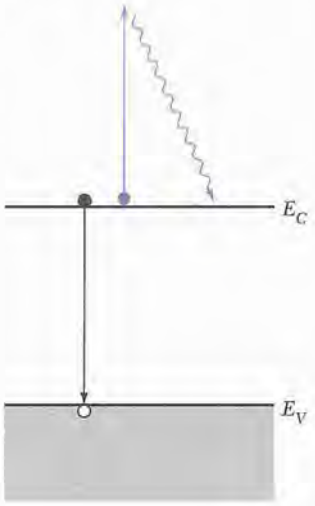


Fig. 14 Auger recombination.

3.3.4 Auger Recombination

The Auger recombination process occurs by the transfer of the energy and momentum released by the recombination of an electron-hole pair to a third particle that can be either an electron or a hole. The example of Auger recombination process is shown in Fig. 14. A second electron in the conduction band absorbs the energy released by the direct recombination. After the Auger process, the second electron becomes an energetic electron. It loses its energy to the lattice by scattering events. Usually, Auger recombination is important when the carrier concentration is very high as a result of either high doping or high injection level. Because the Auger process involves three particles, the rate of Auger recombination can be expressed as

$$R_{Aug} = Bn^2p \text{ or } Bnp^2. \tag{53}$$

The proportionality constant B has a strong temperature dependence.

3.4 CONTINUITY EQUATION

In the previous sections we considered individual effects such as drift due to an electric field, diffusion due to a concentration gradient, and recombination of carriers through intermediate-level recombination centers. We now consider the overall effect when drift, diffusion, and recombination occur simultaneously in a semiconductor material. The governing equation is called the *continuity equation*.

To derive the one-dimensional continuity equation for electrons, consider an infinitesimal slice with a thickness dx located at x shown in Fig. 15. The number of electrons in the slice may increase due to the *net* current flow into the slice and the *net* carrier generation in the slice. The overall rate of electron increase is the algebraic sum of four components: the number of electrons flowing into the slice at x, minus the number of electrons flowing out at $x + dx$, plus the rate at which electrons are generated, minus the rate at which they are recombined with holes in the slice.

The first two components are found by dividing the currents at each side of the slice by the charge of an electron. The generation and recombination rates are designated by

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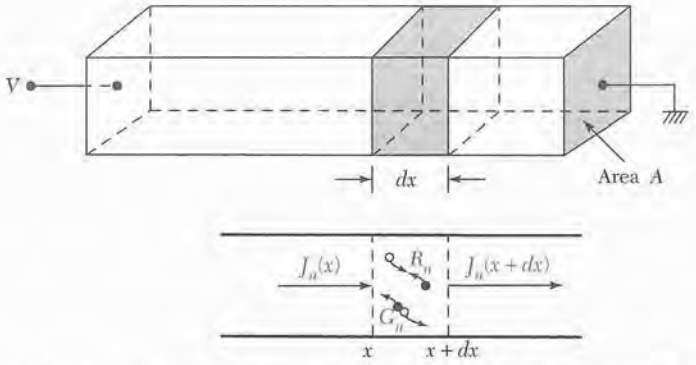


Fig. 15 Current flow and generation-recombination processes in an infinitesimal slice of thickness dx.

G_n and R_n, respectively. The overall rate of change in the number of electrons in the slice is then

$$\frac{\partial n}{\partial t}Adx = \left[\frac{J_n(x)A}{-q} - \frac{J_n(x+dx)A}{-q}\right] + (G_n - R_n)Adx, \tag{54}$$

where A is the cross-sectional area and Adx is the volume of the slice. Expanding the expression for the current at $x + dx$ in Taylor series yields

$$J_n(x+dx) = J_n(x) + \frac{\partial J_n}{\partial x}dx + \ldots \tag{55}$$

We thus obtain the basic *continuity equation* for electrons:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + (G_n - R_n). \tag{56}$$

A similar continuity equation can be derived for holes, except that the sign of the first term on the right-hand side of Eq. 56 is changed because of the positive charge associated with a hole:

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + (G_p - R_p). \tag{57}$$

We can substitute the current expressions from Eqs. 31 and 32 and the recombination expressions from Eq. 43 into Eqs. 56 and 57. For the one-dimensional case under low-injection condition, the continuity equations for minority carriers (i.e., n_p in a p-type semiconductor or p_n in an n-type semiconductor) are

$$\boxed{\frac{\partial n_p}{\partial t} = n_p\mu_n\frac{\partial \mathscr{E}}{\partial x} + \mu_n\mathscr{E}\frac{\partial n_p}{\partial x} + D_n\frac{\partial^2 n_p}{\partial x^2} + G_n - \frac{n_p - n_{po}}{\tau_n},} \tag{58}$$

$$\boxed{\frac{\partial p_n}{\partial t} = -p_n\mu_p\frac{\partial \mathscr{E}}{\partial x} - \mu_p\mathscr{E}\frac{\partial p_n}{\partial x} + D_p\frac{\partial^2 p_n}{\partial x^2} + G_p - \frac{p_n - p_{no}}{\tau_p}.} \tag{59}$$

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In addition to the continuity equations, Poisson's equation

$$\boxed{\frac{d\mathscr{E}}{dx} = \frac{\rho_s}{\varepsilon_s}} \tag{60}$$

must be satisfied, where ε_s is the semiconductor dielectric permittivity and ρ_s is the space charge density given by the algebraic sum of the charge carrier densities and the ionized impurity concentrations, $q(p - n + N_D^+ - N_A^-)$.

In principle, Eqs. 58 through 60 together with appropriate boundary conditions have a unique solution. Because of the algebraic complexity of this set of equations, in most cases the equations are simplified with physical approximations before a solution is attempted. We solve the continuity equations for three important cases.

3.4.1 Steady-State Injection from One Side

Figure 16*a* shows an *n*-type semiconductor where excess carriers are injected from one side as a result of illumination. It is assumed that light penetration is negligibly small (i.e., the assumptions of zero field and zero generation for $x > 0$). At steady state there is a

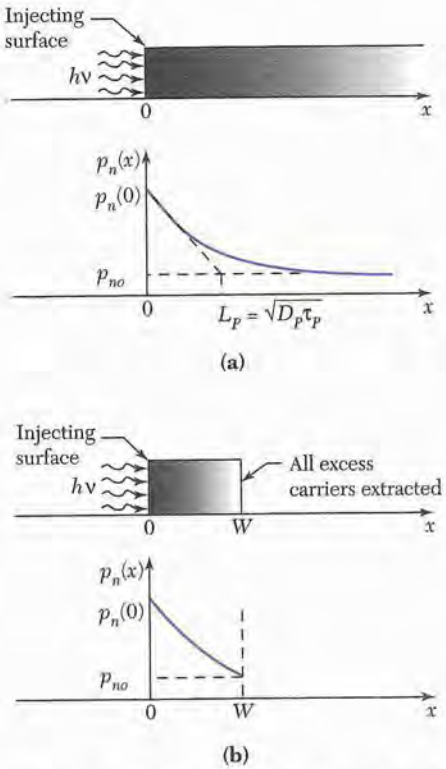


Fig. 16 Steady-state carrier injection from one side. (*a*) Semiinfinite sample. (*b*) Sample with thickness *W*.

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concentration gradient near the surface. From Eq. 59 the differential equation for the minority carriers inside the semiconductor is

$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{no}}{\tau_p}. \tag{61}$$

The boundary conditions are $p_n(x = 0) = p_n(0) =$ constant value and $p_n(x \to \infty) = p_{no}$. The solution of $p_n(x)$ is

$$p_n(x) = p_{no} + [p_n(0) - p_{no}]e^{-x/L_p}. \tag{62}$$

The length L_p is equal to $\sqrt{D_p \tau_p}$ and is called the *diffusion length*. Figure 16*a* shows the variation of the minority carrier density, which decays with a characteristic length given by L_p.

If we change the second boundary condition as shown in Fig. 16*b* so that all excess carriers at $x = W$ are extracted, that is, $p_n(W) = p_{no}$, then we obtain a new solution for Eq. 61:

$$p_n(x) = p_{no} + [p_n(0) - p_{no}] \left[\frac{\sinh\left(\frac{W - x}{L_p}\right)}{\sinh(W / L_p)} \right]. \tag{63}$$

The current density at $x = W$ is given by the diffusion current expression, Eq. 32 with $\mathscr{E} = 0$:

$$J_p = -qD_p \frac{\partial p_n}{\partial x}\Big|_w = q[p_n(0) - p_{no}] \frac{D_p}{L_p} \frac{1}{\sinh(W/L_p)}. \tag{64}$$

3.4.2 Minority Carriers at the Surface

When surface recombination is introduced at one end of a semiconductor sample under illumination (Fig. 17), the hole current density flowing into the surface from the bulk of

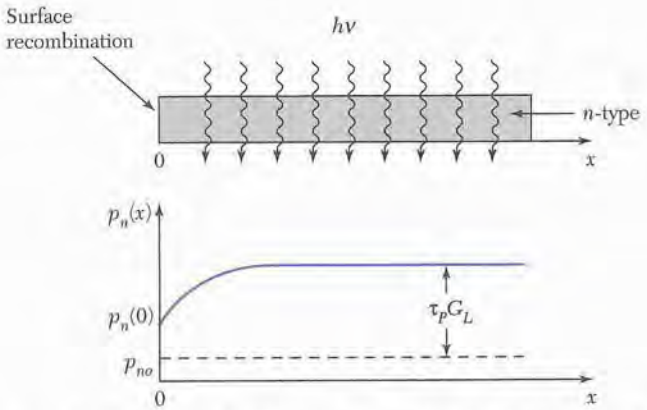


Fig. 17 Surface recombination at $x = 0$. The minority carrier distribution near the surface is affected by the surface recombination velocity.[6]

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the semiconductor is given by qU_s. In this example, it is assumed that the sample is uniformly illuminated with uniform generation of carriers. The surface recombination leads to a lower carrier concentration at the surface. This gradient of hole concentration yields a diffusion current density that is equal to the surface recombination current. Therefore, the boundary condition at $x = 0$ is

$$qD_p \left.\frac{dp_n}{dx}\right|_{x=0} = qU_s = qS_{lr}[p_n(0) - p_{no}]. \tag{65}$$

The boundary condition at $x = \infty$ is given by Eq. 45*a*. At steady state the differential equation is

$$\frac{\partial p_n}{\partial t} = 0 = D_p \frac{\partial^2 p_n}{\partial x^2} + G_L - \frac{p_n - p_{no}}{\tau_p}. \tag{66}$$

The solution of the equation, subject to the boundary conditions above, is[6]

$$p_n(x) = p_{no} + \tau_p G_L \left(1 - \frac{\tau_p S_{lr} e^{-x/L_p}}{L_p + \tau_p S_{lr}}\right). \tag{67}$$

A plot of this equation for a finite S_{lr} is shown in Fig. 17. When $S_{lr} \to 0$, then $p_n(x) \to p_{no} + \tau_p G_L$, as obtained previously (Eq. 45*a*). When $S_{lr} \to \infty$, then

$$p_n(x) = p_{no} + \tau_p G_L (1 - e^{-x/L_p}). \tag{68}$$

From Eq. 68 we can see that at the surface the minority carrier density approaches its thermal equilibrium value p_{no}.

3.4.3 The Haynes–Shockley Experiment

One of the classic experiments in semiconductor physics is the demonstration of drift and diffusion of minority carriers, first made by Haynes and Shockley.[7] The experiment allows independent measurement of the minority carrier mobility μ and diffusion coefficient D. The basic setup of the Haynes–Shockley experiment is shown in Fig. 18*a*. The voltage source, V_1, establishes an electric field in the $+x$ direction in the n-type semiconductor bar. Excess carriers are produced and effectively injected into the semiconductor bar at contact (1) by a pulse. Contact (2) will collect a fraction of the excess carriers as they drift through the semiconductor. After a pulse, the transport equation is given by Eq. 59 by setting $G_p = 0$ and $\partial \mathscr{E}/\partial x = 0$ (i.e., the applied field is constant across the semiconductor bar):

$$\frac{\partial p_n}{\partial t} = -\mu_p \mathscr{E} \frac{\partial p_n}{\partial x} + D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{no}}{\tau_p}. \tag{69}$$

If no field is applied along the sample, the solution is given by

$$p_n(x, t) = \frac{N}{\sqrt{4\pi D_p t}} \exp\left(-\frac{x^2}{4D_p t} - \frac{t}{\tau_p}\right) + p_{no}, \tag{70}$$

where N is the number of electrons or holes generated per unit area. Figure 18*b* shows this solution as the carriers diffuse away from the point of injection and recombine.

If an electric field is applied along the sample, the solution is in the form of Eq. 70, except that x is replaced by $x - \mu_p \mathscr{E} t$ (Fig. 18*c*). Thus, all the excess carriers move with the drift velocity $\mu_p \mathscr{E}$. At the same time, the carriers diffuse outward and recombine as in the field-free case.

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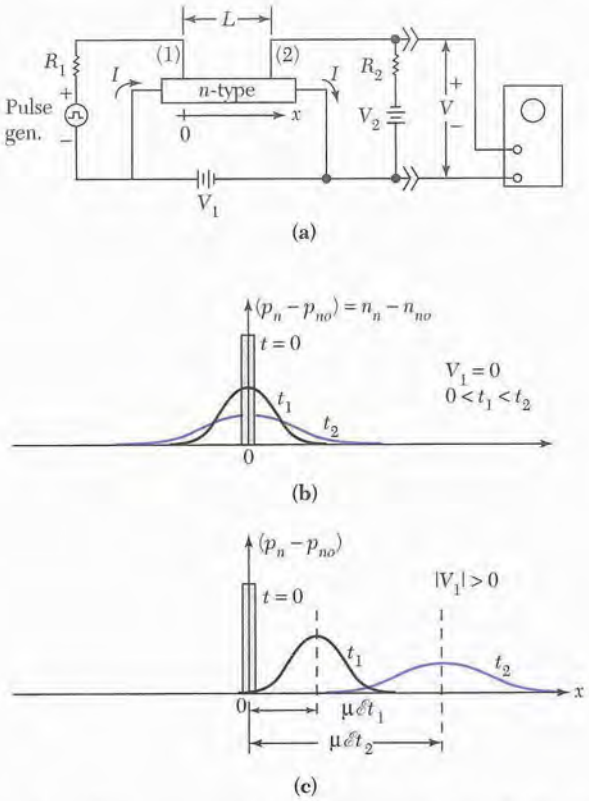


Fig. 18 The Haynes–Shockley experiment. (*a*) Experimental setup. (*b*) Carrier distributions without an applied field. (*c*) Carrier distributions with an applied field.[7]

EXAMPLE 7

In a Haynes–Shockley experiment, the maximum amplitudes of the minority carriers at $t_1 = 100$ μs and $t_2 = 200$ μs differ by a factor of 5. Calculate the minority carrier lifetime.

SOLUTION When an electric field is applied, the minority carrier distribution is given by

$$\Delta p \equiv p_n - p_{no} = \frac{N}{\sqrt{4\pi D_p t}} \exp\left(-\frac{(x-\mu_p \mathscr{E} t)^2}{4D_p t} - \frac{t}{\tau_p}\right).$$

At the maximum amplitude

$$\Delta p = \frac{N}{\sqrt{4\pi D_p t}} \exp\left(-\frac{t}{\tau_p}\right).$$

Therefore

$$\frac{\Delta p(t_1)}{\Delta p(t_2)} = \frac{\sqrt{t_2}}{\sqrt{t_1}} \frac{\exp(-t_1/\tau_p)}{\exp(-t_2/\tau_p)} = \sqrt{\frac{200}{100}} \exp\left(\frac{200-100}{\tau_p\,(\mu s)}\right) = 5$$

$$\therefore \tau_p = \frac{200-100}{\ln(5/\sqrt{2})} = 79\ \mu s.$$

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3.5 THERMIONIC EMISSION PROCESS

In previous sections, we considered carrier transport phenomena inside the bulk semiconductor. At the semiconductor surface, carriers may recombine with the recombination centers due to the dangling bonds of the surface region. In addition, if the carriers have sufficient energy, they may be "thermionically" emitted into the vacuum. This is called the *thermionic emission process*.

Figure 19*a* shows the band diagram of an isolated n-type semiconductor. The electron affinity, $q\chi$, is the energy difference between the conduction band edge and the vacuum level in the semiconductor; and the work function, $q\phi_s$, is the energy between the Fermi level and the vacuum level in the semiconductor. From Fig. 19*b*, it is clear that an electron can be thermionically emitted into the vacuum if its energy is above $q\chi$.

The electron density with energies above $q\chi$ can be obtained from an expression similar to that for the electron density in the conduction band (Eqs. 9 and 16 of Chapter 2) except that the lower limit of the integration is $q\chi$ instead of E_C:

$$n_{th} = \int_{q\chi}^{\infty} n(E)dE = N_C \exp\left[-\frac{q(\chi + V_n)}{kT}\right], \tag{71}$$

where N_C is the effective density of states in the conduction band, and V_n is the difference between the bottom of the conduction band and the Fermi level.

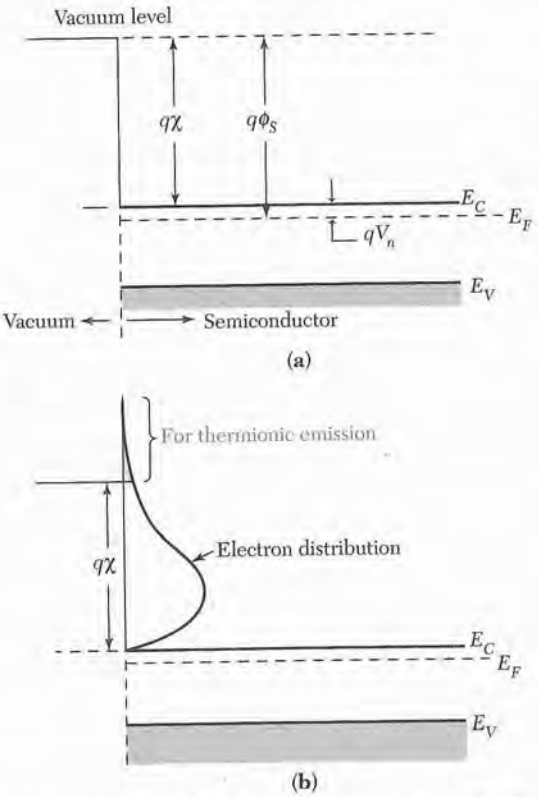


Fig. 19 (*a*) The band diagram of an isolated n-type semiconductor. (*b*) The thermionic emission process.

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EXAMPLE 8

Calculate the thermionically emitted electron density, n_{th}, at room temperature for an n-type silicon sample with an electron affinity of $q\chi$ = 4.05 eV and qV_n = 0.2 eV. If we reduce the effective $q\chi$ to 0.6 eV, what is n_{th}?

SOLUTION

$$n_{th}\,(4.05\text{ eV}) = 2.86 \times 10^{19} \exp\left(-\frac{4.05+0.2}{0.0259}\right) = 2.86 \times 10^{19} \exp\left(-164\right)$$

$$\cong 10^{-52} \approx 0$$

$$n_{th}\,(0.6\text{ eV}) = 2.86 \times 10^{19} \exp\left(-\frac{0.8}{0.0259}\right) = 2.86 \times 10^{19} \exp\left(-30.9\right)$$

$$= 1 \times 10^{6}\text{ cm}^{-3}.$$

From the above example, we see that at 300 K there is no emission of electrons into vacuum for $q\chi$ = 4.05. However, if we can lower the effective electron affinity to 0.6 eV, a substantial number of electrons can be thermionically emitted. The thermionic emission process is of particular important for metal-semiconductor contacts to be considered in Chapter 7.

3.6 TUNNELING PROCESS

Figure 20*a* shows the energy band diagram when two isolated semiconductor samples are brought close together. The distance between them is d, and the potential barrier height qV_0 is equal to the electron affinity $q\chi$. If the distance is sufficient small, the electrons in the left-side semiconductor may transport across the barrier and move to the right-side semiconductor, even if the electron energy is much less than the barrier height. This process is associated with the *quantum tunneling phenomenon*.

Based on Fig. 20*a*, we have redrawn the one-dimensional potential barrier diagram in Fig. 20*b*. We first consider the transmission (or tunneling) coefficient of a particle (e.g., electron) through this barrier. In the corresponding classic case, the particle is always reflected if its energy E is less than the potential barrier height qV_0. We see that in the quantum case, the particle has finite probability to transmit or "tunnel" through the potential barrier.

The behavior of a particle (e.g., a conduction electron) in the region where $qV(x)$ = 0 can be described by the Schrödinger equation:

$$-\frac{\hbar^2}{2m_n}\frac{d^2\psi}{dx^2} = E\psi \tag{72}$$

or

$$\frac{d^2\psi}{dx^2} = -\frac{2m_nE}{\hbar^2}\psi, \tag{73}$$

where m_n is the effective mass, $\hbar$ is the reduced Planck constant, E is the kinetic energy, and ψ is the wave function of the particle. The solutions are

$$\psi(x) = Ae^{jkx} + Be^{-jkx} \qquad x \le 0, \tag{74}$$

$$\psi(x) = Ce^{jkx} \qquad x \ge d, \tag{75}$$

where $k \equiv \sqrt{2m_nE/\hbar^2}$. For $x \le 0$, we have an incident-particle wave function (with amplitude A) and a reflected wave function (with amplitude B); for $x \ge d$, we have a transmitted wave function (with amplitude C).

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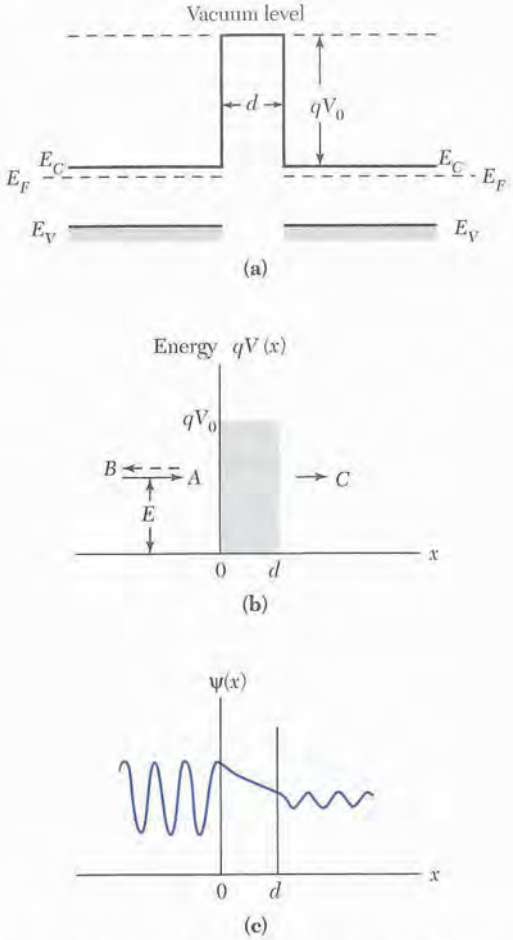


Fig. 20 (*a*) The band diagram of two isolated semiconductors with a distance *d*. (*b*) One-dimensional potential barrier. (*c*) Schematic representation of the wave function across the potential barrier.

Inside the potential barrier, the wave equation is given by

$$-\frac{\hbar^2}{2m_n}\frac{d^2\psi}{dx^2}+qV_0\psi=E\psi \tag{76}$$

or

$$\frac{d^2\psi}{dx^2}=\frac{2m_n(qV_0-E)}{\hbar^2}\psi. \tag{77}$$

The solution for $E < qV_0$ is

$$\psi(x)=Fe^{\beta x}+Ge^{-\beta x}, \tag{78}$$

where $\beta \equiv \sqrt{2m_n(qV_0-E)/\hbar^2}$. A schematic representation of the wave functions across the barrier is shown in Fig. 20*c*. The continuity of ψ and $d\psi/dx$ at $x = 0$ and $x = d$, which is required by the boundary conditions, provides four relations between the five coefficients (A, B, C, F, and G). We can solve for $(C/A)^2$, which is the *transmission coefficient:*

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$$\left(\frac{C}{A}\right)^2 = \left[1 + \frac{(qV_0 \sinh \beta d)^2}{4E(qV_0 - E)}\right]^{-1} \tag{79}$$

The transmission coefficient decreases monotonically as E decreases. When $\beta d >> 1$, the transmission coefficient becomes quite small and varies as

$$\boxed{\left(\frac{C}{A}\right)^2 \sim \exp(-2\beta d) = \exp\left[-2d\sqrt{2m_n(qV_0 - E)/\hbar^2}\right]} \tag{80}$$

To have a finite transmission coefficient, we require a small tunneling distance d, a low-potential barrier qV_0, and a small effective mass. These results will be used for tunnel diodes in Chapter 8.

3.7 HIGH-FIELD EFFECTS

At low electric-fields, the drift velocity is linearly proportional to the applied field. We assume that the time interval between collision, τ_c, is independent of the applied field. This is a reasonable assumption as long as the drift velocity is small compared with the thermal velocity of carriers, which is about 10^7 cm/s for silicon at room temperature.

As the drift velocity approaches the thermal velocity, its field dependence on the electric field will begin to depart from the linear relationship given in Section 3.1. Figure 21 shows the measured drift velocities of electrons and holes in silicon as a function of the electric field. It is apparent that initially the field dependence of the drift velocity is linear, corresponding to a constant mobility. As the electric field is further increased, the drift velocity increases less rapidly. At sufficiently large fields, the drift velocity approaches a saturation velocity. The experimental results can be approximated by the empirical expression[8]

$$v_n,\ v_p = \frac{v_s}{[1 + (\mathscr{E}_0/\mathscr{E})^\gamma]^{1/\gamma}}, \tag{81}$$

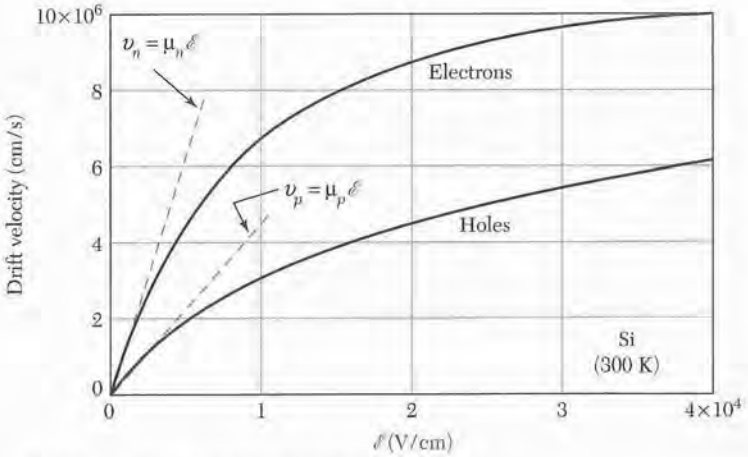


Fig. 21 Drift velocity versus electric field in Si.[8]

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where v_s is the saturation velocity (10^7cm/s for Si at 300 K), $\mathscr{E}_0$ is a constant, equal to 7×10^3 V/cm for electrons and 2×10^4 V/cm for holes in high-purity silicon materials, and γ is 2 for electrons and 1 for holes. Velocity saturation at high fields is particularly likely for field-effect transistors (FETs) with very short channels. Even moderate voltage can result in a high field along the channel. This effect is discussed in Chapter 6.

The high-field transport in n-type gallium arsenide is different from that of silicon.[9] Figure 22 shows the measured drift velocity versus field for n-type and p-type gallium arsenide. The results for silicon are also shown in this log-log plot for comparison. Note that for n-type GaAs, the drift velocity reaches a maximum, then decreases as the field further increases. This phenomenon is due to the energy band structure of gallium arsenide that allows the transfer of conduction electrons from a high-mobility energy minimum (called a valley) to low-mobility, higher-energy satellite valleys, that is, electron transfer from the central valley to the satellite valleys along the [111] direction shown in Chapter 2.

To understand this phenomenon, consider the simple two-valley model of n-type gallium arsenide shown in Fig. 23. The energy separation between the two valleys is ΔE = 0.31 eV. The lower valley's electron effective mass is denoted by m_1, the electron mobility by μ_1, and the electron density by n_1. The upper-valley quantities are denoted by m_2, μ_2, and n_2, respectively; and the total electron concentration is given by $n = n_1 + n_2$. The steady-state conductivity of the n-type GaAs can be written as

$$\sigma = q(\mu_1 n_1 + \mu_2 n_2) = qn\overline{\mu}, \tag{82}$$

where the average mobility is

$$\overline{\mu} \equiv (\mu_1 n_1 + \mu_2 n_2)/(n_1 + n_2). \tag{83}$$

The drift velocity is then

$$v_n = \overline{\mu}\mathscr{E}. \tag{84}$$

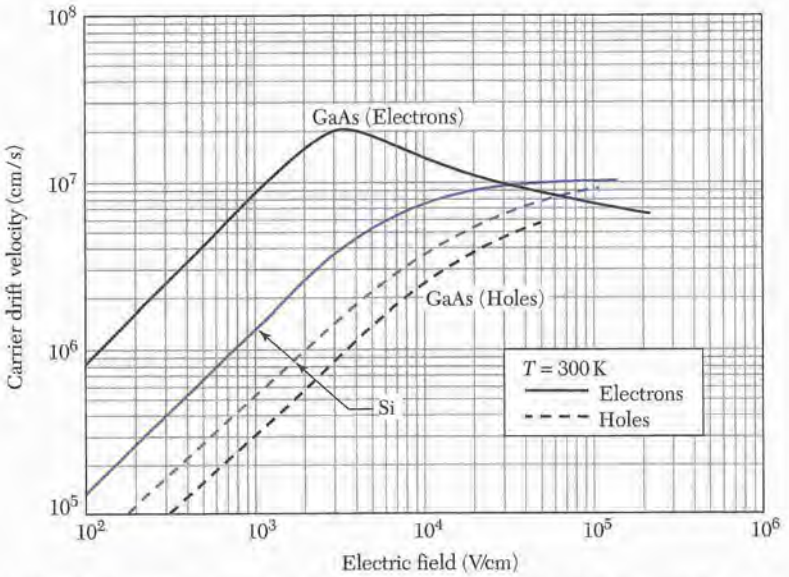


Fig. 22 Drift velocity versus electric field in Si and GaAs. Note that for n-type GaAs, there is a region of negative differential mobility.[8,9]

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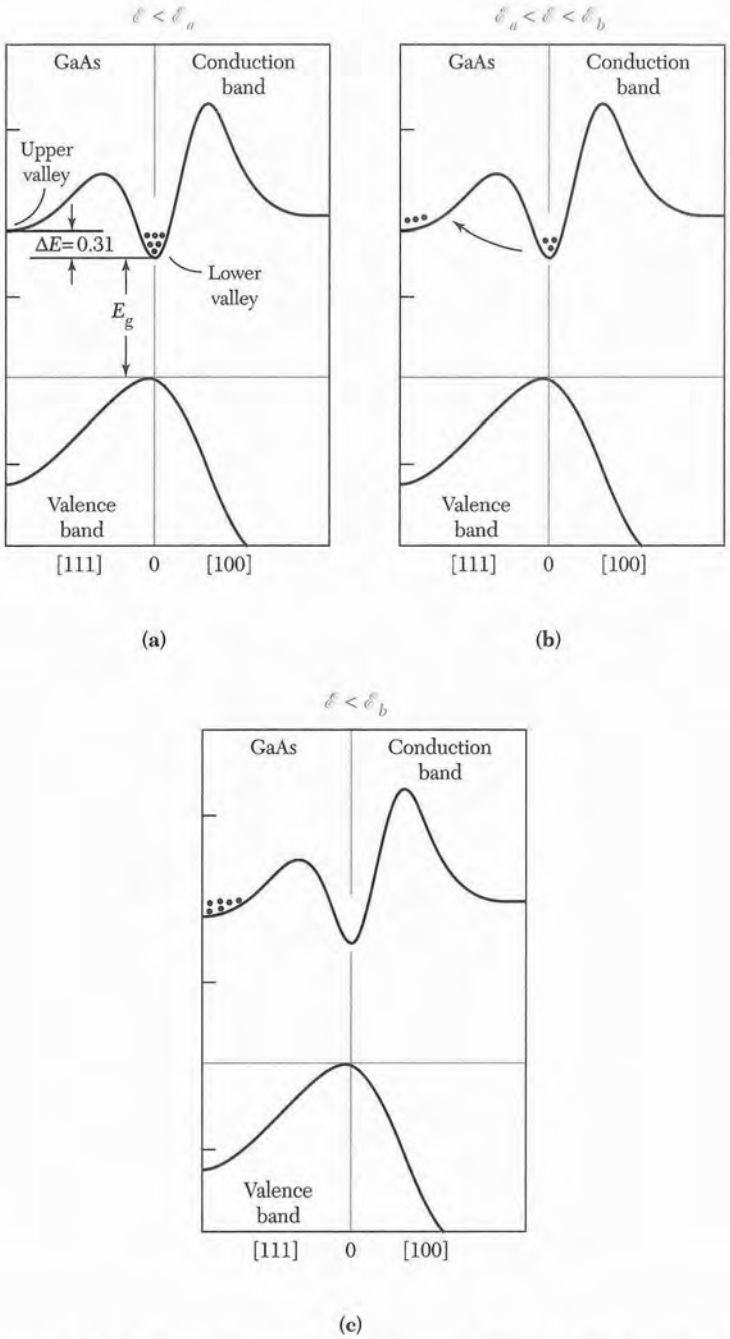


Fig. 23 Electron distributions under various conditions of electric fields for a two-valley semiconductor.

For simplicity we make the following assignments for the electron concentrations in the various ranges of electric-field values illustrated in Fig. 23. In Fig. 23*a*, the field is low and all electrons remain in the lower valley. In Fig. 23*b*,the field is higher and some electrons gain sufficient energies from the field to move to the higher valley. In Fig. 23*c*, the field is high enough to transfer all electrons to the higher valley. Thus, we have

$$\begin{aligned} n_1 &\cong n \quad \text{and} \quad n_2 \cong 0 \quad \text{for } 0 < \mathscr{E} < \mathscr{E}_a, \\ n_1 + n_2 &\cong n \quad \text{for } \mathscr{E}_a < \mathscr{E} < \mathscr{E}_b, \\ n_1 &\cong 0 \quad \text{and} \quad n_2 \cong n \quad \text{for } \mathscr{E} > \mathscr{E}_b \end{aligned} \tag{85}$$

Using these relations, the effective drift velocity takes on the asymptotic values

$$\begin{aligned} v_n &\cong \mu_1 \mathscr{E} \quad \text{for } 0 < \mathscr{E} < \mathscr{E}_a, \\ v_n &\cong \mu_2 \mathscr{E} \quad \text{for } \mathscr{E} > \mathscr{E}_b. \end{aligned} \tag{86}$$

If $\mu_1 \mathscr{E}_a$ is larger than $\mu_2 \mathscr{E}_b$, there is a region in which the drift velocity decreases with an increasing field between $\mathscr{E}_a$ and $\mathscr{E}_b$, as shown in Fig. 24. Because of the characteristics of the drift velocity in *n*-type gallium arsenide, this material is used in microwave transferred-electron devices discussed in Chapter 8.

When the electric field in a semiconductor is increased above a certain value, the carriers gain enough kinetic energy to generate electron-hole pairs by an *avalanche process* that is shown schematically in Fig. 25. Consider an electron in the conduction band (designated by 1). If the electric field is high enough, this electron can gain kinetic energy before it collides with the lattice. On impact with the lattice, the electron imparts most of its kinetic energy to break a bond, that is, to ionize a valence electron from the valence band to the conduction band and thereby generate an electron-hole pair (designated by 2 and 2′). Similarly, the generated pair now begins to accelerate in the field and collides with the lattice as indicated in the figure. In turn, they will generate other electron-hole pairs (e.g., 3 and 3′, 4 and 4′), and so on. This process is called the avalanche process; it is also referred to as the *impact ionization* process. This process will result in breakdown in *p–n* junction, which is discussed in Chapter 4.

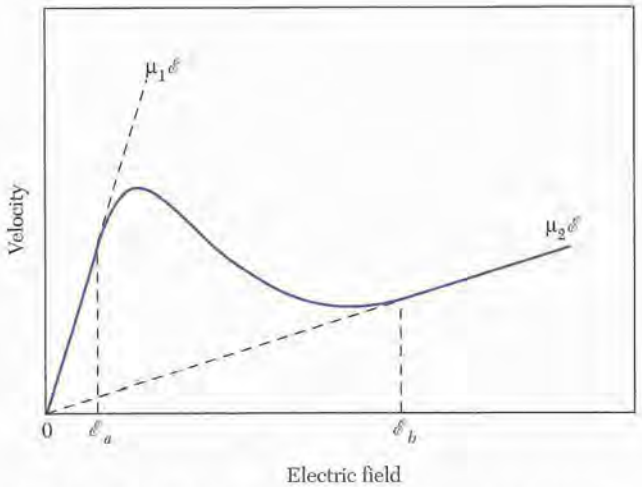


Fig. 24 One possible velocity-field characteristic of a two-valley semiconductor.

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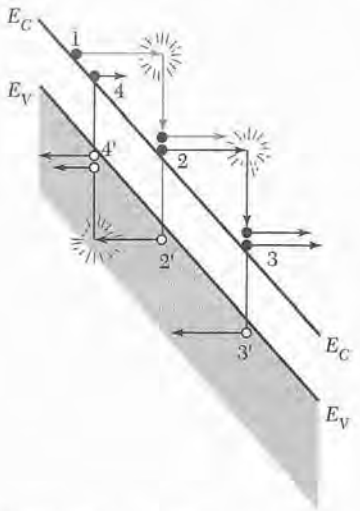


Fig. 25 Energy band diagram for the avalanche process.

To gain some ideas about the ionization energy involved, let us consider the process leading to 2–2′ shown in Fig. 25. Just prior to the collision, the fast-moving electron (no. 1) has a kinetic energy $\frac{1}{2} m_1 v_s^2$ and a momentum $m_1 v_s$, where m_1 is the effective mass and v_s is the saturation velocity. After collision, there are three carriers: the original electron plus an electron–hole pair (no. 2 and no. 2′). If we assume that the three carriers have the same effective mass, the same kinetic energy, and the same momentum, the total kinetic energy is $\frac{3}{2} m_1 v_f^2$, and the total momentum is $3m_1 v_f$, where v_f is the velocity after collision. To conserve both energy and momentum before and after the collision, we require that

$$\frac{1}{2} m_1 v_s^2 = E_g + \frac{3}{2} m_1 v_f^2 \tag{87}$$

and

$$m_1 v_s = 3 m_1 v_f, \tag{88}$$

where in Eq. 87 the energy E_g is the bandgap corresponding to the minimum energy required to generate an electron-hole pair. Substituting Eq. 88 into Eq. 87 yields the required kinetic energy for the ionization process:

$$E_0 = \frac{1}{2} m_1 v_s^{\ 2} = 1.5 E_g. \tag{89}$$

It is obvious that E_0 must be larger than the bandgap for the ionization process to occur. The actual energy required depends on the band structure. For silicon, the value for E_0 is 3.6 eV (3.2 E_g) for electrons and 5.0 eV (4.4 E_g) for holes.

The number of electron-hole pairs generated by an electron per unit distance traveled is called the *ionization rate* for the electron, α_n. Similarly, α_p is the ionization rate for the holes. The measured ionization rates for silicon and gallium arsenide are shown[9] in Fig. 26. We note that both α_n and α_p are strongly dependent on the electric field. For a substantially large ionization rate (say 10^4 cm^{-1}), the corresponding electric field is $\geq 3 \times 10^5$ V/cm for silicon and $\geq 4 \times 10^5$ V/cm for gallium arsenide. The electron-hole pair generation rate G_A from the avalanche process is given by

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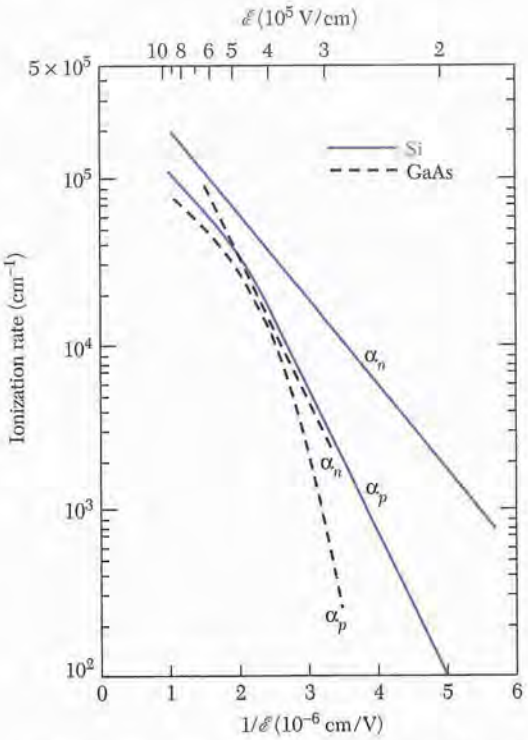


Fig. 26 Measured ionization rates versus reciprocal field for Si and GaAs.[9]

$$G_A = \frac{1}{q}\left(\alpha_n |J_n| + \alpha_p |J_p|\right), \tag{90}$$

where J_n and J_p are the electron and hole current densities, respectively. This expression can be used in the continuity equation for devices operated under an avalanche condition.

SUMMARY

Various transport processes are at work in semiconductor devices. These include drift, diffusion, generation, recombination, thermionic emission, tunneling, and impact ionization.

One of the key transport processes is the carrier drift under the influence of an electric field. At low fields, the drift velocity is proportional to the electric field. This proportionality constant is called mobility. Another key transport process is the carrier diffusion under the influence of the carrier concentration gradient. The total current is the sum of the drift and diffusion components.

Excess carriers in a semiconductor cause a nonequilibrium condition. Most semiconductor devices operate under nonequilibrium conditions. Carriers can be generated by various means such as forward biasing a p–n junction, incident light, and impact ionization. The mechanism that restores equilibrium is the recombination of the excess minority carriers with the majority carriers by direct band-to-band recombination or via localized energy states in the forbidden energy gap. The governing equation for the rate of change of charge carriers is the continuity equation.

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Among other transport processes, thermionic emission occurs when carriers in the surface region gain enough energy to be emitted into the vacuum level. Another, the tunneling process, is based on the quantum tunneling phenomena that results in the transport of electrons across a potential barrier even if the electron energy is less than the barrier height.

As the electric field becomes higher, the drift velocity departs from its linear relationship with the applied field and approaches a saturation velocity. This effect is particularly important in the study of short-channel field-effect transistors discussed in Chapter 6. When the field exceeds a certain value, the carriers gain enough kinetic energy to generate electron-hole pair by colliding with the lattice and breaking a bond. This effect is particularly important in the study of p–n junctions. The high field accelerates these new electron-hole pairs, which collide with the lattice to create more electron-hole pairs. As this process, called impact ionization or the avalanche process, continues, the p–n junction breaks down and conducts a large current. The junction breakdown is discussed in Chapter 4.

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PROBLEMS (* INDICATES DIFFICULT PROBLEMS)

FOR SECTION 3.1 CARRIER DRIFT

1. Find the resistivities of intrinsic Si and intrinsic GaAs at 300 K.
2. Assume that the mobility of electrons in silicon at T = 300 K is $\mu_n = 1300$ cm^2/V-s. Also assume that the mobility is mainly limited by lattice scattering. Determine the electron mobility at (a) $T = 200$ K and (b) $T = 400$ K.
3. Two scattering mechanisms exist in a semiconductor. If only the first mechanism is present, the mobility will be 250 cm^2/V-s. If only the second mechanism is present, the mobility will be 500 cm^2/V-s. Determine the mobility when both scattering mechanisms exist at the same time.
4. Find the electron and hole concentrations, mobilities, and resistivities of silicon samples at 300 K, for each of the following impurity concentrations: (a) 5×10^{15} boron atoms/cm^3; (b) 2×10^{16} boron atoms/cm^3 and 1.5×10^{16} arsenic atoms/cm^3; and (c) 5×10^{15} boron atoms/cm^3, 10^{17} arsenic atoms/cm^3, and 10^{17} gallium atoms/cm^3.
5. Consider a compensated n-type silicon at $T = 300$ K, with a conductivity of $\sigma = 16$ $(\Omega\text{-cm})^{-1}$ and an acceptor doping concentration of 10^{17} cm^{-3}. Determine the donor

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concentration and the electron mobility. (A compensated semiconductor is one that contains both donor and acceptor impurity atoms in the same region.)

6. For a semiconductor with a constant mobility ratio $b \equiv \mu_n/\mu_p > 1$ independent of impurity concentration, find the maximum resistivity ρ_m in terms of the intrinsic resistivity ρ_i and the mobility ratio.

7. A four-point probe (with probe spacing of 0.5 mm) is used to measure the resistivity of a p-type silicon sample. Find the resistivity of the sample if its diameter is 200 mm and its thickness is 50 μm. The contact current is 1 mA, and the measured voltage between the inner two probes is 10 mV.

8. Given a silicon sample of unknown doping, Hall measurement provides the following information: $W = 0.05$ cm, $A = 1.6 \times 10^{-3}$ cm^2 (refer to Fig. 8), $I = 2.5$ mA, and the magnetic field is 30 nT (1 T = 10^{-4} Wb/cm^2). If a Hall voltage of +10 mV is measured, find the Hall coefficient, conductivity type, majority carrier concentration, resistivity, and mobility of the semiconductor sample.

9. A semiconductor is doped with N_D ($N_D >> n_i$) and has a resistance R_1. The same semiconductor is then doped with an unknown amount of acceptors N_A ($N_A >> N_D$), yielding a resistance of 0.5 R_1. Find N_A in terms of N_D if D_n/D_p=50.

10. Consider a semiconductor that is nonuniformly doped with donor impurity atoms $N_D(x)$. Show that the induced electric field in the semiconductor in thermal equilibrium is given by $\mathscr{E}(x) = -\left(\frac{kT}{q}\right)\frac{1}{N_D(x)}\frac{dN_D(x)}{dx}$

FOR SECTION 3.2 CARRIER DIFFUSION

11. An intrinsic Si sample is doped with donors from one side such that $N_D = N_0 \exp(-ax)$. (a) Find an expression for the built-in field $\mathscr{E}(x)$ at equilibrium over the range for which $N_D >> n_i$. (b) Evaluate $\mathscr{E}(x)$ when $a = 1\ \mu m^{-1}$.

12. An n-type Si slice of a thickness L is inhomogeneously doped with phosphorus donor whose concentration profile is given by $N_D(x) = N_0 + (N_L - N_0)(x/L)$. What is the formula for the electric potential difference between the front and the back surfaces when the sample is at thermal and electric equilibria regardless of how the mobility and diffusivity varies with position? What is the formula for the equilibrium electric field at a plane x from the front surface for a constant diffusivity and mobility?

FOR SECTION 3.3 GENERATION AND RECOMBINATION PROCESS

13. Calculate the electron and hole concentration under steady-state illumination in an n-type silicon with $G_L = 10^{16}$ cm^{-3}s^{-1}, $N_D = 10^{15}$ cm^{-3}, and $\tau_n = \tau_p = 10$ μs.

14. An n-type silicon sample has 2×10^{16} arsenic atoms/cm^3, 2×10^{15} bulk recombination centers/cm^3, and 10^{10} surface recombination centers/cm^2. (a) Find the bulk minority carrier lifetime, the diffusion length, and the surface recombination velocity under low-injection conditions. The values of σ_p and σ_s are 5×10^{-15} and 2×10^{-16} cm^2, respectively. (b) If the sample is illuminated with uniformly absorbed light that creates 10^{17} electron-hole pairs/cm^3-s, what is the hole concentration at the surface?

15. Assume that an n-type semiconductor is uniformly illuminated, producing a uniform excess generation rate G. Show that in steady state the change in the semiconductor conductivity is given by $\Delta\sigma = q(\mu_n + \mu_p)\tau_p G$.

FOR SECTION 3.4 CONTINUITY EQUATION

16. The total current in a semiconductor is constant and is composed of electron drift current and hole diffusion current. The electron concentration is constant and equal to 10^{16} cm^{-3}.

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The hole concentration is given by

$$p(x) = 10^{15} \exp\left(\frac{-x}{L}\right) \text{ cm}^{-3} \qquad (x \geq 0),$$

where $L = 12$ μm. The hole diffusion coefficient is $D_p = 12$ cm²/s and the electron mobility is $\mu_n = 1000$ cm²/V-s. The total current density is $J = 4.8$ A/cm². Calculate (a) the hole diffusion current density versus x, (b) the electron current density versus x, and (c) the electric field versus x.

*17. Excess carriers are injected on one surface of a thin slice of n-type silicon with thickness W and extracted at the opposite surface where $p_n(W) = p_{no}$. There is no electric field in the region $0 < x < W$. Derive the expression for current densities at the two surfaces.

18. In Prob. 17, if carrier lifetime is 50 μs and $W = 0.1$ mm, calculate the portion of injected current that reaches the opposite surface by diffusion ($D = 50$ cm²/s).

*19. An n-type semiconductor has excess carrier holes 10^{14} cm⁻³, and a bulk minority carrier lifetime 10^{-6} s in the bulk material, and a minority carrier lifetime 10^{-7} s at the surface. Assume zero applied electric field and let $D_p = 10$ cm²/s. Determine the steady-state excess carrier concentration as a function of distance from the surface ($x = 0$) of the semiconductor.

FOR SECTION 3.5 THERMIONIC EMISSION PROCESS

20. A metal, with a work function $\phi_m = 4.2$ V, is deposited on an n-type silicon semiconductor with affinity $\chi = 4.0$ V and $E_g = 1.12$ eV. What is the potential barrier height seen by electrons in the metal moving into the semiconductor?

21. Consider a tungsten filament with metal work function ϕ_m inside a high vacuum chamber. Show that if a current is passed through the filament to heat it up sufficiently, the electrons with enough thermal energy will escape into the vacuum and the resulted thermionic current density is

$$J = A^* T^2 \exp\left(\frac{-q\phi_m}{kT}\right)$$

where A^* is $4\pi qmk^2 / h^3$ and m is free electron mass. The definite integral

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \left(\frac{\pi}{a}\right)^{1/2}.$$

FOR SECTION 3.6 TUNNELING PROCESS

22. Consider a electron with an energy of 2 eV impinging on a potential barrier with 20 eV and a width of 3 Å. What is the tunneling probability?

23. Evaluate the transmission coefficient for an electron of energy 2.2 eV impinging on a potential barrier of height 6.0 eV and thickness 10^{-10} meters. Repeat the calculation for a barrier thickness of 10^{-9} meters.

FOR SECTION 3.7 HIGH FIELD EFFECTS

24. Use the velocity-field relations for Si and GaAs shown in Fig. 22 to determine the transit time of electrons through a 1 μm distance in these materials for an electric field of (a) 1 kV/cm and (b) 50 kV/cm.

25. Assume that a conduction electron in Si ($\mu_n = 1350$ cm²/V-s) has a thermal energy kT, related to its mean thermal velocity by $E_{th} = m_0 v_{th}^2/2$. This electron is placed in an electric field of 100 V/cm. Show that the drift velocity of the electron in this case is small compared to its thermal velocity. Repeat for a field of 10^4 V/cm, using the same value of μ_n. Comment on the actual mobility effects at this higher value of field.

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CHAPTER 4

p–n Junction

- 4.1 BASIC FABRICATION STEPS
- 4.2 THERMAL EQUILIBRIUM CONDITION
- 4.3 DEPLETION REGION
- 4.4 DEPLETION CAPACITANCE
- 4.5 CURRENT-VOLTAGE CHARACTERISTICS
- 4.6 CHARGE STORAGE AND TRANSIENT BEHAVIOR
- 4.7 JUNCTION BREAKDOWN
- 4.8 HETEROJUNCTION
- SUMMARY

In the preceding chapters we considered the carrier concentrations and transport phenomena in homogeneous semiconductor materials. In this chapter we discuss the behavior of single-crystal semiconductor material containing both p- and n-type regions that form a p–n junction. Most modern p–n junctions are made by planar technology to be described in Section 4.1.

A p–n junction serves an important role both in modern electronic applications and in understanding other semiconductor devices. It is used extensively in rectification, switching, and other operations in electronic circuits. It is a key building block for the bipolar transistor and thyristor (Chapter 5), as well as for metal-oxide-semiconductor field-effect transisitors (MOSFETs) (Chapter 6). Given proper biasing conditions or when exposed to light, p–n junction also functions as either microwave (Chapter 8) or photonic device (Chapter 9).

We also consider a related device—the heterojunction, which is a junction formed between two dissimilar semiconductors. It has many unique features that are not readily available from the conventional p–n junctions. The heterojunction is an important building block for heterojunction bipolar transistors (Chapter 5), modulation doped field-effect transistors (Chapter 7), quantum-effect devices (Chapter 8), and photonic devices (Chapter 9).

Specifically, we cover the following topics:

- The formation of a p–n junction physically and electrically.
- The behavior of the junction depletion layer under voltage biases.
- The current transport in a p–n junction and the influence of the generation and recombination processes.
- The charge storage in a p–n junction and its influence on the transient behavior.

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- The avalanche multiplication in a p–n junction and its impact on the maximum reverse voltage.
- The heterojunction and its basic characteristics.

4.1 BASIC FABRICATION STEPS

Today, planar technology is used extensively for integrated circuit (IC) fabrication. Figures 1 and 2 show the major steps of a planar process. These steps include oxidation, lithography,

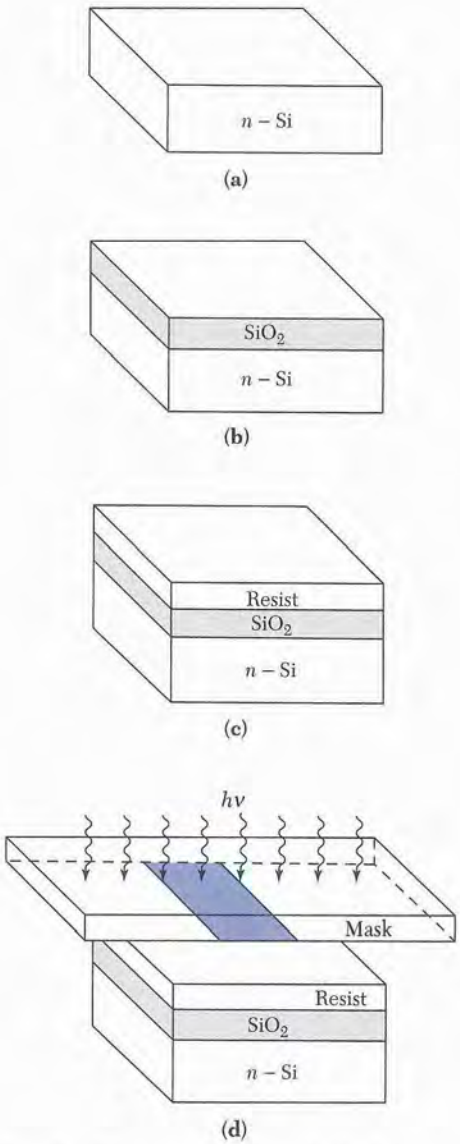


Fig. 1 (*a*) A bare n-type Si wafer. (*b*) An oxidized Si wafer by dry or wet oxidation. (*c*) Application of resist. (*d*) Resist exposure through the mask.

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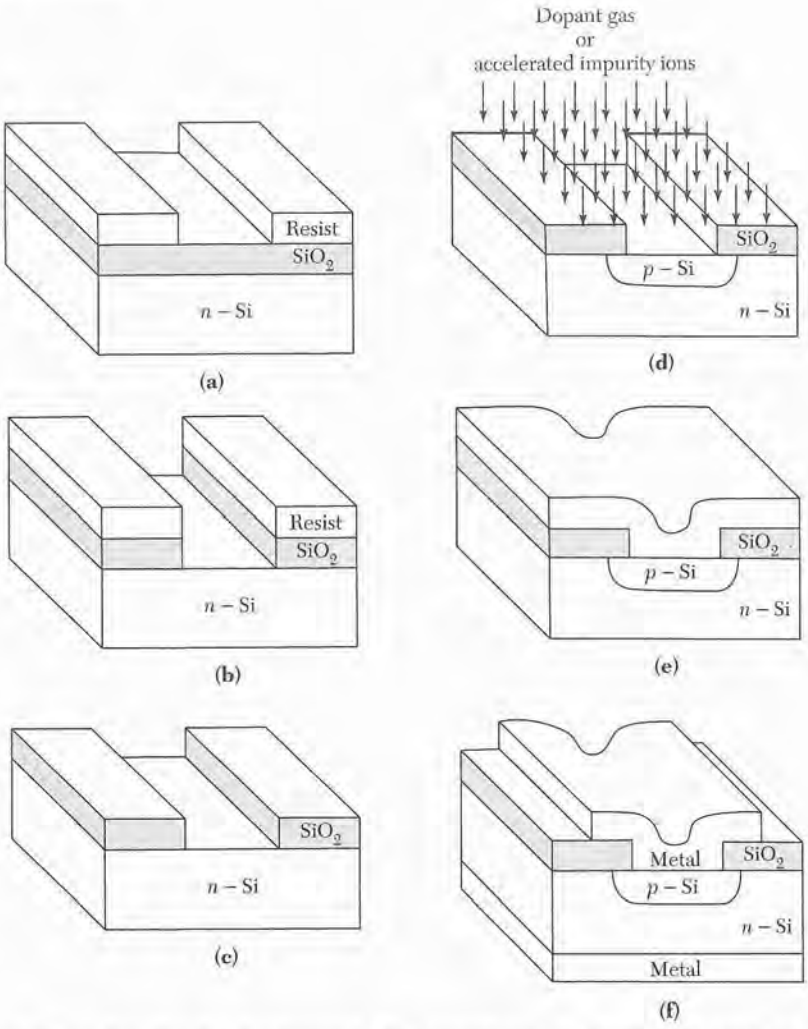


Fig. 2 (a) The wafer after the development. (b) The wafer after SiO_2 removal. (c) The final result after a complete lithography process. (d) A p–n junction is formed in the diffusion or implantation process. (e) The wafer after metallization. (f) A p–n junction after the complete processes.

ion implanation, and metallization. We describe these steps briefly in this section. More detailed discussions can be found in Chapters 10–14.

4.1.1 Oxidation

The development of a high-quality silicon dioxide (SiO_2) has helped to establish the dominance of Si in the production of commercial ICs. Generally, SiO_2 functions as an insulator in a number of device structures or as a barrier to diffusion or implantation during device fabrication. In the fabrication of a p–n junction (Fig. 1), the SiO_2 film is used to define the junction area.

There are two SiO_2 growth methods, dry and wet oxidation, depending on whether dry oxygen or water vapor is used. Dry oxidation is usually used to form thin oxides in a

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device structure because of its good Si-SiO_2 interface characteristics, whereas wet oxidation is used for thicker layers because of its higher growth rate. Figure 1*a* shows a section of a bare Si wafer ready for oxidation. After the oxidation process, a SiO_2 layer is formed all over the wafer surface. For simplicity, Fig. 1*b* shows only the upper surface of an oxidized wafer.

4.1.2 Lithography

Another technology, called photolithography, is used to define the geometry of the p–n junction. After the formation of SiO_2, the wafer is coated with an ultraviolet (UV)-light–sensitive material called a photoresist, which is spun on the wafer surface by a high-speed spinner. After spinner (Fig. 1*c*), the wafer is baked at about 80°–100°C to drive the solvent out of the resist and to harden the resist for improved adhesion. Figure 1*d* shows the next step, which is to expose the wafer through a patterned mask using an UV-light source. The exposed region of the photoresist-coated wafer undergoes a chemical reaction depending on the type of resist. The area exposed to light become polymerized and difficult to remove in an etchant.* The polymerized region remains when the wafer is placed in a developer, whereas the unexposed region (under the opaque area) dissolves and washes away. Figure 2*a* shows the wafer after the development. The wafer is again baked to 120°–180°C for 20 minutes to enhance the adhesion and improve the resistance to the subsequent etching process. Then, an etch using buffered hydrofluoric acid (HF) removes the unprotected SiO_2 surface (Fig. 2*b*). Last, the resist is stripped away by a chemical solution or an oxygen plasma system. Figure 2*c* shows the final result of a region without oxide (a window) after the lithography process. The wafer is now ready for forming the p–n junction by a diffusion or ion-implantation process.

4.1.3 Diffusion and Ion Implantation

In the diffusion method, the semiconductor surface not protected by the oxide is exposed to a source with a high concentration of opposite-type impurity. The impurity moves into the semiconductor crystal by solid-state diffusion. In the ion-implantation method, the intended impurity is introduced into the semiconductor by accelerating the impurity ions to a high-energy level and then implanting the ions in the semiconductor. The SiO_2 layer serves as barrier to impurity diffusion or ion implantation. After the diffusion or implantation process, the p–n junction is formed as shown in Fig. 2*d*. Due to lateral diffusion of impurities or lateral straggle of implanted ions, the width of the p region is slightly wider than the window opening.

4.1.4 Metallization

After diffusion or ion-implantation process, a metallization process is used to form ohmic contacts and interconnections (Fig. 2*e*). Metal films can be formed by physical vapor deposition and chemical vapor deposition. The lithography process is again used to define the front contact which is shown in Fig. 2*f*. A similar metallization step is done on the back contact without using a lithography process. Normally a low-temperature ($\leq$500°C) anneal would also be performed to promote low-resistance contacts between the metal layers

* This is a negative photoresist. We can also use positive photoresist. Lithography and photoresists are considered in detail in Chapter 12.

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and the semiconductor. With the completion of the metallization, the p–n junctions become functional.

4.2 THERMAL EQUILIBRIUM CONDITION

The most important characteristic of p–n junctions is that they rectify, that is, they allow current to flow easily in only one direction. Figure 3 shows the current-voltage characteristics of a typical silicon p–n junction. When we apply "forward bias" to the junction (i.e., positive voltage on the p-side), the current increases rapidly as the voltage increases. However, when we apply a "reverse bias," virtually no current flows initially. As the reverse bias is increased the current remains very small until a critical voltage is reached, at which point the current suddenly increases. This sudden increase in current is referred to as the junction breakdown. The applied forward voltage is usually less than 1 V, but the reverse critical voltage, or breakdown voltage, can vary from just a few volts to many thousands of volts depending on the doping concentration and other device parameters.

4.2.1 Band Diagram

In Fig. 4*a*, we see two regions of p- and n-type semiconductor materials that are uniformly doped and physically separated before the junction is formed. Note that the Fermi level E_F is near the valence band edge in the p-type material and near the conduction band edge in the n-type material. While p-type material contains a large concentration of holes with few electrons, the opposite is true for n-type material.

When the p- and n-type semiconductors are jointed together, the large carrier concentration gradients at the junction cause carrier diffusion. Holes from the p-side diffuse into the n-side, and electrons from the n-side diffuse into the p-side. As holes continue to leave the p-side, some of the negative acceptor ions (N_A^-) near the junction are left uncompensated, since the acceptors are fixed in the semiconductor lattice, whereas the holes are mobile. Similarly, some of the positive donor ions (N_D^+) near the junction are left uncompensated as the electrons leave the n-side. Consequently, a negative space charge

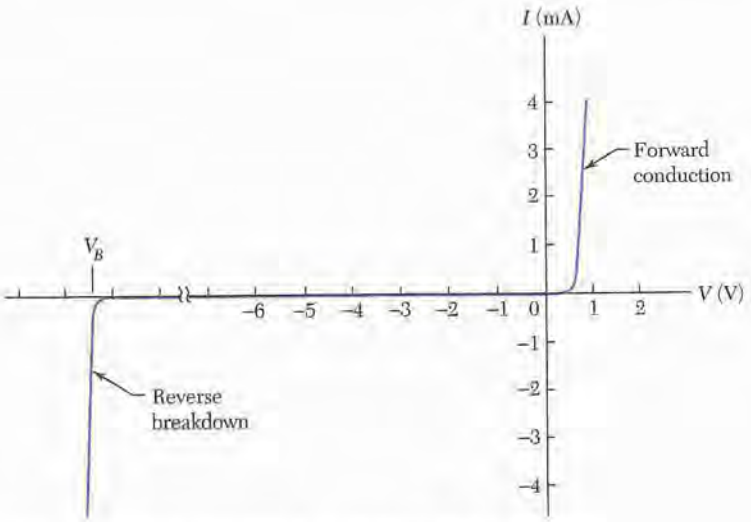


Fig. 3 Current-voltage characteristics of a typical silicon p–n junction.

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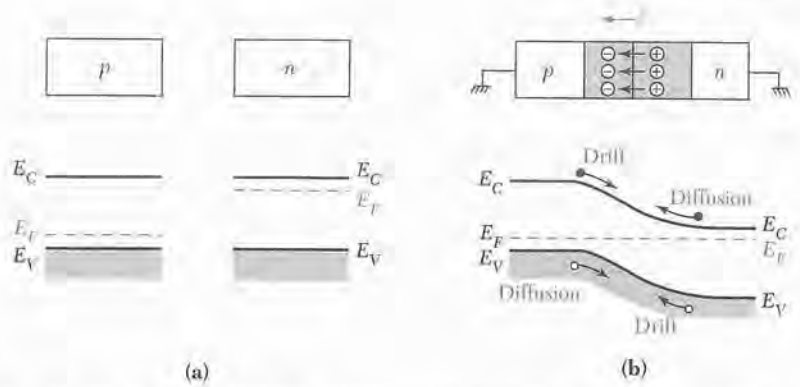


Fig. 4 (*a*) Uniformly doped p-type and n-type semiconductors before the junction is formed. (*b*) The electric field in the depletion region and the energy band diagram of a p–n junction in thermal equilibrium.

forms near the p-side of the junction and a positive space charge forms near the n-side. This space charge region creates an electric field that is directed from the positive charge toward the negative charge, as indicated in the upper illustration of Fig. 4*b*.

The electric field is in the direction opposite to the diffusion current for each type of charge carrier. The lower illustration of Fig. 4*b* shows that the hole diffusion current flows from left to right, whereas the hole drift current due to the electric field flows from right to left. The electron diffusion current also flows from left to right, whereas the electron drift current flows in the opposite direction. Note that because of their negative charge, electrons diffuse from right to left, opposite the direction of electron current.

4.2.2 Equilibrium Fermi Levels

At thermal equilibrium, that is, the steady-state condition at a given temperature without any external excitations, the individual electron and hole current flowing across the junctions are identically zero. Thus, for each type of carrier the drift current due to the electric field must exactly cancel the diffusion current due to the concentration gradient. From Eq. 32 in Chapter 3,

$$\begin{aligned} J_p &= J_p(\text{drift}) + J_p(\text{diffusion}) \\ &= q\mu_p p\mathscr{E} - qD_p \frac{dp}{dx} \\ &= q\mu_p p\left(\frac{1}{q}\frac{dE_i}{dx}\right) - kT\mu_p \frac{dp}{dx} = 0, \end{aligned} \tag{1}$$

where we have used Eq. 8 of Chapter 3 for the electric field and the Einstein relation $D_p = (kT/q)\mu_p$. Substituting the expression for hole concentration

$$p = n_i e^{(E_i - E_F)/kT} \tag{2}$$

and its derivative

$$\frac{dp}{dx} = \frac{p}{kT}\left(\frac{dE_i}{dx} - \frac{dE_F}{dx}\right) \tag{3}$$

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into Eq. 1 yields the net hole current density

$$J_p = \mu_p p \frac{dE_F}{dx} = 0 \tag{4}$$

or

$$\frac{dE_F}{dx} = 0. \tag{5}$$

Similarly, we obtain for the net electron current density

$$\begin{aligned} J_n &= J_n(\text{drift}) + J_n(\text{diffusion}) \\ &= q\mu_n n\mathscr{E} + qD_n \frac{dn}{dx} \\ &= \mu_n n \frac{dE_F}{dx} = 0. \end{aligned} \tag{6}$$

Thus, for the condition of zero net electron and hole currents, the Fermi level must be constant (i.e., independent of x) throughout the sample as illustrated in the energy band diagram of Fig. 4*b*.

The constant Fermi level required at thermal equilibrium results in a unique space charge distribution at the junction. We repeat the one-dimensional p–n junction and the corresponding equilibrium energy band diagram in Figs. 5*a* and 5*b*, respectively. The unique space charge distribution and the electrostatic potential ψ are given by Poisson's equation:

$$\boxed{\frac{d^2\psi}{dx^2} \equiv -\frac{d\mathscr{E}}{dx} = -\frac{\rho_s}{\varepsilon_s} = -\frac{q}{\varepsilon_s}\left(N_D - N_A + p - n\right).} \tag{7}$$

Here we assume that all donors and acceptors are ionized.

In regions far away from the metallurgical junction, charge neutrality is maintained and the total space charge density is zero. For these neutral regions we can simplify Eq. 7 to

$$\frac{d^2\psi}{dx^2} = 0 \tag{8}$$

and

$$N_D - N_A + p - n = 0. \tag{9}$$

For a p-type neutral region, we assume $N_D = 0$ and $p >> n$. The electrostatic potential of the p-type neutral region with respect to the Fermi level, designated as ψ_p in Fig. 5*b*, can be obtained by setting $N_D = n = 0$ in Eq. 9 and by substituting the result ($p = N_A$) into Eq. 2:

$$\psi_p \equiv -\frac{1}{q}\left(E_i - E_F\right)\Big|_{x \le -x_p} = -\frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right). \tag{10}$$

Similarly, we obtain the electrostatic potential of the n-type neutral region with respect to the Fermi level:

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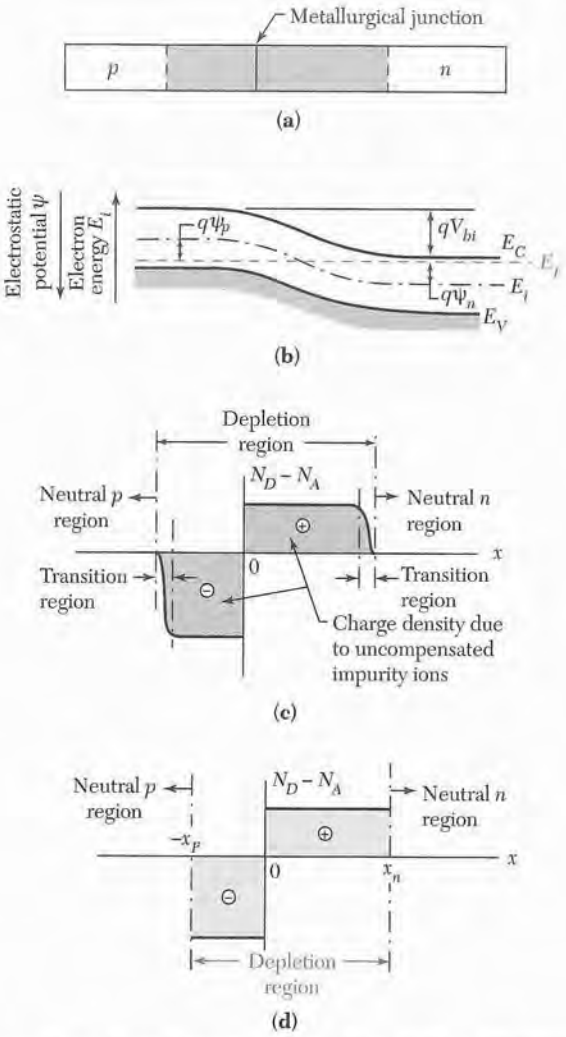


Fig. 5 (*a*) A *p–n* junction with abrupt doping changes at the metallurgical junction. (*b*) Energy band diagram of an abrupt junction at thermal equilibrium. (*c*) Space charge distribution. (*d*) Rectangular approximation of the space charge distribution.

$$v_n \equiv -\frac{1}{q}\left(E_i - E_F\right)\Big|_{x \geq x_n} = \frac{kT}{q}\ln\left(\frac{N_D}{n_i}\right). \tag{11}$$

The total electrostatic potential difference between the *p*-side and the *n*-side neutral regions at thermal equilibrium is called the *built-in potential* V_{bi}:

$$\boxed{V_{bi} = \psi_n - \psi_p = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right).} \tag{12}$$

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4.2.3 Space Charge

Moving from a neutral region toward the junction, we encounter a narrow transition region, shown in Fig. 5*c*. Here the space charge of impurity ions is partially compensated by the mobile carriers. Beyond the transition region we enter the completely depleted region where the mobile carrier densities are zero. This is called the *depletion region* (also called the space-charge region). For typical p–n junctions in silicon and gallium arsenide, the width of each transition region is small compared with the width of the depletion region. Therefore, we can neglect the transition region and represent the depletion region by the rectangular distribution shown in Fig. 5*d*, where x_p and x_n denote the depletion layer widths of the p- and n-sides for the completely depleted region with $p = n = 0$. Equation 7 becomes

$$\frac{d^2\psi}{dx^2} = \frac{q}{\varepsilon_s}\left(N_A - N_D\right). \tag{13}$$

The magnitudes of $|\psi_p|$ and ψ_n as calculated from Eqs.10 and 11 are plotted in Fig. 6 as a function of the doping concentration of silicon and gallium arsenide. For a given doping concentration, the electrostatic potential of gallium arsenide is higher because of its smaller intrinsic concentration n_i.

EXAMPLE 1

Calculate the built-in potential for a silicon p–n junction with $N_A = 10^{18}$ cm^{-3} and $N_D = 10^{15}$ cm^{-3} at 300 K.

SOLUTION From Eq. 12 we obtain

$$V_{bi} = (0.0259)\ln\left[\frac{10^{18}\times 10^{15}}{(9.65\times 10^{9})^2}\right] = 0.774 \text{ V}.$$

Also from Fig. 6,

$$V_{bi} = \psi_n + |\psi_p| = 0.30 \text{ V} + 0.47 \text{ V} = 0.77 \text{ V}.$$

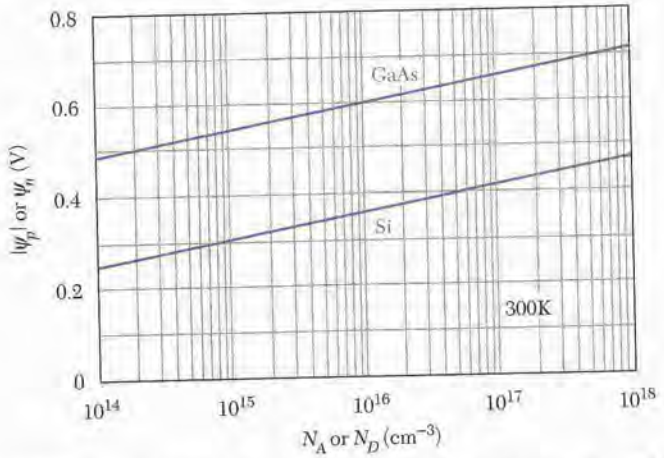


Fig. 6 Built-in potentials on the p-side and n-side of abrupt junctions in Si and GaAs as a function of impurity concentration.

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4.3 DEPLETION REGION

To solve Possion's equation, Eq.13, we must know the impurity distribution. In this section we consider two important cases—the abrupt junction and the linearly graded junction. Figure 7*a* shows an *abrupt junction*, that is, a p–n junction formed by shallow diffusion or low-energy ion implantation. The impurity distribution of the junction can be approximated by an abrupt transition of doping concentration between the n- and p-type regions. Figure 7*b* shows a linearly graded junction. For either deep diffusions or high-energy ion implantations, the impurity profiles may be approximated by linearly graded junctions, that is, the impurity distribution varies linearly across the junction. We consider the depletion regions of both types of junction.

4.3.1 Abrupt Junction

The space charge distribution of an abrupt junction is shown in Fig. 8*a*. In the depletion region, free carriers are totally depleted so that Possion's equation, Eq.13, simplifies to

$$\frac{d^2\psi}{dx^2} = +\frac{qN_A}{\varepsilon_s} \qquad \text{for} \quad -x_p \le x < 0, \tag{14a}$$

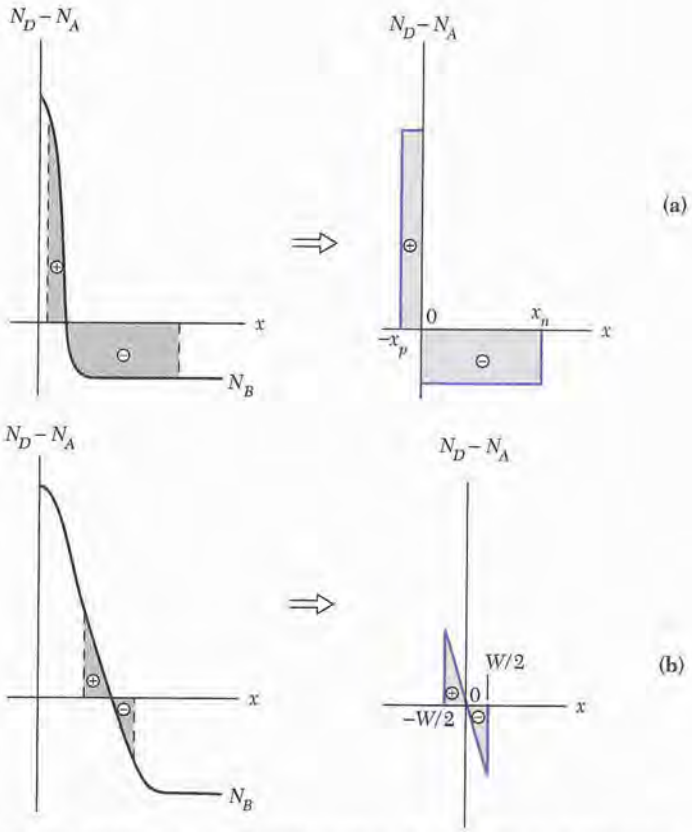


Fig. 7 Approximate doping profiles. (*a*) Abrupt junction. (*b*) Linearly graded junction.

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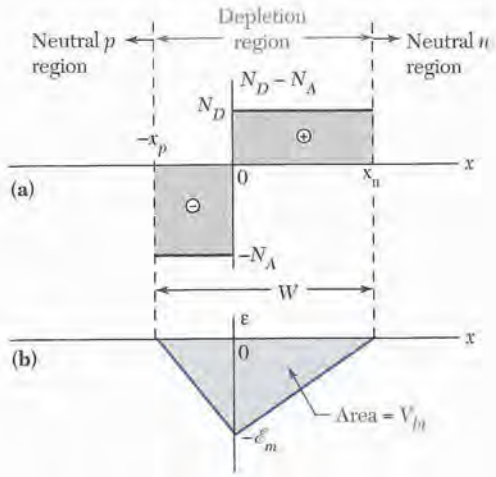


Fig. 8 (*a*) Space charge distribution in the depletion region at thermal equilibrium. (*b*) Electric-field distribution. The shaded area corresponds to the built-in potential.

$$\frac{d^2\psi}{dx^2} = -\frac{qN_D}{\varepsilon_s} \qquad \text{for} \quad 0 < x \le x_n. \tag{14b}$$

The overall space charge neutrality of the semiconductor requires that the total negative space charge per unit area in the *p*-side must precisely equal the total positive space charge per unit area in the *n*-side:

$$N_A x_p = N_D x_n. \tag{15}$$

The total depletion layer width W is given by

$$W = x_p + x_n. \tag{16}$$

The electric field shown in Fig. 8*b* is obtained by integrating Eqs. 14*a* and 14*b*, which gives

$$\mathscr{E}(x) = -\frac{d\psi}{dx} = -\frac{qN_A(x + x_p)}{\varepsilon_s} \qquad \text{for} \quad -x_p \le x < 0 \tag{17a}$$

and

$$\mathscr{E}(x) = -\mathscr{E}_m + \frac{qN_D x}{\epsilon_s} = \frac{qN_D}{\epsilon_s}(x - x_n) \qquad \text{for} \quad 0 < x \le x_n, \tag{17b}$$

where $\mathscr{E}_m$ is the maximum field that exists at $x = 0$ and is given by

$$\mathscr{E}_m = \frac{qN_D x_n}{\varepsilon_s} = \frac{qN_A x_p}{\varepsilon_s}. \tag{18}$$

Integating Eqs. 17*a* and 17*b* over the depletion region gives the total potential variation, namely, the built-in potential V_{bi}:

$$V_{bi} = -\int_{-x_p}^{x_n} \mathscr{E}(x)dx = -\int_{-x_p}^{0} \mathscr{E}(x)dx\Big|_{p\text{-side}} - \int_{0}^{x_n} \mathscr{E}(x)dx\Big|_{n\text{-side}}$$

$$= \frac{qN_A x_p^2}{2\varepsilon_s} + \frac{qN_D x_n^2}{2\varepsilon_s} = \frac{1}{2}\mathscr{E}_m W. \tag{19}$$

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Therefore, the area of the field triangle in Fig. 8*b* corresponds to the built-in potential.

Combining Eqs. 15 to 19 gives the total depletion layer width as a function of the built-in potential,

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right) V_{bi}}. \tag{20}$$

When the impurity concentration on one side of an abrupt junction is much higher than that of the other side, the junction is called a *one-sided abrupt junction* (Fig. 9*a*). Figure 9*b* shows the space charge distribution of a one-sided abrupt p^+–n junction, where $N_A >> N_D$. In this case, the depletion layer width of the p-side is much smaller than the n-side (i.e., $x_p << x_n$), and the expression for W can be simplified to

$$W \cong x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}}. \tag{21}$$

The expression for the electric-field distribution is the same as Eq. 17b:

$$\mathscr{E}(x) = -\mathscr{E}_m + \frac{qN_B x}{\varepsilon_s}, \tag{22}$$

where N_B is the lightly doped bulk concentration (i.e., N_D for a p^+–n junction). The field decreases to zero at $x = W$. Therefore,

$$\mathscr{E}_m = \frac{qN_B W}{\varepsilon_s} \tag{23}$$

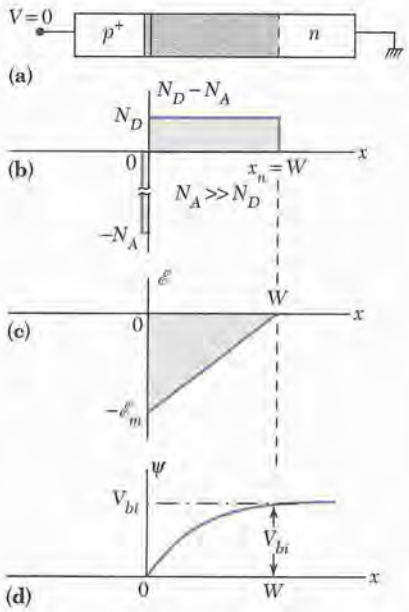


Fig. 9 (*a*) One-sided abrupt junction (with $N_A >> N_D$) in thermal equilibrium. (*b*) Space charge distribution. (*c*) Electric-field distribution. (*d*) Potential distribution with distance, where V_{bi} is the built-in potential.

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and

$$\mathscr{E}(x) = \frac{qN_B}{\varepsilon_s}(-W + x) = -\mathscr{E}_m\left(1 - \frac{x}{W}\right), \tag{24}$$

which is shown in Fig. 9*c*.

Integrating Possion's equation once more gives the potential distribution

$$\psi(x) = -\int_0^x \mathscr{E}\,dx = \mathscr{E}_m\left(x - \frac{x^2}{2W}\right) + \text{constant}. \tag{25}$$

With zero potential in the neutral p-region as a reference, or $\psi(0) = 0$, and employing Eq. 19,

$$\boxed{\psi(x) = \frac{V_{bi}x}{W}\left(2 - \frac{x}{W}\right).} \tag{26}$$

The potential distribution is shown in Fig. 9*d*.

EXAMPLE 2

For a silicon one-sided abrupt junction with $N_A = 10^{19}$ cm^{-3} and $N_D = 10^{16}$ cm^{-3}, calculate the depletion layer width and the maximum field at zero bias ($T = 300$ K).

SOLUTION From Eqs. 12, 21, and 23, we obtain

$$V_{bi} = 0.0259\ln\left[\frac{10^{19}\times 10^{16}}{\left(9.65\times 10^{9}\right)^2}\right] = 0.895\ \text{V},$$

$$W \cong \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}} = 3.41\times 10^{-5} = 0.343\ \mu\text{m},$$

$$\mathscr{E}_m = \frac{qN_B W}{\varepsilon_s} = 0.52\times 10^{4}\ \text{V/cm}.$$

The previous discussions are for a p–n junction at thermal equilibrium without external bias. The equilibrium energy band diagram, shown again in Fig. 10*a*, illustrates that the total electrostatic potential across the junction is V_{bi}. The corresponding potential energy difference from the p-side to the n-side is qV_{bi}. If we apply a positive voltage V_F to the p-side with respect to the n-side, the p–n junction becomes forward-biased, as shown in Fig. 10*b*. The total electrostatic potential across the junction decreases by V_F, that is, it is replaced with $V_{bi} - V_F$. Thus, forward bias reduces the depletion layer width.

By contrast, as shown in Fig. 10*c*, if we apply positive voltage V_R to the n-side with respect to the p-side, the p–n junction now becomes reverse-biased and the total electrostatic potential across the junction increases by V_R, that is, it is replaced by $V_{bi} + V_R$. Here, we find that reverse bias increases the depletion layer width. Substituting these voltage values in Eq. 21 yields the depletion layer widths as a function of the applied voltage for a one-sided abrupt junction:

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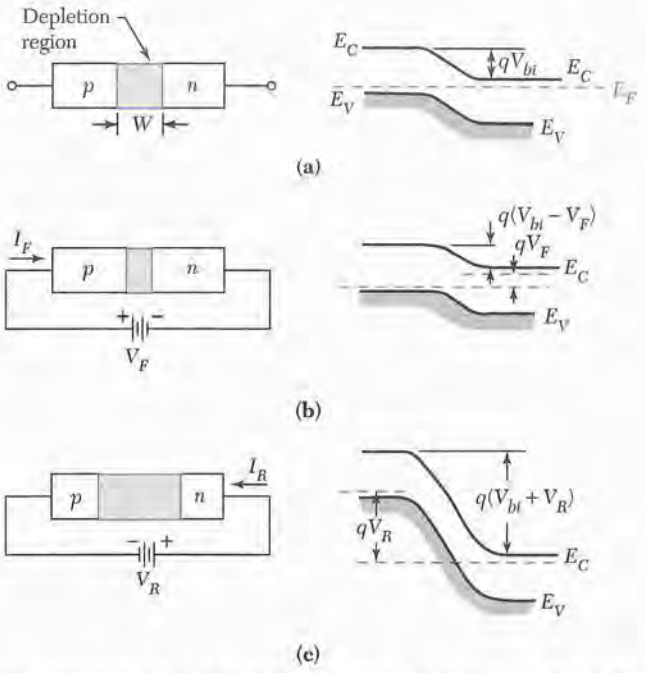


Fig. 10 Schematic representations of depletion layer width and energy band diagrams of a p–n junction under various biasing conditions. (a) Thermal-equilibrium condition. (b) Forward-bias condition. (c) Reverse-bias condition.

$$W = \sqrt{\frac{2\varepsilon_s\left(V_{bi} - V\right)}{qN_B}}, \tag{27}$$

where N_B is the lightly doped bulk concentration, and V is positive for forward bias and negative for reverse bias. Note that the depletion layer width W varies as the square root of the total electrostatic potential difference across the junction.

4.3.2 Linearly Graded Junction

We first consider the case of thermal equilibrium. The impurity distribution for a linearly graded junction is shown in Fig. 11a. The Possion equation for the case is

$$\frac{d^2\psi}{dx^2} = \frac{-d\mathscr{E}}{dx} = \frac{-\rho_s}{\varepsilon_s} = \frac{-q}{\varepsilon_s}ax \qquad -\frac{W}{2} \le x \le \frac{W}{2}, \tag{28}$$

where a is the impurity gradient (in cm^{-4}) and W is the depletion-layer width.

We have assumed that mobile carriers are negligible in the depletion region. By integrating Eq. 28 once with the boundary conditions that the electric field is zero at $\pm W/2$, we obtain the electric-field distribution shown in Fig. 11b

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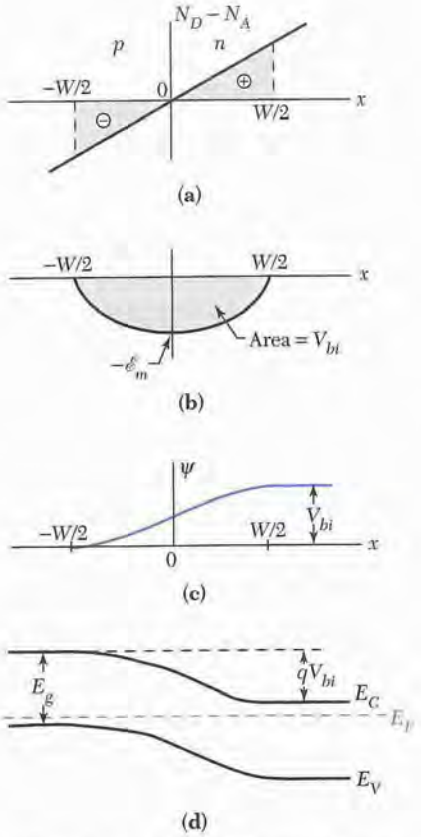


Fig. 11 Linearly graded junction in thermal equilibrium. (*a*) Impurity distribution. (*b*) Electric-field distribution. (*c*) Potential distribution with distance. (*d*) Energy band diagram.

$$\mathscr{E}(x) = -\frac{qa}{\varepsilon_s}\left[\frac{(W/2)^2 - x^2}{2}\right], \tag{29}$$

The maximum field at $x = 0$ is

$$\mathscr{E}_m = \frac{qaW^2}{8\varepsilon_s}. \tag{29a}$$

Integrating Eq. 28 once again yields both the potential distribution and the corresponding energy band diagram shown in Figs. 11*c* and 11*d*, respectively. The built-in potential and the depletion layer width are given by

$$V_{bi} = \frac{qaW^3}{12\varepsilon_s} \tag{30}$$

and

$$W = \left(\frac{12\varepsilon_s V_{bi}}{qa}\right)^{1/3}. \tag{31}$$

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Since the values of the impurity concentrations at the edges of the depletion region ($-W/2$ and $W/2$) are the same and both are equal to $aW/2$, the built-in potential for a linearly graded junction may be expressed in a form similar to Eq. 12*:

$$V_{bi} = \frac{kT}{q}\ln\left[\frac{(aW/2)(aW/2)}{n_i^2}\right] = \frac{2kT}{q}\ln\left(\frac{aW}{2n_i}\right). \tag{32}$$

Solving the transcendental equation that results when W is eliminated from Eqs. 31 and 32 yields the built-in potential as a function of a. The results for silicon and gallium arsenide linearly graded junctions are shown in Fig. 12.

When either forward or reverse bias is applied to the linearly graded junction, the variations of the depletion layer width and the energy band diagram will be similar to those shown in Fig. 10 for abrupt junctions. However, the depletion layer width will vary as $(V_{bi} - V)^{1/3}$, where V is positive for forward bias and negative for reverse bias.

EXAMPLE 3

For a silicon linearly graded junction with a impurity gradient of 10^{20} cm^{-4}, the depletion-layer width is 0.5 μm. Calculate the maximum field and built-in voltage (T = 300 K) :

SOLUTION From Eq. 29a and Eq. 32, we obtain

$$\mathscr{E}_m = \frac{qaW^2}{8\varepsilon_s} = \frac{1.6\times10^{-19}\times10^{20}\times\left(0.5\times10^{-4}\right)^2}{8\times11.9\times8.85\times10^{-14}} = 4.75\times10^3 \text{ V / cm},$$

$$V_{bi} = \frac{2kT}{q}\left(\frac{aW}{2n_i}\right) = 2\times0.0259\left(\frac{10^{20}\times0.5\times10^{-4}}{2\times9.65\times10^9}\right) = 0.645 \text{ V}.$$

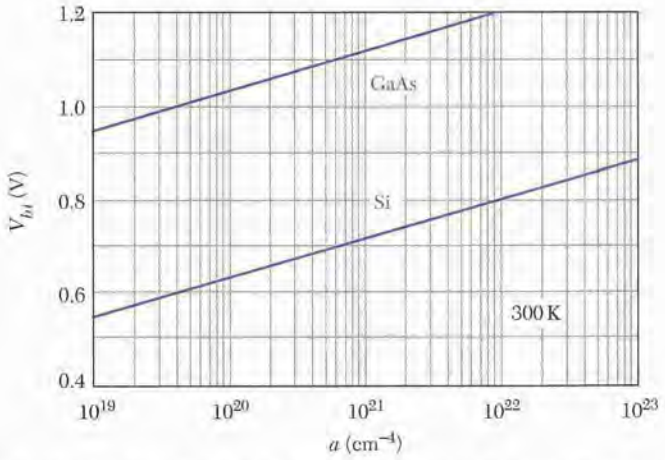


Fig. 12 Built-in potential for a linearly graded junction in Si and GaAs as a function of impurity gradient.

* Based on an accurate numerical technique, the built-in potential is given by $V_{bi} = \frac{2}{3}\frac{kT}{q}\ln\left(\frac{a^2\varepsilon_s kT/q}{8qn_i^3}\right)$.

For a given impurity gradient, the V_{bi} is smaller than that calculated from Eq.32 by about 0.05 – 0.1 V.

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4.4 DEPLETION CAPACITANCE

The junction depletion layer capacitance per unit area is defined as $C_j = dQ/dV$, where dQ is the incremental change in depletion layer charge per unit area for an incremental change in the applied voltage dV.*

Figure 13 illustrates the depletion capacitance of a p–n junction with an arbitrary impurity distribution. The charge and electric-field distributions indicated by the solid lines correspond to a voltage V applied to the n-side. If this voltage is increased by an amount dV, the charge and field distributions will expand to those regions bounded by the dashed lines. In Fig. 13b, the incremental charge dQ corresponds to the colored area between the two charge distribution curves on either side of the depletion region. The incremental space charges on the n- and p-sides of the depletion region are equal but with opposite charge polarity, thus maintaining overall charge neutrality. This incremental charge dQ causes an increase in the electric field by an amount $d\mathscr{E} = dQ/\varepsilon_s$ (from Possion's equation). The corresponding change in the applied voltage dV, represented by the cross-hatched area in Fig. 13c is approximately $Wd\mathscr{E}$, which equals WdQ/ε_s. Therefore, the depletion capacitance per unit area is given by

$$C_j \equiv \frac{dQ}{dV} = \frac{dQ}{W\dfrac{dQ}{\varepsilon_s}} = \frac{\varepsilon_s}{W} \tag{33}$$

or

$$\boxed{C_j = \frac{\varepsilon_s}{W} \quad \text{F/cm}^2.} \tag{33a}$$

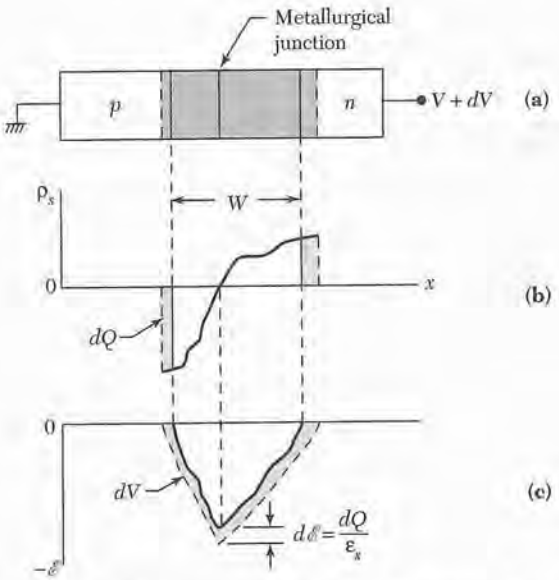


Fig. 13 (a) p–n junction with an arbitrary impurity profile under reverse bias. (b) Change in space charge distribution due to change in applied bias. (c) Corresponding change in electric-field distribution.

*The capacitance is also referred to as the transition region capacitance.

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4.4.1 Capacitance-Voltage Characteristics

Equation 33 for the depletion capacitance per unit area is the same as the standard expression for a parallel-plate capacitor where the spacing between the two plates represents the depletion-layer width. The equation is valid for any arbitrary impurity distribution.

In deriving Eq. 33 we have assumed that only the variation of the space charge in the depletion region contributes to the capacitance. This certainly is a good assumption for the reverse-bias condition. For forward biases, however, a large current can flow across the junction corresponding to a large number of mobile carriers present within the neutral region. The incremental change of these mobile carriers with respect to the biasing voltage contributes an additional term, called the diffusion capacitance, which is considered in Section 4.6.

For a one-sided abrupt junction, we obtain, from Eqs. 27 and 33,

$$C_j = \frac{\varepsilon_s}{W} = \sqrt{\frac{q\varepsilon_s N_B}{2(V_{bi} - V)}} \tag{34}$$

or

$$\boxed{\frac{1}{C_j^2} = \frac{2(V_{bi} - V)}{q\varepsilon_s N_B}}. \tag{35}$$

It is clear from Eq. 35 that a plot of $1/C_j^2$ versus V produces a straight line for a one-sided abrupt junction. The slope gives the impurity concentration N_B of the substrate, and the intercept (at $1/C_j^2 = 0$) gives V_{bi}.

EXAMPLE 4

For a silicon one-sided abrupt junction with $N_A = 2 \times 10^{19}$ cm^{-3} and $N_D = 8 \times 10^{15}$ cm^{-3}, calculate the junction capacitance at zero bias and reversed bias of 4 V ($T = 300$ K).

SOLUTION From Eqs. 12, 27, and 34, we obtain at zero bias

$$V_{bi} = 0.0259 \ln \frac{2 \times 10^{19} \times 8 \times 10^{15}}{\left(9.65 \times 10^{9}\right)^2} = 0.906 \text{ V},$$

$$W\big|_{V=0} \equiv \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}} = \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-14} \times 0.906}{1.6 \times 10^{-19} \times 8 \times 10^{15}}} = 3.86 \times 10^{-5} = 0.386 \ \mu\text{m},$$

$$C_j\big|_{V=0} = \frac{\varepsilon_s}{W\big|_{V=0}} = \sqrt{\frac{q\varepsilon_s N_B}{2V_{bi}}} = 2.728 \times 10^{-8} \text{ F/cm}^2.$$

From Eqs. 27 and 34, we can obtain at reverse bias of 4 V:

$$W\big|_{V=-4} \equiv \sqrt{\frac{2\varepsilon_s (V_{bi} - V)}{qN_D}} = \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-14} \times (0.906 + 4)}{1.6 \times 10^{-19} \times 8 \times 10^{15}}} = 8.99 \times 10^{-5} = 0.899 \ \mu\text{m},$$

$$C_j\big|_{V=-4} = \frac{\varepsilon_s}{W\big|_{V=-4}} = \sqrt{\frac{q\varepsilon_s N_B}{2(V_{bi} - V)}} = 1.172 \times 10^{-8} \text{ F/cm}^2.$$

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4.4.2 Evaluation of Impurity Distribution

The capacitance-voltage characteristics can be used to evaluate an arbitrary impurity distribution. We consider the case of p^+–n junction with a doping profile on the n-side, as shown in Fig. 14*b*. As before, the incremental change in depletion layer charge per unit area dQ for an incremental change in the applied voltage dV is given by $qN(W)\,dW$ (i.e., the shaded area in Fig. 14*b*). The corresponding change in applied voltage (shaded area in Fig. 14*c*) is

$$dV \equiv \left(d\mathscr{E}\right) W = \left(\frac{dQ}{\varepsilon_s}\right) W = \frac{qN(W)\,dW^2}{2\varepsilon_s}. \tag{36}$$

By substituting W from Eq. 33, we obtain an expression for the impurity concentration at the edge of the depletion region:

$$N(W) = \frac{2}{q\varepsilon_s}\left[\frac{1}{d\left(1/C_j^{\,2}\right)/dV}\right]. \tag{37}$$

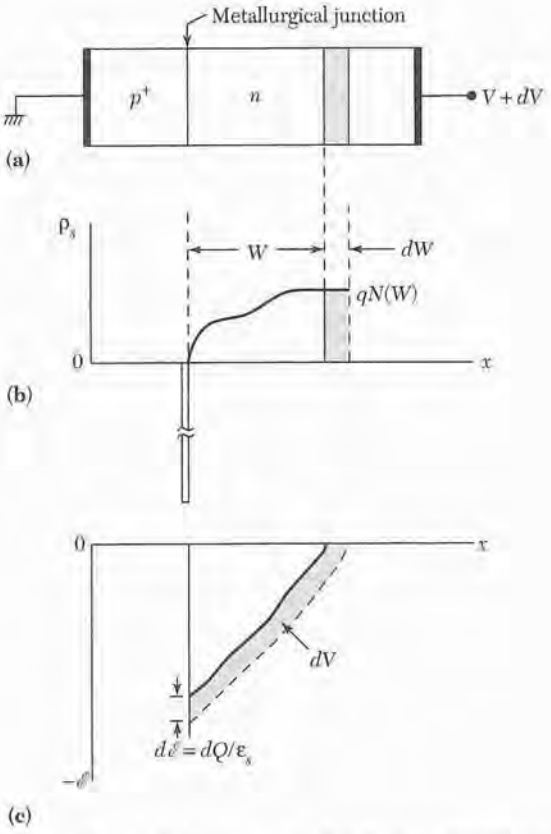


Fig. 14 (*a*) p^+–n junction with an arbitary impurity distribution. (*b*) Change in space charge distribution in the lightly doped side due to a change in applied bias. (*c*) Corresponding change in the electric-field distribution.

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Thus, we can measure the capacitance per unit area versus reverse-bias voltage and plot $1/C_j^{\,2}$ versus V. The slope of the plot, that is, $d\,(1/C_j^{\,2})/dV$, yields $N(W)$. Simultaneously, W is obtained from Eq. 33. A series of such calculations produces a complete impurity profile. This approach is referred to as the C–V method for measuring impurity profiles.

For a linearly graded junction, the depletion layer capacitance is obtained from Eqs. 31 and 33:

$$C_j = \frac{\varepsilon_s}{W} = \left[\frac{qa\varepsilon_s^{\,2}}{12\left(V_{bi} - V\right)}\right]^{1/3} \text{F/cm}^2. \tag{38}$$

For such a junction we can plot $1/C^3$ versus V and obtain the impurity gradient and V_{bi} from the slope and the intercept, respectively.

4.4.3 Varactor

Many circuit applications employ the voltage-variable properties of reverse-biased p–n junctions. A p–n junction designed for such a purpose is called a *varactor*, which is a shortened form of variable reactor. As previously derived, the reverse-biased depletion capacitance is given by

$$C_j \propto \left(V_{bi} + V_R\right)^{-n} \tag{39}$$

or

$$C_j \propto \left(V_R\right)^{-n} \quad \text{for} \quad V_R >> V_{bi}, \tag{39a}$$

where $n = \frac{1}{3}$ for a linearly graded junction and $n = \frac{1}{2}$ for an abrupt junction. Thus, the voltage sensitivity of C (i.e., variation of C with V_R) is greater for an abrupt junction than for a linearly graded junction. We can further increase the voltage sensitivity by using a hyperabrupt junction having an exponent n (Eq. 39) greater than $\frac{1}{2}$.

Figure 15 shows three p^+–n doping profiles with the doner distribution $N_D(x)$ given by $B(x/x_0)^m$, where B and x_0 are constants, $m = 1$ for a linearly graded junction, $m = 0$ for an abrupt junction, and $m = -\frac{3}{2}$ for a hyperabrupt junction. The hyperabrupt profile can

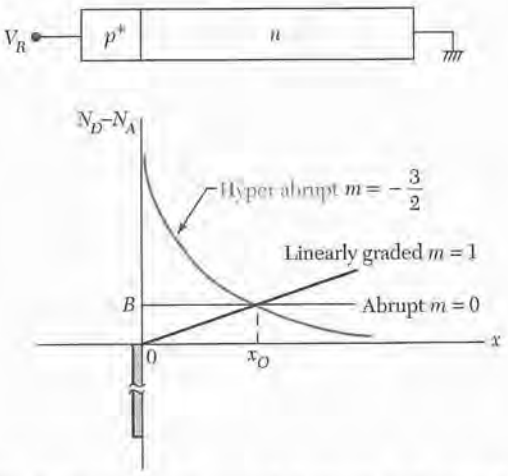


Fig. 15 Impurity profiles for hyperabrupt, one-sided abrupt, and one-sided linearly graded junctions.

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be achieved by epitaxial growth techniques discussed in Chapter 10. To obtain the capacitance-voltage relationship, we solve Possion's equation:

$$\frac{d^2\psi}{dx^2} = -B\left(\frac{x}{x_0}\right)^m. \tag{40}$$

Integrating Eq. 40 twice with appropriate boundary conditions gives the dependence of the depletion layer width on the reverse bias:

$$W \propto \left(V_R\right)^{1/(m+2)} \tag{41}$$

Therefore,

$$C_j = \frac{\varepsilon_s}{W} \propto \left(V_R\right)^{-1/(m+2)} \tag{42}$$

Comparing Eq. 42 with Eq. 39a yields $n = 1/(m + 2)$. For hyperabrupt junctions with $n > ½$, m must be a negative number.

By choosing different values for m, we can obtain a wide variety of C_j-versus-V_R dependencies for specific applications. One interesting example, shown in Fig. 15, is the case for $m = -½$. For this case, $n = 2$. When this varactor is connected to an inductor L in a resonant circuit, the resonant frequency varies linearly with the voltage applied to the varactor:

$$\omega_r = \frac{1}{\sqrt{LC_j}} \propto \frac{1}{\sqrt{V_R^{-n}}} = V_R \qquad \text{for } n = 2. \tag{43}$$

4.5 CURRENT-VOLTAGE CHARACTERISTICS

A voltage applied to a p–n junction will disturb the precise balance between the diffusion current and drift current of electrons and holes. Under forward bias, the applied voltage reduces the electrostatic potential across the depletion region, as shown in the middle of Fig. 16. The drift current is reduced in comparison to the diffusion current. We have an enhanced hole diffusion from the p-side to the n-side and electron diffusion from the n-side to the p-side. Therefore, minority carrier injections occur, that is, electrons are injected into the p-side, whereas holes are injected into the n-side. Under reverse bias, the applied voltage increases the electrostatic potential across the depletion region as shown in the middle of Fig. 16*b*. This greatly reduces the diffusion currents, resulting in a small reverse current. In this section, we first consider the ideal current-voltage characteristics. We then discuss departures from these ideal characteristics due to generation and recombination and other effects.

4.5.1 Ideal Characteristics

We now derive the ideal current-voltage characteristics based on the following assumptions: (a) The depletion region has abrupt boundaries and, outside the boundaries, the semiconductor is assumed to be neutral; (b) the carrier densities at the boundaries are related by the electrostatic potential difference across the junction. (c) the low-injection condition, that is, the injected minority carrier densities, are small compared with the majority carrier densities; in other words, the majority carrier densities are changed negligibly at the boundaries of neutral regions by the applied bias; and (d) neither genera-

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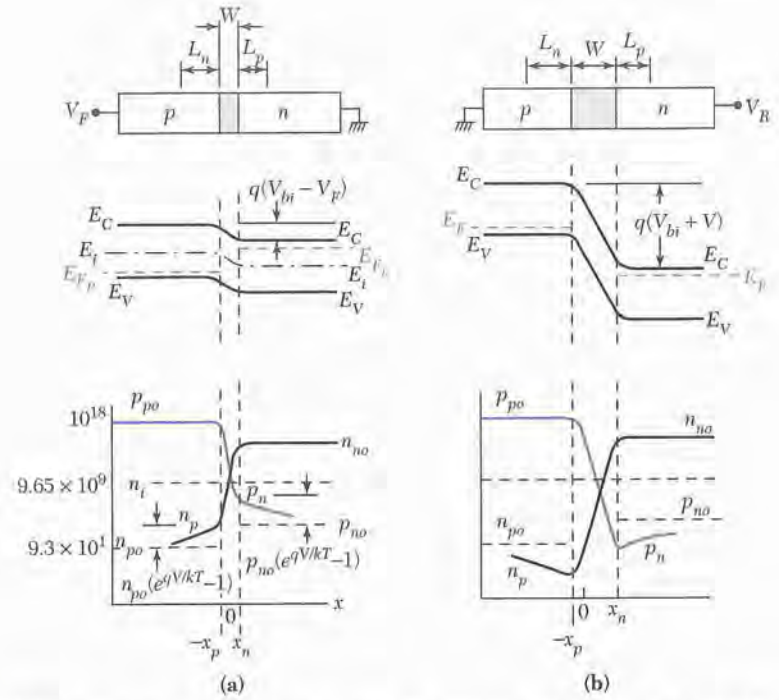


Fig. 16 Depletion region, energy band diagram and carrier distribution. (*a*) Forward bias. (*b*) Reverse bias.

tion nor recombination current exists in the depletion region, and the electron and hole currents are constant throughout the depletion region. Departures from these idealized assumptions are considered in the next section.

At thermal equilibrium, the majority carrier density in the neutral regions is essentially equal to the doping concentration. We use the subscripts n and p to denote the semiconductor type and the subscript o to specify the condition of thermal equilibrium. Hence, n_{no} and n_{po} are the equilibrium electron densities in the n- and p-sides, respectively. The expression for the built-in potential in Eq. 12 can be rewritten as

$$V_{bi} = \frac{kT}{q} \ln \frac{p_{po} n_{no}}{n_i^2} = \frac{kT}{q} \ln \frac{n_{no}}{n_{po}}, \tag{44}$$

where the mass action law $p_{po} n_{po} = n_i^2$ has been used. Rearranging Eq. 44 gives

$$n_{no} = n_{po} e^{qV_{bi}/kT}. \tag{45}$$

Similarly, we have

$$p_{po} = p_{no} e^{qV_{bi}/kT}. \tag{46}$$

We note from Eqs. 45 and 46 that the electron density and the hole density at the two boundaries of the depletion region are related through the electrostatic potential difference V_{bi} at thermal equilibrium. From our second assumption we expect that the same relation holds when the electrostatic potential difference is changed by an applied voltage.

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When a forward bias is applied, the electrostatic potential difference is reduced to $V_{bi} - V_F$; but when a reverse bias is applied, the electrostatic potential difference is increased to $V_{bi} + V_R$. Thus, Eq. 45 is modified to

$$n_n = n_p e^{q(V_{bi}-V)/kT}, \tag{47}$$

where n_n and n_p are the nonequilibrium electron densities at the boundaries of the depletion region in the n- and p-sides, respectively, with V positive for forward bias and negative for reverse bias. For the low-injection condition, the injected minority carrier density is much smaller than the majority carrier density; therefore, $n_n \equiv n_{no}$. Substituting this condition and Eq. 45 into Eq. 47 yields the electron density at the boundary of the depletion region on the p-side ($x = -x_p$):

$$n_p = n_{po} e^{qV/kT} \tag{48}$$

or

$$n_p - n_{po} = n_{po}\left(e^{qV/kT} - 1\right). \tag{48a}$$

Similarly, we have

$$p_n = p_{no} e^{qV/kT} \tag{49}$$

or

$$p_n - p_{no} = p_{no}\left(e^{qV/kT} - 1\right) \tag{49a}$$

at $x = x_n$ for the n-type boundary. Figures 16*a* and 16*b* show band diagrams and carrier concentrations in a p–n junction under forward-bias and reverse-bias conditions, respectively. Note that the minority carrier densities at the boundaries ($-x_p$ and x_n) increase substantially above their equilibrium values under forward bias, whereas they decrease below their equilibrium values under reverse bias. Equations 48 and 49 define the minority carrier densities at the boundaries of depletion region. These equations are the most important boundary conditions for the ideal current-voltage characteristics.

Under our idealized assumptions, no current is generated within the depletion region; all currents come from the neutral regions. In the neutral n-region, there is no electric field, thus the steady-state continuity equation reduces to

$$\frac{d^2 p_n}{dx^2} - \frac{p_n - p_{no}}{D_p \tau_p} = 0. \tag{50}$$

The solution of Eq. 50 with the boundary conditions of Eq. 49 and $p_n(x = \infty) = p_{no}$ gives

$$p_n - p_{no} = p_{no}\left(e^{qV/kT} - 1\right) e^{-(x-x_n)/L_p}, \tag{51}$$

where L_p, which is equal to $\sqrt{D_p \tau_p}$, is the diffusion length of holes (minority carriers) in the n-region. At $x = x_n$,

$$J_p(x_n) = -qD_p \frac{dp_n}{dx}\bigg|_{x_n} = \frac{qD_p p_{no}}{L_p}\left(e^{qV/kT} - 1\right). \tag{52}$$

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Similarly, we obtain for the neutral p-region

$$n_p - n_{po} = n_{po}\left(e^{qV/kT} - 1\right) e^{(x+x_p)/L_n} \tag{53}$$

and

$$J_n(-x_p) = qD_n \left.\frac{dn_p}{dx}\right|_{-x_p} = \frac{qD_n n_{po}}{L_n}\left(e^{qV/kT} - 1\right), \tag{54}$$

where L_n, which is equal to $\sqrt{D_n \tau_n}$, is the diffusion length of electrons. The minority carrier densities (Eqs. 51 and 53) are shown in the middle of Fig. 17.

The graphs illustrate that the injected minority carriers recombine with the majority carriers as the minority carriers move away from the boundaries. The electron and hole currents are shown at the bottom of Fig. 17. The hole and electron currents at the boundaries are given by Eqs. 52 and 54, respectively. The hole diffusion current will decay

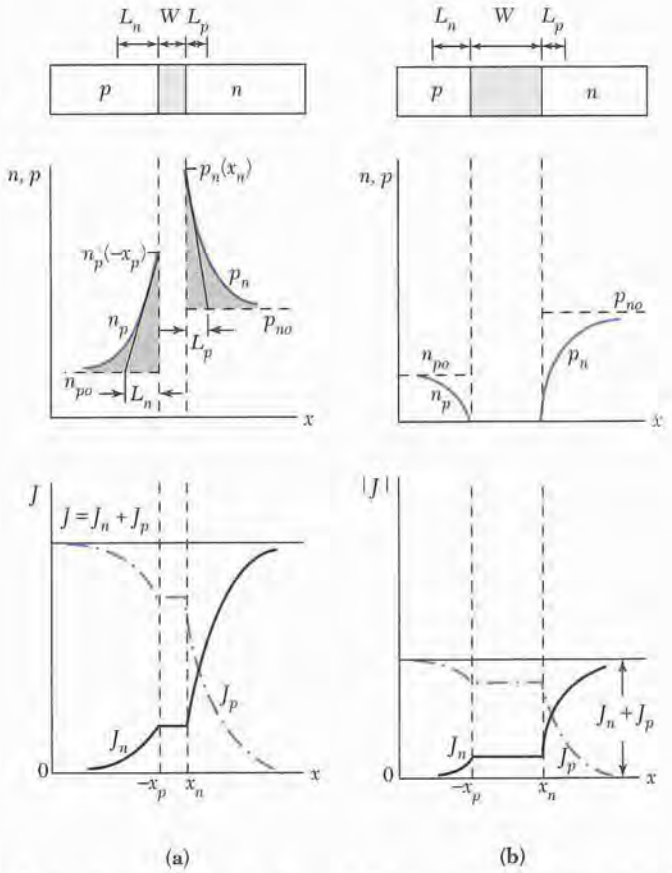


Fig. 17 Injected minority carrier distribution and electron and hole currents. (*a*) Forward bias. (*b*) Reverse bias. The figure illustrates idealized currents. For practical devices, the currents are not constant across the space charge layer.

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exponentially in the n-region with diffusion length L_p, and the electron diffusion current will decay exponentially in the p-region with diffusion length L_n.

The total current is constant throughout the device and is the sum of Eqs. 52 and 54:

$$\boxed{J = J_p(x_n) + J_n(-x_p) = J_s\left(e^{qV/kT} - 1\right),} \tag{55}$$

$$\boxed{J_s \equiv \frac{qD_p p_{no}}{L_p} + \frac{qD_n n_{po}}{L_n},} \tag{55a}$$

where J_s is the saturation current density. Equation 55 is the *ideal diode equation*.[1] The ideal current-voltage characteristic is shown in Figs. 18*a* and 18*b* in the Cartesian and

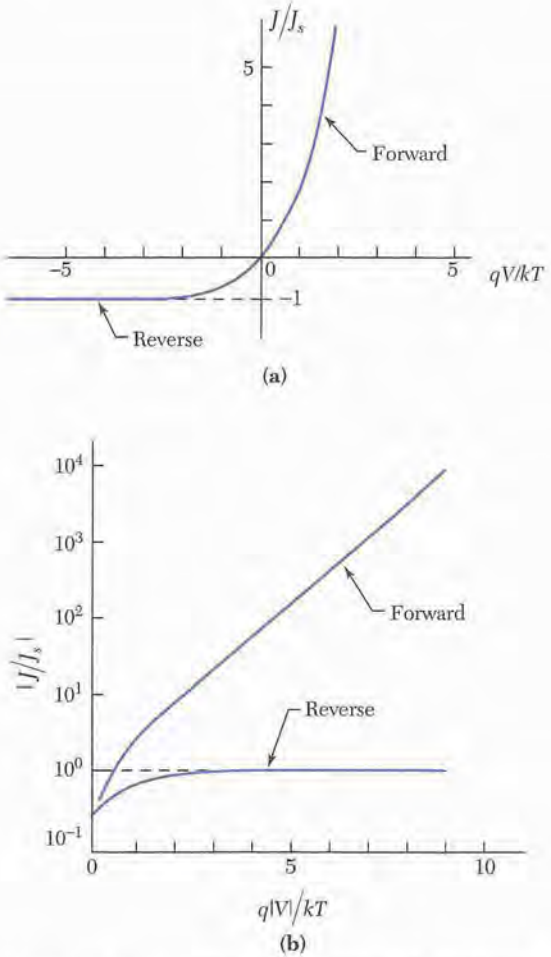


Fig. 18 Ideal current-voltage characteristics. (*a*) Cartesian plot. (*b*) Semilog plot.

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semilog plots, respectively. In the forward direction with positive bias on the p-side, for $V \geq 3kT/q$, the rate of current increase is constant, as shown in Fig. 18*b*. At 300 K for every decade change of current, the voltage change for an ideal diode is 60 mV (= 2.3 kT/q). In the reverse direction, the current density saturates at $-J_s$.

EXAMPLE 5

Calculate the ideal reverse saturation current in a Si p–n junction diode with a cross-sectional area of 2×10^{-4} cm^2. The parameters of the diode are

$N_A = 5 \times 10^{16}$ cm^{-3}, $N_D = 10^{16}$ cm^{-3}, $n_i = 9.65 \times 10^9$, cm^{-3}
$D_n = 21$ cm^2/s, $D_p = 10$ cm^2/s, $\tau_p = \tau_n = 5 \times 10^{-7}$ s.

SOLUTION From Eq. 55*a* and $L_p = \sqrt{D_p \tau_p}$, we can obtain

$$J_s = \frac{qD_p p_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n} = qn_i^2 \left(\frac{1}{N_D} \sqrt{\frac{D_p}{\tau_p}} + \frac{1}{N_A} \sqrt{\frac{D_n}{\tau_n}} \right),$$

$$= 1.6 \times 10^{-19} \times \left(9.65 \times 10^9\right)^2 \left(\frac{1}{10^{16}} \sqrt{\frac{10}{5 \times 10^{-7}}} + \frac{1}{5 \times 10^{16}} \sqrt{\frac{21}{5 \times 10^{-7}}} \right),$$

$$= 8.58 \times 10^{-12} \text{ A/cm}^2.$$

From the cross-sectional area $A = 2 \times 10^{-4}$ cm^2, we obtain

$$I_s = A \times J_s = 2 \times 10^{-4} \times 8.58 \times 10^{-12} = 1.72 \times 10^{-15} \text{ A}.$$

4.5.2 Generation-Recombination and High-Injection Effects

The ideal diode equation, Eq. 55, adequately describes the current-voltage characteristics of germanium p–n junctions at low current densities. For silicon and gallium arsenide p–n junctions, however, the ideal equation can only give qualitative agreement because of the generation or recombination of carriers in the depletion region.

Consider the reverse-bias condition first. Under reverse bias, carrier concentrations in the depletion region fall far below their equilibrium concentrations. The dominant generation-recombination processes discussed in Chapter 3 are those of electron and hole emissions through bandgap generation-recombination centers. The capture processes are not important because their rates are proportional to the concentration of free carriers, which is very small in the reverse-biased depletion region.

The two emission processes operate in the steady state by alternately emitting electrons and holes. The rate of electron-hole pair generation can be obtained from Eq. 48 of Chapter 3 with the conditions $p_n < n_i$ and $n_n < n_i$:

$$G = -U = \left[\frac{\sigma_p \sigma_n \upsilon_{th} N_t}{\sigma_n \exp\left(\frac{E_t - E_i}{kT} \right) + \sigma_p \exp\left(\frac{E_i - E_t}{kT} \right)} \right] n_i$$

$$\equiv \frac{n_i}{\tau_g}, \tag{56}$$

where τ_g, the generation lifetime, is the reciprocal of the expression in the square brackets. We can arrive at an important conclusion about electron-hole generation from this

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expression. Let us consider a simple case where $\sigma_n = \sigma_p = \sigma_o$. For this case, Eq. 56 reduces to

$$G = \frac{\sigma_o \upsilon_{th} N_t n_i}{2\cosh\left(\dfrac{E_t - E_i}{kT}\right)}. \tag{57}$$

The generation rate reaches a maximum value at $E_t = E_i$ and falls off exponentially as E_t moves in either direction away from the middle of the bandgap. Thus, only those centers with an energy level of E_t near the intrinsic Fermi level can contribute significantly to the generation rate.

The current due to generation in the depletion region is

$$J_{gen} = \int_0^W qGdx \cong qGW = \frac{qn_i W}{\tau_g} \tag{58}$$

where W is the depletion layer width. The total reverse current for a p^+–n junction, that is, for $N_A >> N_D$ and for $V_R > 3kT/q$, can be approximated by the sum of both the diffusion current in the neutral regions and the generation current in the depletion region:

$$\boxed{J_R \cong q\sqrt{\frac{D_p}{\tau_p}}\frac{n_i^2}{N_D} + \frac{qn_i W}{\tau_g}} \tag{59}$$

For semiconductors with large values of n_i, such as germanium, the diffusion current dominates at room temperature, and the reverse current follows the ideal diode equation. But if n_i is small, such as for silicon and gallium arsenide, the generation current in the depletion region may dominate.

EXAMPLE 6

Consider the Si p–n junction diode in Example 5 and assume $\tau_g = \tau_p = \tau_n$, calculate the generation current density for a reverse bias of 4 V.

SOLUTION From Eq. 20, we obtain

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} + V)} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)\left(\frac{kT}{q}\ln\frac{N_A N_D}{n_i^2} + V\right)}$$

$$= \sqrt{\frac{2\times 11.9\times 8.85\times 10^{-14}}{1.6\times 10^{-19}}\left(\frac{5\times 10^{16} + 10^{16}}{5\times 10^{16}\times 10^{16}}\right)\left(0.0259\ \ln\frac{5\times 10^{16}\times 10^{16}}{(9.65\times 10^9)^2} + V\right)}$$

$$= 3.97\times\sqrt{0.758 + V}\times 10^{-5}\ \text{cm}.$$

Hence the generation current density is

$$J_{gen} = \frac{qn_i W}{\tau_g} = \frac{1.6\times 10^{-19}\times 9.65\times 10^9}{5\times 10^{-7}}\times 3.97\times\sqrt{0.758 + V}\times 10^{-5}\ \text{A/cm}^2$$

$$= 1.22\times\sqrt{0.758 + V}\times 10^{-7}\ \text{A/cm}^2.$$

If we apply a reversed bias of 4 V, the generation current density is 2.66×10^{-7} A/cm^2.

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Under forward bias, the concentrations of both electrons and holes exceed their equilibrium values. The carriers will attempt to return to their equilibrium values by recombination. Therefore, the dominant generation-recombination processes in the depletion region are the capture processes. From Eq. 49 we obtain

$$p_n n_n \cong p_{no} n_{no} e^{qV/kT} = n_i^2 e^{qV/kT} \tag{60}$$

Substituting Eq. 60 in Eq. 48 of Chapter 3 and assuming $\sigma_n = \sigma_p = \sigma_o$ yields

$$U = \frac{\sigma_o \upsilon_{th} N_t n_i^2 \left(e^{qV/kT} - 1\right)}{n_n + p_n + 2n_i \cosh \dfrac{E_i - E_t}{kT}}. \tag{61}$$

In either recombination or generation, the most effective centers are those located near E_i. As practical examples, gold and copper yield effective generation-recombination centers in silicon where the values of $E_t - E_i$ are 0.02 V for gold and −0.02 eV for copper. In gallium arsenide, chromium gives an effective center with an $E_t - E_i$ value of 0.08 eV.

Equation 61 can be simplified for the case $E_t = E_i$:

$$U = \sigma_o \upsilon_{th} N_t \frac{n_i^2 \left(e^{qV/kT} - 1\right)}{n_n + p_n + 2n_i} \tag{62}$$

For a given forward bias, U reaches its maximum value at a location in the depletion region either where the denominator $n_n + p_n + 2n_i$ is a minimum or where the sum of the electron and hole concentrations, $n_n + p_n$, is at its minimum value. Since the product of these concentrations is a constant given by Eq. 60, the condition $d(p_n + n_n) = 0$ leads to

$$dp_n = -dn_n = \frac{p_n n_n}{p_n^2} dp_n \tag{63}$$

or

$$p_n = n_n \tag{64}$$

as the condition for the minimum. This condition exists at the location in the depletion region, where E_i is halfway between E_{Fp} and E_{Fn}, as illustrated in the middle of Fig. 16*a*. Here, the carrier concentrations are

$$p_n = n_n = n_i e^{qV/2kT} \tag{65}$$

and, therefore,

$$U_{\max} = \sigma_o \upsilon_{th} N_t \frac{n_i^2 \left(e^{qV/kT} - 1\right)}{2n_i \left(e^{qV/2kT} + 1\right)}, \tag{66}$$

For $V > 3kT/q$,

$$U_{\max} \cong \frac{1}{2} \sigma_o \upsilon_{th} N_t n_i e^{qV/2kT}. \tag{67}$$

The recombination current is then

$$J_{rec} = \int_0^W qU dx \cong \frac{qW}{2} \sigma_o \upsilon_{th} N_t n_i e^{qV/2kT} = \frac{qWn_i}{2\tau_r} e^{qV/2kT}, \tag{68}$$

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where τ_r is the effective recombination lifetime given by $1/(\sigma_o \upsilon_{th} N_t)$. The total forward current can be approximated by the sum of Eqs. 55 and 68. For $p_{no} >> n_{po}$ and $V > 3kT/q$ we have

$$J_F = q\sqrt{\frac{D_p}{\tau_p}}\frac{n_i^2}{N_D}e^{qV/kT} + \frac{qWn_i}{2\tau_r}e^{qV/2kT}. \tag{69}$$

In general, the experimental results can be represented empirically by

$$J_F \approx \exp\left(\frac{qV}{\eta kT}\right), \tag{70}$$

where the factor η is called the *ideality factor*. When the ideal diffusion current dominates, $\eta = 1$; whereas when the recombination current dominates, $\eta = 2$. When both currents are comparable, η has a value of between 1 and 2.

Figure 19 shows the measured forward characteristics of a silicon and gallium arsenide p–n junction at room temperature.[2] At low current levels, recombination current dominates and $\eta = 2$. At higher current levels, diffusion current dominates and η approaches 1.

At even higher current levels, we notice that the current departs from the ideal $\eta = 1$ situation and increases more gradually with forward voltage. This phenomenon is associated with two effects: series resistance and high injection. We first consider the series resistance effect. At both low- and medium-current levels, the IR drop across the neutral regions is usually small compared with kT/q (26 mV at 300 K), where I is the forward current and R is the series resistance. For example, for a silicon diode with $R =$

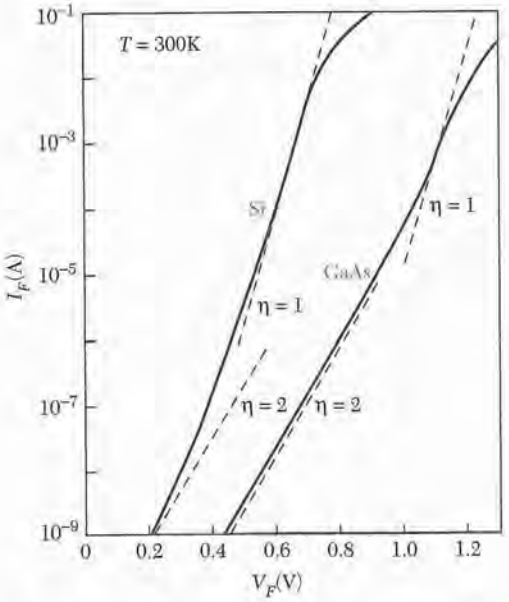


Fig. 19 Comparison of the forward current-voltage characteristics of Si and GaAs diodes[2] at 300 K. Dashed lines indicate slopes of different ideality factors η.

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1.5 ohms, the IR drop at 1 mA is only 1.5 mV. However, at 100 mA, the IR drop becomes 0.15 V, which is six times larger than kT/q. This IR drop reduces the bias across the depletion region; therefore, the current becomes

$$I \cong I_s \exp\left[\frac{q(V - IR)}{kT}\right] = \frac{I_s \exp(qV/kT)}{\exp\left[\frac{q(IR)}{kT}\right]} \tag{71}$$

and the ideal diffusion current is reduced by the factor $\exp[q(IR)/kT]$.

At high-current densities, the injected minority carrier density is compariable to the majority concentration, that is at the n-side of the junction $p_n\,(x = x_n) \cong n_n$. This is the high-injection condition. By substituting the high-injection condition in Eq. 60, we obtain $p_n\,(x = x_n) \cong n_i \exp\,(qV/2kT)$. Using this as a boundary condition, the current becomes roughly proportional to $\exp\,(qV/2kT)$. Thus, the current increases at a slower rate under the high-injection condition.

4.5.3 Temperature Effect

Operating temperature has a profound effect on device performance. In both the forward-bias and reverse-bias conditions, the magnitudes of the diffusion and the recombination-generation currents depend strongly on temperature. We consider the forward-bias case first. The ratio of hole diffusion current to the recombination is given by

$$\frac{I_{\text{diffusion}}}{I_{\text{recombination}}} = 2\frac{n_i}{N_D}\frac{L_p}{W}\frac{\tau_r}{\tau_p}e^{qV/2kT} \approx \exp\left(-\frac{E_g - qV}{2kT}\right). \tag{72}$$

This ratio depends on both the temperature and the semiconductor bandgap. Figure 20*a* shows the temperature dependence of the forward characteristics of a silicon diode. At

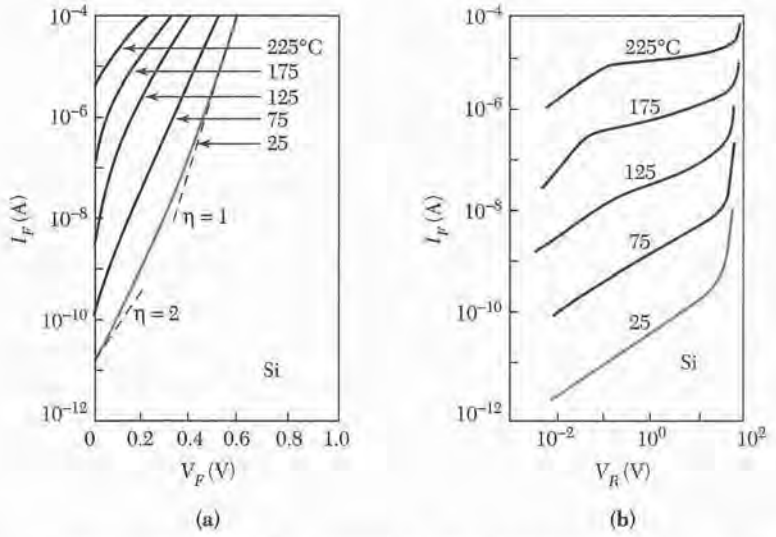


Fig. 20 Temperature dependence of the current-voltage characteristics of a Si diode[2]. (*a*) Forward bias. (*b*) Reverse bias.

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room temperature for small forward voltages, the recombination current generally dominates, whereas at higher forward voltages the diffusion current usually dominates. At a given forward bias, as the temperature increases, the diffusion current will increase more rapidly than the recombination current. Therefore, the ideal diode equation will be followed over a wide range of forward biases as the temperature increases.

The temperature dependence of the saturation current density J_s (Eq. 55a) for a one-sided p^+–n junction in which diffusion current dominates is given by

$$J_s \cong \frac{qD_p p_{no}}{L_p} \approx n_i^2 \approx \exp\left(-\frac{E_g}{kT}\right). \tag{73}$$

Thus, the activation energy obtained from the slope of a plot of J_s versus $1/T$ corresponds to the energy bandgap E_g.

In the reverse-bias condition for a p^+–n junction, the ratio of the diffusion current to the generation current is

$$\frac{I_{\text{diffusion}}}{I_{\text{generation}}} = \frac{n_i L_p}{N_D W}\frac{\tau_g}{\tau_p}. \tag{74}$$

This ratio is proportional to the intrinsic carrier density n_i. As the temperature increases, the diffusion current eventually dominates. Figure 20*b* shows the effects of temperature on the reverse characteristics of a silicon diode. At low temperatures, the generation current dominates and the reverse current varies as $\sqrt{V_R}$ in accordance with Eq. 58 for an abrupt junction (i.e., $W \sim \sqrt{V_R}$). As the temperature increases beyond 175°C, the current demonstrates a saturation tendency for $V_R \geq 3kT/q$, at which point the diffusion current becomes dominant.

4.6 CHARGE STORAGE AND TRANSIENT BEHAVIOR

Under forward bias, electrons are injected from the n-region into the p-region and holes are injected from the p-region into the n-region. Once injected across the junction, the minority carriers recombine with the majority carriers and decay exponentially with distance, as shown in Fig. 17*a*. These minority-carrier distributions lead to current flow and to charge storage in the p-n junction. We consider the stored charge, its effect on junction capacitance, and the transient behavior of the p–n junction due to sudden changes of bias.

4.6.1 Minority-Carrier Storage

The charge of injected minority carriers per unit area stored in the neutral n-region can be found by integrating the excess holes in the neutral region, shown as the shaded area in the middle of Fig. 17*a*, using Eq. 51:

$$\begin{aligned} Q_p &= q\int_{x_n}^{\infty}\left(p_n - p_{no}\right)\,dx, \\ &= q\int_{x_n}^{\infty} p_{no}\left(e^{qV/kT}-1\right)\,e^{-\left(x-x_n\right)/L_p}\,dx, \\ &= qL_p p_{no}\left(e^{qV/kT}-1\right). \end{aligned} \tag{75}$$

A similar expression can be obtained for the stored electrons in the neutral p-region. The number of stored minority carriers depends on both the diffusion length and the charge

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density at the boundary of the depletion region. We can express the stored charge in terms of the injected current. From Eqs. 52 and 75, we have

$$Q_p = \frac{L_p^2}{D_p} J_p(x_n) = \tau_p J_p(x_n) \tag{76}$$

Equation 76 states that the amount of stored charge is the product of the current and lifetime of the minority carriers. This is because the injected holes diffuse farther into the n-region before recombining if their lifetime is longer; thus, more holes are stored.

EXAMPLE 7

For an ideal abrupt silicon p^+–n junction with $N_D = 8 \times 10^{15}$ cm^{-3}, calculate the stored minority carriers per unit area in the neutral n-region when a forward bias of 1V is applied. The diffusion length of the holes is 5 μm.

SOLUTION From Eq. 75, we obtain

$$Q_p = qL_p p_{no}\left(e^{qV/kT} - 1\right) = 1.6\times10^{-19} \times 5\times10^{-4}\,\text{cm} \times \frac{\left(9.65\times10^{9}\right)^2}{8\times10^{15}} \times \left(e^{\frac{1}{0.0259}} - 1\right)$$

$$= 4.69\times10^{-2}\ \text{C/cm}^2.$$

4.6.2 Diffusion Capacitance

The depletion-layer capacitance considered previously accounts for most of the junction capacitance when the junction is reverse biased. When the junction is forward biased, there is an additional significant contribution to junction capacitance from the rearrangement of the stored charges in the neutral regions. This is called the *diffusion capacitance*, denoted C_d, a term derived from the ideal-diode case in which minority carriers move across the neutral region by diffusion.

The diffusion capacitance of the stored holes in the neutral n-region is obtained by applying the definition $C_d = AdQ_p/dV$ to Eq. 75:

$$C_d = \frac{Aq^2 L_p p_{no}}{kT} e^{qV/kT}, \tag{77}$$

where A is the device cross-section area. We may add the contribution to C_d of the stored electrons in the neutral p-region in cases of significant storage. For a p^+–n junction, however, $n_{po} << p_{no}$, and the contribution to C_d of the stored electrons becomes insignificant. Under reverse bias (i.e., V is negative), Eq. 77 shows that C_d is inconsequential because of negligible minority-carrier storage.

In many applications we prefer to represent a p–n junction by an equivalent circuit. In addition to diffusion capacitance C_d and depletion capacitance C_j, we must include conductance to account for the current through the device. In the ideal diode the conductance can be obtained from Eq. 55:

$$G = \frac{AdJ}{dV} = \frac{qA}{kT} J_s e^{qV/kT} = \frac{qA}{kT}\left(J + J_s\right) \cong \frac{qI}{kT}. \tag{78}$$

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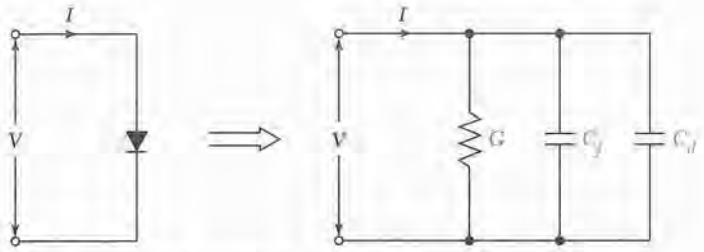


Fig. 21 Small-signal equivalent circuit of a p–n junction.

The diode equivalent circuit is shown in Fig. 21, where C_j stands for the total depletion capacitance (i.e., the result in Eq. 33 times the device area A). For low-voltage, sinusoidal excitation of a diode that is biased quiescently (i.e., at dc), the circuit shown in Fig. 21 provides adequate accuracy. Therefore, we refer to it as the diode small-signal equivalent circuit.

4.6.3 Transient Behavior

For switching applications, the forward-to-reverse-bias transition must be nearly abrupt and the transient time short. Figure 22*a* shows a simple circuit where a forward current I_F flows through a p–n junction. At time $t = 0$, switch S is suddenly thrown to the right and an initial reverse current $I_R \equiv V/R$ flows. The transient time t_{off}, plotted in Fig. 22*b*, is the time required for the current to reach 10% of the initial reverse current I_R.

The transient time may be estimated as follows. Under the forward-bias condition, the stored minority carriers in the n-region for a p^+–n junction is given by Eq. 76:

$$Q_p = \tau_p J_p = \tau_p \frac{I_F}{A}, \tag{79}$$

where I_F is the total forward current and A is the device area. If the average current flowing during the turn-off period is $I_{R,\text{ ave}}$, the turn-off time is the length of time required to remove the total stored charge Q_p:

$$t_{\text{off}} \equiv \frac{Q_p A}{I_{R,\text{ave}}} = \tau_p \left(\frac{I_F}{I_{R,\text{ave}}} \right). \tag{80}$$

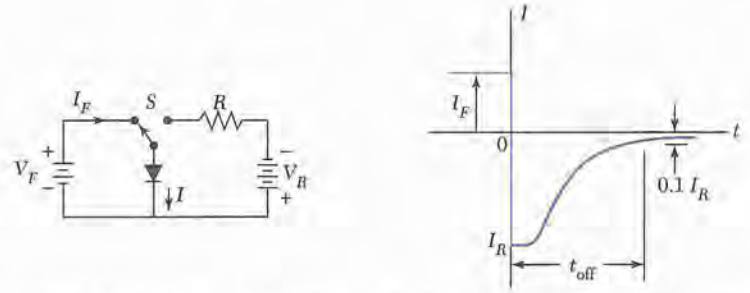


Fig. 22 Transient behavior of a p–n junction. (*a*) Basic switching circuit. (*b*) Transient response of the current switched from forward bias to reverse bias.

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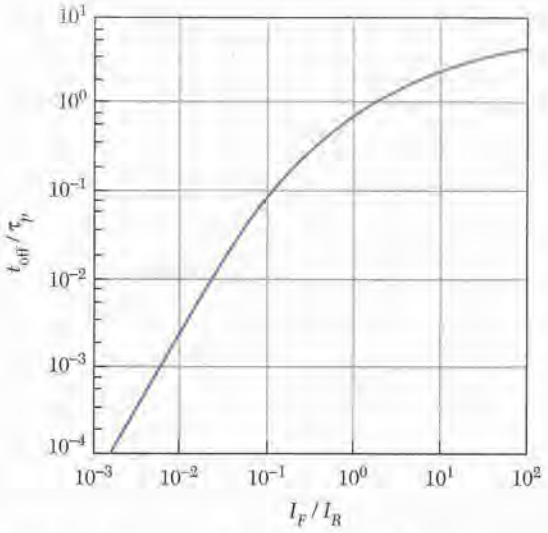


Fig. 23 Normalized transient time versus the ratio of forward current to reverse current.[3]

Thus the turn-off time depends on both the ratio of forward to reverse currents and the lifetime of the minority carriers. The result of a more precise turn-off time calculation[3] accounting for the time-dependent minority-carrier diffusion problem is shown in Fig. 23. For fast-switching devices, we must reduce the lifetime of the minority carriers. Therefore, recombination-generation centers that have energy levels located near mid-bandgap, such as gold in silicon, are usually introduced.

4.7 JUNCTION BREAKDOWN

When a sufficiently large reverse voltage is applied to a p–n junction, the junction breaks down and conducts a very large current. Although the breakdown process is not inherently destructive, the maximum current must be limited by an external circuit to avoid excessive junction heating. Two important breakdown mechanisms are the tunneling effect and avalanche multiplication. We consider the first mechanism briefly and then discuss avalanche multiplication in detail, because avalanche breakdown imposes an upper limit on the reverse bias for most diodes. Avalanche breakdown also limits the collector voltage of a bipolar transistor (Chapter 5) and the drain voltage of a MOSFET (Chapter 6). In addition, the avalanche multiplication mechanisms can generate microwave power, as in an IMPATT diode (Chapter 8), and detect optical signals, as in an avalanche photodetector (Chapter 9).

4.7.1 Tunneling Effect

When a high electric field is applied to a p–n junction in the reverse direction, a valence electron can make a transition from the valence band to the conduction band, as shown in Fig. 24*a*. This process, in which an electron penetrates through the energy bandgap, is called tunneling.

The tunneling process is discussed in Chapter 3. Tunneling occurs only if the electric field is very high. The typical field for silicon and gallium arsenide is about 10^6 V/cm or higher. To achieve such a high field, the doping concentrations for both p- and n-regions

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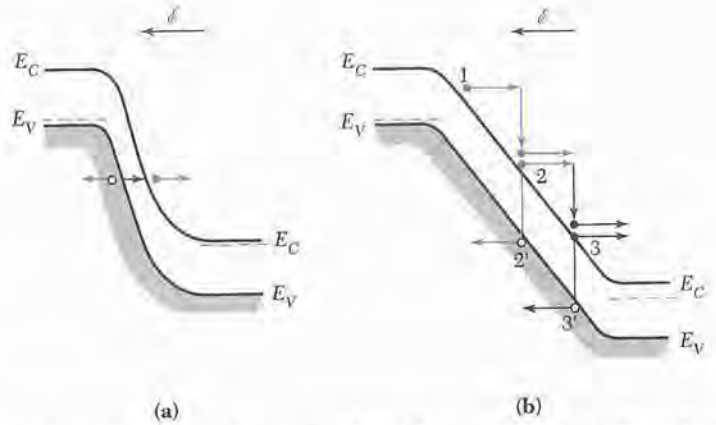


Fig. 24 Energy band diagrams under junction-breakdown conditions. (*a*) Tunneling effect. (*b*) Avalanche multiplication.

must be quite high ($>5 \times 10^{17}$ cm^{-3}). The breakdown mechanisms for silicon and gallium arsenide junctions with breakdown voltages of less than about $4E_g/q$, where E_g is the bandgap, are the result of the tunneling effect. For junctions with breakdown voltages in excess of $6E_g/q$, the breakdown mechanism is the result of avalanche multiplication. At voltages between 4 and $6E_g/q$, the breakdown is due to a mixture of both avalanche multiplication and tunneling.[4]

4.7.2 Avalanche Multiplication

The avalanche multiplication process is illustrated in Fig. 24*b*. The p–n junction such as a p^+–n one-sided abrupt junction with a doping concentration of $N_D \equiv 10^{17}$ cm^{-3} or less is under reverse bias. This figure is essentially the same as Fig. 25 in Chapter 3. A thermally generated electron in the depletion region (designated by 1) gains kinetic energy from the electric field. If the field is sufficiently high, the electron can gain enough kinetic energy that on collision with an atom, it can break the lattice bonds, creating an electron-hole pair (2 and 2′). These newly created electron and hole both acquire kinetic energy from the field and create additional electron-hole pairs (e.g., 3 and 3′). These in turn continue the process, creating other electron-hole pairs. This process is therefore called *avalanche multiplication*.

To derive the breakdown condition, we assume that a current I_{no} is incident at the left-hand side of the depletion region of width W, as shown in Fig. 25. If the electric field in the depletion region is high enough to initiate the avalanche multiplication process, the electron current I_n will increase with distance through the depletion region to reach a value $M_n I_{no}$ at W, where M_n, the multiplication factor, is defined as

$$M_n \equiv \frac{I_n(W)}{I_{no}}. \tag{81}$$

Similarly, the hole current I_p increases from $x = W$ to $x = 0$. The total current $I = (I_p + I_n)$ is constant at steady state. The incremental electron current at x equals the number of electron-hole pairs generated per second in the distance dx:

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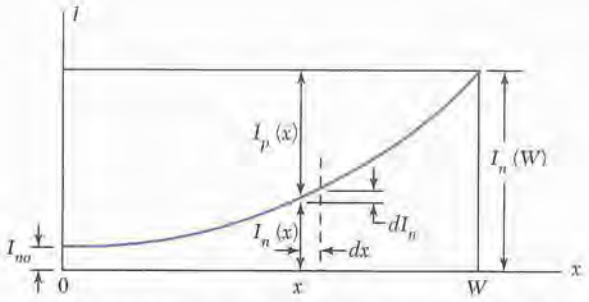


Fig. 25 Depletion region in a p–n junction with multiplication of an incident current.

$$d\left(\frac{I_n}{q}\right)=\left(\frac{I_n}{q}\right)\left(\alpha_n dx\right)+\left(\frac{I_p}{q}\right)\left(\alpha_p dx\right) \tag{82}$$

or

$$\frac{dI_n}{dx}+\left(\alpha_p-\alpha_n\right)I_n=\alpha_p I, \tag{82a}$$

where α_n and α_p are the electron and hole ionization rates, respectively. If we use the simplified assumption that $\alpha_n=\alpha_p=\alpha$, the solution of Eq. 82a is

$$\frac{I_n(W)-I_n(0)}{I}=\int_0^W \alpha dx. \tag{83}$$

From Eqs. 81 and 83, we have

$$1-\frac{1}{M_n}=\int_0^W \alpha dx. \tag{83a}$$

The avalanche breakdown voltage is defined as the voltage where M_n approaches infinity. Hence, the breakdown condition is given by

$$\boxed{\int_0^W \alpha dx=1.} \tag{84}$$

From both the breakdown condition described above and the field dependence of the ionization rates, we may calculate the critical field (i.e., the maximum electric field at breakdown) at which the avalanche process takes place. Using measured α_n and α_p (Fig. 26 in Chapter 3) the critical field $\mathscr{E}_c$ are calculated for silicon and gallium arsenide one-sided abrupt junctions and shown in Fig. 26 as functions of the impurity concentration of the substrate. Also indicated is the critical field for the tunneling effect. It is evident that tunneling occurs only in semiconductors having high doping concentrations.

With the critical field determined, we may calculate the breakdown voltages. As discussed previously, voltages in the depletion region are determined from the solution of Possion's equation:

$$\boxed{V_B(\text{breakdown voltage})=\frac{\mathscr{E}_c W}{2}=\frac{\varepsilon_s \mathscr{E}_c^2}{2q}\left(N_B\right)^{-1}} \tag{85}$$

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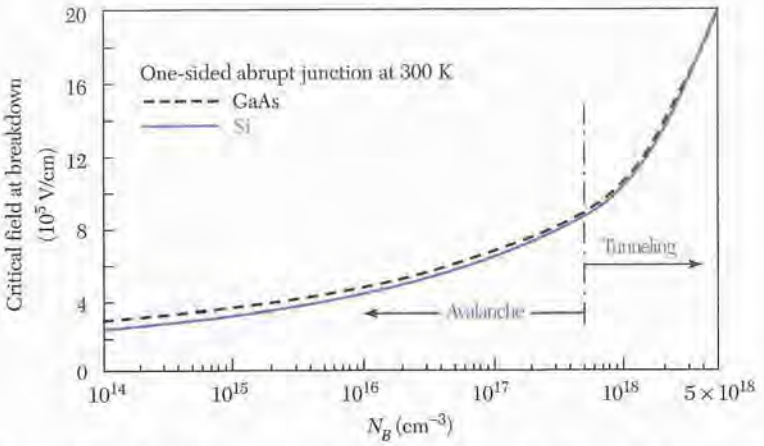


Fig. 26 Critical field at breakdown versus background doping for Si and GaAs one-sided abrupt junctions.[5]

for one-sided abrupt junctions and

$$V_B = \frac{2\mathscr{E}_c W}{3} = \frac{4\mathscr{E}_c^{3/2}}{3}\left(\frac{2\varepsilon_s}{q}\right)^{1/2}(a)^{-1/2} \tag{86}$$

for linearly graded junctions, where N_B is the background doping of the lightly doped side, ε_s is the semiconductor permittivity, and a is the impurity gradient. Since the critical field is a slowly varying function of either N_B or a, the breakdown voltage, as a first-order approximation, varies as N_B^{-1} for abrupt junctions and as $a^{-1/2}$ for linearly graded junctions.

Figure 27 shows the calculated avalanche breakdown voltages for silicon and gallium arsenide junctions.[5] The dash-dot line (to the right) at high dopings or high-impurity gradients indicates the onset of the tunneling effect. Gallium arsenide has higher breakdown voltages than silicon for a given N_B or a, mainly because of its larger bandgap. The larger the bandgap, the larger the critical field must be for sufficient kinetic energy to be gained between collisions. As Eqs. 85 and 86 demonstrate, the larger critical field, in turn, gives rise to higher breakdown voltage.

The inset of Fig. 28 shows the space-charge distribution of a diffused junction with a linear gradient near the surface and a constant doping inside the semiconductor. The breakdown voltage lies between the two limiting cases of abrupt junction and linearly graded junction considered previously.[6] For large a and low N_B, the breakdown voltage of the diffused junctions is given by the abrupt junction results shown on the bottom line in Fig. 28, whereas for small a and high N_B, V_B is given by the linearly graded junction results indicated by the parallel lines in Fig. 28.

EXAMPLE 8

Calculate the breakdown voltage for a Si one-sided p^+–n abrupt junctions with $N_D = 5 \times 10^{16}$ cm^{-3}.

SOLUTION From Fig. 26, we see that the critical field at breakdown for a Si one-sided abrupt junction is about 5.7×10^5 V/cm. Then from Eq. 85, we obtain

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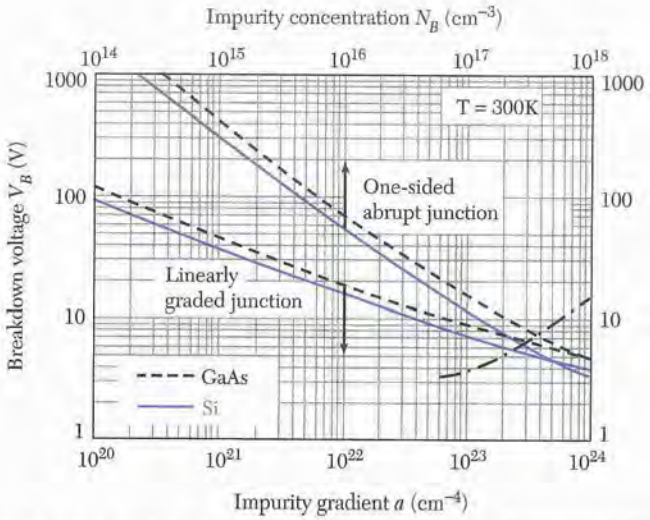


Fig. 27 Avalanche breakdown voltage versus impurity concentration for one-sided abrupt junction and avalanche breakdown voltage versus impurity gradient for linearly graded junction in Si and GaAs. Dash-dot line indicates the onset of the tunneling mechanism.[5]

$$V_B(\text{breakdown voltage}) = \frac{\mathscr{E}_c W}{2} = \frac{\varepsilon_s \mathscr{E}_c^2}{2q}\left(N_B\right)^{-1},$$

$$= \frac{11.9 \times 8.85 \times 10^{-14} \times \left(5.7 \times 10^5\right)^2}{2 \times 1.6 \times 10^{-19}} \left(5 \times 10^{16}\right)^{-1}$$

$$= 21.4 \text{ V}.$$

In Figs. 27 and 28 we assume that the semiconductor layer is thick enough to support the reverse-biased depletion layer width W_m at breakdown. If the semiconductor

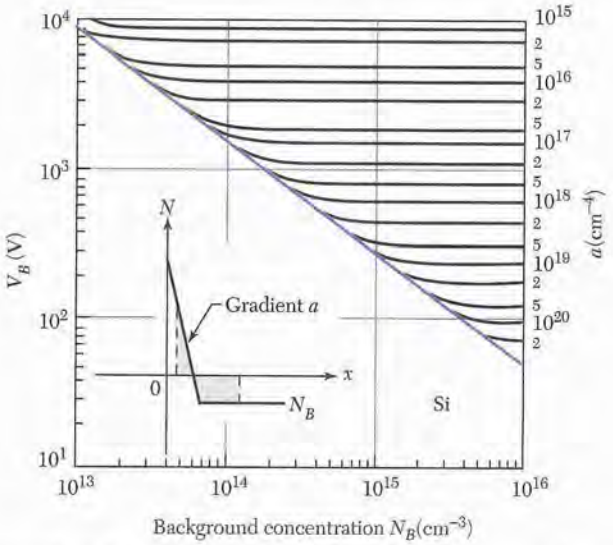


Fig. 28 Breakdown voltage for diffused junctions. Inset shows the space charge distribution.[6]

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layer W is smaller than W_m, as shown in the inset of Fig. 29, the device will be punched through; that is, the depletion layer will reach the n–n^+ interface prior to breakdown. Increase the reverse bias further and the device will break down. The critical field $\mathscr{E}_c$ is essentially the same as that shown in Fig. 26. Therefore, the breakdown voltage V_B' for the punch-through diode is

$$\frac{V_B'}{V_B} = \frac{\text{shaded area in Fig. 29 inset}}{\left(\mathscr{E}_c W_m\right)/2},$$

$$= \left(\frac{W}{W_m}\right)\left(2 - \frac{W}{W_m}\right). \tag{87}$$

Punch-through occurs when the doping concentration N_B becomes sufficiently low, as in a p^+–π–n^+ or p^+–ν–n^+ diode, where π stands for a lightly doped p-type and ν stands for a lightly doped n-type semiconductor. The breakdown voltages for such diodes calculated from Eqs. 85 and 87 are shown in Fig. 29. For a given thickness, the breakdown voltage approaches a constant value as the doping decreases.

EXAMPLE 9

For a GaAs p^+–n one-sided abrupt junction with $N_D = 8 \times 10^{14}$ cm^{-3}, calculate the depletion width at breakdown. If the n-type region of this structure is reduced to 20 μm, calculate the breakdown voltage.

SOLUTION From Fig. 27, we can find the breakdown voltage (V_B) is about 500 V, which is much larger than the built-in voltage (V_{bi}). And from Eq. 27, we obtain

$$W = \sqrt{\frac{2\varepsilon_s\left(V_{bi} - V\right)}{qN_B}} \cong \sqrt{\frac{2 \times 12.4 \times 8.85 \times 10^{-14} \times 500}{1.6 \times 10^{-19} \times 8 \times 10^{14}}} = 2.93 \times 10^{-3} \quad = 29.3\ \mu\text{m}.$$

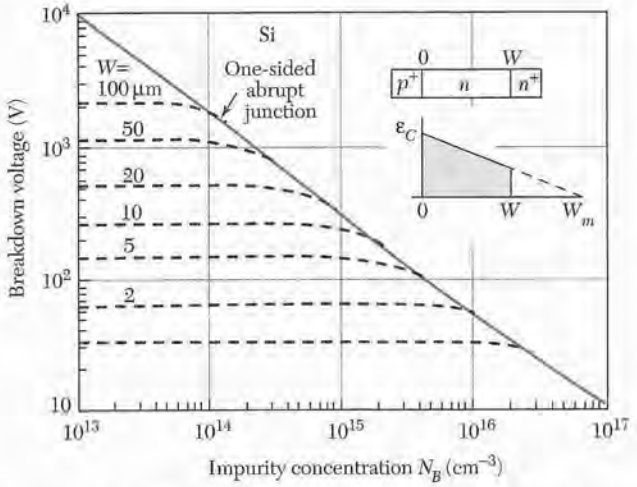


Fig. 29 Breakdown voltage for p^+–π–n^+ and p^+–ν–n^+ junctions. W is the thickness of the lightly doped p-type (π) or the lightly doped n-type (ν) region.

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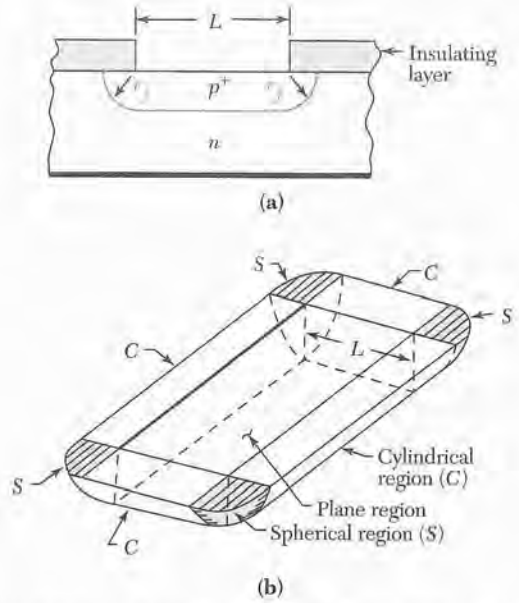


Fig. 30 (*a*) Planar diffusion process that forms junction curvature near the edge of the diffusion mask, where r_j is the radius of curvature. (*b*) Cylindrical and spherical regions formed by diffusion through a rectangular mask.

When the n-type region reduces to 20 μm, the punch-through will occur first. From Eq. 87, we can obtain

$$\frac{V_B'}{V_B} = \frac{\text{shaded area in Fig. 29 inset}}{(\mathscr{E}_c W_m)/2} = \left(\frac{W}{W_m}\right)\left(2 - \frac{W}{W_m}\right),$$

$$V_B' = V_B\left(\frac{W}{W_m}\right)\left(2 - \frac{W}{W_m}\right) = 500 \times \left(\frac{20}{29.3}\right)\left(2 - \frac{20}{29.3}\right) = 449 \text{ V}.$$

Another important consideration of breakdown voltage is the junction curvature effect.[7] When a p–n junction is formed by diffusion through a window in the insulating layer on a semiconductor, the impurities diffuse downward and sideways (see Chapter 13). Hence, the junction has a plane (or flat) region with nearly cylindrical edges, as shown in Fig. 30*a*. If the diffusion mask contains sharp corners, the corner of the junction will acquire the roughly spherical shape shown in Fig. 30*b*. Because the spherical or cylindrical regions of the junction have a higher field intensity, they determine the avalanche breakdown voltage. The calculated results for silicon one-sided abrupt junctions are shown in Fig. 31. The solid line represents the plane junctions considered previously. Note that as the junction radius r_j becomes smaller, the breakdown voltage decreases dramatically, especially for spherical junctions at low impurity concentrations.

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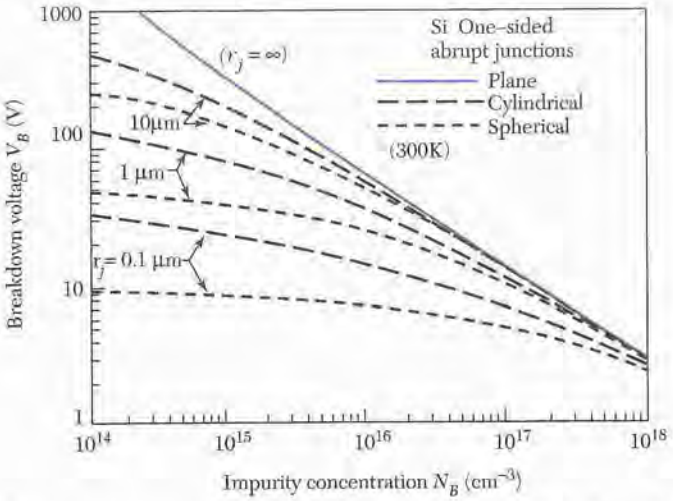


Fig. 31 Breakdown voltage versus impurity concentration for one-sided abrupt doping profile with cylindrical and spherical junction geometries,[7] where r_j is the radius of curvature indicated in Fig. 30.

4.8 HETEROJUNCTION

A heterojunction is defined as a junction formed between two dissimilar semiconductors. Figure 32*a* shows the energy band diagram of two isolated pieces of semiconductors prior to the formation of a heterojunction. The two semiconductors are assumed to have different energy bandgaps E_g, different dielectric permittivities ε_s, different work function $q\phi_s$, and different electron affinities $q\chi$. The work function is defined as the energy required to remove an electron from Fermi level E_F to a position just outside the material (the vacuum level). The electron affinity is the energy required to remove an electron from the bottom of the conduction band E_C to the vacuum level. The difference in energy of the conduction band edges in the two semiconductors is represented by ΔE_C, and the difference in energy in the valence band edges is represented by ΔE_V. From Fig. 32*a*, ΔE_C and ΔE_V can be expressed by

$$\Delta E_C = q(\chi_2 - \chi_1) \tag{88a}$$

and

$$\Delta E_V = E_{g1} + q\chi_1 - (E_{g2} + q\chi_2) = \Delta E_g - \Delta E_C, \tag{88b}$$

where ΔE_g is the energy band difference and $\Delta E_g = E_{g1} - E_{g2}$.

Figure 32*b* shows the equilibrium band diagram of an ideal abrupt heterojunction formed between these semiconductors.[8] In this diagram it is assumed that there is a negligible number of traps or generation-recombination centers at the interface of the two dissimilar semiconductors. Note that this assumption is valid only when heterojunctions are formed between semiconductors with closely matched lattice constants. Therefore, we must choose lattice-matched materials to satisfy the assumption.[§] For example, the $Al_xGa_{1-x}As$ materials, with x from 0 to 1, is the most important material for heterojunc-

[§]The lattice-mismatched epitaxy, also called the strained-layer epitaxy, is considered in Section 10.6.

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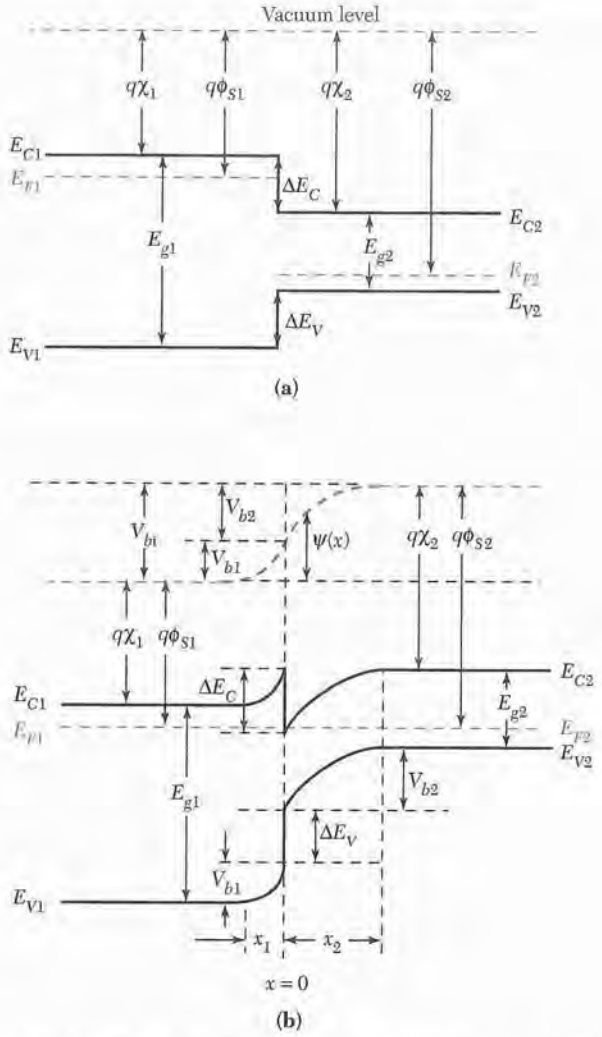


Fig. 32 (*a*) Energy band diagram of two isolated semiconductors. (*b*) Energy band diagram of an ideal n–p heterojunction at thermal equilibrium.

tions. When $x = 0$, we have GaAs, with a bandgap of 1.42 eV and a lattice constant of 5.6533 Å at 300 K. When $x = 1$, we have AlAs, with a bandgap of 2.17 eV and a lattice constant of 5.6605 Å. The bandgap for the ternary $Al_xGa_{1-x}As$ increases with x; however, the lattice constant remains essentially a constant. Even for the extreme cases where $x = 0$ and $x = 1$, the lattice constant mismatch is only 0.1%.

There are two basic requirements in the construction of the energy band diagram: (a) the Fermi level must be the same on both sides of the interface in thermal equilibrium, and (b) the vacuum level must be continuous and parallel to the band edges. Because of these requirements, the discontinuity in conduction band edges ΔE_C and valence band edges ΔE_V will be unaffected by doping as long as the bandgap E_g and electron affinity $q\chi$ are not functions of doping (i.e., as in nondegenerate semiconductors). The total built-in potential V_{bi} can be expressed by

$$V_{bi} = V_{b1} + V_{b2}, \tag{89}$$

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where V_{b1} and V_{b2} are the electrostatic potentials at equilibrium in semiconductors 1 and 2, respectively.

Under the conditions that the potential and the *free-carrier flux density* (defined as the rate of free-carrier flow through a unit area) are continuous at the heterointerface, we can derive the depletion widths and capacitance from the Poisson equation using the conventional depletion approximation. One boundary condition is the continuity of electric displacement, that is, $\varepsilon_1 \mathscr{E}_1 = \varepsilon_2 \mathscr{E}_2$, where $\mathscr{E}_1$ and $\mathscr{E}_2$ are the electric fields at the interface ($x = 0$) in semiconductors 1 and 2, respectively. V_{b1} and V_{b2} are given by

$$V_{b1} = \frac{\varepsilon_2 N_2 \left(V_{bi} - V\right)}{\varepsilon_1 N_1 + \varepsilon_2 N_2}, \tag{90a}$$

$$V_{b2} = \frac{\varepsilon_1 N_1 \left(V_{bi} - V\right)}{\varepsilon_1 N_1 + \varepsilon_2 N_2}, \tag{90b}$$

where N_1 and N_2 are the doping concentrations in semiconductors 1 and 2, respectively. The depletion widths x_1 and x_2 can be obtained by

$$x_1 = \sqrt{\frac{2\varepsilon_1 \varepsilon_2 N_2 \left(V_{bi} - V\right)}{q N_1 \left(\varepsilon_1 N_1 + \varepsilon_2 N_2\right)}} \tag{91a}$$

and

$$x_2 = \sqrt{\frac{2\varepsilon_1 \varepsilon_2 N_1 \left(V_{bi} - V\right)}{q N_2 \left(\varepsilon_1 N_1 + \varepsilon_2 N_2\right)}} \tag{91b}$$

EXAMPLE 10

Consider an ideal abrupt heterojunction with a built-in potential of 1.6 V. The impurity concentrations in semiconductor 1 and 2 are 1×10^{16} donors/cm^3 and 3×10^{19} acceptors/cm^3, and the dielectric constants are 12 and 13, respectively. Find the electrostatic potential and depletion width in each material at thermal equilibrium.

SOLUTION From Eq. 90, the electrostatic potentials of a heterojunction at thermal equilibrium or $V = 0$ are

$$V_{b1} = \frac{13 \times \left(3 \times 10^{19}\right) \times 1.6}{12 \times \left(1 \times 10^{16}\right) + 13 \times \left(3 \times 10^{19}\right)} = 1.6 \text{ V}$$

and

$$V_{b2} = \frac{12 \times \left(1 \times 10^{16}\right) \times 1.6}{12 \times \left(1 \times 10^{16}\right) + 13 \times \left(3 \times 10^{19}\right)} = 4.9 \times 10^{-4} \text{ V}.$$

The depletion widths can be calculated by Eq. 91:

$$x_1 = \sqrt{\frac{2 \times 12 \times 13 \times \left(8.85 \times 10^{-14}\right) \times \left(3 \times 10^{19}\right) \times 1.6}{\left(1.6 \times 10^{-19}\right) \times \left(1 \times 10^{16}\right) \times \left(12 \cdot 1 \times 10^{16} + 13 \cdot 3 \times 10^{19}\right)}} = 4.608 \times 10^{-5} \text{ cm},$$

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$$x_p = \sqrt{\frac{2\times 12\times 13\times\left(8.85\times 10^{-14}\right)\times\left(1\times 10^{16}\right)\times 1.6}{\left(1.6\times 10^{-19}\right)\times\left(3\times 10^{19}\right)\times\left(12\cdot 1\times 10^{16}\ +\ 13\cdot 3\times 10^{19}\right)}} = 1.536\times 10^{-8}\ \text{cm}.$$

We see that most of the built-in potential is in the semiconductor with a lower doping concentration. The depletion width there is also much wider.

SUMMARY

A p–n junction is formed when a p-type and an n-type semiconductor are brought into intimate contact. The p–n junction, in addition to being a device that is used in many application, is the basic building block for other semiconductor devices. Therefore, an understanding of junction theory serves as the foundation to understanding other semiconductor devices.

Most modern p–n junctions are fabricated using "planar technology." This technology includes the thermal oxidation process to grow an oxide layer on the semiconductor surface, the lithography process to open an window in the oxide, the diffusion or ion implantaion process to form a p–n junction in the window area, and the metallization process to provide contacts to connect the junction to other circuit elements. Planar technology is covered in detail in Chapters 10–14.

When a p–n junction is formed, there are uncompensated negative ions (N_A^-) on the p-side and uncompensated positive ions (N_D^+)on the n-side. Therefore, a depletion region (i.e., depletion of mobile carriers) is formed at the junction. This region, in turn, creates an electric field. At thermal equilibrium, the drift current due to the electric field is exactly balanced by the diffusion current due to concentration gradients of the mobile carriers on the two sides of the junction. When a positive voltage is applied to the p-side with respect to the n-side, a large current will flow through the junction. However, when a negative voltage is applied, virtually no current flows. This "rectifying" behavior is the most important characteristic of p–n junctions.

The basic equations presented in Chapters 2 and 3 have been used to develop the ideal static and dynamic behaviors of p–n junctions. We derived expressions for the depletion region, the depletion capacitance, and the ideal current-voltage characteristics of p–n junctions. However, practical devices depart from these ideal characteristics because of carrier generation and recombination in the depletion layers, high injection under forward bias, and series-resistance effects. The theory and methods of calculating the effects of these departures from the ideal are discussed in detail. We also considered other factors that influence p–n junctions, such as minority-carrier storage, diffusion capacitance, and transient behavior in high-frequency and switching applications.

A limiting factor in the operation of p–n junctions is junction breakdown—especially that due to avalanche multiplication. When a sufficiently large reverse voltage is applied to a p–n junction, the junction breaks down and conducts a very large current. Therefore, the breakdown voltage imposes an upper limit on the reverse bias for p–n junctions. We derived equations for the breakdown condition of the p–n junction and have shown the effect of device geometry and doping on the breakdown voltage.

A related device is the heterojunction formed between two dissimilar semiconductors. We obtained expressions for its electrostatic potentials and depletion widths. These expressions are simplified to that for a conventional p–n junction when these two semiconductors become identical.

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PROBLEMS (* DENOTES DIFFICULT PROBLEMS)

FOR SECTION 4.3 DEPLETION REGION

*1. A diffused silicon p–n junction has a linearly graded junction on the p-side with $a = 10^{19}$ cm^{-4}, and a uniform doping of 3×10^{14} cm^{-3} on the n-side. If the depletion layer width of the p-side is 0.8μm at zero bias, find the total depletion layer width, built-in potential, and maximum field at zero bias.

*2. Sketch the potential distribution in the Si p–n junction in Prob. 1.

3. For an ideal silicon p–n abrupt junction with $N_A = 10^{17}$ cm^{-3} and $N_D = 10^{15}$ cm^{-3}, (a) calculate V_{bi} at 250, 300, 350, 400, 450, and 500 K and plot V_{bi} versus T; (b) comment on your result in terms of energy band diagram; and (c) find the depletion layer width and the maximum field at zero bias for $T = 300$ K.

4. Determine the n-type doping concentration to meet the following specifications for a Si p–n junction:

   $N_A = 10^{18}$ cm^{-3}, $\mathscr{E}_{max} = 4 \times 10^5$ V/cm at $V_R = 30$ V, $T = 300$ K.

FOR SECTION 4.4 DEPLETION CAPACITANCE

*5. An abrupt p–n junction has a doping concentration of 10^{15}, 10^{16}, or 10^{17} cm^{-3} on the lightly doped n-side and of 10^{19} cm^{-3} on the heavily doped p-side. Obtain series of curves of $1/C^2$ versus V, where V ranges from –4 V to 0 V in steps of 0.5 V. Comment on the slopes and the interceptions at the voltage axis of these curves.

6. For a silicon linearly graded junction with a impurity gradient of 10^{20} cm^{-4}, calculate the built-in potential and the junction capacitance at reverse bias of 4 V ($T = 300$ K).

7. A one-sided p^+–n Si junction at 300 K is doped with $N_A = 10^{19}$cm^{-3}. Design the junction so that $C_j = 0.85$ pF at $V_R = 4.0$ V.

FOR SECTION 4.5 CURRENT-VOLTAGE CHARACTERISTICS

8. Assume that the p–n junction considered in Prob. 3 contains 10^{15} cm^{-3} generation-recombination centers located 0.02 eV above the intrinsic Fermi level of silicon with $\sigma_n = \sigma_p = 10^{-15}$ cm^2. If $v_{th} \cong 10^7$ cm/s, calculate the generation and recombination current at –0.5 V.

9. Consider a Si p–n junction with n-type doping concentration of 10^{16} cm^{-3} and is forward biased with V = 0.8 V at 300 K. Calculate the minority-carrier hole concentration at the edge of the space charge region.

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10. Calculate the applied reverse-bias voltage at which the ideal reverse current in a p–n junction diode at $T = 300$ K reaches 95% of its reverse saturation current value.

11. Design the Si p–n diode such that $J_n = 25$ A/cm^2 and $J_p = 7$ A/cm^2 at $V_a = 0.7$ V. The remaining parameters are given in Ex. 5.

12. An ideal silicon p–n junction has $N_D = 10^{18}$ cm^{-3}, $N_A = 10^{16}$ cm^{-3}, $\tau_p = \tau_n = 10^{-6}$ s, and a device area of 1.2×10^{-5} cm^2. (*a*) Calculate the theoretical saturation current at 300 K. (*b*) Calculate the forward and reverse currents at ±0.7 V.

13. In Prob. 12, assume the widths of the two sides of the junction are much greater than the respective minority-carrier diffusion length. Calculate the applied voltage at a forward current of 1 mA at 300 K.

14. A silicon p^+–n junction has the following parameters at 300 K: $\tau_p = \tau_g = 10^{-6}$ s , $N_D = 10^{15}$ cm^{-3}, $N_A = 10^{19}$ cm^{-3}. (*a*) Plot diffusion current density, J_{gen}, and total current density versus applied reverse voltage. (*b*) Repeat the above results for $N_D = 10^{17}$ cm^{-3}.

FOR SECTION 4.6 CHARGE STORAGE AND TRANSIENT BEHAVIOR

15. For an ideal abrupt silicon p^+–n junction with $N_D = 10^{16}$ cm^{-3}, find the stored minority carriers per unit area in the neutral n-region when a forward bias of 1 V is applied. The length of neutral region is 1 μm and the diffusion length of the holes is 5 μm.

FOR SECTION 4.7 JUNCTION BREAKDOWN

16. For a silicon p^+–n one-sided abrupt junction with $N_D = 10^{15}$ cm^{-3}, find the depletion layer width at breakdown. If the n-region is reduced to 5 μm, calculate the breakdown voltage and compare your result with Fig. 29.

17. Design an abrupt Si p^+–n junction diode that has a reverse breakdown voltage of 130 V and has a forward-bias current of 2.2 mA at $V_a = 0.7$ volt. Assume τ_{po}=10^{-7}s.

18. In Fig. 20*b*, the avalanche breakdown voltage increases with increasing temperature. Give a qualitative argument for the result.

19. If $\alpha_n = \alpha_p = 10^4(\mathscr{E}/4 \times 10^5)^6$ cm^{-1} in gallium arsenide, where $\mathscr{E}$ is in V/cm, find the breakdown voltage of (*a*) a p-i-n diode with an intrinsic-layer width of 10μm and (*b*) p^+–n junction with a doping of 2×10^{16} cm^{-3} for the lightly doped side.

20. Consider that a Si p–n junction at 300 K with a linearly doping profile varies from $N_A = 10^{18}$ cm^{-3} to $N_D = 10^{18}$cm^{-3} over a distance of 2 μm. Calculate the breakdown voltage.

FOR SECTION 4.8 HETEROJUNCTION

21. For the ideal heterojunction in Ex. 10, find the electrostatic potential and depletion width in each material for applied voltage of 0.5 V and –5 V.

22. For an n-type GaAs/ p-type $Al_{0.3}Ga_{0.7}As$ heterojunction at room temperature, $\Delta E_C = 0.21$ eV. Find the total depletion width at thermal equilibrium when both sides have impurity concentration of 5×10^{15} cm^{-3}. (Hint: the bandgap of $Al_xGa_{1-x}As$ is given by $E_g(x) = 1.424 + 1.247x$ eV, and the dielectric constant is $12.4 - 3.12x$. Assume N_C and N_V are the same for $Al_xGa_{1-x}As$ with $0 < x < 0.4$.

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CHAPTER 5

Bipolar Transistor and Related Devices

- 5.1 THE TRANSISTOR ACTION
- 5.2 STATIC CHARACTERISTICS OF BIPOLAR TRANSISTOR
- 5.3 FREQUENCY RESPONSE AND SWITCHING OF BIPOLAR TRANSISTOR
- 5.4 THE HETEROJUNCTION BIPOLAR TRANSISTOR
- 5.5 THE THYRISTOR AND RELATED POWER DEVICES
- SUMMARY

The transistor (contraction for *transfer resistor*) is a multijunction semiconductor device. Normally, the transistor is integrated with other circuit elements for voltage gain, current gain, or signal-power gain. The bipolar transistor, also called the bipolar junction transistor (BJT), is one of the most important semiconductor devices. It has been used extensively in high-speed circuits, analog circuits, and power applications. Bipolar devices are semiconductor devices in which both electrons and holes participate in the conduction process. This is in contrast to the field-effect devices, discussed in Chapter 6 and 7, in which predominantly only one kind of carrier participates.

The bipolar transistor was invented by a research team at Bell Laboratories[1] in 1947. The device had two metal wires with sharp points making contact with a germanium substrate (see Fig. 3 in Chapter 1). The first transistor was primitive by today's standards, yet it revolutionized the electronics industry and changed our way of life.

For modern bipolar transistors, we have replaced the germanium with silicon and replaced the point contacts with two closely coupled p–n junctions in the form of p-n-p or n-p-n structures. We consider the transistor action of the coupled junctions and derive the static characteristics from the minority carrier distributions in the regions. We also discuss the frequency response and switching behavior of the transistor. In addition, we briefly consider the heterojunction bipolar transistor in which one or both p–n junctions are formed between dissimilar semiconductors.

In the final section, a related bipolar device called a thyristor is introduced. The basic thyristor has three closely coupled p–n junctions in the form of a p-n-p-n structure.[2] The device exhibits bistable characteristics and can be switched between a high-impedance "off" state and a low-impedance "on" state. The name thyristor is derived from *gas thyratron*, which is a gas-filled tube with similar bistable characteristics. Because of the two stable state (on and off) and the low power dissipation in these states, thyristors are useful in many applications. We consider the physical operation of the thyristor and a few

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related switching devices. Furthermore, the various thyristor types and their applications are briefly introduced.

Specifically, we cover the following topics:

- The current gain and modes of operation of bipolar transistors.
- The cutoff frequency and switching time of a bipolar transistor.
- The advantages of heterojunction bipolar transistor.
- The power handling capability of thyristor and related bipolar devices.

5.1 THE TRANSISTOR ACTION

A perspective view of a discrete p-n-p bipolar transistor is shown Fig. 1. The transistor is formed by starting with a p-type substrate. An n-type region is thermally diffused through an oxide window into the p-type substrate. A very heavily doped p^+ region is then diffused into the n-type region. Metallic contacts are made to the p^+- and n-regions through the windows opened in the oxide layer and to the p-region at the bottom. The details of transistor fabrication processes are considered in later chapters.

An idealized, one-dimensional structure of a p-n-p bipolar transistor is shown in Fig. 2*a*. Normally, the bipolar transistor has three separately doped regions and two p–n junctions. The heavily doped p^+-region is called the *emitter* (defined as symbol E in the figure). The narrow central n-region, with moderately doped concentration, is called the *base* (symbol B). The width of the base is small compared with the minority-carrier diffusion length. The lightly doped p-region is called the *collector* (symbol C). The doping concentration in each region is assumed to be uniform. It should be noted that the concepts developed for the p–n junction can be applied directly to the transistor.

Figure 2*b* illustrates the circuit symbol for a p-n-p transistor. The current components and voltage polarities are shown in the figure. The arrows of the various currents indicate the direction of current flow under normal operating conditions (also called the *active mode*). The + and – signs are used to define the voltage polarities. We can also denote the voltage polarity by a double subscript on the voltage symbol. In the active mode, the emitter-base junction is forward biased ($V_{EB} > 0$) and the base-collector junction

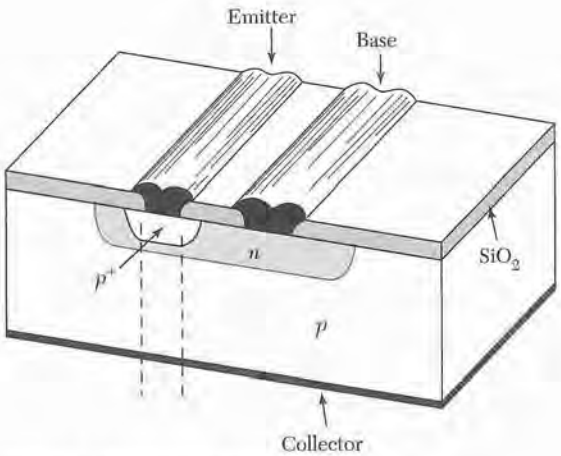


Fig. 1 Perspective view of a silicon p-n-p bipolar transistor.

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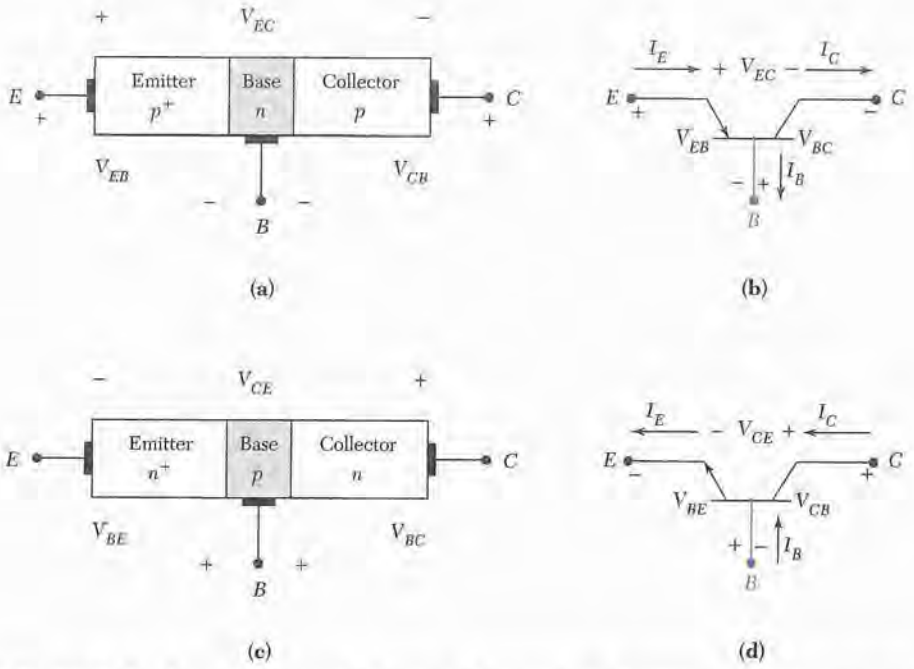


Fig. 2 (*a*) Idealized one-dimensional schematic of a *p*-*n*-*p* bipolar transistor and (*b*) its circuit symbol. (*c*) Idealized one-dimensional schematic of an *n*-*p*-*n* bipolar transistor and (*d*) its circuit symbol.

is reverse biased ($V_{CB} > 0$). According to Kirchhoff's circuit laws, there are only two independent currents for this three-terminal device. If two currents are known, the third current can be obtained.

The *n*-*p*-*n* bipolar transistor is the complementary structure of the *p*-*n*-*p* bipolar transistor. The structure and circuit symbol of an ideal *n*-*p*-*n* transistor are shown in Figs. 2*c* and 2*d*, respectively. The *n*-*p*-*n* structure can be obtained by interchanging *p* for *n* and *n* for *p* from the *p*-*n*-*p* transistor. As a result the current flow and voltage polarity are all reversed. In subsequent sections, we concentrate on the *p*-*n*-*p* type because the direction of minority-carrier (hole) flow is the same as current flow. It provides a more intuitive base for understanding the mechanisms of charge transport. Once we understand the *p*-*n*-*p* transistor, we need only to reverse the polarities and conduction types to describe the *n*-*p*-*n* transistor.

5.1.1 Operation in the Active Mode

Figure 3*a* show an idealized *p*-*n*-*p* transistor in thermal equilibrium, that is, where all three leads are connected together or all are grounded. The depletion regions near the two junctions are illustrated by colored areas. Figure 3*b* shows the impurity densities in the three doped regions, where the emitter is more heavily doped than the collector. However, the base doping is less than the emitter doping, but greater than the collector doping. Figure 3*c* shows the corresponding electric-field profiles in the two depletion regions.

Figure 3*d* illustrates the energy band diagram, which is a simple extension of the thermal-equilibrium situation for the *p*–*n* junction as applied to a pair of closely coupled

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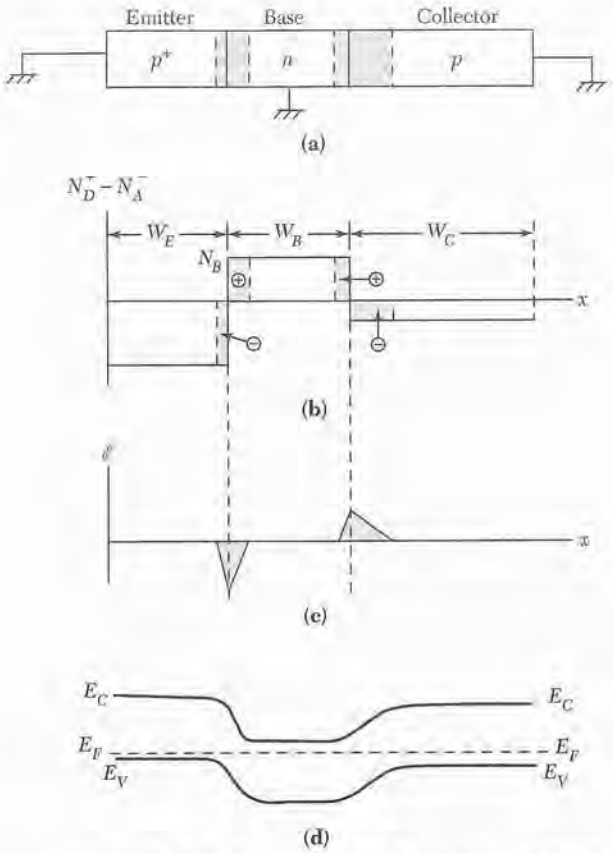


Fig. 3 (*a*) A *p*-*n*-*p* transistor with all leads grounded (at thermal equilibrium). (*b*) Doping profile of a transistor with abrupt impurity distributions. (*c*) Electric-field profile. (*d*) Energy band diagram at thermal equilibrium.

p^+-n and n-p junctions. The results obtained for the p–n junction in Chapter 4 are equally applicable to the emitter-base and base-collector junctions. At thermal equilibrium there is no net current flow, hence the Fermi level is a constant in the regions.

Figure 4 shows the corresponding cases when the transistor in Fig. 3 is biased in the active mode. Figure 4*a* is a schematic of the transistor connected as an amplifier with the *common-base configuration*, that is, the base lead is common to the input and output circuits.[3] Figures 4*b* and 4*c* show the charge densities and the electric fields, respectively, under biasing conditions. Note that the depletion layer width of the emitter-base junction is narrower and the collector-base junction is wider, compared with the equilibrium case shown in Fig. 3.

Figure 4*d* shows the corresponding energy band diagram under the active mode. Since the emitter-base junction is forward biased, holes are injected (or emitted) from the p^+ emitter into the base and electrons are injected from the n base into the emitter. Under the ideal-diode condition, there is no generation-recombination current in the depletion region; these two current components constitute the total emitter current. The collector-base junction is reverse biased, and a small reverse saturation current will flow across the junction. However, if the base width is sufficiently narrow, the holes injected from the emitter can diffuse through the base to reach the base-collector depletion edge

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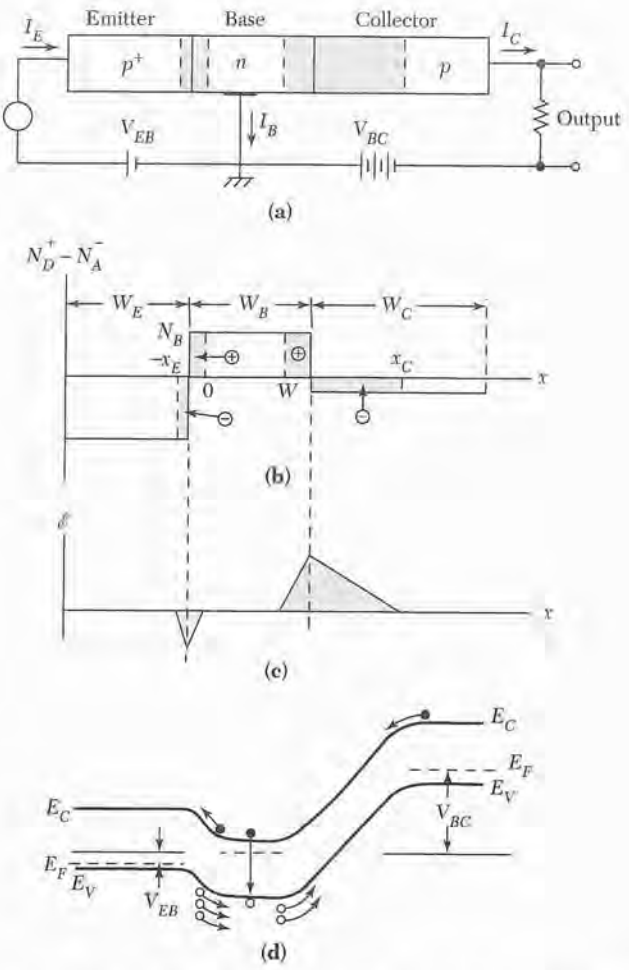


Fig. 4 (*a*) The transistor shown in Fig. 3 under the active mode of operation.[3] (*b*) Doping profiles and the depletion regions under biasing conditions. (*c*) Electric-field profile. (*d*) Energy band diagram.

and then "float up" into the collector (recall the "bubble analogy"). This transport mechanism gives rise to the terminology of *emitter*, which emits or injects carriers, and of *collector*, which collects these carriers injected from a nearby junction. If most of the injected holes can reach the collector without recombining with electrons in the base region, then the collector hole current will be very close to the emitter hole current.

Therefore, carriers injected from a nearby emitter junction can result in a large current flow in a reverse-biased collector junction. This is the *transistor action*, and it can be realized only when the two junctions are physically close enough to interact in the manner described. Therefore, the two junctions are called the *interacting p–n junctions*. If, on the other hand, the two junctions are so far apart that all the injected holes are recombined in the base before reaching the base-collector junction, then the transistor action is lost and the *p-n-p* structure becomes merely two diodes connected back to back.

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5.1.2 Current Gain

Figure 5 shows the various current components in an ideal p-n-p transistor biased in the active mode. Note that we assume that there are no generation-recombination currents in the depletion regions. The holes injected from the emitter constitute the current I_{Ep}, which is the largest current component in a well-designed transistor. Most of the injected holes will reach the collector junction and give rise to the current I_{Cp}. There are three base current components, labeled I_{BB}, I_{En}, and I_{Cn}. I_{BB} corresponds to electrons that must be supplied by the base to replace electrons recombined with the injected holes (i.e., $I_{BB} = I_{Ep} - I_{Cp}$). I_{En} corresponds to the current arising from electrons being injected from the base to the emitter. However, I_{En} is not desirable, as is shown later. It can be minimized by using heavier emitter doping (Section 5.2) or a heterojunction (Section 5.4). I_{Cn} corresponds to thermally generated electrons that are near the base-collector junction edge and drift from the collector to the base. As indicated in the figure, the direction of the electron current is opposite the direction of the electron flow.

We can now express the terminal currents in terms of the various current components described above:

$$I_E = I_{Ep} + I_{En}, \tag{1}$$

$$I_C = I_{Cp} + I_{Cn}, \tag{2}$$

$$I_B = I_E - I_C = I_{En} + (I_{Ep} - I_{Cp}) - I_{Cn}. \tag{3}$$

An important parameter in the characterization of bipolar transistors is the *common-base current gain* α_0. This quantity is defined by

$$\alpha_0 \equiv \frac{I_{Cp}}{I_E}. \tag{4}$$

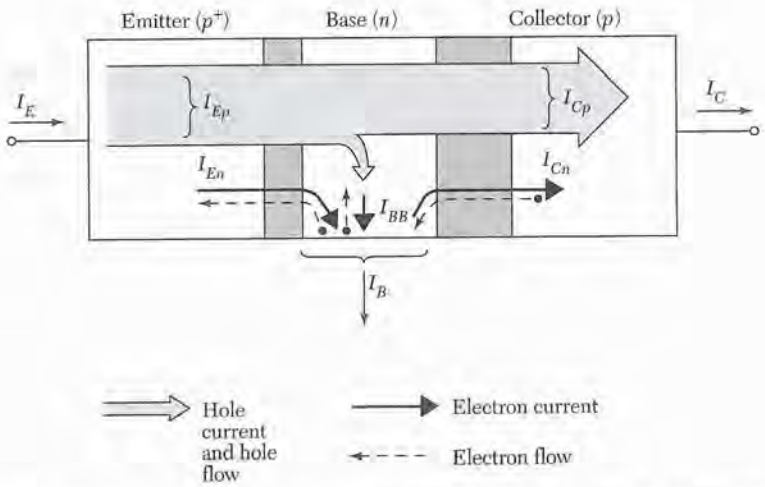


Fig. 5 Various current components in a p-n-p transistor under active mode of operation. The electron flow is in the opposite direction to the electron current.

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Substituting Eq. 1 into Eq. 4 yields

$$\alpha_0 = \frac{I_{Cp}}{I_{Ep} + I_{En}} = \left(\frac{I_{Ep}}{I_{Ep} + I_{En}}\right)\left(\frac{I_{Cp}}{I_{Ep}}\right). \tag{5}$$

The first term on the right-hand side is called the *emitter efficiency* γ, which is a measure of the injected hole current compared with the total emitter current:

$$\boxed{\gamma \equiv \frac{I_{Ep}}{I_E} = \frac{I_{Ep}}{I_{Ep} + I_{En}}.} \tag{6}$$

The second term is called the *base transport factor* α_T, which is the ratio of the hole current reaching the collector to the hole current injected from the emitter:

$$\boxed{\alpha_T \equiv \frac{I_{Cp}}{I_{Ep}}.} \tag{7}$$

Therefore, Eq. 5 becomes

$$\boxed{\alpha_0 = \gamma\alpha_T} \tag{8}$$

For a well-designed transistor, because I_{En} is small compared with I_{Ep} and I_{Cp} is close to I_{Ep}, both γ and α_T approach unity. Therefore, α_0 is close to 1.

We can express the collector current in terms of α_0. The collector current can be described by substituting Eqs. 6 and 7 into Eq. 2:

$$I_C = I_{Cp} + I_{Cn} = \alpha_T I_{Ep} + I_{Cn} = \gamma\alpha_T\left(\frac{I_{Ep}}{\gamma}\right) + I_{Cn} = \alpha_0 I_E + I_{Cn}, \tag{9}$$

where I_{Cn} corresponds to the collector-base current flowing with the emitter open-circuited ($I_E = 0$). We designate I_{Cn} as I_{CBO}, where the first two subscripts (CB) refer to the two terminals between which the current (or voltage) is measured and the third subscript (O) refers to the state of the third terminal with respect to the second. In the present case, I_{CBO} designates the leakage current between the collector and the base with the emitter-base junction open. The collector current for the common-base configuration is then given by

$$\boxed{I_C = \alpha_0 I_E + I_{CBO}.} \tag{10}$$

EXAMPLE 1

For an ideal p-n-p transistor, the current components are given by $I_{Ep} = 3$ mA , $I_{En} = 0.01$ mA, $I_{Cp} = 2.99$ mA, and $I_{Cn} = 0.001$ mA. Determine (a) the emitter efficiency γ, (b) the base transport factor α_T, (c) the common-base current gain α_0, and (d) I_{CBO}.

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SOLUTION

(a) Using Eq. 6, the emitter efficiency is

$$\gamma = \frac{I_{Ep}}{I_{Ep} + I_{En}} = \frac{3}{3 + 0.01} = 0.9967.$$

(b) The base transport factor can be obtained from Eq. 7:

$$\alpha_T = \frac{I_{Cp}}{I_{Ep}} = \frac{2.99}{3} = 0.9967.$$

(c) The common-base current gain is given by Eq. 8:

$$\alpha_0 = \gamma\alpha_T = 0.9967 \times 0.9967 = 0.9934.$$

(d) $I_E = I_{Ep} + I_{En} = 3 + 0.01 = 3.01$ mA

$I_C = I_{Cp} + I_{Cn} = 2.99 + 0.001 = 2.991$

Using Eq. 10, we find

$$I_{CBO} = I_C - \alpha_0 I_E = 2.991 - 0.9934 \times 3.01 = 0.87 \ \mu\text{A}.$$

5.2 STATIC CHARACTERISTICS OF BIPOLAR TRANSISTOR

In this section, we study the static current-voltage characteristics for an ideal transistor and derive equations for the terminal currents. The current equations are based on the minority-carrier concentration in each region and therefore are described by semiconductor parameters such as doping and minority-carrier lifetime.

5.2.1 Carrier Distribution in Each Region

To derive the current-voltage expression for an ideal transistor, we assume the following:

1. The device has uniform doping in each region.
2. The hole drift current in the base region as well as the collector saturation current is negligible.
3. There is low-level injection.
4. There are no generation-combination currents in the depletion regions.
5. There are no series resistances in the device.

Basically, we assume that holes are injected from the emitter into the base under forward-biased condition. These holes then diffuse across the base region and reach the collector junction. Once we determine the minority-carrier distribution (i.e., holes in the n-type base region), we can obtain the current from the minority-carrier gradient.

Base Region

Figure 4*c* shows the electric-field distributions across the junction depletion regions. The minority-carrier distribution in the neutral base region can be described by the field-free, steady-state continuity equation

$$D_p\left(\frac{d^2 p_n}{dx^2}\right) - \frac{p_n - p_{no}}{\tau_p} = 0, \tag{11}$$

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where D_p and τ_p are the diffusion constant and the lifetime of minority carriers, respectively. The general solution of Eq. 11 is

$$p_n(x) = p_n + C_1 e^{x/L_p} + C_2 e^{-x/L_p}, \tag{12}$$

where $L_p = \sqrt{D_p \tau_p}$ is the diffusion length of holes. The constants C_1 and C_2 can be determined by the boundary conditions for the active mode:

$$p_n(0) = p_{no} e^{qV_{EB}/kT} \tag{13a}$$

and

$$p_n(W) = 0, \tag{13b}$$

where p_{no} is the equilibrium minority-carrier concentration in the base, given by $p_{no} = n_i^2/N_B$, and N_B denotes the uniform donor concentration in the base. The first boundary condition (Eq. 13*a*) states that under forward bias, the minority-carrier concentration at the edge of the emitter-base depletion region ($x = 0$) is increased above the equilibrium value by the exponential factor $e^{qV_{EB}/kT}$. The second boundary condition (Eq. 13*b*) states that under reverse bias, the minority carrier concentration at the edge of the base-collector depletion region ($x = W$) is zero.

Substituting Eq. 13 into the general solution expressed in Eq. 12 yields

$$p_n(x) = p_{no}\left(e^{qV_{EB}/kT} - 1\right)\left[\frac{\sinh\left(\frac{W-x}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}\right] + p_{no}\left[1 - \frac{\sinh\left(\frac{x}{L_p}\right)}{\sinh\left(\frac{W}{L_p}\right)}\right]. \tag{14}$$

The sinh function, sinh (Λ), can be approximately expressed by Λ when $\Lambda << 1$. For example, when $\Lambda < 0.3$, the difference between sinh(Λ) and Λ is less than 1.5 percent. Therefore, when $W/L_p << 1$, the distribution equation can be simplified as

$$\boxed{p_n(x) = p_{no} e^{qV_{EB}/kT}\left(1 - \frac{x}{W}\right) = p_n(0)\left(1 - \frac{x}{W}\right).} \tag{15}$$

The distribution approaches a straight line. The approximation is reasonable because the width of the base region is designed to be much smaller than the diffusion length of the minority carrier. Figure 6 shows a linear minority-carrier distribution in a typical transistor operated under active mode. Note that assuming linear minority-carrier distribution can simplify the derivation of current-voltage characteristics. Therefore, we use the assumption hereafter to derive equations for the current-voltage characteristics.

Emitter and Collector Regions

The minority-carrier distributions in the emitter and collector can be obtained in a manner similar to the one used to obtain the distributions for the base region. In Fig. 6, the boundary conditions in the neutral emitter and collector regions are

$$n_E(x = -x_E) = n_{EO} e^{qV_{EB}/kT} \tag{16}$$

and

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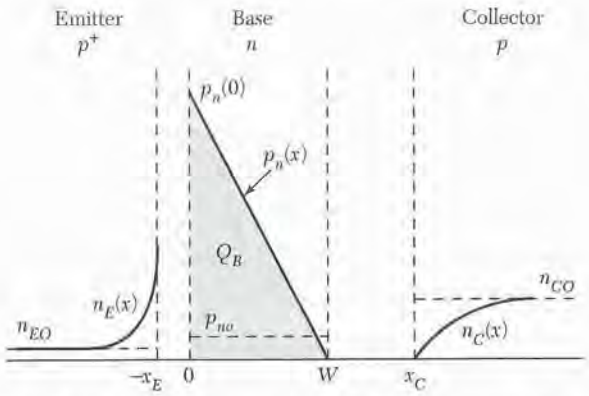


Fig. 6 Minority carrier distribution in various regions of a p-n-p transistor under the active mode of operation.

$$n_C(x = x_C) = n_{CO}\, e^{-q|V_{CB}|} = 0, \tag{17}$$

where n_{EO} and n_{CO} are the equilibrium electron concentrations in the emitter and collector, respectively. We assume that the emitter depth and the collector depth are much larger than their corresponding diffusion lengths L_E and L_C, respectively. Substituting these boundary conditions into expressions similar to Eq. 12 yields

$$n_E(x) = n_{EO} + n_{EO}\left(e^{qV_{EB}/kT} - 1\right) e^{\frac{x+x_E}{L_E}} \qquad x \le -x_E, \tag{18}$$

$$n_C(x) = n_{CO} - n_{CO}\, e^{-\frac{x-x_C}{L_C}} \qquad x \ge x_C\ . \tag{19}$$

5.2.2 Ideal Transistor Currents for Active Mode Operation

Once the minority-carrier distributions are known, the various current components shown in Fig. 6 can be calculated. The hole current I_{Ep}, injected from the emitter at $x = 0$, is proportional to the gradient of the minority carrier concentration. For $W/L_p << 1$, the hole current I_{Ep} can be expressed by using Eq. 15:

$$I_{Ep} = A\left(-qD_p \left.\frac{dp_n}{dx}\right|_{x=0}\right) \equiv \frac{qAD_p p_{no}}{W}\, e^{qV_{EB}/kT}\ . \tag{20}$$

Similarly, the hole current collected by the collector at $x = W$ is

$$I_{Cp} = A\left(-qD_p \left.\frac{dp_n}{dx}\right|_{x=W}\right)$$

$$\equiv \frac{qAD_p p_{no}}{W}\, e^{qV_{EB}/kT}\ , \tag{21}$$

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Note that I_{Ep} is equal to I_{Cp} when $W/L_p << 1$. The electron current I_{En}, which is due to electron flow from the base to the emitter, and I_{Cn}, which is due to electron flow from the collector to the base, are

$$I_{En} = A\left(-qD_E \frac{dn_E}{dx}\bigg|_{x=-x_E}\right) = \frac{qAD_E n_{EO}}{L_E}\left(e^{qV_{EB}/kT} - 1\right), \tag{22}$$

$$I_{Cn} = A\left(-qD_C \frac{dn_C}{dx}\bigg|_{x=x_C}\right) = \frac{qAD_C n_{CO}}{L_C}, \tag{23}$$

where D_E and D_C are the diffusion constants in the emitter and collector, respectively.

The terminal currents can now be obtained from these equations. The emitter current is the sum of Eqs. 20 and 22:

$$\boxed{I_E = a_{11}\left(e^{qV_{EB}/kT} - 1\right) + a_{12}} \tag{24}$$

where

$$a_{11} \equiv qA\left(\frac{D_p p_{no}}{W} + \frac{D_E n_{EO}}{L_E}\right), \tag{25}$$

$$a_{12} \equiv \frac{qAD_p p_{no}}{W}. \tag{26}$$

The collector current is the sum of Eqs. 21 and 23:

$$\boxed{I_C = a_{21}\left(e^{qV_{EB}/kT} - 1\right) + a_{22}.} \tag{27}$$

where

$$a_{21} \equiv \frac{qAD_p p_{no}}{W}, \tag{28}$$

$$a_{22} \equiv qA\left(\frac{D_p p_{no}}{W} + \frac{D_C n_{CO}}{L_C}\right). \tag{29}$$

Note that $a_{12} = a_{21}$. The base current for the ideal transistor is the difference between the emitter current (I_E) and the collector current (I_C). Therefore, the base current can be obtained by subtracting Eq. 27 from Eq. 24:

$$\boxed{I_B = \left(a_{11} - a_{21}\right)\left(e^{qV_{EB}/kT} - 1\right) + \left(a_{12} - a_{22}\right).} \tag{30}$$

From these discussions, we see that the currents in the three terminals of a transistor are mainly determined by the minority carrier distribution in the base region. Once

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we derive the current components, the common-base current gain α_0 can be obtained by using Eqs. 6 through 8.

EXAMPLE 2

An ideal p^+-n-p transistor has impurity concentrations of 10^{19}, 10^{17}, and 5×10^{15} cm^{-3} in the emitter, base, and collector regions, respectively; the corresponding lifetimes are 10^{-8}, 10^{-7}, and 10^{-6} s. Assume that an effective cross section area A is 0.05 mm^2 and the emitter-base junction is forward-biased to 0.6 V. Find the common-base current gain of the transistor. Note that the other device parameters are $D_E = 1$ cm^2/s, $D_p = 10$ cm^2/s, $D_C = 2$ cm^2/s, and $W = 0.5$ μm.

SOLUTION In the base region,

$$L_p = \sqrt{D_p \tau_p} = \sqrt{10 \cdot 10^{-7}} = 10^{-3} \text{ cm},$$

$$p_{no} = n_i^2 / N_B = \left(9.65 \times 10^9\right)^2 / 10^{17} = 9.31 \times 10^2 \text{ cm}^{-3}.$$

Similarly, in the emitter region, $L_E = \sqrt{D_E \tau_E} = 10^{-4}$ cm and $n_{EO} = n_i^2 / N_E = 9.31$ cm^{-3}.
Since $W/L_p = 0.05 << 1$, the current components are given by

$$I_{Ep} = \frac{1.6 \times 10^{-19} \times 5 \times 10^{-4} \times 10 \cdot 9.31 \times 10^2}{0.5 \times 10^{-4}} \times e^{0.6/0.0259} \times 10^{-4} \text{ A} = 1.7137 \times 10^{-4} \text{ A},$$

$$I_{Cp} = 1.7137 \times 10^{-4} \text{ A},$$

$$I_{En} = \frac{1.6 \times 10^{-19} \times 5 \times 10^{-4} \times 1 \times 9.31}{10^{-4}} \left(e^{0.6/0.0259} - 1\right) = 8.5687 \times 10^{-8} \text{ A}.$$

Therefore, the common-base current gain α_0 is

$$\alpha_0 = \frac{I_{Cp}}{I_{Ep} + I_{En}} = \frac{1.7137 \times 10^{-4}}{1.7137 \times 10^{-4} + 8.5687 \times 10^{-8}} = 0.9995.$$

For the case of $W/L_p << 1$, we can simplify the emitter efficiency from Eqs. 20 and 22:

$$\gamma \equiv \frac{I_{Ep}}{I_{Ep} + I_{En}} \equiv \frac{\frac{D_p p_{no}}{W}}{\frac{D_p p_{no}}{W} + \frac{D_E n_{EO}}{L_E}} = \frac{1}{1 + \frac{D_E}{D_P} \frac{n_{EO}}{p_{no}} \frac{W}{L_E}} \tag{31}$$

or

$$\gamma = \frac{1}{1 + \frac{D_E}{D_p} \cdot \frac{N_B}{N_E} \cdot \frac{W}{L_E}}, \tag{31a}$$

where $N_B (= n_i^2 / p_{no})$ is the impurity doping in the base and $N_E (= n_i^2 / n_{EO})$ is the impurity doping in the emitter. This equation shows that to improve γ, we should decrease the ratio N_B/N_E, that is, there should be much heavier doping in the emitter than in the base. This is the reason why we use p^+-doping in the emitter.

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5.2.3 Modes of Operation

A bipolar transistor has four modes of operation, depending on the voltage polarities on the emitter-base junction and the collector-base junction. Figure 7 shows the V_{EB} and V_{CB} voltages for the four modes of operations of a p-n-p transistor. The corresponding minority carrier distributions are also shown. So far in this chapter we have considered the *active mode* of transistor operation. In the active mode, the emitter-base junction is forward biased and the base-collector junction is reverse biased.

In the *saturation mode*, both junctions are forward biased, leading to the nonzero minority-carrier distribution at the edge of each depletion region. Therefore, the boundary condition at $x = W$ becomes $p_n(W) = p_{no}e^{qV_{CB}/kT}$ instead of the one given by Eq. 13*b*. The saturation mode corresponds to small biasing voltage and large output current, that is, the transistor is in a conducting state and acts as a closed (or on) switch.

In the *cutoff mode*, both junctions are reverse biased. The boundary conditions of Eq. 13 become $p_n(0) = p_n(W) = 0$. The cutoff mode corresponds to the open (or off) state of the transistor as a switch.

The fourth mode of operation is the *inverted mode*, which is sometimes called the inverted active mode. In this mode, the emitter-base junction is reverse biased and the collector-base junction is forward biased. The inverted mode corresponds to the case where the collector acts like the emitter and the emitter acts like a collector. In such condition, the device is used backward. However, the current gain for the inverted mode is generally lower than that for the active mode. It is because of poor "emitter efficiency" resulting from low collector doping with respect to the base doping (Eq. 31).

The current-voltage relationships for the various modes of operation can be obtained by following the same procedures used for the active mode, with an appropriate change

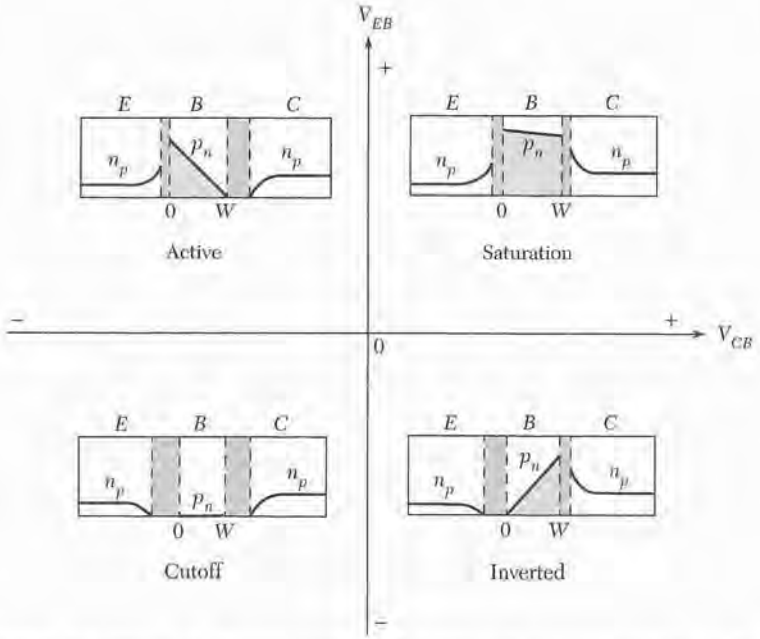


Fig. 7 Junction polarities and minority carrier distributions of a p-n-p transistor under four modes of operation.

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in the boundary conditions as in Eq. 13. The general expressions applicable to all modes of operations are

$$I_E = a_{11}\left(e^{qV_{EB}/kT} - 1\right) - a_{12}\left(e^{qV_{CB}/kT} - 1\right) \tag{32a}$$

and

$$I_C = a_{21}\left(e^{qV_{EB}/kT} - 1\right) - a_{22}\left(e^{qV_{CB}/kT} - 1\right) \tag{32b}$$

where the coefficients a_{11}, a_{12}, a_{21}, and a_{22} are given by Eqs. 25, 26, 28, and 29, respectively. Note that in Eqs. 32*a* and 32*b* the biasing voltages for the junctions can be positive or negative depending on the mode of operation.

5.2.4 Current-Voltage Characteristics of Common-Base and Common-Emitter Configurations

Using Eq. 32, we can obtain the current-voltage characteristics for a transitor in common-base configuration. We note that in this configuration, V_{EB} and V_{BC} are the input and output voltages, and I_E and I_C are the input and output currents, respectively.

However, in circuit applications the common-emitter configuration is most often used, where the emitter lead is common to the input and output circuits. The general expressions of the currents, shown in Eq. 32, are also applicable to the common-emitter configuration. In this case, to generate the current-voltage characteristics, V_{EB} and I_B are the input parameters and V_{EC} and I_C are the output parameters.

Common-Base Configuration

Figure 8*a* shows the common-base configuration of a *p-n-p* transistor. Figure 8*b* shows the measured results of output current-voltage characteristics for the common-base configuration. The various modes of operation are indicated on the figure. Note that the collector current is practically equal to emitter current (i.e., $\alpha_0 \cong 1$) and virtually independent of V_{BC}. This is in close agreement with the ideal transistor behavior given by Eqs. 10 and 27. The collector current remains practically constant, even down to zero volts for V_{BC}, where the holes are still extracted by the collector. This is indicated by the hole distributions

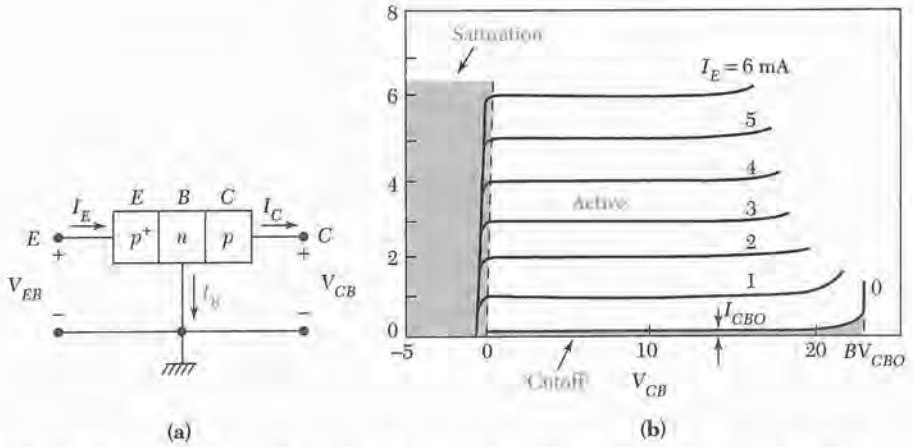


Fig. 8 (*a*) Common-base configuration of a *p-n-p* transistor. (*b*) Its output current-voltage characteristics.

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shown in Fig. 9*a*. Since the hole gradient at $x = W$ changes only slightly from $V_{BC} > 0$ to $V_{BC} = 0$, the collector current remains essentially the same over the entire active mode of operation. To reduce the collector current to zero, we have to apply a small forward bias, about 1 V for silicon, to the base-collector junction (in the saturation mode), as shown in Fig. 9*b*. The forward bias will sufficiently increase the hole density at $x = W$ to make it equal to that of the emitter at $x = 0$ (see the horizontal line in Fig. 9*b*). Therefore, the hole gradient at $x = W$ as well as the collector current will be reduced to zero.

Common-Emitter Configuration

Figure 10*a* shows the common-emitter configuration for a *p-n-p* transistor. The collector current for the common-emitter configuration can be obtained by substituting Eq. 3 into Eq. 10:

$$I_C = \alpha_0 \left(I_B + I_C \right) + I_{CBO} \quad . \tag{33}$$

Solving for I_C, we obtain

$$I_C = \frac{\alpha_0}{1 - \alpha_0} I_B + \frac{I_{CBO}}{1 - \alpha_0} \quad . \tag{34}$$

We now designate β_0 as the *common-emitter current gain*, which is the incremental change of I_C with respect to an incremental change of I_B. From Eq. 34, we obtain

$$\boxed{\beta_0 \equiv \frac{\Delta I_C}{\Delta I_B} = \frac{\alpha_0}{1 - \alpha_0}} \quad . \tag{35}$$

We can also designate I_{CEO} as

$$I_{CEO} \equiv \frac{I_{CBO}}{1 - \alpha_0} \quad . \tag{36}$$

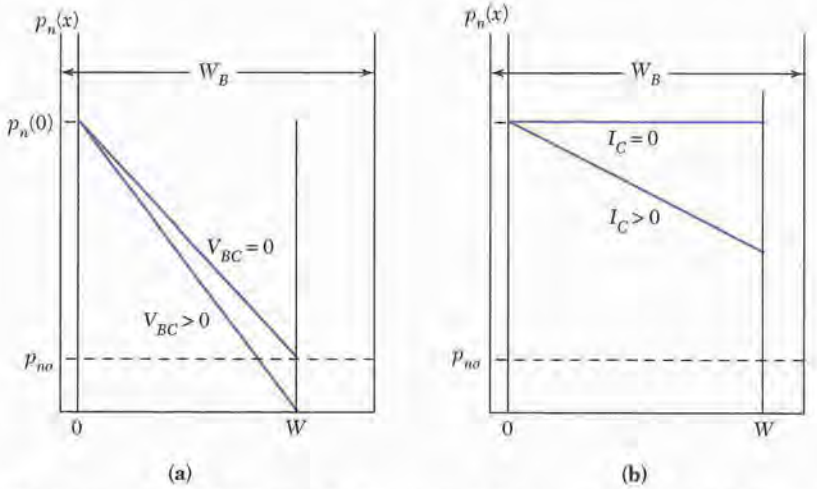


Fig. 9 Minority carrier distributions in the base region of a *p-n-p* transistor. (*a*) Active mode for $V_{BC} = 0$ and $V_{BC} > 0$. (*b*) Saturation mode with both junctions forward biased.

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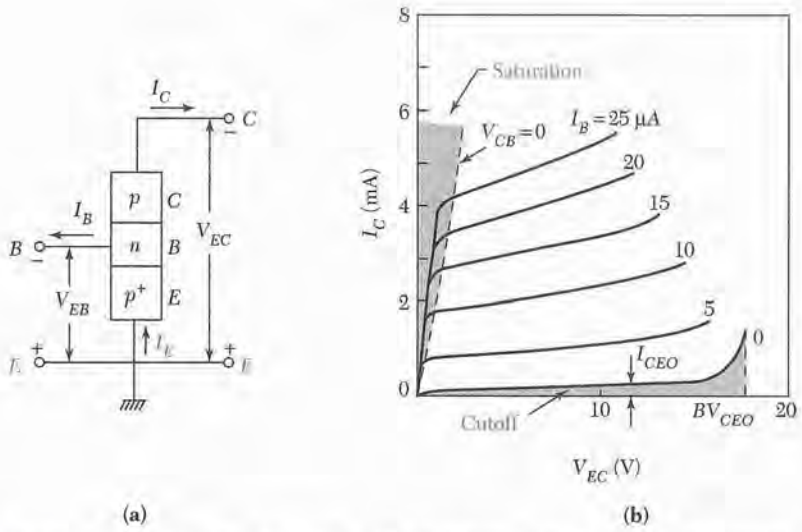


Fig. 10 (*a*) Common-emitter configuration of a *p-n-p* transistor. (*b*) Its output current-voltage characteristics.

This current corresponds to the collector-emitter leakage current for $I_B = 0$. Equation 34 becomes

$$\boxed{I_C = \beta_0 I_B + I_{CEO} .} \tag{37}$$

Because the value of α_0 is generally close to unity, β_0 is much larger than 1. For example, if $\alpha_0 = 0.99$, β_0 is 99; and if α_0 is 0.998, β_0 is 499. Therefore, a small change in the base current can give rise to a much larger change in the collector current. Figure 10*b* shows the measured results of output current-voltage characteristics with various input base currents. Note that the figure shows nonzero collector-emitter leakage current I_{CEO} when $I_B = 0$.

EXAMPLE 3

Refering to Ex. 1, find the common-emitter current gain β_0. Express I_{CEO} in terms of β_0 and I_{CBO} and find the value of I_{CEO}.

SOLUTION The common-base current gain α_0 in Example 1 is 0.9934. Hence, we can obtain β_0 by

$$\beta_0 = \frac{0.9934}{1 - 0.9934} = 150.5.$$

Equation 36 can be expressed by

$$I_{CEO} = \left(\frac{\alpha_0}{1 - \alpha_0} + 1\right) I_{CBO}$$
$$= (\beta_0 + 1) I_{CBO} .$$

Therefore, $$I_{CEO} = (150.5 + 1) \times 0.87 \times 10^{-6} = 1.32 \times 10^{-4} \text{ A}.$$

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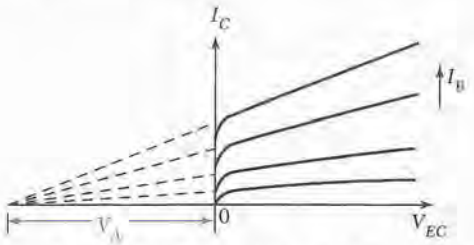


Fig. 11 Schematic diagram of the Early effect and Early voltage V_A. The collector currents for different base currents meet at $-V_A$.

In an ideal transistor with the common-emitter configuration, the collector current for a given I_B is expected to be independent of V_{EC} for $V_{EC} > 0$. This is true when we assume that the neutral base width (W) is constant. However, since the width of the space charge region extending into the base region varies with the base-collector voltage, the base width is a function of the base-collector voltage. The collector current, therefore, is dependent on V_{EC}. As the base-collector reverse-bias voltage increases, the base width will be reduced. The reduced base width causes the gradient in the minority-carrier concentration to increase, which causes an increase in the diffusion current. As a result, β_0 will be increased. Figure 11 shows pronounced slopes, and I_C increasing with increasing V_{EC}. This deviation is known as the *Early effect*[4] or the *base width modulation*. By extrapolating the collector currents and intersecting the V_{EC} axis, we can obtain the voltage V_A, which is called the *Early voltage*.

5.3 FREQUENCY RESPONSE AND SWITCHING OF BIPOLAR TRANSISTOR

In Section 5.2 we discussed four possible modes of operation that depend on the biasing conditions of the emitter-base and collector-base junctions. Generally, in analog or linear circuits the transistors are operated in the active mode only. However, in digital circuits all four modes of operation may be involved. In this section we consider the frequency response and switching characteristics of bipolar transistors.

5.3.1 Frequency Response

High-Frequency Equivalent Circuit

In previous discussions, we were concerned with the static [or direct current (dc)] characteristics of the bipolar transistor. We now study its alternating current (ac) characteristics when a small-signal voltage or current is superimposed upon the dc values. The term small-signal means that the peak values of the ac signal current and voltage are smaller than the dc values. Consider an amplifying circuit shown in Fig. 12*a*, where the transistor is connected in a common-emitter configuration. For a given dc input voltage V_{EB}, a dc base current I_B and dc collector current I_C flow in the transistor. These currents correspond to the operating point shown in Fig. 12*b*. The load line, determined by the applied voltage V_{CC} and the load resistance R_L, intercepts the V_{EC} axis at V_{CC} and has a slope of $(-1/R_L)$. When a small ac signal is superimposed on the input voltage, the base current i_B will vary as a function of time, as illustrated in Fig. 12*b*. This variation, in turn, brings about a corresponding variation in the output current i_C, which is β_0 times larger than the input current variation. Thus, the transistor amplifies the input signal.

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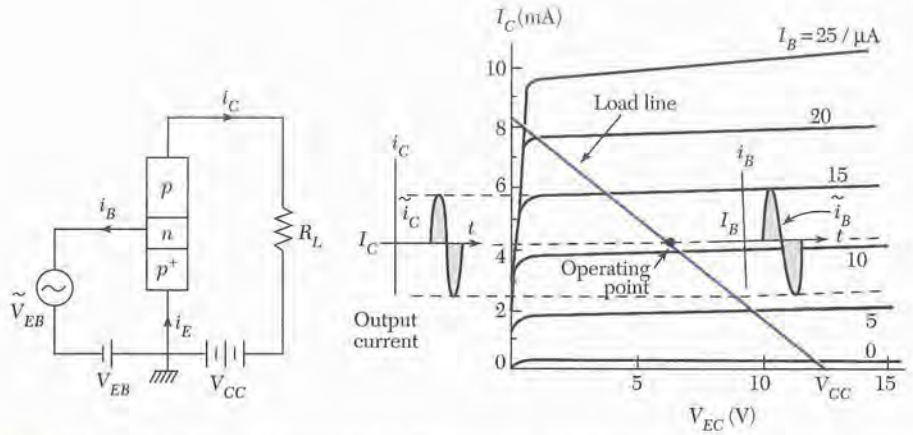


Fig. 12 (*a*) Bipolar transistor connected in the common-emitter configuration. (*b*) Small-signal operation of the transistor circuit.

The equivalent circuit for this low-frequency amplifier is shown in Fig. 13*a*. At higher frequencies, we extend the equivalent circuit by adding the appropriate capacitances. Since the emitter-base junction is forward biased, we expect to have a depletion capacitance C_{EB} and a diffusion capacitance C_d similar to that of a forward-biased p–n junction. For the reverse-biased collector-base junction, we expect to have only a depletion capacitance

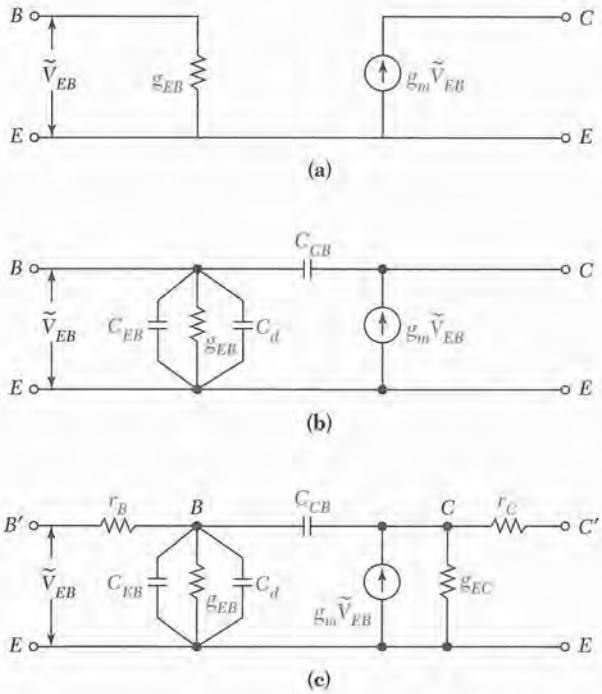


Fig. 13 (*a*) Basic transistor equivalent circuit. (*b*) Basic circuit with the addition of depletion and diffusion capacitances. (*c*) Basic circuit with the addition of resistance and conductance.

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C_{CB}. The high-frequency equivalent circuit with the three added capacitances is shown in Fig. 13*b*. Note that $g_m\ (\equiv \tilde{i}_C/\tilde{v}_{EB})$ is called the *transconductance* and $g_{EB}\ (\equiv \tilde{i}_B/\tilde{v}_{EB})$ is called the *input conductance*. To account for the base width modulation effect, there is a finite output conductance $g_{EC} \equiv \tilde{i}_C/\tilde{v}$. In addition, we have a base resistance r_B and a collector resistance r_C. Figure 13*c* represents the high-frequency equivalent circuit incorporating all of the above elements.

Cutoff Frequency

In Fig. 13*c*, the transconductance g_m and the input conductance g_{EC} are dependent on the common-base current gain. At low frequencies, the current gain is a constant, independent of the operating frequency. However, the current gain will decrease after a certain critical frequency is reached. A typical plot of the current gain versus operating frequency is shown in Fig. 14. The common-base current gain α can be described as

$$\alpha = \frac{\alpha_0}{1 + j\left(f/f_\alpha\right)} \tag{38}$$

where α_0 is the low-frequency (or dc) common-base current gain and f_α is the *common-base cutoff frequency*. At $f = f_\alpha$ the magnitude of α is 0.707 α_0 (3 dB down).

Figure 14 also shows the common-emitter current gain β. From Eq. 38 we have

$$\beta \equiv \frac{\alpha}{1 - \alpha} = \frac{\beta_0}{1 + j\left(f/f_\beta\right)}, \tag{39}$$

where the f_β is the *common-emitter cutoff frequency* and is given by

$$f_\beta = \left(1 - \alpha_0\right) f_\alpha \tag{40}$$

Since $\alpha_0 \approx 1$, f_β is much smaller than f_α. Another cutoff frequency is f_T when $|\beta|$ becomes unity. By setting the magnitude of the right-hand side of Eq. 39 equal to 1, we obtain

$$f_T = \sqrt{\beta_0^{\,2} - 1} f_\beta \equiv \beta_0\left(1 - \alpha_0\right) f_\alpha \equiv \alpha_0 f_\alpha \tag{41}$$

Hence, f_T is very close to but is smaller than f_α.

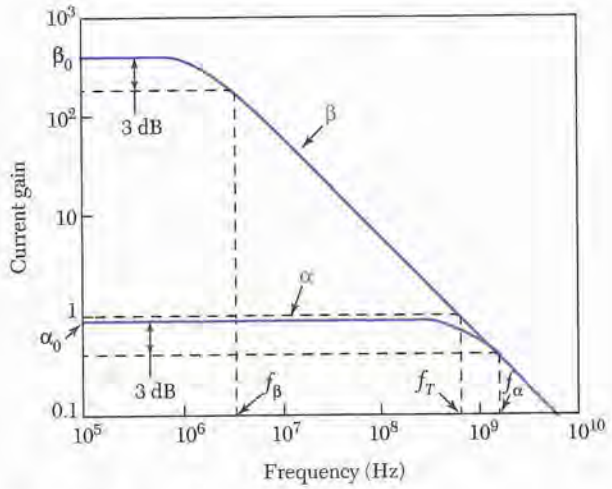


Fig. 14 Current gain as a function of operating frequency.

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The cutoff frequency f_T can also be expressed as $(2\pi\tau_T)^{-1}$ where τ_T is the total time of the carrier transit from the emitter to the collector. τ_T includes the emitter delay time τ_E, the base transit time τ_B,and the collector transit time τ_C. The most important delay time is τ_B. The distance traveled by the minority carriers in the base in a time interval dt is $dx = v(x)\,dt$, where $v(x)$ is the effective minority-carrier velocity in the base. This velocity is related to the current as

$$I_p = qv(x)p(x)A, \tag{42}$$

where A is the device area and $p(x)$ is the distribution of the minority carriers. The transit time τ_B required for a hole to traverse the base is given by

$$\tau_B = \int_0^W \frac{dx}{v(x)} = \int_0^W \frac{q\,p(x)\,A}{I_p}\,dx \tag{43}$$

For a straight-line hole distribution, as given by Eq. 15, the integration of Eq. 43 using Eq. 21 for I_p leads to

$$\boxed{\tau_B = \frac{W^2}{2D_p}.} \tag{44}$$

To improve the frequency response, the transit time of minority carriers across the base must be short. Therefore, high-frequency transistors are designed with a small base width. Because the electron diffusion constant in silicon is about three times larger than that of holes, all high-frequency silicon transistors are of the *n-p-n* type (i,e., the minority carrier in the base is electron). Another way to reduce the base transit time is to use a graded base with a built-in field. For a large doping variation (i.e., high base doping near the emitter and low base doping near the collector), the built-in field in the base will aid the motion of carriers toward the collector and reduce the base transit time.

5.3.2 Switching Transients

For digital applications, a transistor is designed to function as a switch. In these applications, we use a small base current to change the collector current from an *off* condition to an *on* condition (or vice versa), in a very short time. The off condition corresponds to a high-voltage and low-current state, and the on condition corresponds to a low-voltage and high-current state. A basic setup of a switching circuit is shown in Fig. 15*a*, where the emitter-base voltage V_{EB} is suddenly changed from a negative value to a positive value. The output current of the transistor is shown in Fig. 15*b*. The collector current is initially very low because both the emitter-base junction and the collector-base junction are reverse biased. The current will follow the load line through the active region and will finally reach a high current level, where both junctions become forward biased. Thus, the transistor is virtually open-circuited between the emitter and collector terminals in the *off* condition, which corresponds to the cutoff mode, and short-circuited in the *on* condition , which corresponds to the saturation mode. Therefore, a transistor operated in this mode can nearly duplicate the function of an ideal switch.

The switching time is the time required for a transistor to switch from the off condition to the on condition, or vice versa. Figure 16*a* shows that when an input current pulse is applied to the emitter-base terminal at time $t = 0$, the transistor is being turned on. At $t = t_2$, the base current is suddenly switched to zero and the transistor is being turned off. The transient behavior of the collector current I_C can be determined by the

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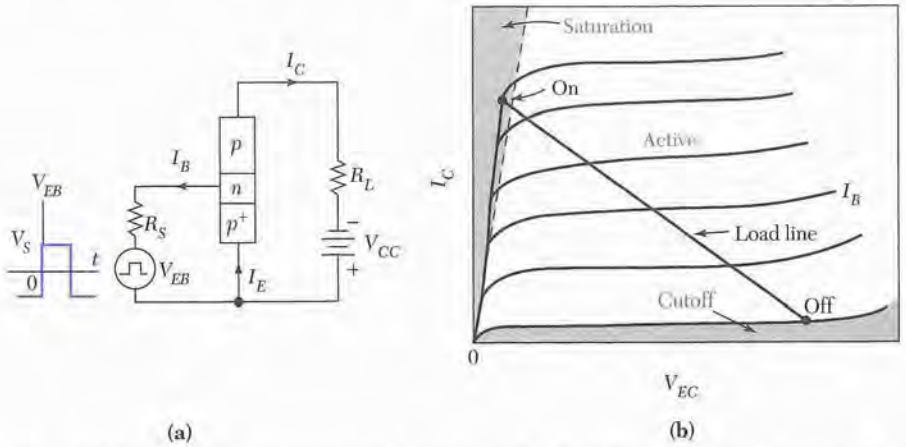


Fig. 15 (*a*) Schematic of a transistor switching circuit. (*b*) Switching operation from cutoff to saturation.

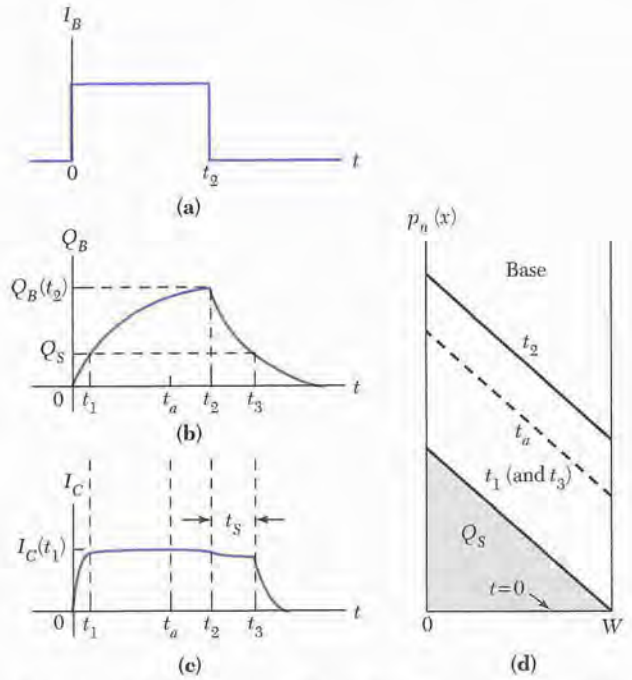


Fig. 16 Transistor switching characteristics. (*a*) Input base current pulse. (*b*) Variations of the base-stored charge with time. (*c*) Variation of the collector current with time. (*d*) Minority-carrier distributions in the base at different times.

variation of the total excess minority carrier charge stored in the base, $Q_B(t)$. A plot of $Q_B(t)$ as a function of time is shown in Fig. 16*b*. During the turn-on transient, the base-stored charge will increase from zero to $Q_B(t_2)$. During the turn-off transient, the base-stored charge will decrease from $Q_B(t_2)$ to zero. For $Q_B(t) < Q_S$, where Q_S is the base charge when $V_{CB} = 0$ (i.e., at the edge of saturation, as shown in Fig. 16*d*), the transistor is in the active mode.

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The variation of I_C with time is plotted in Fig. 16c. In the turn-on transient, the stored base charge reaches Q_S, the charge at the edge of saturation at $t = t_1$. For $Q_B > Q_S$ the device is operated in saturation mode, and both the emitter and collector currents remain essentially constant. Figure 16d shows that for any $t > t_1$ (say $t = t_a$), the hole distribution $p_n(x)$ will be parallel to that for $t = t_1$. Therefore, the gradients at $x = 0$ and $x = W$, as well as the currents, remain the same. In the turn-off transient, since the device is initially in the saturation mode, the collector current remains relatively unchanged until Q_B is reduced to Q_S (Fig. 16d). The time from t_2 to t_3 when $Q_B = Q_S$ is called the *storage time delay* t_S. When $Q_B = Q_S$, the device enters the active mode at $t = t_3$. After that time, the collector current will decay exponentially toward zero.

The turn-on time depends on how fast we can add holes (minority carries in the *p-n-p* transistor) to the base region. The turn-off time depends on how fast we can remove the holes by recombination. One of the most important parameters for switching transistors is the minority carrier lifetime τ_p. One effective method to reduce τ_p for faster switching is to introduce efficient generation-recombination centers near the midgap.

5.4 THE HETEROJUNCTION BIPOLAR TRANSISTOR

We have considered the heterojunction in Sec. 4.8. A heterojunction bipolar transistor (HBT) is a transistor in which one or both *p–n* junctions are formed between dissimilar semiconductors. The primary advantage of an HBT is its high emitter efficiency (γ). The circuit applications of the HBT are essentially the same as those of bipolar transistors. However, the HBT has higher-speed and higher-frequency capability in circuit operation. Because of these features, the HBT has gained popularity in photonic, microwave, and digital applications. For example, in microwave applications, HBT is used in solid-state microwave and millimeter-wave power amplifiers, oscillators, and mixers.

5.4.1 Current Gain in HBT

Let semiconductor 1 be the emitter and semiconductor 2 be the base of a HBT. We now consider the impact of the bandgap difference between these two semiconductors on the current gain of a HBT.

When the base-transport factor α_T is very close to unity, the common-emitter current gain can be expressed from Eqs. 8 and 35 as

$$\beta_0 \equiv \frac{\alpha_0}{1-\alpha_0} \equiv \frac{\gamma\alpha_T}{1-\gamma\alpha_T} = \frac{\gamma}{1-\gamma} \quad (\text{for } \alpha_T = 1). \tag{45}$$

Substituting γ from Eq. 31 in Eq. 45 yields (for *n-p-n* transistors)

$$\beta_0 = \frac{1}{\dfrac{D_E}{D_n}\dfrac{p_{E0}}{n_{p0}}\dfrac{W}{L_E}} \approx \frac{n_{p0}}{p_{E0}} \tag{46}$$

The minority carrier concentrations in the emitter and the base are given by

$$p_{E0} = \frac{n_i^2(\text{emitter})}{N_E(\text{emitter})} = \frac{N_C N_V \exp(-E_{gE}/kT)}{N_E}, \tag{47}$$

$$n_{p0} = \frac{n_i^2(\text{base})}{N_B(\text{base})} = \frac{N'_C N'_V \exp(-E_{gB}/kT)}{N_B}, \tag{48}$$

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where N_C and N_V are the densities of states in the conduction band and the valence band, respectively, and E_{gE} is the bandgap of the emitter semiconductor. N'_C, N'_V, and E_{gB} are the corresponding parameters for the base semiconductor. Therefore

$$\beta_0 \sim \frac{N_E}{N_B} \exp\left(\frac{E_{gE} - E_{gB}}{kT}\right) = \frac{N_E}{N_B} \exp\left(\frac{\Delta E_g}{kT}\right). \tag{49}$$

EXAMPLE 4

A HBT has a bandgap of 1.62 eV for the emitter, and a bandgap of 1.42 eV for the base. A BJT has a bandgap of 1.42 eV for both the emitter and base materials; it has an emitter doping of 10^{18} cm^{-3} and a base doping of 10^{15} cm^{-3}.
(a) If the HBT has the same dopings as the BJT, find the improvement of β_0. (b) If the HBT has the same emitter doping and the same β_0 as the BJT, how much can we increase the base doping of the HBT? Assume that all other device parameters are the same.

SOLUTION

$$\text{(a)} \quad \frac{\beta_0(\text{HBT})}{\beta_0(\text{BJT})} = \frac{\exp\left(\frac{E_{gE} - E_{gB}}{kT}\right)}{1} = \exp\left(\frac{1.62 - 1.42}{0.0259}\right) = \exp\left(\frac{0.2}{0.0259}\right) = \exp(7.722) = 2257.$$

We have an improvement of 2257 times in β_0.

$$\text{(b)} \quad \beta_0(\text{HBT}) = \frac{N_E}{N'_B}\exp(7.722) = \beta_0(\text{BJT}) = \frac{N_E}{N_B}$$

$$\therefore N'_B = N_B \exp(7.722) = 2257 \times 10^{15} = 2.26 \times 10^{18} \text{cm}^{-3}.$$

The base doping of the heterojunction can be increased to 2.26×10^{18} cm^{-3} to maintain the same β_0.

5.4.2 Basic HBT Structures

Most developments of HBT technology are for the $Al_xGa_{1-x}As/GaAs$ material system. Figure 17*a* shows a schematic structure of a basic *n-p-n* HBT. In this device, the *n*-type emitter is formed in the wide bandgap $Al_xGa_{1-x}As$ whereas the *p*-type base is formed in the lower bandgap GaAs. The *n*-type collector and *n*-type subcollector are formed in GaAs with light doping and heavy doping, respectively. To facilitate the formation of ohmic contacts, a heavily doped *n*-type GaAs layer is formed between the emitter contact and the AlGaAs layer. Due to the large bandgap difference between the emitter and the base materials, the common-emitter current gain can be extremely large. However, in homojunction bipolar transistors, there is essentially no bandgap difference; instead the ratio of the doping concentration in the emitter and base must be very high. This is the fundamental difference between the homojunction and the heterojunction bipolar transistors (see Ex. 4).

Figure 17*b* shows the energy band diagram of the HBT under the active mode of operation. The bandgap difference between the emitter and the base will provide band offsets at the heterointerface. In fact, the superior performance of the HBT results directly from the valence-band discontinuity ΔE_V at the heterointerface. ΔE_V increases the valence-band barrier height in the emitter-base heterojunction and thus reduces the injection of holes from the base to the emitter. This effect in the HBT allows the use of a heavily

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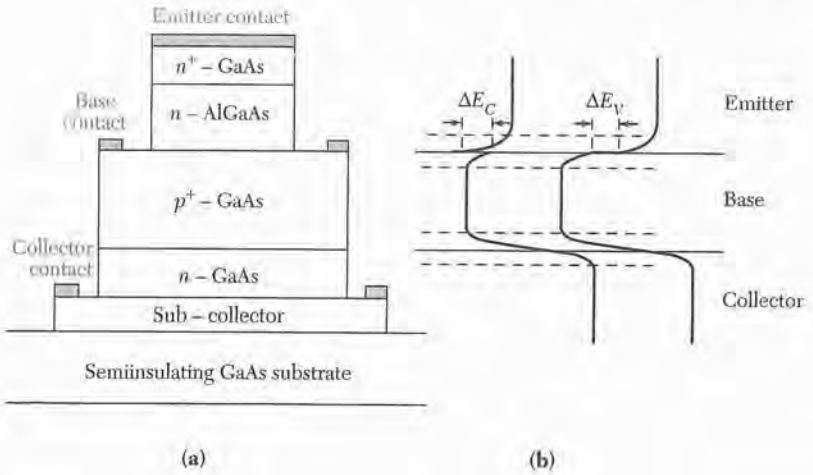


Fig. 17 (*a*) Schematic cross section of an *n-p-n* heterojunction bipolar transistor (HBT) structure. (*b*) Energy band diagram of a HBT operated under active mode.

doped base while maintaining a high emitter efficiency and current gain. The heavily doped base can reduce the base sheet resistance.[5] In addition, the base can be made very thin without concern about the *punch-through* effect in the narrow base region. The punch-through effect arises when the base-collector depletion region penetrates completely through the base and reaches the emitter-base depletion region. A thin base region is desirable because it reduces the base transit time and increases the cutoff frequency.[6]

5.4.3 Advanced HBTs

In recent years the InP-based (InP/InGaAs or AlInAs/InGaAs) material systems have been extensively studied. The InP-based heterostructures had several advantages.[7] The InP/InGaAs structure has very low surface recombination, and because of a higher electron mobility in InGaAs than in GaAs, superior high-frequency performance is expected. A typical performance curve for an InP-based HBT is shown [8] in Fig. 18. A very high cutoff frequency of 254 GHz is obtained. In addition, the InP collector region has higher

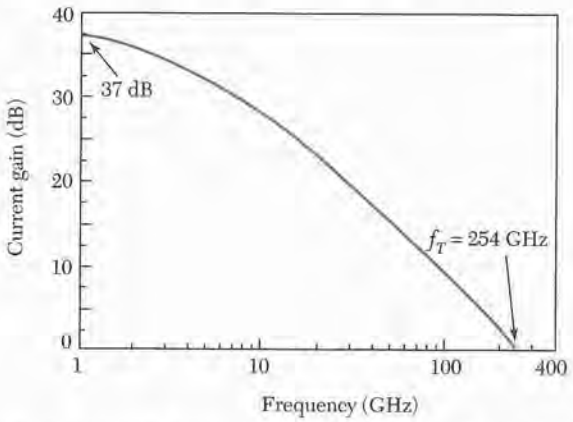


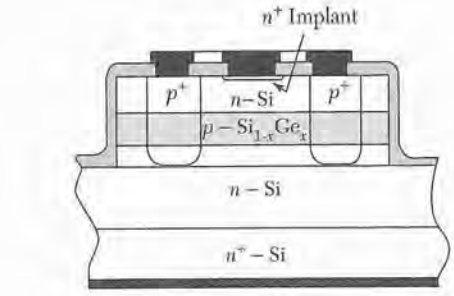
Fig. 18 Current gain as a function of operating frequency for an InP-based HBT.[8]

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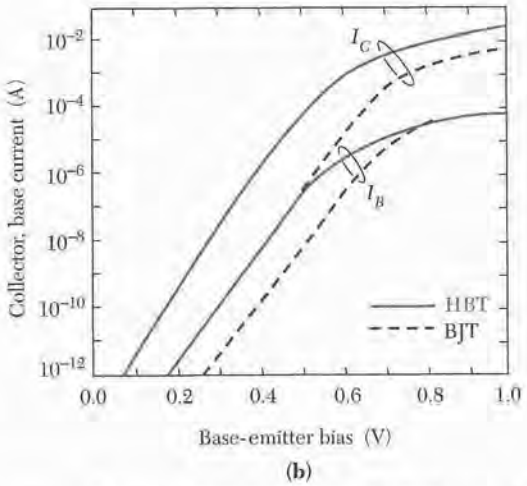
drift velocity at high fields than that in the GaAs collector. The InP collector breakdown voltage is also higher than that in the GaAs.

Another heterojunction is in the Si/SiGe material system. This system has several properties that are attractive for HBT applications. Like AlGaAs/GaAs HBTs, Si/SiGe HBTs have high-speed capability since the base can be heavily doped because of the bandgap difference. The small trap density at the silicon surface minimizes the surface recombination current and ensures a high current gain even at low collector current. Compatability with the standard silicon technology is another attractive feature. Figure 19*a* shows a typical Si/SiGe HBT structure. A comparison of the base and collector currents measured from a Si/SiGe HBT and a Si homojunction bipolar transistor is given by Fig. 19*b*. The results indicate that the Si/SiGe HBT has a higher current gain than the Si homojunction bipolar transistor. Compared with GaAs- and InP-based HBTs, the Si/SiGe HBT, however, has a lower cutoff frequency because of the lower mobilities in Si.

The conduction band discontinuity ΔE_C shown in Fig. 17*b* is not desirable, since the discontinuity will make it necessary for the carriers in the heterojunction to transport by means of thermionic emission across a barrier or by tunneling through it. Therefore, the emitter efficiency and the collector current will suffer. The problems can be alleviated by improved structures such as the graded-layer and the graded-base heterojunctions. Figure 20 shows an energy band diagram in which the ΔE_C is eliminated by a graded



(a)



(b)

Fig. 19 (*a*) Device structure of an *n-p-n* Si/SiGe/Si HBT. (*b*) Collector and base current versus V_{EB} for a HBT and bipolar junction transistor (BJT).

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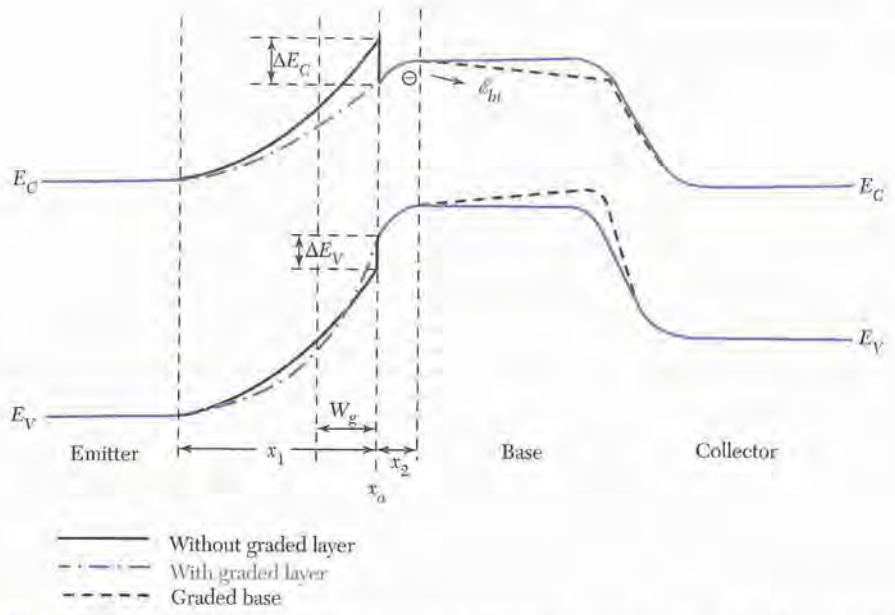


Fig. 20 Energy band diagrams for a heterojunction bipolar transistor with and without graded layer in the junction, and with and without a graded-base layer.

layer placed between the emitter and base heterojunction. The thickness of the graded layer is W_g.

The base region can also have a graded profile, which results in a reduction of the bandgap from the emitter side to the collector side. The energy band diagram of the graded base HBT is illustrated in Fig. 20 (dotted line). Note that there is a built-in electric field $\mathscr{E}_{bi}$ in the quasi-neutral base. It results in a reduction in the minority-carrier transit time and, thus, an increase in the common-emitter current gain and the cutoff frequency of the HBT. $\mathscr{E}_{bi}$ can be obtained, for example, by varying linearly the Al mole fraction x of $Al_xGa_{1-x}As$ in the base from $x = 0.1$ to $x = 0$.

For the design of the collector layer, it is necessary to consider the collector transit time delay and the breakdown voltage requirement. A thicker collector layer will improve the breakdown voltage of the base-collector junction but proportionally increase the transit time. In most devices for high-power applications, the carriers move through the collector at their saturation velocities because very large electric fields are maintained in this layer.

It is possible, however, to increase the velocities by lowering the electric field with certain doping profile in the collector layer. One way is to use p^- collectors with a p^+ pulse-doped layer near the subcollector for an n-p-n HBT. Therefore, electrons entering the collector layer can maintain their higher mobility of the lower valley during most of the collector transit time. Such a device is called a *ballistic collector transistor* (BCT).[9] An energy band diagram of a BCT is shown in Fig. 21. The BCT has been shown to have more favorable frequency response characteristics compared with conventional HBTs over a narrow range of bias voltages. Because of its advantages at relatively low collector voltage and current conditions, the BCT is used for switching applications and microwave-power amplifications.

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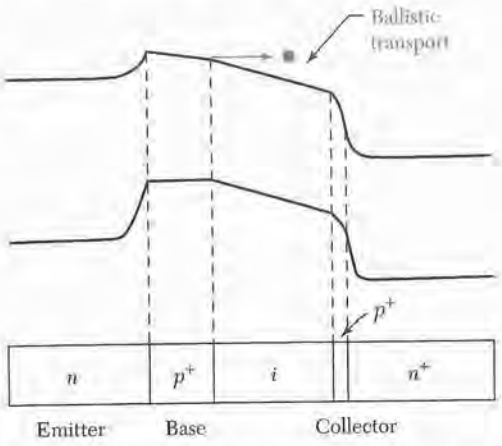


Fig. 21 Energy band diagram for the ballistic collector transistor (BCT).[9]

5.5 THE THYRISTOR AND RELATED POWER DEVICES

The thyristor is an important power device that is designed for handling high voltages and large currents. The thyristor is mainly used for switching applications that require the device to change from an *off* or blocking state to an *on* or conducting state, or vice versa.[10] We have considered the use of bipolar transistors in this application, in which the base current drives the transistor from cutoff to saturation for the on-state, and from saturation to cutoff for the off-state. The operation of a thyristor is intimately related to the bipolar transistor, in which both electrons and holes are involved on the transport processes. However, the switching mechanisms in a thyristor are different from those of a bipolar transistor. Also, because of the device construction, thyristors have a much wider range of current- and voltage-handling capabilities. Thyristors are now available[11] with current ratings from a few milliamperes to over 5000 A and voltage ratings extending above 10,000 V. We first consider the operating principles of basic thyristors and discuss some related high-power and high-frequency thyristors.

5.5.1 Basic Characteristics

Figure 22*a* shows a schematic cross-sectional view of a thyristor structure, which is a four-layer *p-n-p-n* device with three *p–n* junctions in series: $J1$, $J2$, and $J3$. The contact electrode to the outer *p*-layer is called the *anode* and that to the outer *n*-layer is called the *cathode*. This structure without any additional electrode is a two-terminal device and is called the *p-n-p-n* diode. If an additional electrode, called the *gate* electrode, is connected to the inner *p*-layer ($p2$), the resulting three-terminal device is commonly called the *semiconductor-controlled rectifier* (SCR) or *thyristor*.

A typical doping profile of a thyristor is shown in Fig. 22*b*. An *n*-type, high-resistivity silicon wafer is chosen as the starting material ($n1$-layer). A diffusion step is used to form the $p1$- and $p2$-layers simultaneously. Finally, an *n*-type layer is alloyed (or diffused) into one side of the wafer to form the $n2$-layer. Figure 22*c* shows the energy band diagram of a thyristor in thermal equilibrium. Note that at each junction there is a depletion region with a built-in potential that is determined by the impurity doping profile.

The basic current-voltage characteristics of a *p-n-p-n* diode is shown in Fig. 23. It exhibits five distinct regions:

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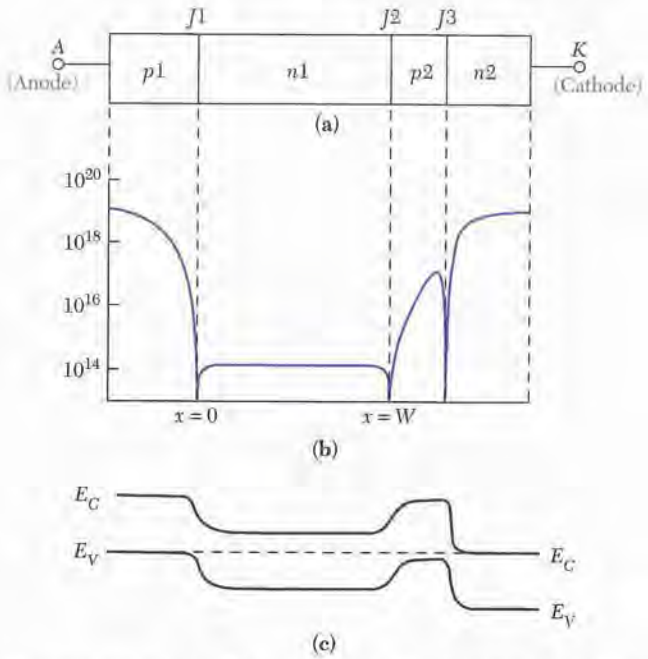


Fig. 22 (*a*) Four-layer *p-n-p-n* diode. (*b*) Typical doping profile of a thyristor. (*c*) Energy band diagram of a thyristor in thermal equilibrium.

0-1: The device is in the forward-blocking or off-state and has very high impedance. Forward breakover (or switching) occurs where $dV/dI = 0$; and at point 1 we define a forward-breakover voltage V_{BF} and a switching current I_S.

1-2: The device is in a negative-resistance region, that is, the current increases as the voltage decreases sharply.

2-3: The device is in the forward-conducting or on-state and has low impedance. At point 2, where $dV/dI = 0$, we define the holding current I_h and holding voltage V_h.

0-4: The device is in the reverse-blocking state.

4-5: The device is in the reverse-breakdown region.

Thus, a *p-n-p-n* diode operated in the forward region is a bistable device that can switch from a high-impedance, low-current off-state to a low-impedance, high-current on-state, or vice versa.

To understand the forward-blocking characteristics, we consider the device as two bipolar transistors, a *p-n-p* transistor and an *n-p-n* transistor, connected in a special way, as shown in Fig. 24. They are connected with the base of one transistor attached to the collector of the other, and vice versa. The relationship between the emitter, collector, and base currents and the dc common-base current gain were given in Eqs. 3 and 10. The base current of the *p-n-p* transistor (transistor 1 with current gain α_1) is

$$\begin{aligned} I_{B1} &= I_{E1} - I_{C1} \\ &= (1 - \alpha_1)I_{E1} - I_1 \\ &= (1-\alpha_1)I - I_1, \end{aligned} \tag{50}$$

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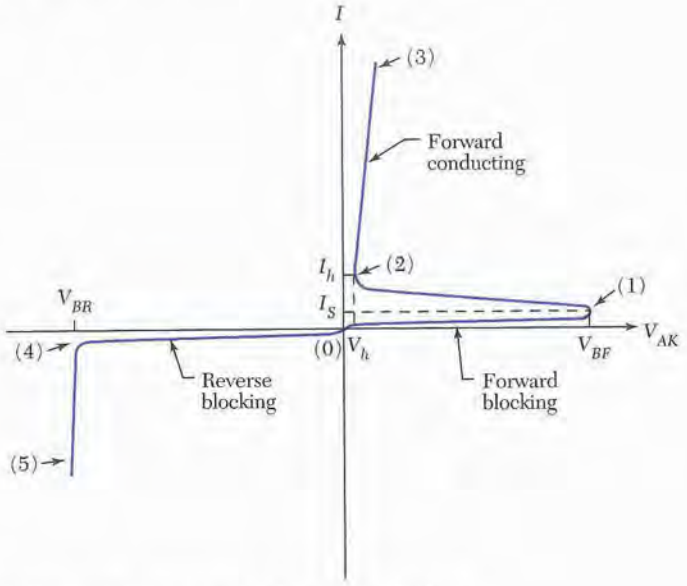


Fig. 23 Current-voltage characteristics of a *p-n-p-n* diode.

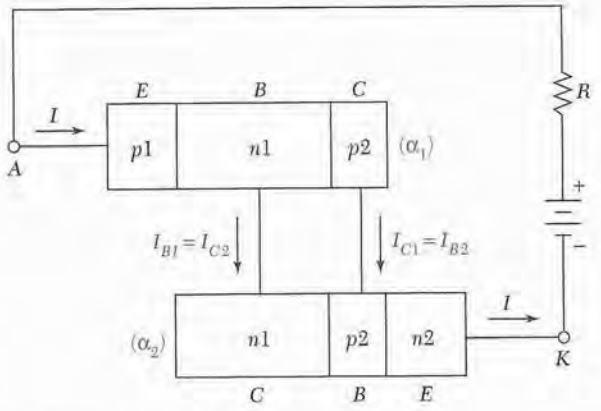


Fig. 24 Two-transistor representation of a thyristor.[2]

where I_1 is the leakage current I_{CBO} for the transistor 1. This base current is supplied by the collector of the *n-p-n* transistor (transistor 2 with the current gain α_2). The collector current of the *n-p-n* transistor is

$$I_{C2} = \alpha_2 I_{E2} + I_2 = \alpha_2 I + I_2, \tag{51}$$

where I_2 is the leakage current I_{CBO} for the transistor 2. By equating I_{B1} and I_{C2}, we obtain

$$\boxed{I = \frac{I_1 + I_2}{1 - (\alpha_1 + \alpha_2)}}. \tag{52}$$

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EXAMPLE 5

Consider a thyristor in which the leakage currents I_1 and I_2 are 0.4 and 0.6 mA, respectively. Explain the forward-blocking characteristics when $(\alpha_1 + \alpha_2)$ is 0.01 and 0.9999.

SOLUTION The current gains are functions of the current I and generally increase with increasing current. At low currents both α_1 and α_2 are much less than 1, and we have

$$I = \frac{0.4 \times 10^{-3} + 0.6 \times 10^{-3}}{1 - 0.01} = 1.01 \text{ mA}$$

In this case, the current flowing through the device is the sum of the leakage currents I_1 and I_2 ($\cong$ 1 mA).

As the applied voltage increases, the current I also increases, as do α_1 and α_2. This in turn causes I to increase further—a regenerative behavior. When $\alpha_1 + \alpha_2 = 0.9999$,

$$I = \frac{0.4 \times 10^{-3} + 0.6 \times 10^{-3}}{1 - 0.9999} = 10 \text{ A} .$$

This value is 10,000 times larger than $I_1 + I_2$. Therefore, as $(\alpha_1 + \alpha_2)$ approaches 1, the current I increases without limit, that is, the device is at forward breakover. ◀

The variations of the depletion layer widths of a p-n-p-n diode biased in different regions are shown in Fig. 25. At thermal equilibrium, Fig. 25*a*, there is no current flowing and the depletion layer widths are determined by the impurity doping profiles. In

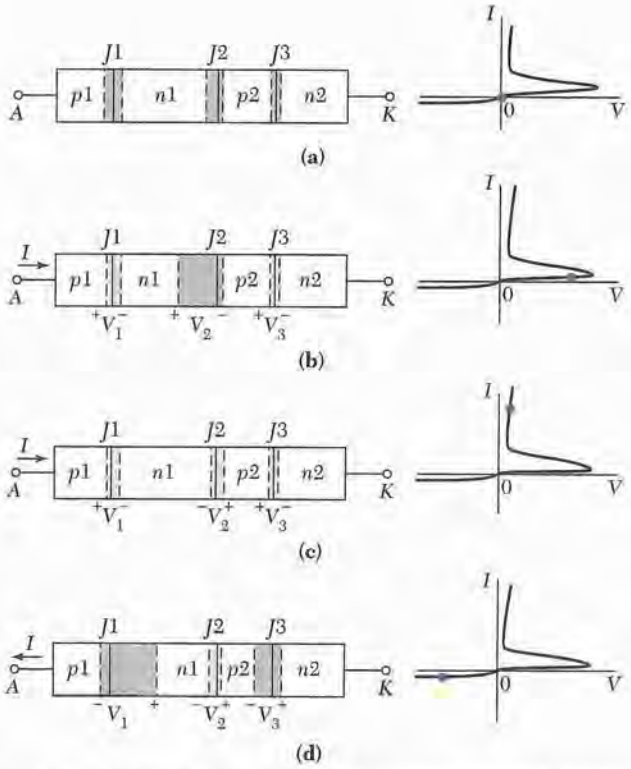


Fig. 25 Depletion layer widths and voltage drops of a thyristor operated under (*a*) equilibrium, (*b*) forward blocking, (*c*) forward conducting, and (*d*) reverse blocking.

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the forward-blocking state, Fig. 25*b*, junction $J1$ and $J3$ are forward biased and $J2$ is reverse biased. Most of the voltage drop occurs across the central junction $J2$. In the forward-conduction state, Fig. 25*c*, all three junctions are forward biased. The two transistors ($p1$-$n1$-$p2$ and $n1$-$p2$-$n2$) are in saturation mode of operation. Therefore, the voltage drop across the device is very low, given by $(V_1 - |V_2| + V_3)$, which is approximately equal to the voltage drop across one forward-biased p–n junction. In the reverse-blocking state, Fig. 25*d*, junction $J2$ is forward biased but both $J1$ and $J3$ are reversed biased. For the doping profile shown in Fig. 22*b*, the reverse-breakdown voltage will be mainly determined by $J1$ because of the lower impurity concentration in the $n1$-region.

Figure 26*a* shows the device configuration of a thyristor that is fabricated by planar processes with a gate electrode connected to the $p2$-region. A cross section of the thyristor along the dashed lines is shown in Fig. 26*b*. The current-voltage characteristic of the thyristor is similar to that of the p-n-p-n diode, except that the gate current I_g causes an increase of $\alpha_1 + \alpha_2$ and results in a breakover at a lower voltage. Figure 27 shows the effect of gate current on the current-voltage characteristics of a thyristor. As the gate current increases, the forward breakover voltage decreases.

A simple application of a thyristor is shown in Fig. 28*a*, where a variable power is delivered to a load from a constant line source. The load R_L may be a light bulb or a heater, such as furnace. The amount of power delivered to the load during each cycle depends on the timing of the gate-current pulses of the thyristor (Fig. 28*b*). If the current pulses are delivered to the gate near the beginning of each cycle, more power will be delivered to the load. However, if the current pulses are delayed, the thyristor will not turn on until later in the cycle, and the amount of power delivered to the load will be substantially reduced.

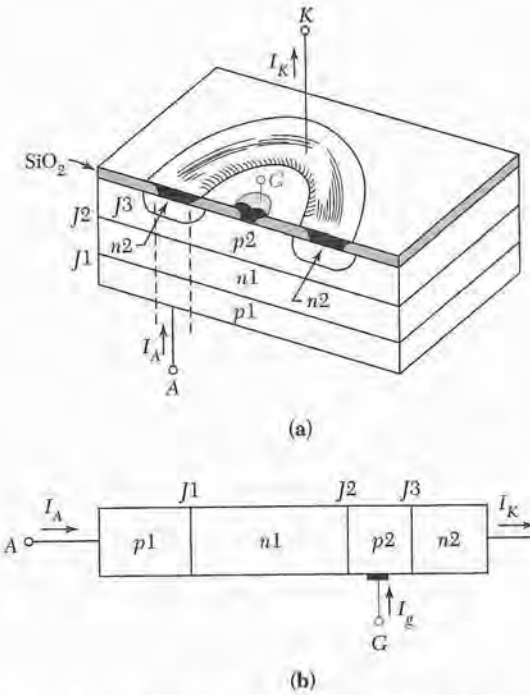


Fig. 26 (*a*) Schematic of a planar three-terminal thyristor. (*b*) One-dimensional cross section of the planar thyristor.

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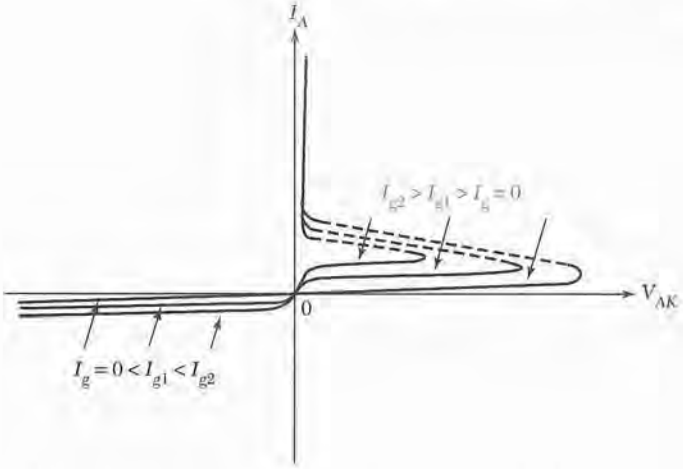


Fig. 27 Affect of gate current on current-voltage characteristics of a thyristor.

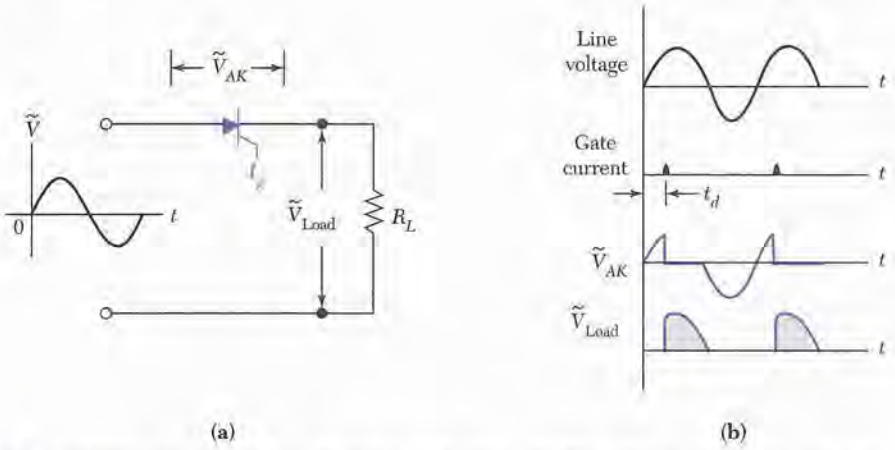


Fig. 28 (*a*) Schematic circuit for a thyristor application. (*b*) Wave forms of voltages and gate current.

5.5.2 Bidirectional Thyristor

A bidirectional thyristor is a switching device that has on- and off-states for positive and negative anode voltages and is therefore useful in ac applications. The bidirectional *p*-*n*-*p*-*n* diode switch is called a *diac* (*di*ode *ac* switch). It behaves like two conventional *p*-*n*-*p*-*n* diodes with the anode of the first diode connected to the cathode of the second, and vice versa. Figure 29*a* illustrates such a structure where *M*1 stands for main terminal 1 and *M*2 for main terminal 2. When we integrate this arrangement into a single two-terminal device, we have a diac, as shown in Fig. 29*b*. The symmetry of this structure will result in identical performance for either polarity of applied voltage.

When a positive voltage is applied to *M*1 with respect to *M*2, junction *J*4 is reverse biased so that the *n*2′ region does not contribute to the functioning of the device. Therefore, the *p*1-*n*1-*p*2-*n*2 layers constitute a *p*-*n*-*p*-*n* diode that produces the forward portion of the *I*-*V* characteristic shown in Fig. 29*c*. If a positive voltage is applied to *M*2, a current

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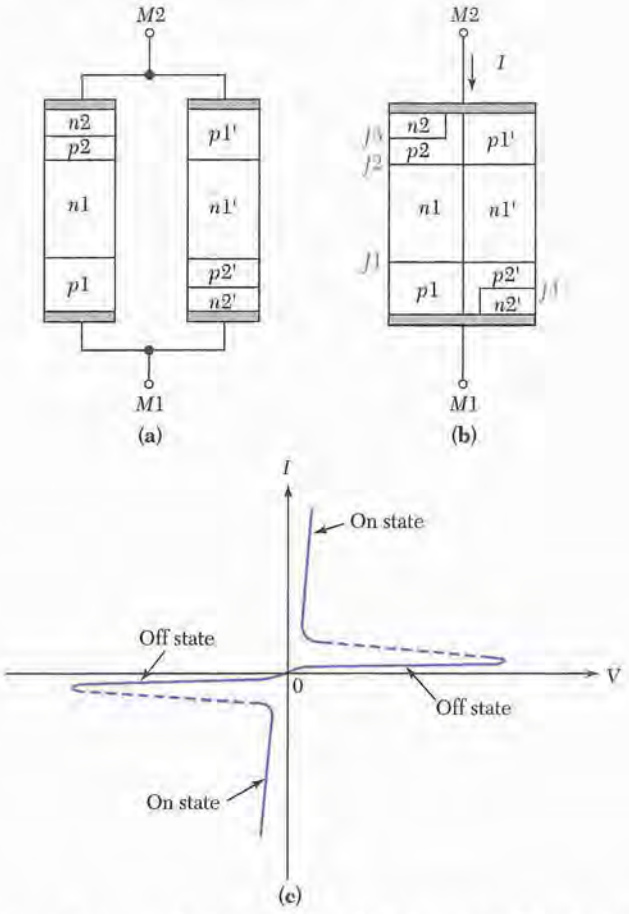


Fig. 29 (*a*) Two reverse-connected *p-n-p-n* diodes. (*b*) Integration of the diodes into a single two-terminal diode ac switch (diac). (*c*) Current-voltage characteristics of a diac.

will conduct in the opposite direction and $J3$ will be reverse biased. Therefore, the $p1'$-$n1'$-$p2'$-$n2'$ layers from the reverse *p-n-p-n* diode produce the reverse portion of the *I-V* characteristics shown in Fig. 29*c*.

A bidirectional three-terminal thyristor is called a *triac* (*tri*ode *ac* switch). The triac can switch the current in either direction by applying a low-voltage, low-current pulse of either polarity between the gate and one of the two main terminals, $M1$ and $M2$, as shown in Fig. 30. The operational principles and the *I-V* characteristics of a triac are similar to those of a diac. By adjusting the gate current, the breakover voltage can be varied in either polarity.

5.5.3 Related Thyristor Types and Applications

In addition to the bidirectional thyristors, there are many other types of thyristors for different applications.[12] The majority of these types are based on the four-layer thyristor structure but with special designs.

The *conventional thyristors* considered in Section 5.5.2 are usually designed to have similar forward and reverse blocking capabilities. Two broad classes exist in the conven-

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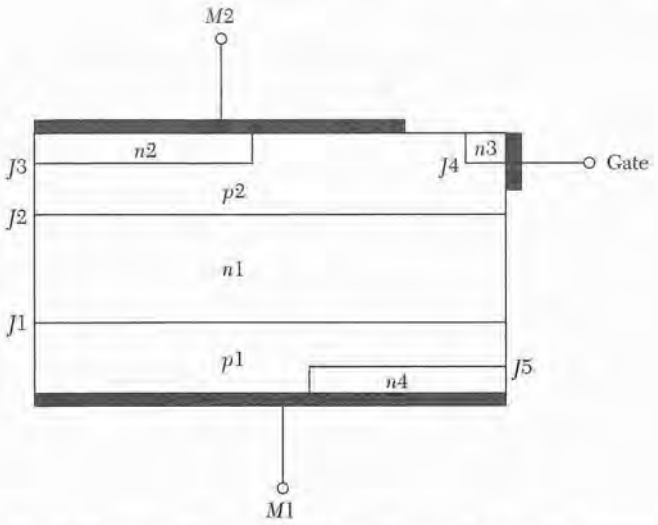


Fig. 30 Cross section of a triode ac switch, a six-layer structure having five p–n junctions.

tional thyristor: one is the converter grade thyristor, and the other is the inverter grade thyristor. The former is used for low frequency and is designed to have the lowest possible on-state voltage drop but can only switch slowly. The latter is designed for high frequency and has a fast turn-off but it generally has a larger on-state voltage drop.

The *asymmetric thyristor* is shown in Fig. 31*a*. It does not have a reverse blocking capability since the n-base contains an additional n^+ layer adjacent to junction $J1$. Figure 31*b* shows the structure of a conventional thyristor for comparison. Note that the

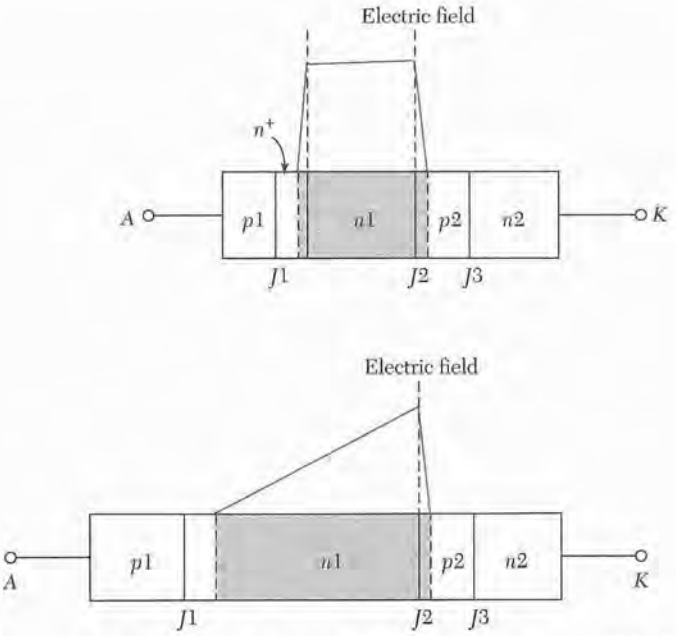


Fig. 31 Comparison of the structure and electric field for the same forward-blocking voltage: (*a*) the asymmetric thyristor and (*b*) the conventional thyristor.

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asymmetric thyristor has a higher-resistivity $n1$-base and an adjecent low-resistivity n^+= layer. In forward blocking the depletion layer on the n-side of the blocking junction spreads to the n^+-layer. However, the depletion layer is then effectively constrained from further spreading by the high-doping level of this layer. Consequently, as shown in Fig. 31, the electric-field distribution is approximately constant for the asymmetric thyristor compared with the triangular distribution for the conventional thyristor. The additional n^+-layer allows the use of a much thinner n-base than the conventional thyristor. The reduced base width greatly helps to reduce the on-state voltage drop and shortens the turn-on time of the device through the narrower base. The turn-off performance is also improved since the narrower n-base results in a lower value of stored charge.

The *gate turn-off* (GTO) thyristor is a device that overcomes one of the basic thyristor's limitations, since the GTO can be turned on and off by the gate control.[13] This is achieved by a tight control on the current gain and by distributing the gate over the whole cathode area. Figure 32 shows a GTO with a negative voltage applied to the p-gate. To turn off the current, a reverse-gate voltage and current are applied instead of reversing the polarity of the main terminals. As a result, the holes from the anode begin to be partially removed by the gate. On the other hand, the electrons injected by the cathode emitter are pushed away from the negative gate voltage toward the active center of the device.

The *light activated thyristor* is a thyristor in which the gate terminal is not electrically contacted but is designed to respond to an optical signal. Usually the optical signal is very weak and consequently the device has a high gain.

The power switching applications of the thyristors cover a broad range of both power and frequency.[10] Figure 33 illustrates the major application areas for thryristors categorized by switching frequency and power. At the low-frequency and high-power end of the spectrum, we have the high-voltage direct current (HVDC) devices, which demand the highest power. On the other hand, for high-frequency and low-power applications, we have the lighting control, ultrasonic generators, high frequency power conversion equipment, and switched-mode power supplies (SMPS). However, the major applications are

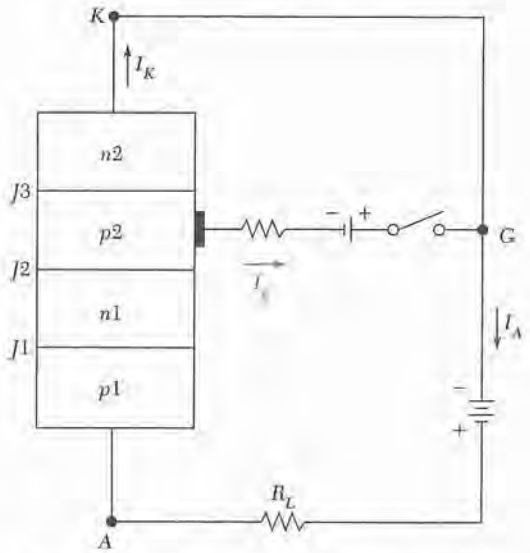


Fig. 32 The gate turn-off thyristor with a negative voltage applied to the gate. The main applications of thyristors.[10]

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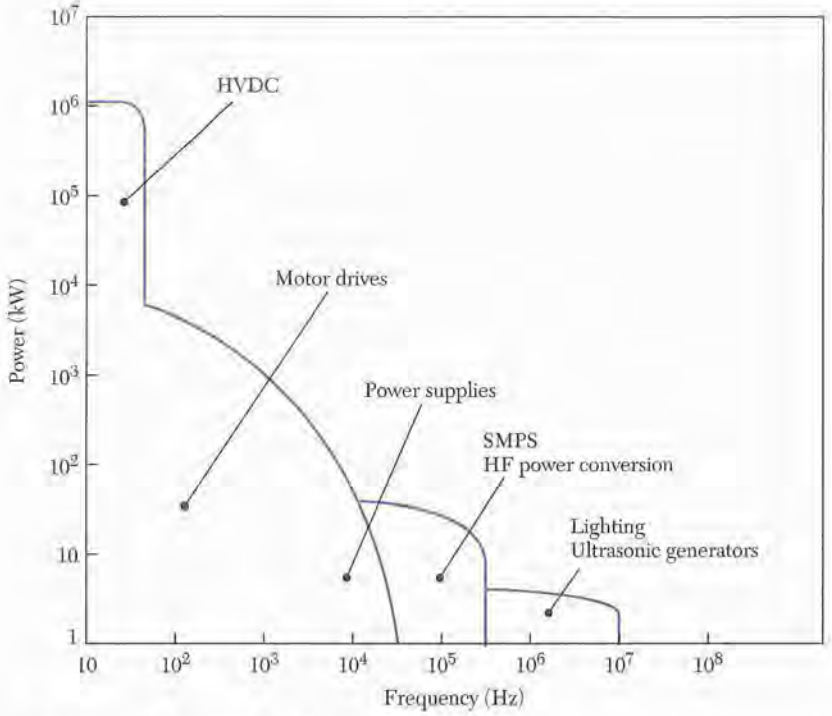


Fig. 33 The main applications of thyristors.[10]

for motor drives and power supplies, which cover a wide range of power and frequency. The motor drive applications, for example, range from small motors used in domestic appliances to the high-power motor drives used in rolling mills.

SUMMARY

The bipolar transistor, developed in 1947, remains one of the most important semiconductor devices. A bipolar transistor is formed when two p–n junctions of the same semiconductor materials are physically close enough to interact. Charge carriers injected from the forward-biased first junction result in a large current flow in the reverse-biased second junction.

We have considered the static characteristics of bipolar transistors, such as the modes of operation and the current-voltage characteristics of the common-emitter configuration. We have also considered the frequency response and switching behavior. A key device parameter of bipolar transistor is the base width, which must be very small compared with the minority-carrier diffusion length to improve the current gain and to increase the cutoff frequency.

Bipolar transistors are used extensively as discrete devices or in integrated circuits for current-gain, voltage-gain, and power-gain applications. They are also used in bipolar-CMOS combination circuits (BiCMOS), covered in Chapters 6 and 14, for high-density, high-speed operations.

The frequency limitations of a conventional bipolar transistor are the result of its low-base doping and relatively wide base. To overcome these limitations, a heterojunction bipolar transistor (HBT) formed between two dissimilar semiconductors can have much

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higher-base doping and a much narrower base. The HBT has, therefore, gained popularity in millimeter-wave and high-speed digital applications.

Another important bipolar device is the thyristor, which is formed of three or more p–n junctions. The thysistor is used mainly for switching applications. These devices can have current ratings from a few milliamperes to over 5000 A and voltage ratings extending above 10,000 V. We have considered the basic characteristic of thyristor operation. In addition, we discussed the bidirectional thyristor (diac and triac) that has on-off states with either positive or negative terminal voltages, the asymmetric thyristor that has very short turn-on and turn-off times, the gate turn-off thyristor (GTO), and the light-activated thyristor. Thyristors can cover a wide range of applications from low-frequency high-current power supplies to high-frequency, low-power applications, including lighting controls, home appliances, and industrial equipment.

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PROBLEMS (* INDICATES DIFFICULT PROBLEMS)

FOR SECTION 5.2 STATIC CHARACTERISTICS OF BIPOLAR TRANSISTOR

1. An n-p-n transistor has a base transport factor α_T of 0.998, an emitter efficiency of 0.997, and an I_{Cp} of 10 nA. (a) Calculate α_0 and β_0 for the device. (b) If $I_B = 0$, what is the emitter current?
2. Given that an ideal transistor has an emitter efficiency of 0.999 and the collector-base leakage current is 10 μA, calculate the active region emitter current due to holes if $I_B = 0$.

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3. A silicon p-n-p transistor has impurity concentrations of 5×10^{18}, 2×10^{17}, and 10^{16} cm^{-3} in the emitter, base, and collector, respectively. The base width is 1.0 μm, and the device cross-sectional area is 0.2 mm^2. When the emitter-base junction is forward biased to 0.5 V and the base-collector junction is reverse biased to 5 V, calculate (a) the neutral base width and (b) the minority carrier concentration at the emitter-base junction.

4. For the transistor in Prob. 3, the diffusion constants of minority carriers in the emitter, base, and collector are 52, 40, and 115 cm^2/s, respectively; and the corresponding lifetimes are 10^{-8}, 10^{-7}, and 10^{-6} s. Find the current components I_{Ep}, I_{Cp}, I_{En}, I_{Cn}, and I_{BB} illustrated in Fig. 5.

5. Using the results obtained from Prob. 3 and 4, (a) find the terminal currents I_E, I_C, and I_B of the transistor; (b) calculate emitter efficiency, base transport factor, common-base current gain, and common-emitter current gain; and (c) comment on how the emitter efficiency and base transport factor can be improved.

6. Referring to the minority carrier concentration shown in Eq. 14, sketch $p_n(x)/p_n(0)$ curves as a function of x with different W/L_p. Show that the distribution will approach a straight line when W/L_p is small enough (say $W/L_p < 0.1$).

*7. For a transistor under the active mode of operation, use Eq. 14 to find the exact solutions of I_{Ep} and I_{Cp}.

8. Derive the expression for total excess minority-carrier charge Q_B, if the transistor is operated under the active mode and $p_n(0) >> p_{no}$. Explain how the charge can be approximated by the triangle area in the base shown in Fig. 6. In addition, using the parameters in Prob. 3, find Q_B.

9. Using Q_B derived from Prob. 8, show that the collector current expressed in Eq. 27 can be approximated by $I_C \cong (2D_p/W^2)Q_B$.

10. Show that the base transport factor α_T can be simplified to $1 - (W^2/2L_p^2)$.

11. If the emitter efficiency is very close to unity, show that the common-emitter current gain β_0 can be given by $2L_p^2/W^2$. (Hint: Use α_T in Prob. 10.)

12. For a p^+-n-p transistor with high emitter efficiency, find the common-emitter current gain β_0. If the base width is 2 μm and the diffusion constant of minority carrier in the base region is 100 cm^2/s, assume that the lifetime of the carrier in the base region is 3×10^{-7} s. (Hint: Refer to β_0 derived in Prob. 11.)

13. A silicon n-p-n bipolar transistor has impurity concentrations of 3×10^{18}, 2×10^{16}, and 5×10^{15} cm^{-3} in the emitter, base, and collector, respectively. Determine the diffusion constants of minority carrier in the three regions by using Einstein's relationship, $D = (kT/q)\mu$. Assume that he mobilities of electrons and holes, μ_n and μ_p, can be expressed as

$$\mu_n = 88 + \frac{1252}{\left(1 + 0.698 \times 10^{-17} N\right)} \text{ and } \mu_p = 54.3 + \frac{407}{\left(1 + 0.374 \times 10^{-17} N\right)} \text{ at } T = 300 \text{ K}.$$

*14. Using the results obtained from Prob. 13, determine the current components in each region with V_{BE} = 0.6V (operated under active mode). The device cross-sectional area is 0.01 mm^2 and the neutral-base width is 0.5 μm. Assume the minority-carrier lifetime in each region is the same and equals to 10^{-6} s.

15. Based on the results obtain from Prob. 14, find the emitter efficiency, base transport factor, common-base current gain, and common-emitter current gain.

16. For an ion implanted n-p-n transistor the net impurity doping in the neutral base is given by $N(x) = N_{AO}e^{-x/l}$, where $N_{AO} = 2 \times 10^{18}$ cm^{-3} and $l = 0.3$ μm. (a) Find the total number of impurities in the neutral-base region per unit area (b) Find the average impurity concentration in the neutral-base region for a neutral-base width of 0.8 μm.

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17. Referring to Problem 16, if $L_E = 1\mu m$, $N_E = 10^{19}\,cm^{-3}$, $D_E = 1\,cm^2/s$, the average lifetime is 10^{-6} s in the base, and the average diffusion coefficient in the base corresponds to the impurity concentration in Prob. 16, find the common-emitter current gain.

18. Estimate the collector current level for the transistor in Probs. 16 and 17 that has an emitter area of $10^{-4}\,cm^2$. The base resistance of the transistor can be expressed as $10^{-3}\,\overline{\rho}_B/W$, where W is the neutral-base width and $\overline{\rho}_B$ is the average base resistivity.

*19. Plot the common-emitter current gain as a function of the base current I_B from 0 to 25 μA at a fixed V_{EC} of 5 V for the transistor shown in Fig. 10*b*. Explain why the current gain is not a constant.

20. The general equations of the emitter and collector currents for the basic Ebers-Moll model [J. J. Ebers and J. L. Moll, "Large-Single Behavior of Junction Transistors," *Proc.IRE.*, **42**,1761(1954)] are

$$I_E = I_{FO}\left(e^{qV_{EB}/kT} - 1\right) - \alpha_R I_{RO}\left(e^{qV_{CB}/kT} - 1\right),$$

$$I_C = \alpha_F I_{FO}\left(e^{qV_{EB}/kT} - 1\right) - I_{RO}\left(e^{qV_{CB}/kT} - 1\right),$$

where α_F and α_R are the *forward common-base current gain* and the *reverse common-base current gain*, respectively. I_{FO} and I_{RO} are the saturation currents of the normally forward- and reverse-biased diodes, respectively. Find α_F and α_R in terms of the constants in Eqs. 25, 26, 28, and 29.

*21. Referring to the transistor in Example 2, find α_F, α_R, I_{FO}, and I_{RO} by using the equations derived in Problem 20.

22. Derive Eq. 32*b* for the collector current starting with the field-free steady-state continuity equation. (Hint: Consider the minority carrier distribution in the collector region.)

FOR SECTION 5.3 FREQUENCY RESPONSE AND SWITCHING OF BIPOLAR TRANSISTOR

23. A Si transistor has D_p of 10 cm^2/s and W of 0.5 μm. Find the cutoff frequencies for the transistor with a common-base current gain α_0 of 0.998. Neglect the emitter and collector delays.

24. If we want to design a bipolar transistor with 5 GHz cutoff frequency f_T, what the neutral base width W will be? Assume D_p is 10 cm^2/s and neglect the emitter and collector delays.

FOR SECTION 5.4 THE HETEROJUNCTION BIPOLAR TRANSISTOR

25. Consider a $Si_{1-x}Ge_x$/Si HBT with $x = 10\%$ in the base region (and 0% in emitter and collector region). The bandgap of the base region is 9.8% smaller than that of Si. If the base current is due to emitter injection efficiency only, what is the expected change in the common-emitter current gain between 0° and 100°C?

26. For an $Al_xGa_{1-x}As$/GaAs HBT, the bandgap of the $Al_xGa_{1-x}As$ is a function of x and can be expressed as $1.424 + 1.247x$eV (when $x \le 0.45$) and $1.9 + 0.125x + 0.143x^2$ eV (when $0.45 < x \le 1$). Plot β_0(HBT)/β_0(BJT) as a function of x.

FOR SECTION 5.5 THE THYRISTOR AND RELATED POWER DEVICES

27. For the doping profile shown in Fig. 22, find the width W (> 10μm) of the $n1$-region so that the thyristor has a reverse blocking voltage of 120 V. If the current gain α_2 for the $n1$-$p2$-$n2$ transistor is 0.4 independent of current, and α_1 of the $p1$-$n1$-$p2$ transistor can be expressed as $0.5\sqrt{L_p/W}\,\ln(J/J_0)$, where L_p is 25 μm and J_0 is 5×10^{-6} A/cm^2, find the cross-sectional area of the thyristor that will switch at a current I_S of 1 mA.

28. For a GTO thyristor shown in Fig. 32, find the minimum gate current I_g to turn off the thyristor. Assume the current gains of the $p1$-$n1$-$p2$ and $n1$-$p2$-$n2$ transistors are α_1 and α_2, respectively.

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CHAPTER 6

MOSFET and Related Devices

- 6.1 THE MOS DIODE
- 6.2 MOSFET FUNDAMENTALS
- 6.3 MOSFET SCALING
- 6.4 CMOS AND BiCMOS
- 6.5 MOSFET ON INSULATOR
- 6.6 MOS MEMORY STRUCTURES
- 6.7 THE POWER MOSFET
- SUMMARY

The metal-oxide-semiconductor field-effect transistor (MOSFET) is composed of an MOS diode and two p–n junctions placed immediately adjacent to the MOS diode. Since its first demonstration in 1960, the MOSFET has developed quickly and has become the most important device for advanced integrated circuits such as microprocessors and semiconductor memories. This is because the MOSFET consumes very low power and has a high yield of working devices. Of particular importance is the fact that MOSFET can be readily scaled down and will take up less space than a bipolar transistor using the same design rule.

Specifically, we cover the following topics:

- The inversion condition and threshold voltage of an MOS diode.
- Basic characteristics of a MOSFET.
- MOSFET scaling and its associated short-channel effects.
- Low-power complementary MOS (CMOS) logic structure.
- MOS memory structures.
- Related devices including charge-coupled device, silicon-on-insulator device, and power MOSFET.

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6.1 THE MOS DIODE

The MOS diode is of paramount important in semiconductor device physics because the device is extremely useful in the study of semiconductor surfaces.[§] In practical application, the MOS diode is the heart of the MOSFET—the most important device for advanced integrated circuits. The MOS diode can also be used as a storage capacitor in integrated circuits and it forms the basic building block for charge-coupled devices (CCD). In this section we consider its characteristics in the ideal case, then we extend our consideration to include the effect of metal-semiconductor work-function difference, interface traps, and oxide charges.[1]

6.1.1 The Ideal MOS Diode

A perspective view of an MOS diode is shown in Fig. 1*a*. The cross section of the device is shown in Fig.1*b*, where d is the thickness of the oxide and V is the applied voltage on the metal field plate. Throughout this section we use the convention that the voltage V is positive when the metal plate is positively biased with respect to the ohmic contact and V is negative when the metal plate is negatively biased with respect to the ohmic contact.

The energy band diagram of an ideal p-type semiconductor MOS at $V = 0$ is shown in Fig. 2. The work function is the energy difference between the Fermi level and the vacuum level (i.e., $q\phi_m$ for the metal and $q\phi_s$ for the semiconductor). Also shown are the electron affinity $q\chi$, which is the energy difference between the conduction band edge and the vacuum level in the semiconductor, and $q\psi_B$, which is the energy difference between the Fermi level E_F and the intrinsic Fermi level E_i.

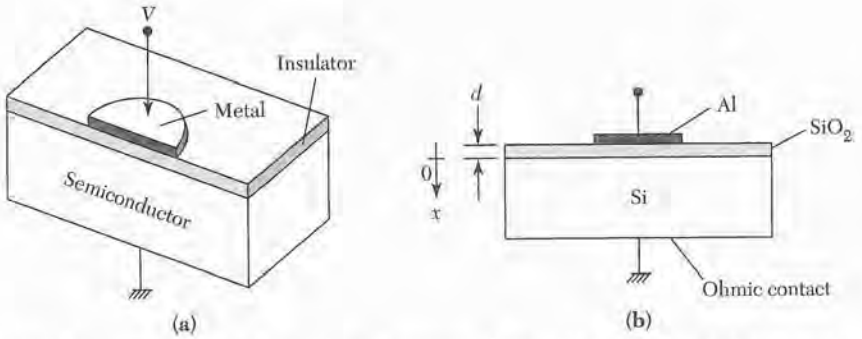


Fig. 1 (*a*) Perspective view of a metal-oxide-semiconductor (MOS) diode. (*b*) Cross-section of an MOS diode.

[§] A more general class of device is the metal-insulator-semiconductor (MIS) diode. However, because in most experimental studies the insulator has been silicon dioxide, in this text the term MOS diode is used interchangeably with MIS diode.

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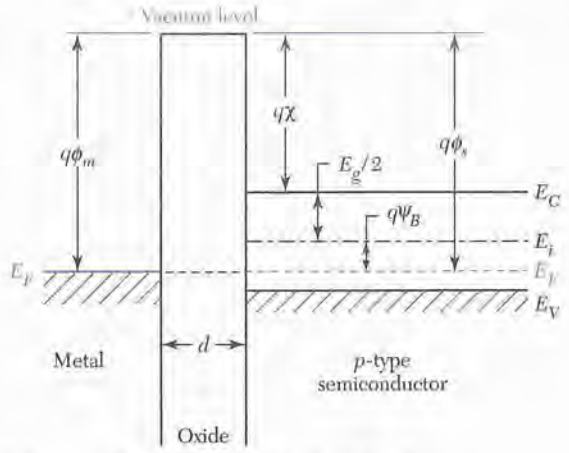


Fig. 2 Energy band diagram of an ideal MOS diode at $V = 0$.

An ideal MOS is defined as follows. (a) At zero applied bias, the energy difference between the metal work function $q\phi_m$ and the semiconductor work function $q\phi_s$ is zero, or the work function difference $q\phi_{ms}$ is zero.[§]

$$q\phi_{ms} \equiv \left(q\phi_m - q\phi_s\right) = q\phi_m - \left(q\chi + \frac{E_g}{2} + q\psi_B\right) = 0, \tag{1}$$

where the sum of the three items in the brackets equals to $q\phi_s$. In other words, the energy band is flat (flat-band condition) when there is no applied voltage. (b) The only charges that exist in the diode under any biasing conditions are those in the semiconductor and those with equal but opposite sign on the metal surface adjacent to the oxide. (c) There is no carrier transport through the oxide under direct current (dc)–biasing conditions, or the resistivity of the oxide is infinite. This ideal MOS diode theory serves as a foundation for understanding practical MOS devices.

When an ideal MOS diode is biased with positive or negative voltages, three cases may exist at the semiconductor surface. For the case of p-type semiconductor, when a negative voltage ($V < 0$) is applied to the metal plate, excess positive carriers (holes) will be induced at the SiO_2-Si interface. In this case, the bands near the semiconductor surface are bent upward, as shown in Fig. 3*a*. For an ideal MOS diode, no current flows in the device regardless of the value of the applied voltage; thus, the Fermi level in the semiconductor will remain constant. Previously, we determined that the carrier density in the semiconductor depends exponentially on the energy difference $E_i - E_F$, that is,

$$p_p = n_i e^{(E_i - E_F)/kT}. \tag{2}$$

The upward bending of the energy band at the semiconductor surface causes an increase in the energy $E_i - E_F$ there, which in turn gives rise to an enhanced concentration, an accumulation of holes near the oxide-semiconductor interface. This is called the *accumulation* case. The corresponding charge distribution is shown on the right side of Fig. 3*a*,

[§] This is for a p-type semiconductor. For an n-type semiconductor, the term $q\psi_B$ will be replaced by $-q\psi_B$.

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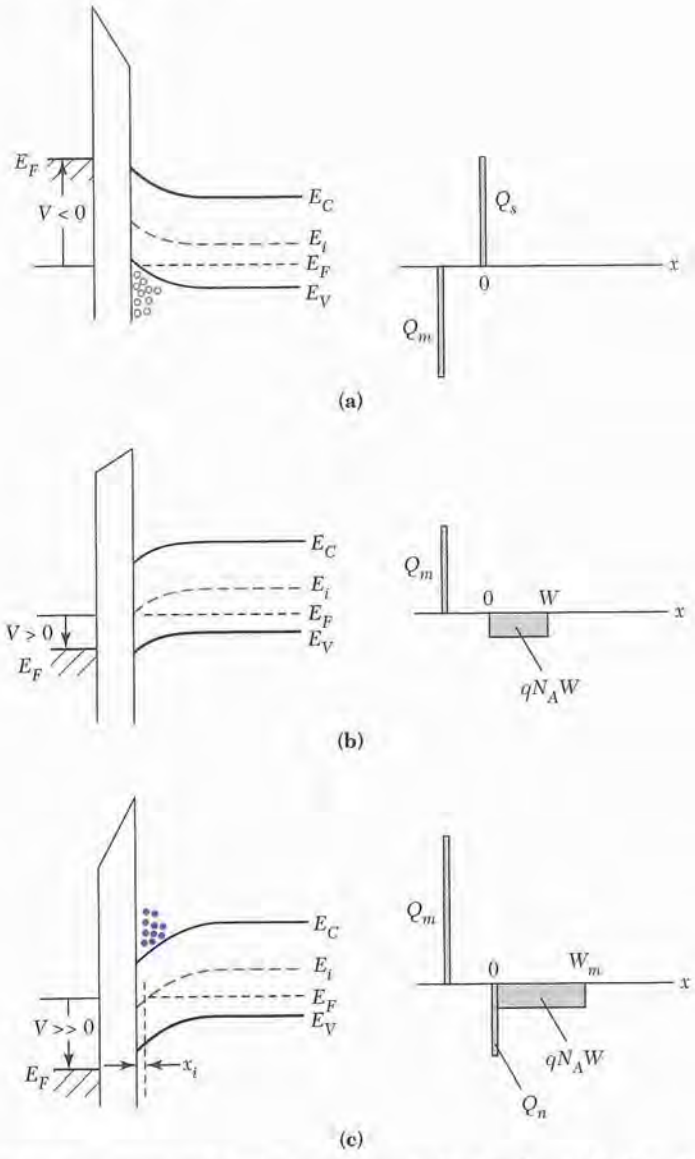


Fig. 3 Energy band diagrams and charge distributions of an ideal MOS diode in (*a*) accumulation, (*b*) depletion, and (*c*) inversion cases.

where Q_s is the positive charge per unit area in the semiconductor and Q_m is the negative charge per unit area ($|Q_m| = Q_s$) in the metal.

When a small positive voltage ($V > 0$) is applied to an ideal MOS diode, the energy bands near the semiconductor surface are bent downward, and the majority carriers (holes) are depleted (Fig. 3*b*). This is called the *depletion* case. The space charge per unit area, Q_{sc}, in the semiconductor is equal to $-qN_AW$, where W is the width of the surface depletion region.

When a larger positive voltage is applied, the energy bands bend downward even more so that the intrinsic level E_i at the surface crosses over the Fermi level, as shown in Fig. 3*c*. That means, the positive gate voltage starts to induce excess negative carriers

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(electrons) at the SiO_2-Si interface. The electron concentration in the semiconductor depends exponentially on the energy difference $E_F - E_i$ and is given by

$$n_p = n_i e^{(E_F - E_i)/kT}. \tag{3}$$

In the case shown in Fig. 3*c*, $(E_F - E_i) > 0$. Therefore, the electron concentration n_p at the interface is larger than n_i, and the hole concentration given by Eq. 2 is less than n_i. The number of electrons (minority carriers) at the surface is greater than holes (majority carriers); the surface is thus inverted. This is called the *inversion* case.

Initially, the surface is in a *weak inversion* condition since the electron concentration is small. As the bands are bent further, eventually the conduction band edge comes close to the Fermi level. The onset of *strong inversion* occurs when the electron concentration near the SiO_2-Si interface is equal to the substrate doping level. After this point most of the additional negative charges in the semiconductor consist of the charge Q_n (Fig. 3*c*) in a very narrow n-type inversion layer $0 \le x \le x_i$, where x_i is the width of the inversion region. Typically, the value of x_i ranges from 1 to 10 nm and is always much smaller than the surface depletion-layer width.

Once strong inversion occurs, the surface depletion-layer width reaches a maximum. This is because when the bands are bent downward far enough for strong inversion to occur, even a very small increase in band bending corresponding to a very small increase in depletion-layer width results in a large increase in the charge Q_n in the inversion layer. Thus, under a strong inversion condition the charge per unit area Q_s in the semiconductor is the sum of the charge Q_n in the inversion layer and the charge Q_{sc} in the depletion region:

$$Q_s = Q_n + Q_{sc} = Q_n - qN_A W_m, \tag{4}$$

where W_m is the maximum width of the surface depletion region.

The Surface Depletion Region

Figure 4 shows a more detailed band diagram at the surface of a p-type semiconductor. The electrostatic potential ψ is defined as zero in the bulk of the semiconductor. At the semiconductor surface, $\psi = \psi_s$; ψ_s is called the surface potential. We can express electron and hole concentrations in Eqs. 2 and 3 as a function of ψ:

$$n_p = n_i e^{q(\psi - \psi_B)/kT}, \tag{5a}$$

$$p_p = n_i e^{q(\psi_B - \psi)/kT}, \tag{5b}$$

where ψ is positive when the band is bent downward, as shown in Fig. 4. At the surface the densities are

$$n_s = n_i e^{q(\psi_s - \psi_B)/kT}, \tag{6a}$$

$$p_s = n_i e^{q(\psi_B - \psi_s)/kT}. \tag{6b}$$

From this discussion and with the help of Eq. 6, the following regions of surface potential can be distinguished:

$\psi_s < 0$	Accumulation of holes (bands bend upward).
$\psi_s = 0$	Flat-band condition.
$\psi_B > \psi_s > 0$	Depletion of holes (bands bend downward).

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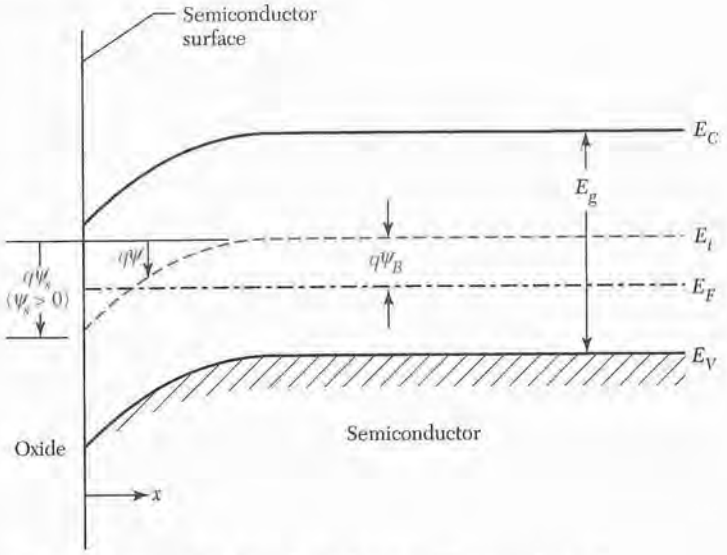


Fig. 4 Energy band diagrams at the surface of a p-type semiconductor.

$\psi_s = \psi_B$	Midgap with $n_s = n_p = n_i$ (intrinsic concentration).
$\psi_s > \psi_B$	Inversion (bands bend downward).

The potential ψ as a function of distance can be obtained by using the one-dimensional Poisson's equation:

$$\frac{d^2\psi}{dx^2} = \frac{-\rho_s(x)}{\varepsilon_s}, \tag{7}$$

where $\rho_s(x)$ is the charge density per unit volume at position x and ε_s is the dielectric permittivity. We use the depletion approximation that we have employed in the study of p–n junctions. When the semiconductor is depleted to a width of W and the charge within the semiconductor is given by $\rho_s = -qN_A$, integration of Poisson's equation gives the electrostatic potential distribution as a function of distance x in the surface depletion region:

$$\psi = \psi_s\left(1 - \frac{x}{W}\right)^2. \tag{8}$$

The surface potential ψ_s is

$$\psi_s = \frac{qN_A W^2}{2\varepsilon_s}. \tag{9}$$

Note that the potential distribution is identical to that for a one-sided n^+-p junction.

The surface is inverted whenever ψ_s is larger than ψ_B. However, we need a criterion for the onset of strong inversion, after which the charges in the inversion layer become significant. A simple criterion is that the electron concentration at the surface is equal to the substrate impurity concentration, i.e., $n_s = N_A$. Since $N_A = n_i e^{q\psi_B/kT}$, from Eq. 6*a* we obtain

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$$\psi_s(inv) \cong 2\psi_B = \frac{2kT}{q}\ln\left(\frac{N_A}{n_i}\right). \tag{10}$$

Equation 10 states that a potential ψ_B is required to bend the energy bands down to the intrinsic condition at the surface ($E_i = E_F$), and bands must then be bent downward by another $q\psi_B$ at the surface to obtain the condition of strong inversion.

As discussed previously, the surface depletion layer reaches a maximum when the surface is strongly inverted. Accordingly, the maximum width of the surface depletion region W_m is given by Eq. 9 in which ψ_s equals $\psi_s(inv)$, or

$$W_m = \sqrt{\frac{2\varepsilon_s\psi_s(inv)}{qN_A}} \cong \sqrt{\frac{2\varepsilon_s(2\psi_B)}{qN_A}} \tag{11}$$

or

$$W_m = 2\sqrt{\frac{\varepsilon_s kT\ln\left(\frac{N_A}{n_i}\right)}{q^2N_A}} \tag{11a}$$

and

$$Q_{sc} = -qN_AW_m \cong -\sqrt{2q\varepsilon_sN_A(2\psi_B)}\,. \tag{12}$$

EXAMPLE 1

For an ideal metal-SiO_2-Si diode having $N_A = 10^{17}$ cm^{-3}, calculate the maximum width of the surface depletion region.

SOLUTION At room temperature $kT/q = 0.026$ V and $n_i = 9.65 \times 10^9$ cm^{-3}, the dielectric permittivity of Si is $11.9 \times 8.85 \times 10^{-14}$ F/cm. From Eq. 11*a*

$$W_m = 2\sqrt{\frac{11.9\times8.85\times10^{-14}\times0.026\ln\left(10^{17}/9.65\times10^9\right)}{1.6\times10^{-19}\times10^{17}}}$$

$$= 10^{-5}\text{ cm} = 0.1\ \mu\text{m}.$$

The relationship between W_m and the impurity concentration is shown in Fig. 5 for silicon and gallium arsenide, where N_B is equal to N_A for p-type and N_D for n-type semiconductors.

Ideal MOS Curves

Figure 6*a* shows the energy band diagram of an ideal MOS diode with band bending identical to that shown in Fig. 4. The charge distribution is shown in Fig. 6*b*. In the absence of any work function differences, the applied voltage will appear partly across the oxide and partly across the semiconductor. Thus,

$$V = V_o + \psi_s, \tag{13}$$

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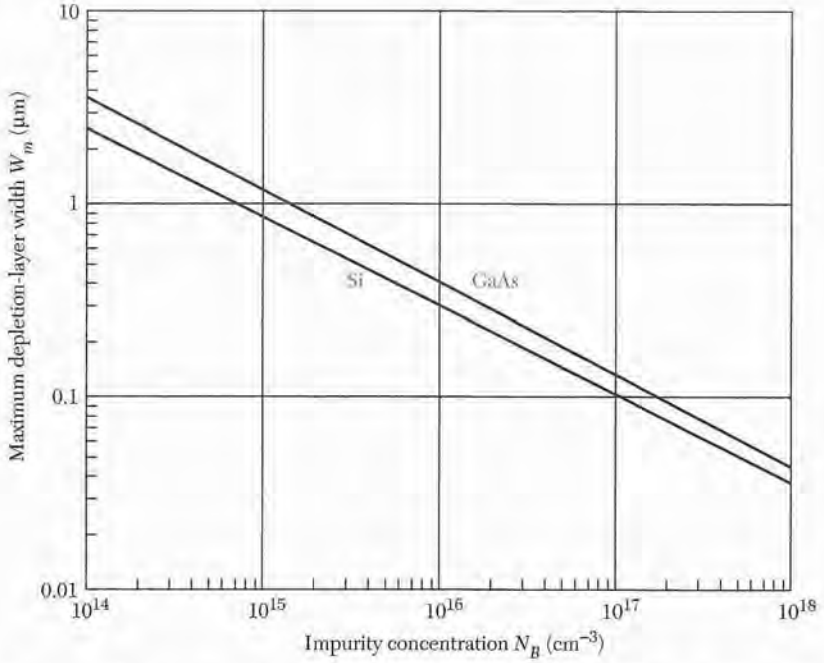


Fig. 5 Maximum depletion-layer width versus impurity concentration of Si and GaAs under strong-inversion condition.

where V_o is the potential across the oxide and is given (Fig. 6*c*) by

$$V_o = \mathscr{E}_o d = \frac{|Q_s|d}{\varepsilon_{ox}} \equiv \frac{|Q_s|}{C_o}, \tag{14}$$

where $\mathscr{E}_o$ is the field in the oxide, Q_s is the charge per unit area in the semiconductor, and C_o ($= \varepsilon_{ox}/d$) is the oxide capacitance per unit area. The corresponding electrostatic potential distribution is shown in Fig. 6*d*.

The total capacitance C of the MOS diode is a series combination (Fig. 7*a*, inset) of the oxide capacitance C_o and the semiconductor depletion-layer capacitance C_j:

$$C = \frac{C_o C_j}{\left(C_o + C_j\right)} \text{ F/cm}^2, \tag{15}$$

where $C_j = \varepsilon_s/W$, the same as for an abrupt p–n junction.

From Eqs. 9, 13, 14, and 15, we can eliminate W and obtain the formula for the capacitance:

$$\frac{C}{C_o} = \frac{1}{\sqrt{1 + \dfrac{2\varepsilon_{ox}^2 V}{qN_A\varepsilon_s d^2}}}, \tag{16}$$

which predicts that the capacitance will decrease with increasing metal-plate voltage while the surface is being depleted. When the applied voltage is negative, there is no depletion region, and we have an accumulation of holes at the semiconductor surface. As a result, the total capacitance is close to the oxide capacitance ε_{ox}/d.

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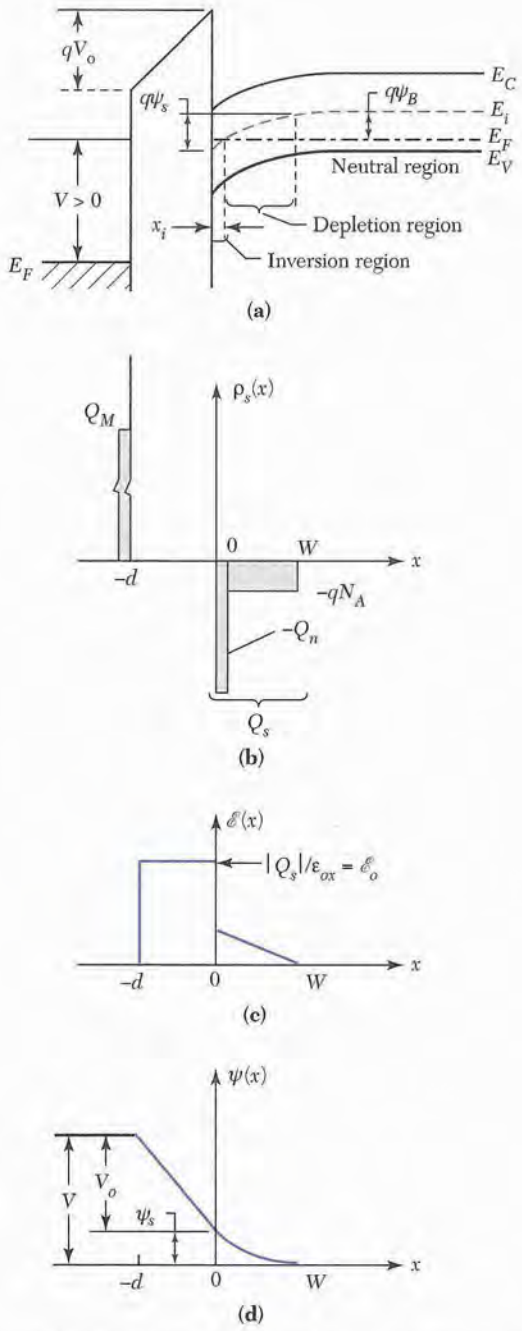


Fig. 6 (*a*) Band diagram of an ideal MOS diode. (*b*) Charge distributions under inversion condition. (*c*) Electric-field distribution. (*d*) Potential distribution.

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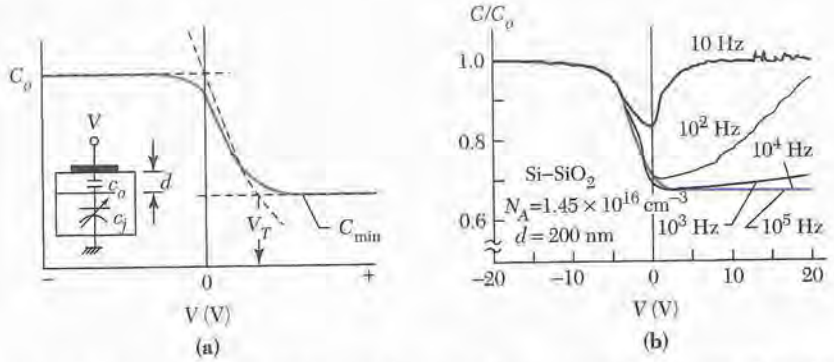


Fig. 7 (*a*) High-frequency MOS *C-V* curve showing its approximated segments (dashed lines). Inset shows the series connection of the capacitors. (*b*) Effect of frequency on the *C-V* curve.[2]

In the other extreme, when strong inversion occurs, the width of the depletion region will not increase with a further increase in applied voltage. This condition takes place at a metal-plate voltage that causes the surface potential ψ_s to reach $\psi_s(inv)$, as given in Eq. 10. Substituting $\psi_s(inv)$ into Eq. 13 and noting that the corresponding charge per unit area is qN_AW_m yields the metal-plate voltage at the onset of strong inversion. This voltage is called the threshold voltage:

$$V_T = \frac{qN_AW_m}{C_o} + \psi_s(inv) \equiv \frac{\sqrt{2\varepsilon_s qN_A(2\psi_B)}}{C_o} + 2\psi_B. \tag{17}$$

Once the strong inversion takes place, the total capacitance will remain at a minimum value given by Eq. 15 with $C_j = \varepsilon_s/W_m$,

$$C_{min} = \frac{\varepsilon_{ox}}{d + \left(\varepsilon_{ox}/\varepsilon_s\right)W_m}. \tag{18}$$

A typical capacitance-voltage characteristics of an ideal MOS diode is shown in Fig. 7*a* based on both the depletion approximation (Eqs. 16–18) and exact calculations (solid curve). Note the close correlation between the depletion approximation and the exact calculations.

Although we have considered only the *p*-type substrate, all of the considerations are equally valid for an *n*-type substrate with the proper changes in signs and symbols (e.g., Q_p for Q_n). The capacitance-voltage characteristics will have identical shapes but will be mirror images of each other, and the threshold voltage is a negative quantity for an ideal MOS diode on an *n*-type substrate.

In Fig. 7*a* we assumed that when the voltage on the metal plate changes, all the incremental charge appears at the edge of the depletion region. Indeed, this happens when the measurement frequency is high. If, however, the measurement frequency is low enough so that generation-recombination rates in the surface depletion region are equal to or faster than the voltage variation, then the electron concentration (minority carrier) can follow the alternating current (ac) signal and lead to charge exchange with the inversion layer in step with the measurement signal. As a result the capacitance in strong inver-

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sion will be that of the oxide layer alone, C_o. Figure 7*b* shows the measured MOS *C-V* curves at different frequencies.[2] Note that the onset of the low-frequency curves occurs at $f \leq 100$ Hz.

EXAMPLE 2

For an ideal metal-SiO_2-Si diode having $N_A = 10^{17}$ cm^{-3} and $d = 5$ nm, calculate the minimum capacitance of the *C-V* curve in Fig. 7*a*. The relative dielectric constant of SiO_2 is 3.9.

SOLUTION

$$C_o = \frac{\varepsilon_{ox}}{d} = \frac{3.9 \times 8.85 \times 10^{-14}}{5 \times 10^{-7}} = 6.90 \times 10^{-7} \text{ F/cm}^2.$$

$$Q_{sc} = -qN_A W_m = -1.6 \times 10^{-19} \times 10^{17} \times (1 \times 10^{-5}) = -1.6 \times 10^{-7} \text{ C/cm}^2.$$

W_m is obtained in Example 1.

$$\psi_s(inv) \approx 2\psi_B = \frac{2kT}{q} \ln\left(\frac{N_A}{n_i}\right) = 2 \times 0.026 \times \ln\left(\frac{10^{17}}{9.65 \times 10^9}\right) = 0.84 \text{ V}.$$

The minimum capacitance C_{min} at V_T is

$$C_{min} = \frac{\varepsilon_{ox}}{d + (\varepsilon_{ox}/\varepsilon_s) W_m} = \frac{3.9 \times 8.85 \times 10^{-14}}{(5 \times 10^{-7}) + (3.9/11.9)(1 \times 10^{-5})}$$

$$= 9.1 \times 10^{-8} \text{ F/cm}^2.$$

Therefore, C_{min} is about 13% of C_o.

6.1.2 The SiO_2-Si MOS Diode

Of all the MOS diodes, the metal-SiO_2-Si is the most extensively studied. The electrical characteristics of the SiO_2-Si system approach those of the ideal MOS diode. However, for commonly used metal electrodes, the work function difference $q\phi_{ms}$ is generally not zero, and there are various charges inside the oxide or at the SiO_2-Si interface that will, in one way or another, affect the ideal MOS characteristics.

The Work Function Difference

The work function of a semiconductor $q\phi_s$, which is the energy difference between the vacuum level and the Fermi level (Fig. 2), varies with the doping concentration. For a given metal with a fixed work function $q\phi_m$ we expect that the work function difference $q\phi_{ms} \equiv (q\phi_m - q\phi_s)$ will vary depending on the doping of the semiconductor. One of the most common metal electrodes is aluminum, with $q\phi_m = 4.1$ eV. Another material also used extensively is the heavily doped polycrystalline silicon (also called polysilicon). The work function for n^+- and p^+-polysilicon are 4.05 and 5.05 eV, respectively. Figure 8 shows the work function differences for aluminum, n^+-, and p^+-polysilicon on silicon as the doping is varied. It is interesting to note that ϕ_{ms} can vary over a 2 V range depending on the electrode materials and the silicon doping concentration.

To construct the energy band diagram of an MOS diode, we start with an isolated metal and an isolated semiconductor with an oxide layer sandwiched between them (Fig. 9*a*). In this isolated situation, all bands are flat; this is the flat-band condition. At thermal equilibrium, the Fermi level must be a constant and the vacuum level must be

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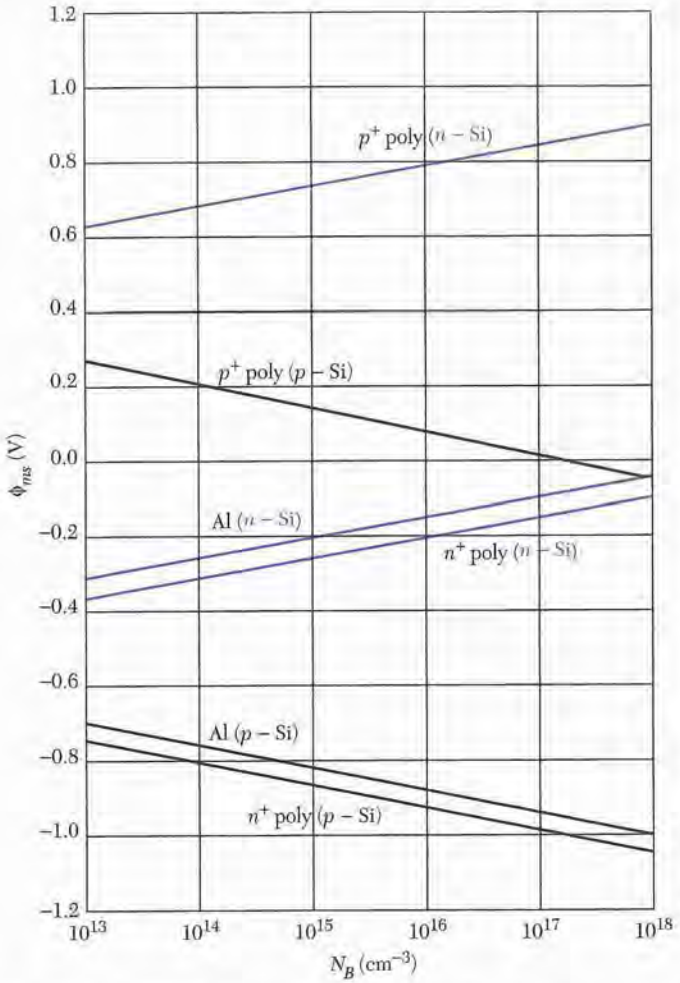


Fig. 8 Work function difference as a function of background impurity concentration for Al, n^+-, and p^+-polysilicon gate materials.

continuous. To accommodate the work function difference, the semiconductor bands bend downward, as shown in Fig. 9*b*. Thus, the metal is positively charged and the semiconductor surface is negatively charged at thermal equilibrium. To achieve the ideal flat-band condition of Fig. 2, we have to apply a voltage equal to the work function difference $q\phi_{ms}$. This corresponds exactly to the situation shown in Fig. 9*a*, where we must apply a negative voltage V_{FB} to the metal, and this voltage is called the flat-band voltage ($V_{FB} = \phi_{ms}$).

Interface Traps and Oxide Charges

In addition to the work function difference, the MOS diode is affected by charges in the oxide and traps at the SiO_2-Si interface. The basic classification of these traps and charges are shown in Fig. 10. They are the interface-trapped charge, fixed-oxide charge, oxide-trapped charge, and mobile ionic charge.[3]

Interface-trapped charges Q_{it} are due to the SiO_2-Si interface properties and dependent on the chemical composition of this interface. The traps are located at the SiO_2-Si

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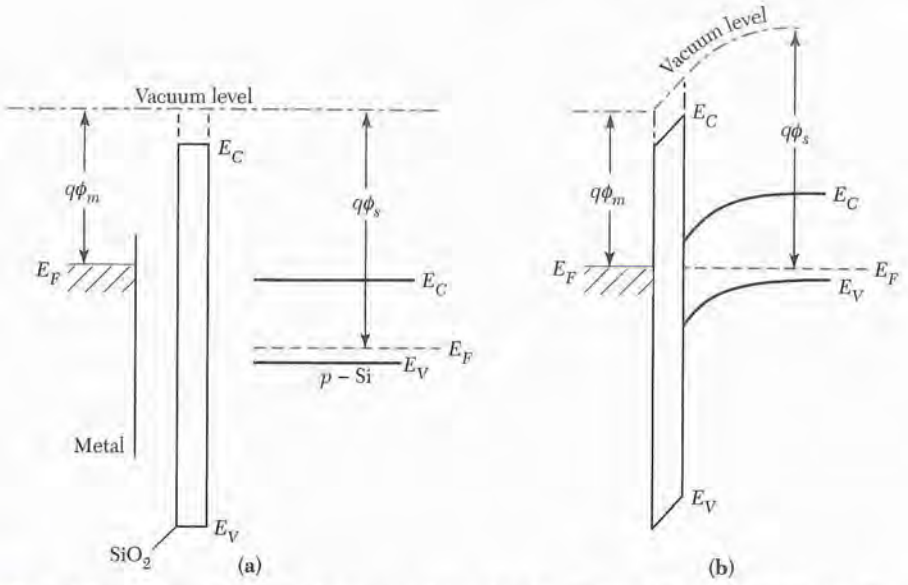


Fig. 9 (*a*) Energy band diagram of an isolated metal and an isolated semiconductor with an oxide layer between them. (*b*) Energy band diagram of an MOS diode in thermal equilibrium.

interface with energy states in the silicon forbidden bandgap. The interface trap density, i.e., number of interface traps per unit area and per eV, is orientation dependent. In ⟨100⟩ orientation, the interface trap density is about an order of magnitude smaller than that in ⟨111⟩ orientation. Present-day MOS diodes with thermally grown silicon dioxide on silicon have most of the interface-trapped charges passivated by low-temperature (450°C) hydrogen annealing. The value of Q_{it}/q for ⟨100⟩-oriented silicon can be as low as $10^{10}\,cm^{-2}$,

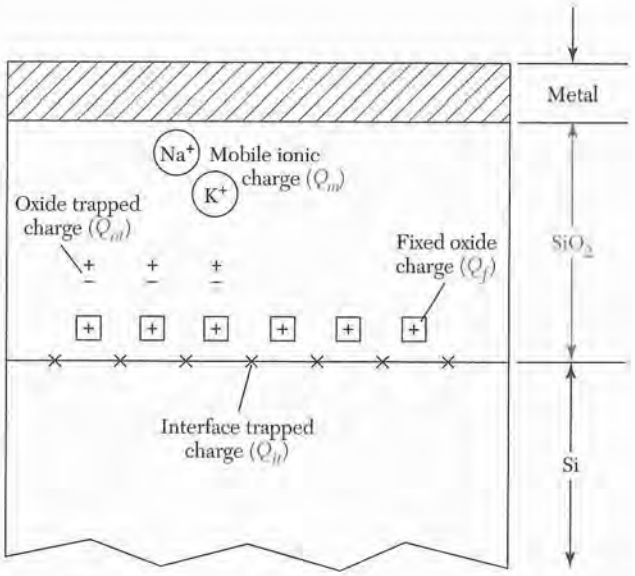


Fig. 10 Terminology for the charges associated with thermally oxidized silicon.[3]

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which amounts to about one interface-trapped charge per 10^5 surface atoms. For <111>-oriented silicon, Q_{it}/q is about $10^{11}\,cm^{-2}$.

The fixed charge Q_f is located within approximately 3 nm of the SiO_2-Si interface. This charge is fixed and cannot be charged or discharged over a wide variation of surface potential ψ_s. Generally, Q_f is positive and depends on oxidation and annealing conditions and on silicon orientation. It has been suggested that when the oxidation is stopped, some ionic silicon is left near the interface. These ions, along with uncompleted silicon bonds (e.g., Si-Si or Si-O bonds) at the surface, may result in the positive interface charge Q_f · Q_f can be regarded as a charge sheet located at the SiO_2-Si interface. Typical fixed-oxide charge densities for a carefully treated SiO_2-Si interface system are about $10^{10}\,cm^{-2}$ for a ⟨100⟩ surface and about $5 \times 10^{10}\,cm^{-2}$ for a ⟨111⟩ surface. Because of the lower values of Q_{it} and Q_f, the ⟨100⟩ orientation is preferred for silicon MOSFETs.

The oxide-trapped charges Q_{ot} are associated with defects in the silicon dioxide. These charges can be created, for example, by X-ray radiation or high-energy electron bombardment. The traps are distributed inside the oxide layer. Most of process-related Q_{ot} can be removed by low-temperature annealing.

The mobile ionic charges Q_m, such as sodium or other alkali ions, are mobile within the oxide under raised-temperature (e.g., >100°C) and high-electric field operations. Trace contamination by alkali metal ions may cause stability problems in semiconductor devices operated under high-bias and high-temperature conditions. Under these conditions mobile ionic charges can move back and forth through the oxide layer and cause shifts of the C-V curves along the voltage axis. Therefore, special attention must be paid to the elimination of mobile ions in device fabrication.

The charges are the effective net charges per unit area in C/cm^2. We now evaluate the influence of these charges on the flat-band voltage. Consider a positive sheet charge per unit area, Q_o, within the oxide, as shown in Fig. 11. This positive sheet charge will

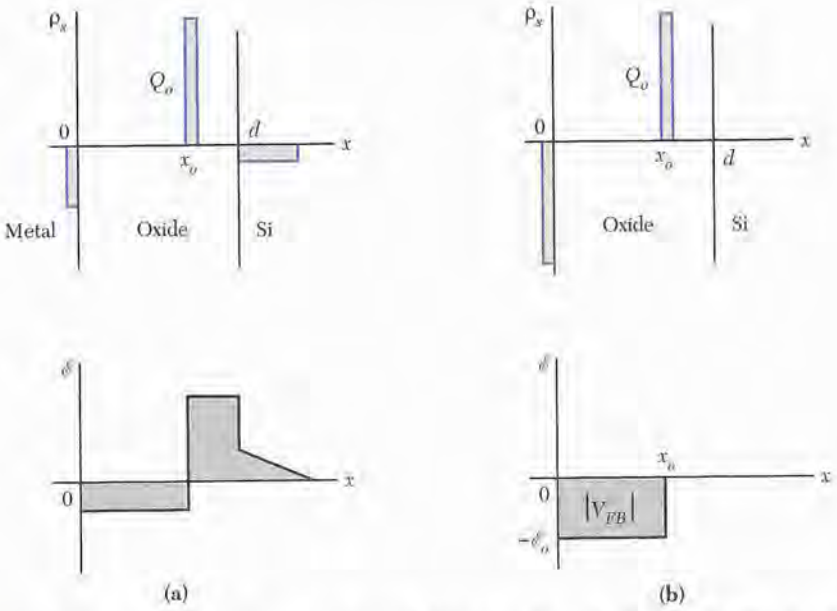


Fig. 11 Effect of a sheet charge within the oxide.[2] (*a*) Condition for $V_G = 0$. (*b*) Flat-band condition.

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induce negative charges partly in the metal and partly in the semiconductor as shown in the upper part of Fig. 11*a*. The resulting field distribution, obtained from integrating Poisson's equation once, is shown in the lower part of Fig. 11*a* where we have assumed that there is no work function difference, or $q\phi_{ms} = 0$.

To reach the flat-band condition (i.e., no charge induced in the semiconductor), we must apply a negative voltage to the metal, as shown in Fig. 11*b*. As the negative voltage increases, more negative charges are put on the metal and thereby the electric-field distribution shift downward until the electric field at the semiconductor surface is zero. Under this condition the area contained under the electric-field distribution corresponds to the flat-band voltage V_{FB}:

$$V_{FB} = -\mathscr{E}_o x_o = -\frac{Q_o}{\varepsilon_{ox}} x_o = -\frac{Q_o}{C_o}\frac{x_o}{d}. \tag{19}$$

The flat-band voltage is, thus, dependent on both the density of the sheet charge Q_o and its location x_o within the oxide. When the sheet charge is located very close to the metal—that is , if $x_o = 0$—it will induce no charges in the silicon and therefore have no effect on the flat-band voltage. On the other hand, when Q_o is located very close to the semiconductor—$x_o = d$, such as the fixed-oxide charge Q_f, it will exert its maximum influence and give rise to a flat-band voltage

$$V_{FB} = -\frac{Q_o}{C_o}\frac{d}{d} = -\frac{Q_o}{C_o}. \tag{20}$$

For the more general case of an arbitrary space charge distribution within the oxide, the flat-band voltage is given by

$$\boxed{V_{FB} = \frac{-1}{C_o}\left[\frac{1}{d}\int_o^d x\rho(x)dx\right]}, \tag{21}$$

where $\rho(x)$ is the volume charge density in the oxide. Once we know $\rho_{ot}(x)$, the volume charge density for oxide-trapped charges, and $\rho_m(x)$, the volume charge density for mobile ionic charges, we can obtain Q_{ot} and Q_m and their corresponding contribution to the flat-band voltage:

$$Q_{ot} \equiv \frac{1}{d}\int_o^d x\rho_{ot}(x)dx, \tag{22a}$$

$$Q_m \equiv \frac{1}{d}\int_o^d x\rho_m(x)dx. \tag{22b}$$

If the value of the work function difference $q\phi_{ms}$ is not zero and if the value of the interface-trapped charges is negligible, the experimental capacitance-voltage curve will be shifted from the ideal theoretical curve by an amount

$$\boxed{V_{FB} = \phi_{ms} - \frac{\left(Q_f + Q_m + Q_{ot}\right)}{C_o}}, \tag{23}$$

The curve in Fig. 12*a* shows the *C-V* characteristics of an ideal MOS diode. Due to nonzero ϕ_{ms}, Q_f, Q_m, or Q_{ot}, the *C-V* curve will be shifted by an amount given by Eq. 23. The

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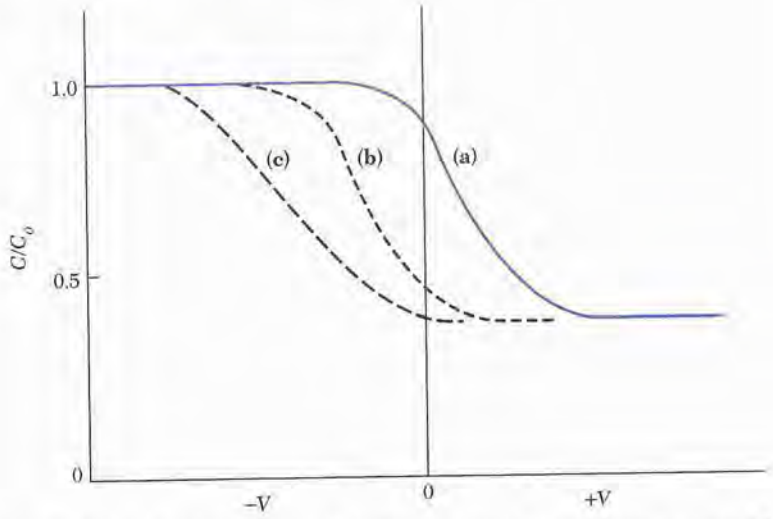


Fig. 12 Effect of a fixed oxide charge and interface traps on the *C*-*V* characteristics of an MOS diode.

parallel shift of the *C*-*V* curve is illustrated in Fig. 12*b*. If, in addition, there are large amounts of interface-trapped charges, the charges in the interface traps will vary with the surface potential. The *C*-*V* curve will be displaced by an amount that itself changes with the surface potential. Therefore, Fig. 12*c* is distorted as well as shifted because of interface-trapped charges.

EXAMPLE 3

Calculate the flat-band voltage for an n^+-polysilicon-SiO_2-Si diode having $N_A = 10^{17}$ cm^{-3} and d = 5 nm. Assume the Q_t and Q_m are negligible in the oxide, and Q_f/q is 5×10^{11} cm^{-2}.

SOLUTION From Fig. 8, ϕ_{ms} is –0.98 V for n^+ polysilicon (p-Si) system with $N_A = 10^{17}$ cm^{-3}. C_o is obtained from Ex. 2.

$$V_{FB} = \phi_{\text{ms}} - \frac{\left(Q_f + Q_m + Q_{ot}\right)}{C_o}$$

$$= -0.98 - \frac{\left(1.6 \times 10^{-19} \times 5 \times 10^{11}\right)}{6.9 \times 10^{-7}} = -1.10 \text{ V}.$$

EXAMPLE 4

Assume that the volume charge density, $\rho_{ot}(x)$, for oxide-trapped charge Q_{ot} in an oxide layer has a triangular distribution. The distribution is described by the function $(10^{18} - 5 \times 10^{23} \times x)$ cm^{-3}, where x is the distance from the location to the metal-oxide interface. The thickness of the oxide layer is 20 nm. Find the change in the flat-band voltage due to Q_{ot}.

SOLUTION From Eqs. 21 and 22*a*,

$$\Delta V_{FB} = \frac{Q_{ot}}{C_o} = \frac{d}{\varepsilon_{ox}} \frac{1}{d} \int_0^{2\times10^{-6}} x\rho_{ot}(x)dx$$

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$$= \frac{1.6\times10^{-19}}{3.9\times8.85\times10^{-14}}\left[\frac{1}{2}\times10^{18}\times\left(2\times10^{-6}\right)^2 - \frac{1}{3}\times5\times10^{23}\times\left(2\times10^{-6}\right)^3\right]$$

$$= \frac{1.6\times10^{-19}\times\left(2\times10^{6} - 1.33\times10^{6}\right)}{3.45\times10^{-13}}$$

$$= 0.31 \text{ V}.$$

6.1.3 The Charge-Coupled Device (CCD)

A schematic view of a CCD is shown[4] in Fig. 13. The basic device consists of a closely spaced array of MOS diodes on a continuous insulator (oxide) layer that covers the semiconductor substrate. A CCD can perform a wide range of electronic functions, including

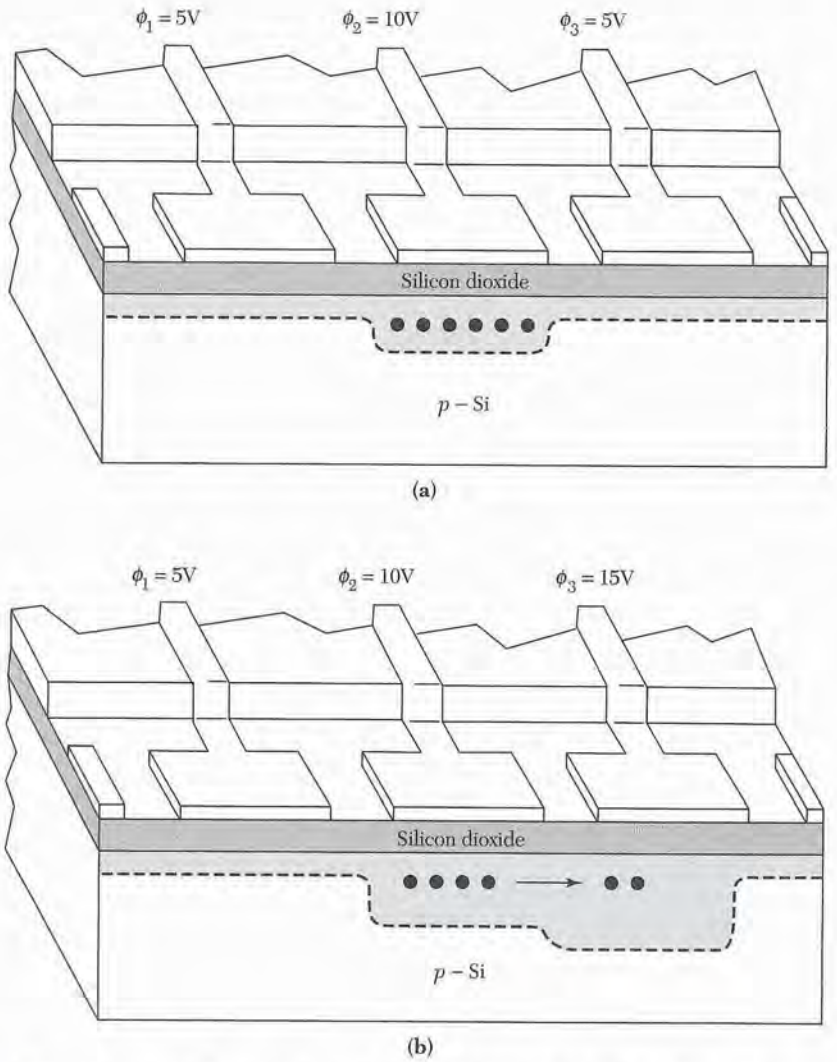


Fig. 13 Cross section of a three-phase charge-coupled device.[4] (*a*) High voltage on ϕ_2. (*b*) ϕ_3 pulsed to a higher voltage for charge transfer.

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image sensing and signal processing. The operating principle of the CCD involves the charge storage and transfer actions controlled by the gate electrodes. Figure 13*a* shows a CCD to which sufficiently large, positive bias pulses have been applied to all the electrodes to produce surface depletion. A slightly higher bias has been applied to the center electrode so that the center MOS structure is under greater depletion and a potential well is formed there, i.e., the potential distribution is shaped like a well because of the larger depletion-layer width under the center electrode. If minority carriers (electrons) are introduced, they will be collected in the potential well. If the potential of the right-hand electrode is increased to exceed that of the central electrode, we obtain the potential distribution shown in Fig. 13*b*. In this case, the minority carriers will be transferred from the central electrode to the right-hand electrode. Subsequently, the potential on the electrodes can be readjusted so that the quiescent storage site is located at the right-hand electrode. By continuing the process, we can transfer the carriers successively along a linear array.

6.2 MOSFET FUNDAMENTALS

The MOSFET has many acronyms, including IGFET (insulating-gate field-effect transistor), MISFET (metal-insulator-semiconductor field-effect transistor), and MOST (metal-oxide-semiconductor transistor). A perspective view for an *n*-channel MOSFET is shown in Fig. 14. It is a four-terminal device and consists of a *p*-type semiconductor substrate in which two n^+ regions, the source and drain, are formed.[§] The metal plate on the oxide is called the gate. Heavily doped polysilicon or a combination of a silicide such as WSi_2 and polysilicon can be used as the gate electrode. The fourth terminal is an ohmic con-

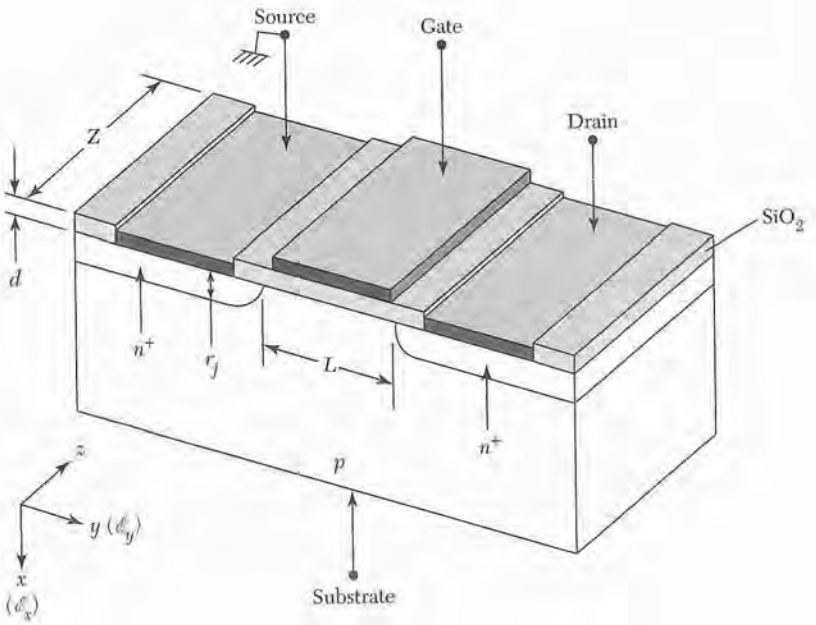


Fig. 14 Perspective view of a metal-oxide-semiconductor field-effect transistor (MOSFET).

[§] For *p*-channel MOSFETs, doping types in substrate and source/drain regions become *n* and p^+, respectively.

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tact to the substrate. The basic device parameters are the channel length L, which is the distance between the two metallurgical n^+-p junctions, the channel width Z, the oxide thickness d, the junction depth r_j, and the substrate doping N_A. Note that the central section of the device corresponds to the MOS diode discussed in Section 6.1.

The first MOSFET was fabricated in 1960 using a thermally oxidized silicon substrate.[5] The device had a channel length of 20 μm and a gate oxide thickness of 100 nm.* Although present-day MOSFETs have been scaled down considerably, the choice of silicon and thermally grown silicon dioxide used in the first MOSFET remains the most important combination. Most of the results in this section are obtained from the Si-SiO_2 system.

6.2.1 Basic Characteristics

The source contact is used as the voltage reference throughout this section. When no voltage is applied to the gate, the source-to-drain electrodes correspond to two p–n junctions connected back to back. The only current that can flow from the source to drain is the reverse-leakage current.† When we apply a sufficiently large positive bias to the gate, the MOS structure is inverted so that a surface inversion layer (or channel) is formed between the two n^+-region. The source and drain are then connected by a conducting surface n-channel through which a large current can flow. The conductance of this channel can be modulated by varying the gate voltage. The substrate contact can be at the reference voltage or is reverse biased with respect to the source; the substrate bias voltage will also affect the channel conductance.

Linear and Saturation Regions

We now present a qualitative discussion of MOSFET operation. Let us consider that a voltage is applied to the gate, causing an inversion at the semiconductor surface (Fig. 15). If a small drain voltage is applied, electrons will flow from the source to the drain (the corresponding current will flow from drain to source) through the conducting channel. Thus, the channel acts as a resistor, and the drain current I_D is proportional to the drain voltage. This is the *linear region*, as indicated by the constant-resistor line in the right-hand diagram of Fig. 15*a*.

When the drain voltage increases, eventually it reaches V_{Dsat}, at which the thickness of the inversion layer x_i near $y = L$ is reduced to zero; this is called the pinch-off point, P (Fig. 15*b*). Beyond the pinch-off point, the drain current remains essentially the same, because for $V_D > V_{Dsat}$, at point P the voltage V_{Dsat} remains the same. Thus, the number of carriers arriving at point P from the source or the current flowing from the drain to the source remains the same. This is the *saturation region*, since I_D is a constant regardless of an increase in the drain voltage. The major change is a decrease of L to the value L' shown in Fig. 15*c*. Carrier injection from P into the drain depletion region is similar to that of carrier injection from an emitter-base junction to the base-collector depletion region of a bipolar transistor.

We now derive the basic MOSFET characteristics under the following ideal conditions. (a) The gate structure corresponds to an ideal MOS diode, as defined in Section 6.1, that is, there are no interface traps, fixed-oxide charges, or work function differences. (b) Only drift current is considered. (c) Carrier mobility in the inversion layer is constant.

* A photograph of the first MOSFET is shown in Fig. 4 of Chapter 1.

† This is true for the n-channel, normally off MOSFET. Other types of MOSFET are discussed in Section 6.2.2.

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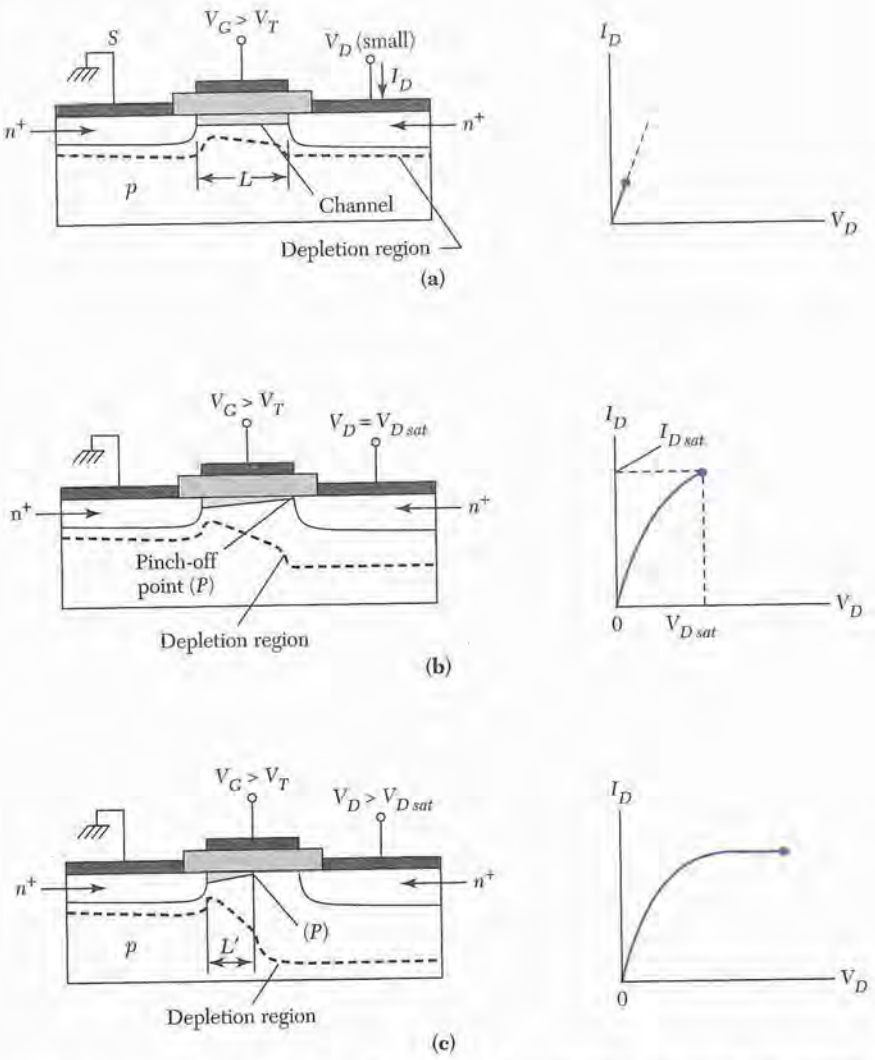


Fig. 15 Operations of the MOSFET and output *I-V* characteristics. (*a*) Low drain voltage. (*b*) Onset of saturation. Point *P* indicates the pinch-off point. (*c*) Beyond saturation.

(d) Doping in the channel is uniform. (e) Reverse-leakage current is negligibly small. (f) The transverse field created by the gate voltage ($\mathscr{E}_x$ in the x-direction, shown in Fig. 14, which is perpendicular to the current flow) in the channel is much larger than the longitudinal field created by the drain voltage ($\mathscr{E}_y$ in the y-direction, which is in parallel to the current flow). The last condition is called the gradual-channel approximation and generally is valid for long-channel MOSFETs. Under this approximation, the charges contained in the surface depletion region of the substrate are induced solely from the field created by the gate voltage.

Figure 16*a* shows MOSFET operated in the linear region. Under the above ideal conditions, the total charge induced in the semiconductor per unit area, Q_s, at a distance y from the source is shown in Fig. 16*b*, which is an enlarged central section of Fig. 16*a*. Q_s is given from Eqs. 13 and 14 by

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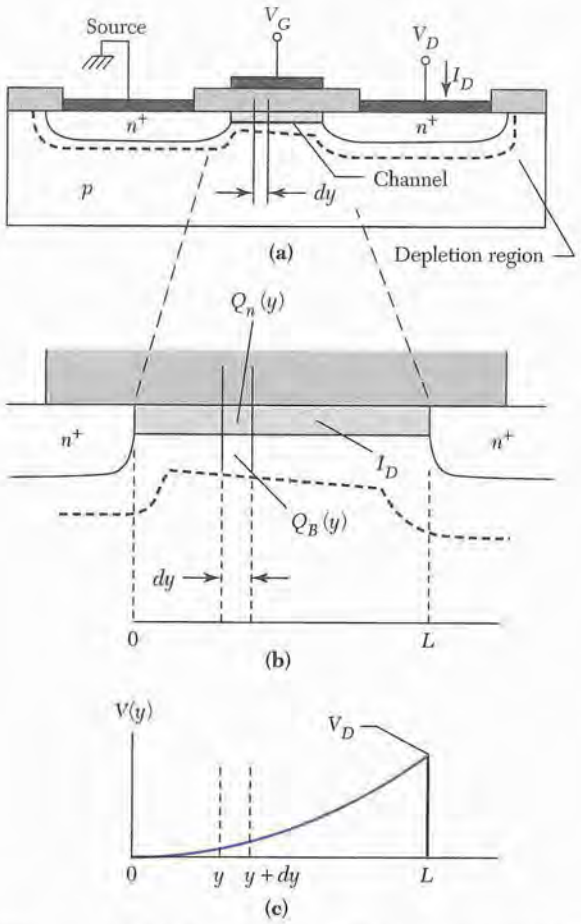


Fig. 16 (*a*) MOSFET operated in the linear region. (*b*) Enlarged view of the channel. (*c*) Drain voltage drop along the channel.

$$Q_s(y) = -\left[V_G - \psi_s(y)\right] C_o, \tag{24}$$

where $\psi_s(y)$ is the surface potential at y and $C_o = \varepsilon_{ox}/d$ is the gate capacitance per unit area. Since Q_s is the sum of the charge in the inversion layer per unit area, Q_n, and the charge in surface depletion region per unit area, Q_{sc}, we can obtain Q_n

$$\begin{aligned} Q_n(y) &= Q_s(y) - Q_{sc}(y), \\ &= -\left[V_G - \psi_s(y)\right] C_o - Q_{sc}(y). \end{aligned} \tag{25}$$

The surface potential $\psi_s(y)$ at inversion can be approximated by $2\psi_B + V(y)$, where $V(y)$ as shown in Fig. 16*c* is the reverse bias between the point y and the source electrode (which is assumed to be grounded). The charge within the surface depletion region $Q_{sc}(y)$ was given previously as

$$Q_{sc}(y) = -qN_A W_m \equiv -\sqrt{2\varepsilon_s q N_A \left[2\psi_B + V(y)\right]}. \tag{26}$$

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Substituting Eq. 26 in Eq. 25 yields

$$Q_n(y) \cong -\left[V_G - V(y) - 2\psi_B \right] C_o + \sqrt{2\varepsilon_s q N_A \left[2\psi_B + V(y) \right]}. \tag{27}$$

The conductivity of the channel at position y can be approximated by

$$\sigma(x) = qn(x)\mu_n(x). \tag{28}$$

For a constant mobility the channel conductance is then given by

$$g = \frac{Z}{L}\int_0^{x_i} \sigma(x)dx = \frac{Z\mu_n}{L}\int_0^{x_i} qn(x)dx. \tag{29}$$

The integral $\int_0^{x_i} qn(x)dx$ corresponds to the total charge per unit area in the inversion layer and is therefore equal to $|Q_n|$, or

$$g = \frac{Z\mu_n}{L}|Q_n|. \tag{30}$$

The channel resistance of an elemental section dy (Fig. 16*b*) is

$$dR = \frac{dy}{gL} = \frac{dy}{Z\mu_n |Q_n(y)|}, \tag{31}$$

and the voltage drop across the elemental section is

$$dV = I_D dR \frac{I_D dy}{Z\mu_n |Q_n(y)|}, \tag{32}$$

where I_D is the drain current, which is independent of y. Substituting Eq. 27 into Eq. 32 and integrating from the source ($y = 0$, $V = 0$) to the drain ($y = L$, $V = V_D$) yield

$$I_D \approx \frac{Z}{L}\mu_n C_o \left\{ \left(V_G - 2\psi_B - \frac{V_D}{2} \right) V_D - \frac{2}{3}\frac{\sqrt{2\varepsilon_s q N_A}}{C_o} \left[\left(V_D + 2\psi_B \right)^{3/2} - \left(2\psi_B \right)^{3/2} \right] \right\}. \tag{33}$$

Figure 17 shows the current-voltage characteristics of an idealized MOSFET based on Eq. 33. For a given V_G, the drain current first increases linearly with drain voltage (the linear region), then gradually levels off, approaching a saturated value (the saturation region). The dashed line indicates the locus of the drain voltage (V_{Dsat}) at which the current reaches a maximum value.

We now consider the linear and saturation regions. For a small V_D, Eq. 33 reduces to

$$\boxed{I_D \cong \frac{Z}{L}\mu_n C_o \left(V_G - V_T \right) V_D \quad \text{for } V_D << (V_G - V_T),} \tag{34}$$

where V_T is the threshold voltage given previously in Eq. 17:

$$\boxed{V_T = \frac{\sqrt{2\varepsilon_s q N_A \left(2\psi_B \right)}}{C_o} + 2\psi_B.} \tag{35}$$

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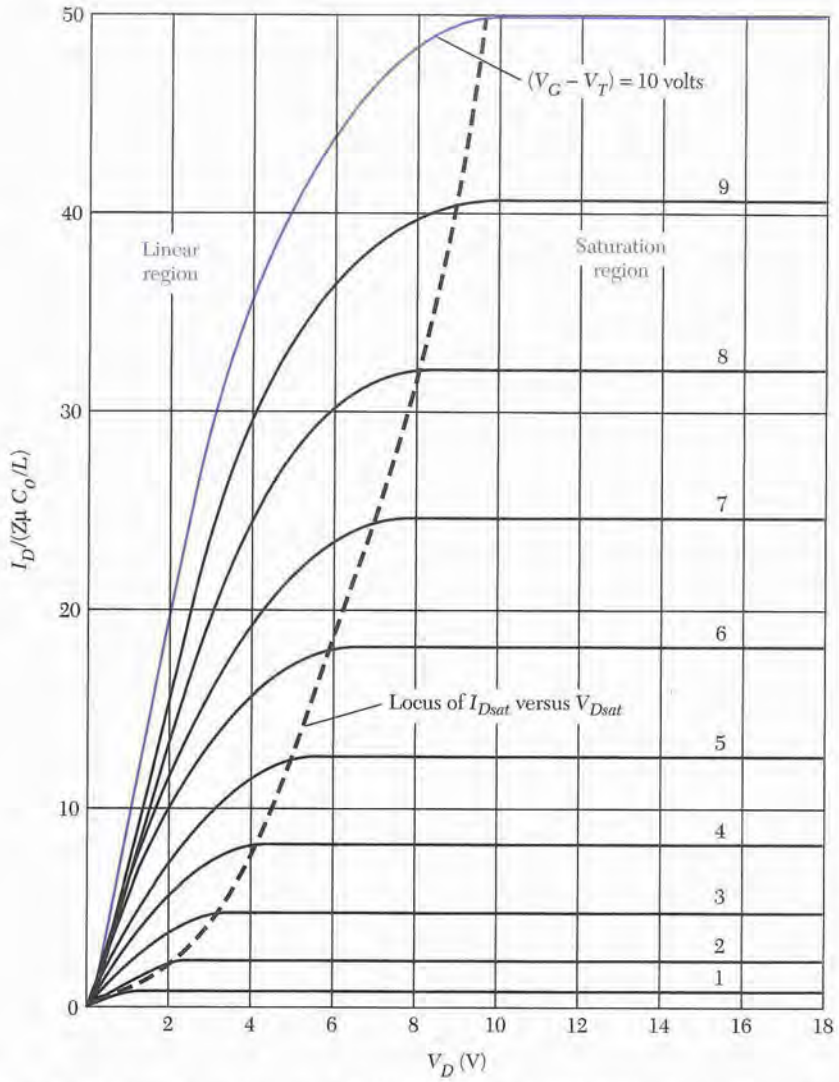


Fig. 17 Idealized drain characteristics of a MOSFET. For $V_D \geq V_{Dsat}$, the drain current remains constant.

By plotting I_D versus V_G (for a given small V_D), the threshold voltage can be deduced from the linearly extrapolated value at the V_G axis. In the linear region, Eq. 34, the channel conductance g_D and the transconductance g_m are given as

$$g_D \equiv \frac{\partial I_D}{\partial V_D}\Big|_{V_G=\text{constant}} \cong \frac{Z}{L}\mu_n C_o\left(V_G - V_T\right), \tag{36}$$

$$g_m \equiv \frac{\partial I_D}{\partial V_G}\Big|_{V_D=\text{constant}} \cong \frac{Z}{L}\mu_n C_o V_D. \tag{37}$$

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When the drain voltage is increased to a point that the charge $Q_n(y)$ in the inversion layer at $y = L$ becomes zero, the number of mobile electrons at the drain are reduced drastically. This point is called pinch-off. The drain voltage and the drain current at this point are designated as V_{Dsat} and I_{Dsat}, respectively. For drain voltages larger than V_{Dsat}, we have the saturation region. We can obtain the value of V_{Dsat} from Eq. 27 under the condition $Q_n(L) = 0$:

$$V_{Dsat} \cong V_G - 2\psi_B + K^2\left(1 - \sqrt{1 + {}^{2V_G}\!/_{K^2}}\right), \tag{38}$$

where $K \equiv \dfrac{\sqrt{\varepsilon_s q N_A}}{C_o}$. The saturation current can be obtained by substituting Eq. 38 into Eq. 33:

$$\boxed{I_{Dsat} \cong \left(\frac{Z\mu_n C_o}{2L}\right)\left(V_G - V_T\right)^2.} \tag{39}$$

The threshold voltage V_T in the saturation region for low substrate doping and thin oxide layers is the same as that from Eq. 35. At higher doping levels, V_T becomes V_G dependent.

For an idealized MOSFET in the saturation region, the channel conductance is zero, and the transconductance can be obtained from Eq. 39:

$$\boxed{g_m \equiv \left.\frac{\partial I_D}{\partial V_G}\right|_{V_D = \text{constant}} = \frac{Z\mu_n \varepsilon_{ox}}{dL}\left(V_G - V_T\right).} \tag{40}$$

EXAMPLE 5

For an n-channel n^+-polysilicon-SiO_2-Si MOSFET with gate oxide = 8 nm, $N_A = 10^{17}$ cm^{-3} and $V_G = 3$V, calculate V_{Dsat}.

SOLUTION

$$C_o = \frac{\varepsilon_{ox}}{d} = \frac{3.9 \times 8.85 \times 10^{-14}}{8 \times 10^{-7}} = 4.32 \times 10^{-7} \text{ F/cm}^2.$$

$$K = \frac{\sqrt{\varepsilon_s q N_A}}{C_o} = \frac{\sqrt{11.9 \times 8.85 \times 10^{-14} \times 1.6 \times 10^{-19} \times 10^{17}}}{4.32 \times 10^{-7}} = 0.3.$$

$2\psi_B = 0.84$ V from Ex. 2.

Therefore, from Eq. 38,

$$V_{Dsat} \cong V_G - 2\psi_B + K^2\left(1 - \sqrt{1 + {}^{2V_G}\!/_{K^2}}\right)$$

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$$= 3 - 0.84 + (0.3)^2\left[1 - \sqrt{1 + \frac{2 \times 3}{(0.3)^2}}\right]$$

$$= 3 - 0.84 - 0.65 = 1.51 \text{ V}.$$

The Subthreshold Region

When the gate voltage is below the threshold voltage and the semiconductor surface is only weakly inverted, the corresponding drain current is called the *subthreshold current*. The subthreshold region is particularly important when the MOSFET is used as a low-voltage, low-power device, such as a switch in digital logic and memory applications, because the subthreshold region describes how the switch turns on and off.

In the subthreshold region, the drain current is dominated by diffusion instead of drift and is derived in the same way as the collector current in a bipolar transistor with homogeneous base doping. If we consider the MOSFET as an *n-p-n* (source-substrate-drain) bipolar transistor, Fig. 16*b*, we have

$$I_D = -qAD_n \frac{\partial n}{\partial y} = qAD_n \frac{n(0) - n(L)}{L}, \tag{41}$$

where A is the channel cross section of the current flow and $n(0)$ and $n(L)$ are the electron densities in the channel at the source and drain, respectively. The electron densities are given by Eq. 5a:

$$n(0) = n_i e^{q(\psi_s - \psi_B)/kT}, \tag{42a}$$

$$n(L) = n_i e^{q(\psi_s - \psi_B - V_D)/kT}, \tag{42b}$$

where ψ_s is the surface potential at the source. Substituting Eq. 42 into Eq. 41 gives

$$I_D = \frac{qAD_n n_i e^{-q\psi_B/kT}}{L}\left(1 - e^{-qV_D/kT}\right) e^{q\psi_s/kT}. \tag{43}$$

The surface potential ψ_s is approximately $V_G - V_T$. Therefore, the drain current will decrease exponentially when V_G becomes less than V_T:

$$\boxed{I_D \sim e^{q(V_G - V_T)/kT}} \tag{44}$$

A typical measured curve for the subthreshold region is shown in Fig. 18. Note the exponential dependence of I_D on $(V_G - V_T)$ for $V_G < V_T$. An important parameter in this region is the *subthreshold swing*, S, which is defined as $[\partial(\log I_D)/\partial V_G]^{-1}$. S is typically 70 ~ 100 mV/decade at room temperature. To reduce the subthreshold current to a negligible value, we must bias the MOSFET a half-volt or more below V_T.

6.2.2 Types of MOSFET

There are basically four types of MOSFETs, depending on the type of inversion layer. If, at zero gate bias, the channel conductance is very low and we must apply a positive voltage to the gate to form the *n*-channel, then the device is a normally off (enhancement)

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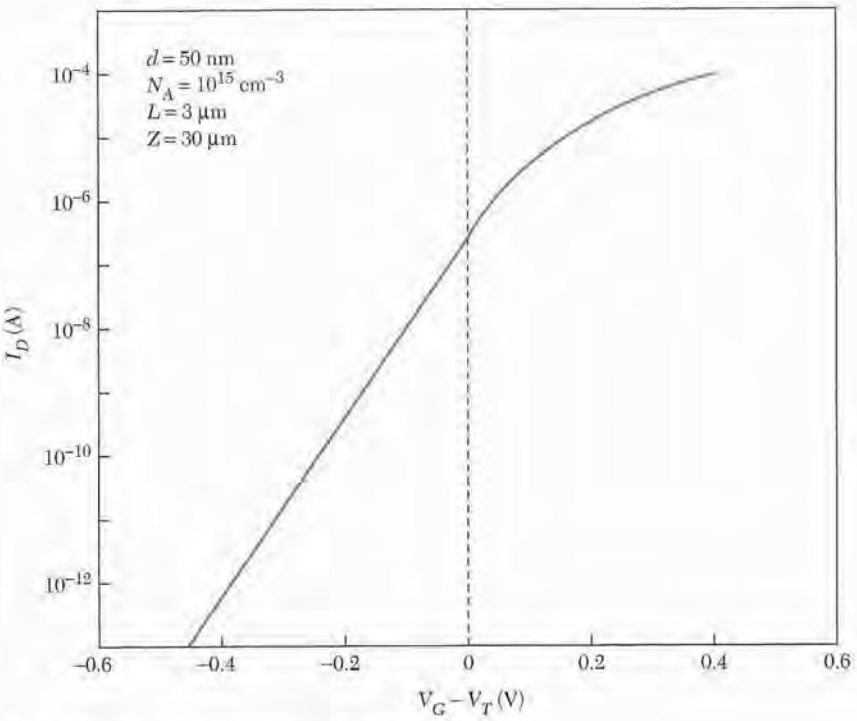


Fig. 18 Subthreshold characteristics of a MOSFET.

n-channel MOSFET. If an n-channel exists at zero bias and we must apply a negative voltage to the gate to deplete carriers in the channel to reduce the channel conductance, then the device is a normally on (depletion) n-channel MOSFET. Similarly, we have the p-channel normally off (enhancement) and normally on (depletion) MOSFETs.

The device cross sections, output characteristics (i.e., I_D versus V_D), and transfer characteristics (i.e., I_D versus V_G) of the four types are shown in Fig. 19. Note that for the normally off n-channel device, a positive gate bias larger than the threshold voltage V_T must be applied before a substantial drain current flows. For the normally on n-channel device, a large current can flow at $V_G = 0$, and the current can be increased or decreased by varying the gate voltage. This discussion can be readily extended to p-channel device by changing polarities.

6.2.3 Threshold Voltage Control

One of the most important parameters of the MOSFET is the threshold voltage. The ideal threshold voltage is given in Eq. 35. However, when we incorporate the effects of the fixed-oxide charge and the difference in work function, there is a flat-band voltage shift. Additionally, substrate bias can also influence the threshold voltage. When a reverse bias is applied between the substrate and the source, the depletion region is widened and the threshold voltage required to achieve inversion must be increased to accommodate the larger Q_{sc}. These factors in turn cause a change in the threshold voltage:

$$V_T \approx V_{FB} + 2\psi_B + \frac{\sqrt{2\varepsilon_s q N_A (2\psi_B + V_{BS})}}{C_o}, \tag{45}$$

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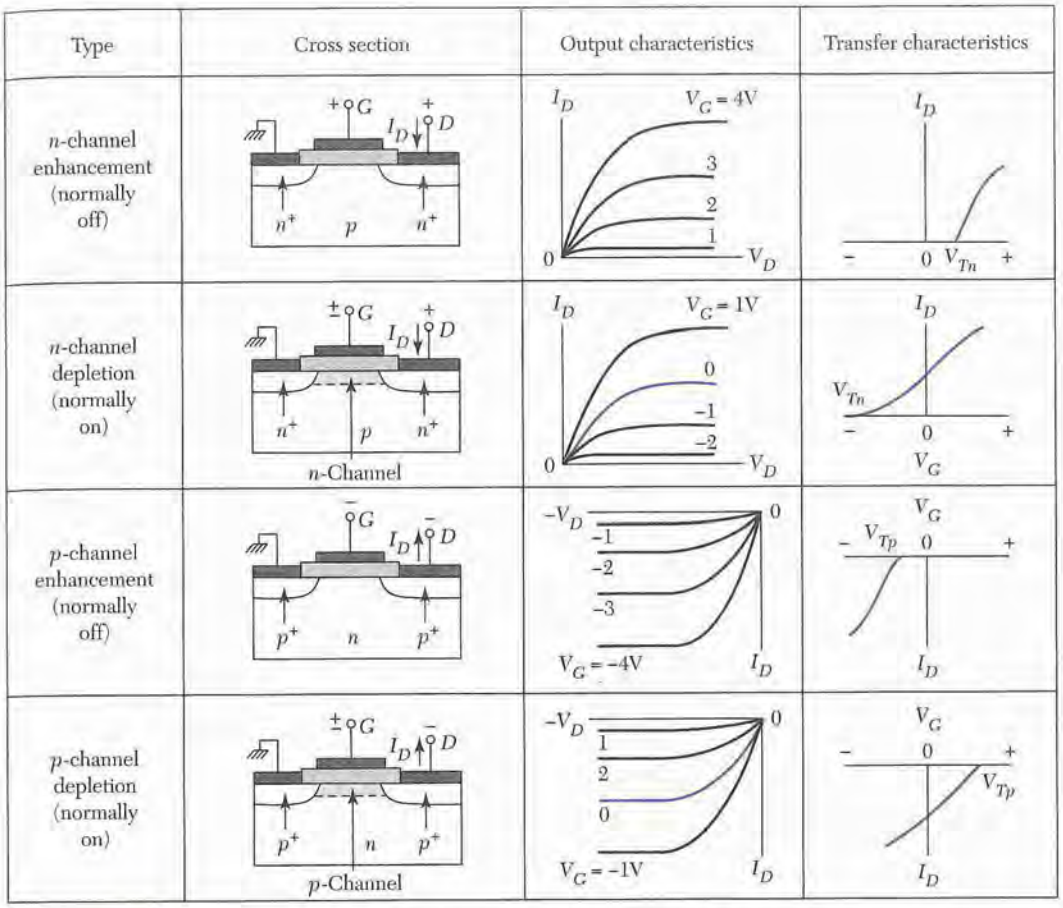


Fig. 19 Cross section, output, and transfer characteristics of four types of MOSFETs.

where V_{BS} is the reverse substrate-source bias. Figure 20 shows the calculated threshold voltage of n-channel (V_{Tn}) and p-channel (V_{Tp}) MOSFETs with n^+-, p^+- polysilicon and mid-gap work function gate electrodes as a function of their substrate doping, assuming d = 5 nm, V_{BS} = 0, and Q_f = 0. Mid-gap gate materials are those with a work function of 4.61 eV, which equals the sum of the electron affinity $q\chi$ and $E_g/2$ of silicon (see Fig. 2).

Precise control of the threshold voltage of MOSFETs in an integrated circuit is essential for reliable circuit operation. Typically, the threshold voltage is adjusted through ion implantation into the channel region. For example, a boron implantation through a surface oxide is often used to adjust the threshold voltage of an n-channel MOSFET (with p-type substrate). Using this method, it is possible to obtain close control of threshold voltage because very precise quantities of impurity can be introduced. The negatively charged boron acceptors increase the doping level of the channel. As a result, V_T increases. Similarly, a shallow boron implant into a p-channel MOSFET can reduce the absolute value of V_T.

EXAMPLE 6

For an n-channel n^+-polysilicon-SiO_2-Si MOSFET with $N_A = 10^{17}$ cm^{-3} and $Q_f/q = 5 \times 10^{11}$ cm^{-2}, calculate V_T for a gate oxide of 5 nm. What is the boron ion dose required to increase V_T to 0.6 V? Assume that the implanted acceptors form a sheet of negative charge at the Si-SiO_2 interface.

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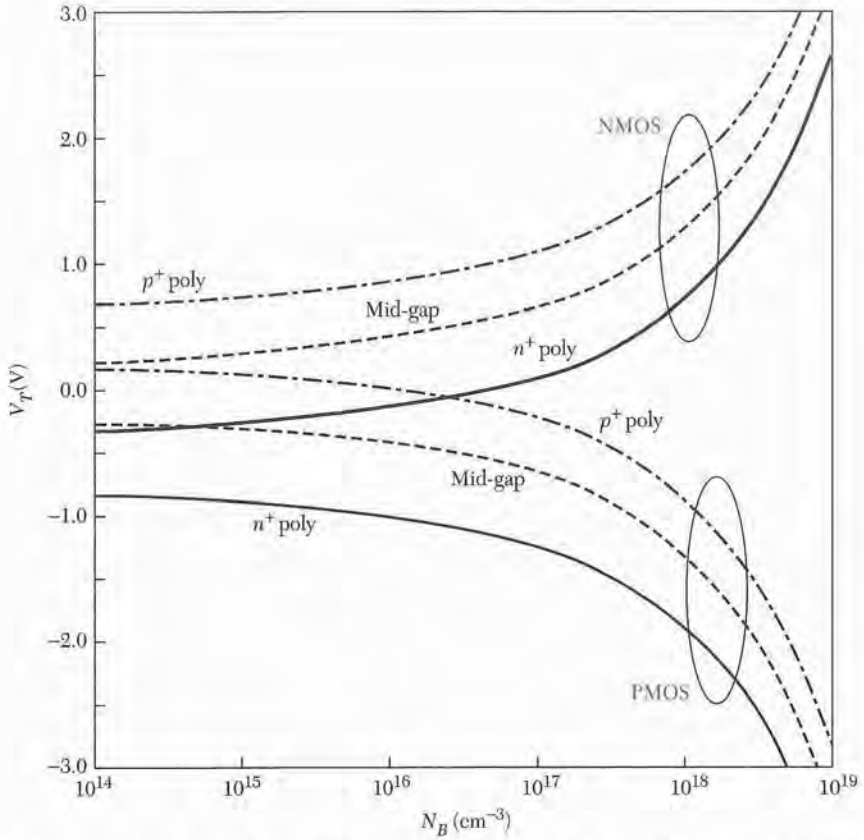


Fig. 20 Calculated threshold voltage of n-channel (V_{Tn}) and p-channel (V_{Tp}) MOSFETs as a function of impurity concentration, for devices with n^+–, p^+–polysilicon, and mid-gap work function gates assuming zero fixed charge. The thickness of the gate oxide is 5 nm. NMOS, n-channel MOSFET; PMOS, p-channel MOSFET.

SOLUTION From the examples in Section 6.1, we have $C_o = 6.9 \times 10^{-7}$ F/cm^2, $2\psi_B = 0.84$ V, and $V_{FB} = -1.1$ V. Therefore, from Eq. 45 (with $V_{BS} = 0$),

$$V_T = V_{FB} + 2\psi_B + \frac{\sqrt{2\varepsilon_s q N_A (2\psi_B)}}{C_o}$$

$$= -1.1 + 0.84 + \frac{\sqrt{2 \times 11.9 \times 8.85 \times 10^{-14} \times 1.6 \times 10^{-19} \times 10^{17} \times 0.84}}{6.9 \times 10^{-7}}$$

$$= -0.02 \text{ V}.$$

The boron charge causes a flat-band shift of qF_B/C_o. Thus,

$$0.6 = -0.02 + \frac{qF_B}{6.9 \times 10^{-7}},$$

$$F_B = \frac{0.62 \times 6.9 \times 10^{-7}}{1.6 \times 10^{-19}} = 2.67 \times 10^{12} \text{ cm}^{-2}.$$

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We can also control V_T by varying the oxide thickness. Threshold voltage becomes more positive for an n-channel MOSFET and more negative for a p-channel MOSFET as the oxide thickness is increased. This is simply due to the reduced field strength at a fixed gate voltage for a thicker oxide. Such an approach is used extensively for isolating transistors fabricated on a chip. Figure 21 shows the cross section of an isolation oxide (also called field oxide) between an n^+ diffusion and an n-well. Details about the field oxide formation and the well technology are given in Chapter 14. The n^+ diffusion region is the source or drain region of a normal n-channel MOSFET. The gate oxide of MOSFET is much thinner than the field oxide. When a conductor line is formed over the field oxide, a parasitic MOSFET, also called a field transistor, results with the n^+ diffusion and n-well regions as the source and drain, respectively. The V_T of the field oxide is typically an order of magnitude larger than that of the thin gate oxide. During circuit operation, the field transistor will not be turned on. Consequently, the field oxide offers good isolation between the n^+ diffusion and n-well regions.

EXAMPLE 7

For an n-channel field transistor with $N_A = 10^{17}$ cm^{-3} and $Q_f/q = 5 \times 10^{11}$ cm^{-2}, calculate V_T for a gate oxide (i.e., the field oxide) of 500 nm.

SOLUTION $C_o = \varepsilon_{ox} / d = 6.9 \times 10^{-9}$ F/cm^2.

From Exs. 2 and 3, we have $2\psi_B = 0.84$ V, and $V_{FB} = -1.1$ V.

Therefore, from Eq.45 (with $V_{BS} = 0$)

$$V_T = V_{FB} + 2\psi_B + \frac{\sqrt{2\varepsilon_s q N_A (2\psi_B)}}{C_o}$$

$$= -1.1 + 0.84 + \frac{\sqrt{2 \times 11.9 \times 8.85 \times 10^{-14} \times 1.6 \times 10^{-19} \times 10^{17} \times 0.84}}{6.9 \times 10^{-9}}$$

$$= 24.12 \text{ V}.$$

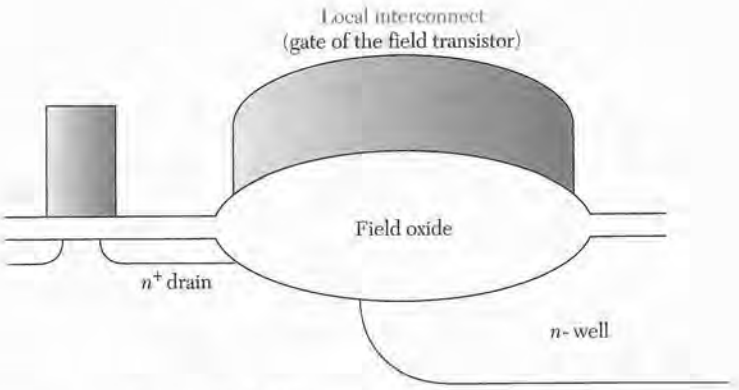


Fig. 21 Cross section of a parasitic field transistor in an n-well structure.

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Substrate bias can also be used to adjust the threshold voltage. According to Eq. 45, the change in threshold voltage due to the substrate bias is

$$\Delta V_T = \frac{\sqrt{2\varepsilon_s q N_A}}{C_o}\left(\sqrt{2\psi_B + V_{BS}} - \sqrt{2\psi_B}\right). \tag{46}$$

If we plot the drain current versus V_G, the intercept at the V_G-axis corresponds to the threshold voltage, Eq. 36. Such a plot is shown in Fig. 22 for three different substrate biases. As the magnitude of the substrate V_{BS} increases from 0 V to 2 V, the threshold voltage also increases from 0.56 V to 1.03 V. The substrate effect can be used to raise the threshold voltage of a marginal enhancement device ($V_T \sim 0$) to a larger value.

EXAMPLE 8

For the MOSFET discussed in Ex. 6 with V_T of −0.02 V, if the substrate bias is increased from zero to 2 V, calculate the change in threshold voltage.

SOLUTION

From Eq. 46,

$$\Delta V_T = \frac{\sqrt{2\varepsilon_s q N_A}}{C_o}\left(\sqrt{2\psi_B + V_{BS}} - \sqrt{2\psi_B}\right)$$

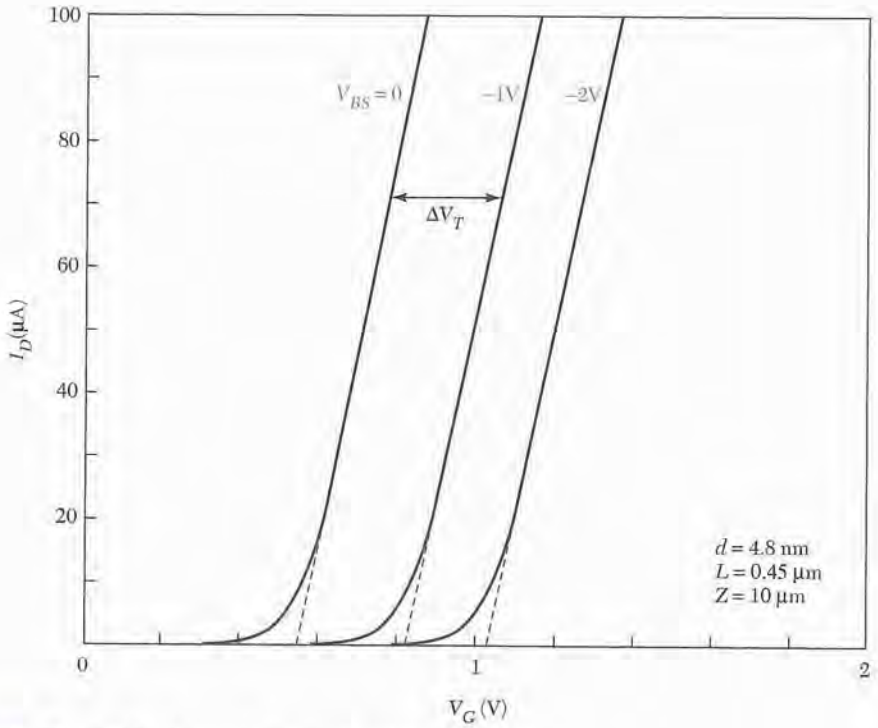


Fig. 22 Threshold voltage adjustment using substrate bias.

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$$= \frac{\sqrt{2 \times 11.9 \times 8.85 \times 10^{-14} \times 1.6 \times 10^{-19} \times 10^{17}}}{6.9 \times 10^{-7}} \left(\sqrt{0.84 + 2} - \sqrt{0.84}\right)$$

$$= 0.27 \times (1.69 - 0.92) = 0.21 \text{ V}.$$

Another way to control V_T is to adjust the work function difference by choosing an appropriate gate material. A number of conducting materials have been proposed, such as W, TiN, and a heavily doped polycrystalline silicon-germanium layer.[6] In deep submicron device fabrication, control of the threshold voltage and device performance becomes more difficult because of the geometric effects encountered in device scaling (see the discussion in the next section). The use of other gate materials to replace the conventional n^+ polysilicon could make device design more flexible in deep submicron fabrication.

6.3 MOSFET SCALING

Scaling down of MOSFET's dimensions is a continuous trend since its inception. Smaller device size enables higher device density in an integrated circuit. In addition, a smaller channel length improves the driving current ($I_D \sim 1/L$) and, thus, the operation performance. As a device's dimensions are reduced, however, influences from the side regions of the channel (i.e., source, drain, and isolation edge) become significant. Device characteristics, therefore, deviate from those derived from long-channel approximation.

6.3.1 Short-Channel Effects

The threshold voltage given in Eq. 45 is derived based on the gradual-channel approximation stated in Section 6.2.1. That is, the charges contained in the surface depletion region of the substrate are induced solely from the field created by the gate voltage. In other words, the third term in Eq. 45 is independent of the lateral fields from the source and drain. As channel length is reduced, however, the fields originating from the source/drain regions may influence the charge distribution and, thus, the device characteristics such as the threshold voltage control and device leakage.

Threshold Voltage Roll-Off in Linear Region

When the channel side effect becomes nonnegligible, the threshold voltage in the linear region usually becomes less positive as channel length decreases for n-channel MOSFETs and less negative as channel length decreases for p-channel MOSFETs. Figure 23 shows an example with $V_{DS} = 0.05$ V of this V_T roll-off phenomenon.[7] Roll-off can be explained by the charge-sharing model,[8] as illustrated in Fig. 24. This figure shows the cross section of an n-channel MOSFET. This device is operated in the linear region ($V_{DS} \leq 0.1$ V). Hence, the depletion region width for the drain junction is almost equal to that for the source junction. Since the channel depletion region overlaps the source and drain depletion regions, charges induced by the field created by the gate bias can be approximated by those within the trapezoidal region.

The threshold voltage shift ΔV_T is due to the reduction of charges in the depletion layer from the rectangular region $L \times W_m$ to the trapezoidal region $(L + L')W_m/2$. ΔV_T is given by (see Prob. 27)

$$\Delta V_T = -\frac{qN_A W_m r_j}{C_o L} \left(\sqrt{1 + \frac{2W_m}{r_j}} - 1 \right), \tag{47}$$

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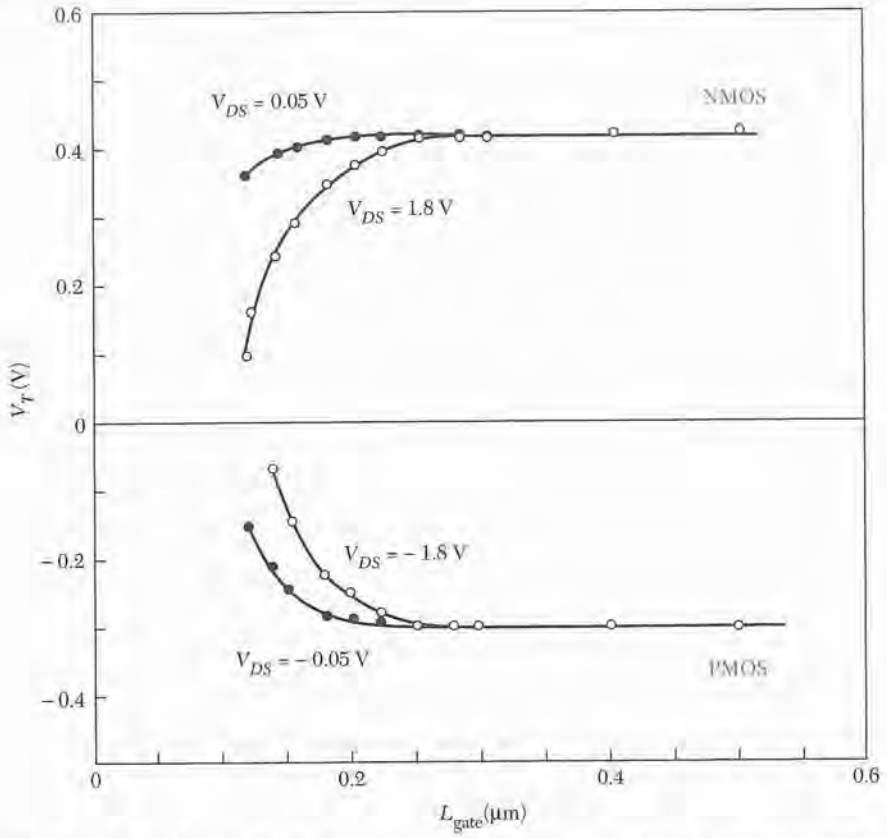


Fig. 23 Threshold voltage roll-off characteristics in a 0.15 μm complementary metal-oxide-semiconductor (CMOS) field-effect transistor technology.[7]

where N_A is the substrate doping concentration, W_m the depletion width, r_j the junction depth, L the channel length, and C_o the gate oxide capacitance per unit area.

For long-channel devices, the charge reduction is smaller, since Δ (Fig. 24) is much smaller than L. For short-channel devices, however, the charges needed to turn on the device are dramatically reduced, since Δ is comparable to L. As can be seen from Eq. 47, for a given set of N_A, W_m, r_j and C_o, the threshold voltage decreases with decreasing channel length.

Drain-Induced Barrier Lowering

When the drain voltage of a short-channel MOSFET increases from the linear region toward the saturation region, its threshold voltage roll-off becomes larger (see Fig. 23). This effect is called drain-induced barrier lowering (DIBL). The surface potentials between source and drain for several n-channel devices with different channel lengths are shown[9] in Fig. 25. The dotted lines are for V_{DS}= 0 and the solid lines for V_{DS}>0. When the gate voltage is below V_T, the p-Si substrate forms a potential barrier between n^+ source and drain and limits the electron flow from source to drain. For a device operated in the saturation region, the depletion-layer width of the drain junction is significantly wider than that of the source junction. In the long-channel case, the increase in depletion-layer region width of the drain junction will not affect the potential barrier height (see Fig. 25, the 1 μm case). Nevertheless, when the channel length is short enough, the increase in drain

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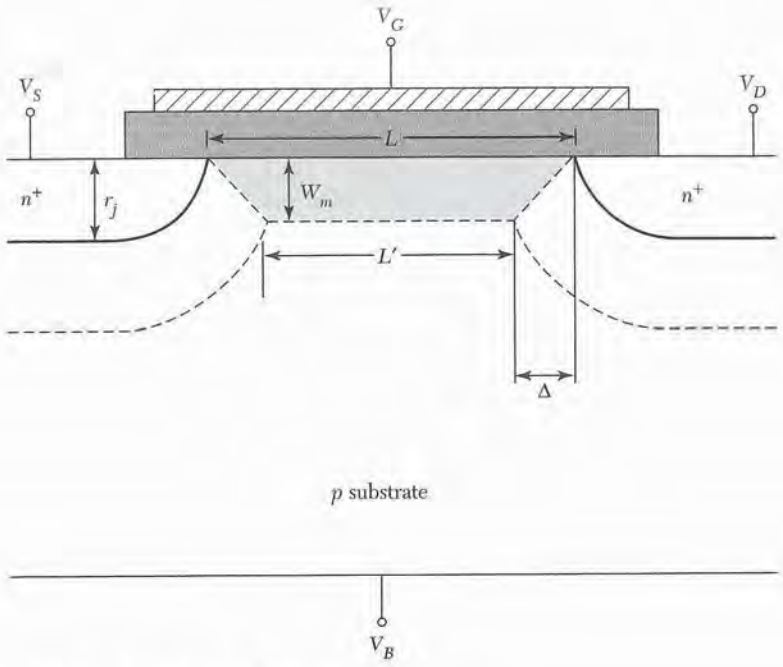


Fig. 24 Schematic of the charge sharing model.[8]

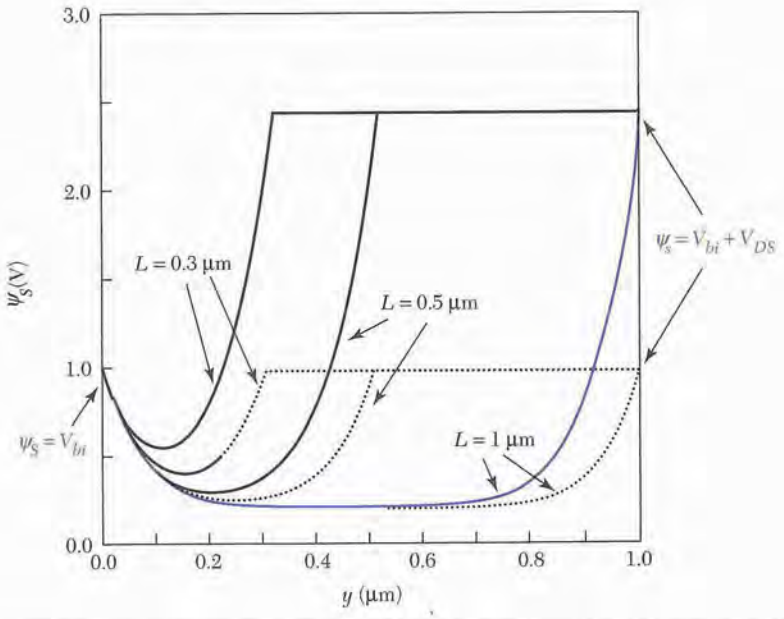


Fig. 25 Calculated surface potential along the channel for n-channel MOSFETs with different channel lengths.[9] The source-channel boundary is at $y = 0$. A low (0.05 V, dotted lines) and a high (1.5 V, solid lines) V_{DS} are applied. Oxide thickness d and substrate doping N_A are10 nm and 10^{16} cm^{-3}, respectively. The substrate bias is 0 V.

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voltage decreases the potential barrier height (the 0.3 and 0.5 μm cases in Fig. 25). This is ascribed to the field penetration at the surface region from the drain to the source when they are too close. Such a barrier-lowering effect leads to a substantial increase in electron injection from the source to the drain. As a result, subthreshold current increases. The threshold voltage, thus, decreases further with increasing drain bias in short-channel devices.

Figure 26 illustrates the subthreshold characteristics of a long and a short n-channel MOSFET at low and high drain bias conditions. The parallel shift in subthreshold current in the short-channel device (Fig. 26*b*) as the drain voltage increases indicates that a significant DIBL effect has been induced.

Bulk Punch-through

DIBL causes the formation of a leakage path at the SiO_2/Si interface. If the drain voltage is large enough, significant leakage current may also flow from drain to source via the bulk of the substrate for a short-channel MOSFET. This is also ascribed to the increase in the depletion-layer region width of the drain junction with increasing drain voltage. In a short-channel MOSFET, the sum of depletion-layer width for source and drain junctions is comparable to the channel length. The depletion region of the drain junction gradually merges with that of the source junction when the drain voltage is increased. As a result, a large leakage current may flow from the drain to the source through the bulk. Figure 27 shows the subthreshold characteristics of a short-channel (L = 0.23 μm) MOSFET. When the drain voltage is increased from 0.1 to 1 V, DIBL is induced with the parallel shift in the subthreshold characteristics similar to that shown in Fig. 26*b*. When the drain voltage is further increased to 4 V, the subthreshold swing is much larger than that for lower drain biases. Consequently, the device has a very high leakage current. This indicates that the bulk punch-through effect is very significant. The gate can no long turn the device completely off and loses control of the drain current. High-leakage current limits device operation for short-channel MOSFETs.

6.3.2 Scaling Rules

As device dimensions are reduced, the short-channel effects must be minimized to maintain normal device and circuit operation. Some guidelines are necessary in scaled-device design. One elegant approach for maintaining the long-channel behavior is to simply reduce all dimensions and voltages by a scaling factor $\kappa(>1)$, so that the internal electric fields are the same as those of a long-channel MOSFET. This approach is called *constant-field scaling*.[10]

Table 1 summarizes the scaling rules of constant-field scaling for various device parameters and circuit performance factors.[11] The circuit performance (speed and power consumption in the on-state) can be enhanced as the device dimensions are scaled down.[§] In practical integrated circuit (IC) manufacturing, however, the electric fields inside the smaller devices are not kept constant but increase to some extent. This is mainly because the voltage factors (e.g., power supply, threshold voltage) cannot be scaled arbitrarily. If the threshold voltage is too small, the leakage level in the off-state (V_G = 0) will increase significantly because of the nonscalable subthreshold swing. Consequently, standby power consumption will also increase.[12] By applying the scaling rules, MOSFETs have been fabricated that have a channel length as short as 20 nm, a very high transconductance (>1000 mS/mm), and a reasonable subthreshold swing (~120 mV/decade).[13]

[§]A comparison of the cutoff frequency for different field-effect transistors (including MOSFET) is shown in Fig. 18 of Chapter 7.

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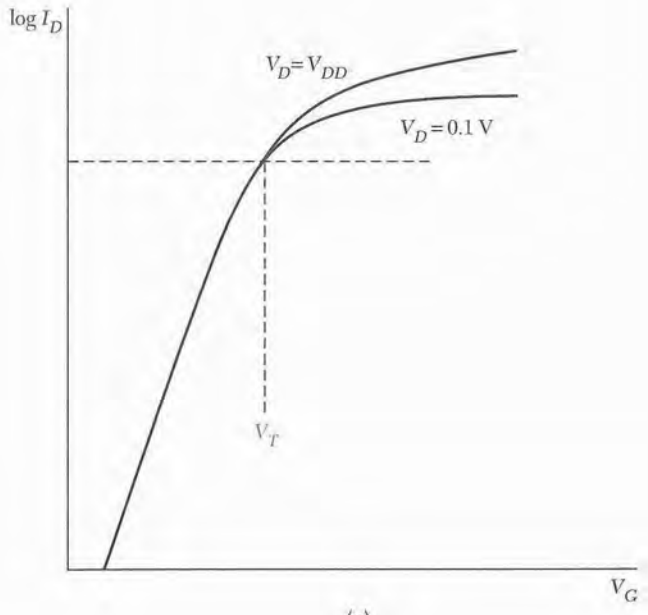


Fig. 26 Subthreshold characteristics of (*a*) a long-channel and (*b*) a short-channel MOSFET.

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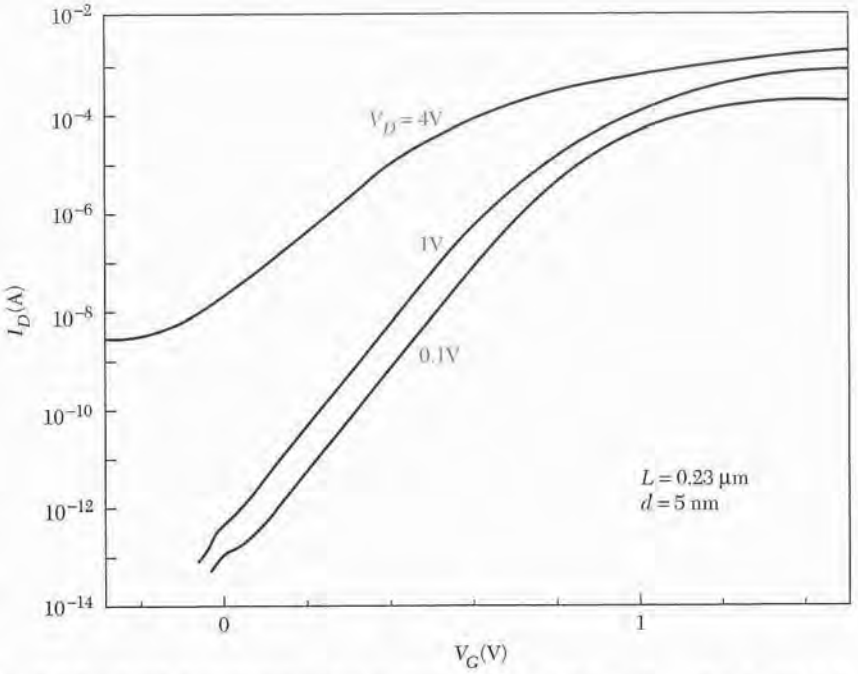


Fig. 27 Subthreshold characteristics of an n-channel MOSFET with V_{DS} = 0.1, 1 , and 4 V.

TABLE 1 Scaling of MOSFET Device and Circuit Parameters

Determinant	MOSFET device and circuit parameters	Multiplying factor ($\kappa > 1$)
Scaling assumptions	Device dimensions (d, L, W, r_j)	$1/\kappa$
	Doping concentration (N_A, N_D)	κ
	Voltage (V)	$1/\kappa$
Derived scaling behavior of device parameters	Electric field ($\mathscr{E}$)	1
	Carrier velocity (v)	1
	Depletion-layer width (W)	$1/\kappa$
	Capacitance ($C = \varepsilon A/d$)	$1/\kappa$
	Inversion-layer charge density (Q_n)	1
	Current, drift (I)	$1/\kappa$
	Channel resistance (R)	1
Derived scaling behavior of circuit parameters	Circuit delay time ($\tau \sim CV/I$)	$1/\kappa$
	Power dissipation per circuit ($P \sim VI$)	$1/\kappa^2$
	Power-delay product per circuit ($P\tau$)	$1/\kappa^3$
	Circuit density ($\sim 1/A$)	κ^2
	Power density (P/A)	1

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6.4 CMOS AND BiCMOS

Complementary MOS (CMOS) refers to a complementary p-channel and n-channel MOSFET pair. CMOS logic is the most popular technology utilized in present-day integrated circuit design. The main reasons for the success of CMOS are low power consumption and good noise immunity. In fact, currently only CMOS technology is used in advanced integrated-circuit manufacturing due to the low power-dissipation requirement.

6.4.1 The CMOS Inverter

A CMOS inverter, which is the basic element of CMOS logic circuits, is shown in Fig. 28. In a CMOS inverter, the gates of the p- and n-channel transistors are connected and serve as the input node to the inverter. The drains of the two transistors are also connected and serve as the output node to the inverter. The source and substrate contacts of n-channel MOSFETs are grounded, whereas those of p-channel MOSFETs are connected to the power supply (V_{DD}). Note that both p-channel and n-channel MOSFETs are enhancement-type transistors. When the input voltage is low (e.g., $V_{in} = 0$, $V_{GSn} = 0 < V_{Tn}$), the n-channel MOSFET is off.[§] The p-channel MOSFET, however, is on, since $|V_{GSp}| \cong V_{DD} > |V_{Tp}|$ (V_{GSp} and V_{Tp} are negative). Consequently, the output node is charged to V_{DD} through the p-channel MOSFET. When the input voltage goes high so that the gate voltage equals V_{DD}, the n-channel MOSFET is turned on, since $V_{GSn} = V_{DD} > V_{Tn}$, and the p-channel MOSFET is turned off, since $|V_{GSp}| \cong 0 < |V_{Tp}|$. Therefore, the output node is discharged to ground through the n-channel MOSFET.

For more detailed understanding of the operation of the CMOS inverter, we can first plot the output characteristics of the transistors. This is given in Fig. 29, in which I_p and I_n are shown as a function of output voltage (V_{out}). I_p is the current of p-channel MOSFET in the direction from the source (connected to V_{DD}) to the drain (output node). I_n is the current of n-channel MOSFET in the direction from the drain (output node) to the source (connected to ground). It is noted that the increase in input voltage (V_{in}) tends to increase I_n but decrease I_p at a fixed V_{out}. In steady state, however, I_n should be equal to I_p. For a given V_{in}, we can determine the corresponding V_{out} from the intercept of $I_n(V_{in})$ and $I_p(V_{in})$, as shown in Fig. 29. The V_{in}-V_{out} curve, as shown in Fig. 30, is called the transfer curve of the CMOS inverter.[11]

An important characteristic of the CMOS inverter is that when the output is in a steady logic state, i.e., $V_{out} = 0$ or V_{DD}, only one transistor is on. The current flow from the power supply to ground is thus very low and is equal to the leakage current of the off device. In fact, there is significant current conduction only during the short transient period when the two devices are temporarily on. Therefore the power consumption is very low in the static state compared with other types of logic circuits, such as n-channel MOSFETs, bipolar, etc.

6.4.2 Latch-up

In order to fabricate both p-channel and n-channel MOSFETs in the same chip for CMOS applications, extra doping and diffusion steps are needed to form the "well" or "tub" in the substrate. The doping type in the well is different from that of the surrounding

[§] V_{GSn} and V_{GSp} represent the voltage difference between the gate and the source for n- and p-channel MOSFETs, respectively.

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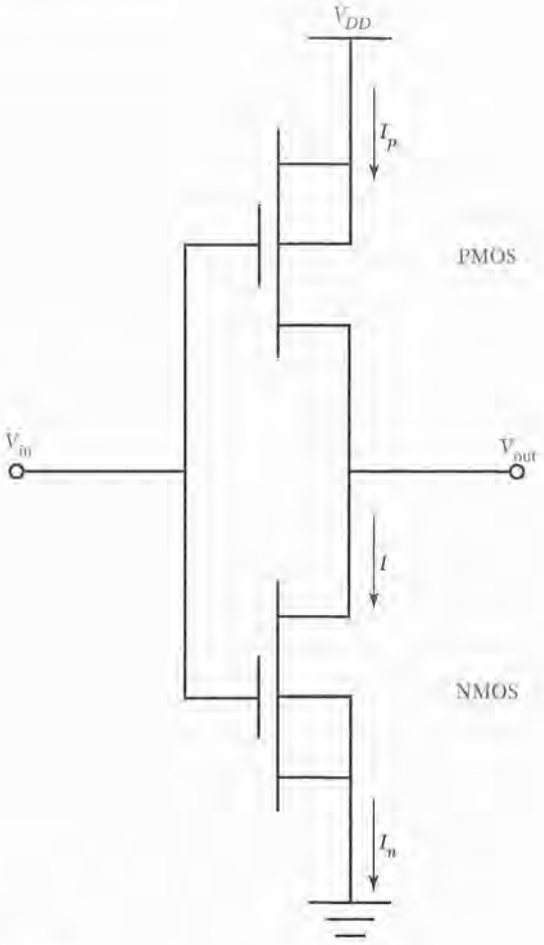


Fig. 28 The CMOS inverter.

substrate. Typical well types are the p-well, n-well, and twin well. Details of the well technology are given in Chapter 14. Figure 31 shows a cross-sectional view of a CMOS inverter fabricated using p-well technology. In this figure, the p-channel and n-channel MOSFETs are fabricated in the n-type Si substrate and the p-well region, respectively.

A major problem related to the well structure in CMOS circuits is the latch-up phenomenon. The cause of latch-up is the action of the parasitic p-n-p-n diode in the well structure. As shown in Fig. 31, the parasitic p-n-p-n diode consists of a lateral p-n-p and a vertical n-p-n bipolar transistors. The p-channel MOSFET's source, n-substrate, and p-well correspond to the emitter, base, and collector of the lateral p-n-p bipolar transistor, respectively. The n-channel MOSFET's source, p-well, and n-substrate are the emitter, base, and collector of the vertical n-p-n bipolar transistor, respectively. The equivalent circuit of the parasitic components is illustrated in Fig. 32. R_S and R_W are the series resistance in the substrate and the well, respectively. The base of each transistor is driven by the collector of the other to form a positive feedback loop. This configuration is similar to the thyristor discussed in Chapter 5. Latch-up is induced when the current gain product of the two bipolar transistors, $\alpha_{npn}\alpha_{pnp}$, is larger than 1. When latch-up occurs, a large current will flow from the power supply (V_{DD}) to the ground contact. This can interrupt

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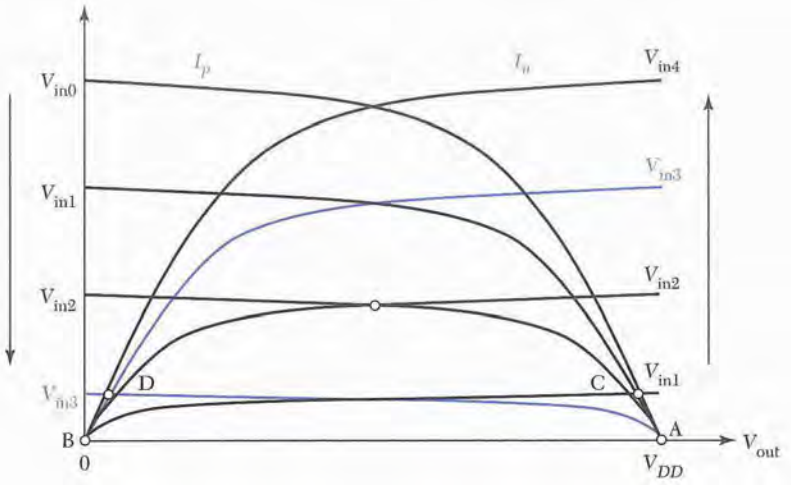


Fig. 29 I_p and I_n as a function of V_{out}. The intercepts of I_p and I_n (circled) represent the steady-state operation points of the CMOS inverter.[11] The curves are labeled by the input voltages: $0 = V_{in0} < V_{in1} < V_{in2} < V_{in3} < V_{in4} = V_{DD}$.

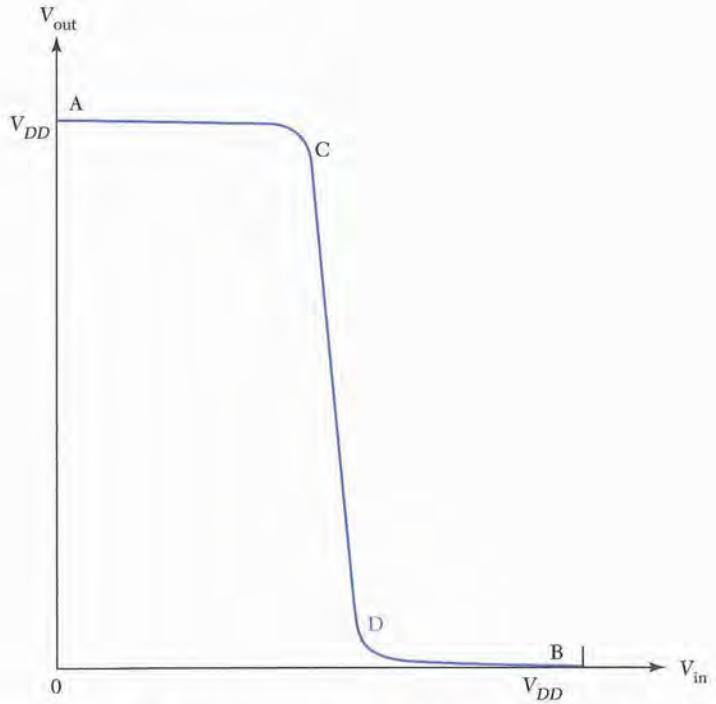


Fig. 30 Transfer curve of a CMOS inverter.[11] Points labeled A, B, C, and D correspond to those points labeled in Fig. 29.

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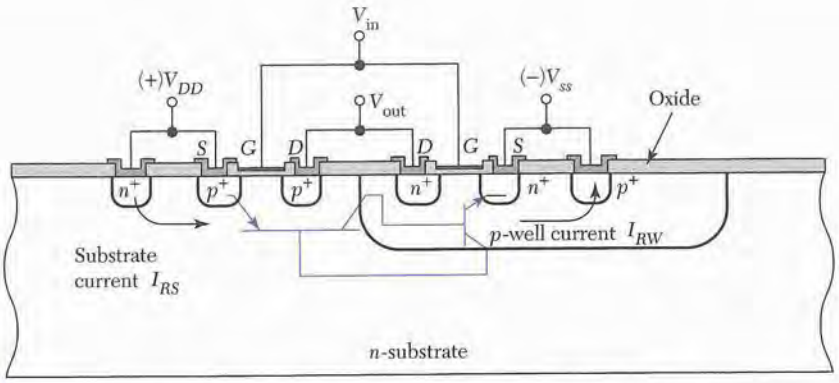


Fig. 31 Cross section of a CMOS inverter fabricated with p-well technology.

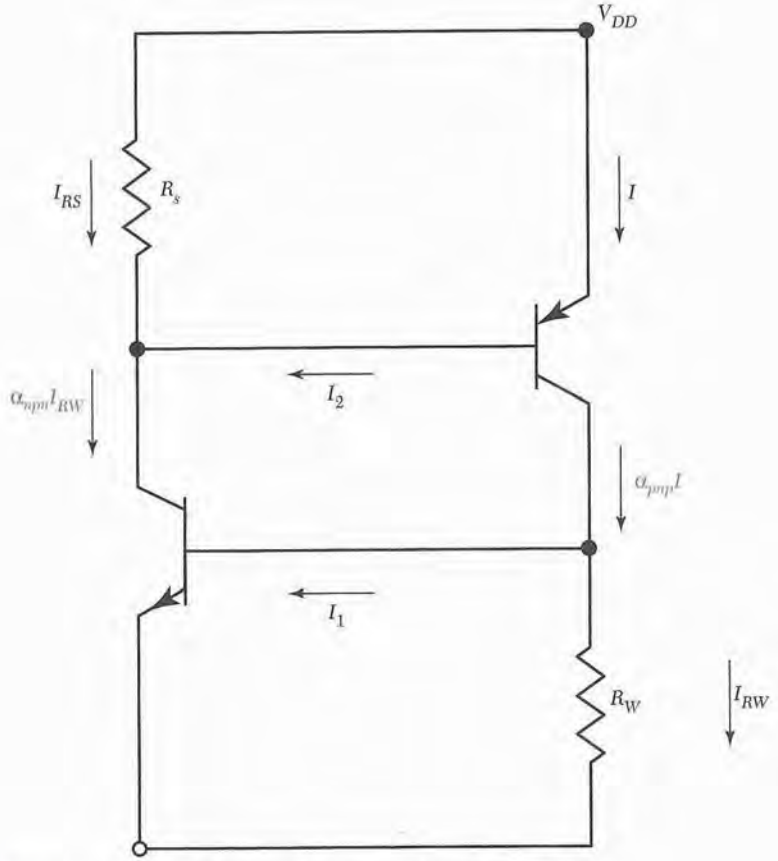


Fig. 32 Equivalent circuit of the p-well structure shown in Fig. 31.

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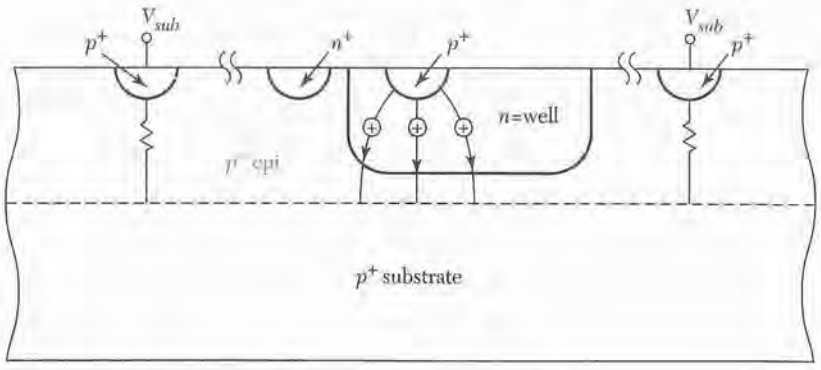


Fig. 33 Prevention of latch-up with a heavily doped substrate.[14]

normal circuit operation and even destroy the chip itself because of the high power dissipation.

To avoid latch-up, the current gains of the parasitic bipolar transistors must be reduced. One method is to use gold doping or neutron irradiation to lower the minority carrier lifetimes. However, this approach is difficult to control. Besides, it also causes an increase of the leakage current. A deeper well structure or high-energy implantation to form retrograde wells can also reduce the current gain of vertical bipolar transistor by raising the impurity concentration in the base. In the retrograde well, the peak of the well doping concentration is located within the substrate away from the surface.

Another way to reduce latch-up is to use a heavily doped substrate with devices fabricated on a lightly doped epitaxial layer, as shown[14] in Fig. 33. The heavily doped substrate provides a highly conductive path to collect the current. The current then is drained away through the surface contacts.

Latch-up can also be avoided with the trench isolation scheme. A process for forming trench isolation is discussed in Chapter 14. This approach can eliminate latch-up because the n-channel and p-channel MOSFETs are physically isolated by the trench.

6.4.3 BiCMOS

CMOS has the advantages of low power dissipation and high device density, which make it suitable for fabricating complex circuits. However, CMOS suffers from low drive capability compared with bipolar technology, limiting its circuit performance. BiCMOS is a technology that integrates both CMOS and bipolar device structures in the same chip. A BiCMOS circuit contains mostly CMOS devices, with a relatively small number of bipolar devices. The bipolar devices have better switching performance than their CMOS counterparts without consuming too much extra power. However, this performance enhancement is achieved at the expense of extra manufacturing complexity, longer fabrication time, and higher cost. The fabrication processes for BiCMOS are discussed in Chapter 14.

6.5 MOSFET ON INSULATOR

For certain applications, MOSFETs are fabricated on an insulating substrate rather than on a semiconductor substrate. The characteristics of these transistors are, however, similar to those of a MOSFET. Usually, we call such devices thin film transistors (TFT) if

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the channel layer is an amorphous or a polycrystalline silicon. If the channel layer is a monocrystalline silicon, we call it silicon-on-insulator (SOI).

6.5.1 The Thin Film Transistor (TFT)

Hydrogenated amorphous silicon (a-Si:H) and polysilicon are the two most popular materials for TFT fabrication. They are usually deposited on an insulating substrate such as a glass, quartz, or Si substrate with a thin SiO_2 capping layer.

The hydrogenated amorphous silicon TFT is an important device in electronic applications that require a large area, such as liquid crystal displays (LCD) and contact imaging sensors (CIS). The a-Si:H materials are usually deposited with a plasma-enhanced chemical vapor deposition (PECVD) system. Since the deposition temperature is low (typically 200°–400°C), inexpensive substrate materials such as glass can be used. The role played by the hydrogen atoms contained in the a-Si:H is to passivate dangling bonds in the amorphous silicon matrix and thus reduce the defect density. Without hydrogen passivation, the gate voltage can not adjust the Fermi level at the insulator and the a-Si interface, since the level is pinned by the large amount of defects.

The a-Si:H TFT is usually fabricated using the inverted staggered structure, as shown in Fig. 34. The inverted staggered structure is with a bottom-gate scheme. A metal gate can be used since the post process temperature is low (< 400°C). A dielectric layer such as silicon nitride or silicon dioxide, also deposited by PECVD, is often used as the gate dielectric. An undoped a-Si:H layer is subsequently deposited to form the channel. The source and drain of the TFT are formed with an in situ–doped n^+ a-Si:H layer complying with the requirement of low process temperature. A dielectric layer that serves as an etch-stop for patterning of n^+ a-Si:H is often used. Device characteristics of TFTs with the bottom-gate structure are usually better than those with the top-gate structure. This is because the a-Si:H channel could be damaged by plasma during PECVD gate-dielectric deposition of top-gated TFTs. In addition, the source/drain formation process is easier for the bottom-gate structure. A typical subthreshold characteristic of the a-Si:H TFT is shown in Fig. 35. Because of the amorphous matrix presents in the channel material, its carrier mobility is usually very low (< 1 cm^2/V-s).

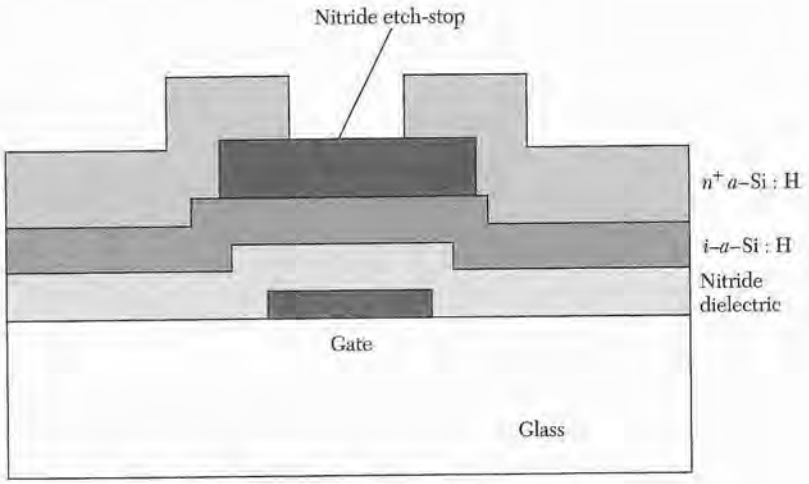


Fig. 34 A typical a-Si:H thin film transistor (TFT) structure.

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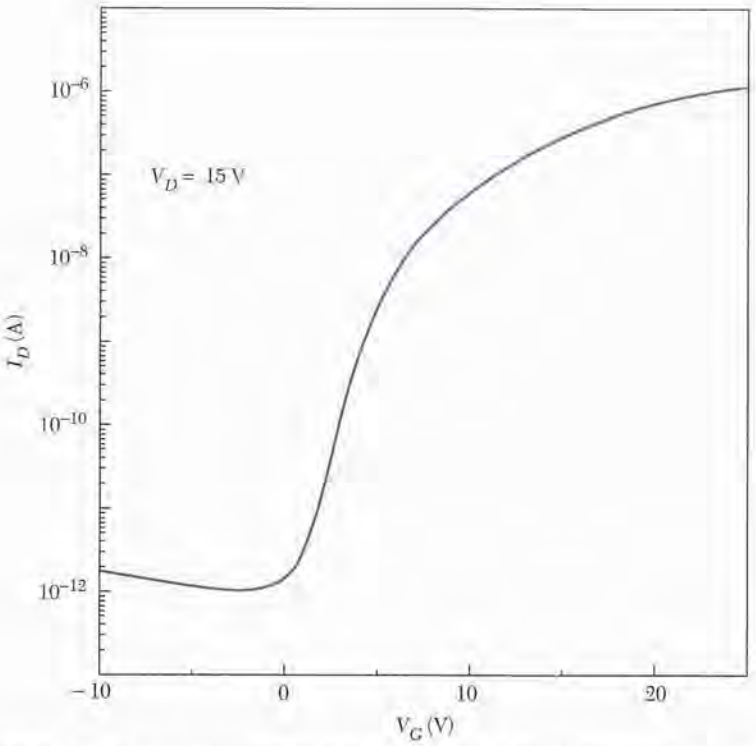


Fig. 35 Subthreshold characteristics of an a-Si:H TFT (L/Z = 10/60 μm/μm). The field-effect carrier mobility is 0.23 cm²/ V-s.

The polysilicon TFT uses a thin polysilicon as the channel layer. Polysilicon is a material consisting of many Si grains. Within the grain are the monocrystalline Si lattices. The orientations of two side-by-side grains are, however, different from each other. The interface between the two grains is called the grain boundary. Polysilicon TFT exhibits much higher carrier mobility and thus better drive capability than a-Si:H TFT because of higher crystallinity. Carrier mobility of these devices typically ranges from 10 to several hundred cm²/V-s, depending on the grain size and process conditions. Polysilicon is usually deposited with low-pressure CVD (LPCVD). The grain size of polysilicon is an important factor in determining the TFT performance, since the carrier mobility generally decreases with decreasing grain size. This is mainly because of the large number of defects contained in the grain boundaries that impede the transport of carriers.

The defects at the grain boundary could also affect the threshold voltage and subthreshold swing of the device. When gate voltage is applied to induce an inversion layer in the channel, these defects act as traps and impede the movement of the Fermi level in the forbidden gap. To alleviate these drawbacks, a hydrogenation step is often adopted after device fabrication. The hydrogenation treatment is usually done in a plasma reactor. Hydrogen atoms or ions generated in the plasma diffuse into the grain boundaries and passivate these defects. After hydrogenation, there is significant improvement in device performance .

Unlike a-Si:H TFT, polysilicon TFT is usually fabricated with the top-gate structure, as shown in Fig. 36. A self-aligned implant is used to form the source/drain. One main limitation of polysilicon TFT manufacturing is the high process temperature (> 600°C). Consequently, expensive substrates such as quartz are usually needed to tolerate the high

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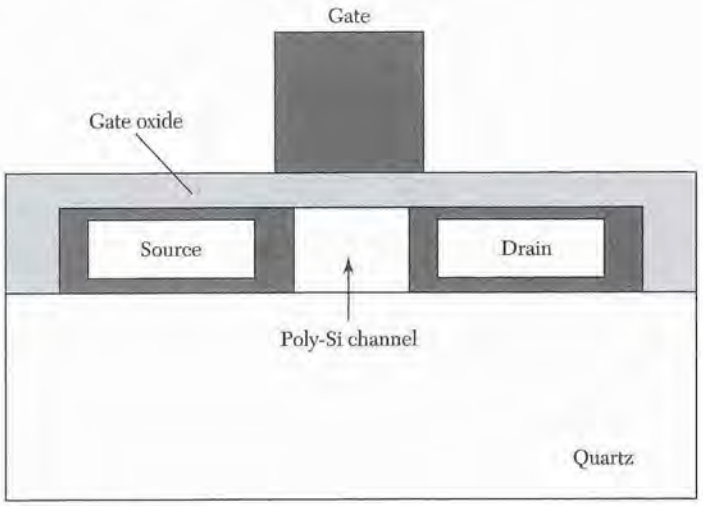


Fig. 36 A polysilicon TFT structure.

process temperatures. This makes polysilicon TFT less attractive than a-Si:H TFT in production for low-end applications because of higher cost. Laser crystallization of Si is a potential way to overcome the problem. In this method, an a-Si layer is deposited first on a glass substrate at low temperatures by PECVD or LPCVD. A high-power laser source is then used to irradiate the a-Si. The energy is absorbed by the a-Si and melting occurs locally in the a-Si layer. After cooling, the a-Si turns into polysilicon with very large grain size ($\geq 100\ \mu m$). Very high carrier mobility, approaching that of crystalline Si MOSFETs, can be obtained using this method.

6.5.2 Silicon-on-Insulator (SOI) Devices

Many SOI devices have been proposed, including silicon-on-sapphire (SOS), silicon-on-spinel, silicon-on-nitride, and silicon-on-oxide.[15] Figure 37 shows a schematic diagram of an SOI CMOS built on silicon dioxide. Compared with CMOS built on a bulk Si substrate, also called bulk CMOS, SOI's isolation scheme is simplified and does not need complicated well structures. Device density can thus be increased. The latch-up phenomenon inherent in bulk CMOS circuits is also eliminated. The parasitic junction capac-

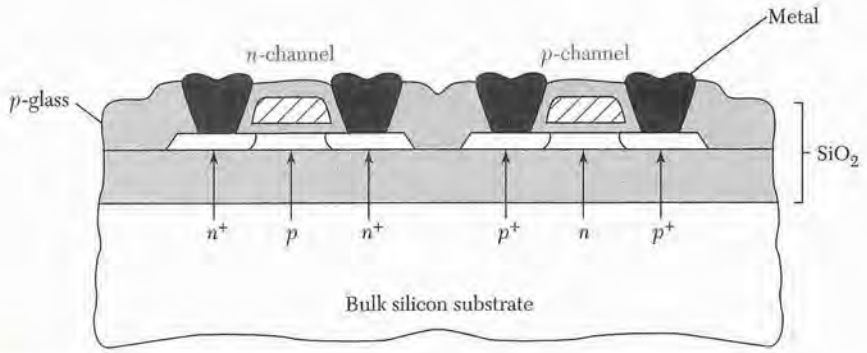


Fig. 37 Cross section of the silicon-on-insulator (SOI).

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itance in the source and drain regions can be significantly reduced with the insulating substrate. Additionally, significant improvement over bulk CMOS in radiation-damage toleration is achieved in SOI. This is because of the small volume of Si available for electron-hole pair generation by radiation. This property is particularly important for space applications.

Depending on the thickness of the Si channel layer, SOI can be classified into partially depleted (PD) and fully depleted (FD) types. PD-SOI uses a thicker Si channel layer so that the depletion width of the channel does not exceed the thickness of Si layer. Device design and performance of a PD-SOI is similar to that of bulk CMOS. One major difference is the floating substrate used in SOI devices. During device operation, a high field near the drain could induce impact ionization there. Majority carriers, holes in the p-substrate for an n-channel MOSFET, generated by impact ionization will be stored in the substrate, since there is no substrate contact to drain away these charges. Therefore, the substrate potential will be changed and will result in a reduction of the threshold voltage. This, in turn, may cause an increase or a kink in the current-voltage characteristics. The kink phenomenon is shown[15] in Fig. 38. This float-body or kink effect is especially dramatic for n-channel devices, because of the higher impact-ionization rate of electrons. The kink effect can be eliminated by forming a substrate contact to the source of the transistor. This will, however, complicate the device layout and process flow.

FD-SOI uses a Si layer thin enough so that the channel of the transistor is completely depleted before threshold is reached. This allows the device to be operated at a lower field. In addition, the kink effect caused by high-field impact ionization can be eliminated. FD-SOI is very attractive for low-power applications. Nevertheless, the FD-SOI's characteristics are sensitive to the Si thickness variation. If an FD-SOI circuit is built on a wafer with nonuniform Si thickness, its operation will be unstable.

EXAMPLE 9

Calculate the threshold voltage for an n-channel SOI device having $N_A = 10^{17}$ cm^{-3}, $d = 5$ nm, and $Q_f/q = 5 \times 10^{11}$ cm^{-2}. Si thickness, d_{Si}, for the device is 50 nm.

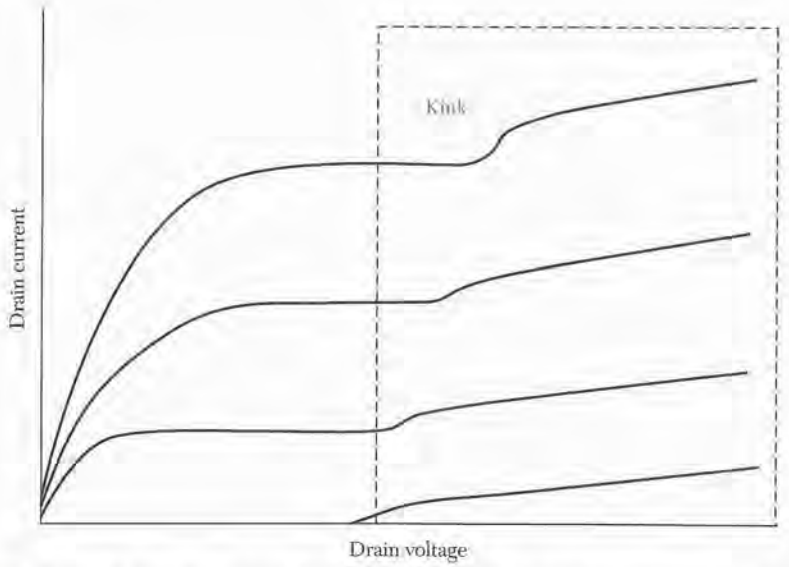


Fig. 38 The kink effect in the output characteristics of an n-channel SOI MOSFET.[15]

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SOLUTION From Ex. 1, the maximum depletion width, W_{m}, for a bulk NMOS device is 100 nm. Therefore, the SOI device is a fully depleted type. Since the width of the depletion region is now the Si thickness, W_m used in Eq. 17 and Eq. 45 for calculating the threshold voltage should be replaced by d_{Si}:

$$V_T = V_{FB} + 2\psi_B + \frac{qN_A d_{Si}}{C_o}.$$

From Exs. 2 and 3, we have $C_o = 6.9 \times 10^{-7}$ F/cm^2, $V_{FB} = -1.1$ V, and $2\psi_B = 0.84$ V.

Therefore,

$$V_T = -1.1 + 0.84 + \frac{1.6\times10^{-19}\times10^{17}\times5\times10^{-6}}{6.9\times10^{-7}} = -0.14 \text{ V}.$$

6.6 MOS MEMORY STRUCTURES

Semiconductor memories can be classified as volatile and nonvolatile memories. Volatile memories such as dynamic random access memories (DRAMs) and static random access memories (SRAMs) lose their stored information if the power supply is switched off. Nonvolatile memories, on the other hand, can retain the stored information. Currently, DRAM and SRAM are extensively used in personal computers and work stations, mainly because of DRAM's attributes of high density and low cost, and SRAM's attribute of high speed. The nonvolatile memory is used extensively in portable electronics systems such as the cellular phone, digital camera, and smart IC cards, mainly because of its attributes of low-power consumption and nonvolatility.

6.6.1 DRAM

Modern DRAM technology consists of a cell array using a storage cell structure shown[16] in Fig. 39. The cell includes an MOSFET and an MOS capacitor [i.e., one transistor/one capacitor (1T/1C) cell]. The MOSFET acts as a switch to control the writing, refreshing, and read-out actions of the cell. The capacitor is used for charge storage. During the write cycle, the MOSFET is turned on so that the logic state in the bit line is transferred to the storage capacitor. For practical applications, charges stored in the capacitor will be gradually lost because of the small but nonnegligible leakage current of the

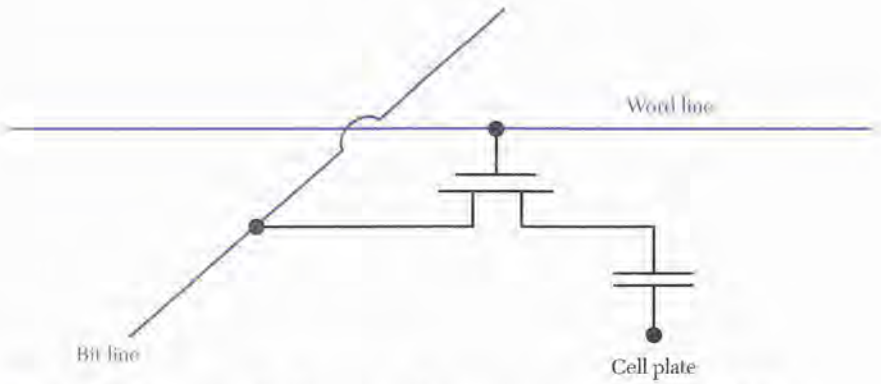


Fig. 39 Basic configuration of a dynamic random access memory (DRAM) cell.[16]

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storage node. Consequently, the operation of DRAM is "dynamic," since the data need to be "refreshed" periodically within a fixed interval, typically 2–50 ms.

The 1T/1C DRAM cell has the advantages of very simple and small area construction. In order to increase the storage density of a chip, aggressive scaling of the cell size is necessary. However, this will degrade the storage capability of the capacitor, since the capacitor electrode area will be diminished as well. To solve this problem, three-dimension (3-D) capacitor structures are required. Some novel 3-D capacitor structures are discussed in Chapter 14. The utilization of high dielectric-constant materials to replace conventional oxide-nitride composite layers (dielectric constant: 4 ~ 6) as the capacitor dielectric materials can also be employed to increase the capacitance.

6.6.2 SRAM

SRAM is a matrix of static cells using a bistable flip-flop structure to store the logic state, as shown in Fig. 40. The flip-flop consists of two cross-coupled CMOS inverters (T1,T3 and T2,T4). The output of the inverter is connected to the input node of the other inverter. This configuration is called "latched." Two additional n-channel MOSFETs, T5 and T6, with their gates connected to the word line, are used to access the SRAM cell. The operation of the SRAM is static since the logic state is sustained as long as the power is applied. Therefore SRAM does not have to be refreshed. The two p-channel MOSFETs (T1 and T2) in the inverters are used as the load transistors. There is essentially no dc current flow through the cell, except during switching. In some situations, p-channel polysilicon TFTs or polysilicon resistors are used instead of bulk p-channel MOSFETs. These

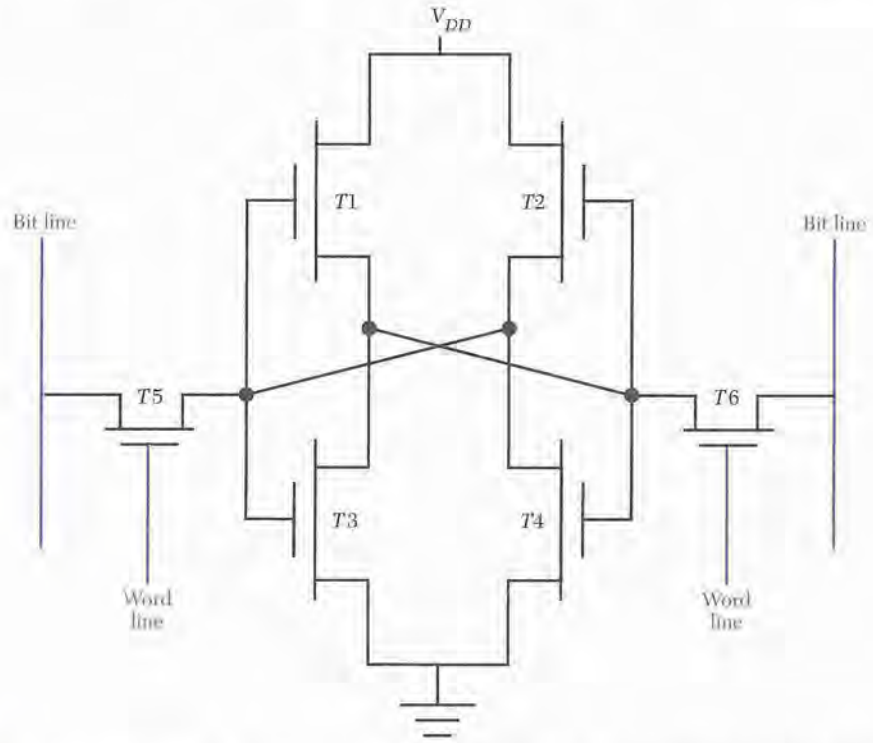


Fig. 40 Configuration of a CMOS SRAM cell. T1 and T2 are load transistors (p-channel). T3 and T4 are drive transistors (n-channel). T5 and T6 are access transistors (n-channel).

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polysilicon load devices can be fabricated over the bulk n-channel MOSFETs. 3-D integration can effectively reduce the cell area and, therefore, increases the storage capacity of the chip.

6.6.3 Nonvolatile Memory

When the gate electrode of a conventional MOSFET is modified so that semipermanent charge storage inside the gate is possible, the new structure becomes a nonvolatile memory device. Since the first nonvolatile memory device proposed[17] in 1967, various device structures have been made. Nonvolatile memory devices have been used extensively in ICs such as the erasable-programmable read-only memory (EPROM), electrically erasable-programmable read-only memory (EEPROM), and flash memory.

Most of the devices used for present-day nonvolatile memories have a floating-gate structure (Fig. 41*a*). Charges are injected from the silicon substrate or the drain to the floating gate across the first insulator. They are then stored in the floating gate. This pro-

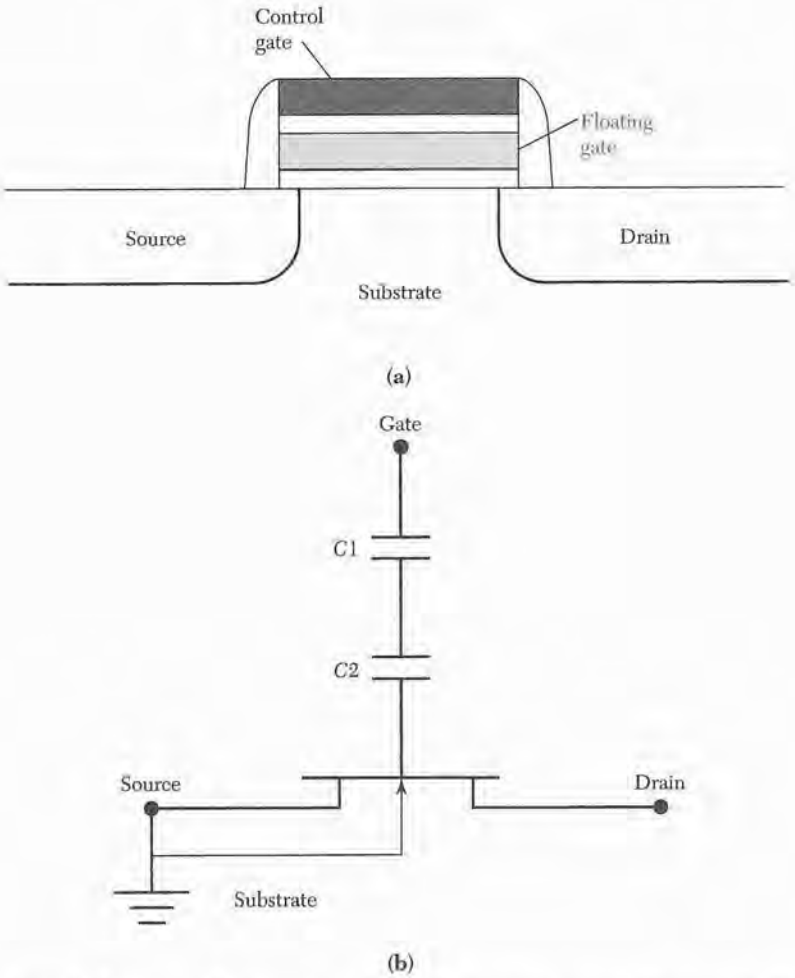


Fig. 41 Nonvolatile semiconductor memory.[17] (*a*) Floating-gate nonvolatile memory. (*b*) Equivalent circuit of the nonvolatile memory.

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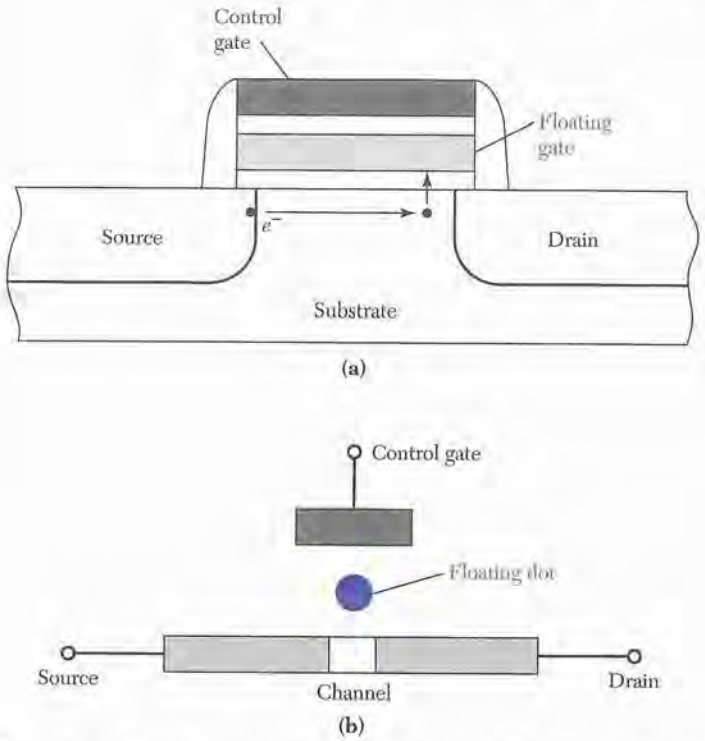


Fig. 42 (*a*) Illustration of hot electron injection in an *n*-channel, floating-gate nonvolatile memory. (*b*) Single-electron memory cell.[18]

cess is called "programming." The equivalent circuit for the floating-gate device can be represented by two capacitors in series for the gate structure, as illustrated in Fig. 41*b*. The stored charge gives rise to a threshold voltage shift, and the device is switched to a high-voltage state (e.g., logic 1). For a well-designed memory device, the charge retention time can be over 10 years. To erase the stored charge and return the device to a low-threshold voltage state (e.g., logic 0), an electrical bias or other means, such as UV light, can be used.

In floating-gate devices, the programming can be done by either hot carrier injection or a tunneling process. Figure 42*a* shows the hot electron injection scheme in an *n*-channel floating-gate device. The hot electrons are "hot" because they are heated to a high-energy state by the high field near the drain. Some of the hot electrons with energy higher than the barrier height of SiO_2/Si conduction band (~3.2 eV) can then surmount the barrier and are injected into the floating gate. Several types of floating-gate devices are differentiated by the erase mechanisms. In the EPROM, which has only a floating gate but no control gate, erasing is done by UV irradiation, since the UV light can excite the stored charges into the conduction band of gate oxide. The EPROM has the advantage of a small cell area due to the 1T/cell structure. Nevertheless, its erase scheme necessitates the use of an expensive package with a quartz window. In addition, the erasing time is long.

The EEPROM uses tunneling process to erase the stored charges. Unlike the EPROM device, in which all cells are erased during erasing, a cell in an EEPROM can be erased only when it is "selected." This function is accomplished through the selective transistor

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contained in each cell. Such "byte-erasable" characteristics make the use of EEPROM more flexible. However, the 2T (one selective transistor plus one storage transistor)/cell feature of EEPROM limits its storage capacity.

The storage cells for a flash memory are divided into several sectors (or blocks). The erasing scheme is performed on one selected sector with the tunneling process. During erasing, all cells in the selected sector are erased simultaneously. The erasing speed is much faster than EPROM and the 1T/cell feature makes the storage capacity of the flash memory higher than EEPROM.

A related device structure is the single-electron memory cell (SEMC), which is a limiting case of the floating gate stucture.[18] By reducing the length of the floating gate to ultrasmall dimensions, say 10 nm, we obtain the SEMC. A cross-sectrional view of a SEMC is shown in Fig. 42*b*. The floating dot corresponds to the floating gate in Fig. 42*a*. Because of its small size the capacitance is also very small (~1 aF). When an electron tunnels into the floating dot, because of the small capacitance, a large tunneling barrier can arise to prevent the transfer of another electron. SEMC is an ultimate floating gate memory cell, since we need only one electron for information storage. A single-electron memory with densities as high as 256 terabits (256×10^{12} bits) that can be operated at room temperature has been projected.

6.7 THE POWER MOSFET

The input impedance of MOS devices is very high because of the insulating SiO_2 between the gate and semiconductor channel. This feature makes the MOSFET an attractive candidate in power-device applications. Because of the high-input impedance, the gate leakage is very low, and thus, the power MOSFET does not require complex input drive circuitry compared with bipolar devices. In addition, the switching speed of the power MOSFET is much faster than that of the power bipolar device. This is because the unipolar characteristics of MOS operation do not involve storage or recombination of minority carriers during turn-off.

Figure 43 shows three basic power MOSFET structures.[19] Unlike the MOSFET structure used in ULSI circuits, the power MOSFETs employ a vertical structure with the source and drain at the top and bottom surfaces of the wafer, respectively. This vertical scheme has the benefit of large channel width and reduced field crowding at the gate. These properties are important for power applications.

Figure 43*a* is the V-MOSFET in which the gate has a V-shaped groove. The V-shaped groove can be formed by preferential wet-etching using a KOH solution. When the gate voltage is larger than the threshold voltage, an inversion channel is induced at the surface along the edge of the V-shaped groove and forms the conductive path between source and drain. One main limitation of V-MOSFET development is related to process control. The high field at the tip of the V-shaped groove may lead to current crowding there and degrade device performance.

Figure 43*b* shows the cross-section of the U-MOSFET. It is similar to the V-MOSFET. The U-shaped trench is formed by reactive ion etching, and the electric fields at the bottom cornors are substantially lower than that at the tip of the V-shaped grove. Another power MOSFET is the D-MOSFET, shown in Fig. 43*c*. The gate is formed at the top surface and then serves as a mask for the subsequent double-diffusion process. The double diffusion process (this is the reason why it is call "D"-MOSFET) is used to form the p-base and n^+ source portions. The advantages of D-MOSFET are its short drift time across the p-base region and the avoidance of high-field corners.

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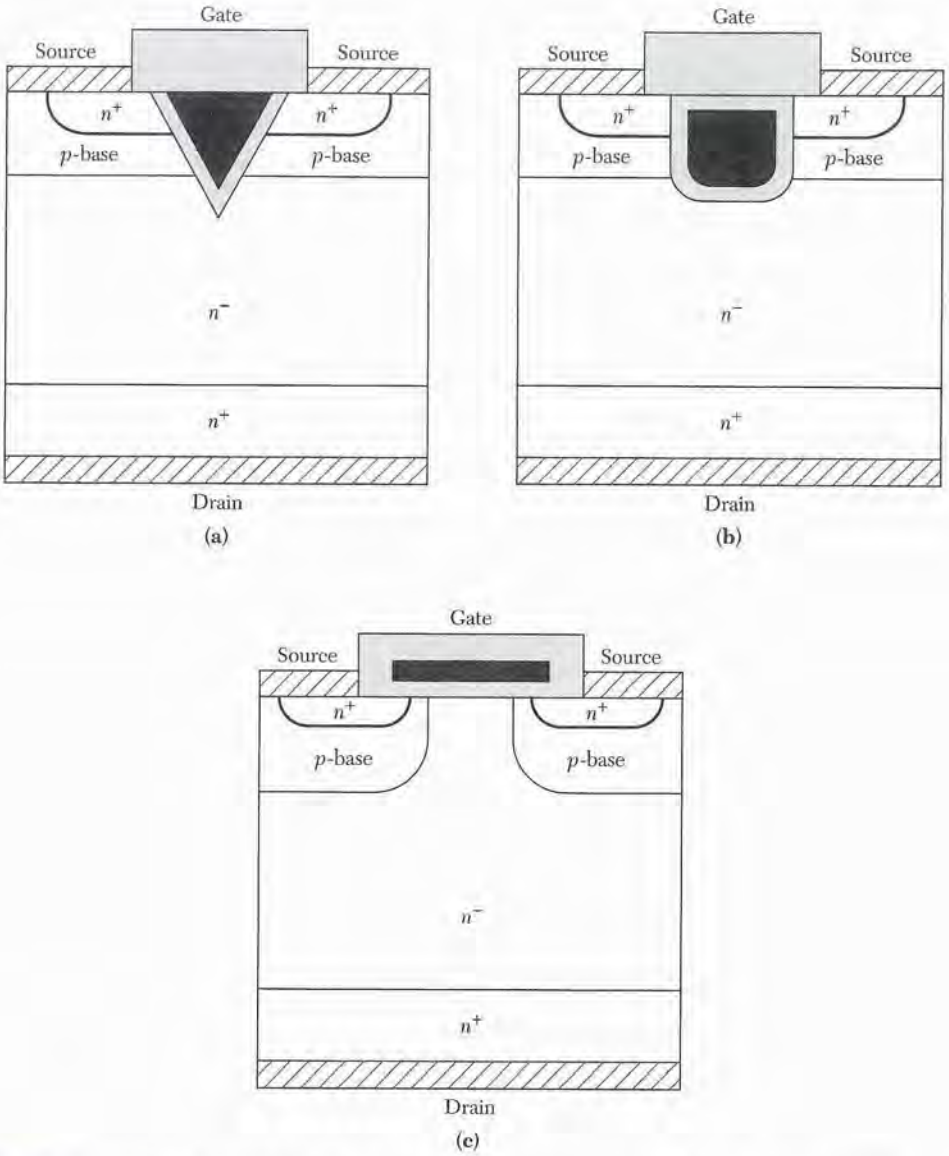


Fig. 43 (*a*) V-shaped MOS (VMOS), (*b*) U-shaped MOS (UMOS), and (*c*) double-diffused MOS (DMOS) power device structures.[19]

There is an n^- drift region in the drain for all the three power MOSFET structures. The doping concentration of n^- drift region is lower than p-base region. Therefore, when a positive voltage is applied to the drain and the drain/p-base junction is reverse biased, most of the depletion width will be developed across the n^- drift region. Therefore, the doping level and width of the n^- drift region are important parameters that determine the drain blocking voltage capability. On the other hand, there exists a parasitic n-p-n^--n^+ device in the power MOSFET structures. To prevent the action of the bipolar transistor during power MOSFET operation, the p-base and n^+ source (emitter) are shorted, as shown in Fig. 43. This can keep the p-base at a fixed potential.

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SUMMARY

In this chapter, we introduced the basic characteristics and the operational principles of the MOSFET. The core component of the MOSFET is the MOS diode. Charge distributions at the oxide/semiconductor interface (accumulation, depletion, and inversion) in an MOS device can be controlled by the gate voltage. The MOSFET is formed when a source and a drain are placed adjacent to the MOS diode. Output current (i.e., drain current) is controlled by varying the gate and drain voltages. The threshold voltage is the main parameter that determines the on–off characteristics of a MOSFET. Adjustment of the threshold voltage can be done by choosing suitable substrate doping, oxide thickness, substrate bias, and gate materials (see Eq. 45).

The Si MOSFET is the most important device for advanced integrated-circuit (IC) applications. Its success is mainly ascribed to the high-quality SiO_2 material and the stable Si/ SiO_2 interface properties. To meet the stringent requirement of low power consumption in an IC chip, CMOS technology is currently the only viable solution and is widely implemented. Superior performance in power dissipation can be understood from the discussion of the CMOS inverter given in Section 6.4.

Scaling down of device dimensions is a continuing trend in CMOS technology to increase the device density, operating speed, and functionality of a chip. The short-channel effects, however, cause deviations in device operation and require attention in device scaling. Optimization of device structural parameters depends on the main requirement of the applications, such as minimized power consumption or faster operating speed.

TFT and SOI are MOSFET devices fabricated on insulating substrates, in contrast to the conventional MOSFETs fabricated on a bulk Si substrate. TFT uses an amorphous or a polycrystalline semiconductor as the active channel layer. The carrier mobility of the TFT is degraded by the presence of a large number of defects in the channel. However, TFT can be applied to a large-area substrate, which is difficult for bulk MOS technology, e.g., the switching element of pixels in a large-area flat-panel display. TFT can also be used as the load devices in the SRAM cell. SOI MOSFETs use a monocrystalline Si channel layer. Compared with bulk-MOS devices, SOI devices provide lower parasitic junction capacitance and improved resistance to radiation damage. SOI is also more attractive for low-power, high-speed applications.

MOSFETs have been employed for semiconductor memory applications, including DRAM, SRAM, and nonvolatile memory. These products share a significant portion of the IC market. Owing to the aggressive shrinkage of device size, the storage capacity of the MOS memories improves rapidly. For example, the DRAM density has been doubled every 18 months and the single-electron memory has been projected to reach a multiterabit level. Finally, we discussed three power MOSFETs. These devices use a vertical structure to allow higher operating voltage and current.

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19. B. J. Baliga, *Power Semiconductor Devices*, PWS Publishers, Boston, 1996.

PROBLEMS (* DENOTES DIFFICULT PROBLEMS)

FOR SECTION 6.1 THE MOS DIODE

1. Plot the band diagram of an ideal MOS diode with n-type substrate at $V_G = V_T$.
2. Plot the band diagram of an n^+-polysilicon-gated MOS diode with p-type substrate at $V_G = 0$.
3. Plot the band diagram of an n^+-polysilicon–gated MOS diode with p-type substrate at flat-band condition.
4. Plot (a) the charge distribution, (b) electric-field distribution, and (c) potential distribution of an ideal MOS diode with n-type substrate under inversion.
5. For a metal-SiO_2-Si capacitor having $N_A = 5 \times 10^{16}$ cm^{-3}, calculate the maximum width of surface depletion region.
6. For a metal-SiO_2-Si capacitor having $N_A = 5 \times 10^{16}$ cm^{-3} and $d = 8$ nm, calculate the minimum capacitance on the C-V curve.

*7. For an ideal Si-SiO_2 MOS diode with $d = 5$ nm, $N_A = 10^{17}$ cm^{-3}, find the applied voltage and the electric field at the interface required to make the silicon surface intrinsic.

8. For an ideal Si-SiO_2 MOS diode with $d = 10$ nm, $N_A = 5 \times 10^{16}$ cm^{-3}, find the applied voltage and the electric field at the interface required to bring about strong inversion.

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*9. Assume that the oxide trapped charge Q_{ot} in an oxide layer has an uniform volume charge density, $\rho_{ot}(y)$, of $q \times 10^{17}$ cm^{-3}, where y is the distance from the location of the charge to the metal-oxide interface. The thickness of the oxide layer is 10 nm. Find the change in the flat-band voltage due to Q_{ot}.

10. Assume that the oxide trapped charge Q_{ot} in an oxide layer is a charge sheet with an area density of 5×10^{11} cm^{-2} locating solely at $y = 5$ nm. The thickness of the oxide layer is 10 nm. Find the change in the flat-band voltage due to Q_{ot}.

11. Assume that the oxide trapped charge Q_{ot} in an oxide layer has a triangular distribution: $\rho_{ot}(y) = q \times (5 \times 10^{23} \times y)$ cm^{-3}. The thickness of oxide layer is 10 nm. Find the change in the flat-band voltage due to Q_{ot}.

12. Assume that initially there is a sheet of mobile ions at the metal-SiO_2 interface. After a long period of electrical stressing under a high positive gate voltage and raised temperature condition, the mobile ions completely drift to the SiO_2-Si interface. This leads to a change of 0.3 V in the flat-band voltage. The thickness of oxide layer is 10 nm. Find the area density of Q_m.

FOR SECTION 6.2 MOSFET FUNDAMENTALS

13. Derive Eq. 34 from Eq. 33 in the text assuming $V_D << (V_G - V_T)$.

*14. Derive the *I-V* characteristics of a MOSFET with the drain and gate connected together and the source and substrate grounded. Can one obtain the threshold voltage from these characteristics?

15. Consider a long-channel MOSFET with $L = 1$ μm, $Z = 10$ μm, $N_A = 5 \times 10^{16}$ cm^{-3}, $\mu_n = 800$ cm^2/ V-s, $C_o = 3.45 \times 10^{-7}$ F/cm^2, and $V_T = 0.7$ V. Find V_{Dsat} and I_{Dsat} for $V_G = 5$ V.

16. Consider a submicron MOSFET with $L = 0.25$ μm, $Z = 5$ μm, $N_A = 10^{17}$ cm^{-3}, $\mu_n = 500$ cm^2/ V-s, $C_o = 3.45 \times 10^{-7}$ F/cm^2, and $V_T = 0.5$ V. Find the channel conductance for $V_G = 1$ V and $V_D = 0.1$ V.

17. For the device stated in Prob. 16, find the transconductance.

18. An *n*-channel, n^+-polysilicon-SiO_2-Si MOSFET has $N_A = 10^{17}$ cm^{-3}, $Q_f/q = 5 \times 10^{10}$ cm^{-2}, and $d = 10$ nm. Calculate the threshold voltage.

19. For the device stated in Prob. 18, boron ions are implanted to increase the threshold voltage to +0.7 V. Find the implant dose, assume that the implanted ions form a sheet of negative charges at the Si-SiO_2 interface.

20. A *p*-channel, n^+-polysilicon-SiO_2-Si MOSFET has $N_D = 10^{17}$ cm^{-3}, $Q_f/q = 5 \times 10^{10}$ cm^{-2}, and $d = 10$ nm. Calculate the threshold voltage.

21. For the device stated in Prob. 20, boron ions are implanted to decrease the value of threshold voltage to −0.7 V. Find the implant dose, assuming that the implanted ions form a sheet of negative charges at the Si-SiO_2 interface.

22. For the device stated in Prob. 20, if the n^+ poly-Si gate is replaced by p^+ poly-Si gate, what will the threshold voltage be?

23. A field transistor with a structure similar to Fig. 21 in the text has $N_A = 10^{17}$ cm^{-3}, $Q_f/q = 10^{11}$ cm^{-2}, and an n^+ polysilicon local interconnect as the gate electrode. If the requirement for sufficient isolation between device and well is $V_T > 20$ V, calculate the minimum field oxide thickness.

24. A MOSFET has a threshold voltage of $V_T = 0.5$ V, a subthreshold swing of 100 mV/decade, and a drain current of 0.1 μA at V_T. What is the subthreshold leakage current at $V_G = 0$?

25. For the device stated in Prob. 24, calculate the reverse substrate-source voltage required to reduce the leakage current by one order of magnitude. ($N_A = 5 \times 10^{17}$ cm^{-3}, $d = 5$ nm).

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FOR SECTION 6.3 MOSFET SCALING

26. When the linear dimensions of MOSFET are scaled down by a factor of 10 based on the constant field scaling, what is the scaling factor for the corresponding switching energy?

*27. Based on the charge-sharing model, Fig. 24, show that the threshold voltage roll-off is given by Eq. 47.

FOR SECTION 6.4 CMOS AND BiCMOS

28. Describe the pros and cons of BiCMOS.

FOR SECTION 6.5 MOSFET ON INSULATOR

29. For an n-channel FD-SOI device having $N_A = 5 \times 10^{17}$ cm^{-3} and $d = 4$ nm, calculate the maximum allowable thickness for Si channel layer (d_{Si}).

30. For an n-channel SOI device with n$^+$-polysilicon gate having $N_A = 5 \times 10^{17}$ cm^{-3}, $d =$ 4 nm, and $d_{Si} = 30$ nm, calculate the threshold voltage. Assume that Q_f, Q_{ot}, and Q_m are all zero.

31. For the device stated in Prob. 29, calculate the range of V_T distribution if the thickness variation of d_{Si} across the wafer is ±5 nm.

FOR SECTION 6.6 MOS MEMORY STRUCTURES

32. What is the capacitance of a DRAM capacitor if it is planar, 1 μm × 1 μm, with an oxide thickness of 10 nm? Calculate the capacitance if the same surface area is used for a trench that is 7 μm deep and with the same oxide thickness.

33. A DRAM must operate with a minimum refresh time of 4 ms. The storage capacitor in each cell has a capacitance of 50 fF and is fully charged to 5 V. Find the worst-case leakage current (i.e., during the refresh cycle 50% of the stored charge is lost) that the dynamic node can tolerate.

34. A floating-gate nonvolatile memory has an initial threshold voltage of –2 V, and a linear-region drain conductance of 10 μmhos at a gate voltage of –5 V. After a write operating, the drain conductance increases to 40 μmhos at the same gate voltage. Find the threshold voltage shift.

FOR SECTION 6.7 THE POWER MOSFET

35. A power MOSFET has an n^+-polysilicon gate and a p-base with $N_A = 10^{17}$ cm^{-3}. Gate oxide thickness $d = 100$ nm. Calculate the threshold voltage.

36. For the device stated in Prob. 35, calculate the effect of a positive fixed charge density of 5×10^{11} cm^{-3} on the threshold voltage.

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CHAPTER 7

MESFET and Related Devices

- 7.1 METAL-SEMICONDUCTOR CONTACTS
- 7.2 MESFET
- 7.3 MODFET
- SUMMARY

The metal-semiconductor field-effect transistor (MESFET) has current-voltage characteristics similar to that of a metal-oxide-semiconductor field-effect transistor (MOSFET). However, it uses a metal-semiconductor rectifying contact instead of a MOS structure for the gate electrode. In addition, the source and drain contacts of MESFET are ohmic,[§] whereas in a MOSFET they are p–n junctions.

Like other field-effect devices, a MESFET has a negative temperature coefficient at high current levels; that is, the current decreases as temperature increases. This characteristic leads to more uniform temperature distribution and the device is, therefore, thermally stable, even when the active area is large or when many devices are connected in parallel. Furthermore, because MESFETs can be made from compound semiconductors with high electron mobilities, such as GaAs and InP, they have higher switching speeds and higher cutoff frequencies than do silicon MOSFETs.

The basic building block of a MESFET is the metal-semiconductor contact. This contact is electrically similar to a one-sided abrupt p–n junction, yet it can be operated as a majority carrier device with inherently fast response. There are two types of metal-semiconductor contacts: the rectifying and the nonrectifying or ohmic types. In this chapter, we begin with the two types of contacts and then consider the basic characteristics and microwave performance of the MESFET. In the last section we discuss modulation-doped field-effect transistor (MODFET), which is similar to the device configuration of a MESFET but can offer even higher-speed performance.

Specifically, we cover the following topics:

- Rectifying metal-semiconductor contact and its current-voltage characteristics.
- Ohmic metal-semiconductor contact and its specific contact resistance.
- The MESFET and its high-frequency performance.

[§] The concept of rectifying was discussed in Chapter 4. The concept of ohmic contact is presented in Section 7.1.

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- The MODFET and its two-dimensional electron gas.
- A comparison of three field-effect transistors—MOSFET, MESFET, and MODFET.

7.1 METAL-SEMICONDUCTOR CONTACTS

The first practical semiconductor device was the metal-semiconductor contact in the form of a point contact rectifier, that is, a metallic whisker pressed against a semiconductor. The device found many applications beginning in 1904. In 1938, Schottky, suggested that the rectifying behavior could arise from a potential barrier as a result of the stable space charges in the semiconductor.[1] The model arising from this concept is known as the Schottky barrier. Metal-semiconductor contacts can also be nonrectifying; that is, the contact has a negligible resistance regardless of the polarity of the applied voltage. This type of contact is called an ohmic contact. All semiconductor devices as well as integrated circuits need ohmic contact to make connections to other devices in an electronic system. We consider the energy band diagram and the current-voltage characteristics of both the rectifying and ohmic metal-semiconductor contacts.

7.1.1 Basic Characteristics

The characteristics of point contact rectifiers were not reproducible from one device to another. They have been largely replaced by metal-semiconductor contacts fabricated by planar processes (see Chapters 10–14). A schematic diagram of such a device is shown in Fig. 1*a*. To fabricate the device, a window is opened in an oxide layer, and metal layer is deposited in a vacuum system. The metal layer covering the window is subsequently defined by a lithographic step. We consider a one-dimensional structure of the metal-semiconductor contact shown in Fig. 1*b*, which corresponds to the central section in Fig. 1*a*, between the dashed lines.

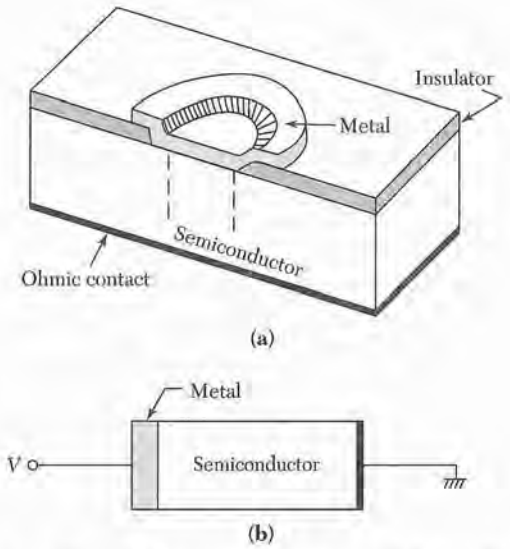


Fig. 1 (*a*) Perspective view of a metal-semiconductor contact fabricated by the planar process. (*b*) One-dimensional structure of a metal-semiconductor contact.

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Figure 2*a* shows the energy band diagram of an isolated metal adjacent to an isolated n-type semiconductor. Note that the metal work function $q\phi_m$ is generally different from the semiconductor work function $q\phi_s$. The work function is defined as the energy difference between the Fermi level and the vacuum level. Also shown is the electron affinity $q\chi$, which is the energy difference between the conduction band edge and the vacuum level in the semiconductor. When the metal makes intimate contact with the semiconductor, the Fermi levels in the two materials must be equal at thermal equilibrium. In addition, the vacuum level must be continuous. These two requirements determine a unique energy band diagram for the ideal metal-semiconductor contact, as shown in Fig. 2*b*.

For this ideal case, the barrier height $q\phi_{Bn}$ is simply the difference between the metal work function and the semiconductor electron affinity[§]:

$$q\phi_{Bn} = q\phi_m - q\chi. \tag{1}$$

Similarly, for the case of an ideal contact between a metal and a *p*-type semiconductor, the barrier height $q\phi_{Bp}$ is given by

$$q\phi_{Bp} = E_g - (q\phi_m - q\chi), \tag{2}$$

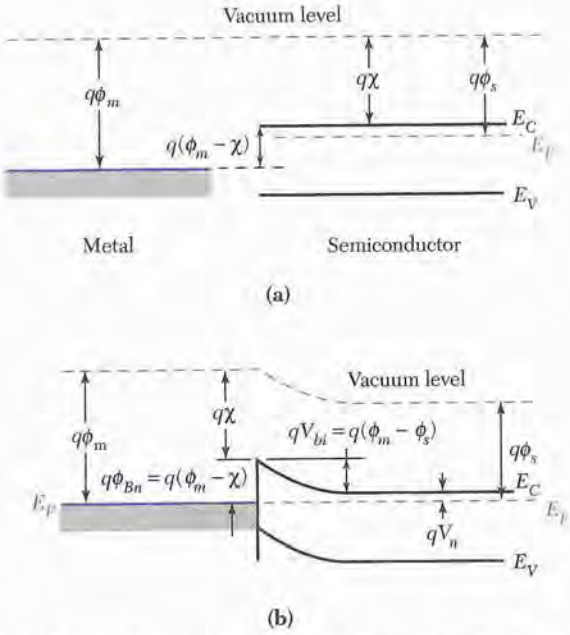


Fig. 2 (*a*) Energy band diagram of an isolated metal adjacent to an isolated n-type semiconductor under thermal nonequilibrium condition. (*b*) Energy band diagram of a metal-semiconductor contact in thermal equilibrium.

§ Both $q\phi_{Bn}$ (in electron volts) and ϕ_{Bn} (in volts) are referred to as the barrier height.

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where E_g is the bandgap of the semiconductor. Therefore, for a given semiconductor and for any metal, the sum of the barrier heights on n-type and p-type substrates is expected to be equal to the bandgap:

$$\boxed{q\left(\phi_{Bn}+\phi_{Bp}\right)=E_g.} \tag{3}$$

On the semiconductor side in Fig. 2*b*, V_{bi} is the built-in potential that is seen by electrons in the conduction band trying to move into the metal.

$$V_{bi}=\phi_{Bn}-V_n\ . \tag{4}$$

The qV_n is the distance between the bottom of the conduction band and the Fermi level. Similar results can be given for the p-type semiconductor.

Figure 3 shows the measured barrier heights for n-type silicon[2] and n-type gallium arsenide.[3] Note that $q\phi_{Bn}$ increases with increasing $q\phi_m$. However, the dependence is not as strong as predicted by Eq. 1. This is because in practical Schottky diodes, the disruption of the crystal lattice at the semiconductor surface produces a large number of surface energy states located in the forbidden bandgap. These surface states can act as donors or acceptors that influence the final determination of the barrier height. For silicon and gallium arsenide, Eq. 1 generally underestimated the n-type barrier height and Eq. 2 overestimates the p-type barrier height. The sum of $q\phi_{Bn}$ and $q\phi Bp$, however, is in agreement with Eq. 3.

Figure 4 shows the energy band diagrams for metals on both n-type and p-type semiconductors under different biasing conditions. Consider the n-type semiconductor first. When the bias voltage is zero, as shown in the left side of Fig. 4*a*, the band diagram is under a thermal equilibrium condition. The Fermi levels for both materials are equal. If we apply a positive voltage to the metal with respect to the n-type semiconductor, the semiconductor-to-metal barrier height decreases as shown on the left side of Fig. 4*b*. This is a forward bias. When a forward bias is applied, electrons can move easily from the semiconductor into the metal because the barrier has been reduced by a voltage V_F. For a

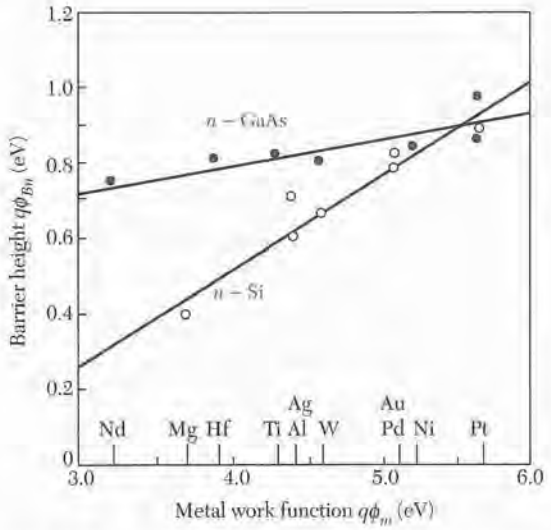


Fig. 3 Measured barrier height for metal-silicon and metal-gallium arsenide contacts.[2,3]

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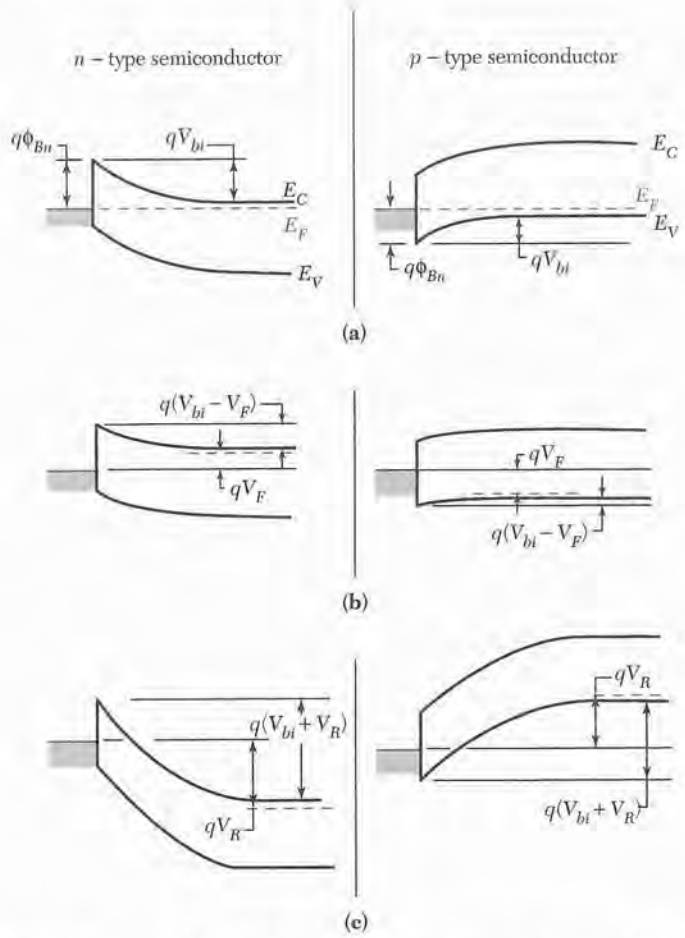


Fig. 4 Energy band diagrams of metal n-type and p-type semiconductors under different biasing conditions: (*a*) thermal equilibrium; (*b*) forward bias; and (*c*) reverse bias.

reverse bias (i.e., a negative voltage is applied to the metal), the barrier has been increased by a voltage V_R, as depicted on the left side of Fig. 4*c*. It is more difficult for electrons to flow from the semiconductor into the metal. We have similar results for p-type semiconductor, however, the polarities must be reversed. In the following derivations, we consider only the metal–n-type semiconductor contact. The results are equally applicable to a p-type semiconductor with an appropriate change of polarities.

The charge and field distributions for a metal-semiconductor contact are shown in Fig. 5*a* and 5*b*, respectively. The metal is assumed to be a perfect conductor; the charge transferred to it from the semiconductor exists in a very narrow region at the metal surface. The extent of the space charge in the semiconductor is W, i.e., $\rho_s = qN_D$ for $x < W$ and $\rho_s = 0$ for $x > W$. Thus, the charge distribution is identical to that of a one-sided abrupt p^+-n junction.

The magnitude of the electric field is decreasing linearly with distance. The maximum electric field $\mathscr{E}_m$ is located at the interface. The electric field distribution is then given by

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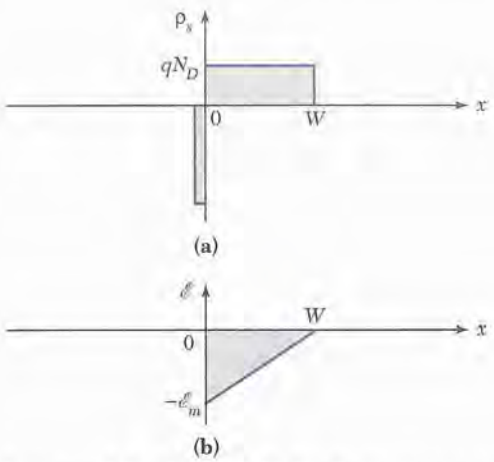


Fig. 5 (*a*) Charge distribution and (*b*) electric-field distribution in a metal-semiconductor contact.

$$\left|\mathscr{E}(x)\right| = \frac{qN_D}{\varepsilon_s}(W - x) = \mathscr{E}_m - \frac{qN_D}{\varepsilon_s}x, \tag{5}$$

$$\mathscr{E}_m = \frac{qN_D W}{\varepsilon_s}, \tag{6}$$

where ε_s is the dielectric permittivity of the semiconductor. The voltage across the space-charge region, which is represented by the area under the field curve in the Fig. 5*b*, is given by

$$V_{bi} - V = \frac{\mathscr{E}_m W}{2} = \frac{qN_D W^2}{2\varepsilon_s}. \tag{7}$$

The depletion-layer width W is expressed as

$$W = \sqrt{2\varepsilon_s(V_{bi} - V)/qN_D}, \tag{8}$$

and the space-charge density, Q_{SC}, in the semiconductor is given as

$$Q_{SC} = qN_D W = \sqrt{2q\varepsilon_s N_D(V_{bi} - V)} \quad \text{C/cm}^2, \tag{9}$$

where the voltage V equal to $+V_F$ for forward bias and to $-V_R$ for reverse bias. The depletion-layer capacitance C per unit area can be calculated by using Eq. 9 :

$$C = \left|\frac{\partial Q_{sc}}{\partial V}\right| = \sqrt{\frac{q\varepsilon_s N_D}{2(V_{bi} - V)}} = \frac{\varepsilon_s}{W} \quad \text{F/cm}^2 \tag{10}$$

and

$$\boxed{\frac{1}{C^2} = \frac{2(V_{bi} - V)}{q\varepsilon_s N_D} \left(\text{F/cm}^2\right)^{-2}}. \tag{11}$$

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We can differentiate $1/C^2$ with respect to V. Rearranging terms we obtain :

$$N_D = \frac{2}{q\varepsilon_s}\left[\frac{-1}{d(1/C^2)/dV}\right]. \tag{12}$$

Thus, measurements of the capacitance C per unit area as a function of voltage can provide the impurity distribution from Eq. 12. If N_D is constant throughout the depletion region, we should obtain a straight line by plotting $1/C^2$ versus V. Figure 6 is a plot of the measured capacitance versus voltage for tungsten-silicon and tungsten-gallium arsenide Schottky diodes.[4] From Eq. 11, the intercept at $1/C^2 = 0$ corresponds to the built-in potential V_{bi}. Once V_{bi} is determined, the barrier height ϕ_{Bn} can be calculated from Eq. 4.

EXAMPLE 1

Find the donor concentration and the barrier height of the tungsten-silicon Schottky diode shown in Fig. 6.

SOLUTION The plot of $1/C^2$ versus V is a straight line, which implies that the donor concentration is constant throughout the depletion region. We find

$$\frac{d\left(1/C^2\right)}{dV} = \frac{6.2\times10^{15} - 1.8\times10^{15}}{-1-0} = -4.4\times10^{15}\ \frac{\left(\text{cm}^2/\text{F}\right)^2}{\text{V}},$$

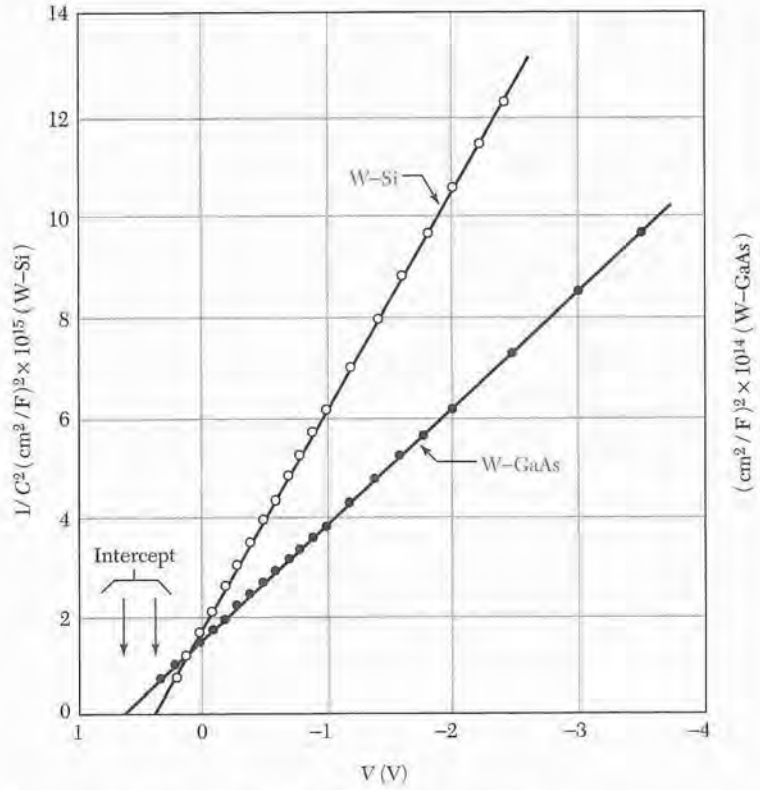


Fig. 6 $1/C^2$ versus applied voltage for W-Si and W-GaAs diodes.[4]

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From Eq. 12,

$$N_D = \left[\frac{2}{1.6\times10^{-19}\times(11.9\times8.85\times10^{-14})}\right]\times\left(\frac{1}{4.4\times10^{15}}\right) = 2.7\times10^{15}\ \text{cm}^{-3},$$

$$V_n = 0.0259\times\ln\left(\frac{2.86\times10^{19}}{2.7\times10^{15}}\right) = 0.24\ \text{V}.$$

Since the intercept V_{bi} is 0.42 V, then the barrier height is $\phi_{Bn} = 0.42 + 0.24 = 0.66$ V.

7.1.2 The Schottky Barrier

A Schottky barrier refers to a metal-semiconductor contact having a large barrier height (i.e., ϕ_{Bn} or $\phi_{Bp} >> kT$) and a low doping concentration that is less than the density of states in the conduction band or valence band.

The current transport in a Schottky barrier is due mainly to majority carrier, in contrast to a p–n junction, where current transport is due mainly to minority carriers. For Schottky diodes operated at moderate temperature (e.g., 300 K), the dominate transport mechanism is thermionic emission of majority carriers from the semiconductor over the potential barrier into the metal.

Figure 7 illustrates the thermionic emission process.[5] At thermal equilibrium (Fig. 7*a*), the current density is balanced by two equal and opposite flows of carriers, thus there is zero net currents. Electrons in the semiconductor tend to flow (or emit) into the metal, and there is an opposing balanced flow of electrons from metal into the semiconductor. These current components are proportional to the density of electrons at the boundary.

As discussed in Section 3.5 of Chapter 3, at the semiconductor surface an electron can be thermionically emitted into the metal if its energy is above the barrier height. Here the semiconductor work function $q\phi_s$ is replaced by $q\phi_{Bn}$, and

$$n_{th} = N_C \exp\left(-\frac{q\phi_{Bn}}{kT}\right), \tag{13}$$

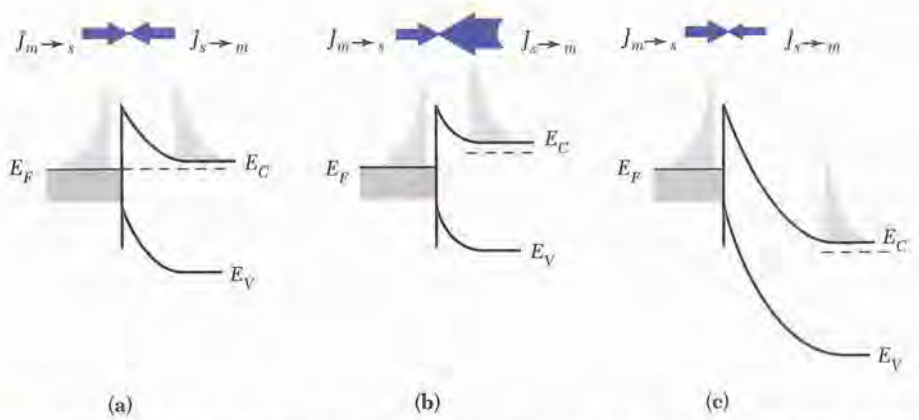


Fig. 7 Current transport by the thermionic emission process. (*a*) Thermal equilibrium; (*b*) forward bias; and (*c*) reverse bias.[5]

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where N_C is the density of states in the conduction band. At thermal equilibrium we have

$$|J_{m\to s}| = |J_{s\to m}| \propto n_{th} \tag{14}$$

or

$$|J_{m\to s}| = |J_{s\to m}| = C_1 N_C \exp\left(-\frac{q\phi_{Bn}}{kT}\right), \tag{14a}$$

where $J_{m\to s}$ is the current from the metal to the semiconductor, $J_{s\to m}$ is the current from the semiconductor to the metal, and C_1 is a proportionality constant.

When a forward bias V_F is applied to the contact (Fig. 7*b*), the electrostatic potential difference across the barrier is reduced, and the electron density at the surface increases to

$$n_{th} = N_C \exp\left[-\frac{q(\phi_{Bn} - V_F)}{kT}\right]. \tag{15}$$

The current $J_{s\to m}$ that results from the electron flow out of the semiconductor is therefore altered by the same factor (Fig. 7*b*). The flux of electrons from the metal to the semiconductor, however, remains the same because the barrier ϕ_{Bn} remains at its equilibrium value. The net current under forward bias is then

$$\begin{aligned} J &= J_{s\to m} - J_{m\to s} \\ &= C_1 N_C \exp\left[-\frac{q(\phi_{Bn} - V_F)}{kT}\right] - C_1 N_C \exp\left(-\frac{q\phi_{Bn}}{kT}\right) \\ &= C_1 N_C e^{-q\phi_{Bn}/kT}\left(e^{qV_F/kT} - 1\right). \end{aligned} \tag{16}$$

Using the same argument for the reverse-bias condition(see Fig. 7*c*), the expression for the net current is identical to Eq. 16 except that V_F is replaced by $-V_R$.

The coefficient C_1N_C is found to be equal to A^*T^2, where A^* is called the *effective Richardson constant* (in units of A/K^2-cm^2), and T is the absolute temperature. The value of A^* depend on the effective mass and are equal to 110 and 32 for n- and p-type silicon, respectively, and 8 and 74 for n- and p-type gallium arsenide, respectively.[6]

The current-voltage characteristic of a metal-semiconductor contact under thermionic emission condition is then

$$\boxed{J = J_s\left(e^{qV/kT} - 1\right),} \tag{17}$$

$$\boxed{J_s = A^*T^2 e^{-q\phi_{Bn}/kT},} \tag{17a}$$

where J_s is the saturation current density and the applied voltage V is positive for forward bias and negative for reverse bias. Experimental forward I-V characteristics of two Schottky diodes[4] are shown in Fig. 8. By extrapolating the forward I-V curve to $V = 0$, we can find J_s. From J_s and Eq. 17*a* we can obtain the barrier height.

In addition to the majority carrier (electron) current, a minority-carrier (hole) current exists in a metal n-type semiconductor contact because of hole injection from the

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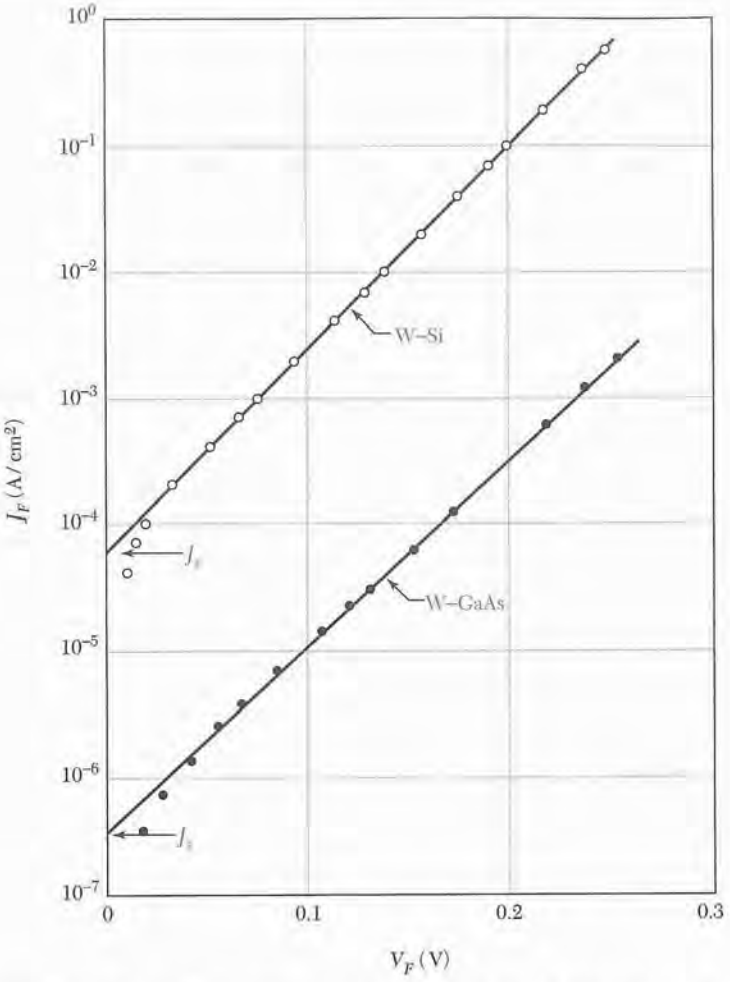


Fig. 8 Forward current density versus applied voltage of W-Si and W-GaAs diodes.[4]

metal to the semiconductor. The hole injection is the same as in a p^+-n junction, which is described in Chapter 4. The current density is given by

$$J_p = J_{po}\left(e^{qV/kT} - 1\right), \tag{18}$$

where

$$J_{po} = \frac{qD_p n_i^2}{L_p N_D}. \tag{18a}$$

Under normal operating conditions, the minority-carrier current is orders of magnitude smaller than the majority-carrier current. Therefore, a Schottky diode is a unipolar device (i.e., predominately only one type of carrier participates in the conduction process).

EXAMPLE 2

For a tungsten-silicon Schottky diode with $N_D = 10^{16}$ cm^{-3}, find the barrier height and depletion-layer width from Fig. 8. Compare the saturation current J_s with J_{po}, assuming that the minority-carrier lifetime in Si is 10^{-6} s.

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SOLUTION From Fig. 8, we have $J_s = 6.5 \times 10^{-5}$ A/cm^2. The barrier height can be obtained from Eq. 17*a*:

$$\phi_{Bn} = 0.0259 \times \ln\left(\frac{110 \times 300^2}{6.5 \times 10^{-5}}\right) = 0.67 \text{ V}.$$

This result is in the close agreement with the *C*-*V* measurement (see Fig. 6 and Ex. 1).

The built-in potential is given by $\phi_{Bn} - V_n$, where

$$V = 0.0259 \times \ln\left(\frac{N_C}{N_D}\right) = 0.0259 \ln\left(\frac{2.86 \times 10^{19}}{1 \times 10^{16}}\right) = 0.17 \text{ V}.$$

Therefore,

$V_{bi} = 0.67 - 0.17 = 0.50$ V.

The depletion-layer width at thermal equilibrium is given by Eq. 8 with $V = 0$:

$$W = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_D}} = 2.6 \times 10^{-5} \text{ cm}.$$

To calculate the minority-carrier current density J_{po}, we need to know D_p, which is 10 cm^2/s for $N_D = 10^{16}$ cm^{-3}, and L_p which is $\sqrt{D_p \tau_p} = \sqrt{10 \times 10^{-6}} = 3.1 \times 10^{-3}$ cm. Therefore,

$$J_{po} = \frac{qD_p n_i^2}{L_p N_D} = \frac{1.6 \times 10^{-19} \times 10 \times \left(9.65 \times 10^9\right)^2}{\left(3.1 \times 10^{-3}\right) \times 10^{16}} = 4.8 \times 10^{-12} \text{ A/cm}^2.$$

The ratio of the two current densities is

$$\frac{J_s}{J_{po}} = \frac{6.5 \times 10^{-5}}{4.8 \times 10^{-12}} = 1.3 \times 10^7.$$

From the comparison, we see that the majority-carrier current is over seven orders of magnitude greater than the minority-carrier current. ◀

7.1.3 The Ohmic Contact

An ohmic contact is defined as a metal-semiconductor contact that has a negligible contact resistance relative to the bulk or series resistance of the semiconductor. A satisfactory ohmic contact should not significantly degrade device performance and can pass the required current with a voltage drop that is small compared with the drop across the active region of the device.

A figure-of-merit for ohmic contacts is the specific contact resistance R_C, defined as

$$\boxed{R_C \equiv \left(\frac{\partial J}{\partial V}\right)_{V=0}^{-1} \Omega\text{-cm}^2.} \tag{19}$$

For metal-semiconductor contacts with low doping concentrations, the thermionic-emission current dominates the current transport, as given by Eq. 17. Therefore,

$$R_C = \frac{k}{qA^* T} \exp\left(\frac{q\phi_{Bn}}{kT}\right). \tag{20}$$

Equation 20 shows that a metal-semiconductor contact with a low barrier height should be used to obtain a small R_C.

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For contacts with high doping concentration, the barrier width becomes very narrow, and the tunneling current becomes dominate. The tunneling current, as described in the upper inset of Fig. 9, is proportional to the tunneling probability, which is given in Section 3.6 of Chapter 3:

$$I \sim \exp\left[-2W\sqrt{2m_n\left(q\phi_{Bn}-qV\right)/\hbar^2}\right], \tag{21}$$

where W is the depletion-layer width, which can be approximated as $\sqrt{(2\varepsilon_s/qN_D)(\phi_{Bn}-V)}$, m_n is the effective mass, and $\hbar$ is the reduced Planck constant. Substituting W into Eq. 21, we obtain

$$I \sim \exp\left[-\frac{C_2\left(\phi_{Bn}-V\right)}{\sqrt{N_D}}\right], \tag{22}$$

where C_2 equal $4\sqrt{m_n\varepsilon_s}/\hbar$. The specific contact resistance for high dopings is thus

$$R_C \sim \exp\left(\frac{C_2\phi_{Bn}}{\sqrt{N_D}}\right) = \exp\left(\frac{4\sqrt{m_n\varepsilon_s}\,\phi_{Bn}}{\sqrt{N_D}\,\hbar}\right). \tag{23}$$

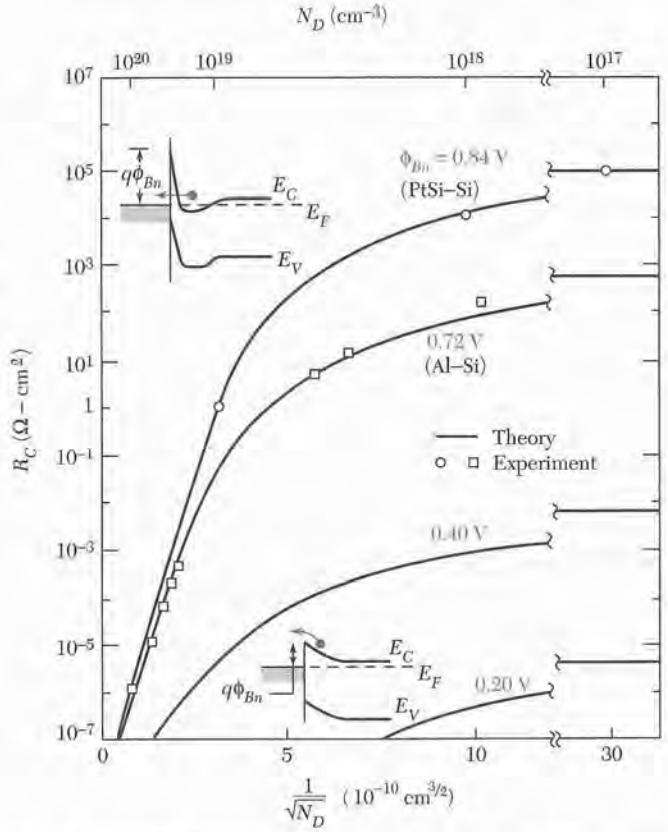


Fig. 9 Calculated and measured values of specific contact resistance. Upper inset shows the tunneling process. Lower inset shows thermionic emission over the low barrier.[6]

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Equation 23 shows that in the tunneling range the specific contact resistance depends strongly on doping concentration and varies exponentially with the factor $\phi_{Bn}/\sqrt{N_D}$.

The calculated values of R_C are plotted[6] in Fig. 9 as a function of $1/\sqrt{N_D}$. For $N_D \geq 10^{19}\,\text{cm}^{-3}$, R_C is dominated by the tunneling process and decreases rapidly with increased doping. On the other hand, for $N_D \leq 10^{17}\,\text{cm}^{-3}$, the current is due to thermionic emission, and R_C is essentially independent of doping. Also shown in Fig. 9 are experimental data for platinum silicide-silicon (PtSi-Si) and aluminum-silicon (Al-Si) diodes. They are in close agreement with the calculated values. Figure 9 shows that a high doping concentration, a low barrier height, or both must be used to obtain a low value of R_C. These two approaches are used for all practical ohmic contacts.

EXAMPLE 3

An ohmic contact has an area of $10^{-5}\,\text{cm}^2$ and a specific contact resistance of $10^{-6}\,\Omega\text{-cm}^2$. The ohmic contact is formed in an n-type silicon. If $N_D = 5 \times 10^{19}\,\text{cm}^{-3}$, and $\phi_{Bn} = 0.8$ V, and the electron effective mass is $0.26\,m_0$, find the voltage drop across the contact when a forward current of 1A flows through it.

SOLUTION The contact resistance for the ohmic contact is

$$\frac{R_C}{A} = 10^{-6}\,\Omega - \text{cm}^2 / 10^{-5}\,\text{cm}^2 = 10^{-1}\,\Omega,$$

$$C_2 = 4\sqrt{m_n \varepsilon_s}/\hbar = \frac{4\sqrt{0.26 \times 9.1 \times 10^{-31} \times \left(1.05 \times 10^{-10}\right)}}{1.05 \times 10^{-34}}$$

$$= 1.9 \times 10^{14}\left(\text{m}^{-3/2}/\text{V}\right).$$

From Eq. 22,

$$I = I_0 \exp\left[-\frac{C_2\left(\phi_{Bn} - \text{V}\right)}{\sqrt{N_D}}\right],$$

$$\left.\frac{\partial I}{\partial \text{V}}\right|_{\text{V}=0} = \frac{A}{R_C} = I_0\left(\frac{C_2}{\sqrt{N_D}}\right)\exp\left(\frac{-C_2\phi_{Bn}}{\sqrt{N_D}}\right)$$

or

$$I_0 = \frac{A}{R_C}\left(\frac{\sqrt{N_D}}{C_2}\right)\exp\left(\frac{C_2\phi_{Bn}}{\sqrt{N_D}}\right)$$

$$= 10 \times \left(\frac{\sqrt{5 \times 10^{19} \times 10^6}}{1.9 \times 10^{14}}\right)\exp\left(\frac{1.9 \times 10^{14} \times 0.8}{\sqrt{5 \times 10^{19} \times 10^6}}\right)$$

$$= 8.13 \times 10^8\ \text{A}.$$

at I = 1A, we have

$$\phi_{Bn} - \text{V} = \frac{\sqrt{N_D}}{C_2}\ln\left(\frac{I_0}{I}\right) = 0.763\text{V}$$

or

V = 0.8 – 0.763 =0.037 V = 37 mV.

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Therefore, there is a negligibly small voltage drop across the ohmic contact. However, the voltage drop may become significant when the contact area is reduced to 10^{-8} cm^2 or smaller.

7.2 MESFET

7.2.1 Device Structures

The metal-semiconductor field-effect transistor (MESFET) was proposed[7] in 1966. The MESFET has three metal-semiconductor contacts—one Schottky barrier for the gate electrode and two ohmic contacts for the source and drain electrodes. A perspective view of a MESFET is illustrated in Fig. 10*a*. The basic device parameters include L, the gate length, Z, the gate width, and a the thickness of the epitaxial layer. Most MESFETs are made of n-type III-V compound semiconductors, such as gallium arsenide, because of their high electron mobilities, which help to minimize series resistances, and because of their high saturation velocities, which result in the increase in the cutoff frequency.

Practical MESFETs are fabricated by using epitaxial layers on semiinsulating substrates to minimize parasitic capacitances. In Fig. 10*a*, the ohmic contacts are labeled source and drain, and the Schottky barrier is labeled as gate. A MESFET is often described in terms of the gate dimensions. If the gate length (L) is 0.5 μm and the gate width (Z)

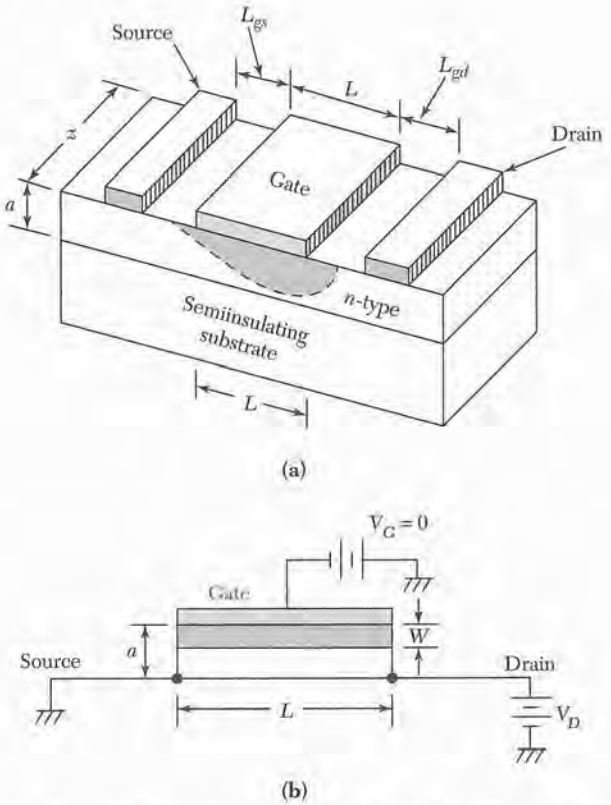


Fig. 10 (*a*) Perspective view of a metal-semiconductor field-effect transistor (MESFET). (*b*) Cross section of the gate region of a MESFET.

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is 300 μm, the device is referred to as a 0.5 × 300 μm device. A microwave- or millimeter-wave device typically has a gate length in the range 0.1–1.0 μm. The thickness a of the epitaxial layer is typically one-third to one-fifth of the gate length. The spacing between the electrodes is one to four times that of the gate length. The current handling capability of a MESFET is directly proportional to the gate width Z because the cross-sectional area available for channel current is proportional to Z.

7.2.2 Principles of Operation

To understand the operation of a MESFET, we consider the section under the gate, Fig. 10*b*. The source is grounded, and the gate and drain voltage are measured with respect to the source. Under normal operating conditions, the gate voltage is zero or reverse biased and the drain voltage is zero or forward biased; that is, $V_G \leq 0$ and $V_D \geq 0$. Since the channel is n-type material, the device is referred to as an n-channel MESFET. Most applications use the n-channel MESFET rather than the p-channel MESFET because of higher carrier mobility in n-channel devices.

The resistance of the channel is given by

$$R = \rho \frac{L}{A} = \frac{L}{q\mu_n N_D A} = \frac{L}{q\mu_n N_D Z(a - W)}, \tag{24}$$

where N_D is the donor concentration, A is the cross-section area for current flow and equals $Z(a - W)$, and W is the width of the depletion region of the Schottky barrier.

When no gate voltage is applied and V_D is small, as shown in Fig. 11*a*, a small drain current I_D flows in the channel. The magnitude of the current is given by V_D/R, where R is the channel resistance given in Eq. 24. Therefore, the current varies linearly with the drain voltage. Of course, for any given drain voltage, the voltage along the channel increases from zero at the source to V_D at the drain. Thus, the Schottky barrier becomes increasingly reverse biased as we proceed from the source to the drain. As V_D is increased, W increases, and the average cross-sectional area for current flow is reduced. The channel resistance R also increases. As a result, the current increases at a slower rate.

As the drain voltage is further increased, eventually the depletion region touches the semiinsulating substrate as shown in Fig. 11*b*. This happens when $W = a$ at the drain. We can obtain the corresponding value of the drain voltage, called the *saturation voltage*, V_{Dsat} from Eq. 7 where $V = -V_{Dsat}$:

$$V_{Dsat} = \frac{qN_D a^2}{2\varepsilon_s} - V_{bi} \quad \text{for } V_G = 0. \tag{25}$$

At this drain voltage, the source and the drain are *pinched off* or completely separated by a reverse-biased depletion region. The location P in Fig. 11*b* is called the pinch-off point. At this point, a large drain current called the *saturation current* I_{Dsat} can flow across the depletion region. This is similar to the situation caused by injecting carriers into a reverse-biased depletion region such as the collector-base depletion region of a bipolar transistor.

Beyond the pinch-off point, as V_D is increased further, the depletion region near the drain will expand and point P will move toward the source, as indicated in Fig. 11*c*. However, the voltage at point P remains the same, V_{Dsat}. Thus, the number of electrons per unit time arriving from the source to point P, and hence the current flowing in the channel, remain the same because the potential drop in the channel from source to point

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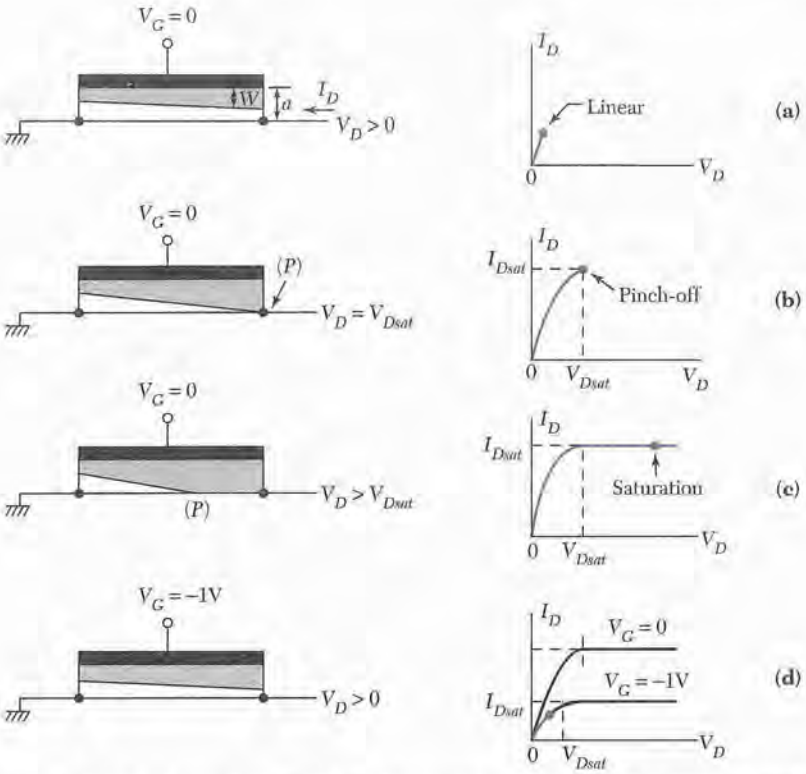


Fig. 11 Variation of the depletion-layer width and output characteristics of a MESFET under various biasing conditions. (*a*) $V_G = 0$ and a small V_D. (*b*) $V_G = 0$ and at pinch-off. (*c*) $V_G = 0$ at post pinch-off ($V_D > V_{Dsat}$). (*d*) $V_G = -1$V and a small V_D.

P does not change. Therefore, for drain voltages larger than V_{Dsat}, the current remains essentially at the value I_{Dsat} and is independent of V_D.

When a gate voltage is applied to reverse bias the gate contact, the depletion-layer width *W* increases. For a small V_D, the channel again acts as a resistor but its resistance is higher because the cross-sectional area available for current flow is decreased. As indicated in Fig. 11*d*, the initial current is smaller for $V_G = -1$ V than for $V_G = 0$. When V_D is increased to a certain value, the depletion region again touches the semiinsulating substrate. The value of this V_D is given by

$$V_{Dsat} = \frac{qN_D a^2}{2\varepsilon_s} - V_{bi} - V_G. \tag{26}$$

For an *n*-channel MESFET, the gate voltage is negative with respect to the source, so we use the absolute value of V_G in Eq. 26 and in subsequent equations. We see from Eq. 26 that the application of a gate voltage V_G reduces the drain voltage required for the onset of pinch-off by an amount equal to V_G.

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7.2.3 Current-Voltage Characteristics

We now consider a MESFET before the onset of pinch-off, as shown in Fig. 12*a*. The drain voltage variation along the channel is shown in Fig. 12*b*. The voltage drop across an elemental section dy of the channel is given by

$$dV = I_D dR = \frac{I_D dy}{q\mu_n N_D Z\left[a - W(y)\right]}, \tag{27}$$

where we have used Eq. 24 for dR and we have replaced L by dy. The depletion-layer width at distance y from the source is given by

$$W(y) = \sqrt{\frac{2\varepsilon_s[V(y) + V_G + V_{bi}]}{qN_D}}. \tag{28}$$

The drain current I_D is a constant, independent of y. We can rewrite Eq. 27 as

$$I_D dy = q\mu_n N_D Z\left[a - W(y)\right] dV. \tag{29}$$

The differentiation of the drain voltage dV is obtained from Eq. 28:

$$dV = \frac{qN_D}{\varepsilon_s} W dW. \tag{30}$$

Substituting dV into Eq. 29 and integrating from $y = 0$ to $y = L$ yields

$$I_D = \frac{1}{L}\int_{W_1}^{W_2} q\mu_n N_D Z\left(a - W\right)\frac{qN_D}{\varepsilon_s} W dW$$

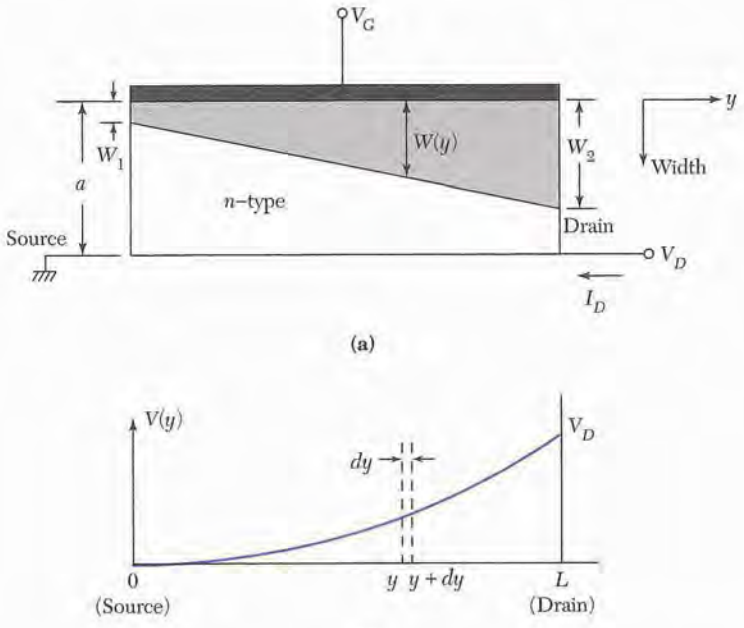


Fig. 12 (*a*) Expanded view of the channel region. (*b*) Drain voltage variation along the channel.

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$$= \frac{Z\mu_n q^2 N_D^2}{2\varepsilon_s L}\left[a\left(W_2^2 - W_1^2\right) - \frac{2}{3}\left(W_2^3 - W_1^3\right)\right]$$

or

$$\boxed{I = I_P\left[\frac{V_D}{V_P} - \frac{2}{3}\left(\frac{V_D + V_G + V_{bi}}{V_P}\right)^{3/2} + \frac{2}{3}\left(\frac{V_G + V_{bi}}{V_P}\right)^{3/2}\right]}, \tag{31}$$

where

$$\boxed{I_P \equiv \frac{Z\mu_n q^2 N_D^2 a^3}{2\varepsilon_s L}} \tag{31a}$$

and

$$\boxed{V_P \equiv \frac{qN_D a^2}{2\varepsilon_s}}. \tag{31b}$$

The voltage V_P is called the pinch-off voltage, that is, the total voltage $(V_D + V_G + V_{bi})$ at which $W_2 = a$.

In Fig. 13 we show the *I-V* characteristics of a MESFET having a pinch-off voltage of 3.2 V. The curves shown are calculated for $0 \le V_D \le V_{Dsat}$ using Eq. 31. Beyond V_{Dsat} the current is taken to be constant in accordance with our previous discussion. Note that there are three different regions in the current-voltage characteristics. When V_D is small,

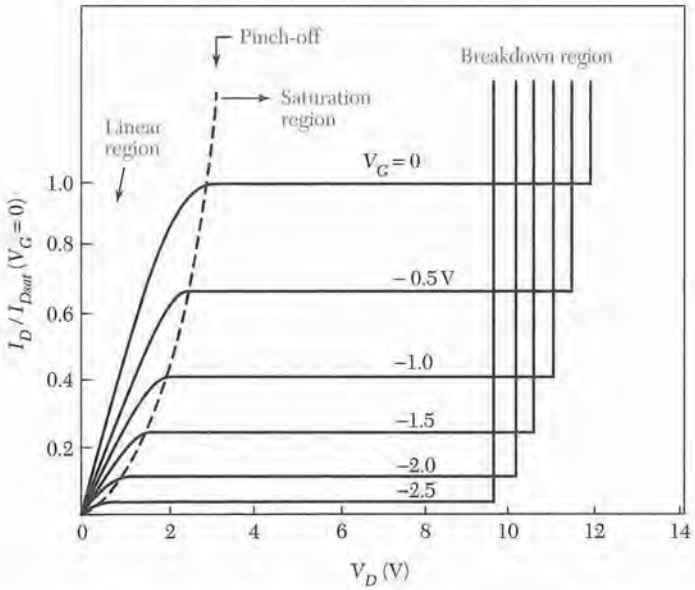


Fig. 13 Normalized ideal current-voltage characteristics of a MESFET with $V_P = 3.2$ V.

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the cross-section area of the channel is essentially independent of V_D and the I-V characteristics are ohmic or linear. We refer to this region of operation as the linear region. For $V_D \geq V_{Dsat}$, the current saturates at I_{Dsat}. We refer to this region of operation as the saturation region. As the drain voltage is further increased, avalanche breakdown of the gate-to-channel diode occurs, and the drain current suddenly increases. This is the breakdown region.

In the linear region where $V_D << V_G$, Eq. 31 can be expended to give

$$I_D \cong \frac{I_P}{V_P}\left[1 - \sqrt{\left(\frac{V_G + V_{bi}}{V_P}\right)}\right]V \tag{32}$$

An important parameter of a MESFET is the transconductance g_m, which represents the change of drain current at a given drain voltage on a change in gate voltage. From Eq. 32, we obtain

$$g_m = \frac{\partial I_D}{\partial V_G}\bigg|_{V_D} = \frac{I_P}{2V_P^2}\sqrt{\frac{V_P}{V_G + V_{bi}}}\,V_D. \tag{33}$$

In the saturation region, the drain current can be calculated from Eq. 31 by evaluating the current at the pinch-off point, that is, by setting $V_P = V_D + V_G + V_{bi}$:

$$I_{Dsat} = I_P\left[\frac{1}{3} - \left(\frac{V_G + V_{bi}}{V_P}\right) + \frac{2}{3}\left(\frac{V_G + V_{bi}}{V_P}\right)^{3/2}\right]. \tag{34}$$

The corresponding saturation voltage is given by

$$V_{Dsat} = V_P - V_G - V_{bi}. \tag{35}$$

The transconductance in the saturation region can be obtained from Eq. 34:

$$g_m = \frac{I_P}{V_P}\left(1 - \sqrt{\frac{V_G + V_{bi}}{V_P}}\right) = \frac{Z\mu_n q N_D a}{L}\left(1 - \sqrt{\frac{V_G + V_{bi}}{V_P}}\right). \tag{36}$$

In the breakdown region, the breakdown voltage occurs at the drain end of the channel, where the reverse voltage is the highest:

$$V_B \text{ (breakdown voltage)} = V_D + |V_G|. \tag{37}$$

For example, in Fig. 13 the breakdown voltage is 12 V for $V_G = 0$. At $|V_G| = 1$, the breakdown voltage is still 12 V and the drain voltage at breakdown is $(V_B - |V_G|)$ or 11 V.

EXAMPLE 4

Consider an n-channel GaAs MESFET at $T = 300$ K with a gold contact. Assume the barrier height is 0.89 V. The n-channel doping is 2×10^{15} cm^{-3} and the channel thickness is 0.6 μm. Calculate the pinch-off voltage and the built-in potential. The dielectric constant of GaAs is 12.4.

SOLUTION The pinch-off voltage is

$$V_p = \frac{qN_D}{2\varepsilon_s}a^2 = \frac{(1.6 \times 10^{-19})(2 \times 10^{15})}{2 \times 12.4 \times (8.85 \times 10^{-14})} \times (0.6 \times 10^{-4})^2 = 0.53 \text{ V}.$$

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The difference between the conduction band and the Fermi level is given by

$$V_n = \frac{kT}{q}\ln\left(\frac{N_C}{N_D}\right) = 0.026\ln\left(\frac{4.7\times10^{17}}{2\times10^{15}}\right) = 0.14 \text{ V}.$$

The built-in potential is

$$V_{bi} = \phi_{Bn} - V_n = 0.89 - 0.14 = 0.75 \text{ V}.$$

So far we have considered only a normally on (or depletion mode) device; that is, the device has a conductive channel at $V_G = 0$. For high-speed, low-power applications, the normally off device is preferred. This device does not have a conductive channel at $V_G = 0$; that is, the built-in potential V_{bi} of the gate contact is sufficient to deplete the channel region. This is possible, for example, in a gallium arsenide MESFET with a very thin epitaxial layer on a semiinsulating substrate. For a normally off MESFET, a positive bias must be applied to the gate before channel current begins to flow. The required voltage, called the *threshold voltage* V_T, is given by

$$V_T = V_{bi} - V_P \tag{38a}$$

or

$$V_{bi} = V_T + V_P, \tag{38b}$$

where V_P is the pinch-off voltage defined in Eq. 31*b*. Near the threshold voltage, the drain current in the saturation region can be obtained by substituting V_{bi} of Eq. 38*b* in Eq. 34 and by using the Taylor series expansion assuming $(V_G - V_T)/V_p << 1$. We obtain

$$I_{Dsat} = I_P\left\{\frac{1}{3} - \left[1 - \left(\frac{V_G - V_T}{V_P}\right)\right] + \frac{2}{3}\left[1 - \left(\frac{V_G - V_T}{V_P}\right)\right]^{3/2}\right\}$$

or

$$\boxed{I_{Dsat} \approx \frac{Z\mu_n\varepsilon_s}{2aL}(V_G - V_T)^2.} \tag{39}$$

In deriving Eq. 39 we used a negative sign for V_G to account for its polarity.

The basic current-voltage characteristics of normally on and normally off devices are similar. Figure 14 compares these two modes of operation. The main difference is the shift of threshold voltage along the V_G axis. The normally off device (Fig. 14*b*) has no current conduction at $V_G = 0$, and the current varies as in Eq. 39 when $V_G > V_T$. Since the built-in potential of the gate is less than about 1 V, the forward bias on the gate is limited to about 0.5 V to avoid excessive gate current.

The transconductance for a normally off device can be obtained from Eq. 39:

$$g_m = \frac{dI_{Dsat}}{dV_G} = \frac{Z\mu_n\varepsilon_s}{aL}(V_G - V_T). \tag{40}$$

7.2.4 High-Frequency Performance

For high-frequency applications of MESFETs, an important figure of merit is the cutoff frequency f_T, which is the frequency at which the MESFET can no longer amplify

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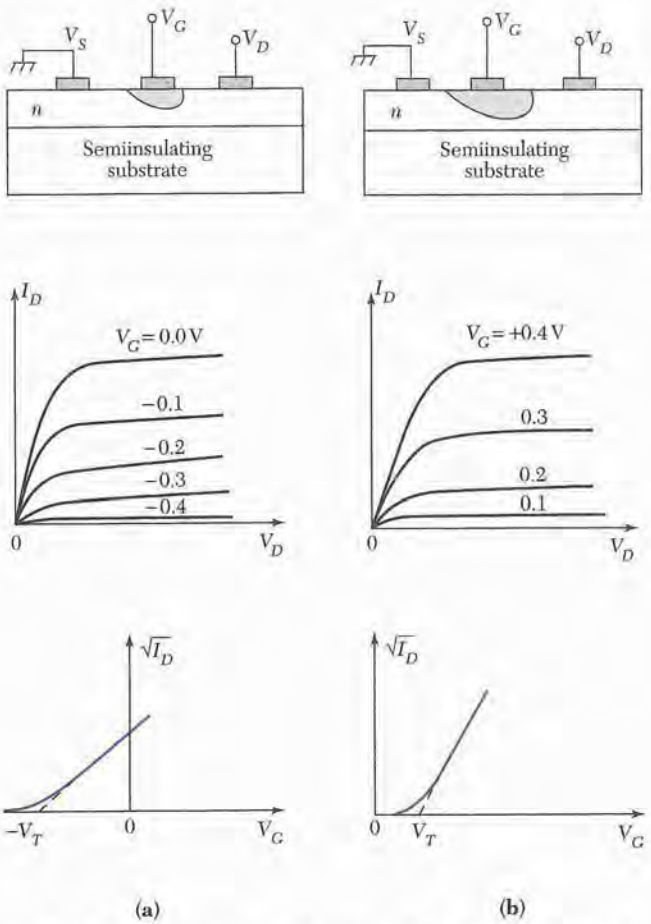


Fig. 14 Comparison of *I-V* characteristics. (*a*) Normally on MESFET. (*b*) Normally off MESFET.

the input signal. The small-signal input current is the product of the gate admittance and the small-signal gate voltage, assuming that the device has negligibly small series resistance:

$$\tilde{i}_{\text{in}} = 2\pi f C_G \tilde{v}_g . \tag{41}$$

where C_G is the gate capacitance equal to $ZL(\varepsilon_s/\overline{W})$ and $\overline{W}$ is the average depletion-layer width under the gate electrode. The small-signal output current is obtainable from the definition of the transconductance:

$$g_m = \frac{\partial I_C}{\partial V_G} = \frac{\tilde{i}_{\text{out}}}{\tilde{v}_g} \tag{42}$$

or

$$\tilde{i}_{\text{out}} = g_m \tilde{v}_g . \tag{42a}$$

Equating Eqs. 41 and 42*a*, we obtain the cutoff frequency

$$f_T = \frac{g_m}{2\pi C_G} < \frac{I_P / V_P}{2\pi ZL\left(\varepsilon_s / \overline{W}\right)} \approx \frac{\mu_n q N_D a^2}{2\pi \varepsilon_s L^2}, \tag{43}$$

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where we have used Eq. 36 for g_m. From Eq. 43 we see that to improve high-frequency performance, we should use a MESFET having high carrier mobility and short channel length. This is the reason that n-channel MESFET, which has higher electron mobility, is preferred.

These derivations are based on the assumption that the carrier mobility in the channel is a constant independent of the applied field. However, for very-high-frequency operations, the longitudinal field, i.e., the electric field directed from the source to the drain, is sufficiently high that the carriers travel at their saturation velocity.

Under these conditions, the saturation channel current is given by

$$\begin{aligned} I_{Dsat} &= (\text{ area for carrier transport }) \times qnv_s, \\ &= Z(a-W)qN_D v_s. \end{aligned} \tag{44}$$

The transconductance is then

$$g_m = \frac{\partial I_{Dsat}}{\partial V_G} = \frac{\partial I_{Dsat}}{\partial W} \cdot \frac{\partial W}{\partial V_G} = \left[qN_D v_s Z(-1)\right]\left(\frac{1}{-qN_D W/\varepsilon_s}\right) \tag{45}$$

or

$$g_m = Zv_s\varepsilon_s / W. \tag{45a}$$

In Eq. 45, we obtain $\partial W/\partial V_G$ from Eq. 28.

From Eq. 45*a*, we can obtain the cutoff frequency under saturation-velocity condition:

$$\boxed{f_T = \frac{g_m}{2\pi C_G} = \frac{Zv_s\varepsilon_s / W}{2\pi ZL(\varepsilon_s / W)} = \frac{v_s}{2\pi L}.} \tag{46}$$

Therefore, to increase f_T, we must reduce the gate length L and to employ a semiconductor with a high velocity. Figure 15 shows the electron drift velocity versus electric field

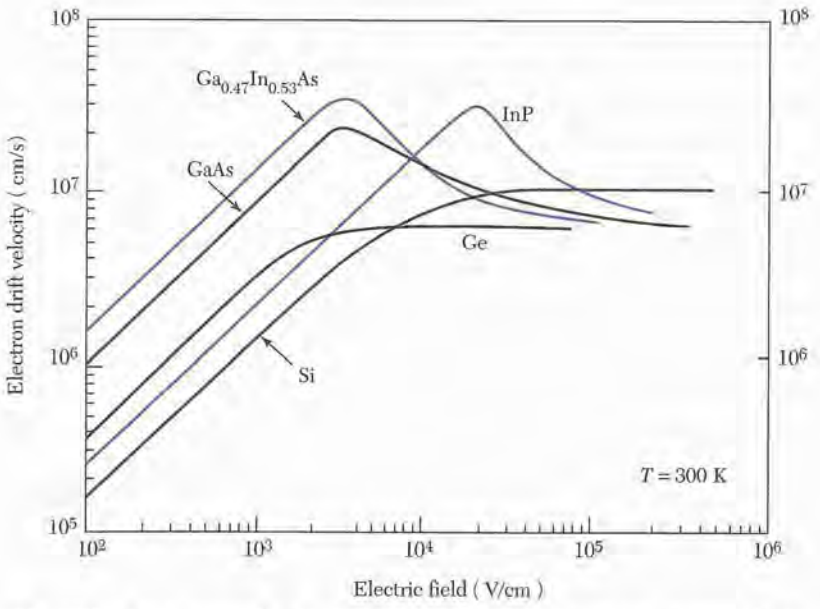


Fig. 15 The drift velocity versus the electric field for electrons in various semiconductor materials.[8]

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for five semiconductors.[8] Note that GaAs has an average velocity[§] of 1.2×10^7 cm/s and a peak velocity of 2×10^7 cm/s, which are 20%–100% higher than the saturation velocity of silicon. Also note that $Ga_{0.47}In_{0.53}As$ and InP have higher average and peak velocities than GaAs. Consequently, the cutoff frequencies of these semiconductors will be higher than that from GaAs.

7.3 MODFET

7.3.1 MODFET Fundamentals

The modulation-doped field-effect transistor (MODFET) is a heterostructure field-effect device. Other names commonly applied to the device include high electron mobility transistor (HEMT), two-dimensional electron gas field-effect transistor (TEGFET), and selectively doped heterostructure transistor (SDHT). Frequently, it is referred to by a general name of heterojunction field-effect transistor (HFET).

Figure 16 shows a perspective view of a conventional MODFET. The special features of a MODFET are its heterojunction structure under the gate, and the modulation doped layers. For the device in Fig. 16, AlGaAs is the wide bandgap semiconductor, whereas GaAs is the narrow bandgap semiconductor. The two semiconductors are modulation doped, i.e., the AlGaAs is doped, except for a narrow region d_o, which is undoped, whereas the GaAs is undoped. Electrons in the AlGaAs will diffuse to the undoped GaAs, where a conduction channel can be formed at the surface of the GaAs.

Figure 17*a* shows the band diagram of a MODFET in thermal equilibrium condition. Similar to a standard Schottky barrier, $q\phi_{Bn}$ is the barrier height of the metal on the

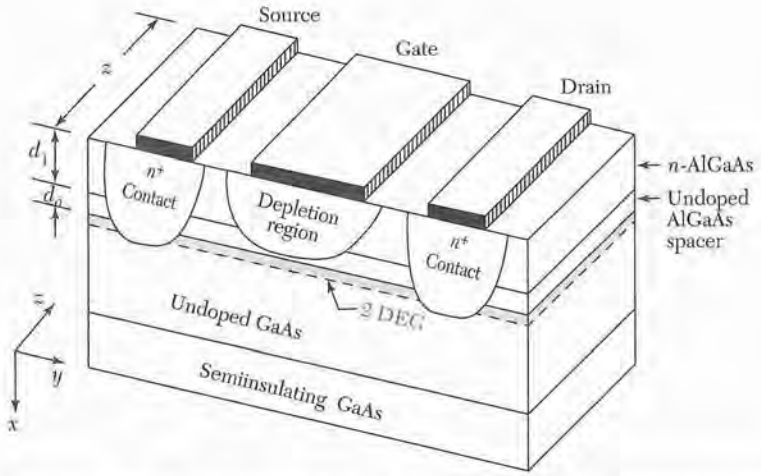


Fig. 16 Perspective view of a conventional modulation-doped field-effect transistor (MODFET) structure.

[§]The average velocity is defined as $\bar{v} \equiv \left[\frac{1}{L}\int_0^L \frac{dx}{v(x)}\right]^{-1}$. If $v(x)$ is a constant v_0, then $\bar{v} = v_0$.

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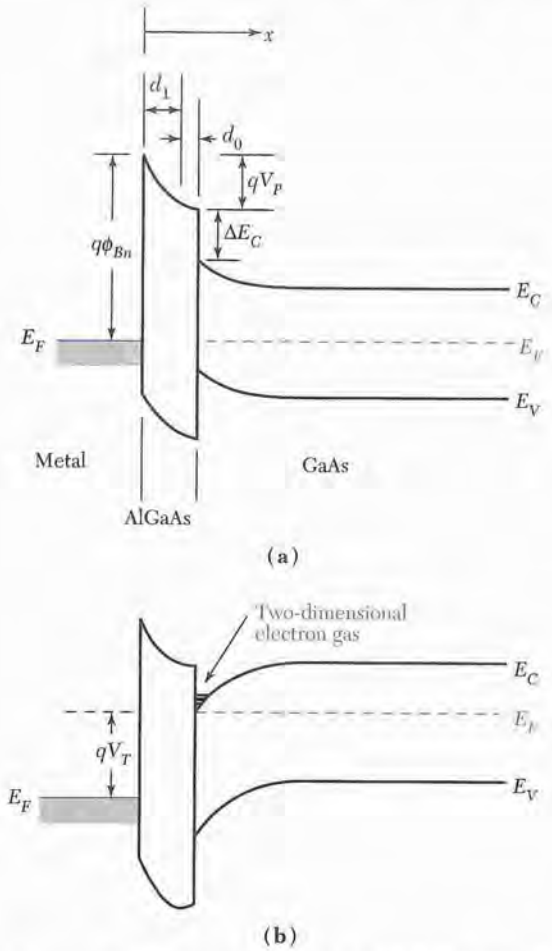


Fig. 17 Energy band diagrams for a normally-off MODFET at (*a*) thermal equilibrium, and (*b*) the onset of threshold. d_1 and d_0 are the doped and undoped regions, respectively.[9]

wide-bandgap semiconductor.[9] ΔE_C is the conduction band discontinuity for the heterojunction structure, and V_P is the pinch-off voltage given by

$$V_P = \frac{q}{\varepsilon_s}\int_0^d N_D(x)x\,dx = \frac{qN_D d_1^2}{2\varepsilon_s}, \tag{47}$$

where d_1 is the thickness of the doped region in AlGaAs and ε_s is the dielectric permittivity.

A key parameter for the operation of a MODFET is the threshold voltage V_T, which is the gate bias at which the channel starts to form between the source and drain. With reference to Fig. 17*b*, V_T corresponds to the situation when the bottom of the conduction band at the GaAs surface coincides with the Fermi level:

$$V_T = \phi_{Bn} - \frac{\Delta E_C}{q} - V_P. \tag{48}$$

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The threshold voltage V_T can be adjusted by using different values for ϕ_{Bn} and V_p. However, ΔE_C is fixed for a given set of semiconductors. Figure 17*b* has a positive V_T, and the MODFET is an enhancement-mode device (normally off), as opposed to a depletion-mode device with a negative V_T (normally on).

When the gate voltage is larger than V_T, a charge sheet $n_s(y)$ is capacitively induced by the gate at the heterojunction interface. The charge sheet is similar to the charge Q_n/q in the inversion layer of a MOSFET (see Section 6.1):

$$n_s(y) = \frac{C_i\left[V_G - V_T - V(y)\right]}{q}, \tag{49}$$

where

$$C_i = \frac{\varepsilon_s}{d_1 + d_0 + \Delta d}, \tag{49a}$$

d_1 and d_0 are the doped and undoped AlGaAs thickness (Fig. 16), and Δd is the channel thickness or the thickness of the inversion layer, estimated to be about 8 nm. $V(y)$ is the channel potential with respect to the source. It varies along the channel from 0 to the drain bias V_D, similar to that shown in Fig. 12*b*. The charge sheet is also called a *two-dimensional electron gas*. This is because that the electrons in the inversion layer are confined in the x-direction by ΔE_C on the left side and by the potential distribution of the conduction band on the right side (Fig. 17*b*). However, these electrons can make two-dimensional movements: in the y-direction from the source to the drain and in the z-direction parallel to the channel width (Fig. 16).

Equation 49 shows that a negative gate bias will reduce the two-dimensional electron gas. If, on the other hand, a positive V_G is applied, n_s will increase.

EXAMPLE 5

Consider an AlGaAs/GaAs heterojunction with n-AlGaAs doped to 2×10^{18} cm^{-3} and a thickness of 40 nm. Assume the undoped spacer layer is 3 nm and the Schottky barrier height is 0.85 V and $\frac{\Delta E_C}{q} = 0.23$ V. The dielectric constant of the AlGaAs is 12.3. Calculate the two-dimensional electron gas concentration for such heterojunction at $V_G = 0$.

SOLUTION

$$V_p = \frac{qN_D d_1^2}{2\varepsilon_s} = \frac{1.6\times10^{-19}\times2\times10^{18}\times(40\times10^{-7})^2}{2\times12.3\times8.85\times10^{-14}} = 2.35 \text{ V}.$$

The threshold voltage is

$$V_T = \phi_{Bn} - \frac{\Delta E_C}{q} - V_p = 0.85 - 0.23 - 2.35 = -1.73 \text{ V}.$$

Therefore, the device is a normally on MODFET.

The two-dimensional electron gas at the source for $V_G = 0$ is

$$n_s = \frac{12.3\times8.85\times10^{-14}}{1.6\times10^{-19}\times(40+3+8)\times10^{-7}} \times \left[0-(-1.73)\right] = 2.29\times10^{12} \text{ cm}^{-2}.$$

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7.3.2 Current-Voltage Characteristics

The current-voltage characteristics of a MODFET can be obtained by using the gradual channel approximation similar to that of a MOSFET. The current at any point along the channel is given by

$$\begin{aligned} I &= Zq\mu_n n_s \mathscr{E}_y \\ &= Z\mu_n C_i \Big[V_G - V_T - V(y) \Big] \frac{dV(y)}{dy}. \end{aligned} \tag{50}$$

Since the current is constant along the channel, integrating Eq. 50 from source to drain ($y = 0$ to $y = L$) gives

$$I = \frac{Z}{L} \mu_n C_i \left[\left(V_G - V_T \right) V_D - \frac{V_D^2}{2} \right]. \tag{51}$$

The output characteristics for an enhancement-mode MODFET are similar to that shown in Fig. 14*b*. In the linear region where $V_D << (V_G - V_T)$, Eq. 51 can be reduced to

$$\boxed{I = \frac{Z}{L} \mu_n C_i \left(V_G - V_T \right) V_D.} \tag{52}$$

For a large drain voltage, the charge sheet $n(y)$ at the drain is reduced to zero. This is the pinch-off condition previously discussed and shown in Fig. 11*b*. From Eq. 49, we obtain the saturation voltage V_{Dsat}, at which $n_s(y = L) = 0$,

$$V_{Dsat} = V_G - V_T, \tag{53}$$

and the saturation current is obtainable from Eqs. 51 and 53,

$$\boxed{I = \frac{Z\mu_n C_i}{2L} \left(V_G - V_T \right)^2 = \frac{Z\mu_n \varepsilon_s}{2L(d_1 + d_0 + \Delta d)} (V_G - V_T)^2.} \tag{54}$$

Note that this equation is very similar to Eq. 39. A similar expression can be obtained for the transconductance given in Eq. 40.

For high-speed operations, the longitudinal field (i.e., the field along the channel) is sufficiently high to cause carrier velocity saturation. The current in the velocity-saturation region is

$$\boxed{\begin{aligned} I_{sat} &= Zv_s q n_s \\ &\cong Zv_s C_i (V_G - V_T). \end{aligned}} \tag{55}$$

The transconductance becomes

$$g_m = \frac{\partial I_{sat}}{\partial V_G} = Zv_s C_i, \tag{56}$$

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Note that I_{sat} is independent of gate length and g_m is independent of both gate length and gate voltage in the velocity-saturation regime.

7.3.3 Cutoff Frequency

The speed of a MODFET is measured by the cutoff frequency

$$f_T = \frac{g_m}{2\pi(\text{total capacitance})} = \frac{Zv_sC_i}{2\pi(ZLC_i + C_p)}$$
$$= \frac{v_s}{2\pi\left(L + C_p / ZC_i\right)}, \tag{57}$$

where C_p is the parasitic capacitance. To improve f_T, we should consider a semiconductor with large v_s, a gate structure with an ultrashort gate length, and a device configuration with minimum parasitic capacitance.

A comparison of the cutoff frequencies of various FETs is shown in Fig. 18. The cutoff frequency f_T is plotted against the channel or gate length.[8,10] Note that for a given length, a silicon n-type MOSFET has the lowest f_T; this is because of the relatively low

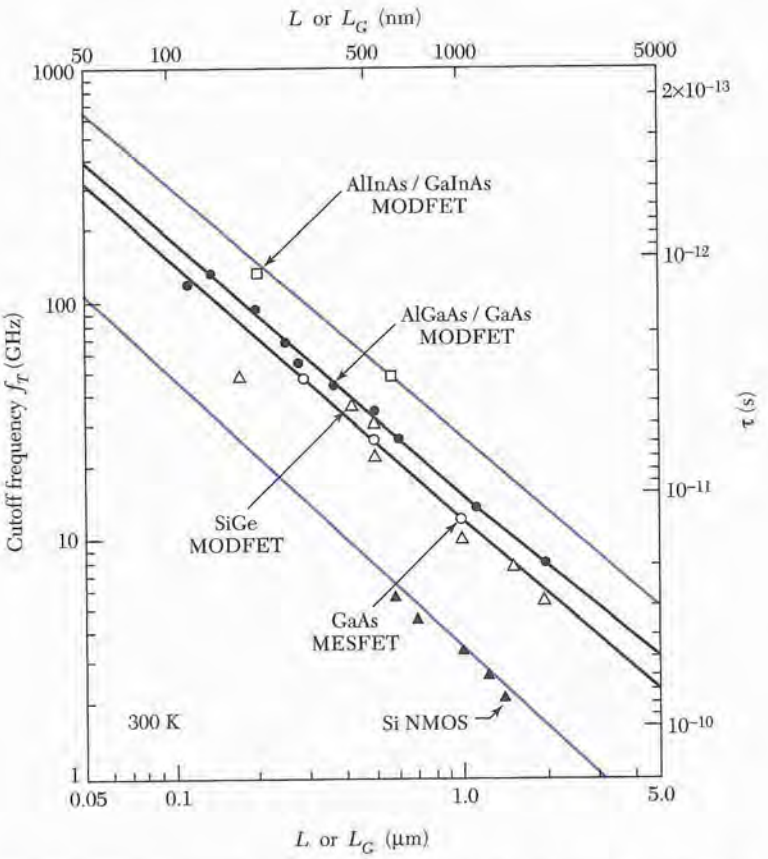


Fig. 18 Cutoff frequency versus channel or gate length for five different field-effect transistors.[8,10]

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mobility and a low average velocity of electron in silicon. GaAs MESFET has an f_T about three times higher than does silicon MOSFET.

Also shown are three MODFETs. The conventional GaAs MODFET (i.e., AlGaAs-GaAs structure) has an f_T about 30% higher than that of the GaAs MESFET. For the pseudomorphic SiGe MODFET (i.e., Si-SiGe structure where the SiGe lattice is slightly shrinked to match the silicon lattice), the best devices have an f_T comparable to the GaAs MODFET. SiGe MODFETs are attractive because they can be processed in a silicon fabrication facility. For even higher cutoff frequencies, we have the $Al_{0.48}In_{0.52}As$-$Ga_{0.47}In_{0.53}As$ MODFET formed on a InP substrate. The superior performance is mainly due to the high electron mobility in $Ga_{0.47}In_{0.53}As$ and its high average and peak velocities. It is anticipated that at a gate length of 50 nm f_T can be as high as 600 GHz.

SUMMARY

When a metal makes intimate contact with a semiconductor, it forms a *metal-semiconductor contact*. There are two types of contacts. The first type is the rectifying contact, also called the Schottky barrier contact, which has a relatively large barrier height and is formed on a semiconductor with a relatively low-doping concentration. The potential and field distribution in a Schottky barrier are identical to those of a one-sided abrupt p–n junction. However, the current transport in a Schottky barrier is by thermionic emission and, therefore, has an inherent fast response.

The second type is the ohmic contact, which is formed on a degenerate semiconductor in which carrier transport is by the tunneling process. An ohmic contact can pass the required current with a very small voltage drop across it. All semiconductor devices and integrated circuits need ohmic contacts to make connections to other devices in an electronic system.

Metal-semiconductor contacts are building blocks for MESFET and MODFET devices. By employing a Schottky barrier as the gate electrode and two ohmic contacts as the source and drain electrodes, we form a MESFET. This three-terminal device is important for high-frequency application, especially for monolithic microwave integrated circuits (MMIC). Most MESFETs are made with n-type III-V compound semiconductors because of their high electron mobilities and high average drift velocities. Of particular importance is GaAs, because of its relatively mature technology and the availability of high-quality GaAs wafers.

The MODFET is a device with enhanced high-frequency performance. This device structure is similar to that of a MESFET except there is a heterojunction under the gate. A two-dimensional electron gas, i.e., a conductive channel, is formed at the heterojunction interface, and electrons with high mobility and high average drift velocity can be transported from the source through the channel to the drain.

The output characteristics of all field-effect transistors (FETs) are similar. They all have a linear region at low-drain biases. As the bias increases, the output current eventually saturates, and at a sufficiently high voltage, avalanche breakdown occurs at the drain. Depending on whether it requires a positive- or negative-threshold voltage, FET can be either normally off (enhancement mode) or normally on (depletion mode).

The cutoff frequency f_T is a figure of merit for the high-frequency performance of an FET. For a given length, the silicon MOSFET (n-type) has the lowest f_T, and the GaAs MESFET has an f_T about three times higher than that of silicon. The conventional GaAs MODFET and the pseudomorphic SiGe MODFET have an f_T about 30% higher than that of the GaAs MESFET. For even higher cutoff frequencies, we have the GaInAs MODFET, which has a projected f_T of 600 GHz at a gate length of 50 nm.

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PROBLEMS (* DENOTES DIFFICULT PROBLEMS)

FOR SECTION 7.1 METAL-SEMICONDUCTOR CONTACTS

1. Calculate the theoretical barrier height and built-in potential in a metal-semiconductor diode for zero applied bias. Assume the metal work function is 4.55 eV, the electron affinity is 4.01 eV, and $N_D = 2 \times 10^{16}$ cm^{-3} at 300 K.
2. (a) Find the donor concentration and barrier height of the W-GaAs Schottky barrier diode shown in Fig. 6. (b) Compare the barrier height with that obtained from the saturation current density of 5×10^{-7} A/cm^2 shown in Fig. 8. (c) For a reverse bias of –1 V, calculate the depletion-layer width W, the maximum field, and the capacitance.
3. Copper is deposited on a carefully prepared n-type silicon substrate to form an ideal Schottky diode. $\phi_m = 4.65$ eV, the electron affinity is 4.01 eV, $N_D = 3 \times 10^{16}$ /cm^3, and $T =$ 300 K. Calculate the barrier height, the built-in potential, the depletion-layer width, and the maximum field at a zero bias.
4. The capacitance of a Au-n-type GaAs Schottky barrier diode is given by the relation $1/C^2 = 1.57 \times 10^5 - 2.12 \times 10^5 V_a$, where C is expressed in μF and V_a is in volts. Taking the diode area to be 10^{-1} cm^2, calculate the built-in potential, the barrier height, the dopant concentration, and the work function.
5. Calculate the value of V_{bi} and ϕ_m in an ideal metal-Si Schottky barrier contact. Assume the barrier height is 0.8 eV, $N_D = 1.5 \times 10^{16}$ cm^{-3}, and $q\chi = 4.01$ eV.
6. In a metal-Si Schottky barrier contact, the barrier height is 0.75 eV and $A^* = 110$ A/cm^2 – K^2. Calculate the ratio of the injected hole current to the electron current at 300 K, assuming $D_p = 12$ cm^2 s^{-1}, $L_p = 1 \times 10^{-3}$ cm, and $N_D = 1.5 \times 10^{16}$ cm^{-3}.

FOR SECTION 7.2 MESFET

7. Given $\phi_{Bn} = 0.9$ eV and $N_D = 10^{17}$ cm^{-3}, find the minimum value of the thickness of the epitaxial layer for a GaAs MESFET to be a depletion mode device (i.e., $V_T < 0$).
8. Assume the doping in a GaAs MESFET is $N_D = 7 \times 10^{16}$ cm^{-3} and the dimensions are $a =$ 0.3 μm, $L = 1.5$ μm, $Z = 5$ μm, $\mu_n = 4500$ cm^2/V-s, and $\phi_{Bn} = 0.89$ V. Calculate the ideal value of g_m for $V_G = 0$, and $V_D = 1$ V.
9. The n-channel GaAs MESFET shown in Fig. 10 has a barrier height $\phi_{Bn} = 0.9$ V, $N_D =$ 10^{17} cm^{-3}, $a = 0.2$ μm, $L = 1$ μm, and $Z = 10$ μm. (a) Is this an enhancement or depletion-

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mode device? (b) Find the threshold voltage (the enhancement mode indicates $V_T > 0$; the depletion mode indicates $V_T < 0$).

10. An n-channel GaAs MESFET has a channel doping $N_D = 2 \times 10^{15}\,\text{cm}^{-3}$, $\phi_{Bn} = 0.8$ V, $a = 0.5\,\mu\text{m}$, $L = 1\,\mu\text{m}$, $\mu_n = 4500\,\text{cm}^2/\text{V-s}$, and $Z = 50\,\mu\text{m}$. Find the pinch-off potential, threshold voltage, and the saturation current at $V_G = 0$.

11. The barrier height ϕ_{Bn} of two GaAs n-channel MESFETs are the same and equal to 0.85 V. The channel doping in device 1 is $N_D = 4.7 \times 10^{16}\,\text{cm}^{-3}$, and that in device 2 is $N_D = 4.7 \times 10^{17}\,\text{cm}^{-3}$. Determine the channel thickness required in each device such that the threshold voltage is zero for each device.

FOR SECTION 7.3 MODFET

12. For an abrupt AlGaAs/GaAs heterojunction with the n-AlGaAs layer doped to 3×10^{18} cm^{-3}, the Schottky barrier is 0.89 V and the heterojunction conduction-band edge discontinuity ΔE_C is 0.23 eV. Calculate the thickness of the doped AlGaAs layer d_1 so that the threshold voltage is −0.5 V. Assume the permittivity of the AlGaAs is 12.3.

*13. Find the thickness of the undoped spacer layer d_0, such that the two-dimensional electron gas concentration of an AlGaAs/GaAs heterojunction is $1.25 \times 10^{12}\,\text{cm}^{-2}$ at zero gate bias. Assume that the n-AlGaAs is doped to $1 \times 10^{18}\text{cm}^{-3}$ and has a thickness d_1 of 50 nm, the Schottky barrier height is 0.89 V, and $\Delta E_C/q = 0.23$ V. The permittivity of the AlGaAs is 12.3.

14. Consider a AlGaAs/GaAs HFET with a 50 nm n-AlGaAs and a 10 nm undoped AlGaAs spacer. Assume the threshold voltage is −1.3 V, N_D is $5 \times 10^{17}\text{cm}^{-3}$, $\Delta E_C = 0.25$ eV, the channel width is 8 nm, and the permittivity of AlGaAs is 12.3. Calculate the Schottky barrier height and the two-dimensional electron gas concentration at $V_G = 0$.

15. The AlGaAs/GaAs has a two-dimensional electron gas concentration of $1 \times 10^{12}\text{cm}^{-2}$, the spacer is 5 nm, the channel width is 8 nm, the pinch-off voltage is 1.5 V, $\Delta E_C/q = 0.23$ V, and the doping concentration of AlGaAs is $10^{18}\,\text{cm}^{-3}$. The Schottky barrier height is 0.8 V. Find the thickness of the doped AlGaAs and the threshold voltage.

16. Consider an n-AlGaAs–intrinsic GaAs abrupt heterojunction. Assume that the AlGaAs is doped to $N_D = 3 \times 10^{18}\text{cm}^{-3}$ and has a thickness of 35 nm (there is no spacer). Let $\phi_{Bn} = 0.89$ V, and assume that $\Delta E_C = 0.24$ eV and the dielectric constant is 12.3. Calculate (a) V_P and (b) n_s for $V_G = 0$.

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CHAPTER 8

Microwave Diodes, Quantum-Effect, and Hot-Electron Devices

- 8.1 BASIC MICROWAVE TECHNOLOGY
- 8.2 TUNNEL DIODE
- 8.3 IMPATT DIODE
- 8.4 TRANSFERRED-ELECTRON DEVICES
- 8.5 QUANTUM-EFFECT DEVICES
- 8.6 HOT-ELECTRON DEVICES
- SUMMARY

Many semiconductor devices discussed in the previous chapters can be operated in the microwave region (0.1 ~ 3000 GHz). However, two-terminal devices generate the highest power levels per device area in system applications, especially at higher frequencies. Additionally, pulsed operation of these devices overcomes thermal limits and increases peak rf (radio frequency) power levels by more than an order of magnitude.[1] In this chapter, we introduce some basic microwave technology and consider some special two-terminal microwave devices including the tunnel diode, the IMPATT diode, the transferred-electron device, and the resonant tunneling diode.

In the past two decades, we have witnessed considerable research and effort in the development of device structures that could exploit quantum-effect and hot-electron phenomena to enhance circuit performances. Speed is often cited as a primary benefit that quantum-effect devices (QEDs) and hot-electron devices (HEDs) can offer. The tunneling process, on which most QEDs rely, is an intrinsically fast process. In HEDs, carriers under ballistic transport can move at velocities considerably in excess of their equilibrium thermal velocity. However, a more significant advantage of QEDs and HEDs is their higher functionality. These devices can perform relatively complex circuit functions with a greatly reduced device count, replacing large numbers of transistors or passive circuit components.[1] The basic device structures and operating principles of QEDs and HEDs are discussed in this chapter.

Specifically, we cover the following topics:

- Advantages of millimeter-wave devices over those operated at lower frequencies.
- The quantum tunneling phenomenon and its related devices—tunnel diode, resonant tunneling diode (RTD), and unipolar resonant tunneling transistor.

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- The IMPATT diode—the most powerful semiconductor source of millimeter-wave power.
- The transferred-electron device and its transit-time domain mode.
- The real-space–transfer transistor and its advantages as a functional device.

8.1 BASIC MICROWAVE TECHNOLOGY

The microwave frequencies cover the range from about 0.1 GHz (10^8 Hz) to 3000 GHz with corresponding wavelengths from 300 cm to 0.01 cm. For frequencies from 30 to 300 GHz, we have the millimeter-wave band because the wavelength is between 10 and 1 mm. For even higher frequencies, we have the submillimeter-wave band. The microwave frequency range is usually grouped into different bands.[2] The bands and the corresponding frequency ranges as designated by the Institute of Electrical and Electronics Engineers (IEEE) are listed in Table 1. It is recommended that both the band and the corresponding frequency range be used when referring to microwave devices.

The development of microwave technology was driven by the demands of short-wavelength radio (and later radar) systems. The history of microwaves started with the first experiments of Heinrich Hertz around 1887. Hertz used a spark transmitter that produced signals in a very broad frequency band, and he selected from these a frequency around 420 MHz with an antenna that measured half a wavelength for his experiments. The rapid development of wireless communication products has led to an explosion in microwave technology. Since the introduction of cellular telephone service in the 1980s, there has been rapid growth of those systems as well as mobile paging devices and a variety of wireless "data communication" services, under the broad heading of personal communication services (PCS). In addition to these terrestrial communication systems, the field of satellite-based video, telephone, and data communication systems has also grown rapidly. These systems use the microwave frequencies from several hundred MHz to well over 60 GHz—the millimeter-wave region.[3]

Millimeter-wave technology offers many advantages for communications and radar systems, such as radio astronomy, clear-air turbulence detection, nuclear spectroscopy, air-traffic–control beacons, and weather radar. The advantages of millimeter waves over lower microwave and infrared systems include light-weight, small-size, broad bandwidths (several GHz), operation in adverse weather conditions, and narrow beamwidths with

TABLE 1 IEEE Microwave Frequency Bands

Designation	Frequency range (GHz)	Wavelength (cm)
VHF	0.1–0.3	300.00–100.00
UHF	0.3–1.0	100.00–30.00
L band	1.0–2.0	30.00–15.00
S band	2.0–4.0	15.00–7.50
C band	4.0–8.0	7.50–3.75
X band	8.0–13.0	3.75–2.31
Ku band	13.0–18.0	2.31–1.67
K band	18.0–28.0	1.67–1.07
Ka band	28.0–40.0	1.07–0.75
Millimeter	30.0–300.0	1.00–0.10
Submillimeter	300.0–3000.0	0.10–0.01

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high resolution. The principal frequencies of interest in the millimeter-wave band are centered around 35, 60, 94, 140, and 220 GHz.[4] The reason for choosing these specific frequencies is mainly the atmospheric absorption of horizontally propagated millimeter waves, as shown in Fig. 1. The atmospheric "windows" where absorption is at a local minimum are found at about 35, 94, 140, and 220 GHz. The absorption peak due to O_2 at 60 GHz can be used for secure communication systems.

Ordinary electronic components behave differently at microwave frequencies than they do at lower frequencies. The distributed effects must be taken into account at microwave frequencies because at these frequencies the wavelength approximates the physical size of the components. At microwave frequencies, a thin film resistor, for example, looks like a complex *RLC* network with distributed L and C values and a different R value. These distributed components have immense significance at microwave frequencies, even though they can be ignored at lower frequencies. At microwave frequencies, capacitors and inductors are often realized by transmission-line segments. Transmission lines are often used for microwave circuit interconnections as well. Transmission lines are actually complex networks containing the equivalent of all of the three basic electrical components: resistor, capacitor, and inductor. Planar transmission lines are the mainstay of modern microwave circuit technology. Such lines consist of one or more flat conductors on a thin dielectric substrate with a ground surface.

Figure 2 shows several basic types of planar transmission lines: microstrip, coplanar waveguide (CPW) stripline, and suspended-substrate stripline (SSSL).[5] Microstrip is the most common type of transmission line. The coplanar waveguide is somewhat lossier, that is, the loss in signal propagation is larger, but it minimizes the parasitic inductance of ground connections. The characteristic impedance Z_0 of a transmission line is given by

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \text{ ohms}, \tag{1}$$

where R is resistance per unit length in ohms, G is conductance per unit length in siemens, L is inductance per unit length in henrys, C is capacitance per unit length in farads, and

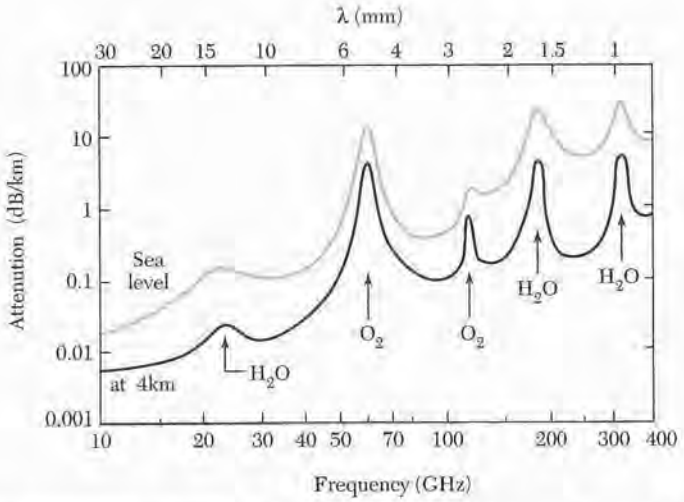


Fig. 1 Average atmosphere absorption of millimeter waves.[4] The upper curve is at the sea level; the lower curve is at 4 km above the sea level.

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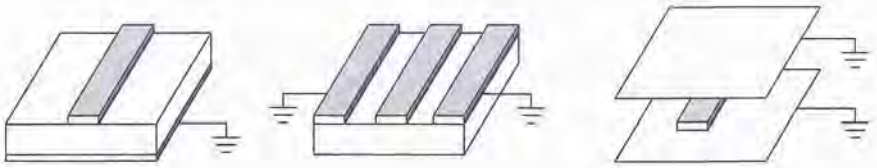


Fig. 2 Basic types of planar transmission lines: (*a*) microstrip, (*b*) coplanar waveguide stripline, and (*c*) suspended-substrate stripline.

ω is angular frequency in radians per second. In microwave circuits, the resistances are typically very low compared with the reactances, so Eq. 1 can be simplified to

$$Z_0 = \sqrt{\frac{L}{C}}. \tag{2}$$

The characteristic impedance for a specific type transmission line is a function of the conductor geometry (size, spacing) and the dielectric constant of the insulating material used between the conductors.

EXAMPLE 1

Find the characteristic impedance of a nearly lossless transmission line (R is very small) that has a unit-length inductance of 10 nH and a unit-length capacitance of 4 pF.

SOLUTION

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{10\times10^{-9}}{4\times10^{-12}}} = \sqrt{2.5\times10^{3}} = 50\ \Omega.$$

At lower microwave frequencies, resonant circuits can be made using inductive and capacitive elements. For millimeter-wave and higher frequency, however, the values of LC components at resonance are too small for practical implementation. Other methods for providing resonant circuits must be used. One popular solution is the resonant cavity, also called the tuned cavity.

The resonant cavity[2] is a metal-walled chamber made of low-resistivity material enclosing a good dielectric (for example, vacuum, dry air, or dry nitrogen). It is analogous to a section of waveguide with both ends shorted and has a means for injecting energy into or extracting energy from the cavity. As shown in Fig. 3, the cavity supports both TE (transverse electric) and TM (transverse magnetic) modes of propagation. The electromagnetic wave is confined by the wall of the cavity. Energy stored in the electric field represents the capacitance, whereas energy stored in the magnetic field represents the inductance. Thus, both elements of the LC tuned resonant tank circuit are present in the cavity. The resonant mode occur in a cavity at those frequencies for which the length d along the Z axis is a half-wavelength (see Fig. 3*a*). The alphanumeric system for designating modes in a cavity is $Tx_{m,n,p}$, where x is E for electric dominant modes and M for magnetic dominant modes, m is the number of half-wavelengths in the a dimension, n is the number of half-wavelengths in the b dimension, and p is the number of half-wavelengths in the

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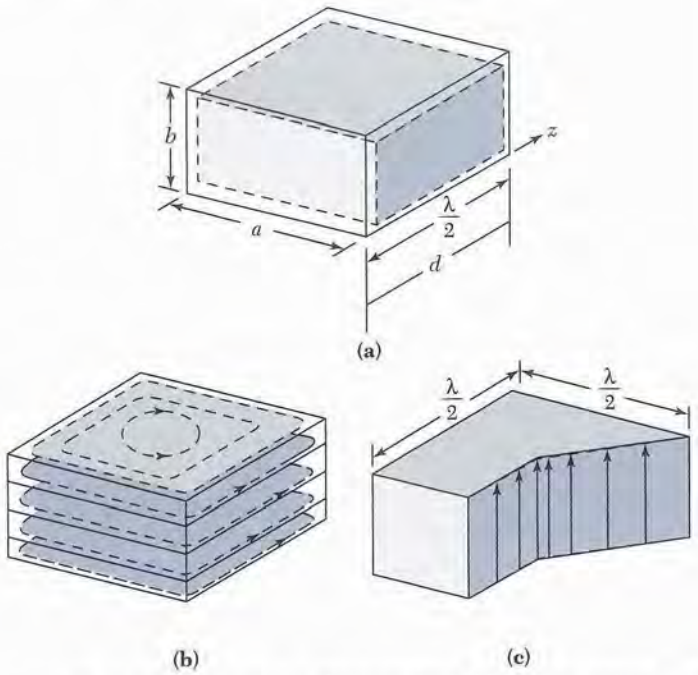


Fig. 3 Resonant cavity: (*a*) resonator shape, (*b*) magnetic field pattern, and (*c*) electric field pattern.[2]

d dimension. The general mode dependent equation for the resonant frequency of the cavity is

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}, \tag{3}$$

where μ and ε are the permeability and the permittivity of the material in the cavity. For vacuum, $\mu = \mu_0$ and $\varepsilon = \varepsilon_0$, and $\sqrt{\mu_0\varepsilon_0} = c^{-1}$, where c is the speed of light in vacuum. We can rewrite Eq. 3 as

$$f_r = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}. \tag{4}$$

EXAMPLE 2

For a cavity of the dimensions $a = 5$ cm (0.05 m), $b = 2.5$ cm (0.025 m), and $d = 10$ cm (0.1 m), find the resonant frequency in the dominant TE_{101} mode.

SOLUTION

$$f_r = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

$$= \frac{c}{2}\sqrt{(20)^2 + 0 + (10)^2}$$

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$$= \frac{3 \times 10^8 \text{ m/s}}{2} \times \sqrt{500} = 3.354 \text{ GHz}.$$

8.2 TUNNEL DIODE

The tunnel diode is associated with the quantum tunneling phenomena.[6] The tunneling time across the device is very short, permitting its use well into the millimeter-wave region. Because of its mature technology, the tunnel diode is used in special low-power microwave applications, such as local oscillators and frequency-locking circuits.

A tunnel diode consists of a simple p–n junction in which both the p- and n-sides are degenerate (i.e., very heavily doped with impurities). Figure 4 shows a typical static current-voltage characteristic of a tunnel diode under four different bias conditions. The I-V characteristic is the result of two current components: tunneling current and thermal current.

When there is no voltage applied to the diode, it is in thermal equilibrium (V = 0). Because of the high dopings, the depletion region is very narrow and the tunneling distance d is quite small (5–10 nm). The dopings also cause the Fermi levels to be located within the allowed bands. The amount of degeneracy, qV_p and qV_n, shown at the far left of Fig. 4, is typically 50–200 meV.

When a forward bias is applied, there exists a band of energy states that is occupied on the n side and a corresponding band of energy states that is available and unoccupied on the p side. The electrons can tunnel from the n-side to the p-side. When the applied

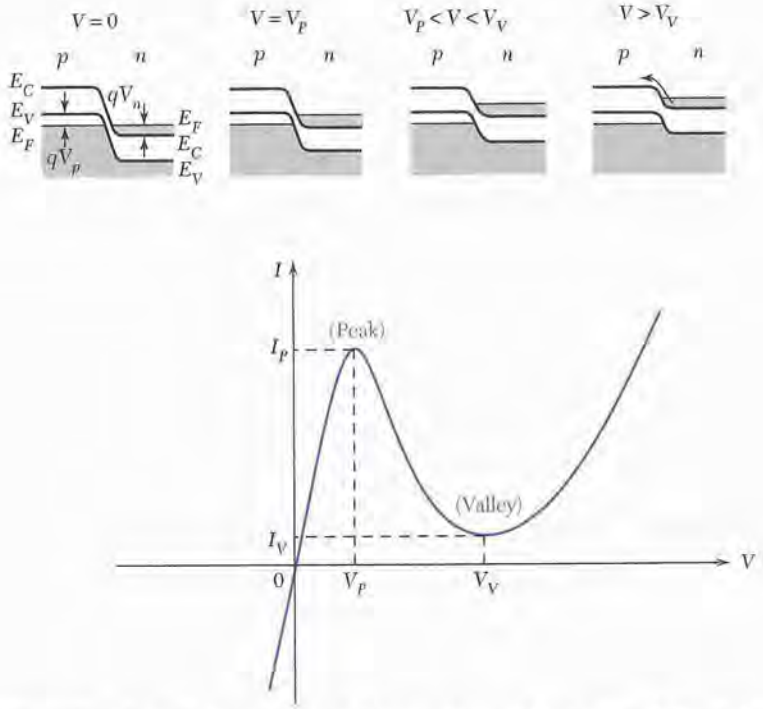


Fig. 4 Static current-voltage characteristics of a typical tunnel diode. I_P and V_P are the peak current and peak voltage, respectively. I_V and V_V are the valley current and valley voltage, respectively. The upper figures show the band diagrams of the device at different bias voltages.

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bias equals approximately $(V_p + V_n)/3$, the tunneling current reaches its peak value I_P and the corresponding voltage is called the peak voltage V_P. When the forward voltage is further increased, there are fewer available unoccupied states on the p-side ($V_P < V < V_V$, where V_V is the valley voltage) and the current decreased. Eventually, the band is "uncrossed," and at this point the tunneling current can no longer flow. With still further increase of the voltage, the normal thermal current will flow (for $V > V_V$).

From this discussion we expect that in the forward direction the tunneling current increases from zero to a peak current I_P as the voltage increases. With a further increase in voltage, the current then decreases to zero when $V = V_n + V_p$, where V is the applied forward voltage. The decreasing portion after the peak current in Fig. 4 is the negative differential resistance region. The values of the peak current I_P and the valley current I_V determine the magnitude of the negative resistance. For this reason their ratio I_P/I_V is used as a figure of merit for tunnel diode.

An empirical form for the I-V characteristics is given by

$$I = I_P\left(\frac{V}{V_P}\right)\exp\left(1-\frac{V}{V_P}\right)+I_0\exp\left(\frac{qV}{kT}\right), \tag{5}$$

where the first term is the tunnel current and I_P and V_P are the peak current and peak voltage, respectively, as shown in Fig. 4. The second term is the normal thermal current. The negative differential resistance can be obtained from the first term in Eq. 5:

$$R=\left(\frac{dI}{dV}\right)^{-1}=-\left[\left(\frac{V}{V_P}-1\right)\frac{I_P}{V_P}\exp\left(1-\frac{V}{V_P}\right)\right]^{-1}. \tag{6}$$

Figure 5 shows a comparison of the typical current-voltage characteristics of Ge, GaSb, and GaAs tunnel diodes at room temperature. The current ratios of I_P/I_V are 8:1 for Ge and 12:1 for GaSb and GaAs. Because of its smaller effective mass (0.042 m_0) and small bandgap (0.72 eV), the GaSb tunnel diode has the largest negative resistance of the three devices.

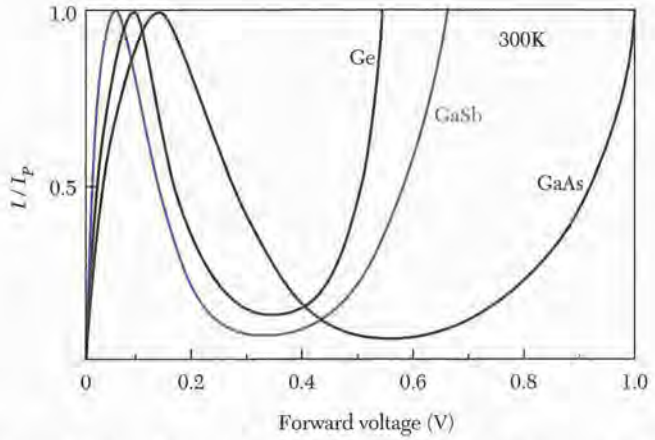


Fig. 5 Typical current-voltage characteristics of Ge, GaSb, and GaAs tunnel diodes at room temperature.

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8.3 IMPATT DIODE

The name IMPATT stands for impact ionization avalanche transit-time. IMPATT diodes employ impact ionization and transit-time properties of semiconductor devices to produce a negative resistance at microwave frequencies. The IMPATT diode is one of the most powerful solid-state sources of microwave power. At present, the IMPATT diode can generate the highest cw (continuous wave) power output of all solid-state devices at millimeter-wave frequencies—above 30 GHz. It is extensively used in radar systems and alarm systems. There is one noteworthy difficulty in IMPATT applications: the noise is high because of random fluctuations of the avalanche multiplication processes.

8.3.1 Static Characteristics

The IMPATT diode family includes many different p–n junction and metal-semiconductor devices. The first IMPATT oscillation was obtained from a simple silicon p–n junction diode biased into reverse avalanche breakdown and mounted in a microwave cavity.[7] Figure 6*a* shows the doping profile and electric-field distribution at avalanche breakdown of a one-sided abrupt p–n junction. Because of the strong dependence of the ionization rate on the electric field, most of the avalanche multiplication processes occurs in a narrow region near the highest field between 0 and x_A (shaded area). x_A is the width of the avalanche region, the distance over which 95% of the contribution to the ionization integrand is obtained.

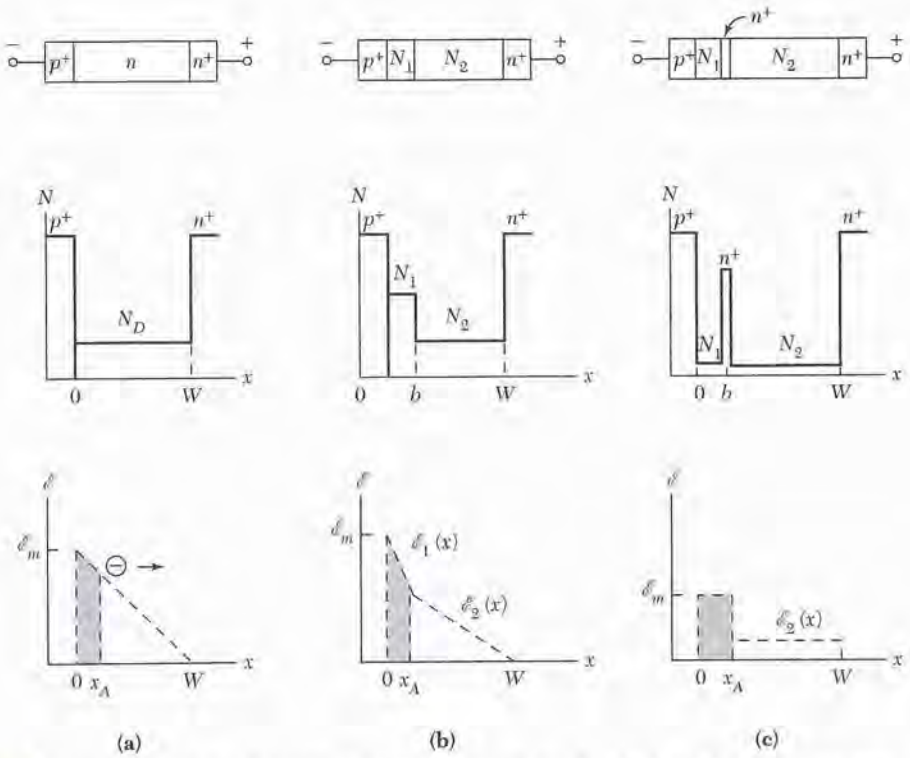


Fig. 6 Doping profiles and electric-field distributions at avalanche breakdown of three single-drift IMPATT diodes: (*a*) one-sided abrupt p–n junction; (*b*) hi-lo structure; and (*c*) lo-hi-lo structure.

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Figure 6*b* shows the hi-lo structure, in which a high doping N_1 region is followed by a lower doping N_2 region. With proper choices of the doping N_1 and its thickness b, the avalanche region can be confined within the N_1 region. Figure 6*c* is the lo-hi-lo structure in which a "clump" of donor atoms is located at $x = b$. Since a nearly uniform high-field region exists from $x = 0$ to $x = b$, the avalanche region x_A is equal to b, and the maximum field can be much lower than that for a hi-lo structure.

The breakdown voltage V_B (including the built-in potential V_{bi}) is given by the area underneath the electric-field versus distance plot (Fig. 6). For the one-sided abrupt junction (Fig. 6*a*), V_B is simply given by $\mathscr{E}_m W/2$. For the hi-lo diode and the lo-hi-lo diode, the breakdown voltages are given, respectively, by

$$V_B(\text{hi - lo}) = \left(\mathscr{E}_m - \frac{qN_1 b}{2\varepsilon_s}\right)b - \frac{1}{2}\left(\mathscr{E}_m - \frac{qN_1 b}{2\varepsilon_s}\right)(W - b), \tag{7}$$

$$V_B(\text{lo - hi - lo}) = \mathscr{E}_m b + \left(\mathscr{E}_m - \frac{qQ}{\varepsilon_s}\right)(W - b), \tag{8}$$

where Q in Eq. 8 is the number of impurities/cm^2 in the clump. The maximum field at breakdown for a hi-lo diode with a given N_1 is the same as the value of the one-sided abrupt junction with the same N_1. The maximum field of a lo-hi-lo structure can be calculated from the ionization coefficient. These structures are single-drift IMPATT diodes because only one type of charge carriers, electrons, traverses the drift region. If, on the other hand, we form a p^+-p-n-n^+ structure, we have a double-drift IMPATT diode in which both electrons and holes participate in device operation over two separate drift regions, i.e., electrons move to the right side and holes move to the left side from the avalanche region. Similar approaches can be used to obtained breakdown voltages for various double-drift diodes.

8.3.2 Dynamic Characteristics

We now use the lo-hi-lo structure, shown in Fig. 6*c*, to discuss the injection delay and transit-time effect of the IMPATT diode. When a reverse direct current (dc) voltage V_B is applied to the diode so that the critical field for avalanche $\mathscr{E}_c$ is just reached (Fig. 7*a*), avalanche multiplication will begin. An alternating current (ac) voltage is superimposed onto this dc voltage at $t = 0$. This voltage is shown in Fig 7*e*. Holes generated in the avalanche region move to the p^+-region, and electrons enter the drift region. As the applied ac voltage goes positive, more electrons are generated in the avalanche region, as shown by the dotted line in Fig 7*b*. The electron pulse keeps increasing as long as the electrical field is above $\mathscr{E}_c$. Therefore, the electron pulse reaches its peak value not at $\pi/2$ when the voltage is maximum, but at π (Fig. 7*c*). The important consequence is that there is a $\pi/2$ phase delay inherent in the avalanche process itself, that is, the injected-carrier density (electron pulse) lags the ac voltage by 90°.

An additional delay is provided by the drift region. Once the applied voltage drops below V_B ($\pi \le \omega t \le 2\pi$), the injected electrons will drift toward the n^+-contact (Fig. 7*d*) with a saturation velocity, provided the field across the drift region is sufficiently high.

The situation described above is illustrated by the injected carriers in Fig. 7*f*. By comparing Figs. 7*e* and 7*f*, we note that the peak value of the ac field (or voltage) occurs at $\pi/2$, but the peak of the injected carrier density occurs at π. The injected carriers then traverse the drift region at saturation velocity, thereby introducing the transit-time delay.

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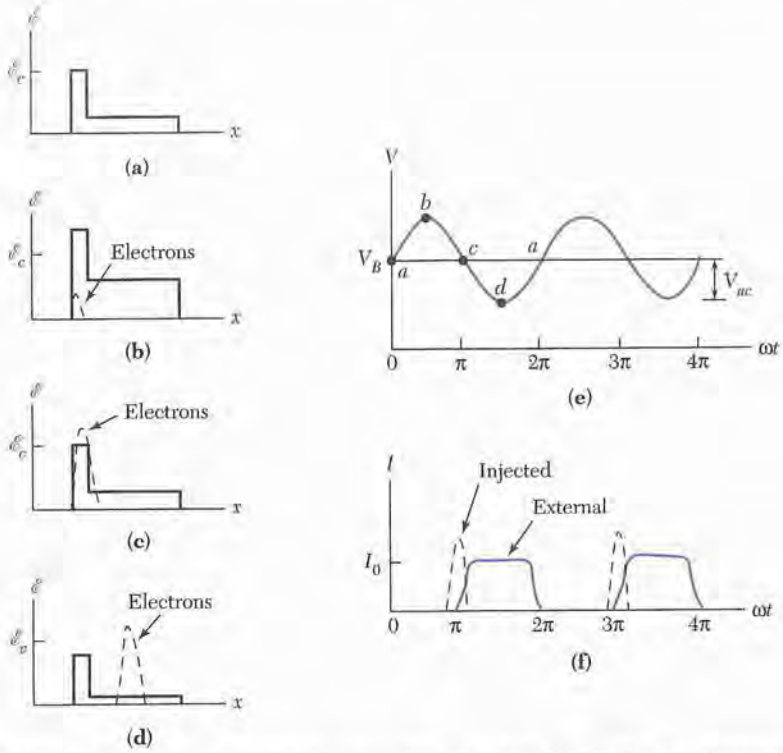


Fig. 7 Field distributions and generated-carrier densities of an IMPATT diode during an ac cycle at four intervals of time (*a*–*d*); (*e*) the ac voltage, and (*f*) the injected and external current.[7]

The induced external current is also shown in Fig. 7*f*. Comparing the ac voltage and the external current shows that the diode exhibits a negative resistance characteristic.

The injected carriers (electron pulse) will traverse the length W of the drift region during the negative half-cycle if we choose the transit time to be one-half the oscillation period, that is,

$$\frac{W - x_A}{v_s} = \frac{1}{2}\left(\frac{1}{f}\right) \tag{9}$$

or

$$\boxed{f = \frac{v_s}{2\,(W - x_A)},} \tag{10}$$

where v_s is the saturation velocity, which is 10^7 cm/s for silicon at 300 K.

EXAMPLE 3

Consider a lo-hi-lo silicon IMPATT diode (p^+-i-n^+-i-n^+) having $b = 1$ μm and $W = 6$ μm. If the field at breakdown is 3.3×10^5 V/cm, $Q = 2.0 \times 10^{12}$ charges/cm^2, find the dc breakdown voltage, the field in the drift region, and the operating frequency.

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SOLUTION From Eq. 8 we can calculated the breakdown voltage:

$$V_B = 3.3\times10^5\times10^{-4} + \left(3.3\times10^5 - \frac{1.6\times10^{-19}\times2.0\times10^{12}}{11.9\times8.85\times10^{-14}}\right)\times\left(5\times10^{-4}\right),$$

$$= 33 + 13 = 46 \text{ V}.$$

The field in the drift region is $\dfrac{13}{5\times10^{-4}} = 2.6\times10^4$ V / cm.

The drift field is high enough for the injected carriers to maintain their saturation velocity. Therefore

$$f = \frac{v_s}{2(W - x_A)} = \frac{10^7}{2\times(6-1)\times10^{-4}} = 10^{10} \text{ Hz} = 10 \text{ GHz}.$$

We can also estimate the dc-to-ac power conversion efficiency of the IMPATT diode using Figs. 7*e* and 7*f*. The dc power input is the product of the average dc voltage and the average dc current, that is, $V_B(I_0/2)$. The ac power output can be estimated by assuming that the maximum ac voltage swing to be 1/2 V_B, that is, $V_{ac} = V_B/2$, and the external current is zero between $0 \le \omega t \le \pi$ and is I_0 between $\pi \le \omega t \le 2\pi$. Therefore, the microwave power-generating efficiency η is

$$\eta = \frac{\text{ac power output}}{\text{dc power input}} = \frac{\int_0^{2\pi}\left(V_{ac}\sin\omega t\right) I\, d(\omega t)}{\left(V_B\dfrac{I_0}{2}\right)2\pi}$$

$$= \frac{\displaystyle\int_\pi^{2\pi}\left(\frac{V_B}{2}\sin\omega t\right) I_0\, d(\omega t)}{V_B I_0 \pi} = \frac{1}{\pi} = 32\%. \tag{11}$$

State-of-the-art IMPATT diodes have cw power capabilities up to 3 W at 30 GHz with over 22% efficiency, up to 1 W at 100 GHz with 10% efficiency, and 50 mW at 250 GHz with 1% efficiency.[8] The substantial reduction in power and efficiency at higher frequencies is due to difficulties in device fabrication and circuit optimization. The reduction is also caused by the nonoptimal transit-time delays introduced by the finite time required to transfer energy to the carriers and the tunneling process for the very narrow depletion-layer width.

8.4 TRANSFERRED-ELECTRON DEVICES

The transferred-electron effect was first observed in 1963. In the first experiment,[9] a microwave output was generated when a dc electric field that exceeded a critical threshold value of several thousand volts per centimeter was applied across a short *n*-type sample of GaAs or InP. The transferred-electron device (TED) is an important microwave device. It is used extensively as a local oscillator and power amplifier covering the microwave frequency range from 1 to 150 GHz. The power output and efficiency of TEDs are generally lower than that of IMPATT diodes. However, TEDs have lower noise, lower operating voltages, and relatively easier circuit designs. The TEDs have matured to become

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important solid-state microwave sources used in detection systems, remote controls, and microwave test instruments.

8.4.1 Negative Differential Resistance

In Chapter 3 we considered the transferred-electron effect, that is, the transfer of conduction electrons from a high-mobility energy valley to low-mobility higher-energy satellite valleys. Based on the analysis in Chapter 3, the current density as shown in Fig. 8 takes on asymptotic values

$$J \cong qn\mu_1 \mathscr{E} \quad \text{for} \quad 0 < \mathscr{E} < \mathscr{E}_a, \tag{12}$$

$$J \cong qn\mu_2 \mathscr{E} \quad \text{for} \quad \mathscr{E} > \mathscr{E}_b. \tag{13}$$

If $\mu_1 \mathscr{E}_a$ is larger than $\mu_2 \mathscr{E}_b$, there is a region of negative differential resistance (NDR)[10] between $\mathscr{E}_a$ and $\mathscr{E}_b$ as shown in Fig. 8. Also shown are the threshold field $\mathscr{E}_T$ corresponding to the onset of the NDR, the threshold current density J_T, the valley field $\mathscr{E}_V$, and the valley current density J_V. The NDR region is between $\mathscr{E}_T$ and $\mathscr{E}_V$.

For the transferred-electron mechanism to give rise to the NDR, certain requirements must be met. (a) The lattice temperature must be low enough that in the absence of an electric field most of the electrons are in the lower valley (the conduction band minimum), that is, the energy separation between the two valleys $\Delta E > kT$. (b) In the lower valley the electrons must have high mobility and small effective mass, whereas in the upper satellite valleys the electrons must have low mobility and large effective mass. (c) The energy separation between the two valleys must be smaller than the semiconductor bandgap (i.e., $\Delta E < E_g$) so that avalanche breakdown does not begin before the transfer of electrons into the upper valleys.

Of the semiconductors satisfying these requirements, n-type gallium arsenide and n-type indium phosphide are the most widely studied and used. The measured room-temperature velocity field characteristics for these semiconductors are shown in Fig. 15 of Chapter 7. The threshold field $\mathscr{E}_T$ is 3.2 kV/cm for gallium arsenide and 10.5 kV/cm for indium phosphide. The peak velocity V_p is about 2.2×10^7 cm/s for gallium arsenide and 2.5×10^7 cm/s for indium phosphide. The maximum negative differential mobility

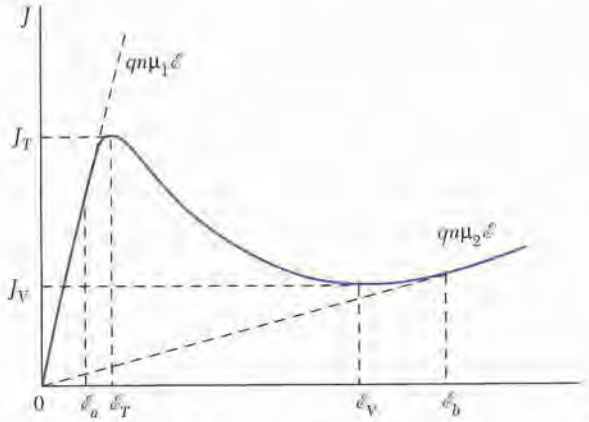


Fig. 8 The current versus electric-field characteristic of a two-valley semiconductor. $\mathscr{E}_T$ is the threshold field and $\mathscr{E}_V$ is the valley field.

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(i.e., $dv/d\mathscr{E}$) is about –2400 cm²/V-s for gallium arsenide and –2000 cm²/V-s for indium phosphide.

A semiconductor exhibiting NDR is inherently unstable because a random fluctuation of carrier density at any point in the semiconductor produces a momentary space charge that will grow exponentially with time. The one-dimensional continuity equation is given by

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J}{\partial x}. \tag{14}$$

If there is a small local fluctuation of the majority carriers from the uniform equilibrium concentration n_0, the locally created space charge density is $n - n_0$. Poisson's equation and the current density equation are

$$\frac{\partial \mathscr{E}}{\partial x} = \frac{-q(n-n_0)}{\varepsilon_s}, \tag{15}$$

$$J = qn_0\overline{\mu}\mathscr{E} + qD\frac{\partial n}{\partial x}, \tag{16}$$

where $\overline{\mu}$ is the average mobility (defined by Eq. 83 in Chapter 3), ε_s is the dielectric permittivity, and D is the diffusion constant. Differentiating Eq. 16 with respect to x and inserting Poisson's equation yields

$$\frac{1}{q}\frac{\partial J}{\partial x} = -\frac{n-n_0}{\varepsilon_s/qn_0\overline{\mu}} + D\frac{\partial^2 n}{\partial x^2}. \tag{17}$$

Substituting this expression into Eq. 14 gives

$$\frac{\partial n}{\partial t} = -\frac{n-n_0}{\varepsilon_s/qn_0\overline{\mu}} + D\frac{\partial^2 n}{\partial x^2}. \tag{18}$$

We can solve Eq. 18 by separation of variables, that is, let $n(x,t) = n_1(x)n_2(t)$. For the temporal response, the solution of Eq. 18 is

$$n - n_0 = (n-n_0)_{t=0}\exp\left(\frac{-t}{\tau_R}\right), \tag{19}$$

where τ_R is the *dielectric relaxation time* given by

$$\tau_R = \frac{\varepsilon_s}{qn_0\overline{\mu}}. \tag{20}$$

τ_R represents the time constant for the decay of the space charge to neutrality if the mobility $\overline{\mu}$ is positive. However, if the semiconductor exhibits NDR, any charge imbalance will grow with a time constant equal to $|\tau_R|$.

8.4.2 Device Operation

The TEDs require very pure and uniform materials with a minimum of deep impurity levels and traps. Modern TEDs almost always have epitaxial layers on n^+-substrates deposited by various epitaxial techniques. Typical donor concentrations range from 10^{14} to 10^{16} cm^{-3} and typical device lengths range from a few microns to several hundred microns. A TED having an epitaxial n-layer on n^+-substrate and an ohmic n^+-contact to the cathode electrode is shown in Fig. 9*a*. Also shown are the energy band diagram at

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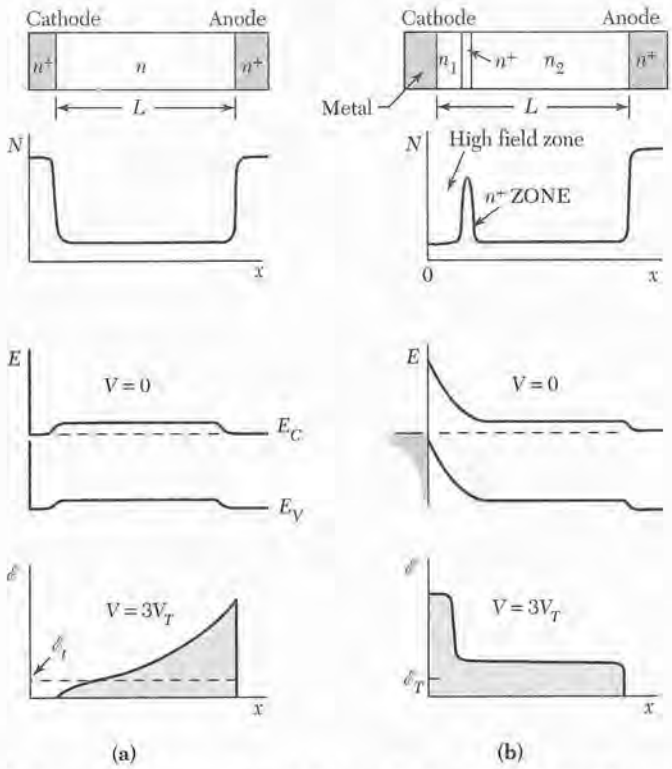


Fig. 9 Two cathode contacts for transferred-electron devices (TEDs). (*a*) Ohmic contact and (*b*) two-zone Schottky barrier contact.

thermal equilibrium and the electric-field distribution when a voltage $V = 3V_T$ is applied to the device where V_T is the product of the threshold field $\mathscr{E}_T$ and the device length L. For such an ohmic contact there is always a low-field region near the cathode, and the field is nonuniform across the device length.

To improve device performance, we use the two-zone cathode contact instead of the n^+-ohmic contact. The two-zone cathode contact consists of a high-field zone and an n^+-zone (Fig. 9*b*). This configuration is similar to that of a lo-hi-lo IMPATT diode. Electrons are "heated" in the high-field zone and subsequently injected into the active region, which has a uniform field. This structure has been used successfully over a wide temperature range with high efficiency and high power output.

The operational characteristics of a TED depends on five factors: doping concentration and doping uniformity in the device, length of the active region, cathode contact characteristics, type of circuit, and operating bias voltage.

We have shown that for a device with NDR, the initial space charge will grow exponentially with time (Eq. 19) and that the time constant is given by Eq. 20:

$$|\tau_R| = \frac{\varepsilon_s}{qn_0|\mu_-|}, \tag{21}$$

where μ_- is the negative differential mobility. If Eq. 19 remains valid throughout the entire transit time of the space-charge layer, the maximum growth factor would be $\exp(L/v|\tau_R|)$, where L is the length of the active region and v is the average drift velocity

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of the space-charge layer. For large space-charge growth, this growth factor must be greater than unity, making $L/v|\tau_R| > 1$, or

$$n_0 L > \frac{\varepsilon_s v}{q|\mu_-|} \approx 10^{12}\,\text{cm}^{-2} \tag{22}$$

for GaAs and InP.

An important mode of operation for the TED is the transit-time domain mode. When there are positive and negative charges separated by a small distance, we have a dipole formation (also called a domain)[10] as shown in Figs. 10*a* and 10*b*. From Poisson's equa-

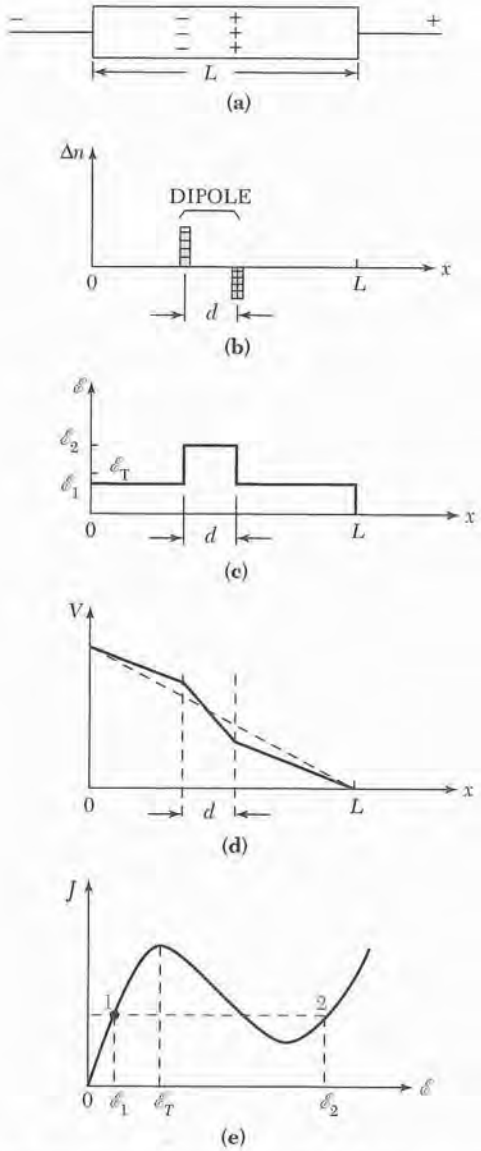


Fig. 10 Formation of a domain (dipole layer) in a medium that has a negative differential resistivity.[10]

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tion, we find that the electric field inside the dipole would be greater than the field on either side of it (Fig. 10*c*). The corresponding voltage variation across the device can be obtained by integrating the Poisson's equation once more, as shown in Fig. 10*d*. Because of the NDR, the current in the low-field region would be greater than that in the high-field region. The two field values tend toward the equilibrium level outside the NDR region, where the high and low currents are the same, Fig. 10*e*. The dipole has now reached a stable configuration. The dipole layer moves through the active region and disappears at the anode, at which time the field begins to rise uniformly across the device through the threshold (i.e., $\mathscr{E} > \mathscr{E}_T$), thus forming a new dipole, and the process repeats itself. The time required for the domain to travel from the cathode to anode is L/v, where L is the active device length and v is the average velocity. The corresponding frequency for the transit time domain mode is $f = v/L$.

Figure 11 shows a simulated time-dependent behavior of a domain in a gallium arsenide TED 100 μm long with a doping of 5×10^{14} cm^{-3} ($n_0L = 5 \times 10^{12}$ cm^{-2}).[11] The time between successive vertical displays of $\mathscr{E}(x,t)$ is $16\tau_R$, where τ_R is the low-field dielectric relaxation time from Eq. 20 ($\tau_R = 1.5$ ps for this device). Each time a domain is absorbed at the anode, the current in the external circuit increases and a new domain is nucleated at the cathode contact where the largest doping fluctuation and space-charge perturbation usually exist.

State-of-the-art TED diodes have cw power capabilities up to 0.5 W at 30 GHz with 15% efficiency, up to 0.2 W at 100 GHz with 7% efficiency, and 70 mW at 150 GHz with 1% efficiency. The power output of TED is lower than that of IMPATT; however, TED has much lower noise (e.g., 20 dB less at 135 GHz).[8]

EXAMPLE 4

A GaAs TED is 10 μm long and is operated in the transit-time domain mode. Find the minimum electron density n_0 required and the time between current pulses.

SOLUTION For transit-time domain mode, we require $n_0L \geq 10^{12}$ cm^{-2}:

$$n_0 \geq 10^{12} / L = 10^{12} / 10 \times 10^{-4} = 10^{15} \text{ cm}^{-3}.$$

The time between current pulses is the time required for the domain to travel from the cathode to the anode:

$$t = L/v = 10 \times 10^{-4} / 10^7 = 10^{-10} \text{s} = 0.1 \text{ ns}.$$

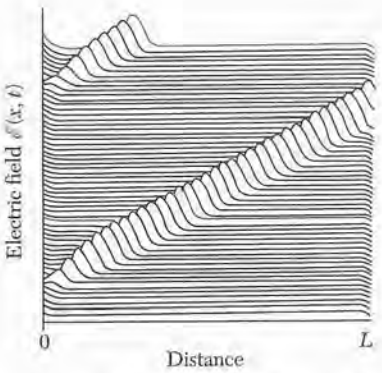


Fig. 11 Numerical simulation of the time-dependant behavior of a cathode-nucleated TED for the transit-time domain mode.[11]

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8.5 QUANTUM-EFFECT DEVICES

A quantum-effect device (QED) uses quantum mechanical tunneling to provide controlled carrier transport. In such a device, the active layer thickness is very small, on the order of 10 nm. These small dimensions give rise to a quantum size effect that can alter the band structures and enhance device transport properties. The basic QED is the resonant tunneling diode (RTD), discussed in Section 8.5.1. Many novel current-voltage characteristics can be obtained by combining a RTD with conventional devices considered in previous chapters. QEDs are of particular importance because they can serve as *functional devices*, that is, they can performance a give circuit function with a greatly reduced number of components.

8.5.1 Resonant Tunneling Diode

Figure 12 shows the band diagram of a RTD. It has a semiconductor double-barrier structure containing four heterojunctions, a GaAs/AlAs/GaAs/AlAs/GaAs structure, and one quantum well in the conduction band. There are three important device parameters for a RTD—the energy barrier height E_0 , which is the conduction band discontinuity, the energy barrier thickness L_B, and the quantum well thickness L_W.

We now concentrate on the conduction band of a RTD as shown[12] in Fig.13*a*. If the well thickness L_W is sufficiently small (on the order of 10 nm or less), a set of discrete energy levels will exist inside the well (such as E_1, E_2, E_3, and E_4 in Fig.13*a*). If the barrier thickness L_B is also very small, resonant tunneling will occur. When an incident electron has an energy E that exactly equals one of the discrete energy levels inside the well, it will tunnel through the double barrier with a unity (100%) transmission coefficient.

The transmission coefficient decreases rapidly as the energy E deviates from the discrete energy levels. For example, an electron with an energy 10 meV higher or lower than the level E_1 will result in 10^5 times reduction in the transmission coefficients, as depicted in Fig.13*b*. The transmission coefficient can be calculated by solving the one-dimensional Schroedinger equations in the five regions in Fig. 13*a* (I, II, III, IV, V). Since

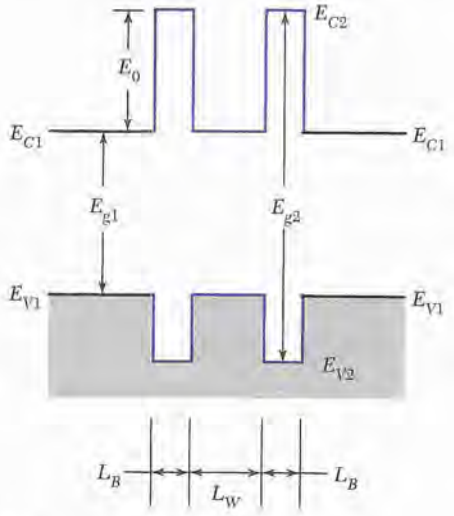


Fig. 12 Band diagram of a resonant-tunneling diode.

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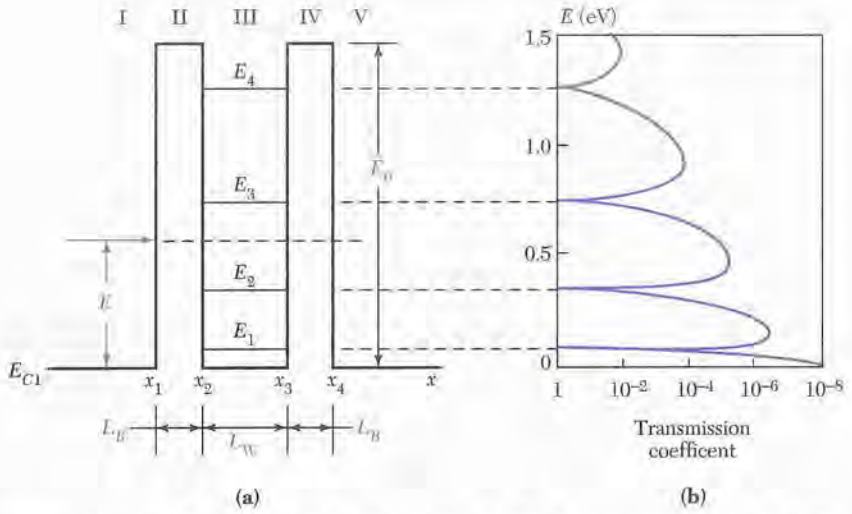


Fig. 13 (*a*) Schematic illustration of AlAs/GaAs/AlAs double-barrier structure with a 2.5 nm barrier and a 7 nm well. (*b*) Transmission coefficient versus electron energy for the structure.[12]

the wavefunctions and their first derivatives at each potential discontinuity must be continuous, we can obtain the transmission coefficient T_t. Appendix J shows the calculation of the transmission coefficient for the RTD.

The energy levels, E_n, at which the transmission coefficient exhibits its first and second resonant peaks in GaAs/AlAs RTD are shown in Fig. 14*a* as a function of barrier thickness L_B with the well thickness L_W as a parameter.[13] It is apparent that E_n is essentially independent of L_B but is dependent on L_W. The calculated width of the peak ΔE_n (i.e., the full width at the half-maximum point of the transmission coefficient where $T_t = 0.5$) is shown in Fig. 14*b* as function of L_B and L_W. For a given L_W, the width ΔE_n decreases exponentially with L_B.

The cross section of a RTD is shown[13] in Fig. 15. The alternating GaAs/AlAs layers are grown sequentially by molecular beam epitaxy (MBE) on an n^+ GaAs substrate (the MBE process is discussed in Chapter 10). The barrier thicknesses are 1.7 nm and the well thickness is 4.5 nm. The active regions are defined with ohmic contacts. The top contact is used as a mask to isolate the region under the contact by etching mesas.

The measured current-voltage characteristic of the above RTD is shown in Fig. 16. Also shown are the band diagrams for varies dc biases. Note that the *I*-*V* curve is similar to that of a tunnal diode (Fig. 4). At thermal equilibrium, $V = 0$, the energy diagram is similar to that in Fig. 13*a* (here only the lowest energy level E_1 is shown). As we increase the applied voltage, the electrons in the occupied energy states near the Fermi level to the left side of the first barrier tunnel into the quantum well. The electrons subsequently tunnel through the second barrier into the unoccupied states in the right side. Resonance occurs when the energy of the injected electrons becomes approximately equal to the energy level E_1, where the transmission probability is maximum. This is illustrated by the energy diagram for $V = V_1 = V_P$, where the conduction band edge on the left side is lined up with E_1. The magnitude of the peak voltage must be at least $2E_1/q$ but is usually larger because of additional voltage drops in the accumulation and depletion regions:

$$V_P > \frac{2E_1}{q}. \tag{23}$$

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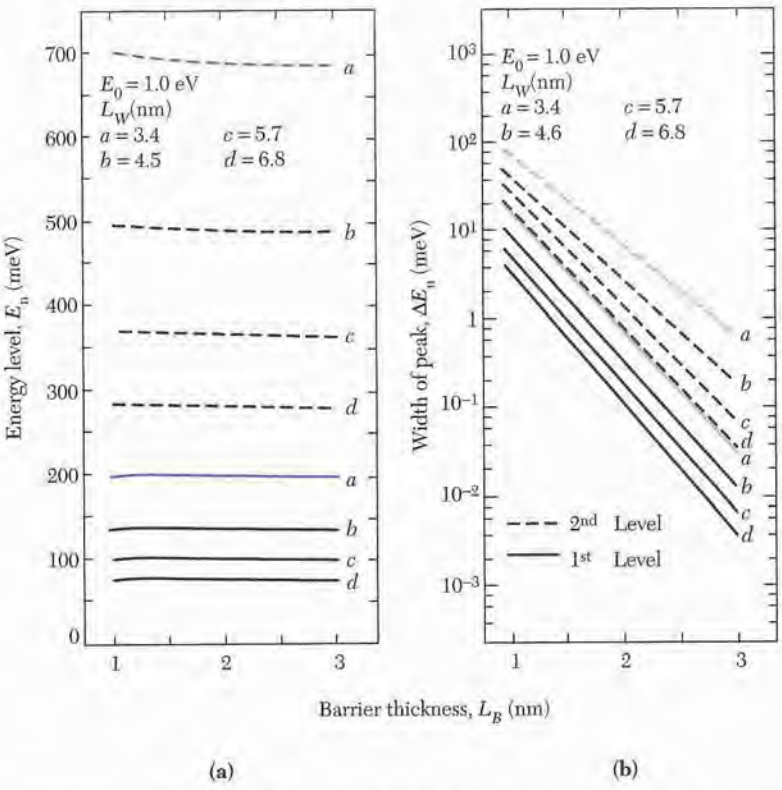


Fig. 14 (*a*) Calculated energy of electrons at which the transmission coefficient shows the resonant peak in an AlAs/GaAs/AlAs structure as a function of barrier thickness for various well thicknesses. (*b*) Full width at half maximum of the transmission coefficient versus barrier thickness for the first and second resonant peak.[13]

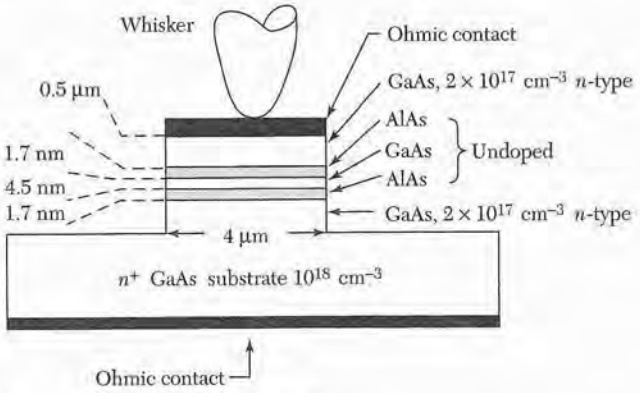


Fig. 15 A mesa-type resonant tunneling diode.[13]

When the voltage is further increased, that is, at $V = V_2$, the conduction band edge is above E_1 and the number of electrons that can tunnel decreases, resulting in a small current. The valley current I_V is due mainly to the excess current components, such as electrons that tunnel via an upper valley in the barrier. At room temperature and higher,

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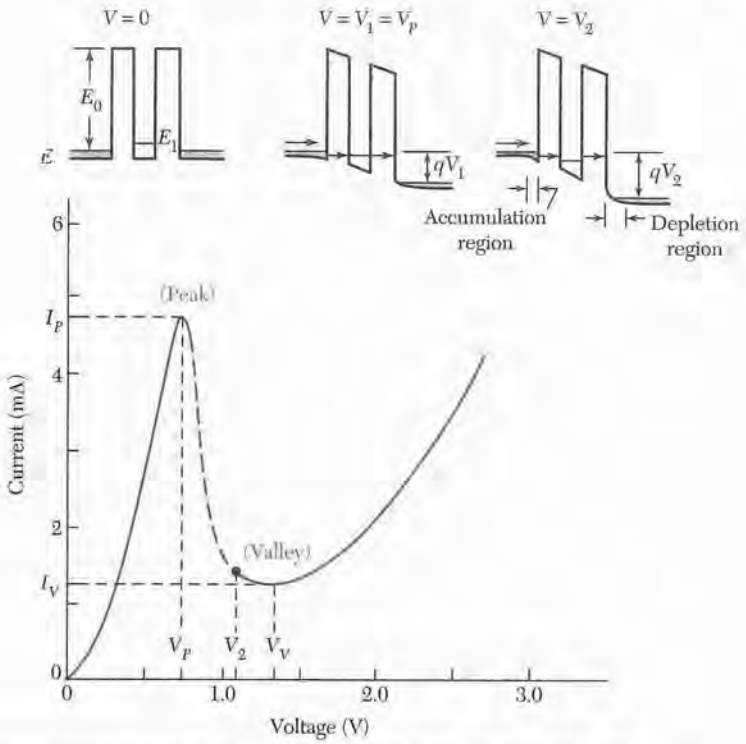


Fig. 16 Measured current–voltage characteristics[13] of the diode in Fig. 15.

there are other components due to tunneling current associated with either lattice vibrations or impurity atoms. To minimize the valley current, we must improve the quality of the heterojunction interfaces and eliminate impurities in the barrier and well regions. For even higher applied voltages, $V > V_V$, we have the thermionic current component I_{th}, due to electrons injected through higher discrete energy levels in the well or thermionically injected over the barriers. The current I_{th} increases monotonically with increasing voltage similar to that of a tunnel diode. To reduce I_{th}, we should increase the barrier height and design a diode that operates at relatively low bias voltages.

RTDs can be operated at very high frequencies because of their smaller parasitics. In the RTD the main contribution to the capacitance is from the depletion region (refer to the band diagram for $V = V_2$ in Fig. 16). Since the doping density there can be much lower than in a degenerate p–n junction, the depletion capacitance is much smaller. The cutoff frequency for a RTD can reach the THz (10^{12} Hz) range. It can be used in ultra fast pulse-forming circuits, in THz radiation detection systems, and in oscillators to generate THz signals.

EXAMPLE 5

Find the ground energy level and the corresponding width of the peak for the RTD in Fig. 15. Compare the peak voltage V_P in Fig. 16 with $2E_1/q$.

SOLUTION From Fig. 14 we find that the ground energy level for $L_W = 4.5$ nm is at 140 meV, and the width of the peak ΔE_1 is about 1 meV. In Fig. 16, V_P is 700 mV, which is larger than 280 mV ($2E_1/q$). The difference (420 mV) is due to voltage drops at the accumulation and the depletion regions.

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8.5.2 Unipolar Resonant Tunneling Transistor

A schematic band diagram of a unipolar resonant tunneling transistor [14] is shown in Fig. 17. The structure consists of a resonant tunneling (RT) double barrier placed between the GaAs emitter and base layers. The RT structure was made of a GaAs quantum well 5.6 nm thick sandwiched between two $Al_{0.33}Ga_{0.67}As$ barriers 5 nm thick. High-energy electrons can be injected from the emitter through the RT into the base region. The electrons are then transported through the n^+-base region 100 nm thick before being collected at the 300 nm thick $Al_{0.2}Ga_{0.8}As$ collector barrier. The barriers and the quantum well are undoped whereas the emitter, base, and collector layers are n-type, doped to 1×10^{18} cm^{-3}.

The operation of the device in the common-emitter configuration with a fixed collector-emitter voltage V_{CE} is shown in the band diagrams of Fig.17. When the base-emitter voltage V_{BE} is zero (Fig. 17*a*), there is no electron injection; hence, the emitter and collector currents are zero even with a positive V_{CE}. A peak in the emitter and collector currents occurs when V_{BE} is equal to $2E_1/q$, where E_1 is the energy of the first resonant level in the quantum well (Fig. 17*b*). With a further increase in V_{BE}, RT is quenched (Fig. 17*c*) with a corresponding drop in the collector current. The current-voltage characteristics is shown in Fig. 17*d*. Note that a current peak occurs around V_{BE} = 0.4 V. If we connect two inputs A and B to the base terminal (Fig. 17*e*), the device can performance

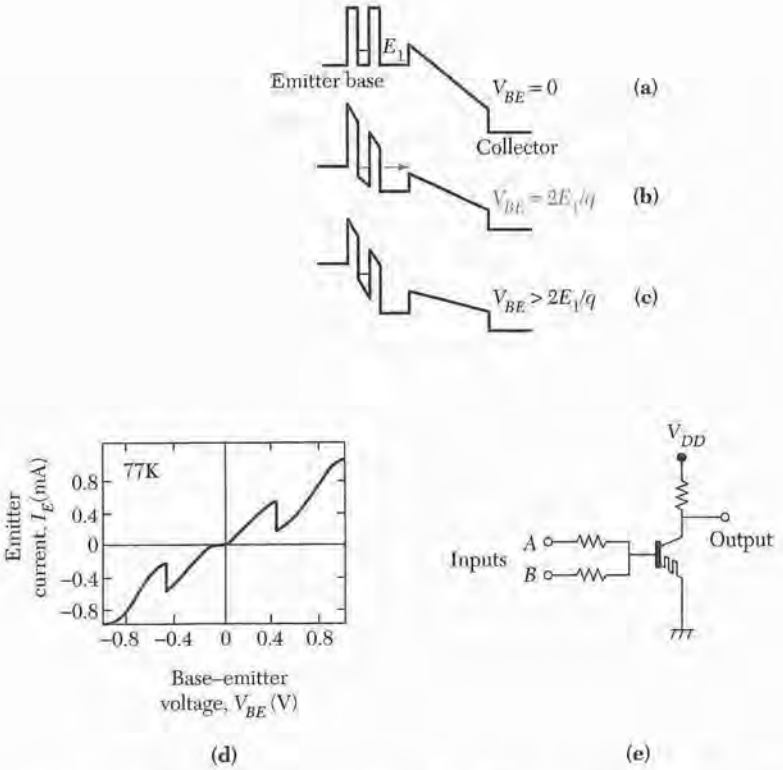


Fig. 17 Band diagrams of a unipolar resonant tunneling (RT) transistor[14] at (*a*) $V_{BE} = 0$, (*b*) $V_{BE} = 2E_1/q$ (maximum RT current), (*c*) $V_{BE} > 2E_1/q$ (RT quenched), (*d*) base-emitter current–voltage characteristics measured at 77 K, and (*e*) an exclusive NOR circuit.

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an exclusive NOR logic function[§], that is, the output voltage will be high, if A and B are both high or both low; otherwise the output voltage will be low. To perform the same function, we need eight conventional MESFETs. Therefore, many quantum-effect devices are useful as functional devices.

8.6 HOT-ELECTRON DEVICES

Hot electrons are electrons with kinetic energies substantially above kT, where k is the Boltzmann's constant and T is the lattice temperature. As the dimensions of semiconductor devices shrink and the internal fields rise, a large fraction of carriers in the active regions of the device during its operation is in states of high kinetic energy. At a given point in time and space the velocity distribution of carriers may be narrowly peaked, in which case one speaks about "ballistic" electron packets. At other times and locations, the electron ensemble can have a broad velocity distribution, similar to a conventional Maxwellian distribution but with an effective electron temperature T_e larger than the lattice temperature T.

Over the years, many hot electron devices have been studied. We now consider two important devices—the hot-electron heterojunction bipolar transistor and the real-space–transfer transistor.

8.6.1 Hot-Electron HBT

Hot-electron injection is enabled in heterojunction bipolar transistors (HBTs) by designing structures with a wider-bandgap emitter, shown[15] in Fig. 18 for an AlInAs/GaInAs HBT lattice matched to InP. There are several advantages of the hot-electron effect. Electrons are injected by thermionic emission over the emitter-base barrier at an energy $\Delta E_C = 0.5$ eV above the conduction band-edge in the p-GaInAs base. Here the purpose

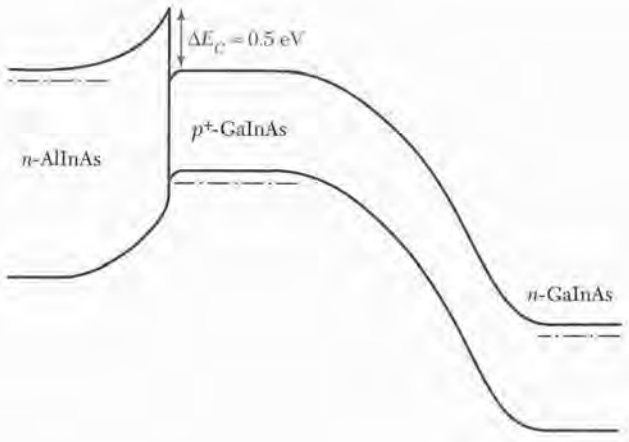


Fig. 18 Energy band diagram of a hot electron heterojunction bipolar transistor.[15]

[§]Exclusive NOR logic function: when the two inputs are both high or both low, the output is high. Otherwise, the output is low.

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of the ballistic injection is to shorten the base traversal time by replacing the relatively slow diffusion motion by faster ballistic propagation.

8.6.2 Real-Space–Transfer Transistor

The original real-space–transfer (RST) structure, illustrated in Fig. 19*a*, is a heterostructure with alternate doped wide-gap AlGaAs and undoped narrow-gap GaAs layers. In thermal equilibrium the mobile electrons reside in the undoped GaAs quantum wells and are spatially separated from their parent donors in AlGaAs layers. [16] If the power input into the structure exceeds the rate of energy loss by the system to the lattice, then the carriers "heat up" and undergo partial transfer into the wide-gap layer where they may have a different mobility (Fig. 19*b*). If the mobility in layer 2 is much lower, negative differential resistance will occur in the two-terminal circuit (Fig. 19*c*). There is a strong analogy to the transferred-electron effect, based on the momentum-space intervalley transfer, therefore the name real-space transfer.

Figure 20 shows a schematic cross-section and the corresponding band diagram of a three-terminal RST transistor (RSTT) implemented in a GaInAs/AlInAs material system.[17,18] The source and drain contacts are to an undoped high-mobility $Ga_{0.47}In_{0.53}As$ (E_g = 0.75 eV with μ_n = 13800 cm²/V-s) channel and the collector contact is to a doped $Ga_{0.47}In_{0.53}As$ conducting layer. This layer is separated from the channel by a larger bandgap material. (i.e., an $Al_{0.48}In_{0.52}As$ with E_g = 1.45 eV). At V_D = 0 an electron density is induced in the source-drain channel by a sufficiently positive collector bias V_C with respect to the

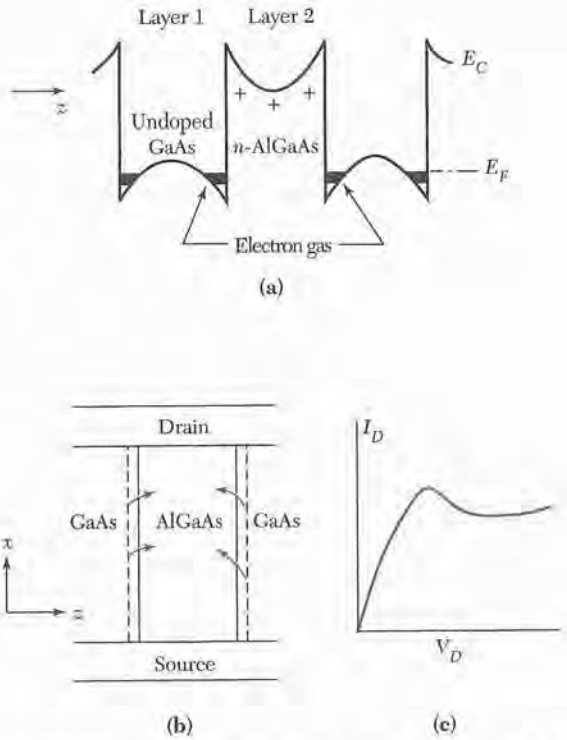


Fig. 19 (*a*) A heterostructure with alternate GaAs and AlGaAs layers. (*b*) Electrons, heated by an applied electric field, transfer into the wide-gap layers. (*c*) If the mobility in layer 2 is lower, the transfer results in a negative differential conductivity.[16]

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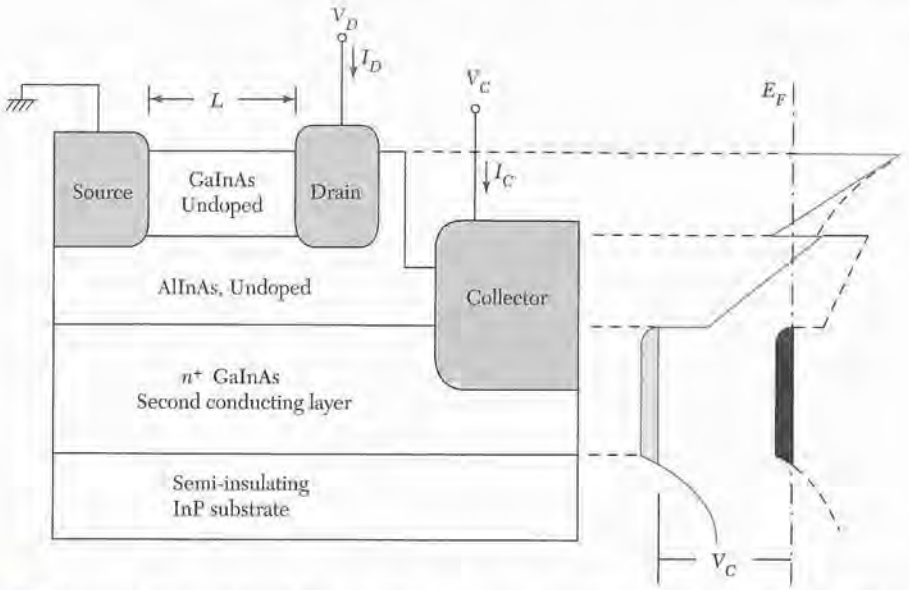


Fig. 20 Cross section and band diagram of a real-space–transfer transistor in a GaInAs/AlInAs material system.[17,18]

ground source, but no collector current I_C flows because of the AlInAs barrier. As V_D increases, however, a drain current I_D begins to flow and the channel electrons heat up to some effective temperature T_e (V_D). This electron temperature determines the RST current injected over the collector barrier. The injected electrons are swept into the collector by the V_C–induced electric field, giving rise to I_C. Transistor action results from control of the electron temperature T_e in the source-drain channel, which modulates I_C flowing into the collector electrode.

The drain current I_D and the collector current I_C versus drain voltage V_D at a fixed V_C = 3.9 V are shown[19] in Fig. 21. On the I_D-V_D characteristics, the RSTT shows pronounced negative differentical resistance, with a peak-to-valley ratio that reaches 7000 at 300 K. In the I_C-V_D characteristics, the collector current increases approximately linearly and eventually reaches a saturation value similar to a field-effect transistor.

The RSTT can be used as a conventional high-speed transistor with high transconductance, $g_m \equiv \partial I_C/\partial V_D$ (at fixed V_C), and a high cutoff frequency f_T. In addition, the RSTT is another useful functional device for logic circuits. This is because the source and drain contacts of an RSTT are symmetrical. A single device, such as that shown in Fig. 20, can perform an exclusive OR (XOR) logic function[§] because the collector current I_C flows if the source and drain are at different logic values, regardless of which is "high." With additional input terminals a single RSTT can perform more complex functions. For example, three input terminals permit an implementation of an OR-NAND gate.* Figure 22*a* shows such an implementation using a Si/SiGe material system.[20] Depending on whether

[§]Exclusive OR logic function: when one of the two inputs, but not both are high, the output is high.

*OR logic function: irrespective of the number of inputs, the output is high if any of the inputs is high. NAND logic function: irrespective of the number of inputs, the output is high at all times except when all inputs are high.

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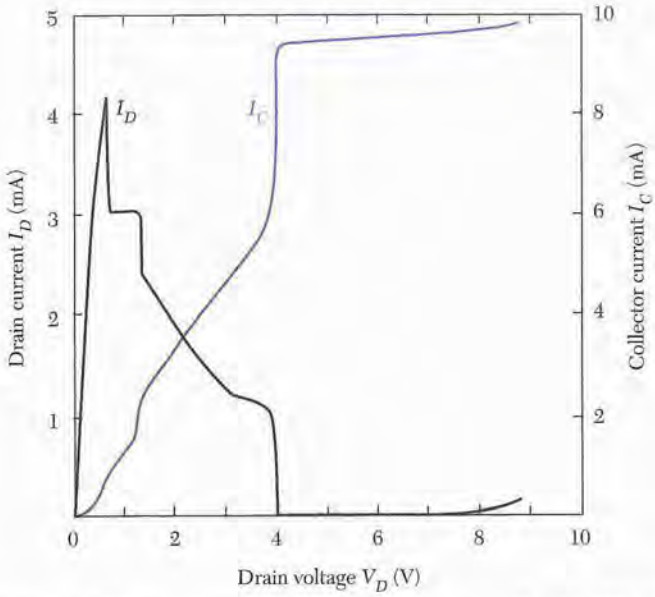
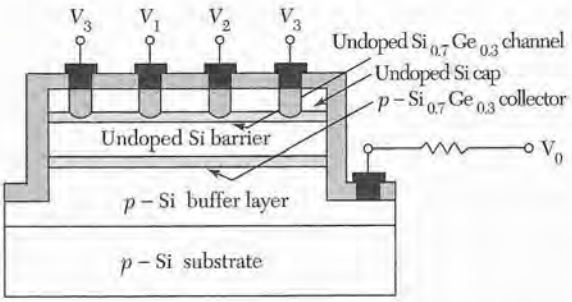


Fig. 21 Experimental real-space–transfer transistor characteristics[19] at $T = 300$ K. Drain current I_D and collector current I_C versus drain voltage V_D at a fixed collector voltage $V_C = 3.9$ V.



(a)

V_1	V_2	V_3	V_0	Logic
0	0	0	0	
1	0	0	1	OR
0	1	0	1	
1	1	0	1	
0	0	1	1	
1	0	1	1	NAND
0	1	1	1	
1	1	1	0	

(b)

Fig. 22 (*a*) A Si/SiGe RSTT OR-NAND gate with three inputs. The channel length between different inputs is 1 μm, and the device width is 50 μm. (*b*) Truth table for OR-NAND logic operation.[20]

the control input is high or low, the output current behaves as either a NAND or an OR function of the other two inputs. A truth table for the output and the logic functions is shown in Fig. 22*b*.

SUMMARY

Diodes associated with tunneling phenomenon (such as the tunnel diode), the avalanche breakdown (IMPATT diode), and momentum-space transfer of electrons (TED) are devices that are used at microwave frequencies. Those two-terminal devices have relatively simple construction and have much less parasitic resistance and capacitance than do their

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three-terminal counterparts. These microwave diodes can operate in the millimeter-wave band (30–300 GHz) and some devices can operate in the submillimeter-wave band (>300 GHz). Among microwave devices, the IMPATT diode is the most extensively used semiconductor device for millimeter-wave power applications. However, the TED is often used in local oscillators and amplifiers because it has lower noise and can be operated at lower voltages than the IMPATT diode.

In this chapter, we also considered the quantum-effect and hot-electron devices. Quantum effects become important when the device dimensions are reduced to about 10 nm. A key quantum-effect device is the resonant tunneling diode (RTD), which is a heterostructure having a double barrier and a quantum well. If the incoming carrier energy is equal to a discrete energy level in the quantum well, the tunneling probability through the double barrier becomes 100%. This effect is called resonant tunneling. Microwave detectors that operate up to the THz (10^{12} Hz) range have been made using a RTD. By combining RTD with conventional devices we can obtain many novel characteristics. One example is the unipolar resonant tunneling transistor, which can perform a given logic function with a greatly reduced number of components.

HEDs can be classified into two groups—ballistic devices and the RST devices, depending on the type of a hot-electron ensemble employed in the operation. Ballistic devices, such as the hot-electron heterojunction bipolar transistor, have the potential for ultrahigh-speed operation. In ballistic devices, high–kinetic-energy electrons are injected over the emitter-base barrier by thermionic emission. This "ballistic propagation" greatly reduces the transit time through the base. In RST devices, electrons in a narrow-gap semiconductor can gain energy from the input power and undergo transfer into a wide-gap semiconductor, giving rise to a NDR characteristic. These devices such as the RSTT have high transconductance and a high cutoff frequency. RSTTs are also used in logic circuits, where they permit a lower component count for a given function than do other devices.

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PROBLEMS (* DENOTES DIFFICULT PROBLEMS)

FOR SECTION 8.1 BASIC MICROWAVE TECHNOLOGY

1. For a nearly lossless transmission line (R is very small) with a 75 Ω characteristic impedance, if this transmission line has a unit length capacitance of 2 pF, what is the unit length inductance of this transmission line?

2. For a cavity of the dimensions a = 10 cm (0.1 m), b = 5 cm (0.05 m), and d = 25 cm (0.25 m), find the resonant frequency in the dominant TE_{101} mode.

FOR SECTION 8.2 TUNNEL DIODE

3. Find the depletion-layer capacitance and depletion-layer width at 0.25 V forward bias for a GaAs tunnel diode doped to 10^{19} cm^{-3} on both sides, using the abrupt junction approximation and assuming $V_n = V_p = 0.03$ V.

4. The current-voltage characteristic of a GaSb tunnel diode can be expressed by the empirical form of Eq. 5. with $I_P = 10$ mA, $V_P = 0.1$ V, and $I_0 = 0.1$ nA. Find the largest negative differential resistance and the corresponding voltage.

FOR SECTION 8.3 IMPATT DIODE

5. The variation of electric field in the depletion region due to avalanche-generated space charge gives rise to an incremental resistance for abrupt p^+-n diode. The incremental resistance is called the space-charge resistance, R_{SC}, and is given by $(1/I)\int_0^W \Delta\mathscr{E}dx$, where $\Delta\mathscr{E}$ is given by

$$\Delta\mathscr{E}(W) = \frac{\int_0^W \rho_S dx}{\varepsilon_S} = \frac{IW}{A\varepsilon_S v_S}.$$

(a) Find R_{SC} for a p^+-n Si IMPATT diode with $N_D = 10^{15}$ cm^{-3}, W = 12 μm, and A = 5×10^{-4} cm^2. (b) Find the total applied dc voltage for a current density of 10^3 A/cm^2.

6. A GaAs IMPATT diode is operated at 10 GHz with a dc bias of 100 V and an average biasing current ($I_0/2$) of 100 mA. (a) If the power-generating efficiency is 25% and the thermal resistance of the diode is 10° C/W, find the junction temperature rise above the

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room temperature. (b) If the breakdown voltage increases with temperature at a rate of 60 mV/°C, find the breakdown voltage of the diode at room temperature.

7. Consider a GaAs single drift lo-hi-lo IMPATT diode shown in Fig. 6*c* with an avalanche region width (where the electric field is constant) of 0.4 μm and a total depletion width of 3 μm. The n^+ clump has a charge Q of $1.5 \times 10^{12}/\text{cm}^2$. (a) Find the breakdown voltage of the diode and the maximum field at breakdown. (b) Is the field in the drift region high enough to maintain the velocity saturation of electrons? (c) Find the operating frequency.

*8. A silicon n^+-p-π-p^+ IMPATT diode has a p-layer 3 μm thick and a π-layer (low-doping p-layer) 9 μm thick. The biasing voltage must be high enough to cause avalanche breakdown in the p-region and velocity saturation in the π region. (a) Find the minimum required biasing voltage and the doping concentration of the p-region. (b) Estimate the transit time of the device.

FOR SECTION 8.4 TRANSFERRED-ELECTRON DEVICES

9. An InP TED is 1 μm long with a cross-section area of 10^{-4} cm^2 and is operated in the transit-time mode. (a) Find the minimum electron density n_0 required for transit-time mode. (b) Find the time between current pulses. (c) Calculate the power dissipated in the device if it is biased at one-half the threshold.

*10. (a) Find the effective density of states in the upper valley N_{CU} of the GaAs conduction band. The upper-valley effective mass is 1.2 m_0. (b) The ratio of electron concentrations between the upper and lower valleys is given by $(N_{CU}/N_{CL})\exp(-\Delta E/kT_e)$, where N_{CL} is the effective density of states in the lower valley, ΔE = 0.31 eV is the energy difference, and T_e is the effective electron temperature. Find the ratio at T_e = 300 K. (c) When electrons gain kinetic energies from the electric field, T_e increases. Find the concentration ratio for T_e = 1500 K.

FOR SECTION 8.5 QUANTUM-EFFECT DEVICES

11. Modecular beam epitaxy interfaces are typically abrupt to within one or two monolayers (one monolayer = 0.28 nm in GaInAs) because of terrace formation in the growth plane. Estimate the energy level broadening for the ground and first excited-electron states of a 15 nm GaInAs quantum well bound by thick AlInAs barriers. (Hint: assume the case of two-monolayer thickness fluctuation and an infinity deep quantum well. The electron effective mass in GaInAs is 0.0427 m_0.)

12. Find the first excited level and the corresponding width of the peak ΔE_2 for a RTD with AlAs (2 nm)/GaAs (6.78 nm)/AlAs (2 nm). If we want to maintain the same energy level but increase the width ΔE_2 by a factor of 10, what should be the thicknesses of AlAs and GaAs?

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CHAPTER 9

Photonic Devices

- 9.1 RADIATIVE TRANSITIONS AND OPTICAL ABSORPTION
- 9.2 LIGHT-EMITTING DIODE
- 9.3 SEMICONDUCTOR LASER
- 9.4 PHOTODETECTOR
- 9.5 SOLAR CELL
- SUMMARY

Photonic devices are devices in which the basic particle of light—the photon—plays a major role. In this chapter, we consider four groups of photonic devices: *light-emitting diodes* (LEDs) and *lasers* (*l*ight *a*mplification by *s*timulated *e*mission of *r*adiation), which convert electrical energy to optical energy; *photodetectors*, which electrically detect optical signals; and *solar cells*, which convert optical energy into electrical energy.

Specifically, we cover the following topics:

- Basic interactions between a photon and an electron.
- Generation of photons by spontaneous emission for the conventional and organic LEDs.
- Generation of photons by stimulated emission for heterostructure lasers.
- Absorption of photons to create electron-hole pairs for photodetectors.
- Absorption of photons to convert them to electrical energy for solar cells.

9.1 RADIATIVE TRANSITIONS AND OPTICAL ABSORPTION

Figure 1 shows the electromagnetic spectrum of the optical region. The detectable range of light by the human eye extends only from approximately 0.4 μm to 0.7 μm. Figure 1 also shows the major color bands from violet to red in the expanded scale. The ultraviolet region includes wavelengths from 0.01 μm to 0.4 μm, and the infrared region extends from 0.7 μm to 1,000 μm. In this chapter, we are primarily interested in the wavelength range from near-ultraviolet (~0.3 μm) to near-infrared (~1.5 μm).

Figure 1 also shows the photon energy on a separate horizontal scale. To convert the wavelength to photon energy, we use the relationship

$$\lambda = \frac{c}{\nu} = \frac{hc}{h\nu} = \frac{1.24}{h\nu(\text{eV})} \ \mu\text{m}, \tag{1}$$

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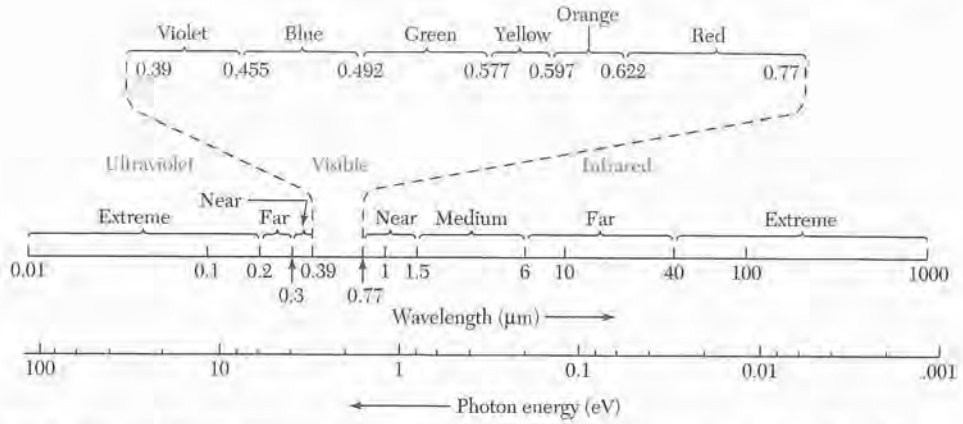


Fig. 1 Chart of the electromagnetic spectrum from the ultraviolet region to the infrared region.

where c is the speed of light in vacuum, ν is the frequency of light, h is Planck's constant, and $h\nu$ is the energy of a photon, and is measured in electron volts. For example, a 0.5 μm green light corresponds to a photon energy of 2.48 eV.

9.1.1 Radiative Transitions

There are basically three processes for interaction between a photon and an electron in a solid: absorption, spontaneous emission, and stimulated emission. We use a simple system to demonstrate these processes.[1] Consider two energy levels E_1 and E_2 of an atom, where E_1 corresponds to the ground state and E_2 corresponds to the excited state (Fig. 2). Any transition between these states involves the emission or absorption of a photon with frequency ν_{12} given by $h\nu_{12} = E_2 - E_1$. At room temperature, most of the atoms in a solid are at the ground state. This situation is disturbed when a photon of energy exactly equal to $h\nu_{12}$ impinges on the system. An atom in state E_1 absorbs the photon and thereby goes to the excited state E_2. The change in the energy state is the *absorption* process, shown in Fig. 2*a*. The excited state of the atom is unstable. After a short time, without any external stimulus, it makes a transition to the ground state, giving off a photon of energy $h\nu_{12}$. This process is called *spontaneous emission* (Fig. 2*b*). When a photon of energy $h\nu_{12}$ impinges on an atom while it is in the excited state (Fig. 2*c*), the atom can be stimulated to make a transition to the ground state and gives off a photon of energy $h\nu_{12}$, which is in phase with the incident radiation. This process is called *stimulated emission*. The radiation from stimulated emission is monochromatic because each photon has precisely an energy $h\nu_{12}$ and is coherent because all photons emitted are in phase.

The dominant operating process for LEDs is spontaneous emission, for the laser diodes (LDs) it is stimulated emission, and for the photodetectors and the solar cell it is absorption.

Let us assume that the instantaneous populations of E_1 and E_2 are n_1 and n_2, respectively. Under a thermal equilibrium condition and for $(E_2 - E_1) > 3\,kT$, the population is given by Boltzmann distribution:

$$\frac{n_2}{n_1} = e^{-(E_2 - E_1)/kT} = e^{-h\nu_{12}/kT}. \tag{2}$$

The negative exponent indicates that n_2 is less than n_1 in thermal equilibrium; that is, most electrons are at the lower energy level.

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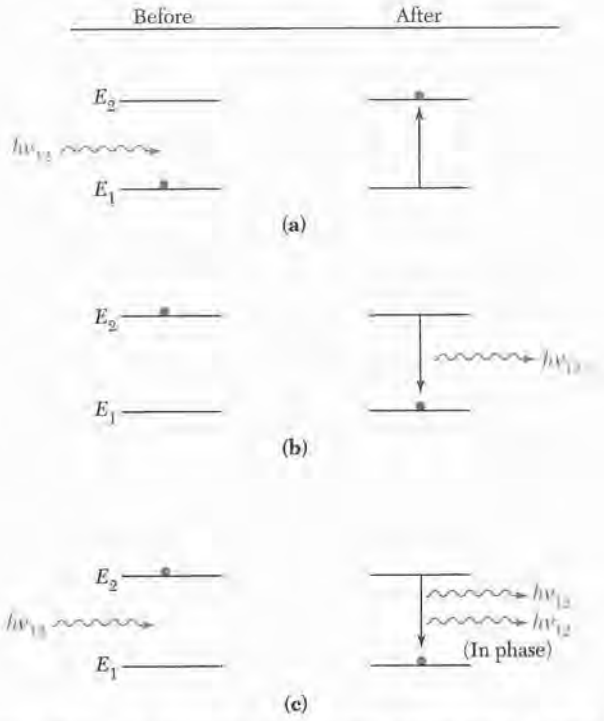


Fig. 2 The three basic transition processes between two energy levels. [1] Black dots indicated the state of the atom. The initial state is at the left; the final state, after the transition, is at the right. (*a*) Absorption. (*b*) Spontaneous emission. (*c*) Stimulated emission.

In steady state, the stimulated-emission rate (i.e., the number of stimulated-emission transitions per unit time) and the spontaneous-emission rate must be balanced by the rate of absorption to maintain the population n_1 and n_2 constant. The stimulated-emission rate is proportional to the photon-field energy density $\rho\,(h\nu_{12})$, which is the total energy in the radiation field per unit volume per unit frequency. Therefore, the stimulated-emission rate can be written as $B_{21}n_2\rho(h\nu_{12})$, where n_2 is the number of electrons in the upper level and B_{21} is a proportionality constant. The spontaneous-emission rate is proportional only to the population of the upper level and can be written as $A_{21}n_2$, where A_{21} is a constant. The absorption rate is proportional to the electron population at the lower level and to $\rho\,(h\nu_{12})$; this rate can be written as $B_{12}n_1\rho(h\nu_{12})$, where B_{12} is a proportionality constant. Therefore, we have at steady state

stimulated-emission rate + spontaneous-emission rate = absorption rate,

or

$$B_{21}n_2\rho\,(h\nu_{12}) + A_{21}n_2 = B_{12}n_1\rho\,(h\nu_{12}). \tag{3}$$

From Eq. 3 we observe that

$$\frac{\text{stimulated} - \text{emission rate}}{\text{spontaneous} - \text{emission rate}} = \frac{B_{21}}{A_{21}}\rho\left(h\nu_{12}\right). \tag{4}$$

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To enhance stimulated emission over spontaneous emission, we must have a very large photon-field energy density $\rho\,(h\nu_{12})$. To achieve this density, an optical resonant cavity is used to increase the photon field. We also observe from Eq. 3 that

$$\frac{\text{stimulated} - \text{emission rate}}{\text{absorption rate}} = \frac{B_{21}}{B_{12}}\left(\frac{n_2}{n_1}\right). \tag{5}$$

If the stimulated emission of photon is to dominate over the absorption of photons, we must have higher electron density in the upper level than in the lower level. This condition is called *population inversion*, since under an equilibrium condition the reverse is true. In Section 9.3 on semiconductor lasers, we consider various ways to have a large photon-field energy density and achieve population inversion, so that the stimulated emission becomes dominant over both spontaneous emission and absorption.

9.1.2 Optical Absorption

Figure 3 shows the basic transitions in a semiconductor. When the semiconductor is illuminated, photons are absorbed to create electron-hole pairs, as shown in Fig. 3*a*, if the photon energy is equal to the bandgap energy, that is, $h\nu$ equals E_g. If $h\nu$ is greater than E_g, an electron-hole pair is generated and, in addition, the excess energy $(h\nu - E_g)$ is dissipated as heat, as shown in Fig. 3*b*. Both processes, (Figs. *a* and *b*) are called *intrinsic transitions* (or band-to-band transitions). On the other hand, for $h\nu$ less than E_g, a photon will be absorbed only if there are available energy states in the forbidden bandgap due to chemical impurities or physical defects, as shown in Fig. 3*c*. That process is called *extrinsic transition*. This discussion also is generally true for the reverse situation. For example, an electron at the conduction band edge combining with a hole at the valence band edge will result in the emission of a photon with energy equal to that of the bandgap.

Assume that a semiconductor is illuminated from a light source with $h\nu$ greater than E_g and a photon flux of Φ_0 in units of photons per square centimeter per second. As the photon flux travels through the semiconductor, the fraction of the photons absorbed is proportional to the intensity of the flux. Therefore, the number of photons absorbed within

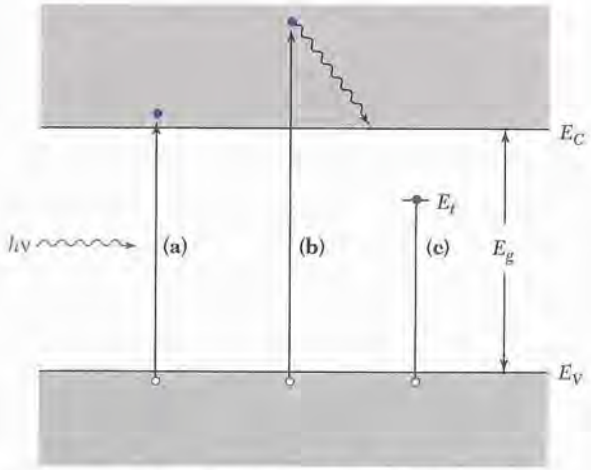


Fig. 3 Optical absorption for (*a*) $h\nu = E_g$, (*b*) $h\nu > E_g$, and (*c*) $h\nu < E_g$.

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an incremental distance Δx (Fig. 4*a*) is given by $\alpha\Phi(x)\Delta x$, where α is a proportionality constant defined as the absorption coefficient. From the continuity of photon flux as shown in Fig. 4*a*, we obtain

$$\Phi(x+\Delta x)-\Phi(x)=\frac{d\Phi(x)}{dx}\Delta x=-\alpha\Phi(x)\Delta x$$

or

$$\frac{d\Phi(x)}{dx}=-\alpha\Phi(x). \tag{6}$$

The negative sign indicates a decreasing intensity of the photon flux due to absorption. The solution of Eq. 6 with the boundary condition $\Phi(x) = \Phi_0$, at $x = 0$ is

$$\Phi(x)=\Phi_0\ e^{-\alpha x}. \tag{7}$$

The fraction of photon flux that exits from the other end of the semiconductor at $x = W$ (Fig. 4*b*) is

$$\boxed{\Phi(W)=\Phi_0 e^{-\alpha W}.} \tag{8}$$

The absorption coefficient α is a function of $h\nu$. Figure 5 shows the measured optical absorption coefficient for some important semiconductors that are used for photonic devices.[2] Also shown is the absorption coefficient for amorphous silicon (dashed curve), which is an important material for solar cells. The absorption coefficient decreases rapidly at the cutoff wavelength λ_c that is,

$$\lambda_c=\frac{1.24}{E_g}\ \mu\text{m} \tag{9}$$

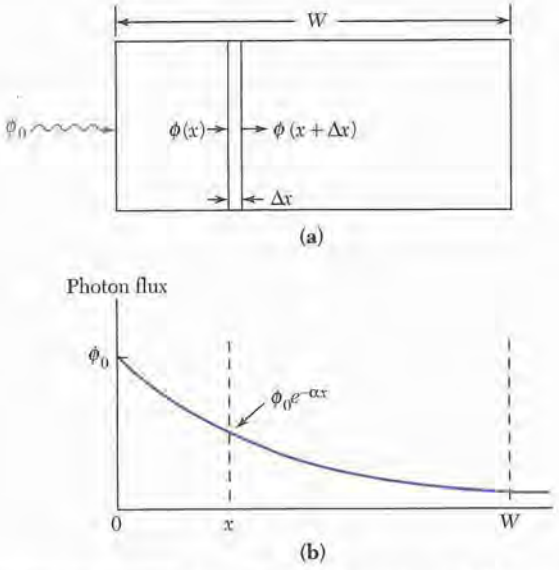


Fig. 4 Optical absorption. (a) Semiconductor under illumination. (b) Exponential decay of photon flux.

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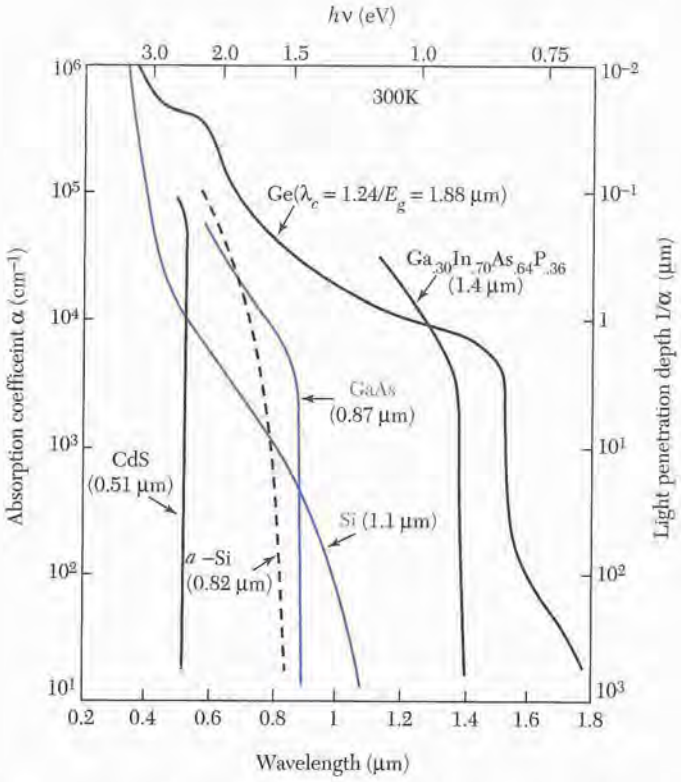


Fig. 5 Optical absorption coefficients for various semiconductor materials.[2] The value in the parenthesis is the cutoff wavelength.

because the optical band-to-band absorption becomes negligible for $h\nu < E_g$, or $\lambda > \lambda_c$.

EXAMPLE 1

A single-crystal silicon sample 0.25 μm-thick is illuminated with a monochromatic light (single frequency) having an $h\nu$ of 3 eV. The incident power is 10 mW. Find the total energy absorbed by the semiconductor per second, the rate of excess thermal energy dissipated to the lattice, and the number of photons per second given off from recombination by intrinsic transitions.

SOLUTION From Fig. 5 the absorption coefficient α is 4×10^4 cm^{-1}. The energy absorbed per second is

$$\Phi_0(1 - e^{-\alpha W}) = 10^{-2}\left[1 - \exp\left(-4 \times 10^4 \times 0.25 \times 10^{-4}\right)\right]$$

$$= 0.0063 \text{ J/s} = 6.3 \text{ mW}.$$

The portion of each photon's energy that is converted to heat is

$$\frac{h\nu - E_g}{h\nu} = \frac{3 - 1.12}{3} = 62\%.$$

Therefore, the amount of energy dissipated per second to the lattice is

$$62\% \times 6.3 = 3.9 \text{ mW}.$$

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Since the recombination radiation accounts for 2.4 mW (i.e., 6.3 mW –3.9 mW) at 1.12 eV/ photon, the number of photons per second from recombination is

$$\frac{2.4\times10^{-3}}{1.6\times10^{-19}\times1.12}=1.3\times10^{16}\ \text{photons / s.}$$

9.2 LIGHT-EMITTING DIODES

Light-emitting diodes (LEDs) are p-n junctions that can emit spontaneous radiation in ultraviolet, visible, or infrared regions. The visible LED has a multitude of applications as an information link between electronic instruments and their users. The infrared LED is useful in opto-isolators and for optical-fiber communication.

9.2.1 Visible LEDs

Figure 6 shows the relative eye response as a function of wavelength (or the corresponding photon energy). The maximum sensitivity of the eye is at 0.555 μm. The eye response falls to nearly zero at the extremes of the visible spectrum at about 0.4 and 0.7 μm. For normal vision at the peak response of the eye, 1 W of radiant energy is equivalent to 683 lumen.

Since the eye is only sensitive to light with a photon energy $h\nu$ equal to or greater than 1.8 eV (≤0.7 μm), semiconductors of interest must have an energy bandgap larger than this limit. Figure 6 also shows the bandgaps of various semiconductors. Table 1 lists the semiconductors used to produce light in the visible and infrared parts of the spectrum. Among all the semiconductors shown, the most important materials for visible LEDs are the alloy $GaAs_{1-y}P_y$ and $Ga_xIn_{1-x}N$ III-V compound systems. An alloy III-V compound is formed when more than one group III element is distributed randomly on group III

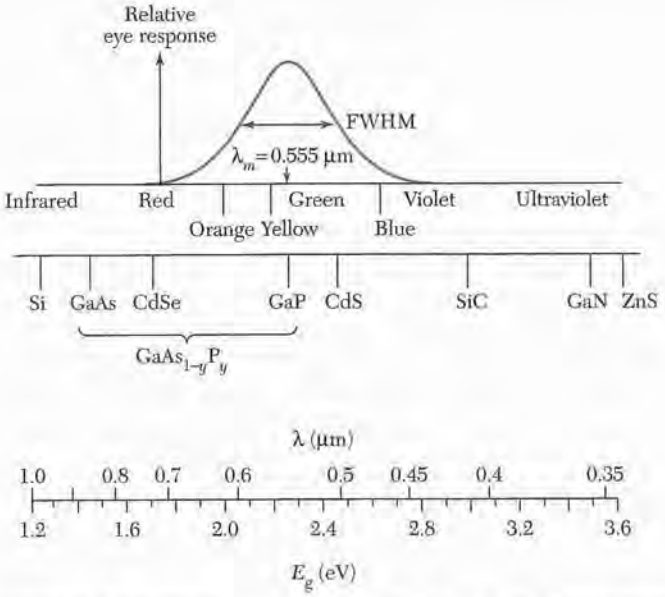


Fig. 6 Semiconductors of interest as visible LEDs. Figure includes relative response of the human eye.

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TABLE 1 Common III-V materials used to produce LEDs and their emission wavelengths.

Material	Wavelength (nm)
InAsSbP/InAs	4200
InAs	3800
GaInAsP/GaSb	2000
GaSb	1800
$Ga_xIn_{1-x}As_{1-y}P_y$	1100-1600
$Ga_{0.47}In_{0.53}As$	1550
$Ga_{0.27}In_{0.73}As_{0.63}P_{0.37}$	1300
GaAs:Er,InP:Er	1540
Si:C	1300
GaAs:Yb,InP:Yb	1000
$Al_xGa_{1-x}As$:Si	650-940
GaAs:Si	940
$Al_{0.11}Ga_{0.89}As$:Si	830
$Al_{0.4}Ga_{0.6}As$:Si	650
$GaAs_{0.6}P_{0.4}$	660
$GaAs_{0.4}P_{0.6}$	620
$GaAs_{0.15}P_{0.85}$	590
$(Al_xGa_{1-x})_{0.5}In_{0.5}P$	655
GaP	690
GaP:N	550-570
$Ga_xIn_{1-x}N$	340,430,590
SiC	400-460
BN	260,310,490

lattice sites (e.g., gallium sites) or more than one group V element are distributed randomly on group V lattice sites (e.g., arsenic sites). The notation used is $A_xB_{1-x}C$ or $AC_{1-y}D_y$ for *ternary* (three elements) compounds and $A_xB_{1-x}C_yD_{1-y}$ for *quaternary* (four elements) compounds, where A and B are the group III elements, C and D are the group V elements, and x and y are the mole fractions, that is, the ratios of the number of atoms of a given species to the total number of group III or group V atoms in the alloy compound.

Figure 7*a* shows the energy gap for $GaAs_{1-y}P_y$ as a function of the mole fraction y. For $0 < y < 0.45$, the bandgap is direct and increases from $E_g = 1.424$ eV at $y = 0$ to $E_g = 1.977$ eV at $y = 0.45$. For $y > 0.45$, the bandgap is indirect. Figure 7*b* shows the corresponding energy-momentum plots for selected alloy compositions.[3] As indicated, the conduction band has two minima. The one along $p = 0$ is the direct minimum, and the one along $p = p_{max}$ is the indirect minimum. Electrons in the direct minimum of the conduction band and holes at the top of the valence band have equal momenta ($p = 0$). But electrons in the indirect minimum of the conduction band and holes at the top of the valence band have different momenta. The radiative transition mechanisms are found predominantly in direct-bandgap semiconductors, such as gallium arsenide and $GaAs_{1-y}P_y$ ($y < 0.45$), since the momentum is conserved. The photon energy is approximately equal to the bandgap energy of the semiconductor.

However, for $GaAs_{1-y}P_y$ with y greater than 0.45 and gallium phosphide, which are indirect-bandgap semiconductors, the probability for radiative transitions is very small, since lattice interactions or other scattering agents must participate in the process to

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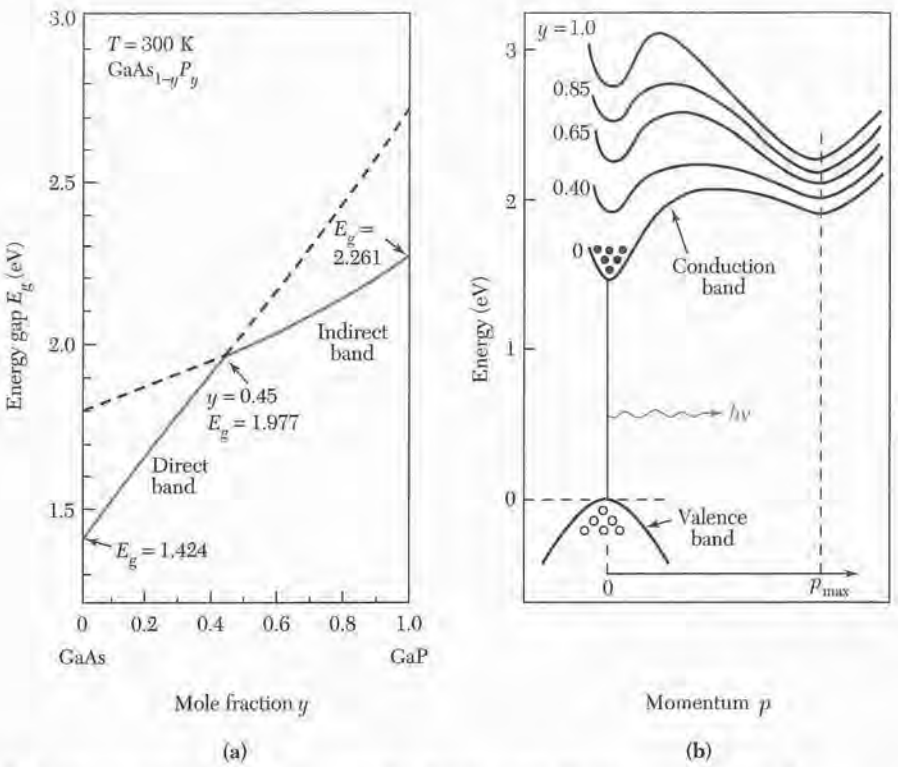


Fig. 7 (a) Compositional dependence for the direct- and indirect-energy bandgap for $GaAs_{1-y}P_y$. (b) The alloy compositions shown correspond to red (y = 0.4), orange (0.65), yellow (0.85), and green light (1.0). [3]

conserve momentum. Therefore, for indirect-bandgap semiconductors, special recombination centers are incorporated to enhance the radiative processes. Incorporating nitrogen into the crystal lattice can form efficient radiative recombination centers in $GaAs_{1-y}P_y$. When nitrogen is introduced, it replaces phosphorous atoms in the lattice sites. The outer electronic structure of nitrogen is similar to that of phosphorus (both are group V elements in the periodic table), but the electronic core structures of these atoms are different. This difference results in the creation of an electron trap level close to the bottom of the conduction band. A recombination center is thus produced, and it is called an *isoelectronic center*. This recombination center can greatly enhance the probability of radiative transition in indirect-bandgap semiconductors.

Figure 8 shows the quantum efficiency, the number of photons generated per electron-hole pair, versus alloy composition for $GaAs_{1-y}P_y$ with and without the isoelectronic impurity nitrogen.[4] The efficiency without nitrogen drops sharply in the composition range $0.4 < y < 0.5$ because the bandgap changes from direct to indirect at $y = 0.45$. The efficiency with nitrogen is considerably higher for $y > 0.5$ but nevertheless decreases steadily with an increasing y because of the increasing separation between the direct and indirect bandgap (Fig. 7*b*).

Figure 9 shows the basic visible LED structures in the flat-diode configuration.[5] Figure 9*a* shows the cross section of a direct-bandgap LED, which emits red light, fabricated on gallium arsenide substrate. Figure 9*b* is a indirect bandgap LED, which emits orange, yellow, or green light fabricated on gallium phosphide substrates. A graded-alloy

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