
Semiconductor Materials and Modelling

Tom Markvart, School of Engineering Sciences,
University of Southampton, UK
Luis Castañer, Universidad Politecnica de Catalunya,
Barcelona, Spain

1	Introduction	96
2	Semiconductor Band Structure	96
3	Carrier Statistics in Semiconductors	99
4	The Transport Equations	100
5	Carrier Mobility	102
6	Carrier Generation by Optical Absorption	104
6.1	Band-to-Band Transitions	104
6.2	Free-Carrier Absorption	106
7	Recombination	107
7.1	Bulk Recombination Processes	108
7.2	Surface Recombination	110
7.3	Minority-Carrier Lifetime	111
8	Radiation Damage	112
9	Heavy Doping Effects	114
10	Properties of Hydrogenated Amorphous Silicon	115
	Acknowledgements	118
	References	118

1 Introduction

Solar cell modelling is fundamental to a detailed understanding of the device operation, and a comprehensive model requires a detailed knowledge of the material parameters. A brief overview of the semiconductor properties relevant to solar cell operation is given in this chapter, including the semiconductor band structure and carrier statistics, transport and optical properties, recombination processes, material aspects of radiation damage to solar cells in space and effects observed under heavy doping, with special attention given to the properties of hydrogenated amorphous silicon. The principal semiconductor parameters encountered in photovoltaic applications are summarised in Tables 1 and 2. The refractive indices of materials used for antireflection coating can be found in Table 3.

Numerous computer programs that use the material parameters to model solar cell operation have been developed over the years, and several are now available commercially:

- PC1D developed by P.A. Basore and colleagues at the University of New South Wales, Australia, is the standard one-dimensional simulator used by the PV community.
- ATLAS, a Device Simulation Software by SILVACO International, uses physical models in two and three dimensions. It includes the Luminous tool which computes ray tracing and response of solar cells. It allows the use of monochromatic or multi-spectral sources of light.
- MEDICI by Technology Modelling Associates models the two-dimensional distribution of potential and carrier concentration in a semiconductor device. It also includes an Optical device advanced application module where photogeneration can be computed for multi-spectral sources.

Some programs are available free of charge. These include, for example, SimWindows which can be downloaded from <http://www-ocs.colorado.edu/SimWindows/simwin.html>.

A discussion of the main principles of the numerical techniques can be found in specialised texts which deal with the modelling of solar cells (see, for example, the review [1]) or with the modelling of semiconductor devices in general [2].

2 Semiconductor Band Structure

The energy gap (or band gap) E_g and its structure as a function of the wave vector are key characteristics of the semiconductor material and of fundamental importance to the operation of the solar cell (see Figure 1). The principal features of interest are the temperature variation of the band gap energy E_g and the magnitude of wave vector associated with low-energy transitions.

The variation of the band gap with temperature can be described by an expression originally suggested by Varshni [10]

Table 1 Properties of the principal semiconductors with photovoltaic applications (all at 300 K). Bandgap: d = direct; i = indirect. Crystal structure: dia = diamond, zb = zinc blende, ch = chalcopyrite. The refractive index is given at the wavelength 590 nm (2.1 eV) unless otherwise stated. Principal sources of data: c-Si [20]; GaAs [3]; InP [4, 41]; a-Si [5] and Section 10; CdTe [6]; CIS [7]; $\text{Al}_x\text{Ga}_{1-x}\text{As}$ [9]. Details of the absorption coefficient and refractive index, including the wavelength dependence, can be found in references [27] and [28]. χ is the electron affinity and TEC stands for thermal expansion coefficient

	E_g (eV)	Crystal structure	ε	n	χ (eV)	Lattice const. (Å)	Density (g/cm^{-3})	TEC (10^{-6}K^{-1})	Melting point (K)	Comments
c-Si	1.12 (i)	dia	11.9	3.97	4.05	5.431	2.328	2.6	1687	Cell material (Part IIb)
GaAs	1.424 (d)	zb	13.18	3.90	4.07	5.653	5.32	6.03	1510	Cell material (Ch. II d-1)
InP	1.35 (d)	zb	12.56	3.60	4.38	5.869	4.787	4.55	1340	Cell material (Ch. II d-1)
a-Si	~ 1.8 (d)	–	~ 11	3.32						Cell material, invariably Si:H alloy, sometimes also alloyed with germanium
CdTe	1.45–1.5 (d)	zb	10.2	2.89*	4.28	6.477	6.2	4.9	1365	or carbon (Chs. II c-1, -2)
CuInSe_2 (CIS)	0.96–1.04 (d)	ch			4.58			6.6	~ 1600	Cell material (Ch. II c-3)
$\text{Al}_x\text{Ga}_{1-x}\text{As}$ ($0 \leq x \leq 0.45$)	1.424 + 1.247x (d)	zb	13.18 – 3.12x		4.07–1.1x	5.653 + 0.0078x	5.36 – 1.6x	6.4–1.2x		Cell material, often alloyed with gallium (Ch. II c-4)
$(0.45 < x \leq 1)$	1.9+0.125x + 0.143x ² (i)				3.64 – 0.14x					Window layer for GaAs solar cells

* At 600 nm.

Table 2 The energy gap E_g , refractive index n and the electron affinity χ of transparent conducting semiconductors used as window layers in thin-film solar cells

Material	E_g (eV)	n	χ
CdS	2.42	2.5	4.5
ZnS	3.58	2.4	3.9
$Zn_{0.3}Cd_{0.7}S$	2.8		4.3
ZnO	3.3	2.02	4.35
$In_2O_3:Sn$	3.7–4.4		4.5
$SnO_2:F$	3.9–4.6		4.8

Table 3 The refractive index at 590 nm (2.1 eV) of the common materials used for antireflection coating. The full wavelength dependence of most of these substances can be found in references [27] and [28]

Material	n
MgF_2	1.38
SiO_2	1.46
Al_2O_3	1.76
Si_3N_4	2.05
Ta_2O_5	2.2
ZnS	2.36
SiO_x	1.8–1.9
TiO_2	2.62

$$E_g(T) = E_{g0} - \frac{\alpha T^2}{T + \beta} \quad (1)$$

where T is the absolute temperature and the parameters α and β are given in Table 4.

The current produced by solar cells is generated by optical transitions across the band gap. Two types of such transitions can be distinguished: direct transitions where the momentum of the resulting electron–hole pair is very close to zero, and indirect transitions where the resulting electron hole pair has a finite momentum. The latter transitions require the assistance of a phonon (quantum of lattice vibration). Thus, there are two types of semiconductors:

- **Direct gap semiconductors** where the top of the valence band and the bottom of the conduction band lie at the Γ point of the first Brillouin zone (i.e., at zero wave vector $\mathbf{k} = 0$).
- **Indirect gap semiconductors** where the minima of the conduction band (in general, more than one) lie at a another point of the first Brillouin zone, with a different value of the wave vector $\mathbf{k}$.

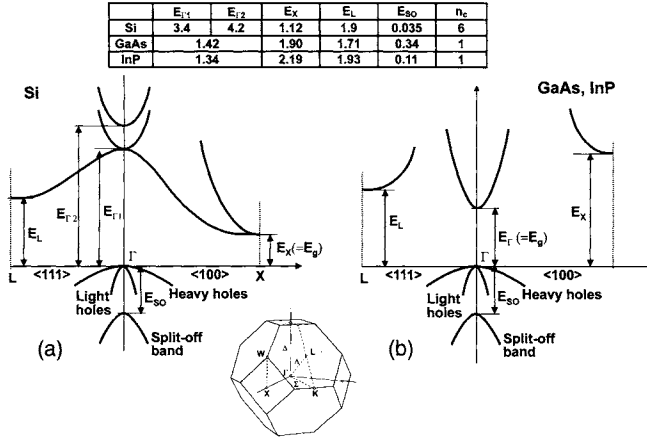


Figure 1 The energy gaps in Si (a), and GaAs and InP (b) as functions of the wave vector $\mathbf{k}$. The inset shows the Brillouin zone of the corresponding face-centred cubic crystal lattice.

Table 4 The parameters E_{g0} , α and β in Equation (1). Sources of data: (a) Thurmond [11]; (b) Varshni [10]

	$E_g (T = 0 \text{ K}), \text{ eV}$	$\alpha \times 10^{-4}, \text{ eV/K}^2$	$\beta, \text{ K}$
Si (a)	1.17	4.730	636
GaAs (a)	1.52	5.405	204
InP (b)	1.42	4.906	327

The optical absorption in indirect-gap semiconductors is considerably weaker than in direct gap semiconductors, as shown in Figure 2 on the example of silicon and gallium arsenide.

3 Carrier Statistics in Semiconductors

In thermal equilibrium the temperature and electrochemical potential (the Fermi level, denoted by E_F) are constant throughout the device. The product of the electron concentration n and the hole concentration p is then independent of doping and obeys the mass action law

$$np = n_i^2 = \mathcal{N}_c \mathcal{N}_v \exp\left(-\frac{E_g}{k_B T}\right) \quad (2)$$

where k_B is the Boltzmann constant, n_i is the electron (or hole) concentration in the intrinsic semiconductor, and the effective densities of states $\mathcal{N}_c$ and $\mathcal{N}_v$ are given by

$$\mathcal{N}_c = 2 \left(\frac{2\pi m_e k_B T}{h^2} \right)^{3/2} \quad \mathcal{N}_v = 2 \left(\frac{2\pi m_h k_B T}{h^2} \right)^{3/2} \quad (3)$$

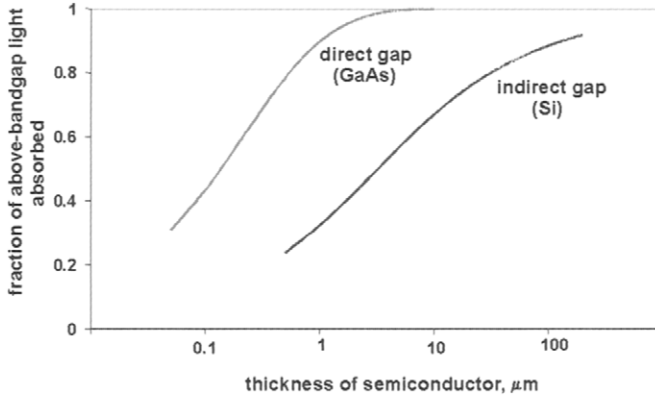


Figure 2 A comparison of the difference in strength of the optical absorption in direct and indirect semiconductors, illustrated on the examples of silicon and gallium arsenide.

Here, h is the Planck constant and m_e and m_h are the electron and hole density-of-states effective masses (see Table 6). In Equation (3), the effective mass m_e includes a factor which allows for several equivalent minima of the conduction band in indirect-gap semiconductors (see Figure 1).

Equation (2) does not hold for a solar cell in operation. Current flow as well as electron transitions between different bands and/or other quantum states are induced by differences and gradients of the electrochemical potentials, and temperature gradients may also exist. It is then usual to assign a quasi-Fermi level to each band which describes the appropriate type of carriers. Thus, electrons in the conduction band will be described by the quasi-Fermi level E_{Fn} , and holes by E_{Fp} . It is convenient to define also the appropriate potentials ϕ_n and ϕ_p by

$$\begin{aligned} E_{Fn} &= -q\phi_n \\ E_{Fp} &= -q\phi_p \end{aligned} \quad (4)$$

The use of quasi-Fermi levels to describe solar cell operation was already mentioned in Section 4.1 of Chapter IIa-1. The formalism leads to expressions for electron and hole concentrations which are summarised in Table 5.

4 The Transport Equations

The electron and hole current densities J_n and J_p are governed transport by the transport equations

$$\begin{aligned} J_n &= q\mu_n n \mathcal{E} + qD_n \nabla n \\ J_p &= q\mu_p p \mathcal{E} - qD_p \nabla p \end{aligned} \quad (5)$$

Table 5 Carrier concentration in degenerate and non-degenerate semiconductors. $F_{1/2}$ denotes the integral $F_{1/2}(z) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} \frac{x^{1/2}}{1 + \exp(x-z)} dx$

Non-degenerate semiconductors			General expressions
	In terms of the band parameters	In terms of the parameters of intrinsic semiconductor	
n	$\mathcal{N}_c \exp\left(\frac{E_{Fn} - E_c}{k_B T}\right)$	$n_i \exp\left\{\frac{q(\psi - \varphi_n)}{k_B T}\right\}$	$\mathcal{N}_c F_{1/2}\left(\frac{E_{Fn} - E_c}{k_B T}\right)$
p	$\mathcal{N}_v \exp\left(\frac{E_v - E_{Fp}}{k_B T}\right)$	$n_i \exp\left\{\frac{q(\varphi_p - \psi)}{k_B T}\right\}$	$\mathcal{N}_v F_{1/2}\left(\frac{E_v - E_{Fp}}{k_B T}\right)$

Table 6 The densities of states in the conduction and valence band ($\mathcal{N}_c$ and $\mathcal{N}_v$, respectively), the intrinsic carrier concentration n_i and the density-of-states effective masses m_e and m_h (all at 300 K) in Si, GaAs and InP. m_o denotes the free-electron mass. Data from references [20, 35]; intrinsic carrier concentration in Si from reference [12]

	$\mathcal{N}_c$ (cm ⁻³)	$\mathcal{N}_v$ (cm ⁻³)	n_i (cm ⁻³)	m_e/m_o	m_h/m_o
Si	2.8×10^{19}	1.04×10^{19}	1.00×10^{10}	1.08	0.55
GaAs	4.7×10^{17}	7.0×10^{18}	1.79×10^6	0.063	0.53
InP	5.7×10^{17}	1.1×10^{19}	1.3×10^7	0.08	0.6

where n and p are the electron and hole the concentrations, μ_n and μ_p are the electron and hole mobilities, D_n and D_p are the electron and hole diffusion constants, and $\mathcal{E}$ is the electric field. The first term in each equation is due to drift in the electric field $\mathcal{E}$, and the second term corresponds to carrier diffusion. With the help of the quasi-Fermi levels, Equations (5) can be written as

$$\begin{aligned} J_n &= -q\mu_n n \nabla \phi_n \\ J_p &= -q\mu_p p \nabla \phi_p \end{aligned} \quad (6)$$

Equations (6) are valid for a semiconductor with a homogeneous composition but position-dependent dopant concentration is included. A generalisation to semiconductors with position-dependent band gap can be found, for example, in [13].

In a region where space charge exists (for example, in the junction), the Poisson equation is needed to link the electrostatic potential ψ with the charge density ρ :

$$\nabla \cdot \mathcal{E} = \frac{\rho}{\varepsilon} \quad (7)$$

with

$$\rho = q(p - n + N_D - N_A) \quad (8)$$

$$\mathcal{E} = -\nabla \cdot \psi \quad (9)$$

where ϵ is the static dielectric constant of the semiconductor (see Table 1).

In non-degenerate semiconductors, the diffusion constants are related to mobilities by the *Einstein relations*

$$D_n = \frac{kT}{q} \mu_n; \quad D_p = \frac{kT}{q} \mu_p \quad (10)$$

The generalisation of the Einstein relations to degenerate semiconductors is discussed, for example, in [14].

The conservation of electrons and holes is expressed by the *continuity equations*

$$\begin{aligned} \frac{\partial n}{\partial t} &= G - U + \frac{1}{q} \nabla \cdot \mathbf{J}_n \\ \frac{\partial p}{\partial t} &= G - U + \frac{1}{q} \nabla \cdot \mathbf{J}_p \end{aligned} \quad (11)$$

where G and U are the generation and recombination rates which may be different for electrons and holes if there are transitions into or from localised states.

5 Carrier Mobility

In weak fields, the drift mobilities in Equations (5) represent the ratio between the mean carrier velocity and the electric field. The mobilities – which are generally different for majority and minority carriers – depend on the concentration of charged impurities and on the temperature. For silicon, these empirical dependencies are generally expressed in the Caughey–Thomas form [15]:

$$\mu = \mu_{min} + \frac{\mu_0}{1 + \left(\frac{N}{N_{ref}} \right)^\alpha} \quad (12)$$

The values of the various constants for majority and minority carriers are given in Tables 7 and 8. A full model which includes the effects of lattice scattering, impurity scattering, carrier-carrier scattering and impurity clustering effects at high concentration is described in [16] where the reader can find further details.

In strong fields, the mobility of carriers accelerated in an electric field parallel to the current flow is reduced since the carrier velocity saturates. The field dependence of the mean velocity v (only quoted reliably for majority carriers) is given by

Table 7 The values of parameters in Equation (12) for majority-carrier mobility in silicon (from reference [17]; $T_n = T/300$)

	$\mu_{min} = AT_n^{-\beta}$		$\mu_0 = BT^{-\beta_2}$		$N_{ref} = CT_n^{\beta_3}$		$\alpha = DT_n^{-\beta_4}$	
	A	β_1	B	β_2	C	β_3	D	β_4
Electrons	88		7.4×10^8		1.26×10^{17}			
Holes	54.3	0.57	1.36×10^8	2.33	2.35×10^{17}	2.4	0.88	0.146

Table 8 The values of parameters in Equation (12) for minority-carrier mobility in silicon (from references [18, 19])

	μ_{min}	μ_0	N_{ref}	α
Electrons	232	1180	8×10^{16}	0.9
Holes	130	370	8×10^{17}	1.25

$$v = \frac{\mu_{lf}}{\left(1 + \left(\frac{\mu_{lf}\mathcal{E}}{v_{sat}}\right)^\beta\right)^{1/\beta}} \quad (13)$$

where μ_{lf} is the appropriate low field value of the mobility (Equation (12)), the parameter $\beta = 2$ for electrons and $\beta = 1$ for holes, and v_{sat} is the saturation velocity, identical for electrons and holes [20]:

$$v_{sat} = \frac{2.4 \times 10^7}{1 + 0.8e^{T/600}} \quad (14)$$

In gallium arsenide, the empirical fitting of the mobility data is of a more complex nature, and the reader is referred to the reference [21] for more information regarding the majority-carrier mobility. The temperature and dopant concentration dependences of the minority-carrier mobility are shown in Figures 3 and 4.

The electric field dependence of carrier mobility in GaAs is different from silicon as the velocity has an ‘overshoot’ which is normally modelled by the following equation [24]:

$$\mu_n = \frac{\mu_{lf}\mathcal{E} + v_{sat}(\mathcal{E}/\mathcal{E}_0)^\beta}{1 + (\mathcal{E}/\mathcal{E}_0)^\beta} \quad (15)$$

where μ_{lf} is the appropriate low-field mobility, $\mathcal{E}_0 = 4 \times 10^3$ V/cm, β equals 4 for electrons and 1 for holes, and v_{sat} is given by [25]

$$v_{sat} = 11.3 \times 10^6 - 1.2 \times 10^4 T \quad (16)$$

where T is the temperature in K.

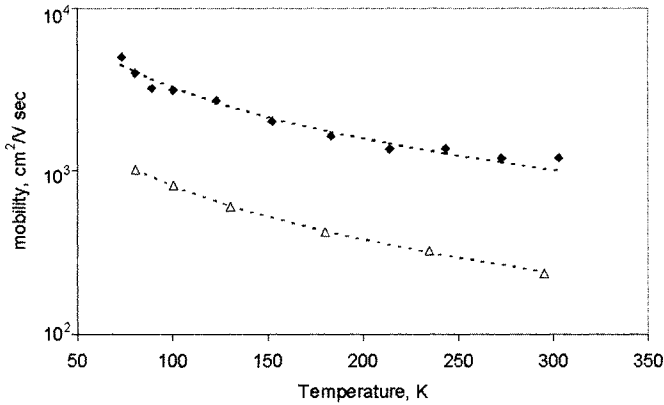


Figure 3 The temperature dependence of the minority-carrier mobility in GaAs. Full symbols: electron mobility in p-type GaAs ($N_A = 4 \times 10^{18} \text{ cm}^{-3}$); data from [22]. Empty symbols: hole mobility in n-type GaAs ($N_D = 1.8 \times 10^{18} \text{ cm}^{-3}$) [23]. The dashed lines show fits to the data with expressions

$$\mu_n = \frac{337\,100}{T} - 116; \quad \mu_p = \frac{85\,980}{T} - 49.84;$$

where T is the temperature in K.

The majority carrier mobility in indium phosphide reported in reference [26] is shown in Figure 5.

6 Carrier Generation by Optical Absorption

6.1 Band-to-Band Transitions

The principal means of carrier generation in solar cells is the absorption of light. For a planar slab (Figure 6), a photon which enters the semiconductor generates $g(x) \delta x$ electron hole pairs in a thin layer at depth $x \rightarrow x + \delta x$. The generation function $g(x)$ is given by

$$g(x) = \alpha(\lambda) \exp\{-\alpha(\lambda)x\} \quad (17)$$

where $\alpha(\lambda)$ is the absorption coefficient, shown in Figure 7 for a number of semiconductors with photovoltaic applications. The generation rate G per unit volume which appears in Equation (11) is related to the generation function g in Equation (17) by $G = g/A$, where A is the illuminated area of the sample.

A useful formula for the absorption coefficient of silicon is provided by the expression of Rajkanan et al. [29]:

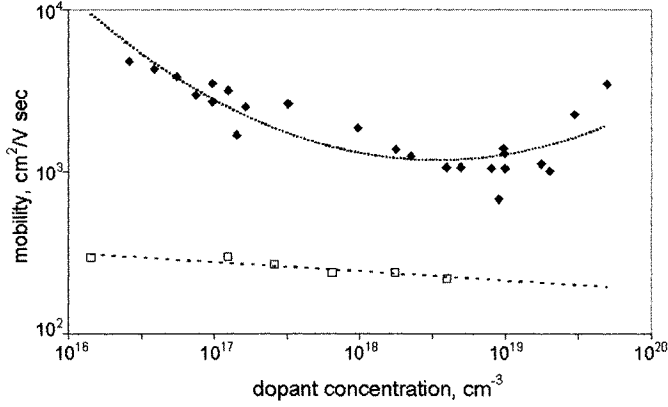


Figure 4 The dopant concentration dependence of the room-temperature minority-carrier mobility in GaAs. Full symbols: electron mobility in p-type GaAs; data from [21]. Empty symbols: hole mobility in n-type GaAs [23]. The dashed lines show fits to the data with the expressions

$$\log \mu_n = 0.16(\log N_A)^2 - 5.93 \log N_A + 58$$

$$\log \mu_p = -0.0575 \log N_D + 3.416$$

where N_D and N_A are the donor and acceptor concentrations in cm^{-3} .

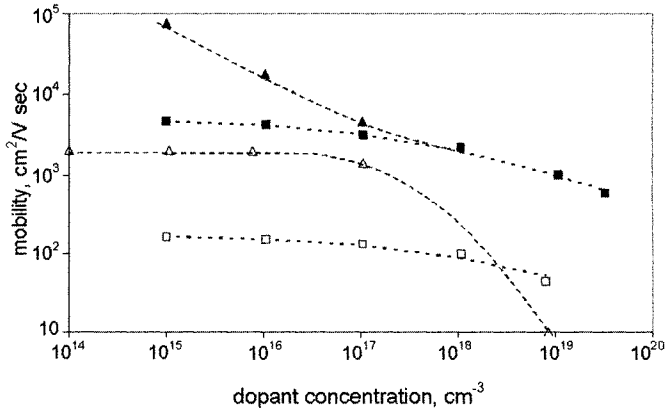


Figure 5 The majority-carrier mobilities in InP [26]. Full symbols correspond to electron mobility, empty symbols to hole mobility. Room temperature (300 K) data are shown by squares; data at 77 K by triangles. The room-temperature data were fitted with a Caughey–Thomas-type expression with parameters shown in Table 9.

$$\alpha(T) = \sum_{\substack{i=1,2 \\ j=1,2}} C_i A_j \left[\frac{\{h\nu - E_{gj}(T) + E_{pi}\}^2}{\{\exp(E_{pi}kT) - 1\}} + \frac{\{h\nu - E_{gj}(T) - E_{pi}\}^2}{\{1 - \exp(-E_{pi}kT)\}} \right] + A_d \{h\nu - E_{gd}(t)\}^{1/2} \quad (18)$$

Table 9 Parameters in Equation (12) which were used to fit the room-temperature electron and hole mobilities in InP

	μ_{min}	μ_0	N_{ref}	α
Electrons	0	4990	4.02×10^{17}	0.433
Holes		168	1.24×10^{18}	

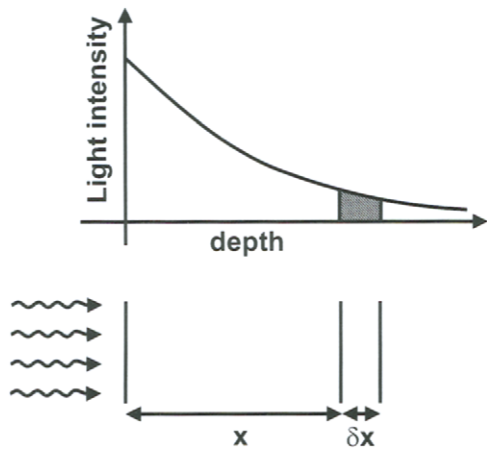


Figure 6 The geometry used to discuss the light absorption in semiconductors.

where $h\nu$ is the photon energy, $E_{g1}(0) = 1.1557$ eV, $E_{g2}(0) = 2.5$ eV and $E_{gd}(0) = 3.2$ eV are the two lowest indirect and the lowest direct band gap, respectively (used here as parameters to obtain a fit to the spectrum), $E_{p1} = 1.827 \times 10^{-2}$ eV and $E_{p2} = 5.773 \times 10^{-2}$ eV are the Debye frequencies of the transverse optical and transverse acoustic phonons, respectively, $C_1 = 5.5$, $C_2 = 4.0$, $A_1 = 3.231 \times 10^2$ cm⁻¹ eV⁻², $A_2 = 7.237 \times 10^3$ cm⁻¹ eV⁻² and $A_1 = 1.052 \times 10^6$ cm⁻¹ eV⁻². The temperature variation of the band gaps is described by Equation (1) where the original Varshni coefficients $\alpha = 7.021 \times 10^{-4}$, eV/K² and $\beta = 1108$ K are used for all three band gaps E_{g1} , E_{g2} and E_{gd} .

6.2 Free-Carrier Absorption

In regions with high-carrier concentration (due to doping or strong illumination, for example) photon absorption can also occur by electron transitions with initial and final states inside the same band. This free-carrier absorption does not generate electron–hole pairs and competes with the band-to-band transitions that produce the photogenerated current which were discussed above. Free carrier absorption might be significant in the case of photon energies near the band gap, and should not be included in the absorption coefficient α in front of the exponential in Equation (17). Figure 8 illustrates the different phenomena that occur near the band edge for high doping concentrations.

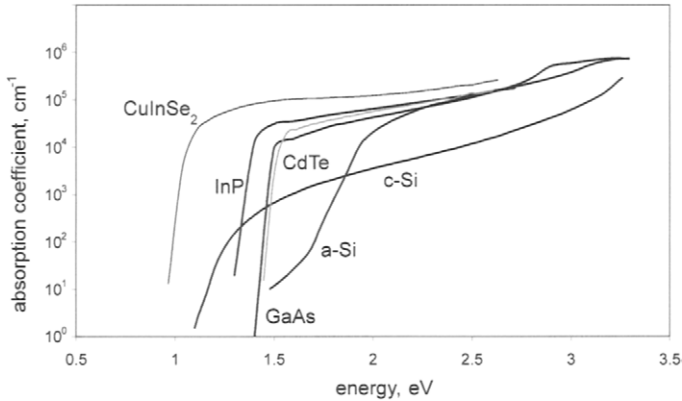


Figure 7 The absorption coefficients of the principal semiconductors used in the manufacture of solar cells. Full details of these and other semiconductors can be found in references [27] and [28]. Further details of silicon absorption are given in the text.

Table 10. The constants for the free carrier absorption coefficient in Equation (19)

	K_1	a	K_2	b
Si	2.6×10^{-27}	3	2.7×10^{-24}	2
GaAs	4×10^{-29}	3	—	—
InP	5×10^{-27}	2.5	—	—

Based on experimental data of [30] and [31], the PC1D model, for example, uses the following expression for the absorption coefficient due to free carrier absorption:

$$\alpha_{FC} = K_1 n \lambda^a + K_2 p \lambda^b \quad (19)$$

where λ is in nanometres and the empirically determined constants K_1 , K_2 , a , and b are given in Table 10.

7 Recombination

Recombination processes can be classified in a number of ways. Most texts distinguish between bulk and surface recombination, and between band-to-band recombination as opposed to transitions with the participation of defect levels within the band gap. Recombination processes can also be classified according to the medium which absorbs the energy of the recombining electron–hole pair: radiative recombination (associated with photon emission), or the two principal non-radiative mechanisms by Auger and multi-phonon transitions, where the recombination energy is absorbed by a free charge carrier or by lattice vibrations, respectively. An opposite process to Auger recombination (where an electron hole pair is generated rather than consumed) is called impact ionisation.

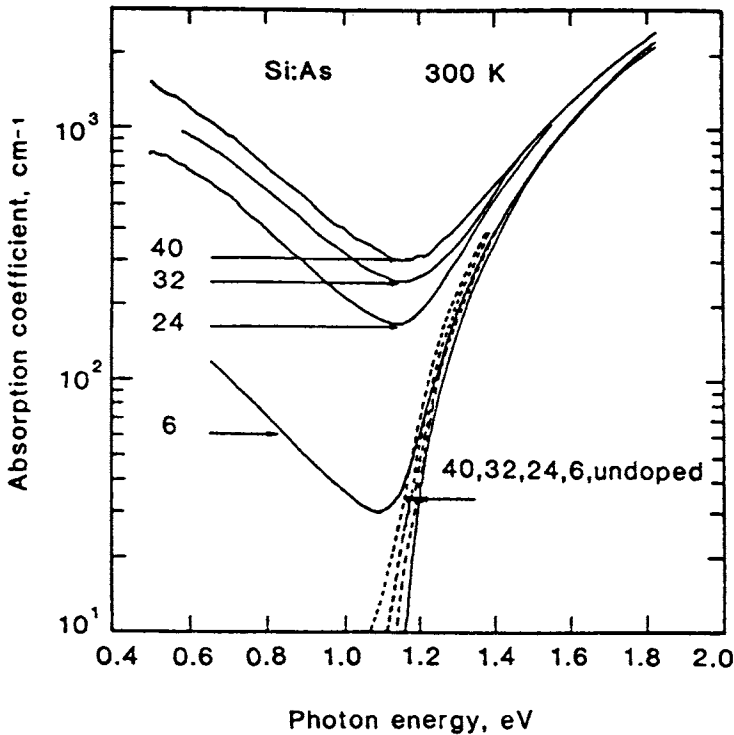


Figure 8 The effects observed near the absorption edge at high doping concentrations, illustrated here on the example of *n*-type silicon (from reference [30], as adapted by M.A. Green, *Silicon Solar Cells – See Bibliography*).

The following sections give a brief summary of the main recombination mechanisms with relevance to solar cell operation which are summarised schematically in Figure 9.

7.1 Bulk Recombination Processes

A detailed discussion of the variety of the bulk recombination mechanisms can be found in [32]. Here, we confine ourselves to a brief overview of the radiative, Auger and defect-assisted recombination processes which are most frequently encountered in practical operation of solar cells. These processes are depicted schematically in Figure 9 which also shows the notation used to describe the relevant parameters.

The rate of *band-to-band radiative recombination* can be written in the form

$$U_{rad} = B(np - n_i^2) \quad (20)$$

where the coefficient B is sometimes written as R/n_i^2 . Radiative transitions between a free electron and a localised state within the band gap may also be important in certain situations, for example in novel concepts such as the

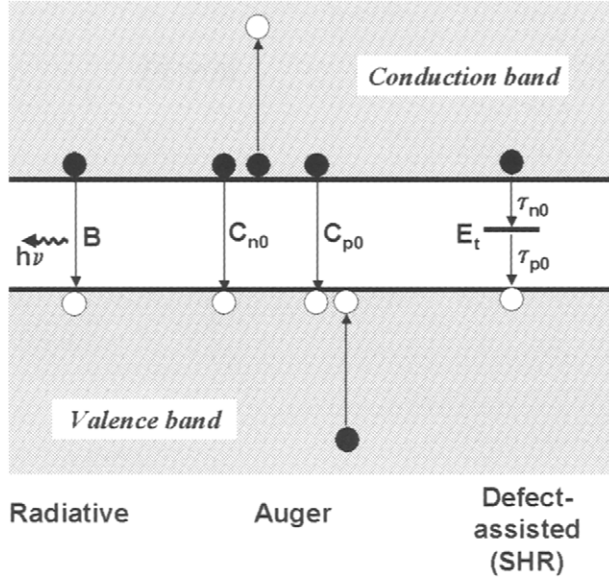


Figure 9 A schematic diagram of the principal recombination processes in semiconductors, and the notation for the rate constant adopted in this book. The direction of arrows indicates electron transitions.

impurity photovoltaic effect. The values of B for silicon, gallium arsenide and indium phosphide can be found in Table 11. In other situations, the radiative recombination coefficient B can be determined from optical absorption using the detailed balance argument due to van Roosbroeck and Shockley [33].

The rate of *band-to-band Auger recombination* can be written as

$$U_{Auger} = (C_{p0}p + C_{n0}n)(np - n_i^2) \quad (21)$$

where the first bracket gives the two, usually most important, Auger terms. Table 11 gives values of the coefficients C_{p0} and C_{n0} for Si, GaAs and InP.

The *recombination rate via defects* of concentration N_t with a level at energy E_t within the band gap is described by the Shockley–Read–Hall formula [36, 37]:

$$U_{SHR} = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)} \quad (22)$$

where

$$n_1 = n_i \exp\left(\frac{E_t - E_i}{k_B T}\right) \quad p_1 = p_i \exp\left(\frac{E_t - E_i}{k_B T}\right) \quad (23)$$

and τ_n , τ_p are parameters, proportional to the defect concentration N_t , which are characteristic for the particular defect and energy level. At low injection, τ_n and τ_p assume the meaning of minority carrier lifetimes. With an appropriate

Table 11 The coefficients B of the radiative recombination rate (Equation (20)) and C_{n0} and C_{p0} of the Auger recombination rate (Equation (21)). Sources of data: (a) [34]; (b) [32]; (c) [35]; (d) [26]

	$B, \text{cm}^3 \text{s}^{-1}$	$C_{n0}, \text{cm}^6 \text{s}^{-1}$	$C_{p0}, \text{cm}^6 \text{s}^{-1}$
Si	1.8×10^{-15} (a)	2.8×10^{-31} (b)	0.99×10^{-31} (b)
GaAs	7.2×10^{-10} (a)	$\sim 10^{-30}$ (c)	
InP	6.25×10^{-10} (d)	$\sim 9 \times 10^{-31}$ (c)	

dependence on the doping concentration and temperature, τ_n and τ_p are also used extensively in material and device modelling (see reference [20], Section 1.5.3 and Section 7.3 below for a fuller discussion).

7.2 Surface Recombination

Surface recombination velocity is an important parameter which affects the dark saturation current and the quantum efficiency of solar cells. Similarly to dislocations and planar defects such as grain boundaries, surfaces (and interfaces in general) introduce band of electronic states in the band gap which can be ascribed to broken (or strained) bonds and impurities. A complete characterisation of surface recombination must also take into account the surface charge which may give rise to band bending. To achieve optimal operation, surface recombination is reduced by a passivating or window layer which prevents minority carriers from reaching the surface. Passivation of silicon surface by an oxide layer, or the deposition of a thin ‘window’ layer of GaAlAs on GaAs solar cells are just two examples of such practice.

For an oxidised silicon surface, surface recombination velocity is strongly dependent on the surface roughness, contamination, ambient gases used during oxidation and the annealing conditions. Under identical process parameters, however, one can identify trends in the dependence of the surface recombination velocity on the surface doping concentration. Cuevas et al. [38] proposed the following analytical relationship between surface recombination velocity and doping concentration:

$$S = 70 \text{ cm/s for } N < 7 \times 10^{17} \text{ cm}^{-3}$$

$$S = 70 \left(\frac{N}{7 \times 10^{17}} \right) \text{ for } N > 7 \times 10^{17} \text{ cm}^{-3} \quad (24)$$

Equation (24) models several experimental results such as those reported in [39].

In gallium arsenide, the surface recombination velocity is very high (of the order of 10^6 cm/s). The deposition of a thin layer of GaAlAs, however, reduces the recombination velocity at the interface to $10\text{--}10^3$ cm/s (see reference [40], p. 41). Coutts and Yamaguchi [41] quote the values of $10^3\text{--}2 \times 10^4$ cm/s and 1.5×10^5 cm/s for the surface recombination velocity in n- and p-type InP, respectively.

7.3 Minority-Carrier Lifetime

Under low injection – a regime of particular importance for solar cell operation – the majority-carrier concentration can be assumed excitation independent, and the effect of recombination can be discussed in terms of minority-carrier lifetime. In p-type material, for example, and the recombination rate can be written as

$$U = \frac{1}{\tau_n}(n - n_0) \quad (25)$$

where τ_n is the minority-carrier (electron) lifetime. An analogous equation can be written for the hole lifetime τ_p in n-type material. The inverse of the lifetime – the rate constant – is a sum of the different contributions to the lifetime:

$$\frac{1}{\tau} = \frac{1}{\tau_{rad}} + \frac{1}{\tau_{Auger}} + \frac{1}{\tau_{SRH}} \quad (26)$$

where τ stands for τ_n or τ_p , as appropriate. This additive nature of the recombination rate constant is also useful when discussing the radiation damage (see Section 8).

The effect of lifetime on transport properties by carrier diffusion can be discussed in terms of the diffusion length defined by

$$L = \sqrt{D\tau} \quad (27)$$

where D is the diffusion constant for the minority carriers in question. If, however, drift in electric field $\mathcal{E}$ is the dominant transport mechanism, it is appropriate to define the drift length as

$$\ell_n = \mathcal{E}\tau_n\mu_n; \quad \ell_p = \mathcal{E}\tau_p\mu_p \quad (28)$$

for electrons and holes as minority carriers. This parameter plays an important role in the analysis of p-i-n junction solar cells (see Section 4.3 in Chapter IIa-1).

The wealth of available data for crystalline silicon has made it possible to arrive at a consensus as to the magnitude as well as the temperature and doping concentration dependence of the contributions (26) to minority carrier lifetime [42] The contribution to lifetime due to defects, when combined with recombination in intrinsic material, has been empirically observed to follow the equations

$$\begin{aligned} \frac{1}{\tau_{n,SRH}} &= \left(\frac{1}{2.5 \times 10^{-3}} + 3 \times 10^{-13} N_D \right) \left(\frac{300}{T} \right)^{1.77} \\ \frac{1}{\tau_{p,SRH}} &= \left(\frac{1}{2.5 \times 10^{-3}} + 11.76 \times 10^{-13} N_A \right) \left(\frac{300}{T} \right)^{0.57} \end{aligned} \quad (29)$$

where the first term in the brackets applies for recombination in intrinsic semiconductor. Similarly, the contribution to Equation (26) by Auger recombination can be described by the expressions

$$\begin{aligned}\frac{1}{\tau_{n,Auger}} &= 1.83 \times 10^{-31} p^2 \left(\frac{T}{300} \right)^{1.18} \\ \frac{1}{\tau_{p,Auger}} &= 2.78 \times 10^{-31} n^2 \left(\frac{T}{300} \right)^{0.72}\end{aligned}\quad (30)$$

Although the concept of minority carrier lifetime is most commonly applied to bulk recombination, a similar notion can be relevant for surface processes. For example, the effective lifetime observed of minority carriers with uniform concentration in a wafer can be written as

$$\frac{1}{\tau_{eff}} = \frac{1}{\tau_{SRH}} + \frac{2W}{A} S \quad (31)$$

where S is the value of the recombination velocity, W is the wafer thickness and A is the area of the sample.

8 Radiation Damage

Solar cells which operate on board of a satellite in an orbit which passes through the van Allen belts are subjected to fluxes of energetic electrons and protons trapped in the magnetic field of the earth and by fluxes of particles associated with high solar activity [43, 44] (see Chapter IIId-2). When slowed down in matter, most of the energy of the incident proton or electron is dissipated by interaction with the electron cloud. A relatively small fraction of this energy is dissipated in collisions with the nuclei, resulting in the formation of a lattice defect when energy in excess of a minimum threshold value is transferred to the nucleus [45, 46]. A proton with energy in excess of this threshold causes the most damage (typically, of the order of 10–100 atomic displacements) near the end of its range in the crystal. At high energy, on the other hand, a proton creates simple point defects, and the displacement rate decreases with increasing proton energy. In contrast, the displacement rate by electrons increases rapidly with energy and approaches a constant value at higher energies. Electron damage can usually be assumed to be uniform throughout the crystal.

Some defects which are thus created act as recombination centres and reduce the minority-carrier lifetime τ . Thus, it is the lifetime τ (or, equivalently, the diffusion length L) which is the principal quantity of concern when the cell is subjected to the particle radiation in space. Under low injection, the reduction of the diffusion length L is described by the Messenger–Spratt equation (see, for example, [47], p. 151):

$$\frac{1}{L^2} = \frac{1}{L_0^2} + K_L \phi \quad (32)$$

where L_0 is the diffusion length in the unirradiated cell, ϕ is the particle fluence (integrated flux), and K_L is a (dimensionless) diffusion-length damage constant characteristic for the material and the type of irradiation.

The damage constant K_L generally depends on the type of dopant and, even in single-junction cells, a different damage constant should therefore be introduced for each region of the cell. Moreover, care should be exercised when dealing with low-energy protons (with energy of the order of 0.1–1 MeV) which are stopped near the surface and may create non-uniform damage near the junction. Figures 10 and 11 show the doping concentration and energy dependence of the damage constant K_L for electrons and protons in p-type silicon. Details of the damage constants for other materials can be found in references [43], [44], [48] and [49].

Damage constants are usually quoted for 1 MeV electrons, and converted to other energies and particles (such as protons) by using the concept of damage equivalent. Conversion tables are available in the Solar Radiation Handbooks which will suit most circumstances. Radiation damage equivalence works well when applied to uniform damage throughout the cell but care should be exercised in the case of non-uniform damage, for example, for low-frequency protons.

The reduction of the diffusion length describes usually the most significant part of the damage but changes in other parameters on irradiation also sometimes need to be considered. The dark diode current I_0 may increase as a result of compensation by the radiation-induced defects. This, however, occurs

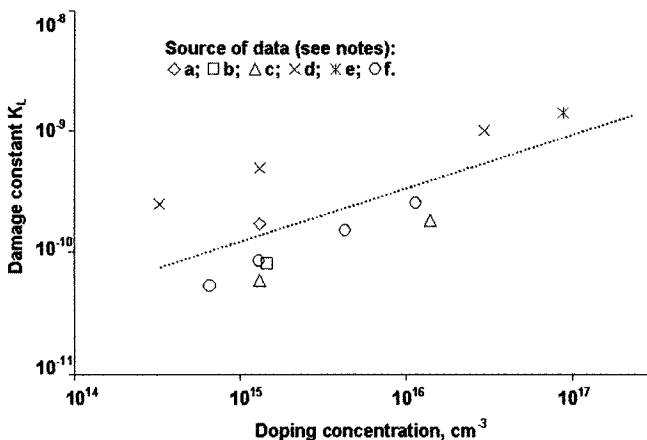


Figure 10 The radiation damage constant K_L for 1 MeV electrons in p-type silicon as a function of the dopant concentration. Source of data: (a) [50]; (b) [51]; (c) [52]; (d) [53]; (e) [54]; (f) [55]. The dashed line shows a fit to the data with the formula $K_L = 3.43 \times 10^{-17} N_A^{0.436}$, where N_A is the acceptor concentration in cm^{-3} .

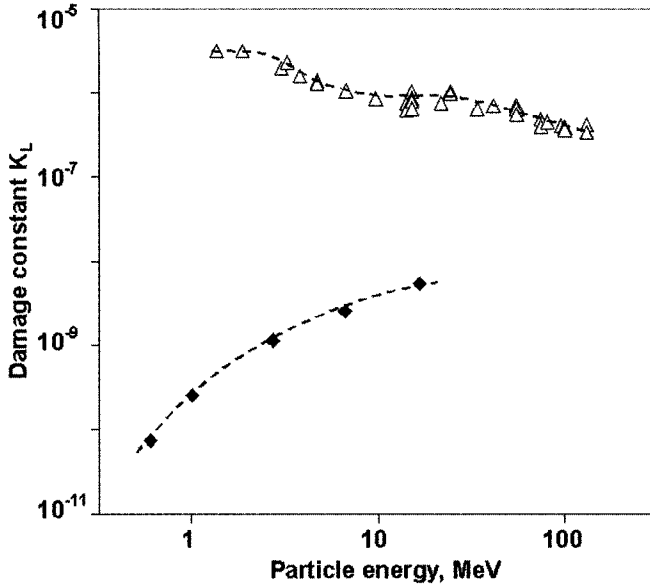


Figure 11 The energy dependence of the damage constant K_L in 1 Ω cm p-type silicon by electron (full symbols, [55]) and protons (empty symbols, [56]).

usually at a fluence orders of magnitude higher than that which degrades τ or L . Other source of degradation may be defects created in the depletion region, at the interfaces between different regions of the cell, or at the external surfaces.

9 Heavy Doping Effects

At high doping densities the picture of independent electrons interacting with isolated impurities becomes insufficient to describe the electron properties of semiconductors. As doping concentration increases, an impurity band is formed from the separate Coulomb wave functions located at individual doping impurities. This band gradually merges with the parent band and eventually gives rise to a tail of localised states. Electron–electron interaction becomes important, and the exchange and correlation energy terms have to be taken into account for a satisfactory description of the optical parameters and semiconductor device operation. An early overview of this multi-faceted physical problem can be found in [57].

In practical situations, this complex phenomenon is usually described in terms of band-gap narrowing, with a possible correction to effective masses and band anisotropy. Different manifestations of this effect are normally found by optical measurements (including absorption and low-temperature luminescence) and from data which pertain to device operation. Parameters which have been inferred from the latter are usually referred to as apparent band gap narrowing denoted by ΔE_g . For n-type material, for example,

$$\Delta E_g = kT \ln \left(\frac{p_0 N_D}{n_i^2} \right) \quad (33)$$

where p_0 is the minority-carrier (hole) concentration and N_D is the dopant (donor) concentration.

The experimental data obtained by various methods have been reviewed in [58] where the reader can find references to much of the earlier work. Jain et al. [59, 60] obtain a fit for band-gap narrowing in various materials using a relatively simple and physically transparent expression which, however, is less easy to apply to device modelling. Values for silicon which are now frequently used in semiconductor devices modelling are based on the work of Klaasen et al. [61] After reviewing the existing experimental data and correcting for the new mobility models and a new value for the intrinsic concentration, Klaasen et al. show that the apparent band gap narrowing of n- and p-type silicon can be accurately modelled by a single expression:

$$\Delta E_g(\text{meV}) = 6.92 \left[\ln \left(\frac{N_{dop}}{1.3 \times 10^{17}} \right) + \sqrt{\left(\ln \frac{N_{dop}}{1.3 \times 10^{17}} \right)^2 + 0.5} \right] \quad (34)$$

where N_{dop} is the dopant concentration

For gallium arsenide, Lundstrom et al. [62] recommend the following formula based on fitting empirical data:

$$\Delta E_g = AN_{dop}^{1/3} + k_B T \ln \{ F_{1/2}(E_F) \} - E_F$$

where E_F is the Fermi energy, the function $F_{1/2}$ is defined in the caption to Table 5, and

$$A = \begin{cases} 2.55 \times 10^{-8} \text{eV} & (p - \text{GaAs}) \\ 3.23 \times 10^{-8} \text{eV} & (n - \text{GaAs}) \end{cases}$$

10 Properties of Hydrogenated Amorphous Silicon¹

In hydrogenated amorphous silicon (a-Si:H), the effective band gap between the conduction and valence band edges is around 1.8 eV but a thermal shrinking of the band gap with temperature has been reported [63]:

$$E_g = E_{go} - \gamma(T - T_o) \quad \gamma \approx 5k_B \quad (35)$$

¹ A rigorous description of the band structure and charge carrier transport in amorphous silicon is a complex matter well beyond the scope of this Handbook. This section gives a simplified picture in terms of effective parameters which has been used in success in semiconductor device modelling.

For statistical calculations, the corresponding effective densities of states can be approximated by $\mathcal{N}_c \approx \mathcal{N}_v \approx 4 \times 10^{19} \text{ cm}^{-3}$. In contrast to crystalline silicon, the conduction and valence bands show evidence of exponential tails of localised states within the band gap [64]:

$$\begin{aligned} D_{Ct} &= D_{Co} \exp\left(\frac{E - E_C}{kT_C}\right) & D_{Co} &\approx 0.8 \times 10^{21} \text{ cm}^{-3} & kT_C &\approx 30 \text{ meV} \\ D_{Vt} &= D_{Vo} \exp\left(\frac{E_V - E}{kT_V}\right) & D_{Vo} &\approx 1.1 \times 10^{21} \text{ cm}^{-3} & kT_V &\approx 50 \text{ meV} \end{aligned} \quad (36)$$

In addition, there is a Gaussian distribution of dangling bond states (states corresponding to non-saturated silicon bonds) around mid gap (Figure 12):

$$D_{DB} = \frac{N_{DB}}{\sqrt{2\pi}w} \exp\left(-\frac{(E - E_{DB})^2}{2w^2}\right) \quad w \approx 100 \text{ meV} \quad E_{DB} = \frac{E_C + E_V}{2} \quad (37)$$

In device-quality intrinsic a-Si:H films, the density of dangling bonds N_{DB} ranges from 10^{15} to 10^{16} cm^{-3} .

The carrier mobility in the localised states (band tails and dangling bonds) is negligible. The accepted values for extended states in the valence and conduction band are $\mu_p = 0.5 \text{ cm}^2/\text{V-s}$ and $\mu_n = 10 \text{ cm}^2/\text{V-s}$, respectively. As in other

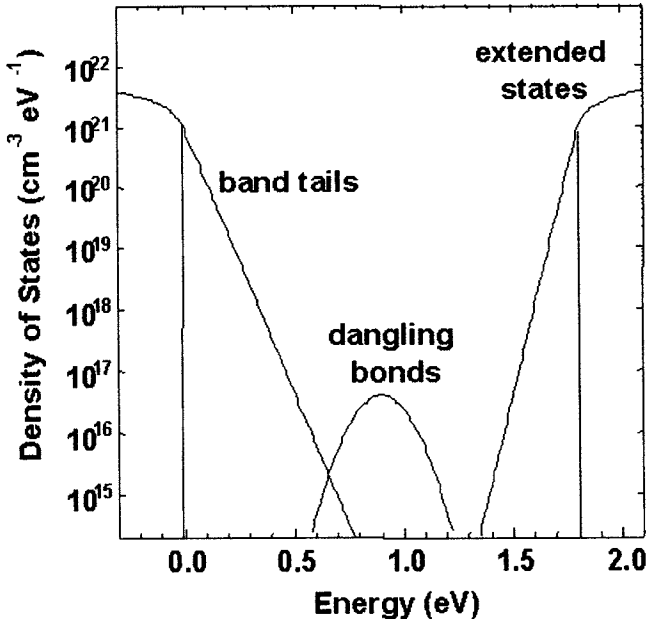


Figure 12 Typical density of states in a-Si:H. The states in the band tails behave as carrier traps whereas dangling bonds are recombination centres. Only those carriers in the extended states contribute to the electron transport [65].

semiconductors, the electron transport is given by the drift-diffusion equations (5) where the Einstein relations (10) apply.

The optical absorption coefficient shows three different zones (Figure 13):

- Band-to-band transitions (Tauc plot) for photon energies in excess of the band gap (> 1.8 eV).
- Transitions involving tail states (Urbach's front) for photon energies in the range 1.5–1.8 eV.
- Transitions involving dangling bonds for photon energies below 1.5 eV.

Once the optical absorption coefficient is known, the carrier generation can be easily calculated according to Equation (17) but it is important to note that only photons with energies in excess of the band gap yield useful electron-hole pairs for photovoltaic conversion.

Finally, the dominant recombination mechanism in intrinsic a-Si:H is given by the modified Shockley–Read–Hall equation, as applied to the amphoteric distribution of dangling bonds:

$$U = v_{th} \left(n\sigma_n^o + p\sigma_p^o \right) \frac{N_{DB}}{1 + \frac{p\sigma_p^o}{n\sigma_n^+} + \frac{n\sigma_n^o}{p\sigma_p^-}} \quad (38)$$

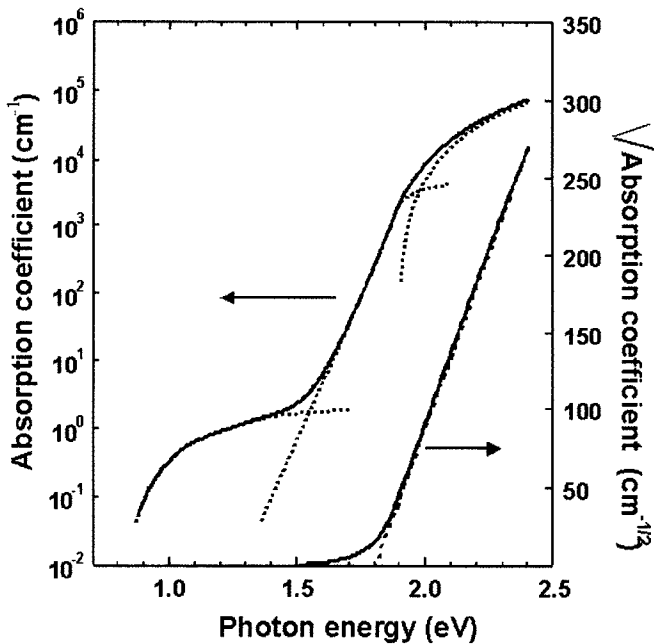


Figure 13 Typical optical absorption coefficient in a-Si:H. Three different regions can be observed corresponding to band-to-band absorption, transitions involving tail states and defect-associated absorption [65].

Table 12 Recombination parameters in amorphous silicon

$\nu_{th} \text{ (cm} \cdot \text{s}^{-1}\text{)}$	$\sigma_n^o \text{ (cm}^2\text{)}$	$\sigma_p^o \text{ (cm}^2\text{)}$	$\sigma_n^+ \text{ (cm}^2\text{)}$	$\sigma_p^- \text{ (cm}^2\text{)}$
10^7	10^{-15}	$\sigma_n^o/3$	$50\sigma_n^o$	$50\sigma_p^o$

where the parameters are given in Table 12 [66].

Acknowledgements

We are grateful to J. Puigdollers and C. Voz for providing graphs of the absorption coefficient and density of states in amorphous silicon.

References

- [1] Schwartz, R.J. 1990. The application of numerical techniques to the analysis and design of solar cells, in T.J. Coutts and J.D. Meakin, Eds., *Current Topics in Photovoltaics*, Vol. 4, p. 25.
- [2] Selberherr, S. 1984. *Analysis and Simulation of Semiconductor Devices*, Springer, Vienna, New York.
- [3] Brozel, M.R. and Stillman, G.E., Eds. 1996. *Properties of Gallium Arsenide* (3rd edition), IEE/INSPEC, Institution of Electrical Engineers, London.
- [4] Pearsall, T.P., Ed. 2000. *Properties, Processing and Applications of Indium Phosphide*, INSPEC/IEE, London.
- [5] Shur, M. 1990. *Physics of Semiconductor Devices*, Prentice Hall, Englewood Cliffs, NJ.
- [6] Chu, T.L. 1988. Cadmium telluride solar cells, in T.J. Coutts and J.D. Meakin, Eds., *Current Topics in Photovoltaics*, Vol. 3, p. 235.
- [7] Kazmerski, L.L. and Wagner, S. 1985. Cu-ternary chalcopyrite solar cells, in Coutts, T.J. and Meakin, J.D., Eds., *Current Topics in Photovoltaics*, Vol. 1, p. 41.
- [8] *Handbook of Physics and Chemistry of Solids* (82nd edition), CRC Press, Boca Raton, 2001.
- [9] Adachi, S. 1985. GaAs, AlAs and $\text{Al}_x\text{Ga}_{1-x}\text{As}$: material parameters for use in research and device applications, *J. Appl. Phys.*, Vol. 58, p. R1.
- [10] Varshni, Y.P. 1967. Temperature dependence of the energy gap in semiconductors, *Physica*, Vol. 34, p. 149.
- [11] Thurmond, C.D. 1975. The standard thermodynamic functions for the formation of electrons and holes in Ge, Si, GaAs and GaP, *J. Electrochem. Soc.*, Vol. 122, p. 1133.
- [12] Sproul A.B. and Green, M.A. 1993. Intrinsic carrier concentration and minority-carrier mobility of silicon from 77-K to 300-K, *J. Appl. Phys.*, Vol. 73, pp. 1214–1225.
- [13] Marshak A.H. and van Vliet, K.M. 1978. Electrical currents in solids with position dependent band structure, *Solid-State Electron.*, Vol. 21, p. 417.

- [14] Smith, R.R. 1978. *Semiconductors* (2nd edition), Cambridge University Press, Cambridge.
- [15] Caughey D.M. and Thomas, R.E. 1967. Carrier mobilities in silicon empirically related to doping and field, *Proc. IEEE*, Vol. 55, p. 2192.
- [16] Klaassen, D.B.M. 1992. A unified mobility model for device simulation – I. Model equations and concentration dependence, *Solid-State Electron.*, Vol. 35, pp. 953–959.
- [17] Arora, N.D., Hauser, T.R. and Roulston, D.J. 1982. Electron and hole mobilities in silicon as a function of concentration and temperature, *IEEE Trans. Electron Devices*, Vol. ED-29, p. 292.
- [18] Swirhun, S.E., Kwark Y.-H. and Swanson, R.M. 1986. Measurement of electron lifetime, electron mobility and band-gap narrowing in heavily doped p-type silicon, *IEDM'86*, pp. 24–27.
- [19] del Alamo, J., Swirhun, S. and Swanson, R.M. 1985. Simultaneous measurement of hole lifetime, hole mobility and band-gap narrowing in heavily doped n-type silicon, *IEDM'85*, pp. 290–293.
- [20] Sze, S.M. 1981. *Physics of Semiconductor Devices* (2nd edition), John Wiley & Sons, New York.
- [21] Lancefield, D. 1996. Electron mobility in GaAs: overview, in Brozel, M.R. and Stillman, G.E., Eds., *Properties of Gallium Arsenide* (3rd edition), IEE/INSPEC, Institution of Electrical Engineers, London, p. 41.
- [22] Harmon, E.S., Lovejoy, M.L., Lundstrom M.S. and Melloch, M.R. 1996. Minority electron mobility in doped GaAs, in Brozel, M.R. and Stillman, G.E., Eds., *Properties of Gallium Arsenide* (3rd edition), IEE/INSPEC, Institution of Electrical Engineers, London, p. 81.
- [23] Lovejoy, M.L., Melloch, M.R. and Lundstrom, M.S. 1996. Minority hole mobility in GaAs, in Brozel, M.R. and Stillman, G.E., Eds., *Properties of Gallium Arsenide* (3rd edition), IEE/INSPEC, Institution of Electrical Engineers, London, p. 123.
- [24] Barnes, J.J., Lomax R.J. and Haddad, G.I. 1976. Finite element simulation of GaAs MESFET's with lateral doping profiles and submicron gates, *IEEE Trans. Electron Devices*, Vol. ED-23, p. 1042.
- [25] Littlejohn, M.A., Hauser, J.R. and Glisson, T.H. 1977. Velocity-field characteristics of GaAs with $\Gamma_6^c - L_6^c - X_6^c$ ordering, *J. Appl. Phys.*, Vol. 48, p. 4587.
- [26] Ahrenkiel, R.K. 2000. Minority carrier lifetime in InP, in T.P. Pearsall, Ed., *Properties, Processing and Applications of Indium Phosphide*, INSPEC/IEE, London.
- [27] Palik, E.D., Ed., 1985. *Handbook of Optical Constants of Solids*, Academic Press Handbook Series, Orlando.
- [28] Palik, E.D., Ed., 1991. *Handbook of Optical Constants of Solids II*, Academic Press, San Diego.
- [29] Rajkanan, K., Singh, R., and Shewchun, J. 1979. Absorption coefficient of silicon for solar cell calculations, *Solid-State Electron.*, Vol. 22, p. 793.
- [30] Schmid, P.E. 1981. Optical absorption in heavily doped silicon, *Phys. Rev.*, Vol. B23, p. 5531.

- [31] Fan, H.Y. 1967. In R.K. Willardson and A.C. Beer, Eds., *Semiconductors and Semimetals*, Vol. 3, Academic Press, p. 409.
- [32] Landsberg, P.T. 1991. *Recombination in Semiconductors*, Cambridge University Press.
- [33] van Roosbroeck, W. and Shockley, W. 1954. Photon-radiative recombination of electrons and holes in germanium, *Phys. Rev.*, Vol. 94, p. 1558.
- [34] Pilkuhn, M.H., Light emitting diodes, in Moss, T.S., Ed., Vol. 4, *Handbook of Semiconductors*, North Holland, p. 539.
- [35] Levinstein, M., Rumyantsev, S. and Shur, M., Eds., 1996, 1999. *Handbook Series on Semiconductor Parameters*, Vols. 1 and 2, World Scientific, London. See also <http://www.ioffe.rssi.ru/SVA/NSM/Semicond/>.
- [36] Shockley, W. and Read, W.T. 1952. Statistics of the recombination of holes and electrons, *Phys. Rev.*, Vol. 87, p. 835.
- [37] Hall, R.N. 1952. Electron hole recombination in germanium, *Phys. Rev.*, Vol. 87, p. 387.
- [38] Cuevas, A., Giroult-Matlakowski, G., Basore, P.A., du Bois C., and King, R. 1994. Extraction of the surface recombination velocity of passivated phosphorus doped emitters, *Proc. 1st World Conference on Photovoltaic Energy Conversion*, Hawaii, pp. 1446–1449.
- [39] King, R.R., Sinton, R.A. and Swanson, R.M. 1990. Studies of diffused emitters: saturation current, surface recombination velocity and quantum efficiency, *IEEE Trans. Electron Devices*, Vol. ED-37, p. 365.
- [40] Andreev, V.M., Grilikhes, V.A. and Rumyantsev, V.D. 1997. *Photovoltaic Conversion of Concentrated Sunlight*, Wiley, Chichester.
- [41] Coutts, T.J. and Yamaguchi, M. 1988. Indium phosphide based solar cells: a critical review of their fabrication, performance and operation, in T.J. Coutts and J.D. Meakin, Eds., *Current Topics in Photovoltaics*, Vol. 3, p. 79.
- [42] Klaassen, D.B.M. 1992. A unified mobility model for device simulation – II. Temperature dependence of carrier mobility and lifetime, *Solid-State Electron.*, Vol. 35, p. 961.
- [43] Tada, H.Y., Carter, J.R., Anspaugh B.E. and Downing, R.G. 1982. *Solar-Cell Radiation Handbook*, JPL Publication 82-69, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA.
- [44] Anspaugh, B.E. 1996. *GaAs Solar Cell Radiation Handbook*, JPL Publication 96-9, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA.
- [45] Kinchin, G.H. and Pease, R.S. 1955. The displacement of atoms in solids by radiation, *Rep. Prog. Phys.*, Vol. 18, p. 1.
- [46] Seitz, F. and Koehler, J.S. 1956. Displacement of atoms during irradiation, *Solid State Physics*, Vol. 2, p. 307.
- [47] Hovel, H.J. 1975. Semiconductor solar cells, in R.K. Willardson and A.C. Beer, Eds., *Semiconductors and Semimetals*, Vol. 11, Academic Press, New York.

- [48] Markvart, T. 1990. Radiation damage in solar cells, *J. Materials Science: Materials in Electronics*, Vol. 1, p. 1.
- [49] Yamaguchi, M. and Ando, K. 1988. Mechanism for radiation resistance of InP solar cells, *J. Appl. Phys.*, Vol. 63, p. 5555.
- [50] Smits, F.M. 1963. *IEEE Trans. Nucl. Sci.*, Vol. NS-10, p. 88.
- [51] Meulenberg, A. and Treble, F.C. 1973. Damage in silicon solar cells from 2 to 155 MeV protons, *Proc. 10th IEEE Photovoltaic Specialists Conf.*, Palo Alto, p. 359.
- [52] Rosenzweig, W. 1962. Diffusion length measurement by means of ionizing radiation, *Bell. Syst. Tech. J.*, Vol. 41, p. 1573–1588.
- [53] Wilsey, N.D. 1972. *Proc. 9th IEEE Photovoltaic Specialists Conf.*, Silver Springs, p. 338.
- [54] Minahan, J.A. and Green, M.J. 1985. *Proc. 18th IEEE Photovoltaic Specialists Conf.*, p. 350.
- [55] Downing, R.G, Carter, J.R. Jr. and Denney, J.M. 1964. The energy dependence of electron damage in silicon, *Proc. 4th IEEE Photovoltaic Specialists Conf.*, Vol. 1, p. A-5-1.
- [56] Rosenzweig, W., Smits, F.M., and Brown, W.L. 1964. Energy dependence of proton irradiation damage in silicon, *J. Appl. Phys.*, Vol. 35, p. 2707.
- [57] Keyes, R.W. 1977. The energy gap of impure silicon, *Comm. Solid State Phys.*, Vol. 7(6), p. 149.
- [58] Wagner, J. and delAlamo, J.A. 1988. Band-gap narrowing in heavily doped silicon: a comparison of optical and electrical data, *J. Appl. Phys.*, Vol. 63, p. 425.
- [59] Jain, S.C. and Roulston, D.J. 1991. A simple expression for bandgap narrowing in heavily doped Si, Ge, GaAs and $\text{Ge}_x\text{Si}_{1-x}$ strained layers, *Solid-State Electron.*, Vol. 34, p. 453.
- [60] Jain, S.C., McGregor, J.M., Roulston D.J. and Balk, P. 1992. Modified simple expression for bandgap narrowing in n-type GaAs, *Solid-State Electron.*, Vol. 35, p. 639.
- [61] Klaassen, D.B.M., Slotboom, J.W and de Graaf, H.C. 1992. Unified apparent bandgap narrowing in n and p-type silicon, *Solid-State Electron.*, Vol. 35, p. 125.
- [62] Lundstrom, M.S., Harmon, E.S. and Melloch, M.R. 1996. Effective bandgap narrowing in doped GaAs, in Brožel, M.R. and Stillman, G.E., Eds., *Properties of Gallium Arsenide* (3rd edition), IEE/INSPEC, Institution of Electrical Engineers, London, p. 186.
- [63] Tsang, C. and Street, R.A. 1979. *Phys. Rev.*, Vol. B19, p. 3027.
- [64] Fritzsche, H., Ed. 1989. *Amorphous Silicon and Related Materials*, University of Chicago.
- [65] Puigdollers, J. and Voz, C., personal communication.
- [66] Street, R.A. 1991. *Hydrogenated Amorphous Silicon*, Cambridge University Press.