

W. T. WELFORD

Optics

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Preface to the third edition

In revising and enlarging the second edition I have taken the opportunity to update one or two areas; notably, the speed of light is, as foreshadowed in the second edition, no longer a measured quantity but an internationally *defined* number of metres per second and I have made the appropriate change in Chapter 1; it is interesting but perhaps futile to wonder how physicists such as Galileo and Michelson would have regarded this development. For the rest, I have added a chapter on the detection of radiation but otherwise the book is as before.

Imperial College
February 1988

W.T.W.

Preface to the second edition

In revising and enlarging the first edition I was greatly helped by comments and criticisms from several colleagues, but in particular Dr M. E. Barnett, Dr G. A. Brooker, and Dr L. J. Cox; needless to say, they may not recognize the results of some of their criticisms and they are in no way to be held responsible for what is in this second edition. The principal changes are, the addition of a short section on the speed of light, several additional sections on aspects of geometrical optics, some expansion of the chapter on laser light, and, lastly, a whole new chapter on optical light guides. This latter is, of course, pitched at a very elementary level and is not to be regarded as even the briefest introduction to, say, the use of optical fibres for communication; rather, it is intended to illustrate many of the ideas and principles developed in the previous seven chapters in a context which is of wide interest.

Imperial College London
May 1980

W.T.W.

Preface to the first edition

Present-day physics courses are under increasing pressure, on the one hand to keep up with developments in fundamental physics and on the other to cover a broad range of topics appropriate to the interests of students who may never become professional physicists. Thus the time available for optics in the first or second year of an undergraduate course, as for other branches of physics, decreases, and this has influenced my choice of topics in this book; I have been very selective and, as can be seen from the contents list, I have chosen material which is either basic to the development of the optics of the visible spectrum or which has interesting links with other kinds of optics or other branches of physics. Some may be concerned about what is *not* to be found in this book, e.g. measurement of the speed of light, group velocity, standing waves, the envelope function for diffraction gratings, refractometry, Fresnel diffraction, and phase-change effects in interferometry. These omissions might have been dictated anyway by the agreed size of the Oxford Physics Series texts, but I do not plead this as an excuse. The book as it stands is intended as a reasonable selection of topics to be presented to undergraduates, perhaps in their first term at University and certainly having to cope with many other new things at the same time.

I have tried to stress physical arguments, and in order to reduce the mathematical complexity I have introduced the concept of a complex amplitude in the first chapter. I have also used the formalism of Fourier-transform theory freely, since this illuminates and simplifies every branch of physics in which waves appear; this may seem rather extreme for an elementary text, but since simple experiments with lasers are most easily discussed in terms of Fourier transforms it seems almost certain that students will meet the transform in their laboratory work and will grasp the basic ideas even if they have not been presented with a systematic formulation. However, Sections 5.5 and 6.6 contain some more difficult Fourier-transform material, which could be omitted in very early courses. The main definitions and theorems of Fourier-transform theory needed are given, without proofs, in the Appendix.

Some of the problems at the end of each chapter amplify the text by introducing simple extensions of the main discussion

I should like to thank my colleagues Dr M. E. Barnett and Dr R. W. Smith for their help with this book, mostly given unknowingly; many of their ideas about the teaching of optics have gone into it. Also I am very grateful to Professor E. J. Burge, who read the first draft, gave very valuable criticism, and made many useful suggestions, and to Miss Lesley Harwood, who prepared the index; and I thank the staff of Oxford University Press for their help during publication.

The quotations from James Joyce's *Ulysses* are by kind permission of the Society of Authors, as the literary representative of the Estate of James Joyce, and of The Bodley Head, as publishers.

Imperial College London
1975

W.T.W.

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8. Optical light guides

Draw out a glass rod to a thin fibre in a bunsen burner flame and break the ends of the fibre from the thicker parts. If one end is applied to a light source such as a lamp bulb or a laser the light is conducted down the fibre and it shines out brightly at the other end. This simple experiment illustrates the basis of two modern technologies. In this chapter we shall apply the ideas developed in the earlier parts of this book to such light guides.

8.1. The acceptance angle of a light guide

The mechanism of transmission of light along a guide is, in the geometrical optics approximation, total internal reflection as described in Section 2.2. Figure 8.1 shows a cylinder of material of refractive index n ; if a ray of light

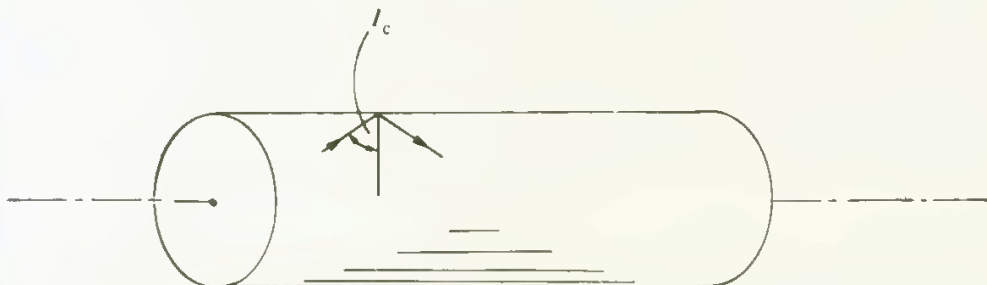


FIG. 8.1. A solid light guide of cylindrical cross-section; the light is confined to the guide by total internal reflection.

travelling in a plane through the axis is incident on the wall at an angle greater than the critical angle I_c given by $\sin I_c = 1/n$ it will be totally internally reflected; it will carry on in this way if the guide is straight until it reaches the end of the guide. Rays in planes through the axis are called meridian rays; skew rays do not lie in an axial plane but equally it can be seen that a skew ray once reflected at an angle greater than the critical angle will carry on in a path with successive points of incidence lying on a helix on the surface of the guide, as in Fig. 8.2.

It is essential to have a very clean perfectly polished surface without

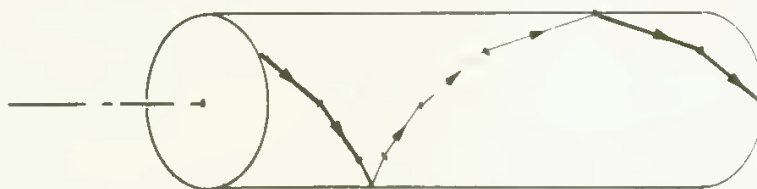


FIG. 8.2. A skew ray propagated inside a light guide.

scratches on glass to achieve total internal reflection without losses. This is not a very serious worry with prisms such as that shown in Fig. 2.5, since the light is only reflected once and small losses do not matter, but in a long light guide a given ray may be reflected very many times, so that losses due to surface defects must be minimized; the experiment described at the beginning of the chapter will illustrate this because specks of dust and scratches on the fibre will appear bright when light is transmitted through it. Such surface losses are prevented by a protective coating or *cladding*, as in Fig. 8.3, of

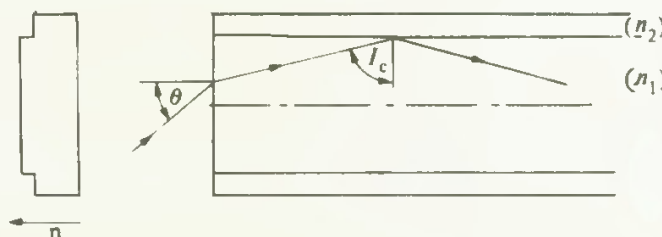


FIG. 8.3. A protective cladding for a light guide; a meridian ray is totally internally reflected at the interface between the guide and the cladding. The diagram on the left indicates the refractive index variation across a diameter.

refractive index say n_2 which is lower than the index n_1 of the guide. Then there will be a critical angle I_c given by

$$\sin I_c = n_2/n_1 \quad (8.1)$$

for total internal reflection at the interface and damage to the outer surface of the protective layer will not matter.

Figure 8.3 can be used to calculate the *acceptance angle* of the light guide. This is the maximum angle of incidence θ on a square cut end face for which any ray is totally internally reflected at the interface and therefore transmitted without loss. The diagram shows a meridian ray and it can be seen that the angle of refraction of this ray on entering the end face is $\pi/2 - I_c$. Thus on applying Snell's law (Section 2.2) we find for the sine of the acceptance angle θ ,

$$\sin \theta = \sqrt{(n_1^2 - n_2^2)}. \quad (8.2)$$

This quantity is called the numerical aperture of the light guide, by analogy with the use of the same term for image-forming systems such as the

microscope (Section 5.4). The above argument was developed for a meridian ray; it can be seen that any skew ray incident on the end face at an angle less than θ will be transmitted. Thus all rays entering inside a solid angle $\pi \sin^2 \theta$ will be transmitted and the numerical aperture can be taken as a measure of the capacity of the guide to transmit light power. For example for a core refractive index of 1.52 the cladding index might be 1.48, giving a numerical aperture 0.35, or an acceptance angle θ of 20° .

At this point some reservations should be made. First, it can easily be seen that although θ as given by eqn (8.2) is the limiting angle for meridian rays, yet skew rays incident at greater angles to the end face can be transmitted, depending on their distance of closest approach to the axis of the fibre. Thus the guide could transmit more light flux than might be supposed from the value of the numerical aperture. Secondly our argument has been in terms of geometrical optics, and while this is accurate enough to predict the main properties of guides with lateral dimensions which are large compared to the wavelength of the light it will not serve for smaller guides; for these a treatment based on electromagnetic theory would be needed (see, e.g. Midwinter 1979).

Returning now to the properties of the guide we see that if the image of a light source is focused onto the end of the guide as in Fig. 8.4, so that it

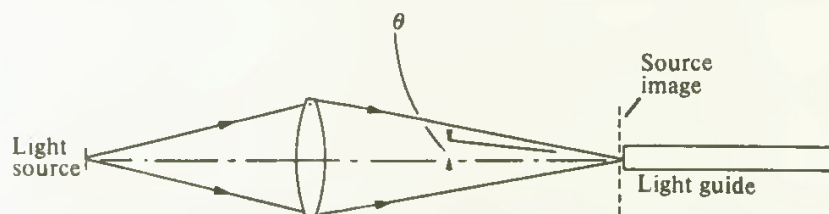


FIG. 8.4. Illuminating a light guide. The image of the source should be at least as large as the end of the guide and the convergence angle of the illuminating beam should be at least as great as the acceptance angle of the guide.

illuminates all of the end, and if the convergence angle of the light from the lens is greater than θ , the acceptance angle of the guide, then the guide will accept as much light flux from the source as it can. Let the radiance of the source (Section 2.6) be $B \text{ W mm}^{-2} \text{ sr}^{-1}$, and let it be a Lambertian radiator; this means that the flux emitted in a direction making an angle ϕ with the surface of the source is proportional to $\cos \phi$; many thermal sources radiate approximately in this way but not, of course, lasers. If the diameter of the core of the guide is $2a$ then the flux collected by the guide is, neglecting the extra skew rays mentioned above, and neglecting also reflection losses at the entry face

$$\pi^2 a^2 B \sin^2 \theta \text{ W}; \quad (8.3)$$

this expression is proved in Problem 8.2.

8.2. Graded index guides

The clad guides described in Section 8.1 can be of any diameter, depending on the application, and in fact some are made as rigid rods several millimetres in diameter. For optical communication purposes (see Section 8.4) there are advantages in having very small effective diameters, i.e. comparable with the wavelength of the light, and these have a different structure from the guides described in Section 8.1. The *graded index* guide is a cylinder in which the refractive index decreases smoothly from the axis radially outwards. The refractive index distribution may be represented, as in Fig. 8.5, by an equation such as

$$\begin{aligned} n &= n_0 - \alpha r^2 & (r < r_0) \\ n &= n_0 - \alpha r_0^2 & (r > r_0) \end{aligned} \quad (8.4)$$

The geometrical optics of a graded index fibre of this kind is more complicated than for the simple fibre of Fig. 8.3; these latter may be called, correspondingly, step index fibres. However, if we consider rays in a meridian or axial plane we can see how a guiding effect is obtained. Consider in Fig. 8.6

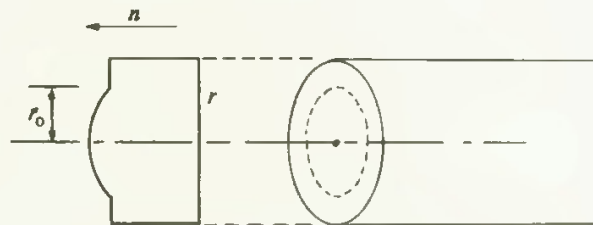


FIG. 8.5. Refractive index profile of a graded index light guide.

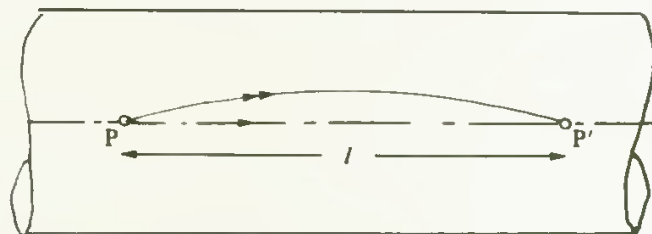


FIG. 8.6. Focusing effect of a graded index guide.

an axial segment PP' of length l say; the optical path length from P to P' along the axis is $n_0 l$, from eqn (8.4). Now consider a second path from P to P' as indicated by the double arrows; the geometrical length of this path is greater than l but from eqn (8.4) it passes through a region of lower refractive index than the axial path, so it is possible that the decrease in index combined with the increase in geometrical distance may make the optical path length from P to P' the same along the double arrowed ray as for the axial path. Thus as in Section 2.3, we can invoke Fermat's principle and claim that P and P' could

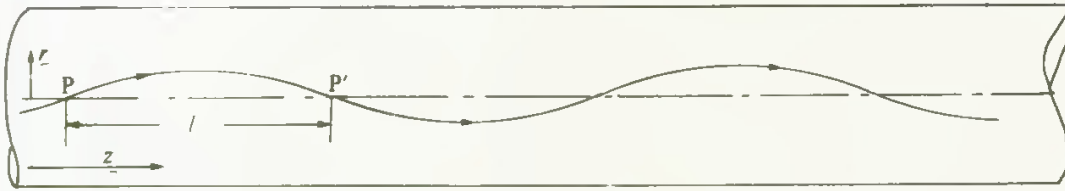


FIG. 8.7. Path of a meridional ray in a graded index guide.

be object and image, since the optical path length from P to P' is constant for all rays within a certain angle. We can quantify this as follows.

According to our reasoning above a meridional ray should follow a periodic path as in Fig. 8.7. Let the length of each half cycle be l ; then we can represent the path by

$$r = a \sin (\pi z/l), \quad (8.5)$$

where z is the axial coordinate and a is a constant for the particular ray chosen. An element of length ds along this ray is

$$\begin{aligned} ds &= \sqrt{(dz^2 + dr^2)} \\ &= \left\{ 1 + \left(\frac{\pi a}{l} \cos \frac{\pi z}{l} \right)^2 \right\}^{1/2} dz. \end{aligned}$$

Thus the optical path length along the ray is, from Section 2.2,

$$\int n ds = \int_0^l (n_0 - \alpha r^2) \left\{ 1 + \left(\frac{\pi a}{l} \cos \frac{\pi z}{l} \right)^2 \right\}^{1/2} dz. \quad (8.6)$$

Now let a be small so that only rays close to the axis, i.e. paraxial rays, are considered; specifically, we neglect powers of a above the square. Then we can substitute for r in eqn (8.6) from eqn (8.5), expand the square root and simplify to obtain

$$\int n ds = \int_0^l \left(n_0 - \alpha a^2 \sin^2 \frac{\pi z}{l} \right) \left\{ 1 + \frac{1}{2} \left(\frac{\pi a}{l} \right)^2 \cos^2 \frac{\pi z}{l} \right\} dz.$$

Then using $\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$ we finally obtain, on again neglecting a^4 , etc.

$$\int n ds = \int_0^l \left(n_0 + a^2 \left\{ \frac{n_0 \pi^2}{4l^2} - \frac{\alpha}{2} \right\} + a^2 \left\{ \frac{n_0 \pi^2}{4l^2} + \frac{\alpha}{2} \right\} \cos \frac{2\pi z}{l} \right) dz. \quad (8.7)$$

It can be seen that when the integration is carried out the third term will vanish at both limits, since it gives $\sin 0$ and $\sin \pi$. The first term will give simply $n_0 l$, the optical path length along the axis. Now if the trajectory of eqn (8.5) is really a physically possible ray of those rays which form an image of P

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at P' the whole integral must be constant for varying a and equal to $n_0 l$, i.e. the second term must vanish, or,

$$\frac{n_0 \pi^2}{4l^2} = \frac{\alpha}{2}.$$

This therefore must give the distance between successive intersections of the axis:

$$l = \pi \sqrt{\left(\frac{n_0}{2\alpha}\right)}. \quad (8.8)$$

The above analysis applies only to meridian rays and the theory for rays which are skew to the axis is considerably longer. In fact it is found that the graded index fibre will confine skew rays to a region near the axis and thus it acts in a way similar to the step index fibre.

The limiting acceptance angle for a meridian ray can be taken as that for which the ray just penetrates the region of uniform refractive index. For a ray entering at the axis this can be found by treating the continuously varying index as a sequence of thin shells of uniform index, as in Fig. 8.8. Then by applying Snell's law we find for the acceptance angle

$$\sin \theta = \sqrt{(n_0^2 - (n_0 - \alpha r_0^2)^2)}$$

which is analogous to eqn (8.2). This, however, applies only to a ray incident on the end face at the axial point. Meridian rays not incident at the axial point would have a smaller acceptance angle whereas skew rays usually have a larger acceptance angle. Thus the concept of numerical aperture for a graded index fibre is not so simple as for a step index fibre.

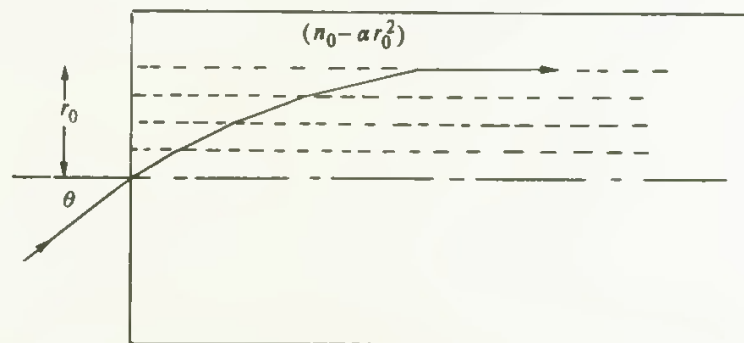


FIG. 8.8. The limiting acceptance angle θ for a graded index fibre; the ray is calculated to be parallel to the axis when it reaches the region of uniform refractive index $n_0 - \alpha r_0^2$.

8.3. Light guides for image transport

There are three main groups of applications for light guides, (a) for transporting light simply for illumination; (b) for transporting images; and

(c) as communication channels. The first application needs no discussion as the principle is obvious; in this section we explain the use of guides for transporting images. If a bundle of fibres is made as in Fig. 8.9 and a real image is projected onto one end then it is obvious that, provided the arrangement of the fibres is kept the same at both ends, the image will appear at the other end of the bundle. Such fibre bundles are made either flexible or rigid; in the second case the individual fibres, usually of step index form, are fused together as a solid glass rod.

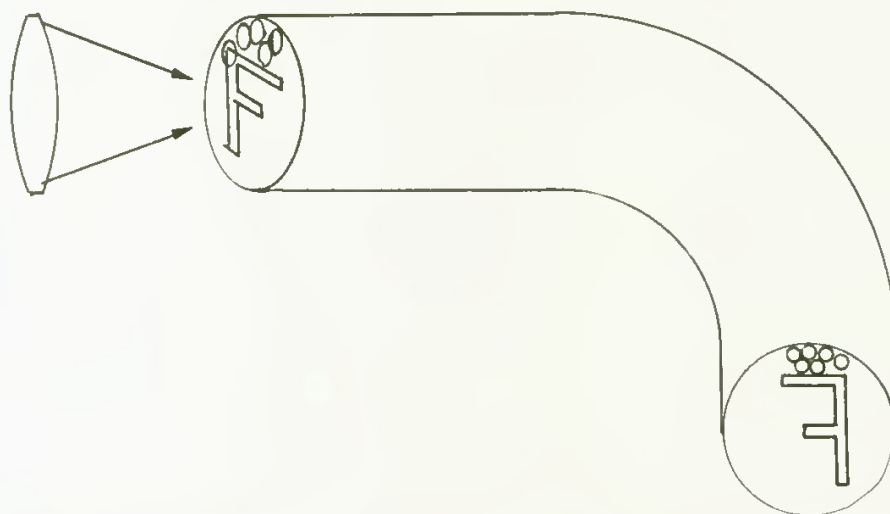


FIG. 8.9. Image transport by a fibre bundle; the diameter of the individual fibres is not shown to scale.

In order to get the maximum image brightness the convergence angle α from the lens which projects the initial real image should equal the acceptance angle of the fibres, as given by eqn (8.2). For commercially made fibres this acceptance angle ranges between about 10° and 30° ; the latter value corresponds to a lens with aperture ratio $F/1$. This leads to applications where large light collecting power is essential.

Consider, for example, an oscilloscope used for recording a single very rapid event detected as, say, a voltage pulse lasting for 10^{-8} s. The trace on the oscilloscope screen must be photographed and a camera lens of very great light collecting power is needed. A typical arrangement would be as in Fig. 8.10, which shows the phosphor inside the necessarily thick faceplate of the cathode ray tube, the camera lens, and the film. It is difficult and expensive to obtain a camera lens with a large enough collecting angle to give an adequate exposure for rapidly occurring single traces and this has led to the development of the fibre-optics faceplate. Figure 8.11 shows a plate composed of short step index fibres vacuum sealed in a matrix of black glass; this plate is used as the faceplate of the cathode ray tube and the phosphor is deposited on the inner surface. Light from the electron beam trace on the

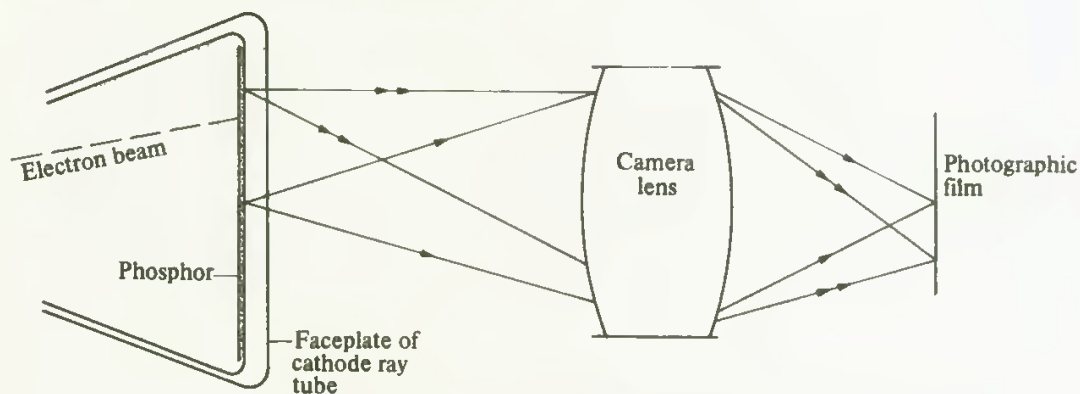


FIG. 8.10. Taking a photograph of a single trace on an oscilloscope screen.

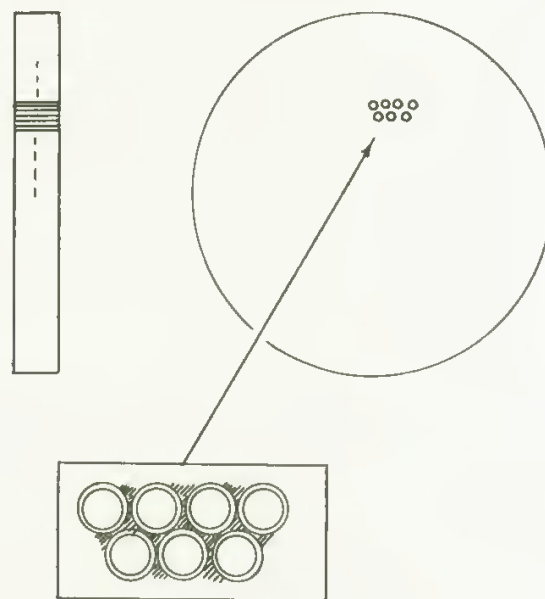


FIG. 8.11. Structure of a fibre-optic face-plate for a cathode ray tube.

phosphor is collected with the full numerical aperture of the fibres and a piece of photographic film pressed against the outside will receive the image. With this system the light flux collected per unit area of the phosphor is proportional to the square of the numerical aperture of the fibres multiplied by a factor A which allows for the packing of the fibres. There is no distortion or other image-forming aberration and the resolving power depends only on the spacing of the fibres. For example, let the individual fibre axes be arranged in a honeycomb pattern at centre spacing d and let the diameter of the high index core of each fibre be $2a$. Then it can be seen that the factor to allow for packing is

$$A = \frac{2\pi}{\sqrt{3}} (a/d)^2. \quad (8.10)$$

The effect on resolving power is indicated in Fig. 8.12; a bright narrow line imaged on one side of the face plate is broadened to a width which may be as large as $d + 2a$ or as small as $2a$, depending on the position and orientation of the line relative to the fibres.

The image transport principle used in the fibre optics face plate is applied in other devices, for example as a way of eliminating field curvature in conventional lens systems and for transporting images through narrow tubes over long distances for, e.g. inspecting inaccessible cavities in machines and chemical apparatus.

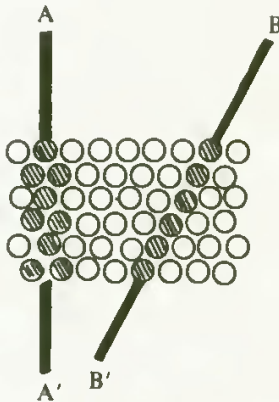


FIG. 8.12. Resolving power of a fibre-optic face-plate; the line AA' is broadened more than BB' since it overlaps more fibres. In practice the arrangement of the fibres would not be as regular as in the diagram and the broadening would be irregular.

8.4. Light guides for communication

It was first suggested in 1966 that step-index light guides should be used for transmitting signals, i.e. as communication channels. The principle is, with hindsight, obvious enough, to send light down the fibre with the intensity modulated in time to give either a digital or an analogue signal. However, it was only at about that time that light sources and detectors began to be available with characteristics suitable to make such a system competitive with electrical communication along wires. In order to understand the reasons for this we have to introduce some ideas about communications.

Sounds are heard at frequencies between approximately 50 Hz and 15 kHz and they are transmitted as analogue signals, i.e. variations of voltage proportional to the instantaneous sound pressure, along telephone wires; at suitable intervals there are amplifiers, called *repeaters*, to keep the signal strength up. There would be no difficulty in modulating the output of a light-emitting diode (LED) of the kind used in hand calculators for example, at frequencies in this range, sending the light down a fibre light guide, detecting it with a photoelectric detector and then converting the electrical signal into sound in the usual way. However, this would be much more expensive and

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complicated than ordinary telephony. To see how light guide transmission could be competitive we note that sound covers a range of frequencies of, say, 15 kHz and this can be transmitted not directly in this frequency range but as a *modulation* of a higher frequency, the *carrier* frequency. Thus in ordinary amplitude modulation (AM) radio the carrier may have a frequency of, say, 1 MHz and the amplitude of this carrier would be varied according to the audio signal. For a carrier frequency ν_0 and a single audio tone of frequency ν the modulated carrier takes the form

$$V(t) = V_0(1 + m \cos 2\pi\nu t) \cos 2\pi\nu_0 t. \quad (8.11)$$

In this equation m represents the strength of the modulation. Equation (8.11) can be rewritten in the form

$$V(t) = V_0 \cos 2\pi\nu_0 t + \frac{1}{2}mV_0[(\cos 2\pi(\nu + \nu_0)t + \cos 2\pi(\nu - \nu_0)t)]$$

or in complex notation, using the convention explained in Section 1.3 that only the real part is to be taken,

$$V(t) = V_0 e^{-2\pi i\nu_0 t} + \frac{1}{2}mV_0(e^{-2\pi i(\nu + \nu_0)t} + e^{-2\pi i(\nu - \nu_0)t}) \quad (8.12)$$

We can now use our Fourier transform ideas again and say that this signal is represented in the transform domain, i.e. the temporal frequency domain, by

$$\tilde{V}(\nu) = V_0\delta(\nu_0) + \frac{1}{2}mV_0(\delta(\nu + \nu_0) + \delta(\nu - \nu_0)) \quad (8.13)$$

and this is shown in Fig. 8.13. The audio signals can occupy a relatively small range of frequency covering perhaps ± 15 kHz on either side of the carrier

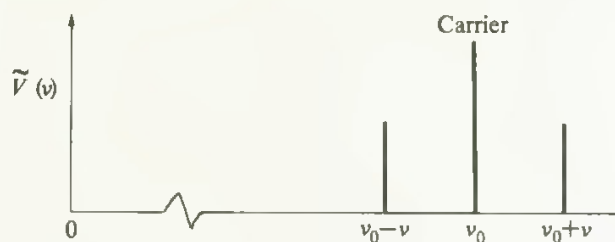


FIG. 8.13. A carrier wave of frequency ν_0 , amplitude modulated at frequency ν , shown in the frequency domain.

frequency ν_0 . This range is called the *bandwidth*. The idea of a bandwidth of temporal frequencies needed to transmit certain kinds of signals is paralleled exactly by the ideas introduced in Chapter 5 of a range of spatial frequencies needed to transmit certain detail in forming the image of an object. For television a bandwidth of order 10 MHz is needed, as can easily be seen by estimating the number of individual points in a 625 line television picture and recalling that the picture is reproduced 25 times a second; the used carrier frequencies range up to about 1000 MHz, but this probably represents the

practical limit for commercial radio communication. The number of different radio or television programmes which can be broadcast simultaneously in the same area can be found by plotting their carrier frequencies and bandwidths on a frequency abscissa as in Fig. 8.14; if neighbouring bands do not overlap they can be received separately without interference or *crosstalk*, provided the receiver is tuned to pick up only the band of frequencies corresponding to the

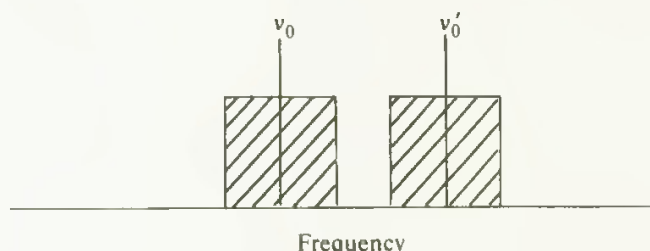


FIG. 8.1.4. Carrier frequencies ν_0 and ν'_0 with their accompanying required bandwidths.

desired programme. Similarly several telephone conversations can be transmitted simultaneously over one line by modulating different carrier frequencies for each; the maximum carrier frequency for a telephone line is a few MHz, so a few hundred separate telephone conversations can be carried simultaneously.

Returning to light guides, it will be recalled from Chapter 1 that visible light has frequencies in the range $4.3\text{--}7.5 \times 10^{14}$ Hz. Thus by selecting suitably spaced carrier frequencies in this range a single light guide might transmit about 10^{11} separate telephone conversations or 10^8 separate television pictures simultaneously! The reality is, so far, not quite so spectacular: from what was said above it can be seen that the range of carrier frequencies and the required bandwidth are two key ideas in discussing any communication system but three further questions which arise in considering light guides as communication channels are (a) can we have enough close-spaced carriers over the frequency range of the light; (b) can a light guide transmit the signals without degrading them enough to make them unrecognizable; and (c) could the signals, if undegraded, be separated again without crosstalk? The answer to all these questions is 'No' at present. Nevertheless, light guides do offer the possibility of substantial advantages in increasing the number of separate signals carried simultaneously, as well as in other ways to be mentioned below.

Consider first the spacing of the carrier frequencies; for 10^8 separate carriers we should require the frequency of each carrier to be stable to within about ± 1 MHz; this is attainable by the most elaborate stabilization methods for a few gas lasers but by no means for 10^8 separate and arbitrarily chosen wavelengths in the visible spectrum. Equally, even supposing all these separate highly stabilized carriers could be obtained they would have to be

sorted out at the receiving end of the light guide (or at intermediate switching or amplifying stations) by filters or spectroscopes of resolving power which is at present quite unattainable under the required working conditions. Thus in practice there are two possible systems, one being to modulate the light wave at a reasonable frequency, say 10^9 Hz, and use this frequency as the carrier; several carriers with suitably spaced frequencies could be used to carry simultaneously different communication channels. The second possibility is to use digital transmission and this is the method which is being most actively explored at present; in digital transmission the analogue signal, sound, television or whatever, is pre-coded into digital form by an analogue-to-digital converter. This will, for example, record the light intensity at a given point in a picture as being any of, say 64 different values in the range 0, 1, 2, . . . 63 and encode it in the binary scale as a 6-bit number. A similar process is carried out with sound intensities, sampled at sufficiently close intervals of time; the signal is then transmitted as a series of 0s and 1s, as in a computer, represented as dark and light periods of the LED at the transmitting end of the light guide. Rates of modulation and detection approaching 10^9 bits per second are possible with available LEDs and photodetectors, which corresponds to an available bandwidth of about 1000 M Hz; this is still very large for a single light guide even if it is disappointing compared to the actual frequency of the light. We thus have to ask whether a fibre could transmit at such frequencies. In other words, if a single bit representing a 1 enters the fibre as in Fig. 8.15(a) will it emerge as in (b), in which case no great confusion will occur, or as in (c), when it would be

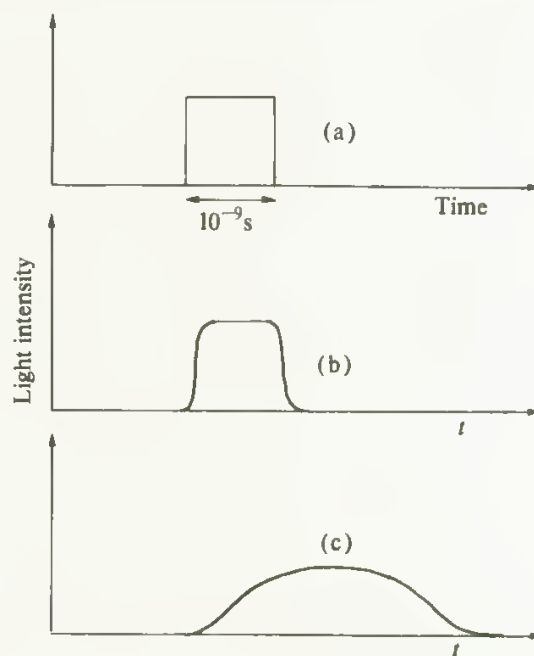


FIG. 8.15. Degradation of a light pulse representing a single bit on transmission through a light guide: (a) input pulse, (b) slightly degraded, (c) degraded unusably.

impossible to use the light guide as a communication channel at the postulated *bit rate*.

We can formalize this in terms of the concepts of Section 5.5, but with one time dimension instead of two space dimensions. We assume the input signal to be a delta function of time, i.e. an indefinitely short burst of light, and the output from the light guide will be spread in time into the *impulse function*, the analogue of the point spread function for an optical system. The time spread of the impulse function gives a measure of the duration of signal which can be transmitted without degradation, or alternatively the Fourier transform of the impulse function gives the *transfer function*, which shows how periodic signals of different frequencies are degraded.

There are two main contributing factors to the size of the impulse function for a light guide. The first, known technically as modal dispersion, can be understood in terms of geometrical optics: in Section 8.2 we found a path for a paraxial meridian ray in a graded index fibre and it was mentioned that there could be aberrations for other rays, i.e. the optical path lengths along the fibre following different rays are not the same; but the optical path length along a ray is a measure of the time taken for light to travel the ray path (Section 2.3), so that parts of a signal travelling different ray paths will arrive at different times, producing a spread out impulse function. The second factor is simply material dispersion, the fact that the refractive index of the fibre material varies with wavelength, i.e. the speed of a light pulse varies with wavelength. Since the light used in practice has an appreciable spread of wavelengths this again contributes to the width of the impulse function.

There is a third feature of light guides which is crucial to their performance, namely *attenuation*. In a light guide only several metres long there may be a serious loss of light by absorption and scattering in the material of the guide and it is this which decides the maximum length of guide which can be used as a communication link without a repeater to restore the signal. Attenuation is expressed in decibels† per unit length. The best optical fibres obtainable at the time of writing are claimed to have attenuation as low as one or two db per kilometre. The attenuation varies with wavelength and for most fibres there is a minimum of attenuation at a wavelength of about 1.3 μm , i.e. in the near infrared region. Figure 8.16 shows, with some detail omitted, a typical attenuation curve.

Figure 8.16 suggests that to minimize the effects of attenuation we should use an infrared source of wavelength 1.3 μm . At present both LEDs and solid state lasers of different kinds which emit in the near infrared are used but there is as yet no clear best choice. Solid-state lasers of suitable design emit over a relatively narrow wavelength range of about 2 nm, so that the effect of material dispersion in a fibre is likely to be small, whereas LEDs emit over

† A decibel is a logarithmic unit such that 10 db is an attenuation by a factor 0.1, 20 db = 0.01, and generally n db corresponds to attenuation $10^{-0.1n}$.

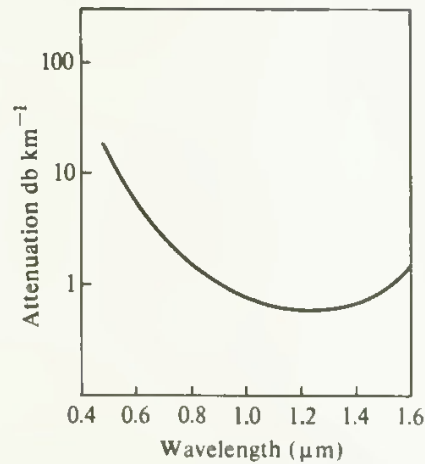


FIG. 8.16. Attenuation curve for a fibre light guide; the curve is drawn to show the general trend for many fibre materials but it does not correspond to a particular material.

perhaps 50 to 100 nm. On the other hand LEDs are cheaper, simpler to run, and more reliable than lasers.

The effect of attenuation is to limit the length of fibre which can be used. Figure 8.17a suggests a typical digital light signal of the form 101001 to be transmitted; ignoring the effect of the impulse function of the light guide, we should naively expect the output voltage from the photoelectric detector at the exit end to have a similar form. However, in a quantum detection process (Section 1.4) the output will actually consist of individual pulses corresponding to the detection of individual photons and if the signal has been attenuated in the fibre the pulses might appear individually as in Fig. 8.17b, i.e. we should have a *noisy* output. Figure 8.17b does not show only the signal photon noise, as it is called; we have indicated also some noise at the times corresponding to zeroes in the signal; this noise is from dark current in the photodetector, i.e. stray electrons produced when there is no light, and from a variety of other sources of noise. Figure 8.17 illustrates the concept of the signal-to-noise ratio, i.e. the ratio of the mean output signal to the mean (or

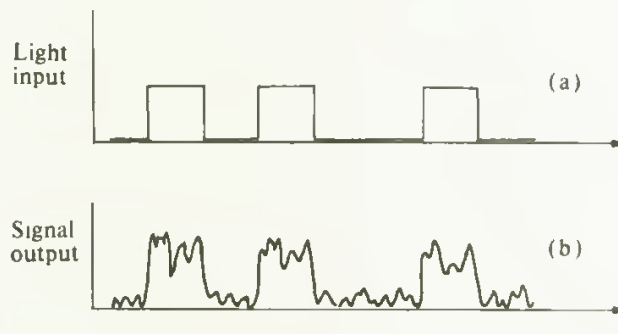


FIG. 8.17. The effect of noise: (a) the input signal 101001, (b) the output signal from the photodetector, showing noise from various sources.

strictly root-mean-square) noise. Clearly with a signal-to-noise ratio which is too unfavourable 1s will sometimes be taken for 0s and vice versa. We can make a very simplified calculation as follows.

Let W be the light power in watts corresponding to the signal 1, let this signal last for time t and let the mean frequency of the light be ν . The energy in a single 1 is Wt and this therefore contains $Wt/h\nu$ photons on average. Thus $\eta Wt/h\nu$ photoelectrons will be produced on average for each 1, if η is the efficiency of the detector. Now it is known that the mean-square fluctuation in the number of photoelectrons produced in a given time is equal to the mean so that the root mean square variation in the number of photoelectrons produced for a 1 will be $(\eta Wt/h\nu)^{1/2}$ and the signal-to-noise ratio is therefore

$$S/N = \left(\frac{\eta Wt}{h\nu} \right)^{1/2}. \quad (8.14)$$

For a further discussion of noise see section 9.2. However, eqn (8.14) shows the essential features that the signal-to-noise ratio is proportional to the square root of the signal strength W and inversely proportional to the square root of the bandwidth $(1/t)$. We should note also that as given by eqn (8.14) the signal-to-noise ratio refers to current or voltage output from the photodetector. For some purposes the ratio of signal-power to noise-power obtained from the photodetector is used and then the square of the expression in eqn (8.14) would apply. Either way it can be seen that the effect of attenuation in reducing the light power W is to reduce the signal-to-noise ratio at the output from the fibre and a point is reached at which the digital signal must be refreshed by detecting the light signal, re-shaping it electronically and re-transmitting it for a further stage. The distance between such repeater stations depends on the fibre attenuation and, of course, on the strength of the input signal. This introduces the concept of coupling light guides to light sources and detectors, briefly touched on in Section 8.1.

We saw in Section 2.6 that the light transmitting power of an optical system is proportional to the square of the Lagrange invariant, a quantity sometimes called the *étendue*. For a step-index guide the Lagrange invariant is the product of its numerical aperture (Section 8.1) and the radius of the core. Then in order to couple a light source as efficiently as possible to the guide we must ensure that all of this *étendue* is used, i.e. that rays from the source enter at all angles up to the full acceptance angle and at all parts of the core cross-section. If the source is a light-emitting diode with a luminous area much greater than the core diameter this can be achieved simply by butting the source against the end of the guide. If there are objections to this because, for example, the source may be encapsulated in a glass or plastic cover we have to use a *condenser*. This is a lens or other image-forming system, which focuses an image of the source onto the end of the guide, as in Fig. 8.18. The lens must have a large enough diameter and small enough focal length to ensure that the core is covered by the source image formed by beams with convergence angles

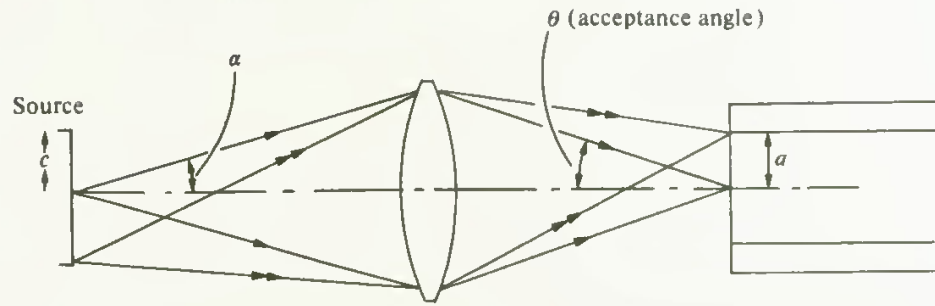


FIG. 8.18. Imaging a light source on to the core of a step-index guide; if the image just covers the core and if the cone of rays just matches the acceptance angle of the guide the maximum possible flux is transmitted.

equal to or greater than the acceptance angle. Let the radiance (Section 2.6) of the source be B . Then as in Chapter 2 the condenser collects $\pi^2 B c^2 \alpha^2$ watts from a portion of the source of diameter $2c$. The magnification of the condenser is α/θ and if c is equal to $a\theta/\alpha$, where $2a$ is the core diameter, all this power will enter the guide. No more power can be collected by the guide from a source of radiance B and for the acceptance angle θ : for if we try to image an area of diameter $4c$ onto the core the convergence angle will become $\alpha/2$ instead of α in order to adjust the magnification, and no gain will have been achieved. A similar point was made in Section 2.6 in connection with the luminance of images and in Section 8.1 when the numerical aperture of a guide was defined.

Finally we stress that this calculation is very approximate. First, we have already noted that step index fibres transmit skew rays incident at larger angles than the formal acceptance angle θ as given by eqn (8.2), whereas on the other hand graded index fibres do *not* transmit some meridian rays at the angle given by eqn (8.9). Secondly the geometrical optics model is an approximation and to get a better calculation it would be necessary to calculate the transmission of light according to electromagnetic theory. For this, however, we refer to Midwinter (1979).

Problems

- 8.1. A step index light guide has refractive index 1.54 and the cladding has index 1.52. Calculate the acceptance angle. What index would be required for the cladding to give an acceptance angle of 25° ?
- 8.2. Show that a guide of circular cross-section with radius a and acceptance angle θ can, within the approximations of Chapter 8, transmit a flux $\pi a^2 B \sin^2 \theta$ from a Lambertian source of radiance B .
- 8.3. A graded index guide has the radial refractive index distribution $1.52 - 2r^2$ with r in mm up to a radius $r_0 = 0.2$. Find its acceptance angle and the distance between successive intersections of the axis for a meridian ray.
- 8.4. A short section of the graded index guide in Problem 8.3 is to be used as an image-forming system. How long should it be if (a) an object at infinity is to be imaged at the exit end of the guide or (b) if it is to be an afocal system?

This book covers basic classical optics—geometrical and physical—at a level suitable for the first or second year of a degree course. The introductory chapter discusses the properties of electromagnetic waves in the optical region and compares them with other regions of the spectrum. The following chapters deal with the geometrical optics model, the propagation of waves (interference and diffraction), polarization, image formation, and optical instruments, including interferometers and spectroscopes. The later chapters on laser light and optical fibres show the applications of the ideas developed earlier in the book. For this third edition of the book, the author has not only updated the existing text but also added a new chapter on the detection of light. The book aims to show how the properties of light fit into the general scheme of physics, and how these properties are used in many of the instruments that are among the basic tools of science.

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