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SOLAR CELLS

Operating Principles, Technology, and System Applications

MARTIN A. GREEN
*University of New South Wales
Australia*

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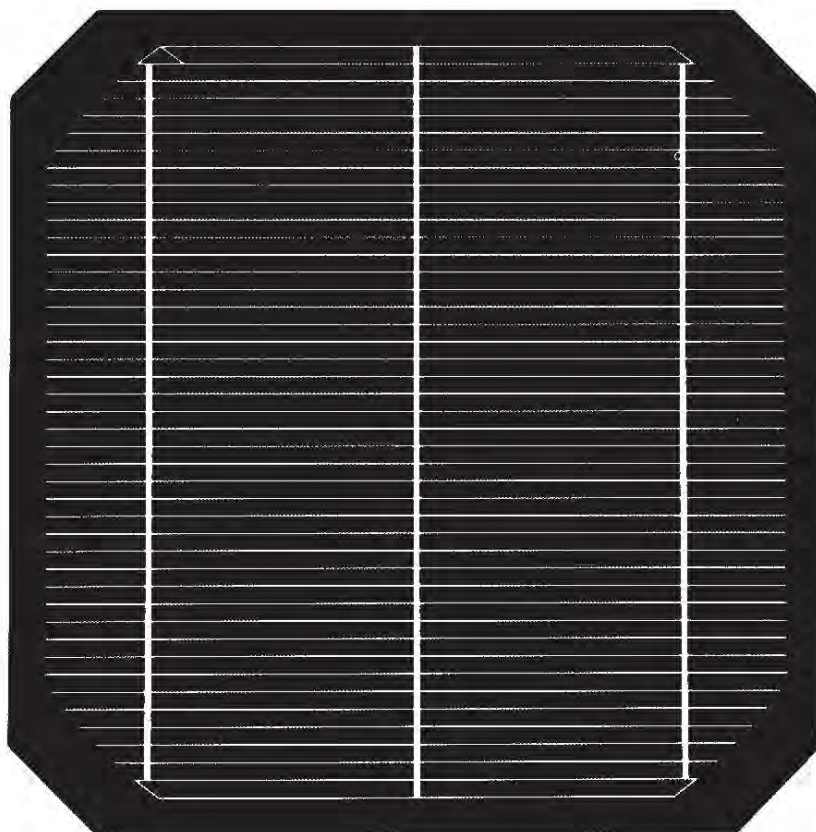
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This solar cell is made from a thin wafer of the semiconductor silicon, about 10 cm square and only a fraction of a millimeter thick. When the cell is illuminated, it converts the energy of the photons in the incident light into electrical energy. Under bright sunshine, the cell can supply a current of up to 3 A at a voltage of about $\frac{1}{2}$ V to an electrical load connected between the metallic contact grid apparent here and a second contact at the rear of the cell. (Photograph courtesy of Motorola, Inc.)

PREFACE

When sunlight strikes a solar cell, the incident energy is converted directly into electricity without any mechanical movement or polluting by-products. Far from being a laboratory curiosity, solar cells have been used for over two decades, initially for providing electrical power for spacecraft and more recently for terrestrial systems. There are very real prospects that the manufacturing technology for these cells can be improved dramatically in the near future. This would allow solar cells to be produced at prices where they could make significant contributions to world energy demands.

This book concentrates on providing descriptions of the basic operating principles and design of solar cells, of the technology used currently to produce cells and the improved technology soon to be in operation, and of considerations of importance in the design of systems utilizing these cells. Accordingly, the early chapters of the book review the properties of sunlight, the relevant properties of the semiconductor material from which the cells are constructed, and the interaction between these two elements. The next group of chapters treat in some detail the factors important in the design of solar cells,

current technology for fabricating them, and probable technological developments in the future. The final chapters deal with system applications, ranging from the small systems commercially available at present to residential and central power systems that may be available in the future.

The book is intended primarily for the increasing numbers of engineers and scientists attracted to this rapidly expanding field. As such, it is suitable for use as a textbook for both undergraduate and graduate courses. A deliberate attempt has been made not to exclude the material contained within from those readers who are entering the field through a different route. For example, a rather pictorial review of the properties of semiconductors relevant to the understanding of solar cell operation is included. Although this may serve as a quick review for many readers, for other readers it may provide a framework on which the material in subsequent chapters can be supported. Irrespective of background, working through the text and associated exercises would place the reader in a very strong position for future activity in this area.

I would like to acknowledge the large number of people, too numerous to mention individually, who have stimulated my interest in solar cells over the last decade. I would particularly like to thank Andy Blakers, Bruce Godfrey, Phill Hart, and Mike Willison for their suggestions and indirect encouragement in this venture. Special thanks are due to Gelly Galang for her help in preparing the manuscript and to John Todd and Mike Willison for preparing photographs incorporated into the text. Finally, I would like to thank Judy Green for her support and encouragement during the fairly intense period in which this book was developed.

Martin A. Green

Chapter 1

SOLAR CELLS AND SUNLIGHT

1.1 INTRODUCTION

Solar cells operate by converting sunlight directly into electricity using the electronic properties of a class of material known as semiconductors. In the following chapters, this elegant energy-conversion process will be examined starting from the basic physical principles of solar cell operation. From this basis, the mathematical equations quantifying the energy transformation are developed. This is followed by a description of the technology used to produce present commercial solar cells, based predominantly on a particular semiconductor, silicon. Improvements in this technology, as well as alternative technologies that hold the promise of significantly lower cost, are then described. Finally, the design of solar cell systems is discussed, ranging from small power supplies for remote-area use to possible future residential and central power-generating plants.

In this chapter, the history of solar cell development is outlined briefly, followed by a review of the properties of the sun and its radiation.

1.2 OUTLINE OF SOLAR CELL DEVELOPMENT

Solar cells depend upon the *photovoltaic effect* for their operation. This effect was reported initially in 1839 by Becquerel, who observed a light-dependent voltage between electrodes immersed in an electrolyte. It was observed in an all-solid-state system in 1876 for the case of selenium. This was followed by the development of photo-cells based on both this material and cuprous oxide. Although a silicon cell was reported in 1941, it was not until 1954 that the forerunner of present silicon cells was announced. This device represented a major development because it was the first photovoltaic structure that converted light to electricity with reasonable efficiency. These cells found application as power sources in spacecraft as early as 1958. By the early 1960s, the design of cells for space use had stabilized, and over the next decade, this was their major application. Reference 1.1 is a good source of more detailed material up to this stage.

The early 1970s saw an innovative period in silicon cell development, with marked increases in realizable energy-conversion efficiencies. At about the same time, there was a reawakening of interest in terrestrial use of these devices. By the end of the 1970s, the volume of cells produced for terrestrial use had completely outstripped that for space use. This increase in production volume was accompanied by a significant reduction in solar cell costs. The early 1980s saw newer device technologies being evaluated at the pilot production stage, poised to enable further reduction in costs over the coming decade. With such cost reductions, a continual expansion of the range of commercial applications is ensured for this approach to utilizing the sun's energy.

1.3 PHYSICAL SOURCE OF SUNLIGHT

Radiant energy from the sun is vital for life on our planet. It determines the surface temperature of the earth as well as supplying virtually all the energy for natural processes both on its surface and in the atmosphere.

The sun is essentially a sphere of gas heated by a nuclear fusion reaction at its center. Hot bodies emit electromagnetic radiation with a wavelength or spectral distribution determined by the body's temperature. For a perfectly absorbing or "black" body, the spectral distribution of the emitted radiation is given by *Planck's radiation law* (Ref. 1.2). As indicated in Fig. 1.1, this law indicates that as a body is heated, not only does the total energy of the electromagnetic

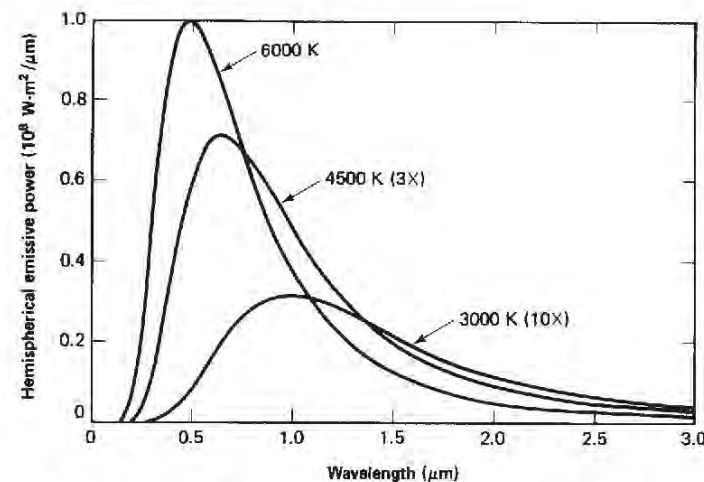


Figure 1.1. Planckian black-body radiation distributions for different black-body temperatures.

radiation emitted increase, but the wavelength of peak emission decreases. An example of this within most of our ranges of experience is that metal, when heated, glows red and then yellow as it gets hotter.

Temperatures near the sun's center are estimated to reach a warm 20,000,000 K. However, this is *not* the temperature that determines the characteristic electromagnetic radiation emission from the sun. Most of the intense radiation from the sun's deep interior is absorbed by a layer of negative hydrogen ions near the sun's surface.

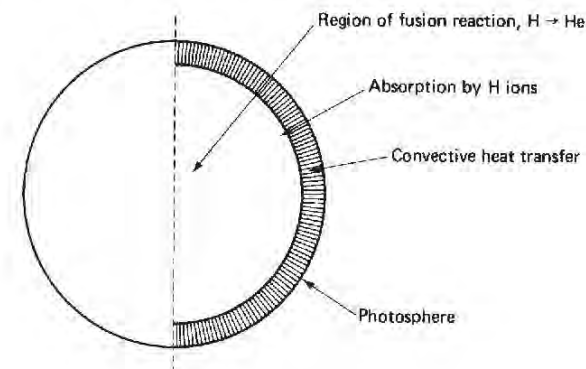


Figure 1.2. Principal features of the sun.

These ions act as continuous absorbers over a great range of wavelengths. The accumulation of heat in this layer sets up convective currents that transport the excess energy through the optical barrier (Fig. 1.2). Once through most of this layer, the energy is reradiated into the relatively transparent gases above. The sharply defined level where convective transport gives way to radiation is known as the *photosphere*. Temperatures within the photosphere are much cooler than at the sun's interior but are still a very high 6000 K. The photosphere radiates an essentially continuous spectrum of electromagnetic radiation closely approximating that expected from a black body at this temperature.

1.4 THE SOLAR CONSTANT

The radiant power per unit area perpendicular to the direction of the sun outside the earth's atmosphere but at the mean earth-sun distance is essentially constant. This radiation intensity is referred to as the *solar constant* or, alternatively, *air mass zero (AM0) radiation*, for reasons that will soon become apparent.

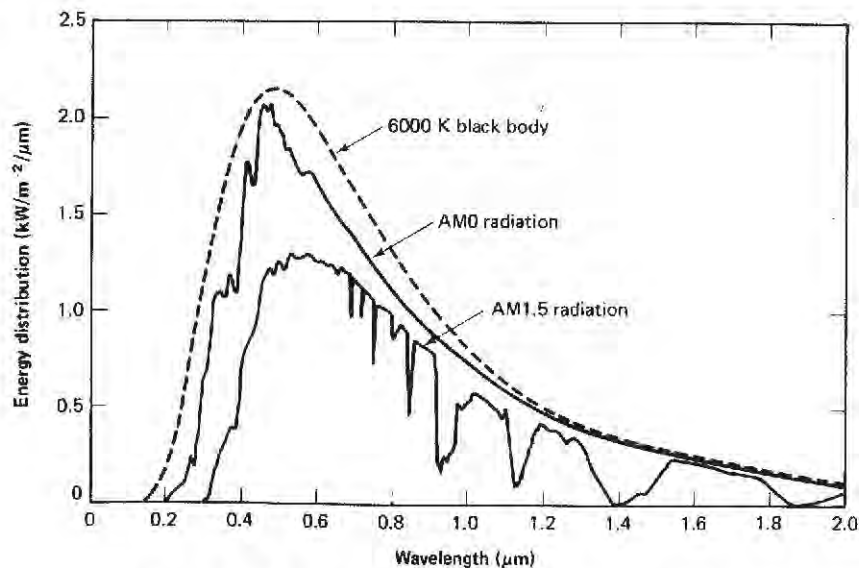


Figure 1.3. Spectral distribution of sunlight. Shown are the cases of AM0 and AM1.5 radiation together with the radiation distribution expected from the sun if it were a black body at 6000 K.

The presently accepted value of the solar constant in photovoltaic work is 1.353 kW/m^2 . This value has been determined by taking a weighted average of measurements made by equipment mounted on balloons, high-altitude aircraft, and spacecraft (Ref. 1.3). As indicated by the two uppermost curves in Fig. 1.3, the spectral distribution of AM0 radiation differs from that of an ideal black body. This is due to such effects as differing transmissivity of the sun's atmosphere at different wavelengths. Currently accepted values for this distribution are tabulated in Ref. 1.3. A knowledge of the exact distribution of the energy content in sunlight is important in solar cell work because these cells respond differently to different wavelengths of light.

1.5 SOLAR INTENSITY AT THE EARTH'S SURFACE

Sunlight is attenuated by at least 30% during its passage through the earth's atmosphere. Causes of such attenuation are (Ref. 1.4):

1. Rayleigh scattering or scattering by molecules in the atmosphere. This mechanism attenuates sunlight at all wavelengths but is most effective at short wavelengths.
2. Scattering by aerosols and dust particles.
3. Absorption by the atmosphere and its constituent gases—oxygen, ozone, water vapor, and carbon dioxide, in particular.

A typical spectral distribution of sunlight reaching the earth's surface is shown by the lower curve of Fig. 1.3, which also indicates the absorption bands associated with molecular absorption.

The degree of attenuation is highly variable. The most important parameter determining the total incident power under clear conditions is the length of the light path through the atmosphere. This is shortest when the sun is directly overhead. The ratio of any actual path length to this minimum value is known as the *optical air mass*. When the sun is directly overhead, the optical air mass is unity and the radiation is described as *air mass one (AM1) radiation*. When the sun is an angle θ to overhead, the air mass is given by

$$\text{Air mass} = \frac{1}{\cos \theta} \quad (1.1)$$

Hence, when the sun is 60° off overhead, the radiation is AM2. The easiest way to estimate the air mass is to measure the length of the shadow s cast by a vertical structure of height h . Then

$$\text{Air mass} = \sqrt{1 + \left(\frac{s}{h}\right)^2} \quad (1.2)$$

With increasing air mass but with other atmospheric variables constant, the energy reaching the earth is attenuated at all wavelengths, with attenuation in the vicinity of the absorption bands of Fig. 1.3 becoming even more severe.

Hence, as opposed to the situation outside the earth's atmosphere, terrestrial sunlight varies greatly both in intensity and spectral composition. To allow meaningful comparison between the performances of different solar cells tested at different locations, a terrestrial standard has to be defined and measurements referred to this standard. Although the situation is in a state of flux, the most widely used terrestrial standard at the time of writing is the AM1.5 distribution of Table 1.1, also plotted as the terrestrial curve in Fig. 1.3. In the photovoltaic program of the U.S. government, this distribution, essentially scaled up so that the total power density content is 1 kW/m^2 , was incorporated as a standard in 1977 (Ref. 1.5). The latter power density is close to the maximum received at the earth's surface.

1.6 DIRECT AND DIFFUSE RADIATION

The composition of terrestrial sunlight is further complicated by the fact that, as well as the component of radiation directly from the sun, atmospheric scattering gives rise to a significant indirect or *diffuse* component. Even in clear, cloudless skies, the diffuse component can account for 10 to 20% of the total radiation received by a horizontal surface during the day.

For less sunny days, the percentage of radiation on a horizontal surface that is diffuse generally increases. From observed data (Ref. 1.6), the following statistical trends can be discerned. For days on which there is a notable lack of sunshine, most of the radiation will be diffuse. This will be true, in general, for days on which the total radiation received is up to one-third that which would be received on a clear, sunny day at the same time of the year. For days between the sunny and cloudy extremes mentioned above, where about one-half of clear-day radiation is received, about 50% of this generally will be diffuse. Poor weather will not only cause some

Table 1.1 SOLAR SPECTRUM—AIR MASS 1.5*

Wave-length (μm)	W/($\text{m}^2 \cdot \mu\text{m}$)	Wave-length (μm)	W/($\text{m}^2 \cdot \mu\text{m}$)	Wave-length (μm)	W/($\text{m}^2 \cdot \mu\text{m}$)	Wave-length (μm)	W/($\text{m}^2 \cdot \mu\text{m}$)
0.295	0	0.595	1262.61	0.870	843.02	1.276	344.11
0.305	1.32	0.605	1251.79	0.875	835.10	1.288	345.69
0.315	20.86	0.615	1255.43	0.8875	817.12	1.314	284.24
0.325	113.48	0.625	1240.19	0.900	807.83	1.335	175.28
0.335	182.23	0.635	1243.79	0.9075	793.87	1.384	2.42
0.345	234.43	0.645	1233.96	0.915	778.97	1.432	30.06
0.355	286.01	0.655	1188.32	0.925	217.12	1.457	67.14
0.365	355.88	0.665	1228.40	0.930	163.72	1.472	59.89
0.375	386.80	0.675	1210.08	0.940	249.12	1.542	240.85
0.385	381.78	0.685	1200.72	0.950	231.30	1.572	226.14
0.395	492.18	0.695	1181.24	0.955	255.61	1.599	220.46
0.405	751.72	0.6983	973.53	0.965	279.69	1.608	211.76
0.415	822.45	0.700	1173.31	0.975	529.64	1.626	211.26
0.425	842.26	0.710	1152.70	0.985	496.64	1.644	201.85
0.435	890.55	0.720	1133.83	1.018	585.03	1.650	199.68
0.445	1077.07	0.7277	974.30	1.082	486.20	1.676	180.50
0.455	1162.43	0.730	1110.93	1.094	448.74	1.732	161.59
0.465	1180.61	0.740	1086.44	1.098	486.72	1.782	136.55
0.475	1212.72	0.750	1070.44	1.101	500.57	1.862	2.01
0.485	1180.43	0.7621	733.08	1.128	100.86	1.955	39.43
0.495	1253.83	0.770	1036.01	1.131	116.87	2.008	72.58
0.505	1242.28	0.780	1018.42	1.137	108.68	2.014	80.01
0.515	1211.01	0.790	1003.58	1.144	155.44	2.057	72.57
0.525	1244.87	0.800	988.11	1.147	139.19	2.124	70.29
0.535	1299.51	0.8059	860.28	1.178	374.29	2.156	64.76
0.545	1273.47	0.825	932.74	1.189	383.37	2.201	68.29
0.555	1276.14	0.830	923.87	1.193	424.85	2.266	62.52
0.565	1277.74	0.835	914.95	1.222	382.57	2.320	57.03
0.575	1292.51	0.8465	407.11	1.236	383.81	2.338	53.57
0.585	1284.55	0.860	857.46	1.264	323.88	2.356	50.01

*Total energy content = 832 W/m^2 .
Source: Ref. 1.5.

regions of the world to receive low levels of solar radiation but will also cause a significant proportion of it to be diffuse.

Diffuse sunlight generally has a different spectral composition from direct sunlight. Generally, it will be richer in the shorter or "blue" wavelengths, giving rise to further variability in the spectral composition of light received by a solar cell system. Uncertainty in the distribution of diffuse radiation from different directions in the sky introduces other uncertainties when calculating radiation levels on inclined surfaces from data generally recorded on horizontal surfaces. A common assumption is that diffuse light is isotropic (uniform in all directions), although the region of the sky surrounding the sun is the most intense source of this radiation.

Photovoltaic systems based on concentrated sunlight can generally only accept rays spanning a limited range of angles. Hence, they usually have to track the sun to utilize the direct component of sunlight, with the diffuse component wasted. This tends to offset the advantage gained by such tracking systems of intercepting maximum power density by always being normal to the sun's rays.

1.7 APPARENT MOTION OF THE SUN

The earth spins daily on an imaginary axis orientated in a fixed direction relative to the plane of the earth's yearly orbit about the sun. The angle this direction makes with the orbital plane is the solar declination ($23^{\circ}27'$). Perhaps less familiar are the details of the apparent motion of the sun relative to a fixed observer on earth resulting from the relationship described above.

This apparent motion is indicated in Fig. 1.4 for an observer at latitude 35° north. On any given day, the plane of the sun's apparent orbit lies at an angle equal to the latitude from the observer's vertical. At the equinoxes (March 21 and September 23), the sun rises due east and sets due west, so that the altitude of the sun at solar noon on these days equals 90° minus the latitude. At the summer and winter solstices (June 21 and December 22, respectively, for the northern hemisphere, the opposite for the southern), the altitude at solar noon has increased or decreased by the declination of the earth ($23^{\circ}27'$).

1.8 SOLAR INSOLATION DATA

The ideal situation for the design of photovoltaic systems would be when there were detailed records of the solar insolation at the site

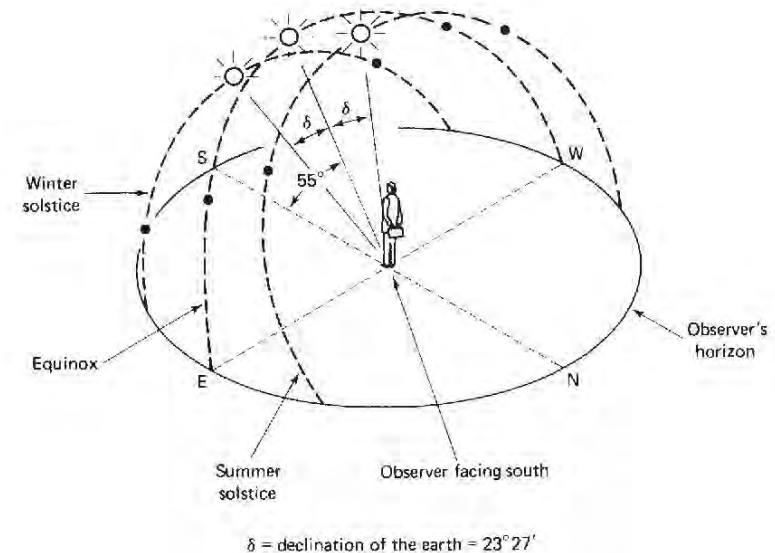


Figure 1.4. Apparent motion of the sun relative to a fixed observer at latitude 35° in the northern hemisphere. The path of the sun is shown at the equinoxes and the summer and winter solstices. The position of the sun is shown at solar noon on each of these days. The shaded circles represent the sun's position 3 h before and after solar noon.

selected for installation. Not only would data on the direct and diffuse components of light be desirable, but data on corresponding ambient temperatures as well as wind speed and direction could be used to advantage. Although there are stations at various locations around the world that do monitor all these parameters, present economies favor the use of photovoltaic systems in remote regions of the world where it is unlikely that such information is available.

The available insolation at a given location depends not only on gross geographical features such as latitude, altitude, climatic classification, and prevailing vegetation, but it also depends strongly upon local geographical features. Although unable to incorporate the latter features, maps of solar insolation distribution are available for different parts of the world. These have usually been prepared by combining measured insolation data with data estimated from a large network of stations around the world monitoring hours of sunshine.

The information most generally available is the average daily total or *global radiation* on a horizontal surface. A widely used source

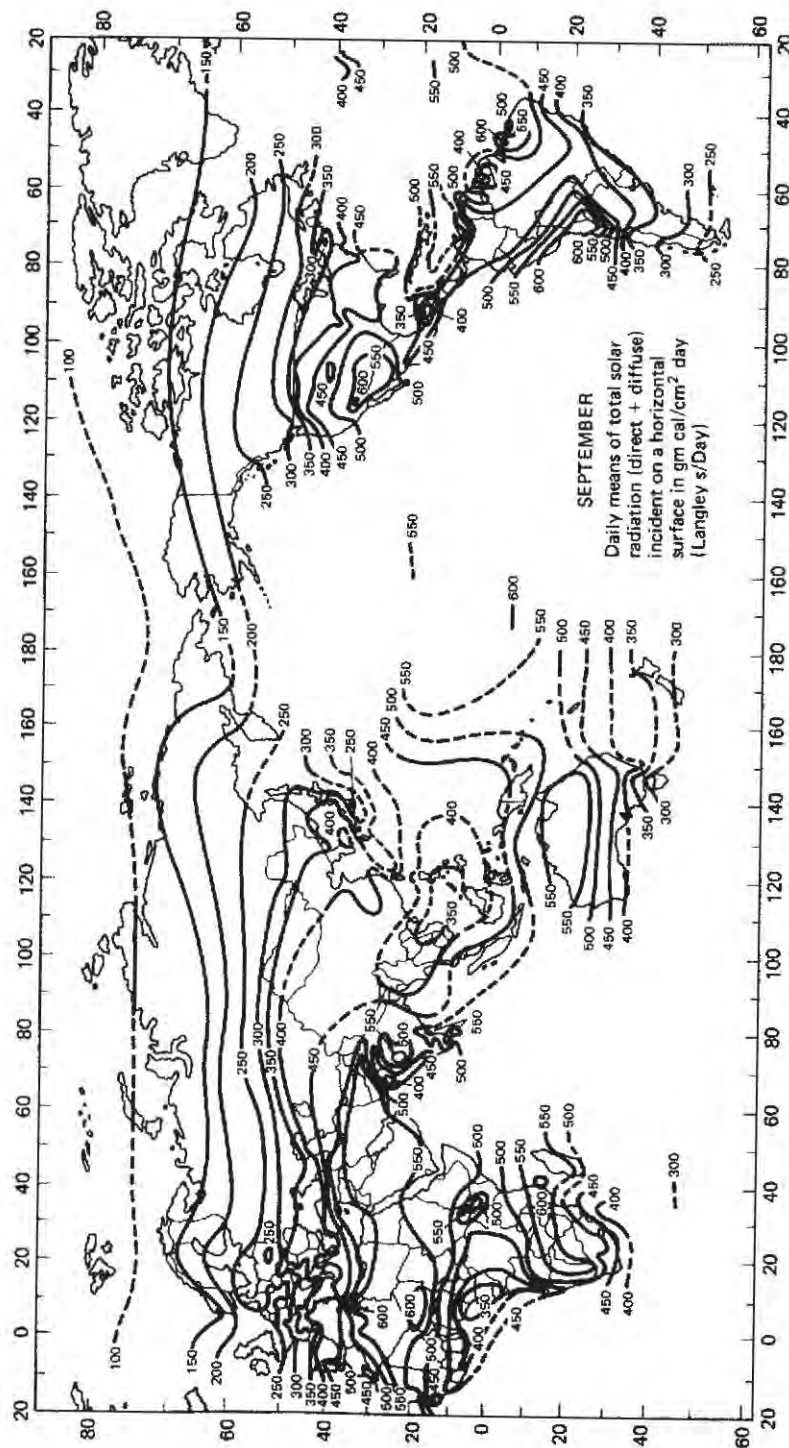


Figure 1.5. Worldwide distribution of solar energy during the month of September. The contours are daily means of global solar radiation incident on a horizontal surface expressed in langleys. A langley equals 1 cal/cm^2 . To convert to MJ/m^2 , multiply by 0.0418. To convert to kWh/m^2 , multiply by 0.0116. The distribution of solar energy during September is roughly indicative of annual average daily radiation for a given site. Similar curves for other months of the year appear in Ref. 1.7. (After Ref. 1.7.)

for such data is Ref. 1.7. This lists, for each month of the year, average daily global radiation on a horizontal surface for hundreds of insolation monitoring stations around the world. It also lists this information estimated from sunshine-hour records, taking into account climatic and vegetation data for several hundred other locations. This information has been incorporated into a sequence of world maps showing contours of constant insolation for each month of the year. Such contours are illustrated in Fig. 1.5 for a month of equinox, September. This month corresponds approximately to average insolation levels throughout the year for most locations.

1.9 SUMMARY

Although sunlight outside the earth's atmosphere is relatively constant, the situation at the earth's surface is more complex. Terrestrial sunlight varies dramatically and unpredictably in availability, intensity, and spectral composition. On clear days, the length of the sunlight's path through the atmosphere or the optical air mass is an important parameter. The indirect or diffuse component of sunlight can be particularly important for less ideal conditions. Reasonable estimates of global radiation (direct plus diffuse) received annually on horizontal surfaces are available for most regions of the world. However, there are uncertainties involved in using this for a specific site because of the large deviations that can be caused by local geographical conditions and approximations involved in conversion to radiation on inclined surfaces.

EXERCISES

- 1.1. Estimate the solar constant for Mercury and Mars given that the mean distances from the sun to Earth, Mercury, and Mars are 150, 58, and 228 million kilometers, respectively.
- 1.2. The sun is at an altitude of 30° to the horizontal. What is the corresponding air mass?
- 1.3. Calculate the sun's altitude at solar noon on June 21 at Sydney (latitude 34°S), San Francisco (latitude 38°N), and New Delhi (latitude 29°N).
- 1.4. The global radiation at solar noon on a summer solstice in Albuquerque, New Mexico (latitude 35°N), is 60 mW/cm^2 . Assume that 30% of this is diffuse radiation and make the approximations that the ground surrounding the module is nonreflecting and the diffuse radiation is uniformly dis-

tributed across the sky. Estimate the radiation intensity on a flat surface facing south at an angle of 45° to the horizontal.

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Chapter 2

REVIEW OF SEMICONDUCTOR PROPERTIES

2.1 INTRODUCTION

In Chapter 1, the properties of sunlight were reviewed. It is now appropriate to look at the properties of the other important component in photovoltaic solar energy conversion, semiconducting material.

The aim of this chapter is *not* to treat the properties of semiconductors rigorously from fundamentals. Rather, it is to highlight those properties of semiconductors that are important in the design and operation of solar cells. As such, the chapter may suffice for quick revision for readers already acquainted with these properties while containing adequate information to allow those not as well acquainted to establish a framework on which subsequent material can be supported. To strengthen this framework, readers in the latter category are referred to one of the many textbooks specifically directed at treating semiconductor properties more fundamentally (Refs. 2.1 to 2.4).

2.2 CRYSTAL STRUCTURE AND ORIENTATIONS

Most of the photovoltaic materials described in this book are *crystalline*, at least on a microscopic scale. Ideal crystalline material is characterized by an orderly, periodic arrangement of the atoms of which it is composed.

In such an orderly arrangement, it is possible to build up the entire crystal structure by repeatedly stacking a small subsection. The smallest such section with which this is possible is known as a *primitive cell*. Such primitive cells naturally contain all the information required to reconstruct the locations of atoms in the crystal but often have awkward shapes. Consequently, it can be more convenient to work with a larger *unit cell* which also contains this information but generally has a simpler shape. For example, Fig. 2.1(a) shows the unit cell for an atomic arrangement known as face-centered-cubic and Fig. 2.1(b) shows the corresponding primitive cell. The directions defining the outline of the unit cell are orthogonal, whereas this is not the case for the primitive cell. The length of the edge of the unit cell is known as the *lattice constant*.

The orientation of planes within the crystal can be expressed in terms of the unit cell structure by using the system of *Miller indices*. The vectors defining the outline of the unit cell are used as the basis of a coordinate system as in Fig. 2.1(a). A plane of the orientation in question is imagined passing through the origin of the coordinate system. Then the next plane parallel to this which passes through atomic sites along each of the coordinate axis is considered. An ex-

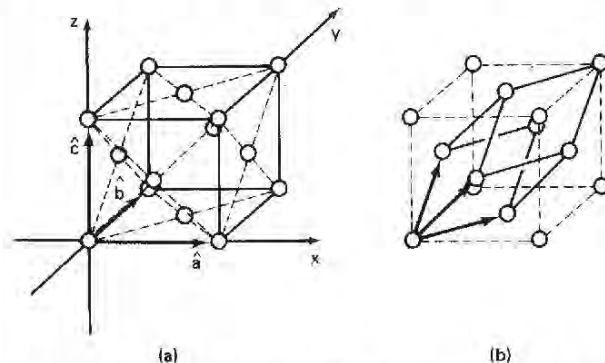


Figure 2.1. (a) Unit cell for the face-centered-cubic atomic arrangement. The unit cell is selected in this case so that the directions defining its outline are orthogonal. The vectors $\hat{a}$, $\hat{b}$, and $\hat{c}$ are unit vectors in each of these directions. (b) Primitive cell for the same atomic arrangement.

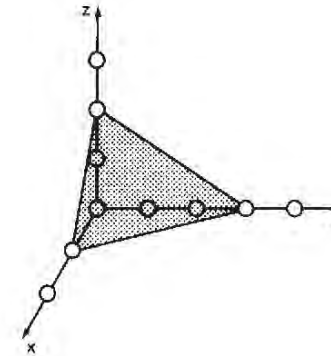


Figure 2.2. Sketch of a plane in a crystal described by the Miller indices $(6\ 2\ 3)$.

ample is shown in Fig. 2.2. The intercepts in this case along each of the axes are 1, 3, and 2 atoms from the origin. Taking inverses gives 1 , $\frac{1}{3}$, and $\frac{1}{2}$. The smallest integrals with the same ratio are 6, 2, and 3. This plane is then expressed in Miller indices as the $(6\ 2\ 3)$ plane. Negative intercepts are indicated by a bar over the top of the corresponding index (e.g., -2 would be written as $\bar{2}$).

Directions within the crystal are expressed in a condensed form of vector notation. A vector in the direction of interest is scaled so it is expressed in the form $h\hat{a} + k\hat{b} + l\hat{c}$, where $\hat{a}$, $\hat{b}$, and $\hat{c}$ are unit vectors along each of the axes of the coordinate system as in Fig. 2.1(a) and h , k , and l are integers. This direction is then described as the $[h\ k\ l]$ direction. The use of square brackets distinguishes directions from Miller indices. Note that for cubic unit cells, the $[h\ k\ l]$ direction is perpendicular to the $(h\ k\ l)$ plane.

Finally, there are planes within the crystal structure which are equivalent. For example, for the face-centered-cubic lattice of Fig. 2.1(a), differences between the (100) , (010) , and (001) planes depend only on the choice of origin. Collectively, the corresponding set of equivalent planes is known as the $\{100\}$ set, with braces reserved for this use.

Figure 2.3(a) shows the atomic arrangement found in many of the semiconductors important in solar cell technology. This is the arrangement for silicon (Si) crystals as well as for crystals of gallium arsenide (GaAs) and cadmium sulfide (CdS). The latter are *compound* semiconductors involving more than one type of atom in the crystal structure. The arrangement shown is generally referred to as the *diamond lattice* or *zincblende lattice* (for compound semiconductors such as GaAs). The unit cell is cubic, as indicated. Figure

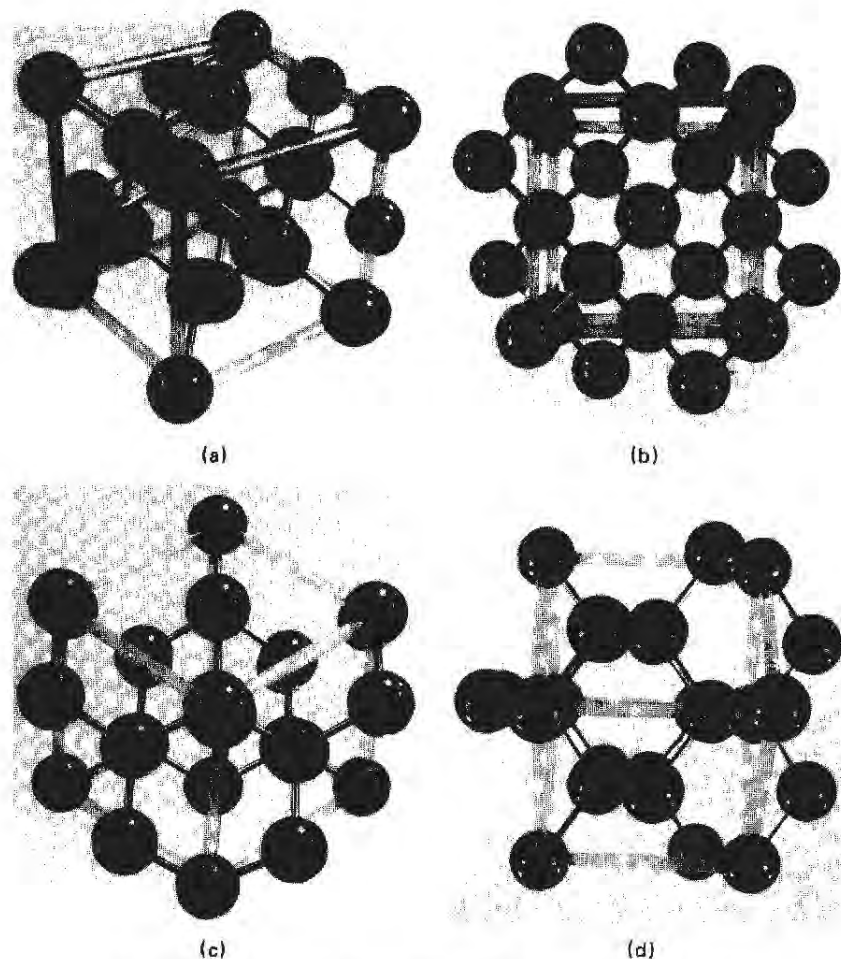


Figure 2.3. (a) Model of the diamond lattice which indicates the atomic structure of many of the semiconductors of importance in solar cells. Also shown is the outline of the unit cell. (b) View of the same structure looking in the [100] direction. (c), (d) Views in the [111] and [110] directions, respectively.

2.3(b)–(d) show atomic arrangements looking at the lattice from selected directions. These highlight the substantial differences in the physical arrangement of atoms in different directions which give rise to directional variations of importance in solar cell work (e.g., see Exercise 2.2).

2.3 FORBIDDEN ENERGY GAPS

An electron in free space has an essentially continuous range of energy values that it can attain. The situation in a crystal can be quite different.

Electrons associated with isolated atoms have a well-defined set of discrete energy levels available to them. As several atoms are brought closer together, the original levels spread out into bands of allowed energy as indicated in Fig. 2.4. When the atoms are in ordered arrangements as in crystals, there will be characteristic distances between them. Figure 2.4(a) shows the case of a crystal where the characteristic separation between atoms, d , is such that the crystal has bands of energies allowed to electrons (corresponding to the atomic energy levels) separated by bands of forbidden energy. A different situation is shown in Fig. 2.4(b), where the bands have overlapped to give virtually a continuum of allowed energies at the value of d characteristic of a different crystalline material.

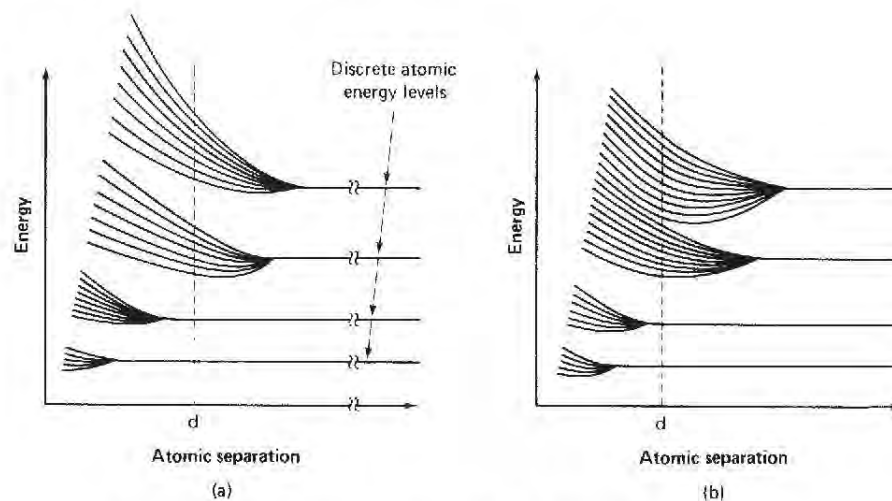


Figure 2.4. Schematic indicating how the discrete energies allowed to electrons in an isolated atom split up into bands of allowed energies when a number of similar atoms are brought together in a crystal:

- (a) For this case, d , the characteristic spacing of atoms in a crystal, is such that there are bands of energies allowed to electrons separated by bands of forbidden energy.
- (b) In this case, d is such that the uppermost bands have overlapped.

2.4 PROBABILITY OF OCCUPATION OF ALLOWED STATES

At low temperatures, electrons in a crystal occupy the lowest possible energy states.

At first sight, it might be expected that the equilibrium state of a crystal would be one in which the electrons are all in the lowest allowed energy level. However, this is *not* the case. A fundamental physical theorem, the *Pauli exclusion principle*, implies that each allowed energy level can be occupied by, at most, two electrons, each of opposite “spin.” This means that, at low temperatures, all available states in the crystal up to a certain energy level will be occupied by two electrons. This energy level is known as the *Fermi level* (E_F).

As the temperature increases, some electrons gain energy in excess of the Fermi level. The probability of occupation of an allowed electron state of any given energy E can be calculated from statistical considerations for this more general case, taking into account the constraints imposed by the Pauli exclusion principle (Refs. 2.1 to 2.4). The result is the *Fermi-Dirac distribution function* $f(E)$, given by

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}} \quad (2.1)$$

where k is a constant known as *Boltzmann’s constant* and T is the absolute temperature. This function is plotted in Fig. 2.5. Near absolute zero, $f(E)$ is essentially unity, as expected, up to an energy equal to E_F , and zero above E_F . As the temperature increases, there is a smearing out of the distribution, with states of energy higher than E_F having a finite probability of occupation, and states of energy below E_F having a finite probability of being empty.

It is now possible to describe the differences among metals, insulators, and semiconductors in terms of electronic band structure. Metals have an electronic structure such that E_F lies within an allowed band (Fig. 2.6). The cause of this may be that there are insufficient electrons available to fill an available band if the band structure is as shown in Fig. 2.4(a), or alternatively, that there are overlapping bands as in Fig. 2.4(b). Insulators have one band fully occupied by electrons and a large energy gap between this band and the next highest band, which is devoid of electrons at low temperatures. From the discussion in the earlier part of this section, it follows that E_F must lie within the forbidden band (Fig. 2.6).

A band in which there are no electrons obviously cannot make

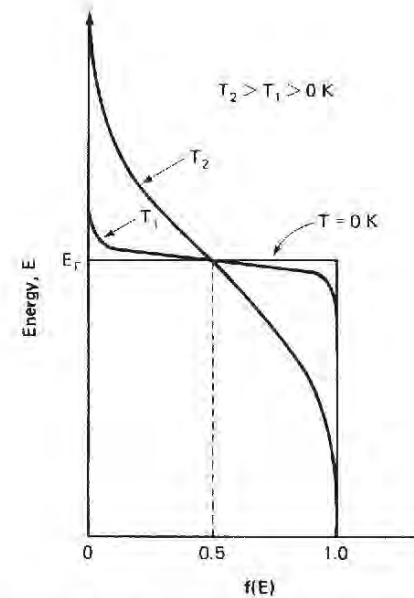


Figure 2.5. Fermi-Dirac distribution function. States above the Fermi level, E_F , have a low probability of being occupied by electrons, whereas those below are likely to be so occupied.

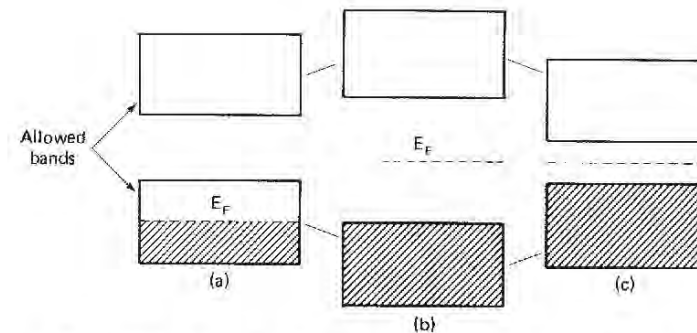


Figure 2.6. Diagrams showing the way in which allowed states are occupied by electrons in:

- (a) A metal.
- (b) An insulator.
- (c) A semiconductor.

any contribution to current flow in the crystal. More surprisingly, *neither can a completely full band*. To contribute to such flow, an electron must extract energy from the applied field. In a completely filled band, this is not possible. There are no vacant allowed energy levels in the vicinity into which an electron can be excited. Hence, an insulator does not conduct electricity, whereas a metal, with an abundance of such levels, does.

A semiconductor is just an insulator with a narrow forbidden band gap. At low temperatures, it does not conduct. At higher temperatures, there is sufficient smearing out of the Fermi-Dirac distribution function to ensure that some levels in the originally completely filled band (*valence band*) are now vacant and some in the next-highest band (*conduction band*) are occupied. The electrons in the conduction band, with an abundance of unoccupied energy states in the vicinity, can contribute to current flows. Since there are now unoccupied levels in the valence band, an additional contribution also comes from electrons in this band.

2.5 ELECTRONS AND HOLES

A very simple but reasonably good analogy to current flow processes in a semiconductor is provided by an idealized two-level parking station (Fig. 2.7). Consider the case where the bottom level of this station is completely filled with cars and the top level completely empty, as in Fig. 2.7(a). Then there is no room for any car to move. If one car is moved from the first to the second level as

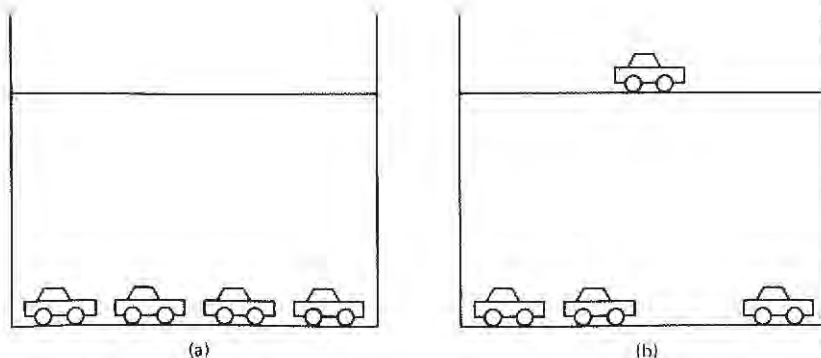


Figure 2.7. Simple "parking-station" analog of conduction processes in a semiconductor:

- (a) No movement possible.
- (b) Movement possible on both upper and lower levels.

in Fig. 2.7(b), the car on the second level is free to move as much as desired. This corresponds to an electron in the conduction band in a semiconductor. There will now be a vacant position on the lower level. Cars adjacent to this position can move into it, leaving a new vacant position behind. Hence, car motion is now possible on the lower level as well. This motion corresponds to the motion of electrons in the valence band. Instead of regarding the motion on the lower level as the result of the movements of a number of cars, it can be more simply described as the motion of the single vacant position. Similarly, in a crystal it is easier to think in terms of the motion of vacant states in the valence band. In many situations the correct motion of the vacancy can be predicted if it is regarded as a physical particle of positive charge commonly called a *hole*. Hence, current flow in a semiconductor can be regarded as being due to the sum of the motion of electrons in the conduction band and holes in the valence band.

2.6 DYNAMICS OF ELECTRONS AND HOLES

The motion of electrons and holes in semiconductors in response to applied forces differs from that of particles in free space. In addition to the applied force, there is always the effect of the periodic forces of the crystal atoms. However, results of quantum mechanical calculations indicate that, in most situations of interest herein, concepts developed for particles in free space can be applied to electrons and holes in semiconductors, with some modifications.

For example, in the case of electrons in a crystal lying within the conduction band, Newton's law becomes

$$F = m_e^* a = \frac{dp}{dt} \quad (2.2)$$

where F is the applied force, m_e^* an "effective mass" of the electron which incorporates the effect of the periodic force of the lattice atoms, and p is known as the crystal momentum analogous to free-space momentum.

For a free electron, energy and momentum are related by a parabolic law,

$$E = \frac{p^2}{2m} \quad (2.3)$$

For carriers in semiconductors, the situation can be more complex. In some semiconductors, an analogous law holds for electrons in the conduction band at energies close to the minimum, E_c , in this band:

$$E - E_c = \frac{p^2}{2m_e^*} \quad (2.4)$$

A similar relationship holds for holes near the maximum energy, E_v , in the valence band:

$$E_v - E = \frac{p^2}{2m_h^*} \quad (2.5)$$

The foregoing relationships are indicated in Fig. 2.8. Such semiconductors are known as *direct-band-gap semiconductors*, and the most important technologically is the compound semiconductor GaAs.

In other semiconductors, the minimum of the conduction band can be at a finite value of crystal momentum, obeying a relationship:

$$E - E_c = \frac{(p - p_0)^2}{2m_e^*} \quad (2.6)$$

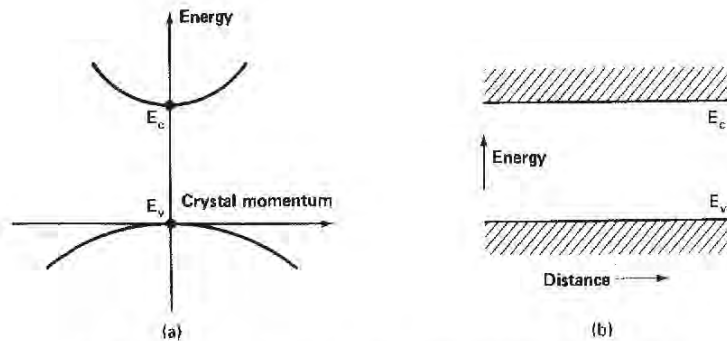


Figure 2.8. (a) Energy-crystal momentum relationships near the band edges for electrons in the conduction band and holes in the valence band of a direct-band-gap semiconductor. (b) Corresponding spatial representation of allowed energies in a semiconductor.

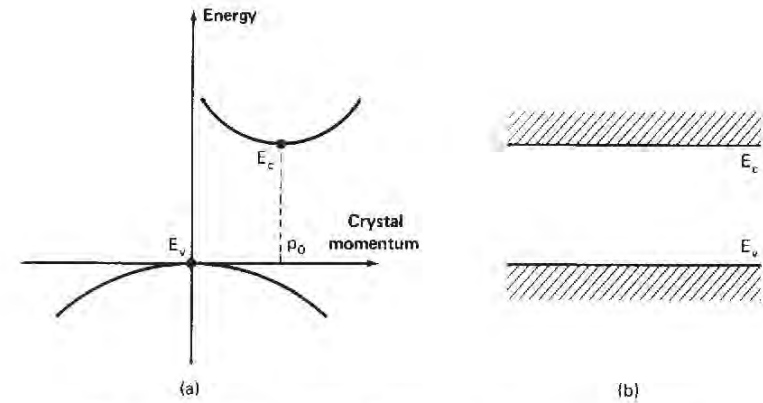


Figure 2.9. Energy-crystal momentum relationships near the band edges for an indirect-band-gap semiconductor. Also shown is the energy band spatial representation.

The valence band can exhibit a similar relationship:

$$E_v - E = \frac{(p - p'_0)^2}{2m_h^*} \quad (2.7)$$

If $p_0 = p'_0$, the semiconductor has a direct band-gap. However, if $p_0 \neq p'_0$, the band-gap is called *indirect*. The most common elemental semiconductors, Ge and Si, both are indirect-band-gap materials. In each case, $p'_0 = 0$ but p_0 is finite. Such a situation is shown in Fig. 2.9.

Note that the common representation of energy relations in semiconductor devices where energy is plotted as a function of distance (as also indicated in Figs. 2.8 and 2.9) does not differentiate between direct- and indirect-band-gap semiconductors.

2.7 ENERGY DENSITY OF ALLOWED STATES

The number of allowed states per unit volume in a semiconductor is obviously zero for energies corresponding to the forbidden gap and nonzero in the allowed bands. The question arises as to just how many states for electrons are distributed within the allowed bands.

An answer can be found reasonably simply (Refs. 2.1 to 2.4), at least for energies near the edges of the allowed bands, where car-

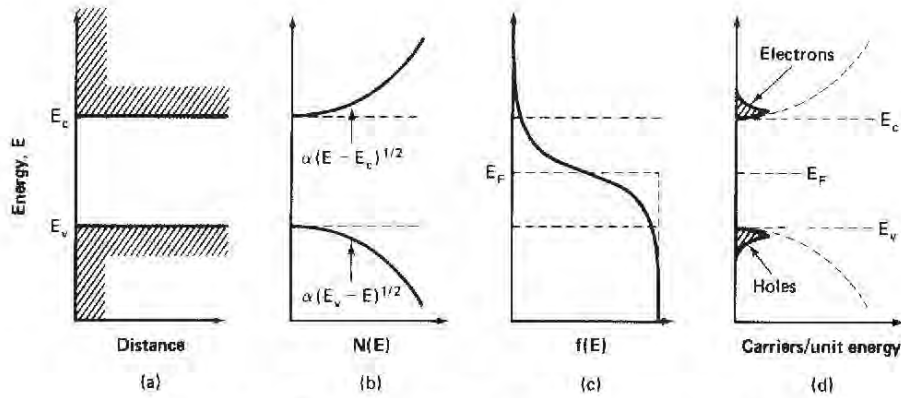


Figure 2.10. (a) Band representation of a semiconductor. (b) Corresponding energy density of allowed states for electrons. (c) Probability of occupation of these states. (d) Resulting energy distribution of electrons and holes. Note that most are clustered near the edge of the respective band.

riers can be treated similarly to free carriers. The number of allowed states per unit volume and energy, $N(E)$, at an energy E near the conduction-band edge (in the absence of anisotropy) is given by

$$N(E) = \frac{8\sqrt{2}\pi m_e^{*3/2}}{h^3} (E - E_c)^{1/2} \quad (2.8)$$

where h is Planck's constant. A similar expression holds for energies near the valence-band edge. These distributions of allowed states are shown in Fig. 2.10(b).

2.8 DENSITIES OF ELECTRONS AND HOLES

Knowing the density of allowed states [Eq. (2.8)] and the probability of occupation of these states [Eq. (2.1)], it is now possible to calculate the actual energy distribution of electrons and holes. The results are shown schematically in Fig. 2.10.

Owing to the nature of the Fermi-Dirac distribution function, most of the electrons in the conduction band and holes in the valence band are clustered near the band edges. The total number in either band can be found by performing an integration. The number of electrons in the conduction band per unit volume of the crystal,

n , is given by

$$n = \int_{E_c}^{E_{c \max}} f(E) N(E) dE \quad (2.9)$$

Since E_c is many kT larger than E_F , $f(E)$ for the conduction band reduces to

$$f(E) \approx e^{-(E - E_F)/kT} \quad (2.10)$$

and the upper limit, $E_{c \max}$, can be replaced by infinity with little error. Therefore,

$$\begin{aligned} n &= \int_{E_c}^{\infty} \frac{8\sqrt{2}\pi m_e^{*3/2}}{h^3} (E - E_c)^{1/2} e^{(E_F - E)/kT} dE \\ &= \frac{8\sqrt{2}\pi}{h^3} m_e^{*3/2} e^{E_F/kT} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{-E/kT} dE \end{aligned} \quad (2.11)$$

Changing the variable of integration to $x = (E - E_c)/kT$ gives

$$n = \frac{8\sqrt{2}\pi}{h^3} (m_e^* kT)^{3/2} e^{(E_F - E_c)/kT} \int_0^{\infty} x^{1/2} e^{-x} dx \quad (2.12)$$

The integral in this expression is in standard form and equals $\sqrt{\pi}/2$. Hence,

$$n = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} e^{(E_F - E_c)/kT} \quad (2.13)$$

$$n = N_c e^{(E_F - E_c)/kT} \quad (2.14)$$

where N_c is a constant at fixed T known as the *effective density of states in the conduction band* and is defined by comparing Eqs. (2.13) and (2.14). Similarly, the total number of holes in the valence band per unit volume of the crystal is given by

$$p = N_v e^{(E_v - E_F)/kT} \quad (2.15)$$

with N_V , the effective density of states in the valence band, similarly defined.

For the idealized case of a pure and perfect semiconductor without surfaces, n equals p because each electron in the conduction band leaves a vacancy or hole in the valence band. Hence,

$$n = p = n_i \quad (2.16)$$

$$\begin{aligned} np &= n_i^2 = N_C N_V e^{(E_v - E_c)/kT} \\ &= N_C N_V e^{-E_g/kT} \end{aligned} \quad (2.17)$$

where n_i is known as the “intrinsic concentration” and E_g is the width of the forbidden gap between the conduction and valence bands. Note also from Eq. (2.16) that

$$N_C e^{(E_F - E_c)/kT} = N_V e^{(E_v - E_F)/kT} \quad (2.18)$$

which gives

$$E_F = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln \left(\frac{N_V}{N_C} \right) \quad (2.19)$$

Hence, the Fermi level in a pure and perfect semiconductor lies close to midgap, being offset only by differences in the effective density of states in the conduction and valence bands.

2.9 BOND MODEL OF A GROUP IV SEMICONDUCTOR

Some of the more fundamental of the semiconductor properties described up to now can be looked at from a different viewpoint for a class of semiconductors represented by those belonging to group IV of the periodic table of chemical elements. Although the following “bond model” description is not universally valid for all semiconducting material, it does allow the effects of impurities upon the electronic properties of semiconductors to be introduced in a simple manner.

The characteristic lattice structure of a semiconductor from group IV of the periodic table was shown in Fig. 2.3. A schematic two-dimensional representation of the silicon lattice is shown in Fig. 2.11(a). Each silicon atom is bonded to four neighbors by covalent bonds. Each covalent bond requires two electrons. Silicon has four

valence electrons, so each covalent bond shares an electron originating from the central atom and one originating from the neighboring atom.

For the case shown in Fig. 2.11(a), the semiconductor cannot conduct electricity. However, at higher temperatures, some electrons in the covalent bond can obtain enough energy to break free from the bond as shown in Fig. 2.11(b). In this case, the electrons released are free to move about throughout the crystal and can contribute to current flows. Electrons in covalent bonds in the vicinity of the broken bond can also move into the location left vacant in this bond, leaving another broken bond behind. This process also contributes to current flow.

Reverting to the terminology of previous sections, an electron released from a covalent bond can be recognized as being in the conduction band, whereas those associated with covalent bonds are in the valence band. A broken bond can be identified as a hole in the valence band. The minimum energy required to release an electron from a covalent bond is then equal to the width of the forbidden band gap in the semiconductor.

The bond model is particularly useful for discussing the effects of impurities in silicon upon its electronic properties. In the

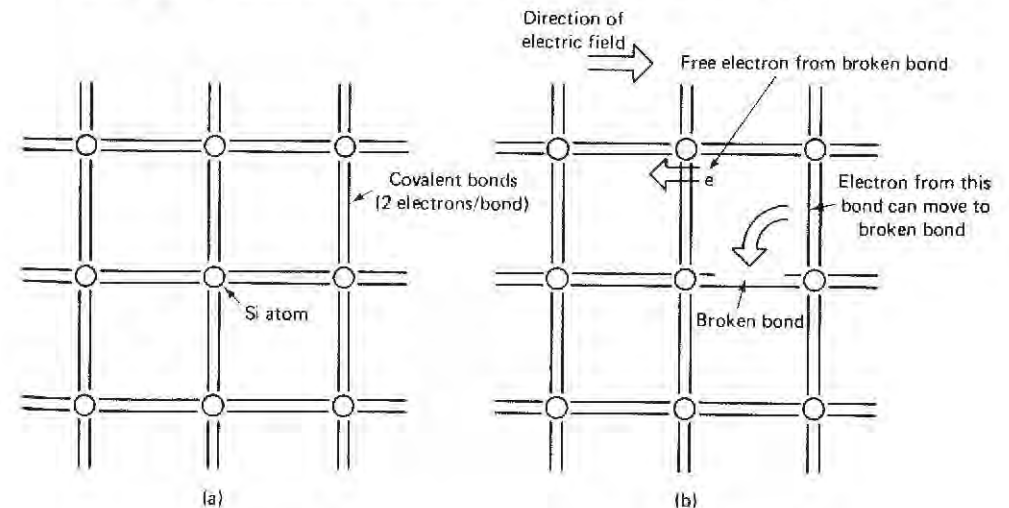


Figure 2.11. Schematic representation of the silicon crystal lattice.

- (a) No covalent bonds broken.
- (b) One covalent bond broken, showing the motion of the released electron as well as the motion of a nearby bonded electron into the position left vacant.

next section, the effects of very specialized impurities known as *dopants* are described.

2.10 GROUP III AND V DOPANTS

Impurity atoms can be incorporated in a crystal structure in two ways. They can occupy positions squeezed in between the atoms of the host crystal, in which case they are known as *interstitial impurities*. Alternatively they can substitute for an atom of the host crystal, maintaining the regular atomic arrangement in the crystal structure, in which case they are known as *substitutional impurities*.

Atoms from groups III and V of the periodic table act as substitutional impurities in silicon. A portion of the lattice where a group V impurity (e.g., phosphorus) has replaced a silicon atom is shown in Fig. 2.12. Four of the valence electrons are used up in covalent bonds, but the fifth is in a different situation. It is not in a covalent bond, so it is not in the valence band. For the case shown, it is tied to the group V atom and so is not free to move through the lattice. Hence, it is not in the conduction band either.

It might be expected that only a small amount of energy is required to release this extra electron compared to that required to free electrons locked up in covalent bonds. This is in fact the case. A rough estimate of the energy required can be found by noting the similarity to an electron tied to a hydrogen atom. The expression for the ionization energy (the energy required to release the electron) in

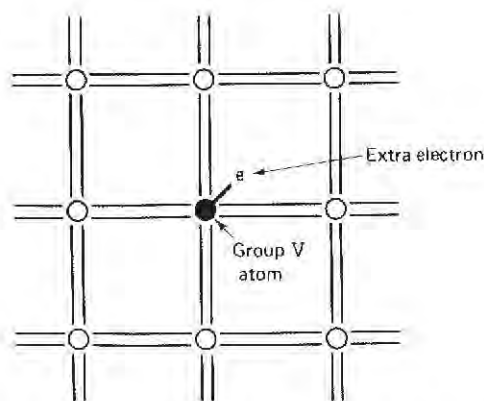


Figure 2.12. Portion of the silicon lattice where a group V atom has replaced a silicon atom.

the latter case is (Refs. 2.1 to 2.4)

$$E_i = \frac{m_0 q^4}{8\epsilon_0^2 h^2} = 13.6 \text{ electron volts (eV)} \quad (2.20)$$

where m_0 is the electron's rest mass, q the electronic charge, and ϵ_0 the permittivity of free space. The extra electron orbits the group V atom, which has one unneutralized positive charge. The expression for the ionization energy in this case is therefore similar. The radius of the orbit turns out to be much larger than the interatomic distance, so ϵ_0 in Eq. (2.20) should be replaced by the permittivity of silicon ($11.7\epsilon_0$). Since the orbiting electron experiences the periodic forces of the silicon lattice, the electron's mass also should be replaced by an effective mass ($m_e^*/m_0 \simeq 0.2$ for silicon). Hence, the energy required to free the extra electron is given by

$$E_i' \approx \frac{13.6(0.2)}{(11.7)^2} \approx 0.02 \text{ eV} \quad (2.21)$$

This is much less than the band-gap energy of silicon of 1.1 eV. A free electron is in the conduction band. Hence, the extra electron tied to the group V atom lies at an energy E_i' below the edge of the conduction band, as illustrated in Fig. 2.13(a). Note that this places an allowed energy level within the "forbidden" gap.

In an analogous way, a group III impurity does not have enough valence electrons to satisfy the four covalent bonds. This gives rise to a hole tied to the group III atom. The energy required to release the hole is similar to that given by Eq. (2.21). Hence, a group III atom gives rise to an allowed energy level for electrons in the forbidden gap just above the valence-band edge, as shown in Fig. 2.13(b).

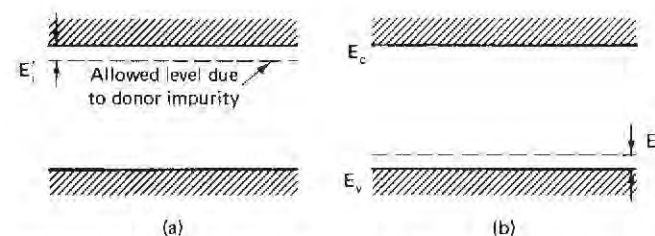


Figure 2.13. (a) Energy of the allowed state introduced into the forbidden gap by a group V substitutional impurity. (b) Corresponding energy state for a group III impurity.

2.11 CARRIER DENSITIES

Since the energy required to release the extra electron from a group V atom is small, it is not unexpected that, at room temperature, most of these electrons have acquired this energy. Hence, most have left the group V atom, with its net positive charge, behind and are free to move through the crystal. Since group V atoms donate electrons to the conduction band, they are known as *donors*. A more quantitative idea of the number of electrons that have obtained the small amount of energy required can be obtained by referring to Fig. 2.14. Note that the form of the Fermi-Dirac distribution function indicates that donor levels have only a small probability of being occupied.¹ This means that most electrons have left the donor site and are in the conduction band.

The total number of electrons in the conduction band and holes in the valence band in this case can be found by considering the condition for charge neutrality in the semiconductor:

$$p - n + N_D^+ = 0 \quad (2.22)$$

where p is the hole density in the valence band, n the density of conduction band electrons, and N_D^+ the density of ionized donors (i.e., positive charges left behind when the electron departs). The other important equation comes from Eq. (2.17):

$$np = n_i^2 \quad (2.23)$$

This relation is more general than for the case of pure semiconductors previously discussed. Equations (2.14), (2.15), and (2.22), in conjunction with the Fermi-Dirac distribution function, can be solved to give precise values for n , p , and N_D^+ under general conditions. However, for most cases of interest in this book, the approximate but much simpler method of solution outlined below will give results of more than adequate accuracy.

Since the vast majority of donors will be ionized, N_D^+ will be nearly equal to the total density of donors, N_D . From Eq. (2.22), n will be greater than p and, in fact, very much greater when N_D be-

¹The statistics governing the occupation of the donor level are actually slightly different from those governing the occupation of levels within the allowed band. Once a donor level is occupied by a single electron of either "spin," the effective positive charge on the central donor atom is neutralized and there is no attraction that would allow occupation by a second electron of opposite spin. This results in an expression for the probability of occupation which differs slightly from the Fermi-Dirac function. That difference is not important in this book.

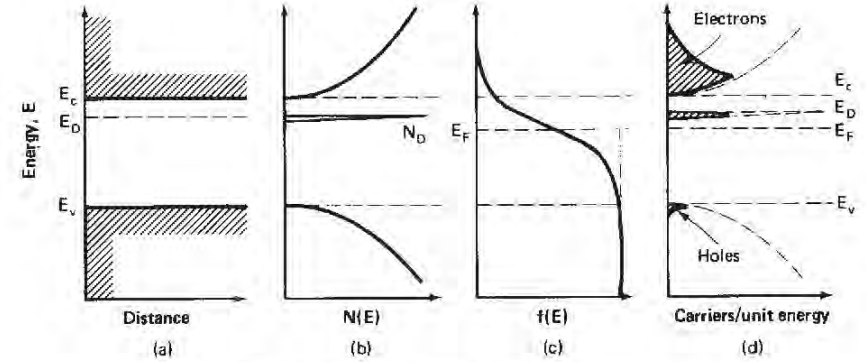


Figure 2.14. (a) Band representation of a group IV semiconductor with a group V substitutional impurity of density N_D per unit volume. (b) Corresponding energy density of allowed states. (c) Probability of occupation of these states. (d) Resulting energy distributions of electrons and holes. (The case shown would correspond to quite high temperatures. At more moderate temperatures the probability of occupation of donor states by electrons would be even smaller than shown.)

comes large. Hence, the approximate solution is

$$\begin{aligned} N_D^+ &\approx N_D \\ n &\approx N_D \\ p &\approx \frac{n_i^2}{N_D} \ll n \end{aligned} \quad (2.24)$$

An analogous situation occurs with group III impurities. These very easily give up their excess hole to the valence band or equivalently accept an electron from this band. Consequently, they are known as *acceptors*. An ionized acceptor has a net negative charge. Hence,

$$p - n - N_A^- = 0 \quad (2.25)$$

where N_A^- is the density of ionized acceptors.

The approximate solution in this case is

$$\begin{aligned} N_A^- &\approx N_A \\ p &\approx N_A \\ n &\approx \frac{n_i^2}{N_A} \ll p \end{aligned} \quad (2.26)$$

2.12 LOCATION OF FERMİ LEVEL IN DOPED SEMICONDUCTORS

The equations for electron and hole densities derived in Eqs. (2.14) and (2.15) apply to more general cases than that for pure semiconductors. For the case of material doped with donors (commonly called *n-type material*), these become

$$n = N_D = N_C e^{(E_F - E_C)/kT} \quad (2.27)$$

or, equivalently,

$$E_F - E_C = kT \ln \left(\frac{N_D}{N_C} \right) \quad (2.28)$$

Similarly, for material doped with acceptors (*p-type material*),

$$p = N_A = N_V e^{(E_V - E_F)/kT} \quad (2.29)$$

$$E_V - E_F = kT \ln \left(\frac{N_A}{N_V} \right) \quad (2.30)$$

As the semiconductor material becomes more heavily doped, the Fermi level E_F moves away from midgap and approaches the conduction band for *n*-type material or the valence band for *p*-type material, as shown in Fig. 2.15.

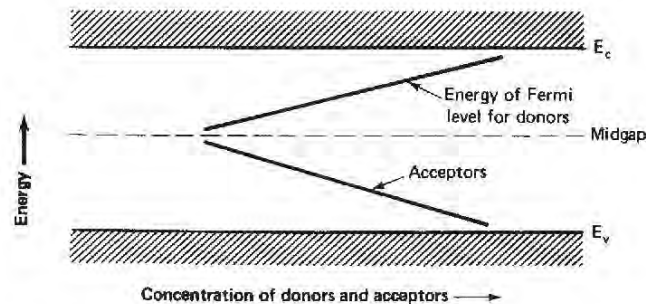
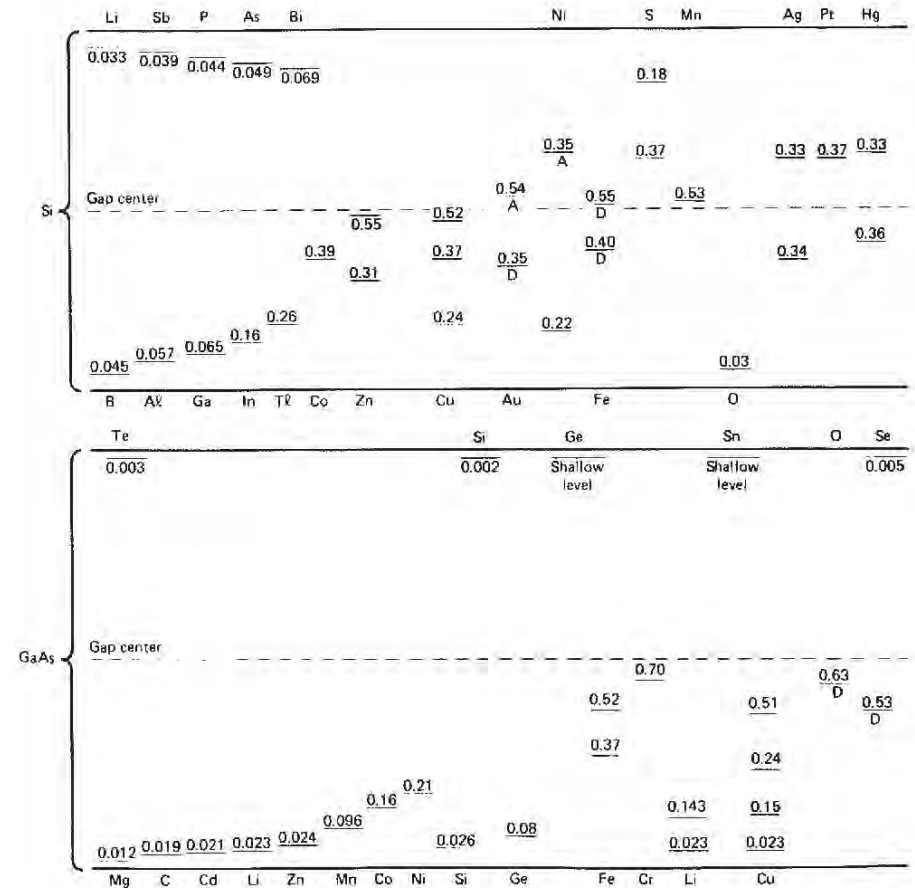


Figure 2.15. Energy of the Fermi level as a function of the concentration of donors and acceptors.

2.13 EFFECT OF OTHER TYPES OF IMPURITIES

Our theoretical understanding of the properties of impurities in silicon other than those from groups III and V is less developed, although the practical effects of such impurities are well known.

Just as group III and V impurities introduce an allowed energy level into the forbidden band gap of silicon, so do more general impurities. This is indicated in Fig. 2.16, which shows the al-



lowed energy levels introduced by a range of impurities in the silicon as well as the compound semiconductor GaAs. Some impurities introduce multiple energy levels, as shown. Crystal defects act in a similar way to introduce allowed levels into the forbidden gap.

Impurities, particularly those which introduce energy levels near the middle of the band gap, generally degrade the properties of semiconductor devices. Impurity concentrations in the starting material used in the fabrication of these devices are therefore kept as low as technology will allow—generally less than 1 part per billion.

2.14 CARRIER TRANSPORT

2.14.1 Drift

Under the influence of an applied electric field, ξ , a randomly moving free electron would have an acceleration $a = \xi/m$ in a direction opposite to the field, with its velocity in this direction increasing with time. The electron in a crystal structure is in a different situation. It moves with a different mass and will not continue accelerating for very long. It will eventually collide with a lattice atom, or an impurity atom, or a defect in the crystal structure. Such a collision will tend to randomize the electron's motion. In other words, it will tend to reduce the excess velocity that the electron picked up in the applied field. The "average" time between collisions is called the *relaxation time*, t_r . This will be determined by the random thermal velocity of electrons, which is generally much larger than field imparted velocities. The average velocity increase of electrons between collisions caused by the field is called the *drift velocity* and is given by

$$v_d = \frac{1}{2}at = \frac{1}{2} \frac{qt_r}{m_e^*} \xi \quad (2.31)$$

for electrons in the conduction band. (The factor of 2 disappears if t_r is averaged over all electron velocities.) The electron carrier mobility is defined by the ratio

$$\mu_e = \frac{v_d}{\xi} = \frac{qt_r}{m_e^*} \quad (2.32)$$

The corresponding current density flow due to conduction band electrons will be

$$J_e = qnv_d = q\mu_e n\xi \quad (2.33)$$

An analogous equation for holes in the valence band is

$$J_h = q\mu_h p\xi \quad (2.34)$$

The total current flow is just the sum of these two components. Hence, the conductivity, σ of the semiconductor can be identified as

$$\sigma = \frac{1}{\rho} = \frac{J}{\xi} = q\mu_e n + q\mu_h p \quad (2.35)$$

where ρ is the resistivity.

Although the analysis resulting in Eq. (2.32) is simplistic, it does allow an intuitive understanding of how the carrier mobilities, μ_n and μ_p , change with changes in the density of dopants, temperature, and electric field strength.

For relatively pure semiconductors of good crystallographic quality, the collisions that randomize the carrier velocities will involve the atoms of the host crystal. However, ionized dopants are very effective scatterers because of their associated net charge. Consequently, as the semiconductor becomes more heavily doped, the average time between collisions and hence the mobility will decrease. For good-quality silicon, empirical expressions relating the carrier mobilities to the level of dopants N (in cm^{-3}) are (Ref. 2.5)

$$\begin{aligned} \mu_e &= 65 + \frac{1265}{1 + (N/8.5 \times 10^{16})^{0.72}} \text{ cm}^2/\text{V-s} \\ \mu_h &= 47.7 + \frac{447.3}{1 + (N/6.3 \times 10^{16})^{0.76}} \text{ cm}^2/\text{V-s} \end{aligned} \quad (2.36)$$

Less specialized impurities as well as crystal defects will decrease mobilities further, for similar reasons.

Increasing temperature will increase the vibration of the host atoms, making them larger "targets," again decreasing the average time between collisions as well as the carrier mobilities. This effect becomes less pronounced at high doping levels, where ionized dopants are effective carrier scatterers.

Increasing the strength of the electric field will eventually increase the drift velocities of carriers to values where they will become comparable to the random thermal velocities. Hence, the total velocity of electrons will ultimately increase with field strength, decreasing the time between collisions and the mobility.

2.14.2 Diffusion

Apart from motion by drift, carriers in semiconductors can also flow by diffusion. It is a well-known physical effect that any excess concentration of particles such as gas molecules will tend to dissipate itself unless constrained. The basic cause of this effect is the random thermal velocity of the particles involved.

The flux of particles is proportional to the negative of the concentration gradient (Fig. 2.17). Since current is proportional to the flux of charged particles, the current density corresponding to a one-dimensional concentration gradient of electrons is

$$J_e = qD_e \frac{dn}{dx} \quad (2.37)$$

where D_e is a constant known as the diffusion constant. Similarly, for holes

$$J_h = -qD_h \frac{dp}{dx} \quad (2.38)$$

Note the sign difference between Eqs. (2.37) and (2.38), which is due to the opposite types of charges involved. Drift and diffusion processes are fundamentally related and the mobilities and diffusion constants are not independent. They are interconnected by the *Einstein relations*

$$D_e = \frac{kT}{q} \mu_e \quad \text{and} \quad D_h = \frac{kT}{q} \mu_h \quad (2.39)$$

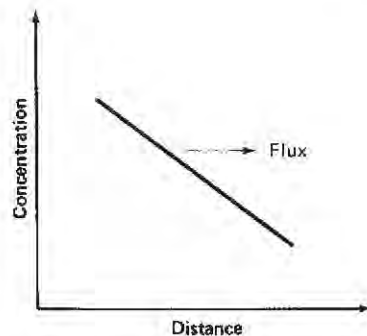


Figure 2.17. Diffusive flux of carriers in the presence of a concentration gradient.

kT/q is a parameter that will appear often in relation to solar cells. It has the dimensions of voltage and the value of 26 mV at room temperature, a value worth committing to memory!

2.15 SUMMARY

The major points arising from this chapter are the following. Semiconductors have an electronic structure such that one band of allowed states virtually completely occupied by electrons (the valence band) is separated by a forbidden energy gap from the next band of allowed states, which is virtually devoid of electrons (the conduction band). Current flow in semiconductors is due to both motion of electrons in the conduction band and the effective motion of vacancies or holes in the valence band. In many situations, electrons in the conduction band and holes in the valence band can be regarded as free particles, provided that an "effective" mass is used to include the effect of periodic forces of the host atoms in the crystal. Most conduction-band electrons have energies close to that of the conduction-band edge, whereas most holes have energies close to that of the valence-band edge.

Semiconductors can be divided into "direct"- and "indirect"-band-gap types, depending on the form of the relationship between the energy of electrons in the conduction band and their crystal momentum.

Specialized impurities known as dopants, when introduced into semiconductors, can control the relative concentrations of electrons in the conduction band of a semiconductor and holes in the valence band. Carriers in these bands can flow by drift and diffusion when the appropriate perturbations are present.

In Chapter 3, additional electronic processes occurring within semiconductors when disturbed by light are described. From the fundamental mechanisms discussed in this and the next chapter, a single system of self-consistent equations will be synthesized. This system will be used in later chapters to establish the principles of solar cell design.

EXERCISES

- 2.1. For a crystal with a cubic unit cell, indicate on a sketch of the cell the following crystal planes: (a) (100); (b) (010); (c) (110); (d) (111).
- 2.2. (a) Silicon solar cell performance can be improved by selectively etch-

ing the cell surface to reduce reflection losses. Figure 7.6 shows a silicon crystal surface originally orientated parallel to the (100) plane which has been attacked by a chemical etch that etches at different rates in different directions through the crystal. This exposes the square-based pyramids shown. Given that the sides of the pyramids are all members of the {111} equivalent set of planes, find the angle between opposite faces of the pyramids.

- (b) A fraction R of light incident normally on the original silicon surface was reflected. Neglecting dependencies upon angle of incidence and wavelength, show that fraction reflected after the selective etch is reduced to *slightly less than* R^2 .
- 2.3. One method of introducing impurities into silicon in controlled quantities is a technique known as *ion implantation*. Ions of the desired impurity are accelerated to high velocity and directed at the silicon surface. If the ions impinged parallel to each of the crystal directions shown in Fig. 2.3(b)-(d), in which case would you expect the ions to penetrate the greatest distance into the silicon?
- 2.4. An allowed state for an electron in a semiconductor lies at an energy equal to 0.4 eV above the Fermi level. What probability has this state of being occupied by an electron under thermal equilibrium conditions at 300 K?
- 2.5. Assuming that the effective masses of electrons and holes are equal to the free electron mass, calculate the effective density of states in the conduction and valence bands for silicon at 300 K. Assuming a band gap of 1.1 eV, find the intrinsic concentration in silicon at this temperature.
- 2.6. (a) Silicon is uniformly doped with 10^{22} phosphorus atoms/m³. Assuming that all these donor impurities are ionized, estimate the concentration of electrons and holes in this material under thermal equilibrium at 300 K. Hence, calculate the energy of the Fermi level in this material below the conduction-band edge.
 (b) Given that the donor level for phosphorus lies 0.045 eV below the conduction-band edge, calculate the probability that this level is occupied by an electron and hence check on the assumption that all donors are ionized. (Use $N_C = 3 \times 10^{25} \text{ m}^{-3}$, $N_V = 10^{25} \text{ m}^{-3}$, and $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$.)
- 2.7. Using Eq. (2.36) for the electron and hole mobilities in silicon, estimate the resistivity of the silicon specimen of Exercise 2.6.
- 2.8. Estimate the average time between collisions with the host atoms for electrons in the conduction band of lightly doped silicon.
- 2.9. An electric field of 10^4 V/m is applied to a specimen of silicon at 300 K doped with 10^{22} donors/m³. Given that the thermal velocity is 10^5 m/s , compare the drift and thermal velocities for conduction-band electrons. At what value of the field strength will these be comparable?
- 2.10. In a section of silicon at 300 K, the field strength is zero and conduction band electrons have a concentration that varies from 10^{22} per m²

to 10^{21} per m² over a distance of $1 \mu\text{m}$. Assuming a linear variation of electrons, calculate the corresponding current density.

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Chapter 3

GENERATION, RECOMBINATION, AND THE BASIC EQUATIONS OF DEVICE PHYSICS

3.1 INTRODUCTION

In Chapters 1 and 2, the relevant properties of sunlight and semiconductors have been reviewed. In the present chapter, the interaction between these two basic components of a solar photovoltaic system is examined.

This leads to the concepts of generation and recombination of excess carriers within the semiconductor material and a description of the physical mechanisms involved. Finally, the material discussed in connection with semiconductor properties is combined into a basic set of equations capable of describing the ideal properties of most semiconductor devices, including solar cells.

3.2 INTERACTION OF LIGHT WITH SEMICONDUCTOR

Figure 3.1 shows a ray of monochromatic light incident perpendicularly to a flat section of semiconductor. A certain fraction of the

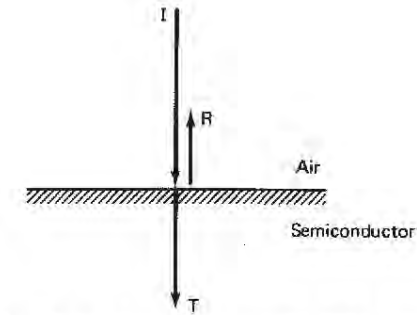


Figure 3.1. Ray of monochromatic light incident on a semiconductor.

incident power, R , will be reflected and the remainder, T , transmitted into the semiconductor.

The transmitted light can be absorbed within the semiconductor by using its energy to excite electrons from occupied low-energy states to unoccupied higher-energy states. Since there are a large number of occupied states within the valence band of a semiconductor separated by the forbidden band from largely unoccupied states in the conduction band, absorption is particularly likely when the energy of the photons making up the light is larger than the forbidden band gap, E_g , of the semiconductor.

An absorbing material has an *index of refraction*, $\hat{n}_c$, which is a complex number. This index can be written as $\hat{n}_c = \hat{n} - i\hat{k}$, where $\hat{k}$ is known as the *extinction coefficient*. The two components of this complex number for silicon are shown in Fig. 3.2 as a function of the wavelength of the incident light. The fraction of light reflected for normal incidence is given by (Refs. 3.1 and 3.2)

$$R = \frac{(\hat{n} - 1)^2 + \hat{k}^2}{(\hat{n} + 1)^2 + \hat{k}^2} \quad (3.1)$$

Substituting the appropriate values for silicon shows that for all wavelengths of interest in solar cell work, over 30% of the incident light is reflected. This is clearly undesirable from the point of view of making efficient solar cells. Antireflection coatings and other techniques (see Exercise 2.2) are used in solar cells to reduce this figure as much as possible.

The transmitted light is attenuated as it passes through the semiconductor. The rate of absorption of light is proportional to the intensity (the flux of photons) for a given wavelength. This common

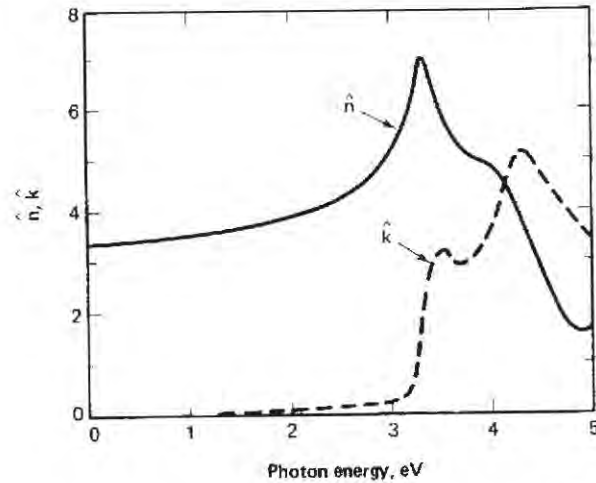


Figure 3.2. Real and (negative) imaginary parts of the refractive index for silicon. [After H. R. Phillip and E. A. Taft, *Physical Review* 120 (1960), 37-38.]

physical occurrence leads to an exponential decay in intensity of monochromatic light as it passes through the semiconductor, described mathematically as

$$I(x) = I(x_0) e^{-\alpha(x-x_0)} \quad (3.2)$$

where α is a function of wavelength and is known as the *absorption coefficient*. This parameter is important in solar cell design because it determines how far below the surface of the cell light of a given wavelength is absorbed.

The absorption coefficient α and the extinction coefficient $\hat{k}$ are not unrelated. Describing the light by a plane wave of frequency f propagating in the x direction with velocity v , the associated electric field strength is (Ref. 3.2)

$$\xi = \xi_0 \exp \left[i 2 \pi f \left(t - \left(\frac{x}{v} \right) \right) \right] \quad (3.3)$$

The velocity in the semiconductor is related to the velocity in vacuum, c , by

$$v = \frac{c}{\hat{n}_c} \quad (3.4)$$

Hence,

$$\frac{1}{v} = \frac{\hat{n}}{c} - \frac{i \hat{k}}{c} \quad (3.5)$$

Substituting into Eq. (3.3) gives

$$\xi = \xi_0 \exp(i 2 \pi f t) \exp \left(-\frac{i 2 \pi x \hat{n}}{c} \right) \exp \left(-\frac{2 \pi f \hat{k} x}{c} \right) \quad (3.6)$$

The last term in Eq. (3.6) is an attenuation factor. Power will attenuate as the square of the electric field strength. Comparing Eqs. (3.2) and (3.6) gives the relationship

$$\alpha = \frac{4 \pi f \hat{k}}{c} \quad (3.7)$$

3.3 ABSORPTION OF LIGHT

3.3.1 Direct-Band-Gap Semiconductor

Fundamental absorption refers to the annihilation or absorption of photons by the excitation of an electron from the valence band up into the conduction band, leaving a hole in the valence band. Both energy and momentum must be conserved in such a transition. A photon has quite a large energy (hf) but a small momentum (h/λ).

The form of the absorption process for a direct-band-gap semiconductor is shown in the energy-momentum sketch of Fig. 3.3. Because the photon momentum is small compared to the crystal momentum, the latter essentially is conserved in the transition. The energy difference between the initial and final state is equal to the energy of the original photon:

$$E_f - E_i = hf \quad (3.8)$$

In terms of the parabolic bands described in Chapter 2,

$$\begin{aligned} E_f - E_c &= \frac{p^2}{2m_e^*} \\ E_i - E_v &= \frac{p^2}{2m_h^*} \end{aligned} \quad (3.9)$$

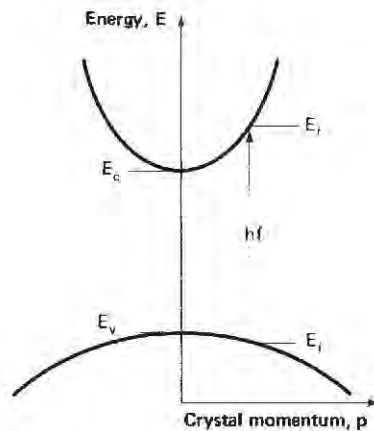


Figure 3.3. Energy-crystal momentum diagram of a direct-band-gap semiconductor, showing the absorption of a photon by the excitation of an electron from the valence to the conduction band.

Therefore, the specific value of crystal momentum at which the transition occurs is given by

$$hf - E_g = \frac{p^2}{2} \left(\frac{1}{m_e^*} + \frac{1}{m_h^*} \right) \quad (3.10)$$

As the photon energy hf increases, so does the value of crystal momentum at which the transition occurs (Fig. 3.3). The energy away from the band edge of both the initial and final states also increases. The probability of absorption depends on the density of electrons at the energy corresponding to the initial state as well as the density of empty states at the final energy. Since both these quantities increase with energy away from the band edge, it is not surprising that the absorption coefficient increases rapidly with increasing photon energy above E_g . A simple theoretical treatment gives the result (Ref. 3.2)

$$\alpha(hf) \approx A^* (hf - E_g)^{1/2} \quad (3.11)$$

where A^* is a constant having the numerical value of 2×10^4 when α is expressed in cm^{-1} and hf and E_g are in electron volts (eV). A comparison of this form of expression and experimental results for GaAs, a direct-band-gap semiconductor, is shown in Fig. 3.4. There is reasonable agreement at the higher values of the absorption coefficient.

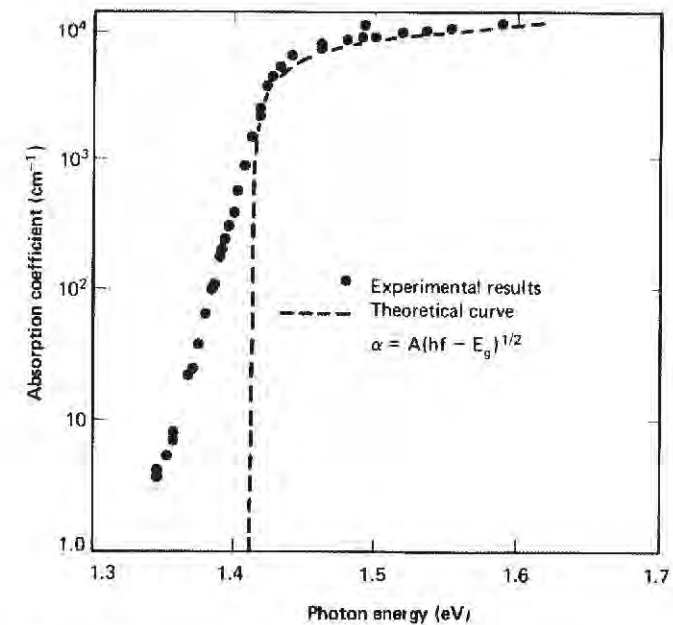


Figure 3.4. Absorption coefficient of GaAs as a function of photon energy. [After T. S. Moss and T. D. F. Hawkins, *Infrared Physics* 1, (1961), 111.]

Since the light intensity drops to $1/e$ of its value in passing a distance $1/\alpha$ through the semiconductor, Eq. (3.11) shows that sunlight of photon energy greater than E_g is absorbed within the first few microns of entering a direct-band-gap semiconductor.

3.3.2 Indirect-Band-Gap Semiconductor

In the case of an indirect-band-gap semiconductor, the minimum energy in the conduction band and the maximum energy in the valence band occur at different values of crystal momentum (Fig. 3.5). Photon energies much larger than the forbidden gap are required to give direct transitions of electrons from the valence to the conduction band, the process described in Section 3.3.1.

However, transitions can occur at lower energies by a two-step process involving not only photons and electrons but also a third particle, a *phonon*. In the same way as light can be thought of as either waves or particles, so can the coordinated vibration of the atoms making up the crystal structure. A phonon is just a *quantum* or fundamental particle corresponding to the coordinated vibration.

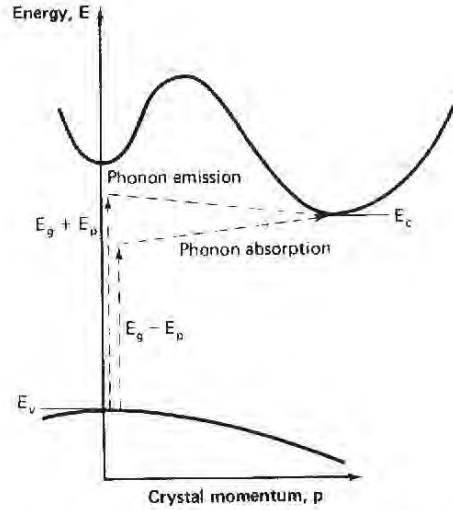


Figure 3.5. Energy-crystal momentum diagram of an indirect-band-gap semiconductor, showing the absorption of photons by two step processes involving phonon emission or absorption.

As opposed to photons, phonons have low energy but relatively high momentum.

This difference is explained by noting the relationship between phonons and sound, which propagates through a solid by coordinated atomic vibrations. The large difference between the velocity of light and sound in a solid is associated with the difference between the ratio of energy and momentum for the corresponding fundamental particles.

As indicated in the energy-momentum sketch of Fig. 3.5, an electron can make a transition from the maximum energy in the valence band to the minimum energy in the conduction band in the presence of photons of suitable energy by the emission or absorption of a phonon of the required momentum. Hence, the minimum photon energy required to excite an electron from the valence to the conduction band is

$$hf = E_g - E_p \quad (3.12)$$

where E_p is the energy of an absorbed phonon with the required momentum.

Since the indirect-band-gap absorption process requires that an extra "particle" be involved, the probability of light being absorbed by this process is much less than in the direct-band-gap case. Hence,

the absorption coefficient is low and light can pass a reasonable distance into the semiconductor prior to absorption. An analysis of the theoretical value of the absorption coefficient gives the results (Ref. 3.2)

$$\alpha_a(hf) = \frac{A(hf - E_g + E_p)^2}{\exp(E_p/kT) - 1} \quad (3.13)$$

for a transition involving phonon absorption, and

$$\alpha_e(hf) = \frac{A(hf - E_g - E_p)^2}{1 - \exp(-E_p/kT)} \quad (3.14)$$

for one involving phonon emission. Since both phonon emission and absorption are possible for $hf > E_g + E_p$, the absorption coefficient is then

$$\alpha(hf) = \alpha_a(hf) + \alpha_e(hf) \quad (3.15)$$

The absorption coefficient for silicon is shown in Fig. 3.6 as a function of the wavelength of incident light at different temperatures. The weak absorption region at wavelengths greater than $0.5 \mu\text{m}$ corresponds to indirect-band-gap processes. Below $0.4 \mu\text{m}$, the absorption coefficient increases very rapidly in a manner that could be attributed to direct gap absorption. An empirical expression involving terms of the form of Eqs. (3.11), (3.13), and (3.14) has been found to describe these experimental results to a high degree of accuracy over the photon energy range 1.1 to 4.0 eV and for the temperature range 20 to 500 K. This takes the form (Ref. 3.3)

$$\alpha(hf, T) = \sum_{\substack{i=1,2 \\ j=1,2}} A_{ij} \left\{ \frac{[hf - E_{gj}(T) + E_{pi}]^2}{\exp(E_{pi}/kT) - 1} + \frac{[hf - E_{gj}(T) - E_{pi}]^2}{1 - \exp(-E_{pi}/kT)} \right\} + A_d [hf - E_{gd}(T)]^{1/2} \quad (3.16)$$

where the values of the constants A_{ij} , E_{gj} , and E_{pi} are as given in Table 3.1.

3.3.3 Other Absorption Processes

Light absorption within a semiconductor is by no means limited to the processes discussed so far. It has already been indicated

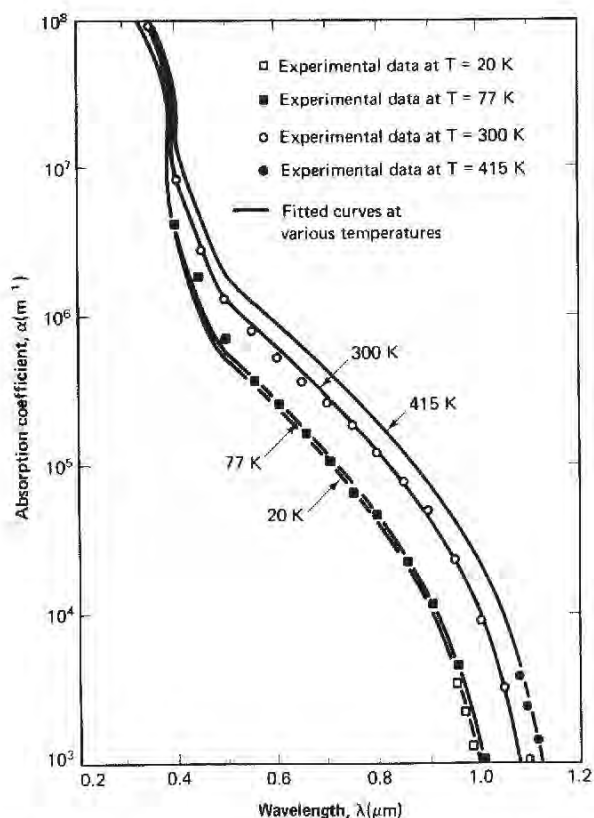


Figure 3.6. Absorption coefficient of silicon as a function of wavelength of light at different temperatures. (After Ref. 3.3.)

that absorption can occur at high-enough photon energy by excitation across the direct forbidden gap of an indirect-band-gap semiconductor such as silicon. Similarly, two-step absorption involving emission or absorption of phonons can also occur in direct-band-gap semiconductors, as indicated in Fig. 3.7(a). This occurs in parallel with the much stronger direct absorption process discussed in Section 3.3.1.

Similarly, a carrier can be excited to a higher energy in its respective band by a photon with the emission or absorption of a phonon as in Fig. 3.7(b). This process is relatively weak but, as might be expected, is strongest at long wavelengths when carrier concentrations are large. Although not important in solar cell work, it does

Table 3.1. VALUE OF CONSTANTS FOR THE EMPIRICAL EXPRESSION FOR THE ABSORPTION COEFFICIENT OF SILICON

Quantity	Numerical value
$E_{g1}(0)^*$	1.1557 eV
$E_{g2}(0)^*$	2.5 eV
$E_{gd}(0)^*$	3.2 eV
E_{p1}	1.827×10^{-2} eV
E_{p2}	5.773×10^{-2} eV
A_{11}	$1.777 \times 10^3 \text{ cm}^{-1}/\text{eV}^2$
A_{12}	$3.980 \times 10^4 \text{ cm}^{-1}/\text{eV}^2$
A_{21}	$1.292 \times 10^3 \text{ cm}^{-1}/\text{eV}^2$
A_{22}	$2.895 \times 10^4 \text{ cm}^{-1}/\text{eV}^2$
A_d	$1.052 \times 10^6 \text{ cm}^{-1}/\text{eV}^{1/2}$

* $E_g(T) = E_g(0) - [\beta T^2/(T + \gamma)]$ with $\beta = 7.021 \times 10^{-4}$ eV/K and $\gamma = 1108$ K.
Source: Ref. 3.3.

demonstrate a case where absorption occurs without the generation of an electron-hole pair.

Light absorption can also occur by the excitation of transitions between the allowed bands in a semiconductor and energy levels introduced into the forbidden gap by impurities as shown in Fig. 3.8.

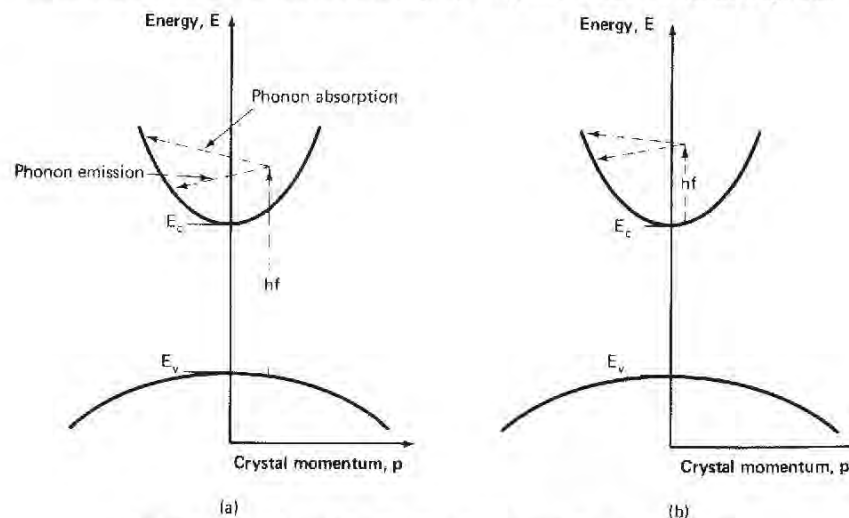


Figure 3.7. (a) Two-step photon absorption processes in direct-band-gap semiconductor. (b) Free carrier absorption in the conduction band which does not result in the generation of electron-hole pairs.

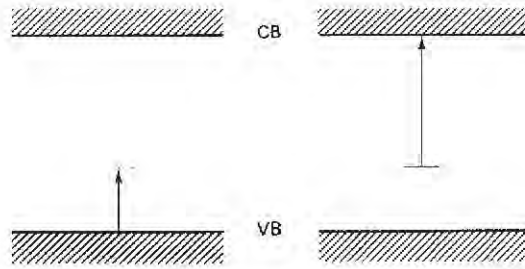


Figure 3.8. Light absorption by carrier excitation between bands and energy levels within the forbidden gap.

Finally, two processes will be briefly mentioned which may produce second-order effects in solar cells. The Franz-Keldysh effect (Ref. 3.2) arises in the presence of a strong electric field such as that which occurs in some regions of solar cells. This effect shifts the absorption edge to lower energies, having the same effect as a reduction of the width of the forbidden band gap. High-dopant-impurity concentrations also affect the absorption edge. A reduction in the width of the forbidden gap again occurs in the presence of such concentrations.

3.4 RECOMBINATION PROCESSES

3.4.1 Relaxation to Equilibrium

Light of appropriate wavelength shining on a semiconductor creates electron-hole pairs. The concentrations of carriers in illuminated material will therefore be in excess of their values in the dark. If the light is switched off, these concentrations decay back to their equilibrium values. The process by which this decay occurs is known as *recombination*. Three different recombination mechanisms will be described in the following sections. These mechanisms can occur in parallel, in which case the recombination rate is just the sum of those for the individual processes.

3.4.2 Radiative Recombination

Radiative recombination is just the reverse of the absorption process described in Section 3.3. An electron occupying a higher-energy state than it would under thermal equilibrium makes a tran-

sition to an empty lower-energy state with all (or most) of the energy difference between states emitted as light. All the mechanisms considered for absorption have inverse radiative recombination processes (Fig. 3.9). Radiative recombination occurs more rapidly in direct-band-gap semiconductors than in indirect types because a two-step process involving a phonon is required for the latter.

The total radiative recombination rate, R_R , is proportional to the product of the concentration of occupied states (electrons) in the conduction band and that of unoccupied states in the valence band (holes):

$$R_R = Bnp \quad (3.17)$$

where B is a constant for a given semiconductor. Because of the relationship between optical absorption and this recombination process, B can be calculated from the semiconductor's absorption coefficient (Ref. 3.2).

In thermal equilibrium when $np = n_i^2$, this recombination rate is balanced by an equal and opposite generation rate. In the absence of generation by external stimuli, the net recombination rate corresponding to Eq. (3.17), U_R , is given by the total recombination rate minus the equilibrium generation rate:

$$U_R = B(np - n_i^2) \quad (3.18)$$

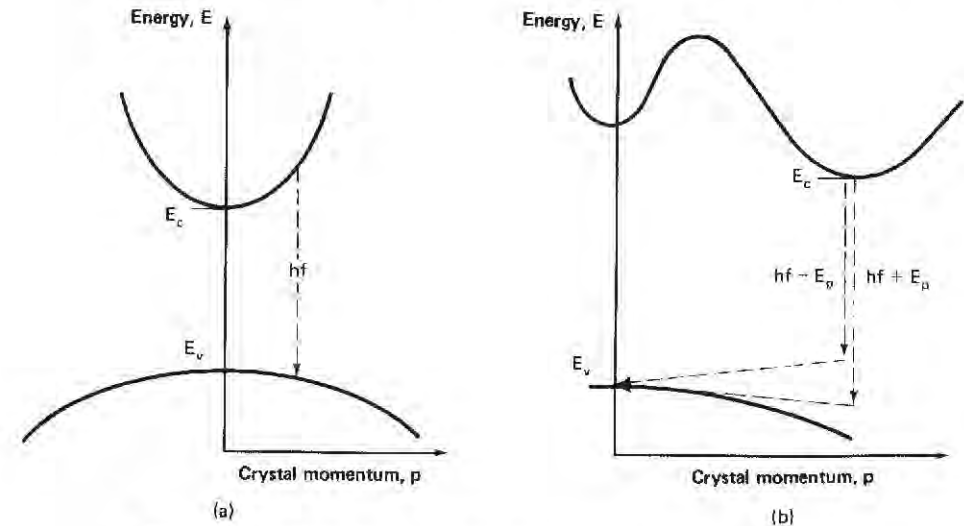


Figure 3.9. Radiative recombination in semiconductors:

- (a) Direct band gap.
- (b) Indirect band gap.

With any recombination mechanism, it is possible to define associated *carrier lifetimes*, τ_e and τ_h , for electrons and holes:

$$\begin{aligned}\tau_e &= \frac{\Delta n}{U} \\ \tau_h &= \frac{\Delta p}{U}\end{aligned}\quad (3.19)$$

where U is the net recombination rate and Δn and Δp are the disturbances of the respective carriers from their equilibrium values, n_0 and p_0 .

For the radiative recombination mechanism with $\Delta n = \Delta p$, the characteristic lifetime determined from Eq. (3.18) is (Ref. 3.2)

$$\tau = \frac{n_0 p_0}{B n_i^2 (n_0 + p_0)} \quad (3.20)$$

For silicon, B has the value of about $2 \times 10^{-15} \text{ cm}^3/\text{s}$ (Ref. 3.2).

As might be expected, radiative recombination lifetimes are much smaller in direct-band-gap material than in indirect material. This process forms the basis of commercial semiconductor lasers and light-emitting diodes using GaAs and its alloys. For silicon, other recombination mechanisms are far more important.

3.4.3 Auger Recombination

In the Auger (pronounced something like “oh-shay”) effect, the electron recombining with the hole gives the excess energy to a second electron (either in the conduction or valence band) instead of emitting light. This process is illustrated in Fig. 3.10. This second electron then relaxes back to its original energy by emitting phonons. Auger recombination is just the reverse process of the more familiar impact ionization effect, where a high-energy electron collides with an atom, breaking a bond and creating an electron-hole pair. The characteristic lifetime τ associated with the Auger process is (Ref. 3.2)

$$\frac{1}{\tau} = Cnp + Dn^2 \quad (3.21)$$

or

$$\frac{1}{\tau} = Cnp + Dp^2$$

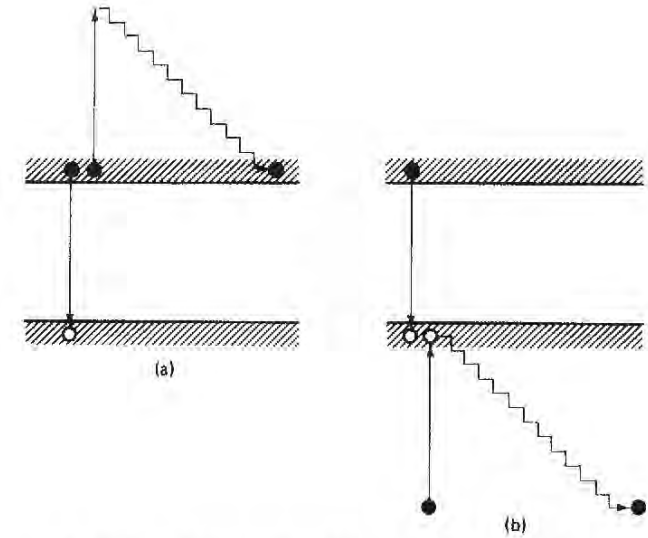


Figure 3.10. Auger recombination with associated excess energy given to an electron in:

- (a) Conduction band.
- (b) Valence band.

for materials with an abundance of electrons and holes, respectively. The first term on the right in each case describes electron excitation in the minority carrier band, and the second describes it in the majority carrier band. Auger recombination is particularly effective in relatively highly doped material due to this second term. For good-quality silicon, this is the dominant recombination process for impurity levels greater than 10^{17} cm^{-3} . The experimental results for the variation of lifetime with increasing dopant density for high-quality silicon shown in Fig. 3.11 indicate the rapid decrease at high dopant density due to Auger processes.

3.4.4 Recombination through Traps

It has been indicated in Chapter 2 that impurities and defects in semiconductors can give rise to allowed energy levels within the otherwise forbidden gap. These defect levels create a very efficient two-step recombination process whereby electrons relax from conduction-band energies to the defect level and then relax to the valence band, annihilating a hole as shown in Fig. 3.12(a).

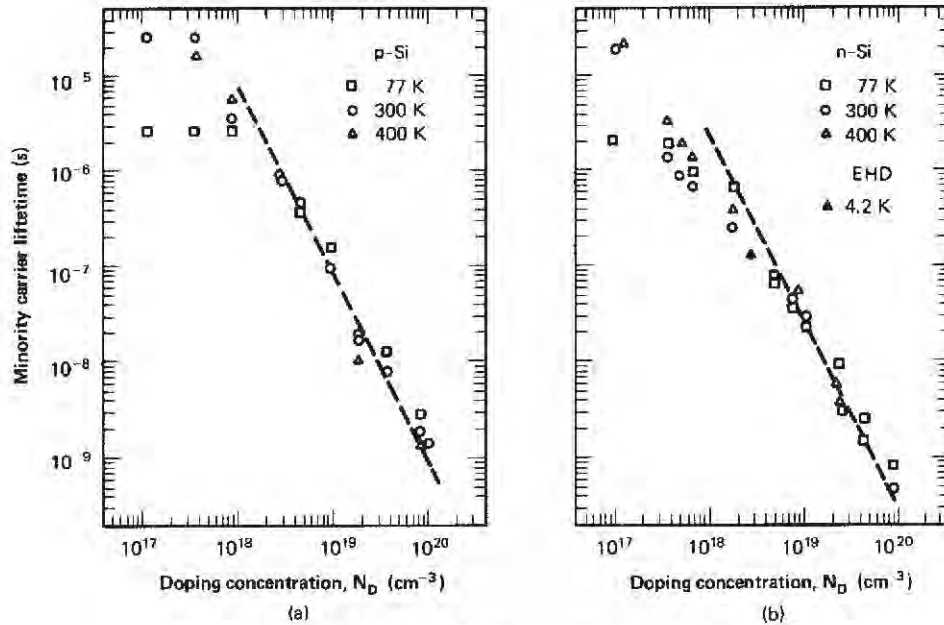


Figure 3.11. Experimental recombination lifetimes in high-quality silicon. The dashed lines represent the quadratic dependence predicted theoretically.

(a) *p*-type silicon

(b) *n*-type silicon

[After J. Dziewior and W. Schmid, *Applied Physics Letters* 31 (1977), 346-348.]

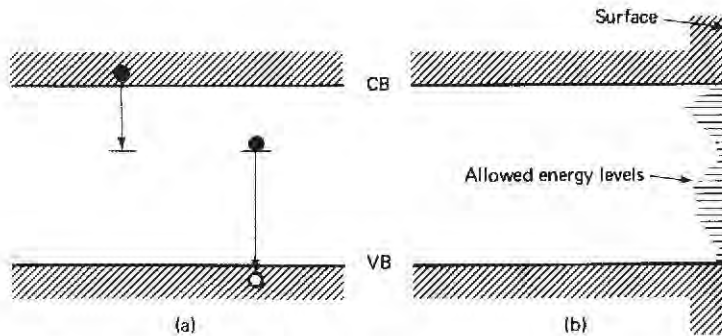


Figure 3.12. (a) Two-step recombination process via a trapping level within the forbidden gap of a semiconductor. (b) Surface states lying within the forbidden gap at the surface of a semiconductor.

The analysis of the dynamics of this process is straightforward but lengthy (Ref. 3.4). The result is that the net recombination-generation rate by traps, U_T , can be written

$$U_T = \frac{np - n_i^2}{\tau_{h0}(n + n_1) + \tau_{e0}(p + p_1)} \quad (3.22)$$

where τ_{h0} and τ_{e0} are lifetime parameters whose values depend on the type of trap and the volume density of trapping defects. n_1 and p_1 are parameters arising from the analysis which introduce a dependency of the recombination rate upon the energy of the trapping level, E_t :

$$n_1 = N_C \exp\left(\frac{E_t - E_c}{kT}\right) \quad (3.23)$$

$$n_1 p_1 = n_i^2 \quad (3.24)$$

Equation (3.23) is very similar in form to the expression for the electron concentration in terms of the Fermi-level energy as given by Eqs. (2.14) and (2.15). If τ_{e0} and τ_{h0} are of the same order of magnitude, it is not difficult to show that U will have its peak value when $n_1 \approx p_1$. This occurs if the defect level lies near the middle of the forbidden band gap. Therefore, impurities that introduce energy levels near midgap are very effective recombination centers.

3.4.5 Recombination at Surfaces

Surfaces represent rather severe defects in the crystal structure and are the site of many allowed states within the forbidden gap, as indicated in Fig. 3.12(b). Recombination can therefore occur very efficiently at surfaces by the mechanism described in 3.4.4. The net recombination rate per unit area, U_A , for a single-level surface state takes a form similar to Eq. (3.22):

$$U_A = \frac{S_{e0} S_{h0} (np - n_i^2)}{S_{e0}(n + n_1) + S_{h0}(p + p_1)} \quad (3.25)$$

where S_{e0} and S_{h0} are surface-recombination velocities. Again, surface-state levels lying near midgap are the most effective recombination centers.

3.5 BASIC EQUATIONS OF SEMICONDUCTOR-DEVICE PHYSICS

3.5.1 Introduction

In previous sections, the relevant properties of semiconductors have been reviewed. This material will now be consolidated by synthesizing from it a set of basic equations capable of describing the operation of semiconductor devices. Solution of these equations allows the ideal characteristics of most semiconductor devices, including solar cells, to be determined. The equations will be written in one-dimensional form, with variations in the other two spatial dimensions neglected. Their three-dimensional form is similar except that spatial derivatives are replaced by the divergence operator for vector quantities (electric field, current density) and by the gradient operator for scalar quantities (concentrations, potentials).

3.5.2 Poisson's Equation

The first equation in the system may be familiar from electrostatics. It is Poisson's equation, one of Maxwell's equations (Ref. 3.5) which relates the divergence of the electric field to the space charge density, ρ . In one dimension, it takes the form

$$\frac{d\xi}{dx} = \frac{\rho}{\epsilon} \quad (3.26)$$

where ϵ is the material's permittivity. This equation is a differentiated form of Gauss's law, which may be more familiar.

Looking at the contributors to charge density in a semiconductor, electrons in the conduction band contribute a negative charge whereas holes give a positive charge. A donor impurity that is ionized (i.e., has its extra electron removed) has a net positive charge due to the unneutralized extra positive charge at the nucleus. Similarly, an ionized acceptor contributes a negative charge. Hence,

$$\rho = q(p - n + N_D^+ - N_A^-) \quad (3.27)$$

where p and n are the densities of holes and electrons, and N_D^+ and N_A^- are the densities of ionized donors and acceptors. Less specialized impurities and defects can also act as charge-storage centers, so corresponding terms should be included in Eq. (3.27). However, the volume density of such nonidealities is kept as small as possible in solar cell work, making the charge contributions relatively insignificant.

As mentioned in Chapter 2, most donors and acceptors are ionized under normal conditions, so that

$$\begin{aligned} N_D^+ &\approx N_D \\ N_A^- &\approx N_A \end{aligned} \quad (3.28)$$

where N_D and N_A are the *total* density of donors and acceptors.

3.5.3 Current-Density Equations

In Chapter 2, it was seen that electrons and holes could contribute to current flow by drift and diffusion processes. Hence, the expressions for the total current densities of electrons and holes, J_e and J_h , become

$$\begin{aligned} J_e &= q\mu_e n\xi + qD_e \frac{dn}{dx} \\ J_h &= q\mu_h p\xi - qD_h \frac{dp}{dx} \end{aligned} \quad (3.29)$$

The mobilities and diffusion constants are related through the Einstein relationships [$D_e = (kT/q)\mu_e$; $D_h = (kT/q)\mu_h$].

3.5.4 Continuity Equations

The final equations in the system are "bookkeeping"-type equations which merely keep track of the number of electrons and holes in a system and ensure that none materialize "out of thin air."

Referring to the elemental volume of length δx and cross-sectional area A of Fig. 3.13, it can be stated that the net rate of increase of electrons in this volume is the rate at which they enter minus the rate at which they exit plus the rate at which they are generated in this volume minus the rate at which they recombine.

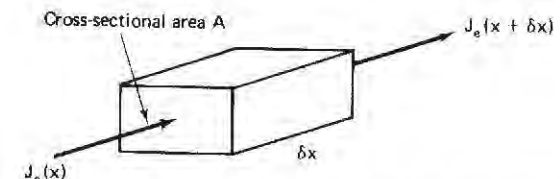


Figure 3.13. Elemental volume for deriving the continuity equations for electrons.

But the rates of entering and exiting are proportional to the current densities at the respective faces of the elemental volume. Hence,

$$\begin{aligned} \text{rate of entering—rate exiting} &= \frac{A}{q} \{-J_e(x) - [-J_e(x + \delta x)]\} \\ &= \frac{A}{q} \frac{dJ_e}{dx} \delta x \end{aligned} \quad (3.30)$$

$$\begin{aligned} \text{rate of generation—rate of recombination} \\ &= A \delta x (G - U) \end{aligned} \quad (3.31)$$

where G is the net generation rate by external processes such as illumination by light, and U is the net recombination rate. Under steady-state conditions the net rate of increase must be zero, so that

$$\frac{1}{q} \frac{dJ_e}{dx} = U - G \quad (3.32)$$

Similarly, for holes

$$\frac{1}{q} \frac{dJ_h}{dx} = -(U - G) \quad (3.33)$$

3.5.5 Equation Set

The set of basic equations is

$$\begin{aligned} \frac{d\xi}{dx} &= \frac{q}{\epsilon} (p - n + N_D - N_A) \\ J_e &= q\mu_e n \xi + qD_e \frac{dn}{dx} \\ J_h &= q\mu_h p \xi - qD_h \frac{dp}{dx} \\ \frac{1}{q} \frac{dJ_e}{dx} &= U - G \\ \frac{1}{q} \frac{dJ_h}{dx} &= -(U - G) \end{aligned} \quad (3.34)$$

Subsidiary relations are also required for U and G . Expressions for these terms depend on the specific processes involved.

The Equations (3.34) form a coupled set of nonlinear differential equations for which it is not possible to find general analytical solutions. They can be solved numerically on a digital computer to give ideal properties for a range of possible semiconductor-device configurations. Examples of this technique applied to solar cells are contained in Refs. 3.6 to 3.8. It is also possible to get good solutions to these equations much more simply and with more insight into the physical principles involved by making a series of well-thought-out approximations. This technique is demonstrated in Chapter 4.

3.6 SUMMARY

Light made up of photons of energy larger than the forbidden band gap can be absorbed in a semiconductor by creating electron-hole pairs. In direct-band-gap semiconductor, the light is absorbed quickly. In indirect-band-gap materials, the emission or absorption of a phonon also is required for photon energies near that of the band gap. Hence, indirect-band-gap material absorbs weakly for such energies but becomes strongly absorbing for higher energies where direct transitions are also possible.

Recombination of carrier concentrations in excess of equilibrium values can occur through a variety of processes. Radiative recombination is the reverse of light absorption and is an important mechanism for direct-band-gap semiconductors. Auger recombination is important at high doping concentrations, whereas recombination through traps caused by impurities and defects is important for indirect-band-gap semiconductors and for those with an undeveloped supporting technology. These recombination processes occur in parallel. The total recombination rate is just the sum of the individual rates. The inverse of the net recombination lifetime is the sum of the inverses of the individual lifetimes. Recombination also occurs particularly effectively at semiconductor surfaces.

The end result of our review of the properties of semiconductors and the starting point for the analysis of the properties of solar cells is a system of coupled differential equations relating the spatial distributions of the quantities important in determining the internal operation of solar cells. Techniques for obtaining solutions to these equations are discussed in Chapter 4.

EXERCISES

- 3.1. Monochromatic light is normally incident on a flat silicon surface. Using the data of Fig. 3.1, calculate the fraction reflected at the following

wavelengths: (a) 1000 nm; (b) 400 nm; (c) 300 nm. [Note: A relationship worth remembering between photon energy (hf) and wavelength in vacuum is $\lambda \text{ (}\mu\text{m)} = 1.24/hf \text{ (eV).}$]

- 3.2. (a) Monochromatic light of photon flux N photons per unit area per second is incident on a semiconductor surface and a fraction R is reflected. If the absorption coefficient of the semiconductor at this wavelength is α , what is an expression for the photon flux as a function of the distance, x , it progresses through the semiconductor?
- (b) Given that each photon absorbed creates an electron-hole pair, calculate an expression in terms of the parameters above for the generation rate, G , of these pairs again as a function of distance through the semiconductor.
- 3.3. The peak photon flux in terrestrial sunlight arrives at wavelengths around 700 nm. Using the data of Figs. 3.4 and 3.6, compare the depth in Si and GaAs at which the photon flux at this wavelength has been reduced to 10% of its value immediately upon entering the respective semiconductors.
- 3.4. For a particular sample of semiconductor, the radiative recombination lifetime for minority carriers is calculated as 100 μs , the Auger recombination lifetime as 50 μs , and the lifetime for trapping processes as 10 μs . Assuming that no other process is available for recombination, what is the net lifetime in this material?
- 3.5. A sample of n -type silicon is illuminated by light so that the electron density is constant at 10^{22} per m^3 and the hole density is also constant at 10^{15} per m^3 . Determine the effectiveness of recombination by traps as a function of trap energy by calculating the recombination rate of electrons and holes for the case where the trap is assumed located at each of the following energies below the conduction-band edge: (a) 0.03 eV; (b) 0.3 eV; (c) 0.5 eV; (d) 0.8 eV; (e) 1.0 eV. Assume the density and capture cross sections of the trap in each case is such that τ_{e0} and τ_{h0} both equal 1 μs . Use the values $N_C = 3 \times 10^{25} \text{ m}^{-3}$, $n_i = 1.5 \times 10^{16} \text{ m}^{-3}$, and $hT/q = 26 \text{ mV}$.
- 3.6. In terms of the electronic properties of semiconductors, explain why the absorption coefficient increases with increasing photon energy for energies near the semiconductor band gap.

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Chapter 4

p-n JUNCTION DIODES

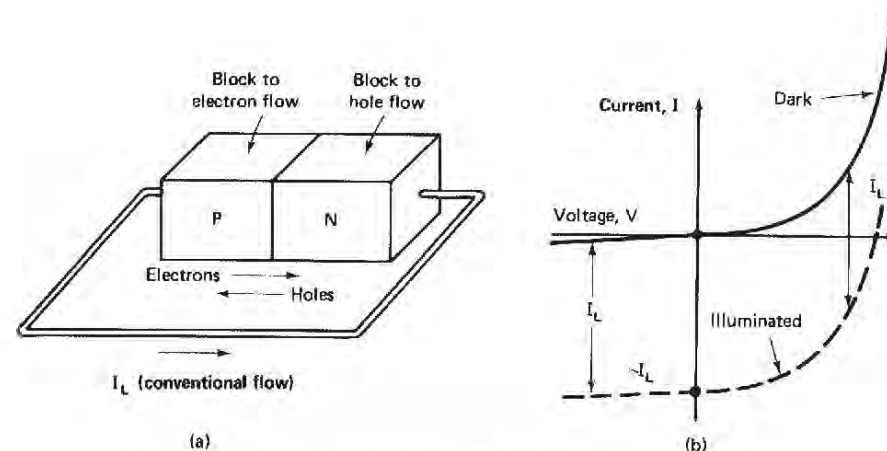


Figure 4.1. (a) Asymmetrical properties of a *p-n* junction diode. These cause a net current flow in an external lead connecting the *p*-type and *n*-type regions when the junction is illuminated. (b) This light-generated current is superimposed upon the normal rectifying current-voltage characteristics of the diode. This results in a region in the fourth quadrant where electrical power can be extracted from the device.

4.1 INTRODUCTION

Regions of semiconductors doped with donor impurities have an increased number of electrons in the conduction band at normal temperatures and are known as *n-type material*. Those doped with acceptors are known as *p-type*. The most common solar cells are essentially just very large area *p-n junction diodes*, where such a diode is formed by making a junction between *n*-type and *p*-type regions. In the present chapter, the basic properties of such a junction will be analyzed both when in the dark and when illuminated.

The basic device requirement for photovoltaic energy conversion is an electronic asymmetry in the semiconductor structure. Figure 4.1(a) demonstrates that a *p-n* junction has the required asymmetry. *n*-Type regions have large electron densities but small hole densities. Hence, electrons flow readily through such material but holes find it very difficult. Exactly the opposite is true for *p*-type material. When illuminated, excess electron-hole pairs are generated by light throughout the material. The inherent asymmetry in the

carrier transport properties encourages a flow of generated electrons from the *p*-type region to the *n*-type, and a flow of holes in the opposite direction. When the illuminated *p-n* junction is electrically shorted, a current will flow in the short-circuiting lead. It will be shown in this chapter that this light-generated current is superimposed upon the normal diode-rectifying characteristics to give an operating region where power can be extracted from the cell as indicated in Fig. 4.1(b).

4.2 ELECTROSTATICS OF *p-n* JUNCTIONS

Consider isolated pieces of *n*-type and *p*-type semiconducting material as shown in Fig. 4.2. If these are brought together in a conceptual experiment, it would be expected that electrons would flow from regions of high concentration (*n*-type side) to regions of low concentration (*p*-type side) and similarly for holes. However, electrons leaving the *n*-type side will create a charge imbalance in this side by exposing ionized donors (positive charge). Similarly, holes leaving the *p*-type side will expose negative charge. These exposed charges

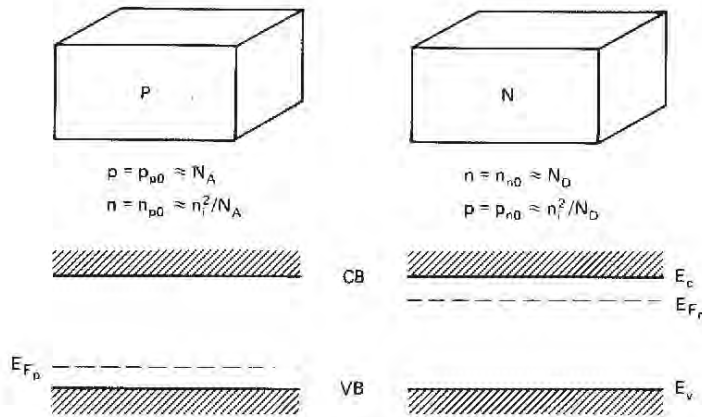


Figure 4.2. Isolated pieces of *p*-type and *n*-type semiconductor material with corresponding energy-band diagrams.

will set up an electric field that will oppose the natural diffusion tendency of the electrons and holes and an equilibrium situation will be obtained.

The characteristics of the equilibrium situation can be found by considering Fermi levels. *A system in thermal equilibrium can have only one Fermi level.*

Far enough away from the metallurgical junction, conditions could be expected to remain unperturbed from those in isolated material. Referring to Fig. 4.3, this means there must be a transition region near the junction in which a potential change, ψ_0 , occurs. The value of ψ_0 can be found from this figure:

$$q\psi_0 = E_g - E_1 - E_2 \quad (4.1)$$

The expressions for E_1 and E_2 were derived in Eqs. (2.28) and (2.30) and are also indicated in Fig. 4.3. Hence,

$$\begin{aligned}
 q\psi_0 &= E_g - kT \ln \left(\frac{N_V}{N_A} \right) - kT \ln \left(\frac{N_C}{N_D} \right) \\
 &= E_g - kT \ln \left(\frac{N_C N_V}{N_A N_D} \right)
 \end{aligned} \quad (4.2)$$

but, from Eq. (2.17),

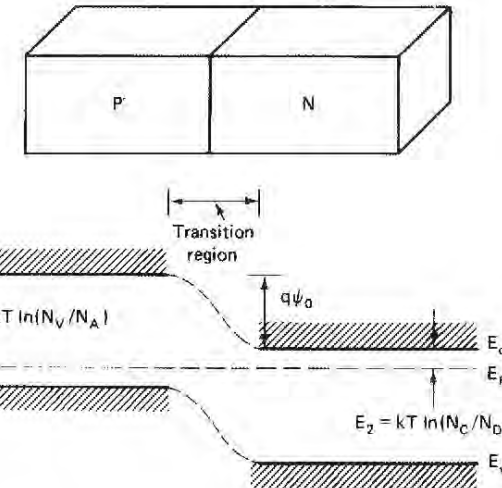


Figure 4.3. A *p-n* junction formed by bringing the isolated *p*-type and *n*-type regions together. Also shown is the corresponding energy-band diagram at thermal equilibrium.

$$n_i^2 = N_C N_V \exp \left(-\frac{E_g}{kT} \right)$$

Therefore,

$$\psi_0 = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right) \quad (4.3)$$

An applied voltage, V_a , will change the potential difference between the two sides of the diodes by V_a . Hence, the potential across the transition region will become $(\psi_0 - V_a)$.

It is instructive to plot the carrier concentrations corresponding to Fig. 4.3. These concentrations depend on the exponential of the energy difference between the Fermi level and the respective band. The resulting plot is shown in Fig. 4.4 on a logarithmic scale. A corresponding plot of the space-charge density, ρ , as given by Eq. (3.27), is shown as the dashed line of Fig. 4.5(a). The rapid change of ρ near the edge of the depletion region leads to *approximation 1*, the *depletion approximation*.

In this approximation, the device is divided into two types of regions: *quasi-neutral regions* where the space-charge density is as-

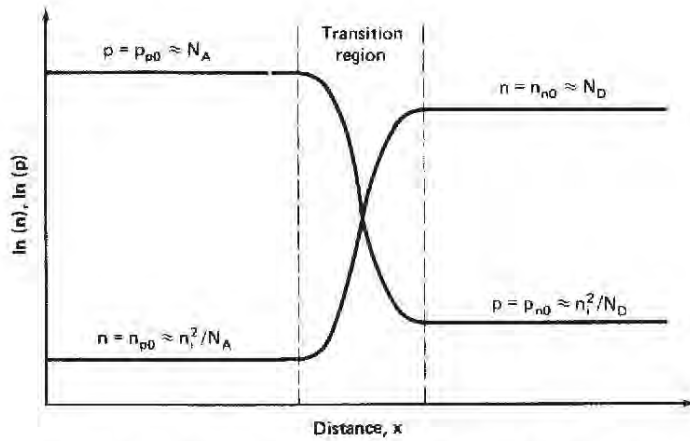


Figure 4.4. Plot of the natural logarithms of the electron and hole concentrations corresponding to Fig. 4.3. Since these concentrations depend exponentially on the energy between the Fermi level and the respective band, the shapes of the distributions on a logarithmic scale are linearly related to that of Fig. 4.3.

sumed zero throughout and a *depletion region* where the carrier concentrations are assumed so small that the only contribution to space-charge density comes from the ionized dopants. This approximation essentially just sharpens up the space-charge distribution, as indicated by the solid line in Fig. 4.5(a).

With this approximation, it is a simple matter to find the electric field and potential distribution across the depletion region, as shown in Fig. 4.5(b) and (c). The space-charge distribution is just integrated successively, remembering that electric field strength is the negative gradient of potential. The results for the maximum field strength in the depletion region, $\xi_{\max}$, the width of the depletion region, W , and the distance this region extends on either side of the junction, l_n and l_p , are (Ref. 4.1)

$$\begin{aligned}\xi_{\max} &= - \left[\frac{2q}{\epsilon} (\psi_0 - V_a) \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2} \\ W = l_n + l_p &= \left[\frac{2\epsilon}{q} (\psi_0 - V_a) \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \right]^{1/2} \quad (4.4) \\ l_p &= W \frac{N_D}{N_A + N_D} \quad l_n = W \frac{N_A}{N_A + N_D}\end{aligned}$$

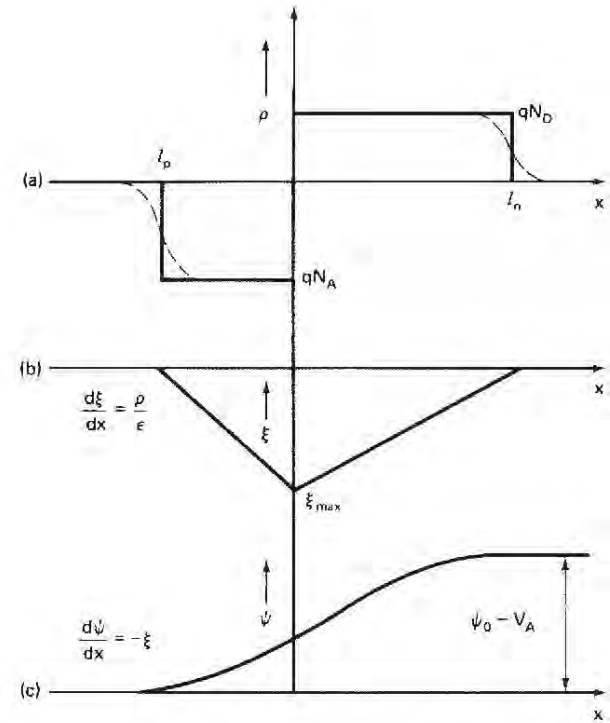


Figure 4.5. (a) Space-charge density corresponding to Fig. 4.4. The dashed line shows the actual distribution while the solid line shows the assumed distribution in the depletion approximation. (b) Corresponding electrical field strength. (c) Corresponding potential distribution.

4.3 JUNCTION CAPACITANCE

It is a very easy matter to detect the presence of a depletion region in a *p-n* diode and to measure its width. In the depletion approximation, a change in applied voltage will cause a change in stored charge right at the edges of the region, as indicated in Fig. 4.6. This is identical to the situation of a parallel-plate capacitor of plate separation, W . Hence, the depletion-region capacitance, C , is

$$C = \frac{\epsilon A}{W} \quad (4.5)$$

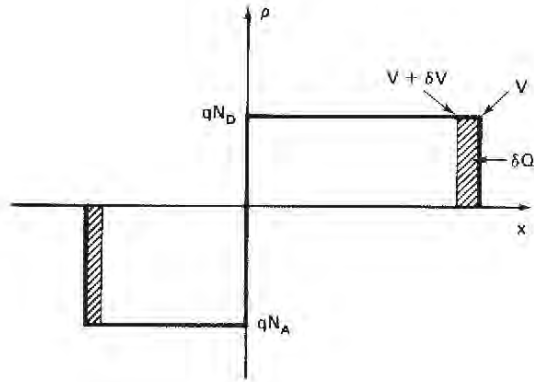


Figure 4.6. Change in the charge stored in the depletion region when the applied voltage is incrementally altered (depletion approximation).

where W is given by Eq. (4.4). If one side of the diode is heavily doped, Eq. (4.5) reduces to:

$$\frac{C}{A} = \left[\frac{q \epsilon N}{2(\psi_0 - V_a)} \right]^{1/2} \quad (4.6)$$

where N is the smaller of N_A and N_D . Under reverse bias, depletion-region capacitance dominates the total diode capacitance. Hence, measuring C as a function of reverse bias to a diode or solar cell and plotting $1/C^2$ versus V_a will allow N , the doping density on the lightly doped side of the diode, to be found. A similar technique can also be used (Ref. 4.2) to calculate the spatial variation of the dopant density in cases where this quantity is not constant.

4.4 CARRIER INJECTION

The next calculation involves finding the concentration of carriers at the edge of the depletion region as a function of bias. Referring to Fig. 4.7, values are sought for the concentrations n_{pa} and p_{nb} .

At zero bias, their values are already known (Fig. 4.4):

$$\begin{aligned} p_{nb} &= p_{n0} = p_{p0} \exp \left(-\frac{q\psi_0}{kT} \right) \approx \frac{n_i^2}{N_D} \\ n_{pa} &= n_{p0} = n_{n0} \exp \left(-\frac{q\psi_0}{kT} \right) \approx \frac{n_i^2}{N_A} \end{aligned} \quad (4.7)$$

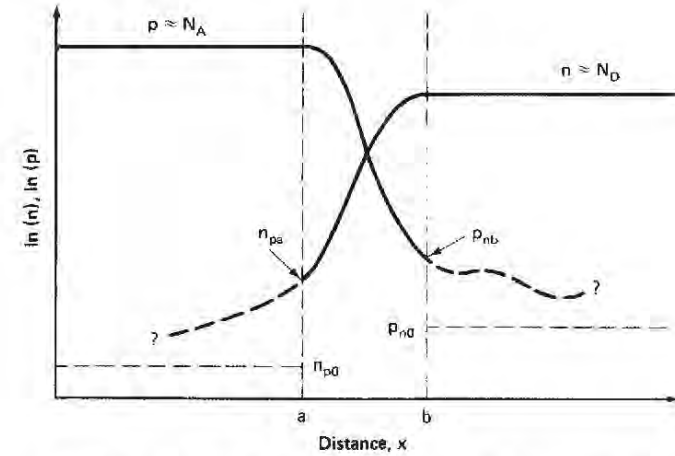


Figure 4.7. Plot of carrier concentrations when a voltage is applied to the p - n junction. In the text, expressions are found for the minority carrier concentrations n_{pa} and p_{nb} at the edge of the junction depletion region. Subsequently, the precise form of the distributions shown dashed are also calculated.

Within the depletion region, there are present both the highest electric field strengths and concentration gradients. The net current flow through such regions is actually the small difference between two large terms. For holes,

$$J_h = q\mu_h p \xi - qD_h \frac{dp}{dx} \quad (4.8)$$

Both the drift and diffusion terms are large but opposing. At zero bias they balance. At moderate bias points, the net current flow is a small difference between these two much larger terms. This leads to *approximation 2*—that, within depletion regions,

$$q\mu_h p \xi \approx qD_h \frac{dp}{dx} \quad (4.9)$$

In other words,

$$\xi \approx \frac{kT}{q} \frac{1}{p} \frac{dp}{dx} \quad (4.10)$$

making use of Einstein's relationship between μ_h and D_h . Integrating the negative of both sides of Eq. (4.10) across the depletion region gives

$$\begin{aligned}\psi_0 - V_a &= -\frac{kT}{q} \ln p \Big|_a^b \\ &= \frac{kT}{q} \ln \frac{p_{pa}}{p_{nb}}\end{aligned}\quad (4.11)$$

or, rearranging,

$$p_{nb} = p_{pa} e^{-q\psi_0/kT} e^{qV_a/kT} \quad (4.12)$$

But from space-charge neutrality at point a and introducing *approximation 3*, that only cases where the carriers in the minority have a much lower concentration than those in the majority will be considered ($p_{pa} \gg n_{pa}$, $n_{na} \gg p_{na}$), gives

$$\begin{aligned}p_{pa} &= N_A + n_{pa} \quad (\text{where } n_{pa} \text{ is small}) \\ &\approx p_{p0} \approx p_{n0} e^{q\psi_0/kT}\end{aligned}\quad (4.13)$$

Hence,

$$\begin{aligned}p_{nb} &= p_{n0} e^{qV_a/kT} = \frac{n_i^2}{N_D} e^{qV_a/kT} \\ n_{pa} &= n_{p0} e^{qV_a/kT} = \frac{n_i^2}{N_A} e^{qV_a/kT}\end{aligned}\quad (4.14)$$

Therefore, the concentration of carriers in the minority (*minority carriers*) at the edge of the depletion region increases exponentially with applied voltage. The process by which this concentration is controlled by the bias across the junction is known as *minority-carrier injection*.

4.5 DIFFUSIVE FLOW IN QUASI-NEUTRAL REGIONS

Carriers can flow by drift and diffusion. If a uniformly doped region of semiconductor material is quasi-neutral (space-charge density ap-

proximately zero) and minority-carrier flows are not insignificant, minority carriers will flow predominantly by diffusion.

Proof (reductio ad absurdum): Consider n -type quasi-neutral material where $n \gg p$ and the case where minority-carrier flows are not insignificant (i.e., the case where $J_e \gg J_h$, with the symbol $\gg$ is used to mean "not a lot greater than")

We have

$$\begin{aligned}J_e &= q\mu_e n \xi + qD_e \frac{dn}{dx} \\ J_h &= q\mu_h p \xi - qD_h \frac{dp}{dx}\end{aligned}\quad (4.15)$$

$$p - n + N_D \approx 0 \quad (\text{quasi-neutral})$$

Differentiating the latter equation, remembering that N_D is constant, yields

$$\frac{dp}{dx} \approx \frac{dn}{dx} \quad (4.16)$$

Assume that the drift component of minority-carrier flow is *not* negligible, that is,

$$|q\mu_h p \xi| \ll \left| qD_h \frac{dp}{dx} \right| \quad (4.17)$$

Since $n \gg p$, it follows, applying Eqs. (4.17) and then (4.16), that

$$|q\mu_h n \xi| \gg \left| qD_h \frac{dp}{dx} \right| \gg \left| qD_h \frac{dn}{dx} \right|$$

Equivalently,

$$|q\mu_e n \xi| \gg \left| qD_e \frac{dn}{dx} \right| \quad (4.18)$$

Also, since μ_e and μ_h are of comparable magnitude,

$$|q\mu_e n \xi| \gg |q\mu_h p \xi| \quad (4.19)$$

Combining Eqs. (4.17) to (4.19) leads to the conclusion:

$$J_e \gg J_h$$

This violates one of the original conditions. Therefore, the assumption of Eq. (4.17) is incorrect. This leads to the result

$$|q\mu_h p \xi| \ll \left| qD_h \frac{dp}{dx} \right|$$

That is, *minority carriers in quasi-neutral regions flow predominantly by diffusion, subject to the conditions previously mentioned. Hence, approximation 4 is*

$$\begin{aligned} J_h &= -qD_h \frac{dp}{dx} & (\text{n-type quasi-neutral region}) \\ J_e &= qD_e \frac{dn}{dx} & (\text{p-type quasi-neutral region}) \end{aligned} \quad (4.20)$$

Basically, the small number of minority carriers compared to majority carriers shields them from the effect of an electric field. In the following sections, the relevance of this in connection with finding current flows in p-n diodes will become apparent.

4.6 DARK CHARACTERISTICS

4.6.1 Minority Carriers in Quasi-Neutral Regions

To summarize progress to date, it has been shown that a reasonable approximation for analysis is to divide the diode up into depletion and quasi-space-charge neutral regions. It has been found that the minority-carrier concentration at the edge of the depletion region depends exponentially on the voltage applied to the diode. This information is summarized in Fig. 4.7. Moreover, it has been shown that, when quasi-neutral regions are uniformly doped and majority carrier currents small, minority carriers flow primarily by diffusion. This will allow the distributions shown dashed in Fig. 4.7 to be calculated.

On the n-type side of the diode,

$$J_h = -qD_h \frac{dp}{dx} \quad (4.21)$$

while the continuity equation gives

$$\frac{1}{q} \frac{dJ_h}{dx} = -(U - G) \quad (4.22)$$

Explicit expressions for the recombination rate were given in Chapter 3 for several recombination mechanisms. From the definition of carrier lifetimes of Eq. (3.19), the recombination rate in the n-type region can be put in the form

$$U = \frac{\Delta p}{\tau_h} \quad (4.23)$$

where Δp is the excess concentration of minority-carrier holes equal to the total concentration, p_n , minus the equilibrium concentration, p_{n0} . τ_h is the minority-carrier lifetime which can be regarded as a constant, at least for small disturbances from equilibrium. Combining the three equations above gives

$$D_h \frac{d^2 p_n}{dx^2} = \frac{p_n - p_{n0}}{\tau_h} - G \quad (4.24)$$

In the dark, $G = 0$. Also, $d^2 p_{n0}/dx^2 = 0$. Hence, Eq. (4.24) simplifies to

$$\frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{L_h^2} \quad (4.25)$$

where

$$L_h = \sqrt{D_h \tau_h} \quad (4.26)$$

L_h has the dimensions of length and is known as the *diffusion length*. It will become apparent that this is a very important parameter in solar cell work. The general solution to Eq. (4.25) is

$$\Delta p = A e^{x/L_h} + B e^{-x/L_h} \quad (4.27)$$

The constants A and B can be found by applying two boundary conditions as follows:

1. At $x = 0$, $p_{nb} = p_{n0} e^{qV/kT}$.
2. p_n finite as $x \rightarrow \infty$. Therefore, $A = 0$.

These boundary conditions give the particular solution:

$$p_n(x) = p_{n0} + p_{n0} [e^{qV/kT} - 1] e^{-x/L_h} \quad (4.28)$$

Similarly,

$$n_p(x') = n_{p0} + n_{p0} [e^{qV/kT} - 1] e^{-x'/L_e} \quad (4.29)$$

where x' is defined in Fig. 4.8(b).

These solutions for minority-carrier concentrations throughout the device are plotted in Fig. 4.8(a). In the quasi-neutral regions, the majority-carrier concentrations must have a corresponding change in their distributions to maintain space-charge neutrality, also as shown in Fig. 4.8(a). Even though the absolute changes are the same, the relative changes in the majority changes are very much smaller, as indicated in the logarithmic plot of Fig. 4.8(b).

4.6.2 Minority-Carrier Currents

It is a very simple matter to calculate minority-carrier current flows once the carrier distributions are known. Since currents are

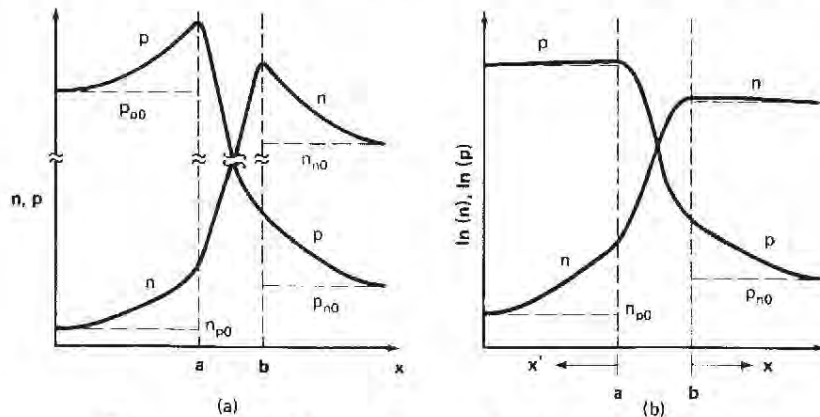


Figure 4.8. (a) Linear plot of the distributions of carriers throughout the p - n junction diode under forward bias. (b) Corresponding semilogarithmic plot. Note the differences with respect to majority carriers.

diffusive in quasi-neutral regions, on the n -type side:

$$J_h = -qD_h \frac{dp}{dx} \quad (4.30)$$

Substituting Eq. (4.28) gives

$$J_h(x) = \frac{qD_h p_{n0}}{L_h} (e^{qV/kT} - 1) e^{-x/L_h} \quad (4.31)$$

Similarly, in the p -type region,

$$J_e(x') = \frac{qD_e n_{p0}}{L_e} (e^{qV/kT} - 1) e^{-x'/L_e} \quad (4.32)$$

The current distributions resulting from these expressions are plotted in Fig. 4.9(a). In order to calculate the total current flow in the diode, it is necessary to know both the electron and hole components at the same point. Considering current flows in the depletion region, the continuity equations give

$$\frac{1}{q} \frac{dJ_e}{dx} = U - G = -\frac{1}{q} \frac{dJ_h}{dx} \quad (4.33)$$

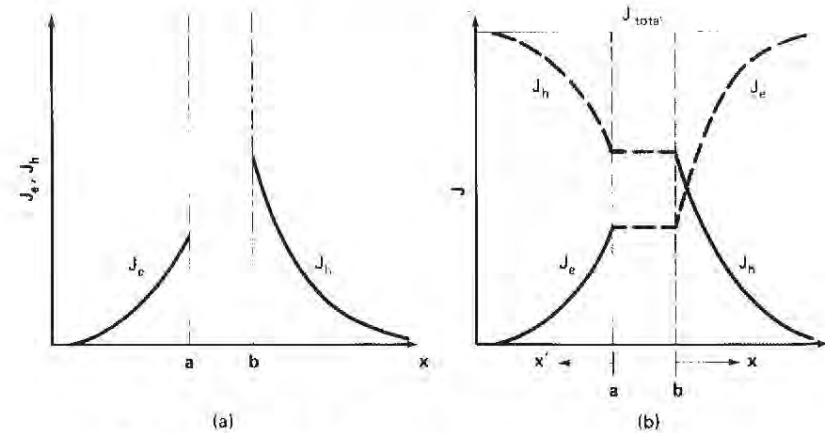


Figure 4.9. (a) Minority-carrier current densities in a p - n junction diode corresponding to Fig. 4.8. (b) Distribution of minority, majority, and total current densities in the diode, neglecting recombination in the depletion region.

Hence, the magnitude of the change in current across the depletion region is

$$\delta J_e = |\delta J_h| = q \int_{-W}^0 (U - G) dx \quad (4.34)$$

W is generally much less than L_e and L_h , the characteristic decay lengths of J_e and J_h . This indicates that Fig. 4.9(a) is grossly out of proportion. Since W is small, a reasonable approximation is to assume that the integral involved in Eq. (4.34) is negligible, so that $\delta J_e = |\delta J_h| \approx 0$. It follows that J_e and J_h are essentially constant across the depletion region as shown in Fig. 4.9(b). This approximation, *approximation 5*, would appear more reasonable if W were drawn to scale. The total current can now be found since both J_e and J_h are known at all points in the depletion region. Hence,

$$\begin{aligned} J_{\text{total}} &= J_e \big|_{x'=0} + J_h \big|_{x=0} \\ &= \left(\frac{qD_e n_{p0}}{L_e} + \frac{qD_h p_{n0}}{L_h} \right) (e^{qV/kT} - 1) \end{aligned} \quad (4.35)$$

Since J_{total} is constant with position, it is now possible to complete the distributions of J_e and J_h throughout the diode as shown by the dashed lines of Fig. 4.9(b).

The result of the analysis has been the derivation of the ideal diode law:

$$I = I_0 (e^{qV/kT} - 1) \quad (4.36)$$

The important point as far as this book is concerned is the expression derived for the saturation current density:

$$I_0 = A \left(\frac{qD_e n_i^2}{L_e N_A} + \frac{qD_h n_i^2}{L_h N_D} \right) \quad (4.37)$$

where A is the cross-sectional area of the diode.

4.7 ILLUMINATED CHARACTERISTICS

Next the characteristics of a p - n diode when illuminated will be explored. For mathematical simplicity, an idealized case will be assumed where the generation rate of electron-hole pairs by the illumination

will be assumed uniform throughout the device. This would correspond closely to a specific physical situation where the cell was illuminated by long-wavelength light consisting of photons of energy close to that of the semiconductor band gap. Such light would be only weakly absorbed and the volume generation rate of electron-hole pairs would be approximately constant over the distances characteristically involved. It is emphasized that this case of uniform generation rate *does not correspond to the situation in actual solar energy conversion*. More realistic situations will be treated using a different approach in subsequent chapters.

Problem: Derive the ideal current-voltage characteristics of a p - n junction diode when illuminated by light such that the volume rate of generation of electron-hole pairs by the light, G , is constant throughout the device.

The analysis closely parallels that of the diode in the dark. To consolidate this material, the reader is encouraged to attempt to work through the problem without reference to the worked solution that follows.

Worked Solution: The reader should first be satisfied that approximations 1 through 4 and the intermediate results leading from them are equally valid regardless of whether the device is illuminated or not. This being the case, Eq. (4.24) is still valid, but in this case G is not zero but a constant. Hence, on the n -type side:

$$\frac{d^2 \Delta p}{dx^2} = \frac{\Delta p}{L_h^2} - \frac{G}{D_h} \quad (4.38)$$

Since G/D_h is constant, the corresponding general solution is

$$\Delta p = G\tau_h + Ce^{x/L_h} + De^{-x/L_h} \quad (4.39)$$

The boundary conditions remain unchanged from the analysis of the diode in the dark. This gives the particular solution

$$p_n(x) = p_{n0} + G\tau_h + [p_{n0}(e^{qV/kT} - 1) - G\tau_h] e^{-x/L_h} \quad (4.40)$$

with a similar expression for $n_p(x')$ as plotted in Fig. 4.10.

The corresponding current density is

$$J_h(x) = \frac{qD_h p_{n0}}{L_h} (e^{qV/kT} - 1) e^{-x/L_h} - qGL_h e^{-x/L_h} \quad (4.41)$$

with a similar expression for $J_e(x')$.

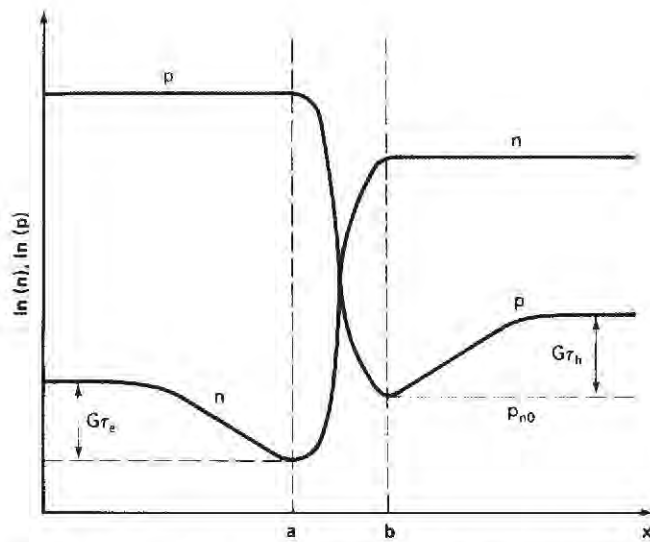


Figure 4.10. Distribution of carriers through a p - n junction when short-circuited under infrared illumination (generation rate assumed uniform through diode).

Again neglecting the effect of recombination in the depletion region (approximation 5) but including the effect of generation in this region in this case gives the change in current density across this region as

$$|\delta J_e| = |\delta J_h| = qGW \quad (4.42)$$

Hence, proceeding as before gives the following result for the current-voltage characteristics:

$$I = I_0 (e^{qV/kT} - 1) - I_L \quad (4.43)$$

where I_0 retains the value of Eq. (4.37) and I_L has the value

$$I_L = qAG(L_e + W + L_h) \quad (4.44)$$

This result is plotted in Fig. 4.11. Note that the illuminated characteristics are merely the dark characteristics shifted down by a current I_L . This gives a region in the fourth quadrant of this plot where power can be extracted from the diode.

Note that the form of Eq. (4.44) suggests a conclusion that will be confirmed later. The light-generated current, I_L , has a value

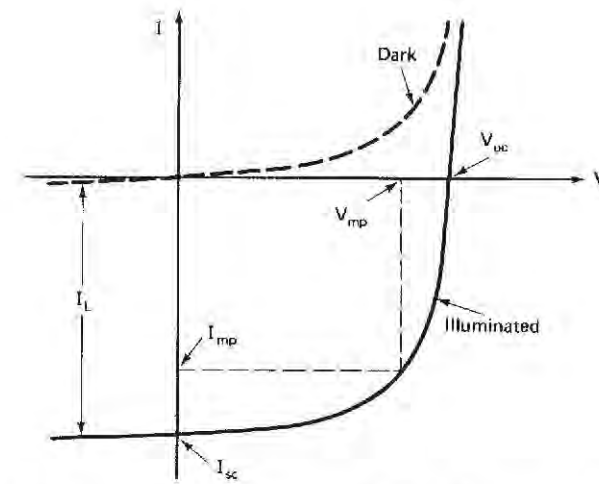


Figure 4.11. Terminal properties of a p - n junction diode in the dark and when illuminated.

equal to that expected if all the carriers generated by light in the depletion region of the diode and within a minority carrier diffusion length on either side were to contribute to it. The depletion region and the generally much larger volume of material lying within a diffusion length of either side of it is indeed the “active” collection region of a p - n junction solar cell.

4.8 SOLAR CELL OUTPUT PARAMETERS

Three parameters are usually used to characterize solar cell outputs (Fig. 4.11).

One of these is the *short-circuit current*, I_{sc} . Ideally, this is equal to the light-generated current I_L . A second parameter is the *open-circuit voltage*, V_{oc} . Setting I to zero in Eq. (4.43) gives the ideal value:

$$V_{oc} = \frac{kT}{q} \ln \left(\frac{I_L}{I_0} + 1 \right) \quad (4.45)$$

V_{oc} is determined by the properties of the semiconductor by virtue of its dependence on I_0 . The power output for any operating point in the fourth quadrant is equal to the area of the rectangle indicated in Fig. 4.11. One particular operating point (V_{mp} , I_{mp}) will

maximize this power output. The third parameter, the fill factor, FF, is defined as

$$FF = \frac{V_{mp} I_{mp}}{V_{oc} I_{sc}} \quad (4.46)$$

It is a measure of how “square” the output characteristics are. For cells of reasonably efficiency, it has a value in the range 0.7 to 0.85. Ideally, it is a *function only of the open-circuit voltage*, V_{oc} . Defining a normalized voltage, v_{oc} , as $V_{oc}/(kT/q)$, the ideal (maximum) value of FF is shown in Fig. 4.12. An empirical expression describing this relationship to about four significant digits for $v_{oc} > 10$ is (see also Section 5.4.4)

$$FF = \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1} \quad (4.47)$$

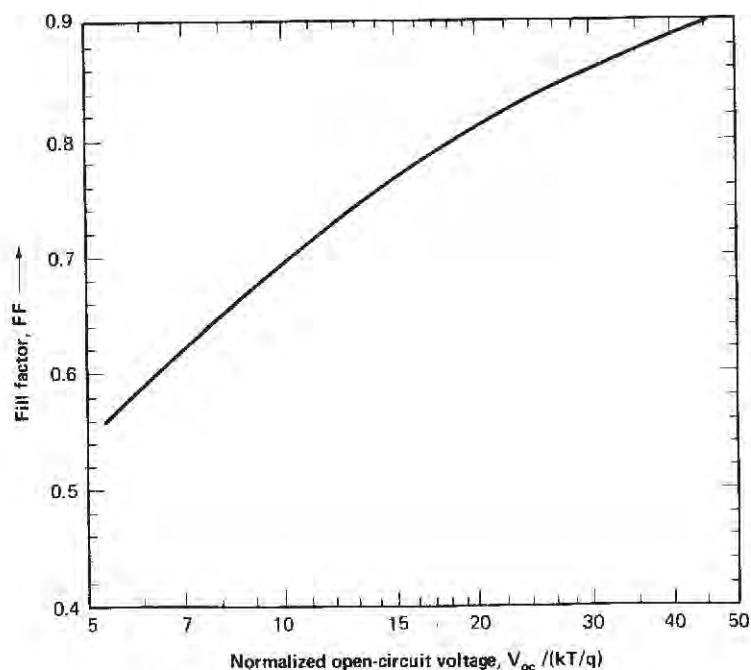


Figure 4.12. Ideal value of the fill factor as a function of the open-circuit voltage normalized to the thermal voltage, kT/q .

The energy-conversion efficiency, η , is then given by

$$\eta = \frac{V_{mp} I_{mp}}{P_{in}} = \frac{V_{oc} I_{sc} FF}{P_{in}} \quad (4.48)$$

where P_{in} is the total power in the light incident on the cell. Energy-conversion efficiencies of commercial solar cells generally lie in the 12 to 14% range.

4.9 EFFECT OF FINITE CELL DIMENSIONS ON I_0

The diode saturation current, I_0 , determines V_{oc} as indicated by Eq. (4.45). In deriving Eq. (4.37) for I_0 , it was implicitly assumed that the diode extended an unlimited distance on either side of the junction. Actual devices will not. A solar cell of finite dimensions is indicated in Fig. 4.13.

This modifies the value of the saturation current, I_0 . The modified value will depend on the surface-recombination velocities (Section 3.4.5) at the exposed surfaces. Two limiting cases are of interest: (1) when this velocity is very high, approaching infinity; and (2) when it is very low, approaching zero. In the former case the excess minority carrier concentration is zero at the surface. In the latter, the minority carrier current flow into the surface is zero. Applying these as boundary conditions allows the modified expression for I_0 to be found (Exercise 4.3). It takes the form¹

$$I_0 = A \left(\frac{qD_e n_i^2}{L_e N_A} * F_P + \frac{qD_h n_i^2}{L_h N_D} * F_N \right) \quad (4.49)$$

If the surface on the p -type side of the device is a high recombination velocity surface, F_P has the form

$$F_P = \coth \left(\frac{W_P}{L_e} \right) \quad (4.50)$$

¹General expressions for F_N and F_P are (Ref. 4.3)

$$F_N = \frac{S_h \cosh(W_N/L_h) + D_h/L_h \sinh(W_N/L_h)}{D_h/L_h \cosh(W_N/L_h) + S_h \sinh(W_N/L_h)}$$

$$F_P = \frac{S_e \cosh(W_P/L_e) + D_e/L_e \sinh(W_P/L_e)}{D_e/L_e \cosh(W_P/L_e) + S_e \sinh(W_P/L_e)}$$

where S_e and S_h are the surface-recombination velocities of the respective surfaces as introduced in Section 3.4.5.

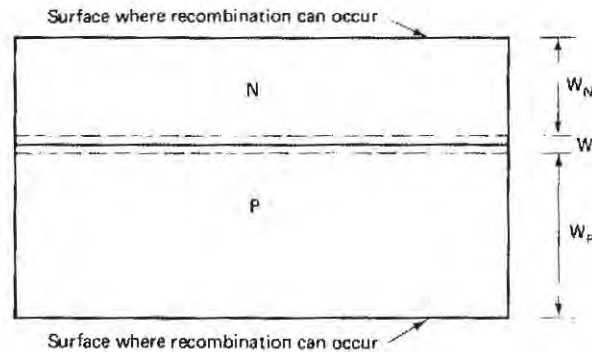


Figure 4.13. Basic solar cell, defining important dimensions.

where W_P is defined in Fig. 4.13. A corresponding expression will hold for F_N if the corresponding surface is also a high-recombination-velocity surface. If it happens to be a *low-recombination-velocity* surface, then F_N would be given by

$$F_N = \tanh\left(\frac{W_N}{L_n}\right) \quad (4.51)$$

A similar expression would hold for F_P if the p -type region were also to have a low-recombination-velocity surface.

Note that the smallest value of I_0 and hence the largest V_{oc} will occur if both surfaces possess low recombination velocities.

4.10 SUMMARY

By making a series of approximations, the general system of equations describing solar cell operation can be reduced to a tractable form. This approach has allowed the ideal form of the dark and illuminated characteristics of solar cells to be found.

Under illumination, the dark current-voltage curves ideally are displaced downward by the light-generated current. The active region of a solar cell for collecting the latter current is the junction depletion region and the volume of cell material within a minority carrier diffusion length of this region.

The parameters used to characterize the terminal properties of a solar cell are the short-circuit, I_{sc} , the open-circuit voltage, V_{oc} , and the fill factor, FF. The conditions at the surfaces of a solar cell,

necessarily of finite extent, influence the open-circuit voltage and will be shown in subsequent chapters also to affect the short-circuit current.

EXERCISES

- 4.1. A p - n junction diode is uniformly doped with 10^{24} dopants/ m^3 on the p -type side and 10^{22} dopants on the n -type side. At 300 K, calculate the maximum electric field strength, the width of the depletion region, and the junction capacitance per unit area for the following bias conditions: (a) zero bias; (b) 0.4 V forward bias; (c) 10 V reverse bias.
- 4.2. The derivation of Section 4.4 assumes that in quasi-neutral regions, carriers in the minority have a much lower concentration than majority carriers. Find an expression for the maximum applied voltage at which this can be valid.
- 4.3. (a) Consider the case of a cell of finite dimensions as shown in Fig. 4.13. Derive expressions for the electron concentration on the p -type side in the dark as a function of applied voltage for the two extreme cases where the back surface has a very high recombination velocity and a very low recombination velocity. Sketch and compare these distributions when the width of the p -type region is very much less than a minority-carrier diffusion length.
(b) Referring to this sketch, indicate which distribution would give the smallest contribution to the diode saturation current and hence the larger open-circuit voltage under illumination if other parameters of the diode are identical.
- 4.4. The following is a further example of the analysis technique of this chapter, which is not as demanding mathematically. Consider a cell of finite dimensions which are much smaller than corresponding minority-carrier diffusion lengths. The recombination velocities along the front and rear surface of this cell are given as very high, assumed infinite. In this case, it will be a good approximation to neglect bulk recombination compared to surface recombination (i.e., the recombination rate U can be assumed zero throughout bulk regions). Find an expression for the saturation current density of this diode and for the short-circuit current for the *special case* where the generation rate of electron-hole pairs by light, G , is constant throughout the cell.
- 4.5. When the cell temperature is 300 K, a certain silicon cell of 100-cm^2 area gives an open-circuit voltage of 600 mV and a short-circuit current output of 3.3 A under 1-kW/m^2 illumination. Assuming that the cell behaves ideally, what is its energy-conversion efficiency at the maximum power point?

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Chapter

5

EFFICIENCY LIMITS, LOSSES, AND MEASUREMENT

5.1 INTRODUCTION

Sunlight incident on a solar cell creates electron-hole pairs within the semiconducting material making up the cell. The cell has an asymmetrical electronic structure which causes the electrons and holes making up these generated electron-hole pairs to separate and create current flow in a load connected between the cell terminals. In the present chapter, energy-conversion efficiency limits are discussed for this process as well as the effects of various nonidealities on efficiency. Techniques for measuring the efficiency of photovoltaic devices are also described.

5.2 EFFICIENCY LIMITS

5.2.1 General

In Chapter 4, it was seen that three parameters could be used to characterize the performance of a p - n junction solar cell. These

are the open-circuit voltage (V_{oc}), the short-circuit current (I_{sc}), and the fill factor (FF) (Fig. 4.11). It was also indicated that the maximum value of the fill factor is a function of V_{oc} . Hence, only the ideal limits of I_{sc} and V_{oc} need to be examined.

5.2.2 Short-Circuit Current

It is relatively easy to calculate the upper limit to the short-circuit current obtainable from any selected solar cell material. Under ideal conditions, each photon incident on the cell of energy greater than the band gap gives rise to one electron flowing in the external circuit.¹ Hence, to calculate the maximum I_{sc} , the photon flux in sunlight must be known. This can be calculated from the energy distribution of sunlight (Chapter 1) by dividing the energy content at a given wavelength by the energy of an individual photon (hf or hc/λ) of this wavelength. The results of such a calculation are shown in Fig. 5.1(a) for AM0 radiation and the standard terrestrial AM1.5 radiation mentioned in Chapter 1.

The maximum I_{sc} is then found by integrating these distributions from low wavelengths up to the maximum wavelength for which electron-hole pairs can be generated for a given semiconductor. [A relationship between photon energy in eV and its wavelength in micrometers worth remembering is E (eV) = $1.24/\lambda$ (μm). For silicon, the band gap is about 1.1 eV, so that λ corresponding to this is 1.13 μm .] The resulting upper limits on short-circuit current density are shown in Fig. 5.1(b).

It is not surprising that as the band gap decreases, the short-circuit current density increases. More photons have the energy required to create an electron-hole pair as the band gap becomes smaller.

5.2.3 Open-Circuit Voltage and Efficiency

The fundamental limitations on the open-circuit voltages of solar cells are not as clearly defined. In Chapter 4, it was shown that, for an ideal p - n junction cell, V_{oc} was given by

$$V_{oc} = \frac{kT}{q} \ln \left(\frac{I_L}{I_0} + 1 \right) \quad (5.1)$$

¹Very high energy photons of energy several times the band-gap energy can create an electron-hole pair where the electron has sufficient energy above the conduction-band edge to create a second electron-hole pair by *impact ionization* (Section 3.4.3). There are not many such photons in sunlight, so this mechanism is not of significance in solar cells.

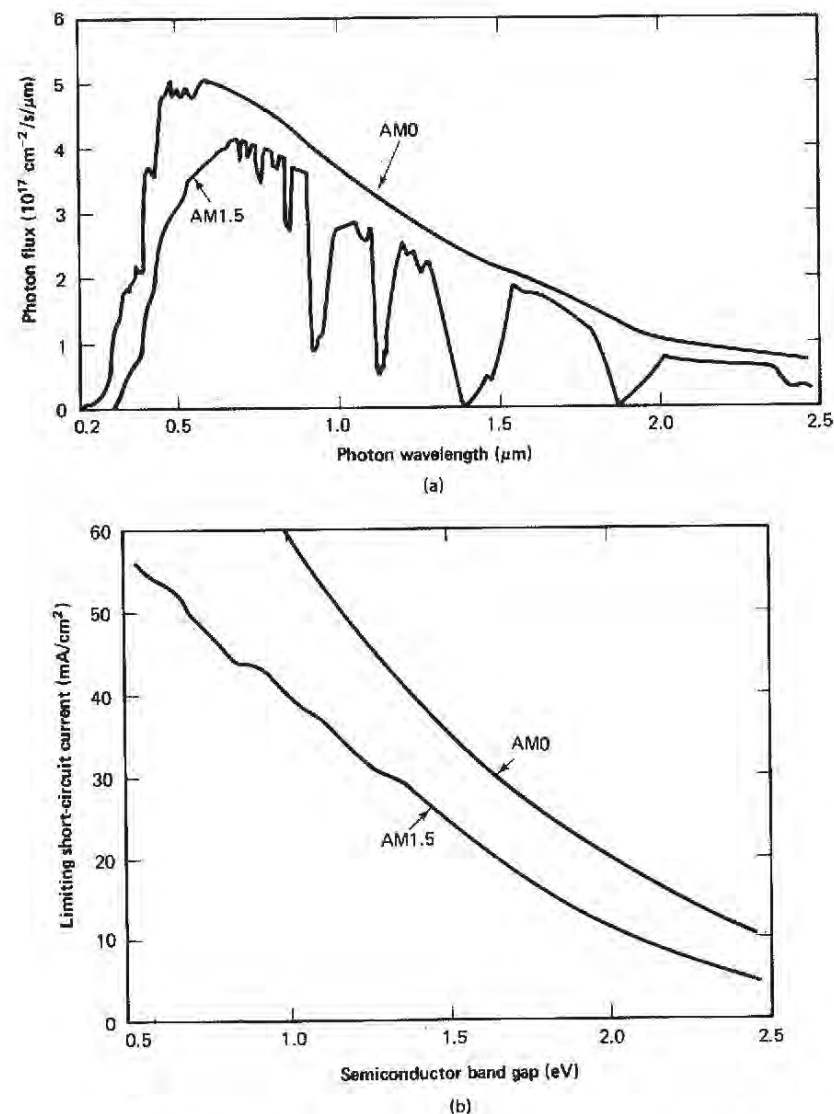


Figure 5.1. (a) Photon flux in sunlight corresponding to the AM0 and AM1.5 energy distributions given in Fig. 1.3. (b) Corresponding upper limits on the short-circuit current density as a function of the energy band gap of the solar cell material.

where I_L is the light-generated current and I_0 is the diode saturation current calculated as

$$I_0 = A \left(\frac{qD_e n_i^2}{L_e N_A} + \frac{qD_h n_i^2}{L_h N_D} \right) \quad (5.2)$$

I_0 needs to be as small as possible for maximum V_{oc} . One approach to calculating upper limits on V_{oc} (and hence efficiency) is to assign favorable values to the semiconductor parameters in Eq. (5.2) while still keeping them within the range expected to be required to produce good solar cells (Ref. 5.1). For silicon, this gives a maximum V_{oc} of about 700 mV. The corresponding maximum fill factor is 0.84. Combining with the results of the preceding section for I_{sc} allows the maximum energy-conversion efficiency to be found.

The parameter in Eq. (5.2) which depends most strongly on the choice of semiconductor material is the square of the intrinsic concentration, n_i^2 . From Chapter 2,

$$n_i^2 = N_C N_V \exp \left(-\frac{E_g}{kT} \right) \quad (5.3)$$

A reasonable estimate of the minimum value of the saturation current density as a function of band gap from Eq. (5.2) is

$$I_0 = 1.5 \times 10^5 \exp \left(-\frac{E_g}{kT} \right) \text{ A/cm}^2 \quad (5.4)$$

This relationship ensures that the maximum value of V_{oc} decreases with decreasing band gap. This trend is opposite from that observed for I_{sc} . It follows that there will be an optimum-band-gap semiconductor for highest efficiency.

This is demonstrated in Fig. 5.2, which shows maximum energy-conversion efficiency calculated as outlined above as a function of band gap. The peak efficiency occurs for a band gap in the range 1.4 to 1.6 eV and increases from 26 to 29% as the air mass increases from 0 to 1.5. Silicon's band gap is lower than optimum, but maximum efficiencies are still relatively high. GaAs has a near-optimal band gap (1.4 eV).

A major contributor to these relatively low maximum efficiencies is the fact that each photon absorbed creates one electron-hole pair regardless of its energy. The electron and hole quickly relax back to the band edges, emitting phonons (Fig. 5.3). Even though

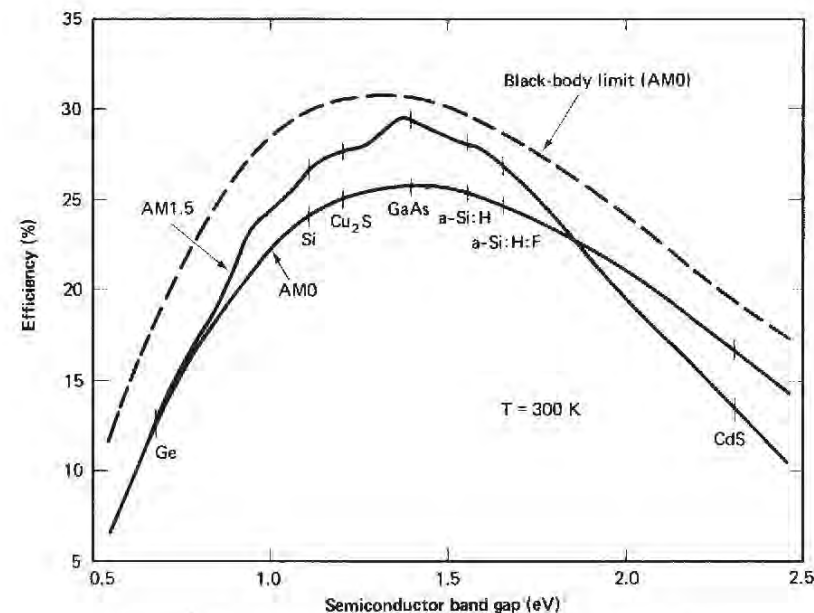


Figure 5.2. Solar cell efficiency limits as a function of the band gap of the cell material. The solid lines are semiempirical limits for AM0 and AM1.5 illumination; the dashed line is based on thermodynamic considerations for black-body solar cells under AM0 radiation.

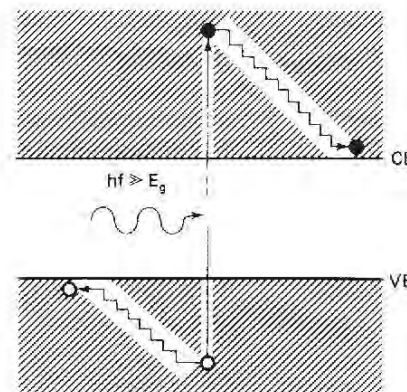


Figure 5.3. One of the major loss mechanisms in solar cells. An electron-hole pair created by a high-energy photon quickly "thermalizes" or relaxes back to the edges of the respective carrier bands. The energy wasted is dissipated as heat.

the photon's energy may be much larger than the band gap, the resulting electron and hole effectively are separated by only the latter energy. This effect alone limits the maximum achievable efficiency to about 44% (Ref. 5.2). The other major contributor is that even though the generated carriers are separated by a potential corresponding to the band-gap energy, p - n junction cells are inherently capable of giving a voltage output only a fraction of this potential. For example, for silicon, the maximum value of this fraction is $0.7/1.1 \approx 60\%$.

The foregoing discussion is limited to the case of an individual cell exposed directly to sunlight. Experimentally, devices of this type based on GaAs have exceeded 20% efficiency. There exist techniques mentioned later whereby photovoltaic system efficiencies can be enhanced further. An efficiency of 28.5% was reported in 1978 (Ref. 5.3) for a system involving multiple cells. Despite the low maximum-efficiency values, solar cells remain the most efficient way yet demonstrated of converting sunlight to electricity.

5.2.4 Efficiency Limits for Black-Body Cells

The limitation of the previous approach to calculating the maximum V_{oc} is its empirical nature. There is a small probability that an unanticipated change in materials technology and/or solar cell design could produce higher efficiency cells than the limits of Fig. 5.2. A more fundamental approach is developed in Ref. 5.2 for the case of a black-body solar cell. Such a body absorbs all radiation incident on it. It is plausible that such a cell would have an efficiency limit at least as high as any non-black-body types.

Black bodies emit radiation with a spectral distribution characteristic of their temperature (Chapter 1). Hence, a black-body solar cell in equilibrium emits photons. For photons of energy larger than the band gap, the source of these photons is predominantly radiative recombination events in the semiconductor. In thermal equilibrium, these events will be balanced by an equal generation rate. A lower limit to the rate of recombination at thermal equilibrium is then the total number of photons emitted per unit time with energy larger than the band gap. In a cell with minimum recombination, the recombination rate can be shown to increase exponentially with bias. This leads to an equation identical to the ideal diode law already discussed for solar cells in the dark, with the value of I_0 equal to the electronic charge multiplied by the equilibrium rate of recombination throughout the cell.

Performing the appropriate calculations for silicon gives the minimum value of I_0 , which corresponds to a maximum possible V_{oc}

of 850 mV for a black-body silicon solar cell. Calculations for different band-gap semiconductors are shown as the dashed curve in Fig. 5.2. This places the upper efficiency limit for a directly exposed individual cell above 30%.

5.3 EFFECT OF TEMPERATURE

Since the operating temperature of solar cells in the field can vary over wide extremes, it is necessary to understand the effect of temperature on performance.

The short-circuit current of solar cells is not strongly temperature-dependent. It tends to increase slightly with increasing temperature. This can be attributed to increased light absorption, since semiconductor band gaps generally decrease with temperature. The other cell parameters, the open-circuit voltage and the fill factor, both decrease.

The relation between short-circuit current and open-circuit voltage is

$$I_{sc} = I_0 (e^{qV_{oc}/kT} - 1) \quad (5.5)$$

Neglecting the small negative term, this can be written as

$$I_{sc} = AT^\gamma e^{-E_{g0}/kT} e^{qV_{oc}/kT} \quad (5.6)$$

where A is independent of temperature, E_{g0} is the linearly extrapolated zero temperature band gap of the semiconductor making up the cell, and γ includes the temperature dependencies of the remaining parameters determining I_0 . Its value generally lies in the range 1 to 4. Differentiating gives, with $V_{g0} = E_{g0}/q$,

$$\begin{aligned} \frac{dI_{sc}}{dT} = & AT^\gamma e^{q(V_{oc} - V_{g0})/kT} + AT^\gamma \left(\frac{q}{kT} \right) \left[\frac{dV_{oc}}{dT} \right. \\ & \left. - \left(\frac{V_{oc} - V_{g0}}{T} \right) \right] e^{q(V_{oc} - V_{g0})/kT} \end{aligned} \quad (5.7)$$

Neglecting dI_{sc}/dT in comparison with more significant terms results in the expression

$$\boxed{\frac{dV_{oc}}{dT} = - \frac{V_{g0} - V_{oc} + \gamma(kT/q)}{T}} \quad (5.8)$$

This predicts an approximately linear decrease in V_{oc} with increasing temperature. Substituting values for silicon ($V_{g0} \sim 1.2$ V, $V_{oc} \sim 0.6$ V, $\gamma \sim 3$, $T = 300$ K) gives

$$\frac{dV_{oc}}{dT} = -\frac{1.2 - 0.6 + 0.078}{300} \text{ V/}^\circ\text{C} \quad (5.9)$$

$$= -2.3 \text{ mV/}^\circ\text{C} \quad (5.10)$$

This agrees well with the experimental results.² Hence, for silicon, V_{oc} decreases by about 0.4% per $^\circ\text{C}$. The ideal fill factor depends on the value of V_{oc} normalized to kT/q . Hence, the fill factor also decreases with increasing temperature.

The dominant variation is that of V_{oc} . This causes the power output and efficiency to decrease with increasing temperature. For silicon, the power output decreases by 0.4 to 0.5% per $^\circ\text{C}$. This dependency is reduced for larger band-gap material. For example, GaAs cells are only about half as sensitive to increasing temperature as are silicon cells.

5.4 EFFICIENCY LOSSES

5.4.1 General

A schematic cross section of an actual p - n junction solar cell is shown in Fig. 5.4. Actual devices are significantly less efficient than the ideal limits discussed, owing to various additional loss mechanisms. In later chapters, it will be shown how solar cells are designed to achieve the best trade-off between the loss mechanisms discussed in the following sections.

5.4.2 Short-Circuit Current Losses

There are three types of losses in solar cells which could be described as being of an "optical" nature:

1. It was mentioned in Section 3.2 that bare silicon is quite reflective. The antireflection (AR) coating of Fig. 5.4 reduces such reflection losses to about 10%.

²This can be attributed largely to the fact that the form of Eq. (5.8) is valid for conditions more general than those for which it was derived here.

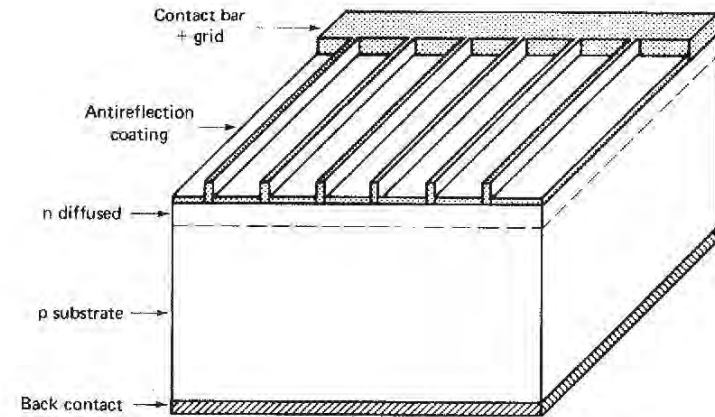


Figure 5.4. Major features of a solar cell. Dimensions in the vertical direction are exaggerated compared to lateral dimensions for the purposes of illustration.

2. The necessity of making electrical contact to both p - and n -type regions of solar cells generally results in a metal grid contact on the side of the cell exposed to sunlight. This blocks 5 to 15% of the incoming light.
3. Finally, if the cell is not thick enough, some of the light of appropriate energy that does get coupled into the cell will pass straight out the back. This determines the minimum thickness of semiconductor material required. Indirect-band-gap semiconductors require more material than direct-gap materials, as indicated by the calculated results for Si and GaAs in Fig. 5.5.

Another source of I_{sc} loss is recombination in the bulk semiconductor and at surfaces. It was indicated in Chapter 4 that only electron-hole pairs generated near the p - n junction itself contribute to I_{sc} . Carriers generated well away from the junction have a high probability of recombining before they complete the trip from the point of generation to the device terminals.

5.4.3 Open-Circuit Voltage Losses

The fundamental process determining V_{oc} is recombination in the semiconductor. This is brought out by the approach to calculating limits on V_{oc} in Section 5.2.4. The lower the recombination

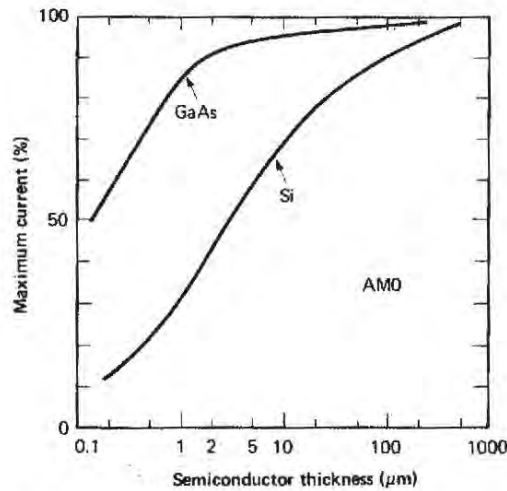


Figure 5.5. Effect of cell thickness on the percentage of full short-circuit current that may ideally be generated by a solar cell. Note the difference between the direct-band-gap semiconductor (GaAs) and indirect-band-gap silicon.

rate in the semiconductor, the higher is V_{oc} . Both bulk and surface recombination are important.

One important effect that can limit V_{oc} is recombination through trapping levels in depletion regions. This recombination mechanism is particularly effective in such regions. Referring back to the expression for this mechanism (Chapter 3), we obtain

$$U = \frac{np - n_i^2}{\tau_{h0}(n + n_1) + \tau_{e0}(p + p_1)} \quad (5.11)$$

This rate will have its peak value when both n_1 and p_1 are small and both n and p are small. Both these conditions occur for traps near midgap located within the depletion region. In the analysis of the dark characteristics of p - n diodes in Chapter 4, recombination in depletion regions was neglected on the basis of the depletion-layer width, W , being very small (approximation 5). However, the enhanced value of recombination in this region can make it quite important in some situations.

Including such depletion-region recombination adds an additional term to the dark current-voltage characteristics, which then become

$$I = I_0(e^{qV/kT} - 1) + I_W(e^{qV/2kT} - 1) \quad (5.12)$$

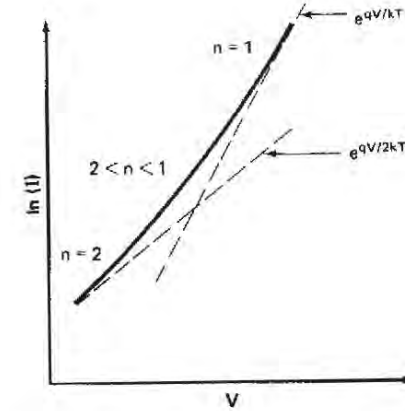


Figure 5.6. Semilogarithmic plots of the dark current-voltage characteristics of a p - n junction diode in the dark, including the effect of recombination in the depletion region.

where I_0 has the same value as before and I_W has the value (Ref. 5.4)

$$I_W = \frac{qAn_i\pi}{2\sqrt{\tau_{e0}\tau_{h0}}} \frac{kT}{q\xi_{max}} \quad (5.13)$$

where ξ_{max} is the maximum electric field strength in the junction. Its value is given by Eq. (4.4) for junctions uniformly doped on each side.

These characteristics are plotted on a semilogarithmic plot in Fig. 5.6. The second term of Eq. (5.12) dominates at low currents, the first at high currents.

It is possible to write Eq. (5.12) in the form

$$I = I'_0(e^{qV/nkT} - 1) \quad (5.14)$$

where n is known as the *ideality factor*. It varies with current level as does I'_0 . From Eq. (5.12), n decreases from 2 at low currents to 1 at higher currents.³ Since the illuminated solar cell characteristics are those of Fig. 5.6 shifted down into the fourth quadrant, it can be seen that the presence of this additional depletion-region recombination current can act to decrease V_{oc} .

³An additional region where n again approaches 2 can be obtained at high currents when minority carrier concentrations approach those of the majority carriers in some regions of the device (Ref. 5.5).

Other technology-dependent limits on V_{oc} are discussed in later chapters.

5.4.4 Fill Factor Losses

Recombination in the depletion region can also reduce the fill factor. If the ideality factor, n , of the preceding section is greater than unity, the fill factor is that calculated for the ideal case (Fig. 4.12) at a voltage V_{oc}/n . This will give a lower value than when n equals unity.

Defining the normalized voltage, v_{oc} , as $V_{oc}/(nkT/q)$ in this more general case, the empirical expression for the fill factor given in Chapter 4 remains valid with an accuracy of about four significant digits for $v_{oc} > 10$:

$$FF_0 = \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1} \quad (5.15)$$

Solar cells generally have a parasitic series and shunt resistance associated with them, as indicated in the solar cell equivalent circuit of Fig. 5.7. There are several physical mechanisms responsible for these resistances. The major contributors to the series resistance, R_S , are the bulk resistance of the semiconductor material making up the cell, the bulk resistance of the metallic contacts and interconnections, and the *contact resistance* between the metallic contacts and the semiconductor. The shunt resistance, R_{SH} , is caused by leakage across the p - n junction around the edge of the cell and in nonperipheral regions in the presence of crystal defects and precipitates of foreign impurities in the junction region. Both types of parasitic resistances act to reduce the fill factor, as indicated in Fig. 5.8. Very high values of R_S and very low values of R_{SH} can also reduce I_{sc} and V_{oc} , respectively, as indicated.

The magnitude of the effect of R_S and R_{SH} on the fill factor can be found by comparing their values to the *characteristic resistance*

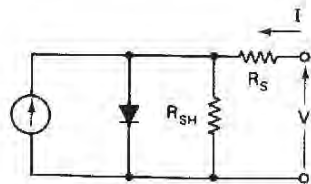


Figure 5.7. Equivalent circuit of a solar cell.

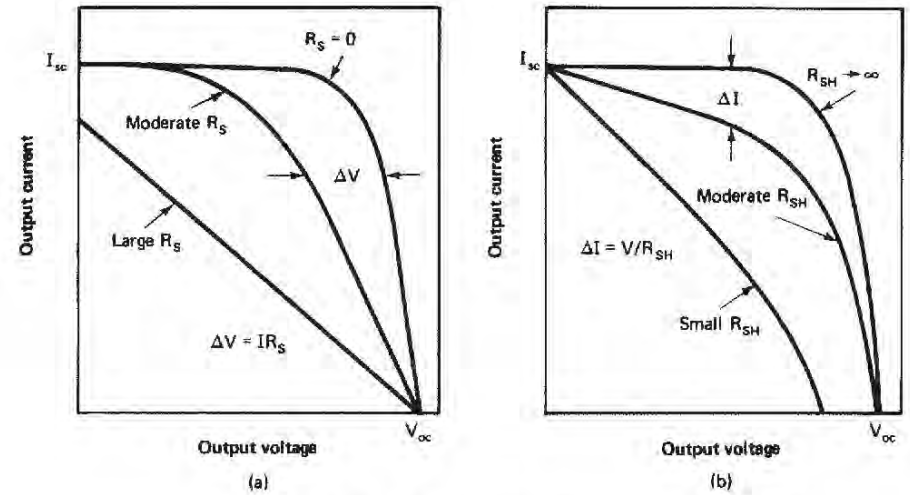


Figure 5.8. Effect of parasitic resistances on the output characteristics of solar cells:

- (a) Effect of series resistance, R_S .
- (b) Effect of a shunt resistance, R_{SH} .

of a solar cell defined as (Ref. 5.6)

$$R_{CH} = \frac{V_{oc}}{I_{sc}} \quad (5.16)$$

If R_S is a lot less than this quantity or R_{SH} a lot larger, there will be little effect upon the fill factor. Defining a normalized resistance, r_s , as R_S/R_{CH} , an approximate expression for the fill factor in the presence of series resistance is (precise values are given in Fig. 5.9)

$$FF = FF_0(1 - r_s) \quad (5.17)$$

where FF_0 represents the ideal fill factors in the absence of parasitic resistance as approximated quite closely by Eq. (5.15). This expression is accurate to close to two significant digits for $v_{oc} > 10$ and $r_s < 0.4$. Defining a normalized shunt resistance, r_{sh} , as R_{SH}/R_{CH} , a corresponding expression for the effect of shunt resistance, also involving the normalized voltage $v_{oc} = V_{oc}/(nkT/q)$, is (exact values are again given in Fig. 5.9)

$$FF = FF_0 \left\{ 1 - \frac{(v_{oc} + 0.7)}{v_{oc}} \frac{FF_0}{r_{sh}} \right\} \quad (5.18)$$

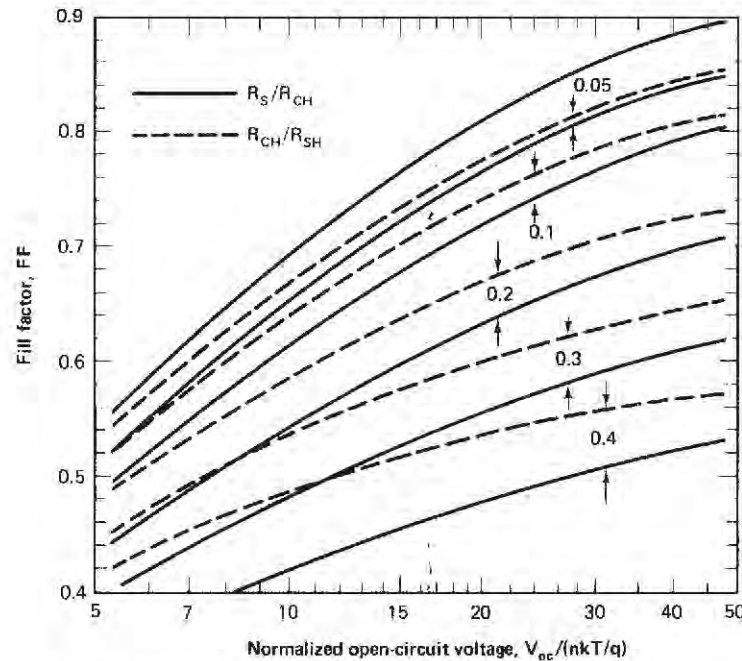


Figure 5.9. General curves for solar cell fill factors as a function of normalized open-circuit voltage. The solid curves show the fill factor as a function of normalized series resistance, R_s/R_{CH} , where $R_{CH} = V_{oc}/I_{sc}$. The dashed curves show the effect of shunt resistance. The normalized parameter in this case is R_{CH}/R_{SH} . These curves allow the fill factor to be found for any combination of open-circuit voltage, temperature, and ideality factor, together with any value of either series or shunt resistance.

This expression is accurate to close to three significant digits for $v_{oc} > 10$ and $r_{sh} > 2.5$. When both series and shunt parasitic resistance is important, an approximate expression for the fill factor over a more limited range of parameters is Eq. (5.18) with FF_0 replaced by FF as calculated from Eq. (5.17).

5.5 EFFICIENCY MEASUREMENT

It may seem a relatively simple matter to measure solar cell efficiency by measuring the power in the incident sunlight using a pyranometer and the electrical power the cell is generating at the maximum power point. The difficulty with this approach is that the performance of the cell measured in this way will depend greatly on the precise spec-

tral content of the sunlight, which varies with air mass, water-vapor content, turbidity, and so on. Combined with typical uncertainties in pyranometer calibration ($\sim \pm 5\%$), this approach would make comparison difficult between the performance of devices measured at other than the same time and place.

An alternative approach is to use a method based on calibrated reference cells. A central test authority calibrates reference cells under standard illumination conditions. The performance of the cell under test is then measured relative to the reference cell. For this technique to be accurate, two conditions must be satisfied:

1. The response of the reference and test cells to different wavelengths of light (*spectral response*) must be similar within specified limits.
2. The spectral content of the illumination source used for the comparative testing must approximate that of the standard illumination, again within specified limits.

The first condition usually dictates that reference and test cells be made from the same semiconductor material using a similar processing technique. With both conditions satisfied, all measurements are referenced to the standard illumination conditions at the calibration center.

A technique similar to that outlined above has been used in the photovoltaic program of the U.S. Department of Energy (Ref. 5.7). In this case the standard sunlight distribution to which measurements are referenced is the AM1.5 distribution of Fig. 1.3. Recommended illumination sources for testing are natural sunlight (with certain constraints on the presence of clouds, air mass, and the rate of variation of intensity), an appropriately filtered xenon lamp, or an ELH lamp. The latter is an inexpensive tungsten-filament projector lamp incorporating a wavelength-sensitive reflector which allows infrared wavelengths to pass out the rear of the lamp. This enhances the visible content of the output beam, whose spectral content is then reasonably close to sunlight. The illumination source must give a collimated beam of uniform intensity at the test plane and be stable during the time of measurement, all within specified limits.

A typical experimental arrangement for measuring solar cell output characteristics is shown in Fig. 5.10(a). A *four-point contacting* scheme is desirable in which voltage and current leads to the cell under test are kept separate. This eliminates effects due to the series resistance of the test leads and associated contact resistances. The cells are mounted on a temperature-controlled block. Both 25°C and

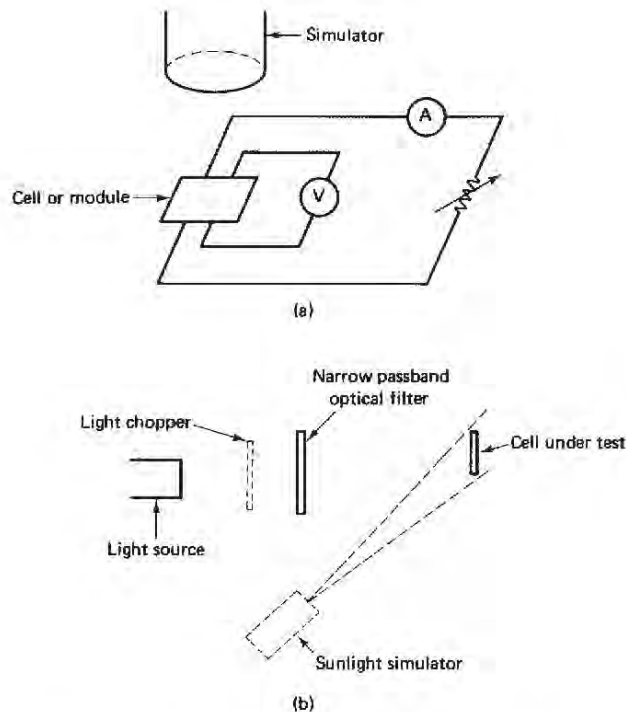


Figure 5.10. (a) Experimental configuration for testing solar cells and modules. (b) Possible setup for measuring the spectral response. The bias light source together with chopped monochromatic light is necessary for cells with a nonlinear spectral response.

28°C are standard temperatures for solar cell measurements. The lamp intensity is adjusted to give the desired intensity as measured by a reference cell. By varying the load resistance, the characteristics of the test cell can then be measured.

The *spectral response* of a test cell can also be measured by direct comparison with the output of a cell with calibrated spectral response. The simplest technique is to use a steady-state source of monochromatic light from a monochromator or that obtained by passing white light through narrow-band optical filters as in Fig. 5.10(b). Since the response of cells to light of increasing intensity may not always be linear, a preferred method is to use a white light source approximating sunlight to bias the cell and measure the incremental response to a small superimposed alternating component of monochromatic light.

5.6 SUMMARY

The upper limit to the efficiency of solar cells lies in the range 26 to 29% and occurs for band gaps in the range 1.4 to 1.6 eV. Several factors combine to ensure that actual solar cells have efficiencies somewhat below ideal. Some relate to coupling the light into the cells, others to excessive recombination in the semiconductor bulk and at surfaces, and still others are due to parasitic resistive effects.

Solar cell efficiencies decrease with increasing temperature, primarily due to the temperature sensitivity of the open-circuit voltage.

Preferred experimental methods for measuring solar cell performance are based on the use of calibrated reference cells to eliminate variables associated with more direct techniques.

EXERCISES

- 5.1. (a) A solar cell is uniformly illuminated by monochromatic light of wavelength 700 nm and intensity 20 mW/cm². What is the corresponding incident photon flux and the upper limit to the short-circuit current output of the cell if its band gap is 1.4 eV?
(b) What would be the corresponding current if the band gap were 2.0 eV?
- 5.2. When illuminated by the AM1.5 radiation of Table 1.1, the maximum current density obtainable in a given solar cell design is 40 mA/cm². If the maximum open-circuit voltage at 300 K is 0.5 V, what is the upper limit to cell efficiency at this temperature?
- 5.3. A solar cell is uniformly illuminated by monochromatic light of intensity 100 mW/cm². The minimum value of cell saturation current density at 300 K is 10⁻¹¹ A/cm². Calculate upper limits to the efficiency of conversion of this light to electrical power at this temperature if the wavelength of light is: (a) 450 nm; (b) 900 nm. Assume that the photon energy is larger than the band gap in each case. (c) Explain the difference in the efficiency values obtained.
- 5.4. Calculate and sketch the upper limit to the *spectral sensitivity* (short-circuit current/power in incident monochromatic light) as a function of wavelength for a silicon solar cell.
- 5.5. Silicon cells typically have an open-circuit voltage of 0.6 V and GaAs cells, about 1.0 V. Compare the theoretical temperature dependencies of open-circuit voltages for these cells at 300 K on both an absolute and a relative basis. (The effective zero temperature band gaps are 1.2 and 1.57 eV, respectively.)
- 5.6. From Fig. 5.5, compare the thickness of Si and GaAs required to enable 75% or maximum current output to be obtained under AM0 illumination.

- 5.7. A solar cell has nearly ideal characteristics, with the ideality factor equal to 1. A second cell has characteristics dominated by recombination in the depletion region and an ideality factor of 2. If both give an open-circuit voltage of 0.6 V at 300 K, compare their ideal fill factors.
- 5.8. A solar cell at 300 K has an open-circuit voltage of 550 mV, a short-circuit current of 2 A, and an ideality factor of 1.3. Calculate the fill factor under the following conditions: (a) series resistance is $0.08\ \Omega$, shunt resistance large. (b) series resistance negligible, shunt resistance is $1\ \Omega$. (c) series resistance is $0.08\ \Omega$, shunt resistance is $2\ \Omega$. (d) series resistance is $0.02\ \Omega$, shunt resistance is $1\ \Omega$.

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Chapter 6

STANDARD SILICON SOLAR CELL TECHNOLOGY

6.1 INTRODUCTION

After the development of the first reasonably efficient silicon cells in 1953, these cells found a major application in generating electricity for spacecraft. The first such use was in Vanguard I in 1958. Since then, a small but growing industry has developed to supply cells for the ever-increasing number of communication satellites and other space craft. Stringent requirements on performance and reliability led to the development of a standard cell-processing sequence which remained virtually unchanged throughout the 1960s and early 1970s.

Since 1973, increased interest in renewable energy resources has led to several companies currently producing cells specifically for terrestrial use. A list of several solar cell manufacturers is given in Table 6.1. Initially these tended to carry over the standard technology developed for space cells. Although different requirements for terrestrial use have since produced some significant changes in the techniques used to fabricate cells, the standard technology will be described in this chapter to provide a base for discussing these changes and probable future developments.

Table 6.1. MANUFACTURERS OF SOLAR CELLS (circa 1980)

<i>United States</i>	
Applied Solar Energy Corporation 15251 East Don Julian Road City of Industry, CA 91746	Solar Power Corporation 20 Cabot Road Woburn, MA 01801
Arco Solar, Inc. 20554 Plummer Street Chatsworth, CA 91311	Solarex Corporation 1335 Piccard Drive Rockville, MD 20850
Motorola, Inc. Solar Energy Department Phoenix, AZ 95008	Solec International, Inc. 12533 Chadron Avenue Hawthorne, CA 90250
Photon Power, Inc. 10767 Gateway West El Paso, TX 79935	Solenergy Corporation 23 North Avenue Wakefield, MA 01880
Photowatt International, Inc. 21012 Lassen Street Chatsworth, CA 91311	Spectrolab, Inc. 12484 Gladstone Avenue Sylmar, CA 91342
SES Incorporated Tralee Industrial Park Newark, DE 19711	Spire Corporation Patriots Park Bedford, MA 01730
<i>Europe</i>	
AEG Telefunken Discrete Component Dept. P.O. Box 1109 7100 Heilbronn, W. Germany	RTC (Philips Group) Route de la Delivrande 14000 Caen-Cedex, France
<i>Japan</i>	
Japan Solar Energy Co. 11-17 Kogahonmachi Fushimiku, Kyoto	Sharp Corp. Engineering Division 2613.1 Ichinomoto Tenri-Shi, Nara
Matsushita Electric Kadoma, Osaka	
<i>Australia</i>	<i>India</i>
Tideland Energy Pty. Ltd. P.O. Box 519 Brookvale, N.S.W. 2100	Central Electronics Ltd. Site 4, Industrial Area Sahibabad, U.P. 201005

The standard technology for making cells can be broken down into the following stages:

1. Reduction of sand to metallurgical-grade silicon.
2. Purification of metallurgical-grade silicon to semiconductor-grade silicon.
3. Conversion of semiconductor-grade silicon to single-crystal silicon wafers.
4. Processing of single-crystal silicon wafers into solar cells.
5. Solar cell encapsulation into weatherproof solar cell modules.

6.2 SAND TO METALLURGICAL-GRADE SILICON

Silicon is the second most abundant element in the earth's crust. The source material for the extraction of silicon is silicon dioxide, the major constituent of sand. However, in the present commercial extraction process, the crystalline form of silicon dioxide, quartzite, is used. This material is reduced in large arc furnaces, as illustrated in Fig. 6.1, by carbon (in the form of a mixture of wood chips, coke,

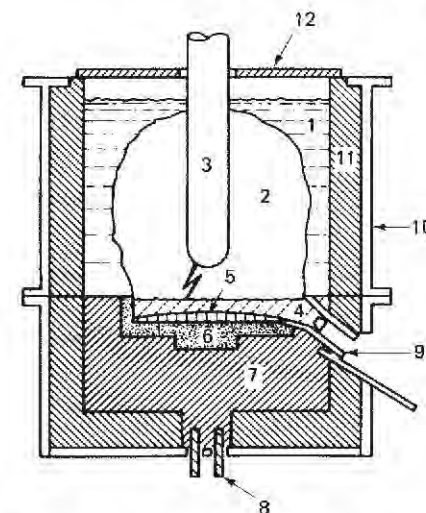


Figure 6.1. Cross section showing typical features of an arc furnace as used to produce metallurgical-grade silicon: 1, carbon and quartz; 2, cavity; 3, electrode; 4, silicon; 5, silicon carbide; 6, hearth; 7, electrode paste; 8, copper electrodes; 9, tapping spout; 10, cast-iron shell; 11, ceramic; 12, graphite lid. (After Ref. 6.1, © 1976 IEEE.)

Table 6.2. TYPICAL CONCENTRATIONS OF IMPURITIES IN METALLURGICAL-GRADE SILICON

Impurity	Concentration range (parts per million, atomic)
Al	1500-4000
B	40-80
Cr	50-200
Fe	2000-3000
Mn	70-100
Ni	30-90
P	20-50
Ti	160-250
V	80-200

and coal) to produce silicon according to the reaction (Ref. 6.1)



Silicon is periodically poured from the furnace and blown with oxygen or oxygen/chlorine mixtures to further purify it. It is then poured into shallow troughs, where it solidifies and is subsequently broken into chunks.

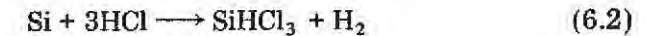
Of the order of 1 million metric tons of this metallurgical-grade silicon (MG-Si) are produced globally each year, primarily for use in the steel and aluminum industries. It is generally 98 to 99% pure, with the major impurities being iron and aluminum, as indicated by the typical breakdown of Table 6.2. The reduction process is reasonably energy-efficient. The total processing energy requirements are similar to those required for related metals such as aluminum or titanium. The material is also quite inexpensive. A very small portion of the total MG-Si production is further purified to *semiconductor grade* (SeG) for the electronics industry, which uses few thousand metric tons of this material annually.

6.3 METALLURGICAL-GRADE SILICON TO SEMICONDUCTOR-GRADE SILICON

For use in solar cells as well as other semiconductor devices, silicon must be much purer than metallurgical grade. The standard approach to purifying it is known as the Siemens process (Ref. 6.2). The MG-Si is converted to a volatile compound that is condensed and

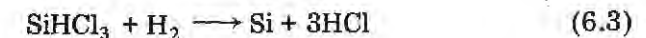
refined by fractional distillation. Ultrapure silicon is then extracted from this refined product.

The detailed processing sequence is that a bed of fine MG-Si particles is fluidized with HCl in the presence of a Cu catalyst to promote the reaction



The gases emitted are passed through a condenser and the resulting liquid subjected to multiple fractional distillation to produce SeG-SiHCl₃ (trichlorosilane). This is a source material for the silicone industry.

To extract SeG-Si, the SeG-SiHCl₃ is reduced by hydrogen when mixtures of the gases are heated. Silicon is deposited in a fine-grained polycrystalline form onto an electrically heated silicon rod according to the reaction



The latter step not only requires a lot of energy but also has a low yield (~37%). This is the main reason for the massive increase in the energy required to produce SeG-Si compared to MG-Si discussed in Section 6.7. The cost increase during this transition is even larger. Consequently, the purification of MG-Si has been a prime target for improved technology.

6.4 SEMICONDUCTOR-GRADE POLYSILICON TO SINGLE-CRYSTAL WAFERS

For the semiconductor electronics industry, silicon must not only be very pure, but it must also be in single-crystal form with essentially zero defects in the crystal structure. The major method used to produce such material commercially is the *Czochralski process* illustrated in Fig. 6.2. The SeG polycrystalline silicon is melted in a crucible with trace levels of one of the dopants required in the completed device added. For solar cells, boron, a *p*-type dopant, is normally used. Using a seed crystal and with very close temperature control, it is possible to pull from the melt a large cylindrical single crystal of silicon. Crystals of diameter in excess of 12.5 cm and 1 to 2 m in length are routinely grown in this manner.

As seen in Section 5.4.2, silicon solar cells need only be 100 μm or so thick to absorb most of the appropriate wavelengths in

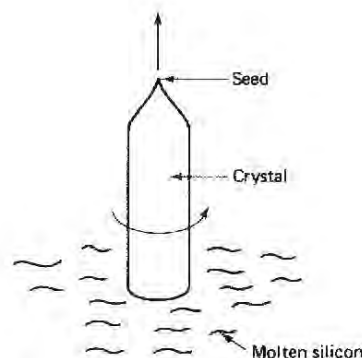


Figure 6.2. Essential features of the Czochralski process for the growth of large cylindrical ingots of single-crystal silicon.



Figure 6.3. Slicing of thin wafers from a cylindrical ingot. The techniques used for this slicing process are described and compared in Ref. 6.3. About half the ingot is wasted as kerf or cutting loss in this process.

sunlight. Therefore, the large single crystal is sliced up into wafers which are as thin as possible, as indicated in Fig. 6.3. With present wafering technology (Ref. 6.3), it is difficult to cut wafers from the large crystals previously described which are any thinner than $300\text{ }\mu\text{m}$ and still retain reasonable yields. More than half the silicon is wasted as kerf or cutting loss in the process. The low overall yield in going from SeG-Si to single-crystal wafers is another weak link in the standard silicon technology.

6.5 SINGLE-CRYSTAL WAFERS TO SOLAR CELLS

After etching the silicon wafers (to remove damage from the wafering process) and cleaning them, additional impurities are introduced into the cell in a controlled manner by a high-temperature impurity diffusion process.

It was mentioned in the preceding section that, in standard solar cell technology, boron was normally added to the melt in the Czochralski process which then results in *p*-type wafers. To form

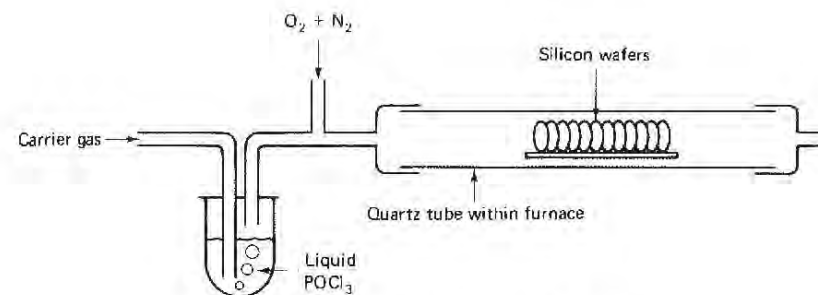


Figure 6.4. Phosphorus diffusion process.

a solar cell, *n*-type impurities must be introduced to give a *p-n* junction. Phosphorus is the impurity generally used. In the most common process, a carrier gas is bubbled through phosphorus oxychloride (POCl_3), mixed with a small amount of oxygen, and passed down a heated furnace tube in which the wafers are stacked as illustrated in Fig. 6.4. This grows an oxide layer on the surface of the wafers containing phosphorus. At the temperatures involved (800 to 900°C), the phosphorus diffuses from the oxide into the silicon. After about 20 min, the phosphorus impurities override the boron impurities in the regions near the surface of the wafer to give a thin, heavily doped *n*-type region shown in Fig. 6.5(a). In subsequent processing, the oxide layer is removed, as are the junctions at the side and back of the cell to give the structure of Fig. 6.5(b).

Metal contacts are then attached to both the *n*-type and the *p*-type regions. In the standard technology, the process used is known as *vacuum evaporation*. The metal to be deposited is heated in a vacuum to a high-enough temperature to cause it to melt and vaporize. It will then condense on any cooler parts of the vacuum system in direct line of sight, including the solar cells. The back contact is normally deposited over the entire back surface, while the top

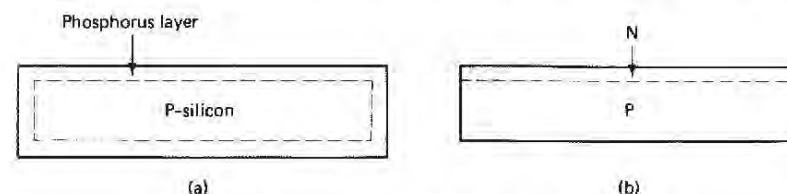


Figure 6.5. Distribution of phosphorus impurities:
(a) Immediately after diffusion process.
(b) After etching of back and side of wafer.

contact is required in the form of a grid. Two techniques are available for defining such a pattern. One is to use a metal *shadow mask* (Fig. 6.6). Alternatively, the metal can be deposited over the entire front surface of the cell and subsequently etched away from unwanted regions using a photographic technique known as *photolithography*.

The contact is usually made up of the three separate layers of metal. A thin layer of titanium is used as the bottom layer for good adherence to silicon. The top layer is silver for low resistance and solderability. Sandwiched between these two is a layer of palladium which prevents an undesirable reaction between the Ti and Ag layers in the presence of moisture. After deposition, the contacts are sintered at 500 to 600°C to give good adherence and low contact resistance. Finally, a thin antireflection (AR) coating is deposited on the top of the cell by the same vacuum evaporation process.

Yields of about 90% from starting wafers to completed terrestrial cells can be obtained. All the processes above are done on a *batch* basis, where a batch of perhaps 40 to 100 wafers moves through the processing sequence at the same time. This makes the processing very labor-intensive. In addition, the vacuum evaporation equipment is expensive compared to its throughput. Moreover,

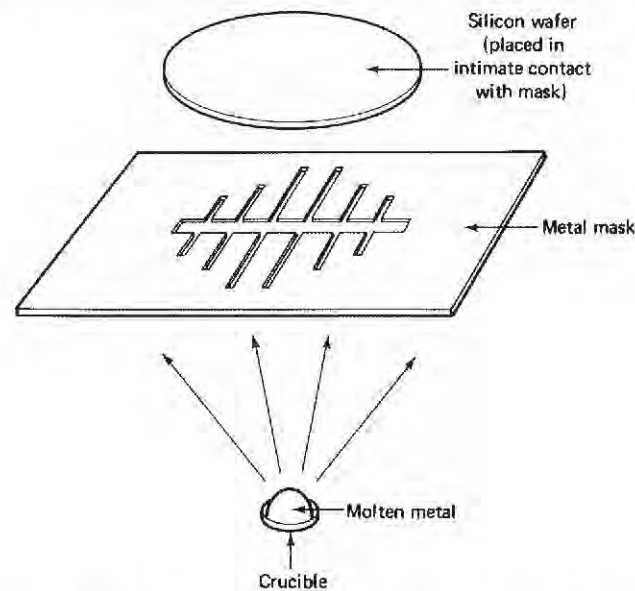


Figure 6.6. Essential features of vacuum evaporation of metallic layers as well as the use of a metal mask to define the top metal grid pattern.

because of the nature of the evaporation process, only a small fraction of the material being evaporated ends up where desired. This is quite extravagant when dealing with material as expensive as silver.

6.6 SOLAR CELLS TO SOLAR CELL MODULES

6.6.1 Module Construction

Solar cells require encapsulation not only for mechanical protection but also to provide electrical isolation and a degree of chemical protection. Encapsulation provides mechanical rigidity to support the brittle cells and their flexible interconnections. It also provides protection from mechanical damage as may be caused by hail, birds, and objects dropped or thrown onto the module. Encapsulation protects metallic contacts and interconnections from corrosive elements in the atmosphere. Finally, it provides electrical isolation of the voltages generated by the panel. These may reach voltages as high as 1500 V above ground in some systems. The durability of the encapsulation will determine the ultimate operating life of the module, which ideally should be 20 years or more.

Additional features an encapsulation scheme must possess are ultraviolet (UV) stability, tolerance to temperature extremes and thermal shocks without stressing cells to fracture, resistance to abrasion as might occur during dust storms, self-cleaning ability, ability to keep cell temperature low to minimize power loss, and low cost.

Several different approaches to module design are possible. One essential component is a structural layer to provide rigidity. This layer can be either at the back or front of the module as indicated in Fig. 6.7. Cells can be either bonded directly to this layer and encapsulated in a flexible potting or enclosed in a laminate supported by it. The final layer, if at the rear of the module, acts as a moisture barrier. If at the top, it has the additional duties of imparting self-cleaning properties and improving impact resistance. Some form of moisture seal is incorporated around the edges of the module.

For the *structural back* configuration of Fig. 6.7(a) and (b), the most common materials for the structural back have been anodized aluminum, porcelainized steel, epoxy board, or window glass. Wood composites such as chipboard may prove to be the cheapest overall possibilities for this layer. For the *structural front configuration* of Fig. 6.7(c) and (d), glass is the obvious choice for the structural layer. It combines excellent weatherability with low cost and good self-cleaning properties. Most designs use a toughened or tem-

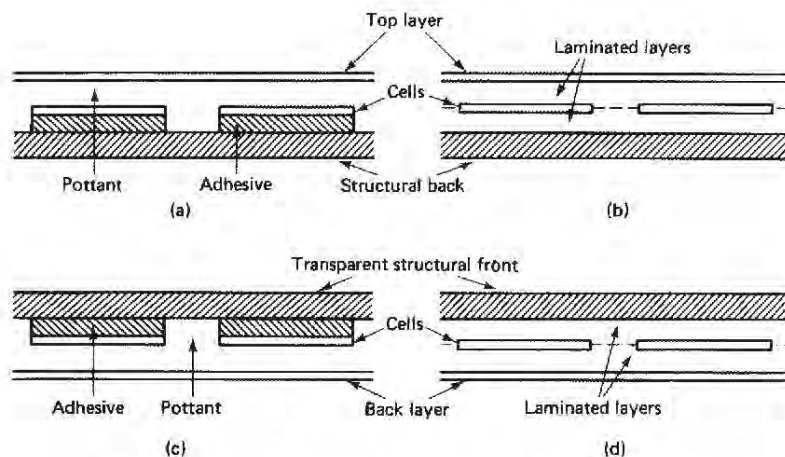


Figure 6.7. Schematic diagrams showing possible approaches to the encapsulation of solar cells:

- (a) Structural back configuration.
- (b) Structural back configuration with laminated layers.
- (c) Structural front configuration.
- (d) Structural front configuration with laminated layers.

pered glass with a low iron content for good light transmission. For the adhesive and pottant layers, silicones have been widely used. They have good UV stability, low light absorption, and are appropriately elastic to alleviate thermal stresses in the module, but they are expensive. Polyvinyl butyral (PVB) and ethylene/vinyl acetate (EVA) have also been used by several manufacturers for the corresponding layers in the laminated approach.

The top layer of the *structural back configuration* serves to impart self-cleaning properties and in some designs acts as a moisture barrier. Low-iron glass again has been a popular choice. Polymers such as acrylics have also been used. Some manufacturers have concentrated on producing a moisture-resistant cell, thereby relaxing encapsulation requirements. These may use a soft silicone encapsulant and a harder silicone overlying layer to improve self-cleaning. The most popular choices for the back layer in the structural front approach have been Mylar or Tedlar to act as a moisture barrier. However, all polymers are permeable to moisture to some extent. A solution to this has been to use a thin foil of aluminum or stainless steel embedded between layers of an appropriate polymer.

If the back layer is white, it is possible to increase the module output marginally using the *zero-depth concentration effect* (Ref. 6.4). Some of the light striking regions of the module between cells

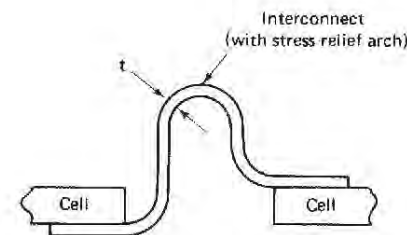


Figure 6.8. Stress-relief loop, as generally required in the metallic interconnections between cells to prevent fatigue due to cyclic thermal and wind loading stresses. For maximum effect, the thickness of the interconnect, t , should be small and the height of the arch high as discussed in Ref. 6.5.

will be scattered by the back layer and channeled by the glass superstrate to active regions of the module. This can give a boost in the module output, particularly when the packing density of cells is low.

Another important area of module design concerns the interconnections between the cells. It is common practice to use multiple interconnects for redundancy. This increases the module tolerance to interconnect failure (by corrosion or fatigue) as well as to cracked cells. Cyclic stresses are set up in the interconnects due to differentials in temperature expansion coefficients and wind loading. A stress-relief loop as indicated in Fig. 6.8 is generally required in the interconnect (Ref. 6.5).

6.6.2 Cell Operating Temperature

Different module designs will cause cells encapsulated within them to reach different temperatures under identical operating conditions. Since the cell performance is adversely affected by increasing temperature (Section 5.3), the modules operating at cooler temperature will have a relative performance boost.

Rather than comparing the performance of different modules at the same temperature, it is more appropriate to compare the performance at different temperatures. In each case, this would be the temperature that the cells would reach under typical operating conditions. If a standard set of operating conditions (insolation level, wind velocity and direction, ambient temperature, electrical loading of the cells) is defined, it is possible to associate a particular temperature, the *nominal operating cell temperature* (NOCT), with each module type. Experimental techniques have been developed (Ref.

6.6) for calculating this temperature from field data at nonstandard operating conditions.

Field data indicate that the operating temperature of the solar cell above ambient is roughly proportional to the intensity of the incident sunlight, provided that wind velocity is not excessive. As a rule of thumb, most commercial modules allow the cell temperature to rise about 30°C above ambient under full sun irradiation (100 mW/cm²) when mounted on an open frame. Hence, an approximate expression for the cell temperature is

$$T_{\text{cell}}(^{\circ}\text{C}) = T_{\text{ambient}}(^{\circ}\text{C}) + 0.3 \times \text{intensity (in mW/cm}^2\text{)} \quad (6.4)$$

When roof-mounted, cell operating temperatures would tend to be higher.

6.6.3 Module Durability

The durability of the solar cell encapsulation will ultimately determine the operating life of a solar cell system, since there is no “wear-out” mechanism associated with cell operation in the terrestrial environment.

The types of module degradation that have been observed in the field in the past have been:

1. Breakage of cells due to excessive mechanical stress caused by thermal fluctuations or, more directly, by hail damage or vandalism.
2. Corrosion of metallization.
3. Delamination of different layers of the encapsulation.
4. Discoloring of encapsulating material.
5. Accumulation of dirt on modules with “soft” top surfaces.
6. Breakage of interconnects due to inadequate stress relief.

With increasing field experience, module design has steadily improved to the stage where a 20-year operating life is feasible. Accelerated life-testing of new module designs is normally carried out by subjecting them to the types of stress below:

1. Thermal cycling
2. High humidity

3. Prolonged ultraviolet radiation
4. Cyclic pressure loading

Combinations of these stresses will often enhance module degradation. Other types of qualification testing that may be carried out include:

1. Impact testing
2. Abrasion resistance
3. Self-cleaning properties
4. Flexibility (to test for mounting onto warped surfaces)
5. Electrical insulation (particularly after accelerated life-testing)

Although dirt accumulation has caused severe degradation in some locations for modules with soft top surfaces (Ref. 6.7), this is not a major problem with glass-faced modules. Self-cleaning by rain and wind keeps the power loss from this effect to less than 10%. Since cells can operate from scattered light, modules have been found to give a reasonable percentage of their peak output even when deliberately covered with sufficient dust so that individual cells in the module can barely be distinguished (Ref. 6.8).

6.6.4 Module Circuit Design

The electrical circuit aspects of the way solar cells are interconnected within solar modules can have a substantial effect on the field performance and operating life of the modules.

When solar cells are connected together, mismatches in their operating characteristics ensure that the output power of the combination is less than the sum of the maximum output power of the constituents. The difference, the *mismatch loss*, is the most significant for cells connected in series.

Even more important than the loss in power is the potential for overheating in the poorest cell of a series string. Figure 6.9 shows the output characteristic of the lowest current output cell in a series string together with the combined output of the rest of the cells. The voltage across each of these components must be equal and opposite in sign when the module is short-circuited. The module short-circuit current can be found by reflecting one of the curves of Fig. 6.9 in the current axis as shown and finding its intercept with the other. Note that, in this condition, the poorest cell is reversed-biased

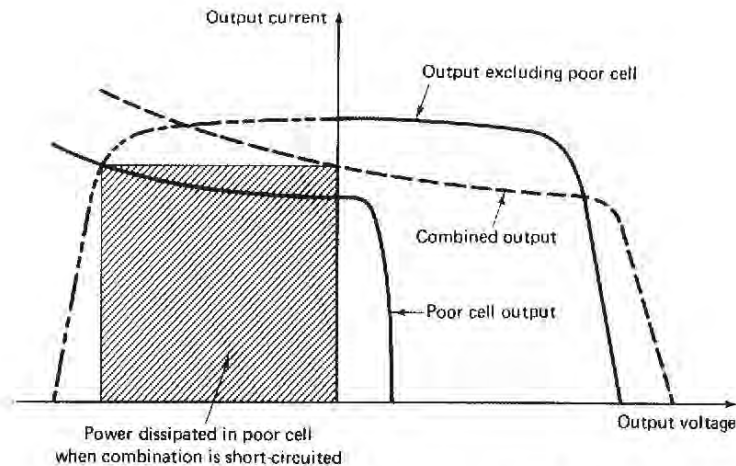


Figure 6.9. Effect of a cell with mismatched output in a series connection of cells. Under short-circuit, the cell with poor output becomes reverse-biased and can dissipate substantial amounts of power. The current output of the series string is determined by that of the poorest cell.

and power equal to the shaded area shown is being dissipated in it. It is apparent from Fig. 6.9 that power up to the maximum generating capability of the rest of the series string can be dissipated in the poorest cell under some conditions. Such effects can cause an excessive temperature rise in localized regions of the poorest cell, which can damage the cell encapsulation and cause the eventual deterioration of an entire module. The same effect can be caused by partial shadowing of some cells in a module or by cracked cells.

It is not difficult to show that the open-circuit voltage of a series string is the sum of those of the individual cells. Another property apparent from Fig. 6.9 is that the short-circuit current will be determined by that of the lowest output current cell in the string. It follows that severe mismatch in the short-circuit currents can cause the current-generating capabilities of the better cells in the string to be entirely wasted. Although similar *mismatch losses* also occur in parallel connections of cells, they are not nearly as severe.

Two techniques are available for lessening the severity of the foregoing effects (Ref. 6.9). One is a technique known as *series-parallel* and the other is by the use of *bypass diodes*. Figure 6.10 defines the terminology used in describing such module circuit design techniques.

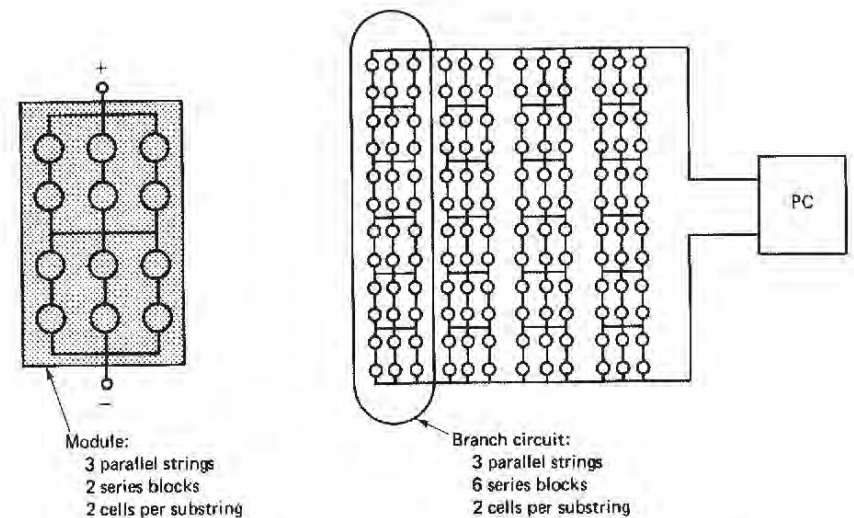


Figure 6.10. Terminology used in module circuit design. The block labeled PC is power conditioning equipment. (After Ref. 6.9, © 1980 IEEE.)

By increasing the numbers of *series blocks* and *parallel strings* per module or *branch circuit*, tolerance to cell mismatch, cracked cells, and partial shadowing is increased. This is the technique known as *series-parallel*. An alternative approach is to use a *bypass diode* connected across one or more *series blocks* in the module. The bypass diode becomes forward-biased when a series block becomes reversed-biased. This limits the power dissipated in this block as well as providing a low-resistance path for the module or *branch string* current.

6.7 ENERGY ACCOUNTING

It is naturally important that any power-generating device used on a large scale delivers more energy over its operating life than that invested in constructing, commissioning, and maintaining it. How do silicon cells fabricated using the standard technology described in this chapter fare in this regard?

The extraction of metallurgical-grade silicon from quartzite is quite an energy efficient process. Taking into account the energy required to mine, deliver, and prepare the raw materials for this

process together with the processing energy required, the equivalent of about 24 kWh of electrical energy [kWh(e)] is required to produce 1 kg of this material (Ref. 6.1). This is roughly in the same area as that for aluminum [19 kWh(e)/kg] or titanium [46 kWh(e)/kg] calculated on the same basis.

The Siemens process for purifying this to semiconductor-grade quality is costly, inefficient, and energy-intensive. This has made it an obvious target for replacement in future silicon cell technology. On the same basis as above, the energy content of semiconductor-grade silicon is 621 kWh(e)/kg (Ref. 6.1).

The conversion of this pure silicon to single-crystal sheet form via the Czochralski process and subsequent slicing of the cylindrical ingots produced by this process does not make good use of the semiconductor-grade silicon. Sheet is produced at the rate of about 0.4 m²/kg of this material. The major reason for this low yield is the nonideal nature of the cutting process, which wastes half the starting material and produces wafers thicker than required from photovoltaic considerations. The energy content of silicon wafers on the same basis as before is 1700 kWh(e)/m².

Cell processing and encapsulation add an estimated 250 kWh(e)/m². Assuming a 90% yield from wafers to completed modules gives a total energy content of 2170 kWh(e)/m² of cell area.

The time the cell would have to operate to pay back this energy invested in it depends on where it is deployed. With the equivalent of 5 h of peak sunshine per day on average and 12% encapsulated cell efficiency, the annual amount of energy generated would be 219 kWh(e)/m². Hence, the energy payback time would be just under 10 years. Less direct energy inputs such as the energy required to make the machines used in cell manufacture plus the energy required to market and install systems together with the energy content of associated power storage and conditioning equipment would increase this payback time further.

However, the important point is that the same processing steps that have precluded the widespread use of silicon cells in the past on economic grounds are also those which contribute most significantly to the energy content of the cells. Improved technology for producing silicon cells as described in Chapter 7 not only improves the economic viability but also reduces the energy required to manufacture a solar cell dramatically. With such technology, energy payback times are reduced from 10 years for the inefficient processing sequence outlined in the present chapter to a fraction of a year.

6.8 SUMMARY

The standard solar cell processing technology as developed for space cells and initially used for terrestrial cells has been described as have encapsulation schemes for packaging the cells in weatherproof modules. Several areas have been identified where improvements in the standard technology are desirable. Improved technologies for producing the cells are described in Chapter 7.

Cells produced for space and those produced in the past for terrestrial use with the standard processing described in this chapter have substantial amounts of energy invested in them compared to their generating capabilities. This is not the case for the improved technologies of Chapter 7.

EXERCISES

- 6.1. Draw a block diagram showing the major steps involved in converting quartzite to silicon solar cells.
- 6.2. For solar modules made up of 12% efficient silicon cells, estimate the difference in the operating temperature of cells under bright sunshine when the module is open-circuited and when it is delivering maximum power.
- 6.3. One solar cell has an open-circuit voltage of 0.55 V and a short-circuit current of 1.3 A. A second cell has values of 0.60 V and 1.0 A for these parameters. Assuming that both cells obey the ideal diode law, calculate the open-circuit voltage and short-circuit current of the combination when the cells are connected: (a) in parallel; (b) in series.
- 6.4. A hypothetical solar module consists of 40 identical solar cells, each giving an open circuit voltage of 0.60 V and a short-circuit current of 3 A under bright sunshine. The module is short-circuited under bright sunshine and one cell partially shaded. Assuming that the cells obey the ideal diode law and neglecting temperature effects, find the power dissipated in the shaded cell as a function of the fractional shading of the cell.

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Chapter 7

IMPROVED SILICON CELL TECHNOLOGY

7.1 INTRODUCTION

In Chapter 6, the standard technology used in the past to produce silicon solar cells was described. In converting quartzite, the source material, to encapsulated cells, several steps were seen to be costly and energy-intensive.

Activity worldwide is being directed at reducing these costs. The present chapter describes some of the most promising new silicon technology for each of the processing steps outlined in Chapter 6. Most of this technology is at an advanced stage of development. Some aspects are being evaluated at the pilot production stage, whereas others have already been incorporated into commercial products.

7.2 SOLAR-GRADE SILICON

It was seen in Chapter 6 that solar cells presently use the ultrapure silicon produced for the semiconductor industry. However, in tran-

sistors and integrated circuits, the emphasis is on silicon quality, and material costs are relatively unimportant. For solar cells, it is worthwhile considering a trade-off between performance and cost.

As mentioned in Section 3.4.4, impurities in solar cells generally introduce allowed levels into the forbidden gap and thereby act as recombination centers. It was seen in Section 5.4.2 that an increased density of such centers will decrease the cell efficiency. Figure 7.1 shows the experimental results for a range of different metallic impurities, when each is the only impurity present apart from the dopant (Ref. 7.1). Although some metallic impurities (Ta, Mo, Nb, Zr, W, Ti, and V) can reduce cell performance when present in extremely small concentrations, others can be present in concentrations in excess of $10^{15}/\text{cm}^3$ before becoming a problem. This is about 100 times higher than impurity levels in semiconductor-grade silicon (SeG-Si). It makes it likely that an alternative, less expensive process might produce a less pure *solar-grade* silicon (SoG-Si) which still gives cells of adequate performance. Several alternative processes appear capable of producing silicon of quality not appreciably lower than Se-G but at a fraction of the cost of conventional technology.

One of the more promising is that developed by the Union Carbide Corporation. It involves the preparation of silane (SiH_4) from

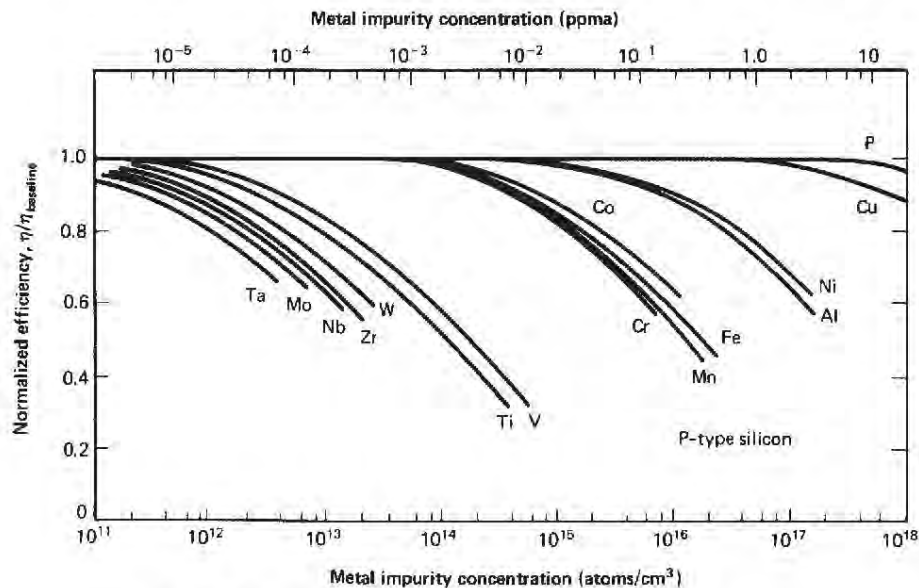


Figure 7.1. Effect of different secondary impurities on the performance of silicon solar cells (After Ref. 7.1, ©1978 IEEE.)

metallurgical-grade silicon and the subsequent deposition of silicon from silane (Ref. 7.2). Analysis of this process indicates that it is capable of producing silicon at one-fifth the cost of the present commercial process using only one-sixth of the processing energy. An alternative process at an advanced stage of development is that developed by Batelle Columbus Laboratories. It is based on the zinc reduction of silicon tetrachloride (Ref. 7.3). It is probable that improved processes such as these will replace the conventional Siemens process for producing silicon not only for the solar cell industry but also for the semiconductor electronics industry in general.

7.3 SILICON SHEET

7.3.1 Sheet Requirements

Having produced purified silicon, it is then necessary to convert it into the form of thin sheets of good crystallographic quality for use as solar cells. To obtain the full photovoltaic output from the material, it only has to be $100\ \mu\text{m}$ or so thick. In the past, the approach used has been to form a large single-crystal ingot using the Czochralski (CZ) process and then slice thin wafers from this ingot. This approach is very inefficient in converting bulk silicon into large areas of solar cells. Not only is over half the silicon lost in sawing it into wafers, but cutting limitations result in thicker wafers than essential. Also, the wafers are circular, which means that they cannot be packed very densely when encapsulated into solar modules unless trimmed to a square or hexagonal shape.

7.3.2 Ingot Technologies

The Czochralski approach is an example of an *ingot technology*. The fundamental limitation of such a technology is that the ingots produced have to be sliced up into wafers with the disadvantages already mentioned.

The standard Czochralski process can be modified to operate on a semicontinuous basis with resultant cost savings (Ref. 7.4). The fact that the process produces cylindrical ingots¹ is also a disadvantage in solar cell applications.

A simpler approach to producing ingots, particularly those of square cross section, is to use a process similar to casting. This will

¹With modifications, ingots with an approximately square cross section can be prepared by the Czochralski process (Ref. 7.5).

generally result in a polycrystalline ingot which is not ideal for solar cell applications. However, with careful control over the conditions under which molten silicon solidifies, silicon of large grain size can be formed. With a suitable "mold" material, such large-grained material can produce solar cells of good performance (Ref. 7.6).

With a suitable seed and a method of controlling the rate of solidification as in the heat-exchanger method (Ref. 7.7), it is possible to produce essentially single-crystal ingots of quite massive proportions with a "casting" approach. The performance of cells fabricated from such material has been comparable to that of cells made of good-quality Czochralski material. Studies indicate that this approach has a marked economic advantage over even the advanced Czochralski techniques (Ref. 7.8).

7.3.3 Ribbon Silicon

The limitations of the ingot approach can be avoided if the silicon can be formed directly into sheets or ribbons. Several techniques have been developed for achieving this.

The first to be developed to the stage of producing commercial cells is the *edge-defined film-fed growth* (EFG) method, as illustrated in Fig. 7.2. This technique bears some relationship to the Czochralski process except that the shape of the crystal pulled is defined by a graphite "die." As a consequence, it is possible to obtain crystals in the form of thin ribbons directly from the melt (Ref. 7.9). Very high production rates of silicon can be obtained by pulling several ribbons simultaneously from the same melt (Ref. 7.10).

The major problem with this technique relates to the quality of the material produced. The process produces material of relatively poor crystallographic quality compared to the Czochralski process. Owing to the nature of the growth process, impurities introduced into the molten silicon from the die, crucible, and surrounding regions of the growth furnace are incorporated into the growing ribbon. (In other crystal growth processes, most of these are preferentially rejected from the growing crystal to the molten silicon.) In addition, the molten silicon tends to react with the graphite die. This can result in silicon carbide precipitates in the ribbon which can disrupt its growth and degrade the properties of subsequently fabricated cells.

The dendritic web method illustrated in Fig. 7.3 overcomes some of these disadvantages. By controlling temperature gradients, it is possible to encourage the growth of parallel dendrites into the molten material. As these are pulled out of the melt, a film of molten silicon is trapped between them (Ref. 7.11). This subsequently solid-

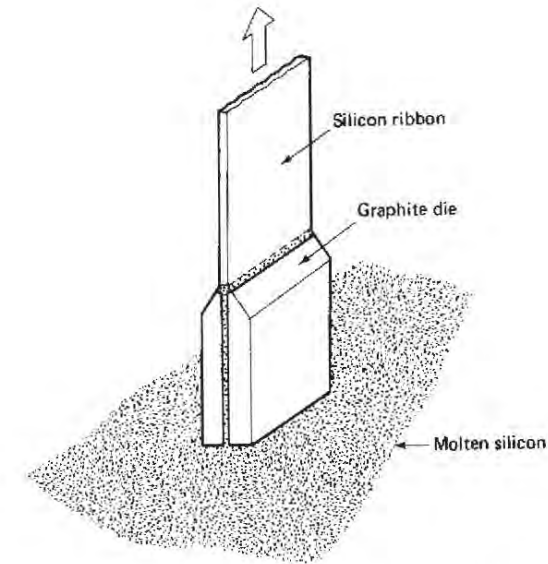


Figure 7.2. Essential features of the edge-defined film-fed growth method for producing ribbon crystals of silicon. The molten silicon moves up the interior of the graphite die by capillary action. The ribbon shape is defined by the shape of the top of the die.

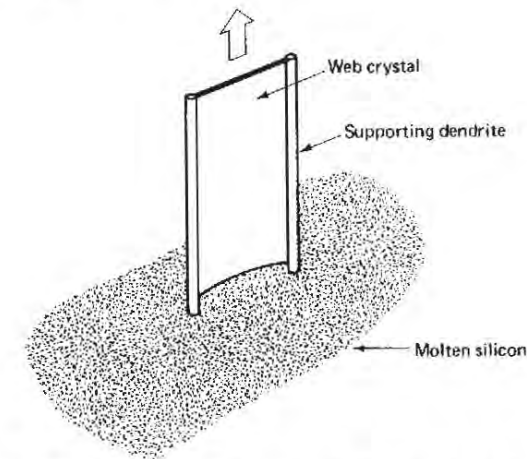


Figure 7.3. Dendritic web approach to producing silicon ribbon. No die is required. The shape of the ribbon is controlled by thermal gradients in the molten silicon. The dendrites down the edge of the ribbon solidify first. As these are drawn out of the melt, a thin layer of initially molten silicon is trapped between them.

ifies to give a thin ribbon of silicon bounded by the thicker dendrites. These can be removed and recycled after cell fabrication on the ribbon section. No die is required and material properties are nearly as good as these obtained from the Czochralski process. The relatively low production rate of silicon crystal material by this process has been its major disadvantage.

Relatively small samples of silicon ribbon produced by both the EFG and dendritic web processes are compared in Fig. 7.4. Note the corrugated surface finish typical of material produced by the EFG process and the mirror finish characteristic of dendritic web material.



Figure 7.4. Comparison of the physical appearance of EFG and dendritic web silicon ribbon crystals. The dendrites are removed from the latter after cell processing. (Ribbon courtesy of Japan Solar Energy Corporation and Westinghouse Research Laboratories.)

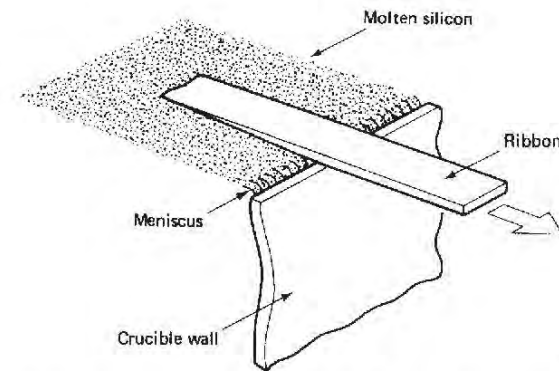


Figure 7.5. Horizontal or low-angle growth method for producing silicon ribbon. Ribbons can be produced at very high rates, but dimensional control has been a problem in the past.

Several other ribbon growth processes are at a less advanced stage of development. One distinguished by very high rates of ribbon production is the *horizontal or low-angle growth method* of Fig. 7.5. The major developmental problems with this technique have related to control of the ribbon dimensions (Ref. 7.12). Casting of silicon directly into sheet or ribbon form has also been explored (Refs. 7.6 and 7.13).

7.4 CELL FABRICATION AND INTERCONNECTION

The disadvantage of the standard technique for fabricating cells on silicon wafers as described in Chapter 6 is that it is a "batch"-orientated technique. As such, it is capable of only relatively low throughput and also is extravagant in the use of materials, particularly metal for forming the cell contacts. Several approaches are now being used in commercial devices which overcome these limitations and, as a bonus, result in cells of higher performance.

One major development has been the use of silicon wafers with *textured surfaces*. Using selective etching, it is possible to form small pyramids on the surface of silicon wafers (Ref. 7.14). The appearance of these pyramids greatly magnified under the scanning electron microscope is shown in Fig. 7.6. Light reflected from the side of one of these pyramids is reflected downward, getting a second chance of being coupled into the cell. With the use of an antireflection coating, reflection losses can be reduced to less than a few percent. As shown



Figure 7.6. Appearance of a textured silicon surface under a scanning electron microscope. The peaks are typically $10\ \mu\text{m}$ high and are the tops of square-based pyramids. The sides of these pyramids are intersecting (111) planes within the crystal structure of the silicon.

in Chapter 8, this technique can give acceptable performance even without antireflection coatings.

In the previous standard process, the p - n junction was formed by diffusing impurities on a batch basis. A more cost-effective approach is to spray layers containing the required dopant onto the wafer surface and to diffuse the impurity in using a continuous-belt process (Ref. 7.15). An alternative approach is to use a technique known as *ion implantation* (Ref. 7.16). Ions of the dopant are accelerated to high velocities and directed at the wafer surface. These are embedded in the silicon wafer near the surface they impact. A subsequent annealing step removes the damage inflicted on the silicon lattice by this process and also “activates” the dopants in an electronic sense. Pulsing by an electron beam or laser is an energy-efficient annealing approach. A very tidy process for the high-volume production of solar cells based on ion implantation is indicated in Fig. 7.7.

Further improvement in solar cell performance has been obtained by paying attention to obtaining a low-surface-recombination-velocity contact to the rear of the cell. As indicated in Chapter 5, this improves the open-circuit voltage of the cell and it also marginally increases the current output. The use of what is somewhat

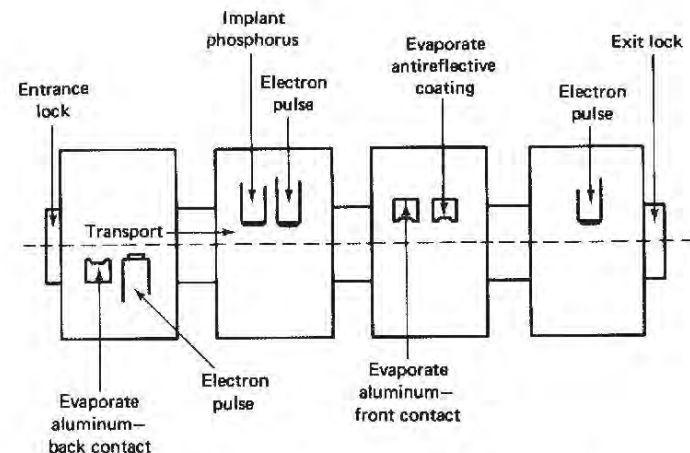


Figure 7.7. Schematic diagram of a continuous vacuum process for producing solar cells based on ion-implanted junctions, evaporated metal contacts, and pulsed electron beam annealing. (After Ref. 7.16, © 1976 IEEE.)

inappropriately called a *back surface field* (BSF) is one technique for achieving low effective recombination velocities at the rear of the cell. As indicated in Fig. 7.8, a heavily doped region is included right at the back contact to the cell. From theoretical considerations, the interface between this region and the more lightly doped bulk regions can be shown to act as a low-recombination-velocity surface. Not only does this technique improve voltage and current output as mentioned, but it also makes it easier to make a low-resistance contact to the silicon at the rear of the cell. In practice, the most effective technique for producing this “back surface field” has been to screen print an alu-

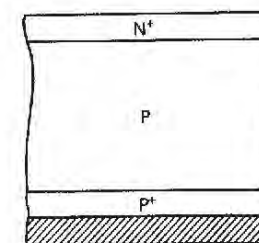


Figure 7.8. Sketch of the N^+PP^+ solar cell. The heavily doped P^+ region at the rear of the cell blocks minority carrier flows, making the PP^+ junction plane an effective low-recombination-velocity surface.

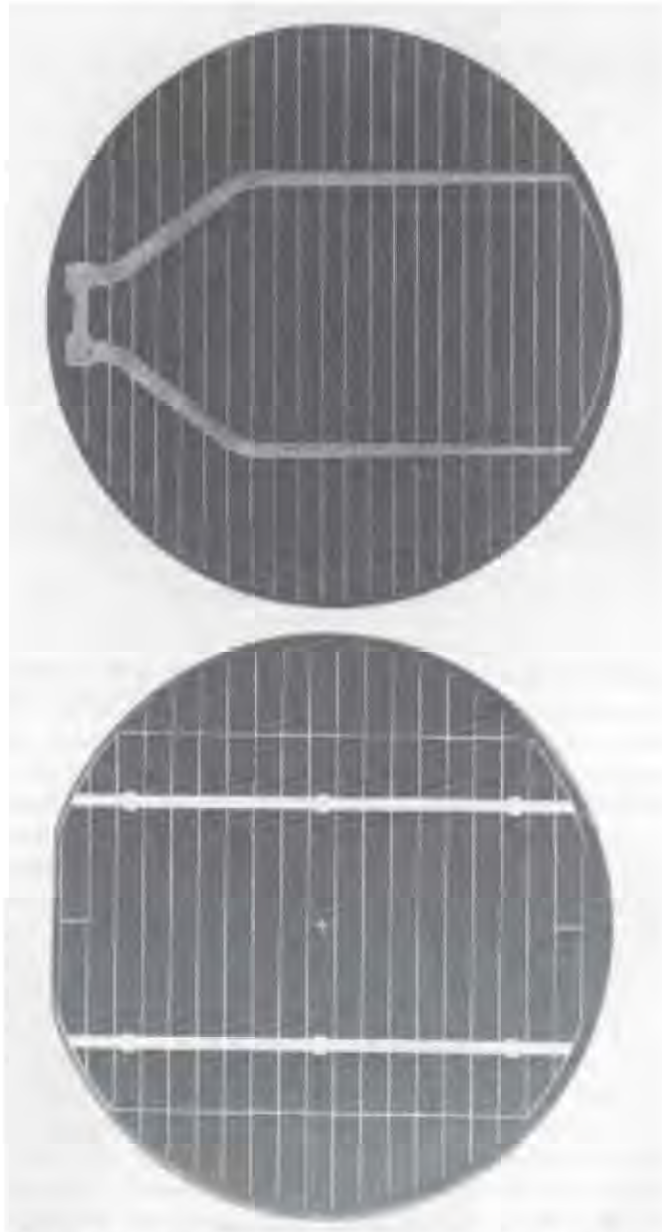


Figure 7.9. Solar cells of 10 cm in diameter with solder-dipped (upper) and screen-printed (lower) top-layer metallization.

minum-based paste onto the rear of the cell and alloy the aluminum into the silicon in a subsequent firing step (Ref. 7.15).

Metallization was one of the weak links in the standard solar cell processing sequence of Chapter 6. Two lower-cost processes used in commercial cells are screen printing and electroplating. Both of these prevent wastage of metal and eliminate the requirement for expensive vacuum equipment. In the former, a paste containing the metal is printed onto the silicon wafer through a mask and subsequently fired to remove “binders” in the paste and to lower the metal resistivity. Silver pastes were the first to be used commercially although nickel, aluminum, and copper pastes may be lower-cost alternatives. In the plating approach, a pattern is etched through an insulating layer on the cell surface and the desired metal layer is plated through this. More than one type of metal layer generally is plated sequentially, since few metals by themselves possess the properties of good adherence to silicon, corrosion resistance, low resistivity, and low cost. A final step in this approach may be “solder dipping” to cover the plated metal by a layer of solder for corrosion protection and lower series resistance. Figure 7.9 compares the physical appearance of cells with solder-dipped and screen-printed top-layer metallization.

Spraying appears to be the most cost-effective method for applying the antireflection coating, although with textured surfaces this layer may not be required. Automated machinery has been developed for connecting the cells together and for their encapsulation into solar modules (Ref. 7.17).

7.5 ANALYSIS OF CANDIDATE FACTORIES

Several alternative approaches to producing silicon cells have been outlined in previous sections. To provide common ground for the comparison of economic aspects of these approaches, a costing technique known as SAMICS (Solar Array Manufacturing Industry Costing Standards) has been developed (Ref. 7.18). Using a computer program based on this technique, some of the best documented and validated costing has been done of the processing sequences involved in fabricating solar modules.

Large-capacity factories have been designed based on different combinations of the technologies described in this chapter and the cost of the resulting solar modules compared using the SAMICS methodology. The results have been that several combinations of these technologies appear capable of producing solar modules at costs which

would make them competitive energy generators in some of the large-scale applications described in Chapter 14. The EFG and dendritic web ribbon approaches as well as HEM (heat-exchanger method) with subsequent wafer slicing all appear viable silicon sheet technologies, with the advanced Czochralski approaches lagging somewhat behind (Ref. 7.8).

As an example of an automated factory capable of producing low-cost silicon modules, the characteristics of one of the first to be analyzed by the SAMICS approach will be described (Ref. 7.19). The product resulting from this factory is a 1.2×1.2 m module made up of 192 cells fabricated on EFG ribbon silicon cut to a size of 10×7.5 cm as shown in Fig. 7.10. The module efficiency is 11.4%.

The processing sequence used to produce the module is shown in Fig. 7.11. The Union Carbide process is used to refine metallurgical-grade silicon. Silicon sheet is prepared from this material by the EFG process and cut to size. The major stages in fabricating cells on this material are the application of a back surface field, texture etching, ion implantation of the junction followed by pulse annealing, and the screen printing of contacts. The cells are then encapsulated and tested.

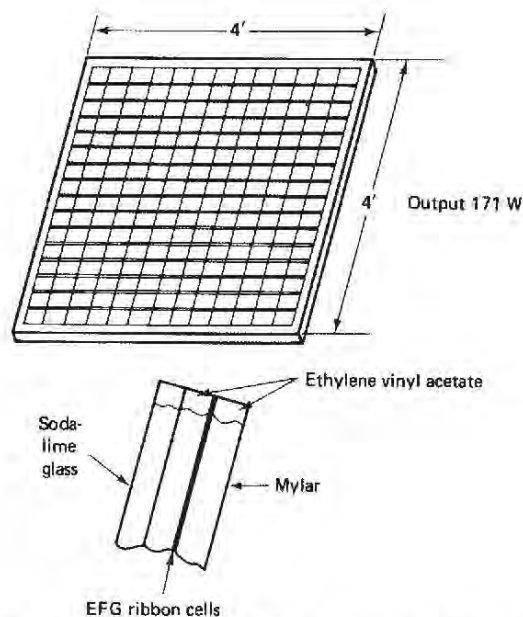


Figure 7.10. Solar cell module produced by the candidate factory described in the text. (After Ref. 7.19.)

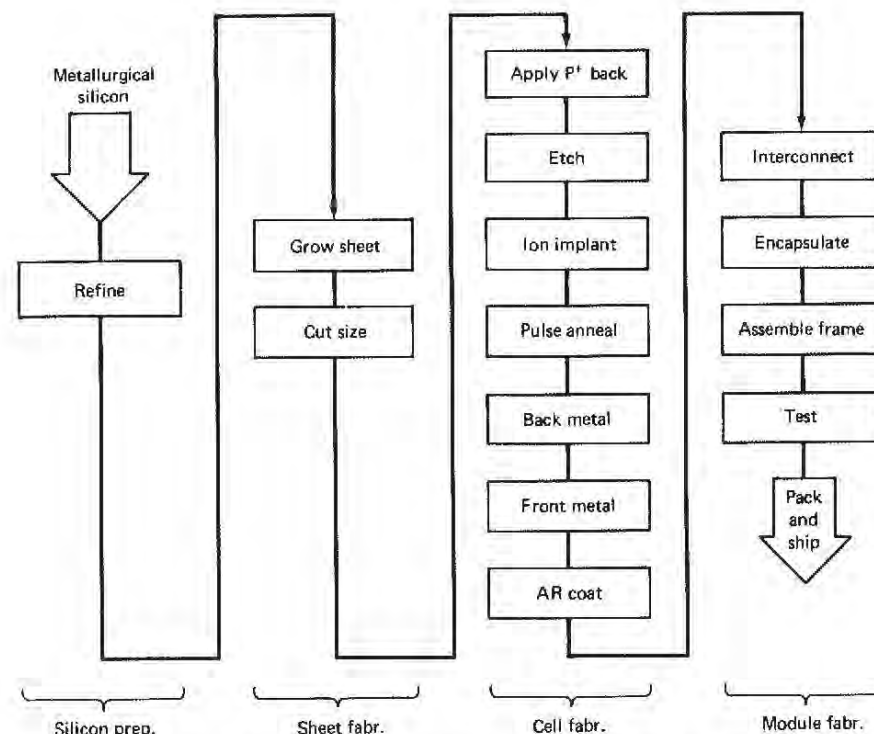
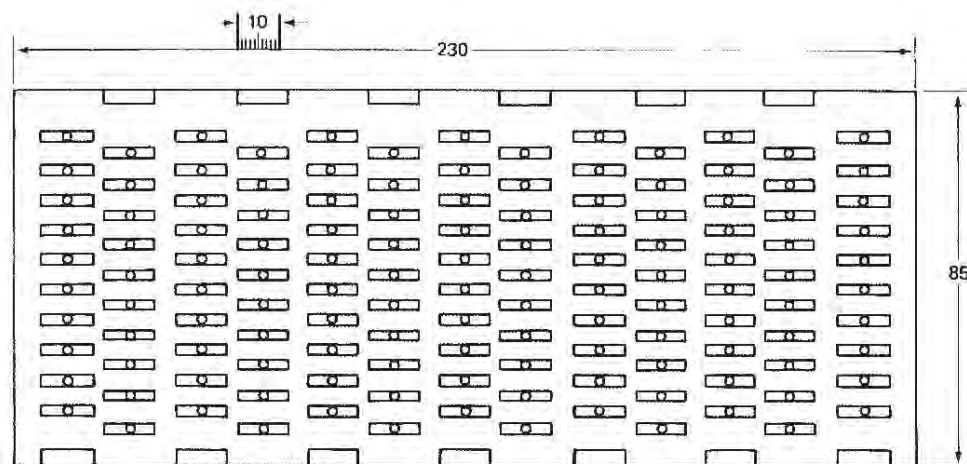


Figure 7.11. Processing sequence used to produce the module of Figure 7.10. (After Ref. 7.19.)

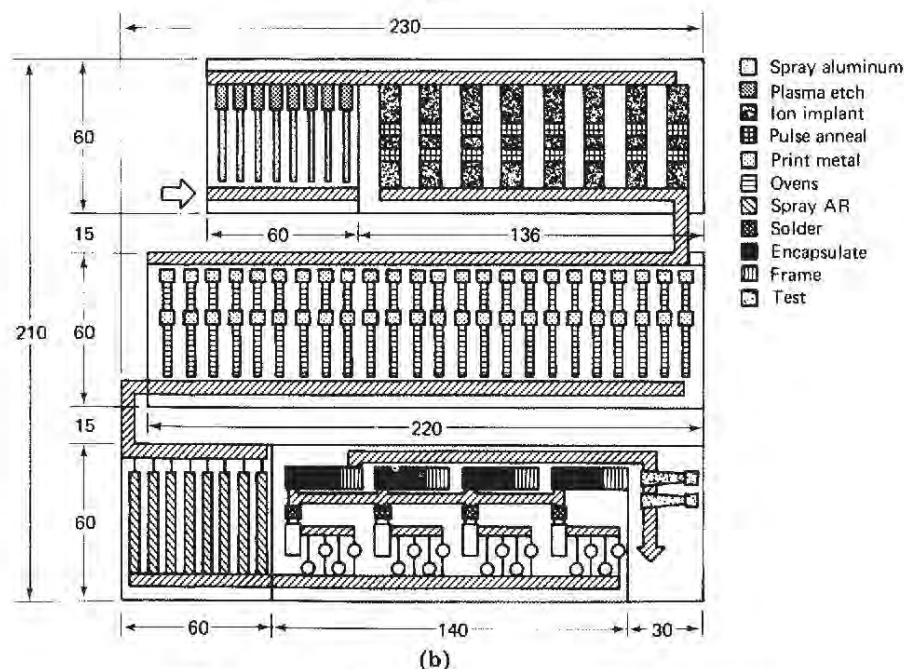
ADM	Silicon refinement 40,800 ft ²	Production rate (W/yr)	250,000,000
		Labor force (all shifts)	1,152 direct 529 indirect
Sheet, cell, and module fabrication 91,630 ft ²		Factory area (ft ²)	
		Silicon refinement	40,800
		Sheet growth	19,550
		Cell fabrication	31,200
		Module fabrication	10,200
		Warehouse	9,779
		Misc. (aisles, shops, cafeteria, etc.)	20,901
		Capital equipment (\$)	
		Silicon refinement	19,400,000 *
		Sheet growth	14,820,000
		Cell and module fabrication	8,219,000
		* Union Carbide	
		Energy payback time sheet, cell, and module 0.179 year	

Figure 7.12. Calculated space, labor, and capital requirements (1975 \$US) to produce cells in a volume of 250 MW_p/yr. The calculated selling price of the module (F.O.B. factory) was \$0.46 per W_p in the same monetary units. (After Ref. 7.19.)



130 pairs of 5-ribbon machines with melt replenisher between 10 pairs with maintenance bench per column
One operator per pair

(a)



(b)

Figure 7.13. (a) Physical layout of the ribbon crystal puller for the candidate factory. Each puller produces five ribbons simultaneously using the EFG approach (dimensions are in feet). (b) Physical layout of cell processing, encapsulation, and testing sections of the plant. (After Ref. 7.19.)

To achieve the low costs mentioned, production volume has to be an estimated 250 MW_p/yr. The labor requirements and area requirements for different steps in the module fabrication are shown in Fig. 7.12. The operating time of the modules to pay back the processing energy invested in them for this processing sequence is a very low 65 days.

To give an idea of the size and physical layout of different sections of the candidate 250 MW_p/yr factory, Fig. 7.13(a) shows a possible layout for the sheet-preparation section of the factory. Figure 7.13(b) shows the sections dealing with cell fabrication, encapsulation, and testing.

7.6 SUMMARY

Several advanced technologies have been developed for producing low-cost silicon solar cell modules. These eliminate the deficiencies in the standard processing sequence described in Chapter 6.

Direct growth of silicon ribbons eliminates wafer slicing, which is the weak link in any ingot technology. The main requirements on techniques for fabricating cells on silicon wafers are that they be capable of a high degree of automation and that they do not consume excessive materials. Several processing sequences satisfy these criteria. Economic analysis of candidate factories based on advanced processing sequences show that it should be possible to produce silicon solar cell modules at prices that would make them competitive with conventional sources of electricity, at least in *some* large-scale applications. A "thin-film" technology as described in Chapter 9 places less severe demands on the quantity of photovoltaic material required. For this reason, approaches other than those based on crystalline silicon, as described in this chapter, will prove even less expensive in the long term.

EXERCISES

- 7.1. The volume density of silicon atoms in a silicon crystal is about $5 \times 10^{28} \text{ m}^{-3}$. According to Fig. 7.1, how many parts per billion of the following impurities can be tolerated in standard solar cells without degrading performance by more than 10%? (a) Mo; (b) Ti; (c) Cu.
- 7.2. (a) The best performance in the near term expected to be able to be maintained in a production setting with advanced silicon ingot slicing techniques is to produce wafers 250 μm thick with a cutting or kerf loss

of 150 μm . With such a technology, what is the maximum area of silicon that can be obtained from each kilogram of silicon starting material, assuming that the material lost in cutting is not reclaimed?

- (b) What is the corresponding figure for a ribbon technology that produces ribbons 100 μm thick?
- (c) If the material produced as above results in cells of 16% and 12% energy-conversion efficiency, respectively, under bright sunlight (1 kW/m^2), find a value for the maximum power-generating capacity of each of the foregoing technologies expressed as peak watts/kilograms.
- (d) If pure silicon were to be produced at the present volume of metallurgical-grade silicon (~1000 million kilograms annually), what would be the maximum possible generating capacity from one year's production for solar cells made by each of the foregoing approaches?

- 7.3. Calculate the maximum packing density (cell area to module area) that can be obtained for circular cells packed into rectangular modules.

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Chapter 8

DESIGN OF SILICON SOLAR CELLS

8.1 INTRODUCTION

In previous chapters, standard and improved technologies for producing silicon solar cells have been described. In the present chapter, considerations relevant to the detailed design of silicon cells will be discussed. Answers will be found to questions such as: What is the optimum level of dopants on either side of the junction? Where is the best location for the junction? What is the best shape for the top contact to the cell? How can optical losses from the cell be minimized?

Although answers to these questions will be provided specifically for silicon cells, parallel considerations will apply to cells made from other materials discussed in Chapter 10.

8.2 MAJOR CONSIDERATIONS

8.2.1 Collection Probability of Generated Carriers

A spatially dependent parameter, the *collection probability*, can be defined as the *probability a light-generated minority carrier*

has of contributing to the short-circuit current of a solar cell. This is a function of the position the carrier is generated within the cell. This parameter will be seen to be critical in determining the physical design of solar cells.

To find the collection probability, the artificial situation shown in Fig. 8.1(a) will be analyzed. Generation of electron-hole pairs by light will be assumed to occur only at points lying on a single plane throughout the cell. For the case where symmetry allows a "one-dimensional" analysis, the generation rate as a function of distance through the cell will be an impulse function, as indicated in Fig. 8.1(b).

The aim of the analysis will be to find the proportion of electrons generated at the point x_1 which contribute to the corresponding current flowing in the cell under short-circuit. During the analysis, it will be seen that no nonlinearities are involved and, by the superposition principle, the results can be applied to cases where the form of the generation rate corresponds more closely to practical situations. The analysis closely parallels that of Section 4.6.

In region 1 of Fig. 8.1(b), the generation rate is zero everywhere except at the single point x_1 right at the edge of the region. The differential equation that the excess minority carriers, Δn , must satisfy is therefore similar to Eq. (4.25):

$$\frac{d^2 \Delta n}{dx^2} = \frac{\Delta n}{L_e^2} \quad (8.1)$$

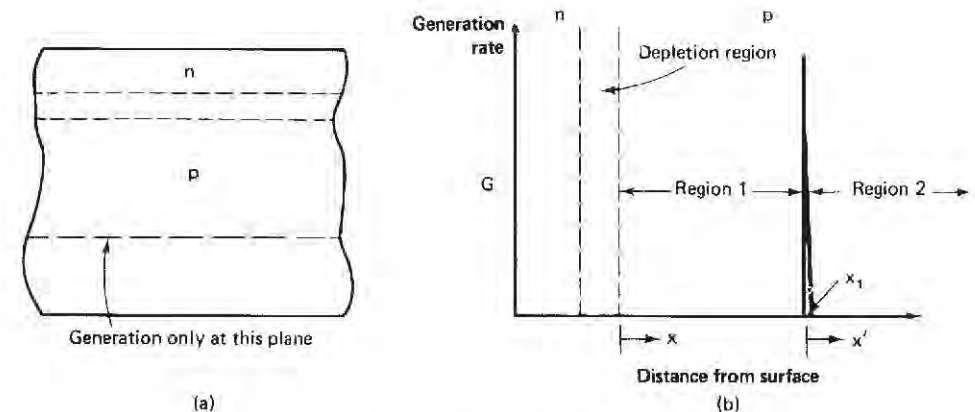


Figure 8.1. Idealized carrier-generation conditions used to calculate collection probabilities. An expression for the fraction of carriers generated at point x_1 which contribute to the cell short-circuit current is found in the text.

where L_e is the diffusion length. The general solution is, as before,

$$\Delta n = Ae^{x/L_e} + Be^{-x/L_e} \quad (8.2)$$

Boundary conditions are applied to assign values to the constants A and B . Under short-circuit conditions, the excess electron concentration at $x = 0$ is zero, since its value is determined by the voltage across the junction. Hence, $A = -B$ and

$$\Delta n = A(e^{x/L_e} - e^{-x/L_e}) = 2A \sinh\left(\frac{x}{L_e}\right) \quad (8.3)$$

Similarly, in region 2 of Fig. 8.1(b),

$$\Delta n = Ce^{x'/L_e} + De^{-x'/L_e}$$

where the origin of the x' direction is at the point x_1 . In this case, the excess concentration must be finite as x' becomes large. Hence, $C = 0$ and

$$\Delta n = De^{-x'/L_e} \quad (8.4)$$

At the position x_1 , the two solutions given by Eqs. (8.3) and (8.4) must match up since the electron concentration is continuous. Hence,

$$D = 2A \sinh\left(\frac{x_1}{L_e}\right) \quad (8.5)$$

On the n -type side of the device, the generation rate is zero throughout since the only generation occurs at x_1 . Also, the excess hole concentration Δp is zero at the edge of the depletion region when the cell is short-circuited. It follows that Δp is zero throughout this region. The resulting distributions of electrons and holes are shown in Fig. 8.2(a). Since minority carriers in uniformly doped quasi-neutral regions flow predominantly by diffusion (Section 4.5), minority-carrier current flows can be readily calculated by differentiating the foregoing distributions. The results are illustrated in Fig. 8.2(b).

The discontinuity in the electron current density at the point x_1 arises from the generation of carriers at this point. The size of the discontinuity is just the electronic charge multiplied by the rate of generation at this point. Assuming little change in currents across

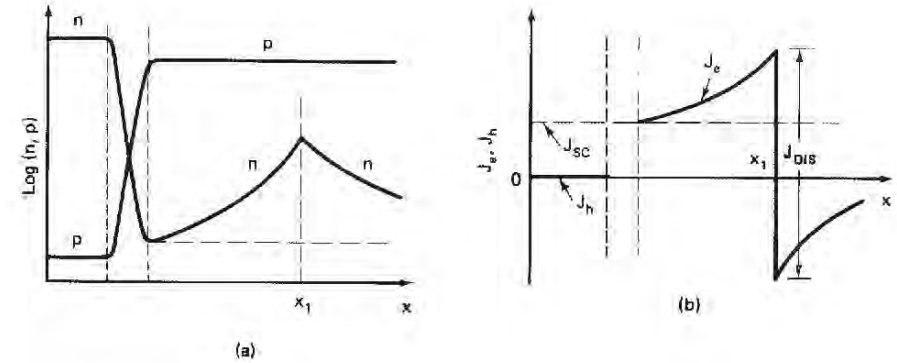


Figure 8.2. (a) Carrier distributions through the solar cell for the generation conditions of Fig. 8.1. (b) Corresponding minority carrier current distributions.

the depletion region (as previously discussed in Sections 4.6.2 and 5.4.3), the total current density in the device is equal to the electron current density at $x = 0$. The probability of collection (f_c) is just the ratio of the rate of flow of carriers in the external circuit to the rate of generation. Hence,

$$f_c = \frac{J_{sc}}{J_{DIS}} \quad (8.6)$$

However, in the p -type region,

$$J_e = qD_e \frac{dn}{dx} \quad (8.7)$$

Hence, in region 1,

$$J_e = \frac{2qD_e A}{L_e} \cosh\left(\frac{x}{L_e}\right) \quad (8.8)$$

which gives, at $x = 0$,

$$J_{sc} = \frac{2qD_e A}{L_e} \quad (8.9)$$

J_{DIS} can be found by evaluating the two expressions for current flow on either side of the discontinuity (J_{e-} and J_{e+}):

$$J_{e-} = \frac{2qD_e A}{L_e} \cosh\left(\frac{x_1}{L_e}\right) \quad (8.10)$$

$$J_{e+} = \frac{-qD_e D}{L_e} = \frac{-2qD_e A}{L_e} \sinh\left(\frac{x_1}{L_e}\right) \quad (8.11)$$

The latter equation follows from Eqs. (8.5) and (8.7). Hence,

$$\begin{aligned} J_{\text{DIS}} &= J_{e-} - J_{e+} \\ &= \frac{2qD_e A}{L_e} e^{x_1/L_e} \end{aligned} \quad (8.12)$$

It follows that

$$f_c = e^{-x_1/L_e} \quad (8.13)$$

The collection probability decreases exponentially with increasing distance of the point of generation away from the edge of the junction-depletion region. The characteristic decay length is just the minority carrier diffusion length. Since the foregoing analysis is linear, this conclusion is valid regardless of the distribution of the generation rate of carriers throughout the device.

A sketch of the probability of collection as a function of distance through a solar cell is shown in Fig. 8.3. As postulated earlier (Section 4.7), the depletion region and the region of the cell lying

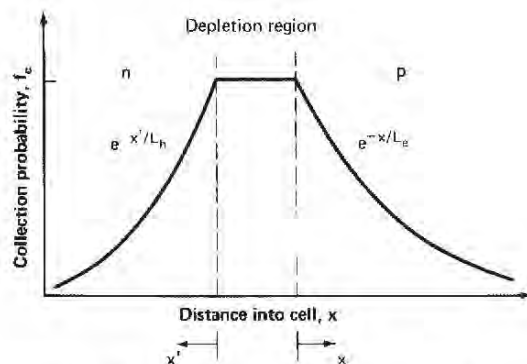


Figure 8.3. Calculated collection probability of generated minority carriers as a function of the point of generation in the solar cell.

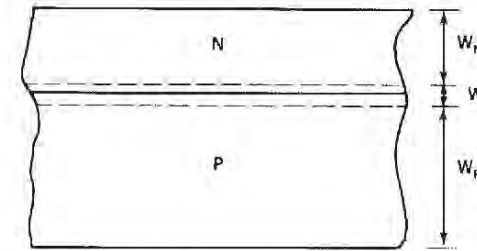


Figure 8.4. Critical dimensions in a cell of finite extent.

within a minority-carrier diffusion length of it are the regions where generated carriers have the highest chance of contributing to current flows (i.e., of being collected). A minority carrier generated outside these regions has an excellent chance of recombining before reaching the junction and subsequently the device terminals.

The foregoing analysis has assumed implicitly that both the *n*- and *p*-type regions extend much further than a diffusion length from the junction. For actual cells of finite extent as shown in Fig. 8.4, the collection probability will be modified. For example, if the surface on the *p*-type side of the device has a high recombination velocity, the expression corresponding to Eq. (8.13) is

$$f_c = \frac{\sinh[(W_P - x)/L_e]}{\sinh(W_P/L_e)} \quad (8.14)$$

If it has a low surface-recombination velocity, then

$$f_c = \frac{\cosh[(W_P - x)/L_e]}{\cosh(W_P/L_e)} \quad (8.15)$$

Both these approach the expression of Eq. (8.13) for $W_P \gg L_e$.

8.2.2 Junction Depth

Exposed surfaces such as between grid lines at the top of the solar cell are generally high-recombination-velocity surfaces. Interfaces between ohmic metal contacts and semiconductor are also generally regions of high recombination velocity. It has already been shown in Section 7.4 that one way of creating an effective low-recombination-velocity interface was by using a back surface field (i.e., by forming a junction between highly doped and lightly doped material of the same dopant type).

In view of the above and Eqs. (8.14) and (8.15), the collection probability of generated carriers as a function of distance from the surface of a p - n junction solar cell has the form shown in Fig. 8.5(a). Two features are important. A back surface field improves the probability of collection of carriers generated near the back contact and hence improves the short-circuit current of the cell. The probability of collection of carriers generated near the top surface of the solar cell is generally low.

Considering the actual generation rate of carriers in a semiconductor when illuminated by sunlight, the highest rate of generation will occur right at the semiconductor surface. For monochromatic light, the generation rate is given by

$$G = (1 - R) \alpha N e^{-\alpha x} \quad (8.16)$$

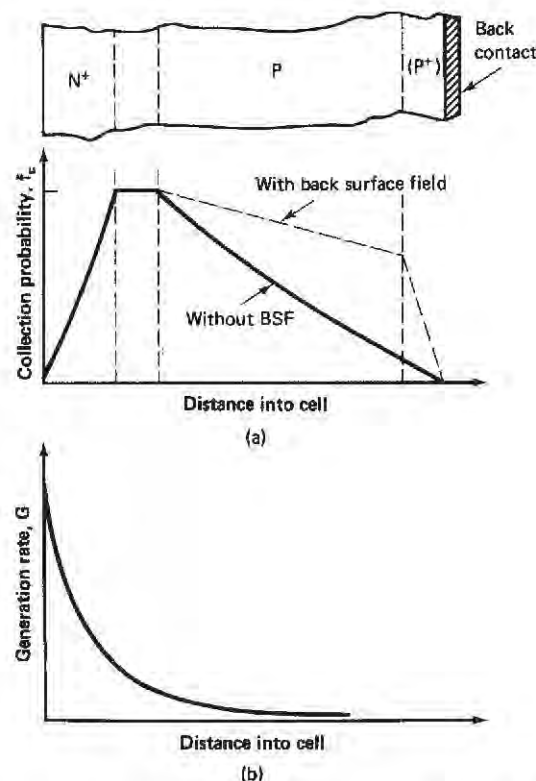


Figure 8.5. (a) Collection probability for a cell of finite extent with and without a back surface field. (b) Schematic diagram showing the generation rate of electron-hole pairs as a function of distance into the cell when illuminated by sunlight.

where x is the distance below the surface, α the absorption coefficient, N the incident photon flux, and R the fraction reflected. Under sunlight, the generation rate becomes

$$G(x) = \int_0^{\lambda_{\max}} [1 - R(\lambda)] \alpha(\lambda) N'(\lambda) e^{-\alpha(\lambda)x} d\lambda \quad (8.17)$$

where $N'(\lambda)$ is the incident flux per unit wavelength. The approximate form of this weighted exponential is shown in Fig. 8.5(b). The generation rate has a very strong peak near the surface precisely where the probability of collection is low. *This deficiency obviously is minimized if the junction is as close to the surface as possible.*

8.2.3 Lateral Resistance of Top Layer

Current flows in the bulk of the device are generally perpendicular to the surface of the cell as indicated in Fig. 8.6(a). To extract current via a contact that partially covers the top of the cell, current must flow laterally through the top layer of cell material. For a uniformly doped n -type layer, the resistivity of this layer is given by (Section 2.14)

$$\rho = \frac{1}{q\mu_e N_D} \quad (8.18)$$

A quantity more appropriate for describing the lateral resistance of this layer is the "sheet resistivity" (ρ_s), which is the resistivity divided by the layer thickness, t :

$$\rho_s = \frac{1}{q\mu_e N_D t} \quad (8.19)$$

For nonuniformly doped layers, the product $\mu_e N_D t$ is replaced by the integral $\int_0^t \mu_e(x) N_D(x) dx$. Sheet resistivities have the dimensions of ohms but are normally expressed as ohms/square ($\Omega/\square$).

The sheet resistivity determines the minimum spacing required between the grid lines of the top contact. Referring to Fig. 8.6(b), the resistive power loss due to lateral current flow is readily calculable. The incremental power loss in the section dy is given by

$$dP = I^2 dR \quad (8.20)$$

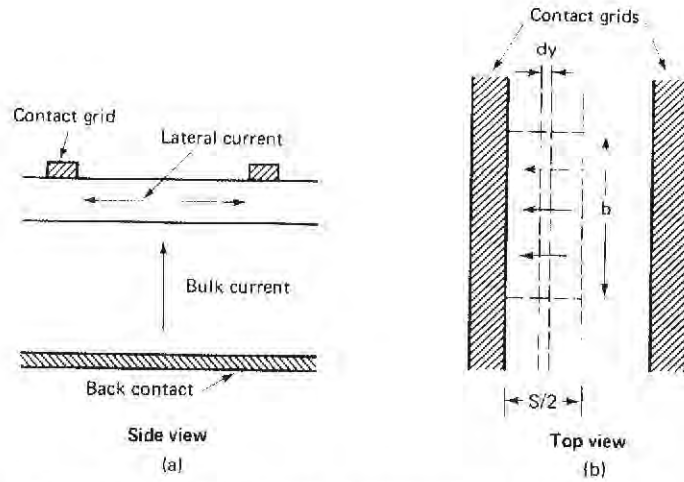


Figure 8.6. (a) Direction of current flow in different regions of a p-n junction solar cell. (b) Calculating power loss due to the lateral resistance of the top layer.

where dR is just $\rho_s dy/b$, and I , the lateral current flow, is zero at the midpoint between grating lines and increases linearly to its maximum at the grating line under uniform illumination. Hence,

$$I = Jby \quad (8.21)$$

where J is the current density in the device. The total power loss is the integral of the incremental losses:

$$P_{\text{loss}} = \int I^2 dR = \int_0^{S/2} \frac{J^2 b^2 y^2 \rho_s dy}{b} = \frac{J^2 b \rho_s S^3}{24} \quad (8.22)$$

At the maximum power point, the generated power from the region under consideration is $V_{\text{mp}} J_{\text{mp}} bS/2$. Hence, the fractional power loss at this point is

$$p = \frac{P_{\text{loss}}}{P_{\text{mp}}} = \frac{\rho_s S^2 J_{\text{mp}}}{12 V_{\text{mp}}} \quad (8.23)$$

This allows the minimum spacing S to be found for a given set of cell parameters. For example, for a typical commercial silicon cell, $\rho_s = 40 \Omega/\square$, $J_{\text{mp}} = 30 \text{ mA/cm}^2$, and $V_{\text{mp}} = 450 \text{ mV}$. For less

than 4% power loss due to lateral resistance effects,

$$S^2 < \frac{12 p V_{\text{mp}}}{\rho_s J_{\text{mp}}}$$

That is,

$$S < \left(\frac{12 \times 0.04 \times 0.45}{40 \times 0.03} \right)^{1/2} \text{ cm} < 4 \text{ mm} \quad (8.24)$$

This agrees with the grid spacings found on commercial silicon cells. Those with any larger spacing have a smaller sheet resistivity, and those with smaller spacing have a larger sheet resistivity. In practice, the major factor determining the sheet resistivity is the depth of the junction in Eq. (8.19). In fact, the resolution limits of the technology used to produce the grid pattern effectively sets the lower limit to the depth of the junction beneath the cell surface. To achieve minimum sheet resistivity of this layer, it is doped as high as practical.

8.3 DOPING OF THE SUBSTRATE

The substrate is uniformly doped during its preparation from molten material. The level of doping used is determined by the criteria which will now be discussed.

To get maximum I_{sc} once the junction depth is specified, the key parameter is the diffusion length in the substrate material. This is determined primarily by the minority-carrier lifetime in this region ($L_e = \sqrt{D_e \tau_e}$). In Section 3.4, three different recombination mechanisms were identified which determine the value of this quantity. For all of these, the general trend is a decrease in lifetime with increasing doping level. This is indicated in Fig. 8.7, which shows this dependency and the relative values of the lifetimes as caused by the different processes. For recombination by traps, the ideal expression for the variation of lifetime with doping follows from Eq. (3.22). It has the form

$$\tau_{nT} = \tau_{n0} \left(1 + \frac{m_1}{N_A} \right) \quad (8.25)$$

where m_1 is approximately equal to the larger of the parameters p_1 and $\tau_{p0} n_1 / \tau_{n0}$. Its value is determined by the energy and perhaps the