
PETERSON'S STRESS CONCENTRATION FACTORS

Second Edition

WALTER D. PILKEY



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CHAPTER 1

DEFINITIONS AND DESIGN RELATIONS

1.1 NOTATION

A = area (or point)

a = radius of hole

a = material constant for evaluating notch sensitivity factor

a = major axis of ellipse

a = half crack length

b = minor axis of ellipse

d = diameter (or width)

D = larger diameter

H = larger width

h = thickness

K = stress concentration factor

K_e = effective stress concentration factor

K_f = fatigue notch factor for normal stress

K_{fs} = fatigue notch factor for shear stress

K_t = theoretical stress concentration factor for normal stress

K_{t2}, K_{t3} = stress concentration factors for two, three dimensional problems

K_{tf} = estimated fatigue notch factor for normal stress

K_{tsf} = estimated fatigue notch factor for shear stress

K_{tg} = stress concentration factor with the nominal stress based on gross area

K_{tn} = stress concentration factor with the nominal stress based on net area

$K_{tx}, K_{t\theta}$ = stress concentration factors in x, θ directions

2 DEFINITIONS AND DESIGN RELATIONS

K_{ts} = stress concentration factor for shear stress
 K'_t = stress concentration factor using a theory of failure
 K_I = mode I stress intensity factor
 l, m = direction cosines of normal to boundary
 L_b = limit design factor for bending
 M = moment
 n = factor of safety
 p = pressure
 p_x, p_y = surface forces per unit area in x, y directions
 P = load, force
 $\bar{p}_{Vx}, \bar{p}_{Vy}, \bar{p}_{Vr}$ = body forces per unit volume in x, y, r directions
 q = notch sensitivity factor
 r = radius of curvature of hole, arc, notch
 R = radius of hole, circle, or radial distance
 t = depth of groove, notch
 T = torque
 u, v, w = displacements in x, y, z directions (or in r, θ, x directions in cylindrical coordinates)
 x, y, z = rectangular coordinates
 r, θ = polar coordinates
 r, θ, x = cylindrical coordinates
 $\epsilon_r, \epsilon_\theta, \epsilon_x, \gamma_{rx}$ = strain components
 ν = Poisson's ratio
 ρ = mass density
 ρ' = material constant for evaluating notch sensitivity factor
 σ = normal stress
 σ_a = alternating normal stress amplitude
 σ_n = normal stress based on net area
 σ_{eq} = equivalent stress
 σ_f = fatigue strength (endurance limit)
 σ_{max} = maximum normal stress
 σ_{nom} = nominal or reference normal stress
 σ_0 = static normal stress
 $\sigma_x, \sigma_y, \tau_{xy}$ = stress components
 $\sigma_r, \sigma_\theta, \tau_{r\theta}, \tau_{\theta x}$ = stress components
 σ_y = yield strength (tension)
 σ_{ut} = ultimate tensile strength
 σ_{uc} = ultimate compressive strength
 $\sigma_1, \sigma_2, \sigma_3$ = principal stresses
 τ = shear stress
 τ_a = alternating shear stress
 τ_f = fatigue limit in torsion

τ_y = yield strength in torsion
 $\tau_{\max}$ = maximum shear stress
 τ_{nom} = nominal or reference shear stress
 τ_0 = static shear stress
 ω = angular rotating speed

1.2 STRESS CONCENTRATION

The elementary stress formulas used in the design of structural members are based on the members having a constant section or a section with gradual change of contour (Fig. 1.1). Such conditions, however, are hardly ever attained throughout the highly stressed region of actual machine parts or structural members. The presence of shoulders, grooves, holes, keyways, threads, and so on, results in modifications of the simple stress distributions of Fig. 1.1 so that localized high stresses occur as shown in Figs. 1.2 and 1.3. This localization of high stress is known as *stress concentration*, measured by the *stress concentration factor*. The stress concentration factor K can be defined as the ratio of the peak stress in the body (or stress in the perturbed region) to some other stress (or stresslike quantity) taken as a

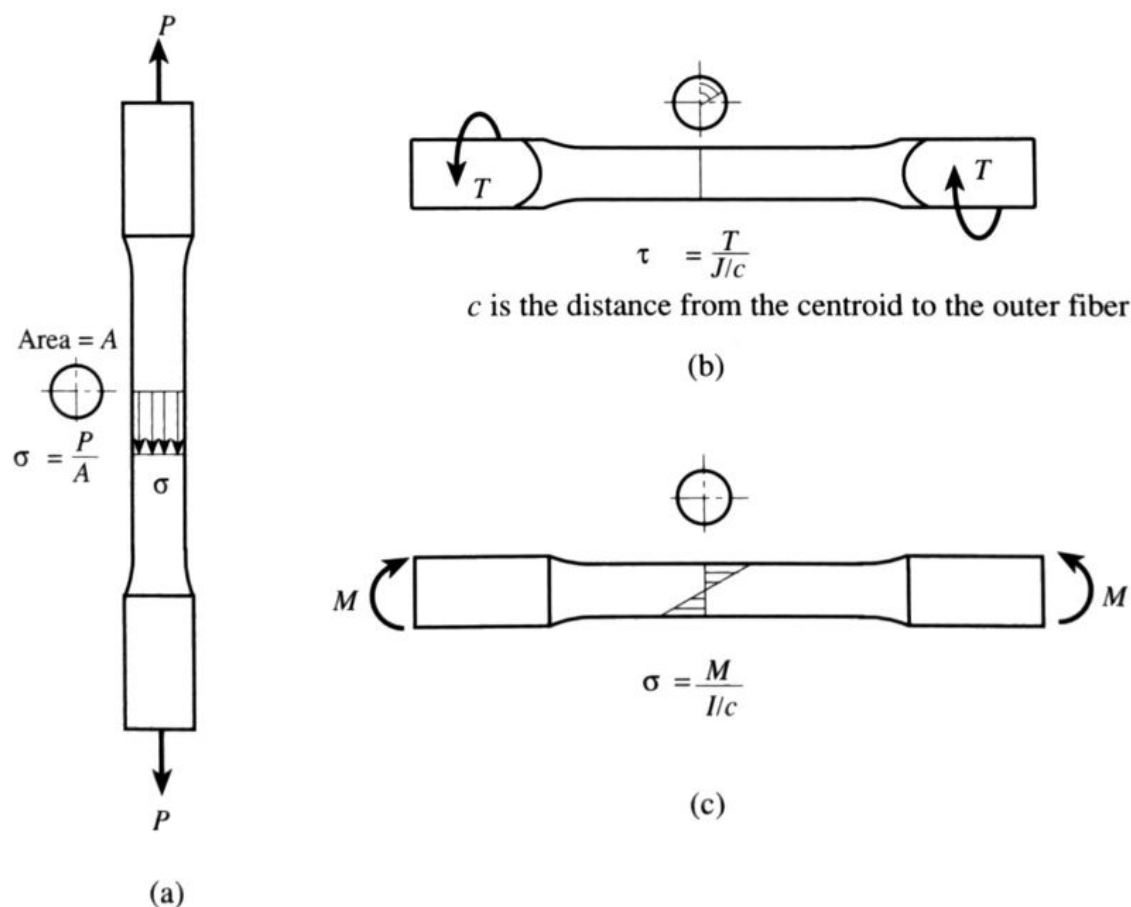


Figure 1.1 Elementary stress cases for specimens of constant cross section or with a gradual cross-sectional change: (a) Tension; (b) torsion; (c) bending.

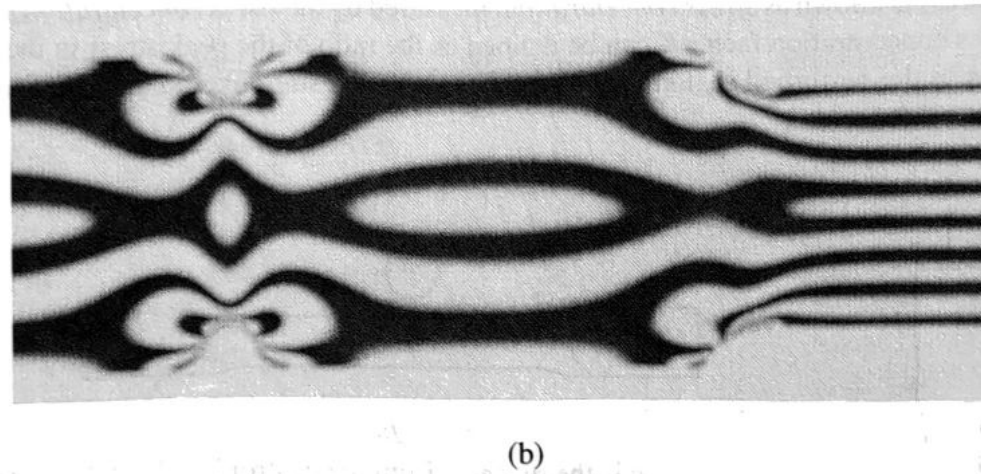
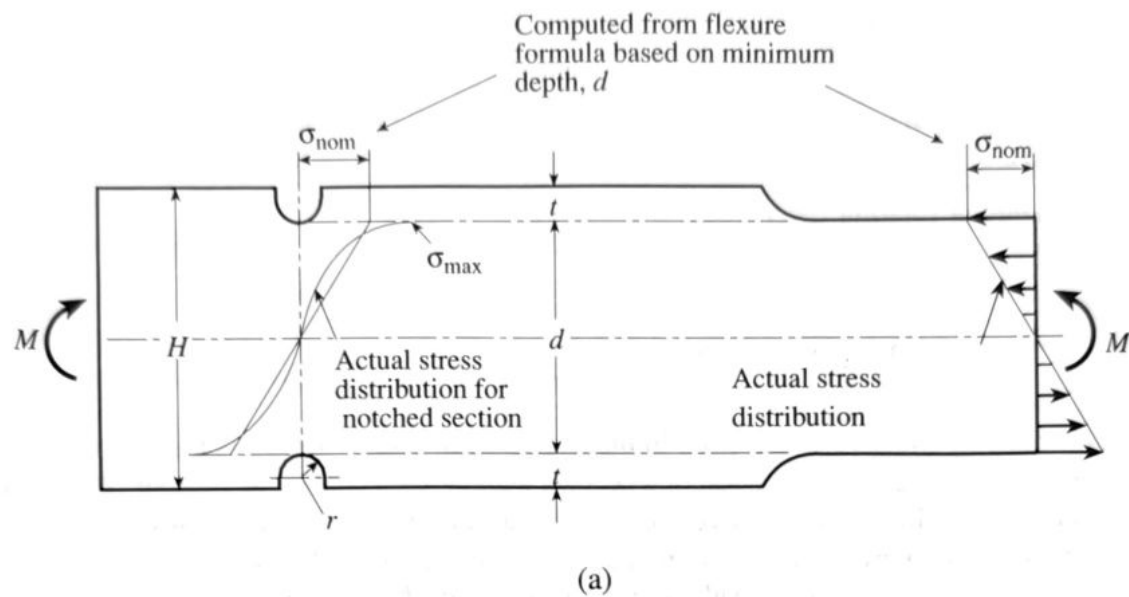


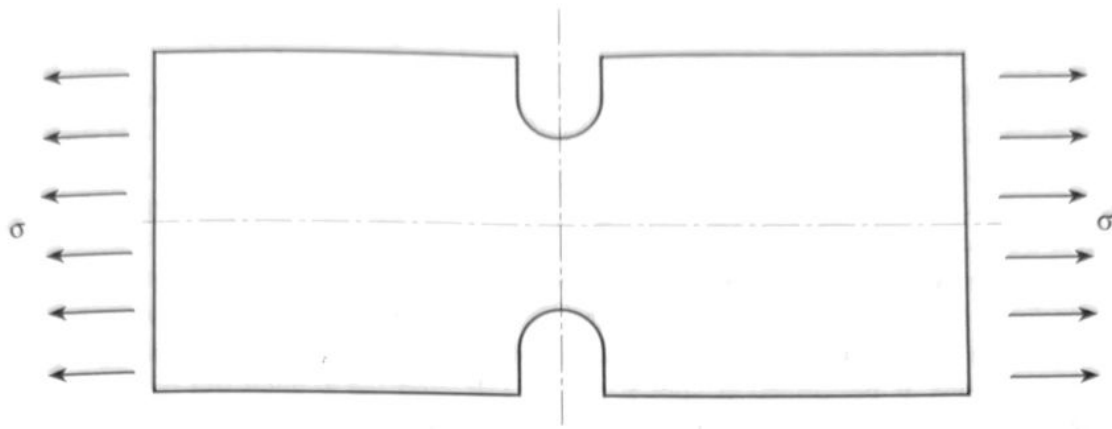
Figure 1.2 Stress concentrations introduced by a notch and a cross-sectional change which is not gradual: (a) Bending of specimen; (b) photoelastic fringe photograph (Peterson 1974).

reference stress:

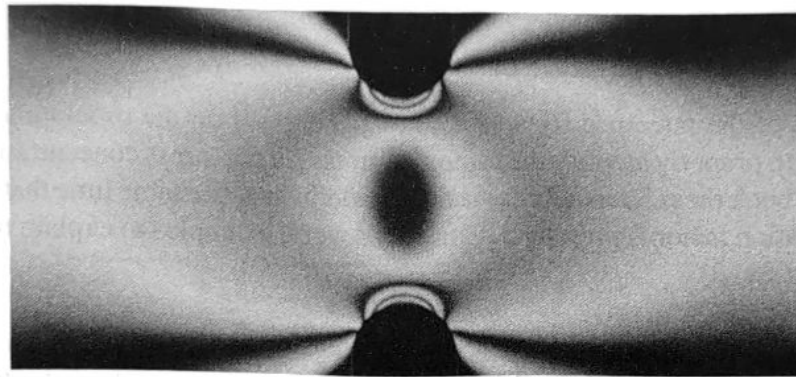
$$K_t = \frac{\sigma_{\max}}{\sigma_{\text{nom}}} \quad \text{for normal stress (tension or bending)} \quad (1.1)$$

$$K_{ts} = \frac{\tau_{\max}}{\tau_{\text{nom}}} \quad \text{for shear stress (torsion)} \quad (1.2)$$

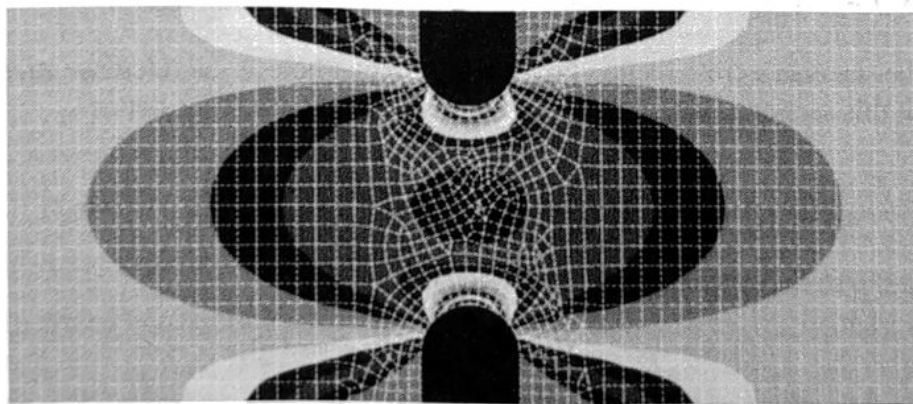
where the stresses $\sigma_{\max}$, $\tau_{\max}$ represent the maximum stresses to be expected in the member under the actual loads and the *nominal stresses* σ_{nom} , τ_{nom} are reference normal and shear stresses. The subscript *t* indicates that the stress concentration factor is a theoretical factor. That is to say, the peak stress in the body is based on the theory of elasticity, or it is derived from a laboratory stress analysis experiment. The subscript *s* of Eq. (1.2) is often ignored. In the case of the theory of elasticity, a two-dimensional stress distribution of a homogeneous elastic body under known loads is a function only of the body geometry and is not dependent on the material properties. This book deals primarily with elastic stress concentration



(a)



(b)



(c)

Figure 1.3 Tension bar with notches: (a) The specimen; (b) photoelastic fringe photograph (Doz. Dr.-Ing. habil. K. Fethke, Universität Rostock); (c) finite element solution (Guy Nerad, University of Virginia).

factors. In the plastic range one must consider separate stress and strain concentration factors that depend on the shape of the stress-strain curve and the stress or strain level. Sometimes K_t is also referred to as a *form factor*. The subscript t distinguishes factors derived from theoretical or computational calculations, or experimental stress analysis methods such as photoelasticity, or strain gage tests from factors obtained through mechanical damage tests such as impact tests. For example, the fatigue notch factor K_f is determined using a fatigue test. It will be described later.

The universal availability of powerful, effective computational capabilities, usually based on the finite element method, has altered the use of and the need for stress concentration factors. Often a computational stress analysis of a mechanical device, including highly stressed regions, is performed as shown in Fig. 1.3c, and the explicit use of stress concentration factors is avoided. Alternatively, a computational analysis can provide the stress concentration factor, which is then available for traditional design studies. The use of experimental techniques such as photoelasticity (Fig. 1.3b) to determine stress concentration factors has been virtually replaced by the more flexible and more efficient computational techniques.

1.2.1 Selection of Nominal Stresses

The definitions of the reference stresses σ_{nom} , τ_{nom} depend on the problem at hand. It is very important to properly identify the reference stress for the stress concentration factor of interest. In this book the reference stress is usually defined at the same time that a particular stress concentration factor is presented. Consider several examples to explain the selection of reference stresses.

Example 1.1 Tension Bar with Hole Uniform tension is applied to a bar with a single circular hole, as shown in Fig. 1.4a. The maximum stress occurs at point A, and the stress distribution can be shown to be as in Fig. 1.4a. Suppose that the thickness of the plate is h , the width of the plate is H , and the diameter of the hole is d . The reference stress could be defined in two ways:

- Use the stress in a cross section far from the circular hole as the reference stress. The area at this section is called the *gross cross-sectional area*. Thus define

$$\sigma_{\text{nom}} = \frac{P}{Hh} = \sigma \quad (1)$$

so that the stress concentration factor becomes

$$K_{tg} = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}} = \frac{\sigma_{\text{max}}}{\sigma} = \frac{\sigma_{\text{max}} Hh}{P} \quad (2)$$

- Use the stress based on the cross section at the hole, which is formed by removing the circular hole from the gross cross section. The corresponding area is referred to as the *net cross-sectional area*. If the stresses at this cross section are uniformly distributed and equal to σ_n :

$$\sigma_n = \frac{P}{(H - d)h} \quad (3)$$

CHAPTER 2

NOTCHES AND GROOVES

2.1 NOTATION

Definition:

Panel. A thin flat element with in-plane loading. This is a sheet that is sometimes referred to as a membrane.

Symbols:

a = width of a notch or semimajor axis of an ellipse

b = distance between notch centers or semiminor axis of an ellipse

c = distance between notch centers

d = minimum diameter (for three-dimensional) or minimum width (for two-dimensional) of member

D = diameter of member

h_o = minimum thickness of a thin element

h = thickness of a thin element

H = height or width of member

K_t = stress concentration factor

K_{tg} = stress concentration factor with the nominal stress based on gross area

K_{tn} = stress concentration factor with the nominal stress based on net area

L = length of member

M = bending moment

m = bending moment per unit length

P = total applied in-plane force

r = radius of a notch, notch bottom radius

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T = torque

t = depth of a notch

α = open angle of notch

ν = Poisson's ratio

σ = normal stress

τ = shear stress

2.2 STRESS CONCENTRATION FACTORS

The U-shaped notch or circumferential groove is a geometrical shape of considerable interest in engineering. It occurs in machine elements such as in turbine rotors between blade rows and at seals. Other examples include in a variety of shafts (Fig. 2.1) as a shoulder relief groove or as a retainer for a spring washer.

The round-bottomed V-shaped notch or circumferential groove, and to a lesser extent the U-shaped notch, is a conventional contour shape for stress concentration test pieces in the areas of fatigue, creep-rupture, and brittle fracture. A threaded part may be considered an example of a multigrooved member.

As mentioned in Chapter 1, two basic K_t factors may be defined: K_{tg} , based on the larger (gross) section of width H and K_{tn} , based on the smaller (net) section of width d , see sketch and definitions on Chart 2.3. For tension (Chart 2.3), $K_{tg} = \sigma_{\max}/\sigma$, where $\sigma = P/hH$ and $K_{tn} = \sigma_{\max}/\sigma_{\text{nom}}$, where $\sigma_{\text{nom}} = P/hd$. Since design calculations are usually based on the critical section (width d) where $\sigma_{\max}$ is located, K_{tn} is the generally used factor. Unless otherwise specified, K_t refers to K_{tn} in this chapter.

Neuber (1958) found the theoretical stress concentration factors for the deep hyperbolic notch (Fig. 2.2a) and the shallow elliptical notch (Fig. 2.2b) in infinitely wide members under tension, bending and shear. These results will be given in this chapter. For finite width members, Neuber introduced the following ingenious simple relation for notches

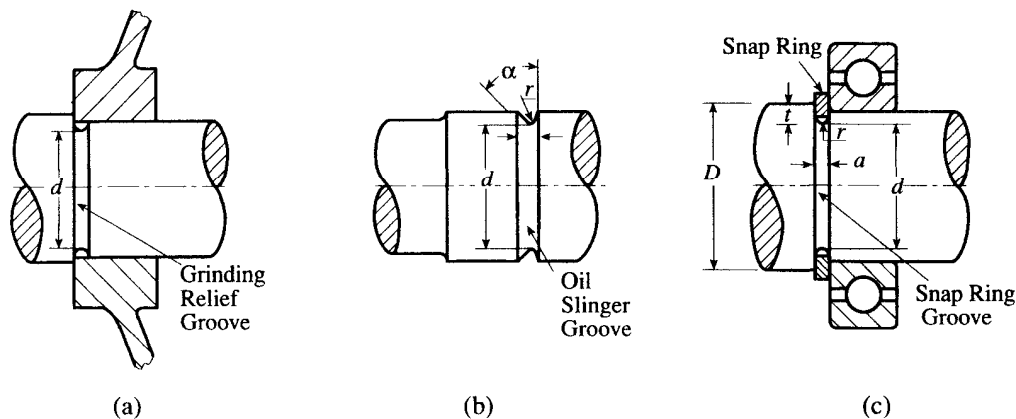


Figure 2.1 Examples of grooved shafts.

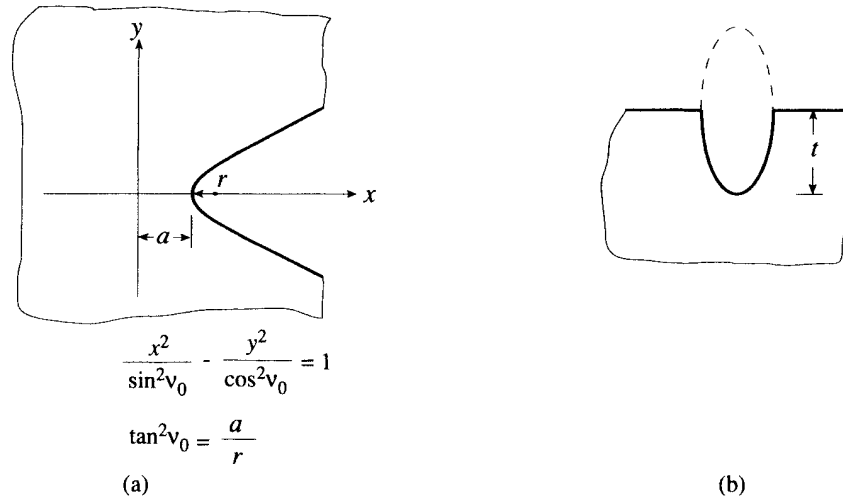


Figure 2.2 Notches: (a) Deep hyperbolic; (b) shallow elliptical.

with arbitrary shapes:

$$K_{tn} = 1 + \sqrt{\frac{(K_{te} - 1)^2 (K_{th} - 1)^2}{(K_{te} - 1)^2 + (K_{th} - 1)^2}} \quad (2.1)$$

where K_{te} is the stress concentration factor for a shallow elliptical notch (with the same t/r as for the notch in the finite width member) in a semi-infinitely wide member, K_{th} is the stress concentration factor for a deep hyperbolic notch (with the same r/d as for the notch in the finite width member) in an infinitely wide member, t is the notch depth, r is the notch radius (minimum contour radius), d is a minimum diameter (three-dimensional) or minimum width (two-dimensional) of the member.

Consider, for example, a flat bar with opposite notches of arbitrary shape, with notch depth t , notch radius r , and a minimum bar width d . If K_{te} is the stress concentration factor of a shallow elliptical notch of Chart 2.2 with the same t/r and K_{th} is the stress concentration factor of a deep hyperbolic notch of Chart 2.1 with the same r/d , then K_{tn} of Eq. (2.1) is an estimate of the stress concentration factor of the flat bar with opposite notches.

Equation (2.1) is not exact. Recent investigations have provided more accurate values for the parameter ranges covered by the investigations, as will be presented in the following sections. If the actual member being designed has a notch or groove that is either very deep or shallow, the Neuber approximation will be close. However, for values of d/H in the region of $1/2$, the Neuber K_t can be as much as 12% too low, which is on the unsafe side from a design standpoint. More accurate values have been obtained over the most used ranges of parameters. These form the basis of some of the charts presented here. However, when a value for an extreme condition such as a very small or large r/d is sought, the Neuber method is the only means of obtaining a useful factor. Some charts covering the extreme ranges are also included in this book.

Another use of the charts of Neuber factors is in designing a test piece for maximum K_t , as detailed in Section 2.10.

The K_t factors for the flat members covered in this chapter are for two-dimensional states of stress (plane stresses) and apply strictly to very thin sheets, or more strictly to where $h/r \rightarrow 0$, where h = element thickness and r = notch radius. As h/r increases, a state of plane strain is approached, in which case the stress at the notch surface at the middle of the element thickness increases and the stress at the element surface decreases. Some guidance may be obtained by referring to the introductory remarks at the beginning of Chapter 4.

The K_t factor for a notch can be lowered by use of a reinforcing bead (Suzuki 1967).

2.3 NOTCHES IN TENSION

2.3.1 Opposite Deep Hyperbolic Notches in an Infinite Thin Element; Shallow Elliptical, Semicircular, U-Shaped or Keyhole-Shaped Notches in Semi-Infinite Thin Elements; Equivalent Elliptical Notch

In Chart 2.1, K_{tn} values are given for the deep ($d/H \rightarrow 0$) hyperbolic notch in an infinite thin element (Neuber 1958; Peterson 1953). Chart 2.2 provides K_{tg} values for an elliptical or U-shaped notch in a semi-infinite thin element (Seika 1960; Bowie 1966; Barrata and Neal 1970). For the higher t/r values, K_{tg} for the U-notch is up to 1% higher than for the elliptical notch. For practical purposes the solid curve of Chart 2.2 covers both cases.

The semicircular notch ($t/r = 1$) in a semi-infinite thin element has been studied by a number of investigators. Ling (1967) has provided the following summary of K_t factors:

1936	Maunsell	3.05
1940	Isibasi	3.06
1941	Weinel	3.063
1948	Ling	3.065
1965	Yeung	3.06
1965	Mitchell	3.08

Similar to the “equivalent elliptical hole” in an infinite panel (see Section 4.5.1), an “equivalent elliptical notch” in a semi-infinite thin element may be defined as an elliptical notch that has the same t/r and envelops the notch geometry under consideration. All such notches, U-shaped, keyhole (circular hole connected to edge by saw cut), and so on have very nearly the same K_t as the equivalent elliptical notch. The “equivalent elliptical notch” applies for tension. It is not applicable for shear.

Stress concentration factors have been approximated by splitting a thin element with a central hole axially through the middle of the hole (Fig. 2.3) and using the K_t for the hole to represent the resulting notches. From Eq. (1.90) the stress concentration factor for an elliptical hole in a thin element is

$$K_t = 1 + 2\sqrt{\frac{t}{r}} \quad (2.2)$$

where t is the semiaxis which is perpendicular to the tensile loading. Chart 2.2 shows this K_t also. The factors for the U-shaped slot (Isida 1955) are practically the same. A comparison of the curves for notches and holes in Chart 2.2 shows that the preceding approximation can be in error by as much as 10% for the larger values of t/r .

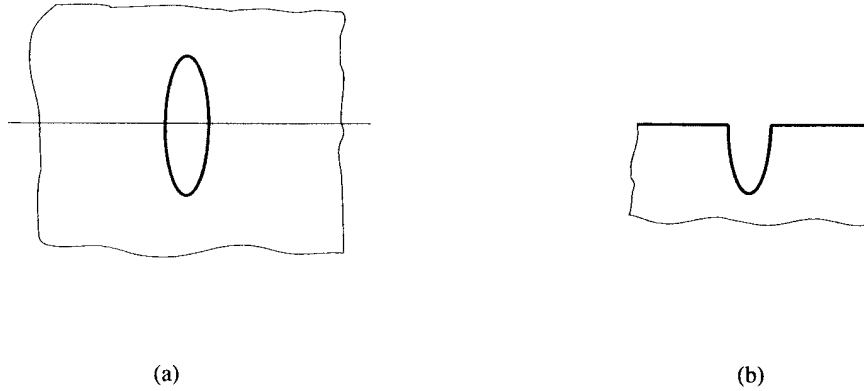


Figure 2.3 Equivalent notch from splitting a thin element with a hole: (a) Thin element with a hole; (b) Half of thin element with a notch.

2.3.2 Opposite Single Semicircular Notches in a Finite-Width Thin Element

For the tension case of opposite semicircular notches in a finite-width thin element, K_{tg} and K_m factors (Isida 1953; Ling 1968; Appl and Koerner 1968; Hooke 1968) are given in Chart 2.3. The difference between K_{tg} and K_m is illustrated in Chart 2.3.

Consider a bar of constant width H and a constant force P . As notches are cut deeper (increasing $2r/H$), K_m decreases, reflecting a decreasing stress concentration (peak stress divided by average stress across d). This continues until $2r/H \rightarrow 1$, in effect a uniform stress tension specimen. Factor K_{tg} increases as $2r/H$ increases, reflecting the increase in $\sigma_{\max}$ owing to the loss of section. Slot (1972) found that with $2r/H = 1/2$ in a strip of length $1.5H$, good agreement was obtained with the stress distribution for σ applied at infinity.

2.3.3 Opposite Single U-Shaped Notches in a Finite-Width Thin Element

Strain gage tests (Kikukawa 1962), photoelastic tests (Flynn and Roll 1966), and mathematical analysis (Appl and Koerner 1969) provide consistent data for the opposite U-shaped notches in a flat bar (two-dimensional case) of Chart 2.4. An important check is provided by including in Chart 2.4 the curve representing mathematical results (Isida 1953; Ling 1968) for the semicircular notch (Chart 2.3), a special case of the U-notch. The agreement is excellent for values of $H/d \leq 2$. Photoelastic results (Wilson and White 1973) for $H/d = 1.05$ are also in good agreement.

Barrata (1972) has compared empirical formulas for K_m with experimentally determined values and concludes that the following two formulas are satisfactory for predictive use.

Barrata and Neal (1970):

$$K_m = \left(0.780 + 2.243 \sqrt{\frac{t}{r}} \right) \left[0.993 + 0.180 \left(\frac{2t}{H} \right) - 1.060 \left(\frac{2t}{H} \right)^2 + 1.710 \left(\frac{2t}{H} \right)^3 \right] \left(1 - \frac{2t}{H} \right) \quad (2.3)$$

Heywood (1952):

$$K_m = 1 + \left[\frac{t/r}{1.55 (H/d) - 1.3} \right]^n \quad (2.4)$$

$$n = \frac{H/d - 1 + 0.5\sqrt{t/r}}{H/d - 1 + \sqrt{t/r}}$$

with t the depth of a notch, $t = (H - d)/2$.

Referring to Chart 2.4, Eq. (2.3) gives values in good agreement with the solid curves for $r/d < 0.25$. Equation (2.4) is in better agreement for $r/d > 0.25$. For the dashed curves (not the dot-dash curve for semicircular notches), Eq. (2.3) gives lower values as r/d increases. The tests on which the formulas are based do not include parameter values corresponding to the dashed curves, which are uncertain owing to their determination by interpolation of r/d curves having H/d as abscissae. In the absence of better basic data, the dashed curves, representing higher values, should be used for design.

In Chart 2.4 the values of r/d are from 0 to 0.3, and the values of H/d are from 1 to 2, covering the most widely used parameter ranges. There is considerable evidence (Kikukawa 1962; Flynn and Roll 1966; Appl and Koerner 1969) that for greater values of r/d and H/d , the K_t versus H/d curve for a given r/d does not flatten out but reaches a peak value and then decreases slowly toward a somewhat lower K_t value as $H/d \rightarrow \infty$. The effect is small and is not shown on Chart 2.4.

In Chart 2.4 the range of parameters corresponds to the investigations of Kikukawa (1962), Flynn and Roll (1966), and Appl and Koerner (1969). For smaller and larger r/d values, the Neuber values (Eq. 2.1, Charts 2.5 and 2.6), although approximate, are the only wide-range values available and are useful for certain problems. The largest errors are at the midregion of d/H . For shallow or deep notches the error becomes progressively smaller. Some specific photoelastic tests (Liebowitz et al. 1967) with $d/H \approx 0.85$ and r/H varying from ≈ 0.001 to 0.02 gave higher K_m values than does Chart 2.5.

2.3.4 "Finite-Width Correction Factors" for Opposite Narrow Single Elliptical Notches in a Finite-Width Thin Element

For the very narrow elliptical notch, approaching a crack, "finite-width correction" formulas have been proposed by Dixon (1962), Westergaard (1939), Irwin (1958), Bowie (1963), Brown and Strawley (1966), and Koiter (1965). Plots for opposite narrow edge notches are given in Peterson (1974).

The formula (Barrata and Neal 1970; Brown and Strawley 1966), based on Bowie's results for opposite narrow elliptical notches in a finite width thin element (Fig. 2.4), is satisfactory for values of $2t/H < 0.5$:

$$\frac{K_{Ig}}{K_{I\infty}} = 0.993 + 0.180 \left(\frac{2t}{H} \right) - 0.160 \left(\frac{2t}{H} \right)^2 + 1.710 \left(\frac{2t}{H} \right)^3 \quad (2.5)$$

where t is the crack length and $K_{I\infty}$ is K_t for $H = \infty$.

The following Koiter (1965) formula covers the entire $2t/H$ range. For the lower $2t/H$ range agreement with Eq. (2.5) is good. For the mid- $2t/H$ range, somewhat higher values

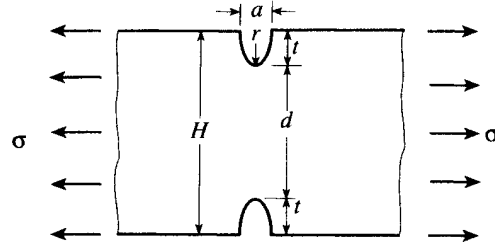


Figure 2.4 Opposite narrow edge notch.

than the Eq. (2.5) values are obtained.

$$\frac{K_{tg}}{K_{t\infty}} = \left[1 - 0.50 \left(\frac{2t}{H} \right) - 0.0134 \left(\frac{2t}{H} \right)^2 + 0.081 \left(\frac{2t}{H} \right)^3 \right] \left[1 - \frac{2t}{H} \right]^{-1/2} \quad (2.6)$$

These gross area factors K_{tg} are related to the net area factors K_{tn} by

$$\frac{K_{tn}}{K_{t\infty}} = \frac{K_{tg}}{K_{t\infty}} \left(1 - \frac{2t}{H} \right) \quad (2.7)$$

2.3.5 Opposite Single V-Shaped Notches in a Finite-Width Thin Element

Stress concentration factors $K_{t\alpha}$ have been obtained (Appl and Koerner 1969) for the flat tension bar with opposite V notches as a function of the V angle, α (Chart 2.7). The Leven-Frocht (1953) method of relating $K_{t\alpha}$ to the K_{tu} of a corresponding U notch as used in Chart 2.7, shows that for $H/d = 1.66$ the angle has little effect up to 90° . For $H/d = 3$ it has little effect up to 60° . In comparing these results with Chart 2.28, where the highest $H/d = 1.82$, the agreement is good, even though the two cases are different (symmetrical notches, in tension [Chart 2.7]; notch on one side, in bending [Chart 2.28]).

2.3.6 Single Notch on One Side of a Thin Element

Neuber (1958) has obtained approximate K_{tn} values for a semi-infinite thin element with a deep hyperbolic notch, wherein tension loading is applied along a midline through the minimum section (Chart 2.8). Chart 2.9 presents K_{tn} curves based on photoelastic tests (Cole and Braun 1958). Corresponding Neuber K_{tn} factors obtained by use of Chart 2.8 and of Eq. (2.1) are on the average 18% lower than the K_{tn} factors of Chart 2.9.

The curve for the semicircular notch is obtained by noting that for this case $r = H - d$ or $H/d = 1 + r/d$ and that $K_{tn} = 3.065$ at $r/d \rightarrow 0$.

2.3.7 Notches with Flat Bottoms

Chart 2.10 gives stress concentration factors K_{tn} for opposing notches in finite width thin elements, with flat bottoms (Neuber 1958; Hetényi and Liu 1956; Sobey 1965).

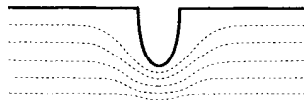
Chart 2.11 provides factors for a rectangular notch on the edge of a wide (semi-infinite) flat panel in tension (Rubenchik 1975; ESDU 1981). The maximum stress $\sigma_{\max}$ occurs at points A of the figure in Chart 2.11. When $a = 2r$, the notch base is semicircular, and two points A coincide at the base of the notch.

2.3.8 Multiple Notches in a Thin Element

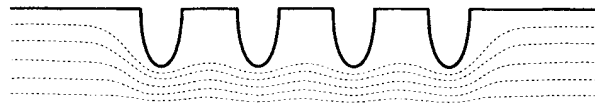
It has long been recognized that a single notch represents a higher degree of stress concentration than a series of closely spaced notches of the same kind as the single notch. Considered from the standpoint of flow analogy, a smoother flow is obtained in Figs. 2.5*b* and 2.5*c*, than in Fig. 2.5*a*.

For the infinite row of semicircular edge notches, factors have been obtained mathematically (Atsumi 1958) as a function of notch spacing and the relative width of a bar, with results summarized in Charts 2.12 and 2.13. For infinite notch spacing, the K_{tn} factors are in agreement with the single-notch factors of Isida (1953) and Ling (1968), Chart 2.3.

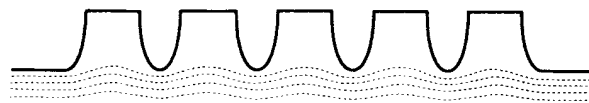
For a specific case (Slot 1972) with $r/H = 1/4$ and $b/a = 3$, good agreement was obtained with the corresponding Atsumi (1958) value.



(a)



(b)



(c)

Figure 2.5 Multiple notches.

An analysis (Weber 1942) of a semi-infinite panel with an edge of wave form of depth t and minimum radius r gives $K_{tn} = 2.13$ for $t/r = 1$ and $b/a = 2$, which is in agreement with Chart 2.12.

Stress concentration factors K_{tn} are available for the case of an infinite row of circular holes in a panel stressed in tension in the direction of the row (Weber 1942; Schulz 1942, 1943–45). If we consider the panel as split along the axis of the holes, the K_{tn} values should be nearly the same (for the single hole, $K_{tn} = 3$; for the single notch, $K_{tn} = 3.065$). The K_{tn} curve for the holes as a function of b/a fits well (slightly below) the top curve of Chart 2.12.

For a finite number of multiple notches (Fig. 2.5b), the stress concentration of the intermediate notches is considerably reduced. The maximum stress concentration occurs at the end notches (Charts 2.14 and 2.15, Durelli et al. 1952), and this is also reduced (as compared to a single notch) but to a lesser degree than for the intermediate notches. Sometimes a member can be designed as in Fig. 2.5c, resulting in a reduction of stress concentration as compared to Fig. 2.5b.

Factors for two pairs of notches (Atsumi 1967) in a square pattern ($b/H = 1$) are included in Chart 2.13.

Photoelastic tests have been made for various numbers (up to six) of semicircular notches (Durelli et al. 1952; Hetényi 1943). The results (Charts 2.14, 2.15, and 2.16) are consistent with the recent mathematical factors for the infinite row (Atsumi 1958). Chart 2.16 provides, for comparison, K_{tg} for a groove that corresponds to a lower K_{tg} limit for any number of semicircular notches with overall edge dimension c .

For the Aero thread shape (semicircular notches with $b/a = 1.33$), two-dimensional photoelastic tests (Hetényi 1943) of six notches gave K_t values of 1.94 for the intermediate notch and 2.36 for the end notch. For the Whitworth thread shape (V notch with rounded bottom), the corresponding photoelastic tests (Hetényi 1943) gave K_t values of 3.35 and 4.43, respectively.

Fatigue tests (Moore 1926) of threaded specimens and specimens having a single groove of the same dimensions showed considerably higher strength for the threaded specimens.

2.4 DEPRESSIONS IN TENSION

2.4.1 Hemispherical Depression (Pit) in the Surface of a Semi-infinite Body

For a semi-infinite body with a hemispherical depression under equal biaxial stress (Fig. 2.6), $K_t = 2.23$ was found (Eubanks 1954) for Poisson's ratio $\nu = 1/4$. This is about 7% higher than for the corresponding case of a spherical cavity ($K_t = 2.09$ for $\nu = 1/4$), (Chart 4.73). Moreover the semicircular edge notch in tension ($K_t = 3.065$) (Chart 2.2) is about 2% higher than the circular hole in tension ($K_t = 3$) (Chart 4.1 and Eq. 4.18).

2.4.2 Hyperboloid Depression (Pit) in the Surface of a Finite-Thickness Element

The hyperboloid depression simulates the type of pit caused by meteoroid impact of an aluminum panel (Denardo 1968). For equal biaxial stress (Fig. 2.7) values of K_t in the range of 3.4 to 3.8 were obtained for individual specific geometries as reported by Reed and Wilcox (1970). The authors point out that the K_t values are higher than for complete penetration in the form of a circular hole ($K_t = 2$; see Eq. 4.17).

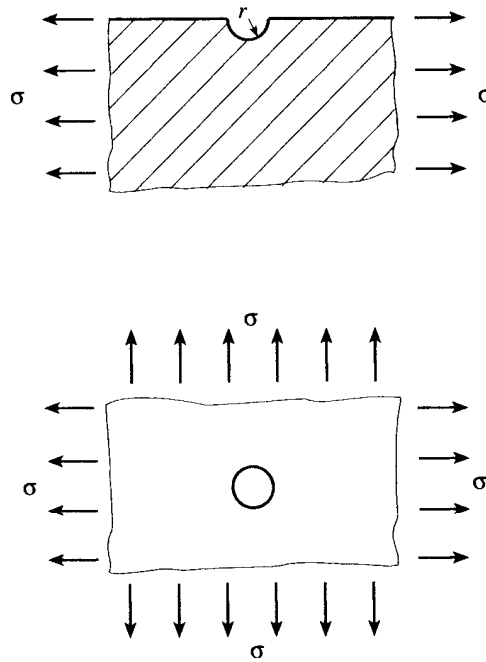


Figure 2.6 A semi-infinite body with a hemispherical depression under equal biaxial stress.

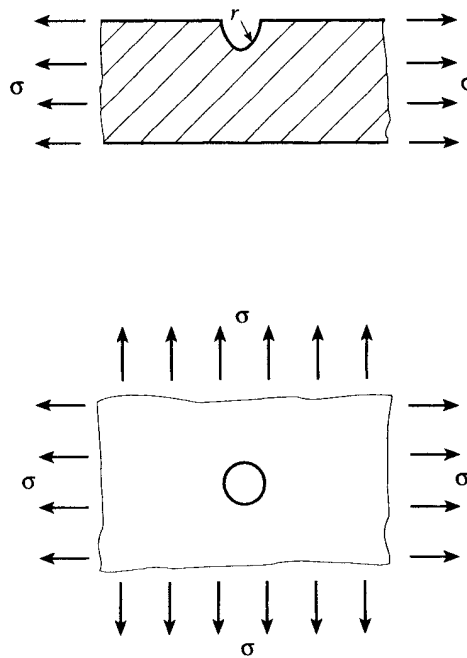


Figure 2.7 Hyperbolical depression in the surface of a finite thickness panel under equal biaxial stress.

2.4.3 Opposite Shallow Spherical Depressions (Dimples) in a Thin Element

The geometry of opposite dimples in a thin element has been suggested for a test piece in which a crack forming at the thinnest location can progress only into a region of lower stress (Cowper 1962). Dimpling is often used to remove a small surface defect. If the depth is small relative to the thickness (h_o/h approaching 1.0 in Chart 2.17), the stress increase is small. The $K_{tg} = \sigma_{\max}/\sigma$ values are shown for uniaxial stressing in Chart 2.17. These values also apply for equal biaxial stressing.

The calculated values of Chart 2.17 are for a shallow spherical depression having a diameter greater than four or five times the thickness of the element. In terms of the variables given in Chart 2.17, the spherical radius is

$$r = \frac{1}{4} \left[\frac{d^2}{(h - h_o)} + (h - h_o) \right] \quad (2.8)$$

for $d \geq 5h$, $r/h_o > 25$. For such a relatively large radius, the stress increase for a thin section ($h_o/h \rightarrow 0$) is due to the thinness of section rather than stress concentration per se (i.e., stress gradient is not steep).

For comparison, for a groove having the same sectional contour as the dimple, K_{tg} is shown by the dashed line on Chart 2.17, the K_{tg} values being calculated from the K_{tn} values of Chart 2.6. Note that in Chart 2.6 the K_{tn} values for $r/d = 25$ represent a stress concentration of about 1%. The K_{tg} factors therefore essentially represent the loss of section.

Removal of a surface defect in a thick section by means of creating a relatively shallow spherical depression results in negligible stress concentration, on the order of 1%.

2.5 GROOVES IN TENSION

2.5.1 Deep Hyperbolic Groove in an Infinite Member (Circular Net Section)

Exact K_t values for Neuber's solution (Neuber 1958; Peterson 1953) are given in Chart 2.18 for a deep hyperbolic groove in an infinite member. Note that Poisson's ratio has only a relatively small effect.

2.5.2 U-Shaped Circumferential Groove in a Bar of Circular Cross Section

The K_{tn} values for a bar of circular cross section with a U groove (Chart 2.19) are obtained by multiplying the K_{tn} of Chart 2.4 by the ratio of the corresponding Neuber three-dimensional K_{tn} (Chart 2.18) over two-dimensional K_{tn} values (Chart 2.1). This is an approximation. However, after comparison with the bending and torsion cases, the results seem reasonable.

The maximum stress occurs at the bottom of the groove. Cheng (1970) has by a photoelastic test obtained $K_{tn} = 1.85$ for $r/d = 0.209$ and $D/d = 1.505$. The corresponding K_{tn} from Chart 2.19 is 1.92, which agrees fairly well with Cheng's value, which he believes to be somewhat low.

Approximate K_{tn} factors, based on the Neuber (1958) formula, are given in Chart 2.20 for smaller r/d values and in Chart 2.21 for larger r/d values (e.g., test specimens).

2.5.3 Flat-Bottom Grooves

Chart 2.22 gives K_{tn} for flat-bottom grooves under tension based on the Neuber formula (Peterson 1953; ESDU 1981).

2.6 BENDING OF THIN BEAMS WITH NOTCHES

2.6.1 Opposite Deep Hyperbolic Notches in an Infinite Thin Element

Exact K_{tn} values of Neuber's solution (Neuber 1958; Peterson 1953) are presented in Chart 2.23 for infinite thin elements subject to in-plane moments with opposite deep hyperbolic notches.

2.6.2 Opposite Semicircular Notches in a Flat Beam

Chart 2.24 provides K_{tn} values determined mathematically (Ling 1967; Isida 1953) for a thin beam element with opposite semicircular notches. Slot (1972) found that with $r/H = 1/4$ a strip of length $1.5H$, good agreement was obtained with the stress distribution for M applied at infinity.

2.6.3 Opposite U-Shaped Notches in a Flat Beam

The stress concentration factor K_{tn} for opposite U-shaped notches in a finite-width thin beam element is given in Chart 2.25. These curves are obtained by increasing the photoelastic K_{tn} values (Frocht 1935), which as in tension are known to be low, to agree with the semicircular notch mathematical values of Chart 2.24, which are assumed to be accurate. Photoelastic (Wilson and White 1973) and numerical results (Kitagawa and Nakade 1970) are in good agreement. Approximate K_{tn} values for extended r/d values are given in Charts 2.26 and 2.27.

2.6.4 V-Shaped Notches in a Flat Beam Element

The effect of notch angle on the stress concentration factors is presented in Chart 2.28 for a bar in bending with a V-shaped notch on one side (Leven and Frocht 1953). The K_{tn} value is for a U notch. $K_{t\alpha}$ is for a notch with inclined sides having an included angle α but with all other dimensions the same as for the corresponding U notch case. The curves of Chart 2.28 are based on data from specimens covering a H/d range up to 1.82. Any effect of H/d up to this value is sufficiently small that single α curves are adequate. For larger H/d values, the α curves may be lower (see Chart 2.7).

2.6.5 Notch on One Side of a Thin Beam

Chart 2.29 provides K_{tn} for bending of a semi-infinite thin element with a deep hyperbolic notch (Neuber 1958). In Chart 2.30a, K_{tn} curves based on photoelastic tests (Leven and Frocht 1953) are given. Corresponding Neuber K_{tn} factors obtained by use of Chart 2.29 and Eq. (2.1) are on the average 6% higher than the K_{tn} factors of Chart 2.30a.

The curve for the semicircular notch is obtained by noting that for this case $H/d = 1 + r/d$ and that $K_{tn} = 3.065$ at $r/d \rightarrow 0$.

In Chart 2.30*b*, finite-width correction factors are given for a bar with a notch on one side. The full curve represents a crack (Wilson 1970) and the dashed curved a semicircular notch (Leven and Frocht 1953). The correction factor for the crack is the ratio of the stress-intensity factors. In the small-radius, narrow-notch limit, the ratio is valid for stress concentration (Irwin 1960; Paris and Sih 1965). Note that the end points of the curves are 1.0 at $t/H = 0$ and $1/K_{t\infty}$ at $t/H = 1$. The $1/K_{t\infty}$ values at $t/H = 1$ for elliptical notches are obtained from $K_{t\infty}$ of Chart 2.2. These $1/K_t$ values at $t/H = 1$ are useful in sketching in approximate values for elliptical notches.

If $K_{tg}/K_{t\infty}$ (not shown in Chart 2.30*b*) is plotted, the curves start at 1.0 at $t/H = 0$, dip below 1.0, reach a minimum in the $t/H = 0.10$ to 0.15 range, and then turn upward to go to infinity at $t/H = 1.0$. This means that for bending the effect of the nominal stress gradient is to cause $\sigma_{\max}$ to decrease slightly as the notch is cut into the surface, but beyond a depth of $t/H = 0.25$ to 0.3 the maximum stress is greater than for the infinitely deep bar. The same effect, only of slight magnitude, was obtained by Isida (1953) for the bending case of a bar with opposite semicircular notches (Chart 2.24; K_{tg} not shown). In tension, since there is no nominal stress gradient, this effect is not obtained.

Chart 2.31 gives K_{tn} for various impact specimens.

2.6.6 Single or Multiple Notches with Semicircular or Semielliptical Notch Bottoms

From work on propellant grains, it is known (Tsao, Ching, and Okubo 1965) "that invariably the stress concentration factor for an optimized semielliptic notch is significantly lower than that for the more easily formed semicircular notch." Photoelastic tests (Tsao, Ching, and Okubo 1965; Nishioka and Hisemitsu 1962; Ching, Okabu, and Tsao 1968) were made on beams in bending, with variations of beam and notch depth, notch spacing, and semielliptical notch bottom shape. The ratio of beam depth to notch depth H/t (notch on one side only) varied from 2 to 10. Chart 2.32 provides results for $H/t = 5$.

For the single notch with $t/a = 2.666$, the ratio of K_{tn} for the semicircular bottom to the K_{tn} for the optimum semielliptical bottom, $a/b = 2.4$, is 1.25 (see Chart 2.32). In other words, a considerable stress reduction (20% in this case) results from using a semielliptical notch bottom instead of a semicircular notch bottom. As can be found from Chart 2.32, even larger stress reductions can be obtained for multiple notches.

Although these results are for a specific case of a beam in bending, it is reasonable to expect that, in general, a considerable stress reduction can be obtained by use of the semielliptical notch bottom. The optimum a/b of the semiellipse varies from 1.8 to greater than 3, with the single notch and the wider spaced multiple notches averaging at about 2 and the closer spaced notches increasing toward 3 and greater.

Other uses of the elliptical contour are found in Chart 4.59 for the slot end, where the optimum a/b is about 3 and in Chart 3.9, for the shoulder fillet.

2.6.7 Notches with Flat Bottoms

Chart 2.33 offers stress concentration factors K_{tn} for thin beams with opposite notches with flat bottoms (Neuber 1958; Sobey 1965). For a shaft with flat-bottom grooves in bending and/or tension, stress concentration factors K_{tn} are given in Chart 2.34 (Rubenchik 1975; ESDU 1981).

2.7 BENDING OF PLATES WITH NOTCHES

2.7.1 Various Edge Notches in an Infinite Plate in Transverse Bending

Stress concentration factors K_t for opposite deep hyperbolic notches in a thin plate (Neuber 1958; Lee 1940) are given in Chart 2.35. The factors were obtained for transverse bending.

The bending of a semi-infinite plate with a V-shaped notch or a rectangular notch with rounded corners (Shioya 1959) is covered in Chart 2.36. At $r/t = 1$, both curves have the same K_t value (semicircular notch). Note that the curve for the rectangular notch has a minimum K_t at about $r/t = 1/2$.

In Chart 2.37, K_t factors are given for the elliptical notch (Shioya 1960). For comparison, the corresponding curve for the tension case from Chart 2.2 is shown. This reveals that the tension K_t factors are considerably higher.

In Chart 2.38 the K_t factor for an infinite row of semicircular notches is given as a function of the notch spacing (Shioya 1963). As the notch spacing increases, the K_t value for the single notch is approached asymptotically.

2.7.2 Notches in Finite-Width Plate in Transverse Bending

Approximate values have been obtained by the Neuber method (Neuber 1958; Peterson 1953) which makes use of the exact values for the deep hyperbolic notch (Lee 1940) and the shallow elliptical notch (Shioya 1960) in infinitely wide members and modifies these for finite-width members by using a second-power relation that has the correct end conditions. The results are shown for the thin plate in Chart 2.39.

No direct results are available for intermediate thicknesses. If we consider the tension case as representing maximum values for a thick plate in bending, we can use Chart 2.4 for $t/h \rightarrow 0$. For the thin plate ($t/h \rightarrow \infty$) use is made of Chart 2.39, as described in the preceding paragraph. (For intermediate thickness ratios, some guidance can be obtained from Chart 4.83; see also values on Chart 4.85 in the region of $b/a = 1$.)

2.8 BENDING OF SOLIDS WITH GROOVES

2.8.1 Deep Hyperbolic Groove in an Infinite Member

Stress concentration factors K_t for Neuber's exact solution (Neuber 1958; Peterson 1953) are given in Chart 2.40 for the bending of an infinite three-dimensional solid with a deep hyperbolic groove. The net section on the groove plane is circular.

2.8.2 U-Shaped Circumferential Groove in a Bar of Circular Cross Section

The K_t values of Chart 2.41 for a U-shaped circumferential groove in a bar of circular cross section are obtained by the method used in the tension case (see Section 2.5.2). Approximate K_t factors for small r/d values are given in Chart 2.42 and for large r/d values (e.g., test specimens), in Chart 2.43.

Example 2.1 Design of a Shaft with a Circumferential Groove Suppose that we wish to estimate the bending fatigue strength of the shaft shown in Fig. 2.8 for two materials: an axle steel (normalized 0.40% C), and a heat-treated 3.5% nickel steel. These materials

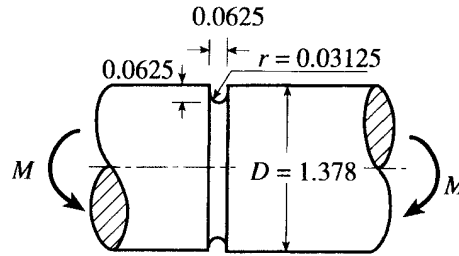


Figure 2.8 Grooved shaft.

will have fatigue strengths (endurance limits) of approximately 30,000 and 70,000 lb/in.², respectively, when tested in the conventional manner, with no stress concentration effects, in a rotating beam machine.

First we determine K_m . From Fig. 2.8, $d = 1.378 - (2)0.0625 = 1.253$ in. and $r = 0.03125$ in. We calculate $D/d = 1.10$ and $r/d = 0.025$. From Chart 2.41 we find that $K_m = 2.90$.

From Fig. 1.31 we obtain, for $r = 0.03125$ in., a q value of 0.76 for the axle steel and 0.93 for the heat-treated alloy steel.

Substituting in Eq. (1.53), for axle steel,

$$K_{tf} = 1 + 0.76(2.90 - 1) = 2.44 \quad (2.9)$$

$$\sigma_{tf} = \frac{30,000}{2.44} = 12,300 \text{ lb/in.}^2$$

for heat-treated alloy steel,

$$K_{tf} = 1 + 0.93(2.90 - 1) = 2.77 \quad (2.10)$$

$$\sigma_{tf} = \frac{70,000}{2.77} = 25,200 \text{ lb/in.}^2$$

This tells us that we can expect strength values of approximately 12,000 and 25,000 lb/in.² under fatigue conditions for the shaft of Fig. 2.8 when the shaft is made of normalized axle steel and quenched-and-tempered alloy steel (as specified), respectively. These are not working stresses, since a factor of safety must be applied that depends on type of service, consequences of failure, and so on. Different factors of safety are used throughout industry depending on service and experience. The strength of a member, however, is not, in the same sense, a matter of opinion or judgment and should be estimated in accordance with the best methods available. Naturally a test of the member is desirable whenever possible. In any event, an initial calculation is made, and this should be done carefully and include all known factors.

2.8.3 Flat-Bottom Grooves in Bars of Circular Cross Section

Stress concentration factors for a bar of circular cross section with flat bottomed grooves are presented in Chart 2.44 (Peterson 1953; Sobey 1965; ESDU 1981).

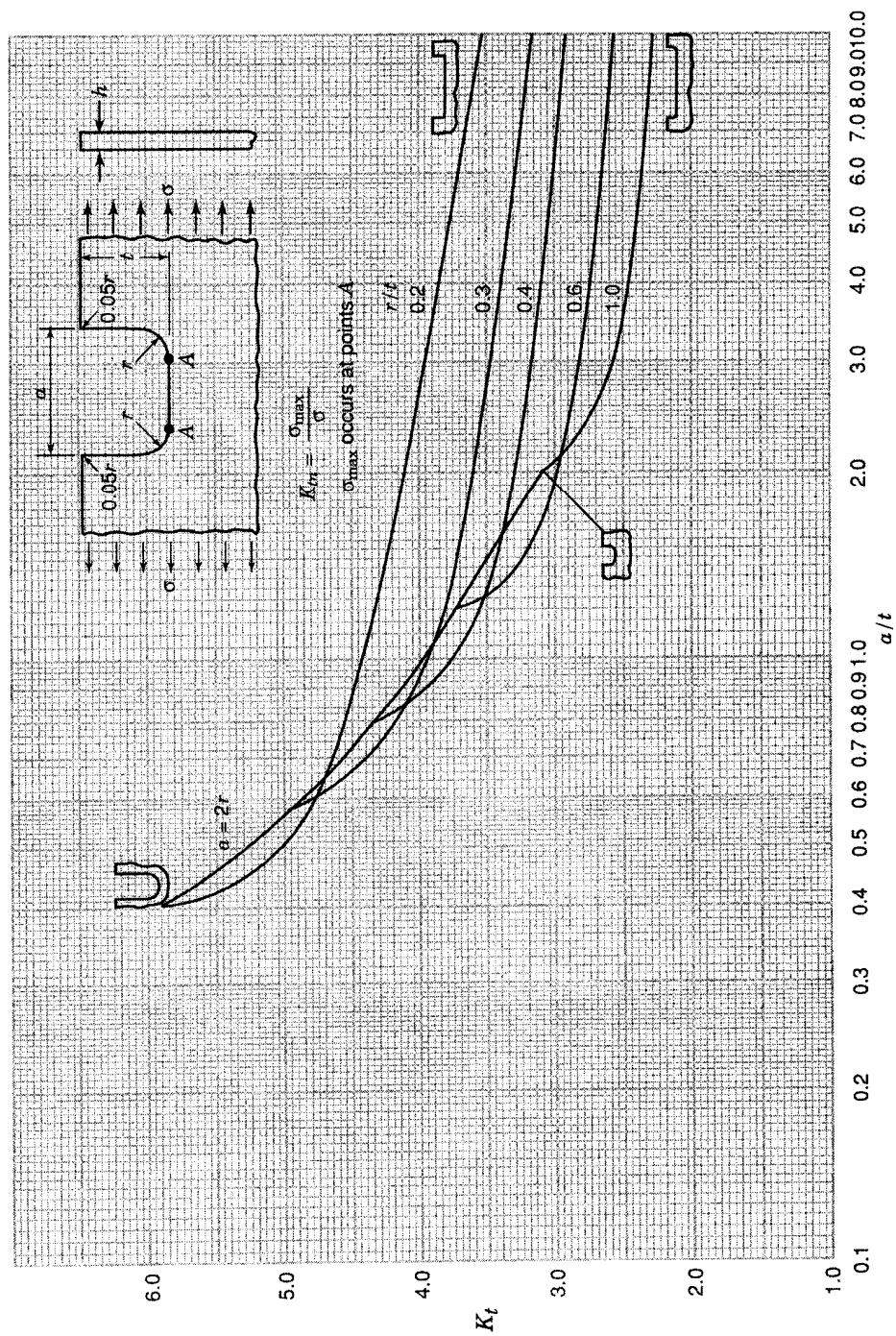


Chart 2.11 Stress concentration factors K_t for notches with flat bottoms in semi-infinite flat elements in tension (Rubenchik 1975; ESDU 1981).

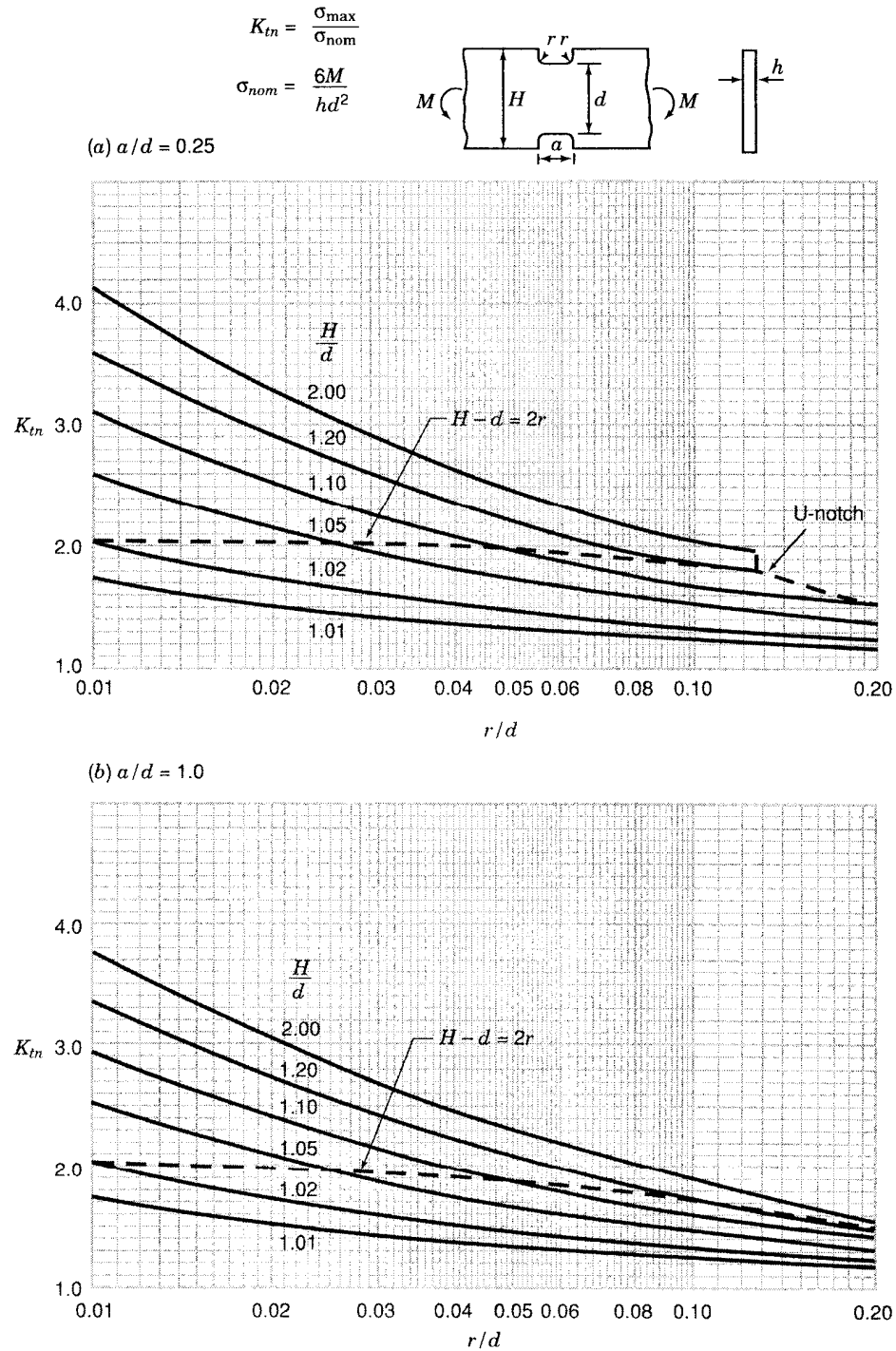


Chart 2.33 Stress concentration factors K_{tn} for thin beam in bending with opposite notches with flat bottoms (Neuber 1958; Sobey 1965; ESDU 1981). (a) $a/d = 0.25$; (b) $a/d = 1.0$.