

On the Diversity, Bandwidth, and Performance of Digital Transmission over Frequency-Selective Slow Fading Channels

Wern-Ho Sheen, *Member, IEEE*, Chun-Chieh Tseng, *Student Member, IEEE*, and Chia-Shu Wang

Abstract—The issues of diversity reception, bandwidth utilization, and performance evaluation of digital transmission over slowly time-varying frequency-selective fading channels are investigated, with a transmit pulse design that is more general than previously considered. In particular, the tradeoffs among performance, bandwidth, and complexity are addressed. Three types of transmit pulses are considered and compared, including the raised cosine (RC), square-root RC (SRRC), and Gaussian pulses. Union bounds on the symbol and burst error rate are derived for maximum likelihood sequence estimation (MLSE) based on exact pairwise error probabilities for burst-mode transmission. Explicit and implicit diversity are employed to improve the system performance. The results show that the implicit diversity is flexible and easy to exploit and is a good complement to the space or other explicit diversity to further improve the system performance.

Index Terms—Diversity, frequency-selective fading, maximum likelihood sequence estimation.

I. INTRODUCTION

EFFICIENT digital signaling over time-varying multipath fading channels is of great interest in today's wireless communications [1]–[11]. Time selectivity, frequency selectivity, and multipath fading are the major impediments to a high bit rate data transmission over such a channel. Time selectivity requires fast convergent algorithms at the receiver for adaptive signal processing. It may also distort the transmitted symbol and induce an error floor. Frequency selectivity causes intersymbol interference (ISI) in the time domain and needs to be compensated by an equalizer in order to obtain satisfactory performance. Multipath fading results in a very low received signal power in deep fades and may significantly increase the required average signal-to-noise ratio (SNR) to achieve a prespecified error performance. Optimum and suboptimum detectors of digital signals over time-varying multipath fading channels have been investigated extensively in the literature. In [6], the optimum receiver with maximum likelihood sequence estimation (MLSE) and a proper front-end processor was developed under the assumption of perfect channel state information (CSI). In [7] and [8], on the other hand, the optimum receiver was developed for the case of no CSI, based on MLSE with extended Kalman and FIR filters, respectively, and, in [9]–[11] suboptimum receivers with different front-end processors were

proposed with per-survivor processing for CSI estimation. In addition, diversity reception such as space, frequency, or time diversity¹ has been used to lower the required SNR for a prespecified error rate. Space diversity is implemented by using multiple antennas at the receiver, usually located at one-quarter wavelength intervals. Consequently, its use may be limited by the physical size of the receiver and may need multiple radio frequency (RF) modules depending on the combining method. Frequency diversity is implemented by using multiple carriers to carry the same information with frequency separation larger than the channel coherence bandwidth in order to obtain good performance. This may not be possible in some applications, and also it needs multiple RF modules which increase the receiver complexity and cost. Channel coding can be regarded as a sophisticated form of time diversity and is very ineffective for a slow fading channel.

In this paper, the issues of diversity reception, bandwidth utilization, and performance evaluation of digital transmission over slowly time-varying frequency-selective fading channels are investigated, with a transmit pulse design that is more general than previously considered. In particular, the tradeoffs among performance, bandwidth, and complexity are addressed with the optimum MLSE receiver. Three transmit pulses are considered, namely, the raised cosine (RC), square root RC (SRRC), and Gaussian pulses. Upper bounds derived with exact pairwise error probabilities are used for evaluating the error rates that are difficult to obtain by computer simulations. Explicit and implicit diversity are employed to improve the system performance. Numerical results show that the implicit diversity is flexible and easy to exploit and is a good complement to the space or other explicit diversity to further improve the system performance.

This paper is organized as follows. Following this Introduction, the system and channel models are described in Section II. Section III presents the performance evaluation of MLSE for burst-mode transmission. Section IV gives numerical results and discussions. Finally, our conclusions are given in Section V.

II. SYSTEM AND CHANNEL MODELS

The equivalent base-band model for the considered digital communication system with multiple explicit diversity receptions is depicted in Fig. 1. $\{a_i\}$ denotes the information symbols

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The authors are with the Department of Electrical Engineering, National Chung Cheng University, Chia-Yi 621, Taiwan, R.O.C. (e-mail: whsheen@ee.ccu.edu.tw).

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¹The conventional space, frequency, or time diversity will be called as explicit diversity.

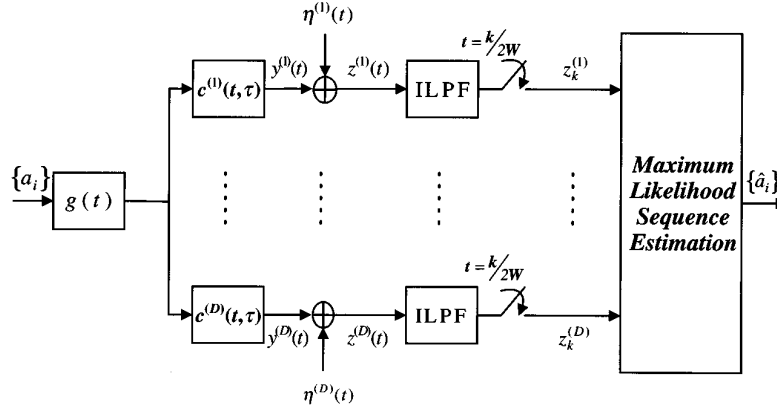


Fig. 1. The base-band model for frequency-selective fading channels with diversity reception.

transmitted for every T_s s, $g(t)$ is an arbitrary transmit pulse assumed to be bandlimited to B , and the channel is a wide-sense stationary uncorrelated scattering (WSSUS) Rayleigh fading channel corrupted by additive white Gaussian noise (AWGN). The channel impulse response for the d th diversity branch is $c^{(d)}(t, \tau)$ which denotes the response at time t when an impulse is applied at $t - \tau$. In practice, $c^{(d)}(t, \tau) \approx 0$ for $\tau \geq \tau_m$, the maximum excessive delay spread. For Rayleigh fading channels, $c^{(d)}(t, \tau)$ is a zero-mean complex-valued Gaussian process. At the receiver, the received signal $z^{(d)}(t)$ is passed through an ideal low-pass filter (ILPF) with a bandwidth of $W \geq B$ [3]. If W is large enough to accommodate the effect of Doppler spread, then the set $\{z^{(d)}(t)\}$ is a set of sufficient statistics for the optimum detection of the received signal [3], [4], [10]. For slowly time-varying fading channels, the Doppler spread is considered very small.² Note that other front-end processors like the ones in [4] and [6] can also be used to obtain the sufficient statistics. Since D orders of explicit diversity are assumed, the channel has D diversity branches; each has the same fading characteristics, but is statistically independent of one another. The explicit diversity can be obtained by using space or other forms of diversity.

For the d th branch, the received signal $z^{(d)}(t)$ is given by

$$z^{(d)}(t) = \sum_i a_i h^{(d)}(t, t - iT_s) + \eta^{(d)}(t) \quad (1)$$

where $h^{(d)}(t, \xi)$ is the overall impulse response given by

$$h^{(d)}(t, \xi) = \int_{-\infty}^{\infty} c^{(d)}(t, \tau) g(\xi - \tau) d\tau \quad (2)$$

and $\eta^{(d)}(t)$ is a zero-mean complex-valued white Gaussian noise with two-side power spectral density of N_0 W/Hz.

As in Fig. 1, the samples of the received signals are taken every $1/2W$ s, while symbols are transmitted every T_s s. For ease of implementation, the bandwidth W of the ILPF is often

adjusted such that there are α samples per symbol, i.e., $\alpha = 2WT_s$, where α is a positive integer. Further, let

$$b_k = \begin{cases} a_{k/\alpha}, & \text{for } k/\alpha \text{ integer} \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

Then an equivalent discrete model which incorporates the transmit pulse, fading channel, ideal low-pass filter, and sampler can be devised, as shown in Fig. 2, where

$$z_k^{(d)} = \sum_{j=0}^{L_c} h_{k,j}^{(d)} b_{k-j} + \eta_k^{(d)} \quad (4)$$

$$h_{k,j}^{(d)} \triangleq h^{(d)}(t, \xi)|_{t=k/2W, \xi=j/2W} \quad (5)$$

$L_c = L_e + L_g$ is the last tap with a significant power (so later taps may be ignored for practical purpose) and $\{\eta_k^{(d)}\}$ are i.i.d. random variables with

$$\sigma_\eta^2 \triangleq \frac{1}{2} \cdot E[|\eta_k|^2] = 2N_0W. \quad (6)$$

From Fig. 2, it is clear that the equivalent discrete system is a finite-state machine and can be described by a trellis diagram with each path in the trellis representing a different transmitted block of information symbols. The state is defined as

$$s_i = (a_{i-1}, a_{i-2}, \dots, a_{i-L}) \quad (7)$$

where $L \triangleq \lfloor L_c/\alpha \rfloor$ and $\lfloor x \rfloor$ denotes the largest integer smaller than x . Let q be the size of the signal constellation. Then the trellis has a total of $N_S = q^L$ states. Furthermore, assuming that there are N information symbols in a burst, then without considering the preamble symbols (they are irrelevant to the detection), the length of the trellis needed to be considered is $N + L$ symbols, where L symbols are appended to the tail of the information block for detecting the last L information symbols. In the literature, a wide variety of search algorithms have been proposed to search for the most probable transmitted block

²Here, by slowly time-varying, we mean that the channel remains unchanged during a burst of transmission.

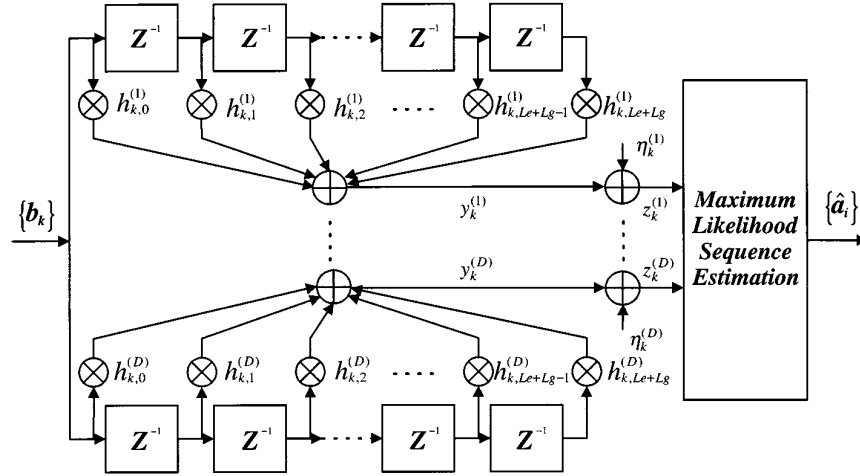


Fig. 2. The equivalent discrete time model.

of information symbols. In the following, only the exhaustive search MLSE via the Viterbi algorithm will be considered. For our system, it can be shown that the branch metric is

$$\gamma_i = - \sum_{d=1}^D \sum_{j=0}^{\alpha-1} \left| z_{i\alpha+j}^{(d)} - \sum_{l=0}^L h_{i\alpha+j,l\alpha+j}^{(d)} a_{i-l} \right|^2 \quad (8)$$

with perfect CSI, and the path metric is

$$\Gamma = \sum_i \gamma_i. \quad (9)$$

Before proceeding further, three important observations are in order. First, since W is large enough to accommodate the effect of Doppler spread, the set of sample values $\{z_k^{(d)}\}$ is a set of sufficient statistics, and MLSE is optimal under the assumption of perfect CSI. For burst-mode transmission over a slowly fading channel, it is plausible that a very accurate estimation of CSI can be obtained with a preamble. Second, the channel taps $\{h_{k,j}^{(d)}, j = 0 \dots L_c\}$ form a set of correlated zero-mean colored Gaussian processes which can be generated from the independent colored Gaussian processes $v_{k,p}$, $p = 1 \dots P$, $P \leq L_c$ through a unitary transformation [4]. The power of $v_{k,p}$ is the p th (nonnegligible) eigenvalue of the autocovariance matrix as defined in (32). Therefore, P may suitably be defined as the effective diversity order of the overall channel. Note that with explicit diversity, the total effective diversity order becomes $D \cdot P$. Third, unlike the case previously considered in the literature, there is no restriction imposed on the transmit pulse $g(t)$ in this study. Take the RC pulse as an example

$$g(t) = \frac{\sin \pi t/T}{\pi t/T} \cdot \frac{\cos \beta \pi t/T}{1 - 4\beta^2 t^2/T^2} \quad (10)$$

where β is the rolloff factor and $B = (1 + \beta)/2T$. In previous studies, T is usually selected as the symbol duration. This is necessary in a frequency-nonselective channel in order to avoid ISI. Nevertheless, in a frequency-selective channel, there is no necessity that T and T_s are commensurable, i.e., T_s/T is not necessarily an integer. Hence, for a fixed B , β and T can be

selected, based on a tradeoff between performance and complexity, as will be illustrated in Section IV. Likewise, B and T_s are not necessarily commensurable; any B can be selected independent of T_s . As shown in Table I, a larger B always gives a larger effective diversity order P , the effective diversity order, and hence gives a better performance for a fixed maximum excessive delay spread. In addition, with a smaller β , a larger P may be obtained and/or the total received power may be distributed more evenly over the independent paths. This will improve the system performance, as will be shown in Section IV. Finally, a larger B may reduce the receiver's complexity as shown in Table II. In fact, one of the aims of this study is to show the tradeoff between diversity, complexity, performance, and bandwidth for the considered system.

III. SYMBOL AND BURST ERROR RATE

A. Upper Bounds

Let $\mathbf{a} = [a_0, a_2, \dots, a_{N-1}]$ and $\hat{\mathbf{a}} = [\hat{a}_0, \hat{a}_2, \dots, \hat{a}_{N-1}]$ be the transmitted and detected symbol vectors, respectively. Define the error vector $\boldsymbol{\varepsilon} = \mathbf{a} - \hat{\mathbf{a}}$ and $\mathcal{G}$ be the set of error vectors that contain no more than $L-1$ consecutive zeros amid nonzero elements. Then, the symbol error rate (SER) P_s is bounded by [5], [13], [15], [16]

$$\text{SER} \leq \frac{1}{N} \sum_{\boldsymbol{\varepsilon} \in \mathcal{G}} w(\boldsymbol{\varepsilon}) P_r[\Gamma(\mathbf{a} - \boldsymbol{\varepsilon}) \geq \Gamma(\mathbf{a})] \cdot P_r[\mathcal{X}(\boldsymbol{\varepsilon})] \quad (11)$$

where $w(\boldsymbol{\varepsilon})$ is the number of symbol errors associated with $\boldsymbol{\varepsilon}$, and $\mathcal{X}$ is the set of all possible transmitted vectors that can have $\boldsymbol{\varepsilon}$ as the error vector. Let $M(\boldsymbol{\varepsilon})$ be the positions within the N information symbols where the error can begin. The upper bound can then be simplified as

$$\text{SER} \leq \frac{1}{N} \sum_{\boldsymbol{\varepsilon} \in \mathcal{G}_0} M(\boldsymbol{\varepsilon}) w(\boldsymbol{\varepsilon}) P_r[\Gamma(\mathbf{a} - \boldsymbol{\varepsilon}) \geq \Gamma(\mathbf{a})] \cdot P_r[\mathcal{X}(\boldsymbol{\varepsilon})] \quad (12)$$

where $\mathcal{G}_0 \subset \mathcal{G}$ is the set of error vector with $\varepsilon_0 \neq 0$. It is easy to show that $M(\boldsymbol{\varepsilon}) = N + L - I(\boldsymbol{\varepsilon}) + 1$, where $I(\boldsymbol{\varepsilon})$ is the length of

TABLE I
EIGENVALUES OF AUTOCOVARANCE MATRICES FOR THE CONSIDERED EXPONENTIAL MIP ($\zeta_M = \max\{\zeta_j\}$)

Raised-Cosine (RC)				Square-Root RC (SRRC)				Gaussian	
$B = 1.5/T_s$		$B = 3.0/T_s$		$B = 1.5/T_s$		$B = 3.0/T_s$		$B = 1.5/T_s$	$B = 3.0/T_s$
$\beta = 0.1$	$\beta = 1.0$	$\beta = 0.1$	$\beta = 1.0$	$\beta = 0.1$	$\beta = 1.0$	$\beta = 0.1$	$\beta = 1.0$		
$\zeta_M = 0.93$	$\zeta_M = 1.3$	$\zeta_M = 1.1$	$\zeta_M = 2.0$	$\zeta_M = 0.34$	$\zeta_M = 1.1$	$\zeta_M = 0.19$	$\zeta_M = 0.53$	$\zeta_M = 1.4$	$\zeta_M = 1.8$
ζ/ζ_M	ζ/ζ_M	ζ/ζ_M	ζ/ζ_M	ζ/ζ_M	ζ/ζ_M	ζ/ζ_M	ζ/ζ_M	ζ/ζ_M	ζ/ζ_M
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.74	0.50	0.83	0.65	0.74	0.57	0.84	0.70	0.55	0.66
0.56	0.28	0.70	0.45	0.56	0.37	0.73	0.53	0.33	0.48
0.41	0.14	0.60	0.33	0.41	0.24	0.63	0.42	0.19	0.36
0.29	0.05	0.51	0.24	0.29	0.13	0.53	0.32	0.09	0.28
0.17	0.02	0.43	0.17	0.19	0.05	0.45	0.26	0.05	0.21
0.05	0.0	0.36	0.11	0.06	0.01	0.38	0.21	0.0	0.16
0.0	0.0	0.30	0.06	0.0	0.0	0.32	0.15	0.0	0.11
0.0	0.0	0.25	0.03	0.0	0.0	0.27	0.10	0.0	0.07
0.0	0.0	0.21	0.01	0.0	0.0	0.23	0.05	0.0	0.04
0.0	0.0	0.16	0.0	0.0	0.0	0.19	0.02	0.0	0.03
0.0	0.0	0.04	0.0	0.0	0.0	0.06	0.01	0.0	0.01

TABLE II
NUMBER OF SYSTEM STATES FOR VARIOUS β 's AND B 's

β	RC		SRRC		Gaussian	
	$B = 1.5/T_s$	$B = 3.0/T_s$	$B = 1.5/T_s$	$B = 3.0/T_s$	$B = 1.5/T_s$	$B = 3.0/T_s$
	(N_S)	(N_S)	(N_S)	(N_S)	(N_S)	(N_S)
0.1	64	16	128	16	4	4
0.5	8	4	16	4		
1.0	4	4	16	4		

the corresponding error event. By letting $w(\varepsilon) = 1$ and $N = 1$, where we obtain the upper bound for burst (word) error rate (WER) P_B as follows:

$$\text{WER} \leq \sum_{\varepsilon \in \mathcal{G}_0} M(\varepsilon) P_r[\Gamma(\mathbf{a} - \varepsilon) \geq \Gamma(\mathbf{a})] \cdot P_r[\mathcal{X}(\varepsilon)]. \quad (13)$$

Note that not like the case of continuous-mode transmission, there are only finite terms in the upper bounds (11) and (13) for the burst-mode transmission.

B. Pairwise Error Probability

By definition, given the channel impulse response $\mathbf{h}$, the pairwise error event probability is calculated by

$$\begin{aligned} & P_r\{\Gamma(\mathbf{a} - \varepsilon) \geq \Gamma(\mathbf{a}) | \mathbf{h}\} \\ &= P_r\left\{ \sum_{d=1}^D \sum_{i=0}^{I(\varepsilon)-1} \sum_{j=0}^{\alpha-1} \left| z_{i\alpha+j}^{(d)} - \sum_{l=0}^L h_{i\alpha+j, l\alpha+j}^{(d)} \hat{a}_{i-l} \right|^2 \right. \\ &\quad \left. \leq \sum_{d=1}^D \sum_{i=0}^{I(\varepsilon)-1} \sum_{j=0}^{\alpha-1} \left| z_{i\alpha+j}^{(d)} - \sum_{l=0}^L h_{i\alpha+j, l\alpha+j}^{(d)} a_{i-l} \right|^2 \middle| \mathbf{h} \right\} \\ &= P_r\left\{ \sum_{d=1}^D \sum_{i=0}^{I(\varepsilon)-1} \sum_{j=0}^{\alpha-1} 2\Re\left[\delta_{i,j}^{(d)*} \eta_{i,j}^{(d)}\right] \right. \\ &\quad \left. \leq - \sum_{d=1}^D \sum_{i=0}^{I(\varepsilon)-1} \sum_{j=0}^{\alpha-1} \left| \delta_{i,j}^{(d)} \right|^2 \middle| \mathbf{h} \right\} \end{aligned} \quad (14)$$

$$\delta_{i,j}^{(d)} = \sum_{l=0}^L h_{i\alpha+j, l\alpha+j}^{(d)} \varepsilon_{i-l} \quad (15)$$

$$\eta_{i,j}^{(d)} = \eta_{i\alpha+j}^{(d)} \quad (16)$$

and $\Re[\cdot]$ denotes the real part of $\cdot$. Since the $\{\eta_{i,j}^{(d)}\}$ are i.i.d. zero mean complex-valued Gaussian random variables with variance $2N_0W$, then the term in (14)

$$\sum_{d=1}^D \sum_{i=0}^{I(\varepsilon)-1} \sum_{j=0}^{\alpha-1} 2\Re\left[\delta_{i,j}^{(d)*} \eta_{i,j}^{(d)}\right]$$

is also a zero-mean Gaussian random variable with variance equal to

$$8N_0W \sum_{d=1}^D \sum_{i=0}^{I(\varepsilon)-1} \sum_{j=0}^{\alpha-1} \left| \delta_{i,j}^{(d)} \right|^2.$$

Therefore, (14) can be represented in the following form:

$$P_r\{\Gamma(\mathbf{a} - \varepsilon) \geq \Gamma(\mathbf{a}) | \mathbf{h}\} = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{\Delta^2}{16N_0W}}\right) \quad (17)$$

where $\text{erfc}(x)$ is the complementary error function defined by

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt \quad (18)$$

and

$$\Delta^2 = \sum_{d=1}^D \sum_{i=0}^{I(\varepsilon)-1} \sum_{j=0}^{\alpha-1} \left| \delta_{i,j}^{(d)} \right|^2. \quad (19)$$

For time-variant multipath fading channels, we define a new random variable φ as

$$\begin{aligned} \varphi &\triangleq \frac{\Delta^2}{16N_0W} \\ &= \frac{1}{16N_0W} \sum_{d=1}^D \sum_{i=0}^{I(\varepsilon)-1} \sum_{j=0}^{\alpha-1} \left| \sum_{l=0}^L h_{i\alpha+j, l\alpha+j}^{(d)} \varepsilon_{i-l} \right|^2. \end{aligned} \quad (20)$$

Then, (17) can be rewritten as

$$P_r\{\Gamma(\mathbf{a} - \varepsilon) \geq \Gamma(\mathbf{a}) | \mathbf{h}\} = \frac{1}{2} \operatorname{erfc}(\sqrt{\varphi}). \quad (21)$$

Recall that the channels under consideration are slowly time varying, hence, it is reasonable that the channel remains fixed during several tens of symbol durations. Thus, the pairwise error event probability, given in (21), can be averaged over the fading channel statistics, i.e.,

$$P\{\Gamma(\mathbf{a} - \varepsilon) \geq \Gamma(\mathbf{a})\} = \int_0^\infty \frac{1}{2} \operatorname{erfc}(\sqrt{\varphi}) p_\varphi(\varphi) d\varphi \quad (22)$$

where $p_\varphi(\varphi)$ is the probability density function of the random variable φ . In order to calculate the integral (22), we define the new random variable³

$$\varphi_d \triangleq \sum_{i=0}^{I(\varepsilon)-1} \sum_{j=0}^{\alpha-1} \left| \sum_{l=0}^L h_{i\alpha+j, l\alpha+j}^{(d)} \varepsilon_{i-l} \right|^2 \quad (23)$$

where the time index with the channel tap gains has been omitted. Then, (20) becomes

$$\varphi = \frac{1}{16N_0W} \sum_{d=1}^D \varphi_d. \quad (24)$$

Furthermore, it can be shown that (23) is a quadratic form, given by

$$\varphi_d = \mathbf{h}_d^H \mathbf{\Pi} \mathbf{h}_d \quad (25)$$

where

$$\mathbf{h}_d = \left(h_0^{(d)}, \dots, h_{\alpha(L+1)-1}^{(d)} \right)^T \quad (26)$$

with T denoting the matrix transpose, and

$$\mathbf{\Pi} = \sum_{i=0}^{I(\varepsilon)-1} \mathbf{\Pi}_i \quad (27)$$

is a $(L+1)\alpha$ order symmetric matrix with

$$\mathbf{\Pi}_i = \begin{bmatrix} \pi_i(0,0) & \pi_i(0,1) & \cdots & \pi_i(0,L) \\ \pi_i(1,0) & \pi_i(1,1) & \cdots & \pi_i(1,L) \\ \vdots & \vdots & \ddots & \vdots \\ \pi_i(L,0) & \pi_i(L,1) & \cdots & \pi_i(L,L) \end{bmatrix} \quad (28)$$

³The time index will be omitted in the sequel.

and

$$\pi_i(m, n) = \begin{bmatrix} \varepsilon_{i-m}^* \varepsilon_{i-n} & 0 & \cdots & \cdots & 0 \\ 0 & \varepsilon_{i-m}^* \varepsilon_{i-n} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & \varepsilon_{i-m}^* \varepsilon_{i-n} \end{bmatrix} \quad (29)$$

is a $\alpha \times \alpha$ matrix. As evident, $p_{\varphi_d}(\varphi_d)$ must be found before calculating (22). Let $\Phi_{\varphi_d}(s)$ be the characteristic function of the random variable φ_d . By definition

$$\Phi_{\varphi_d}(s) = \int_0^\infty e^{-s\varphi_d} p_{\varphi_d}(\varphi_d) d\varphi_d \quad (30)$$

where $p_{\varphi_d}(\varphi_d)$ is the probability density function of random variable φ_d . Then from [17]

$$\begin{aligned} \Phi_{\varphi_d}(s) &= \frac{1}{\det(\mathbf{I} + 2s\mathbf{R}\mathbf{\Pi})} \\ &= \frac{1}{\prod_{l=1}^{(L+1)\alpha} (1 + 2s\lambda_l)} \end{aligned} \quad (31)$$

where $\mathbf{R}$ is the covariance matrix of the random vector $\mathbf{h}_d$, i.e.,

$$\mathbf{R} \triangleq \frac{1}{2} \cdot E\{\mathbf{h}_d \mathbf{h}_d^H\} \quad (32)$$

with H denoting the Hermitian transpose, $\mathbf{I}$ is the $(L+1)\alpha \times (L+1)\alpha$ identity matrix, and $\{\lambda_l\}$ are the eigenvalues of the matrix $\mathbf{R}\mathbf{\Pi}$. Since the fading statistics on the D different diversity branches are mutually independent, the received signals $\{z_k^{(d)}, d = 1, \dots, D\}$ are also statistically independent. Hence, the characteristic function of the random variable φ is directly given by

$$\Phi_\varphi(s) = \frac{1}{\prod_{l=1}^{(L+1)\alpha} (1 + s\varpi_l)^D}. \quad (33)$$

It follows from (33) that the probability density function $p_\varphi(\varphi)$ is given as

$$p_\varphi(\varphi) = \sum_{l=1}^{(L+1)\alpha} \sum_{d=1}^D \frac{a_{l,d}}{\varpi_l^d} \varphi^{d-1} \frac{\exp[-\varphi/\varpi_l]}{(d-1)!} \quad (34)$$

where

$$\begin{aligned} a_{l,d} &= \frac{1}{(D-d)! \varpi_l^{D-d}} \\ &\cdot \left\{ \frac{d^{D-d}}{d^{D-d}} [1 + s\varpi_l]^D \cdot \Phi_\varphi(s) \right\}_{s=\frac{-1}{\varpi_l}} \end{aligned} \quad (35)$$

and

$$\varpi_l = \frac{\lambda_l}{8N_0W}. \quad (36)$$

Then from [12], we have the pairwise error event probability given by

$$\begin{aligned} P\{\Gamma(\mathbf{a} - \varepsilon) \geq \Gamma(\mathbf{a})\} &= \sum_{l=1}^{(L+1)\alpha} \sum_{d=1}^D a_{l,d} \left[\left(\frac{1 - \mu_l}{2} \right)^d \right. \\ &\quad \times \left. \sum_{m=0}^{d-1} \binom{d-1+m}{m} \left(\frac{1 + \mu_l}{2} \right)^m \right] \end{aligned} \quad (37)$$

where

$$\mu = \sqrt{\frac{\varpi_l}{1 + \varpi_l}}. \quad (38)$$

C. Evaluation of Upper Bounds

The error state diagram and stack algorithm in [5] can be modified to evaluate the upper bounds (12) and (13).

1) *Error State Diagram:* Define error states as $\mathbf{v} = \mathbf{s}_i^{(n)} - \mathbf{s}_i^{(n')}$, where $\mathbf{s}_i^{(n)}$ and $\mathbf{s}_i^{(n')}$ denote legitimate system states. The error state diagram is constructed as follows. The initial node and the final node are zero-error states, but the intermediate nodes are nonzero error states. A directed line from $\mathbf{v}_l$ to $\mathbf{v}_m$ for some l and m represents the transition from $\mathbf{v}_l$ to $\mathbf{v}_m$ which is denoted by $(\mathbf{v}_l, \mathbf{v}_m)$. Since traveling from the initial node to the final node, passing through an intermediate zero-error state is not possible, the error state diagram can enumerate the set of error vectors $\mathcal{G}_0$. The next step is to define an appropriate transition gain for each transition $(\mathbf{v}_l, \mathbf{v}_m)$ so that (12) and (13) can be evaluated. Let b_{lm} denote the transition gain associated with the transition $(\mathbf{v}_l, \mathbf{v}_m)$. Define b_{lm}

$$b_{lm} = P_{lm} X_1 X_2^{u_{lm}} X_3^{\Pi_{lm}} \quad (39)$$

where we have the following.

- X_1 , X_2 , and X_3 are dummy variables. X_1 is used to count the length of error events.
- P_{lm} is the fraction of correct symbols a_i such that the transition $(\mathbf{v}_l, \mathbf{v}_m)$ is possible (or the corresponding error symbol ε_i is possible).
- $u_{lm} = 1$, if there is an error associated with the transition $(\mathbf{v}_l, \mathbf{v}_m)$, and SER is to be evaluated; otherwise $u_{lm} = 0$.
- Π_{lm} is the branch distance matrix given by (28), but here it is emphasized that it is a function of the transition $(\mathbf{v}_l, \mathbf{v}_m)$.

From the definition of b_{lm} in (39), the path gain for a particular path in the error-state diagram is given by

$$\prod_{\{(l,m)\}} P_{lm} \cdot X_1 \cdot X_2^{\sum_{\{(l,m)\}} u_{lm}} \cdot X_3^{\sum_{\{(l,m)\}} \Pi_{lm}} \quad (40)$$

where $\{(l,m)\}$ denotes the set of state transitions associated with the path under consideration. Also, for the path starting with the initial node and terminating at the final node

$$P\{\mathcal{X}(\varepsilon)\} = \prod_{\{(l,m)\}} P_{lm} \quad (41)$$

$$w(\varepsilon) = \sum_{\{(l,m)\}} u_{lm} \quad (42)$$

$$\Pi = \sum_{\{(l,m)\}} \Pi_{lm} \quad (43)$$

and

$$M(\varepsilon) = N + L + 1 - \sum_{\{(l,m)\}} 1. \quad (44)$$

2) *The Stack Algorithm:* To calculate (12) and (13), the following stack algorithm will be employed. The basic idea is that a stack is constructed and maintained with each path (entry) containing the following information; terminal node, $\prod_{\{(l,m)\}} P_{lm}$, $\sum_{\{(l,m)\}} u_{lm}$, the length of path, $\sum_{\{(l,m)\}} \Pi_{lm}$, and the intermediate bit error probability P_I . Here, P_I is calculated by

$$P_I = \frac{M(\varepsilon)}{N} \prod_{\{(l,m)\}} P_{lm} \cdot \sum_{\{(l,m)\}} u_{lm} \cdot \check{P}\{\Gamma(\mathbf{a} - \varepsilon) \geq \Gamma(\mathbf{a})\} \quad (45)$$

where $\check{P}\{\Gamma(\mathbf{a} - \varepsilon) \geq \Gamma(\mathbf{a})\}$ is calculated by using (37) along with the eigenvalues associated with the matrix $\sum_{\{(l,m)\}} \Pi_{lm}$. The complete algorithm is given in Fig. 3 and is described as follows.

- *Step 1:* Load the stack with the initial node, set $P_S = 0$ or $P_B = 0$.
- *Step 2:* Determine if the length of the top path is larger than $N + L$. If it is, then go to *Step 7*; otherwise go to *Step 3*.
- *Step 3:* Determine if the top path terminates at the final node. If it does, then go to *Step 4*; otherwise go to *Step 5*.
- *Step 4:* If $P_I < \rho \cdot P_S$ (or P_B), then output P_S (or P_B) and terminate the algorithm; otherwise $P_S = P_S + P_I$ (or $P_B = P_B + P_I$) and go to *Step 5*.
- *Step 5:* Extend the top path and calculate $\prod_{\{(l,m)\}} P_{lm}$, $\sum_{\{(l,m)\}} u_{lm}$, the path length, $\sum_{\{(l,m)\}} \Pi_{lm}$, and P_I for all successors of the top path.
- *Step 6:* Insert these successors extended from the top path and rearrange the stack in the order of decreasing intermediate bit error probability P_I (from top to bottom).
- *Step 7:* Delete the top path.
- *Step 8:* Determine if the stack is empty. If it does, then output P_S (or P_B) and stop; otherwise go to *Step 2*.

Note that with $\rho > 0$, SER (or WER) evaluated with this algorithm may only be a truncated upper bound. Theoretically, however, ρ can always be selected small enough to obtain the desired accuracy.

IV. NUMERICAL RESULTS

In this section, BPSK system is used for numerical results. Three transmit pulses are considered, including the RC, SRRC, and Gaussian pulses. For BPSK, $q = 2$ and $a_i \in \{1, -1\}$. In all examples, $N = 100$, $B = W$, and L_c is selected to contain 99% of the total power. The method in [4] of using unitary transformations is employed to produce the desired correlations between channel taps in the simulations [4]. Only the results for the exponential MIP (with $\tau_m = 2T_s$)

$$\begin{aligned} \phi_c(\tau) &\triangleq 1/2 \cdot E[|c(t; \tau)|^2] \\ &= K \cdot \exp[-\tau/T_s] \quad 0 \leq \tau \leq \tau_m \end{aligned} \quad (46)$$

are shown here, where K is the normalized constant such that

$$\int_0^{\tau_m} \phi_c(\tau) d\tau = 1. \quad (47)$$

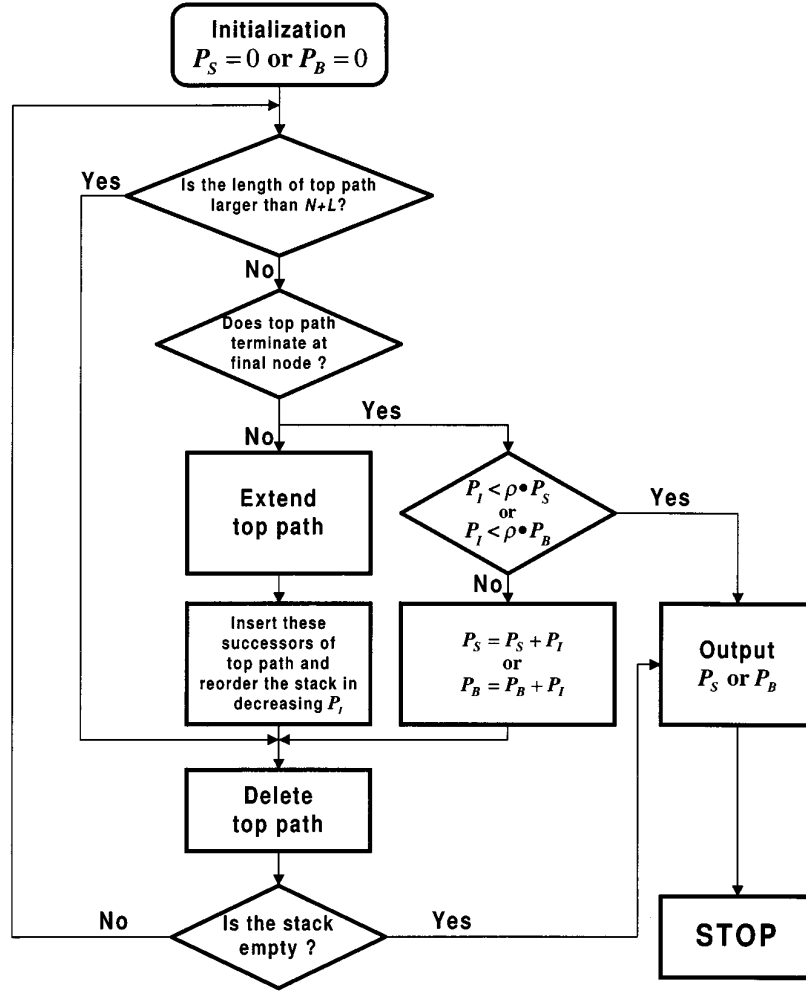


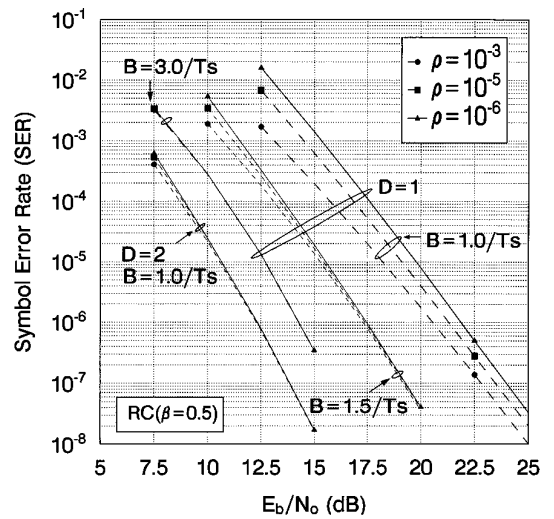
Fig. 3. The stack algorithm.

The results obtained with this exponential MIP is about 0.2 dB inferior to those obtained with the rectangular MIP. In all figures, the received SNR is defined as

$$\frac{E_b}{N_0}(\text{dB}) \triangleq 10 \log \frac{E[|a_i|^2] \cdot \int_{-\infty}^{\infty} |h^d(t, \tau)|^2 d\tau}{2N_0}. \quad (48)$$

Fig. 4 shows an example of selecting ρ in the stack algorithm for the evaluation of the upper bounds (12) and (13). As can be seen, the truncated upper bound evaluated with $\rho = 10^{-6}$ provides a good approximation to the true upper bound for bit error rate smaller than 10^{-4} , except for the case $D = 1$ with $B = 1/T_s$, where a smaller ρ needs to be used.⁴ As expected, a smaller ρ is required for a low SNR and/or a channel with a smaller effective diversity order. $\rho = 10^{-6}$ is used for all the analytical results in the following. Figs. 5 and 6 compare the simulation and analytical results for RC pulse with $\beta = 0.5$. Similar results are observed for other pulses and parameters. Also, the upper bounds are tighter at a higher SNR and/or for a channel with a larger effective diversity order. In addition, from simulation results in the figure, it is observed that with no explicit

⁴Unfortunately, we are not able to use smaller ρ in our evaluation in this case due to the memory limitation in our work stations.

Fig. 4. Selection of ρ in the stack algorithm.

diversity ($D = 1$), a significant performance improvement is obtained by using the implicit path diversity at the expense of

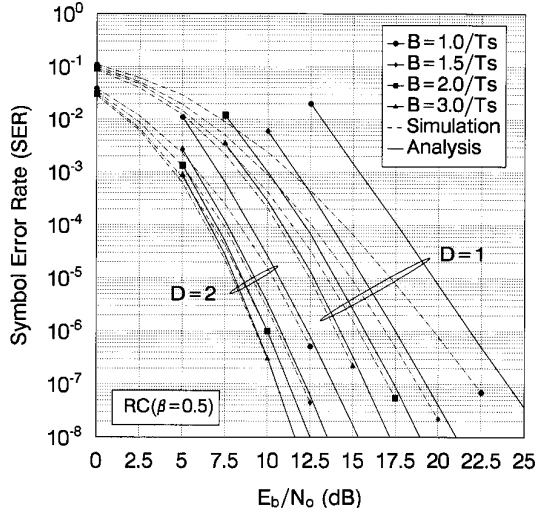
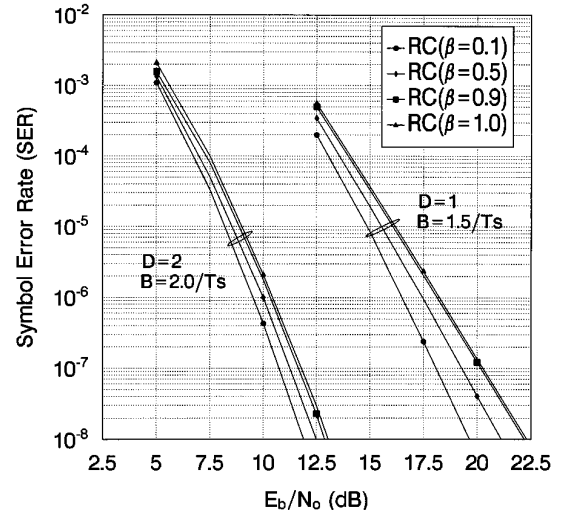
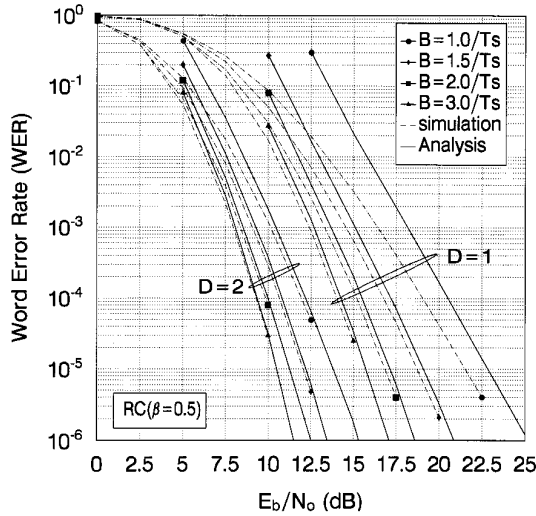
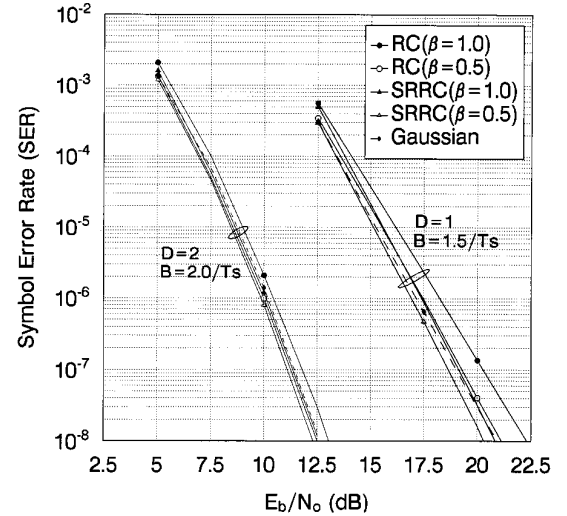
Fig. 5. Simulation and analytical SER for RC pulse with $\beta = 0.5$.Fig. 7. Effects of β 's on the SER for RC pulse.Fig. 6. Simulation and analytical WER for RC pulse with $\beta = 0.5$.

Fig. 8. Comparison of SER with different transmit pulses.

a larger bandwidth. For example, at $SER = 10^{-6}$, approximately 6- and 5-dB improvements are obtained by using three and two times the bandwidth, respectively. For $D = 2$, as expected, the improvement with the implicit diversity is less significant than for the $D = 1$ case. Nevertheless, a 2–2.5-dB improvement is still possible by using two times the bandwidth at $SER = 10^{-6}$. Clearly, the explicit diversity is more effective in increasing the system performance. This is because the effective diversity order is increased by D times when D explicit diversity is used. As a result, the implicit path diversity may not be designed to replace the explicit diversity, rather it may serve as a complement to the explicit diversity to further improve the performance. From Table II, it is worthy noting that a larger bandwidth often reduces the system state number N_S .

Fig. 7 shows an example of the effects of β 's on SER for RC pulse. (Similar results are observed for SRRC pulse.) As mentioned, better performance is obtained with a smaller β due to a larger effective diversity order P and/or due to the total

channel power being distributed more evenly over the independent paths which are used to generate the overall channel response. In the figure, with $B = 1.5/T_s$, about 2 dB of improvement (at $SER = 10^{-6}$) is obtained by using $\beta = 0.1$ instead of 1.0. Unfortunately, as shown in Table II, a smaller β often results in a larger number of system states N_S and hence increases the system complexity. Hence, one needs to trade performance off against complexity. In some cases, however, it is possible to have a smaller β , but with the same number of system states, as shown in Table II. Fig. 8 shows the relative performance of the considered three transmit pulses with various pulse parameters. In the figure, the bandwidth for Gaussian pulse is its 99% bandwidth. As seen, the performance difference due to using different pulses can exceed 1 dB, with different selections of pulse parameters. In addition, from Table I and Fig. 8, it is observed that better performance is obtained if the overall received power is distributed more evenly over the independent paths used to generate the overall channel response.

V. CONCLUSION

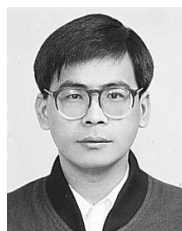
In this paper, the issues of diversity reception, bandwidth utilization, and performance evaluation of digital transmission over slowly time-varying frequency-selective fading channels are investigated, with a transmit pulse design that is more general than previously considered. Three transmit pulses, namely, the RC, SRRC, and Gaussian pulses have been used as examples. Upper bounds on the error rate are derived for maximum likelihood sequence estimation for burst-mode transmission. The bounds are useful for performance estimation at the error rates that would be difficult to estimate by using computer simulations. Numerical results show that good tradeoffs among performance, bandwidth, and complexity are always feasible with an appropriate design on the transmit pulses and by combining explicit and implicit diversity.

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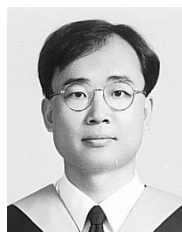
REFERENCES

- [1] R. D'avella, L. Moreno, and M. Sant'agostino, "An adaptive MLSE receiver for TDMA digital mobile radio," *IEEE J. Select. Areas Commun.*, vol. 7, pp. 122–129, Jan. 1989.
- [2] J. G. Proakis, "Adaptive equalization for TDMA digital mobile radio," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 333–341, May 1991.
- [3] K. Pahlavan and J. W. Matthews, "Performance of adaptive matched filter receivers over fading multipath channels," *IEEE Trans. Commun.*, vol. 38, pp. 2106–2113, Dec. 1991.
- [4] H. Meyr, M. Moeneclaey, and S. A. Fechtel, *Digital Communication Receivers*. New York: Wiley, 1998.
- [5] W.-H. Sheen and G. L. Stüber, "MLSE equalization and decoding for multipath-fading channels," *IEEE Trans. Commun.*, vol. 39, pp. 1455–1464, Oct. 1991.
- [6] B. Hart and P. Taylor, "Extended MLSE diversity receiver for the time- and frequency-selective channel," *IEEE Trans. Commun.*, vol. 45, pp. 322–333, Mar. 1997.
- [7] Q. Dai and E. Shwedyk, "Detection of band-limited signals over frequency selective Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 42, pp. 941–950, Feb./Mar./Apr. 1994.
- [8] X. Yu and S. Pasupathy, "Innovations-based MLSE for Raleigh fading channels," *IEEE Trans. Commun.*, vol. 43, pp. 1534–1544, Feb./Mar./Apr. 1995.
- [9] R. Raheli, A. Polydoros, and C. K. Tzou, "Per-survivor processing: A general approach to MLSE in uncertain environments," *IEEE Trans. Commun.*, vol. 43, pp. 354–364, Feb./Mar./Apr. 1995.
- [10] K. M. Chugg and A. Polydoros, "MLSE for an unknown channel—Part I: Optimality considerations," *IEEE Trans. Commun.*, vol. 45, pp. 836–846, July 1996.
- [11] —, "MLSE for an unknown channel—Part II: Tracking performance," *IEEE Trans. Commun.*, vol. 45, pp. 949–958, Aug. 1996.
- [12] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.
- [13] G. D. Forney, Jr., "Maximum likelihood sequence estimation of digital sequence in the presence of intersymbol interference," *IEEE Trans. Inform. Theory*, vol. IT-18, pp. 363–378, May 1972.
- [14] —, "The Viterbi algorithm," *Proc. IEEE*, vol. 61, pp. 268–278, Mar. 1973.
- [15] G. J. Foschini, "Performance bound for maximum-likelihood reception of digital data," *IEEE Trans. Inform. Theory*, vol. IT-21, pp. 47–50, Jan. 1975.
- [16] S. Verdú, "Maximum likelihood sequence detection for intersymbol interference channels: A new upper bound on error probability," *IEEE Trans. Inform. Theory*, vol. IT-33, pp. 62–68, Jan. 1987.
- [17] M. Schwartz, W. R. Bennett, and S. Stein, *Communication Systems and Techniques*. New York: McGraw-Hill, 1966.



Wern-Ho Sheen (S'89–M'91) received the B.S. degree from the National Taiwan Institute of Technology, Taiwan, R.O.C., in 1982, the M.S. degree from the National Chiao Tung University, Taiwan, in 1984, and the Ph.D. degree from the Georgia Institute of Technology, Atlanta, in 1991.

From 1984 to 1993, he was with Telecommunication Laboratories, Taiwan, where he was mainly involved in the projects of personal communications and basic rate ISDN. Since 1993, he has been with the Department of Electrical Engineering, National Chung Cheng University, Chia-Yi, Taiwan, where he is currently a Professor. His research interests include adaptive signal processing, spread-spectrum communications, and personal and mobile radio systems.



Chun-Chieh Tseng (S'99) received the B.S. degree in electronics from the National Taiwan Institute of Technology, Taiwan, R.O.C., in 1992 and the M.S. degree from the National Chung Cheng University, Chia-Yi, Taiwan, in 1996. He is currently working toward the Ph.D. degree at National Chung Cheng University.

His research interests include personal and mobile radio systems and spread-spectrum communications.



Chia-Shu Wang received the B.S. degree from the National Chung Cheng Institute of Technology, Taiwan, R.O.C., and the M.S. degree from National Chung Cheng University, Chia-Yi, Taiwan, in 1992 and 1997, respectively, both in electrical engineering.

His research interests include the system design and implementation of personal and mobile radio communications.