

Frequency-Diversity Spread-Spectrum Communication System to Counter Bandlimited Gaussian Interference

Ghassan Kawas Kaleh

Abstract—A spread spectrum system to counter bandlimited Gaussian interference is proposed. The optimum receiver for this system is easy to build. Frequency diversity which allows the receiver to distinguish unjammed signal replicas from their jammed versions is used. The system can also resist bandlimited partial-time jamming. The only choice left to a smart jammer to maximize the error probability is to spread its signal like the communicator. The optimum receiver, which jointly performs symbol detection and interference rejection, is derived. Side information needed by this receiver can easily be estimated. However, if the interference bandwidth is narrow compared to the signal bandwidth, side information on noise and interference levels is not needed by a simpler and near-optimum receiver. Bit-error probability is evaluated for quaternary phase shift keying (QPSK) modulation and compared to that of direct sequence spread spectrum. We also propose the use of polyphase filters for simple system implementation.

I. INTRODUCTION

CONSIDER the transmission of information symbols over an additive white Gaussian noise (AWGN) channel in the presence of bandlimited Gaussian interference. This interference may be due to intentional jamming [1]–[3] or due to narrowband communication links that coexist with our signal [4]. It is well-known that spread spectrum (SS) signaling is an efficient measure to counter bandlimited interference. There are two main types: frequency hopping (FH) and direct sequence (DS). However, FH-SS is vulnerable to partial-band interference, and DS-SS to bandlimited partial-time (or burst) jamming. The performance can be improved by introducing time or frequency diversity. More improvement can be gained by using channel coding instead, because diversity is a mere repetition code. For simplicity, the analysis in this paper is limited to uncoded systems. Also, since an uncoded FH-SS system is vulnerable to the type of interference that we deal with here, the performance of our proposed system will be compared to the DS-SS system only.

It is generally admitted, see, e.g., [2, (8.2.40)], that in an uncoded DS-SS system, the jamming margin (i.e., the maximum jamming-to-signal power ratio that can be tolerated) can be improved by increasing the bandwidth expansion

factor, then called *processing gain*. We emphasize the fact that this result is valid if the demodulator is the simple conventional correlator. This receiver, which is optimum in the presence of AWGN, is suboptimum in the presence of nonwhite Gaussian interference. The optimum receiver is very complicated since it may include a cascade of a noise whitening filter, a matched filter [5] and then an optimum detector, e.g., the Viterbi sequence detector, which takes into account the presence of intersymbol interference introduced by the whitening filter. However, since a DS-SS signal is robust to the effect of intersymbol interference, the optimum detector can be replaced by a simple threshold symbol-by-symbol detector with negligible degradation. Also, the whitening filter is equivalent to the best [6] interference-rejection filter [7]–[9]. The interference-rejection filter should be adaptive and, consequently, significant time and energy (when known reference symbols are transmitted) are needed to adjust its parameters. More complicated solutions based on state-space representation are also derived [10]–[12].

We conclude from the previous paragraph that the use of the suboptimum conventional DS-SS correlator receiver is in contradiction with the significant effort that the designer of advanced systems employs in order to optimize every function. Besides, the improvement obtained from interference-rejection techniques requires complicated processing and may lead to energy loss. Therefore, we aim in this paper to remedy these drawbacks by proposing a waveform whose optimal receiver in the presence of white Gaussian noise and nonwhite (bandlimited) Gaussian interference is easy to build. Obviously, this optimum receiver performs joint symbol detection and interference rejection. In the proposed system, the key parameter is not the processing gain, but rather the fraction of bandwidth occupied by the interference.

The proposed waveform is obtained as depicted in Fig. 1. The information symbols are encoded by a block or convolutional channel encoder. Then, the coded symbols are interleaved and arranged in form of blocks of N symbols. Each symbol in the i th block, say the n th symbol, is multiplied by a secret chip symbol $\gamma_{i,n}$ and the resulting number modulates the amplitude of a pulse function $\psi_n(t)$. The sum of the N modulated pulses forms the transmitted signal. The role of chip symbols is to hide the signal from a possible exploiter and to allow the use of code-division multiple access (CDMA). Functions $\{\psi_n(t); n = 0, 1, \dots, N-1\}$ should be orthogonal; they may overlap or not in the time domain or in the frequency

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The author is with the Ecole Nationale Supérieure des Télécommunications, 46, rue Barrault, 75634 Paris Cedex 13, France.

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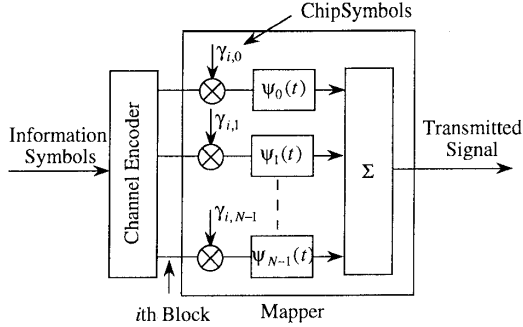


Fig. 1. Generation of the proposed baseband waveform. Pulses $\{\Psi_n(t); n \in \{0, 1, \dots, N-1\}\}$ have disjoint frequency support.

domain. However, in the presence of bandlimited interference which occupies an unknown frequency interval, we require that these functions have disjoint frequency support. This constraint allows us to limit the effect of the interference to a subset of the coded symbols. If the interference level is much larger than the background noise, the receiver can easily distinguish jammed symbols from unjammed symbols. Moreover, if side information on interference and noise levels is available, optimum soft decoding is possible. Otherwise, jammed symbols are erased and unjammed symbols contribute to reliable decoding. If Rayleigh fading diversity is also desired, then guard frequency intervals larger than the coherence bandwidth of the channel should separate the subchannels.

In order to simplify the initial analysis of the proposed scheme, we consider in this paper only the case where a trivial repetition channel code $(N, 1)$ is used. This is equivalent to N -order diversity where each information symbol is associated with N replicas. We can define the replica of symbol a_i as $a_i \gamma_{i,n}$ because the chip symbol $\gamma_{i,n}$ is known by the receiver. Since the replicas $\{a_i \gamma_{i,n}; n = 0, 1, \dots, N-1\}$, transmitted via N nonoverlapping functions $\{\Psi_n(f); n = 0, 1, \dots, N-1\}$, will use N subchannels with disjoint frequency support, the system is equivalent to an N -order frequency-diversity system. As this scheme implies the use of a larger bandwidth than in single transmission, we call it a frequency-diversity spread spectrum (FD-SS) system. It will still be considered as *uncoded* because no coding gain is obtained from the repetition code on the AWGN channel.

Diversity is used in communications to combat Rayleigh fading. It has also been proposed, in a form different from what we advocate here, to improve the performance of conventional SS systems in the presence of partial-time or partial-band jamming [1]–[3]. Recently, Kondo and Milstein [13] proposed the transmission of M replicas of a wideband DS-SS signal over M disjoint frequency bands. Each replica is received by a correlator and the outputs of the M correlators are combined. A necessary condition for the optimality of the receiver is that each correlator sees white interference and/or noise. This condition is not satisfied in general unless the interference has as wide a bandwidth as the DS-SS replica. Our proposed waveform is different in that each replica is an *unspread* narrowband signal. Consequently, its matched

filter has sufficiently narrow bandwidth to isolate the effect of narrowband interference.

It is useful to relate our proposed waveform to multicarrier signaling [14]. This technique, also called orthogonal frequency-division multiplex (OFDM), was devised to reduce the processing effort by the use of the fast Fourier transform. OFDM has been proposed in several forms for CDMA signaling, see e.g., [15]–[17]. Only [15] deals with bandlimited interference that we examine here. The waveform of [17] is similar to the DS-SS waveform with the slight modification that the conventional signature is replaced by its DFT. These schemes differ from ours in that their corresponding functions $\{\Psi_n(f)\}$ introduced above are sinc functions with overlapping frequency support. Having disjoint frequency support is a crucial point in our proposed system. Otherwise, the bank of matched filters at the receiver would have overlapping frequency support. Consequently, the effect of a partial-band interference would appear at the output of all filters. Side information on the presence or absence of interference would not be as precise as with disjoint filters. Also, all the outputs would be correlated; instead of being a simple diagonal matrix, the optimum combiner would correspond to a nondiagonal matrix. In addition, side information on the value of its entries would be difficult to obtain.

The outline of the paper is as follows. FD-SS signal, interference and noise models are described in Section II. In Section III, we derive the optimal detector which minimizes the sequence error probability and delivers sufficient statistics to a decision device. The detector for uncoded symbols is discussed in Section IV for quaternary phase shift keying (QPSK) modulation. Its probability of error is derived and compared with that of a DS-SS system. Then, we describe in Section V a simple method of implementation using polyphase filters. Conclusions are summarized in the last section.

Notation: In this paper, the asterisk $*$ denotes complex conjugation. The Fourier transform of $x(t)$ is denoted $X(f)$. A random variable is written with an uppercase character and a sample of it with a lowercase. $E[\cdot]$ means expectation. The size of a finite set $\mathcal{J}$ is $|\mathcal{J}|$. Real and imaginary parts are written $\Re[\cdot]$ and $\Im[\cdot]$, respectively. Orthogonality and norm $\|\cdot\|$ are in the sense of L_2 -space. $Q(x) = \int_x^\infty \exp(-u^2/2) du / \sqrt{2\pi}$.

II. SYSTEM MODEL

Consider the transmission of a sequence of data symbols $\{a_i; i \in \mathcal{I}\}$ with period T , where $\mathcal{I}$ is the time index set $\{0, 1, \dots, I-1\}$. In FD-SS, the transmitter baseband signal is modeled by

$$x(t) = \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{I}} a_i \gamma_{i,n} \psi_n(t - iT) \\ \psi_n(t) = g(t) e^{j2\pi n F t} \quad (1)$$

where $\mathcal{N}$ denotes the set $\{0, 1, \dots, N-1\}$. The set $\{\gamma_{i,n}; n \in \mathcal{N}\}$ is the signature, i.e., a segment from a real or complex pseudo-random signal. We assume that $|\gamma_{i,n}| = 1$. The pulse $g(t)$ is real, of finite energy ($\|g\|^2 < \infty$) and satisfies the

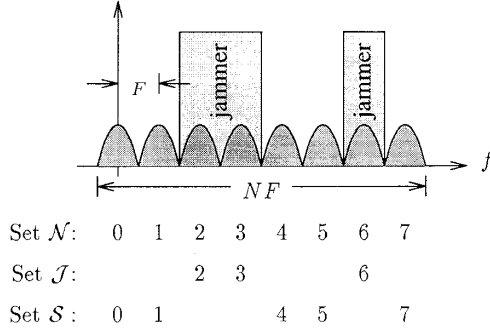


Fig. 2. Signal and interference spectrum, $N = 8, \eta = 3/8$.

following orthogonality condition:

$$\int_{-\infty}^{\infty} g(t - iT)g(t - jT) dt = \delta_{i,j} \|g\|^2 \quad (2)$$

where $\delta_{i,j} = 1$ if $i = j$, and zero otherwise. The Fourier transform of $g(t)$, $G(f)$, is nonzero only for $|f| < (1 + \alpha)/(2T)$, where α is the roll off parameter that takes values between zero and one. The parameter F should guarantee that the Fourier transforms $\Psi_n(f) = G(f - nF)$ and $\Psi_m(f)$ have disjoint support if $n \neq m$. We choose $F = (1 + \alpha)/T$. Let us denote by $\mathcal{F}_n$ the support of $\Psi_n(f)$, i.e.,

$$\mathcal{F}_n = \{f | nF - F/2 \leq f < nF + F/2\}. \quad (3)$$

The above property, along with orthogonality (2), implies the following orthogonality relation:

$$\int_{-\infty}^{\infty} \psi_n(t - iT)\psi_m^*(t - jT) dt = \delta_{n,m} \delta_{i,j} \|g\|^2. \quad (4)$$

Thus, the set $\{\psi_n(t - iT); n \in \mathcal{N}, i \in \mathcal{I}\}$ is an orthogonal basis for the signal space.

Fig. 2 shows the set $\{\Psi_n(f); n \in \mathcal{N}\}$. The bandwidth of the bandpass signal associated with $x(t)$ is $W_x = N(1 + \alpha)/T$. Each element $(a_i \gamma_{i,n})$ modulates a pulse $\psi_n(t - iT)$ which occupies the frequency interval $\mathcal{F}_n$. Thus, $x(t)$ is the sum of N frequency-multiplexed replicas where the n th signal replica is transmitted over a channel with a band $\mathcal{F}_n$. In OFDM schemes [15]–[17], the filter $g(t)$ is simply a rectangle of duration T .

The energy per symbol a_i is an important parameter given by

$$\begin{aligned} E_s &= \frac{1}{2} E[|a_i|^2] \|g\|^2 \sum_{n \in \mathcal{N}} |\gamma_{i,n}|^2 \\ &= \frac{1}{2} E[|a_i|^2] \|g\|^2 N. \end{aligned} \quad (5)$$

The baseband interference signal, denoted $u(t)$, is a sample function of a stationary complex Gaussian process, independent of the signal, with zero mean and power spectral density $\Phi_u(f)$. In order to *simplify* the presentation, we assume that $\Phi_u(f)$ is constant over a given jammed frequency interval of the type $\mathcal{F}_n$ defined in (3), i.e.,

$$\Phi_u(f) = \begin{cases} 2J_{0,n}, & \text{if } f \in \mathcal{F}_n, n \in \mathcal{N} \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

This model implies that the bandwidth $(1 + \alpha)/T$ of a replica is equal to the expected minimum bandwidth of the interference. Therefore, T is a design parameter which determines the coding rate in a coded system.

The baseband background noise, denoted $v(t)$, is a sample function of a second-order and zero mean complex Gaussian process, independent of signal and interference and with power spectral density $2N_0$.

III. OPTIMUM RECEIVER

The received baseband signal in a coherent receiver is

$$z(t) = x(t) + u(t) + v(t). \quad (7)$$

We derive here the optimum detector for (7). The optimality criterion is the minimum error probability when deciding on the sequence $\{a_i; i \in \mathcal{I}\}$. Since the total noise in the observation is Gaussian but not white, the familiar detector includes a cascade of noise whitening filter and a matched filter [5]. Our approach is different.

Proposition 1: Given $z(t)$, the following statistics are sufficient for the optimal detection of the sequence $\{a_i; i \in \mathcal{I}\}$

$$z_{i,n} = \int_{-\infty}^{\infty} z(t)\psi_n^*(t - iT) dt; \quad n \in \mathcal{N}; \quad i \in \mathcal{I}. \quad (8)$$

Also,

$$z_{i,n} = \|g\|^2 a_i \gamma_{i,n} + y_{i,n}; \quad n \in \mathcal{N}; \quad i \in \mathcal{I} \quad (9)$$

where $\{y_{i,n}; n \in \mathcal{N}; i \in \mathcal{I}\}$ are samples of independent Gaussian variables with zero mean and variance

$$E[|Y_{i,n}|^2] = 2(J_{0,n} + N_0) \|g\|^2; \quad n \in \mathcal{N}, \quad i \in \mathcal{I}. \quad (10)$$

Proof: Let us decompose the observation $Z(f)$ into a sum of orthogonal signals by passing it through a bank of resolution filters $\{Q_n(f); n \in \mathcal{N}\}$, where

$$Q_n(f) = \begin{cases} 1, & \text{if } f \in \mathcal{F}_n \\ 0, & \text{otherwise.} \end{cases}$$

Thus, by defining

$$P_n(f) = Z(f)Q_n(f) \quad (11)$$

we have

$$z(t) = \sum_{n \in \mathcal{N}} p_n(t)$$

where, from (1), (7), and (11)

$$p_n(t) = \sum_{i \in \mathcal{I}} a_i \gamma_{i,n} \psi_n(t - iT) + w_n(t); \quad n \in \mathcal{N}$$

and $w_n(t)$ is due to interference $u(t)$ and noise $v(t)$.

Every signal $p_n(t)$ is the sum of an information-carrying signal which belongs to a signal subspace, $\mathcal{X}_n$, spanned by the orthogonal basis $\{\psi_n(t - iT); i \in \mathcal{I}\}$, and an independent noise $w_n(t)$ which is Gaussian with constant power spectral density $2(J_{0,n} + N_{0,n})$. Therefore, given $p_n(t)$, the orthogonal projection of $p_n(t)$ on $\mathcal{X}_n$ constitutes sufficient statistics for the detection of symbols $\{a_i; i \in \mathcal{I}\}$, [18]. This projection

coincides with the orthogonal projection of $z(t)$ on $\mathcal{X}_n$ because the difference $z(t) - p_n(t)$ is orthogonal to $\mathcal{X}_n$. Thus, we can associate every $p_n(t)$ with the following set of sufficient statistics:

$$z_{i,n} = \int_{-\infty}^{\infty} z(t) \psi_n^*(t - iT) dt; \quad i \in \mathcal{I}.$$

Now, observing $\{p_n(t); n \in \mathcal{N}\}$ is equivalent to observing the received signal $z(t) = \sum_{n \in \mathcal{N}} p_n(t)$. Therefore, we can associate $z(t)$ with the sufficient statistics defined in (8).

Using (1), (4), and (7) in (8) one can easily prove (9). Also, we obtain from (1), (7), (8), and (9) that

$$\begin{aligned} y_{i,n} &= \int_{-\infty}^{\infty} [u(t) + v(t)] \psi_n^*(t - iT) dt; \\ &\quad n \in \mathcal{N}, \quad i \in \mathcal{I} \\ E[Y_{i,n} Y_{j,m}^*] &= \int_{\mathcal{F}_n \cap \mathcal{F}_m} [\Phi_u(f) + 2N_0] \Psi_n^*(f) \\ &\quad \cdot \Psi_m(f) \exp[\sqrt{-1} 2\pi(i-j)Tf] df \\ &= 2(J_{0,n} + N_0) \|g\|^2 \delta_{n,m} \delta_{i,j}; \\ &\quad m, n \in \mathcal{N}; \quad i, j \in \mathcal{I} \end{aligned}$$

which completes the proof. $\square$

Let $\mathcal{M}$ be the set of messages carried by the symbol sequence. With every $m \in \mathcal{M}$ we associate an *a priori* probability $P[m]$ and a sequence $\{a_i^m; i \in \mathcal{I}\}$. Given the sufficient statistics (8), the optimum detector [18] maximizes

$$P[m] p[\{z_{i,n}; n \in \mathcal{N}, i \in \mathcal{I}\} | \{a_i^m; i \in \mathcal{I}\}]$$

over $\mathcal{M}$, where $p[\cdot | \cdot]$ denotes conditional probability density.

Proposition 2: Given $z(t)$, the following statistics are sufficient for the optimal detection of the sequence $\{a_i; i \in \mathcal{I}\}$:

$$\Lambda_i = \sum_{n \in \mathcal{N}} \frac{1}{1 + (J_{0,n}/N_0)} z_{i,n} \gamma_{i,n}^*; \quad i \in \mathcal{I}. \quad (12)$$

Also

$$\Lambda_i = a_i \frac{2E_s}{E[|a_i|^2]} \beta + \sqrt{\frac{2E_s}{E[|a_i|^2]}} \beta b_i \quad (13)$$

$$\beta = \frac{1}{N} \sum_{n \in \mathcal{N}} \frac{1}{1 + (J_{0,n}/N_0)}. \quad (14)$$

The noise samples $\{b_i; i \in \mathcal{I}\}$ are drawn from independent Gaussian variables with zero mean and $E[|B_i|^2] = 2N_0$.

Proof: The independence of noise components $\{Y_{i,n}\}$ implies the factorization

$$\begin{aligned} p[\{z_{i,n}; n \in \mathcal{N}, i \in \mathcal{I}\} | \{a_i^m; i \in \mathcal{I}\}] \\ = \prod_{i \in \mathcal{I}} p[\{z_{i,n}; n \in \mathcal{N}\} | a_i^m]. \end{aligned}$$

This implies the possibility of computing each factor separately from other factors. Using (9) and the Gaussian probability density formula, one finds that in the expression of $p[\{z_{i,n}; n \in \mathcal{N}\} | a_i^m]$, the observation appears only in its exponent and as follows:

$$\sum_{n \in \mathcal{N}} \frac{1}{2(J_{0,n} + N_0)} |z_{i,n} - a_i \gamma_{i,n}|^2 |g|^2.$$

By simplifying we find that the sufficient statistics $\{z_{i,n}; n \in \mathcal{N}\}$ appear only in Λ_i .

To prove (13), we replace in (12) $z_{i,n}$ by its value from (9) and use (5). It is easy to verify that $E[|B_i|^2] = 2N_0$. $\square$

Sufficient statistics $\{\Lambda_i; i \in \mathcal{I}\}$ are decision variables to be processed by a decision device such as a soft channel decoder.

IV. DETECTION OF UNCODED QPSK SYMBOLS

A. Optimum Receiver

Uncoded data symbols are independent and, consequently, the *a priori* message probability can be factored as $P[m] = \prod_{i \in \mathcal{I}} P[a_i^m]$, where $P[a_i^m]$ is the *a priori* probability of a symbol at time i in the message m . Assuming that each symbol of the alphabet has an equal probability, the optimum decision consists of choosing the symbol that maximizes the likelihood function $p[\{z_{i,n}; n \in \mathcal{N}\} | a_i^m]$. We only consider the case of QPSK modulation in which $\Re[a_i]$ and $\Im[a_i]$ belong to $\{1, -1\}$.

Also, in order to obtain clear results, we simplify our interference model by assuming that $J_{0,n}$ takes binary values J_0 or zero, without requiring that jammed frequency intervals be adjacent, see Fig. 2. We thus partition the set of replicas $\mathcal{N}$ as $\mathcal{N} = \mathcal{J} \cup \mathcal{S}$, $\mathcal{J} \cap \mathcal{S} = \emptyset$, where

$$\begin{aligned} \mathcal{J} &= \{n | n \in \mathcal{N}, J_{0,n} = J_0\} \\ \mathcal{S} &= \{n | n \in \mathcal{N}, J_{0,n} = 0\}. \end{aligned}$$

We also define the key parameter η as the fraction of replicas hit by the interference

$$\eta = \frac{|\mathcal{J}|}{|\mathcal{N}|}, \quad 0 \leq \eta \leq 1.$$

This is also the ratio of the interference effective bandwidth $|\mathcal{J}|(1 + \alpha)/T$ to the signal bandwidth W_x . Thus, the decision variable Λ_i and the parameter β become

$$\Lambda_i = \sum_{n \in \mathcal{J}} \frac{1}{1 + (J_0/N_0)} z_{i,n} \gamma_{i,n}^* + \sum_{n \in \mathcal{S}} z_{i,n} \gamma_{i,n}^*; \quad i \in \mathcal{I} \quad (15)$$

$$\beta = \frac{1}{1 + (J_0/N_0)} \eta + (1 - \eta). \quad (16)$$

The variance (10) of the noise $\{y_{i,n}; n \in \mathcal{N}; i \in \mathcal{I}\}$ that contaminates the statistics (9) becomes

$$E[|Y_{i,n}|^2] = \begin{cases} (J_0 + N_0) \|g\|^2, & \text{if } n \in \mathcal{J}, i \in \mathcal{I} \\ 2N_0 \|g\|^2, & \text{if } n \in \mathcal{S}, i \in \mathcal{I}. \end{cases} \quad (17)$$

Proposition 3: For the optimal detector of uncoded QPSK symbols, we have the following:

- 1) The decision on a_i is $\hat{a}$ if and only if $\hat{a}$ maximizes $\Re[\hat{a}^* \Lambda_i]$ over the symbol alphabet.
- 2) The bit-error probability is

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}} \beta\right). \quad (18)$$

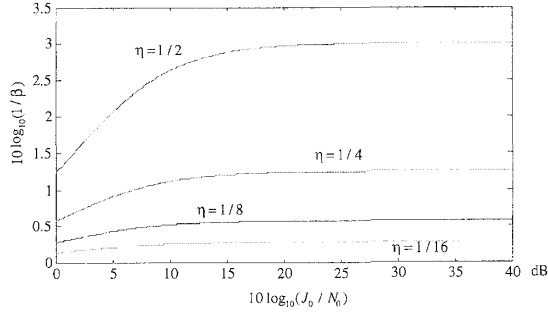


Fig. 3. Performance degradation of the jammer channel with respect to the unjammed AWGN channel.

- 3) Given a fixed average jamming power J , the worst jammer has a bandwidth equal to the signal bandwidth W_x . The corresponding maximum error probability is

$$P_{e,\max} = Q\left(\sqrt{\frac{2E_b}{N_0 + (J/W_x)}}\right). \quad (19)$$

Proof:

- 1) Maximize $p[\{z_{i,n}; n \in \mathcal{N}\} | a_i^m]$ over the symbol alphabet using the associated statistic Λ_i given by (13).
- 2) P_e is easy to derive using the above decision rule and (13).
- 3) Replacing J_0 by its value $J/(\eta W_x)$ in (16) yields

$$\beta = \beta(\eta) = 1 - \frac{J}{N_0 W_x + (J/\eta)}.$$

This is a decreasing function over the interval $[0, 1]$. It reaches its minimum value $\beta(1) = N_0/(J_0 + N_0)$ at $\eta = 1$. With $\beta = \beta(1)$, (18) becomes (19). $\square$

The decision variable (15) indicates that there are two levels of side information: the jammed subset $\mathcal{J}$ and the ratio J_0/N_0 . Notice from (9) and (17) that $\mathcal{J}$ is easily identified in $\mathcal{N}$ by comparing $|z_{i,n}|^2$ or its time average to a threshold. Also, samples $z_{i,n}$ can be used to estimate J_0/N_0 .

Expression (18) indicates that the performance degradation due to interference is represented by the parameter β . Therefore, to achieve an error probability on the jammed channel equal to that on the unjammed AWGN channel, the required E_b should increase by a factor $1/\beta$. Fig. 3 shows this performance degradation as a function of J_0/N_0 for different values of η . The degradation is low and is upper-bounded by $1/(1-\eta)$ at which it becomes insensitive to the increase in J_0/N_0 . Also, it decreases when η decreases. Notice that even moderately large values of η guarantee acceptable degradations.

B. Suboptimum Receiver

Side information J_0/N_0 is not needed by a suboptimum receiver which erases jammed statistics $\{z_{i,n}; n \in \mathcal{J}, i \in \mathcal{I}\}$ and, thus, neglects the first sum in (15). Its decision variable is

$$\Lambda'_i = \sum_{n \in \mathcal{S}} z_{i,n} \gamma_{i,n}^*; \quad i \in \mathcal{I}.$$

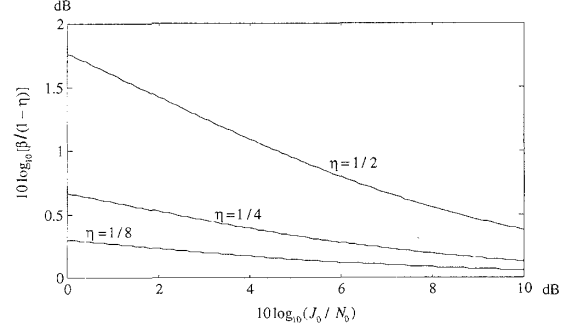


Fig. 4. Performance degradation of the suboptimum receiver with respect to the optimum receiver.

Notice that this receiver does not exist if $\eta = 1$; the set $\mathcal{S}$ being void in this case. The signal energy in Λ'_i is a fraction $1 - \eta$ of the total energy. Therefore, with respect to the unjammed AWGN channel, the degradation in the E_b/N_0 ratio due to bandlimited interference is $10 \log(1 - \eta)$ dB. It is easy to show that the bit-error probability for QPSK becomes

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}(1 - \eta)}\right). \quad (20)$$

Compared to the optimum error probability (18), this represents the following performance degradation:

$$\frac{\beta}{(1 - \eta)} = 1 + \frac{1}{1 + (J_0/N_0)} \frac{\eta}{(1 - \eta)}.$$

Fig. 4 shows this degradation as a function of J_0/N_0 for different values of η . The degradation decreases when J_0/N_0 increases and when η decreases. The suboptimum receiver should not be used when the majority of replicas are jammed.

C. Comparison with DS-SS System

We first derive the bit-error probability of an uncoded QPSK DS-SS system as a function of the parameters defined above; mainly η and J_0 . Recall that the conventional receiver is a matched filter and a symbol-by-symbol detector which fails to take into account the nonwhite characteristics of the bandlimited interference.

Using the DS-SS signal model, the received signal is

$$z'(t) = \sum_{i \in \mathcal{I}} a_i \sum_{n=0}^{N-1} \gamma_{i,n} c(t - iT - nT_c) + u(t) + v(t)$$

where a_i is a QPSK symbol and $\{c(t - nT_c)\}$ are orthogonal. Let $h_i(t) = \sum_{n=0}^{N-1} \gamma_{i,n} c(t - iT - nT_c)$. The output of a matched filter $h_i(-t)$, whose input is $z'(t)$, at $t = 0$ is the decision variable

$$z'_i = a_i \|h_i\|^2 + u'_i + v'_i$$

where $\{|h_i|^2 = N|c|^2 = 2E_b, i \in \mathcal{I}\}$, v'_i is due to noise, $E[|v'_i|^2] = 2N_0|c|^2$, and u'_i is the interference component

$$u'_i = \sum_{n=0}^{N-1} \gamma_{i,n} u'_{i,n}$$

$$u'_{i,n} = \int_{-\infty}^{\infty} u(t) c(t - iT - nT_c) dt.$$

Its variance depends on the shape of $c(t)$ and on the spectrum location of the interference signal. For the sake of simplicity, we assume that $c(t)$ has a square-root Nyquist shape with a roll off factor α equal to zero. This implies that $|C(f)|$ is constant over the signal band, and

$$E[|u'_{i,n}|^2] = \int_{-\infty}^{\infty} \Phi_u(f) |C(f)|^2 df$$

$$= 2J_0 \sum_{n \in \mathcal{I}} \int_{\mathcal{F}_n} |C(f)|^2 df = 2J_0 |c|^2 \eta.$$

This result, along with the whiteness of $\{\gamma_{i,n}\}$, yields

$$E[|u'_i|^2] = \sum_{n=0}^{N-1} |\gamma_{i,n}|^2 E[|u'_{i,n}|^2] = 2J_0 |h_i|^2 \eta$$

because $|h_i|^2 = N|c|^2$. Given z'_i , the decision on a_i is $\hat{a}$ if and only if $\hat{a}$ maximizes $\Re[\hat{a}^* z'_i]$ over the symbol alphabet. It can easily be shown that the bit-error probability is

$$P_e(DS) = Q\left(\sqrt{\frac{2E_b}{N_0} \beta_{DS}}\right), \quad \beta_{DS} = \frac{1}{1 + \eta J_0/N_0}. \quad (21)$$

This shows that the effect of spreading is to reduce J_0 by a factor η . It is easy to demonstrate that the same expression can be obtained with our FD-SS signal if the decision variable (15) is replaced by $\sum_{n \in \mathcal{N}} z_{i,n} \gamma_{i,n}^*$ which does not distinguish reliable statistics $z_{i,n}$ from jammed ones.

We can now compare (21) to (18), the performance of the optimum receiver of the proposed FD-SS system. To achieve equal error probability in both FD-SS and DS-SS systems, the improvement in the required bit energy in the FD-SS system with respect to the DS-SS system is given by the ratio

$$\frac{\beta}{\beta_{DS}} = 1 + \eta(1 - \eta) \frac{(J_0/N_0)^2}{1 + (J_0/N_0)}.$$

There is no improvement when $\eta = 1$; in this case both systems represent quaternary biorthogonal signaling over AWGN channel. Otherwise, this ratio is always larger than one. It is maximum when $\eta = 1/2$ and increases linearly with J_0/N_0 for $J_0 \gg N_0$.

However, the comparison is more fair if suboptimum receivers are used in both systems. Therefore, we presently compare (21) to (20). For a given error probability, the improvement in the required bit energy in the FD-SS system with respect to the DS-SS system is given by the ratio

$$\frac{1 - \eta}{\beta_{DS}} = 1 + \eta[(1 - \eta)(J_0/N_0) - 1] = (1 - \eta) \left(1 + \frac{J}{W_x N_0}\right).$$

This ratio is larger than one if $(J_0/N_0) > 1/(1 - \eta)$, which is satisfied by the condition on the validity of the FD-SS

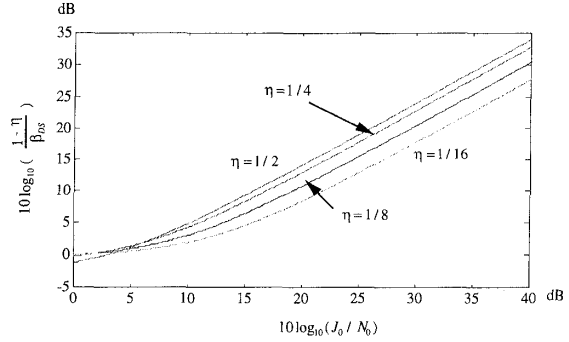


Fig. 5. Performance improvement of the proposed FD-SS system using a suboptimum receiver with respect to the conventional uncoded DS-SS system.

suboptimum receiver. Fig. 5 shows this improvement as a function of J_0/N_0 for different values of η . It increases linearly with J_0/N_0 and with the jammer power J .

D. Partial-Time Jamming Performance

Partial-band jamming which severely affects FH-SS is actually included in our jammer model. In DS-SS the bandlimited partial-time jamming is a successful counter-measure to reduce the processing gain obtained from spreading. We investigate here its effect on FD-SS system. In this tactic the jamming signal is a bandlimited Gaussian noise with power spectral density $2J_0/\rho$ in the band during a fraction ρ of the time, and zero during the remaining time [1]. Such a nonstationary burst interference was investigated in [7] and [8]. Also, a computationally complex receiver was derived in [12].

Assuming that the stationarity of the jamming signal is maintained, the average probability of error in the presence of a bandlimited partial-time jammer is obtained by replacing J_0 by J_0/ρ during the time the jammer is on

$$P_e = (1 - \rho) Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \rho Q\left(\sqrt{\frac{2E_b}{N_0} \beta'}\right)$$

$$\beta' = \frac{N_0}{J_0/\rho + N_0} \eta + (1 - \eta).$$

Since $\beta' < 1$ this probability can be bounded as follows:

$$P_e < Q\left(\sqrt{\frac{2E_b}{N_0} \beta'}\right) < Q\left(\sqrt{\frac{2E_b}{N_0} (1 - \eta)}\right)$$

which shows that FD-SS is insensitive to partial-time jamming if $\eta \ll 1$. This is also the case of the suboptimum receiver because its error probability is independent of J_0 .

As proved in Proposition 3, the worst stationary jamming occurs when $\eta = 1$. This means that FD-SS forces a stationary jammer to spread his power over the signal bandwidth W_x . However, if nonstationary broadband partial-time jamming is possible, and the communicator cannot increase W_x , uncoded FD-SS becomes vulnerable. An efficient counter-measure would then lie in replacing the degenerate repetition code by an efficient channel code, as mentioned in the introduction.

V. MODULATOR AND DEMODULATOR STRUCTURE

With the help of Fig. 2, we can relate the generation of the baseband FD-SS signal to the digital translation of frequency-division multiplex to time-division multiplex [19], [20]. Demodulation, i.e., obtaining $\{z_{i,n}\}$ from (8), is simply the inverse operation. For simplicity's sake we only present the case where the roll off parameter α is equal to one. The baseband signal has then a frequency support of width $W_x = 2N/T$, and it can be represented by its samples $\{x(kT_s)\}$ at a sampling period $T_s = T/2N$.

A. Modulator

The baseband signal (1) can be generated by interpolating its samples at the Nyquist rate $1/T_s = 2N/T$. Let $\{c_{i,k}\}$ be the inverse DFT of chip symbols $\{\gamma_{i,n}\}$, i.e.,

$$\gamma_{i,n} = \sum_{k \in \mathcal{N}} c_{i,k} e^{-j(2\pi kn/N)}; \quad n \in \mathcal{N}.$$

By substituting the expression of $\gamma_{i,n}$ in (1) we obtain

$$x(t) = \sum_{i \in \mathcal{I}} a_i g(t - iT) \sum_{k \in \mathcal{N}} c_{i,k} \sum_{n \in \mathcal{N}} e^{-j(2\pi kn/N)} e^{j2\pi n F t}$$

or

$$x(t) = \sum_{i \in \mathcal{I}} a_i g(t - iT) \sum_{k \in \mathcal{N}} c_{i,k} r(t - kT_s) \quad (22)$$

where

$$r(t) = e^{j\pi F t(N-1)} \frac{\sin \pi F t N}{\sin \pi F t}$$

$$r(kT_s) = \delta_{0,k}, \quad k \text{ is integer.}$$

Expression (22) shows that the effect of $c_{i,k}$ appears twice in a baud interval. By sampling we obtain

$$x(nT + mT_s) = \sum_{i \in \mathcal{I}} a_i c_{i,m} g(nT - iT + mT_s);$$

$$m = 0, 1, \dots, 2N - 1$$

which can be written as

$$x(nT + mT_s) = \sum_{i \in \mathcal{I}} h_m(iT) a_{n-i} c_{n-i,m};$$

$$m = 0, 1, \dots, 2N - 1 \quad (23)$$

where

$$h_m(iT) = g(mT_s + iT).$$

The filters $\{h_m(iT); m = 0, 1, \dots, 2N - 1\}$ are called polyphase filters [19]. Given $m = m_0$ and assuming that $c_{i,m_0} = 1$ for all i , the input to the filter $h_m(iT)$ is the sequence $\{a_i\}$. The rate at the input and the output of each filter is equal to $1/T$. Fig. 6 illustrates the $2N$ filtering described by (23). The $2N$ outputs per interval T are time-ordered, as shown in the figure, and multiplexed. The multiplexed output rate is $1/T_s$.

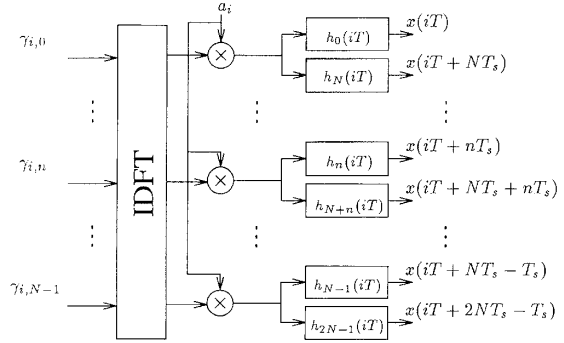


Fig. 6. Modulator structure, IDFT denotes inverse discrete Fourier transform.

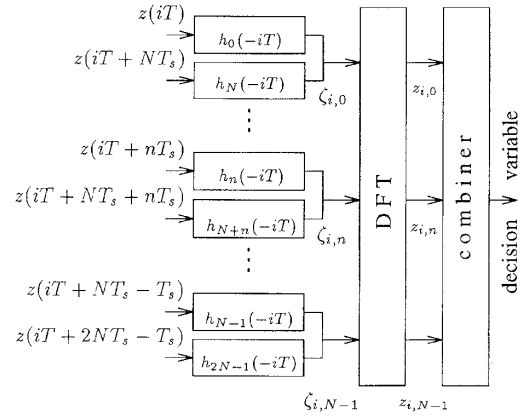


Fig. 7. Demodulator structure.

B. Demodulator

The inner product (8) is implemented by using the samples $z(lT + mNT_s + kT_s)$, $k \in \mathcal{N}$, $m \in \{0, 1\}$, l is integer.

$$z_{i,n} = T_s \sum_l \sum_{m=0}^1 \sum_{k \in \mathcal{N}} z(lT + mNT_s + kT_s)$$

$$\cdot \psi_n^*(lT + mNT_s + kT_s - iT)$$

$$= T_s \sum_{k \in \mathcal{N}} \zeta_{i,k} e^{-j(2\pi kn/N)}$$

where

$$\zeta_{i,k} = \sum_l \sum_{m=0}^1 h_{k+mN}(-iT) z(lT - iT + mNT_s + kT_s).$$

Fig. 7 shows also the demodulator structure. The received signal is sampled and filtered by a bank of $2N$ polyphase filters which are matched to those used in the modulator. The output block $\{\zeta_{i,k}; k \in \mathcal{N}\}$ is transformed by a DFT to obtain $\{z_{i,n}; n \in \mathcal{N}\}$.

Assuming that the number of coefficients in a square-root Nyquist filter is M , MN multiplications per period T are then needed to generate the baseband FD-SS signal. This is equal to the number of multiplications that we need in a bandlimited

conventional DS-SS signal [21], because M multiplications per chip period are then needed. Therefore, DS-SS and FD-SS have the same complexity. However, FD-SS provides the possibility of doing parallel multiplications.

VI. CONCLUSION

We have proposed a frequency-diversity SS digital communications system which has the following features:

- 1) The optimum receiver, which performs joint symbol detection and interference rejection, is simple to implement. Its probability of error, with uncoded QPSK modulation, significantly outperforms that of a conventional DS-SS system.
- 2) Instead of the processing gain which characterizes conventional SS systems, the key parameter here is η , the fraction of signal bandwidth occupied by the interference. There is no need to have a large signal spread in proportion to the interference bandwidth; values of η as large as 1/2 and 1/8 cause degradations of only 3 and 0.58 dB, respectively, with respect to the unjammed AWGN channel. This leads to large spectrum savings and to the possibility to spread mainly via channel coding rather than via repetition.
- 3) A suboptimum receiver does not need side information on interference and noise levels. Its probability of error is independent of J_0 and, compared to the unjammed AWGN channel, it shows a loss factor $(1 - \eta)$.
- 4) Bandlimited partial-time jamming is not an attractive tactic for the jammer. A smart stationary jammer should spread its signal the same as the communicator.
- 5) Since there is no hopping, coherent reception is quite possible. Also, the symbol time synchronization needed here is easier to obtain than the *chip* time synchronization required in the DS-SS system.
- 6) The system can easily be built by using polyphase filters with the same complexity as DS-SS whose chip pulse is a square-root Nyquist pulse.
- 7) The system has the disadvantage of not having constant envelope.

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REFERENCES

- [1] R. E. Blahut, *Digital Transmission of Information*. Reading, MA: Addison-Wesley, 1990.
- [2] J. G. Proakis, *Digital Communications*, 2nd ed. New York: McGraw-Hill, 1989.

- [3] M. K. Simon, J. Omura, R. A. Scholtz, and B. K. Levitt, *Spread Spectrum Communications*, vols. I-III. Rockville, MD: Computer Science Press, 1985.
- [4] R. L. Pickholtz, L. B. Milstein, and D. L. Schilling, "Spread spectrum for mobile communications," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 313-322, May 1991.
- [5] H. L. Van Trees, *Detection, Estimation, and Modulation Theory*, Part I. New York: Wiley, 1968.
- [6] E. Masry, "Closed-form analytical results for the rejection of narrow-band interference in PN spread-spectrum systems—Part II: Linear interpolation filters," *IEEE Trans. Commun.*, vol. COM-33, pp. 10-19, Jan. 1985.
- [7] J. W. Ketchum and J. G. Proakis, "Adaptive algorithms for estimating and suppressing narrow band interference in PN spread-spectrum systems," *IEEE Trans. Commun.*, vol. COM-30, no. 5, pp. 913-924, May, 1982.
- [8] L. M. Li and L. B. Milstein, "Rejection of pulsed CW interference in PN spread-spectrum systems using complex adaptive filter," *IEEE Trans. Commun.*, vol. COM-31, no. 1, pp. 10-20, Jan. 1983.
- [9] L. B. Milstein, "Interference rejection techniques in spread-spectrum communications," in *Proc. IEEE*, vol. 76, no. 6, pp. 657-671, June 1988.
- [10] L. M. Garth and H. V. Poor, "Narrowband interference suppression in impulsive channels," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, no. 1, pp. 15-34, Jan. 1992.
- [11] R. Vijayan and H. V. Poor, "Nonlinear techniques for interference suppression in spread-spectrum systems," *IEEE Trans. Commun.*, vol. COM-38, no. 7, pp. 1060-1065, July 1990.
- [12] D. M. Krinsky, A. H. Haddad, and C. C. Lee, "An adaptive direct-sequence spread-spectrum for burst type interference," *IEEE J. Select. Areas Commun.*, vol. 13, no. 1, pp. 59-70, Jan. 1995.
- [13] S. Kondo and L. B. Milstein, "Multicarrier DS CDMA systems in the presence of partial band interference," in *IEEE MILCOM, Conf. Rec.*, 1994, pp. 588-592.
- [14] J. A. C. Bingham, "Multicarrier modulation for transmission: An idea whose time has come," *IEEE Commun. Mag.*, vol. 28, no. 5, pp. 5-14, May 1990.
- [15] S. Kondo and L. B. Milstein, "On the use of multicarrier direct sequence spread spectrum systems," in *IEEE MILCOM, Conf. Rec.*, 1993, pp. 52-56.
- [16] L. Vandendorpe, "Multitone spread spectrum multiple access communications system in a multipath Rician fading channel," *IEEE Trans. Veh. Technol.*, vol. 44, pp. 327-337, May 1995.
- [17] N. Yee, J.-P. M. G. Linnartz, and G. Fettweis, "Multi-carrier CDMA in indoor wireless radio networks," *IEICE Trans. Commun.*, vol. E77-B, no. 7, pp. 900-904, July 1994.
- [18] D. J. Sakrison, *Communication Theory: Transmission of Waveforms and Digital Information*. New York: Wiley, 1968.
- [19] R. E. Crochiere and L. R. Rabiner, *Multirate Digital Signal Processing*. Englewood-Cliffs, NJ: Prentice-Hall, 1983.
- [20] J. J. Boutros, "Course notes on multicarrier transmission," ENST, Paris, France, 1994.
- [21] A. J. Viterbi, *CDMA: Principles of Spread Spectrum Communication*. Reading, MA: Addison-Wesley, 1995.



Ghassan Kawas Kaleh was born in Syria in 1941. He received the B.Sc. degree in electrical engineering from Cairo University, Egypt, in 1964, the diploma from the Ecole Nationale Supérieure des Télécommunications (ENST), Paris, France, in 1973, and the Docteur Ingénieur degree from the University of Paris VI in 1977.

He is a Professor at the ENST, where he has worked since 1973. His main research interests are in communication theory and signal processing.