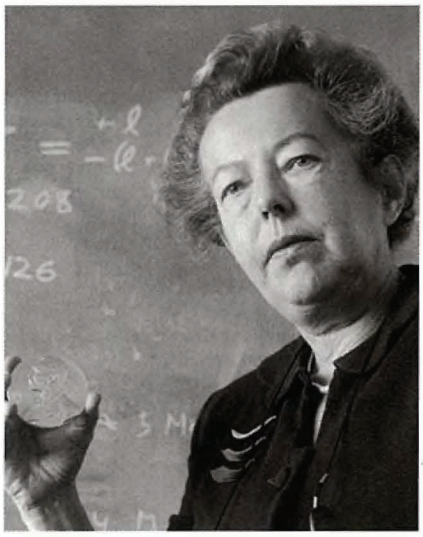
Figure 42.8 also shows the electrostatic potential energy of two protons for comparison. Notice how small the electrostatic energy is in comparison to the potential energy of the strong force. At $r \approx 1.5$ fm, where the strong force is maximum, the aptly named attractive strong force is ≈100 times larger than the repulsive electrostatic force.

A question asked earlier was why the nucleus has neutrons at all. The answer is related to the short range of the strong force. Protons throughout the nucleus exert repulsive electrostatic forces on each other, but, because of the short range of the strong force, a proton feels an attractive force only from the very few other protons with which it is in close contact. Even though the strong force at its maximum is much larger than the electrostatic force, there wouldn't be enough attractive nuclear bonds for an all-proton nucleus to be stable. Because neutrons participate in the strong force but exert no repulsive forces, **the neutrons provide the extra "glue" that holds the nucleus together.** In small nuclei, where most nucleons are in contact, one neutron per proton is sufficient for stability. Hence small nuclei have $N \approx Z$. But as the nucleus grows, the repulsive force increase faster than the binding energy. More neutrons are needed for stability, causing heavy nuclei to have $N > Z$.

42.4 The Shell Model



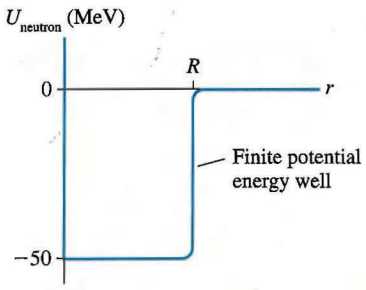
Marie Goeppert-Mayer shows her Nobel Prize.

Figure 42.8 is the potential energy of *two* interacting nucleons. To solve Schrödinger's equation for the nucleus, we would need to know the total potential energy of *all* interacting nucleon pairs within the nucleus, including both the strong force and the electrostatic force. This is far too complex to be a tractable problem.

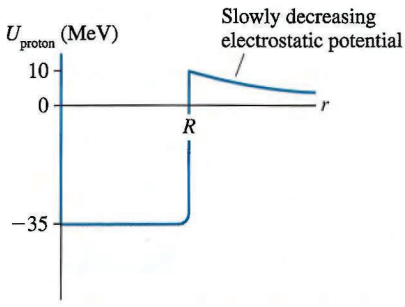
We faced a similar situation with multielectron atoms. Calculating an atom's exact potential energy, including electron-electron repulsion, is exceedingly complicated. To simplify the problem, we made a *model* of the atom in which each electron moves independently with an *average* potential energy due to the nucleus and all other electrons. That model, although not perfect, led to the correct prediction of electron shells and explained the periodic table of the elements.

The **shell model** of the nucleus, using multielectron atoms as an analogy, was proposed in 1949 by Marie Goeppert-Mayer, one of the first prominent women in physics. The shell model considers each nucleon to move independently with an *average* potential energy due to the strong force of all the other nucleons. For the protons, we also have to include the electrostatic potential energy due to the other protons.

Figure 42.9 shows the average potential energy of a neutron and a proton. You can see that, to a good approximation, a nucleon appears to be a particle in a finite potential well, a quantum-mechanics problem you learned how to deal with in Chapter 40.



The average neutron potential energy is due to the strong force.



The average proton potential energy is due to the strong force and the electric force. This potential-well depth is for $Z \approx 30$.

FIGURE 42.9 The average potential energy of a neutron and a proton.

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Several observations are worthwhile:

1. The depth of the neutron's potential-energy well is ≈50 MeV for all nuclei. The radius of the potential-energy well is the nuclear radius $R = r_0 A^{1/3}$.
2. For protons, the positive electrostatic potential energy "lifts" the potential-energy well. The lift varies from essentially none for very light elements to a significant fraction of the well depth for very heavy elements. The potential-energy well shown in the figure would be appropriate for a nucleus with $Z \approx 30$.
3. Outside the nucleus, where the strong force has vanished, a proton's potential energy is $U = (Z-1)e^2/4\pi\epsilon_0 r$ due to its electrostatic interaction with the $(Z-1)$ other protons within the nucleus. This positive potential energy decreases slowly with increasing distance.

The task of quantum mechanics is to solve for the energy levels and wave functions of the nucleons in these potential-energy wells. Once the energy levels are found, we build up the nuclear state, just as we did with atoms, by placing all the nucleons in the lowest energy levels consistent with the Pauli principle. The Pauli principle affects nucleons, just as it did electrons, because they are spin-$\frac{1}{2}$ particles. Each energy level can hold only a certain number of spin-up particles and spin-down particles, depending on the quantum numbers. Additional nucleons have to go into higher energy levels.

Low-Z Nuclei

As an example, we'll consider the energy levels of low-Z nuclei ($Z < 8$). Because these nuclei have so few protons, we can use a reasonable approximation that neglects the electrostatic potential energy due to proton-proton repulsion and considers only the much larger nuclear potential energy. In that case, the proton and neutron potential-energy wells and energy levels are the same.

Figure 42.10 shows the three lowest energy levels and the maximum number of nucleons that the Pauli principle allows in each. Energy values vary from nucleus to nucleus, but the spacing between these levels is several MeV. It's customary to draw the proton and neutron potential-energy diagrams and energy levels back to back. Notice that the radial axis for the proton potential-energy well points to the right, while the radial axis for the neutron potential-energy well points to the left.

Let's apply this model to the $A = 12$ isobar. Recall that an isobar is a series of nuclei with the same number of neutrons and protons. Figure 42.11 shows the energy-level diagrams of ^{12}B, ^{12}C, and ^{12}N. Look first at ^{12}C, a nucleus with six protons and six neutrons. You can see that exactly six protons are allowed in the $n = 1$ and $n = 2$ energy levels. Likewise for the six neutrons. Thus ^{12}C has a closed $n = 2$ proton shell and a closed $n = 2$ neutron shell.

NOTE ▶ Protons and neutrons are different particles, so the Pauli principle is not violated if a proton and a neutron have the same quantum numbers. ◀

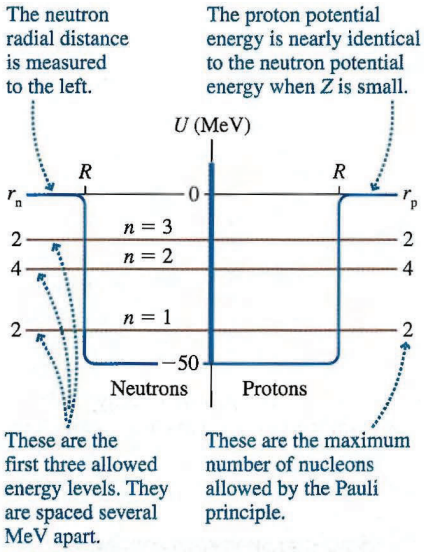


FIGURE 42.10 The three lowest energy levels of a low-Z nucleus. The neutron energy levels are on the left, the proton energy levels on the right.

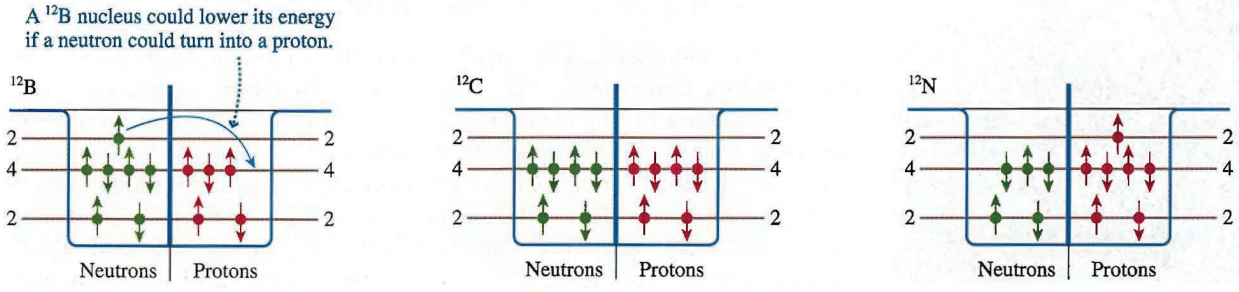


FIGURE 42.11 The $A = 12$ isobar has to place 12 nucleons in the lowest available energy levels.

^{12}N has seven protons and five neutrons. The sixth proton fills the $n = 2$ proton shell, so the seventh proton has to go into the $n = 3$ energy level. The $n = 2$ neutron shell has one vacancy because there are only five neutrons. ^{12}B is just the opposite, with the seventh neutron in the $n = 3$ energy level. You can see from the diagrams that the ^{12}B and ^{12}N nuclei have significantly more energy—by several MeV—than ^{12}C.

In atoms, electrons in higher energy levels decay to lower energy levels by emitting a photon as the electron undergoes a quantum jump. That can't happen here because the higher-energy nucleon in ^{12}B is a neutron whereas the vacant lower energy level is that of a proton. But an analogous process could occur *if* a neutron could somehow turn into a proton. And that's exactly what happens! We'll explore the details in Section 42.6, but both ^{12}B and ^{12}N decay into ^{12}C in the process known as *beta decay*.

^{12}C is just one of three low-Z nuclei in which both the proton and neutron shells are full. The other two are ^{4}He (filling both $n = 1$ shells with $Z = 2, N = 2$) and ^{16}O (filling both $n = 3$ shells with $Z = 8, N = 8$). If the analogy with closed electron shells is valid, these nuclei should be more tightly bound than nuclei with neighboring values of A. And indeed, we've already noted that the curve of binding energy (Figure 42.6) has peaks at $A = 4$, 12, and 16. The shell model of the nucleus satisfactorily explains these peaks. Unfortunately, the shell model quickly becomes much more complex as we go beyond $n = 3$. Heavier nuclei do have closed shells, but there's no evidence for them in the curve of binding energy.

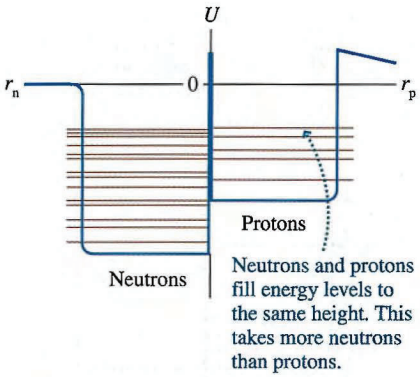


FIGURE 42.12 The proton energy levels are displaced upward in a high-Z nucleus.

High-Z Nuclei

We can use the shell model to give a qualitative explanation for one more observation, although the details are beyond the scope of this text. Figure 42.12 shows the neutron and proton potential-energy wells of a high-Z nucleus. In a nucleus with many protons, the electrostatic potential energy lifts the proton potential-energy well higher than the neutron potential-energy well. Protons and neutrons now have a different set of energy levels.

As a nucleus is "built," by the adding of protons and neutrons, the proton energy well and the neutron energy well must fill to just about the same height. If there were neutrons in energy levels above vacant proton levels, the nucleus would lower its energy by using beta decay to change the neutron into a proton. Similarly, beta decay would change a proton into a neutron if there were a vacant neutron energy level beneath a filled proton level. **The net result of beta decay is to keep the filled levels on both sides at just about the same height.**

Because the neutron potential-energy well starts at a lower energy, *more neutron states* are available than proton states. Consequently, a high-Z nucleus will have more neutrons than protons. This conclusion is consistent with our observation in Figure 42.2 that $N > Z$ for heavy nuclei.



Marie Curie.

42.5 Radiation and Radioactivity

Becquerel's 1896 discovery of "rays" from crystals of uranium prompted a burst of activity. Becquerel was soon joined in France by Marie Curie and Pierre Curie. They focused on isolating the element or elements responsible for the radiation, and, in the process, discovered the element radium.

In England, J. J. Thompson and, especially, his student and protégé Ernest Rutherford worked to identify the unknown rays. Using combinations of electric and magnetic fields, much as Thompson had done in his investigations of cathode rays, they found three distinct types of radiation. Figure 42.13 shows the basic experimental procedure, and Table 42.3 summarizes the results.

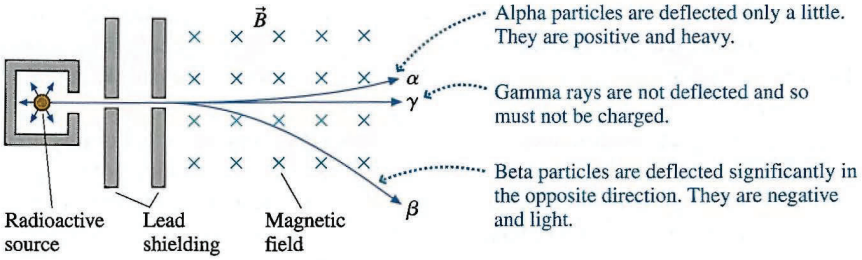


FIGURE 42.13 Identifying radiation by its deflection in a magnetic field.

TABLE 42.3 Three types of radiation

Radiation	Identification	Charge	Stopped by
Alpha, α	^{4}He nucleus	$+2e$	Sheet of paper
Beta, β	Electron	$-e$	Few mm of aluminum
Gamma, γ	High-energy photon	0	Many cm of lead

Within a few years, as Rutherford and others deduced the basic structure of the atom, it became clear that these emissions of radiation were coming from the atomic nucleus. We now define *radioactivity* or *radioactive decay* to be the spontaneous emission of particles or high-energy photons from unstable nuclei as they decay from higher-energy to lower-energy states. Radioactivity has nothing to do with the orbiting valence electrons.

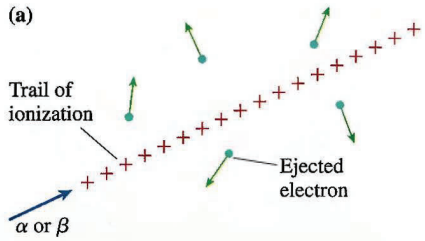
NOTE ▶ The term "radiation" merely means something that is *radiated outward,* similar to the word "radial." Electromagnetic waves are often called *electromagnetic radiation.* Infrared waves from a hot object are referred to as "thermal radiation." Thus it was no surprise that these new "rays" were also called radiation. Unfortunately, the general public has come to associate the word "radiation" with *nuclear radiation,* something to be feared. It is important, when you use the term, to be sure you're not conveying a wrong impression to a listener or a reader. ◀

Ionizing Radiation

Electromagnetic waves, from microwaves through ultraviolet radiation, are *absorbed by* matter. The absorbed energy increases an object's thermal energy and its temperature, which is why objects sitting in the sun get warm.

In contrast to visible-light photon energies of a few eV, the energies of the alpha and beta particles and the gamma-ray photons of nuclear decay are typically in the range 0.1–10 MeV, a factor of roughly 10^6 larger. These energies are much larger than the ionization energies of atoms and molecules. Rather than simply being absorbed and increasing an object's thermal energy, nuclear radiation *ionizes* matter and *breaks* molecular bonds. Nuclear radiation and x rays, which behave much the same in matter, are called **ionizing radiation.**

An alpha or beta particle traveling through matter creates a trail of ionization, as shown in Figure 42.14a. Because the ionization energy of an atom is $\approx$10 eV, a particle with 1 MeV of kinetic energy can ionize $\approx$100,000 atoms or molecules before finally stopping. The low-mass electrons are kicked sideways, but the much more massive positive ions barely move and form the trail. This behavior is the basis for the *cloud chamber* and the *hydrogen bubble chamber,* where microscopic water droplets or hydrogen gas bubbles coalesce around the positive ions to make the trail visible. Figure 42.14b is a picture of the ionization trails of high-energy particles in a bubble chamber. The curvature of the trajectories is due to a magnetic field.



(b)



FIGURE 42.14 Alpha and beta particles create a trail of ionization as they pass through matter. This is the basis for the hydrogen bubble chamber.

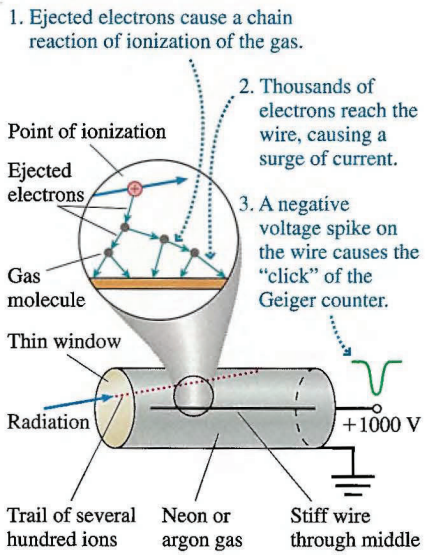


FIGURE 42.15 A Geiger counter.

Ionization is also the basis for the **Geiger counter,** one of the most well-known detectors of nuclear radiation. Figure 42.15 shows how a Geiger counter works. The important thing to remember is that a Geiger counter detects only *ionizing radiation.*

Ionizing radiation damages materials. Ions drive chemical reactions that wouldn't otherwise occur. Broken molecular bonds alter the workings of molecular machinery, especially in large biological molecules. It is through these mechanisms—ionization and bond breaking—that nuclear radiation can cause mutations or tumors. We'll look at the biological issues in Section 42.7.

NOTE ▶ Ionizing radiation causes structural damage to materials, but **irradiated objects do not become radioactive.** Ionization drives chemical processes involving the electrons. An object could become radioactive only if its nuclei were somehow changed, and that does not happen. ◀

STOP TO THINK 42.3 A very bright spotlight shines on a Geiger counter. Does it click?

Nuclear Decay and Half-Lives

Rutherford was the first to find that the number of radioactive atoms in a sample decreases exponentially with time. This is the expected time dependence if the decay is a *random process.* But to say that a process is random doesn't mean there are no patterns. Tossing a coin is a random process because you can't predict what one coin will do. Even so, if you tossed 1000 coins into the air, you'd certainly find very nearly 500 heads and 500 tails. Nuclear decay is similar.

Let r be the probability that one particular nucleus will decay in the next 1 s by emitting an alpha or beta particle or a gamma-ray photon. For example, $r = 0.010\ \text{s}^{-1}$ means that a nucleus has a 1% chance of decay in the next second. Notice that r, which is called the **decay rate,** has units s^{-1}, making it a *rate.*

The probability that a nucleus decays during the small interval of time Δt is

$$\text{Prob(in } \Delta t) = r\Delta t \tag{42.7}$$

For example, a nucleus with $r = 0.010\ \text{s}^{-1}$ has a 0.1% chance of decay (Prob = 0.001) during a 0.1 s interval. If there are N independent nuclei, the number of nuclei expected to decay during Δt is

$$\text{number of decays} = N \times \text{Probability of decay} = rN\Delta t \tag{42.8}$$

This is like saying you expect 500 heads when tossing 1000 coins, each coin with a 50% probability of landing heads up.

Each decay *decreases* the number of radioactive nuclei in the sample, hence the change in the number of radioactive nuclei during Δt is

$$\Delta N = -rN\Delta t \tag{42.9}$$

The negative sign shows that N, the number of nuclei, decreases due to the decays. Finally, if we let $\Delta t \rightarrow dt$, Equation 42.9 becomes

$$\frac{dN}{dt} = -rN \tag{42.10}$$

The *rate of change* in the number of radioactive nuclei depends both on the decay rate (a larger probability of decay per second means more decays per second) and on the number of radioactive nuclei present (more nuclei means that more are available to decay). And dN/dt is negative because N is decreasing.

Equation 42.10 is the same equation we solved in Chapter 31, with different symbols, for the voltage decay in an *RC* circuit. First, separate the variables onto opposite sides of the equation:

$$\frac{dN}{N} = -r\,dt \tag{42.11}$$

We need to integrate this equation, starting from $N = N_0$ nuclei at $t = 0$. Thus

$$\int_{N_0}^{N} \frac{dN}{N} = -r\int_0^t dt \tag{42.12}$$

By carrying out the integrations we find

$$\ln N - \ln N_0 = \ln\left(\frac{N}{N_0}\right) = -rt \tag{42.13}$$

We can now solve for N by taking the exponential of both sides and multiplying by N_0. The result is

$$N = N_0 e^{-rt} \tag{42.14}$$

Equation 42.13 predicts that the number of radioactive nuclei will decrease exponentially, a prediction that has been borne out in countless experiments during the last hundred years.

It is useful to define the **time constant** τ as

$$\tau = \frac{1}{r}$$

With this definition, Equation 42.13 becomes

$$N = N_0 e^{-t/\tau} \tag{42.15}$$

Figure 42.16 shows the decrease of N with time. The number of radioactive nuclei decreases from N_0 at $t = 0$ to $e^{-1}N_0 = 0.368N_0$ at time $t = \tau$. In practical terms, the number decreases by roughly two-thirds during one time constant.

NOTE ▶ An important aspect of exponential decay is that you can choose any instant you wish to be $t = 0$. The number of radioactive nuclei present at that instant is N_0. If at one instant you have 10,000 radioactive nuclei whose time constant is $\tau = 10$ min, you'll have roughly 3680 nuclei 10 min later. The fact that you may have had more than 10,000 nuclei earlier isn't relevant. ◀

Equation 42.14 is useful in the theoretical sense that we can relate τ directly to the probability of decay. But in practice, it's much easier to measure the time at which half of a sample has decayed than the time at which 36.8% has decayed. Let's define the **half-life** $t_{1/2}$ as the time interval in which half of a sample of radioactive atoms decays. The half-life is shown in Figure 42.16.

The half-life is easily related to the time constant τ because we know, by definition, that $N = \frac{1}{2}N_0$ at $t = t_{1/2}$. Thus, according to Equation 42.15

$$\frac{N_0}{2} = N_0 e^{-t_{1/2}/\tau} \tag{42.16}$$

The N_0 cancels, and we can then take the natural logarithm of both sides to find

$$\ln\left(\frac{1}{2}\right) = -\ln 2 = -\frac{t_{1/2}}{\tau} \tag{42.17}$$

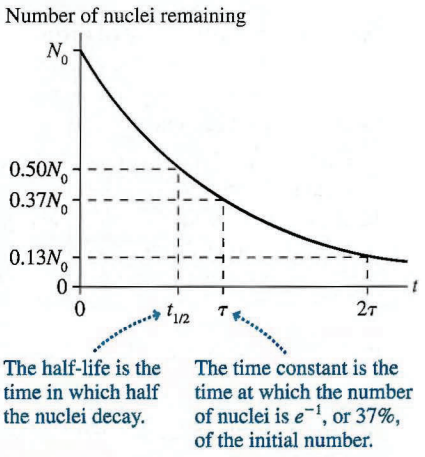


FIGURE 42.16 The number of radioactive atoms decreases exponentially with time.

With one final rearrangement we have

$$t_{1/2} = \tau \ln 2 = 0.693\tau \tag{42.18}$$

We'll leave it as a homework problem for you to show that Equation 42.15 can be written in terms of the half-life as

$$N = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}} \tag{42.19}$$

Thus $N = N_0/2$ at $t = t_{1/2}$, $N = N_0/4$ at $t = 2t_{1/2}$, $N = N_0/8$ at $t = 3t_{1/2}$, and so on. **No matter how many nuclei there are, the number decays by half during the next half-life.**

NOTE ► Half the nuclei decay during one half-life, but don't fall into the trap of thinking that all will have decayed after two half-lives. ◄

Figure 42.17 shows the half-life graphically. This figure also conveys two other important ideas:

1. Nuclei don't vanish when they decay. The decayed nuclei have merely become some other kind of nuclei.
2. The decay process is random. We can predict that half the nuclei will decay in one half-life, but we can't predict which ones.

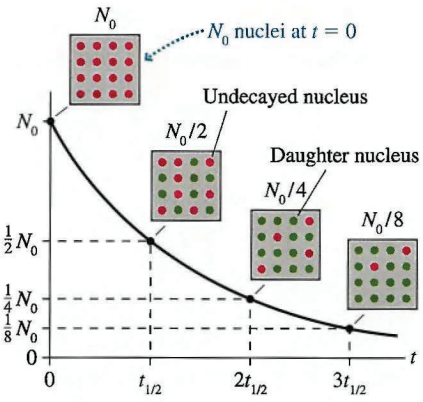


FIGURE 42.17 Half the nuclei decay during each half-life.

Each radioactive isotope, such as ^{14}C, has its own half-life. That half-life doesn't change with time as a sample decays. If you've flipped a coin 10 times and, against all odds, seen 10 heads, you may feel that a tail is overdue. Nonetheless, the probability that the next flip will be a head is still 50%. After 10 half-lives have gone by, $(1/2)^{10} = 1/1024$ of a radioactive sample is still there. There was nothing special or distinctive about these nuclei, and, despite their longevity, each remaining nucleus has exactly a 50% chance of decay during the next half-life.

EXAMPLE 42.2 The decay of iodine

The iodine isotope ^{131}I, which has an eight-day half-life, is used in nuclear medicine. A sample of ^{131}I containing 2.00×10^{12} atoms is created in a nuclear reactor.

a. How many ^{131}I atoms remain 36 hours later when the sample is delivered to a hospital?
b. Although the sample is constantly getting weaker, it remains usable as long as there are at least 5.0×10^{11} ^{131}I atoms. What is the maximum delay before the sample is no longer usable?

MODEL The number of ^{131}I atoms decays exponentially.

SOLVE

a. The half-life is $t_{1/2} = 8$ days $= 192$ hr. After 36 hr have elapsed,

$$N = (2.00 \times 10^{12})\left(\frac{1}{2}\right)^{36/192} = 1.76 \times 10^{12} \text{ nuclei}$$

b. The time after creation at which 5.0×10^{11} ^{131}I atoms remain is given by

$$5.0 \times 10^{11} = 0.50 \times 10^{12} = (2.0 \times 10^{12})\left(\frac{1}{2}\right)^{t/8 \text{ days}}$$

To solve for t, first write this as

$$\frac{0.50}{2.00} = 0.25 = \left(\frac{1}{2}\right)^{t/8 \text{ days}}$$

Now take the logarithm of both sides. Either natural logarithms or base-10 logarithms can be used, but we'll use natural logarithms:

$$\ln(0.25) = -1.39 = \frac{t}{t_{1/2}}\ln(0.5) = -0.693\frac{t}{t_{1/2}}$$

Solving for t gives

$$t = 2.00t_{1/2} = 16 \text{ days}$$

ASSESS The weakest usable sample is one-quarter of the initial sample. You saw in Figure 42.17 that a radioactive sample decays to one-quarter of its initial number in 2 half-lives.

Activity

The **activity** R of a radioactive sample is the number of decays per second, or the decay rate. The decay rate is simply the absolute value of dN/dt, or

$$R = \left|\frac{dN}{dt}\right| = rN = rN_0e^{-t/\tau} = R_0e^{-t/\tau} = R_0\left(\frac{1}{2}\right)^{t/t_{1/2}} \qquad (42.20)$$

where $R_0 = rN_0$ is the activity at $t = 0$. The activity of a sample decreases exponentially along with the number of remaining nuclei.

The SI unit of activity is the **becquerel,** defined as

$$1 \text{ becquerel} = 1 \text{ Bq} \equiv 1 \text{ decay/s or } 1 \text{ s}^{-1}$$

An older unit of activity, but one that continues in widespread use, is the **curie.** The curie was originally defined as the activity of 1 g of radium. Today, the conversion factor is

$$1 \text{ curie} = 1 \text{ Ci} \equiv 3.7 \times 10^{10} \text{ Bq}$$

1 Ci is a substantial amount of radiation. The radioactive samples used in laboratory experiments are typically ≈1 μCi, or, equivalently, ≈40,000 Bq. These samples can be handled with only minor precautions. Larger sources of activity require lead shielding and special precautions to prevent exposure to high levels of radiation.

EXAMPLE 42.3 A laboratory source

The isotope ^{137}Cs is a standard laboratory source of gamma rays. The half-life of ^{137}Cs is 30 years.

a. How many ^{137}Cs atoms are in a 5.0 μCi source?
b. What is the activity of the source 10.0 years later?

MODEL The number of ^{137}Cs atoms decays exponentially.

SOLVE

a. The number of atoms can be found from $N_0 = R_0/r$. The activity in SI units is

$$R = 5.0 \times 10^{-6} \text{ Ci} \times \frac{3.7 \times 10^{10} \text{ Bq}}{1 \text{ Ci}} = 1.85 \times 10^5 \text{ Bq}$$

To find the decay rate, first convert the half-life to seconds:

$$t_{1/2} = 30 \text{ years} \times \frac{3.15 \times 10^7 \text{ s}}{1 \text{ year}} = 9.45 \times 10^8 \text{ s}$$

Then

$$r = \frac{1}{\tau} = \frac{\ln 2}{t_{1/2}} = 7.33 \times 10^{-10} \text{ s}^{-1}$$

Thus the number of ^{137}Cs atoms is

$$N_0 = \frac{R_0}{r} = \frac{1.85 \times 10^5 \text{ Bq}}{7.33 \times 10^{-10} \text{ s}^{-1}} = 2.52 \times 10^{14} \text{ atoms}$$

b. The activity decreases exponentially, just like the number of nuclei. After 10 years,

$$R = R_0\left(\frac{1}{2}\right)^{t/t_{1/2}} = (5.0 \ \mu\text{Ci})\left(\frac{1}{2}\right)^{10/30} = 4.0 \ \mu\text{Ci}$$

ASSESS Although N_0 is a very large number, it is a very small fraction ($\approx 10^{-10}$) of a mole. The sample is about 60 ng (nanograms) of ^{137}Cs.

Radioactive Dating

Many geological and archeological samples can be dated by measuring the decays of naturally occurring radioactive isotopes. Because we have no way to know N_0, the initial number of radioactive nuclei, radioactive dating depends on the use of ratios.

The most well-known dating technique is carbon dating. The carbon isotope ^{14}C has a half-life of 5730 years, so any ^{14}C present when the earth formed 4.5 billion years ago would long since have decayed away. Nonetheless, ^{14}C is present in atmospheric carbon dioxide because high-energy cosmic rays collide with gas molecules high in the atmosphere. These cosmic rays are energetic enough to create ^{14}C nuclei from nuclear reactions with nitrogen and oxygen nuclei. The creation and decay of ^{14}C have reached a steady state in which the ^{14}C/^{12}C ratio is

1.3×10^{-12}. That is, atmospheric carbon dioxide has ^{14}C at the concentration of 1.3 parts per trillion. As small as this is, it's easily measured by modern chemical techniques.

All living organisms constantly exchange carbon dioxide with the atmosphere, so the $^{14}C/^{12}C$ ratio in living organisms is also 1.3×10^{-12}. As soon as an organism dies, the ^{14}C in its tissue begins to decay and no new ^{14}C is added. Objects are dated by comparing the measured $^{14}C/^{12}C$ ratio to the 1.3×10^{-12} value of living material.

Carbon dating is used to date skeletons, wood, paper, fur, food material, and anything else made of organic matter. It is quite accurate for ages to about 15,000 years. Beyond that, the difficulty of measuring such a small ratio and some uncertainties about the cosmic ray flux in the past combine to decrease the accuracy. Even so, items are dated to about 50,000 years with a fair degree of reliability.

Other isotopes with longer half-lives are used to date geological samples. Potassium-argon dating, using ^{40}K with a half-life of 1.25 billion years, is especially useful for dating rocks of volcanic origin.

EXAMPLE 42.4 Carbon dating

Archeologists excavating an ancient hunters' camp have recovered a 5.0 g piece of charcoal from a fireplace. Measurements on the sample find that the ^{14}C activity is 0.35 Bq. What is the approximate age of the camp?

MODEL Charcoal, from burning wood, is almost pure carbon. The number of ^{14}C atoms in the wood has decayed exponentially since the branch fell off a tree. Because wood rots, it is reasonable to assume that there was no significant delay from when the branch fell off the tree and when the hunters burned it.

SOLVE The $^{14}C/^{12}C$ ratio was 1.3×10^{-12} when the branch fell from the tree. We first need to determine the present ratio, then use the known ^{14}C half-life $t_{1/2} = 5730$ years to calculate the time needed to reach the present ratio. The number of ordinary ^{12}C nuclei in the sample is

$$N(^{12}C) = \left(\frac{5.0\text{ g}}{12\text{ g/mol}}\right) 6.02 \times 10^{23}\text{ atoms/mol}$$
$$= 2.5 \times 10^{23}\text{ nuclei}$$

The number of ^{14}C nuclei can be found from the activity to be $N(^{14}C) = R/r$, but we need to determine the ^{14}C decay rate r. After converting the half-life to seconds, $t_{1/2} = 5730$ years $= 1.807 \times 10^{11}$ s, we can compute

$$r = \frac{1}{\tau} = \frac{1}{t_{1/2}/\ln 2} = 3.84 \times 10^{-12}\text{ s}^{-1}$$

Thus

$$N(^{14}C) = \frac{R}{r} = \frac{0.35\text{ Bq}}{3.84 \times 10^{-12}\text{ s}^{-1}} = 9.1 \times 10^{10}\text{ nuclei}$$

and the present $^{14}C/^{12}C$ ratio is $N(^{14}C)/N(^{12}C) = 0.36 \times 10^{-12}$. Because this ratio has been decaying with a half-life of 5730 years, the time needed to reach the present ratio is found from

$$0.36 \times 10^{-12} = 1.3 \times 10^{-12}\left(\frac{1}{2}\right)^{t/t_{1/2}}$$

To solve for t, first write this as

$$\frac{0.36}{1.3} = 0.277 = \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

Now take the logarithm of both sides:

$$\ln(0.277) = -1.28 = \frac{t}{t_{1/2}}\ln(0.5) = -0.693\frac{t}{t_{1/2}}$$

Thus the age of the hunters' camp is

$$t = 1.85t_{1/2} = 10{,}600\text{ years}$$

ASSESS This is a realistic example of how radioactive dating is done.

STOP TO THINK 42.4 A sample starts with 1000 radioactive atoms. How many half-lives have elapsed when 750 atoms have decayed?

a. 0.25
b. 1.5
c. 2.0
d. 2.5

42.6 Nuclear Decay Mechanisms

This section will look in more detail at the mechanisms of the three types of radioactive decay.

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Alpha Decay

An alpha particle, symbolized as α, is a ^{4}He nucleus, a strongly bound system of two protons and two neutrons. An unstable nucleus that ejects an alpha particle will lose two protons and two neutrons, so we can write the decay as

$$^{A}X_Z \rightarrow {}^{A-4}Y_{Z-2} + \alpha + \text{energy} \qquad (42.21)$$

Figure 42.18 shows the alpha-decay process. The original nucleus X is called the **parent nucleus** and the decay-product nucleus Y is the **daughter nucleus.** This reaction can occur only when the mass of the parent nucleus is greater than the mass of the daughter nucleus plus the mass of an alpha particle. This requirement is met for heavy, high-Z nuclei well above the maximum on the Figure 42.6 curve of binding energy. It is energetically favorable for these nuclei to eject an alpha particle because the daughter nucleus is more tightly bound than the parent nucleus.

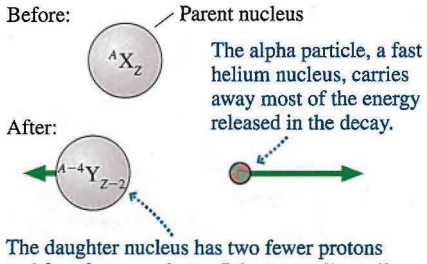


FIGURE 42.18 Alpha decay.

Although the mass requirement is based on the nuclear masses, we can express it—as we did the binding energy equation—in terms of atomic masses. The energy released in an alpha decay, essentially all of which goes into the alpha particle's kinetic energy, is

$$\Delta E \approx K_\alpha = (m_X - m_Y - m_{He})c^2 \qquad (42.22)$$

EXAMPLE 42.5 Alpha decay of uranium

The uranium isotope ^{238}U undergoes alpha decay to ^{234}Th. The atomic masses are 238.0508 u for ^{238}U and 234.0436 u for ^{234}Th. What is the kinetic energy, in MeV, of the alpha particle?

MODEL Essentially all of the energy release ΔE goes into the alpha particle's kinetic energy.

SOLVE The atomic mass of helium, from Table 42.2, is 4.0026 u. Thus

$$K_\alpha = (238.0508\text{ u} - 234.0436\text{ u} - 4.0026\text{ u})c^2$$
$$= \left(0.0046\text{ u} \times \frac{931.5\text{ MeV}/c^2}{1\text{ u}}\right)c^2 = 4.3\text{ MeV}$$

ASSESS This is a typical alpha-particle energy. Notice how the c^2 canceled from the calculation so that we never had to evaluate c^2.

Alpha decay is a purely quantum-mechanical effect. Figure 42.19 shows the potential energy of an alpha particle, where the ^{4}He nucleus of an alpha particle is so tightly bound that we can think of it as existing "prepackaged" inside the parent nucleus. Both the depth of the energy well and the height of the Coulomb barrier are twice that of a proton because the charge of an α particle is $2e$.

Because of the high Coulomb barrier (alpha decay occurs only in high-Z nuclei), there may be one or more allowed energy levels with $E > 0$. Energy levels with $E < 0$ are completely bound, but an alpha particle in an energy level with $E > 0$ can *tunnel* through the Coulomb barrier and escape. That is exactly how alpha decay occurs.

Energy must be conserved, so the kinetic energy of the escaping α particle is the height of the energy level above $E = 0$. That is, potential energy is transformed into kinetic energy as the particle escapes. Notice that the width of the barrier decreases as E increases. The tunneling probability depends very sensitively on the barrier width, as you learned in conjunction with the scanning tunneling microscope. Thus an alpha particle in a higher energy level should have a

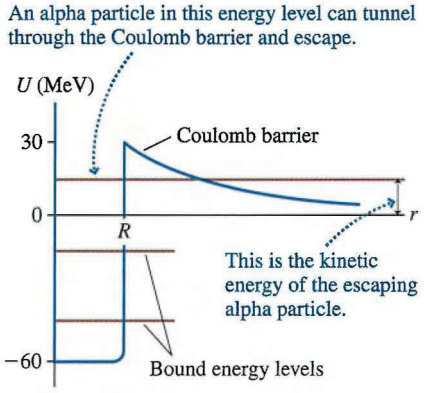


FIGURE 42.19 The potential-energy diagram of an alpha particle in the parent nucleus.

shorter half-life and escape with *more kinetic energy*. The full analysis is beyond the scope of this text, but this prediction is in excellent agreement with measured energies and half-lives.

Beta Decay

Beta decay was initially associated with the emission of an electron e^-, the beta particle. It was later discovered that some nuclei can undergo beta decay by emitting a positron e^+, the antiparticle of the electron, although this decay mode is not as common. A positron is identical to an electron except that it has a positive charge. To be precise, the emission of an electron is called *beta-minus decay* and the emission of a positron is *beta-plus decay*.

A typical example of beta decay occurs in the carbon isotope ^{14}C, which undergoes the beta-decay process $^{14}C \rightarrow {}^{14}N + e^-$. Carbon has $Z = 6$ and nitrogen has $Z = 7$. Because Z increases by 1 but A doesn't change, it appears that a neutron within the nucleus has changed itself into a proton and electron. That is, the basic beta-minus decay process appears to be

$$n \rightarrow p^+ + e^- \tag{42.23}$$

The electron is ejected from the nucleus but the proton is not. Thus the decay process, shown in Figure 42.20a, is

$$^AX_Z \rightarrow {}^AY_{Z+1} + e^- + \text{energy} \qquad \text{(beta-minus decay)} \tag{42.24}$$

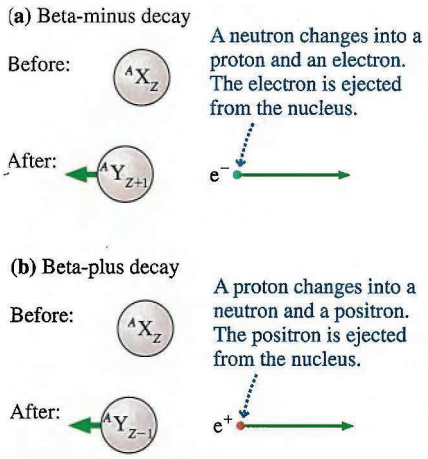


FIGURE 42.20 Beta decay.

Indeed, a free neutron turns out to *not* be a stable particle. It decays with a half-life of approximately 10 min into a proton and an electron. This decay is energetically allowed because $m_n > m_p + m_e$. Furthermore, it conserves charge.

Whether a neutron *within* a nucleus can decay depends not only on the masses of the neutron and proton but also on the masses of the parent and daughter nuclei, because energy has to be conserved for the entire nuclear system. **Beta decay occurs only if $m_X > m_Y$.** ^{14}C can undergo beta decay to ^{14}N because $m(^{14}C) > m(^{14}N)$. But $m(^{12}C) < m(^{12}N)$, so ^{12}C is stable and its neutrons cannot decay.

Beta-plus decay is the conversion of a proton into a neutron and a positron:

$$p^+ \rightarrow n + e^+ \tag{42.25}$$

The full decay process, shown in Figure 42.20b, is

$$^AX_Z \rightarrow {}^AY_{Z-1} + e^+ + \text{energy} \qquad \text{(beta-plus decay)} \tag{42.26}$$

Beta-plus decay does *not* happen for a free proton because $m_p < m_n$. It *can* happen within a nucleus as long as energy is conserved for the entire nuclear system.

In our earlier discussion of Figure 42.11 we noted that the ^{12}B and ^{12}N nuclei could reach a lower energy state if a proton could change into a neutron and vice versa. Now we see that such a change can occur if the energy conditions are favorable. And, indeed, ^{12}B undergoes beta-minus decay to ^{12}C while ^{12}N undergoes beta-plus decay to ^{12}C.

In general, beta decay is a process used by nuclei with too many neutrons or too many protons in order to move closer to the line of stability in Figure 42.4.

NOTE ▶ The electron emitted in beta decay has nothing to do with the atom's valence electrons. The beta particle is created in the nucleus and ejected directly from the nucleus when a neutron is transformed into a proton and an electron. ◀

A third form of beta decay occurs in some nuclei that have too many protons but not enough mass to undergo beta-plus decay. In this case, a proton changes