into a neutron by "capturing" an electron from the innermost shell of orbiting electrons (an $n = 1$ electron). The process is

$$p^+ + \text{orbital } e^- \rightarrow n \tag{42.27}$$

This form of beta decay is called **electron capture,** abbreviated EC. The net result, ${}^{A}X_Z \rightarrow {}^{A}Y_{Z-1}$, is the same as beta-plus decay but without the emission of a positron. Electron capture is the only nuclear decay mechanism that involves the orbital electrons.

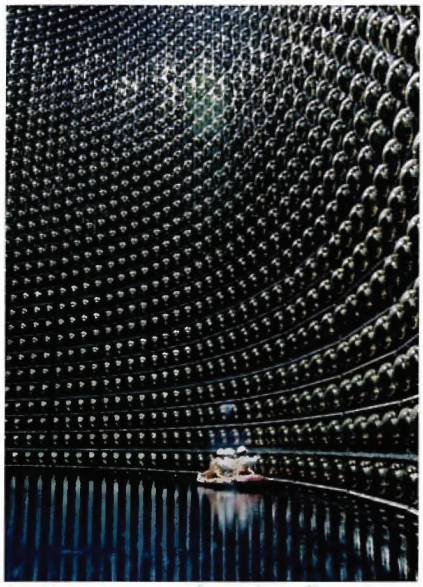
The Weak Interaction

We've presented beta decay as if it were perfectly normal for one kind of matter to change spontaneously into a completely different kind of matter. For example, it would be energetically favorable for a large truck to spontaneously turn into a Cadillac and a VW Beetle, ejecting the Beetle at high speed. But it doesn't happen.

Once you stop to think of it, the process $n \rightarrow p^+ + e^-$ seems ludicrous, not because it violates mass-energy conservation but because we have no idea *how* a neutron could turn into a proton. Alpha decay may be a strange process because tunneling in general goes against our commonsense notions, but it is a perfectly ordinary quantum-mechanical process. Now we're suggesting that one of the basic building blocks of matter can somehow morph into a different basic building block.

To make matters more confusing, measurements in the 1930s found that beta decay didn't seem to conserve either energy or momentum. Faced with these difficulties, the Italian physicist Enrico Fermi made two bold suggestions:

1. A previously unknown fundamental force of nature is responsible for beta decay. This force, which has come to be known as the **weak interaction,** has the ability to turn a neutron into a proton and vice versa.
2. The beta-decay process emits a particle that, at that time, had not been detected. This new particle has to be electrically neutral, in order to conserve charge, and it has to be much smaller than an electron. Fermi called it the **neutrino,** meaning "little neutral one." Energy and momentum really are conserved, but the neutrino carries away some of the energy and momentum of the decaying nucleus. Thus experiments that detect only the electron seem to violate conservation laws.



The Super Kamiokande neutrino detector in Japan looks for the neutrinos emitted from nuclear fusion reactions in the core of the sun.

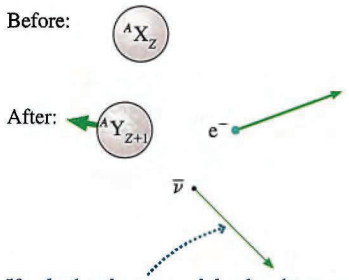
The neutrino is represented by the symbol ν, a lowercase Greek nu. The beta-decay processes that Fermi proposed are

$$\begin{aligned} n &\rightarrow p^+ + e^- + \bar{\nu} \\ p^+ &\rightarrow n + e^+ + \nu \end{aligned} \tag{42.28}$$

The symbol $\bar{\nu}$ is an *antineutrino,* although the reason why one is a neutrino and the other an antineutrino need not concern us here. Figure 42.21 shows that the electron and antineutrino (or positron and neutrino) *share* the energy released in the decay.

The neutrino interacts with matter so weakly that a neutrino can pass straight through the earth with only a very slight chance of a collision. Thousands of neutrinos created by nuclear fusion reactions in the core of the sun are passing through your body every second. Neutrino interactions are so rare that the first laboratory detection did not occur until 1956, over 20 years after Fermi's proposal.

It was initially thought that the neutrino had not only zero charge but zero mass. However, experiments within the last few years have shown that the neutrino mass, although very tiny, is not zero. The best current evidence suggests a mass about one-millionth the mass of an electron. Experiments now underway will attempt to determine a more accurate value. The result will have far more importance than simply understanding beta decay. Neutrinos are the most numerous of all particles in the universe, so it may turn out that the neutrino mass has cosmological significance for the evolution of the universe.



If only the electron and the daughter nucleus are measured, energy and momentum appear not to be conserved. The "missing" energy and momentum are carried away by the undetected antineutrino.

FIGURE 42.21 A more accurate picture of beta decay includes neutrinos.

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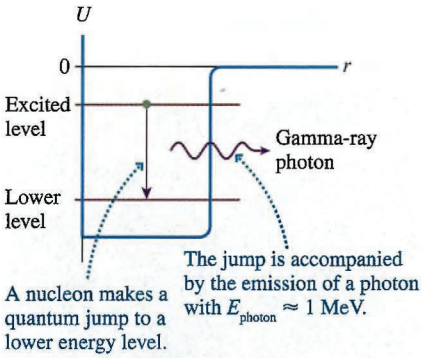


FIGURE 42.22 Gamma decay.

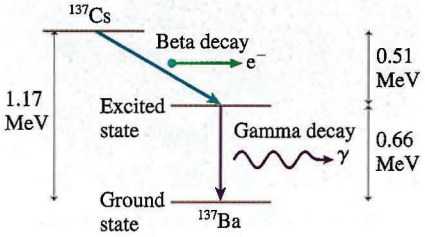


FIGURE 42.23 The decay of ^{137}Cs involves both beta and gamma decay.

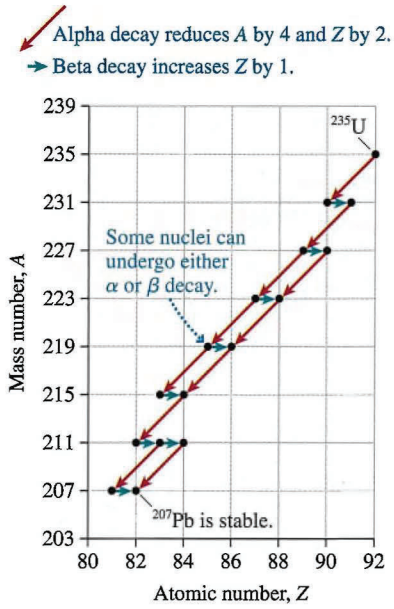


FIGURE 42.24 The nuclear decay series of ^{235}U.

EXAMPLE 42.6 Beta decay of ^{14}C

How much energy is released in the beta-minus decay of ^{14}C?

MODEL The decay is $^{14}\text{C} \rightarrow {}^{14}\text{N} + e^- + \bar{\nu}$.

SOLVE In Appendix C we find $m(^{14}\text{C}) = 14.003\,242$ u and $m(^{14}\text{N}) = 14.003\,074$ u. The mass difference is a mere 0.000 168 u, but this is the mass that is converted into the kinetic energy of the escaping particles. The energy released is

$$E = (\Delta m)c^2 = (0.000\,168\text{ u}) \times (931.5\text{ MeV/u})$$
$$= 0.156\text{ MeV}$$

ASSESS This energy is shared between the electron and the antineutrino.

Gamma Decay

Gamma decay is the easiest form of nuclear decay to understand. You learned that an atomic system can emit a photon with $E_{photon} = \Delta E_{atom}$ when an electron undergoes a quantum jump from an excited energy level to a lower energy level. Nuclei are no different. A proton or a neutron in an excited nuclear state, such as the one shown in Figure 42.22, can undergo a quantum jump to a lower-energy state by emitting a high-energy photon. This is the gamma-decay process.

The spacing between atomic energy levels is only a few eV. Nuclear energy levels, by contrast, are typically 1 MeV apart. Hence gamma-ray photons have $E_{gamma} \approx 1$ MeV. Photons with this much energy have tremendous penetrating power and deposit an extremely large amount of energy at the point where they are finally absorbed.

Nuclei left to themselves are usually in their ground states and thus cannot emit gamma-ray photons. However, alpha and beta decay often leave the daughter nucleus in an excited nuclear state, so gamma emission is usually found to accompany alpha and beta emission.

The cesium isotope ^{137}Cs is a good example. We noted earlier that ^{137}Cs is used as a laboratory source of gamma rays. Actually, ^{137}Cs undergoes beta-minus decay to ^{137}Ba. Figure 42.23 shows the full process. A ^{137}Cs nucleus undergoes beta-minus decay by emitting an electron and an antineutrino, which share between them a total energy of 0.51 MeV. The half-life for this process is 30 years. This leaves the daughter ^{137}Ba nucleus in an excited state 0.66 MeV above the ground state. The excited Ba nucleus then decays within a few seconds to the ground state by emitting a 0.66 MeV gamma-ray photon. Thus a ^{137}Cs sample *is* a source of gamma-ray photons, but the photons are actually emitted by barium nuclei rather than cesium nuclei.

Decay Series

A radioactive nucleus decays into a daughter nucleus. In many cases, the daughter nucleus is also radioactive and decays to produce its own daughter nucleus. The process continues until reaching a daughter nucleus that is stable. The sequence of isotopes, starting with the original unstable isotope and ending with the stable isotope, is called a **decay series.**

Decay series are especially important for very heavy nuclei. As an example, Figure 42.24 shows the decay series of ^{235}U, an isotope of uranium with a 700-million-year half-life. This is a very long time, but it is only about 15% the age of the earth and most (but not all) of the ^{235}U nuclei present when the earth was formed have now decayed. There are many unstable nuclei along the way, but all ^{235}U nuclei eventually end as the ^{207}Pb isotope of lead, a stable nucleus.

Notice that some nuclei can decay by either alpha *or* beta decay. Thus there are a variety of paths that a decay can follow, but they all end at the same point.

STOP TO THINK 42.5 The cobalt isotope ^{60}Co ($Z = 27$) decays to the nickel isotope ^{60}Ni ($Z = 28$). The decay process is

a. Alpha decay. b. Beta-minus decay. c. Beta-plus decay.
d. Electron capture. e. Gamma decay.

42.7 Biological Applications of Nuclear Physics

Nuclear physics has brought both peril and promise to society. Radioactivity can cause tumors. At the same time, radiation can be used to cure some cancers. This section is a brief survey of medical and biological applications of nuclear physics.

Radiation Dose

Nuclear radiation, which is ionizing radiation, disrupts a cell's machinery by altering and damaging the biological molecules. The consequences of this disruption vary from genetic mutations to uncontrolled cell multiplication (i.e., tumors) to cell death.

Beta and gamma radiation can penetrate the entire body and damage internal organs. Alpha radiation has less penetrating ability, but it deposits all its energy in a very small, localized volume. Internal organs are usually safe from alpha radiation, but the skin is very susceptible, as are the lungs if radioactive dust is inhaled.

Biological effects of radiation depend upon the **dose,** the amount of radiation received. Two factors enter into determining the dose. The first is the physical factor of energy absorbed by the body. The second is the biological factor of how tissue reacts to different forms of radiation.

The **rad,** an acronym for **r**adiation **a**bsorbed **d**ose, measures the energy deposited in an irradiated material. It is defined as

$$1 \text{ rad} \equiv 0.010 \text{ J/kg of absorbed energy}$$

The number of rads depends only on the energy absorbed, not at all on the type of radiation or on what the absorbing material is.

Biologists and biophysicists have found that a 1 rad dose of gamma rays and a 1 rad dose of alpha particles have different biological consequences. To account for such differences, the **relative biological effectiveness,** RBE, is defined as the biological effect of a given dose relative to the biological effect of an equal dose of x rays.

Table 42.4 shows the relative biological effectiveness of different forms of radiation. Larger values correspond to larger biological effects. Alpha and beta radiation have a range of values because the biological effect varies with the energy of the particle. Alpha radiation has the largest RBE because the energy is deposited in a smaller volume.

TABLE 42.4 Relative biological effectiveness of radiation

Radiation type	RBE
X rays	1
Gamma rays	1
Beta particles	1–2
Alpha particles	10–20

Combining these two measures, the **biologically equivalent dose** is the product of the energy dose in rads with the relative biological effectiveness. The biologically equivalent dose is measured in **rem,** an acronym for **r**öntgen **e**quivalent in **m**an. (This term is based on historical usage. The *röntgen* is a unit for measuring the ionization ability of x rays, but we need not be concerned with its definition.) To be precise,

$$\text{biologically equivalent dose in rem} \equiv \text{dose in rad} \times \text{RBE}$$

One rem of radiation produces the same biological damage regardless of the type of radiation.

EXAMPLE 42.7 Radiation exposure

A 75 kg laboratory technician working with the radioactive isotope ^{137}Cs receives an accidental 100 mrem (millirem) exposure. ^{137}Cs emits 0.66 MeV gamma-ray photons. How many gamma-ray photons are absorbed in the technician's body?

MODEL The radiation dose is a combination of deposited energy and biological effectiveness. The RBE for gamma rays is 1. Gamma rays are penetrating, so this is a whole-body exposure.

SOLVE The dose in rads is the dose in rems divided by the RBE. In this case, because the RBE = 1, the dose is 100 mrad = 0.10 rad = 0.0010 J/kg. This is a whole-body exposure, so the total energy deposited in the technician's body is 0.075 J. The energy of each absorbed photon is 0.66 MeV, but this value must be converted into joules. The number of photons in 0.075 J is

$$N = \frac{0.075\ \text{J}}{(6.6 \times 10^5\ \text{eV/photon})(1.60 \times 10^{-19}\ \text{J/eV})}$$
$$= 7.1 \times 10^{11}\ \text{photons}$$

ASSESS The energy deposited, 0.075 J, is very small. Radiation does its damage not by thermal effects, which would require substantially more energy, but by ionization.

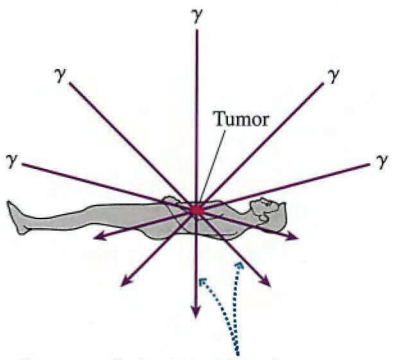
TABLE 42.5 Radiation exposure

Radiation source	Typical exposure (mrem/year)
Natural background	300
Mammogram x ray	80
Chest x ray	30
Dental x ray	3

Table 42.5 shows some basic information about radiation exposure. We are all exposed to a continual *natural background* of radiation from cosmic rays and from naturally occurring radioactive atoms (uranium and atoms in the uranium decay series) in the ground, the atmosphere, and even the food we eat. This background averages about 300 mrem per year, although there are wide regional variations depending on the soil type and the elevation. (Higher elevations have a larger exposure to cosmic rays.)

Medical x rays vary significantly. The average person in the United States receives approximately 60 mrem per year from all medical sources. All other sources, such as fallout from atmospheric nuclear tests many decades ago, nuclear power plants, and industrial uses of radioactivity, amount to <10 mrem per year.

The question inevitably arises, "What is a safe dose?" This remains a controversial topic and the subject of ongoing research. The effects of large doses of radiation are easily observed. The effects of small doses are hard to distinguish from other natural and environmental causes. Thus there's no simple or clear definition of a safe dose. A prudent policy is to avoid unnecessary exposure to radiation but not to worry over exposures less than the natural background. It's worth noting that the μCi radioactive sources used in laboratory experiments provide exposures *much* less than the natural background, even if used on a regular basis.



Gamma radiation is incident along many lines, all of which intersect the tumor.

FIGURE 42.25 Radiation therapy is designed to deliver a lethal dose to the tumor without damaging nearby tissue.

Medical Uses of Radiation

Radiation can be put to good use killing cancer cells. This area of medicine is called *radiation therapy.* Gamma rays are the most common form of radiation, often from the isotope ^{60}Co. As Figure 42.25 shows, the gamma rays are directed along many different lines, all of which intersect the tumor. The goal is to provide a lethal dose to the cancer cells without overexposing nearby tissue. The patient and the radiation source are rotated around each other under careful computer control to deliver the proper dose.

Other tumors are treated by surgically implanting radioactive "seeds" within or next to the tumor. Alpha particles, which are very damaging locally but don't penetrate far, can be used in this fashion.

Radioactive isotopes are also used as *tracers* in diagnostic procedures. This technique is based on the fact that all isotopes of an element have identical chemical behavior. As an example, a radioactive isotope of iodine is used in the diagnosis of certain thyroid conditions. Iodine is an essential element in the body, and it concentrates in the thyroid gland. A doctor who suspects a malfunctioning thyroid gland gives the patient a small dose of sodium iodide in which some of the normal ^{127}I atoms have been replaced with ^{131}I. (Sodium iodide, which is harmless, dissolves in water and can simply be drunk.) The ^{131}I isotope, with a half-life of eight days, undergoes beta decay, and subsequently emits a gamma-ray photon.

The radioactive iodine concentrates within the thyroid gland within a few hours. The doctor then monitors the gamma-ray photon emissions over the next few days to see how the iodine is being processed within the thyroid and how quickly it is eliminated from the body.

Other important radioactive tracers include the chromium isotope ^{51}Cr, which is taken up by red blood cells and can be used to monitor blood flow, and the xenon isotope ^{133}Xe, which is inhaled to reveal lung functioning. Radioactive tracers are *noninvasive,* meaning that the doctor can monitor the inside of the body without surgery.



Radiation therapy is a beneficial use of nuclear physics.

Magnetic Resonance Imaging

The proton, like the electron, has an inherent angular momentum (spin) and an inherent magnetic moment. You can think of the proton as being like a little compass needle that can be in one of two positions, the positions we call spin up and spin down.

A compass needle aligns itself with an external magnetic field. This is the needle's lowest-energy position. Turning a compass needle by hand is like rolling a ball uphill; you're giving it energy, but, like the ball rolling downhill, it will realign itself with the lowest-energy position when you remove your finger. There is, however, an *unstable equilibrium* position in which the needle is anti-aligned with the field. The slightest jostle will cause it to flip around, but the needle will be steady in its upside-down configuration if you can balance it perfectly.

A proton in a magnetic field behaves similarly, but with a major difference: Because the proton's energy is quantized, the proton cannot assume an intermediate position. It's either aligned with the magnetic field (the spin-up orientation) or anti-aligned (spin-down). Figure 42.26a shows these two quantum states. Turning on a magnetic field lowers the energy of a spin-up proton and increases the energy of an anti-aligned, spin-down proton. In other words, the magnetic field creates an *energy difference* between these states.

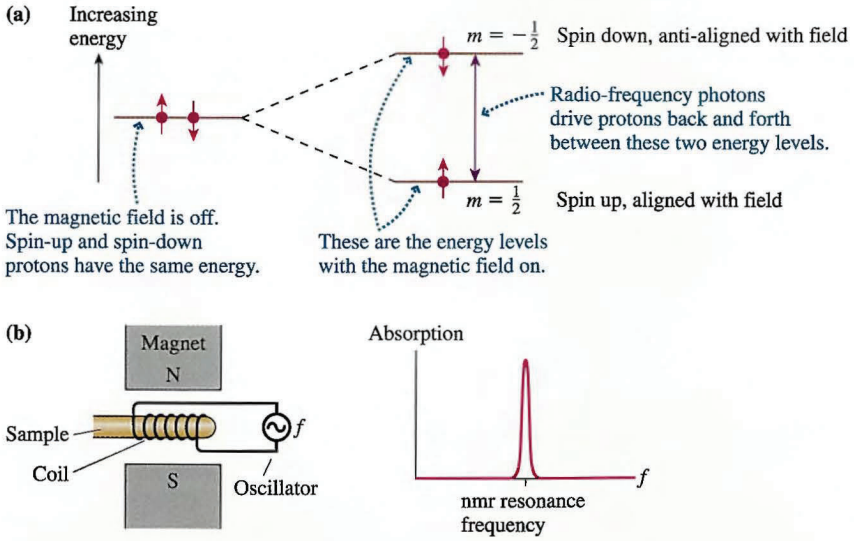


FIGURE 42.26 Nuclear magnetic resonance is possible because spin-up and spin-down protons have slightly different energies in a magnetic field.

The energy difference is very tiny, only about 10^{-7} eV. Nonetheless, photons whose energy matches the energy difference cause the protons to move back and forth between these two energy levels as the photons are absorbed and emitted. In effect, the photons are causing the proton's spin to flip back and forth rapidly. The



FIGURE 42.27 Magnetic resonance imaging shows internal organs in exquisite detail.

photon frequency, which depends on the magnetic field strength, is typically about 100 MHz, similar to FM radio frequencies.

Figure 42.26b shows how this behavior is put to use. A sample containing protons is placed in a magnetic field. A coil is wrapped around the sample, and a variable-frequency AC source drives a current through this coil. The protons absorb power from the coil when its frequency is just right to flip the spin back and forth; otherwise, no power is absorbed. A *resonance* is seen by scanning the coil through a small range of frequencies.

This technique of observing the spin flip of nuclei (the technique works for nuclei other than hydrogen) in a magnetic field is called **nuclear magnetic resonance,** or *nmr*. It has many applications in physics, chemistry, and materials science. Its medical use exploits the fact that tissue is mostly water, and two out of the three nuclei in a water molecule are protons. Thus the human body is basically a sample of protons, with the proton density varying as tissue density varies.

The medical procedure known as **magnetic resonance imaging,** or MRI, places the patient in a spatially varying magnetic field. The variations in the field cause the proton absorption frequency to vary from point to point. From the known shape of the field and measurements of the frequencies that are absorbed, and how strongly, sophisticated computer software can transform the raw data into detailed images such as the one shown in Figure 42.27.

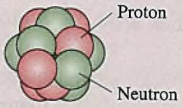
As an interesting footnote, the technique was still being called *nuclear magnetic resonance* when it was first introduced into medicine. Unfortunately, doctors soon found that many patients were afraid of it because of the word "nuclear." Hence the alternative term "magnetic resonance imaging" was coined. It is true that the public perception of nuclear technology is not always positive, but equally true that nuclear physics has made many important and beneficial contributions to society.

SUMMARY

The goal of Chapter 42 has been to understand the physics of the nucleus and some of the applications of nuclear physics.

GENERAL PRINCIPLES

The Nucleus



The nucleus is a small, dense, positive core at the center of an atom.

Z protons: charge $+e$, spin $\frac{1}{2}$

N neutrons: charge 0, spin $\frac{1}{2}$

The mass number is $A = Z + N$

The nuclear radius is $r = r_0 A^{1/3}$, where $r_0 = 1.2$ fm. Typical radii are a few fm.

Nuclear forces

Attractive strong force

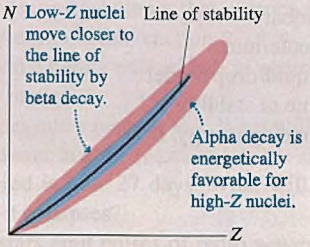
- Acts between any two nucleons
- Is short range, <3 fm
- Is felt between nearest neighbors

Repulsive electric force

- Acts between two protons
- Is long range
- Is felt across the nucleus

Nuclear Stability

Most nuclei are not stable. Unstable nuclei undergo radioactive decay. Stable nuclei cluster along the **line of stability** in a plot of the isotopes.



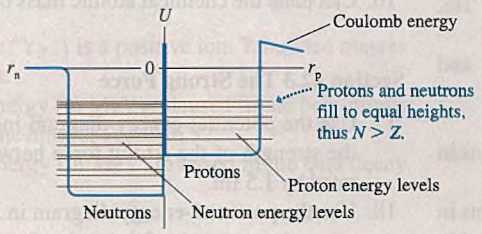
Three mechanisms by which unstable nuclei decay:

Decay	Particle	Mechanism	Energy	Penetration
α	^{4}He nucleus	tunneling	few MeV	low
β	e^-	$n \rightarrow p^+ + e^-$	≈ 1 MeV	medium
	e^+	$p^+ \rightarrow n + e^+$	≈ 1 MeV	medium
γ	photon	quantum jump	≈ 1 MeV	high

IMPORTANT CONCEPTS

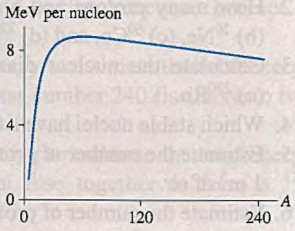
Shell model

Each nucleon moves with an average potential energy due to all other nucleons.



Curve of binding energy

The average binding energy per nucleon has a broad maximum at $A \approx 60$.



APPLICATIONS

Radioactive decay

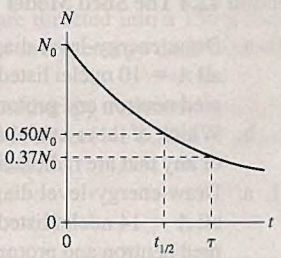
The number of undecayed nuclei decreases exponentially with time t.

$$N = N_0 \exp(-t/\tau)$$
$$= N_0 (1/2)^{t/t_{1/2}}$$

The **time constant** τ is $1/r$, where r is the **decay rate.** The **half-life**

$$t_{1/2} = \tau \ln 2 = 0.693\tau$$

is the time in which half of any sample decays.



Measuring radiation

The **activity** $R = rN$ of a radioactive sample, measured in becquerels or curies, is the number of decays per second.

The radiation **dose** is measured in rad, where

1 rad $\equiv$ 0.010 J/kg of absorbed energy

The **relative biological effectiveness** RBE is the biological effect of a dose relative to the biological effects of x rays. The **biologically equivalent dose** is measured in rem, where rem = rad $\times$ RBE. One rem of radiation produces the same biological effect regardless of the type of radiation.

TERMS AND NOTATION

nuclear physics	curve of binding energy	parent nucleus
nucleon	strong force	daughter nucleus
atomic number, Z	shell model	electron capture
mass number, A	alpha decay	weak interaction
neutron number, N	beta decay	neutrino
isotope	gamma decay	decay series
radioactive	ionizing radiation	dose
stable	Geiger counter	rad
natural abundance	decay rate, r	relative biological effectiveness (RBE)
isobar	time constant, τ	biologically equivalent dose
deuterium	half-life, $t_{1/2}$	rem
liquid-drop model	activity, R	nuclear magnetic resonance
line of stability	becquerel, Bq	magnetic resonance imaging (MRI)
binding energy, B	curie, Ci	

EXERCISES AND PROBLEMS

See Appendix C for data on atomic masses, isotopic abundance, radioactive decay modes, and half-lives.

Exercises

Section 42.1 Nuclear Structure

1. How many protons and how many neutrons are in (a) ^{3}H, (b) ^{40}Ar, (c) ^{40}Ca, and (d) ^{239}Pu?
2. How many protons and how many neutrons are in (a) ^{3}He, (b) ^{20}Ne, (c) ^{60}Co, and (d) ^{226}Ra?
3. Calculate the nuclear diameters of (a) ^{4}He, (b) ^{40}Ar, and (c) ^{220}Rn.
4. Which stable nuclei have a diameter of 8.84 fm?
5. Estimate the number of protons and the number of neutrons in 1 m^3 of air.
6. Estimate the number of protons and the number of neutrons in your body.
7. What would be the mass of a 1.0-cm-diameter marble if it had nuclear density?

Section 42.2 Nuclear Stability

8. Use data in Appendix C to make your own chart of stable and unstable nuclei, similar to Figure 42.4, for all nuclei with $Z \leq 8$. Use a blue or black dot to represent stable isotopes, a red dot to represent isotopes that undergo beta-minus decay, and a green dot to represent isotopes that undergo beta-plus decay or electron-capture decay.
9. a. What is the smallest value of A for which there are two stable nuclei? What are they?
   b. For which values of A less than this are there *no* stable nuclei?
10. Calculate (in MeV) the total binding energy and the binding energy per nucleon for ^{3}H and for ^{3}He.
11. Calculate (in MeV) the total binding energy and the binding energy per nucleon for ^{40}Ar and for ^{40}Ca.
12. Calculate (in MeV) the binding energy per nucleon for ^{3}He and ^{4}He. Which is more tightly bound?
13. Calculate (in MeV) the binding energy per nucleon for ^{12}C and ^{13}C. Which is more tightly bound?
14. Calculate (in MeV) the binding energy per nucleon for (a) ^{14}N, (b) ^{56}Fe, and (c) ^{207}Pb.
15. Calculate the chemical atomic mass of neon.
16. Calculate the chemical atomic mass of magnesium.

Section 42.3 The Strong Force

17. Use the potential-energy diagram in Figure 42.8 to estimate the strength of the strong force between two nucleons separated by 1.5 fm.
18. Use the potential-energy diagram in Figure 42.8 to sketch an approximate graph of the strong force between two nucleons versus the distance r between their centers.
19. What is the ratio of the gravitational potential energy to the nuclear potential energy for two neutrons separated by 1.0 fm?

Section 42.4 The Shell Model

20. a. Draw energy-level diagrams, similar to Figure 42.11, for all $A = 10$ nuclei listed in Appendix C. Show all the occupied neutron and proton levels.
    b. Which of these nuclei are stable? What is the decay mode of any that are radioactive?
21. a. Draw energy-level diagrams, similar to Figure 42.11, for all $A = 14$ nuclei listed in Appendix C. Show all the occupied neutron and proton levels.
    b. Which of these nuclei are stable? What is the decay mode of any that are radioactive?

Section 42.5 Radiation and Radioactivity

22. The barium isotope ^{133}Ba has a half-life of 10.5 years. A sample begins with 1.0×10^{10} ^{133}Ba atoms. How many are left after (a) 2 years, (b) 20 years, and (c) 200 years?
23. The cadmium isotope ^{109}Cd has a half-life of 462 days. A sample begins with 1.0×10^{12} ^{109}Cd atoms. How many are left after (a) 50 days, (b) 500 days, and (c) 5000 days?
24. The radioactive hydrogen isotope ^{3}H is called *tritium.*
    a. What are the decay mode and the daughter nucleus of tritium?
    b. What are the time constant and the decay rate of tritium?
25. How many half-lives must elapse until (a) 90% and (b) 99% of a radioactive sample of atoms has decayed?
26. What is the age in years of a bone in which the $^{14}C/^{12}C$ ratio is measured to be 1.65×10^{-13}?
27. What is the half-life in days of a radioactive sample with 5.0×10^{15} atoms and an activity of 5.0×10^{8} Bq?

Section 42.6 Nuclear Decay Mechanisms

28. Identify the unknown isotope X in the following decays.
    a. $^{234}U \rightarrow X + \alpha$.
    b. $^{32}P \rightarrow X + e^- + \bar{\nu}$
    c. $X \rightarrow {}^{30}Si + e^+ + \nu$
    d. $^{24}Na \rightarrow {}^{24}Mg + e^- + \bar{\nu} \rightarrow X + \gamma$
29. Identify the unknown isotope X in the following decays.
    a. $X \rightarrow {}^{224}Ra + \alpha$
    b. $X \rightarrow {}^{207}Pb + e^- + \bar{\nu}$
    c. $^{7}Be + e^- \rightarrow X + \nu$
    d. $X \rightarrow {}^{60}Ni + \gamma$
30. What is the energy (in MeV) released in the alpha decay of ^{239}Pu?
31. What is the energy (in MeV) released in the alpha decay of ^{228}Th?
32. What is the total energy (in MeV) released in the beta-minus decay of ^{3}H?
    Hint: The daughter $^{A}Y_{Z-1}$ is a positive ion. Tabulated masses are for neutral atom.
33. What is the total energy (in MeV) released in the beta-minus decay of ^{19}O? See the hint for Problem 32.
34. What is the total energy (in MeV) released in the beta decay of a neutron?

Section 42.7 Biological Applications of Nuclear Physics

35. A 50 kg laboratory worker is exposed to 20 mJ of beta radiation with RBE = 1.5. What is the dose in mrem?
36. How many rad of gamma-ray photons cause the same biological damage as 30 rad of alpha radiation with an RBE of 15?
37. 150 rad of gamma radiation are directed into a 150 g tumor during radiation therapy. How much energy does the tumor absorb?
38. The doctors planning a radiation therapy treatment have determined that a 100 g tumor needs to receive 0.20 J of gamma radiation. What is the dose in rads?

Problems

39. a. What initial speed must an alpha particle have to just touch the surface of a ^{197}Au gold nucleus before being turned back?
    b. What is the initial energy (in MeV) of the alpha particle?
    Hint: The alpha particle is not a point particle.
40. Particle accelerators fire protons at target nuclei for investigators to study the nuclear reactions that occur. In one experiment, the proton needs to have 20 MeV of kinetic energy as it impacts a ^{207}Pb nucleus. With what initial kinetic energy (in MeV) must the proton be fired toward the lead target?
    Hint: The proton is not a point particle.
41. Stars are powered by nuclear reactions that fuse hydrogen into helium. The fate of many stars, once most of the hydrogen is used up, is to collapse, under gravitational pull, into a *neutron star.* The force of gravity becomes so large that protons and electrons are fused into neutrons in the reaction $p^+ + e^- \rightarrow n + \nu$. The entire star is then a tightly packed ball of neutrons with the density of nuclear matter.
    a. Suppose the sun collapses into a neutron star. What will its radius be? Give your answer in km.
    b. The sun's rotation period is now 27 days. What will its rotation period be after it collapses?

    Rapidly rotating neutron stars emit pulses of radio waves at the rotation frequency and are known as *pulsars.*
42. The chemical atomic mass of hydrogen, with the two stable isotopes ^{1}H and ^{2}H (deuterium), is 1.00798. Use this value to determine the natural abundance of these two isotopes.
43. You learned in Chapter 41 that the binding energy of the electron in a hydrogen atom is 13.6 eV.
    a. By how much does the mass decrease when a hydrogen atom is formed from a proton and an electron? Give your answer both in atomic mass units and as a percentage of the mass of the hydrogen atom.
    b. By how much does the mass decrease when a helium nucleus is formed from two protons and two neutrons? Give your answer both in atomic mass units and as a percentage of the mass of the helium nucleus.
    c. Compare your answers to parts a and b. Why do you hear it said that mass is "lost" in nuclear reactions but not in chemical reactions?
44. Use the graph of binding energy to estimate the total energy released if a nucleus with mass number 240 fissions into two nuclei with mass number 120.
45. Use the graph of binding energy to estimate the total energy released if three ^{4}He nuclei fuse together to form a ^{12}C nucleus.
46. Could a ^{56}Fe nucleus fission into two ^{28}Al nuclei? Your answer, which should include some calculations, should be based on the curve of binding energy.
47. a. What are the isotopic symbols of all $A = 17$ isobars?
    b. Which of these are stable nuclei?
    c. For those that are not stable, identify both the decay mode and the daughter nucleus.
48. a. What are the isotopic symbols of all $A = 19$ isobars?
    b. Which of these are stable nuclei?
    c. For those that are not stable, identify both the decay mode and the daughter nucleus.
49. Derive Equation 42.19 from Equation 42.15.
50. What energy (in MeV) alpha particle has a de Broglie wavelength equal to the diameter of a ^{238}U nucleus?
51. What is the activity in Bq and in Ci of a 2.0 mg sample of ^{3}H?
52. The activity of an ^{39}Ar sample is 1.25×10^{9} Bq. What is the mass of the sample?

53. The activity of a sample of the cesium isotope ^{137}Cs is 2.0×10^8 Bq. Many years later, after the sample has fully decayed, how many beta particles will have been emitted?
54. A 115 mCi radioactive tracer is made in a nuclear reactor. When it is delivered to a hospital 16 hours later its activity is 95 mCi. The lowest usable level of activity is 10 mCi.
    a. What is the tracer's half-life?
    b. For how long after delivery is the sample usable?
55. The radium isotope ^{223}Ra, an alpha emitter, has a half-life of 11.43 days. You happen to have a 1.0 g cube of ^{223}Ra, so you decide to use it to boil water for tea. You fill a well-insulated container with 100 mL of water at 18° C and drop in the cube of radium.
    a. How long will it take the water to boil?
    b. Will the water have been altered in any way by this method of boiling? If so, how?
56. A sample of 1.0×10^{10} atoms that decay by alpha emission has a half-life of 100 min. How many alpha particles are emitted between $t = 50$ min and $t = 200$ min?
57. A sample contains radioactive atoms of two types, A and B. Initially there are five times as many A atoms as there are B atoms. Two hours later, the numbers of the two atoms are equal. The half-life of A is 0.50 hours. What is the half-life of B?
58. Radioactive isotopes often occur together in mixtures. Suppose a 100 g sample contains ^{131}Ba, with a half-life of 12 days, and ^{47}Ca, with a half-life of 4.5 days. If there are initially twice as many calcium atoms as there are barium atoms, what will be the ratio of calcium atoms to barium atoms 2.5 weeks later?
59. The technique known as potassium-argon dating is used to date old lava flows. The potassium isotope ^{40}K has a 1.28 billion year half-life and is naturally present at very low levels. ^{40}K decays by beta emission into ^{40}Ar. Argon is a gas, and there is no argon in flowing lava because the gas escapes. Once the lava solidifies, any argon produced in the decay of ^{40}K is trapped inside and cannot escape. A geologist brings you a piece of solidified lava in which you find the ^{40}Ar/^{40}K ratio to be 0.12. What is the age of the rock?
60. The half-life of the uranium isotope ^{235}U is 700 million years. The earth is approximately 4.5 billion years old. How much more ^{235}U was there when the earth formed than there is today? Give your answer as the then-to-now ratio.
61. A 75 kg patient swallows a 30 μCi beta emitter that is to be used as a tracer. The isotope's half-life is 5.0 days. The average energy of the beta particles is 0.35 MeV with an RBE of 1.5. Ninety percent of the beta particles are absorbed within the patient's body and 10% escape. What total dose (in mrem) does the patient receive?
62. What dose in rads of gamma radiation must be absorbed by a block of ice at 0°C to transform the entire block to liquid water at 0°C?
63. A chest x ray uses 10 keV photons with an RBE of 0.85. A 60 kg person receives a 30 mrem dose from one x ray that exposes 25% of the patient's body. How many x ray photons are absorbed in the patient's body?
64. The rate at which a radioactive tracer is lost from a patient's body is the rate at which the isotope decays *plus* the rate at which the element is excreted from the body. Medical experiments have shown that stable isotopes of a particular element are excreted with a 6.0 day half-life. A radioactive isotope of the same element has a half-life of 9.0 days. What is the effective half-life of the isotope in a patient's body?
65. The plutonium isotope ^{239}Pu has a half-life of 24,000 years and decays by the emission of a 5.2 MeV alpha particle. Plutonium is not especially dangerous if handled because the activity is low and the alpha radiation doesn't penetrate the skin. However, there are serious health concerns if even the tiniest speck of plutonium is inhaled and lodges deep in the lungs. This could happen following any kind of fire or explosion that disperses plutonium as dust. Let's determine the level of danger.
    a. Soot particles are roughly 1 μm in diameter, and it is known that these particles can go deep into the lungs. How many atoms are in a 1.0-μm-diameter particle of ^{239}Pu? The density of plutonium is 19,800 kg/m^3.
    b. What is the activity, in Bq, of a 1.0-μm-diameter particle?
    c. The activity of the particle is very small, but the penetrating power of alpha particles is also very small. The alpha particles are all stopped, and each deposits its energy in a 50-μm-diameter sphere around the particle. What is the dose, in rem/year, to this small sphere of tissue in the lungs? Use an average RBE of 15 and assume that the tissue density is that of water.
    d. Is this exposure likely to be significant? How does it compare to the natural background of radiation exposure?

Challenge Problems

66. The uranium isotope ^{238}U is naturally present at low levels in many soils. One of the nuclei in the decay series of ^{238}U is the radon isotope ^{222}Rn, which decays by emitting a 5.50 MeV alpha particle with $t_{1/2} = 3.82$ days. Radon is a gas, and it tends to seep from soil into basements. The Environmental Protection Agency recommends that homeowners take steps to remove radon, by pumping in fresh air, if the radon activity exceeds 4 pCi per liter of air.
    a. How many ^{222}Rn atoms are there in 1 m^3 of air if the activity is 4 pCi/L?
    b. The range of alpha particles in air is ≈3 cm. Suppose we model a person as a 180-cm-tall, 25-cm-diameter cylinder with a mass of 65 kg. Only decays within 3 cm of the cylinder can cause exposure, and only ≈50% of the decays direct the alpha particle toward the person. Determine the dose in mrem per year for a person who spends the entire year in a room where the activity is 4 pCi/L. Assume an average RBE of 15.
    c. Does the EPA recommendation seem appropriate? Why or why not?
67. Estimate the stopping distance in air of a 5.0 MeV alpha particle. Assume that the particle loses an average 30 eV per collision.
68. Beta-plus decay is $^A\text{X}_Z \rightarrow {}^A\text{Y}_{Z-1} + \text{e}^+ + \nu$.
    a. Determine the mass threshold for beta-plus decay. That is, what is the minimum atomic mass m_X for which this decay is energetically possible? Your answer will be in terms of the atomic mass m_Y and the electron mass m_e.
    Hint: Start with the nuclear masses, then add an equal number of electrons to both sides of the reaction to get atomic masses.
    b. Can ^{13}N undergo beta-plus decay into ^{13}C? If so, how much energy is released in the decay?

69. All the very heavy atoms found in the earth were created long ago by nuclear fusion reactions in a supernova, an exploding star. The debris spewed out by the supernova later coalesced into the gases from which the sun and the planets of our solar system were formed. Nuclear physics suggests that the uranium isotopes ^{235}U and ^{238}U should have been created in roughly equal numbers. Today, 99.28% of uranium is ^{238}U and only 0.72% is ^{235}U. How long ago did the supernova occur?
70. It might seem strange that in beta decay the positive proton, which is repelled by the positive nucleus, remains in the nucleus while the negative electron, which is attracted to the nucleus, is ejected. To understand beta decay, let's analyze the decay of a free neutron that is at rest in the laboratory. We'll ignore the antineutrino and consider the decay $n \rightarrow p^+ + e^-$. The analysis requires the use of relativistic energy and momentum, from Chapter 36.
    a. What is the total kinetic energy, in MeV, of the proton and electron?
    b. Write the equation that expresses the conservation of relativistic energy for this decay. Your equation will be in terms of the three masses m_n, m_p, and m_e and the relativistic factors γ_p and γ_e.
    c. Write the equation that expresses the conservation of relativistic momentum for this decay. Let v represent speed, rather than velocity, then write any minus signs explicitly.
    d. You have two simultaneous equations in the two unknowns v_p and v_e. To help in solving these, first prove that $\gamma v = (\gamma^2 - 1)^{1/2}c$.
    e. Solve for v_p and v_e. (It's easiest to solve for γ_p and γ_e, then find v from γ.) First get an algebraic expression for each, in terms of the masses. Then evaluate each, giving v as a fraction of c.
    f. Calculate the kinetic energy in MeV of the proton and the electron. Verify that their sum matches your answer to part a.
    g. Now explain why the electron is ejected in beta decay while the proton remains in the nucleus.
71. Alpha decay occurs when an alpha particle tunnels through the Coulomb barrier. Figure CP42.71 shows a simple one-dimensional model of the potential-energy well of an alpha particle in a nucleus with $A \approx 235$. The 15 fm width of this one-dimensional potential-energy well is the *diameter* of the nucleus. Further, to keep the model simple, the Coulomb barrier has been modeled as a 20-fm-wide, 30-MeV-high rectangular potential-energy barrier. The goal of this problem is to calculate the half-life of an alpha particle in the energy level $E = 5.0$ MeV.
    a. What is the kinetic energy of the alpha particle while inside the nucleus? What is its kinetic energy after it escapes from the nucleus?
    b. Consider the alpha particle within the nucleus to be a point particle bouncing back and forth with the kinetic energy you stated in part a. What is the particle's *collision rate*, the number of times per second it collides with the wall of the potential-energy barrier?
    c. What is the tunneling probability P_{tunnel}?
    d. P_{tunnel} is the probability that on any one collision with the wall the alpha particle tunnels through instead of reflecting. The probability of *not* tunneling is $1 - P_{tunnel}$. Hence the probability that the alpha particle is still inside the nucleus after N collisions is $(1 - P_{tunnel})^N \approx 1 - NP_{tunnel}$, where we've used the binomial approximation because $P_{tunnel} \ll 1$. The half-life is the *time* at which half the nuclei have not yet decayed. Use this information to determine (in years) the half-life of this nucleus.

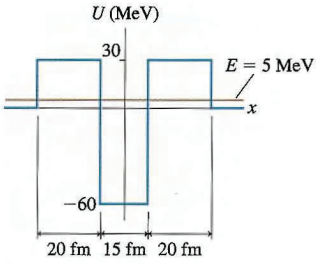


FIGURE CP42.71

STOP TO THINK ANSWERS

Stop to Think 42.1: 3. Different isotopes of an element have different numbers of neutrons but the same number of protons. The number of electrons in a neutral atom matches the number of protons.

Stop to Think 42.2: d. To keep A constant, increasing N by 1 requires decreasing Z by 1.

Stop to Think 42.3: No. A Geiger counter responds only to ionizing radiation. Visible light is not ionizing radiation.

Stop to Think 42.4: c. One-quarter of the atoms are left. This is one-half of one-half, or $(1/2)^2$.

Stop to Think 42.5: b. An increase of Z with no change in A occurs when a neutron changes to a proton and an electron, ejecting the electron.