

UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

AVIDBOTS CORP. and AVIDBOTS USA CORP.,
Petitioners

v.
BRAIN CORPORATION,
Patent Owner.

Case No. IPR2025-01600
U.S. Patent No. 10,001,780

**DECLARATION OF DR. MARY K. BOLIN
IN SUPPORT OF PETITION FOR *INTER PARTES* REVIEW
OF U.S. PATENT NO. 10,001,780**

I, Mary K. Bolin, declare as follows:

1. I am Emeritus Professor at the University of Nebraska, Lincoln Libraries. I have held the following positions: Professor and Catalog and Metadata Librarian, Digital Initiatives and Special Collections Department, University of Nebraska—Lincoln Libraries (2015–2020); Professor and Chair, Discovery and Resource Management and Coordinator of Collection Development, University of Nebraska, Lincoln Libraries (2004–2015); San Jose State University School of Information: Lecturer (2008–present); Head of the Technical Services Department, University of Idaho Library (1993–2003; promoted to Professor in 1999); Head of the Cataloging Department, University of Idaho Library (1986–1993; promoted to Associate Professor and tenured in 1991); and Head of Records Maintenance Section, Cataloging Department, University of Georgia Libraries (1981–1986).

2. Over the course of my work as a librarian, professor, researcher, student research supervisor, and author of numerous scholarly papers and other publications, I have had extensive experience with cataloging records and online library management systems built around Machine-Readable Cataloging (MARC) standards. I also have substantial experience in authenticating printed documents and establishing the date when they were accessible to ordinarily skilled researchers.

3. Attachment A is my full resume.

4. I have been retained by counsel for Avidbots Corp. and Avidbots USA Corp. (“Petitioners”) to evaluate, authenticate, and establish the date of public availability of documents relevant to the above-captioned Petition for Inter Partes Review. For my time performing this service, I am being paid my usual hourly fee. I have no financial interest in this action, or affiliation with Petitioner. My compensation is not dependent upon the outcome of, or my testimony in, this proceeding or any other proceeding, including before the U.S. Patent Office.

A. Documents of Interest

5. The documents that are the subject of this Declaration are the following:

- a) **Document 1.** Castellanos, J. A., Montiel, J. M., Neira, J., & Tardós, J. D., *The SPmap: A probabilistic framework for simultaneous localization and map building*, IEEE Transactions on Robotics and Automation 15(5), 948-952 (2002).
- b) **Document 2.** Khatib, O., *Real-time obstacle avoidance for manipulators and mobile robots*, The International Journal of Robotics Research 5(1), 90-98, (1986).
- c) **Document 3.** Chang, Ya-Chun, Yamamoto, Yoshio, *On-line path planning strategy integrated with collision and deadlock avoidance schemes for wheeled mobile robot in indoor*

environments, *Industrial Robot, An International Journal* 35(5)
421-34 (2008).

B. Methodology

6. I searched for these documents in the OCLC database, Google Scholar, and several subscription databases such as IEEE Xplore, SpringerLink, Emerald, and ACM Digital Library. I also searched the library catalogs of institutions such as the University of Nebraska—Lincoln and other large research libraries. My goal was to find a record in one of these databases that contained a date indicating when these documents were available to the public.

7. *Use of MARC Records.* The “Machine Readable Cataloging” standard, also known as “MARC,” is an industry-wide standard method of organizing library catalog information. The Library of Congress first developed MARC in the 1960s. A MARC-compatible library is one that has a catalog consisting of individual MARC records for each of its works. Today, MARC is the primary communication protocol for the transfer and storage of bibliographic metadata in libraries.

8. OCLC is a database of MARC records used by libraries worldwide. When an OCLC-participating institution acquires a work, it creates a MARC record for this work in its computer catalog system, and OCLC automatically supplies the date of creation. The MARC record creation date reflects the date on which (or shortly after) the item was first acquired and cataloged. Once the MARC record is

created by a cataloger at an OCLC-participating member institution, it is instantaneously available to any OCLC-participating members and, therefore, available to the public.

9. A MARC record comprises several fields, each of which contains specific data about the work. Each field is identified by a standardized, unique, three-digit code corresponding to the type of data that follows. For example, a work's title is recorded in field 245, the primary author of the work is recorded in field 100, an item's International Standard Book Number ("ISBN") is recorded in field 020, an item's Library of Congress call number is recorded in field 050, and the publication date is recorded in field 260 or 264 under the subfield "c." If a work is a periodical, then its publication frequency is recorded in field 310, and the publication dates (e.g., the first and last publication) are recorded in field 362, which is also referred to as the enumeration/chronology field. The library that created the record is recorded in field 040 in subfield "a" with a unique library code. When viewing the MARC record online via OCLC's Worldshare Record Manager database, hovering over this code with the mouse reveals the name of the library. I used this method of "mousing over" the library codes in the OCLC database to identify the originating library for all of the MARC records in this Declaration, as well as the "display holdings" command to see a list of libraries that own the item. In the MARC records in the OCLC

Worldshare Record Manager database, MARC field 040 contains the OCLC symbols of institutions transcribing, creating, and modifying the record.

10. OCLC was created “to establish, maintain and operate a computerized library network and to promote the evolution of library use, of libraries themselves, and of librarianship, and to provide processes and products for the benefit of library users and libraries, including such objectives as increasing availability of library resources to individual library patrons and reducing the rate of rise of library per-unit costs, all for the fundamental public purpose of furthering ease of access to and use of the ever-expanding body of worldwide scientific, literary and educational knowledge and information.”¹ Among other services, OCLC and its members are responsible for maintaining the WorldCat database (<http://www.worldcat.org/>), which is used by independent and institutional libraries throughout the world. OCLC is the international cooperative to which libraries catalog records and others pay to access MARC records. The MARC records referenced in this Declaration, extracted from the OCLC database, were generated in the ordinary course of business. Further, I personally located and downloaded each of these records associated with the document on which this Declaration focuses.

¹ Third Article, Amended Articles of Incorporation of OCLC Online Computer Library Center, Incorporated (available at <http://www.oclc.org/en-US/membership/councils.html>).

11. OCLC provides its members online access to MARC records through its OCLC Worldshare Record Manager database. When an OCLC member institution acquires a work, it creates a MARC record for this work in its computer catalog system in the ordinary course of its business. MARC records created at the Library of Congress are batch-loaded into the OCLC database through a subscription to MARC Distribution Services daily or weekly. Once the MARC record is created by a cataloger at an OCLC member institution or is tape-loaded from the Library of Congress, it is then made available to any other OCLC members online, and, therefore, made available to the public.

12. When an OCLC member institution creates a new record, OCLC automatically supplies the date of creation for that record. The MARC record creation date reflects the date on which, or shortly after which, the item was first acquired or cataloged.

13. The date of creation for the MARC record appears in the fixed field (008), characters 00 through 05. Field 005 is a 16 digit field that records the date characters yyyyymmdd including 4 for the year, 2 for the month, and 2 for the day. The date entered on file never changes. These dates are automatically supplied by the OCLC system and cannot be changed by the user. When the MARC record is created in OCLC, it is available throughout the world to other libraries and in WorldCat, the public version of OCLC Connexion. Thus, the date of creation of the

MARC record by a cataloger at an OCLC member institution reflects when the underlying item is accessible to the public.

14. MARC records also include several fields that include subject matter classification information. An overview of MARC record fields is available through the Library of Congress.² For example, 6XX fields are termed “Subject Access Fields.”³ Among these, for example, is the 650 field, which is the “Subject Added Entry – Topical Term” field.⁴ The 650 field is a “[s]ubject added entry in which the entry element is a topical term.” *Id.* The 650 field entries “are assigned to a bibliographic record to provide access according to generally accepted thesaurus-building rules (e.g., Library of Congress Subject Headings (LCSH), Medical Subject Headings (MeSH)).” *Id.* Thus, a researcher can easily discover material relevant a topic of interest with a search using the terms employed in the MARC Fields 6XX.

15. For Document 2, I performed a search of OCLC (www.oclc.org), which is a large shared database used by libraries around the world to create and exchange cataloging data, using MARC format, which is a universal communications format for that data.

16. I am also familiar with many online catalogs, databases, and search engines. In preparing for this Declaration, I used authoritative information systems

² See <http://www.loc.gov/marc/bibliographic/>.

³ See <http://www.loc.gov/marc/bibliographic/bd6xx.html>.

⁴ See <http://www.loc.gov/marc/bibliographic/bd650.html>.

such as the above-mentioned WorldCat (<https://www.worldcat.org>), the online catalog of the Library of Congress (<https://catalog.loc.gov>), shared research databases such as ResearchGate and CiteSeerX (formerly known as CiteSeer), the online catalog of a number of large research libraries such as the University of Nebraska—Lincoln, IEEEExplore, SpringerLink, Emerald Insights, Sage Journals, and Google Scholar search for records. Where relevant these records are identified and discussed in this Declaration.

17. The IEEE Xplore⁵ research database is a vital resource for undergraduate and graduate students, researchers, scientists, and the public, offering access to high-quality, peer-reviewed publications. IEEE Xplore serves as a repository for IEEE's ebooks, standards, conference proceedings, technical reports, and journal articles, with content dating back to 1988 and selectively covering materials from 1913 to 1987. The database currently includes over 5.5 million full-text documents and is recognized for its rigorous curation and accurate bibliographic records. IEEE Xplore ensures precise reporting of publication dates and event locations in alignment with industry standards for major scholarly and technical journals. Approximately 20,000 new documents are added each month. While

⁵ See <https://ieeexplore.ieee.org/Xplore/home.jsp>.

abstracts are freely available, full-text access requires a subscription or institutional login. The IEEE Xplore database has been online since May in the year 2000.⁶

18. SpringerLink⁷ is a similar research database that is commonly used by students, researchers, scientists, and the public to access high-quality, peer-reviewed publications. SpringerLink serves as an online repository of ebooks, conference papers, and journal articles, including content dating back decades. SpringerLink ensures precise reporting of publication dates and event locations in alignment with industry standards for major scholarly and technical journals. While abstracts are freely available, full-text access requires a subscription or institutional login. The SpringerLink database has been online since the year 1996.

19. Emerald Insights⁸ is a similar research database that is commonly used by students, researchers, scientists, and the public to access high-quality, peer-reviewed publications. Emerald Insights serves as an online repository of ebooks, case studies, and journal articles, including content dating back decades. Emerald Insights ensures precise reporting of publication dates and event locations in alignment with industry standards for major scholarly and technical journals. While abstracts are freely available, full-text access requires a subscription, purchase, or

⁶ See <https://innovate.ieee.org/ieee-xplore-reaches-5-million-documents/>.

⁷ See <https://link.springer.com/>.

⁸ See <https://www.emerald.com/>.

institutional login. The Emerald Insights database has been online since the year 2000.⁹

20. Sage Journals¹⁰ is a similar research database that is commonly used by students, researchers, scientists, and the public to access high-quality, peer-reviewed publications. Sage Journals serves as an online repository of research articles, case reports, scholarly commentary, meeting reports, and other similar content, including content dating back decades. Sage Journals ensures precise reporting of publication dates and event locations in alignment with industry standards for major scholarly and technical journals. While abstracts are freely available, full-text access to some material requires a subscription, purchase, or institutional login. The Sage Journals database has been online since the year 1998.¹¹

21. I am familiar with the Internet Archive, a digital library formally certified by the State of California as a public library. Among other services that the Internet Archive makes available to the general public, is the Wayback Machine, an online archive. The Internet Archive's Wayback Machine service archives webpages as of a certain capture date to track changes in the web over time. The Internet Archive has been in operation as a nonprofit library since 1996 and has hosted the

⁹ See <https://www.emeraldgrouppublishing.com/who-we-are/the-emerald-timeline#:~:text=1967,2008>.

¹⁰ See <https://journals.sagepub.com/>.

¹¹ See <https://www.sagepub.com/about/sage-history-timeline#time-1990>.

Wayback Machine service since its inception in 2001. I consider the Internet Archive's records keeping to be as rigorous and detailed as other formal library records keeping practices such as MARC records and OCLC records. That said, the inability to find a particular webpage on the internet archive is not dispositive or telling of a document's historic availability, because the Internet Archive does not archive all web content all of the time, and for some paywalled databases the Internet Archive cannot access particular webpages.

C. Conclusions Regarding Document 1

22. The following are my conclusions with respect to the authenticity and public availability of Document 1—Castellanos, J. A., Montiel, J. M., Neira, J., & Tardós, J. D., *The SPmap: A probabilistic framework for simultaneous localization and map building*, IEEE Transactions on Robotics and Automation 15(5), 948-952 (2002) (“Castellanos”).

1. Authentication of Document 1

23. Attachment B is true and correct copy of Document 1, which was downloaded from IEEE Xplore. Attachment B is identical to Exhibit 1007 to the Petition for Inter Partes Review.

2. Public Availability of Document 1

24. Attachment B is a PDF of Document 1, downloaded from IEEE Xplore, with a publication date of October 1999, where it was originally published in volume 15(5) of the journal “IEEE Transactions on Robotics and Automation.”

25. Attachment C is a PDF of the search results for Document 1 from the IEEE Xplore database, showing a publication date of October 31, 1999.

26. Attachment D is a declaration from Gordon MacPherson, who is the Director of Board Governance & Policy Development at the IEEE. *Id.* ¶ 1. Mr. MacPherson's declaration confirms that Document 1 was published in the IEEE Xplore digital database as of October 31, 1999.

27. Based on the fact that Document 1 was published in the IEEE Xplore digital database on October 21, 1999, and the IEEE Xplore database was put online a few months later in May 2000, Document 1 was available online as of May 2000, and could have been found by a skilled researcher.

3. *Locating Document 1*

28. In my professional experience as a librarian, academic libraries had access to online periodical databases by 1998.

29. For example, Document 1 could have been found by searching the IEEE Xplore database for any words in the article title, journal title, authors' names and subject terms. This includes terms such as "localization," "map building," and "robotics and automation."

D. Conclusions Regarding Document 2

30. The following are my conclusions with respect to the authenticity and public availability of Document 2—Khatib, O., *Real-time obstacle avoidance for*

manipulators and mobile robots, The international journal of robotics research 5(1), 90-98, (1986).

1. *Authentication of Document 2*

31. Attachment E is true and correct copy of Document 2, which was downloaded from Sage Journals. Attachment E is identical to Exhibit 1006 to the Petition for Inter Partes Review.

2. *Public Availability of Document 2*

32. Attachment E shows a publication in the International Journal of Robotics Research, Volume 5(1) in Spring of 1986. Further, Attachment F shows the availability of Document 2 on Sage Journal's online database.

33. Further, as shown in Attachment G, the PDF of Attachment E (i.e., the version added to Sage Journals) was created on October 31, 2006. In my experience, this indicates that Document 2 was added to Sage Journals on October 31, 2006, or within a day or two of the PDF's creation. It is thus my opinion that Document 2 was available online on Sage Journals on or around October 31, 2006.

34. Document 2 was also published as a chapter in the book *Autonomous Robot Vehicles*, which was published in 1990. Attachment H is a version of Document 2 as published in that book, which I have pulled from a digital copy of the book *Autonomous Robot Vehicles*. I have confirmed that the version of Document 2 published in *Autonomous Robot Vehicles* (that is, Attachment H) is

identical but for pagination to the version published on Sage Journals (that is, Attachment E).

35. Attachment I is a screenshot of a URL on SpringerLink's website from December 21, 2013 accessed via the WayBack Machine's Internet Archive showing the availability of Document 2 (and of the book *Autonomous Robot Vehicles* generally) on SpringerLink's website as of that time. Based on the fact that Document 2 was published on SpringerLink by at least December 21, 2013, Document 2 was available online by *at least* December 2013,¹² and could have been found by a skilled researcher as of that date.

36. As noted above, Document 2 was also published as a chapter in the book *Autonomous Robot Vehicles*, which was published in 1990. Attachment J is MARC record for that book, showing a creation date of January 17, 1990. And Attachment K shows the MARC record for that book from the Library of Congress, showing in the 955 field that this book was added to the collection in January 17, 1990.

37. And even before that date, Document 2 was published in the International Journal of Robotics Research in 1986. *See* Attachment E.

¹² Note that the internet archive does not scan and save all pages of the all websites every day. Thus, it is in my opinion highly likely that Document 2 was on SpringerLink long before 2013, and it was just not archived until the date shown in Attachment I.

38. In sum, Document 2 has been publicly available for decades. It was published in 1986 in the *International Journal of Robotics Research*, it was then published as a chapter in the book *Autonomous Robot Vehicles* in 1990, it was available on Sage Journals on or around October 31, 2006, and it was added to the SpringerLink database by at least December 21, 2013 (and likely earlier).

3. *Locating Document 2*

39. In my professional experience as a librarian, academic libraries had access to online periodical databases by 1998.

40. It is my opinion that someone searching for Document 2 on SpringerLink or Sage Journals would have found the document using any number of basic keyword searches. For example, Document 2 could have been found by searching the SpringerLink database for any words in the article title, journal title, authors' names and subject terms. These include terms such as "obstacle avoidance," "mobile robots," or "Khatib."

41. Further, the 1986 publication date is for the print version of the journal, which could have been located using the catalog of any library that owned the journal *International Journal of Robotics Research*. And likewise, as of 1990 Document 2 could have been found using the catalog of any library that owned the book *Autonomous Robot Vehicles* as of 1990.

E. Conclusions Regarding Document 3

42. The following are my conclusions with respect to the authenticity and public availability of Document 3—Chang, Ya-Chun, Yamamoto, Yoshio, *On-line path planning strategy integrated with collision and dead-lock avoidance schemes for wheeled mobile robot in indoor environments*, Industrial Robot, An International Journal 35(5) 421-34 (2008).

1. Authentication of Document 3

43. Attachment L is true and correct copy of Document 3, which was downloaded from Emerald Insight's online database. Attachment L is identical to Exhibit 1010 to the Petition for Inter Partes Review.

2. Public Availability of Document 3

44. Attachment M is a screenshot of the webpage on Emerald Insight's online database where Document 3 is available for download. At the top of Attachment M is the date August 22, 2008, which is the publication date of this article. Because this article lists a specific publication date (as opposed to just a month or year), and because it was published in 2008—a time when scientific and academic articles were typically published simultaneously both in print journals and in corresponding online databases—this August 22, 2008 date represents both the date when Document 3 was published in the journal “Industrial Robot, An International Journal,” and the date that Document 3 was published on Emerald Insight's online database.

45. My conclusion is buttressed by several additional indicia confirming simultaneous online and print publication. First, Attachment N is a screenshot showing the document properties for Attachment L. The metadata indicates that this PDF was created on August 21, 2008—the day before the article was published. This date being the day before the listed publication date confirms that the document was created in a web accessible format (i.e., a PDF) the day before it was published, which confirms to me that it was published online on its listed publication of August 22, 2008. Second and relatedly, Attachment L (i.e., Document 3, which PDF was created on August 21, 2008) states that the “the current issue and full text archive of this journal is available at [a particular URL].” *See* Attachment L at 1. Thus, Document 3 itself confirms that it was published simultaneously online and in the print journal.

46. In sum, Document 3 was published both in print on the Emerald Insight online database on August 22, 2008, and was publicly available as of that date.

3. *Locating Document 3*

47. In my professional experience as a librarian, academic libraries had access to online periodical databases by 1998.

48. It is my opinion that someone searching for Document 3 on Emerald Insights online database would have found the document using any number of basic keyword searches. For example, Document 3 could have been found by searching

the Emerald Insight database for any words in the article title, journal title, authors' names and subject terms. These include terms such as "mobile robot," "mobile robot in indoor environment," or "path planning."

I hereby declare that all statements made herein of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements are made with the knowledge that willful false statements and the like are punishable by fine, imprisonment, or both, under Section 1001 of Title 18 of the U.S. Code. I state under penalty of perjury under the laws of the United States of America that the foregoing is true and correct.

Executed this ____ day of October, 2025, in Minneapolis, Minnesota.

A handwritten signature in black ink, appearing to read "Mary K. Bolin", is written over a light gray rectangular background.

Mary K. Bolin

ATTACHMENT A

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Lincoln, Nebraska 68503
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Mary K. Bolin, PhD

Professor Emeritus
University Libraries
University of Nebraska—Lincoln
mbolin2@unl.edu

Lecturer,
School of Information
San José State University
mary.bolin@sjsu.edu

EDUCATION

PhD, Educational Studies (Educational Leadership and Higher Education), 2007, University of Nebraska

Dissertation: *A Typology and Discourse Analysis of the Status and Appointments of Librarians at Land Grant Universities* (available at digitalcommons.unl.edu)

MA, English, 1999, University of Idaho

Thesis: *Grace: A Contrastive Analysis of a Biblical Semantic Field* (available at digitalcommons.unl.edu)

MSLS, 1981, University of Kentucky

BA, Linguistics, 1976, University of Nebraska—Lincoln

WORK EXPERIENCE

Center for Digital Research in the Humanities. Fellow, 2019- Emeritus, 2020-

University of Nebraska--Lincoln Libraries: Professor and Catalog and Metadata Librarian, Digital Initiatives and Special Collections, 2015-2020. Emeritus, 2020-

University of Nebraska—Lincoln Libraries: Professor and Chair, Discovery and Resource Management, 2004-2015; Coordinator of Collection Development, 2012-2015

University of Idaho Library: Head, Technical Services Department, 1993-2003. Promoted to Professor in 1999

University of Idaho Library: Head, Cataloging Department, 1986-1993. Promoted to Associate Professor and tenured, 1991

University of Georgia Libraries: Head, Records Maintenance Section, Cataloging Department, 1981-1986

Teaching:

San José State University School of Information:

LIBR 281, Metadata. 2009-

LIBR 248, Introduction to Cataloging and Classification. 2013-

LIBR 202, Information Retrieval. 2008-2013

Associate Coordinator, Gateway PhD program, San Jose State University, 2021-2022

American Library Association:

Cataloging for Non-Catalogers, Online Continuing Education Course, 2017-

Intermediate Cataloging for Non-Catalogers, Online Continuing Education Course, 2018-

Lyrasis:

Webinar on BibFrame, taught twice a year, 2020-

Quick and Clean Cataloging, Parts 1, 2, and 3. 2022-

Independent Study in Idaho:

Library Science C415, Technical Services for Small Libraries, 2004-2013

Library Science C416, Library Services to Rural Libraries, 2004-2013

Library Science C422, Use of the School Library, 1999-2013

Library Science C425/525, Organization and Management of Small Libraries,
1988-2013

Library Science C427, Library or Media Center Practicum, 1995-2013

Gateway PhD Program (San Jose State University and Queensland University of Technology).
SJSU supervisor for two PhD students, 2015-

Gateway PhD Program (San Jose State University and Manchester Metropolitan University).
Associate Coordinator, 2021-2022

Consulting

Expert witness for multiple attorneys employed by Wolf, Greenfield & Sacks, P.C.,
Hughes Hubbard & Reed LLP, and other firms. 2017-

Editing

Language and academic editing for [EditSprings](#), a service for international scholars, 2020-

SCHOLARSHIP

Refereed Publications:

(Publications available from digitalcommons.unl.edu)

Pytlík Zillig, Brian, and Mary K. Bolin. (Forthcoming 2024). “Visualization of Semantic Primes
in Texts from Early Modern English Drama.” *Journal of the Japanese Association for the Digital
Humanities*.

Bolin, Mary K. (2022). “Academic Libraries and Writing Centers: Collaborations at US Public
Research Universities.” *School of Information Student Research Journal*, 11(2). Retrieved from
<https://scholarworks.sjsu.edu/ischoolsrj/vol11/iss2/2>

Bolin, Mary K. (2022). *Refocusing Academic Libraries through Learning and Discourse: The
Idea of a Library*. Oxford, UK: Chandos/Elsevier.

Bolin, Mary K. (2022). “Metadata, Catalog Systems, Linked Data, and the Evolving ILS.” In:
Sandra Hirsh (Ed.), *Information Services Today: An Introduction*. 3rd ed. New York: Rowman
and Littlefield.

Wintermute, Harriet E., and Bolin, Mary K. (2018). "Metadata Generators for Backlog Reduction: Metadata Maker Streamlines Cataloging and Facilitates Transition. *Technical Services Quarterly* 35(2) doi: <https://doi.org/10.1080/07317131.2018.1422879>

Bolin, Mary K. (2018). "Metadata, Catalog Systems, Linked Data, and the Evolving ILS." In: Sandra Hirsh (Ed.), *Information Services Today: An Introduction*. 2nd ed. New York: Rowman and Littlefield.

Bolin Mary K. (2017). *The 21-century Academic Library: Global Patterns of Organization and Discourse*. Oxford: Chandos Publishing. (Chandos Information Professional Series)

Bolin, Mary K. (2017). Linguistics and LIS: A Research Agenda." *SLIS Student Research Journal*, 7:1.
Retrieved from <http://scholarworks.sjsu.edu/slissrj/vol7/iss1/2>

Bolin, Mary K. (2014). "Open Access Web Resources for Library Continuing Education and Training." In S. Hines (Ed.), *Revolutionizing the Development of Library and Information Professionals: Planning for the Future* (pp. 107- 124). Hershey, PA: .
doi:10.4018/978-1-4666-4675-9.ch008.

Bolin, Mary K. (2014). "The Language of Academic Librarianship: The Discourse of Promotion and Tenure. " *Advances in Library Administration and Organization* 32.

Bolin, Mary K. "Librarian Status at US Research Universities: Extending the Typology." *Journal of Academic Librarianship* 34:4 (September 2008)

Bolin, Mary K. "A Typology of Librarian Status at Land Grant Universities." *Journal of Academic Librarianship*, 34: 3 (May 2008)220:230.

Bolin, Mary K. "The Library and the Computer Center: Organizational Patterns at Land Grant Universities." *Journal of Academic Librarianship*, 31:1 (2005)

Eckwright, Gail Z. and Mary K. Bolin. "The Hybrid Librarian: The Affinity of Collection Management with Technical Services and the Organizational Benefits of an Individualized Assignment." *Journal of Academic Librarianship*, 27:6 (2001)452:457.

Bolin, Mary K. "Catalog Design, Catalog Maintenance, Catalog Governance," *Library Collections, Acquisitions, and Technical Services*. 24: (2000)53-63.

Bolin, Mary K. "The Collegial Environment and the Functional Organization," *Journal of Library Administration*. 29:2(1999):49-61.

Bolin, Mary K. "Cataloging Environmental Impact Statements and Related Documents," *Resource Sharing and Library Networks*. 14:1(1999): 21-40.

Bolin, Mary K. "Waiting for the Electrician or Someone like Him: Client-Centered Technical Services" Proceedings of the Association of College and Research Libraries National Conference, Nashville, April 1997.

< http://www.ala.org/Content/ContentGroups/ACRL1/Nashville_1997_Papers/Bolin.htm >

Grants:

Electronic Theses and Dissertations. Seed Grant from University of Idaho Office of Research, 2002. Co PI with Gail Z. Eckwright. \$6500.

Interoperability of Metadata for Thematic Research Collections. Grant to the Center for Digital Research in the Humanities from the Institute of Museum and Library Services 2005-2008.

Project Team Member.

http://unlcms.unl.edu/cas/center-for-digital-research-in-the-humanities/projects/pages/iom_files/OM_Final_Narrative_Report_July_2008.pdf

Other Publications:

Library Philosophy & Practice. Peer-reviewed electronic library science journal. Mary K. Bolin and Gail Z. Eckwright, founders and editors. <http://digitalcommons.unl.edu/libphilprac>

Library Science C415, Technical Services for Small Libraries, through Independent Study in Idaho. 2004.

Library Science C416, Library Services to Rural Libraries, through Independent Study in Idaho. 2004.

Library Science C422, Use of the School Library, through Independent Study in Idaho. Revised 1999.

Library Science C425/525, Organization and Management of Small Libraries through Independent Study in Idaho. Revised 2005.

Library Science C427, Library or Media Center Practicum, Independent Study in Idaho. Revised 1999.

Presentations:

Bolin, Mary K. "BibFrame and the MARC Cataloging Environment: an Introduction." Lyrisis Webinar, June 16, 2020, August 17, 2021.. Recording available <http://lyrisis.adobeconnect.com/paufo9e0dm30/>

Pytlik Zillig, Brian, and Mary K. Bolin. "A Little More than Kin, a Little Less than Kind": the Semantic Field Grace in Early Modern English" Poster Session, Text Encoding Initiative Meeting, Graz, Austria, September 18, 2019

Pytlik Zillig, Brian, and Mary K. Bolin. "Shakespeare and the Enumeration of Semantic Primes." Research Paper Presentation, Text Encoding Initiative Meeting, Tokyo, Japan, September 13, 2018

"Library Philosophy and Practice: Creating and Maintaining an Open-Access E-Journal," Presentation to UNCL meeting (libraries at campuses of the University of Nebraska), Lincoln, Nebraska, March 20, 2008.

Presentation to "Thinking outside the Borders" participants on the open-access electronic journal Library Philosophy and Practice and its role in the international library community. University of Nebraska—Lincoln Libraries, October 24, 2007

Panel discussion on open access electronic journals. GWLA directors' meeting. Linda K. Hall Library, Kansas City, Missouri. April 2007.

"Waiting for the Electrician or Someone Like Him: Client-Centered Technical Services." Association of College and Research Libraries National Conference, Nashville, April 5, 1997.

" 'Some Problems of an Editor': the Idaho Librarian from Manuscript to Published Issue," Joint Annual Conference of the Idaho Library Association and the Washington Library Association, Spokane, Washington, April 6, 1995.

"Automating Idaho's Libraries: a Historical Perspective." Idaho Library Association Annual Conference, Idaho Falls, October 4, 1990.

"Small Library Management." Idaho Library Association Spring Conference, Lewiston, Idaho, May 13, 1989.

"No More 'Good Workers': Excellence and Academic Library Classified Staff." Association of College and Research Libraries 5th Annual Conference, Cincinnati, April 7, 1989.

"Communication to and from Cataloging Departments." Heads of Cataloging Discussion Group, American Library Association Midwinter Meeting, San Antonio, January 1988.

Discussion Leader at the LAMA (Library Administration and Management) Management Exchange. American Library Association Annual Meeting, San Francisco, June 1987.

"Staff Turnover." Heads of Cataloging Discussion Group, American Library Association Midwinter Meeting, Chicago, January 1987.

SELECTED PROFESSIONAL SERVICE

Reviewer. *Resources for College Libraries*. Linguistics bibliography, Spring 2020.

Peer reviewer for ALISE National Conference presentations, Spring 2020.

Peer reviewer for *Journal of Academic Librarianship*, 2012-

Peer reviewer for refereed journal, *Western Nigerian Journal of Medical Sciences* (<http://www.wnjms.org/>), a publication of Ladoke Akintola University of Technology (LAUTECH) Teaching Hospital, 2008-

PhD Reviewer for students from Aligarh Muslim University, India, 2010-

Outside reviewer for numerous library faculty promotion and tenure cases in US universities.

University of Nebraska--Lincoln

Chair, University Libraries Faculty, 2018

Faculty Senate Academic Rights and Responsibilities Committee, 2016- Chair, 2018-19

Faculty Senate – 2004-2008; 2008-2011

Faculty Senate Executive Committee 2004-2008

Academic Planning Committee (APC), 2005-2008; Vice Chair, 2006-2007; Chair, 2007-2008

Aesthetic Review Committee 2006-2007

APC monitor for academic program reviews: Department of Political Science, 2005;
Department of Educational Psychology 2006
Pacific Northwest Library Association:

Editor, *PNLA Quarterly*, December 2002-December 2013

Idaho Library Association:

Editor, *The Idaho Librarian*, September 1990-January 1996

ATTACHMENT B

The SPmap: A Probabilistic Framework for Simultaneous Localization and Map Building

Jose A. Castellanos, J. M. M. Montiel, J. Neira, and J. D. Tardós

Abstract—This article describes a rigorous and complete framework for the simultaneous localization and map building problem for mobile robots: the symmetries and perturbations map (SPmap), which is based on a general probabilistic representation of uncertain geometric information. We present a complete experiment with a LabMate™ mobile robot navigating in a human-made indoor environment and equipped with a rotating two-dimensional (2-D) laser rangefinder. Experiments validate the appropriateness of our approach and provide a real measurement of the precision of the algorithms.

Index Terms—Correlations, probabilistic model, simultaneous localization and map building.

I. INTRODUCTION

Successful path planning and navigation of a mobile robot in a human-made indoor environment requires the availability of both a sufficiently reliable estimation of the current vehicle location, and a sufficiently precise map of the navigation area. *A priori* model maps are rarely available, costly to obtain, and when they are available, they usually introduce inaccuracies in the planning tasks. An automatic construction of the map of the environment in which the robot navigates would be desirable, and it has become an important research direction in today's robotics community.

The precision of the constructed map is highly influenced by the accuracy of the dead-reckoning system of the mobile robot, whose location estimations drift with time. An improved solution would require the relocation of the mobile robot along its trajectory to avoid biases introduced by odometry, hence, an approach based on the simultaneous localization and map building would be necessary (Fig. 1). Exact mathematical approaches to the simultaneous localization and map building problem were originally addressed by Smith *et al.* [1], [2] who introduced the concept of *stochastic map*, a representation of spatial relationships, their uncertainties, and their interdependencies with respect to a base reference. Later, this concept was used in the works of Moutarlier *et al.* [3], Leonard *et al.* [4]–[6], and Hébert *et al.* [7]. Recently, Uhlmann *et al.* [8] have reported interesting work related to the problem of correlations in the simultaneous localization and map building problem.

This article proposes a new probabilistic framework adapted to the problem of simultaneous localization and map building: the symmetries and perturbations map (SPmap) [9] which is based on a general representation of uncertain geometric information. Our main contributions are as follows.

- 1) SPmap represents a rigorous and complete solution to the simultaneous localization and map building problem for mobile robots.
- 2) SPmap consistently formulates the representation and integration of sensorial information gathered by different sensors.

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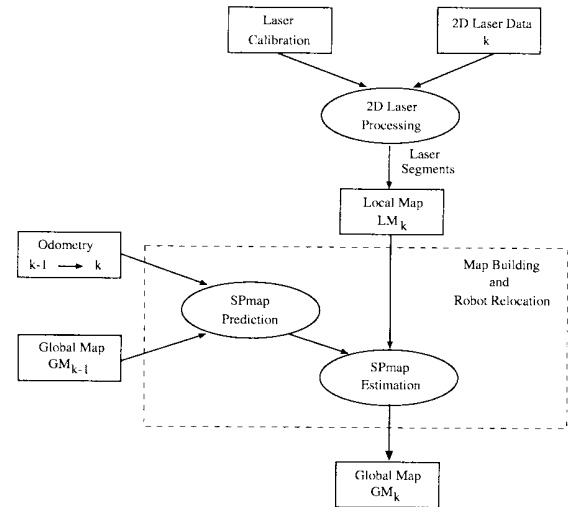


Fig. 1. Simultaneous localization and map building.

- 3) SPmap overcomes the difficulties reported in previous works dealing with singularities in the representation of geometric features.
- 4) SPmap has been experimentally validated by a complete experiment which profited from ground-truth to accurately validate the precision and the appropriateness of the approach.

This article also extends our preliminary results reported in [10] to demonstrate the importance of maintaining the correlations between the estimation of the entities involved in the simultaneous localization and map building problem, to avoid optimistic estimations of uncertainty associated to the precision of the locations of features.

The rest of the article is structured as follows. Section II describes the probabilistic representation of uncertain geometric information. Sections III and IV present our probabilistic framework for the simultaneous localization and map building problem. Experimental results are described in Section V while some conclusions and further work are shown in Section VI.

II. SYMMETRIES AND PERTURBATIONS MODEL

In our feature-based approach, uncertain geometric information is represented using a probabilistic model: the symmetries and perturbation model (SPmodel) [11], [12] which combines the use of probability theory to represent the imprecision in the location of a geometric element, and the theory of symmetries to represent the partiality due to characteristics of each type of geometric element.

In the SPmodel, the location of a geometric element E with respect to a base reference W is given by a *location vector* $\mathbf{x}_{WE} = (x, y, \phi)^T$. The estimation of the location of an element is denoted by $\hat{\mathbf{x}}_{WE}$, and the estimation error is represented locally by a *differential location vector* $\mathbf{d}_E$ relative to the reference attached to the element. Thus, the true location of the element is

$$\mathbf{x}_{WE} = \hat{\mathbf{x}}_{WE} \oplus \mathbf{d}_E \quad (1)$$

where $\oplus$ represents the composition of location vectors. To account for the symmetries of the geometric element, we assign in $\mathbf{d}_E$ a null value to the degrees of freedom corresponding to them, because they do not represent an effective location error. We call *perturbation*

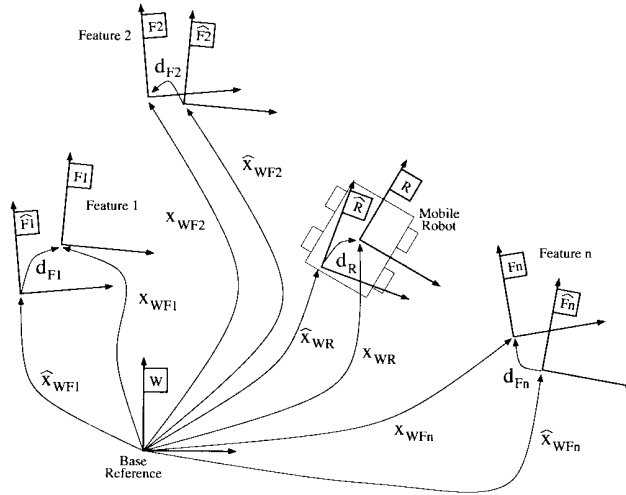


Fig. 2. Representation of the SPmap with generic features and the mobile robot expressed with respect to a base reference W . The location vector of each entity is obtained by the composition of its estimated location vector and its differential location vector.

vector the vector $\mathbf{p}_E$ formed by the non null elements of $\mathbf{d}_E$. Both vectors can be related by a row selection matrix $\mathbf{B}_E$ that we call *self-binding matrix* of the geometric element

$$\mathbf{d}_E = \mathbf{B}_E^T \mathbf{p}_E; \quad \mathbf{p}_E = \mathbf{B}_E \mathbf{d}_E. \quad (2)$$

Then, the *uncertain location* of every geometric entity is represented in the SPmodel by a quadruple $\mathbf{L}_{WE} = (\hat{\mathbf{x}}_{WE}, \hat{\mathbf{p}}_E, \mathbf{C}_E, \mathbf{B}_E)$, where the transformation $\hat{\mathbf{x}}_{WE}$ is an estimation taken as base for perturbations, $\hat{\mathbf{p}}_E$ is the estimated value of the perturbation vector, and $\mathbf{C}_E$ its covariance.

III. SYMMETRIES AND PERTURBATIONS MAP

The symmetries and perturbations map (SPmap) is a complete representation of the environment of the robot which includes the uncertain location of the mobile robot $\mathbf{L}_{WR}$, the uncertain locations of the features obtained from sensor observations $\mathbf{L}_{WF_i}$, $i \in \{1 \dots N_F\}$ and their interdependencies (Fig. 2).

The SPmap can be defined as a quadruple

$$\text{SPmap} = (\hat{\mathbf{x}}^W, \hat{\mathbf{p}}^W, \mathbf{C}^W, \mathbf{B}^W) \quad (3)$$

where $\hat{\mathbf{x}}^W$ is the *estimated location vector* of the SPmap and $\hat{\mathbf{p}}^W$ is the *perturbation vector* of the SPmap

$$\hat{\mathbf{x}}^W = \begin{bmatrix} \hat{\mathbf{x}}_{WR} \\ \hat{\mathbf{x}}_{WF1} \\ \vdots \\ \hat{\mathbf{x}}_{WF_{N_F}} \end{bmatrix}; \quad \hat{\mathbf{p}}^W = \begin{bmatrix} \mathbf{d}_R \\ \mathbf{p}_{F1} \\ \vdots \\ \mathbf{p}_{F_{N_F}} \end{bmatrix}. \quad (4)$$

The true location of the robot and the map features is

$$\mathbf{x}^W = \hat{\mathbf{x}}^W \oplus (\mathbf{B}^W)^T \mathbf{p}^W \quad (5)$$

where the composition operator $\oplus$ applies in this case to each of the components of the vectors, and $\mathbf{B}^W$ is the *binding matrix* of the SPmap, a diagonal matrix formed by the self-binding matrix of the robot and the self-binding matrices of the map features

$$\mathbf{B}^W = \text{diag}(\mathbf{B}_R, \mathbf{B}_{F1}, \dots, \mathbf{B}_{F_{N_F}}). \quad (6)$$

The covariance matrix of the SPmap represents the covariance of the estimation of the robot and the map feature locations, the cross-covariances between the robot and the map features, and finally, the cross-covariances between the map features themselves

$$\mathbf{C}^W = \begin{pmatrix} \mathbf{C}_R & \mathbf{C}_{RF1} & \cdots & \mathbf{C}_{RF_{N_F}} \\ \mathbf{C}_{RF1}^T & \mathbf{C}_{F1} & \cdots & \mathbf{C}_{F1F_{N_F}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_{RF_{N_F}}^T & \mathbf{C}_{F1F_{N_F}} & \cdots & \mathbf{C}_{F_{N_F}} \end{pmatrix}. \quad (7)$$

Note that we may represent any type of geometric entity within this general framework. The proposed approach can include different types of features obtained by different types of sensors, and thus it is suitable to deal with a multisensor system.

IV. INCREMENTAL CONSTRUCTION OF THE SPMAP

This section describes the incremental construction of the SPmap using suboptimal estimation techniques based on the extended Kalman filter [13].

A. Uncertain Displacement of the Mobile Robot

An estimation of the displacement of the mobile robot between two intermediate points along its trajectory can be obtained by dead-reckoning

$$\mathbf{x}_{R_{k-1}R_k} = \hat{\mathbf{x}}_{R_{k-1}R_k} \oplus \mathbf{d}_{R_{k-1}R_k} \quad (8)$$

where $\hat{\mathbf{x}}_{R_{k-1}R_k}$ represents the estimated displacement of the robot and $\mathbf{d}_{R_{k-1}R_k} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{R_{k-1}R_k})$ represents the imprecision in its estimation (i.e., dead-reckoning errors). Other nonrandom systematic errors are not considered because they can be corrected by an appropriate calibration procedure.

Thus, the predicted location $\mathbf{x}_{WR_k}$ of the mobile robot at time k can be calculated by the composition

$$\begin{aligned} \mathbf{x}_{WR_k} &= \mathbf{x}_{WR_{k-1}} \oplus \mathbf{x}_{R_{k-1}R_k} \\ &= \hat{\mathbf{x}}_{WR_{k-1}} \oplus \mathbf{d}_{R_{k-1}} \oplus \hat{\mathbf{x}}_{R_{k-1}R_k} \oplus \mathbf{d}_{R_{k-1}R_k} \\ &= \hat{\mathbf{x}}_{WR_k} \oplus \mathbf{J}_{R_k R_{k-1}} \mathbf{d}_{R_{k-1}} \oplus \mathbf{d}_{R_{k-1}R_k} \end{aligned} \quad (9)$$

where $\mathbf{J}_{R_k R_{k-1}}$ is the Jacobian of the transformation $\hat{\mathbf{x}}_{R_k R_{k-1}}$ between the location vectors of the robot at time k and that at time $k-1$.

After the displacement of the vehicle, only the location of the mobile robot changes as estimated by dead-reckoning, while the location of map features, being static entities, remain the same as the estimated in the previous time instant $k-1$. Nevertheless, the displacement of the mobile robot produces changes in the dependencies existing between the location of the robot and those of the map features. The complexity of this phase grows linearly with the number of features, $\mathcal{O}(N_F)$.

B. Matching Local and Global Maps

The predicted mobile robot location is improved by matching local observations expressed with respect to the robot reference R , with map features represented with respect to the base reference W . At each point of the robot's trajectory it is desirable to obtain as much pairings as possible because they represent the links between new observations and previous stored knowledge of the navigation area.

Fig. 3 exemplifies the pairing between a local observation E obtained at time k and represented with respect to the robot by $\mathbf{L}_{RE} = (\hat{\mathbf{x}}_{RE}, \hat{\mathbf{p}}_E, \mathbf{C}_E, \mathbf{B}_E)$, and a global map feature F available at time $k-1$ and represented with respect to the base reference W

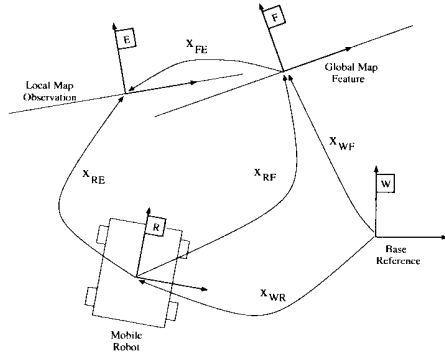


Fig. 3. Matching local observation E with global feature F .

by $\mathbf{L}_{WF} = (\hat{\mathbf{x}}_{WF}, \hat{\mathbf{p}}_F, \mathbf{C}_F, \mathbf{B}_F)$. From their relative location we formulate a nonlinear measurement equation [11]

$$\begin{aligned} \mathbf{f}_{k,m}(\mathbf{d}_R, \mathbf{p}_F, \mathbf{p}_E) &= \mathbf{B}_{FE} \mathbf{x}_{FE} \\ &= \mathbf{B}_{FE} (\ominus \mathbf{x}_{WF} \oplus \mathbf{x}_{WR} \oplus \mathbf{x}_{RE}) \\ &= \mathbf{B}_{FE} (\ominus \mathbf{J}_F^T \mathbf{p}_F \oplus \hat{\mathbf{x}}_{FE} \oplus \mathbf{J}_{ER} \mathbf{d}_R \oplus \mathbf{B}_E^T \mathbf{p}_E) \\ &= \mathbf{0} \end{aligned} \quad (10)$$

where $\mathbf{B}_{FE}$ is the binding matrix of the pairing [11], a row selection matrix which selects the components of $\mathbf{x}_{FE}$ which must be zero. Due to uncertainty, a hypothesis test based on the squared Mahalanobis distance D^2 , validates the compatibility between E and F

$$D^2 = (\mathbf{B}_{FE} \hat{\mathbf{x}}_{FE})^T [\mathbf{B}_{FE} \mathbf{C}(\mathbf{x}_{FE}) \mathbf{B}_{FE}^T]^{-1} (\mathbf{B}_{FE} \hat{\mathbf{x}}_{FE}) \quad (11)$$

where matrix $\mathbf{C}(\mathbf{x}_{FE})$ is computed from the linearization of (10) taking into account the correlations between the map feature F and the robot:

$$\mathbf{C}(\mathbf{x}_{FE}) = \mathbf{J}_{2\oplus} \{\hat{\mathbf{x}}_{FE}, \mathbf{0}\} \mathbf{C}(\mathbf{d}_E^F) \mathbf{J}_{2\oplus}^T \{\hat{\mathbf{x}}_{FE}, \mathbf{0}\} \quad (12)$$

where

$$\begin{aligned} \mathbf{C}(\mathbf{d}_E^F) &= \mathbf{J}_{ER} \mathbf{C}_R \mathbf{J}_{ER}^T + \mathbf{J}_{EF} \mathbf{B}_F^T \mathbf{C}_F \mathbf{B}_F \mathbf{J}_{EF}^T \\ &\quad - \mathbf{J}_{EF} \mathbf{B}_F^T \mathbf{C}_F^T \mathbf{J}_{ER}^T - \mathbf{J}_{ER} \mathbf{C}_R \mathbf{B}_F \mathbf{J}_{EF}^T \\ &\quad + \mathbf{B}_E^T \mathbf{C}_E \mathbf{B}_E. \end{aligned} \quad (13)$$

Under the Gaussianity hypothesis, D^2 follows a χ^2 distribution. For a given significance level α , the local feature E is compatible with the global feature F if $D^2 \leq \chi_{r,\alpha}^2$, with $r = \text{rank}(\mathbf{B}_{FE})$ degrees of freedom, otherwise the matching between the E and F is discarded. In general, when there exist multiple possible pairings for a particular local feature, the pairing with the smallest Mahalanobis distance is chosen.

C. Estimating the SPmap at Time k

The matching between E and F is used both to relocalize the robot at the current trajectory point and, simultaneously, to update the estimated location of the map features. The perturbation vector of the local observation $\mathbf{p}_E \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_E)$ constitutes the measurement used to improve the estimation of $\mathbf{p}_k^W$ through the relation established by

¹Matrices $\mathbf{J}_{1\oplus}$ and $\mathbf{J}_{2\oplus}$ are the Jacobians of the composition of location vectors [1]

$$\begin{aligned} \mathbf{J}_{1\oplus} \{\mathbf{x}_1, \mathbf{x}_2\} &= \frac{\partial(\mathbf{y} \oplus \mathbf{z})}{\partial \mathbf{y}} \bigg|_{\mathbf{y}=\mathbf{x}_1, \mathbf{z}=\mathbf{x}_2} \\ \mathbf{J}_{2\oplus} \{\mathbf{x}_1, \mathbf{x}_2\} &= \frac{\partial(\mathbf{y} \oplus \mathbf{z})}{\partial \mathbf{z}} \bigg|_{\mathbf{y}=\mathbf{x}_1, \mathbf{z}=\mathbf{x}_2}. \end{aligned}$$

(10). Linearization of (10) is done by considering a first order Taylor expansion

$$\begin{aligned} \mathbf{h}_m &= \mathbf{f}_{k,m}(\hat{\mathbf{p}}_k^W, \hat{\mathbf{p}}_E) = \mathbf{B}_{FE} \hat{\mathbf{x}}_{FE} \\ \mathbf{H}_m &= \frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{p}_k^W} \bigg|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{p}}_E)} = (\mathbf{H}_m^R \mathbf{0} \cdots \mathbf{0} \mathbf{H}_m^F \mathbf{0} \cdots \mathbf{0}) \\ \mathbf{H}_m^R &= \frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{d}_R} \bigg|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{p}}_E)} = \mathbf{B}_{FE} \mathbf{J}_{2\oplus} \{\hat{\mathbf{x}}_{FE}, \mathbf{0}\} \mathbf{J}_{ER} \\ \mathbf{H}_m^F &= \frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{p}_F} \bigg|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{p}}_E)} = -\mathbf{B}_{FE} \mathbf{J}_{1\oplus} \{\mathbf{0}, \hat{\mathbf{x}}_{FE}\} \mathbf{B}_F^T \\ \mathbf{G}_m &= \frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{p}_E} \bigg|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{p}}_E)} = \mathbf{B}_{FE} \mathbf{J}_{2\oplus} \{\hat{\mathbf{x}}_{FE}, \mathbf{0}\} \mathbf{B}_E^T. \end{aligned} \quad (14)$$

Integration of local observations LM_k (i.e., new knowledge of the navigation area) into the SPmap known up to the previous time instant GM_{k-1} by using the EKF equations, produces a reestimation of its whole perturbation vector $\mathbf{p}_k^W$ and the uncertainties represented by its covariance matrix $\mathbf{C}_k^W$. Location estimations of the complete set of entities included in the SPmap are reestimated after the integration of new information. Furthermore, correlations between their estimations are also updated. The complexity of this phase grows polynomially with the number of features $\mathcal{O}(N_F^2 M)$, where M is the number of matched observations.

D. Adding Nonmatched Features to the Global Map

Local observations obtained at time k which cannot be paired with any of the global features of GM_{k-1} are interpreted as knowledge about the environment which has not yet been learned. These local observations are added to the SPmap by considering the composition

$$\begin{aligned} \mathbf{x}_{WE} &= \mathbf{x}_{WR} \oplus \mathbf{x}_{RE} \\ &= \hat{\mathbf{x}}_{WE} \oplus \mathbf{J}_{ER} \mathbf{d}_R \oplus \mathbf{B}_E^T \mathbf{p}_E. \end{aligned} \quad (15)$$

The covariance matrix of the SPmap is extended to represent the correlation between the mobile robot and the nonpaired feature E , the cross-correlation between the previous map features and the nonpaired feature E , and the covariance of the nonpaired feature. When the whole local map is composed only of nonpaired observations, the robot's location estimation obtained by odometry cannot be improved.

V. EXPERIMENTING WITH THE SPMAP

This section presents the experimental verification of the previous ideas by considering a LabMateTM mobile robot navigating indoors, and equipped with a two-dimensional (2-D) rotating laser scanner. We also analyze the effects of neglecting correlations between the location estimation of the features. The vehicle was programmed to follow a trajectory (53 m approximately), stopping at regular intervals to take measurements. Complementary information was taken, by hand, with a pair of theodolites which provided real locations of the robot with respect to a base reference.

From the laser readings gathered at each point along the trajectory, a straight-line segment-based local map LM_k was constructed. A segmentation algorithm [14] was used. The resultant 2-D segments (Fig. 4) were expressed with respect to the mobile robot reference frame R by using the SPmodel formulation; also, estimations of their lengths were computed from their endpoints. Typically the number of straight-line segments was kept small for each local map (below ten segments per local map) by considering only meaningful (i.e., long) segments.

Pairings between local observations and previously stored knowledge of the navigation area were obtained by using a 5% significance level. In the following figures, precision of the results has been

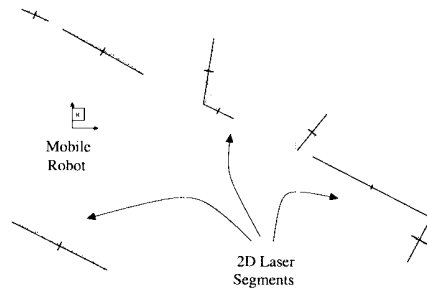


Fig. 4. Straight-line segment representation of a local environment of the vehicle R at the first point of its trajectory. Lateral uncertainty for each detected 2-D laser segment has been represented (magnified three times).

represented by 95% error ellipses (i.e., $\pm 2\sigma$ bounds). This probabilistic matching technique has proven to be simple and effective for laser segments. However, other sensor systems, such as monocular or stereo vision, require more robust matching strategies to properly solve the data association problem.

A. Neglecting Correlations Between Entities

When neglecting correlations, the EKF estimation phase is decomposed into the estimation of the mobile robot location and subsequently the estimation of the map features [10] from a common set of observations. Fig. 5(a) shows the result of this process. It can be seen that this two-step process produces optimistic estimations of uncertainty because the same observations are used twice in the estimation algorithms.

Table I presents the maximum errors and the error growing-rate along the vehicle trajectory when neglecting correlations between features. An increasing discrepancy of the estimated robot location with respect to ground truth was observed (only 5.6% of the estimations were compatible with the real solution). This discrepancy became larger as the vehicle moved to previously learned places in the navigation area, and was due to optimistic estimations of the uncertainty associated to the location of environment features, producing a high rejection rate in the matching process. Thus, when the robot revisited places already learned, very few pairings were obtained and neither the mobile robot was effectively relocalized nor the global map was accurately built. Most of the local map features were directly added to the representation, creating multiple location hypotheses for the same map feature. Later, matchings involving those false hypotheses (i.e., data association errors) induce the final solution to diverge from ground truth.

B. Maintaining Correlations within the SPmap

Fig. 5(b) shows the result of the robot localization and map building process considering the SPmap approach. Even though we simultaneously were building the map and relocating the mobile robot, whenever the vehicle navigated in previously unknown areas, uncertainty continually increased, that is, integration of new observations only reduced the uncertainty growing rate downtown the measurement error of the sensor used, but not further. On the contrary, whenever the vehicle revisited places in the environment already learned, uncertainty decreased, converging to the values of the location uncertainty of the reobserved global map features. Also, the "indirect estimation" effect appeared, that is, location uncertainty decreased for all the features of the SPmap, even for those not visible from the current robot location but statistically correlated to current observations through the off-diagonal elements of the covariance matrix of the SPmap.

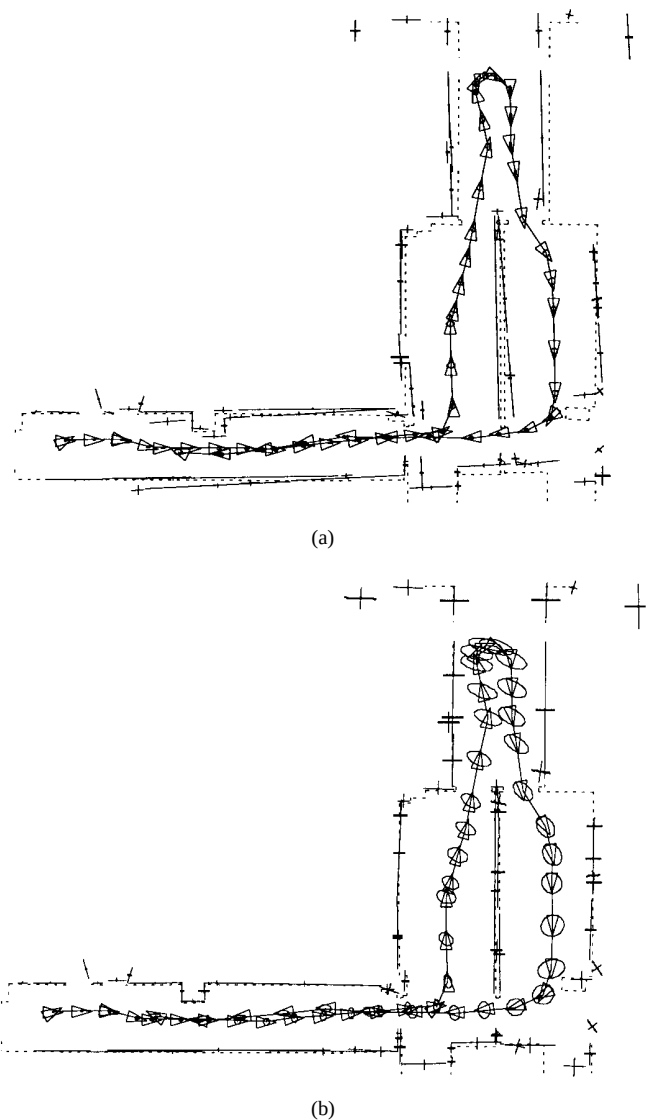


Fig. 5. Estimated robot trajectory and built map (a) neglecting correlations between entities, where the estimated solution diverges from ground-truth, and (b) using the SPmap approach. An *a priori* model map is drawn for reference purposes.

TABLE I
SUMMARY OF THE SOLUTIONS OBTAINED BY THE DIFFERENT APPROACHES:
MAXIMUM ERRORS (x_{max} , y_{max} , AND t_{max}), DISTANCE d_{rel} AND
ANGULAR t_{rel} ERROR GROWING-RATES, AND χ^2 TEST RESULTS

	Odometry	Neg. Corr.	SPmap
x_{max} (mm)	1518.22	416.24	272.09
y_{max} (mm)	2066.69	352.36	208.99
ϕ_{max} (deg)	15.50	2.89	2.05
d_{rel} (mm/m)	46.3	9.5	5.9
ϕ_{rel} (deg/m)	0.293	0.055	0.039
χ^2_{test} (%)	100	5.6	98.1

Table I compares the largest errors obtained by the SPmap approach with the approach neglecting correlations. Clearly, the SPmap approach obtained estimations for the mobile robot localization with an upper bound of around 5.9 mm/m of the total trajectory length for the distance error and 0.04°/m for the orientation error, which represented an order of magnitude below dead-reckoning

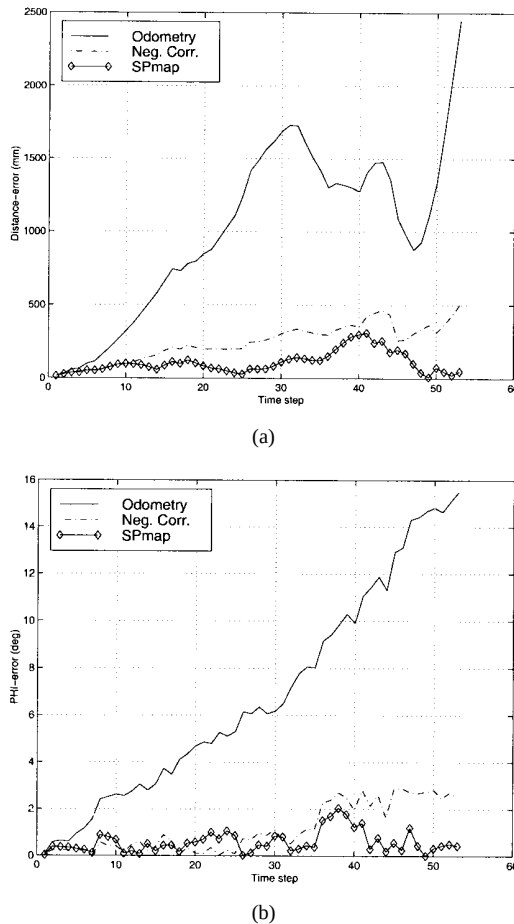


Fig. 6. Comparison of errors along the robot trajectory: (a) distance error and (b) angular error.

errors. Also, the location uncertainty of the map features was not underestimated, with a compatibility of $\chi^2_{\text{test}} \approx 98\%$, therefore, pairings between local observations at time k and previously stored knowledge up to time $k - 1$ were found at each point along the robot trajectory even when the vehicle returned to previously learned places of the navigation area. An average of 74% of the number of available observations were matched with previous known features. Fig. 6 compares the real errors along the robot path, obtained by each approach. The highest errors correspond to odometry-based navigation, while the smallest correspond to the SPmap approach. From the Fig. 6 note how the location uncertainty of the robot decreases when the vehicle revisits previously learned places (i.e., from trajectory point 40 onwards).

VI. CONCLUSION

This article has presented the symmetries and perturbations map (SPmap), a probabilistic framework for the simultaneous localization and map building problem for mobile robots. We have presented a complete experiment, where a LabMate mobile robot equipped with a rotating 2-D laser rangefinder navigated indoors. Experimentation showed the importance of maintaining correlations between entities. Satisfactory results have been obtained concerning the problem of revisiting previously learned places of the navigation area.

Our recent work has motivated us with further extensions of the concept of SPmap, both to extent its applicability to real-life environments and to increase its robustness:

- 1) sensor cooperation to obtain more robust and reliable observations from the navigation area [15];
- 2) increase in the structuration and the semantical contents of the representation toward a topological description where human-language-like instructions could be commanded to the vehicle;
- 3) search for optimal representations of the navigation area to reduce the complexity $\mathcal{O}(N^2)$ of the current approach, when larger environments are considered;
- 4) design of strategies to maintain the constructed map, such as those required to remove features not visible for a long time.

Also, we believe that when the vehicle revisited places of the navigation area already learned after travelling for a long time in unknown areas, the Mahalanobis distance would be insufficient to match local observations with previously stored features in the SPmap. In those cases, new data association mechanisms would be required.

REFERENCES

- [1] R. Smith, M. Self, and P. Cheeseman, "Estimating uncertain spatial relationships in robotics," in *Uncertainty in Artificial Intelligence 2*, J. F. Lemmer and L. N. Kanal, Eds. New York: Elsevier, 1988, pp. 435–461.
- [2] —, "A stochastic map for uncertain spatial relationships," in *Robotics Research, The Fourth Int. Symposium*, O. Faugeras and G. Giralt, Eds. Cambridge, MA: MIT Press, 1988, pp. 467–474.
- [3] P. Moutarlier and R. Chatila, "Stochastic multisensory data fusion for mobile robot location and environment modeling," in *5th Int. Symp. Robot. Res.*, Tokyo, Japan, 1989, pp. 207–216.
- [4] J. J. Leonard and H. F. Durrant-Whyte, "Simultaneous map building and localization for an autonomous mobile robot," in *Proc. 1991 IEEE/RSJ Int. Conf. Intell. Robot. Syst.*, Osaka, Japan, 1991, pp. 1442–1447.
- [5] —, *Directed Sonar Sensing for Mobile Robot Navigation*. London, U.K.: Kluwer, 1992.
- [6] J. J. Leonard, H. F. Durrant-Whyte, and I. J. Cox, "Dynamic map building for an autonomous mobile robot," *Int. Robot. Res.*, vol. 11, no. 4, pp. 286–298, 1992.
- [7] P. Hébert, S. Betgé-Brezetz, and R. Chatila, "Probabilistic map learning: Necessity and difficulties," in *Proc. Int. Workshop Reasoning Uncertainty Robot.*, Amsterdam, The Netherlands, 1995.
- [8] J. K. Uhlmann, S. J. Julier, and M. Csorba, "Nondivergent simultaneous map building and localization using covariance intersection," in *Proc. AeroSense: 11th Int. Symp. Aerospace/Defense Sensing, Simulation Contr.*, Orlando, FL, 1997.
- [9] J. A. Castellanos, "Mobile robot localization and map building: A multisensor fusion approach," Dept. Inform. Eng. Syst., Univ. Zaragoza, Maria de Luna, Spain, May 1998.
- [10] J. A. Castellanos, J. D. Tardós, and G. Schmidt, "Building a global map of the environment of a mobile robot: The importance of correlations," in *Proc. 1997 IEEE Int. Conf. Robot. Automat.*, Albuquerque, NM, Apr. 1997, pp. 1053–1059.
- [11] J. D. Tardós, "Representing partial and uncertain sensorial information using the theory of symmetries," in *Proc. 1992 IEEE Int. Conf. Robot. Automat.*, Nice, France, 1992, pp. 1799–1804.
- [12] J. Neira, J. D. Tardós, J. Horn, and G. Schmidt, "Fusing range and intensity images for mobile robot localization," *IEEE Trans. Robot. Automat.*, vol. 15, pp. 76–84, Feb. 1999.
- [13] T. Bar-Shalom and T. E. Fortmann, *Tracking and Data Association*. New York: Academic, 1988.
- [14] J. A. Castellanos and J. D. Tardós, "Laser-based segmentation and localization for a mobile robot," in *Proceedings of the Sixth International Symposium (ISRAM)*, F. Pin, M. Jamshidi, and P. Dauchez, Eds. New York: ASME, 1996, pp. 101–108.
- [15] J. A. Castellanos, J. M. Martínez, J. Neira, and J. D. Tardós, "Simultaneous map building and localization for mobile robots: A multisensor fusion approach," in *Proc. 1998 IEEE Int. Conf. Robot. Automat.*, Leuven, Belgium, May 1998, pp. 1244–1249.

ATTACHMENT C

The SPmap: a probabilistic framework for simultaneous localization and map building

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This article describes a rigorous and complete framework for the simultaneous localization and map building problem for mobile robots: the symmetries and perturbation map (SPmap), which is based on a general probabilistic representation of uncertain geometric information. We present a complete experiment with a LabMate/sup TM/ mobile robot navigating in a human-made indoor environment and equipped with a rotating 2D laser rangefinder. Experiments validate the appropriateness of our approach and provide a real measurement of the precision of the algorithms.

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ATTACHMENT D



DECLARATION OF GORDON MACPHERSON

I, Gordon MacPherson, am over twenty-one (21) years of age. I have never been convicted of a felony, and I am fully competent to make this declaration. I declare the following to be true to the best of my knowledge, information, and belief:

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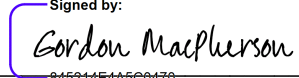
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EXHIBIT A

The SPmap: A Probabilistic Framework for Simultaneous Localization and Map Building

Jose A. Castellanos, J. M. M. Montiel, J. Neira, and J. D. Tardós

Abstract—This article describes a rigorous and complete framework for the simultaneous localization and map building problem for mobile robots: the symmetries and perturbations map (SPmap), which is based on a general probabilistic representation of uncertain geometric information. We present a complete experiment with a LabMate™ mobile robot navigating in a human-made indoor environment and equipped with a rotating two-dimensional (2-D) laser rangefinder. Experiments validate the appropriateness of our approach and provide a real measurement of the precision of the algorithms.

Index Terms—Correlations, probabilistic model, simultaneous localization and map building.

I. INTRODUCTION

Successful path planning and navigation of a mobile robot in a human-made indoor environment requires the availability of both a sufficiently reliable estimation of the current vehicle location, and a sufficiently precise map of the navigation area. *A priori* model maps are rarely available, costly to obtain, and when they are available, they usually introduce inaccuracies in the planning tasks. An automatic construction of the map of the environment in which the robot navigates would be desirable, and it has become an important research direction in today's robotics community.

The precision of the constructed map is highly influenced by the accuracy of the dead-reckoning system of the mobile robot, whose location estimations drift with time. An improved solution would require the relocation of the mobile robot along its trajectory to avoid biases introduced by odometry, hence, an approach based on the simultaneous localization and map building would be necessary (Fig. 1). Exact mathematical approaches to the simultaneous localization and map building problem were originally addressed by Smith *et al.* [1], [2] who introduced the concept of *stochastic map*, a representation of spatial relationships, their uncertainties, and their interdependencies with respect to a base reference. Later, this concept was used in the works of Moutarlier *et al.* [3], Leonard *et al.* [4]–[6], and Hébert *et al.* [7]. Recently, Uhlmann *et al.* [8] have reported interesting work related to the problem of correlations in the simultaneous localization and map building problem.

This article proposes a new probabilistic framework adapted to the problem of simultaneous localization and map building: the symmetries and perturbations map (SPmap) [9] which is based on a general representation of uncertain geometric information. Our main contributions are as follows.

- 1) SPmap represents a rigorous and complete solution to the simultaneous localization and map building problem for mobile robots.
- 2) SPmap consistently formulates the representation and integration of sensorial information gathered by different sensors.

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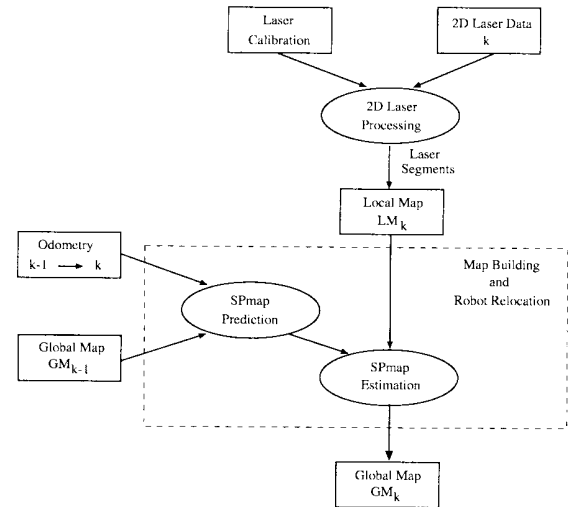


Fig. 1. Simultaneous localization and map building.

- 3) SPmap overcomes the difficulties reported in previous works dealing with singularities in the representation of geometric features.
- 4) SPmap has been experimentally validated by a complete experiment which profited from ground-truth to accurately validate the precision and the appropriateness of the approach.

This article also extends our preliminary results reported in [10] to demonstrate the importance of maintaining the correlations between the estimation of the entities involved in the simultaneous localization and map building problem, to avoid optimistic estimations of uncertainty associated to the precision of the locations of features.

The rest of the article is structured as follows. Section II describes the probabilistic representation of uncertain geometric information. Sections III and IV present our probabilistic framework for the simultaneous localization and map building problem. Experimental results are described in Section V while some conclusions and further work are shown in Section VI.

II. SYMMETRIES AND PERTURBATIONS MODEL

In our feature-based approach, uncertain geometric information is represented using a probabilistic model: the symmetries and perturbation model (SPmodel) [11], [12] which combines the use of probability theory to represent the imprecision in the location of a geometric element, and the theory of symmetries to represent the partiality due to characteristics of each type of geometric element.

In the SPmodel, the location of a geometric element E with respect to a base reference W is given by a *location vector* $\mathbf{x}_{WE} = (x, y, \phi)^T$. The estimation of the location of an element is denoted by $\hat{\mathbf{x}}_{WE}$, and the estimation error is represented locally by a *differential location vector* $\mathbf{d}_E$ relative to the reference attached to the element. Thus, the true location of the element is

$$\mathbf{x}_{WE} = \hat{\mathbf{x}}_{WE} \oplus \mathbf{d}_E \quad (1)$$

where $\oplus$ represents the composition of location vectors. To account for the symmetries of the geometric element, we assign in $\mathbf{d}_E$ a null value to the degrees of freedom corresponding to them, because they do not represent an effective location error. We call *perturbation*

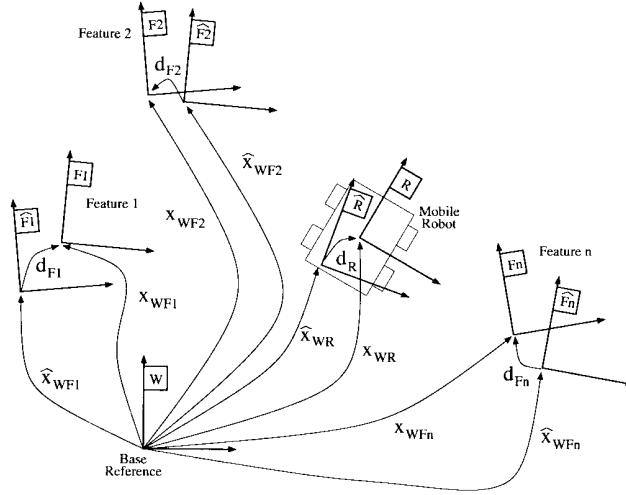


Fig. 2. Representation of the SPmap with generic features and the mobile robot expressed with respect to a base reference W . The location vector of each entity is obtained by the composition of its estimated location vector and its differential location vector.

vector the vector $\mathbf{p}_E$ formed by the non null elements of $\mathbf{d}_E$. Both vectors can be related by a row selection matrix $\mathbf{B}_E$ that we call *self-binding matrix* of the geometric element

$$\mathbf{d}_E = \mathbf{B}_E^T \mathbf{p}_E; \quad \mathbf{p}_E = \mathbf{B}_E \mathbf{d}_E. \quad (2)$$

Then, the *uncertain location* of every geometric entity is represented in the SPmodel by a quadruple $\mathbf{L}_{WE} = (\hat{\mathbf{x}}_{WE}, \hat{\mathbf{p}}_E, \mathbf{C}_E, \mathbf{B}_E)$, where the transformation $\hat{\mathbf{x}}_{WE}$ is an estimation taken as base for perturbations, $\hat{\mathbf{p}}_E$ is the estimated value of the perturbation vector, and $\mathbf{C}_E$ its covariance.

III. SYMMETRIES AND PERTURBATIONS MAP

The symmetries and perturbations map (SPmap) is a complete representation of the environment of the robot which includes the uncertain location of the mobile robot $\mathbf{L}_{WR}$, the uncertain locations of the features obtained from sensor observations $\mathbf{L}_{WF_i}$, $i \in \{1 \dots N_F\}$ and their interdependencies (Fig. 2).

The SPmap can be defined as a quadruple

$$\text{SPmap} = (\hat{\mathbf{x}}^W, \hat{\mathbf{p}}^W, \mathbf{C}^W, \mathbf{B}^W) \quad (3)$$

where $\hat{\mathbf{x}}^W$ is the *estimated location vector* of the SPmap and $\hat{\mathbf{p}}^W$ is the *perturbation vector* of the SPmap

$$\hat{\mathbf{x}}^W = \begin{bmatrix} \hat{\mathbf{x}}_{WR} \\ \hat{\mathbf{x}}_{WF1} \\ \vdots \\ \hat{\mathbf{x}}_{WF_{N_F}} \end{bmatrix}; \quad \hat{\mathbf{p}}^W = \begin{bmatrix} \mathbf{d}_R \\ \mathbf{p}_{F1} \\ \vdots \\ \mathbf{p}_{F_{N_F}} \end{bmatrix}. \quad (4)$$

The true location of the robot and the map features is

$$\mathbf{x}^W = \hat{\mathbf{x}}^W \oplus (\mathbf{B}^W)^T \mathbf{p}^W \quad (5)$$

where the composition operator $\oplus$ applies in this case to each of the components of the vectors, and $\mathbf{B}^W$ is the *binding matrix* of the SPmap, a diagonal matrix formed by the self-binding matrix of the robot and the self-binding matrices of the map features

$$\mathbf{B}^W = \text{diag}(\mathbf{B}_R, \mathbf{B}_{F1}, \dots, \mathbf{B}_{F_{N_F}}). \quad (6)$$

The covariance matrix of the SPmap represents the covariance of the estimation of the robot and the map feature locations, the cross-covariances between the robot and the map features, and finally, the cross-covariances between the map features themselves

$$\mathbf{C}^W = \begin{pmatrix} \mathbf{C}_R & \mathbf{C}_{RF1} & \cdots & \mathbf{C}_{RF_{N_F}} \\ \mathbf{C}_{RF1}^T & \mathbf{C}_{F1} & \cdots & \mathbf{C}_{F1F_{N_F}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_{RF_{N_F}}^T & \mathbf{C}_{F1F_{N_F}} & \cdots & \mathbf{C}_{F_{N_F}} \end{pmatrix}. \quad (7)$$

Note that we may represent any type of geometric entity within this general framework. The proposed approach can include different types of features obtained by different types of sensors, and thus it is suitable to deal with a multisensor system.

IV. INCREMENTAL CONSTRUCTION OF THE SPMAP

This section describes the incremental construction of the SPmap using suboptimal estimation techniques based on the extended Kalman filter [13].

A. Uncertain Displacement of the Mobile Robot

An estimation of the displacement of the mobile robot between two intermediate points along its trajectory can be obtained by dead-reckoning

$$\mathbf{x}_{R_{k-1}R_k} = \hat{\mathbf{x}}_{R_{k-1}R_k} \oplus \mathbf{d}_{R_{k-1}R_k} \quad (8)$$

where $\hat{\mathbf{x}}_{R_{k-1}R_k}$ represents the estimated displacement of the robot and $\mathbf{d}_{R_{k-1}R_k} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{R_{k-1}R_k})$ represents the imprecision in its estimation (i.e., dead-reckoning errors). Other nonrandom systematic errors are not considered because they can be corrected by an appropriate calibration procedure.

Thus, the predicted location $\mathbf{x}_{WR_k}$ of the mobile robot at time k can be calculated by the composition

$$\begin{aligned} \mathbf{x}_{WR_k} &= \mathbf{x}_{WR_{k-1}} \oplus \mathbf{x}_{R_{k-1}R_k} \\ &= \hat{\mathbf{x}}_{WR_{k-1}} \oplus \mathbf{d}_{R_{k-1}} \oplus \hat{\mathbf{x}}_{R_{k-1}R_k} \oplus \mathbf{d}_{R_{k-1}R_k} \\ &= \hat{\mathbf{x}}_{WR_k} \oplus \mathbf{J}_{R_k R_{k-1}} \mathbf{d}_{R_{k-1}} \oplus \mathbf{d}_{R_{k-1}R_k} \end{aligned} \quad (9)$$

where $\mathbf{J}_{R_k R_{k-1}}$ is the Jacobian of the transformation $\hat{\mathbf{x}}_{R_k R_{k-1}}$ between the location vectors of the robot at time k and that at time $k-1$.

After the displacement of the vehicle, only the location of the mobile robot changes as estimated by dead-reckoning, while the location of map features, being static entities, remain the same as the estimated in the previous time instant $k-1$. Nevertheless, the displacement of the mobile robot produces changes in the dependencies existing between the location of the robot and those of the map features. The complexity of this phase grows linearly with the number of features, $\mathcal{O}(N_F)$.

B. Matching Local and Global Maps

The predicted mobile robot location is improved by matching local observations expressed with respect to the robot reference R , with map features represented with respect to the base reference W . At each point of the robot's trajectory it is desirable to obtain as much pairings as possible because they represent the links between new observations and previous stored knowledge of the navigation area.

Fig. 3 exemplifies the pairing between a local observation E obtained at time k and represented with respect to the robot by $\mathbf{L}_{RE} = (\hat{\mathbf{x}}_{RE}, \hat{\mathbf{p}}_E, \mathbf{C}_E, \mathbf{B}_E)$, and a global map feature F available at time $k-1$ and represented with respect to the base reference W

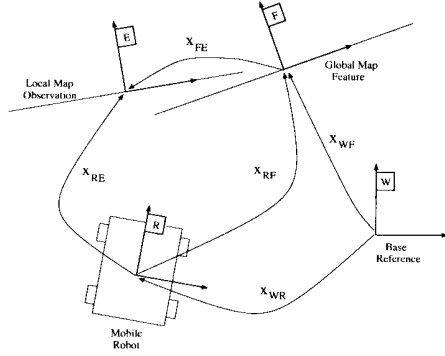


Fig. 3. Matching local observation E with global feature F .

by $\mathbf{L}_{WF} = (\hat{\mathbf{x}}_{WF}, \hat{\mathbf{p}}_F, \mathbf{C}_F, \mathbf{B}_F)$. From their relative location we formulate a nonlinear measurement equation [11]

$$\begin{aligned} \mathbf{f}_{k,m}(\mathbf{d}_R, \mathbf{p}_F, \mathbf{p}_E) &= \mathbf{B}_{FE} \mathbf{x}_{FE} \\ &= \mathbf{B}_{FE} (\ominus \mathbf{x}_{WF} \oplus \mathbf{x}_{WR} \oplus \mathbf{x}_{RE}) \\ &= \mathbf{B}_{FE} (\ominus \mathbf{J}_F^T \mathbf{p}_F \oplus \hat{\mathbf{x}}_{FE} \oplus \mathbf{J}_{ER} \mathbf{d}_R \oplus \mathbf{B}_E^T \mathbf{p}_E) \\ &= \mathbf{0} \end{aligned} \quad (10)$$

where $\mathbf{B}_{FE}$ is the binding matrix of the pairing [11], a row selection matrix which selects the components of $\mathbf{x}_{FE}$ which must be zero. Due to uncertainty, a hypothesis test based on the squared Mahalanobis distance D^2 , validates the compatibility between E and F

$$D^2 = (\mathbf{B}_{FE} \hat{\mathbf{x}}_{FE})^T [\mathbf{B}_{FE} \mathbf{C}(\mathbf{x}_{FE}) \mathbf{B}_{FE}^T]^{-1} (\mathbf{B}_{FE} \hat{\mathbf{x}}_{FE}) \quad (11)$$

where matrix $\mathbf{C}(\mathbf{x}_{FE})$ is computed from the linearization of (10) taking into account the correlations between the map feature F and the robot:

$$\mathbf{C}(\mathbf{x}_{FE}) = \mathbf{J}_{2\oplus} \{\hat{\mathbf{x}}_{FE}, \mathbf{0}\} \mathbf{C}(\mathbf{d}_E^F) \mathbf{J}_{2\oplus}^T \{\hat{\mathbf{x}}_{FE}, \mathbf{0}\} \quad (12)$$

where

$$\begin{aligned} \mathbf{C}(\mathbf{d}_E^F) &= \mathbf{J}_{ER} \mathbf{C}_R \mathbf{J}_{ER}^T + \mathbf{J}_{EF} \mathbf{B}_F^T \mathbf{C}_F \mathbf{B}_F \mathbf{J}_{EF}^T \\ &\quad - \mathbf{J}_{EF} \mathbf{B}_F^T \mathbf{C}_F^T \mathbf{J}_{ER}^T - \mathbf{J}_{ER} \mathbf{C}_R \mathbf{B}_F \mathbf{J}_{EF}^T \\ &\quad + \mathbf{B}_E^T \mathbf{C}_E \mathbf{B}_E. \end{aligned} \quad (13)$$

Under the Gaussianity hypothesis, D^2 follows a χ^2 distribution. For a given significance level α , the local feature E is compatible with the global feature F if $D^2 \leq \chi_{r,\alpha}^2$, with $r = \text{rank}(\mathbf{B}_{FE})$ degrees of freedom, otherwise the matching between the E and F is discarded. In general, when there exist multiple possible pairings for a particular local feature, the pairing with the smallest Mahalanobis distance is chosen.

C. Estimating the SPmap at Time k

The matching between E and F is used both to relocalize the robot at the current trajectory point and, simultaneously, to update the estimated location of the map features. The perturbation vector of the local observation $\mathbf{p}_E \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_E)$ constitutes the measurement used to improve the estimation of $\mathbf{p}_k^W$ through the relation established by

¹Matrices $\mathbf{J}_{1\oplus}$ and $\mathbf{J}_{2\oplus}$ are the Jacobians of the composition of location vectors [1]

$$\begin{aligned} \mathbf{J}_{1\oplus} \{\mathbf{x}_1, \mathbf{x}_2\} &= \frac{\partial(\mathbf{y} \oplus \mathbf{z})}{\partial \mathbf{y}} \bigg|_{\mathbf{y}=\mathbf{x}_1, \mathbf{z}=\mathbf{x}_2} \\ \mathbf{J}_{2\oplus} \{\mathbf{x}_1, \mathbf{x}_2\} &= \frac{\partial(\mathbf{y} \oplus \mathbf{z})}{\partial \mathbf{z}} \bigg|_{\mathbf{y}=\mathbf{x}_1, \mathbf{z}=\mathbf{x}_2}. \end{aligned}$$

(10). Linearization of (10) is done by considering a first order Taylor expansion

$$\begin{aligned} \mathbf{h}_m &= \mathbf{f}_{k,m}(\hat{\mathbf{p}}_k^W, \hat{\mathbf{p}}_E) = \mathbf{B}_{FE} \hat{\mathbf{x}}_{FE} \\ \mathbf{H}_m &= \frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{p}_k^W} \bigg|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{p}}_E)} = (\mathbf{H}_m^R \mathbf{0} \cdots \mathbf{0} \mathbf{H}_m^F \mathbf{0} \cdots \mathbf{0}) \\ \mathbf{H}_m^R &= \frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{d}_R} \bigg|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{p}}_E)} = \mathbf{B}_{FE} \mathbf{J}_{2\oplus} \{\hat{\mathbf{x}}_{FE}, \mathbf{0}\} \mathbf{J}_{ER} \\ \mathbf{H}_m^F &= \frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{p}_F} \bigg|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{p}}_E)} = -\mathbf{B}_{FE} \mathbf{J}_{1\oplus} \{\mathbf{0}, \hat{\mathbf{x}}_{FE}\} \mathbf{B}_F^T \\ \mathbf{G}_m &= \frac{\partial \mathbf{f}_{k,m}}{\partial \mathbf{p}_E} \bigg|_{(\hat{\mathbf{p}}_k^W, \hat{\mathbf{p}}_E)} = \mathbf{B}_{FE} \mathbf{J}_{2\oplus} \{\hat{\mathbf{x}}_{FE}, \mathbf{0}\} \mathbf{B}_E^T. \end{aligned} \quad (14)$$

Integration of local observations LM_k (i.e., new knowledge of the navigation area) into the SPmap known up to the previous time instant GM_{k-1} by using the EKF equations, produces a reestimation of its whole perturbation vector $\mathbf{p}_k^W$ and the uncertainties represented by its covariance matrix $\mathbf{C}_k^W$. Location estimations of the complete set of entities included in the SPmap are reestimated after the integration of new information. Furthermore, correlations between their estimations are also updated. The complexity of this phase grows polynomially with the number of features $\mathcal{O}(N_F^2 M)$, where M is the number of matched observations.

D. Adding Nonmatched Features to the Global Map

Local observations obtained at time k which cannot be paired with any of the global features of GM_{k-1} are interpreted as knowledge about the environment which has not yet been learned. These local observations are added to the SPmap by considering the composition

$$\begin{aligned} \mathbf{x}_{WE} &= \mathbf{x}_{WR} \oplus \mathbf{x}_{RE} \\ &= \hat{\mathbf{x}}_{WE} \oplus \mathbf{J}_{ER} \mathbf{d}_R \oplus \mathbf{B}_E^T \mathbf{p}_E. \end{aligned} \quad (15)$$

The covariance matrix of the SPmap is extended to represent the correlation between the mobile robot and the nonpaired feature E , the cross-correlation between the previous map features and the nonpaired feature E , and the covariance of the nonpaired feature. When the whole local map is composed only of nonpaired observations, the robot's location estimation obtained by odometry cannot be improved.

V. EXPERIMENTING WITH THE SPMAP

This section presents the experimental verification of the previous ideas by considering a LabMate™ mobile robot navigating indoors, and equipped with a two-dimensional (2-D) rotating laser scanner. We also analyze the effects of neglecting correlations between the location estimation of the features. The vehicle was programmed to follow a trajectory (53 m approximately), stopping at regular intervals to take measurements. Complementary information was taken, by hand, with a pair of theodolites which provided real locations of the robot with respect to a base reference.

From the laser readings gathered at each point along the trajectory, a straight-line segment-based local map LM_k was constructed. A segmentation algorithm [14] was used. The resultant 2-D segments (Fig. 4) were expressed with respect to the mobile robot reference frame R by using the SPmodel formulation; also, estimations of their lengths were computed from their endpoints. Typically the number of straight-line segments was kept small for each local map (below ten segments per local map) by considering only meaningful (i.e., long) segments.

Pairings between local observations and previously stored knowledge of the navigation area were obtained by using a 5% significance level. In the following figures, precision of the results has been

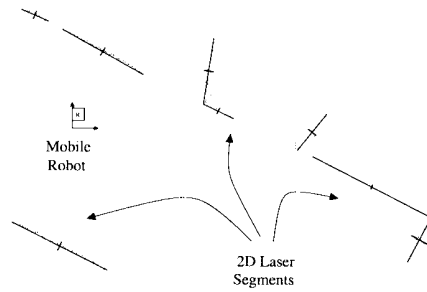


Fig. 4. Straight-line segment representation of a local environment of the vehicle R at the first point of its trajectory. Lateral uncertainty for each detected 2-D segment has been represented (magnified three times).

represented by 95% error ellipses (i.e., $\pm 2\sigma$ bounds). This probabilistic matching technique has proven to be simple and effective for laser segments. However, other sensor systems, such as monocular or stereo vision, require more robust matching strategies to properly solve the data association problem.

A. Neglecting Correlations Between Entities

When neglecting correlations, the EKF estimation phase is decomposed into the estimation of the mobile robot location and subsequently the estimation of the map features [10] from a common set of observations. Fig. 5(a) shows the result of this process. It can be seen that this two-step process produces optimistic estimations of uncertainty because the same observations are used twice in the estimation algorithms.

Table I presents the maximum errors and the error growing-rate along the vehicle trajectory when neglecting correlations between features. An increasing discrepancy of the estimated robot location with respect to ground truth was observed (only 5.6% of the estimations were compatible with the real solution). This discrepancy became larger as the vehicle moved to previously learned places in the navigation area, and was due to optimistic estimations of the uncertainty associated to the location of environment features, producing a high rejection rate in the matching process. Thus, when the robot revisited places already learned, very few pairings were obtained and neither the mobile robot was effectively relocalized nor the global map was accurately built. Most of the local map features were directly added to the representation, creating multiple location hypotheses for the same map feature. Later, matchings involving those false hypotheses (i.e., data association errors) induce the final solution to diverge from ground truth.

B. Maintaining Correlations within the SPmap

Fig. 5(b) shows the result of the robot localization and map building process considering the SPmap approach. Even though we simultaneously were building the map and relocating the mobile robot, whenever the vehicle navigated in previously unknown areas, uncertainty continually increased, that is, integration of new observations only reduced the uncertainty growing rate downtown the measurement error of the sensor used, but not further. On the contrary, whenever the vehicle revisited places in the environment already learned, uncertainty decreased, converging to the values of the location uncertainty of the reobserved global map features. Also, the "indirect estimation" effect appeared, that is, location uncertainty decreased for all the features of the SPmap, even for those not visible from the current robot location but statistically correlated to current observations through the off-diagonal elements of the covariance matrix of the SPmap.

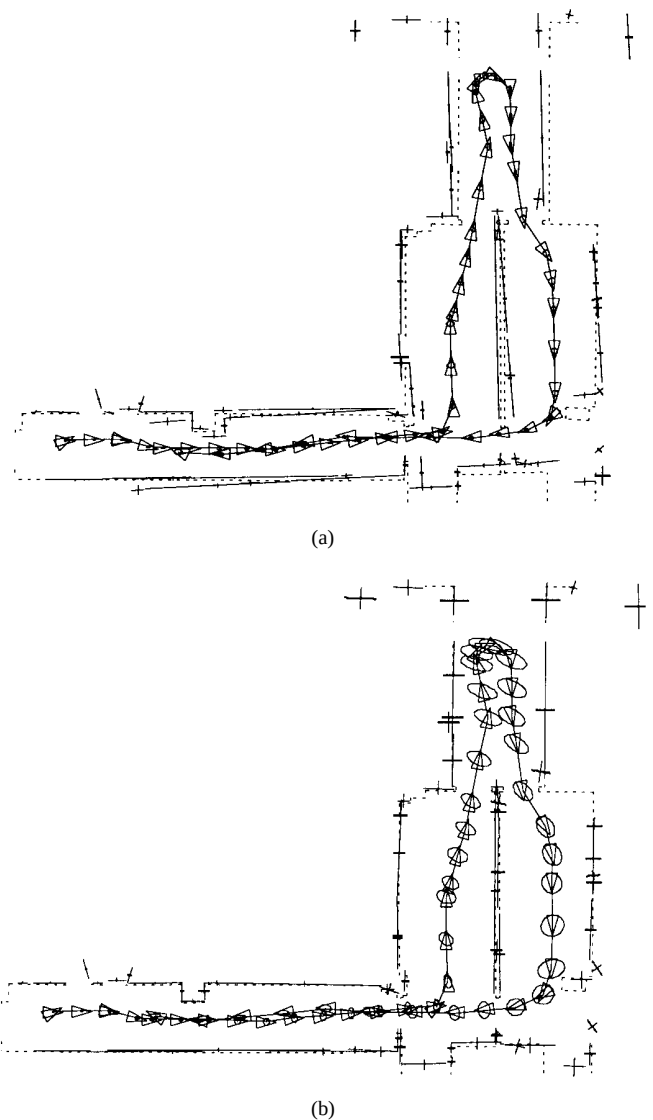


Fig. 5. Estimated robot trajectory and built map (a) neglecting correlations between entities, where the estimated solution diverges from ground-truth, and (b) using the SPmap approach. An *a priori* model map is drawn for reference purposes.

TABLE I
SUMMARY OF THE SOLUTIONS OBTAINED BY THE DIFFERENT APPROACHES:
MAXIMUM ERRORS (x_{max} , y_{max} , AND t_{max}), DISTANCE d_{rel} AND
ANGULAR t_{rel} ERROR GROWING-RATES, AND χ^2 TEST RESULTS

	Odometry	Neg. Corr.	SPmap
x_{max} (mm)	1518.22	416.24	272.09
y_{max} (mm)	2066.69	352.36	208.99
ϕ_{max} (deg)	15.50	2.89	2.05
d_{rel} (mm/m)	46.3	9.5	5.9
ϕ_{rel} (deg/m)	0.293	0.055	0.039
χ^2_{test} (%)	100	5.6	98.1

Table I compares the largest errors obtained by the SPmap approach with the approach neglecting correlations. Clearly, the SPmap approach obtained estimations for the mobile robot localization with an upper bound of around 5.9 mm/m of the total trajectory length for the distance error and 0.04°/m for the orientation error, which represented an order of magnitude below dead-reckoning

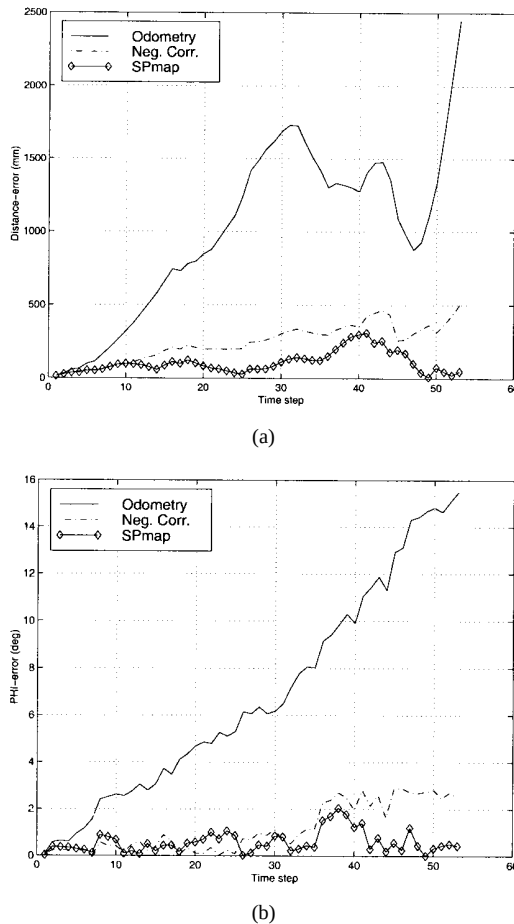


Fig. 6. Comparison of errors along the robot trajectory: (a) distance error and (b) angular error.

errors. Also, the location uncertainty of the map features was not underestimated, with a compatibility of $\chi^2_{\text{test}} \approx 98\%$, therefore, pairings between local observations at time k and previously stored knowledge up to time $k - 1$ were found at each point along the robot trajectory even when the vehicle returned to previously learned places of the navigation area. An average of 74% of the number of available observations were matched with previous known features. Fig. 6 compares the real errors along the robot path, obtained by each approach. The highest errors correspond to odometry-based navigation, while the smallest correspond to the SPmap approach. From the Fig. 6 note how the location uncertainty of the robot decreases when the vehicle revisits previously learned places (i.e., from trajectory point 40 onwards).

VI. CONCLUSION

This article has presented the symmetries and perturbations map (SPmap), a probabilistic framework for the simultaneous localization and map building problem for mobile robots. We have presented a complete experiment, where a LabMate mobile robot equipped with a rotating 2-D laser rangefinder navigated indoors. Experimentation showed the importance of maintaining correlations between entities. Satisfactory results have been obtained concerning the problem of revisiting previously learned places of the navigation area.

Our recent work has motivated us with further extensions of the concept of SPmap, both to extent its applicability to real-life environments and to increase its robustness:

- 1) sensor cooperation to obtain more robust and reliable observations from the navigation area [15];
- 2) increase in the structuration and the semantical contents of the representation toward a topological description where human-language-like instructions could be commanded to the vehicle;
- 3) search for optimal representations of the navigation area to reduce the complexity $\mathcal{O}(N^2)$ of the current approach, when larger environments are considered;
- 4) design of strategies to maintain the constructed map, such as those required to remove features not visible for a long time.

Also, we believe that when the vehicle revisited places of the navigation area already learned after travelling for a long time in unknown areas, the Mahalanobis distance would be insufficient to match local observations with previously stored features in the SPmap. In those cases, new data association mechanisms would be required.

REFERENCES

- [1] R. Smith, M. Self, and P. Cheeseman, "Estimating uncertain spatial relationships in robotics," in *Uncertainty in Artificial Intelligence 2*, J. F. Lemmer and L. N. Kanal, Eds. New York: Elsevier, 1988, pp. 435–461.
- [2] —, "A stochastic map for uncertain spatial relationships," in *Robotics Research, The Fourth Int. Symposium*, O. Faugeras and G. Giralt, Eds. Cambridge, MA: MIT Press, 1988, pp. 467–474.
- [3] P. Moutarlier and R. Chatila, "Stochastic multisensory data fusion for mobile robot location and environment modeling," in *5th Int. Symp. Robot. Res.*, Tokyo, Japan, 1989, pp. 207–216.
- [4] J. J. Leonard and H. F. Durrant-Whyte, "Simultaneous map building and localization for an autonomous mobile robot," in *Proc. 1991 IEEE/RSJ Int. Conf. Intell. Robot. Syst.*, Osaka, Japan, 1991, pp. 1442–1447.
- [5] —, *Directed Sonar Sensing for Mobile Robot Navigation*. London, U.K.: Kluwer, 1992.
- [6] J. J. Leonard, H. F. Durrant-Whyte, and I. J. Cox, "Dynamic map building for an autonomous mobile robot," *Int. Robot. Res.*, vol. 11, no. 4, pp. 286–298, 1992.
- [7] P. Hébert, S. Betgé-Brezetz, and R. Chatila, "Probabilistic map learning: Necessity and difficulties," in *Proc. Int. Workshop Reasoning Uncertainty Robot.*, Amsterdam, The Netherlands, 1995.
- [8] J. K. Uhlmann, S. J. Julier, and M. Csorba, "Nondivergent simultaneous map building and localization using covariance intersection," in *Proc. AeroSense: 11th Int. Symp. Aerospace/Defense Sensing, Simulation Contr.*, Orlando, FL, 1997.
- [9] J. A. Castellanos, "Mobile robot localization and map building: A multisensor fusion approach," Dept. Inform. Eng. Syst., Univ. Zaragoza, Maria de Luna, Spain, May 1998.
- [10] J. A. Castellanos, J. D. Tardós, and G. Schmidt, "Building a global map of the environment of a mobile robot: The importance of correlations," in *Proc. 1997 IEEE Int. Conf. Robot. Automat.*, Albuquerque, NM, Apr. 1997, pp. 1053–1059.
- [11] J. D. Tardós, "Representing partial and uncertain sensorial information using the theory of symmetries," in *Proc. 1992 IEEE Int. Conf. Robot. Automat.*, Nice, France, 1992, pp. 1799–1804.
- [12] J. Neira, J. D. Tardós, J. Horn, and G. Schmidt, "Fusing range and intensity images for mobile robot localization," *IEEE Trans. Robot. Automat.*, vol. 15, pp. 76–84, Feb. 1999.
- [13] T. Bar-Shalom and T. E. Fortmann, *Tracking and Data Association*. New York: Academic, 1988.
- [14] J. A. Castellanos and J. D. Tardós, "Laser-based segmentation and localization for a mobile robot," in *Proceedings of the Sixth International Symposium (ISRAM)*, F. Pin, M. Jamshidi, and P. Dauchez, Eds. New York: ASME, 1996, pp. 101–108.
- [15] J. A. Castellanos, J. M. Martínez, J. Neira, and J. D. Tardós, "Simultaneous map building and localization for mobile robots: A multisensor fusion approach," in *Proc. 1998 IEEE Int. Conf. Robot. Automat.*, Leuven, Belgium, May 1998, pp. 1244–1249.

The SPmap: a probabilistic framework for simultaneous localization and map building

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Text Views**Abstract**[Authors](#)[References](#)[Citations](#)[Keywords](#)[Metrics](#)[More Like This](#)**Abstract:**

This article describes a rigorous and complete framework for the simultaneous localization and map building problem for mobile robots: the symmetries and perturbation map (SPmap), which is based on a general probabilistic representation of uncertain geometric information. We present a complete experiment with a LabMate/sup TM/ mobile robot navigating in a human-made indoor environment and equipped with a rotating 2D laser rangefinder. Experiments validate the appropriateness of our approach and provide a real measurement of the precision of the algorithms.

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ATTACHMENT E

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Real-Time Obstacle Avoidance for Manipulators and Mobile Robots

Abstract

This paper presents a unique real-time obstacle avoidance approach for manipulators and mobile robots based on the artificial potential field concept. Collision avoidance, traditionally considered a high level planning problem, can be effectively distributed between different levels of control, allowing real-time robot operations in a complex environment. This method has been extended to moving obstacles by using a time-varying artificial potential field. We have applied this obstacle avoidance scheme to robot arm mechanisms and have used a new approach to the general problem of real-time manipulator control. We reformulated the manipulator control problem as direct control of manipulator motion in operational space—the space in which the task is originally described—rather than as control of the task's corresponding joint space motion obtained only after geometric and kinematic transformation. Outside the obstacles' regions of influence, we caused the end effector to move in a straight line with an upper speed limit. The artificial potential field approach has been extended to collision avoidance for all manipulator links. In addition, a joint space artificial potential field is used to satisfy the manipulator internal joint constraints. This method has been implemented in the COSMOS system for a PUMA 560 robot. Real-time collision avoidance demonstrations on moving obstacles have been performed by using visual sensing.

1. Introduction

In previous research, robot collision avoidance has been a component of higher levels of control in hierarchical robot control systems. Collision avoidance has been treated as a planning problem, and research in this area has focused on the development of collision-

free path planning algorithms (Lozano-Perez 1980; Moravec 1980; Chatila 1981; Brooks 1983). These algorithms aim at providing the low level control with a path that will enable the robot to accomplish its assigned task free from any risk of collision.

From this perspective, the function of low level control is limited to the execution of elementary operations for which the paths have been precisely specified. The robot's interaction with its environment is then paced by the time cycle of high level control, which is generally several orders of magnitude slower than the response time of a typical robot. This places limits on the robot's real-time capabilities for precise, fast, and highly interactive operations in a cluttered and evolving environment. We will show that it is possible to extend greatly the function of low level control and to carry out more complex operations by coupling environment sensing feedback with the lowest level of control.

Increasing the capability of low level control has been the impetus for the work on real-time obstacle avoidance that we discuss here. Collision avoidance at the low level of control is not intended to replace high level functions or to solve planning problems. The purpose is to make better use of low level control capabilities in performing real-time operations. At this low level of control, the degree or *level of competence* (Brooks 1984) will remain less than that of higher level control.

The *operational space formulation* is the basis for the application of the potential field approach to robot manipulators. This formulation has its roots in the work on end-effector motion control and obstacle avoidance (Khatib and Le Maitre 1978) that has been implemented for an MA23 manipulator at the Laboratoire d'Automatique de Montpellier in 1978. The operational space approach has been formalized by constructing its basic tool, the equations of motion in the operational space of the manipulator end effector.

Details of this work have been published elsewhere (Khatib 1980; Khatib 1983; Khatib 1985). In this paper, we review the fundamentals of the operational space formulation and the artificial potential field concept. We present the integration of this collision avoidance approach into the operational space control system and its real-time implementation. The extension of this work to link collision avoidance is also developed.

2. Operational Space Formulation

An *operational coordinate system* is a set x of m_0 independent parameters describing the manipulator end-effector position and orientation in a frame of reference R_0 . For a nonredundant manipulator, these parameters form a set of configuration parameters in a domain of the operational space and constitute a system of generalized coordinates. The kinetic energy of the holonomic articulated mechanism is a quadratic form of the generalized velocities,

$$T(x, \dot{x}) = \frac{1}{2} \dot{x}^T \Lambda(x) \dot{x}, \quad (1)$$

where $\Lambda(x)$ designates the symmetric matrix of the quadratic form, *i.e.*, the kinetic energy matrix. Using the Lagrangian formalism, the end-effector equations of motion are given by

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = F, \quad (2)$$

where the Lagrangian $L(x, \dot{x})$ is

$$L(x, \dot{x}) = T(x, \dot{x}) - U(x), \quad (3)$$

and $U(x)$ represents the potential energy of the gravity. The symbol F is the operational force vector. These equations can be developed (Khatib 1980; Khatib 1983) and written in the form,

$$\Lambda(x) \ddot{x} + \mu(x, \dot{x}) + p(x) = F, \quad (4)$$

where $\mu(x, \dot{x})$ represents the centrifugal and Coriolis forces and $p(x)$ the gravity forces.

The control of manipulators in operational space is

based on the selection of F as a command vector. In order to produce this command, specific forces Γ must be applied with joint-based actuators. The relationship between F and the joint forces Γ is given by

$$\Gamma = J^T(q) F, \quad (5)$$

where q is the vector of the n joint coordinates and $J(q)$ the Jacobian matrix.

The decoupling of the end-effector motion in operational space is achieved by using the following structure of control,

$$F = \Lambda(x) F^* + \mu(x, \dot{x}) + p(x), \quad (6)$$

where F^* represents the command vector of the decoupled end effector that becomes equivalent to a *single unit mass*.

The extension of the operational space approach to redundant manipulator systems is discussed in Khatib (1980); Khatib (1983). The integration of active force control for assembly operations is presented in Khatib (1985).

3. The Artificial Potential Field Approach

We present this method in the context of manipulator collision avoidance. Its application to mobile robots is straightforward. The philosophy of the artificial potential field approach can be schematically described as follows:

The manipulator moves in a field of forces. The position to be reached is an attractive pole for the end effector and obstacles are repulsive surfaces for the manipulator parts.

We first consider the collision avoidance problem of a manipulator end effector with a single obstacle O . If x_d designates the goal position, the control of the manipulator end effector with respect to the obstacle O can be achieved by subjecting it to the artificial potential field,

$$U_{art}(x) = U_{x_d}(x) + U_O(x). \quad (7)$$

This leads to the following expression of the potential

energy in the Lagrangian (Eq. 3),

$$U(x) = U_{art}(x) + U_g(x), \quad (8)$$

where $U_g(x)$ represents the gravity potential energy. Using Lagrange's (Eq. 2) and taking into account the end-effector dynamic decoupling (Eq. 6), the command vector F_{art}^* of the decoupled end effector that corresponds to applying the artificial potential field U_{art} (Eq. 7) can be written as

$$F_{art}^* = F_{x_d}^* + F_O^*, \quad (9)$$

with

$$\begin{aligned} F_{x_d}^* &= -\text{grad}[U_{x_d}(x)], \\ F_O^* &= -\text{grad}[U_O(x)]. \end{aligned} \quad (10)$$

The symbol $F_{x_d}^*$ is an attractive force allowing the point x of the end effector to reach the goal position x_d , and F_O^* represents a *Force Inducing an Artificial Repulsion from the Surface* (FIRAS, from the French) of the obstacle created by the potential field $U_O(x)$. The symbol $F_{x_d}^*$ corresponds to the proportional term, i.e. $-k_p(x - x_d)$, in a conventional PD servo where k_p is the position gain. The attractive potential field $U_{x_d}(x)$ is simply

$$U_{x_d}(x) = \frac{1}{2}k_p(x - x_d)^2. \quad (11)$$

$U_O(x)$ is selected such that the artificial potential field $U_{art}(x)$ is a positive continuous and differentiable function which attains its zero minimum when $x = x_d$. The articulated mechanical system subjected to $U_{art}(x)$ is stable. Asymptotic stabilization of the system is achieved by adding dissipative forces proportional to $\dot{x}$. Let k_v be the velocity gain; the forces contributing to the end-effector motion and stabilization are of the form,

$$F_m^* = -k_p(x - x_d) - k_v\dot{x}. \quad (12)$$

This command vector is inadequate to control the manipulator for tasks that involve large end-effector motion toward a goal position without path specification. For such a task, it is better for the end effector to move in a straight line with an upper speed limit.

Rewriting Eq. (12) leads to the following expression, which can be interpreted as specifying a desired velocity vector in a pure velocity servo-control,

$$\dot{x}_d = \frac{k_p}{k_v}(x_d - x). \quad (13)$$

Let V_{max} designate the assigned speed limit. The limitation of the end-effector velocity magnitude can then be obtained (Khatib, Llibre, and Mampey 1978),

$$F_m^* = -k_v(\dot{x} - v\dot{x}_d), \quad (14)$$

where

$$v = \min\left(1, \frac{V_{max}}{\sqrt{\dot{x}_d^T \dot{x}_d}}\right). \quad (15)$$

With this scheme, the velocity vector $\dot{x}$ is controlled to be pointed toward the goal position while its magnitude is limited to V_{max} . The end effector will then travel at that speed in a straight line, except during the acceleration and deceleration segments or when it is inside the repulsive potential field regions of influence.

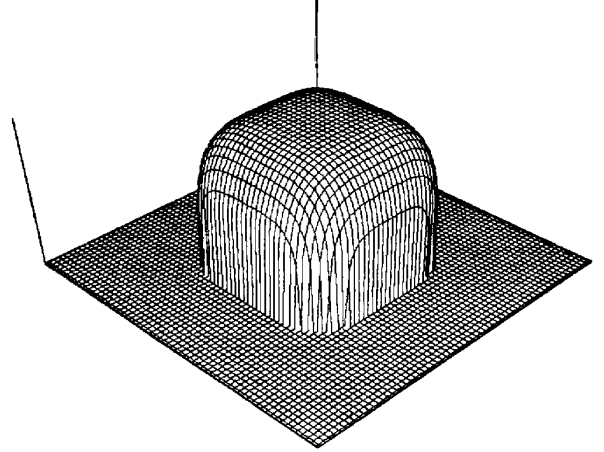
4. FIRAS Function

The artificial potential field $U_O(x)$ should be designed to meet the manipulator stability condition and to create at each point on the obstacle's surface a potential barrier which becomes negligible beyond that surface. Specifically, $U_O(x)$ should be a nonnegative continuous and differentiable function whose value tends to infinity as the end effector approaches the obstacle's surface. In order to avoid undesirable perturbing forces beyond the obstacle's vicinity, the influence of this potential field must be limited to a given region surrounding the obstacle.

Using analytic equations $f(x) = 0$ for obstacle description, the first artificial potential field function used (Khatib and Le Maitre 1978) was based on the values of the function $f(x)$,

$$U_O(x) = \begin{cases} \frac{1}{2} \eta \left(\frac{1}{f(x)} - \frac{1}{f(x_0)} \right)^2 & \text{if } f(x) \leq f(x_0), \\ 0 & \text{if } f(x) > f(x_0). \end{cases} \quad (16)$$

Fig. 1. An n -ellipsoid with $n = 4$.



The region of influence of this potential field is bounded by the surfaces $f(x) = 0$ and $f(x) = f(x_0)$, where x_0 is a given point in the vicinity of the obstacle and η a constant gain. This potential function can be obtained very simply in real time since it does not require any distance calculations. However, this potential is difficult to use for asymmetric obstacles where the separation between an obstacle's surface and equipotential surfaces can vary widely.

Using the shortest distance to an obstacle O , we have proposed (Khatib 1980) the following artificial potential field;

$$U_O(x) = \begin{cases} \frac{1}{2} \eta \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right)^2 & \text{if } \rho \leq \rho_0, \\ 0 & \text{if } \rho > \rho_0, \end{cases} \quad (17)$$

where ρ_0 represents the limit distance of the potential field influence and ρ the shortest distance to the obstacle O . The selection of the distance ρ_0 will depend on the end effector operating speed $V_{\max}$ and on its deceleration ability. End-effector acceleration characteristics are discussed in Khatib and Burdick (1985).

Any point of the robot can be subjected to the artificial potential field. The control of a *Point Subjected to the Potential* (PSP) with respect to an obstacle O is achieved using the FIRAS function,

$$F_{(O, \text{PSP})}^* = \begin{cases} \eta \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2} \frac{\partial \rho}{\partial \mathbf{x}} & \text{if } \rho \leq \rho_0, \\ \mathbf{0} & \text{if } \rho > \rho_0, \end{cases} \quad (18)$$

where $\frac{\partial \rho}{\partial \mathbf{x}}$ denotes the partial derivative vector of the distance from the PSP to the obstacle,

$$\frac{\partial \rho}{\partial \mathbf{x}} = \left[\frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y} \frac{\partial \rho}{\partial z} \right]^T. \quad (19)$$

The joint forces corresponding to $F_{(O, \text{PSP})}^*$ are obtained using the Jacobian matrix associated with this PSP. Observing Eqs. (6) and (9), these forces are given by

$$\Gamma_{(O, \text{PSP})} = J_{\text{PSP}}^T(q) \Lambda(x) F_{(O, \text{PSP})}^*. \quad (20)$$

5. Obstacle Geometric Modelling

Obstacles are described by the composition of *primitives*. A typical geometric model base includes primitives such as a point, line, plane, ellipsoid, parallelepiped, cone, and cylinder. The first artificial potential field (Eq. 16) requires analytic equations for the description of obstacles. We have developed analytic equations representing envelopes which best approximate the shapes of primitives such as a *parallelepiped*, *finite cylinder*, and *cone*.

The surface, termed an n -ellipsoid, is represented by the equation,

$$\left(\frac{x}{a} \right)^{2n} + \left(\frac{y}{b} \right)^{2n} + \left(\frac{z}{c} \right)^{2n} = 1, \quad (21)$$

and tends to a parallelepiped of dimensions (2a, 2b, 2c) as n tends to infinity. A good approximation is obtained with $n = 4$, as shown in Fig. 1. A cylinder of elliptical cross section (2a, 2b) and of length $2c$ can be approximated by the so-called n -cylinder equation,

$$\left(\frac{x}{a} \right)^2 + \left(\frac{y}{b} \right)^2 + \left(\frac{z}{c} \right)^{2n} = 1. \quad (22)$$

The analytic description of primitives is not necessary for the artificial potential field (Eq. 17) since the

Fig. 2. Displacement of a 4 dof manipulator inside an enclosure.

continuity and differentiability requirement is on the shortest distance to the obstacle. The primitives above, and more generally all convex primitives, comply with this requirement.

Determining the orthogonal distance to an n-ellipsoid or to an n-cylinder requires the solution of a complicated system of equations. A variational procedure for the distance evaluation has been developed that avoids this costly computation. The distance expressions for other primitives are presented in Appendices I through III.

6. Robot Obstacle Avoidance

An obstacle O_i is described by a set of primitives $\{P_p\}$. The superposition property (additivity) of potential fields enables the control of a given point of the manipulator with respect to this obstacle by using the sum of the relevant gradients,

$$F_{O_i, psp}^* = \sum_p F_{(P_p, psp)}^* \quad (23)$$

Control of this point for several obstacles is obtained using

$$F_{psp}^* = \sum_i F_{(O_i, psp)}^* \quad (24)$$

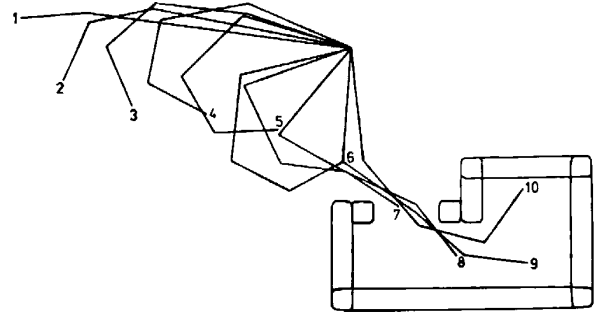
It is also feasible to have different points on the manipulator controlled with respect to different obstacles. The resulting joint force vector is given by

$$\Gamma_{obstacles} = \sum_j J_{pspj}^T(q) \Lambda(x) F_{pspj}^* \quad (25)$$

Specifying an adequate number of PSPs enables the protection of all of the manipulator's parts. An example of a dynamic simulation for a redundant 4 dof manipulator operating in the plane is shown in the display of Fig. 2.

The artificial potential field approach can be extended to *moving obstacles* since stability of the mechanism persists with a continuously time-varying potential field.

The manipulator obstacle avoidance problem has been formulated in terms of *collision avoidance of*



links rather than points. Link collision avoidance is achieved by continuously controlling the link's closest point to the obstacle. At most, n PSPs then have to be considered. Additional links can be artificially introduced or the length of the last link can be extended to account for the manipulator tool or load. In an articulated chain, a link can be represented as the line segment defined by the Cartesian positions of its two neighboring joints. In a frame of reference R , a point $m(x, y, z)$ of the link bounded by $m_1(x_1, y_1, z_1)$ and $m_2(x_2, y_2, z_2)$ is described by the parametric equations,

$$\begin{aligned} x &= x_1 + \lambda(x_2 - x_1), \\ y &= y_1 + \lambda(y_2 - y_1), \\ z &= z_1 + \lambda(z_2 - z_1). \end{aligned} \quad (26)$$

The problem of obtaining the link's shortest distance to a parallelepiped can be reduced to that of finding the link's closest point to a vertex, edge, or face. The analytic expressions of the link's closest point, the distance, and its partial derivatives are given in Appendix I. In Appendices II and III these expressions are given for a cylinder and a cone, respectively.

7. Joint Limit Avoidance

The potential field approach can be used to satisfy the manipulator internal joint constraints. Let $\underline{q}_i$ and $\bar{q}_i$ be respectively the minimal and maximal bounds of the i^{th} joint coordinate q_i . These boundaries can contain q_i by creating barriers of potential at each of the hyperplanes ($q_i = \underline{q}_i$) and ($q_i = \bar{q}_i$). The corresponding joint

forces are

$$\gamma_{q_i} = \begin{cases} \eta \left(\frac{1}{\rho_i} - \frac{1}{\rho_{i(0)}} \right) \frac{1}{\rho_i^2} & \text{if } \rho_i \leq \rho_{i(0)}, \\ 0 & \text{if } \rho_i > \rho_{i(0)}, \end{cases} \quad (27)$$

and

$$\gamma_{\bar{q}_i} = \begin{cases} -\eta \left(\frac{1}{\bar{\rho}_i} - \frac{1}{\bar{\rho}_{i(0)}} \right) \frac{1}{\bar{\rho}_i^2} & \text{if } \bar{\rho}_i \leq \bar{\rho}_{i(0)}, \\ 0 & \text{if } \bar{\rho}_i > \bar{\rho}_{i(0)}, \end{cases} \quad (28)$$

where $\rho_{i(0)}$ and $\bar{\rho}_{i(0)}$ represent the distance limit of the potential field influence. The distances ρ_i and $\bar{\rho}_i$ are defined by

$$\begin{aligned} \rho_i &= q_i - \underline{q}_i, \\ \bar{\rho}_i &= \bar{q}_i - q_i. \end{aligned} \quad (29)$$

8. Level of Competence

The potential field concept is indeed an attractive approach to the collision avoidance problem and much research has recently been focused on its applications to robot control (Kuntze and Schill 1982; Hogan 1984; Krogh 1984). However, the complexity of tasks that can be implemented with this approach is limited. In a cluttered environment, local minima can occur in the resultant potential field. This can lead to a stable positioning of the robot before reaching its goal. While local procedures can be designed to exit from such configurations, limitations for complex tasks will remain. This is because the approach has a *local* perspective of the robot environment.

Nevertheless, the resulting potential field does provide the global information necessary and a collision-free path, if attainable, can be found by linking the absolute minima of the potential. Linking these minima requires a computationally expensive exploration of the potential field. This goes beyond the real-time control we are concerned with here but can be considered as an integrated part of higher level control. Work on high level collision-free path planning based on the potential field concept has been investigated in Buckley (1985).

9. Real-Time Implementation

Finally, the global control system integrating the potential field concept with the operational space approach has the following structure:

$$\Gamma = \Gamma_{motion} + \Gamma_{obstacles} + \Gamma_{joint-limit}, \quad (30)$$

where Γ_{motion} can be developed (Khatib 1983) in the form,

$$\Gamma_{motion} = J^T(q) \Lambda(q) F_m^* + \tilde{B}(q) [\dot{q} \dot{q}] + \tilde{C}(q) [\dot{q}^2] + g(q), \quad (31)$$

where $\tilde{B}(q)$ and $\tilde{C}(q)$ are the $n \times n(n-1)/2$ and $n \times n$ matrices of the joint forces under the mapping into joint space of the end-effector Coriolis and centrifugal forces. The symbol $g(q)$ is the gravity force vector, and the symbolic notations $[\dot{q} \dot{q}]$ and $[\dot{q}^2]$ are for the $n(n-1)/2 \times 1$ and $n \times 1$ column matrices,

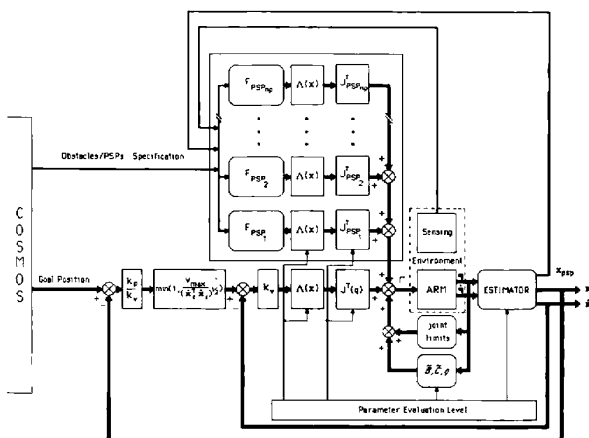
$$\begin{aligned} [\dot{q} \dot{q}] &= [\dot{q}_1 \dot{q}_2 \dot{q}_1 \dot{q}_3 \dots \dot{q}_{n-1} \dot{q}_n]^T, \\ [\dot{q}^2] &= [\dot{q}_1^2 \dot{q}_2^2 \dots \dot{q}_n^2]^T. \end{aligned} \quad (32)$$

In this control structure, dynamic decoupling of the end effector is obtained using the end-effector dynamic parameters (EEDP) $\Lambda(q)$, $\tilde{B}(q)$, $\tilde{C}(q)$ and $g(q)$, which are configuration dependent. In real time, these parameters can be computed at a lower rate than that of the servo control. In addition, the integration of an operational position and velocity estimator allows a reduction in the rate of end-effector position computation, which involves evaluations of the manipulator geometric model. This leads to a two-level control system architecture (Khatib 1985):

- A low rate *parameter evaluation level* that updates the end-effector dynamic coefficients, the Jacobian matrix, and the geometric model.
- A high rate *servo control level* that computes the command vector using the estimator and the updated dynamic coefficients.

The control system architecture is shown in Fig. 3 where np represents the number of PSPs. The Jacobian matrices $J_{psp_i}^T(q)$ have common factors with the end-effector Jacobian matrix $J^T(q)$. Thus, their evaluation does not require significant additional computation.

Fig. 3. Control system architecture.



10. Applications

This approach has been implemented in an experimental manipulator programming system *Control in Operational Space of a Manipulator-with-Obstacles System* (COSMOS). Demonstration of real-time collision avoidance with links and moving obstacles (Khatib et al. 1984) have been performed using a PUMA 560 and a Machine Intelligence Corporation vision module.

We have also demonstrated real-time end-effector motion and active force control operations with the COSMOS system using wrist and finger sensing. These include contact, slide, insertion, and compliance operations (Khatib, Burdick, and Armstrong 1985).

In the current multiprocessor implementation (PDP 11/45 and PDP 11/60), the rate of the servo control level is 225 Hz while the coefficient evaluation level runs at 100 Hz.

11. Summary and Discussion

We have described the formulation and implementation of a real-time obstacle avoidance approach based on the artificial potential field concept. Collision avoidance, generally treated as high level planning, has been demonstrated to be an effective component of low level real-time control in this approach. Further, we have briefly presented our operational space for-

mulation of manipulator control that provides the basis for this obstacle avoidance approach, and have described the two-level architecture designed to increase the real-time performance of the control system.

The integration of this low level control approach with a high level planning system seems to be one of the more promising solutions to the obstacle avoidance problem in robot control. With this approach, the problem may be treated in two stages:

- at high level control, generating a global strategy for the manipulator's path in terms of intermediate goals (rather than finding an accurate collision-free path);
- at the low level, producing the appropriate commands to attain each of these goals, taking into account the detailed geometry and motion of manipulator and obstacle, and making use of real-time obstacle sensing (low level vision and proximity sensors).

By extending low level control capabilities and reducing the high level path planning burden, the integration of this collision avoidance approach into a multi-level robot control structure will improve the real-time performance of the overall robot control system. Potential applications of this control approach include moving obstacle avoidance, collision avoidance in grasping operations, and obstacle avoidance problems involving multimanipulators or multifingered hands.

Appendix I: Link Distance to a Parallelepiped

The axes of the frame of reference R are chosen to be the parallelepiped axes of symmetry. The link's length is l and the dot product is $(\cdot)$.

DISTANCE TO A VERTEX

The closest point m of the line (Eq. 26) to the vertex v is such that

$$\lambda = \frac{(vm_1) \cdot (m_1 m_2)}{l^2}. \quad (A1-1)$$

The link's closest point m is identical to m_1 if $\lambda \leq 0$; it is identical to m_2 if $\lambda \geq 1$ and it is given by Eq. (26) otherwise. The shortest distance is therefore,

$$\rho = \begin{cases} [\rho_1^2 - \lambda^2 l^2]^{1/2}, & \text{if } 0 \leq \lambda \leq 1, \\ \rho_1, & \text{if } \lambda < 0, \\ \rho_2, & \text{if } \lambda > 1, \end{cases} \quad (\text{A1-2})$$

where ρ_1 and ρ_2 are the distance to the vertex from m_1 and m_2 , respectively. The distance partial derivatives are

$$\frac{\partial \rho}{\partial \mathbf{x}} = \left[\frac{x}{\rho} \frac{y}{\rho} \frac{z}{\rho} \right]^T. \quad (\text{A1-3})$$

DISTANCE TO AN EDGE

By a projection in the plane perpendicular to the considered edge (xoy , $yozy$, or zox), this problem can be reduced to that of finding the distance to a vertex in the plane. This leads to expressions similar to those of (A1-1)–(A1-3) with a zero partial derivative of the distance *w.r.t.* the axis parallel to the edge.

DISTANCE TO A FACE

In this case, the distance can be directly obtained by comparing the absolute values of the coordinates of m_1 and m_2 along the axis perpendicular to the face. The partial derivative vector is identical to the unit normal vector of this face.

Appendix II: Link Distance to a Cylinder

The frame of reference R is chosen such that its z -axis is the cylinder axis of symmetry and its origin is the cylinder center of mass. The cylinder radius and height are designated by r and h , respectively.

DISTANCE TO THE CIRCULAR SURFACE

The closest point of the link (Eq. 27) to the circular surface of the cylinder can be deduced from the dis-

tance to a vertex considered in the xoy plane and by allowing for the radius r .

DISTANCE TO THE CIRCULAR EDGES

The closest distance to the cylinder circular edge can be obtained from that of the circular surface by taking into account the relative z -coordinate of m to the circular edge, *i.e.*, $(z + h/2)$ for the base and $(z - h/2)$ for the top. The distance partial derivative vector results from the torus equation,

$$[x^2 + y^2 + (z \pm h/2)^2 - r^2 - \rho^2]^2 = 4r^2[\rho^2 - (z \pm h/2)^2]. \quad (\text{A2-1})$$

This vector is

$$\frac{\partial \rho}{\partial \mathbf{x}} = \left[\zeta \frac{x}{\rho} \zeta \frac{y}{\rho} \frac{z \pm h/2}{\rho} \right]^T, \quad (\text{A2-2})$$

with

$$\zeta = \frac{x^2 + y^2 + (z \pm h/2)^2 - r^2 - \rho^2}{x^2 + y^2 + (z \pm h/2)^2 + r^2 - \rho^2}. \quad (\text{A2-3})$$

The distance to the planar surfaces is straightforward and can be simply obtained as in Appendix I.

Appendix III: Link Distance to a Cone

In this case, the frame of reference R is chosen such that its z -axis is the cone axis of symmetry and its origin is the center of the cone circular base. The cone base radius, height and half angle are represented respectively by r , h , and β .

DISTANCE TO THE CONE-SHAPED SURFACE

The problem of locating $m(x, y, z)$ is identical to that for the cylinder case. The distance can be written as

$$\rho = z \sin(\beta) + (\sqrt{x^2 + y^2} - r) \cos(\beta). \quad (\text{A3-1})$$

The partial derivatives come from the equation,

$$x^2 + y^2 = r_z^2, \quad (\text{A3-2})$$

where

$$r_z = \tan(\beta)[h + \rho \sin(\beta) - z]. \quad (\text{A3-3})$$

They are

$$\frac{\partial \rho}{\partial x} = \left[\frac{x}{r_z \tan(\beta)} \frac{y}{r_z \tan(\beta)} \frac{1}{\sin(\beta)} \right]^T. \quad (\text{A3-4})$$

The problem of the distance to the cone circular edge is identical to that of the cylinder circular edge in Appendix II. The distance to the cone vertex is solved as in Appendix I.

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REFERENCES

- Brooks, R. 1983. Solving the find-path problem by good representation of free space. *IEEE Sys., Man Cyber. SMC*-13:190-197.
- Brooks, R. 1984 Aug. 20-23, Kyoto, Japan. Aspects of mobile robot visual map making. *2nd Int. Symp. Robotics Res.*
- Buckley, C. 1985. *The application of continuum methods to path planning*. Ph.D. Thesis (in progress). Stanford University, Department of Mechanical Engineering.
- Chatila, R. 1981. *Système de navigation pour un robot mobile autonome: modélisation et processus décisionnels*. Thèse de Docteur-Ingénieur. Université Paul Sabatier. Toulouse, France.
- Hogan, N. 1984 June 6-8, San Diego, California. Impedance control: an approach to manipulation. 1984 *Amer. Control Conf.*
- Khatib, O., Llibre, M. and Mampey, R. 1978. Fonction decision-commande d'un robot manipulateur. *Rapport No. 2/7156*. DERA/CERT. Toulouse, France.
- Khatib, O. and Le Maitre, J. F. 1978 September 12-15, Udine, Italy. Dynamic control of manipulators operating in a complex environment. *Proc. 3rd CISM-IFTOMM Symp. Theory Practice Robots Manipulators*, 267-282. Elsevier. 1979.
- Khatib, O. 1980. *Commande dynamique dans l'espace opérationnel des robots manipulateurs en présence d'obstacles*. Thèse de Docteur-Ingénieur. École Nationale Supérieure de l'Aéronautique et de l'Espace (ENSAE). Toulouse, France.
- Khatib, O. 1983 December 15-20, New Delhi. Dynamic control of manipulators in operational space. *6th CISM-IFTOMM Congress Theory Machines Mechanisms*, 1128-1131.
- Khatib, O., et al. 1984 June, *Robotics in three acts* (Film). Stanford University. Artificial Intelligence Laboratory.
- Khatib, O. 1985 September 11-13, Tokyo. The operational space formulation in robot manipulators control. *15th Int. Symp. Indust. Robots*.
- Khatib, O. and Burdick, J. 1985 November, Miami, Florida. Dynamic optimization in manipulator design: the operational space formulation. *ASME Winter Annual Meeting*.
- Khatib, O., Burdick, J., and Armstrong, B. 1985. *Robotics in three acts-Part II* (Film). Stanford University, Artificial Intelligence Laboratory.
- Krogh, B. 1984 August, Bethlehem, Pennsylvania. A generalized potential field approach to obstacle avoidance control. *SME Conf. Proc. Robotics Research: The Next Five Years and Beyond*.
- Kuntze, H. B., and Schill, W. 1982 June 9-11, Paris. Methods for collision avoidance in computer controlled industrial robots. *12th ISIR*.
- Lozano-Perez, T. 1980. Spatial planning: a configuration space approach. *AI Memo 605*. Cambridge, Mass. MIT Artificial Intelligence Laboratory.
- Moravec, H. P. 1980. *Obstacle avoidance and navigation in the real world by a seeing robot rover*. Ph.D. Thesis. Stanford University, Artificial Intelligence Laboratory.

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Abstract

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References

Brooks, R. 1983. Solving the find-path problem by good representation of free space. *IEEE Sys., Man Cyber.* SMC-13:190-197.

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Brooks, R. 1984 Aug. 20-23, Kyoto, Japan. *Aspects of mobile robot visual map making* 2nd Int. Symp. Robotics Res.

[Google Scholar](#)

Buckley, C. 1985. *The application of continuum methods to path planning. Ph.D. Thesis (in progress)*. Stanford University, Department of Mechanical Engineering.

[Google Scholar](#)

Chatila, R. 1981. *Système de navigation pour un robot mobile autonome: modélisation et processus décisionnels* Thèse de Docteur-Ingénieur. Université Paul Sabatier. Toulouse, France.

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Hogan, N. 1984 June 6-8, San Diego, California. *Impedance control: an approach to manipulation*. 1984 Amer. Control Conf.

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Khatib, O., Llibre, M. and Mampey, R. 1978. *Fonction decision-commande d'un robot manipulateur*. Rapport No. 2/7156. DERA/CERT. Toulouse, France.

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Khatib, O. and Le Maitre, J.F. 1978 September 12-15, Udine, Italy. *Dynamic control of manipulators operating in a complex environment*. Proc. 3rd CISM-IFTOMM Symp. Theory Practice of Robots Manipulators, 267-282. Elsevier . 1979.

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. Khatib, O., Burdick, J., and Armstrong, B. 1985. *Robotics in three acts-Part II (Film)*. Stanford University, Artificial Intelligence Laboratory.

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Krogh, B. 1984 August, Bethlehem, Pennsylvania. *A generalized potential field approach to obstacle avoidance control. SME Conf. Proc. Robotics Research: The Next Five Years and Beyond*

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. Kuntze, H.B., and Schill, W. 1982 June 9-11, Paris. *Methods for collision avoidance in computer controlled industrial robots. 12th ISIR*

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. Lozano-Perez, T. 1980. *Spatial planning: a configuration space approach*. AI Memo 605. Cambridge, Mass. MIT Artificial Intelligence Laboratory.

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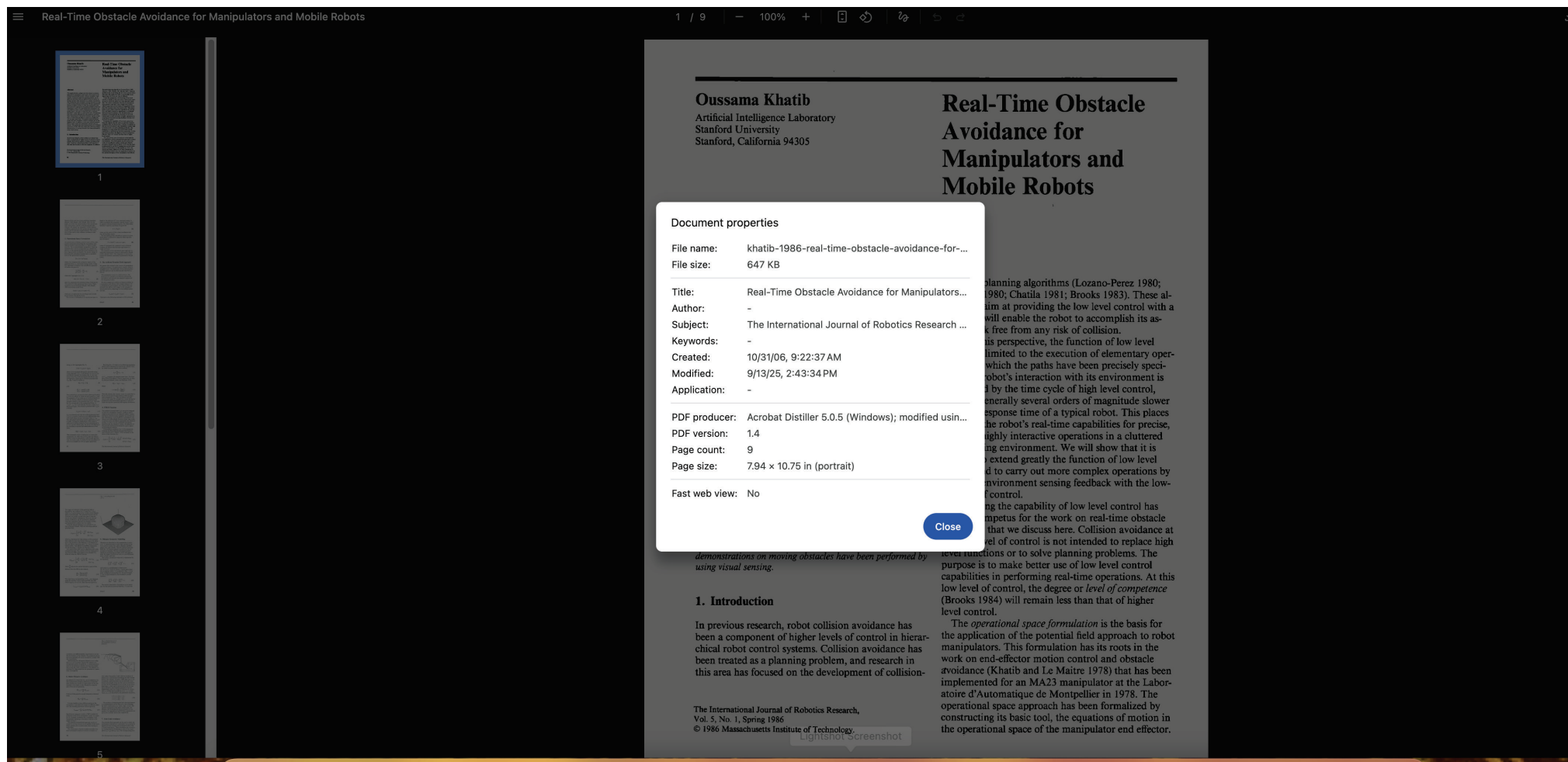
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ATTACHMENT G



ATTACHMENT H

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Real-Time Obstacle Avoidance for Manipulators and Mobile Robots

Abstract

This paper presents a unique real-time obstacle avoidance approach for manipulators and mobile robots based on the artificial potential field concept. Collision avoidance, traditionally considered a high level planning problem, can be effectively distributed between different levels of control, allowing real-time robot operations in a complex environment. This method has been extended to moving obstacles by using a time-varying artificial potential field. We have applied this obstacle avoidance scheme to robot arm mechanisms and have used a new approach to the general problem of real-time manipulator control. We reformulated the manipulator control problem as direct control of manipulator motion in operational space—the space in which the task is originally described—rather than as control of the task's corresponding joint space motion obtained only after geometric and kinematic transformation. Outside the obstacles' regions of influence, we caused the end effector to move in a straight line with an upper speed limit. The artificial potential field approach has been extended to collision avoidance for all manipulator links. In addition, a joint space artificial potential field is used to satisfy the manipulator internal joint constraints. This method has been implemented in the COSMOS system for a PUMA 560 robot. Real-time collision avoidance demonstrations on moving obstacles have been performed by using visual sensing.

1. Introduction

In previous research, robot collision avoidance has been a component of higher levels of control in hierarchical robot control systems. Collision avoidance has been treated as a planning problem, and research in this area has focused on the development of collision-

free path planning algorithms (Lozano-Perez 1980; Moravec 1980; Chatila 1981; Brooks 1983). These algorithms aim at providing the low level control with a path that will enable the robot to accomplish its assigned task free from any risk of collision.

From this perspective, the function of low level control is limited to the execution of elementary operations for which the paths have been precisely specified. The robot's interaction with its environment is then paced by the time cycle of high level control, which is generally several orders of magnitude slower than the response time of a typical robot. This places limits on the robot's real-time capabilities for precise, fast, and highly interactive operations in a cluttered and evolving environment. We will show that it is possible to extend greatly the function of low level control and to carry out more complex operations by coupling environment sensing feedback with the lowest level of control.

Increasing the capability of low level control has been the impetus for the work on real-time obstacle avoidance that we discuss here. Collision avoidance at the low level of control is not intended to replace high level functions or to solve planning problems. The purpose is to make better use of low level control capabilities in performing real-time operations. At this low level of control, the degree or *level of competence* (Brooks 1984) will remain less than that of higher level control.

The *operational space formulation* is the basis for the application of the potential field approach to robot manipulators. This formulation has its roots in the work on end-effector motion control and obstacle avoidance (Khatib and Le Maitre 1978) that has been implemented for an MA23 manipulator at the Laboratoire d'Automatique de Montpellier in 1978. The operational space approach has been formalized by constructing its basic tool, the equations of motion in the operational space of the manipulator end effector.

Details of this work have been published elsewhere (Khatib 1980; Khatib 1983; Khatib 1985). In this paper, we review the fundamentals of the operational space formulation and the artificial potential field concept. We present the integration of this collision avoidance approach into the operational space control system and its real-time implementation. The extension of this work to link collision avoidance is also developed.

2. Operational Space Formulation

An *operational coordinate system* is a set x of m_0 independent parameters describing the manipulator end-effector position and orientation in a frame of reference R_0 . For a nonredundant manipulator, these parameters form a set of configuration parameters in a domain of the operational space and constitute a system of generalized coordinates. The kinetic energy of the holonomic articulated mechanism is a quadratic form of the generalized velocities,

$$T(x, \dot{x}) = \frac{1}{2} \dot{x}^T \Lambda(x) \dot{x} \quad (1)$$

where $\Lambda(x)$ designates the symmetric matrix of the quadratic form, *i.e.*, the kinetic energy matrix. Using the Lagrangian formalism, the end-effector equations of motion are given by

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = F \quad (2)$$

where the Lagrangian $L(x, \dot{x})$ is

$$L(x, \dot{x}) = T(x, \dot{x}) - U(x), \quad (3)$$

and $U(x)$ represents the potential energy of the gravity. The symbol F is the operational force vector. These equations can be developed (Khatib 1980; Khatib 1983) and written in the form,

$$\Lambda(x) \ddot{x} + \mu(x, \dot{x}) + p(x) = F, \quad (4)$$

where $\mu(x, \dot{x})$ represents the centrifugal and Coriolis forces and $p(x)$ the gravity forces.

The control of manipulators in operational space is

based on the selection of F as a command vector. In order to produce this command, specific forces Γ must be applied with joint-based actuators. The relationship between F and the joint forces Γ is given by

$$\Gamma = J^T(q) F, \quad (5)$$

where q is the vector of the n joint coordinates and $J(q)$ the Jacobian matrix.

The decoupling of the end-effector motion in operational space is achieved by using the following structure of control,

$$F = \Lambda(x) F^* + \mu(x, \dot{x}) + p(x), \quad (6)$$

where F^* represents the command vector of the decoupled end effector that becomes equivalent to a *single unit mass*.

The extension of the operational space approach to redundant manipulator systems is discussed in Khatib (1980; Khatib (1983). The integration of active force control for assembly operations is presented in Khatib (1985).

3. The Artificial Potential Field Approach

We present this method in the context of manipulator collision avoidance. Its application to mobile robots is straightforward. The philosophy of the artificial potential field approach can be schematically described as follows:

The manipulator moves in a field of forces. The position to be reached is an attractive pole for the end effector and obstacles are repulsive surfaces for the manipulator parts.

We first consider the collision avoidance problem of a manipulator end effector with a single obstacle O . If x_d designates the goal position, the control of the manipulator end effector with respect to the obstacle O can be achieved by subjecting it to the artificial potential field,

$$U_{art}(x) = U_x(x) + U_O(x) \quad (7)$$

This leads to the following expression of the potential

energy in the Lagrangian (Eq. 3),

$$U(x) = U_{art}(x) + U_g(x), \quad (8)$$

where $U_g(x)$ represents the gravity potential energy. Using Lagrange's (Eq. 2) and taking into account the end-effector dynamic decoupling (Eq. 6), the command vector F_{art}^* of the decoupled end effector that corresponds to applying the artificial potential field U_{art} (Eq. 7) can be written as

$$F_{art}^* = F_{x_d}^* + F_O^*, \quad (9)$$

with

$$\begin{aligned} F_{x_d}^* &= -\text{grad}[U_{x_d}(x)], \\ F_O^* &= -\text{grad}[U_O(x)]. \end{aligned} \quad (10)$$

The symbol $F_{x_d}^*$ is an attractive force allowing the point x of the end effector to reach the goal position x_d , and F_O^* represents a *Force Inducing an Artificial Repulsion from the Surface* (FIRAS, from the French) of the obstacle created by the potential field $U_O(x)$. The symbol $F_{x_d}^*$ corresponds to the proportional term, i.e. $-k_p(x - x_d)$, in a conventional PD servo where k_p is the position gain. The attractive potential field $U_{x_d}(x)$ is simply

$$U_{x_d}(x) = \frac{1}{2}k_p(x - x_d)^2. \quad (11)$$

$U_O(x)$ is selected such that the artificial potential field $U_{art}(x)$ is a positive continuous and differentiable function which attains its zero minimum when $x = x_d$. The articulated mechanical system subjected to $U_{art}(x)$ is stable. Asymptotic stabilization of the system is achieved by adding dissipative forces proportional to $\dot{x}$. Let k_v be the velocity gain; the forces contributing to the end-effector motion and stabilization are of the form,

$$F_m^* = -k_p(x - x_d) - k_v\dot{x}. \quad (12)$$

This command vector is inadequate to control the manipulator for tasks that involve large end-effector motion toward a goal position without path specification. For such a task, it is better for the end effector to move in a straight line with an upper speed limit.

Rewriting Eq. (12) leads to the following expression, which can be interpreted as specifying a desired velocity vector in a pure velocity servo-control,

$$\dot{x}_d = \frac{k_p}{k_v}(x_d - x). \quad (13)$$

Let V_{max} designate the assigned speed limit. The limitation of the end-effector velocity magnitude can then be obtained (Khatib, Llibre, and Mampey 1978),

$$F_m^* = -k_v(\dot{x} - v\dot{x}_d), \quad (14)$$

where

$$v = \min\left(1, \frac{V_{max}}{\sqrt{\dot{x}_d^T \dot{x}_d}}\right). \quad (15)$$

With this scheme, the velocity vector $\dot{x}$ is controlled to be pointed toward the goal position while its magnitude is limited to V_{max} . The end effector will then travel at that speed in a straight line, except during the acceleration and deceleration segments or when it is inside the repulsive potential field regions of influence.

4. FIRAS Function

The artificial potential field $U_O(x)$ should be designed to meet the manipulator stability condition and to create at each point on the obstacle's surface a potential barrier which becomes negligible beyond that surface. Specifically, $U_O(x)$ should be a nonnegative continuous and differentiable function whose value tends to infinity as the end effector approaches the obstacle's surface. In order to avoid undesirable perturbing forces beyond the obstacle's vicinity, the influence of this potential field must be limited to a given region surrounding the obstacle.

Using analytic equations $f(x) = 0$ for obstacle description, the first artificial potential field function used (Khatib and Le Maitre 1978) was based on the values of the function $f(x)$,

$$U_O(x) = \begin{cases} \frac{1}{2} \eta \left(\frac{1}{f(x)} - \frac{1}{f(x_0)} \right)^2 & \text{if } f(x) \leq f(x_0), \\ 0 & \text{if } f(x) > f(x_0). \end{cases} \quad (16)$$

Fig. 1. An n -ellipsoid with $n = 4$.

The region of influence of this potential field is bounded by the surfaces $f(x) = 0$ and $f(x) = f(x_0)$, where x_0 is a given point in the vicinity of the obstacle and η a constant gain. This potential function can be obtained very simply in real time since it does not require any distance calculations. However, this potential is difficult to use for asymmetric obstacles where the separation between an obstacle's surface and equipotential surfaces can vary widely.

Using the shortest distance to an obstacle O , we have proposed (Khatib 1980) the following artificial potential field;

$$U_O(x) = \begin{cases} \frac{1}{2} \eta \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right)^2 & \text{if } \rho \leq \rho_0, \\ 0 & \text{if } \rho > \rho_0, \end{cases} \quad (17)$$

where ρ_0 represents the limit distance of the potential field influence and ρ the shortest distance to the obstacle O . The selection of the distance ρ_0 will depend on the end effector operating speed $V_{\max}$ and on its deceleration ability. End-effector acceleration characteristics are discussed in Khatib and Burdick (1985).

Any point of the robot can be subjected to the artificial potential field. The control of a *Point Subjected to the Potential* (PSP) with respect to an obstacle O is achieved using the FIRAS function,

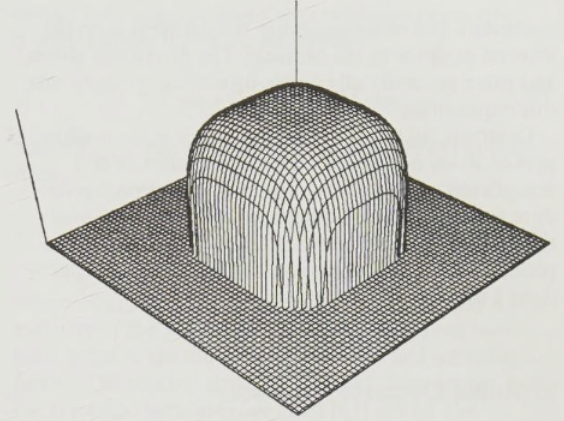
$$F_{(O, \text{psp})}^* = \begin{cases} \eta \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2} \frac{\partial \rho}{\partial x} & \text{if } \rho \leq \rho_0, \\ 0 & \text{if } \rho > \rho_0, \end{cases} \quad (18)$$

where $\frac{\partial \rho}{\partial x}$ denotes the partial derivative vector of the distance from the PSP to the obstacle,

$$\frac{\partial \rho}{\partial x} = \left[\frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y} \frac{\partial \rho}{\partial z} \right]^T. \quad (19)$$

The joint forces corresponding to $F_{(O, \text{psp})}^*$ are obtained using the Jacobian matrix associated with this PSP. Observing Eqs. (6) and (9), these forces are given by

$$\Gamma_{(O, \text{psp})} = J_{\text{psp}}^T(q) \Lambda(x) F_{(O, \text{psp})}^*. \quad (20)$$



5. Obstacle Geometric Modelling

Obstacles are described by the composition of *primitives*. A typical geometric model base includes primitives such as a point, line, plane, ellipsoid, parallelepiped, cone, and cylinder. The first artificial potential field (Eq. 16) requires analytic equations for the description of obstacles. We have developed analytic equations representing envelopes which best approximate the shapes of primitives such as a *parallelepiped*, *finite cylinder*, and *cone*.

The surface, termed an n -ellipsoid, is represented by the equation,

$$\left(\frac{x}{a} \right)^{2n} + \left(\frac{y}{b} \right)^{2n} + \left(\frac{z}{c} \right)^{2n} = 1, \quad (21)$$

and tends to a parallelepiped of dimensions $(2a, 2b, 2c)$ as n tends to infinity. A good approximation is obtained with $n = 4$, as shown in Fig. 1. A cylinder of elliptical cross section $(2a, 2b)$ and of length $2c$ can be approximated by the so-called n -cylinder equation,

$$\left(\frac{x}{a} \right)^2 + \left(\frac{y}{b} \right)^2 + \left(\frac{z}{c} \right)^{2n} = 1. \quad (22)$$

The analytic description of primitives is not necessary for the artificial potential field (Eq. 17) since the

continuity and differentiability requirement is on the shortest distance to the obstacle. The primitives above, and more generally all convex primitives, comply with this requirement.

Determining the orthogonal distance to an n -ellipsoid or to an n -cylinder requires the solution of a complicated system of equations. A variational procedure for the distance evaluation has been developed that avoids this costly computation. The distance expressions for other primitives are presented in Appendices I through III.

6. Robot Obstacle Avoidance

An obstacle O_i is described by a set of primitives (P_p) . The superposition property (additivity) of potential fields enables the control of a given point of the manipulator with respect to this obstacle by using the sum of the relevant gradients,

$$F_{O_i, psp}^* = \sum_p F_{(P_p, psp)}^* \quad (23)$$

Control of this point for several obstacles is obtained using

$$F_{psp}^* = \sum_i F_{(O_i, psp)}^* \quad (24)$$

It is also feasible to have different points on the manipulator controlled with respect to different obstacles. The resulting joint force vector is given by

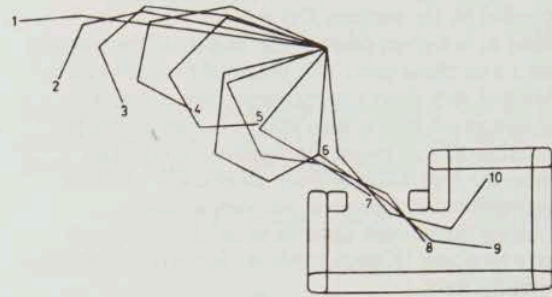
$$\Gamma_{obstacles} = \sum_j J_{pspj}^T(q) \Lambda(x) F_{pspj}^* \quad (25)$$

Specifying an adequate number of PSPs enables the protection of all of the manipulator's parts. An example of a dynamic simulation for a redundant 4 *dof* manipulator operating in the plane is shown in the display of Fig. 2.

The artificial potential field approach can be extended to *moving obstacles* since stability of the mechanism persists with a continuously time-varying potential field.

The manipulator obstacle avoidance problem has been formulated in terms of *collision avoidance of*

Fig. 2. Displacement of a 4 *dof* manipulator inside an enclosure.



links rather than points. Link collision avoidance is achieved by continuously controlling the link's closest point to the obstacle. At most, n PSPs then have to be considered. Additional links can be artificially introduced or the length of the last link can be extended to account for the manipulator tool or load. In an articulated chain, a link can be represented as the line segment defined by the Cartesian positions of its two neighboring joints. In a frame of reference R , a point $m(x, y, z)$ of the link bounded by $m_1(x_1, y_1, z_1)$ and $m_2(x_2, y_2, z_2)$ is described by the parametric equations,

$$\begin{aligned} x &= x_1 + \lambda(x_2 - x_1), \\ y &= y_1 + \lambda(y_2 - y_1), \\ z &= z_1 + \lambda(z_2 - z_1). \end{aligned} \quad (26)$$

The problem of obtaining the link's shortest distance to a parallelepiped can be reduced to that of finding the link's closest point to a vertex, edge, or face. The analytic expressions of the link's closest point, the distance, and its partial derivatives are given in Appendix I. In Appendices II and III these expressions are given for a cylinder and a cone, respectively.

7. Joint Limit Avoidance

The potential field approach can be used to satisfy the manipulator internal joint constraints. Let $\underline{q}_i$ and $\bar{q}_i$ be respectively the minimal and maximal bounds of the i^{th} joint coordinate q_i . These boundaries can contain q_i by creating barriers of potential at each of the hyperplanes ($q_i = \underline{q}_i$) and ($q_i = \bar{q}_i$). The corresponding joint

forces are

$$\gamma_{\underline{q}_i} = \begin{cases} \eta \left(\frac{1}{\underline{\rho}_i} - \frac{1}{\underline{\rho}_{i(0)}} \right) \frac{1}{\underline{\rho}_i^2} & \text{if } \underline{\rho}_i \leq \underline{\rho}_{i(0)}, \\ 0 & \text{if } \underline{\rho}_i > \underline{\rho}_{i(0)}, \end{cases} \quad (27)$$

and

$$\gamma_{\bar{q}_i} = \begin{cases} -\eta \left(\frac{1}{\bar{\rho}_i} - \frac{1}{\bar{\rho}_{i(0)}} \right) \frac{1}{\bar{\rho}_i^2} & \text{if } \bar{\rho}_i \leq \bar{\rho}_{i(0)}, \\ 0 & \text{if } \bar{\rho}_i > \bar{\rho}_{i(0)}, \end{cases} \quad (28)$$

where $\underline{\rho}_{i(0)}$ and $\bar{\rho}_{i(0)}$ represent the distance limit of the potential field influence. The distances $\underline{\rho}_i$ and $\bar{\rho}_i$ are defined by

$$\begin{aligned} \underline{\rho}_i &= q_i - \underline{q}_i, \\ \bar{\rho}_i &= \bar{q}_i - q_i. \end{aligned} \quad (29)$$

8. Level of Competence

The potential field concept is indeed an attractive approach to the collision avoidance problem and much research has recently been focused on its applications to robot control (Kuntze and Schill 1982; Hogan 1984; Krogh 1984). However, the complexity of tasks that can be implemented with this approach is limited. In a cluttered environment, local minima can occur in the resultant potential field. This can lead to a stable positioning of the robot before reaching its goal. While local procedures can be designed to exit from such configurations, limitations for complex tasks will remain. This is because the approach has a *local* perspective of the robot environment.

Nevertheless, the resulting potential field does provide the global information necessary and a collision-free path, if attainable, can be found by linking the absolute minima of the potential. Linking these minima requires a computationally expensive exploration of the potential field. This goes beyond the real-time control we are concerned with here but can be considered as an integrated part of higher level control. Work on high level collision-free path planning based on the potential field concept has been investigated in Buckley (1985).

9. Real-Time Implementation

Finally, the global control system integrating the potential field concept with the operational space approach has the following structure:

$$\Gamma = \Gamma_{motion} + \Gamma_{obstacles} + \Gamma_{joint-limit}, \quad (30)$$

where Γ_{motion} can be developed (Khatib 1983) in the form,

$$\Gamma_{motion} = J^T(q) \Lambda(q) F_m^* + \tilde{B}(q) [\dot{q}\dot{q}] + \tilde{C}(q) [\dot{q}^2] + g(q), \quad (31)$$

where $\tilde{B}(q)$ and $\tilde{C}(q)$ are the $n \times n(n-1)/2$ and $n \times n$ matrices of the joint forces under the mapping into joint space of the end-effector Coriolis and centrifugal forces. The symbol $g(q)$ is the gravity force vector, and the symbolic notations $[\dot{q}\dot{q}]$ and $[\dot{q}^2]$ are for the $n(n-1)/2 \times 1$ and $n \times 1$ column matrices,

$$\begin{aligned} [\dot{q}\dot{q}] &= [\dot{q}_1 \dot{q}_2 \dot{q}_1 \dot{q}_3 \dots \dot{q}_{n-1} \dot{q}_n]^T, \\ [\dot{q}^2] &= [\dot{q}_1^2 \dot{q}_2^2 \dots \dot{q}_n^2]^T. \end{aligned} \quad (32)$$

In this control structure, dynamic decoupling of the end effector is obtained using the end-effector dynamic parameters (EEDP) $\Lambda(q)$, $\tilde{B}(q)$, $\tilde{C}(q)$ and $g(q)$, which are configuration dependent. In real time, these parameters can be computed at a lower rate than that of the servo control. In addition, the integration of an operational position and velocity estimator allows a reduction in the rate of end-effector position computation, which involves evaluations of the manipulator geometric model. This leads to a two-level control system architecture (Khatib 1985):

- A low rate *parameter evaluation level* that updates the end-effector dynamic coefficients, the Jacobian matrix, and the geometric model.
- A high rate *servo control level* that computes the command vector using the estimator and the updated dynamic coefficients.

The control system architecture is shown in Fig. 3 where np represents the number of PSPs. The Jacobian matrices $J_{psp_i}^T(q)$ have common factors with the end-effector Jacobian matrix $J^T(q)$. Thus, their evaluation does not require significant additional computation.

The link's closest point m is identical to m_1 if $\lambda \leq 0$; it is identical to m_2 if $\lambda \geq 1$ and it is given by Eq. (26) otherwise. The shortest distance is therefore,

$$\rho = \begin{cases} [\rho_1^2 - \lambda^2/2]^{1/2}, & \text{if } 0 \leq \lambda \leq 1, \\ \rho_1, & \text{if } \lambda < 0, \\ \rho_2 & \text{if } \lambda > 1, \end{cases} \quad (\text{A1-2})$$

where ρ_1 and ρ_2 are the distance to the vertex from m_1 and m_2 , respectively. The distance partial derivatives are

$$\frac{\partial \rho}{\partial \mathbf{x}} = \left[\frac{x}{\rho} \frac{y}{\rho} \frac{z}{\rho} \right]^T. \quad (\text{A1-3})$$

DISTANCE TO AN EDGE

By a projection in the plane perpendicular to the considered edge (xoy , $yo\bar{z}$, or zox), this problem can be reduced to that of finding the distance to a vertex in the plane. This leads to expressions similar to those of (A1-1)–(A1-3) with a zero partial derivative of the distance w.r.t. the axis parallel to the edge.

DISTANCE TO A FACE

In this case, the distance can be directly obtained by comparing the absolute values of the coordinates of m_1 and m_2 along the axis perpendicular to the face. The partial derivative vector is identical to the unit normal vector of this face.

Appendix II: Link Distance to a Cylinder

The frame of reference R is chosen such that its z -axis is the cylinder axis of symmetry and its origin is the cylinder center of mass. The cylinder radius and height are designated by r and h , respectively.

DISTANCE TO THE CIRCULAR SURFACE

The closest point of the link (Eq. 27) to the circular surface of the cylinder can be deduced from the dis-

tance to a vertex considered in the xoy plane and by allowing for the radius r .

DISTANCE TO THE CIRCULAR EDGES

The closest distance to the cylinder circular edge can be obtained from that of the circular surface by taking into account the relative z -coordinate of m to the circular edge, i.e., $(z + h/2)$ for the base and $(z - h/2)$ for the top. The distance partial derivative vector results from the torus equation,

$$[x^2 + y^2 + (z \pm h/2)^2 - r^2 - \rho^2]^2 = 4r^2[\rho^2 - (z \pm h/2)^2]. \quad (\text{A2-1})$$

This vector is

$$\frac{\partial \rho}{\partial \mathbf{x}} = \left[\zeta \frac{x}{\rho} \zeta \frac{y}{\rho} \frac{z \pm h/2}{\rho} \right]^T, \quad (\text{A2-2})$$

with

$$\zeta = \frac{x^2 + y^2 + (z \pm h/2)^2 - r^2 - \rho^2}{x^2 + y^2 + (z \pm h/2)^2 + r^2 - \rho^2}. \quad (\text{A2-3})$$

The distance to the planar surfaces is straightforward and can be simply obtained as in Appendix I.

Appendix III: Link Distance to a Cone

In this case, the frame of reference R is chosen such that its z -axis is the cone axis of symmetry and its origin is the center of the cone circular base. The cone base radius, height and half angle are represented respectively by r , h , and β .

DISTANCE TO THE CONE-SHAPED SURFACE

The problem of locating $m(x, y, z)$ is identical to that for the cylinder case. The distance can be written as

$$\rho = z \sin(\beta) + (\sqrt{x^2 + y^2} - r) \cos(\beta). \quad (\text{A3-1})$$

The partial derivatives come from the equation,

$$x^2 + y^2 = r_z^2, \quad (\text{A3-2})$$

where

$$r_z = \tan(\beta)[h + \rho \sin(\beta) - z]. \quad (\text{A3-3})$$

They are

$$\frac{\partial \rho}{\partial x} = \left[\frac{x}{r_z \tan(\beta)} \frac{y}{r_z \tan(\beta)} \frac{1}{\sin(\beta)} \right]^T. \quad (\text{A3-4})$$

The problem of the distance to the cone circular edge is identical to that of the cylinder circular edge in Appendix II. The distance to the cone vertex is solved as in Appendix I.

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REFERENCES

- Brooks, R. 1983. Solving the find-path problem by good representation of free space. *IEEE Sys., Man Cyber. SMC*-13:190-197.
- Brooks, R. 1984 Aug. 20-23, Kyoto, Japan. Aspects of mobile robot visual map making. *2nd Int. Symp. Robotics Res.*
- Buckley, C. 1985. *The application of continuum methods to path planning*. Ph.D. Thesis (in progress). Stanford University, Department of Mechanical Engineering.
- Chatila, R. 1981. *Système de navigation pour un robot mobile autonome: modélisation et processus décisionnels*. Thèse de Docteur-Ingénieur. Université Paul Sabatier. Toulouse, France.
- Hogan, N. 1984 June 6-8, San Diego, California. Impedance control: an approach to manipulation. 1984 *Amer. Control Conf.*
- Khatib, O., Llibre, M. and Mampey, R. 1978. Fonction decision-commande d'un robot manipulateur. *Rapport No. 2/7156*. DERA/CERT. Toulouse, France.
- Khatib, O. and Le Maitre, J. F. 1978 September 12-15, Udine, Italy. Dynamic control of manipulators operating in a complex environment. *Proc. 3rd CISM-IFTOMM Symp. Theory Practice Robots Manipulators*, 267-282. Elsevier. 1979.
- Khatib, O. 1980. *Commande dynamique dans l'espace opérationnel des robots manipulateurs en présence d'obstacles*. Thèse de Docteur-Ingénieur. École Nationale Supérieure de l'Aéronautique et de l'Espace (ENSAE). Toulouse, France.
- Khatib, O. 1983 December 15-20, New Delhi. Dynamic control of manipulators in operational space. *6th CISM-IFTOMM Congress Theory Machines Mechanisms*, 1128-1131.
- Khatib, O., et al. 1984 June, *Robotics in three acts* (Film). Stanford University. Artificial Intelligence Laboratory.
- Khatib, O. 1985 September 11-13, Tokyo. The operational space formulation in robot manipulators control. *15th Int. Symp. Indust. Robots*.
- Khatib, O. and Burdick, J. 1985 November, Miami, Florida. Dynamic optimization in manipulator design: the operational space formulation. *ASME Winter Annual Meeting*.
- Khatib, O., Burdick, J., and Armstrong, B. 1985. *Robotics in three acts-Part II* (Film). Stanford University, Artificial Intelligence Laboratory.
- Krogh, B. 1984 August, Bethlehem, Pennsylvania. A generalized potential field approach to obstacle avoidance control. *SME Conf. Proc. Robotics Research: The Next Five Years and Beyond*.
- Kuntze, H. B., and Schill, W. 1982 June 9-11, Paris. Methods for collision avoidance in computer controlled industrial robots. *12th ISIR*.
- Lozano-Perez, T. 1980. Spatial planning: a configuration space approach. *AI Memo 605*. Cambridge, Mass. MIT Artificial Intelligence Laboratory.
- Moravec, H. P. 1980. *Obstacle avoidance and navigation in the real world by a seeing robot rover*. Ph.D. Thesis. Stanford University, Artificial Intelligence Laboratory.

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Abstract

This paper presents a unique real-time obstacle avoidance approach for manipulators and mobile robots based on the artificial potential field concept. Collision avoidance, traditionally considered a high level planning problem, can be effectively distributed between different levels of control, allowing real-time robot operations in a complex environment. This method has been extended to moving obstacles by using a time-varying artificial potential field. We have applied this obstacle avoidance scheme to robot arm mechanisms and have used a new approach to the general problem of real-time manipulator control. We reformulated the manipulator control problem as direct control of manipulator motion in operational space—the space in which the task is originally described—rather than as control of the task's corresponding joint space motion obtained only after geometric and kinematic transformation. Outside the obstacles' regions of influence, we caused the end effector to move in a straight line with an upper speed limit. The artificial potential field approach has been extended to collision avoidance for all manipulator links. In addition, a joint space artificial potential field is used to satisfy the manipulator internal joint constraints. This method has been implemented in the COSMOS system for a PUMA 560 robot. Real-time collision avoidance demonstrations on moving obstacles have been performed by using visual sensing.

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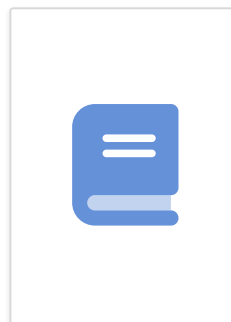
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ATTACHMENT L

On-line path planning strategy integrated with collision and dead-lock avoidance schemes for wheeled mobile robot in indoor environments

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Abstract

Purpose – This paper aims to present a hybrid path planning algorithm which is designed for use of autonomous vehicles in indoor environments. The approach mainly contributes the ability of generating a safe and smooth collision avoidance path for attaining a desired position in an unknown and obstructed environment.

Design/methodology/approach – The hybrid planner is based on potential field method and Voronoi diagram approach, and it is represented with the ability of concurrent map building and autonomous navigation.

Findings – The possibility of controlling the look-ahead distance allows the mobile robot to smartly control the velocity for creating a smooth trajectory autonomously. The dead-lock problem is solved by defining necessary sub-goals between targets on the constructed map.

Originality/value – The system controller (look-ahead control) with the potential field method allows the robot to generate a smooth and safe path for an expected position. Only essential exploration of unknown environment is performed since the approach constrains the mobile robot to explore a safe and sub-optimal route towards a destination.

Keywords Robotics, Navigation, Control systems, Linear motion

Paper type Research paper

1. Introduction

Path planning has been studied extensively in recent years because of its importance to mobile robot control. Most of path planning is executed by calculating a path for a mobile robot to follow, so that it can move from one pose to another without colliding with obstacles. Moreover, in order to allow a robot to perform autonomous tasks in a wide and unknown environment, the ability of building a reliable map of its workspace becomes extremely important. At present, map building and path planning are always being discussed in different areas because of vastness of both fields. During exploration stage the robot localizes its position while generating a map model of its environment via sensory data simultaneously. Based on a reliable and consistent map the robot calculates a path to an expected position as well as updating its position in conjunction with performing a desired motion precisely during a long-term navigation task (Figure 1(a)).

In this paper, we propose a hybrid path planner model which achieves concurrent navigation and topological map building. The hybrid path planner is composed of potential field method (Khatib, 1986) (Volpe and Khosla, 1990) and Voronoi diagram approach. The notion of this approach is inspired by the fact that most of mapping and path search algorithms are not computationally viable over large-scale environment. In contrast, the proposed approach constrains the mobile robot to explore a safe and sub-optimal route to a destination and allows the robot to accomplish environment modeling and safe path finding parallelly when a position of where the robot should attain is explicitly (Figure 1(b)). As a result, computational burden of environment exploration is reduced to a safe path finding for reaching a target position and the approach benefits from saving time during environment exploration if a target position is explicitly defined.

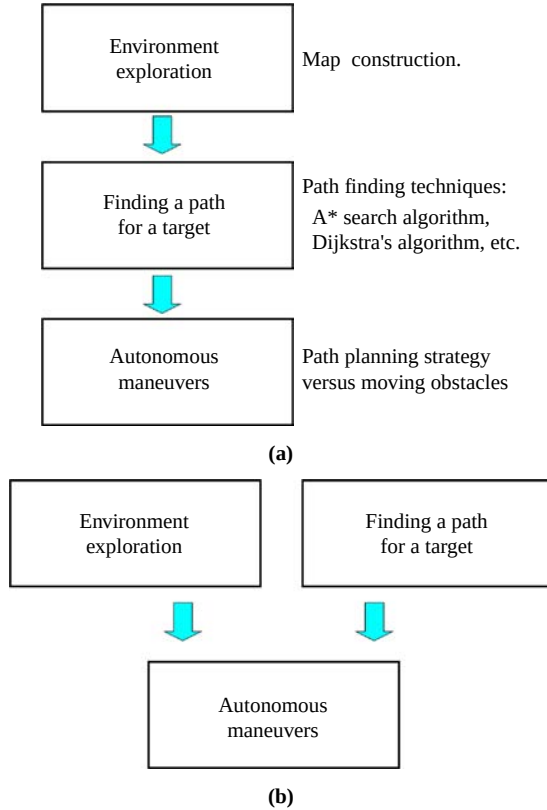
The potential field-based path strategy is straightforward and guarantees a safe navigation under dynamic environment. However, it suffers from a certain limitation (Krogh, 1984) (Lau, 2003) in which the robot will be trapped in positions where attractive and repulsive forces negate each other. There are several conventional solutions for the dead-lock problem. (Connolly, 1992; Connolly and Grupen, 1993 utilized Laplace's heat equation to create a potential field without local minima. Koditschek and Rimon, 1990) developed a local minima free potential field method by designing potential functions that contain only one minimum which is

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Figure 1 Notion of proposed hybrid path planner: (a) general procedure of autonomous navigation; (b) our approach



precisely at the goal position. Also there is a conventional recovery algorithm called wall following method (Borenstein and Koren, 1989) that can be used for solving the dead-lock problem. In our approach, we propose our potential field-based path planner within topological map building ability which enables the robot to perform an exploration towards a desired position and to build a reliable and compact map between targets (a start and goal), as well as helping the robot to avoid dead-lock positions existing in environment during a navigation task.

We also propose a recovery algorithm (Chang and Yamamoto, 2006) for dead-lock problem encountered while the robot is travelling along sub-goals. Three dead-lock avoidance schemes are taken into account in different consequences of positions of obstacles which surround a mobile robot while it is in a dead-lock position. In order to estimate the space between the obstacles and be applicable for modifying repulsive potential shapes, the shapes of the obstacles causing the robot halt are detected by adopting the Hough transformation (Hough, 1962).

The system controller (look-ahead control) used in this research is based on feedback linearization technique in which linearization is achieved between the control inputs and appropriate outputs. By controlling the output equation, the velocity of the mobile robot can be controlled to create a very smooth trajectory during obstacle avoidance. Within Look-ahead control framework, our path planner becomes more powerful and allows the mobile robot to easily maneuver in an environment where obstacles exist.

The paper is organized as follows. Section 2 introduces our system controller. In Section 3, we verify the control method in real experiments. Having covered the control system of the mobile robot, we go into the path planning of the robot in Section 4. Section 5 presents a recovery algorithm for dead-lock problem, and Section 6 concludes with final remarks.

2. System model and controller

2.1 Modeling of system

In this subsection we derive the motion equation of the mobile robot. Motion equation of the mobile robot which is two wheeled can be obtained from the approach proposed in (Yamamoto and Yun (1997). Suppose we choose $\mathbf{M}(q)$ as inertia matrix, $\mathbf{C}(q, \dot{q})$ as velocity vector, $\mathbf{G}(q)$ as gravity vector, $\mathbf{E}$, $\boldsymbol{\tau}$, $\mathbf{A}^T(q)$ and λ as constant matrix, input torque vector, constraint matrix and Lagrange multipliers, respectively. Then we can write the motion equation as:

$$\mathbf{M}(q)\ddot{q} + \mathbf{C}(q, \dot{q}) + \mathbf{G}(q) = \mathbf{E}\boldsymbol{\tau} - \mathbf{A}^T(q)\boldsymbol{\lambda} \quad (1)$$

The system model of the mobile robot is shown in Figure 2 where b represents half the distance between two driving wheels, and r is the radius of the wheel.

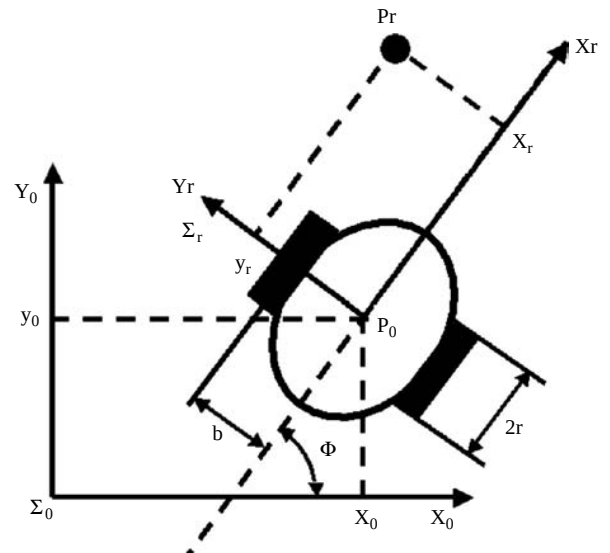
In this kind of mobile robot (two wheeled type), two kinematic constraints are imposed. First, no lateral slip is allowed between the wheels and the floor, and the second one is the no longitudinal slip constraint. According to these two constraints, the mobile robot is subject to two kinds of non-holonomic constraints which can be described by:

$$\dot{x}_0 \sin \phi - \dot{y}_0 \cos \phi = 0 \quad (2)$$

$$\dot{x}_0 \cos \phi + \dot{y}_0 \sin \phi = \frac{r}{2}(\dot{\theta}_r + \dot{\theta}_l) \quad (3)$$

where (x_0, y_0) is the coordinates of the center of the mobile robot, ϕ is the heading angle of the mobile robot measured from X -axis, θ_r and θ_l are the angular positions of the right wheel and the left wheel.

Figure 2 System model



Here, we choose $q = [x_0 \ y_0 \ \theta_r \ \theta_l]^T$ as the generalized coordinate vector. By following the standard approach for dimensional reduction, the motion equation (1) can be transformed as:

$$\mathbf{S}^T \mathbf{M} \mathbf{S} \dot{\eta} + \mathbf{S}^T \mathbf{M} \dot{\mathbf{S}} \eta + \mathbf{S}^T \mathbf{C} = \tau \quad (4)$$

where a distribution matrix $\mathbf{S} = \{S_1, S_2\}$ satisfies $\mathbf{A}(q)\mathbf{S}(q) = 0$. Moreover, the velocity vector $\dot{q}$ also has to be in the same null space of $\mathbf{A}(q)$ while the vector $\eta = [\eta_1 \ \eta_2]^T$ represents an independent pseudo velocity vector which lives in the same null space.

Combining q with η to define $\mathbf{x}$ as the state vector of the mobile robot, from equation (4) we can write the differential of the state vector $\mathbf{x}$ as:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{q} \\ \dot{\eta} \end{bmatrix} = f(\mathbf{x}) + g(\mathbf{x})\tau \quad (5)$$

where:

$$f(\mathbf{x}) = \begin{bmatrix} S\eta \\ -(\mathbf{S}^T \mathbf{M} \mathbf{S})^{-1} (\mathbf{S}^T \mathbf{M} \dot{\mathbf{S}} \eta + \mathbf{S}^T \mathbf{C}) \end{bmatrix}$$

and:

$$g(\mathbf{x}) = \begin{bmatrix} 0 \\ (\mathbf{S}^T \mathbf{M} \mathbf{S})^{-1} \end{bmatrix}$$

2.2 Controller design

The control method using in this study is based on feedback linearization technique in which linearization is achieved between the control inputs and appropriate outputs. The control diagram of the mobile robot is shown in Figure 3.

We choose the output equation at the XY coordinates of the appropriate reference point P_r (refer to Figure 2). The output equation y is:

$$y = \begin{bmatrix} x_0 + x_r \cos \phi - y_r \sin \phi \\ y_0 + x_r \sin \phi + y_r \cos \phi \end{bmatrix} \quad (6)$$

To find the new linearization between the control input and output, we differentiate the output (6) with respect to time until a new input μ appears.

After the second time-differential of y , the new input μ appears while $\dot{y}$ is written as:

$$\dot{y} = \Phi(x)\dot{\eta} + \dot{\Phi}\eta = \Phi(x)\mu + \dot{\Phi}\eta \quad (7)$$

where $\Phi(x)$ is the decoupling matrix described as:

$$\Phi(x) = \begin{bmatrix} (r/2)\cos \phi - Lc \sin \phi & (r/2)\cos \phi + Lc \sin \phi \\ (r/2)\sin \phi - Lc \cos \phi & (r/2)\sin \phi + Lc \cos \phi \end{bmatrix} \quad (8)$$

where L is the distance between the reference point P_r and the center of the mobile robot, and $c = r/2b$.

Also the new input μ is given by:

$$\mu = \Phi^{-1}(x)(\dot{y} - \dot{\Phi}(x)\eta) \quad (9)$$

From equations (7) and (9), we obtain:

$$\dot{y} = u, \quad u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T \quad (10)$$

Equation (10) is our new linearization between the control input and output. Moreover, for stabilizing the output y , the following equation is used:

$$u = \ddot{y} + k_D(\dot{y} - \dot{y}) + k_P(\hat{y} - y) \quad (11)$$

where $\ddot{y}$, $\dot{y}$ and $\hat{y}$ are the targets.

In addition, from (9) we know that the decoupling matrix has to be invertible, namely, Φ has to be a non-singular matrix. Also from equation (8) we obtain:

$$\det(\Phi) = -\frac{r^2}{2b} L.$$

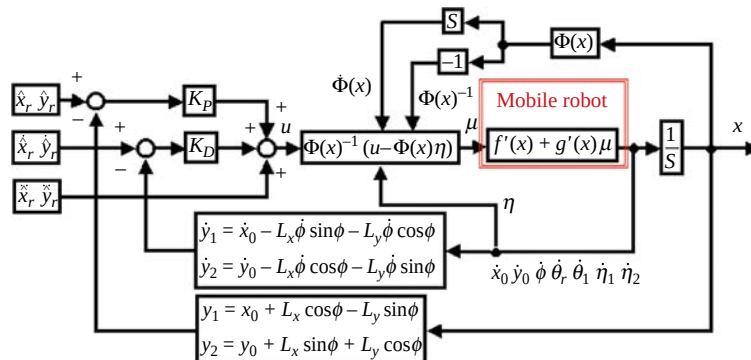
Therefore, $L \neq 0$ has to be satisfied because b and r always are positive.

By using this control system, the motion of the mobile robot is generated so as to compensate any deviation from the desired one, for example, in the sense of manipulability (Yamamoto and Yun, 1994). Moreover, because of the capability of controlling the distance between the center of robot and the reference point, the motion of the mobile robot can be more flexible and easy to control under dynamic environment where moving obstacles exist. In addition, for the sake of simplicity, we choose the reference point on the X_r -axis (Figure 2) in the following implementations.

3. Experimental setup

The mobile robot we use for experiments is an ActivMedia product, Pioneer 3-DX, which has two independent wheels

Figure 3 Control diagram



and eight sonars affixed under the front deck. In order to make our robot more autonomous, having the ability of gathering accurate information about environment becomes extremely important. Therefore, a range finding device, SICK laser, is equipped on our robot. Figure 4 shows Pioneer 3 with SICK laser loaded on the top.

3.1 Implementation of look-ahead control system

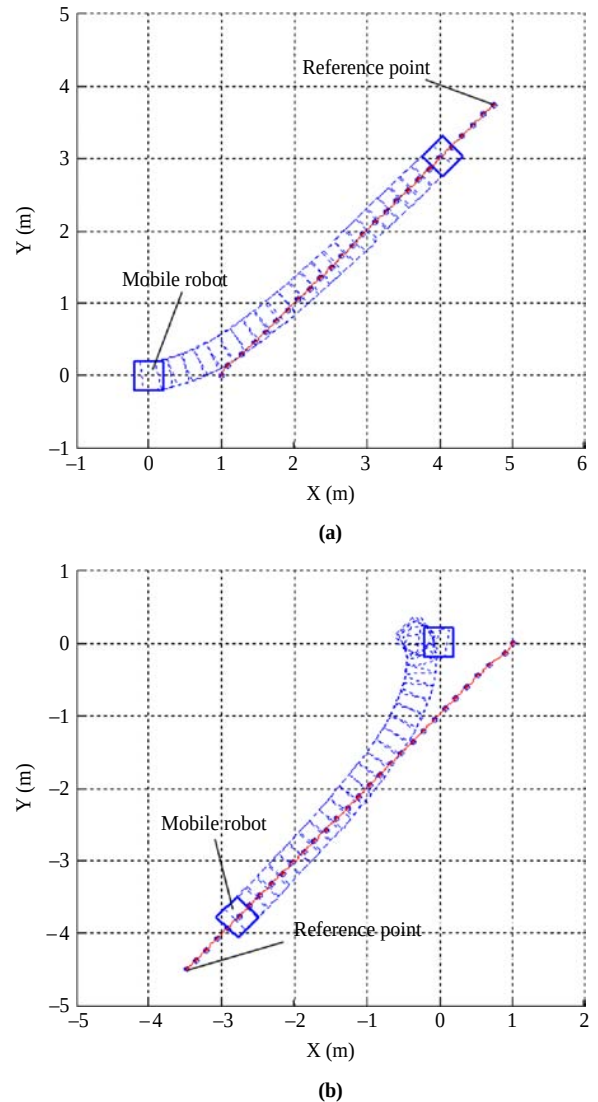
In this subsection we verify our system controller in real robot experiments. There are two kinds of case studies conducted while the mobile robot is moving along a straight line 45° and 225° from a positive X -axis, respectively. In both studies, the mobile robot initially faces to a positive X -axis, and the length of L set in 1 [m], also the velocity of the mobile robot is at 0.15 (m/s). Figure 5(a) and (b) show the experimental results of the trajectory of mobile robot which moves along a straight line 45° and 225° from a positive X -axis in processing time 25 and 30 s, respectively.

Figure 6 plots the heading angle of mobile robot with respect to time in two case studies. Figure 7 represents the occurring error during the mobile robot chases the reference point in both studies. From both studies, it is seen that by using look-ahead control system, the mobile robot will always keep chasing the reference point, which agrees with the three necessary conditions for asymptotic convergence discussed in Brockett (1983). Therefore, as to path planning, instead of scheming the path of the mobile robot directly we plan on calculating the path to reach the target position for the reference. As a result, the mobile robot will attain the target position as we expect by following the motion of the reference point.

Figure 4 Pioneer 3 with SICK laser



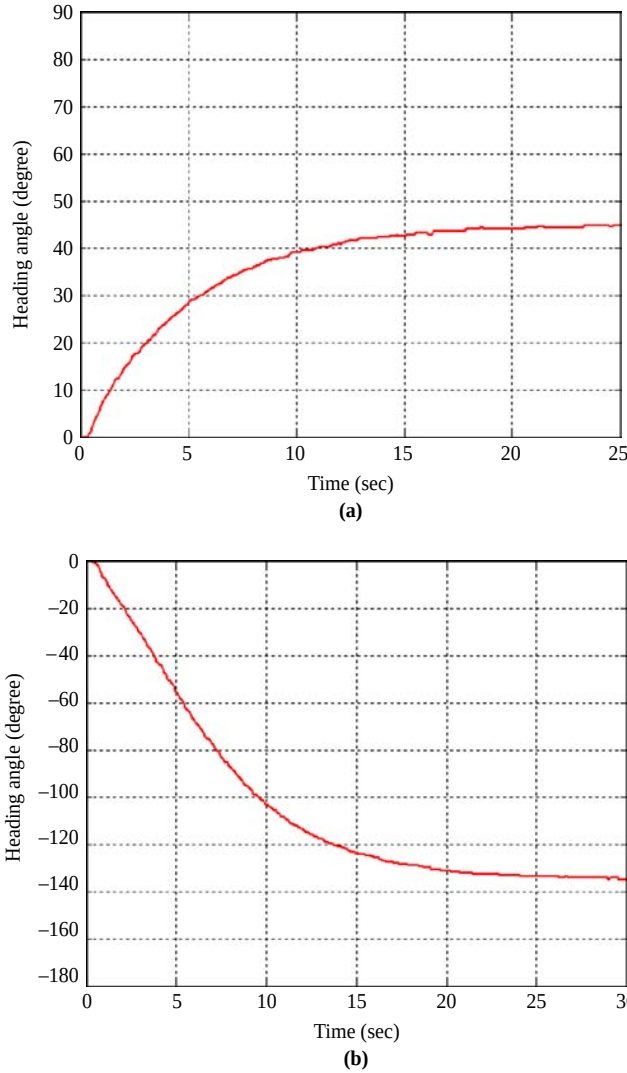
Figure 5 Experimental result: trajectory of mobile robot: (a) 45° ; (b) 225°



4. Concurrent navigation and map building system

As mentioned previously, we propose our path planner with both navigation and map building abilities. Based on our approach, the a priori unknown environment does not need to be neither bounded nor static in terms of building a map model of robot's workspace since the robot is constrained to explore a route towards a destination and is capable of avoiding collisions with moving obstacles during exploration. Based on the constructed map, the robot is able to seamlessly start autonomous maneuvers between any targets in the explored area as well as starting a new exploration to an unvisited region.

The hybrid path planner is comprised of two modules. Section 4.1 introduces the obstacle avoidance module of the hybrid planner. Section 4.2 explains the map building module of the system and shows a demonstration result in real environment.

Figure 6 Experimental result: heading angle of mobile robot: (a) 45°; (b) 225°

4.1 Collision avoidance algorithm

Since the control system we use will cause the mobile robot keep chasing the reference point, instead of scheming the obstacle avoidance of the mobile robot directly, we focus on the path planning of the reference point. In this research, we adopt the potential field method for obstacle avoidance during navigation.

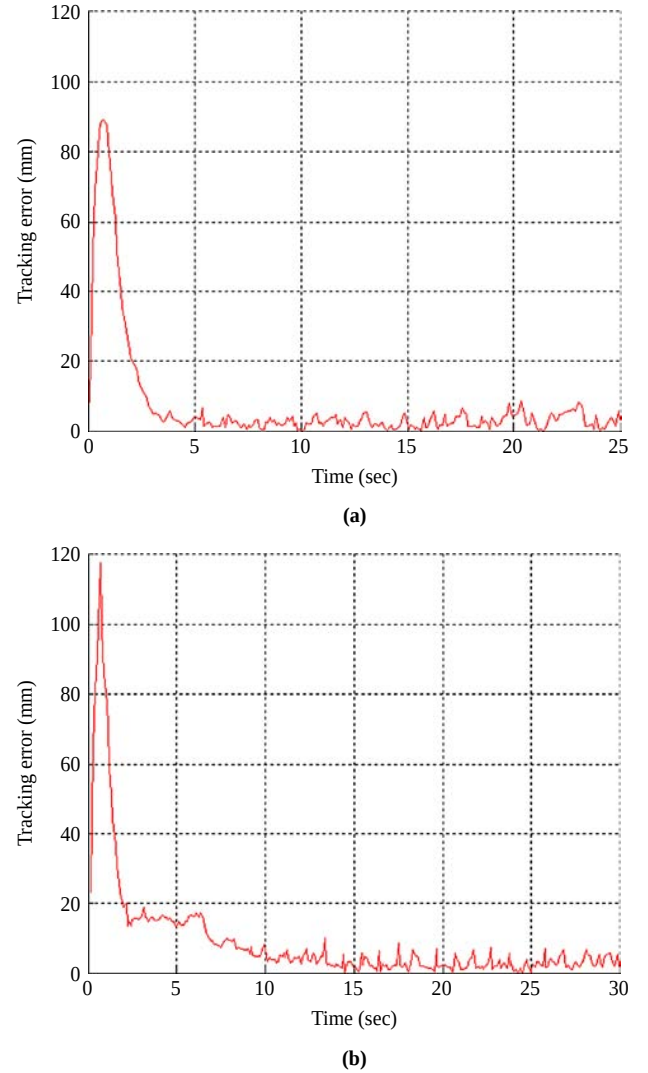
4.1.1 Potential field method

Potential field method is widely used in decades for autonomous mobile robot path planning because of its simplicity. An attractive potential which drives the mobile robot to its destination can be described by:

$$U_{\text{att}}(x) = A\sqrt{(x_r - x_g)^2 + (y_r - y_g)^2} \quad (12)$$

where A is a positive constant, (x_r, y_r) is the coordinates of the reference point, and (x_g, y_g) is the coordinates of the destination.

Also a repulsive potential well which prevents the mobile robot from collisions can be written as:

Figure 7 Experimental result: tracking error: (a) 45°; (b) 225°

$$U_{\text{rep}}(K) = B \frac{e^{-\alpha K}}{K} \quad (13)$$

where B and α are positive constants. K is the variable which acts as a pseudo-distance from the obstacle, becoming zero at the surface and increasing with successive contours away from the surface.

From equations (12) and (13), we obtain the potential energy U which drives the reference point of the mobile robot to the destination, described by:

$$U = U_{\text{att}}(x) + U_{\text{rep}}(K) \quad (14)$$

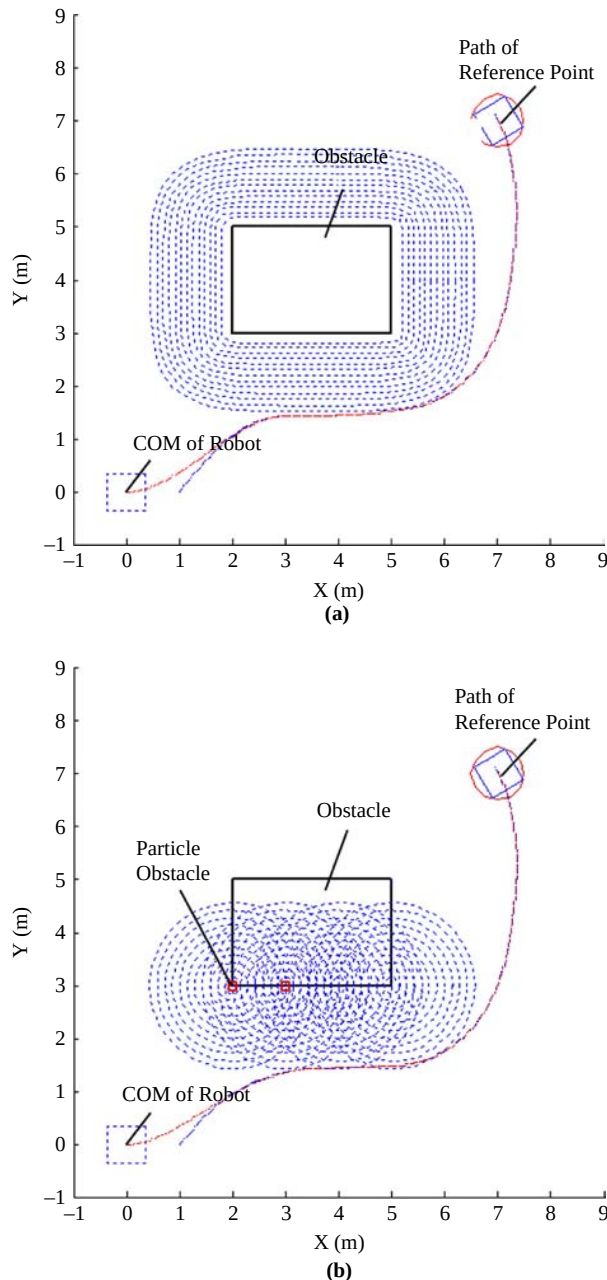
also the momentary force F which the reference point receives can be given by:

$$F = -\nabla U = \left[-\frac{\partial U}{\partial x} \quad -\frac{\partial U}{\partial y} \right]^T \quad (15)$$

In order to perform our avoidance algorithm in real-time, unlike the conventional potential field method we do not take into account whole existing obstacles in environment.

We use the closest sensory measurement from the obstacle surface, and define a smallish particle obstacle at the closest location which generates a small repulsive potential as well. As a result, the generated repulsive potential will vary with time even though there are only static obstacles existing in environment, and the momentary potential fields from smallish particle obstacles will be gradually generated in discrete time which eventually sum up to create almost as the same shape as repulsive potential generated from the original obstacle. Figure 8 shows the comparison between the result of conventional potential field method and the result based on our new approach.

Figure 8 Simulation results for comparison: (a) conventional approach; (b) our approach

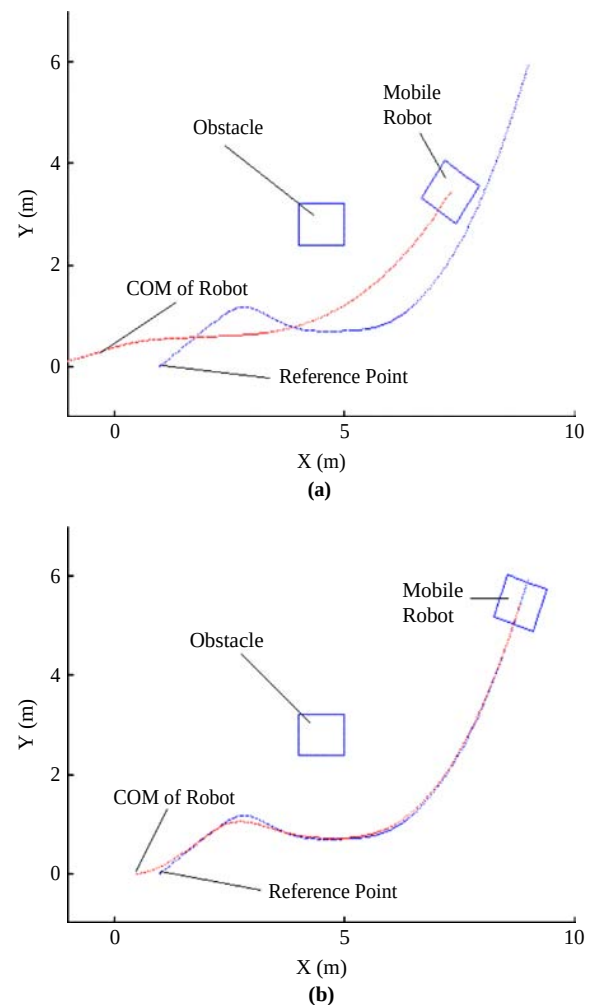


4.1.2 Dynamic control of output equation

As for creating a smooth trajectory in conjunction with obstacle avoidance, we vary the output equation of the control system to control the velocity of the mobile robot. Figure 9(a) and (b) represent the obstacle avoidance path of the mobile robot moving with a large and small look-ahead distance (L), respectively. From both results it can be seen that although the path of the reference point is the same, the motion of the mobile robot can be totally different, and the mobile robot may possibly collide with the obstacle if L is too large (Figure 9(a)). On the other hand, if we always keep L small, the mobile robot takes almost the same path as the reference point at the expense of lack of smoothness, and in general it takes more time for the mobile robot to reach the destination (Figure 9(b)). The term “dynamic control” should not be confused with “dynamic compensation” or “dynamic extension” used in the context of differential geometric control (Isidori, 1995; Nijmeijer and van der Schaft, 1990).

The natural choice is to keep L large when the mobile robot is away from the obstacle, and make it automatically become smaller while the mobile robot is being closer to the obstacle. Moreover, referring to Figure 3, we know that L cannot be 0 because the decoupling matrix $\Phi(x)$ has to be invertible.

Figure 9 Simulation with different look-ahead distances: (a) large L ; (b) small L



Since L cannot become 0, we realize that the mobile robot actually cannot attain the exact target position as we expect. Therefore, we propose:

$$L = \left(L_m + C \frac{\partial U_{\text{rep}}}{\partial K} \right) e^{-1/K_g} + \alpha \quad (16)$$

where L_m and C are positive adjustable constants, and K_g is the distance between the center of the mobile robot and the target position. L changes along with $\partial U_{\text{rep}}/\partial K$ which becomes smaller as the mobile robot comes closer to the obstacle. The exponential function reduces L_m to become as close as 0 while the reference point attains the goal point, and also α keeps L would not become 0 and allows the mobile robot to attain the target position as closely as possible.

4.1.3 Under static and dynamic environment

Two experiments are conducted to verify the proposed path planning method under static and dynamic environment. In both experiments, the mobile robot initially faces to a positive X-axis, the length of the initial look-ahead distance (L) set in 1 [m], and the mobile robot moves at a constant velocity of 0.4 (m/s). Tables I and II show the condition settings of two experiments, respectively.

Figure 10 represents the obstacle avoidance path of the mobile robot while its look-ahead distance varies along with equation (16). Figure 11 shows the look-ahead distance with respect to time. L becomes short when robot is close to an obstacle, and increases gradually to the initial setting when robot is away from obstacle, and eventually varies

Table I Experimental conditions of obstacle avoidance (static case)

Initial robot position	Origin
Initial robot direction	0°
Initial reference point position	$x_r = 1.0[m]$, $y_r = 0.0[m]$
Goal position	Goal _x = 7.0[m], Goal _y = 7.0[m]
Obstacle position	Obs1 _x = 3.0[m], Obs1 _y = 5.0[m] Obs2 _x = 4.0[m], Obs2 _y = 2.0[m] Obs3 _x = 6.0[m], Obs3 _y = 4.5[m]
Obstacle shape	Width = 1.0[m], Height = 0.8[m]
Processing time	30 s

Table II Experimental conditions of obstacle avoidance (dynamic case)

Initial robot position	Origin
Initial robot direction	0°
Initial reference point position	$x_r = 1.0[m]$, $y_r = 0.0[m]$
Goal position	Goal _x = 7.0[m], Goal _y = 7.0[m]
Initial obstacle position	Obs1 _x = 0.0[m], Obs1 _y = 2.0[m] moving direction and velocity moving in a positive X-axis direction at 0.2 (m/s) Obs2 _x = 8.5[m], Obs2 _y = 5.5[m] moving in a negative X-axis direction at 0.2 (m/s) Obs3 _x = 6.0[m], Obs3 _y = 6.0[m] moving in a negative Y-axis direction at 0.2 (m/s)
Obstacle shape	Width = 1.0[m], Height = 0.8[m]
Processing time	30 s

Figure 10 Collision avoidance (static case)

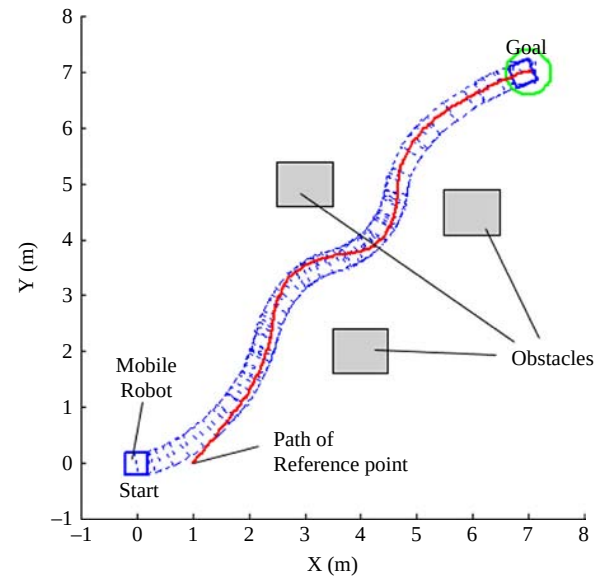
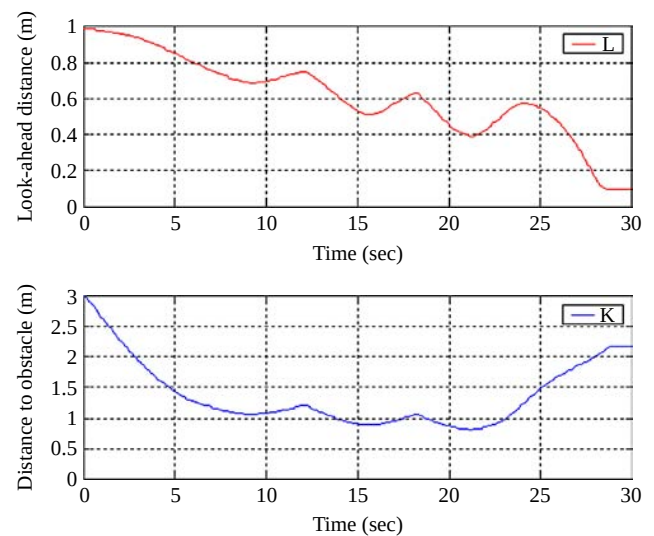


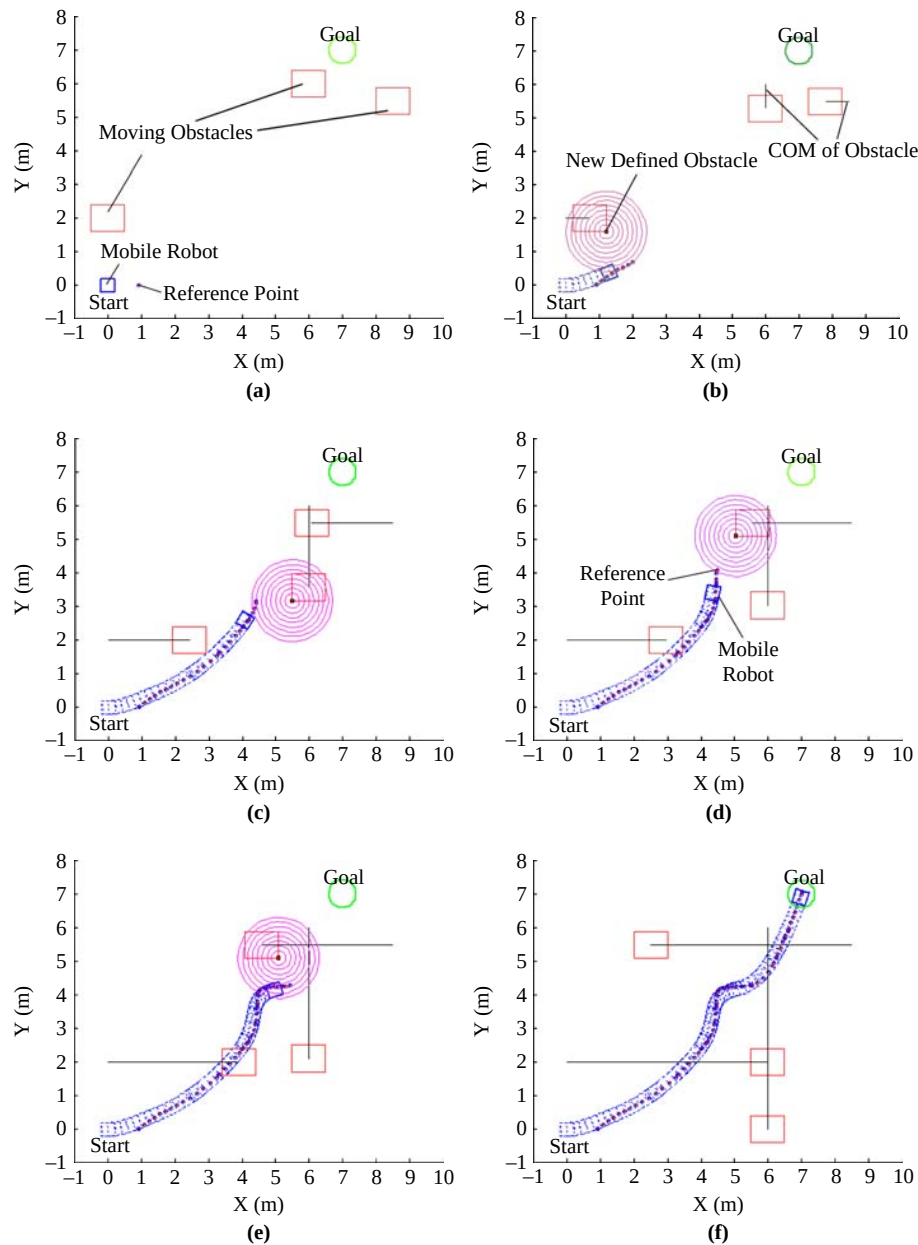
Figure 11 Look-ahead distance (static case)



almost to a constant when the robot is trying to attain the goal position.

Figure 12(a) to (e) thoroughly plot the obstacle avoidance path of the mobile robot in discrete time while it is maneuvered in a dynamic environment where three rectangular shapes of obstacles are moving in different directions. Figure 13 shows the change of the look-ahead distance with respect to time, as well as controlling the velocity of the mobile robot for creating a smooth obstacle avoidance path during a navigation task.

From these results, it is seen that based on our collision avoidance algorithm by predefining start and goal positions the mobile robot can find a safe and smooth path for attaining the target position autonomously whether in a static or dynamic environment.

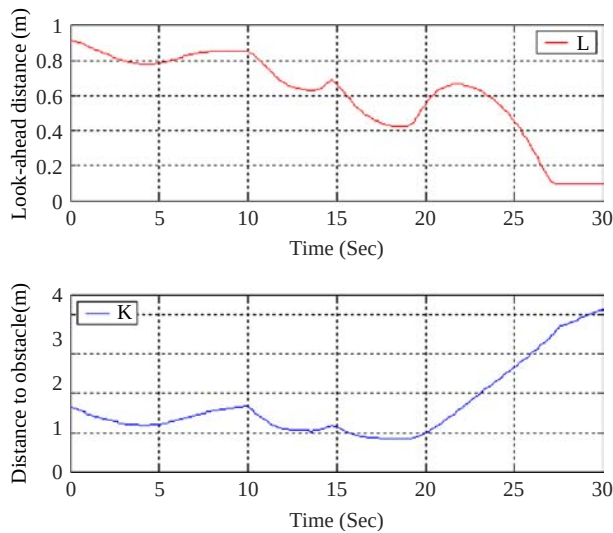
Figure 12 Collision avoidance (dynamic case): (a) 0.0 s; (b) 3.6 s; (c) 14.8 s; (d) 12.0 s; (e) 19.5 s; (f) 20.0 s

4.2 Simultaneous map building

In terms of modeling a configuration space of robot's environment during navigation, we adopt Voronoi diagram approach to enable the robot to generate a map of its workspace while it is moving towards a destination. Topological structure of environment representation can benefit from its compact model that achieves fast route planning and reduces the amount of sensory data. Usage of such technique for environment modeling has been demonstrated in previous approaches (Thrun, 1998; Mahkovic and Slivnik, 2000; Choset and Nagatani, 2001). However, their approaches have three common limitations:

- 1 First, size of the exploration space has to be set up as either a bounded area or a close environment in order to terminate the exploration when all the edges have been explored.

- 2 Secondly, their approaches do not support the obstacle avoidance ability. It limits the robot to trace the constructed Voronoi edges from one to another in a certain range, in order to allow the robot to follow a safe path while it is tracing towards Voronoi edges. As a result, it slows down the exploration process, as well as limiting the robot to perform exploration tasks in only static environments.
- 3 After an exploration process is finished, a graph search algorithm is required to calculate a path for performing autonomous maneuvers between targets defined in the existing exploration. However, most of graph search algorithms are not computationally viable over large-scale environment.

Figure 13 Look-ahead distance (dynamic case)

To cope with these limitations, the proposed hybrid path planner features:

- **Efficient environment exploration:** Instead of performing an exploration in a bounded area, the exploration is constrained to finding a safe path for attaining a target position. Moreover, the mobile robot does not have to trace the generated Voronoi edges one by one during exploration phase since this approach allows the mobile robot to simultaneously avoid collisions with obstacles and generate a smooth obstacle avoidance path while it is moving towards an edge position.
- **Map building in conjunction with finding a sub-optimal path for attaining a desired position:** It is unnecessary to search a safe path on the graph for attaining a target position. After an exploration towards a target, the robot can perform autonomous maneuvers immediately by setting up sub-goals on the constructed map.

4.2.1 Approximate Voronoi diagram

In order to reduce the computational burden of the Voronoi edge calculation, despite of the richness of the information obtained by a laser range finder, we approximate the sensory data to allow the robot to generate an approximate Voronoi structure such as a representation of free space of its environment. Before getting into the details of determining edges for tracing towards a target during an exploration, we explain the algorithm of approximate Voronoi diagram computation in this approach. Figure 14 shows the pseudo-code description for generating approximate Voronoi diagram from the sensory information.

First, the robot gathers information of its vicinity and creates a finite set P of points which approximates the sensory data from each laser scan. Clustering of each point is done by perceiving the change of distance between two successive laser readings. Based on each local scan, the robot accumulates the approximate sensory data for incrementally building an approximate Voronoi structure of its workspace while moving towards a target.

To allow the robot to create only useful Voronoi edges for exploration, the radius of Delaunay circle is constrained as:

$$\text{diameter of robot} < R < \text{a certain limit} \quad (17)$$

This constraint guarantees not only that safe edges allow mobile robot to pass through a narrow passage, but also that none of unnecessary edge caused by a large concave environment is created (Figure 15). Figure 15(a) represents approximate Voronoi diagram which is defined by equation (17). Figure 15(b) shows the motion of mobile robot which traces towards explored Voronoi edges as:

- access an unvisited Voronoi edge which is closest to the goal in the sense of exploring towards goal position;
- during an exploration if no Voronoi edge can be found, the mobile robot automatically switches to the motion of attaining the goal position;
- restart to trace an edge if any new and unvisited Voronoi edge is perceived; and
- the mobile robot avoids colliding with obstacle simultaneously while accessing an edge position.

4.2.2 Tracing edge determination

Figure 16 shows the notion of tracing Voronoi edges incrementally towards a target. By selecting an edge point which is unvisited and is closest to a predefined goal as an edge for tracing, the mobile robot is able to accomplish an exploration towards the target. However, there is a possibility for the robot to fall into a dead-lock while the robot is accessing an edge which is closest to goal and is created by an occlusion (Figure 17).

In order to avoid being trapped in a dead-lock position while the mobile robot is accessing an uncertain Voronoi edge, we allow the robot to access a three-equidistant point which is recorded as a node with departing edges if a node is perceived. In addition, in this approach the node point is created by points belonging to large groups in which the amount of each group is large enough and each group contains over a certain number of members. It allows the mobile robot to perform an exploration task in environment where moving obstacles exist, since a node created by a moving obstacle is unnecessary to be recorded and it may cause unexpected problem if the robot accesses a node generated by a moving obstacle despite of ignoring the motion of the moving obstacle. According to the definition of the node point, it is seen that none of node point is generated by an environment shown in Figure 15.

Once the mobile robot attains a node position, the mobile robot then selects one unexplored edge of the defined node which is closest to the goal and continues tracing. During an exploration if the mobile robot reaches a dead-end, the robot returns back to the closest visited node and restarts to trace another unexplored Voronoi edge which is closest to the goal, as well as starting to explore a new route for attaining the target position.

4.2.3 Sub-goal setting

The proposed hybrid path planner is demonstrated in real environment. The mobile robot initially faces to a positive X-axis, and starts at the origin (0, 0). The position of the goal is predefined as:

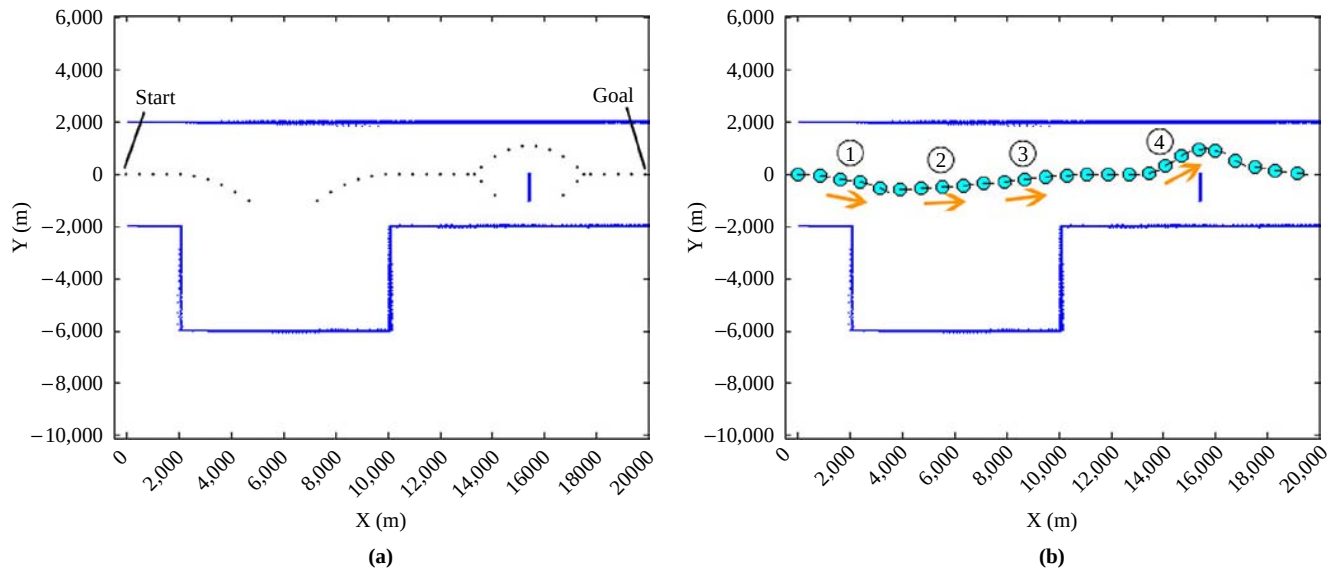
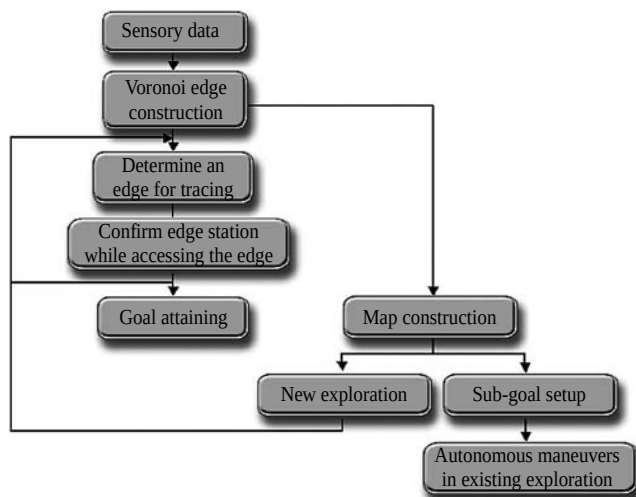
$$\text{Goal}(15.0[m], -37.5[m])$$

Figure 18 shows the real experimental result in which (a) and (b) represent the motion of the mobile robot while it is tracing Voronoi edges towards a target and the accessed Voronoi edges of approximate Voronoi diagram, respectively. It is seen that the mobile robot autonomously generates a smooth and safe

Figure 14 Computation of approximate Voronoi diagram

Compute Approximate Voronoi Diagram (Input: sensory data)

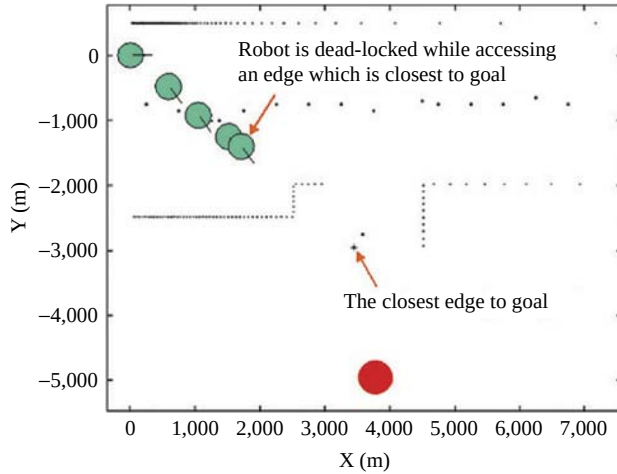
01. for each local scan
02. approximate sensory data and save as $P(x,y, \text{scanning degree})$
03. Clusterrize (P)
04. write new data on data list and save data list as *DataList*.
05. if first scan from laser range finder
06. divide *DataList* into different groups and save as group list (*GroupList*).
07. else
08. divide *DataList* into different groups and save as new group list (*NewGroupList*).
09. Compare groups of *GroupList* to groups of *NewGroupList*
10. if distance between two group are small enough
11. add new data to group of *GroupList*.
12. else
13. add group of *NewGroupList* as new group on *GroupList*.
14. Triangulate (*GroupList*)
15. calculate center of circumcircle for each triangle
16. if radius of circumcircle is in a certain range (here we set as $1000[\text{mm}] < \text{radius} < 3000[\text{mm}]$)
17. Generate AVD (*GroupList*)

Figure 15 Approximate Voronoi diagram defined by constraints: (a) approximate Voronoi diagram; (b) motion of mobile robot**Figure 16** Procedure of autonomous navigation

trajectory by receiving attractive and repulsive forces while it is travelling along the accessed edge positions, and simultaneously avoids possible dead-locks caused by concave spaces existing in environment by exploring and tracing Voronoi edges.

Figure 19 shows the constructed map after the robot explores a route for attaining the goal position. From this result, it can be seen that instead of an exploration of whole free space of the bounded environment, only a safe route exploration is performed when a desired goal position is given. Moreover, in order to reduce the computational burden, the mobile robot stops calculating Voronoi edges and performs autonomous maneuvers in a manner of fully depending on its obstacle collision avoidance ability after an exploration process is accomplished. As a result, after constructing and modeling the environment of robot's workspace, sub-goals are set up at positions of visited node points on the map for immediate applications such as travelling between targets (start and goal positions) in the existing exploration.

Figure 17 Robot is dead-locked while tracing an edge created by an occlusion



5. Recovery algorithm for dead-lock

Through previous section, we explain the notion of our hybrid path planner. We propose a potential field-based avoidance algorithm within map building ability which allows our robot to be able to trace explored Voronoi edges for attaining a goal position simultaneously while avoiding local minimum problem caused by concavities in environment. However, there is still a possibility for the robot to be trapped in a dead-lock position because of the presence of changes to the environment while it travels along setup sub-goals between targets. Here, we propose a recovery algorithm for solving the dead-lock problem. Figure 20(a) and (b) shows the iso-potential contours of two repulsive potential fields in both XY and 3D where the deadlock problem occurred.

From Figure 20, we understand that the dead-lock problem is occurred when multiple potential contours are overlapping, and sum up as one big potential field with concavity that causes the mobile robot facing uphill from the repulsive potential well when an undesirable minimum is created.

5.1 Obstacle detection

In order to conduct our dead-lock avoidance technique for solving such type of deadlock problem, we need to acquire the actual shapes of obstacles that cause the robot to stop. Thus, there is a preliminary requirement that the obstacle recognition ability for dead-lock avoidance has to be taken into account. In this paper, we adopt the Hough transformation for it. For a simple example, suppose there is a rectangular shape of obstacle in environment. The detection of this rectangular obstacle can be done by the line detection. Because it requires that the desired feature be specified in parametric form, a measurement of the obstacle with coordinates x and y can be written as:

$$\rho = x \cos \theta + y \sin \theta \quad (18)$$

where ρ and θ are variables used in parameter space.

After mapping all measurements into parameter space, the shape of the obstacle can be extracted by finding local maxima in the parameter space (i.e. local peaks). Figure 21 shows the result of a real obstacle detection based on the LRF measurements by using this procedure.

5.2 Dead-lock solution

We propose a dead-lock recovery algorithm which is stated as the following procedure:

- Set a criterion, that is, if the mobile robot moves under a limit of speed (almost to zero) except in a goal position, the robot will declare itself to be in a dead-lock position.
- Define shapes and positions of obstacles that surround the robot in a certain area by adopting the obstacle detection

Figure 18 Experimental demonstration of proposed path planner: (a) motion of mobile robot; (b) accessed Voronoi edges

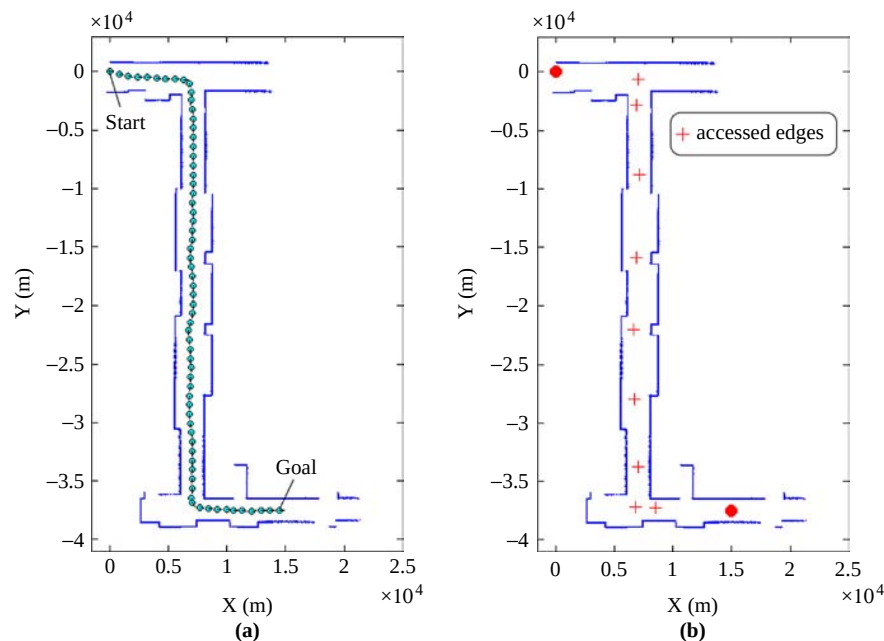
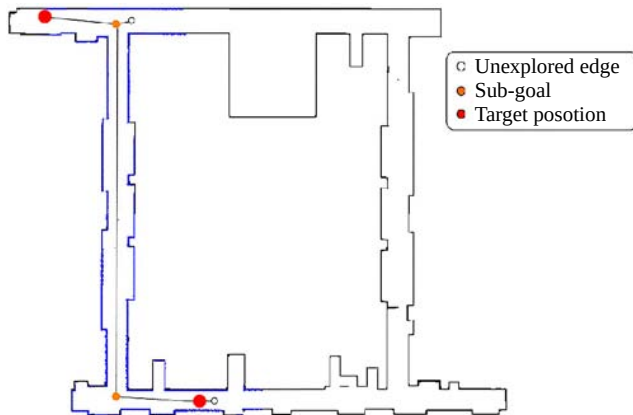


Figure 19 The topological map constructed by the proposed path planner

technique mentioned in previous subsection. Figure 22(a) shows a circular area which is drawn from the center of the robot where r is the radius of the circular area.

- Pick up two closest obstacles and define two reference lines to divide the area into three regions. Line 1 is a reference line that is across the centers of two obstacles. Line 2 is

perpendicular to Line 1 and goes straight across the center of the robot. Areas 1 and 2 indicate the areas that are on the right side and the left side of Line 2, respectively. Area 3 represents the area that is behind the two closest obstacles.

- Based on obstacle information extracted from laser range finder measurements, the robot itself can estimate the space between the obstacles, and choose a suitable avoidance scheme to proceed by going through the decision-making process shown in Figure 22(b).

Three available dead-lock avoidance schemes that are taken into account as decisions:

- Reduce the repulsive potential energy to create smaller potential contours for the robot to pass through (Figure 23(a)).
- Combine the two obstacles into one pseudo-obstacle to make the mobile robot move to the downhill side of the obstacle and take a detour to escape from a dead-lock position. As a result shown in Figure 23(b), to make the mobile robot turn right and detour around a new potential field yielded from the pseudo-obstacle.
- Tilt the merged obstacle for adjustment to let the robot to make a left turn and detour around a new potential field contour.

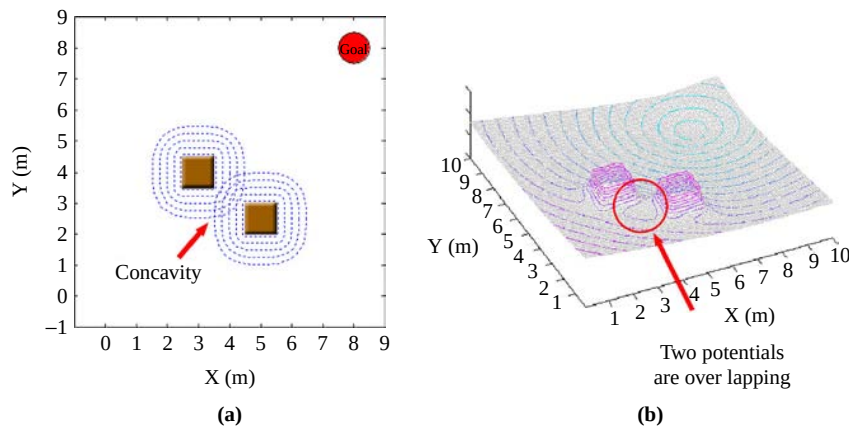
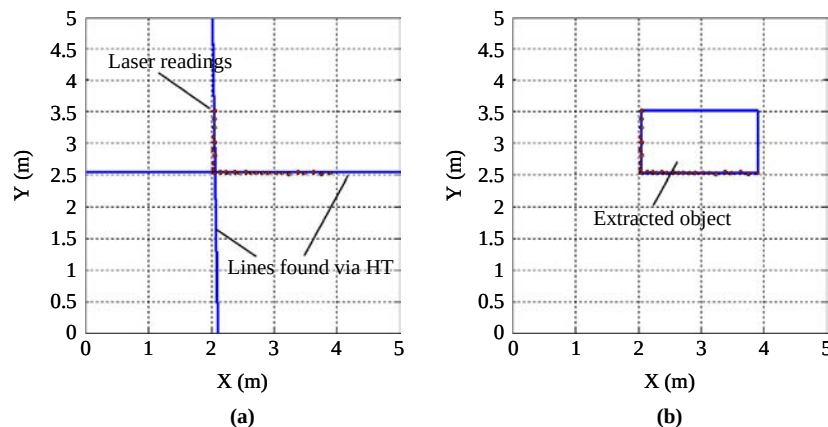
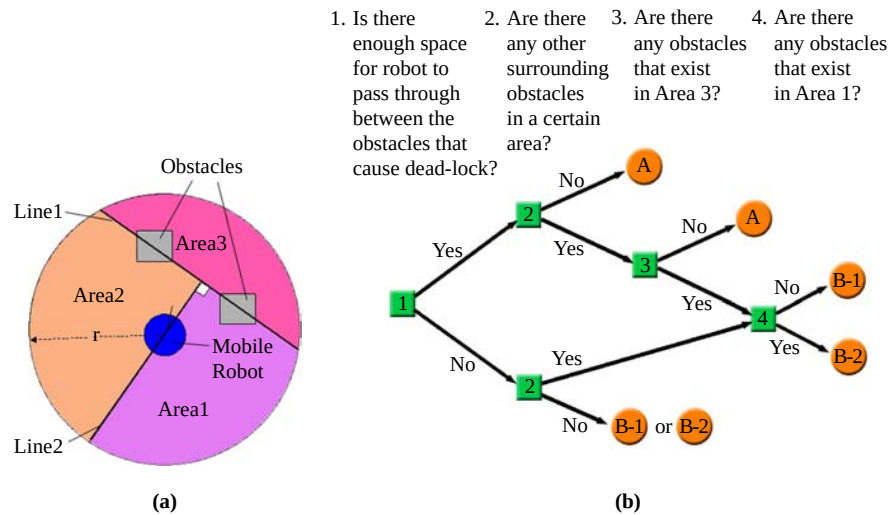
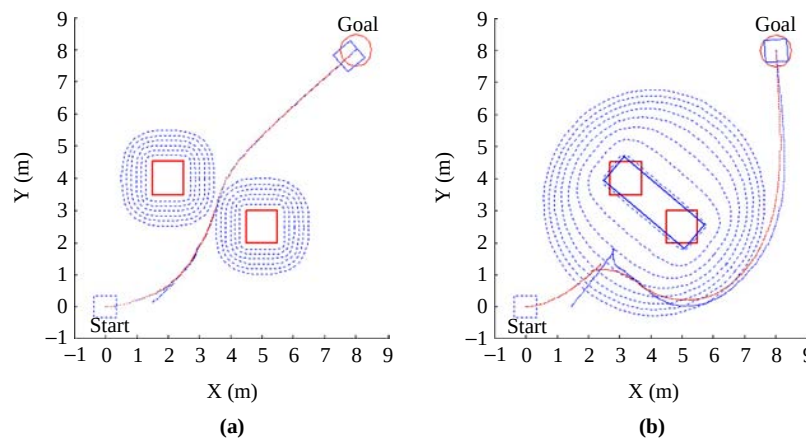
Figure 20 Potential fields causing a dead-lock: (a) XYplane; (b) elevated view**Figure 21** Obstacle detection example: (a) line detection; (b) extracted obstacle

Figure 22 Making decisions for dead-lock avoidance: (a) a dead-lock position; (b) decision-making process**Figure 23** Dead-lock solutions: (a) reduce potential fields; (b) merge of potential fields

5.3 Real experimental results

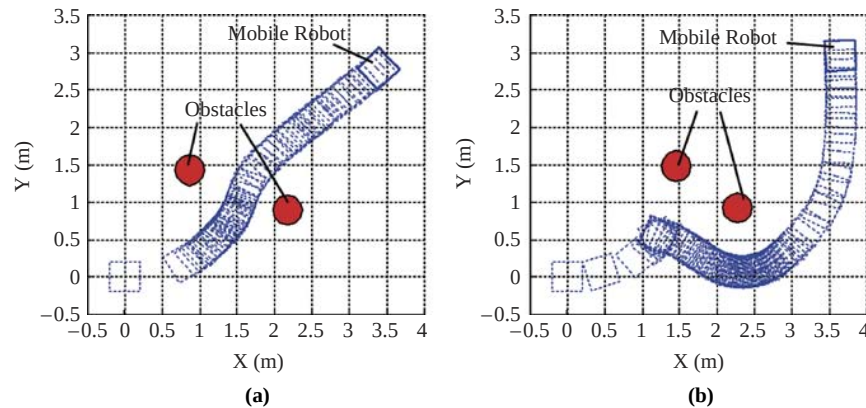
The proposed dead-lock avoidance technique is implemented on the real robot in an environment where two circular obstacles are located as to generate artificial potentials which stop the mobile robot at a dead-lock position. Figure 24 shows the results of applying our technique in different dead-lock conditions (different distances between obstacles causing dead-lock problem). From both results, we can verify that our dead-lock avoidance technique works successfully, and it gives the robot ability of escaping from a dead-lock position and choosing a suitable path for attaining the goal autonomously.

6. Conclusion

A hybrid path planner comprised of ability of concurrent navigation and map building is proposed. The collision avoidance algorithm presented has demonstrated its flexibility in both dynamically controlling the output equation and changing the shape of the repulsive potential field. By using Look-ahead control as the system controller, the possibility of controlling the look-ahead distance (L) allows the mobile

robot to define the suboptimal path and smartly control the velocity for creating a smooth trajectory autonomously. The dead-lock problem is solved by defining necessary sub-goals between targets on the constructed map and a recovery algorithm is proposed for dead-lock avoidance while the robot travels between sub-goals.

Instead of giving a limitation such as setting up a bounded area for autonomous exploration during a navigation mission, we combine our path planner with map building ability to enable the mobile robot to explore a safe path towards a target as reducing the exploration as finding a safe route for attaining the target. As a result, only an essential exploration of unknown environment is performed and it allows the mobile robot to benefit from time saving during exploration for autonomous navigation. Based on collision avoidance algorithm the constructed map information is immediately applicable to autonomous maneuvers for helping the mobile robot to travel between predefined start and goal positions as well as travelling between any targets on the existing map. It should be mentioned that a new Voronoi structure will be naturally

Figure 24 Dead-lock avoidance on real robot: (a) able to pass through; (b) unable to pass through

merged to existing one and active area for further expansion. Thus, the map information can be augmented if the robot starts a new exploration to an unvisited area.

References

- Borenstein, J. and Koren, Y. (1989), "Real-time obstacle avoidance for fast mobile robots", *IEEE Transactions on Systems, Man and Cybernetics*, Vol. 19 No. 5, pp. 1179-87.
- Brockett, R.W. (1983), "Asymptotic stability and feedback stabilization", *Differential Geometric Control Theory*, Vol. 7 No. 11, pp. 1163-73.
- Chang, Y.-C. and Yamamoto, Y. (2006), "Dynamic decision making of mobile robot under obstructed environment", *Proceedings of IEEE/RSJ International Conference on Intelligent Robots and Systems, Beijing, China*.
- Choset, H. and Nagatani, K. (2001), "Topological simultaneous localization and mapping (slam): toward exact localization without explicit localization", *IEEE Transactions on Robotics and Automation*, Vol. 17 No. 2, pp. 125-37.
- Connolly, C.I. (1992), "Applications of harmonic functions to robotics", *Proceedings of IEEE International Symposium on Intelligent Control*, pp. 498-502.
- Connolly, C.I. and Grupen, R.A. (1993), "On the application of harmonic functions to robotics", *Journal of Robotic Systems*, Vol. 10 No. 7, pp. 931-46.
- Hough, P.V.C. (1962), "Method and means for recognizing complex patterns", US Patent 3,069,654.
- Isidori, A. (1995), *Nonlinear Control Systems*, 3rd ed., Springer, New York, NY.
- Khatib, O. (1986), "Real-time obstacle avoidance for manipulators and mobile robots", *International Journal of Robotics Research*, Vol. 5 No. 1, pp. 90-8.
- Koditschek, D.E. and Rimon, E. (1990), "Robot navigation functions on manifolds with boundary", *Advances in Applied Mathematics*, Vol. 11, pp. 412-42.
- Krogh, B.H. (1984), "A generalized potential field approach to obstacle avoidance control", *Proceedings of SME Conference on Robotics Research, Bethlehem*.
- Lau, H. (2003), "Behavioural approach for multi-robot exploration", *Proceedings of Australasian Conference on Robotics and Automation, Brisbane, Australia*.
- Mahkovic, R. and Slivnik, T. (2000), "Constructing the generalized local Voronoi diagram from laser range scanner data", *IEEE Transactions on Systems, Man and Cybernetics*, Vol. 30 No. 6, pp. 710-9.
- Nijmeijer, H. and van der Schaft, A.J. (1990), *Nonlinear Dynamical Control Systems*, Springer, New York, NY.
- Thrun, S. (1998), "Learning metric-topological maps for indoor mobile robot navigation", *Artificial Intelligence*, Vol. 99 No. 1, pp. 21-71.
- Volpe, R. and Khosla, P. (1990), "Manipulator control with superquadric artificial potential functions: theory and experiments", *IEEE Transactions on Systems, Man and Cybernetics*, Vol. 20 No. 6, pp. 1423-36.
- Yamamoto, Y. and Yun, X. (1994), "Coordinating locomotion and manipulation of a mobile manipulator", *IEEE Transactions on Automatic Control*, Vol. 39 No. 6, pp. 1326-32.
- Yamamoto, Y. and Yun, X. (1997), "A modular approach to dynamic modeling of a class of mobile manipulators", *International Journal of Robotics & Automation*, Vol. 12 No. 2, pp. 41-8.

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ATTACHMENT M

RESEARCH ARTICLE | AUGUST 22 2008

On-line path planning strategy integrated with collision and dead-lock avoidance schemes for wheeled mobile robot in indoor environments

Ya-Chun Chang; Yoshio Yamamoto 

[+ Author & Article Information](#)

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Purpose

This paper aims to present a hybrid path planning algorithm which is designed for use of autonomous vehicles in indoor environments. The approach mainly contributes the ability of generating a safe and smooth collision avoidance path for attaining a desired position in an unknown and obstructed environment.

Design/methodology/approach

The hybrid planner is based on potential field method and Voronoi diagram approach, and it is represented with the ability of concurrent map building and autonomous navigation.

Findings

The possibility of controlling the look-ahead distance allows the mobile robot to smartly control the velocity for creating a smooth trajectory autonomously. The dead-lock problem is solved by defining necessary sub-goals between targets on the constructed map.

Originality/value

The system controller (look-ahead control) with the potential field method allows the robot to generate a smooth and safe path for an expected position. Only essential exploration of unknown environment is performed since the approach constrains the mobile robot to explore a safe and sub-optimal route towards a destination.



Keywords: [Robotics](#), [Navigation](#), [Control systems](#), [Linear motion](#)

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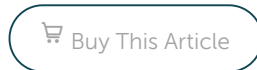
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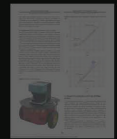
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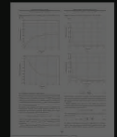
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Research article

On-line path planning strategy integrated with collision and dead-lock avoidance schemes for indoor environments

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robot to perform autonomous tasks in a wide and unknown environment, the ability of building a reliable map of its workspace becomes extremely important. At present, map building and path planning are always being discussed in different areas because of vastness of both fields. During exploration stage the robot localizes its position while generating a map model of its environment via sensory data simultaneously. Based on a reliable and consistent map the robot calculates a path to an expected position as well as updating its position in conjunction with performing a desired motion precisely during a long-term navigation task (Figure 1(a)).

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...er, we propose a hybrid path planner model which
...current navigation and topological map building.
...path planner is composed of potential field method
...6) (Volpe and Khosla, 1990) and Voronoi diagram
...The notion of this approach is inspired by the fact
...f mapping and path search algorithms are not
...nally viable over large-scale environment. In
...contrast, the proposed approach constrains the mobile robot
...to explore a safe and sub-optimal route to a destination and
...allows the robot to accomplish environment modeling and safe
...path finding parallelly when a position of where the robot should
...attain is explicitly (Figure 1(b)). As a result, computational
...burden of environment exploration is reduced to a safe path
...finding for reaching a target position and the approach benefits
...from saving time during environment exploration if a target
...position is explicitly defined.

The potential field-based path strategy is straightforward and guarantees a safe navigation under dynamic environment. However, it suffers from a certain limitation (Krogh, 1984) (Lau, 2003) in which the robot will be trapped in positions where attractive and repulsive forces negate each other. There are several conventional solutions for the dead-lock problem. (Connolly, 1992; Connolly and Grupe, 1993 utilized Laplace's heat equation to create a potential field without local minima. Koditschek and Rimon, 1990) developed a local minima free potential field method by designing