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An Efficient Frequency Offset Estimator for OFDM Systems and Its Performance Characteristics

Yun Hee Kim, Ickho Song, Seokho Yoon, and So Ryoung Park

Abstract—In this paper, we propose a new frequency synchronization algorithm for orthogonal frequency division multiplexing (OFDM) systems requiring only one training symbol, based on a conventional method which requires two training symbols. While the timing synchronization is obtained by using the conventional method, the carrier frequency offset is efficiently estimated by the proposed method. It is shown that the proposed method not only reduces the number of the training symbols but also possesses better performance than the conventional method without increase in complexity.

Index Terms—Frequency synchronization, orthogonal frequency division multiplexing (OFDM).

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is known to be effective in coping with such wireless channel impairments as multipath fading, cochannel interference, and impulsive noise [1], [2]. Moreover, OFDM systems have become practical to implement using fast Fourier transform (FFT) techniques thanks to advances in digital signal processing. Recently, OFDM systems are being applied for fixed and mobile digital transmission such as digital audio and video broadcasting, asymmetric digital subscriber line (ADSL) modems, and wireless LAN. In OFDM systems, synchronization is

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clearly one of the most important steps that should be performed. It is required to set the FFT window within the OFDM symbol not to incur intersymbol interference (ISI). It is also important to synchronize the carrier frequency in the receiver since OFDM systems can only tolerate carrier frequency offset (CFO), which is a fraction of the subcarrier spacing [3].

Many recent studies on OFDM systems are devoted to the frequency synchronization of OFDM signals using blind [4]–[6] and data-aided [3], [7]–[9] schemes. Data-aided schemes are more suitable for the applications requiring fast and reliable synchronization. A maximum likelihood estimator of the CFO was investigated in [3], where the estimation is based on the observation of two consecutive and identical symbols. The estimation range is $\pm(1/2)$ the subcarrier spacing, which can be increased by shortening the training symbol duration at the cost of reduced estimation accuracy. A joint timing and frequency synchronization method is proposed in [8], where two training OFDM symbols are used for both frame/symbol timing and frequency offset estimation. In this method, the frame/symbol timing is found with the first training symbol, which has a repetition within half a symbol period. The fractional small CFO is then corrected and a correlation with the second training symbol is performed to find the remaining large CFO. The method allows a large estimation range of the CFO up to the overall transmission spectrum with a relatively simple structure. A modified version of [8] is proposed in [9], where one training symbol is used for the CFO estimation. The method in [9] can extend the estimation range of the CFO with one symbol compared to that in [8] when only one training symbol is used at the price of much increase in complexity. However, the estimation range is much smaller than that of the method in [8] utilizing two training symbols. Furthermore, a new symbol timing method or another training symbol is required to obtain the symbol timing when the CFO estimator of [9] is utilized.

This paper proposes a new CFO estimation method which can be combined with the conventional timing synchronization algorithm used in [8]: the proposed method requires only one training symbol for both frame/symbol timing and frequency synchronization without performance degradation or increase in complexity. It is also worth noting that the estimation range of the CFO estimator is as wide as that in [8].

II. SYSTEM MODEL

We consider an OFDM system implemented by the inverse fast Fourier transform (IFFT) and FFT, each of size $N = 2^\nu$ for modulation and demodulation. Only N_u subcarriers out of total N subcarriers are used to avoid aliasing effect at the edges of the transmission spectrum. The OFDM samples at the outputs of the IFFT are given by

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N_u-1} X_k e^{j \frac{2\pi k n}{N}}, \quad n = 0, 1, 2, \dots, N-1 \quad (1)$$

where X_k is the QPSK or QAM data symbol transmitted through the k th subcarrier. An OFDM symbol is denoted as $\{x_{N-G}, \dots, x_{N-1}, x_0, x_1, \dots, x_{N-1}\}$, of which the first G samples are guard samples to eliminate ISI. The OFDM samples are pulse-shape filtered at every $1/T_s$ and the filter output is up converted for signal transmission.

The impulse response of the multipath fading channel is assumed to be in the form

$$h(\tau; t) = \sum_{l=0}^{L_p-1} h_l(t) \delta(\tau - \tau_l) \quad (2)$$

where L_p is the number of resolvable multipaths and $h_l(t)$ and τ_l represent the equivalent lowpass impulse response and delay time of the l th path, respectively. It is assumed that $GT_s > T_{\max} = (L_p - 1)T_s$: that is, the guard interval is larger than the maximum delay spread of the channel so that ISI can be completely eliminated.

If the received signal is affected by the CFO f_o , the output signal of the channel is

$$r(t) = e^{j2\pi f_o t} \int_{-\infty}^{\infty} x(t - \tau) h(\tau; t) d\tau + w(t) \quad (3)$$

where $x(t)$ is the baseband OFDM signal and $w(t)$ is a zero-mean complex additive white Gaussian noise (AWGN). The received signal is matched filtered and sampled at the OFDM sample rate $1/T_s$. Assuming that the matched filter is flat within the transmitter bandwidth and the symbol timing is obtained within the guard interval region such that $-GT_s + T_{\max} \leq n_o T_s \leq 0$, the samples belonging to the first effective OFDM symbol, after the timing synchronization is performed, can be written as

$$y_n = e^{j\theta_o} e^{j2\pi f_o n T_s} \sum_{l=0}^{L_p-1} h_l((n + n_o)T_s) x_{n-n_l} + w_n, \quad \text{for } n = 0, 1, \dots, N-1 \quad (4)$$

where $\theta_o = 2\pi f_o n_o T_s$, $n_l = \lfloor -n_o + \tau_l/T_s \rfloor$, and w_n is the sampling output of the AWGN. While we assume that the symbol timing is obtained within the guard interval region to make analysis simpler in this section, we will also consider the case where the symbol timing is erroneously obtained outside the guard interval in Section V.

To clearly see the effect of the CFO on the FFT output, let us assume that the channel is time invariant during one OFDM symbol duration, i.e., $h_l(nT_s) = h_l$, $n = 0, 1, \dots, N-1$. Then the FFT output is given by

$$Y_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y_n e^{-j\frac{2\pi n m}{N}} = e^{j\theta_o} \sum_{k=0}^{N_u-1} H_k X_k \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2\pi n(k-m+c_o)}{N}} + W_m \quad (5)$$

where $H_k = \frac{1}{(1/\sqrt{N})} \sum_{n=0}^{N-1} h_l e^{-j(2\pi k n_l/N)}$, $W_m = \frac{1}{(1/\sqrt{N})} \sum_{n=0}^{N-1} w_n e^{-j(2\pi n m/N)}$, and $\epsilon_o = f_o N T_s$ is the *relative frequency offset* (RFO). The subscripts of Y_m , H_k , X_k , and W_m in (5) and the sequel are all modulo N values. Let the RFO be expressed as $\epsilon_o = 2\epsilon_I + \epsilon_F$, where $2\epsilon_I$ is the *integral part* of the RFO (IFO) that is even and ϵ_F is the *fractional part* of the RFO (FFO) with $|\epsilon_F| < 1$. Then (5) can be rewritten as

$$Y_m = e^{j\theta_o} H_{m-2\epsilon_I} X_{m-2\epsilon_I} e^{j(\frac{N-1}{N})\pi\epsilon_F} \frac{\sin(\pi\epsilon_F)}{N \sin(\frac{\pi\epsilon_F}{N})} + e^{j\theta_o} \times \sum_{k \neq m-2\epsilon_I} H_k X_k \frac{e^{j\pi\epsilon_F} \sin(\pi\epsilon_F)}{e^{j\frac{\pi(m-k-2\epsilon_I)}{N}} N \sin(\frac{\pi(k-m+2\epsilon_I+\epsilon_F)}{N})} + W_m. \quad (6)$$

It is worthy to note that the IFO alters the order of the FFT outputs while the FFO causes the interchannel interference (ICI) in the FFT outputs.

III. A CFO ESTIMATION ALGORITHM

In [8], the frame/symbol timing is found by searching for a training OFDM symbol, which has two identical halves in the time domain and the FFO is also estimated with the metric for the timing synchronization. The training symbol has the property $x_{n+N/2} = x_n$, $n =$

$0, 1, \dots, N/2 - 1$, which is achieved by transmitting the data symbols over the even subcarriers and zeros over the odd subcarriers. The data symbols on the even subcarriers are multiplied by $\sqrt{2}$ in the training symbol to maintain constant OFDM symbol energy. Thus, if we ignore the additive noise, the corresponding two halves of a received OFDM symbol are identical except for a phase shift caused by the CFO, i.e., $y_{n+(N/2)} \approx y_n e^{j\pi\epsilon_o}$. After the timing is obtained, the FFO can be estimated by

$$\hat{\epsilon}_F = \frac{\text{angle} \left(\sum_{n=0}^{N/2-1} y_n^* y_{n+N/2} \right)}{\pi} \quad (7)$$

using the fact that the main difference between the two halves of the first training symbol would be a phase difference of $\phi = \pi\epsilon_o$. Apparently, the IFO cannot be estimated using (7).

We now propose a method to find the IFO using only the training symbol used for the timing and FFO estimation: the conventional method requires another training OFDM symbol to find the IFO. After the FFO of the training OFDM symbol is corrected by multiplying the OFDM samples by $e^{j(2\pi n \hat{\epsilon}_F/N)}$, the FFT outputs of the training symbols assuming perfect correction can be written as

$$Y_m = \sqrt{2} e^{j\theta_o} H_{m-2\epsilon_I} X_{m-2\epsilon_I} + W_m \quad \text{for } m = 0, 1, \dots, N-1 \quad (8)$$

where $X_m = 0$ if m is odd or $m > N_u$. While the ICI may exist in (8), if the FFO estimation error exists as noted already in (6) it is very small at reasonable signal-to-noise ratio (SNR) values so that the effect of the residual CFO error may be ignored.

Let us define a sequence

$$C_k = \frac{X_{2k}}{X_{2k+2}}, \quad \text{for } k = 0, 1, \dots, \frac{N_u}{2} - 2 \quad (9)$$

which has the differential coding structure in the *frequency domain*. Let us assume that the symbol timing is perfectly found and the FFO is compensated without error. The quotient Y_{2k}/Y_{2k+2} is then approximately equal to C_k except that it would be shifted by $2\epsilon_I$ in the frequency domain due to the uncompensated IFO. While the difference between the phases of Y_{2k}/Y_{2k+2} and C_k may exist due to the difference between H_{2k} and H_{2k+2} , it can be ignored since fading components of adjacent subchannels are strongly correlated for OFDM systems with a large number of subcarriers. Therefore, an estimate of ϵ_I is $\hat{g} = g$ which maximizes the metric

$$F(g) = \frac{|\sum_{k \in \mathcal{K}} Y_{2k+2g} C_k^* Y_{2k+2g+2}|^2}{(\sum_{k \in \mathcal{K}} |Y_{2k+2g}|^2)^2} \quad (10)$$

where $\mathcal{K} = \{0, 1, \dots, N_u/2 - 2\}$ and the denominator is used to normalize the metric value. Note that the proposed method has a differential coding between two data symbols transmitted through two adjacent subcarriers in one OFDM symbol, while the conventional method has a differential coding between two data symbols transmitted through the same subcarrier in two adjacent OFDM symbols.

IV. PERFORMANCE OF THE CFO ESTIMATOR IN AWGN

Here, we consider the performance of the proposed CFO estimator when perfect timing recovery is assumed and QPSK symbols are used. The CFO is estimated in two steps, FFO and IFO estimations. First, the performance of the IFO estimator is investigated when the FFO is perfectly estimated, and then the performance when two estimators are used together will be considered.

A. Distribution of the IFO Metric

The mean and variance of the metric $F(g)$ can be obtained using a method similar to that in [8]. In AWGN channel, the FFT outputs, under the assumptions of the perfect timing synchronization and FFO compensation, are given by $Y_m = \sqrt{2}X_{m-2\epsilon_I} + W_m$, if $\text{mod}(m - 2\epsilon_I, N) = 0, 2, \dots, N_u - 2$, and $Y_m = W_m$, otherwise. If we define

$$B(g) = \sum_{k \in \mathcal{K}} Y_{2k+2g} C_k^* Y_{2k+2g+2}^* \quad (11)$$

and

$$A(g) = \sum_{k \in \mathcal{K}} |Y_{2k+2g}|^2 \quad (12)$$

the IFO metric can be written as $F(g) = (|B(g)|^2/A(g)^2)$.

First, let us consider the distribution of $F(g)$ when $g = \epsilon_I$. Then $B_{\text{cor}} = B(\epsilon_I)$ can be written as

$$\begin{aligned} B_{\text{cor}} &= 2 \sum_{k \in \mathcal{K}} |X_{2k}|^2 \\ &+ \sum_{k \in \mathcal{K}} \left(\sqrt{2}X_{2k}^* W_{2k+2\epsilon_I} + \sqrt{2}X_{2k+2} W_{2k+2\epsilon_I+2}^* \right. \\ &\left. + W_{2k+2\epsilon_I}^* W_{2k+2\epsilon_I+2} \frac{X_{2k}^*}{X_{2k+2}} \right). \end{aligned} \quad (13)$$

At reasonable values of SNR and N_u , the real part of B_{cor} is much larger than the imaginary part. Thus, we can ignore $\text{Im}\{B_{\text{cor}}\}$ when we take the magnitude such that

$$\begin{aligned} |B_{\text{cor}}| &\approx 2 \sum_{k \in \mathcal{K}} |X_{2k}|^2 + 2\sqrt{2} \sum_{k \in \mathcal{K}} \text{Re}\{X_{2k} W_{2k+2\epsilon_I}^*\} \\ &+ \sum_{k \in \mathcal{K}} \frac{1}{2\sigma_x^2} \text{Re}\{X_{2k}^* X_{2k+2} W_{2k+2\epsilon_I}^* W_{2k+2\epsilon_I+2}\}. \end{aligned} \quad (14)$$

Let us define that $\text{Re}\{X_{2k}\} = \text{Im}\{X_{2k}\} = \sigma_x^2$, $E\{\text{Re}\{W_m\}\} = E\{\text{Im}\{W_m\}\} = \sigma_w^2$, and $D = N_u/2 - 1$, the number of indices in $\mathcal{K}$. By the central limit theorem, $|B_{\text{cor}}|$ can be approximated as a Gaussian random variable with mean

$$\mu_{|B_{\text{cor}}|} = 2 \sum_{k \in \mathcal{K}} |X_{2k}|^2 = 4D\sigma_x^2 \quad (15)$$

and variance

$$\sigma_{|B_{\text{cor}}|}^2 = D(16\sigma_x^2\sigma_w^2 + 2\sigma_w^4). \quad (16)$$

Similarly, we can express $A_{\text{cor}} = A(\epsilon_I)$ as

$$\begin{aligned} A_{\text{cor}} &= \sum_{k \in \mathcal{K}} [2|X_{2k}|^2 + |W_{2k+2\epsilon_I}|^2 \\ &+ \sqrt{2}(X_{2k}^* W_{2k+2\epsilon_I} + X_{2k} W_{2k+2\epsilon_I}^*)] \end{aligned} \quad (17)$$

whose mean and variance are

$$\mu_{A_{\text{cor}}} = D(4\sigma_x^2 + 2\sigma_w^2) \quad (18)$$

and

$$\sigma_{A_{\text{cor}}}^2 = D(16\sigma_x^2\sigma_w^2 + 4\sigma_w^4) \quad (19)$$

respectively. Note that, since A_{cor} is a Rician random variable with the mean much larger than the standard deviation at reasonable values of SNR and D , it can be approximated as a Gaussian random variable.

Let us define $q_{\text{cor}} = \sqrt{F(\epsilon_I)} = |B_{\text{cor}}|/A_{\text{cor}}$, $u = |B_{\text{cor}}| - \mu_{B_{\text{cor}}}$, and $v = A_{\text{cor}} - \mu_{A_{\text{cor}}}$. Then we have

$$q_{\text{cor}} \approx \frac{\mu_{|B_{\text{cor}}|}}{\mu_{A_{\text{cor}}}} \left(1 + \frac{u}{\mu_{|B_{\text{cor}}|}} - \frac{v}{\mu_{A_{\text{cor}}}} \right) \quad (20)$$

using that $a(1+b)/(1+c) \approx a(1+b-c)$ for $b, c \ll 1$, since $\mu_{|B_{\text{cor}}|} \gg \sigma_{|B_{\text{cor}}|}$ and $\mu_{A_{\text{cor}}} \gg \sigma_{A_{\text{cor}}}$. Here, u and v are both Gaussian since $|B_{\text{cor}}|$ and A_{cor} are approximated to be Gaussian in this paper: it thus follows that q_{cor} is Gaussian. It is easy to see that u and v are zero-mean, $E\{u^2\} = 2D\sigma_w^2(8\sigma_x^2 + \sigma_w^2)$, $E\{v^2\} = 4D\sigma_w^2(4\sigma_x^2 + \sigma_w^2)$, and $E\{uv\} = 16D\sigma_x^2\sigma_w^2$. The mean and variance of q_{cor} are, therefore, given by

$$\mu_{q_{\text{cor}}} = \frac{\mu_{|B_{\text{cor}}|}}{\mu_{A_{\text{cor}}}} = \frac{2\sigma_x^2}{2\sigma_x^2 + \sigma_w^2} \quad (21)$$

and

$$\begin{aligned} \sigma_{q_{\text{cor}}}^2 &= \frac{1}{\mu_{A_{\text{cor}}}^2} [E\{u^2\} - 2\mu_{q_{\text{cor}}}E\{uv\} + \mu_{q_{\text{cor}}}^2E\{v^2\}] \\ &= \frac{8\sigma_x^2\sigma_w^2(1 - \mu_{q_{\text{cor}}})^2 + \sigma_w^4(1 + 2\mu_{q_{\text{cor}}}^2)}{2D(2\sigma_x^2 + \sigma_w^2)^2} \end{aligned} \quad (22)$$

respectively.

Since q_{cor} is Gaussian, $F_{\text{cor}} = F(\epsilon_I) = q_{\text{cor}}^2$ is a noncentral chi-square random variable with degree of freedom 1 and noncentrality parameter $\lambda = \mu_{q_{\text{cor}}}^2/\sigma_{q_{\text{cor}}}^2$ [10]. Hence, the mean and variance of F_{cor} are given by $\mu_{F_{\text{cor}}} = \mu_{q_{\text{cor}}}^2 + \sigma_{q_{\text{cor}}}^2$ and $\sigma_{F_{\text{cor}}}^2 = 2\sigma_{q_{\text{cor}}}^4 + 4\mu_{q_{\text{cor}}}^2\sigma_{q_{\text{cor}}}^2$, respectively.

Next, let us find the distribution of $F(g)$ when $g \neq \epsilon_I$. When $g \neq \epsilon_I$, $B(g)$ is the sum of a number of random variables with different phases. From the central limit theorem, $B_{\text{out}}(g) = B(g \neq \epsilon_I)$ can be approximated as a zero-mean complex Gaussian random variable with

$$\begin{aligned} E\{\text{Re}^2\{B_{\text{out}}(g)\}\} &= E\{\text{Im}^2\{B_{\text{out}}(g)\}\} \\ &= 2(D - d_g)(2\sigma_x^2 + \sigma_w^2)^2 + 2d_g\sigma_w^4 \end{aligned} \quad (23)$$

where $d_g = |g - \epsilon_I|$ such that $1 \leq d_g \leq (N/2) - D$. Since $|B_{\text{out}}(g)|^2 = \text{Re}^2\{B_{\text{out}}(g)\} + \text{Im}^2\{B_{\text{out}}(g)\}$ is the sum of two zero-mean Gaussian random variables squared, we have

$$|B_{\text{out}}(g)|^2 = 2 \left\{ (D - d_g)(2\sigma_x^2 + \sigma_w^2)^2 + d_g\sigma_w^4 \right\} \chi_2^2 \quad (24)$$

where χ_2^2 is a chi-square random variable with degree of freedom 2. The different statistical characteristics of $B_{\text{out}}(g)$ for different g values arise because the guard band exists.

The denominator of $F(g)$, $A_{\text{out}}(g) = A(g \neq \epsilon_I)$, is a noncentral chi-square variable with degree of freedom 1 and noncentrality parameter $\lambda = \mu_{A_{\text{out}}(g)}^2/\sigma_{A_{\text{out}}(g)}^2$, where $\mu_{A_{\text{out}}(g)} = (D - d_g)(4\sigma_x^2 + 2\sigma_w^2) + 2d_g\sigma_w^2$ and $\sigma_{A_{\text{out}}(g)}^2 = (D - d_g)(16\sigma_x^2\sigma_w^2 + 4\sigma_w^4) + 4d_g\sigma_w^4$. Therefore, $F_{\text{out}}(g) = F(g \neq \epsilon_I) = (B_{\text{out}}(g)/A_{\text{out}}(g))^2$ has a doubly noncentral F distribution with degrees of freedom $v_1 = 2$, $v_2 = 1$ and noncentrality parameters $\lambda_1 = 2$, $\lambda_2 = \mu_{A_{\text{out}}(g)}^2/\sigma_{A_{\text{out}}(g)}^2$ [11]. Since the mean of $A_{\text{out}}(g)$ is much larger than the standard deviation at reasonable values of SNR and D , $F_{\text{out}}(g)$ can be approximated as

$$\begin{aligned} F_{\text{out}}(g) &\approx \frac{|B_{\text{out}}(g)|^2}{\mu_{A_{\text{out}}(g)}^2} \\ &= \frac{(D - d_g)(2\sigma_x^2 + \sigma_w^2)^2 + d_g\sigma_w^4}{\{2(D - d_g)(2\sigma_x^2 + \sigma_w^2)^2 + d_g\sigma_w^4\}^2} \chi_2^2. \end{aligned} \quad (25)$$

Note that the worst case occurs when $d_g = 1$ and the metric $F_{\text{out},w}$ in this worst case approximately has the mean $\mu_{F_{\text{out},w}} \approx (1/D)$ and variance $\sigma_{F_{\text{out},w}}^2 \approx (1/D^2)$.

Figs. 1 and 2 show the mean and variance of the proposed and conventional metrics in AWGN channel versus the SNR, $\gamma = (E^2\{y_n\}/E\{w_n^2\}) = (N_u/N)(\sigma_x^2/\sigma_w^2)$, when $g = \epsilon_I$ (Correct in the figures) and $g \neq \epsilon_I$ in the worst case (Incorrect in the figures). The dotted points are the results from a Monte Carlo simulation with 10^4 iterations. It is observed that the results from analysis agree quite well with those from simulation. Even though we do not show in

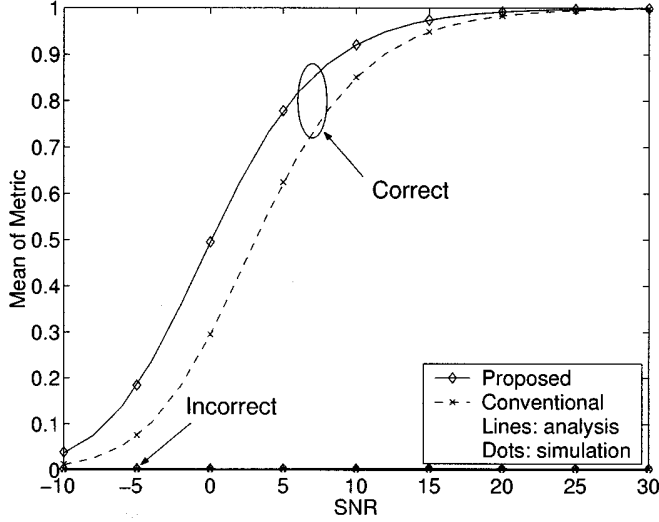


Fig. 1. The mean of the IFO metrics when $N = 1024$, $N_u = 864$, $G = 64$ in AWGN channel.

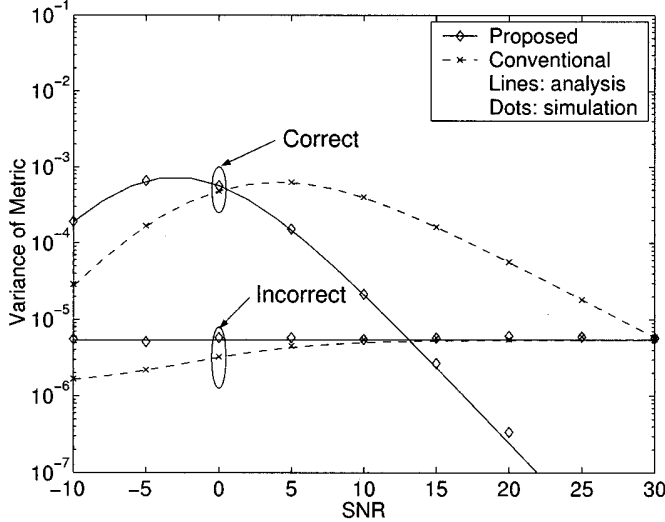


Fig. 2. The variance of the IFO metrics when $N = 1024$, $N_u = 864$, and $G = 64$ in AWGN channel.

the figure, we found that the mean and variance of $F_{\text{out}}(g)$ does not change significantly for large N and N_u .

B. Probability of Missing the Correct IFO

Here, we investigate the probability of missing the correct IFO when a threshold is set to satisfy the required false alarm probability. It is possible to express the problem of finding the correct IFO by a hypothesis testing problem of choosing between a null hypothesis $H_0 : g \neq \epsilon_I$ and an alternative hypothesis $H_1 : g = \epsilon_I$. If the IFO metric is larger than a threshold T , H_1 is accepted; otherwise, H_1 is rejected. The missing probability of the proposed method is

$$P_M = \Pr\{F(g) < T | H_1\} \approx Q\left(\frac{\mu_{q_{\text{cor}}} - \sqrt{T}}{\sigma_{q_{\text{cor}}}}\right) \quad (26)$$

where $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-(y^2/2)} dy$. The probability of falsely finding the IFO, i.e., the false alarm probability, is

$$P_{\text{fa}} = \Pr\{F(g) > T | H_0\} \approx e^{-DT}. \quad (27)$$

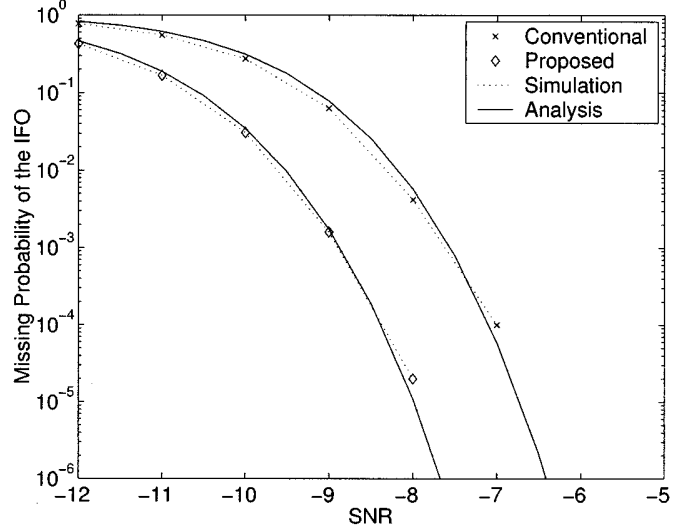


Fig. 3. The probability of missing the correct IFO when $N = 1024$, $N_u = 864$, and $G = 64$ in AWGN channel.

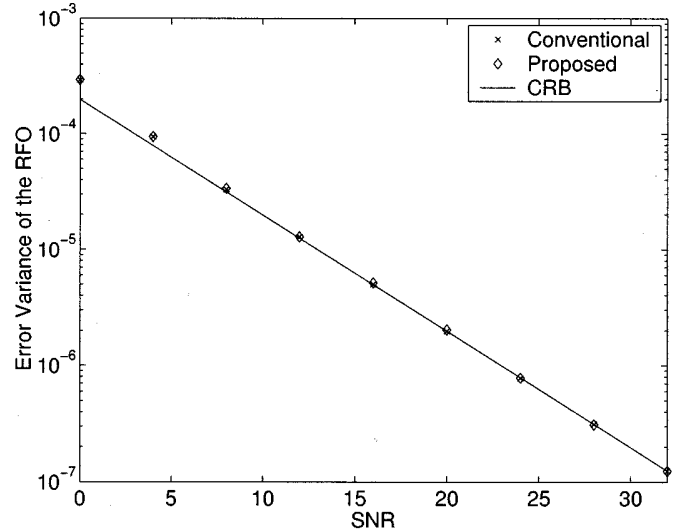


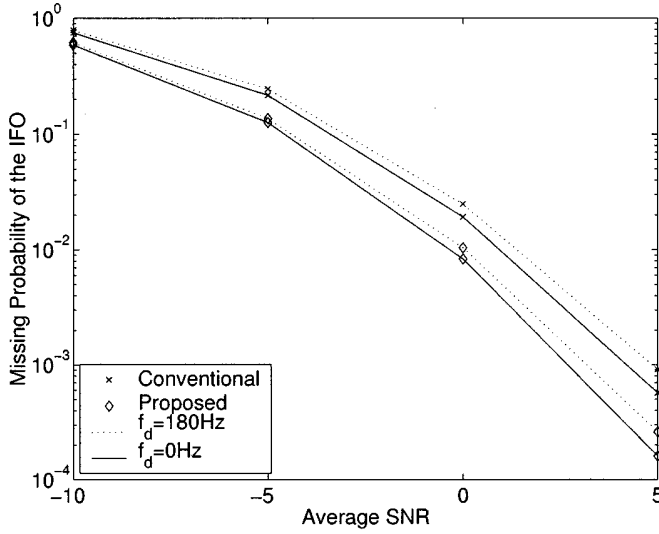
Fig. 4. The error variance of the RFO estimate when $N = 1024$, $N_u = 864$, $G = 64$, and $\epsilon_o = 13.6$ in AWGN channel.

Usually, the threshold is obtained by fixing P_{fa} such that $T_P = -(1/D) \log P_{\text{fa}}$.

Fig. 3 shows the probability of missing the correct IFO when $N = 1024$, $N_u = 864$, $G = 64$, and $\hat{\epsilon}_I$ is obtained by setting the threshold to satisfy $P_{\text{fa}} = 10^{-3}$. It is observed that the results from analysis are close to those from simulation. It implies that the assumptions we made for the pdfs of the IFO metric when $g = \epsilon_I$ and $g \neq \epsilon_I$ are acceptable. It is also observed that the proposed method has much smaller probability of missing the correct IFO than the conventional method. This is because the proposed method utilizes all the symbol energy of one training symbol while the conventional method cannot utilize all the symbol energy since two training symbols must have different structures.

C. The Performance of the Total RFO Estimator

The error variance of the RFO estimate $\hat{\epsilon}_o = 2\hat{\epsilon}_I + \hat{\epsilon}_F$ when the FFO and IFO estimations are performed sequentially is shown in Fig. 4 as a function of the SNR when $N = 1024$, $N_u = 864$, $G = 64$,


 Fig. 5. The probability of missing the correct IFO when $f_d = 0$ and 180 Hz.

and $\epsilon_I = 13.6$. In the figure, ϵ_I is estimated by searching for $\hat{g}$ that maximizes the IFO metric without setting the threshold. Since ϵ_I is found correctly at reasonable values of the SNR in both the proposed and conventional methods, the error variance of the RFO results only from the FFO estimator and thus is the same for both methods. The error variance of the FFO estimator can be found at high SNR as [3], [8]

$$E\{(\hat{\epsilon}_o - \epsilon_o)^2\} = \frac{2}{\pi^2 N \gamma}. \quad (28)$$

This satisfies the Crámer–Rao bound (CRB) as mentioned in [8]. We can see that the proposed method shows almost the same performance as the conventional method even though the former uses only one training OFDM symbol while the latter uses two. We would also like to mention that the proposed method has the same estimation range and complexity as the conventional one.

V. PERFORMANCE IN A MULTIPATH FADING CHANNEL

The performance of the CFO estimator is now investigated in a multipath fading channel by Monte Carlo simulation method. In the following results, we use the Jakes model [12] to simulate the fading channel and we fix $N = 1024$, $N_u = 864$, $G = 64$, $f_c = 5$ GHz, and $T_s = 0.2 \mu s$.

The channel model is a six-path time varying multipath fading channel model of which $L_p = 6$ and $\tau_l = 10T_s$ in (2) and the autocorrelation of the path gains is

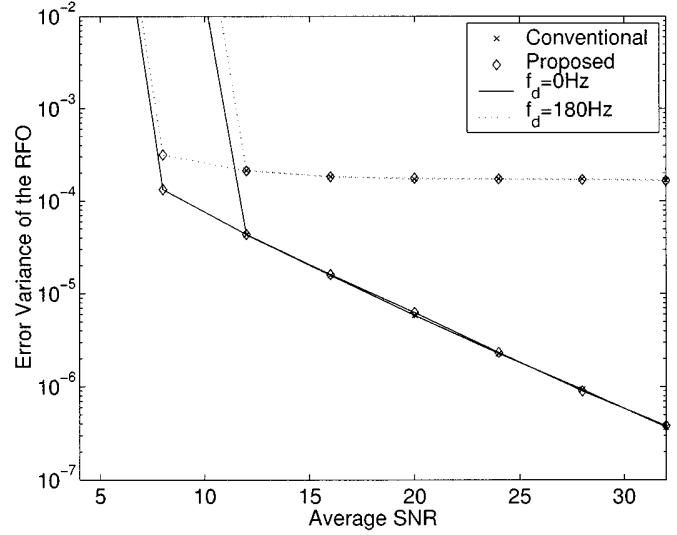
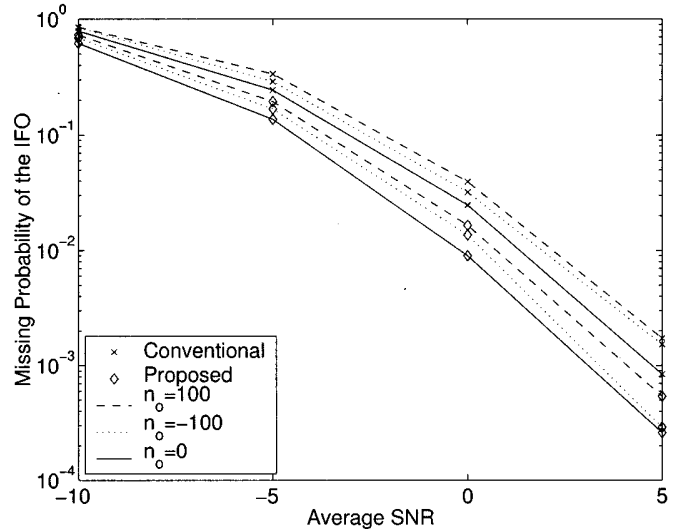
$$E\{h_l(t)h_l^*(t+u)\} = \rho_l J_0(2\pi f_d u) \quad (29)$$

where

- ρ_l power of the l th path;
- $J_0(\cdot)$ zeroth order Bessel function;
- f_d maximum Doppler frequency related to the rate of the channel variation [12].

The exponential multipath intensity profile $\rho_l = \rho \exp(-\tau_l/\bar{\tau})$ is assumed, where $\bar{\tau} = 30T_s$ and ρ is a constant which satisfies $\sum_{l=0}^{L_p-1} \rho_l = 1$.

Fig. 5 shows the probability of missing the correct IFO when $f_d = 0$ Hz and 180 Hz. It is observed that the proposed IFO estimator has better performance than the conventional one in the multipath fading


 Fig. 6. The error variance of the RFO estimate when $\epsilon_o = 13.6$ and $f_d = 0$ and 180 Hz.

 Fig. 7. The probability of missing the correct IFO for some timing errors when $f_d = 180$ Hz.

channel. It is also observed that the missing probability increases when the maximum Doppler frequency increases from 0 to 180 Hz.

Fig. 6 shows the error variance of the RFO estimate when $\epsilon_o = 13.6$ and $f_d = 0$ Hz and 180 Hz. At low SNR values, the mean of the RFO estimator deviates a lot from the RFO since the IFO estimator cannot find the IFO exactly. However, if the SNR is greater than 10 dB, the RFO estimator performs well. It is observed that the error variance of the RFO estimator gets larger as the time variation of the channel increases. Despite the proposed method uses only one training OFDM symbol while the conventional method uses two, the proposed RFO estimator shows slightly better performance than the conventional one in multipath fading channels. Thus we can reduce the overhead due to the training symbols with the proposed method.

Figs. 7 and 8 show the probability of missing the correct IFO and the error variance of the RFO estimate when some timing errors exist, respectively. In these figures, the maximum Doppler frequency is fixed to $f_d = 180$ Hz. It is observed that the results with timing error exhibit similar performance tendency to that observed in the results obtained

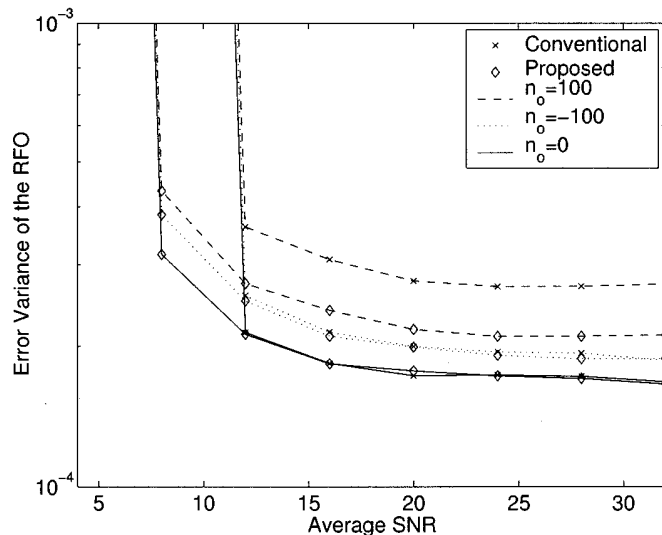


Fig. 8. The error variance of the RFO estimate error for some timing errors when $\epsilon_o = 13.6$ and $f_d = 180$ Hz.

without timing error except that there is a slight increase in the probability of missing the correct IFO and the error variance of the RFO estimate.

VI. CONCLUSION

In this paper, we propose an efficient CFO estimator that uses only one training symbol. The proposed training symbol structure can be used not only for CFO estimation but also for frame/symbol timing synchronization. The performance of the proposed method is presented in terms of the missing probability and the error variance of the CFO estimator. The proposed method is shown to have better performance than the conventional method in AWGN and multipath fading channels without increasing complexity.

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Performance of 2-D RAKE Receiver in a Correlated Frequency-Selective Nakagami-Fading

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Abstract—The bit error rate (BER) performance and the characteristics of a two-dimensional (2-D) RAKE receiver operating in a correlated frequency-selective Nakagami-fading environment are analyzed. Correlated fading between array elements whose fading statistics are identical across the same RAKE branch, as well as an arbitrary number of RAKE branches with arbitrary fading statistics, are assumed. We derived an approximated signal-to-noise ratio (SNR) statistics for one RAKE branch with correlated multiple antennas, which is extended to that for multiple RAKE branches with arbitrary fading statistics, i.e., a 2-D RAKE receiver. The receiver's performance and characteristics are analyzed using the cumulative distribution function of the SNR at the 2-D RAKE receiver output and the BER under various conditions. Numerical results show that the improvement in performance of the 2-D RAKE receiver is brought about by the average SNR and diversity gains, which are identified by two parameters specifying the Gamma distribution of SNR.

Index Terms—Adaptive array, Nakagami fading, spatial correlation, 2-D RAKE receiver.

I. INTRODUCTION

The use of an adaptive array antenna is one prominent technique in improving the performance of wireless communication systems based on the current advancing real-time signal-processing technology. Among the applications of the adaptive array antenna to code-division multiple-access (CDMA) systems, it is the 2-D RAKE receiver that exploits the space- and time-domain structure of the received multipath signal and is known as a very suitable structure for CDMA [1], [2].

A number of studies have analyzed the bit error rate (BER) performance of the 2-D RAKE receiver, which is always limited to the case of correlated Rayleigh fading [1], [2]. In the case of the Nakagami-fading model, analyses are usually limited to maximal ratio combiner (MRC) with correlated branches in nonselective fading channel [3], [4]. Among them, the literature presents a moment generating function (MGF)-based approach for MRC performance with an arbitrary number of branches and any arbitrary branch correlation in a Nakagami-fading environment [4]. It corrects some approximate results and removes the limitation associated with the correlation model

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