Optimal Communication in Bluetooth Piconets

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***Abstract*—Bluetooth is a low-power, low-cost, short-range wireless communication system operating in the 2.4-GHz industrial, scientific, and medical (ISM) band. Bluetooth links use frequency hopping whereby each packet is sent on a single frequency while different packets are sent on different frequencies. Further, there are a limited number of packet sizes. We show that we can exert indirect control over transmission conditions by choosing the packet size transmitted over each frequency as a function of the channel conditions. Our goal then is to provide a packet-size-selection algorithm that can maximize the throughput in a Bluetooth piconet in the presence of lossy wireless channels. We first develop a renewal-theory-based mathematical model of packet transmission in a frequency-hopping system such as a Bluetooth piconet. We use this model to show that a threshold-based algorithm for choosing the packet lengths maximizes the throughput of the system. We provide an algorithm that determines the optimal thresholds efficiently. We show the optimality of this algorithm without using standard optimization techniques, since it is not clear that these techniques would be applicable given the functions involved. Using simulations, we observe that this strategy leads to significantly better throughput as compared to other baseline strategies, even if the assumptions made to prove optimality are relaxed.**

***Index Terms*—Bluetooth, packet-size -selection, scheduling, optimization.**

I. Introduction

THERE has recently been tremendous interest in applications of Bluetooth wireless technology. Bluetooth is a low-power, low-cost, short-range wireless communication system [10], [19]. It enables small portable devices to connect to each other and communicate in an *ad hoc* fashion with nominal speeds of up to 1 Mb/s. Industry analysts have estimated that 13 million bluetooth devices were shipped in 2001, meeting earlier forecasts, and predict that by 2005 there will be over 780 million new Bluetooth devices shipped worldwide [6].

We focus on the use of Bluetooth as a cable-replacement technology. Bluetooth uses frequency hopping and operates in the unlicensed 2.4-GHz industrial, scientific, and medical (ISM) band, which is also used by IEEE 802.11 radios as well as other devices, such as microwave ovens, baby monitors, etc. Thus, the frequencies in this band are subjected to interference from other sources in addition to being subjected to the vagaries of wireless links. We concentrate on providing solutions for enabling efficient communication between Bluetooth devices in the presence of such interference. Rather than resorting to increased error correction, improved power control, or other lower layer techniques, we consider how scheduling packet transmissions and modifying the length of packets in response to the current channel conditions can improve the system throughput.

The basic idea of this paper is as follows. A Bluetooth piconet consists of a *master* device and up to seven active *slaves*. Time is divided into slots and the master decides which slave can communicate in a given slot. Bluetooth uses 79 different frequencies during its frequency-hopping sequence. The master's fixed Bluetooth address determines the hopping sequence used within the piconet; thus, there is a specific frequency associated with a given slot. At any time, each frequency can have a different bit-error rate (BER) associated with it. If the BER for a particular frequency can be estimated, the master can appropriately select the length of the packet sent over that frequency. Intuitively, shorter packets are sent over frequencies with high BER so as to reduce the probability that they are lost. Further, the BER to each slave may be different and time varying, depending on its distance from the master and other factors. The master can select the slaves based on the BERs. Thus, the slaves that have a low BER in a slot may be preferred, thus reducing the packet loss and increasing the throughput.

Bluetooth packets can have three lengths: namely, 366 (DH1), 1622 (DH3), and 2870 bits (DH5) (this includes the packet headers also). These packets consume 1, 3, 5 slots, respectively. Each slot consumes 625 μs. The entire packet is sent at the same frequency, irrespective of the length of the packet, and a new frequency is used only for the next packet. The throughput can be increased significantly by the appropriate selection of packet lengths in accordance with the BER; we use a simple example to illustrate this.

Example: Let the frequency sequence alternate between two frequencies f_1 and f_2. The BERs associated with f_1 and f_2 are 10^{-3} and 10^{-4}, respectively. Consider the following three transmission strategies: 1) transmit all packets as DH5 packets; 2) transmit DH5 packets on frequency f_1 and DH1 on f_2; and 3) transmit DH1 on f_1 and DH5 on f_2. A DH5 packet sent on f_1 experiences a packet-error rate (PER) of 0.94 while a DH5 packet sent on f_2 experiences a PER of 0.249; the corresponding values for a DH1 packet are 0.306 and 0.036. Therefore, the throughputs of the three strategies are 46 550·4, 17 500, 80 312·4 B/s, respectively.

Thus, by judiciously adapting the packet length to channel conditions, we can significantly increase the throughput of the system. We provide algorithms that can choose the packet lengths so as to maximize the throughput for the given channel conditions.

The problem of choosing packet lengths in lossy channels has been investigated widely for systems with only one frequency

Manuscript received March 22, 2004; revised September 5, 2004 and October 10, 2004. This work was supported by National Science Foundation (NSF) Grants: ANI01–06984, NCR02–38340, CNS04–35306. The review of this paper was coordinated by Prof. T. Hou.

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Digital Object Identifier 10.1109/TVT.2004.841554

0018-9545/$20.00 © 2005 IEEE

Zophonos EX2008
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IPR2026-00085

of transmission [8], [11]. The aim has been to trade off the overhead of packet headers with the PER (the tradeoff is that small packets have more overhead than longer ones, but have an increased probability of successful transmission). Our work differs due to the nature of frequency-hopping systems such as Bluetooth. We consider the case whereby the entire packet is sent on a single frequency, but different packets are sent on different frequencies. Hence, we can mitigate the effects of channel conditions by selecting the packet length to be transmitted over a frequency as a function of the channel conditions. As illustrated in the earlier example, by deciding to send larger packets over frequencies with good transmission conditions, we can increase the fraction of time the system experiences good transmission conditions and thereby maximize the bandwidth attained by the system. Due to the frequency hopping in Bluetooth, the BER for each frequency and time slot must be considered in determining the packet lengths, and the optimizations can provide higher throughput than in a system with a single frequency. In addition, packet lengths cannot be chosen arbitrarily, but are restricted to specified values. Finally, due to the master–slave relationship in Bluetooth, the master can select slaves which receive information and decide the lengths of the packets it transmits, thus allowing fine-grained control over the system throughput.

Chiasserini *et al.* propose mechanisms to mitigate interference between 802.11 and Bluetooth networks [5]. The key idea in Bluetooth is to avoid the frequencies used by the 802.11 network. The concept of adaptive frequency hopping has been proposed in the IEEE 802.15 group [20]. This proposal advocates dynamically changing the frequency-hopping sequence in order to minimize interference. The focus is on providing primitives in the Bluetooth stack to make this happen without regard to the actual algorithms to be used. In addition, this requires modification to the existing Bluetooth devices. Our research is complementary in that we propose to use the existing frequency-hopping sequence more efficiently by optimizing the packet lengths. Further, our framework can cater to sources of interference other than IEEE 802.11. The algorithm we propose can also be implemented in conjunction with adaptive frequency hopping [20] to provide a higher efficiency.

In Section II, we provide some background on Bluetooth as well as a brief survey of related work. In Section III, we develop a mathematical model of packet transmission in a Bluetooth piconet with a single slave. We then use this model to show that a threshold-based decision strategy for choosing the length of the packet to be transmitted on a given frequency maximizes the system throughput. Given the three packet lengths that we consider, this means that there are two thresholds. The largest (smallest) packet sizes must be used for BER values below the lower (above the higher) threshold. We then provide an algorithm [throughput optimal threshold selection (TOTS)] that determines the optimal values of the thresholds efficiently. The standard optimization techniques do not apply in this case due to the nature of the functions involved. We then extend our results to multiple active slaves in a piconet. In Section IV, we demonstrate using simulations that the optimum throughput is significantly higher (e.g., around 50% more) than other baseline strategies. We present the proofs in Appendexes.

II. Frequency-Hopping Characteristics in Bluetooth

We describe the relevant technical details of the Bluetooth standard. A set of Bluetooth devices form a group called a piconet, which has one device operating as a master and up to seven devices functioning as active slaves at any given time. A master can communicate with any slave in its piconet. Slaves, on the other hand, can communicate directly only with the master.

The devices in a piconet use frequency hopping to communicate. The clock and Bluetooth device address of the master determine the frequency-hopping pattern used in the piconet. Time is divided into slots of length 625 μs. Slaves communicate to the master using time-division multiple access (TDMA). A slave can transmit only if the master has addressed it in the previous slot. The master transmits in the even-numbered slots and a slave transmits in the odd-numbered slots. For this purpose, packets must occupy an odd number of slots. Hence, each packet spans one, three, or five slots and is transmitted at a single frequency. After each packet is transmitted, the devices retune their radios to the next frequency in the sequence. The sequence involves all 79 (or 23) hop frequencies.

Two types of links are allowed. Synchronous connection-oriented (SCO) links support symmetrical circuit-switched connections and are expected to be used for voice traffic. Asynchronous connectionless (ACL) links are used for bursty data transmissions. The master controls the allocation of the ACL link bandwidth to each slave. We consider only ACL links. We would like to point to [3], [4], [7], [13], [15],[16], [17], [22] for algorithms on scatternet formation, scheduling and cross-layer maximization in Bluetooth networks.

III. System Model and Analysis

Our goal is to provide a packet-size-selection algorithm that can maximize the throughput in a Bluetooth piconet. For simplicity, we consider one-way data transfer from the master to the slave. This happens when the master transmits streaming audio or streaming video or large files. We first develop a mathematical model for capturing the throughput-optimization problem in the Bluetooth scenario. The model must be general enough to capture the various nuances of the problem and at the same time simple enough to cater to a packet-length-selection algorithm that can be executed by a resource constrained wireless device. Subsequently, we use the model to design an algorithm for optimally selecting the packet lengths. Even though we consider the specific case of Bluetooth, the model we develop applies to any frequency-hopping system where a single packet must be transmitted on a single frequency. We initially consider a piconet with a single slave. Later, we extend this model to consider piconets with multiple slaves.

A. Piconets With One Slave

First, we explain the model and subsequently justify it. Consider a Bluetooth piconet with a single slave. The master of the piconet transmits packets to the slave using frequency hopping. The master can choose from three different packet lengths, namely, 366 (DH1), 1622 (DH3), and 2870 bits (DH5), with payloads of 216, 1464, and 2712 bits, respectively. These packets occupy one, three, or five bluetooth slots; each slot

is of length 625 μs. When a slave receives a packet, it sends an acknowledgment packet that occupies one slot and has 126 bits.[1] Thus, a packet and the acknowledgment packet together consume two, four, or six slots. Every data and acknowledgment packet has 18 bits in the header that are $1/3$ FEC protected; that is, each such bit is repeated three times.

Since our objective is to determine the maximum possible throughput, we assume that there is an unlimited supply of packets of each type. This happens if there are multiple sessions (at least three) from a master to every slave with an infinite supply of data for each session. Each session uses a single packet size. Thus, this translates to an unlimited supply of each packet type. Segmentation is still assumed to occur at the L2CAP layer.

The probability of successful transmission of each bit in a given packet or the bit success rate (BSR)[2] is denoted as p. We assume that p is a random variable with cumulative distribution function (cdf) given by $F(p)$. All bits of a packet are assumed to have the same BSR while the BSR associated with different packets are mutually independent. Transmission of a packet is successful if every bit that is not FEC protected is received without error and a minority of the FEC protected bits are in error. For example, if a FEC protected bit has value 1 with each bit being repeated three times, then the packet would contain 111 and an error-free reception would contain 111, 110, 101, or 011. We assume that an acknowledgment packet is received with no error.

Now we justify the model. The BSR associated with a bit strongly depends on the frequency as the transmission condition can be poor in one frequency and good in another. If the frequencies are random variables, then the BSR is also random. In Bluetooth, frequencies in the hopping sequence are generated by a pseudorandom number generator seeded by the masters address and clock. Thus, the frequencies constitute "pseudorandom variables" and this motivates the above model. All the frequencies can, however, be known before the system transmits any packet, since the frequencies are generated by a pseudorandom sequence. If the BSRs associated with these frequencies are known ahead as well, then the entire packet length sequence can be determined using a deterministic optimization. However, the frequency sequences are usually long and contain several different frequencies. Thus, the optimization will be computationally intensive. Besides the storage of the results and the table lookup will require substantial space and time. More importantly, BSR is not a deterministic function of frequency since the transmission conditions vary with time for the same frequency. Hence, the previous knowledge of the frequency sequence may not be useful in determining the BSRs ahead of time. This motivates the modeling of the BSRs as random variables. We do not assume any specific characteristic of the cdf of the random variable BSR.

A packet is transmitted on a single frequency irrespective of its length. The transmission conditions associated with a frequency do not change significantly in a short duration, particularly when the devices do not move or move slowly. The length of a packet is at most five slots (3.125 ms). This motivates the assumption of the same BSR for all bits in a packet. We assume that the BSRs for different packets are mutually independent. This is justified because different packets are transmitted in different frequencies generated as a pseudorandom number sequence and transmission conditions may be quite different for different frequencies. Losses can be bursty in the absence of frequency hopping. Finally, we assume that the acknowledgment is transmitted without loss, since an acknowledgment packet has a small size and large parts of it are FEC protected.

At the start of the transmission, the master decides the packet lengths based on the BSR governing the bits of the packet. The assumption that the transmitter knows the channel conditions has been made elsewhere as well, e.g., [9] and [12]. Using simulations, we will consider the performance of the optimum algorithm when the master decides the packet lengths based on estimates of the BSR.

Now, we demonstrate how this model can be used to provide an algorithm for choosing the packet lengths so as to maximize the throughput. Given the BSR p for the frequency on which a packet is to be transmitted, the master of the piconet decides the packet length $l(p)$ bits, as a function of the BSR, at the beginning of the packet transmission. The time taken to transfer l bits and the acknowledgment is denoted as $\bar{l}$. Here, $\bar{l}$ is 1.25, 2.5, and 3.75 ms for l equaling 366, 1622, and 2870 bits, respectively. The payload for a packet of l bits is $\underline{l}$. Here, $\underline{l}$ is 216, 1464, and 2712 bits for l equaling 366, 1622, and 2870 bits, respectively. Since the BSRs of different packets are mutually independent, lengths of different packets are also mutually independent under the optimal strategy [14]. Thus, the packet-transmission process is a renewal process, with the system renewing itself after every packet transmission [21]. Let $F(p)$ be the cdf for the BSR. Then, the average duration of a single renewal period is the average time taken to transmit a packet and the acknowledgment $E(\bar{l}(p)) = \int_0^1 \bar{l}(p)dF(p)$. There is a reward associated with transmission of a packet. If the packet is transmitted successfully, then the reward is the packet's payload; otherwise, the reward is 0. Thus, the expected reward for a packet is $l(p)p^{l(p)}$ if p is the BSR during the transmission of the packet. We initially consider ACL links without error protection mechanisms. Now, given the cdf $F(p)$, the expected reward for any packet is $E(\underline{l}(p)(p^3+3p^2(1-p))^{18}p^{l(p)-54}) = \int_0^1 \underline{l}(p)(p^3+3p^2(1-p))^{18}p^{l(p)-54}dF(p)$. Note that each packet has 18 FEC protected bits and $l(p)-54$ bits that are not protected by FEC. The FEC protected bits in the packet contribute to $(p^3+3p^2(1-p))^{18}$ and remaining bits contribute to $p^{l(p)-54}$. The throughput of the system is the average reward per unit time $\lim_{t\to\infty} E(R(t))/t$ where $R(t)$ is the total reward obtained before time t. Using the renewal–reward theorem [21], the throughput of a packet length selection rule $l(p)$ is

$$\begin{aligned}\lim_{t\to\infty} & E(R(t))/t \\ &= \frac{E\left(\underline{l}(p)(p^3+3p^2(1-p))^{18}p^{l(p)-54}\right)}{E\{\bar{l}(p)\}} \\ &= \frac{\int_0^1 \underline{l}(p)\left(p^3+3p^2(1-p)\right)^{18}p^{l(p)-54}dF(p)}{\int_0^1 \bar{l}(p)dF(p)}. \qquad (1)\end{aligned}$$

[1]A slave may also send a data packet instead of an acknowledgment, but we do not consider this option.

[2]BSR $= 1 -$ BER, where BER is the bit-error rate.

We next wish to find a packet-length-selection rule $l(p)$ that maximizes $\lim_{t\to\infty} E(R(t))/t$. We first present a generic property of the optimal strategy that holds irrespective of the distribution $F(p)$.

Theorem 1: There exists thresholds p_1^*, p_2^* such that the optimal packet length $l^*(p) = 366$ bits for $p < p_1^*, l^*(p) = 1622$ bits for $p_1^* \le p < p_2^*$ and $l^*(p) = 2870$ bits for $p \ge p_2^*$.

We prove Theorem 1 in Appendix A. We now present the intuition behind Theorem 1. Large packets are more likely to have error in at least one bit. However, large packets must be transmitted if the BSR is high. This is because the BSR does not change until the packet transmission ends and, thus, a larger number of bits will be transmitted under good channel conditions. Small packets must be selected for low BSR, as this exposes fewer bits to poor channel conditions. This motivates a threshold-type decision process irrespective of the distribution $F(p)$.

We summarize the packet-length-selection rule that we call the throughput optimal packet length selection (TOPS) rule.

1) Define a function $f(a,b)$, as shown in the equation at the bottom of the page.
2) If the BSR is less than p_1^*, transmit a DH1 packet. If the BSR is between p_1^* and p_2^*, transmit a DH3 packet. If the BSR is greater than p_2^*, transmit a DH5 packet.

Theorem 2: TOPS attains the maximum possible throughput for a single slave.

Theorem 2 follows from (1) and Theorem 1.

The function $f(a,b)$ can be nonconcave depending on the distribution $F(p)$. The usual gradient search-based optimization algorithms [1] are not guaranteed to converge or attain a global maxima for a nonconcave function. Thus, *the rich body of optimization literature cannot be used to design an efficient algorithm for maximizing* $f(a,b)$. Maximizing $f(a,b)$ is a two-variable optimization that is more difficult than a single-variable optimization. Using the structure of $f(a,b)$, we have devised a *single-variable* optimization-based iterative technique that we refer to as the TOTS algorithm. TOTS is guaranteed to converge to a global maxima of $f(a,b)$ if the individual single-variable optimizations can be solved. The iterative procedure initializes the arguments a, b to 0. Let (p_1, p_2) be the current value of the iterates. First, $f(p_1, b)$ is maximized by varying b in the range $[p_1, 1]$. Let p_2 be the maximum point in the above maximization. Then, the next step is to maximize $f(a, p_2)$ with respect to a in the range $0 \le a \le p_2$. We show that the iterates are guaranteed to converge to a fixed point (p_1^c, p_2^c), i.e., $f(p_1^c, p_2^c) = \max_{0\le a\le p_2^c} f(a, p_2^c)$ and $f(p_1^c, p_2^c) = \max_{p_1^c \le b \le 1} f(p_1^c, b)$. We prove that the fixed point is a global maxima of $f(a,b)$:

Step 1) $p_1^0 = p_2^0 = 0, k = 0$;
Step 2) $k \to k+1$;
Step 3) $p_2^k = \arg\max_{p_1^{k-1} \le p \le 1} f(p_1^{k-1}, p)$,
$p_1^k = \arg\max_{0 \le p \le p_2^k} f(p, p_2^k)$;[3]
Step 4) terminate if $(p_1^k, p_2^k) = (p_1^{k-1}, p_2^{k-1})$; otherwise, go to Step 2).

It is not obvious whether the iterates converge to the optimal thresholds or if there is convergence at all. The following theorem, however, shows that TOTS indeed converges to the optimal thresholds.

Theorem 3: TOTS converges to the optimal thresholds.

We prove Theorem 3 in Appendix B. Note that we do not have analytical bounds on the number of iterations required by TOTS to attain the optimal thresholds. However, our simulations indicate that TOTS attains the optimal thresholds in only few iterations—in all the cases we simulated, TOTS converged in two iterations.

If the distribution function $F(p)$ is known, the optimal thresholds can be computed offline as per TOTS. For unknown or time-varying channels, the optimal thresholds will have to be computed online after estimating the channel conditions. The above algorithm can be easily implemented in resource-constrained wireless devices. Also, the TOPS and TOTS algorithms can easily be adopted to cater to Bluetooth chipsets that do not support either DH3 or DH5 packets. Finally, the renewal reward framework can be generalized to accommodate many other attributes. For example, if error-correction schemes are used for all bits in a packet, as is done for DM5, DM3, and DM1 packets, then a packet can be successfully received as long as at least k bits are correct. The renewal reward framework can capture this by assuming that a packet with payload l attains a reward of l bits as long as k bits are correct. The average reward (numerator) in the throughput expression in (1) must be altered. Refer to the technical report [18] for the formulations and simulations in this case.

B. Multiple Slaves

We next consider a piconet with k slaves $1 < k \le 7$. Now, the master must select the slave and decide the size of the packet before transmitting a packet.

Different slaves may experience different BSRs, even though the same frequency is used for all slaves. This happens due to location-dependent errors in wireless networks. For example, a particular slave may be near a microwave that operates at the frequency of transmission, while other slaves may be away from the microwave. Hence, in any slot we consider different BSRs for different slaves $p_1, \ldots, p_k$. These are random variables governed by a joint cdf $F(p_1, p_2 \ldots, p_k)$. The joint distribution allows for correlation among the BSRs perceived by different

[3]If the maximizations are not unique, and $f(p_1^{k-1}, p_2^{k-1}) = \max_{p_1^{k-1} \le p \le 1} f(p_1^{k-1}, p)$ and $p_2^{k-1} \ge p_1^{k-1}$, set $p_2^k = p_2^{k-1}$. A similar remark applies for p_1^k.

$$\phi(x) = (x^3 + 3x^2(1-x))^{18}$$
$$f(a,b) = \frac{\int_0^a 216\phi(x)x^{312}dF(x) + \int_a^b 1464\phi(x)x^{1568}dF(x) + \int_b^1 2712\phi(x)x^{2816}dF(x)}{\int_0^a 1250dF(x) + \int_a^b 2500dF(x) + \int_b^1 3750dF(x)}$$
$$(p_1^*, p_2^*) = \arg\max_{0\le a\le b\le 1} f(a,b).$$

slaves. As before, we assume that all the bits of a packet experience the same BSR and a packet is lost if one or more non-FEC protected bits and two or more FEC protected bits are in error. Further, the master knows the BSRs $p_1, \ldots, p_k$ for all the slaves at the beginning of transmission of a packet and uses this knowledge to choose the appropriate slave and packet length. Finally, BSRs for different packets are mutually independent.

Since packet lengths depend only on the BSRs and BSRs for different packets are mutually independent, packet transmission is a renewal process. System throughput can be related to the packet-length-selection policy using the renewal reward theorem [21]. Let a policy π choose the slaves as per function $s(p_1, \ldots, p_k)$ and the packet lengths as per $l(p_1, \ldots, p_k)$ bits [duration $\overline{l}(p_1, \ldots, p_k)$ μs, payload $\underline{l}(p_1, \ldots, p_k)$bits], where $p_1, \ldots, p_k$ are the BSRs of slaves $1, 2, \ldots, k$. Recall that $\phi(x) = (x^3 + 3x^2 * (1-x))^{18}$. Now, π's throughput η_π, is given by (2) at the bottom of the page.

Our goal is to design the optimal strategy π^* that maximizes the throughput η_{π^*}. We describe such a strategy next, which we denote as TOPS-M (throughput optimal packet-length-selection rule for multiple slaves). TOPS-M selects the slave with the highest BSR and then decides the packet length as per the channel condition of the selected slave and the transmission thresholds that depend on the joint distribution function. The strategy can be summarized as follows.

- Define a function $f_M(a,b)$ as

$$\begin{aligned} f_M(a,b) &= A/B \\ \text{where } A &= \int_{0 \le \max(p_1,\ldots,p_k) \le a} 216\phi(\max(p_1,\ldots,p_k)) \\ &\quad \times \max(p_1,\ldots,p_k)^{312} dF(p_1,\ldots,p_k) \\ &\quad + \int_{a \le \max(p_1,\ldots,p_k) \le b} 1464\phi(\max(p_1,\ldots,p_k)) \\ &\quad \times \max(p_1,\ldots,p_k)^{1568} dF(p_1,\ldots,p_k) \\ &\quad + \int_{b \le \max(p_1,\ldots,p_k) \le 1} 2712\phi(\max(p_1,\ldots,p_k)) \\ &\quad \times \max(p_1,\ldots,p_k)^{2816} dF(p_1,\ldots,p_k) \end{aligned}$$

and

$$\begin{aligned} B &= \int_{0 \le \max(p_1,\ldots,p_k) \le a} 1250 dF(p_1,\ldots,p_k) \\ &\quad + \int_{a \le \max(p_1,\ldots,p_k) \le b} 2500 dF(p_1,\ldots,p_k) \\ &\quad + \int_{b \le \max(p_1,\ldots,p_k) \le 1} 3750 dF(p_1,\ldots,p_k) \\ (p_1^*, p_2^*) &= \arg \max_{0 \le a \le b \le 1} f_M(a,b). \end{aligned}$$

- Select the slave m for which $m = \arg\max(p_1, \ldots, p_k)$.
- If m's BSR p_m is less than p_1^*, transmit a DH1 packet. If p_m is between p_1^* and p_2^*, transmit a DH3 packet. If p_m is greater than p_2^*, transmit a DH5 packet.

Theorem 4: TOPS-M maximizes the throughput for any number of slaves.

We prove Theorem 4 in Appendix C. The optimum thresholds are different from those computed according to the marginal distribution of the chosen slave. The function $f_M(a,b)$ can be simplified in special cases. For example, when the BSRs of different slaves are independent and identically distributed (i.i.d.) with the marginal distribution function $F(p)$, $f_M(a,b)$ can be computed as shown in (3) at the bottom of the page.

The TOTS algorithm can be used to obtain the optimum thresholds iteratively by using the function $f_M(a,b)$ instead of $f(a,b)$. We refer to this modified algorithm as TOTS-M.

Theorem 5: TOTS-M converges to the optimal thresholds.

We prove Theorem 5 in Appendix D. In a different context, Bhagwat *et al.* showed that the throughput is maximized when the transmission is over the channel with the least probability of error [2]. This paper applies to generic wireless channels, not specifically to Bluetooth, and as such does not contain the packet-length-optimization problem we are focusing on. Nevertheless, the receiver-selection strategy turns out to be identical in both.

IV. Simulation Results

In this section, using Matlab simulations, we compare the performance of the optimal and some benchmark algorithms. We first describe the simulation scenario. We consider a single piconet in which the master transmits messages to the slaves using the user datagram protocol (UDP) protocol at the transport layer. The master has at least three different UDP applications; each application has infinite number of packets and a predetermined Bluetooth packet size. We assume that there is at least one application that has DHx packets, $x = 1, 2, 3$. Thus, even though the segmentation in a Bluetooth network occurs at L2CAP, the master can transmit packets of the desired size by choosing from the appropriate flow. A slave acknowledges a data packet by sending a NULL packet. We average the throughput over sufficient number of trials (each trial consists of 1600 Bluetooth slots) to ensure with 95% confidence that the empirical average does not deviate from the actual average by more than 1%.

The optimality results would hold for any distribution for the BER. Determining the distribution of the BER in Bluetooth networks is beyond the scope of this paper. We, thus, consider two sample distributions, namely, scaled beta and Rayleigh for the

$$\eta_\pi = \frac{\int_0^1 \cdots \int_0^1 \underline{l}(p_1, \ldots p_k) \phi\left(p_{s(p_1,\ldots,p_k)}\right) \left(p_{s(p_1,\ldots,p_k)}\right)^{l(p_1,\ldots p_k)-54} dF(p_1,\ldots,p_k)}{\int_0^1 \cdots \int_0^1 \overline{l}(p_1, \ldots p_k) dF(p_1,\ldots,p_k)}. \tag{2}$$

$$f_M(a,b) = \frac{\int_0^a 216x^{312}\phi(x)d((F(x))^k) + \int_a^b 1464x^{1568}\phi(x)d((F(x))^k) + \int_b^1 2712x^{2816}\phi(x)d((F(x))^k)}{\int_0^a 1250d((F(x))^k) + \int_a^b 2500d((F(x))^k) + \int_b^1 3750d((F(x))^k)}. \tag{3}$$

channel BER. Note that Rayleigh distribution has been widely used to model fading. In the scaled-beta distribution, the BER for a frequency in a slot is $K \times X$ where K is a constant and X is a random variable with beta probability density function (pdf) (parameters A and B).

$$h(x) = \begin{cases} \frac{x^{A-1}(1-x)^{B-1}}{\int_0^1 x^{A-1}(1-x)^{B-1}}, & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

In the figures, we plot the throughput as a function of the upper limit of the BER range K; here, K varies from 10^{-4} to 10^{-2}. In the simulations, we consider $A = 1$ and $B = 5$.

In the Rayleigh distribution, the BER has a Rayleigh (parameter R) pdf, which is described as

$$h(x) = \begin{cases} \frac{x}{R^2} \exp^{-x^2/2R^2}, & x \ge 0 \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

In the figures, we plot the throughput as a function of the Rayleigh parameter R.

In the simulations, we relax the following assumptions we made in the analysis.

- Assumption 1: The retransmissions can be of type DH1, DH3, or DH5 regardless of the length of the original packet.
- Assumption 2: The reverse channel is lossless (i.e., ACKs are not lost).
- Assumption 3: The statistics of the channel, i.e., $F(p_1, \ldots, p_k)$ is known.

We next discuss how we relax these assumptions.

Relaxation 1: Same-Size Retransmissions: When a packet transmitted by the master is lost, the simplest action for the master is to send the same packet. However, channel conditions may have changed and, under the optimal algorithm, the master should perhaps send a packet of different size. But, our simulations consider the more realistic case in which retransmissions are constrained to be of the same size as the original packet.

Relaxation 2: Lossy Reverse Channels: We allow the ACKs to be lost in the simulations.

Relaxation 3: Channel BSR Estimation: In practice, the master does not know the statistics of the channel specified by the joint distribution for the BSRs $F(p_1, \ldots, p_k)$. The master can, however, estimate this distribution using channel-estimation techniques, which have been extensively studied in cellular and other wireless systems. We compare the performance of the optimal and benchmark algorithms when the master estimates the joint distribution using a simple existing technique. We do not propose any new technique for channel estimation. We assume that the master knows the distribution governing the packet losses and determines the *parameters of the distribution* by transmitting DH1 packets initially to the slave and then by observing the fraction of DH1 packets lost for each given slave and frequency. In our experiments, the master transmits DH1 packets for 3200 slots, estimates the channel parameters from the transmission results, computes the optimal thresholds from the estimates, and uses these thresholds in subsequent transmissions. Note that better estimation schemes can be expected to improve the performance of the algorithms proposed as well as to remove the assumption that the packet-loss distribution is known. In addition, the estimation can be done online continuously for slowly time-varying channels. We do not investigate this approach in this paper.

We present simulation results for a piconet with multiple slaves. In all the figures, the curve "optimal" shows the throughput of TOPSM, as measured in the simulations. We also consider the following benchmark strategies for choosing the slave and the packet lengths.

- **Round-robin DHx (rrdhx)**: The master transmits only DHx (where $x = 1, 3, 5$) packets to each slave in a round-robin fashion.
- **Round-robin optimal (rropt)**: The master selects the slaves in round-robin fashion, but determines the packet size based on the channel conditions to the selected slave.

We can accommodate arbitrary correlations among BERs of different slaves by selecting the joint distribution function for the BERs appropriately. However, for simplicity, we assumed that the BER for different slaves are mutually independent and have either a scaled-beta distribution (4) or a Rayleigh distribution (5).

We first consider a piconet with four slaves. In Fig. 1(a), we investigate the case in which the optimal algorithm is operating under ideal conditions. This figure compares the optimal throughput computed using the expressions obtained in the TOPS-M algorithm (plot labeled as optimal_comp) and the throughput of the optimal policy measured using simulations (plot labeled as optimal). As expected, these plots are identical, which validates the analysis. In Fig. 1(b), we consider relaxation 1, i.e., the master repeatedly retransmits the same packet to the same slave, until successful. In Fig. 1(c) and (d), we consider the behavior of the optimal algorithm and relaxation 1, respectively, for a Rayleigh distribution. From these figures, we see that the optimal algorithm has the best throughput under for all BERs. The rropt strategy also has much better throughput than the other benchmark algorithms.

In Fig. 2(a) and (b), we plot the throughput for scaled-beta and Rayleigh distributions, respectively, with relaxation 2 (i.e., when the reverse channel is lossy). In Fig. 2(c), we consider relaxation 3, i.e., when the channel conditions are estimated. In Fig. 2(d), we plot the throughput of different strategies as a function of the number of slaves in the piconet. In this case, we assume Rayleigh distribution with parameter $R = 0.005$. The optimal algorithm has the best throughput irrespective of the number of slaves and its throughput increases with an increase in the number of slaves.

For all these cases, the optimal algorithm outperforms all other policies for all channel conditions; the performance difference is significant (e.g., around 50%) under ideal conditions. The probability of high BER increases and, hence, the throughput of all the strategies decrease with increase in the scale factor K for the scaled-beta distribution and with an increase in the value of the parameter R for Rayleigh distribution.

The optimal strategy trades off fairness for performance gain. For example, the optimal strategy will rarely schedule a slave that has substantially inferior transmission condition as compared to other slaves. If fairness is an issue, rropt can be used

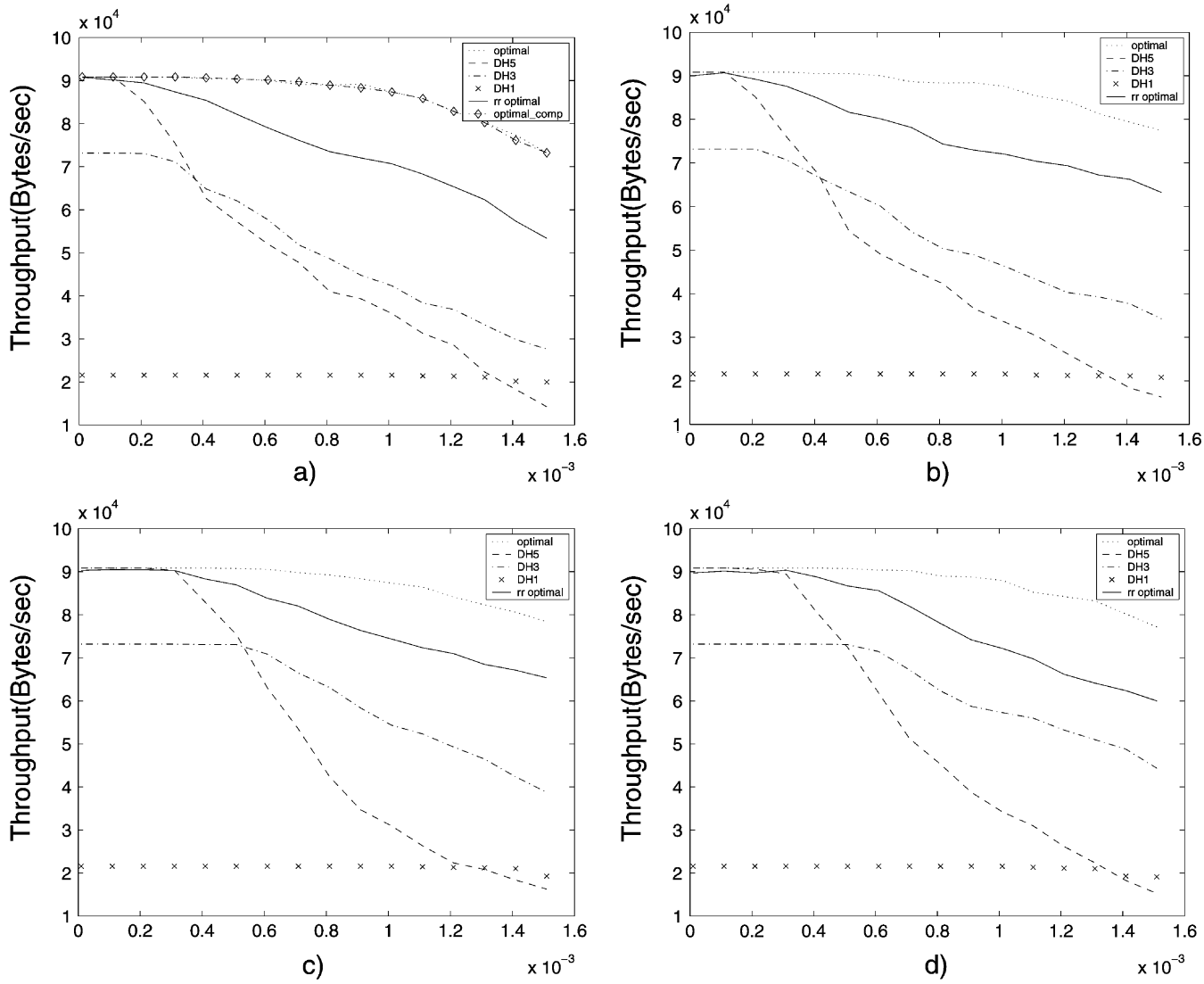

Fig. 1. We plot the throughput in a piconet with four slaves. (a) Ideal conditions for scaled-beta distribution [optimal_comp is the value directly computed from the throughput function in (3)]. (b) Relaxation 1 for scaled-beta distribution. (c) Ideal conditions for Rayleigh distribution. (d) Relaxation 1 for Rayleigh distribution.

instead. Note that rropt schedules slaves in a round-robin order and outperforms other benchmark strategies, since it optimally decides the packet length for the chosen slave.

V. Conclusion

Bluetooth uses frequency hopping and operates in the unlicensed 2.4-GHz ISM band, which is also used by IEEE 802.11 radios as well as other devices such as microwave ovens. Thus, the frequencies in this band will be subjected to interference from other sources in addition to being subjected to the vagaries of wireless links. We concentrate on providing solutions for enabling efficient communication between Bluetooth devices in presence of such interference. We provide algorithms to maximize the throughput under lossy transmission conditions in a piconet with one or more slaves by selecting the packet lengths optimally in accordance with the channel conditions for different frequencies.

We first develop a mathematical model of packet transmission in a frequency-hopping system such as Bluetooth. We use this model to show that a threshold-based algorithm for choosing the packet lengths maximizes the throughput of the system. We provide an algorithm that determines the optimal thresholds efficiently for a given system. We prove the optimality of this algorithm without using standard optimization techniques, since it is not clear that these techniques would be applicable given the functions involved [18]. We relax the analytical assumptions in simulations and demonstrate that the optimal strategy leads to significantly more throughput (e.g., around 50%) as compared to other baseline strategies. We then extend our results to multiple active slaves in a piconet.

Appendix A
Proof of Theorem 1

We first prove that the optimum packet length is a nondecreasing function of the BSR p. The result would follow from the fact that only three packet lengths, namely, 366, 1622, and 2870 bits, are allowed. For simplicity, we consider a discrete random variable p and consider the discrete version of (1). When p is a continuous random variable, the proof uses similar reasoning, but is more complicated.

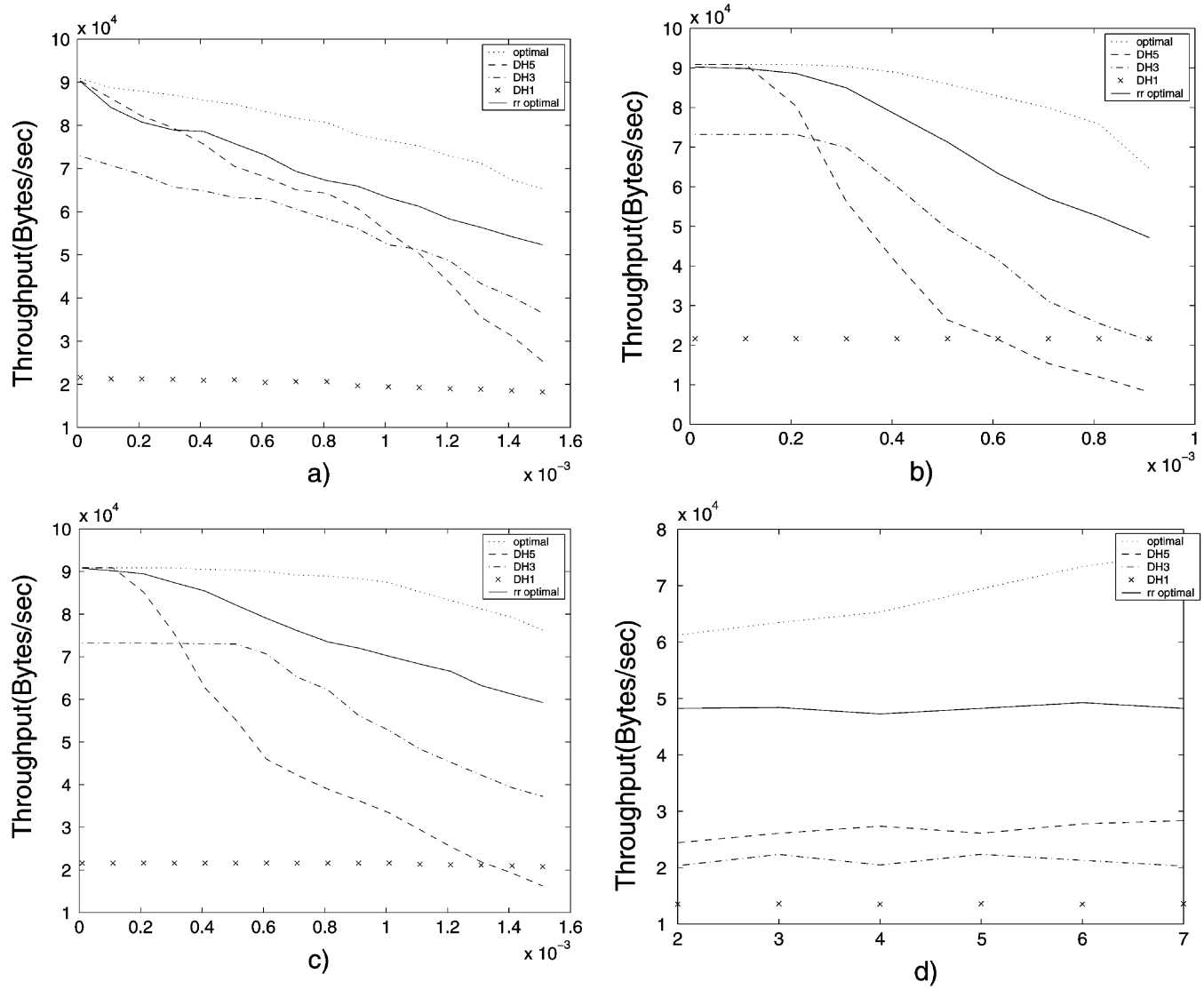

Fig. 2. Throughput in a piconet with four slaves under (a) relaxation 2 for scaled-beta distribution, (b) relaxation 2 for Rayleigh distribution, and (c) relaxation 3 for Rayleigh distribution. (d) Throughput as a function of the number of slaves.

The proof is by contradiction. Let $l(p)$, $\underline{l}(p)$, and $\bar{l}(p)$ denote the length, payload, and duration for the optimal packet length at BSR p. Let $n(p) = \phi(p) \times \underline{l}(p)$ and $m(p) = l(p) + 18$. Let there exist p_1, p_2 such that $l(p_1) > l(p_2)$ while $p_1 < p_2$. Thus, $\bar{l}(p_1) > \bar{l}(p_2), m(p_1) > m(p_2)$. Then, the optimum throughput, denoted by A, is given by

$$A = \frac{\sum_p n(p) p^{m(p)} dF(p)}{\sum_p \bar{l}(p) dF(p)}. \tag{6}$$

Consider the throughputs B, C obtained by the following modifications in the optimum rule: B is the throughput obtained if the packet length is $l(p_1)$ at both p_1, p_2, and C is the throughput obtained if the packet length is $l(p_2)$ at both p_1, p_2. The decision processes are the same in all three policies for all BSRs other than p_1, p_2.

$$\alpha = \sum_{p \neq p_1, p_2} n(p) p^{m(p)} dF(p)$$

$$\beta = \sum_{p \neq p_1, p_2} \bar{l}(p) dF(p)$$

$$A = \frac{n(p_1) p_1^{m(p_1)} dF(p_1) + n(p_2) p_2^{m(p_2)} dF(p_2) + \alpha}{\bar{l}(p_1) dF(p_1) + \bar{l}(p_2) dF(p_2) + \beta}$$

$$B = \frac{n(p_1) p_1^{m(p_1)} dF(p_1) + n(p_1) p_2^{m(p_1)} dF(p_2) + \alpha}{\bar{l}(p_1) dF(p_1) + \bar{l}(p_1) dF(p_2) + \beta}$$

$$C = \frac{n(p_2) p_1^{m(p_2)} dF(p_1) + n(p_2) p_2^{m(p_2)} dF(p_2) + \alpha}{\bar{l}(p_2) dF(p_1) + \bar{l}(p_2) dF(p_2) + \beta}$$

We first show that

$$n(p_1) p_2^{m(p_1)} - n(p_2) p_2^{m(p_2)} > n(p_1) p_1^{m(p_1)} - n(p_2) p_1^{m(p_2)}. \tag{7}$$

Then, using (7) we show that $C > A$. The argument is shown in (8) at the bottom of the next page. This contradicts the fact that A is the throughput of the optimal strategy. $\diamondsuit$

Appendix B
Proof of Theorem 3

This proof is in two steps: Lemma 1 shows that the iterates of the TOTS algorithm converge and Lemma 2 shows that the limiting points are the optimal thresholds.

Lemma 1: $\lim_{k\to\infty} p_i^k$ exists for $i \in \{1,2\}$.

Lemma 2: Let $p_i^c = \lim_{k\to\infty} p_i^k, i \in \{1,2\}$. Then, (p_1^c, p_2^c) is the global maxima of $f(a,b)$.

Theorem 3 follows from Lemmas 1 and 2.

Proof of Lemma 1: Consider the following iterative algorithm.

Step 1) $q_1^0 = q_2^0 = 0, k = 1$.
Step 2) $q_2^k = \arg\max_{q_1^{k-1} \le p \le 1} f(q_1^{k-1}, p)$.[4]
Step 3) if $q_2^k \neq q_2^{k-1}, q_1^k = q_1^{k-1}$, go to Step 5).
Step 4) if $q_2^k = q_2^{k-1}$, then
  a) $q_1^k = \arg\max_{0\le p\le q_2^k} f(p, q_2^k)$;[5]
  b) terminate if $q_1^k = q_1^{k-1}$.
Step 5) $k \to k+1$; go to Step 2).

We make the following observations, which we use in the proof.

- $q_1^k \le q_2^k$,
- $f(q_1^k, q_2^k) \le f(q_1^{k+1}, q_2^{k+1})$.

Observe that the sequence q_i^k can be constructed from the p_i^k in TOTS as follows.

Step 1) $r = 0, k = 0, (q_1^0, q_2^0) = (p_1^0, p_2^0)$.
Step 2) $r \to r+1, k \to k+1$.
Step3) $q_2^r = p_2^k$.
Step 4) if $q_2^r = q_2^{r-1}$, then $q_1^r = p_1^k$, else
  $q_1^r = q_1^{r-1}$;
  $r \to r+1$;
  $(q_1^r, q_2^r) = (p_1^k, p_2^k)$.
Step 5) If TOTS terminates at k, terminate, otherwise, go to Step 2).

Note that $\lim_{k\to\infty} p_i^k$ exists for $i \in \{1,2\}$ if and only if $\lim_{k\to\infty} q_i^k$ exists for $i \in \{1,2\}$. We will show that $q_i^{k+1} \ge q_i^k$ for $i = 1,2$ and $k \ge 0$. Since $q_i^k \in [0,1]$, it follows that $\lim_{k\to\infty} q_i^k$ exists. The result follows.

Let the inequality $q_i^{k+1} \ge q_i^k$ for $i = 1,2$ and $k \ge 0$, be violated at the $k+1$th iteration for the first time for some i. First, consider the case $i = 1$, i.e., $q_1^{k+1} < q_1^k$ and $q_1^m \ge q_1^{m-1}$, $1 \le m \le k$.

Let $f(q_1^k, q_2^k) = x/y$. Since $q_1^{k+1} \neq q_1^k$, we have $q_2^{k+1} = q_2^k$. Thus, $f(q_1^{k+1}, q_2^{k+1}) = (x+\alpha)/(y+\beta)$, where $\alpha = \int_{q_1^{k+1}}^{q_1^k} (1464x^{1568} - 216x^{312}) dF(x)$ and $\beta = \int_{q_1^{k+1}}^{q_1^k} 1250 dF(x)$. We now show that $f(q_1^{k+1}, q_2^{k+1}) > f(q_1^k, q_2^k)$. Since $q_2^k = q_2^{k+1}, f(q_1^k, q_2^k) = f(q_1^k, q_2^{k+1})$. Since $q_1^k \le q_2^k = q_2^{k+1}$, if $f(q_1^k, q_2^{k+1}) = \max_{0\le p\le q_2^{k+1}} f(p, q_2^{k+1}), q_1^{k+1} = q_1^k$. Since $q_1^{k+1} < q_1^k, f(q_1^k, q_2^{k+1}) \neq \max_{0\le p\le q_2^{k+1}} f(p, q_2^{k+1})$. Since $0 \le q_1^k \le q_2^{k+1}, f(q_1^k, q_2^{k+1}) < \max_{0\le p\le q_2^{k+1}} f(p, q_2^{k+1})$. Since $f(q_1^k, q_2^k) = f(q_1^k, q_2^{k+1}), f(q_1^k, q_2^k) < \max_{0\le p\le q_2^{k+1}} f(p, q_2^{k+1})$.

[4]If $f(q_1^{k-1}, q_2^{k-1}) = \max_{q_1^{k-1}\le p\le 1} f(q_1^{k-1}, p)$, and the $\arg\max$ is not unique, $q_2^k = q_2^{k-1}$

[5]If $f(q_1^{k-1}, q_2^k) = \max_{0\le p\le q_2^k} f(p, q_2^k)$, and $q_1^{k-1} \le q_2^k$, and the $\arg\max$ is not unique, $q_1^k = q_1^{k-1}$

$$
\begin{aligned}
n(p_1)p_2^{m(p_1)} - n(p_2)p_2^{m(p_2)} &= n(p_1)p_2^{m(p_2)}\left(p_2^{m(p_1)-m(p_2)} - \frac{n(p_2)}{n(p_1)}\right) \\
&> n(p_1)p_1^{m(p_2)}\left(p_1^{m(p_1)-m(p_2)} - \frac{n(p_2)}{n(p_1)}\right) \\
&\quad (\text{since } p_2 > p_1 \text{ and } m(p_1) > m(p_2)) \\
&= n(p_1)p_1^{m(p_1)} - n(p_2)p_1^{m(p_2)}
\end{aligned}
$$

$$
\begin{aligned}
B &= \frac{n(p_1)p_1^{m(p_1)}dF(p_1) + n(p_2)p_2^{m(p_2)}dF(p_2) + \alpha + n(p_1)p_2^{m(p_1)}dF(p_2) - n(p_2)p_2^{m(p_2)}dF(p_2)}{\bar{l}(p_1)dF(p_1) + \bar{l}(p_2)dF(p_2) + \beta + (\bar{l}(p_1) - \bar{l}(p_2))dF(p_2)} \\
&= A\frac{1 + dF(p_2)\frac{n(p_1)p_2^{m(p_1)} - n(p_2)p_2^{m(p_2)}}{n(p_1)p_1^{m(p_1)}dF(p_1) + n(p_2)p_2^{m(p_2)}dF(p_2) + \alpha}}{1 + dF(p_2)\frac{(\bar{l}(p_1) - \bar{l}(p_2))}{\bar{l}(p_1)dF(p_1) + \bar{l}(p_2)dF(p_2) + \beta}}
\end{aligned}
$$

$$
\text{Since } B \le A, \frac{n(p_1)p_2^{m(p_1)} - n(p_2)p_2^{m(p_2)}}{n(p_1)p_1^{m(p_1)}dF(p_1) + n(p_2)p_2^{m(p_2)}dF(p_2) + \alpha} \le \frac{\bar{l}(p_1) - \bar{l}(p_2)}{\bar{l}(p_1)dF(p_1) + \bar{l}(p_2)dF(p_2) + \beta}. \tag{8}
$$

$$
\begin{aligned}
C &= \frac{n(p_2)p_1^{m(p_2)}dF(p_1) + n(p_1)p_1^{m(p_1)}dF(p_1) - n(p_1)p_1^{m(p_1)}dF(p_1) + n(p_2)p_2^{m(p_2)}dF(p_2) + \alpha}{\bar{l}(p_1)dF(p_1) + \bar{l}(p_2)dF(p_2) + \beta - (\bar{l}(p_1) - \bar{l}(p_2))dF(p_1)} \\
&= A\frac{1 - dF(p_1)\frac{n(p_1)p_1^{m(p_1)} - n(p_2)p_1^{m(p_2)}}{n(p_1)p_1^{m(p_1)}dF(p_1) + n(p_2)p_2^{m(p_2)}dF(p_2) + \alpha}}{1 - dF(p_1)\frac{\bar{l}(p_1) - \bar{l}(p_2)}{\bar{l}(p_1)dF(p_1) + \bar{l}(p_2)dF(p_2) + \beta}} \\
&> A\frac{1 - dF(p_1)\frac{n(p_1)p_2^{m(p_1)} - n(p_2)p_2^{m(p_2)}}{n(p_1)p_1^{m(p_1)}dF(p_1) + n(p_2)p_2^{m(p_2)}dF(p_2) + \alpha}}{1 - dF(p_1)\frac{\bar{l}(p_1) - \bar{l}(p_2)}{\bar{l}(p_1)dF(p_1) + \bar{l}(p_2)dF(p_2) + \beta}} \quad [\text{from } (7)] \\
&\ge A \quad [\text{from } (8)].
\end{aligned}
$$

Since $q_2^k = q_2^{k+1}, f(q_1^{k+1}, q_2^{k+1}) = \max_{0 \le p \le q_2^{k+1}} f(p, q_2^{k+1})$ from 4a). Thus, $f(q_1^{k+1}, q_2^{k+1}) > f(q_1^k, q_2^k)$. Thus,

$$\frac{x+\alpha}{y+\beta} > \frac{x}{y}.$$

Hence

$$\frac{x}{y} < \frac{\alpha}{\beta}. \tag{9}$$

Since $\beta > 0$, it follows from (9) that $\alpha > 0$.

Let q_1^k be reached in the lth iteration for the first time, i.e., $l = \arg\min_{j \ge 0}(q_1^j = q_1^k)$. Clearly, $l \le k$. Also, $l > 0$ as $q_1^0 = 0 \le q_1^{k+1} < q_1^k$. Thus, $q_1^l = \arg\max_{0 \le p \le q_2^l} f(p, q_2^l)$. Let $f(q_1^l, q_2^l) = u/v$. Note that $f(q_1^l, q_2^l) \le f(q_1^k, q_2^k)$, for all $l \le k$. Thus, $u/v \le x/y < \alpha/\beta$. The last inequality follows from (9). Thus, $(u+\alpha)/(v+\beta) > (u/v)$. Thus, $f(q_1^{k+1}, q_2^l) > f(q_1^l, q_2^l)$ and $0 \le q_1^{k+1} < q_1^k = q_1^l \le q_2^l$. This contradicts the fact that $q_1^l = \arg\max_{0 \le p \le q_2^l} f(p, q_2^l)$.

The case for $i = 2$ can be argued similarly. Let q_2^k be reached at l ($l \le k$). It can be shown that $l > 0$. Thus, $q_2^l = \arg\max_{q_1^{l-1} \le p \le 1} f(q_1^{l-1}, p)$. Note that $1 \ge q_2^{k+1} \ge q_1^k \ge q_1^{l-1}$. The first inequality follows since $q_2^{k+1} = \arg\max_{q_1^k \le p \le 1} f(q_1^k, p)$. It can also be shown that $f(q_1^{l-1}, q_2^{k+1}) > f(q_1^{l-1}, q_2^l)$. These contradict that fact that $q_2^l = \arg\max_{q_1^{l-1} \le p \le 1} f(q_1^{l-1}, p)$. The result follows. ♢

We state and prove Lemmas 3–7, which we use in proving Lemma 2.

Lemma 3:

$$p_1^c = \arg\max_{0 \le p \le p_2^c} f(p, p_2^c). \tag{10}$$

$$p_2^c = \arg\max_{p_1^c \le p \le 1} f(p_1^c, p). \tag{11}$$

Proof of Lemma 3: Note that $p_i^c = \lim_{k \to \infty} p_i^k$, $i \in \{1, 2\}$. We will prove (10). The proof for (11) is similar. We assume that the functions $g(y) = \arg\max_{0 \le x \le y} f(x, y)$ and $h(y) = \arg\max_{y \le x \le 1} f(y, x)$ are continuous at p_2^c and p_1^c, respectively.

Let $p_1^c \ne g(p_2^c)$. From the continuity of $g(y)$ at p_2^c there exists $\epsilon > 0$, such that $|g(p_2^c) - g(z)| < 1/2|p_1^c - g(p_2^c)|$ for all $z, |z - p_2^c| < \epsilon$. Since $p_2^c = \lim_{k \to \infty} p_2^k$, there exists k_0, such that for all $k > k_0, |p_2^k - p_2^c| < \epsilon$. Thus, $|g(p_2^c) - g(p_2^k)| < 1/2|p_1^c - g(p_2^c)|$ for all $k > k_0$. Since $p_1^k = g(p_2^k), |g(p_2^c) - p_1^k| < 1/2|p_1^c - g(p_2^c)|$ for all $k > k_0$. Thus, $|p_1^c - p_1^k| > 1/2|p_1^c - g(p_2^c)|$ for all $k > k_0$. This contradicts the fact that $p_1^c = \lim_{k \to \infty} p_1^k$. Thus, $p_1^c = g(p_2^c)$. ♢

Let (p_1^*, p_2^*) be the global maxima of the function $f(a, b), 0 \le a \le b \le 1$. Since $0 \le a \le b \le 1$ is a compact set, $f(a, b)$ has a global maxima in $0 \le a \le b \le 1$. We will assume that $p_2^* < 1, 0 < p_1^c \le p_2^c < 1$.

Lemma 4:

$$2712x^{1248} - 1464 \ge 0 \text{ if } x \in \{p_2^*, p_2^c\} \tag{12}$$

$$\phi(x) \times (2712x^{2816} - 1464x^{1568}) > \phi(y)(2712y^{2816} - 1464y^{1568}) \text{ if } x > y \ge \min(p_2^*, p_2^c). \tag{13}$$

$$\int_{\theta_1}^{\theta_2} \phi(x) \times (2712x^{2816} - 1464x^{1568}) dF(x) \ge 0 \text{ if } \theta_2 \ge \theta_1 \ge \min(p_2^*, p_2^c). \tag{14}$$

The last inequality is strict if and only if $\int_{\theta_1}^{\theta_2} dF(x) > 0$.

Proof of Lemma 4: Relation (12) follows since, otherwise, there exist $\delta_1 > 0$ such that $f(p_1^*, \delta_1 + p_2^*) > f(p_1^*, p_2^*)$ or there exist $\delta_2 > 0$ such that $f(p_1^c, \delta_2 + p_2^c) > f(p_1^c, p_2^c)$. The first contradicts the fact that (p_1^*, p_2^*) is a global maxima of the function $f(a, b)$ and the second contradicts (11) of Lemma 3.

Now we show (13). Note that $\phi(x) \times (2712x^{2816} - 1464x^{1568}) = x^{1568} \times \phi(x) \times (2712x^{1248} - 1464)$. Each term is increasing for $x \ge 0$. The product is increasing for $x \ge \min(p_2^*, p_2^c)$ if each term is nonnegative for $x \ge \min(p_2^*, p_2^c)$. Clearly, $x^{1568} \ge 0$, $\phi(x) \ge 0$, for $x \ge 0$. Since $(2712x^{1248} - 1464)$ is increasing, it is sufficient to show that $(2712x^{1248} - 1464) \ge 0$ at $x = \min(p_2^*, p_2^c)$. This follows from (12).

Now we show (14). From (13) and since $\theta_2 \ge \theta_1 \ge \min(p_2^*, p_2^c)$

$$\begin{aligned}
&\int_{\theta_1}^{\theta_2} \phi(x) \times (2712x^{2816} - 1464x^{1568}) dF(x) \\
&\quad \ge \phi(\min(p_2^*, p_2^c)) \times \Big(2712 \min(p_2^*, p_2^c)^{2816} \\
&\qquad - 1464 \min(p_2^*, p_2^c)^{1568}\Big) \int_{\theta_1}^{\theta_2} dF(x) \\
&\quad = \phi(\min(p_2^*, p_2^c)) \times \min(p_2^*, p_2^c)^{1568} \\
&\quad \times \Big(2712 \min(p_2^*, p_2^c)^{1248} - 1464\Big) \\
&\qquad \times (F(\theta_2) - F(\theta_1)).
\end{aligned}$$

Now (14) follows from (12) and since $\theta_2 \ge \theta_1$. The inequality is strict if $\int_{\theta_1}^{\theta_2} dF(x) > 0$. ♢

Lemma 5: Let $1 > c \ge p_2^*$. Then, $1464c^{1568} - 216c^{312} > 2712c^{2816} - 1464c^{1568}$. If $c = 1$, $1464c^{1568} - 216c^{312} = 2712c^{2816} - 1464c^{1568}$.

Proof of Lemma 5: Clearly, $1464c^{1568} - 216c^{312} = 2712c^{2816} - 1464c^{1568}$ if $c = 1$. We prove by contradiction for $1 > c \ge p_2^*$. Let $1464c^{1568} - 216c^{312} \le 2712c^{2816} - 1464c^{1568}$ for some $c \in [p_2^*, 1)$. Then

$$2712c^{2504} - 2928c^{1256} + 216 \ge 0. \tag{15}$$

From (12) in Lemma 4, $2712(p_2^*)^{1248} - 1464 \ge 0$. Thus $p_2^* \ge (1464/2712)^{1/1248}$ and, thus, $c \ge (1464/2712)^{1/1248}$. Let $h(c) = 2712c^{2504} - 2928c^{1256} + 216$. Note that $h(c) < 0$ for $c \in [(1464/2712)^{1/1248}, 1)$. This contradicts (15). ♢

Lemma 6: For any global maxima (p_1^*, p_2^*) of $f(a, b), p_1^* < p_2^*$.

Proof of Lemma 6: We prove by contradiction. Consider a global maxima (c, c). By assumption, $0 < c < 1$. Let $f(c, c) = x/y$. Then, $f(c - \delta, c) = (x + \alpha_1)/(y + \beta_1)$, where $\alpha_1 = \int_{c-\delta}^{c} \phi(x) \times (1464x^{1568} - 216x^{312}) dF(x)$, $\beta_1 = \int_{c-\delta}^{c} 1250 dF(x)$. Since $f(c - \delta, c) \le f(c, c)$, $(\alpha_1)/(\beta_1) \le (x/y)$. Also, $f(c, c + \delta) = (x - \alpha_2)/(y - \beta_2)$, where $\alpha_2 = \int_{c}^{c+\delta} \phi(x) * (2712x^{2816} - 1464x^{1568}) dF(x), \beta_2 = \int_{c}^{c+\delta} 1250 dF(x)$. Since $f(c, c + \delta) \le f(c, c)$, $(\alpha_2)/(\beta_2) \ge (x/y)$. Thus, $(\alpha_1)/(\beta_1) \le (\alpha_2)/(\beta_2)$ for each $\delta > 0$. From the continuity of the functions $\phi(x), 1464x^{1568} - 216x^{312}$, and $2712x^{2816} - 1464x^{1568}$, $\phi(c) \times (1464c^{1568} - 216c^{312}) \le \phi(c) \times (2712c^{2816} - 1464c^{1568})$. Since $\phi(c) > 0$, as $c > 0$, $(1464c^{1568} - 216c^{312}) \le (2712c^{2816} - 1464c^{1568})$. This contradicts Lemma 5. Thus, (c, c) is not a global maxima. ♢

Lemma 7:

$$1464x^{1256} - 216 \geq 0 \text{ if } x \in \{p_1^*, p_1^c\}. \tag{16}$$

$$\phi(x) \times (1464x^{1568} - 216x^{312}) > \phi(y) \times (1464y^{1568} - 216y^{312}) \text{ if } x > y \geq \min(p_1^*, p_1^c). \tag{17}$$

$$\int_{\theta_1}^{\theta_2} \phi(x) \times (1464x^{1568} - 216x^{312}) dF(x) \geq 0 \text{ if } \theta_2 \geq \theta_1 \geq \min(p_1^*, p_1^c). \tag{18}$$

The last inequality is strict if and only if $\int_{\theta_1}^{\theta_2} dF(x) > 0$.

Lemma 7 can be proven similarly to Lemma 4.

Proof of Lemma 2: The proof is by contradiction. Let (p_1^c, p_2^c) not be a global maxima of $f(a,b)$. Thus, $f(p_1^c, p_2^c) < f(p_1^*, p_2^*)$. Let $f(p_1^c, p_2^c) = (x/y)$.

We divide the proof into the following cases, which cover all possible scenarios:

1) $p_1^* \leq p_1^c \leq p_2^* \leq p_2^c$;
2) $p_1^* < p_2^* \leq p_1^c \leq p_2^c$;
3) $p_1^c \leq p_1^* < p_2^* \leq p_2^c$;
4) $p_1^c \leq p_1^* \leq p_2^c \leq p_2^*$;
5) $p_1^c \leq p_2^c \leq p_1^* < p_2^*$;
6) $p_1^* \leq p_1^c \leq p_2^c \leq p_2^*$.

$\diamondsuit$

Case 1: Here, $p_1^* \leq p_1^c \leq p_2^* \leq p_2^c$.

$$\alpha_1 = \int_{p_1^*}^{p_1^c} \phi(x) \times (1464x^{1568} - 216x^{312}) dF(x)$$

$$\beta_1 = \int_{p_1^*}^{p_1^c} 1250 dF(x)$$

$$\alpha_2 = \int_{p_2^*}^{p_2^c} \phi(x) \times (2712x^{2816} - 1464x^{1568}) dF(x)$$

$$\beta_2 = \int_{p_2^*}^{p_2^c} 1250 dF(x).$$

Note that $f(p_1^*, p_2^*) = (x + \alpha_1 + \alpha_2)/(y + \beta_1 + \beta_2)$. Since $f(p_1^c, p_2^c) < f(p_1^*, p_2^*)$, $(x + \alpha_1 + \alpha_2)/(y + \beta_1 + \beta_2) > (x/y)$. Note that $x \geq 0, y > 0$ for $i \in \{1,2\}$. Also, $\beta_i \geq 0$, $\alpha_i \geq 0$, $i \in \{1,2\}$ from Lemmas 4 and 7. Further, $\alpha_i > 0$ if and only if $\beta_i > 0$ for each i, from Lemmas 4 and 7.

First, consider the case that $(x + \alpha_1)/(y + \beta_1) > (x/y)$. Then, $f(p_1^*, p_2^c) > f(p_1^c, p_2^c)$, and $p_1^* \leq p_2^c$. These contradict (10). Now consider the case that $(x + \alpha_2)/(y + \beta_2) > (x/y)$. Then, $f(p_1^c, p_2^*) > f(p_1^c, p_2^c)$, and $p_1^c \leq p_2^*$. These contradict (11).

Now we will show that either $(x + \alpha_1)/(y + \beta_1) > (x/y)$ or $(x + \alpha_2)/(y + \beta_2) > (x/y)$. The first holds if $\beta_2 = 0$ and the second holds if $\beta_1 = 0$. Let $\beta_i > 0$, $i \in \{1,2\}$. Note that $(\alpha_1 + \alpha_2)/(\beta_1 + \beta_2) > (x/y)$. Let $(\alpha_m)/(\beta_m) = \max((\alpha_1)/(\beta_1), (\alpha_2)/(\beta_2))$. Note that $(\alpha_m)/(\beta_m) \geq (\alpha_1 + \alpha_2)/(\beta_1 + \beta_2)$. Thus, $(\alpha_m)/(\beta_m) > (x/y)$. Thus, $(x + \alpha_m)/(y + \beta_m) > (x/y)$. The result follows.

Case 2: Now we consider the case $p_1^* < p_2^* \leq p_1^c \leq p_2^c$.

$$\alpha_1 = \int_{p_1^*}^{p_2^*} \phi(x) \times (1464x^{1568} - 216x^{312}) dF(x)$$

$$\beta_1 = \int_{p_1^*}^{p_2^*} 1250 dF(x)$$

$$\alpha_2 = \int_{p_2^*}^{p_1^c} \phi(x) \times (1464x^{1568} - 216x^{312}) dF(x)$$

$$\beta_2 = \int_{p_2^*}^{p_1^c} 1250 dF(x)$$

$$\alpha_3 = \int_{p_2^*}^{p_1^c} \phi(x) \times (2712x^{2816} - 1464x^{1568}) dF(x)$$

$$\beta_3 = \int_{p_2^*}^{p_1^c} 1250 dF(x)$$

$$\alpha_4 = \int_{p_1^c}^{p_2^c} \phi(x) \times (2712x^{2816} - 1568x^{1568}) dF(x)$$

$$\beta_4 = \int_{p_1^c}^{p_2^c} 1250 dF(x).$$

Note that $f(p_1^*, p_2^*) = (x + \sum_{i=1}^4 \alpha_i)/(y + \sum_{i=1}^4 \beta_i)$. Here, $\beta_i \geq 0$, $i \in \{1, \ldots, 4\}$. From Lemma 4 and 7, $\alpha_i \geq 0$, $i \in \{1, \ldots, 4\}$, and $\alpha_i > 0$ if and only if $\beta_i > 0$ for each i. Since $f(p_1^*, p_2^*) > f(p_1^c, p_2^c)$, $\beta_i > 0$ for some i. Also, $(\sum_{i=1}^4 \alpha_i)/(\sum_{i=1}^4 \beta_i) > (x/y)$. Let $(\alpha_i/\beta_i)^+ = \alpha_i/\beta_i$ if $\beta_i > 0$ and $(\alpha_i/\beta_i)^+ = 0$ otherwise. Let $(\alpha_m)/(\beta_m) = \max((\alpha_i)/(\beta_i))^+$. Thus, $\beta_m > 0$. Also, $(\alpha_m)/(\beta_m) > (\sum_{i=1}^4 \alpha_i)/(\sum_{i=1}^4 \beta_i)$. Thus, $(\alpha_m)/(\beta_m) > (x/y)$. Hence, $(x + \alpha_m)/(y + \beta_m) > (x/y) = f(p_1^c, p_2^c)$. Note that $\beta_2 = \beta_3$. Also, $\alpha_3 \leq \alpha_2$ from Lemma 5 and since $\phi(x) > 0$ for $x \geq p_2^* > p_1^* \geq 0$. Thus, $(\alpha_3/\beta_3)^+ \leq (\alpha_2/\beta_2)^+$. Thus, $m \in \{1,2,4\}$. If $m = 4$, $f(p_1^c, p_1^c) = (x + \alpha_m)/(y + \beta_m) > f(p_1^c, p_2^c)$, and $p_1^c \leq p_1^c$. This contradicts (11). Thus, $m \in \{1,2\}$.

First consider the case that $\beta_2 > 0$. If $m = 2$, then $f(p_2^*, p_2^c) = (x + \alpha_m)/(y + \beta_m) > f(p_1^c, p_2^c)$ and $p_2^* \leq p_2^c$. This contradicts (10). We next show that $m = 2$. If $\beta_1 = 0$, then $m = 2$. Let $\beta_1 > 0$. From (17) in Lemma 7, $\phi(x) \times (1464x^{1568} - 216x^{312})$ is nondecreasing in the range $p_1^* \leq x \leq p_2^*$. Hence, $\alpha_1 \leq \phi(p_2^*) \times (1464(p_2^*)^{1568} - 216(p_2^*)^{312}) \int_{p_1^*}^{p_2^*} dF(x)$. Thus, $(\alpha_1)/(\beta_1) \leq \phi(p_2^*) \times (1464(p_2^*)^{1568} - 216(p_2^*)^{312})/1250$. For the same reason, $(\alpha_2)/(\beta_2) \geq \phi(p_2^*) \times (1464(p_2^*)^{1568} - 216(p_2^*)^{312})/1250$. Thus, $(\alpha_1)/(\beta_1) \leq (\alpha_2)/(\beta_2)$. The result follows. Thus, there is a contradiction.

Now consider the case when $\beta_2 = 0$. Thus, $m = 1$. Now, $f(p_1^*, p_2^c) = (x + \alpha_1 + \alpha_2)/(y + \beta_1 + \beta_2) = (x + \alpha_m)/(y + \beta_m) > f(p_1^c, p_2^c)$ and $p_1^* \leq p_2^c$. This contradicts (10).

Case 3: Now consider $p_1^c \leq p_1^* < p_2^* \leq p_2^c$.

$$\alpha_1 = \int_{p_2^*}^{p_2^c} \phi(x) \times (2712x^{2816} - 1464x^{1568}) dF(x)$$

$$\beta_1 = \int_{p_2^*}^{p_2^c} 1250 dF(x)$$

$$\alpha_2 = \int_{p_1^c}^{p_1^*} \phi(x) \times (1464x^{1568} - 216x^{312}) dF(x)$$

$$\beta_2 = \int_{p_1^c}^{p_1^*} 1250 dF(x).$$

Note that $f(p_1^*, p_2^*) = (x + \alpha_1 - \alpha_2)/(y + \beta_1 - \beta_2)$. Here, $\beta_i \geq 0, i = 1,2 \geq 0$ and $\alpha_i \geq 0, i = 1,2$ from Lemmas 4 and

7. Also, $\alpha_i > 0$ if and only if $\beta_i > 0, i = 1, 2$, from Lemmas 4 and 7. Since $f(p_1^c, p_2^c) < f(p_1^*, p_2^*)$

$$\frac{x + \alpha_1 - \alpha_2}{y + \beta_1 - \beta_2} > \frac{x}{y}. \tag{19}$$

If $\beta_1 = 0, (x - \alpha_2)/(y - \beta_2) = f(p_1^*, p_2^*)$. Note that $f(p_1^*, p_2^c) = (x - \alpha_2)/(y - \beta_2)$. Thus, $f(p_1^*, p_2^c) > f(p_1^c, p_2^c)$, and $p_1^* \leq p_2^c$. This contradicts (10). Let $\beta_2 = 0$. Now, $(x + \alpha_1)/(y + \beta_1) = f(p_1^*, p_2^*)$. Here, $f(p_1^c, p_2^*) = (x + \alpha_1)/(y + \beta_1)$. Thus, $f(p_1^c, p_2^*) > f(p_1^c, p_2^c)$, and $p_1^c \leq p_2^*$. This contradicts (11).

Now we consider $\beta_i > 0, i = 1, 2$. If $(\alpha_1)/(\beta_1) > (x/y)$ since $\alpha_1 > 0, \beta_1 > 0$, $(x + \alpha_1)/(y + \beta_1) > (x/y)$. Then, $f(p_1^c, p_2^*) > f(p_1^c, p_2^c)$. This contradicts (11). Thus

$$\frac{\alpha_1}{\beta_1} \leq \frac{x}{y}. \tag{20}$$

Also, $(x - \alpha_2)/(y - \beta_2) \leq (x/y)$, else $f(p_1^*, p_2^c) > f(p_1^c, p_2^c)$, which contradicts (10). Thus, since $\beta_2 \geq 0$, $\alpha_2 \geq 0$

$$\frac{\alpha_2}{\beta_2} \geq \frac{x}{y}. \tag{21}$$

From (20) and (21)

$$\frac{\alpha_1}{\beta_1} \leq \frac{\alpha_2}{\beta_2}. \tag{22}$$

First, assume that $\beta_1 - \beta_2 \geq 0$. From (19), $\alpha_1 - \alpha_2 \geq 0$ as well. From (19)

$$\frac{\alpha_1 - \alpha_2}{\beta_1 - \beta_2} > \frac{x}{y}. \tag{23}$$

From (20) and (23)

$$\frac{\alpha_1}{\beta_1} < \frac{\alpha_1 - \alpha_2}{\beta_1 - \beta_2}.$$

Thus

$$\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}.$$

This inequality contradicts (22).

Now assume that $\beta_1 - \beta_2 < 0$. Let $\alpha_1 - \alpha_2 > 0$. Thus, $(\alpha_1)/(\beta_1) > (\alpha_2)/(\beta_2)$. This contradicts (22). Thus, $\alpha_1 - \alpha_2 \leq 0$. From (19), $(x - (\alpha_2 - \alpha_1))/(y - (\beta_2 - \beta_1)) > (x/y)$. Since $\beta_1 - \beta_2 < 0$ and $\alpha_1 - \alpha_2 \leq 0$, it follows that $(\alpha_2 - \alpha_1)/(\beta_2 - \beta_1) < (x/y)$. From (21), $(\alpha_2 - \alpha_1)/(\beta_2 - \beta_1) < (\alpha_2)/(\beta_2)$. Thus, $(\alpha_1)/(\beta_1) > (\alpha_2)/(\beta_2)$, which contradicts (22).

Case 4: Now consider $p_1^c \leq p_1^* \leq p_2^c \leq p_2^*$.

$$\alpha_1 = \int_{p_1^c}^{p_1^*} \phi(x) \times (1464x^{1568} - 216x^{312}) dF(x)$$
$$\beta_1 = \int_{p_1^c}^{p_1^*} 1250 dF(x)$$
$$\alpha_2 = \int_{p_2^c}^{p_2^*} \phi(x) \times (2712x^{2816} - 1464x^{1568}) dF(x)$$
$$\beta_2 = \int_{p_2^*}^{p_2^c} 1250 dF(x).$$

Note that $f(p_1^*, p_2^*) = (x - \alpha_1 - \alpha_2)/(y - \beta_1 - \beta_2)$. The proof is similar to that for Case 1 and is omitted for brevity.

Case 5: Now we consider the case $p_1^c \leq p_2^c \leq p_1^* < p_2^*$.

$$\alpha_1 = \int_{p_1^c}^{p_2^c} \phi(x) \times (1464x^{1568} - 216x^{312}) dF(x)$$
$$\beta_1 = \int_{p_1^c}^{p_2^c} 1250 dF(x)$$
$$\alpha_2 = \int_{p_2^c}^{p_1^*} \phi(x) \times (1464x^{1568} - 216x^{312}) dF(x)$$
$$\beta_2 = \int_{p_2^c}^{p_1^*} 1250 dF(x)$$
$$\alpha_3 = \int_{p_2^c}^{p_1^*} \phi(x) \times (2712x^{2816} - 1464x^{1568}) dF(x)$$
$$\beta_3 = \int_{p_2^c}^{p_1^*} 1250 dF(x)$$
$$\alpha_4 = \int_{p_1^*}^{p_2^*} \phi(x) \times (2712x^{2816} - 1464x^{1568}) dF(x)$$
$$\beta_4 = \int_{p_1^*}^{p_2^*} 1250 dF(x).$$

Note that $f(p_1^*, p_2^*) = (x - \sum_{i=1}^4 \alpha_i)/(y - \sum_{i=1}^4 \beta_i)$. The proof is similar to that for Case 2 is and omitted for brevity.

Case 6: The proof when $p_1^* \leq p_1^c \leq p_2^c \leq p_2^*$ is similar to that in Case 3 and is omitted for brevity. ♢

Appendix C
Proof of Theorem 4

We prove in two steps: Lemma 8 shows that the optimum strategy chooses the slave with the maximum BSR and Lemma 9 shows that the choice of the packet length is a threshold-driven decision process once the slave is selected.

Lemma 8: There exists an optimum transmission strategy that always transmits a packet for the slave with the maximum BSR.

Lemma 9: Let the BSR of the selected slave be p. There exists thresholds p_1^*, p_2^* such that the optimal packet length $l^*(p) = 216$ bits for $p < p_1^*$, $l^*(p) = 1622$ bits for $p_1^* \leq p < p_2^*$ and $l^*(p) = 2870$ bits for $p \geq p_2^*$.

Theorem 4 follows from Lemmas 8 and 9 and from the expression for the throughput of an arbitrary strategy for multiple slaves specified in (2).

Proof of Lemma 8: Consider the optimum strategy π^*, which chooses the slave and packet length as per functions $s^*(p_1, \ldots, p_k)$ and $l^*(p_1, \ldots, p_k)$, respectively. Now consider a policy π that chooses the slave with the maximum BSR and packet length according to the same function $l^*(p_1, \ldots, p_k)$ as π^*. Let the throughputs of these strategies be η_{π^*} and η_π, respectively. These throughputs are specified by the equation at the top of the next page. These equations demonstrate that π also maximizes the throughput. The result follows. ♢

$$\begin{aligned}\eta_{\pi^*} &= \frac{\int_0^1 \cdots \int_0^1 \underline{l}^*(p_1,\ldots p_k)\phi\left(p_{s^*(p_1,\ldots,p_k)}\right)\left(p_{s^*(p_1,\ldots,p_k)}\right)^{l^*(p_1,\ldots p_k)-54} dF(p_1,\ldots,p_k)}{\int_0^1 \cdots \int_0^1 \bar{l}^*(p_1,\ldots p_k) dF(p_1,\ldots,p_k)} \\ &\leq \frac{\int_0^1 \cdots \int_0^1 \underline{l}^*(p_1,\ldots p_k)\phi\left(\max(p_1,\ldots p_k)\right)\left(\max(p_1,\ldots,p_k)\right)^{l^*(p_1,\ldots p_k)-54} dF(p_1,\ldots,p_k)}{\int_0^1 \cdots \int_0^1 \bar{l}^*(p_1,\ldots p_k) dF(p_1,\ldots,p_k)} \\ &= \eta_\pi\end{aligned}$$

Lemma 9 is similar to Theorem 1 for piconets with only one slave. The proof uses the same arguments and is omitted for brevity. The only difference is that the joint distribution function $F(p_1,\ldots,p_k)$ must be used instead of the marginal $F(p)$.

APPENDIX D
PROOF OF THEOREM 5

The proof uses the same arguments as that for Theorem 3. The only differences are that the function $f_M(a,b)$ and the joint distribution function $F(p_1,\ldots,p_k)$ must be used instead of the function $f(a,b)$ and the marginal $F(p)$. Also, the joint integrals must substitute the single integrals. The proof is omitted for brevity.

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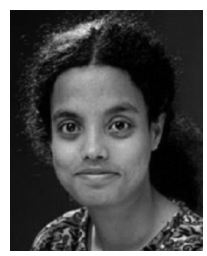

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