

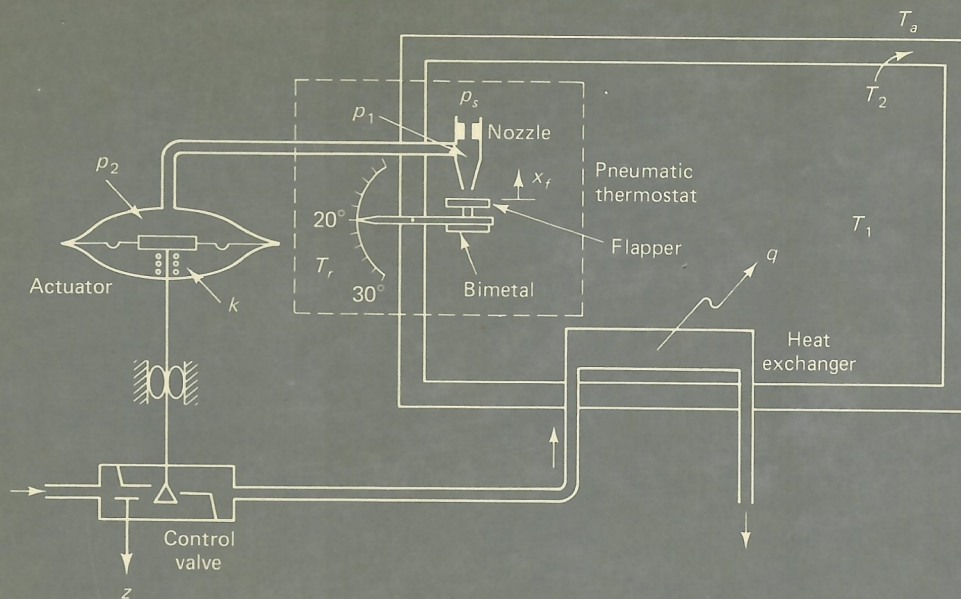
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DESIGN OF CONTROL SYSTEMS

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Library of Congress Cataloging-in-Publication Data

D'SOUZA, A. FRANK.

Design of control systems.

Includes bibliographies and index.

1. Automatic control. I. Title.

TJ213.D766 1988 629.8'312 87-2514

ISBN 0-13-199951-6

Editorial/production supervision and

interior design: Mary Jo Stanley

Cover design: 20/20 Services, Inc.

Manufacturing buyer: Rhett Conklin

© 1988 by Prentice-Hall, Inc.

A Division of Simon & Schuster

Englewood Cliffs, New Jersey 07632

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Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-199951-6 025

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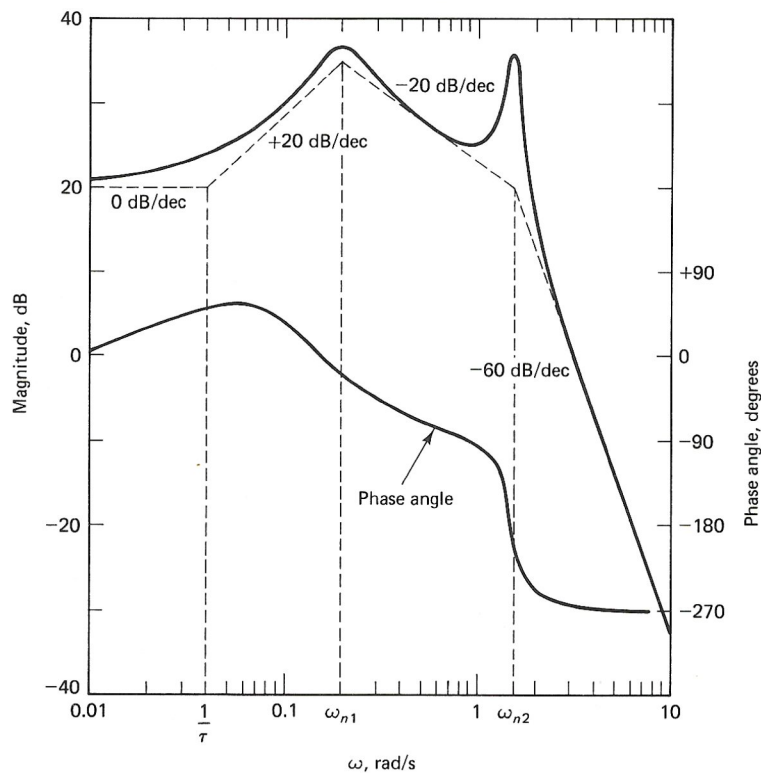


Figure 5.10 Bode diagram for Eqs. (5.44) and (5.45).

5.10 is drawn only by using this information, and corrections due to the damping ratios are made as shown. The phase angle is plotted from Eq. (5.45) and it varies from 0° , as $\omega \rightarrow 0$, to -270° , as $\omega \rightarrow \infty$.

5.4 FILTER CHARACTERISTICS

We have seen that for steady-state frequency response, the amplitude of the output is equal to $a|G(j\omega)|$, where a is the amplitude of the input sinusoid and $G(j\omega)$ is the frequency-response function. Hence, when $|G(j\omega)| > 1$, the input amplitude is magnified, and when $|G(j\omega)| < 1$, it is attenuated by the system dynamics. If $|G(j\omega)| \ll 1$ in a certain range of frequencies, the input amplitude is completely attenuated and the system does not respond in that range of frequencies. The system acts as a filter of those frequencies. The filter characteristics have important applications for the design of control systems and are discussed in the following.

Filter of Low Frequencies. A transfer function that attenuates the amplitude of low-frequency input is called a filter of low frequencies or a high-pass filter. It is noted that the distinction between low and high frequencies is dependent on the

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EXAMPLE 5.4

Let the frequency-response function of a system be given by

$$G(j\omega) = \frac{0.1(j\omega)}{j0.01\omega + 1} \quad (5.46)$$

We get

$$20 \log |G(j\omega)| = 20 \log 0.1 + 20 \log \omega - \frac{20}{2} \log (0.01^2 \omega^2 + 1) \quad (5.47)$$

$$\angle G(j\omega) = 90^\circ - \tan^{-1} 0.01\omega \quad (5.48)$$

The magnitude curve for the expression of Eq. (5.47) is shown in Fig. 5.11.

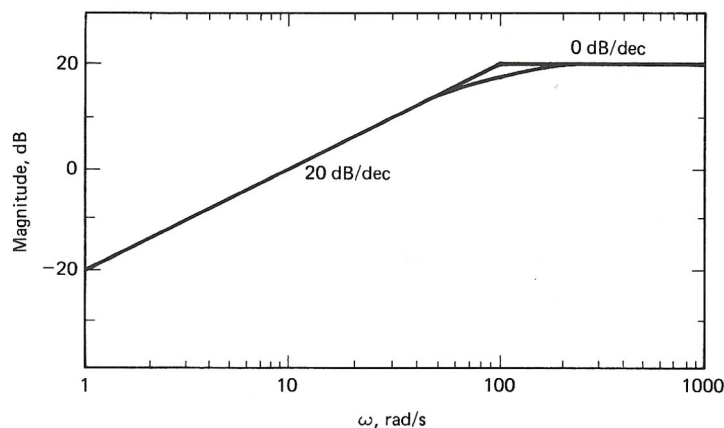
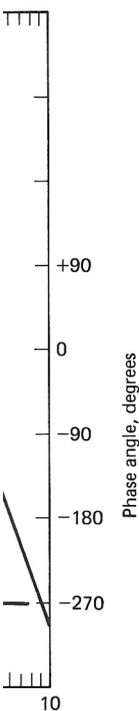


Figure 5.11 Low-frequency filter.

It is seen that this transfer function amplifies the amplitude of frequencies above 10 rad/s and attenuates the amplitude of those below 10 rad/s. At $\omega = 0.1$ rad/s, the attenuation factor corresponding to -40 dB is 0.01. Hence, this transfer function is a filter of low frequencies.

Filter of High Frequencies. A transfer function that attenuates the amplitude of high-frequency inputs is called a filter of high frequencies or a low-pass filter. The filtering of unwanted high-frequency noise and vibrations requires careful consideration. For example, the suspension system of an automobile is required to filter out the high-frequency random undulations of the road surface and not to transmit them to the sprung mass.

EXAMPLE 5.5(a)

The Bode diagram of the open-loop transfer function of the hydraulic speed-control system of Example 3.5 has been considered in Example 5.2. The

frequency-response function is given by Eq. (5.38) and the Bode diagram is shown in Fig. 5.9. It is seen that the amplitude of all frequencies above 3 rad/s is attenuated and the attenuation increases with frequency. At the frequency $\omega = \omega_n = 200$ rad/s, where the quadratic break occurs in Fig. 5.9, the magnitude has a value of nearly -70 dB, which corresponds to an attenuation factor of 0.0003.

Such high frequencies are filtered out and we can discard the quadratic factor in Eq. (5.36) without affecting the accuracy of the transfer function. The order of the transfer function is thus reduced from fourth to second as in Example 4 in Chapter 4. Hence, in a transfer function, the factors with time constants and natural frequencies, which contribute a break in the magnitude curve at frequencies that are filtered out by the remaining factors, can be discarded. The order of the system is thus reduced without affecting the dynamic performance. For example, we have simplified Eq. (2.160) to Eq. (2.161).

Intermediate-Pass Filter. A transfer function that filters out both low- and high-frequency inputs but transmits inputs in the intermediate-frequency range is called an intermediate-pass filter.

EXAMPLE 5.5(b)

The transfer function relating the displacement output to the pressure input of the pneumatic device of Example 2.9 is given by Eq. (2.105). This transfer function is

$$G(s) = \frac{k_g s}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (5.49)$$

Let the numerical values be $k_g = 10$, $\tau_1 = 1$, and $\tau_2 = 0.1$. The frequency-

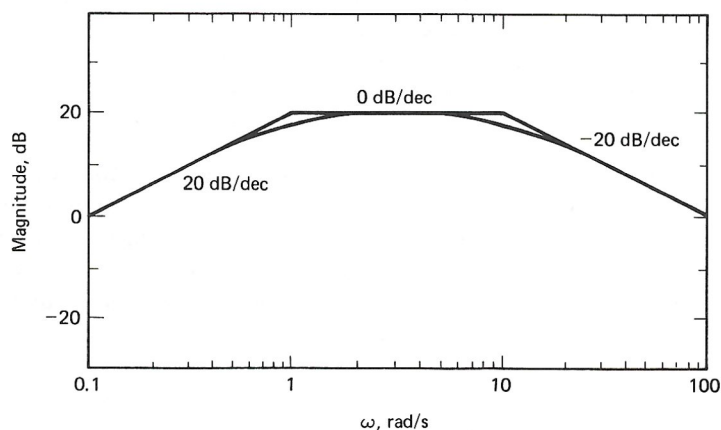


Figure 5.12 Intermediate-pass filter.

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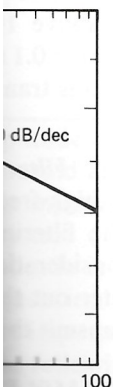
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response function becomes

$$G(j\omega) = \frac{10(j\omega)}{(j\omega + 1)(j0.1\omega + 1)} \quad (5.50)$$

Its magnitude is given by

$$20 \log |G(j\omega)| = 20 + 20 \log \omega - \frac{20}{2} \log (\omega^2 + 1) - \frac{20}{2} \log (0.1^2 \omega^2 + 1) \quad (5.51)$$

The magnitude curve is shown in Fig. 5.12. It is seen that the amplitudes of frequencies in the range from 0.1 to 100 rad/s are amplified. The amplitudes of frequencies outside that range are attenuated.

Frequency-Domain Specification of Instrumentation. The frequency-domain specification of sensors, recorders, and other instruments used to measure and/or record dynamic variables is usually given by stating that the frequency-response curve of the magnitude is flat up to a certain maximum frequency. This signifies that up to that maximum frequency, the dynamics of the instrument or recorder itself can be neglected and its input and output related by only a gain constant. This gain constant can be determined from the steady-state calibration and then the instrument can be used for measurement of dynamic variables.

Beyond the maximum stated frequency, the dynamics of the instrument itself cannot be neglected and its input and output are related by a transfer function that is no longer a gain constant. Hence, the steady-state calibration becomes invalid and the output will be phase shifted from the input and the input amplitude multiplied by the value of $|G(j\omega)|$, where $G(j\omega)$ is the frequency-response function of the instrument. Hence, care is required in the selection of instrumentation and recording equipment to ensure that their frequency response is suitable for the measurement of dynamic variables under consideration. For example, strip chart recorders usually have their frequency response magnitude curve flat up to about 50 Hz, whereas an oscillograph has a flat frequency response up to some kHz range.

EXAMPLE 5.6

The transfer function of the U-tube manometer of Example 2.6 is given by Eq. (2.80) and is shown in the block diagram of Fig. 2.26. Let the parameter values be such that this transfer function becomes

$$G(s) = \frac{0.1}{s^2/5^2 + 2(0.4)s/5 + 1} \quad (5.52)$$

We get

$$20 \log |G(j\omega)| = 20 \log 0.1 - \frac{20}{2} \log [(1 - \omega^2/5^2)^2 + (2(0.4)\omega/5)^2] \quad (5.53)$$

The magnitude curve is shown in Fig. 5.13. The corner frequency is the natural frequency $\omega_n = 5$ rad/s, and because the damping ratio $\zeta = 0.4$, the magnitude curve is flat up to about 1 rad/s. The steady-state gain calibration of the manometer is not valid for measurement of pressure with frequency

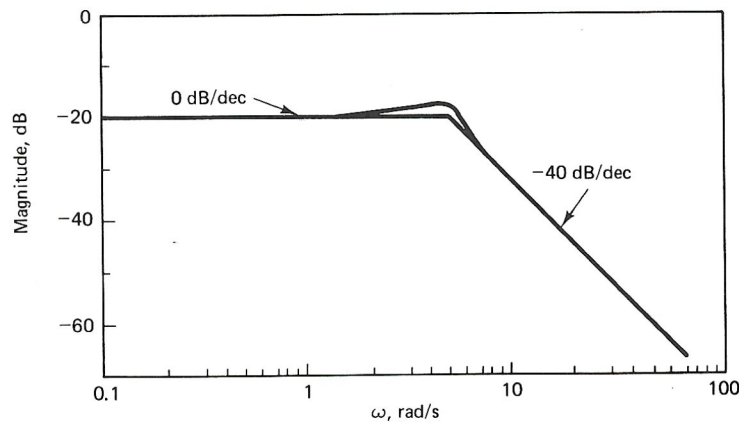


Figure 5.13 Frequency response of a manometer.

content above 1 rad/s. For the measurement of a transient pressure signal that changes fast with time, we need to select some other instrument, such as a strain-gage pressure transducer.

5.5 PERFORMANCE SPECIFICATIONS IN THE FREQUENCY DOMAIN

The transient performance specifications of control systems, such as rise time, maximum overshoot, and settling time, have been discussed in Chapter 4. We now give the performance specifications of control systems in the frequency domain in terms of peak resonance and bandwidth. We consider the closed-loop transfer function $G_c(s)$ of a control system defined by Eq. (5.5). It is assumed for asymptotic stability that all poles of $G_c(s)$ have negative real parts, that is, all roots of the characteristic equation $\det |s\mathbf{I} - \mathbf{A} + \mathbf{BK}| = 0$ from Eq. (5.5) have negative real parts. Hence, $G_c(j\omega)$ is the closed-loop frequency response function. A typical plot of $20 \log |G_c(j\omega)|$ is shown in Fig. 5.14.

Peak Resonance M_p . The peak resonance M_p is defined as the maximum value $|G_c(j\omega)|$ shown in Fig. 5.14. It gives an indication of the relative stability of the closed-loop control system. In the design of control systems, it is not sufficient that all roots of the characteristic equation have negative real parts so that asymptotic stability is ensured. If the roots are located too close to the left of the imaginary axis in the s -plane, then due to wear and tear and changes in the temperature and other conditions, the roots might migrate to the right half of the s -plane, thus giving rise to instability.

A large value of M_p usually corresponds to a large value of the maximum overshoot of the step response. The commonly specified optimum value of M_p is in the range from 1.1 to 1.5, corresponding to the maximum value of $20 \log |G_c(j\omega)|$, lying in the range from 0.83 to 3.52 dB.

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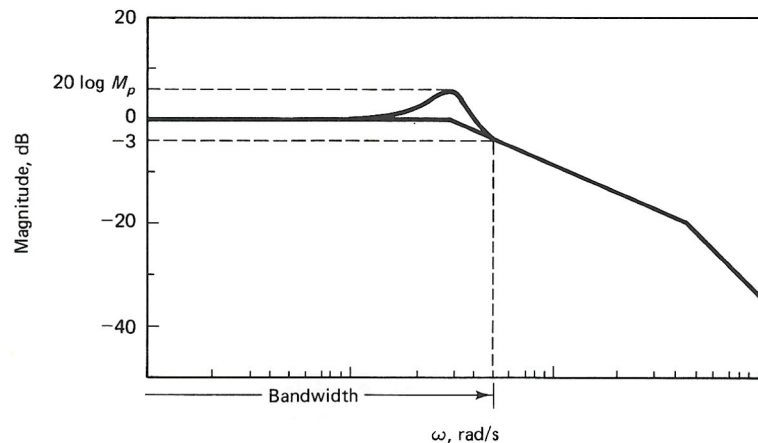
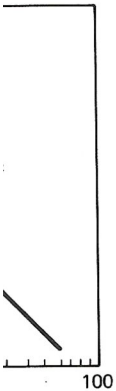


Figure 5.14 Typical magnitude plot of a control system.

Bandwidth. The cutoff frequency ω_c is defined as the frequency at which $20 \log |G_c(j\omega)|$ falls below -3 dB, which corresponds to the magnitude $|G_c(j\omega)|$ dropping below 0.707 . It is an indication of the filter characteristics of the system. The bandwidth is defined as the spread between zero frequency and the cutoff frequency, as shown in Fig. 5.14. In some systems, there may be two cutoff frequencies. In that case, the bandwidth is the spread between the two frequencies. For example, in Fig. 5.12 there are two cutoff frequencies and the bandwidth is the spread from about 0.07 to slightly above 100 rad/s.

A control system may be subjected to high-frequency electrical noise, vibrations, hydraulic and pneumatic pressure fluctuations, and other disturbances. In case the bandwidth is very high, these unwanted disturbances will be transmitted around the loop. An optimum bandwidth is one that transmits the signals unattenuated but filters out the unwanted noise. The bandwidth is also a measure of the transient response. A large bandwidth corresponds to the roots of the characteristic equation that are far to the left of the imaginary axis in the complex s -plane and yields a fast rise time. A small bandwidth yields a time response that is generally slow and sluggish.

EXAMPLE 5.7

The transient response of the first-order control system of Example 3.1 has been studied in Example 4.5. The closed-loop transfer function given by Eq. (4.75) is

$$G_c(s) = \frac{k}{(1/a)s + 1} \quad (5.54)$$

where the time constant $\tau = 1/a$ and the gain constant k are defined by Eq. (4.74). The root of the characteristic equation is $-a$. Letting $k = 0.95$, we get

$$20 \log |G_c(j\omega)| = -0.45 - \frac{20}{2} \log (\omega^2/a^2 + 1) \quad (5.55)$$

The magnitude curve is shown in Fig. 5.15.

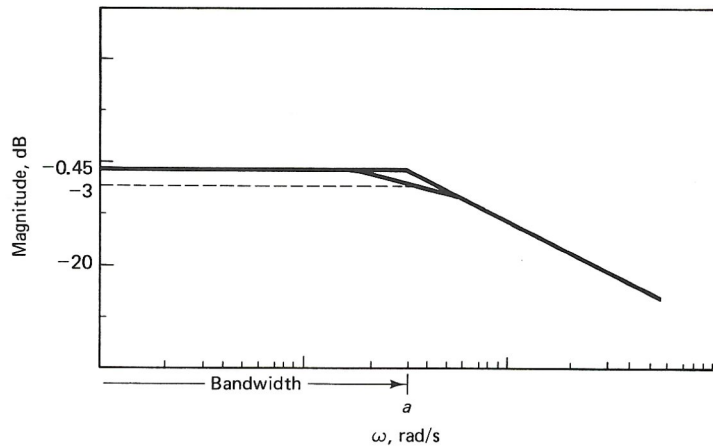


Figure 5.15 First-order control system.

It is seen that this control system does not have a resonance peak and the maximum value of $|G_c(j\omega)|$ is 0.95, which corresponds to -0.45 dB. The bandwidth is close to a rad/s. If the bandwidth is increased by increasing the value of a , it is seen from Fig. 4.5 that the transient response has a fast rise time. On the other hand, if the bandwidth is decreased by decreasing the value of a , the transient response of Fig. 4.5 becomes sluggish.

EXAMPLE 5.8

The transient response of the simplified model of the hydraulic control system of Example 3.5 has been investigated in Example 4.6. The closed-loop transfer function is obtained from Eq. (4.84) as

$$G_c(s) = \frac{1}{s^2/\omega_n^2 + 2\zeta s/\omega_n + 1} \quad (5.56)$$

This second-order transfer function has been studied earlier and its Bode diagram is shown in Fig. 5.7. It is seen from this figure that the peak resonance value M_p depends on the damping ratio ζ and that the bandwidth is close to the natural frequency. Their exact values can be obtained as follows. We have

$$M = \frac{1}{[(1 - \omega^2/\omega_n^2)^2 + (2\zeta\omega/\omega_n)^2]^{1/2}} \quad (5.57)$$

For maximum value of M , we set $dM/d\omega = 0$ and obtain

$$4(\omega/\omega_n)^3 - 4(\omega/\omega_n) + 8\zeta^2\omega/\omega_n = 0$$

The two positive roots of this equation are

$$\omega = 0 \quad (5.58)$$

$$\omega = \omega_n \sqrt{1 - 2\zeta^2} \quad \zeta \leq 0.707 \quad (5.59)$$

Substituting
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5.6 MODELING RESPONSE

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The closed-loop
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Substituting for ω from Eq. (5.59) in Eq. (5.57) and simplifying the resulting equation, we obtain

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad (5.60)$$

which shows that M_p is a function of ζ only for $\zeta < 0.707$. For $\zeta \geq 0.707$, solution of Eq. (5.58) is the valid one and $M_p = 1$. To obtain the bandwidth frequency, we set the right-hand side of Eq. (5.57) equal to 0.707 and solve for ω . The solution yields the cutoff frequency as

$$\omega_c = \omega_n[(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}]^{1/2} \quad (5.61)$$

The bandwidth is the spread from zero frequency to this cutoff frequency. The bandwidth is directly proportional to ω_n . For a fixed value of ω_n , both the bandwidth and the resonance peak M_p increase as ζ decreases. This frequency response can be compared with the unit step response of the system studied in Example 4.5. The maximum overshoot of the unit step response depends upon ζ only. The rise time increases with ζ , whereas the bandwidth decreases with the increase of ζ for a fixed ω_n .

5.6 MODELING FROM EXPERIMENTAL FREQUENCY RESPONSE

The mathematical models of control systems are usually obtained by making several assumptions. It is therefore desirable to validate the models experimentally and at the same time identify the parameter values, such as gain, time constants, natural frequencies, and damping ratios. Sometimes we encounter difficulties in obtaining the transfer function of a system by analytical means. Experimental frequency response is a powerful and practical method of validating a linear model and identifying the parameter values or even developing a linear model solely from experimental results.

A sinusoidal test signal $a \sin \omega t$ is used as input. The output of a nonlinear system usually will contain harmonics. For stable linear systems, the output is $b \sin(\omega t + \phi)$, where $b/a = |G(j\omega)|$ and $\phi = \angle G(j\omega)$. Here, $G(s)$ is the transfer function relating the input and output. A device called a frequency-response analyzer measures the amplitude ratio b/a and the phase angle ϕ as the frequency is swept from a very low value to some maximum value at which the input is completely filtered out. A Bode diagram is obtained by plotting $20 \log b/a = 20 \log |G(j\omega)|$ and $\phi = \angle G(j\omega)$ versus the frequency. Some frequency-response analyzers are equipped with a plotter for machine plotting the Bode diagram. Earlier, we studied the drawing of a Bode diagram for a given transfer function. Now, we have the inverse problem where given an experimentally obtained plot of a Bode diagram, we have to identify the transfer function.

The experimental frequency response is usually conducted on the open-loop system, Eq. (5.7), to identify the transfer function $G_o(s)$ of Eq. (5.8) with