
ANTENNA THEORY

ANALYSIS AND DESIGN

THIRD EDITION

Constantine A. Balanis



A JOHN WILEY & SONS, INC., PUBLICATION

Copyright © 2005 by John Wiley & Sons, Inc. All rights reserved.

Published by John Wiley & Sons, Inc., Hoboken, New Jersey.

Published simultaneously in Canada.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400, fax 978-646-8600, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services please contact our Customer Care Department within the U.S. at 877-762-2974, outside the U.S. at 317-572-3993 or fax 317-572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print, however, may not be available in electronic format.

Library of Congress Cataloging-in-Publication Data is available.

ISBN: 0-471-66782-X

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

Simply stated, *the corresponding far-zone **E**- and **H**-field components are orthogonal to each other and form TEM (to *r*) mode fields.* This is a very useful relation, and it will be adopted in the chapters that follow for the solution of the far-zone radiated fields. The far-zone (far-field) region for a radiator is defined in Figures 2.7 and 2.8. Its smallest radial distance is $2D^2/\lambda$ where D is the largest dimension of the radiator.

3.7 DUALITY THEOREM

When two equations that describe the behavior of two different variables are of the same mathematical form, their solutions will also be identical. The variables in the two equations that occupy identical positions are known as *dual* quantities and a solution of one can be formed by a systematic interchange of symbols to the other. This concept is known as the *duality theorem*.

Comparing Equations (3-2a), (3-3), (3-10), (3-14), and (3-15) to (3-16), (3-17), (3-21), (3-25), and (3-26), respectively, it is evident that they are to each other dual equations and their variables dual quantities. Thus knowing the solutions to one set (i.e., $\mathbf{J} \neq 0, \mathbf{M} = 0$), the solution to the other set ($\mathbf{J} = 0, \mathbf{M} \neq 0$) can be formed by a proper interchange of quantities. The dual equations and their dual quantities are listed, respectively in Tables 3.1 and 3.2 for electric and magnetic sources. Duality only serves as a guide to form mathematical solutions. It can be used in an abstract manner to explain the motion of magnetic charges giving rise to magnetic currents, when compared to their dual quantities of moving electric charges creating electric currents. It must, however, be emphasized that this is purely mathematical in nature since it is known, as of today, that there are no magnetic charges or currents in nature.

3.8 RECIPROCITY AND REACTION THEOREMS

We are all well familiar with the reciprocity theorem, as applied to circuits, which states that “in any network composed of linear, bilateral, lumped elements, if one places a constant **current** (*voltage*) generator between two **nodes** (*in any branch*) and

TABLE 3.1 Dual Equations for Electric (J**) and Magnetic (**M**) Current Sources**

Electric Sources ($\mathbf{J} \neq 0, \mathbf{M} = 0$)	Magnetic Sources ($\mathbf{J} = 0, \mathbf{M} \neq 0$)
$\nabla \times \mathbf{E}_A = -j\omega\mu\mathbf{H}_A$	$\nabla \times \mathbf{H}_F = j\omega\epsilon\mathbf{E}_F$
$\nabla \times \mathbf{H}_A = \mathbf{J} + j\omega\epsilon\mathbf{E}_A$	$-\nabla \times \mathbf{E}_F = \mathbf{M} + j\omega\mu\mathbf{H}_F$
$\nabla^2\mathbf{A} + k^2\mathbf{A} = -\mu\mathbf{J}$	$\nabla^2\mathbf{F} + k^2\mathbf{F} = -\epsilon\mathbf{M}$
$\mathbf{A} = \frac{\mu}{4\pi} \iiint_V \mathbf{J} \frac{e^{-jkR}}{R} dv'$	$\mathbf{F} = \frac{\epsilon}{4\pi} \iiint_V \mathbf{M} \frac{e^{-jkR}}{R} dv'$
$\mathbf{H}_A = \frac{1}{\mu} \nabla \times \mathbf{A}$	$\mathbf{E}_F = -\frac{1}{\epsilon} \nabla \times \mathbf{F}$
$\mathbf{E}_A = -j\omega\mathbf{A}$	$\mathbf{H}_F = -j\omega\mathbf{F}$
$-j\frac{1}{\omega\mu\epsilon} \nabla(\nabla \cdot \mathbf{A})$	$-j\frac{1}{\omega\mu\epsilon} \nabla(\nabla \cdot \mathbf{F})$

TABLE 3.2 Dual Quantities for Electric (J) and Magnetic (M) Current Sources

Electric Sources ($\mathbf{J} \neq 0, \mathbf{M} = 0$)	Magnetic Sources ($\mathbf{J} = 0, \mathbf{M} \neq 0$)
$\mathbf{E}_A$	$\mathbf{H}_F$
$\mathbf{H}_A$	$-\mathbf{E}_F$
$\mathbf{J}$	$\mathbf{M}$
$\mathbf{A}$	$\mathbf{F}$
ϵ	μ
μ	ϵ
k	k
η	$1/\eta$
$1/\eta$	η

places a **voltage** (*current*) meter between any other two **nodes** (*in any other branch*), makes observation of the meter reading, then interchanges the locations of the source and the meter, the meter reading will be unchanged" [3]. We want now to discuss the reciprocity theorem as it applies to electromagnetic theory. This is done best by the use of Maxwell's equations.

Let us assume that within a linear and isotropic medium, but not necessarily homogeneous, there exist two sets of sources $\mathbf{J}_1, \mathbf{M}_1$, and $\mathbf{J}_2, \mathbf{M}_2$ which are allowed to radiate simultaneously or individually inside the same medium at the same frequency and produce fields $\mathbf{E}_1, \mathbf{H}_1$ and $\mathbf{E}_2, \mathbf{H}_2$, respectively. It can be shown [1], [2] that the sources and fields satisfy

$$-\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) = \mathbf{E}_1 \cdot \mathbf{J}_2 + \mathbf{H}_2 \cdot \mathbf{M}_1 - \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_1 \cdot \mathbf{M}_2 \quad (3-60)$$

which is called the *Lorentz Reciprocity Theorem* in differential form.

Taking a volume integral of both sides of (3-60) and using the divergence theorem on the left side, we can write it as

$$\begin{aligned} & - \oint_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{s}' \\ &= \iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2 + \mathbf{H}_2 \cdot \mathbf{M}_1 - \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_1 \cdot \mathbf{M}_2) dv' \end{aligned} \quad (3-61)$$

which is designated as the *Lorentz Reciprocity Theorem* in integral form.

For a source-free ($\mathbf{J}_1 = \mathbf{J}_2 = \mathbf{M}_1 = \mathbf{M}_2 = 0$) region, (3-60) and (3-61) reduce, respectively, to

$$\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) = 0 \quad (3-62)$$

and

$$\oint_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{s}' = 0 \quad (3-63)$$

Equations (3-62) and (3-63) are special cases of the Lorentz Reciprocity Theorem and must be satisfied in source-free regions.

As an example of where (3-62) and (3-63) may be applied and what they would represent, consider a section of a waveguide where two different modes exist with fields $\mathbf{E}_1, \mathbf{H}_1$ and $\mathbf{E}_2, \mathbf{H}_2$. For the expressions of the fields for the two modes to be valid, they must satisfy (3-62) and/or (3-63).

Another useful form of (3-61) is to consider that the fields ($\mathbf{E}_1, \mathbf{H}_1, \mathbf{E}_2, \mathbf{H}_2$) and the sources ($\mathbf{J}_1, \mathbf{M}_1, \mathbf{J}_2, \mathbf{M}_2$) are within a medium that is enclosed by a sphere of infinite radius. Assume that the sources are positioned within a finite region and that the fields are observed in the far field (ideally at infinity). Then the left side of (3-61) is equal to zero, or

$$\oint_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{s}' = 0 \quad (3-64)$$

which reduces (3-61) to

$$\iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2 + \mathbf{H}_2 \cdot \mathbf{M}_1 - \mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_1 \cdot \mathbf{M}_2) dv' = 0 \quad (3-65)$$

Equation (3-65) can also be written as

$$\boxed{\iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2 - \mathbf{H}_1 \cdot \mathbf{M}_2) dv' = \iiint_V (\mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_2 \cdot \mathbf{M}_1) dv'} \quad (3-66)$$

The reciprocity theorem, as expressed by (3-66), is the most useful form.

A close observation of (3-61) reveals that it does not, in general, represent relations of power because no conjugates appear. The same is true for the special cases represented by (3-63) and (3-66). Each of the integrals in (3-66) can be interpreted as a coupling between a set of fields and a set of sources, which produce another set of fields. This coupling has been defined as *Reaction* [4] and each of the integrals in (3-66) are denoted by

$$\langle 1, 2 \rangle = \iiint_V (\mathbf{E}_1 \cdot \mathbf{J}_2 - \mathbf{H}_1 \cdot \mathbf{M}_2) dv \quad (3-67)$$

$$\langle 2, 1 \rangle = \iiint_V (\mathbf{E}_2 \cdot \mathbf{J}_1 - \mathbf{H}_2 \cdot \mathbf{M}_1) dv \quad (3-68)$$

The relation $\langle 1, 2 \rangle$ of (3-67) relates the reaction (coupling) of fields ($\mathbf{E}_1, \mathbf{H}_1$), which are produced by sources $\mathbf{J}_1, \mathbf{M}_1$ to sources ($\mathbf{J}_2, \mathbf{M}_2$), which produce fields $\mathbf{E}_2, \mathbf{H}_2$; $\langle 2, 1 \rangle$ relates the reaction (coupling) of fields ($\mathbf{E}_2, \mathbf{H}_2$) to sources ($\mathbf{J}_1, \mathbf{M}_1$). For reciprocity to hold, it requires that the reaction (coupling) of one set of sources with the corresponding fields of another set of sources must be equal to the reaction (coupling) of the second

set of sources with the corresponding fields of the first set of sources, and vice versa. In equation form, it is written as

$$\langle 1, 2 \rangle = \langle 2, 1 \rangle \quad (3-69)$$

3.8.1 Reciprocity for Two Antennas

There are many applications of the reciprocity theorem. To demonstrate its potential, an antenna example will be considered. Two antennas, whose input impedances are Z_1 and Z_2 , are separated by a linear and isotropic (but not necessarily homogeneous) medium, as shown in Figure 3.3. One antenna (#1) is used as a transmitter and the other (#2) as a receiver. The equivalent network of each antenna is given in Figure 3.4. The internal impedance of the generator Z_g is assumed to be the conjugate of the impedance of antenna #1 ($Z_g = Z_1^* = R_1 - jX_1$) while the load impedance Z_L is equal to the conjugate of the impedance of antenna #2 ($Z_L = Z_2^* = R_2 - jX_2$). These assumptions are made only for convenience.

The power delivered by the generator to antenna #1 is given by (2-83) or

$$P_1 = \frac{1}{2} \text{Re}[V_1 I_1^*] = \frac{1}{2} \text{Re} \left[\left(\frac{V_g Z_1}{Z_1 + Z_g} \right) \frac{V_g^*}{(Z_1 + Z_g)^*} \right] = \frac{|V_g|^2}{8R_1} \quad (3-70)$$

If the transfer admittance of the combined network consisting of the generator impedance, antennas, and load impedance is Y_{21} , the current through the load is $V_g Y_{21}$ and the power delivered to the load is

$$P_2 = \frac{1}{2} \text{Re}[Z_2 (V_g Y_{21}) (V_g Y_{21})^*] = \frac{1}{2} R_2 |V_g|^2 |Y_{21}|^2 \quad (3-71)$$

The ratio of (3-71) to (3-70) is

$$\frac{P_2}{P_1} = 4R_1 R_2 |Y_{21}|^2 \quad (3-72)$$

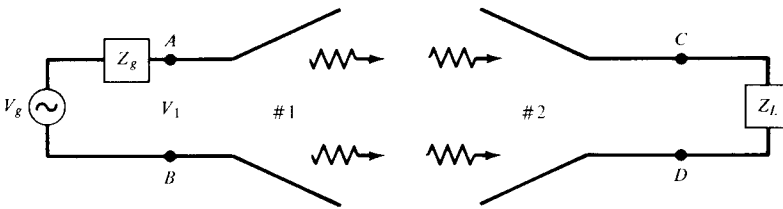


Figure 3.3 Transmitting and receiving antenna systems.

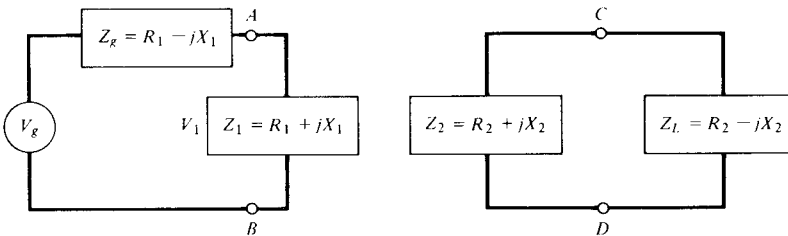


Figure 3.4 Two-antenna system with conjugate loads.

In a similar manner, we can show that when antenna #2 is transmitting and #1 is receiving, the power ratio of P_1/P_2 is given by

$$\frac{P_1}{P_2} = 4R_2R_1|Y_{12}|^2 \quad (3-73)$$

Under conditions of reciprocity ($Y_{12} = Y_{21}$), the power delivered in either direction is the same.

3.8.2 Reciprocity for Antenna Radiation Patterns

The radiation pattern is a very important antenna characteristic. Although it is usually most convenient and practical to measure the pattern in the receiving mode, it is identical, because of reciprocity, to that of the transmitting mode.

Reciprocity for antenna patterns is general provided the materials used for the antennas and feeds, and the media of wave propagation are linear. Nonlinear devices, such as diodes, can make the antenna system nonreciprocal. The antennas can be of any shape or size, and they do not have to be matched to their corresponding feed lines or loads provided there is a distinct single propagating mode at each port. The only other restriction for reciprocity to hold is for the antennas in the transmit and receive modes to be polarization matched, including the sense of rotation. This is necessary so that the antennas can transmit and receive the same field components, and thus total power. If the antenna that is used as a probe to measure the fields radiated by the antenna under test is not of the same polarization, then in some situations the transmit and receive patterns can still be the same. For example, if the transmit antenna is circularly polarized and the probe antenna is linearly polarized, then if the linearly polarized probe antenna is used twice and it is oriented one time to measure the θ -component and the other the ϕ -component, then the sum of the two components can represent the pattern of the circularly polarized antenna in either the transmit or receive modes. During this procedure, the power level and sensitivities must be held constant.

To detail the procedure and foundation of pattern measurements and reciprocity, let us refer to Figures 3.5(a) and (b). The antenna under test is #1 while the probe antenna (#2) is oriented to transmit or receive maximum radiation. The voltages and currents V_1, I_1 at terminals 1–1 of antenna #1 and V_2, I_2 at terminals 2–2 of antenna #2 are related by

$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned} \quad (3-74)$$

where

Z_{11} = self-impedance of antenna #1

Z_{22} = self-impedance of antenna #2

Z_{12}, Z_{21} = mutual impedances between antennas #1 and #2

If a current I_1 is applied at the terminals 1–1 and voltage V_2 (designated as V_{2oc}) is measured at the *open* ($I_2 = 0$) terminals of antenna #2, then an equal voltage V_{1oc}

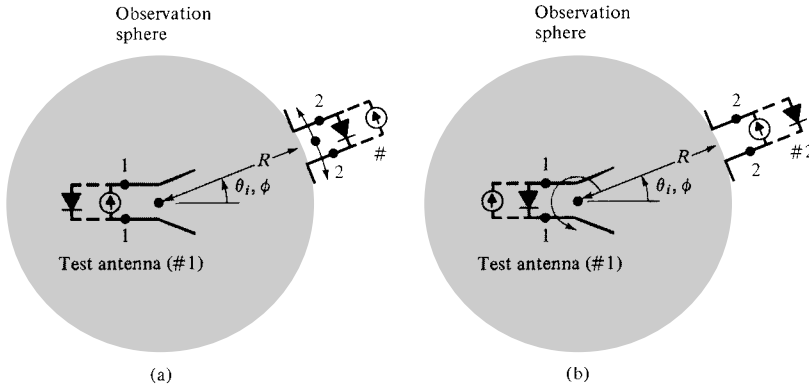


Figure 3.5 Antenna arrangement for pattern measurements and reciprocity theorem.

will be measured at the *open* ($I_1 = 0$) terminals of antenna #1 provided the current I_2 of antenna #2 is equal to I_1 . In equation form, we can write

$$Z_{21} = \left. \frac{V_{2oc}}{I_1} \right|_{I_2=0} \quad (3-75a)$$

$$Z_{12} = \left. \frac{V_{1oc}}{I_2} \right|_{I_1=0} \quad (3-75b)$$

If the medium between the two antennas is linear, passive, isotropic, and the waves monochromatic, then because of reciprocity

$$Z_{21} = \left. \frac{V_{2oc}}{I_1} \right|_{I_2=0} = \left. \frac{V_{1oc}}{I_2} \right|_{I_1=0} = Z_{12} \quad (3-76)$$

If in addition $I_1 = I_2$, then

$$V_{2oc} = V_{1oc} \quad (3-77)$$

The above are valid for any position and any configuration of operation between the two antennas.

Reciprocity will now be reviewed for two modes of operation. In one mode, antenna #1 is held stationary while #2 is allowed to move on the surface of a constant radius sphere, as shown in Figure 3.5(a). In the other mode, antenna #2 is maintained stationary while #1 pivots about a point, as shown in Figure 3.5(b).

In the mode of Figure 3.5(a), antenna #1 can be used either as a transmitter or receiver. In the transmitting mode, while antenna #2 is moving on the constant radius sphere surface, the open terminal voltage V_{2oc} is measured. In the receiving mode, the open terminal voltage V_{1oc} is recorded. The three-dimensional plots of V_{2oc} and V_{1oc} , as a function of θ and ϕ , have been defined in Section 2.2 as *field patterns*. Since the three-dimensional graph of V_{2oc} is identical to that of V_{1oc} (due to reciprocity), the *transmitting* (V_{2oc}) and *receiving* (V_{1oc}) field patterns are also equal. The same conclusion can be arrived at if antenna #2 is allowed to remain stationary while #1 rotates, as shown in Figure 3.5(b).

The conditions of reciprocity hold whether antenna #1 is used as a transmitter and #2 as a receiver *or* antenna #2 as a transmitter and #1 as a receiver. In practice, the

most convenient mode of operation is that of Figure 3.5(b) with the test antenna used as a receiver. Antenna #2 is usually placed in the far-field of the test antenna (#1), and vice versa, in order that its radiated fields are plane waves in the vicinity of #1.

The receiving mode of operation of Figure 3.5(b) for the test antenna is most widely used to measure antenna patterns because the transmitting equipment is, in most cases, bulky and heavy while the receiver is small and lightweight. In some cases, the receiver is nothing more than a simple diode detector. The transmitting equipment usually consists of sources and amplifiers. To make precise measurements, especially at microwave frequencies, it is necessary to have frequency and power stabilities. Therefore, the equipment must be placed on stable and vibration-free platforms. This can best be accomplished by allowing the transmitting equipment to be held stationary and the receiving equipment to rotate.

An excellent manuscript on test procedures for antenna measurements of amplitude, phase, impedance, polarization, gain, directivity, efficiency, and others has been published by IEEE [5]. A condensed summary of it is found in [6], and a review is presented in Chapter 17 of this text.

REFERENCES

1. R. F. Harrington, *Time-Harmonic Electromagnetic Fields*, McGraw-Hill, New York, 1961.
2. C. A. Balanis, *Advanced Engineering Electromagnetics*, John Wiley & Sons, New York, 1989.
3. P. E. Mayes, personal communication.
4. V. H. Rumsey, "The Reaction Concept in Electromagnetic Theory," *Physical Review*, Series 2, Vol. 94, No. 6, June 15, 1954, pp. 1483–1491.
5. *IEEE Standard Test Procedures for Antennas*, IEEE Std 149–1979, IEEE, Inc., New York, 1979.
6. W. H. Kummer and E. S. Gillespie, "Antenna Measurements–1978," *Proc. IEEE*, Vol. 66, No. 4, April 1978, pp. 483–507.

PROBLEMS

- 3.1. If $\mathbf{H}_e = j\omega\epsilon\nabla \times \mathbf{\Pi}_e$, where $\mathbf{\Pi}_e$ is the electric Hertzian potential, show that
 - (a) $\nabla^2 \mathbf{\Pi}_e + k^2 \mathbf{\Pi}_e = j \frac{1}{\omega\epsilon} \mathbf{J}$
 - (b) $\mathbf{E}_e = k^2 \mathbf{\Pi}_e + \nabla(\nabla \cdot \mathbf{\Pi}_e)$
 - (c) $\mathbf{\Pi}_e = -j \frac{1}{\omega\mu\epsilon} \mathbf{A}$
- 3.2. If $\mathbf{E}_h = -j\omega\mu\nabla \times \mathbf{\Pi}_h$, where $\mathbf{\Pi}_h$ is the magnetic Hertzian potential, show that
 - (a) $\nabla^2 \mathbf{\Pi}_h + k^2 \mathbf{\Pi}_h = j \frac{1}{\omega\mu} \mathbf{M}$
 - (b) $\mathbf{H}_h = k^2 \mathbf{\Pi}_h + \nabla(\nabla \cdot \mathbf{\Pi}_h)$
 - (c) $\mathbf{\Pi}_h = -j \frac{1}{\omega\mu\epsilon} \mathbf{F}$
- 3.3. Verify that (3-35) and (3-36) are solutions to (3-34).
- 3.4. Show that (3-42) is a solution to (3-39) and (3-43) is a solution to (3-31).
- 3.5. Verify (3-57) and (3-57a).
- 3.6. Derive (3-60) and (3-61).