

Multirate Filters: An Overview

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Abstract—Multirate filtering techniques are widely used in both sampling rate conversion systems and in constructing filters with equal input and output rates in the case where the use of a conventional method becomes extremely costly. The purpose of this paper is twofold. First, this paper gives a short review on designing proper digital filters that are useful in providing the desired sampling-rate conversion between the output and input signals. Second, this paper shortly reviews on how to apply multirate techniques as well as complementary filters for constructing digital filters in cases where the implementation of a conventional filter becomes, due to a too huge arithmetic complexity and the effects of finite word length effect, so large that this filter is not possible to construct in practice. In the second case, when using both multirate filtering and complementary filters enable one to share the overall filtering task between several simplified low-order sub-filters that operate at the lowest possible sampling rates and have significantly relaxed design constraints. Due to these facts, the resulting filter has significant reductions in the overall complexity as well in finite word-length effects.

Keywords— complementary multirate filters, decimation, halfband filters, interpolation, multirate filters, multistage systems, sampling rate conversion

I. INTRODUCTION

A multirate filter is a digital filter that changes the input sampling rate of the input signal into another desired one. These filters are of an essential importance in communications, image processing, digital audio, and multimedia. During the past three decades, multirate filters have been developed for implementation of digital filters with stringent spectral constraints [1], [2], [3], [4], [5], [8], [9], [10], [27], [38].

The key importance of multirate filtering in modern digital signal processing systems is roughly three-fold. First, it is required when connecting together two digital systems with different sampling rates. Filtering is required to suppress aliasing when reducing the sampling rate, called as decimation, and to remove imaging when increasing the sampling rate, called as interpolation. The use of an appropriate filter enables one to convert a digital signal of a specified sampling rate into another signal with the desired sampling rate without significantly destroying the signal components of interest. In many cases, instead of using a single-stage system, it is more beneficial, in terms of lowered arithmetic complexity, to carry out the overall procedure with a multistage system. In this case, there is a need to design all stages containing a filter and a sampling rate conversion in the manner that the resulting overall system corresponds to a single-stage system. Either

finite-impulse response (FIR) filters or infinite-impulse response (IIR) filters are used for generating the overall system. In some cases, both filter types are in use for building the overall conversion system. The selection of the filter type depends on the criteria at hand. The advantage of using linear-phase FIR filters is that they preserve the waveform of the signal components of interest at the expense of a higher overall complexity compared to their IIR counterparts. However, multirate techniques significantly improve the efficiency of FIR filters that makes them very desirable in practice.

Second, multirate filtering is required in constructing multirate as well wavelet filterbanks. Third, it one of the best approaches together with the proper use of complementary filter pairs for solving complex filtering problems when a single filter operating at a fixed sampling rate is of a significantly high order and suffers from output noise due to multiplication round-off errors and from the high sensitivity to variations in the filter coefficients.

The purpose of this paper is to give a short review on the above-mentioned first and third key advantages of using multirate filtering.

II. ONE-STAGE SAMPLING RATE CONVERSION

The role of a filter in decimation and interpolation is to suppress aliasing and to remove imaging, respectively. The performance of the system for sampling rate conversion depends mainly by filter characteristics. Since an ideal frequency response is not achievable, the choice of an appropriate specification is the first step in designing a proper filter.

Decimating the sampling rate by an integer factor of M means that among the input samples, only the only every M th sample are used as in the output sequence. For avoiding aliasing, there is a need to use a low-pass anti-aliasing filter before down-sampler as shown Fig. 1(a) for suppressing in a proper manner the signal components in the range $[f_s/M, f_s]$, where f_s is the input sampling rate, as well as to preserve the signal in the range $[0, f_s/M]$ as well as possible. When increasing the sampling rate from f_s by a factor L , thereby resulting in the sampling rate of Lf_s , there is a need to insert $L-1$ zero-valued samples between the existing input samples, as shown in Fig. 1(b). In this case, the role of an interpolation filter is to preserve the input signal in the range $[0, f_s]$ and to eliminate the extra images in the range $[f_s, Mf_s]$ as well as possible. The role of filtering in sampling rate conversion is demonstrated in Fig. 2.

The efficiency of FIR filters for sampling rate conversion is significantly improved using the polyphase realization. Filtering is embedded in the decimation/interpolation process and a polyphase structure is used to simultaneously achieve the interpolation/decimation by a given factor but running at a low data rate.

The polyphase structure is obtained when an N^{th} order filter transfer function is decomposed into M (L) polyphase components, $M, L < N$. For FIR filters, polyphase decomposition is obtained simply by inspection of the transfer function [5], [8], [27], and [38]. Figure 3 shows the polyphase structures for an M to 1 decimator and a 1-to- L interpolator.

Due to the polyphase multirate implementation, the number of arithmetic operations in a linear phase FIR filter is decreased by a factor M (or L). An effective method, which leads to high efficiency for a high-order FIR filter is proposed in [28].

For multirate IIR filters, several approaches to polyphase decomposition have been developed [2], [5], [7], [18], [31], and [32]. For a rational conversion factor L/M an efficient decomposition of the N^{th} order IIR filter, based on the method given in [2] is proposed in [32].

Polyphase IIR filters require lower computation rates among the known decimators and interpolators [31]. If a strictly linear phase characteristic is not requested, an IIR filter is an adequate choice. Moreover, an IIR transfer function can be designed to approximate a linear phase in the pass-band [12], [19], [36]. An IIR decimator or interpolator is particularly useful in applications that cannot tolerate a considerably large delay of an adequate FIR decimator or interpolator. For a restricted class of filter specifications, an attractive solution based on all-pass sub-filters can be used leading to very efficient implementation [18], [31]. The most attractive solution is achieved for $M, L=2$ with the IIR half-band filter implemented as a parallel connection two all-pass sub-filters [14], [18], [22], [23], [31].

III. MULTISTAGE FILTERING

For decimation and interpolation filters, and for multirate narrowband filters, additional efficiency may be achieved by cascading several stages, each of them consisting of a sub-filter and down-sampler for decimation and an up-sampler and sub-filter for interpolation [8], [23], and [27]. Design constraints for sub-filters are relaxed if compared to an overall filter. Hence, by using the multistage approach, the total number of coefficients is significantly reduced when compared with the single stage-design. The effects of finite word-length in sub-filters are low in comparison with the single-stage overall filter. When a decimation/interpolation factor is expressible as a power-of-two, the application of half-band filters improves the efficiency of the system. An example of multistage decimator is shown in Fig. 4. In the most general case, there is an extra stage after the last down-sampler. There exist at least two approaches using this alternative. First, when constructing sub-filters as a parallel connection of all-pass sub-filters [31], there is not possible to start a uniform stop-band $[0, f_s/M]$ with M being the overall decimator factor. Using an additional filter stage enables one to solve this problem. In the second

approach, where [34] (see also the references in this paper) provides a short review on this approach.

IV. FILTERS WITH EQUAL INPUT AND OUTPUT RATES

Digital filters with sharp transition bands are difficult, sometimes impossible, to be implemented using conventional structures. A serious problem with a sharp FIR filter is its complexity. The FIR filter length is inversely proportional to transition-width and complexity becomes prohibitively high for sharp filters [5], [8], [27], [33], and [38]. In a very long FIR filter, the finite word-length effects produce a significant derogation of the filtering characteristics in fixed-point implementation [27]. IIR filters with sharp transition bands suffer from extremely high sensitivities of transfer function poles that make them inconvenient for fixed-point implementation [21]. In many practical cases, the multirate approach is the only solution that could be applied for the implementation of a sharp FIR or IIR filter. Thus to design a multirate narrowband FIR or IIR filter, a classical time-invariant filter is replaced with three stages consisting of: (1) a low-pass antialiasing filter and down-sampler, (2) a low-pass kernel filter, and (3) an up-sampler and low-pass anti-imaging filter [5], [8], [23], [27]. The total number of coefficients in a multirate solution is considerably lower than the number of coefficients of a single rate time invariant filter.

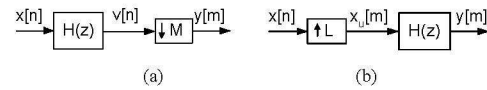


Figure 1. Decimator and interpolator

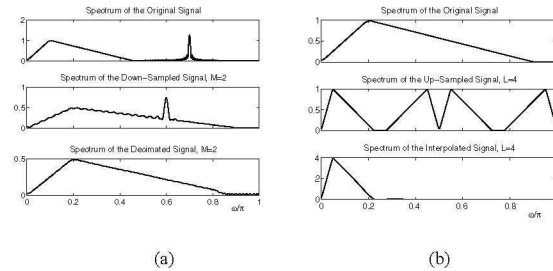


Figure 2. Frequency domain representation: (a) Downsampling and decimation, (b) Upsampling and interpolation.

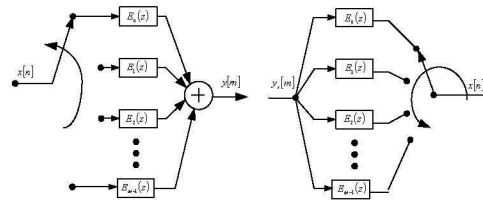


Figure 3. Polyphase structure: Decimator and interpolator

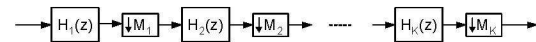


Figure 4. Multistage decimators

A. Multirate complementary filters

This method can be used in designing filters with any pass-band bandwidth. The multirate techniques are included to reduce the computational complexity. Using the complementary property, the multirate, narrow pass-band filter designs can be used to develop high-pass and low-pass filters with wide pass-bands [8], [27], [30]. When the output of a low-pass multirate filter is subtracted from the delayed replica of the input signal, the result is a wideband high-pass filter. The delay has to be selected to exactly equal the group delay of the multirate filter. For a low-pass wideband filter the multirate narrowband high-pass filter has to be used.

Efficient FIR filters with an arbitrary bandwidth can be designed using multirate and complementary filtering [8], [15] [30]. The overall design is evaluated by cascading complementary multirate filtering two-ports composed of two series branches and one parallel branch. The cascade is terminated with a simple kernel filter. One series branch of the cascade is a decimator (filter and down-sampler), while the other is an interpolator (up-sampler and filter). The parallel branch is a delay. The most efficient solution is obtained when half-band filters are used in the cascade.

Recently, the complementary filtering approach is extended to IIR filters [16]. The overall filter makes use of an IIR filter as a kernel filter, the periodic all-pass filters for constructing complementary pair, and linear phase FIR filters for the sampling rate alterations. Figure 5 illustrates a solution of sharp wide band filter.

B. Pipelining/Interleaving techniques

The pipelining/interleaving (P/I) architectures when used in constructing multirate systems can significantly improve the hardware efficiency. The P/I technique is based on the repetitive use of the same filter with the clock rate two or three times higher than a data rate, and with interleaved feedback of the output samples [14]. The resulting filtering is equivalent to the cascade of identical filters.

V. HALF-BAND FILTERS

Half-band filters are basic building blocks in multirate systems. A half-band filter divides the basis band of a discrete-time system in two equal bands with symmetry properties. For a linear phase FIR half-band filter, half of the constants are zero valued when the filter order is an even number [27], [33]. A half-band IIR filter can have fewer multipliers than the FIR filter for the same sharp cutoff specification. An IIR elliptic half-band filter when implemented as a parallel connection of two all-pass branches is an efficient solution [23]. The main disadvantage of elliptic IIR filters is their very nonlinear phase response. To overcome the phase distortion one can use optimization to design an IIR filter with an approximately linear phase response [36] or one can apply the double filtering with the block processing technique for real-time processing [20], [29].

For the appropriate usage of digital filter design software in half-band filter design, it is necessary to calculate the exact relations between the filter design parameters in advance [23].

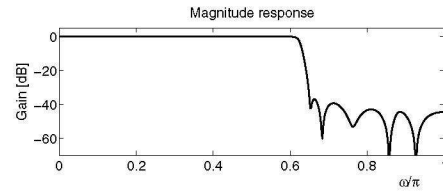


Figure 5. Example of a sharp wideband filter.

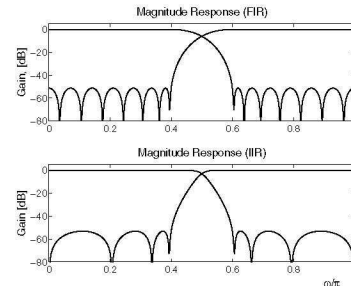


Figure 6. Efficient half-band filter pairs

The accurate FIR half-band filter design methods can be found in [33], [37], and [39]. For the IIR half-band filter design see [21], [23], [24], [35].

VI. COMPLEMENTARY FILTER PAIRS

Complementary filter pairs are used to split the input signal in two adjacent bands, and also are of importance for constructing complex multirate systems and filter banks. A low-pass/high-pass filter pair can be designed to exhibit all-pass complementary, power complementary, or magnitude complementary properties [27]. This solution benefits the possibility of implementing the complementary filter pair at the cost of a single FIR (IIR) filter [8], [27], [33]. In the most applications, half-band filter pairs are used to divide frequency band in two equal sub-bands. Figure 6 shows the example magnitude responses of two efficient complementary filter pairs: (a) low-pass/high-pass linear phase FIR filter pair: filter order $N=26$ implemented with 7 multiplication constants, and 7 delays (memory save implementation); (b) low-pass/high-pass IIR filter pair: filter order $N=7$ implemented with only 3 multiplication constants and 3 double delays. When the sampling rate is to be changed by 2, all the arithmetic operations can be evaluated at the lower rate.

As shown recently, an IIR filter pair with the arbitrary crossover frequency can be easily obtained by simple transformation of the half-band filter prototypes [25], [26]. This new filter pair retains the complementary properties of the start-up half-band filter pair.

VII. MULTIPLIERLESS SOLUTION

The efficiency of multirate filters is greatly improved by simplifying arithmetic operations. This is achieved by replacing a multiplier with a small number of shifters-and-adders. Generally, implementing multiplierless design techniques in

sub-filters, at the cost of a slight derogation of filtering performances, increases the efficiency of the overall multirate filter. For instance, one can use the optimization technique [17], the multiplier block approach [6], or design based on EMQF (Elliptic Minimal Q-Factors) transfer functions [20], [21], and [22]. A well-known solution for large conversion factors in decimation is a cascaded integrated comb (CIC) filter, which performs multiplierless filtering [9], [10], [11].

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