

Land-Vehicle Navigation Using GPS

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Invited Paper

The Global Positioning System (GPS) has made navigation systems practical for a number of land-vehicle navigation applications. Today, GPS-based navigation systems can be found in motor vehicles, farming and mining equipment, and a variety of other land-based vehicles (e.g., golf carts and mobile robots). In Section II of this paper, each of these applications is discussed, and the reader is introduced to some of the issues involved with each one. Beginning in Section III, one particular technical aspect of navigation for land vehicles is discussed. Specifically, the research discussed in this paper presents a quantitative examination of the impact that individual navigation sensors have on the performance of a land-vehicle navigation system. A range of navigation sensor performance levels and their influence on vehicle positioning accuracy are examined. Results show that, for a typical navigation system, positioning error is dominated by the accuracy of the position fixes provided by the GPS receiver when GPS position fixes are available and by the rate gyro's bias drift when GPS position fixes are not available. Furthermore, results show that the accuracy of the GPS position fixes has a significant impact on the relative contributions that each dead-reckoning navigation sensor error makes. The implications of these results for navigation system design and sensor design are discussed.

Keywords—GPS applications, Kalman filtering, land-vehicle navigation, sensitivity analysis.

I. INTRODUCTION

The Global Positioning System (GPS) has made navigation systems practical for a number of land-vehicle navigation applications. Today, GPS-based navigation systems can be found in motor vehicles, farming and mining equipment, and a variety of other land-based vehicles. The purpose of this paper is to introduce the reader to various land-vehicle navigation applications and to present research that explores one particular technical aspect of land-vehicle navigation. In this introductory section, various positioning issues associated with land vehicles will be introduced and the motivation for the research presented in this paper will be given. At the end of this section, the organization of the remainder of the paper will be described.

Manuscript received September 22, 1997; revised January 26, 1998. This work was supported in part by Gyration, Inc. and Daimler-Benz. E. Abbott is with Allied Signal, Redmond, WA 98073 USA. D. Powell is with the Aeronautical and Astronautical Engineering Department, Stanford University, Stanford, CA 94305 USA. Publisher Item Identifier S 0018-9219(99)00427-2.



Fig. 1. Functional schematic of a land-vehicle navigation system.

A. Positioning for Land Vehicles

Simply put, the most basic function of a land-vehicle navigation system is to accurately identify the location of a vehicle. In many existing land-vehicle navigation systems, this is achieved typically by an on-board computer that continuously collects data from sensors that are mounted inside the vehicle. The computer uses the sensor data to compute the vehicle's location and conveys this location to the vehicle's driver by means of an electronic user interface. Examples of positioning sensors that might be utilized in a land-vehicle navigation system include a GPS receiver, a gyroscope, an electronic compass, and a tap into the vehicle's odometer. Fig. 1 shows a simple functional schematic diagram representing the operation of a land-vehicle navigation system.

Although the purpose of GPS is to provide its users with the ability to compute their location in three-dimensional space, in general, a land-vehicle navigation system cannot continuously position a vehicle using a GPS receiver alone, and other navigation aids are necessary. In order to understand why this is so, one must first understand some basic facts about GPS.

In order to compute its location in three-dimensional space, a GPS receiver must be able to lock onto signals from at least four different satellites. Moreover, the receiver must maintain its lock on each satellite's signal for a period of time that is long enough to receive the information encoded in the transmission. Achieving and maintaining a lock on four (or more) satellite signals can be impeded because each signal is transmitted at a frequency (1.575 GHz) that is too high to bend around or pass through solid objects in the signal's path. For this reason, GPS receivers cannot be used indoors. Outdoors, tall buildings, dense foliage, or terrain that stand between a GPS receiver and a GPS

0018-9219/99\$10.00 © 1999 IEEE

satellite will block the satellite's signal. Thus, in urban or heavily foliated environments a GPS receiver may be unable to provide a position fix for indefinitely long periods of time. For this reason, land-vehicle navigation systems in general cannot continuously position a vehicle using a GPS receiver alone.

Even if GPS position fixes are available, however, they contain errors and are accurate to 100 m (95% of the time). This error is unacceptably high—in automobile navigation applications, for example, densely packed urban road networks generally contain roads that are fewer than 100 m apart. It should be noted that the inherent accuracy of the GPS is better than 100 m. However, the signals from the GPS satellites have been degraded intentionally by the U.S. Department of Defense (DoD) for purposes of national security. This performance degradation is known as selective availability (SA), and only DoD-approved users have access to satellite signals without SA. However, a policy statement recently issued by the White House indicates that SA will be turned off before the year 2006 [1], and the accuracy of GPS position fixes will improve significantly. In addition, a more accurate form of GPS positioning known as differential GPS (DGPS) may soon become widespread and free of charge. (In 1997, the U.S. Senate approved funding for a nationwide DGPS under the auspices of the 1998 Transportation Appropriation Bill. This system, scheduled for completion by the year 2000, would make DGPS corrections available free of charge in the 48 contiguous states and Alaska [2].)

Because GPS position fixes are inaccurate and at times may be unavailable altogether, many land-vehicle navigation systems utilize other navigation aids in conjunction with GPS position fixes to enhance overall system performance. These aids usually include some combination of sensors, e.g., low-cost gyroscopes, compasses, an odometer, inclinometers, and/or accelerometers. Any sensors other than GPS that are used to position the vehicle are collectively referred to as a dead-reckoning unit. Dead-reckoning sensors generally cannot be used alone to position a vehicle accurately for indefinitely long periods of time because dead-reckoning sensors, by definition, do not measure absolute position. Without an occasional measurement of absolute position, the error in a position estimate computed using dead-reckoning sensors alone grows without bound. Dead-reckoning sensors are utilized because they accurately measure changes in a vehicle's position over short time periods and can be used alone (for short time periods) if GPS position fixes become unavailable.

The nature of the errors in GPS position fixes is somewhat different than that of the errors in dead-reckoning sensor outputs. The total error in any given GPS position fix contains a number of components, each of which arises from a different error source. Some error components are correlated over long time periods, while others are random and uncorrelated from one position fix to the next. All of the errors, however, are bounded. Therefore, the errors that appear in GPS position fixes and in the outputs of dead-reckoning sensors are complementary in

nature—dead-reckoning sensors smooth out the short-term GPS errors, and GPS fixes calibrate the dead-reckoning sensor drift over long time periods. Proper fusion of the GPS position fixes with the dead-reckoning sensor data can take advantage of these complementary errors, producing positioning performance that is better than could be obtained with either type of data alone.

B. The Research in This Paper

In light of the many possible combinations of navigation aids that can be used in these systems, one is led to question what criteria navigation system designers have used when selecting sensors for use in their vehicle navigation system. One could probably say with some certainty that the set of sensors selected by a design team is heavily influenced by the team's dual goals of maximizing the system's performance while minimizing its total cost. Unfortunately for system designers, however, system cost and performance are usually directly, rather than inversely, related—very accurate sensors may improve the performance of a system, but they tend to cost more than similar, less accurate sensors. Therefore, designers of land-vehicle navigation systems are faced with trading off system cost and performance and must judiciously select that set of sensors deemed to be most cost effective.

This paper includes an examination of the impact that individual navigation sensors have on the performance of a land-vehicle navigation system. The quantitative results of this work reveal the influence that individual navigation sensor error parameters have on navigation system performance. These results should be valuable to navigation system designers because they can be used to identify cost-effective navigation system designs.

C. Organization of This Paper

Before the research described in Section I-B is presented, however, some basic information about land-vehicle navigation systems is in order. The information in the next several sections of the paper is therefore devoted to descriptions of various land-vehicle navigation applications. The purpose of these descriptions is to familiarize the reader with various land-vehicle navigation applications and concepts.

After this, in Sections III–VIII, the research described in Section I-B is presented—namely, the contributions that individual navigation sensors make to overall system performance are quantified. Because the performance characteristics of individual navigation sensors are intimately related to this analysis, this latter part of the paper includes a discussion of several navigation sensors commonly found in existing land-vehicle navigation systems. After that, the analytical method used to quantify how much each navigation sensor contributes to the overall navigation system performance will be presented. Finally, analysis results will be shown for several navigation systems.

II. APPLICATIONS

A. Motor Vehicles

1) *Introduction:* Traditionally, navigation systems have been very large, expensive, and used only in aviation or military applications. However, the presence of the GPS and the recent proliferation of small, low-cost motion sensors have made possible navigation systems that are small and inexpensive enough to be used in consumer products. Commercial consumer-grade navigation systems are, in fact, readily found today in Japan, Europe, and the United States, with one application being automobile navigation systems.

The concept of in-vehicle navigation systems is not new, but implementations of such systems are relatively recent. Programs investigating the possibility of establishing an infrastructure to support widespread navigation for motor vehicles began in the U.S. as early as the late 1960's. However, results from these studies deemed that the supporting infrastructure for such a system would be too expensive, and further study in the United States was dropped until the 1980's [3]. In the late 1980's, the U.S. government, recognizing that parts of the country's road system were taxed nearly to capacity, launched a campaign to promote the application of high-tech solutions to enhance roadway efficiency. Outlined in the National Program Plan for Intelligent Transportation Systems (NPP) [4], this campaign includes a strategy for improving the efficiency of the U.S. highway system over a 20-year period. The NPP's goals include reducing highway congestion, fuel consumption, and the number of traffic accidents by providing drivers with real-time traffic information, route guidance, electronic toll collection, advanced vehicle collision avoidance systems, and automatic notification to authorities in the event of a traffic emergency. These ambitious renovations to the U.S. road system involve a number of diverse technologies, and knowledge of a vehicle's location lies at the heart of many services described in the NPP (e.g., route guidance and emergency response).

In Japan, research efforts in real-time automobile route guidance were begun in the 1970's with the goal of reducing traffic congestion. Throughout the 1970's and 1980's, the Japanese government, in cooperation with industry, was continuously involved in launching initiatives which helped to mature vehicle navigation technology [5]. Today, most Japanese car manufacturers offer factory-installed navigation systems in at least some of their models. Estimates indicate that, by the year 2000, per annum sales of vehicles with factory-installed navigation systems will reach 2.5 million [3].

2) *Example Applications:* Two examples of navigation applications involving motor vehicles include systems for fleet vehicles and systems for individual consumer vehicles. "Fleet tracking" involves the monitoring and efficient dispatch of vehicles that comprise a fleet with a common purpose. Examples of vehicle fleets whose efficiency may be improved with navigation systems include ambulances, police cars, mass transit vehicles, and delivery vehicles.

A typical fleet-tracking system includes navigation and communication equipment in each vehicle and a monitoring station. The equipment in each vehicle is typically comprised of a set of positioning sensors, an on-board computer, and a radio link to the monitoring station. The vehicle's location is determined from the navigation equipment by the on-board computer. The estimated location of the vehicle then is relayed periodically to the monitoring station by means of the radio link. At the monitoring station, the location of each vehicle is displayed on a computer screen that shows a bird's-eye view of a local road map (from a digital map database). Each vehicle typically is represented by an icon in the appropriate location on the screen. Operators at the monitoring center can use their knowledge of each vehicle's location to dispatch members of the fleet efficiently and, if necessary, guide drivers in the field to their destinations.

Navigation in consumer vehicles is slightly different, in that each vehicle's location generally is not monitored. Because the navigation system is entirely contained within the vehicle itself, these systems are sometimes referred to as "autonomous" navigation systems [6]. The basic purpose of these types of systems usually is to guide the driver to a destination that the driver specifies. A typical system includes positioning equipment, an on-board computer, a user-input device, and a graphical electronic display. The computer estimates the vehicle's position based on information from the navigation sensors and conveys the vehicle's location to the driver on the electronic display. The display may show prompts that guide the user from turn to turn (e.g., a right arrow soon before the user is supposed to turn right).

Currently, consumer automobile navigation systems and fleet-tracking systems are available in Europe, Japan, and the United States. As of the publication of [6], there had been approximately 200 fleet tracking and autonomous consumer navigation systems produced worldwide, most of which were made available in Japan.

Some of the potential benefits of these systems are clear. The NPP cites that \$100 billion are lost every year in wasted time and fuel as a result of inefficient travel on U.S. roadways [4]. Improving travel efficiency therefore has significant economic and environmental implications. Moreover, these systems could have lifesaving potential. Some existing systems, for example, combine a GPS receiver with a cellular phone. The system hardware may include a "panic" button that a user in distress can press, causing the cellular phone to relay the vehicle's GPS-derived location to a monitoring station established to accept such incoming distress calls. Operators at the monitoring station can notify authorities as necessary.

3) *Map-Matching:* The navigation sensors in automobile navigation systems frequently include a GPS receiver and some combination of the low-cost motion sensors mentioned in Section I-A. However, in addition to a GPS receiver and dead-reckoning sensors, many automotive navigation systems utilize data from a digital map database to aid in navigation. A digital map database is essentially an

electronic road map—a digitization of a local road network, with each road represented as a collection of points assumed to be connected in a dot-to-dot fashion.

Map databases are frequently used to convey the vehicle's position to the vehicle's driver. However, information in a map database can also be used to improve navigation accuracy if the vehicle is assumed to be traveling on a road stored in the database. The software algorithm that combines the sensor data with the map data to produce a position estimate generally is referred to as a map-matching algorithm. Map-matching algorithms are usually heuristic rules by which sensor data and information from the map database are processed to identify that road on which the vehicle is most likely to be traveling. Map-matching algorithms described in the literature most often involve pattern-matching techniques that attempt to correlate the pattern created by several consecutive position fixes to a similar pattern of connected roads in the local road network. (A few of the many such map-matching algorithms can be found in [7]–[12].) After a successful correlation is made, information about the matched road can be extracted from the database and used to calibrate errors in the navigation sensors.

A map-matching algorithm that performs well can substantially improve the navigation accuracy of an automobile navigation system. Not only can it reduce positioning error to less than that of (stand-alone) GPS position fixes, but it can also be utilized to aid in the calibration of dead-reckoning sensors. Generally, if the algorithm is able to identify the approximate location of the vehicle, then the accuracy of the navigation system's estimated position is limited by the error in the map database. Position error in the database can arise from two sources: 1) error in the digitization when the map database is created and 2) error arising because physical roads are represented in the map database by line segments. The first error means that the position coordinates stored in the map database for an incorrectly digitized road would be different from the position coordinates for the actual physical road; in the presence of this error, a map-matched position fix would be in error by an amount equal to the digitization error. The second error occurs even if the road is correctly digitized. This (second) error arises because multilane roads are effectively represented by lines (which have zero width). A vehicle on a multilane road could move laterally several meters relative to the line that represents the road in the map database. In the presence of this error, a map-matched position could be in error by as much as one-half of the road's total width.

The influence of map-matching on the performance of a navigation system will not be addressed in this paper. The reader is referred to [13] for an examination of this topic.

B. Farming

GPS has been used on farm equipment for the last few years for the purpose optimizing the application of fertilizer. Crop yield is first recorded as a function of the position on the field and subsequent applications of

fertilizer are scheduled to maximize the effectiveness of the fertilizer based on the historical data. These systems require differential GPS for accuracy on the order of a meter but do not require dead-reckoning sensors as foliage and buildings are not present in this environment.

Research is now underway on developing robotic farm equipment. The motivation is to improve yield through denser planting and to enable accurate farm equipment operation with a less well trained operator. These systems require accuracies on the order of a few centimeters; therefore, carrier phase differential GPS (CDGPS) tracking technology is required in order to achieve this. Signal obstruction is less likely to be a problem than the road environment, and therefore, it does not appear as if dead-reckoning sensors will be required for this application. The special features of the GPS system for this application are the procedures required to initialize the carrier wave cycle ambiguities and the need for attitude knowledge to correct for vehicle rocking on the uneven ground. It has been demonstrated [14] that tractor heading control can be maintained to within 1° and line tracking can be carried out to within 2.5 cm (1σ).

C. Heavy Equipment and Mining

Road construction equipment and open-pit mining equipment will likely benefit from a GPS navigation system. Road equipment requires accuracy levels on the order of a few centimeters for positioning of implements; therefore, CDGPS will be required for this application. Furthermore, vehicle attitude will likely be required as well in order to correct for the offset between the roof-mounted antenna and the implement. Cohen [3] showed that GPS-based attitude with multiple antennas has the capability to provide acceptable accuracy for this purpose.

In many environments, there can be some satellite shielding by adjacent buildings or excavation walls. In order to have a satellite based system that remains operational under these conditions, it is necessary to augment the satellites. Research at Stanford by Cohen [3] and others has shown that pseudolites are capable of augmenting the satellites so that CDGPS positioning can be carried out using signals from the pseudolites in addition to those from the satellites. Although the applications to date have concentrated on using the pseudolites for ambiguity resolution and accuracy improvement rather than covering for blanked satellites, the same architecture can be applied to cover for blanked satellites and research at Stanford University is currently in progress on this application [15].

D. Miscellaneous Applications

Golf has attracted interesting applications of GPS. The simplest systems simply tell the golfer how far to the next hole and some characteristics of the terrain ahead. The key to this sort of application is an accurate and complete database of the golf courses. More aggressive applications entail a motorized golf bag that follows the golfer similarly to how a human caddie would. This type

of device requires navigation accuracy at the submeter level in order to navigate narrow paths and to avoid hazardous regions of the course. They require a reference station in the vicinity of the golf course and a data link to the golfer's motorized device that is accessible on all parts of the golf course. Furthermore, this type of system needs to be operational and maintain its accuracy under foliage. Therefore, dead reckoning sensors are required including gyros and odometers in a similar arrangement to the road vehicle application. These kind of systems are currently in an advanced state of development.

III. THE ROLE OF LOW-COST NAVIGATION SENSORS IN LAND-VEHICLE NAVIGATION SYSTEM PERFORMANCE: PRELIMINARY REMARKS

As was mentioned in Section I-A, this paper contains an examination of the contributions that individual navigation sensors make to a navigation system's performance. In this research, the criterion for system "performance" that has been chosen is the rms positioning error achieved by the navigation system. Because a navigation system's ability to position a vehicle accurately is determined by the accuracy of its sensor measurements, then it is really the individual sensor measurement errors that cause the positioning error. Therefore, the contributions that individual sensor measurement errors make to the rms positioning error are quantified.

The remainder of this paper presents this analysis and its results. First, in Section IV, several navigation sensors commonly found in existing land-vehicle navigation systems are discussed. In each discussion, emphasis is placed on each sensor's error characteristics. (The nature of each sensor's measurement errors is an integral part of the analysis presented in subsequent sections of the paper; for this reason, it is important to preface the analysis with these discussions.) Next, in Section V, the theory underlying the analysis is presented. Finally, the results of the analysis are presented in Section VI.

IV. LOW-COST NAVIGATION SENSORS: FUNCTION AND PERFORMANCE CHARACTERISTICS

A. Rate Gyroscopes

Rate gyroscopes, i.e., sensors that measure their rate of rotation about a particular axis, are found in a number of existing land-vehicle navigation systems [16]. Two example low-cost rate gyros are the Murata Gyrostar and the Systron Donner Gyrochip Horizon. Both of these gyros generate an analog signal that is amplified so that 1 V of output corresponds to rotation rate of $45^\circ/\text{s}$. The maximum specified rotation rate for both gyros is $90^\circ/\text{s}$. Both the Murata gyro and the Systron Donner gyro transduce rotation rate using a vibrating element. When the gyro is not rotating, the vibrating element continuously vibrates back and forth within a plane. When the element is subjected to rotation about a particular axis, coriolis forces cause the element to deflect out-of-plane; the amplitude of the out-of-plane

motion is proportional to the rate of rotation. This out-of-plane motion is sensed and filtered by electronics inside the gyro, and the filtered signal serves as the gyro's output. (Excellent discussions of vibrating-element gyroscopes can be found in [17] and [18].)

In vehicle navigation systems, rate gyros are used to measure changes in the vehicle's heading by integrating the rate gyro's output. However, the output of any real rate gyro contains measurement errors that produce errors in the computed heading change. Examples of errors that appear in a typical gyro's output include noise, a (time-varying) bias, scale factor error, g-sensitivity, and cross-axis sensitivity. Because of these errors, the error in the integrated rate gyro output generally will grow without bound. In low-cost rate gyroscopes, measurement errors integrate into heading errors at a rate of roughly $15\text{--}100^\circ$ per hour. The nature of each of these error sources will now be described briefly.

1) *Bias Drift*: As will be demonstrated in a later section, a rate gyro's bias drift and noise have a significant impact on land-vehicle navigation system performance. Generally, it seems true that the bias drift of low-cost gyroscopes is a strong function of temperature. Temperature-dependent performance variations usually are quantified in specification sheets for low-cost gyros and have also been noted by other researchers (e.g., [19] and [18]). It is nevertheless instructive to show the data collected by the authors that confirms these temperature-related phenomena.

To examine the nature of bias drift in two low-cost gyroscopes, the authors collected data from both the Murata Gyrostar and Systron Donner Gyrochip Horizon. Fig. 2, for example, shows data collected concurrently from a Gyrostar and a Gyrochip Horizon. The data shown were collected over a 48-h period, beginning immediately after power was applied to each gyro. The output of each gyro was first filtered with a sixth-order continuous-time low-pass anti-aliasing filter with a cutoff frequency of 20 Hz. (The purpose of this filter was to bandlimit the broadband noise appearing in each gyro's output.) The filtered signal was sampled at 50 Hz; this data was then filtered again (digitally) to reduce the noise in the samples. The sampled data were saved at 1 Hz. Even though filtered twice, the raw data was still somewhat noisy, and this noise obscures the drift in the gyros' outputs. For purposes of clarity, then, the raw data were averaged in blocks of time 1 min long. (Averaging the data really is another form of low-pass filtering and is legitimate because it does not obscure long-term bias drift.) Finally, in order to compare the data from the two gyros more easily, the mean of each set of data (over the entire test) was removed. The modified gyro data is shown in Fig. 2.

As the figure shows, the bias in both gyros drifts nearly identically with time. This suggests that the drift of both biases is caused by the same phenomenon—probably temperature variations. Other tests have been run in which the gyro's output was recorded as the gyro temperature was varied. These tests confirmed the fact that each gyro's bias is a strong function of temperature.

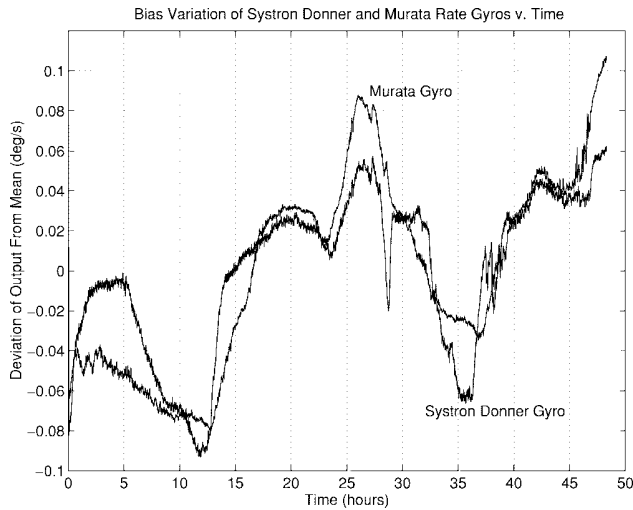


Fig. 2. Forty-eight hours of data from two rate gyros.

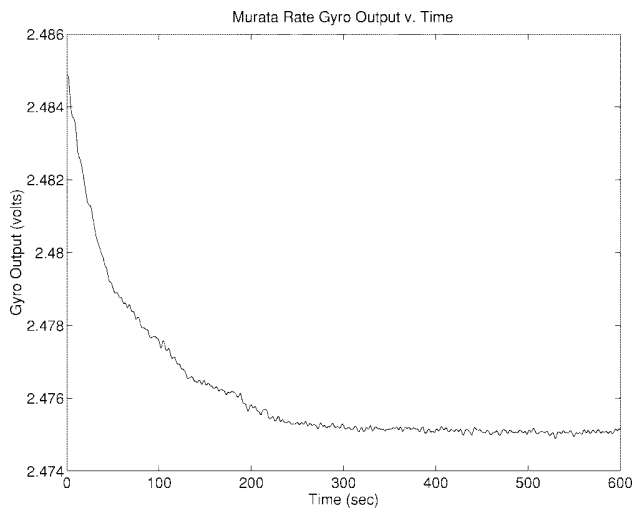


Fig. 3. Murata gyro transient.

Other tests reveal that both gyros suffer from a significant transient bias drift immediately after power is applied to them. Figs. 3 and 4 show the average output of each gyro for several minutes after power was applied. As the figures show, the mean output of both gyros approaches a steady-state value in a roughly exponential fashion. This phenomenon is probably the result of the gyro's self-heating, i.e., when each gyro is turned on, the electronics inside begin to dissipate heat and cause the temperature of the gyro to rise. As the temperature inside the gyro stabilizes, the mean output of the gyro changes at a slower rate. The outputs of both gyros take about the same amount of time to stabilize; however, note that the magnitude of the drift is larger for the Murata gyro than for the Systron Donner gyro. A similar phenomenon was observed (for the Murata gyro) in [19] and (for another gyro) in [18].

2) *Scale Factor Error*: The extent to which a scale factor error contributes to heading error depends on how much the vehicle turns. Theoretically, if a rate gyro's output is integrated directly to measure a change in heading, then

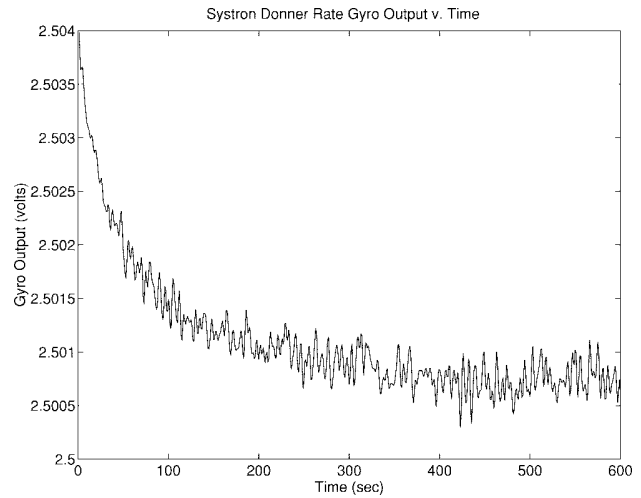


Fig. 4. Systron Donner gyro transient.

the computed heading change will be in error by the same percent as the rate gyro scale factor. For example, a scale factor error of 1.0% will result in a heading error of approximately 0.90° after a 90° turn. If the vehicle never turns, then scale factor error contributes virtually nothing to errors in the heading estimate. Hence, scale-factor errors cause the largest heading error when the vehicle makes turns that sweep through large angles.

3) *Cross-Axis Sensitivity and g-Sensitivity*: The remaining error sources (g-sensitivity and cross-axis sensitivity) probably have less impact on the performance of a land-vehicle's navigation system than the bias drift and noise. The g-sensitivity of a rate gyro causes errors to be introduced into the gyro's output as a result of linear acceleration. (The output of an ideal rate gyro would be entirely insensitive to acceleration.) The influence of this error source on a land-vehicle navigation system's performance obviously will depend on the magnitude and frequency of the acceleration encountered in the automobile. A simple calculation, however, can show that the typical acceleration encountered in a real automobile is quite small and that the error resulting from g-sensitivity can probably be considered "small"—a vehicle accelerating from 0–60 MPH in 10 s accelerates at 0.27 g. Furthermore, experience suggests that most of the time land-vehicles accelerate at even lower rates and often travel at nearly constant speeds. This error is therefore likely to be a very small contributor to the overall navigation error. Cross-axis sensitivity causes errors to be introduced into the gyro's output as a result of rotations about an axis perpendicular to the axis of sensitivity. Although a vehicle rotates almost exclusively about a vertical axis, imperfections in a road may cause a vehicle to achieve pitch and roll rates that are large enough to induce errors into a gyro's output. If this is the case, however, these errors should be considered random disturbances and would probably contribute to overall navigation error in a manner similar to the broadband noise in the gyro's output.

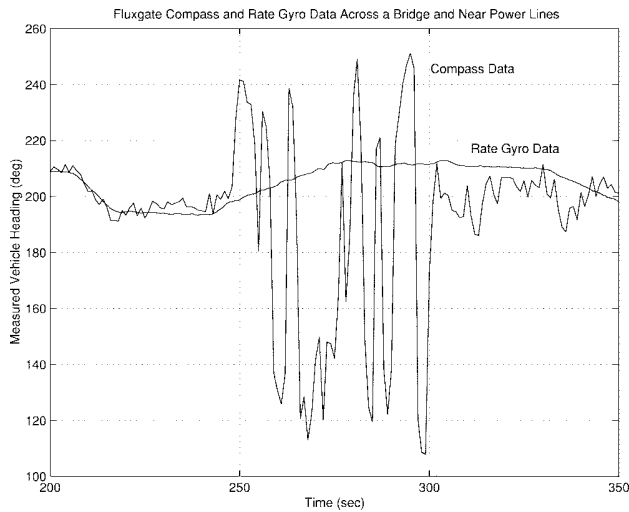


Fig. 5. Compass data taken on a bridge and near power lines.

B. Magnetic Compasses

A magnetic compass is an electronic device that measures its heading relative to magnetic north by measuring the direction of Earth's local magnetic field. Compasses are generally implemented with magnetometers, a Hall effect sensor, or a set of orthogonal coils referred to as a "flux-gate." Generally, it seems true that of the existing compass implementations, the fluxgate compass is most commonly used in existing land-vehicle navigation systems [16].

Accurate heading measurements can be difficult to obtain with a magnetic compass because disturbances in the magnetic field near the compass can induce large errors in the compass' output. Sources of magnetic disturbance encountered in vehicle navigation include power lines, motors (e.g., a fan inside the vehicle), and residual magnetism in local metal structures such as bridges, buildings, and even the vehicle's chassis.

Data collected by the authors from a fluxgate compass have verified that a compass can exhibit very large measurement errors. Fig. 5 shows data collected from a fluxgate compass taken in a vehicle that was being driven across a nearly straight bridge in the vicinity of power lines. Included in the plot is the integral of data that were simultaneously collected from a Systron Donner rate gyro. The integrated gyro data provide a measure of the vehicle's heading history that is independent of the compass reading. As the figure illustrates, the gyro data indicate that the vehicle's heading is changing very little, while the compass data shows swings larger than 100° . Two other independent gyroscopes were sampled concurrently and their data verify this result. Clearly, the compass reading is in error, probably as a result of magnetic disturbances induced by the power lines and the metal in the structure of the bridge.

Because compasses are susceptible to large errors, error-compensation schemes for the fluxgate compass have received significant attention. Some compensation methods depend on having other sensors available, such as an angular velocity sensor [20] or GPS [21]. Other approaches

involve calibrating the compass errors by generating a lookup table of the errors as a function of heading [22]–[24] or involve some type of basic prefiltering technique [25], [26]. Still other approaches involve gimbaling the compass to prevent the compass from tilting relative to a local horizontal plane, thereby avoiding tilt-induced errors in the compass' output [27]. Finally, some fluxgate compass manufacturers specify a method by which the user can calibrate the errors in the compass. Such a calibration process is designed to eliminate systematic measurement errors that are a function of heading. Usually, calibration involves rotating the compass through at least 360° , while digital electronics in the compass generate a lookup table of heading errors. This type of calibration can compensate for systematic errors that affect the gyro at the time of calibration. However, this type of calibration is ineffective against random errors that arise during operation and changes in the systematic errors that occur after calibration.

Because compasses can exhibit unpredictably large errors at unpredictable times, it is difficult to gauge their utility in land-vehicle navigation. If the compass reading is accurate, then the heading measurement can enhance the navigation system's performance significantly. However, if the compass reading contains large errors, then including the compass data in the position calculation can worsen the navigation system's performance. However the compass data is treated in a land-vehicle navigation system, it is clear that compasses can produce readings with very large errors and their utility is questionable.

C. Odometers

An odometer measures the curvilinear distance traveled by a vehicle. Discussions of the use of odometers in particular navigation systems appear in [25], [26], [28], and [29]; detailed error analyses are given in [13] and [28]. In [30] and [31], the authors discuss various error sources in odometer measurements, and empirical data is presented in [31], but no formal analyses are presented.

To understand the nature of odometer measurements, it is necessary to understand some basic facts about an odometer's operation. Typically, an odometer is an electronic device that generates an integer number of digital pulses each time a wheel on the vehicle makes one revolution. (A typical odometer generates one pulse for every 3–20 cm traveled.) In order to convert the number of pulses generated by the odometer (in a particular time interval) to the distance traveled by the vehicle (in that same interval), it is necessary to multiply the number of pulses by an odometer scale factor. The odometer scale factor generally is not constant because it depends on the radius of the vehicle's wheel, and a wheel's radius can change with tire pressure, temperature, tread wear, and the vehicle's speed. (Empirical data that quantify the change in odometer scale factor versus tire pressure, tread wear, and vehicle speed are presented in [31].)

In addition to variations in the odometer scale factor, the output of an odometer contains random errors. These errors arise because the output of the odometer is a pulse

train, implying that distances measured by the odometer are quantized into packets represented by a single pulse. Distances cannot, therefore, be resolved more accurately than the distance corresponding to one pulse. For this reason, an odometer's output contains quantization error. This quantization error can be treated as a random quantity with a zero mean [13].

Naturally, drift in the odometer scale factor and quantization error will introduce error into a position estimate that is computed using odometer information. In Section VI, results will be shown that demonstrate how much each of these two errors contributes to the overall positioning error in an automobile navigation system.

D. GPS Positioning

As of the writing of this paper, the only type of GPS positioning available to civilian users worldwide is unaided positioning corrupted by SA. It is likely, however, that SA will be turned off [1], and free differential corrections may become widely available in the near future. In either case, the accuracy of GPS position fixes available to civilian users would improve significantly. This may have significant impact on land-vehicle navigation system design because the improved positioning accuracy may cause the relative impact of dead-reckoning sensors on overall system performance to change. For example, improved positioning accuracy may improve dead-reckoning sensor calibration and, therefore, may permit navigation system designers to relax requirements on dead-reckoning sensor performance. This possibility is a compelling reason to investigate the impact that each type of GPS positioning (i.e., stand-alone GPS with SA turned on, stand-alone GPS with SA turned off, and DGPS) has on the contributions that dead-reckoning sensors make to overall system performance.

Unlike unaided GPS, the use of DGPS requires a source of differential corrections. As of the writing of this paper, there is no single widely available source of free differential corrections. However, commercial sources of corrections exist, and sources of free corrections exist in restricted geographical areas. Commercial differential corrections are services to which users can subscribe. For a fee, subscribers are given access to differential corrections that are broadcast on a radio frequency in their locale.

Two other sources of differential corrections may become widely available for use by land-vehicle navigation systems in the near future. The U.S. Coast Guard (USCG) has established differential GPS correction stations covering the U.S. coasts and inland waterways. This system provides differential GPS corrections to the marine community free of charge [32]. The radio beacons broadcasting the corrections transmit nondirectionally at a frequency of 285–325 kHz with enough power to reach a user 10–75 mi away [33], [34]. Therefore, although not designed primarily for land-based GPS users, the corrections may be receivable by land-based GPS users in the vicinity of the U.S. coasts and inland waterways. A similar differential correction service may be placed inland to cover the entire continental U.S. [35] (For excellent discussions of the USCG DGPS system

Table 1
RMS Values for Horizontal GPS Positioning Error

GPS Positioning Type	With SA	Without SA	DGPS
RMS Bias Error (m)	41.1	10.2	2.2

and its performance characteristics, the reader is referred to [33], [36], and [37].)

A second source of differential corrections that may become available to land-vehicle navigation systems is the Wide Area Augmentation System (WAAS). The WAAS is a GPS-based navigation system currently being developed by the Federal Aviation Administration for the aviation community. According to current plans, differential GPS corrections would be broadcast free of charge over the entire United States by a set of geosynchronous communications satellites [3, ch. 4].

For purposes of comparison, Table 1 summarizes the approximate rms position error for each type of GPS positioning. The values given in Table 1 represent position error in a horizontal plane. It should be emphasized that the data in this table are approximate and represent the rms error for horizontal positioning. The values in the table were obtained from [3, ch. 1] and [38, ch. 11].

V. QUANTIFYING THE EFFECTS OF INDIVIDUAL SENSORS

In order to quantify the extent to which individual navigation sensor errors contribute to a navigation system's positioning error, it is necessary to first establish a systematic means for performing an analysis. In this section, the analysis methodology that is utilized in this research is described.

Before proceeding with a description of this method, however, a few very important preliminary remarks must be made. It is important to note that the analyses performed in this research were done on several navigation systems that were designed by the authors. At the heart of these navigation systems is a software algorithm that utilizes all of the available sensor data to produce an estimate of the vehicle's position. The analysis methodology utilized in this research is intimately related to this algorithm. In order to understand the methods used to perform the analyses in this research, then, the reader must first understand the navigation system itself, and, in particular, the algorithm that was used to produce the position estimates. Hence, before the theories underlying the analysis can be presented, it is first necessary to discuss the means by which position estimates are produced.

A. Kalman Filters and Their Analysis

As was mentioned in Section I-A, the errors appearing in GPS position fixes and the outputs of dead-reckoning sensors are complementary—GPS position fixes contain bounded errors, some of which are random and uncorrelated from one fix to the next, and dead-reckoning sensors are accurate only for short time periods but drift over long

time periods. Proper fusion of GPS position fixes with dead-reckoning sensor data can take advantage of these complementary errors, producing positioning performance that is better than could be obtained with either type of data alone. One algorithm for combining data with complementary characteristics is known as a Kalman filter. The advantages of the Kalman filter are such that it is well suited for use in fusing GPS position measurements with data from inertial instruments, and, for this reason, it is frequently utilized in the navigation community. Furthermore, the Kalman filter provides a mathematical framework that is conducive to analysis. For these reasons, a Kalman filter was used in this research.

A Kalman filter is a specific formulation of what is more generally known as an estimator. This name is applied to a Kalman filter because the filter attempts to estimate the values of certain physical quantities, given data that bear some known relationship to those quantities. In the field of navigation, for example, Kalman filters frequently are used to estimate the value of the measurement errors that appear in the outputs of navigation sensors. Successful estimation of these sensor errors generally improves navigation system performance because the estimate of each error can be subtracted from the corresponding sensor's output.

Many texts are devoted to a discussion of Kalman filtering ([39], for example), and the reader is referred to one of them for details. For the purposes of this discussion, it is sufficient to know simply that a Kalman filter is a statistically optimal means for estimating quantities whose time histories can be approximately modeled and for which a related measurement is available. In the state-space formulation of the Kalman filter, the quantities being estimated are generally encapsulated in a column matrix known as the filter's state vector. The time history of the state vector can presumably be modeled by linear stochastic differential equations referred to as the filter's model equations. The forcing function for these equations is assumed to be zero-mean white noise with a known spectral density matrix. In addition to the model equations, a measurement of a linear combination of the states is assumed to be available; the relationship between the measurement vector and the state vector is referred to as the measurement equation. The measurements may be corrupted with additive zero-mean white noise with a known spectral density matrix.

In order for a Kalman filter to produce a statistically optimal estimate of its state, the filter's model equations, measurement equations and spectral density matrices must describe exactly the actual dynamical and statistical properties of the system of interest. In other words, the time-history of the system's state must be described exactly by known linear stochastic differential equations driven by white noise with known statistical properties. However, it is frequently the case that the dynamical equations that exactly describe the behavior of a system are not linear or are not known precisely. Moreover, it may be the case that the number of states required to accurately model the system would be so large that the computational requirements of the filter mechanization would exceed available

computational capacity. Under these circumstances, the filter designer must resort to a reduced-state Kalman filter, capturing the essential behavior of the system with fewer states than are required to model it exactly. Whenever the equations in a Kalman filter mechanization do not model the behavior of the physical system at hand exactly for any reason—whether due to imprecise knowledge of the physical system or due to deliberate reduction of the state—the Kalman filter will produce a suboptimal state estimate. The filter is thus referred to as a “suboptimal” filter.

The error in the state estimate produced by a suboptimal filter will be greater than that produced by an optimal filter. The extent to which the performance of a suboptimal filter deviates from optimality may be quantified using a tool known as sensitivity analysis. Sensitivity analysis can be extremely enlightening because it can reveal a great deal about various error mechanisms in a Kalman filter. Moreover, the equations of sensitivity analysis can be formulated in such a way that the contributions that individual error sources make to the total error in a Kalman filter's estimate can be computed. Sensitivity analysis is therefore a useful tool for this research. In Section V-B, the equations of sensitivity analysis will be described briefly.

B. The Mathematics Underlying Sensitivity Analysis

The following discussion will expose the reader to the salient features of the mathematics behind sensitivity analysis. For a more detailed derivation and description of the equations, the reader is referred to [13].

For completeness, it should be noted that the subject of sensitivity analysis has been studied in [40]–[43]. The formulation in [42] is more general than those in [40] and [43] and is more relevant to the work in this paper than the formulation in [41]. The equations that follow are therefore closely related to those appearing in [42]. Although the derivation of the following equations was originally given in [42], [13] presents extensions to the work in [42] that are essential for the complete understanding of sensitivity analysis as it has been applied to the problem at hand.

The derivation begins by assuming that the physical system at hand can be modeled precisely by a set of linear stochastic difference equations. All of the parameters governing these equations are assumed to be known, and the equations should model the true behavior of the system as accurately as is reasonably possible. This set of equations will henceforth be referred to as the “reference” system because a sensitivity analysis quantifies a Kalman filter's performance against (or in reference to) this system. Finally, it should be noted that these equations need not be identical to the model equations of the Kalman filter in that they may include states that the Kalman filter cannot estimate reliably or constants whose true values are not known exactly.

This being said, the derivation may proceed. The reference system's state evolves in time according to the following model:

$$\mathbf{x}_{r,k+1} = \Phi_{r,k} \mathbf{x}_{r,k} + \mathbf{w}_{r,k}. \quad (1)$$

In this equation, the reference system state vector $\mathbf{x}_{r,k}$ has dimensions $n \times 1$, the reference system state transition matrix $\Phi_{r,k}$ has dimensions $n \times n$, and $\mathbf{w}_{r,k}$ is the k th element of an uncorrelated random sequence with covariance matrix $Q_{r,k}$. The quantity $\mathbf{w}_{r,k}$ generally is referred to as process noise. The variable k is a discrete-time index representing time $t = kT$, where T is the discretization period of the system. The subscript “ r ” distinguishes these quantities as being associated with the reference system (as opposed to the Kalman filter).

The initial condition for (1) is denoted $\mathbf{x}_r(0)$ and is a random variable whose distribution is denoted by

$$P_r(0) \equiv E[\mathbf{x}_r(0)\mathbf{x}_r^T(0)] \quad (2)$$

where $E[x]$ is the expectation of x .

The measurement vector $\mathbf{z}_{r,k}$ has dimensions $p \times 1$ and is assumed to be a linear combination of the system states

$$\mathbf{z}_{r,k} = H_{r,k}\mathbf{x}_{r,k} + \mathbf{v}_{r,k} \quad (3)$$

where $H_{r,k}$ is the reference system observation matrix and $\mathbf{v}_{r,k}$ is the k th element of an uncorrelated random sequence with covariance $R_{r,k}$.

The output of the system is defined by the $s \times 1$ column matrix $\mathbf{y}_{r,k}$, which is a linear combination of the system states

$$\mathbf{y}_{r,k} \equiv C_{r,k}\mathbf{x}_{r,k} \quad (4)$$

where $C_{r,k}$ is a matrix of dimension $s \times n$.

The Kalman filter state vector $\mathbf{x}_{f,k}$, which is an $m \times 1$ column matrix, evolves in time according to the equation

$$\mathbf{x}_{f,k+1} = \Phi_{f,k}\mathbf{x}_{f,k} + \mathbf{w}_{f,k} + B_k\mathbf{u}_k \quad (5)$$

where $\Phi_{f,k}$ has dimensions $m \times m$, $\mathbf{w}_{f,k}$ is process noise (an uncorrelated sequence of covariance $Q_{f,k}$), B_k has dimensions $m \times j$, and $\mathbf{u}_k$ is a deterministic input to the filter of dimension $j \times 1$. Note that the subscript “ f ” in this (and subsequent) equations denotes a quantity that is associated with the filter (as opposed to the reference system).

The actual measurements read from the sensors and used by the filter are a linear combination of the reference states, the model for which is given in (3). The filter’s model for the measurement vector, however, may be different from the actual measurement vector. Accordingly, the filter “believes” that the measurement taken is actually modeled by the equation

$$\mathbf{z}_k = H_{f,k}\mathbf{x}_{f,k} + \mathbf{v}_{f,k} \quad (6)$$

where $\mathbf{v}_{f,k}$ is the k th element of a $p \times 1$ uncorrelated sequence with $p \times p$ covariance matrix $R_{f,k}$.

At each timestep, a measurement is taken and the Kalman filter generates an estimate of the state vector. The state estimate is recursively computed using the well-known Kalman filter algorithm [39]. The output of the Kalman filter is defined by the $s \times 1$ column matrix $\mathbf{y}_{f,k}$, which is a linear combination of the estimate of the state

$$\mathbf{y}_{f,k} \equiv C_{f,k}\hat{\mathbf{x}}_{f,k}^+ \quad (7)$$

where $C_{f,k}$ is a matrix of dimension $s \times m$, and $\hat{\mathbf{x}}_{f,k}^+$ is the state estimate after the k th measurement is processed. (The estimate of the state before the k th measurement is processed is denoted $\hat{\mathbf{x}}_{f,k}^-$.)

Finally, we define the error between the output of the reference system and the output of the Kalman filter as

$$\mathbf{e}_k \equiv \mathbf{y}_{r,k} - \mathbf{y}_{f,k} \quad (8)$$

and define the covariance of $\mathbf{e}_k$ as

$$P_{e,k} \equiv E[\mathbf{e}_k\mathbf{e}_k^T]. \quad (9)$$

The goal of this derivation is to arrive at a recursive expression for the covariance matrix $P_{e,k}$. Before proceeding, however, an important constraint on the exogenous input to the Kalman filter $\mathbf{u}_k$ must be established. Namely, it is required that $\mathbf{u}_k$ be the sum of a linear combination of the reference states and an additive noise term that is assumed to be normally distributed and uncorrelated with any states or noise sources in the system. Mathematically, $\mathbf{u}_k$ can be expressed as

$$\mathbf{u}_k = \Gamma_k\mathbf{x}_{r,k} + \psi_k. \quad (10)$$

This assumption is not too restrictive but may require $\mathbf{x}_{r,k}$ to be augmented with states that define the time history of $\mathbf{u}_k$. Note that Γ_k has dimensions $j \times n$ and ψ_k has dimensions $j \times 1$.

With this restriction on $\mathbf{u}_k$ in hand, we may proceed with the derivation. We begin by constructing a single combined state vector $\mathbf{x}_{c,k}$ by stacking $\mathbf{x}_{r,k}$ and $\hat{\mathbf{x}}_{f,k}$

$$\mathbf{x}_{c,k} \equiv \begin{bmatrix} \mathbf{x}_{r,k} \\ \hat{\mathbf{x}}_{f,k} \end{bmatrix} \quad (11)$$

so that

$$\mathbf{x}_{c,k}^- \equiv \begin{bmatrix} \mathbf{x}_{r,k} \\ \hat{\mathbf{x}}_{f,k}^- \end{bmatrix} \quad (12)$$

and

$$\mathbf{x}_{c,k}^+ \equiv \begin{bmatrix} \mathbf{x}_{r,k} \\ \hat{\mathbf{x}}_{f,k}^+ \end{bmatrix}. \quad (13)$$

Utilizing (1), (10), and (12), one can arrive at an expression that describes the propagation of $\mathbf{x}_{c,k}^+$ in time from timestep k to timestep $k+1$

$$\mathbf{x}_{c,k+1}^- = \begin{bmatrix} \Phi_{r,k} & 0 \\ B_k\Gamma_k & \Phi_{f,k} \end{bmatrix} \mathbf{x}_{c,k}^+ + \begin{bmatrix} \mathbf{w}_{r,k} \\ B_k\psi_k \end{bmatrix}. \quad (14)$$

It is convenient to define $\Phi_{c,k}$ and $\mathbf{w}_{c,k}$ so that (14) can be rewritten as follows:

$$\mathbf{x}_{c,k+1}^- = \Phi_{c,k}\mathbf{x}_{c,k}^+ + \mathbf{w}_{c,k} \quad (15)$$

where

$$\Phi_{c,k} \equiv \begin{bmatrix} \Phi_{r,k} & 0 \\ B_k\Gamma_k & \Phi_{f,k} \end{bmatrix} \quad (16)$$

and

$$\mathbf{w}_{c,k} \equiv \begin{bmatrix} \mathbf{w}_{r,k} \\ B_k\psi_k \end{bmatrix}. \quad (17)$$

Utilizing (13) and the well-known Kalman filter measurement update equation [39], one can produce an expression that describes the evolution of $\mathbf{x}_{c,k}^-$ when a measurement is processed at timestep k

$$\mathbf{x}_{c,k}^+ = B_{c,k}\mathbf{x}_{c,k}^- + K_{c,k}\mathbf{v}_{r,k} \quad (18)$$

where

$$B_{c,k} \equiv \begin{bmatrix} I & 0 \\ K_k H_{r,k} & I - K_k H_{f,k} \end{bmatrix} \quad (19)$$

and

$$K_{c,k} \equiv \begin{bmatrix} 0 \\ K_k \end{bmatrix} \quad (20)$$

where K_k is known as the Kalman gain and is computed from the Kalman filter algorithm [39].

The error term $\mathbf{e}_k$ is given by

$$\mathbf{e}_k = C_{c,k}\mathbf{x}_{c,k}^+ \quad (21)$$

where

$$C_{c,k} \equiv [C_{r,k} \quad -C_{f,k}]. \quad (22)$$

The covariance of $\mathbf{x}_{c,k}^-$ is derived from (15) and is given by

$$P_{c,k+1}^- = \Phi_{c,k} P_{c,k}^+ \Phi_{c,k}^T + Q_{c,k} \quad (23)$$

where

$$Q_{c,k} \equiv \begin{bmatrix} Q_{r,k} & 0 \\ 0 & B_k \Psi_k B_k^T \end{bmatrix} \quad (24)$$

and Ψ_k is the $j \times j$ covariance matrix of ψ_k .

The covariance of $\mathbf{x}_{c,k}^+$ is derived from (18) and is given by

$$P_{c,k}^+ = B_{c,k} P_{c,k}^- B_{c,k}^T + K_{c,k} R_{r,k} K_{c,k}^T. \quad (25)$$

Finally, the covariance of the error term $\mathbf{e}_k$ is given by

$$P_{e,k} = C_{c,k} P_{c,k}^+ C_{c,k}^T. \quad (26)$$

The diagonal elements of $P_{e,k}$ represent the mean-square value of the difference between $\mathbf{y}_{r,k}$ (the output of the reference system) and $\mathbf{y}_{f,k}$ (the output of the filter). As such, they represent collectively the “true” mean-square error in the filter’s estimates and are therefore the terms of interest.

Equations (23), (25), and (26) together with the initial condition for $P_{c,k}^-$ form the so-called equations of sensitivity analysis. The power of sensitivity analysis lies in the fact that the diagonal elements of P_e are linear expressions in R_r , Q_r , and $P_r(0)$. Hence, the equations of sensitivity analysis are difference equations that are linear in their forcing functions (R_r and Q_r) and their initial condition ($P_r(0)$). This important result implies that the principle of superposition applies. Therefore, the total effect of all of the terms in R_r , Q_r , and $P_r(0)$ on P_e is equal to the sum of the effects caused by each individual term in R_r , Q_r , and $P_r(0)$. Therefore, each term in R_r , Q_r , and $P_r(0)$ may be played through the sensitivity equations one at a time, with all other terms in these matrices set to zero; the

resulting diagonal terms in P_e represent the contribution that the nonzero term alone makes to the mean-square error in the filter’s state estimate. The sensitivity equations must be run many times—one time for each term in R_r , Q_r , and $P_r(0)$ whose nominal value is nonzero. After the individual contributions of each term in R_r , Q_r , and $P_r(0)$ have been computed, the contributions may be summed to obtain the total mean-square error in the Kalman filter’s estimates.

VI. ANALYSIS ASSUMPTIONS AND RESULTS

A. Assumptions

Before proceeding with a presentation of the results of this research, it is important to describe some of the important assumptions that were made to simplify the analyses. In this section, one of the most important simplifications that was made in the analyses in this research is introduced and justified. (A detailed explanation of this and other assumptions underlying this research can be found in [13].)

As one might imagine, a general analysis of an automobile navigation system bears certain complications that must be simplified in order to obtain meaningful results efficiently. One such complication arises in connection with the effect that the vehicle’s trajectory has on the performance of a navigation system. Simply stated, the vehicle’s motion—e.g., how frequently the vehicle turns, how fast the vehicle moves, how frequently the vehicle stops, etc.—will have an impact on the positioning accuracy of its navigation system. This fact is readily seen when one realizes that certain sensor errors will change as the vehicle moves. For example, a rate gyro scale factor error contributes more to heading error when the vehicle turns, and the magnitude of the errors caused by the odometer depends on the vehicle’s speed. Also, if a vehicle comes to a complete stop, it is possible to accurately estimate a rate gyro’s bias because the vehicle is known to be stationary. As these examples illustrate, the contribution that some sensor errors make to the accuracy of the navigation system’s position estimate can vary with the vehicle’s motion; consequently, analysis results are trajectory-dependent and, therefore, specific to the chosen trajectory. This makes it more difficult to perform a general analysis.

These trajectory dependencies impede efficient analysis because they tie analysis results to a specific trajectory—any analysis results apply only for the assumed trajectory. One obvious analysis methodology, then, is to examine the navigation system’s performance over a wide variety of representative trajectories. This type of analysis would require an examination of many trajectories that the vehicle would be likely to take. However, it is immediately clear that this method would be tedious and almost certainly incomplete.

A much more efficient alternative can be found if one recognizes that vehicles spend most of their time traveling on straight roads. Simple experience tells us that this is true and that turns are, by comparison, infrequent. For this reason, most of the analyses in this research assume that the

Table 2
Comparison of Cross-Track Error and GPS Positioning Error

GPS Positioning Type	SA On	SA Off	DGPS
RMS Steady-state Cross-track Position Error (m)	19.3	7.7	1.1
RMS Cross-track Bias Error in GPS Fixes (m)	20.6	8.0	1.07

vehicle is traveling on a straight road at a constant speed, thereby eliminating trajectory-dependent effects associated with turns and vehicle acceleration. While this solution is not a truly general solution, it reduces the analysis to its simplest form and will produce results that are general except during those occasions when the vehicle turns.

Trajectory-dependent effects on navigation system performance are, therefore, not examined in this paper. However, the authors have produced other results in which trajectory-dependent effects are examined [13].

B. Results: Preliminary Remarks

The results which will be shown in the following sections were obtained by applying sensitivity analysis to several navigation systems, each of which utilized a different set of navigation sensors. Most results pertain to a system utilizing GPS position fixes, a rate gyro, and an odometer. This set of sensors will be referred to as the “baseline” system configuration, and the performance of this system will serve generally as the benchmark to which other systems will be compared.

In addition, the results which follow include examinations of the roles of each navigation sensor both while the GPS position fixes are available and while the position fixes are unavailable. Both of these situations are examined because, as was pointed out in Section I-A, GPS position fixes can be unavailable for indefinitely long periods of time. Under these circumstances, land-vehicle navigation systems must proceed utilizing information from dead-reckoning sensors only. Finally, the results which follow were taken from [13], which contains results for other systems and more involved discussions, as well.

C. The Roles of Various Sensors While GPS Fixes Are Available

First, the ability of the Kalman filter to estimate the vehicle’s position while GPS position fixes are available will be examined. Results will be shown for three navigation systems, each one of which has the baseline sensor set (i.e., GPS fixes, a rate gyro, and an odometer) but utilized a different type of GPS position fix (GPS with SA on, GPS with SA off, and DGPS). For these three navigation systems, the cross-track and along-track components of the position error will be examined separately, beginning with the cross-track error.

Table 2 shows the rms bias error in the GPS position fixes (in the cross-track direction) and the rms error in the cross-track position estimate after the filter reached steady-state. The first row in the table shows the rms value of the error

Table 3
Relative Contributions to Mean-Square Error in Cross-Track Position Estimate

GPS Positioning Type	SA On	SA Off	DGPS
GPS Bias Drift	95.3%	97.4%	92.7%
GPS Measurement Noise	0.9%	1.8%	6.4%
Rate Gyro Bias Drift	3.5%	0.5%	0.1%
Rate Gyro Measurement Noise	0.3%	0.3%	0.8%
Odometer Errors	0.0%	0.0%	0.0%

in the cross-track position estimate after the Kalman filter reached steady state. The data in the three columns in this row show results for each of the three navigation systems at hand. The second row in the table shows the rms value of the bias in the GPS position fixes that were available to the navigation system. The data in this (second) row represent the accuracy of the cross-track position measurements that were available to the filter. The data in the second row serves as a benchmark to which the data in the first row can be compared to determine how much the filter was able to improve upon its position measurements.

As the data show, the filter is not able to reduce the positioning error significantly below that of the RMS value of the bias error in the GPS position fixes. In fact, when DGPS position fixes are utilized, the rms error in the cross-track position estimate is greater than the RMS bias in the position fixes. This occurs because the RMS value of the uncorrelated noise in the DGPS position fixes is large (1.4 m) in comparison to the RMS bias error and, therefore, figures prominently into the cross-track position error. The data in Table 2 suggest that, while GPS position fixes are available, the accuracy of the positioning system is dominated by the accuracy of the GPS position fixes.

Sensitivity analysis results support this conclusion and provide further insight into the filter’s error mechanisms. Table 3 shows the percent contribution that various error sources make to the steady-state mean-square error in the estimate of the vehicle’s cross-track position. The error sources listed in the table have been categorized according to their sensor of origin and their time-correlation. For example, “GPS Bias Drift” refers to the time-correlated errors in the GPS position fixes. The percentages listed in this row of the table represent the fractional contribution that random bias drift in the GPS position fixes makes to the total mean-square cross-track position error. “GPS Measurement Noise” refers to errors in the GPS position fixes that are uncorrelated in time. “Rate Gyro Bias Drift” refers to time-correlated bias errors in the rate gyro’s output, and “Rate Gyro Measurement Noise” refers to errors in the rate gyro’s output that are uncorrelated in time. Finally, the influence of uncorrelated and correlated errors in the odometer readings are grouped together in the row labeled “Odometer Errors.” As the data show, the bias in the GPS position fix contributes more than 90% of the total error in the cross-track position estimate, demonstrating that the positioning error is dominated by the accuracy of the GPS position fixes.

Table 4

Comparison of Along-Track Position Error
and GPS Positioning Error

GPS Positioning Type	SA On	SA Off	DGPS
Steady-state RMS Position Error (m)	13.9	6.5	1.1
RMS Bias Error in GPS Position Fix (m)	20.6	8.0	1.07

Table 5

Relative Contribution to Mean-Square Error
in Along-Track Position Estimate

GPS Positioning Type	SA On	SA Off	DGPS
GPS Bias Drift	85.8%	90.3%	91.9%
GPS Measurement Noise	0.4%	0.8%	4.9%
Rate Gyro Bias Drift	0.0%	0.0%	0.0%
Rate Gyro Measurement Noise	0.0%	0.0%	0.0%
Odometer Bias Drift	13.3%	8.1%	0.4%
Odometer Quantization Noise	0.5%	0.8%	2.8%

The data in this table reveal that the error sources consisting of uncorrelated noise—the GPS measurement noise and the rate gyro measurement noise—become increasingly significant as the accuracy of the position fixes improves. The fractional contribution of the uncorrelated error sources increases because, as the GPS position accuracy improves, the filter is better able to mitigate the effects of the correlated error sources than the uncorrelated error sources. In other words, the filter estimates the correlated errors with increasing accuracy as the GPS measurement accuracy improves, but it is less able to reduce the effects of the uncorrelated noise sources. This occurs because the information in an uncorrelated signal contains no information that can be utilized to predict its future value—the value at one timestep is unrelated to the value at every other timestep. In contrast, the value of a correlated signal at any point in time contains information about the signal at a later point in time. Therefore the filter is better able to take advantage of the improved accuracy in the GPS position fixes when estimating correlated errors.

Results for the along-track error are slightly better. Table 4 shows results similar to those of Table 2, and Table 5 shows results similar to those of Table 3. Table 5, however, includes separate rows for the contributions of the correlated and uncorrelated errors in the measurement from the odometer. As Table 4 shows, the steady-state along-track error is smaller than the steady-state cross-track error when SA is on or off. When DGPS position fixes are utilized, the steady-state along-track error is the same as the steady-state cross-track error. Table 5 shows the percent contribution that each error source makes to the mean-square along-track position error. As the data in this table show, the position error is dominated by the bias in the GPS position fixes.

The data presented in Table 5 exhibit trends that are similar to those in Table 3—uncorrelated errors (from the odometer and position measurements) contribute an increasing fraction of the total error as the GPS position

measurements become more accurate. Not all of the correlated error sources, however, exhibit the same trends as those in Table 3. Specifically, the fractional contribution of the GPS bias drift increases as the position measurements become more accurate. This trend appears because the absolute contribution of the odometer bias error drops rapidly. The filter effectively takes advantage of the improved positioning accuracy to estimate the odometer bias, and, as a result, the error in the odometer bias decreases rapidly. Because the odometer bias is estimated more accurately, the odometer bias drift contributes less to the error in the along-track position estimate. The absolute contribution of the GPS bias drift to the along-track position error also decreases as the position measurements improve, but it does not decrease as rapidly as the contribution of the odometer's bias drift. As a result, the fractional contribution of the GPS bias drift increases.

The data in Tables 3 and 5 also show that the rate gyro errors contribute nothing to along-track position error and that the odometer errors contribute nothing to cross-track position error. (Other results have shown that a compass, like a rate gyro, contributes only to cross-track position error.) Hence, heading sensors are tied to cross-track position and the odometer is tied to along-track position. Perhaps this simple observation is not surprising when one recognizes that a heading measurement is related kinematically to changes in cross-track position and the odometer measurement is related kinematically to changes in along-track position. As a consequence of these kinematic relationships, each dead-reckoning sensor error is “orthogonal” to a component of position error. This result reveals a fundamental error mechanism of the filter and leads to important conclusions. For example, changes in sensor quality will have little or no impact on “orthogonal” components of the positioning error—e.g., using a high-quality gyro will not improve the along-track positioning error. Furthermore, improvements in one component of position accuracy will improve calibration of only the associated dead-reckoning sensors.

D. System Performance Without GPS Position Fixes

The results shown thus far demonstrate that the dead-reckoning sensors do not reduce significantly the position error while GPS position fixes are available. However, if GPS position fixes become unavailable, the subsequent performance of the navigation system is determined by two factors: the sensors' drift characteristics and the accuracy with which dead-reckoning sensor errors were calibrated before the GPS fixes became unavailable. In this section, the performance of various navigation systems without GPS will be examined, and the major contributors to positioning error will be identified.

As has been mentioned, a GPS receiver may not be able to provide a position fix under certain circumstances. In urban or heavily foliated environments, for example, buildings or foliage may prevent a sufficient number of GPS satellite signals from reaching the GPS receiver. As a result,

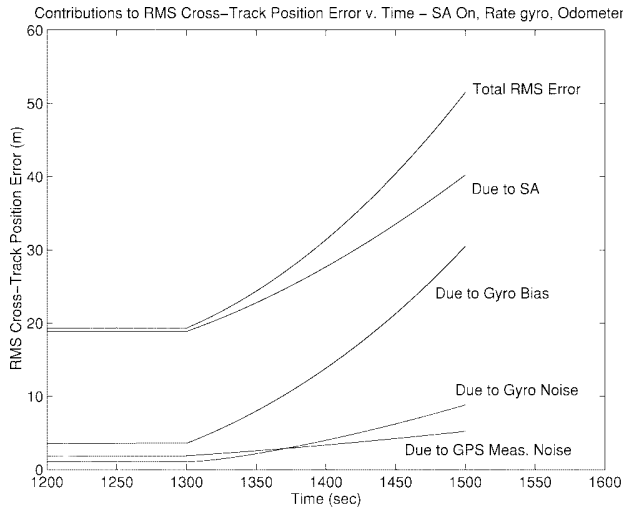


Fig. 6. RMS error in cross-track position after a GPS loss (SA on).

the GPS receiver may be incapable of providing a position fix for an indefinitely long period of time, and the navigation system would have to produce a position estimate based solely on its dead-reckoning sensor data. The results presented in Section VI-C provide clues to the performance to be expected from various navigation systems if GPS position fixes become unavailable. Naturally, one would expect that those systems for which the rate gyro bias, the heading, and the odometer scale factor bias are calibrated more accurately would perform better after GPS position fixes become unavailable. However, this is not conclusive because the performance of each system without GPS also depends on the drift characteristics inherent in each dead-reckoning sensor. Therefore, in this section the influence of sensor drift on system performance will be examined by applying sensitivity analysis to systems for which GPS position fixes are unavailable. The navigation systems to be examined are the same as those of Section VI-C.

Fig. 6 and 7 show the rms cross-track and along-track positioning error versus time for a system in which the GPS position fixes were corrupted with SA. The uppermost curve in each plot represents the total rms position error; the other curves in each figure represent the rms contributions that each error source makes to the position error. After the filter reached (nearly) steady-state (after 1300 s), the GPS position measurement was denied to the Kalman filter, causing the filter to continue with measurements from only the rate gyro and odometer. Note that the time axis (i.e., the ordinate) begins at 1200 s because we are interested only in the results after that time.

As both figures show, the GPS positioning errors (bias and measurement noise together) make the largest absolute contribution to the position error after the GPS measurements have become unavailable. At first glance, this may not appear to make sense. After all, how can GPS measurement errors contribute to positioning error if the GPS position fixes are not available? The answer to this question lies in the realization that the GPS position error

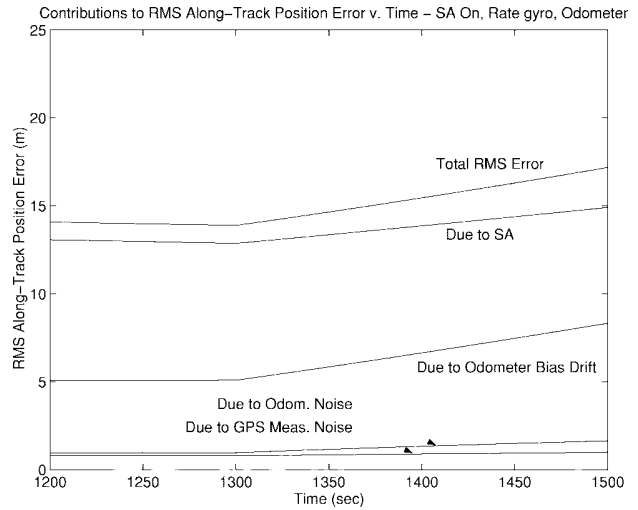


Fig. 7. RMS error in along-track position after a GPS loss (SA on).

contributes to errors in the calibration of the heading, rate gyro bias, and the odometer scale factor bias during the first 1300 s. (This fact was demonstrated in Section VI-C.) Part of the position error that accrues after GPS becomes unavailable is due to the calibration error induced by GPS errors before GPS becomes unavailable. Therefore, the GPS measurement errors cause the position error to increase with time even after the GPS fixes become unavailable because they induce errors in the estimates of the vehicle's heading, the gyro's bias, and the odometer scale factor bias. Hence, the plot demonstrates that position error grows rapidly as a result of the calibration error induced by SA while the GPS position fixes are available.

Comparison of the figures shows that the cross-track position error grows much more rapidly than the along-track position error. The reason for this is revealed by examining the contributions that the individual sensor errors make to the position error. As Fig. 6 shows, the growth rate of the cross-track position error is dominated by the contribution of the rate gyro bias' random drift. Fig. 7 shows that the growth rate of the along-track position error is dominated by the contribution of SA and the odometer scale factor bias drift. Part of the reason that the cross-track position error grows more rapidly than the along-track position error is that the rate gyro bias is two integrations removed from position; this implies that a constant rate gyro bias will cause position error to grow roughly as the square of time. The odometer scale factor bias, on the other hand, is only one integration removed from position, implying that a constant odometer scale factor bias error will cause position error to grow linearly with time. Hence, the magnitude of the sensor errors and their kinematic relationship to position determine the growth rate of the position error.

Because the rate gyro's bias drift dominates the growth rate of the cross-track position error, using a gyro with a lower bias drift rate should improve system performance. However, even when a better gyro is used, the total cross-

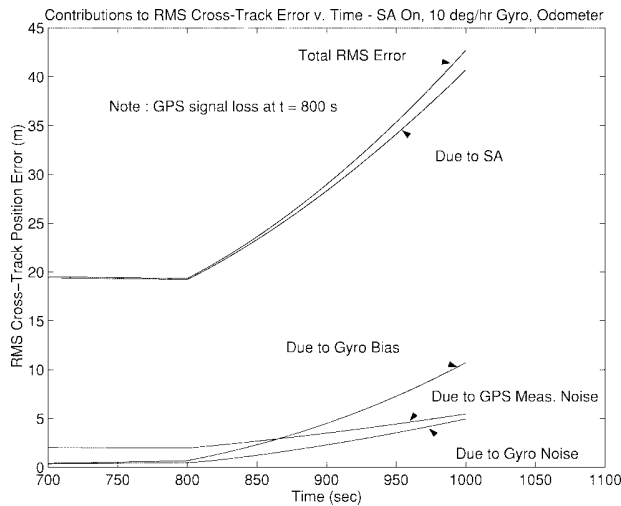


Fig. 8. RMS cross-track position error using a 10°/h gyro.

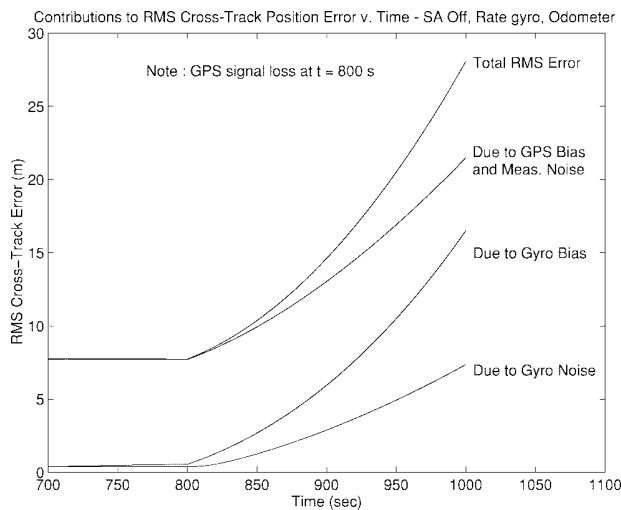


Fig. 9. RMS error in cross-track position after a GPS loss (SA off).

track position error is not reduced significantly. Fig. 8 shows the total rms cross-track position error versus time for a system in which the rate gyro (with a bias drift of approximately 30°/h) was replaced with a better one (having a bias drift of approximately 10°/h). The uppermost curve in the figure is the total RMS cross-track position error, and the remaining curves represent the contributions that individual sensor errors make to the total cross-track position error. As the data show, the position error induced by SA is by far the largest contributor. When compared with Fig. 6, one can see that the contribution of the 10°/h rate gyro is significantly less than that of the 30°/h gyro. However, the contribution of SA remains nearly unchanged, and, ultimately, using the better gyro is of little consequence.

For a system utilizing GPS position fixes with SA off, the relative contributions of the various sensor errors are similar to those when SA is on. Figs. 9 and 10 show data similar to Figs. 6 and 7. As the data show, the growth rate

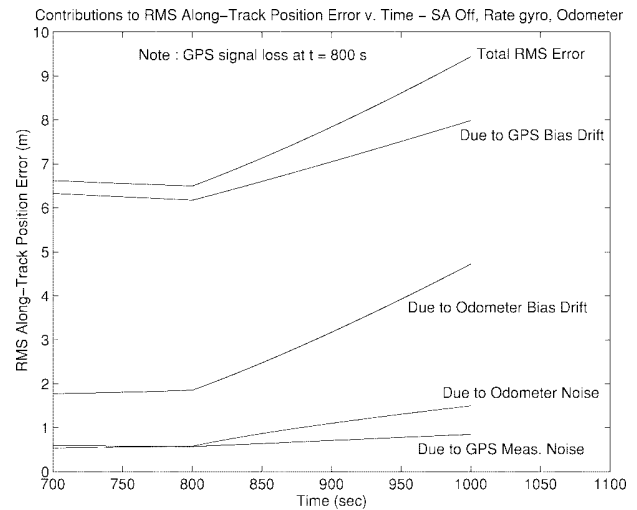


Fig. 10. RMS error in along-track position after a GPS loss (SA off).

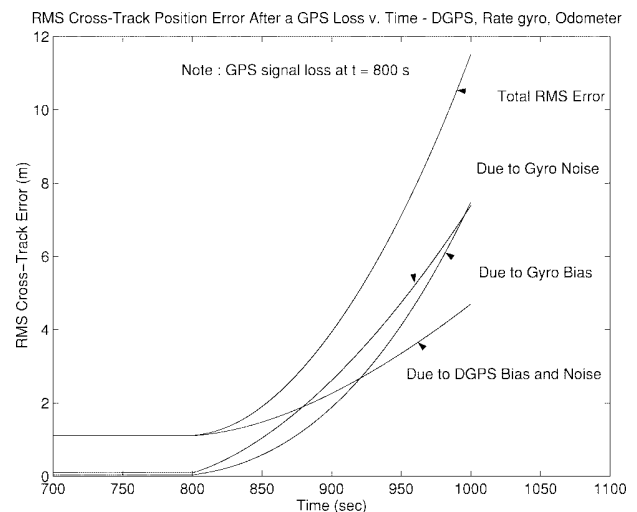


Fig. 11. RMS error in cross-track position after a GPS loss (DGPS).

of the error in the cross-track direction is dominated by the rate gyro's bias drift and is higher than that for the along-track direction.

For a system utilizing DGPS position fixes, results are roughly similar—the rate gyro's bias and noise dominate the cross-track position error growth rate, and the odometer's bias drift dominates the along-track position error growth rate. Figs. 11 and 12 show data similar to Figs. 6 and 7. As the data show, the total contribution of the gyro bias drift to the cross-track position error is substantially less than when stand-alone GPS position fixes (with SA on) are utilized. This occurs because the rate gyro's bias is calibrated more accurately when DGPS is utilized. Also, the relative contribution of the noise in the gyro's output is larger in this case than when SA is on. Interestingly, the contribution of the gyro bias drift and the noise in the gyro output are commensurate. Nevertheless, the contribution of the drift eventually exceeds that of the noise, and the growth

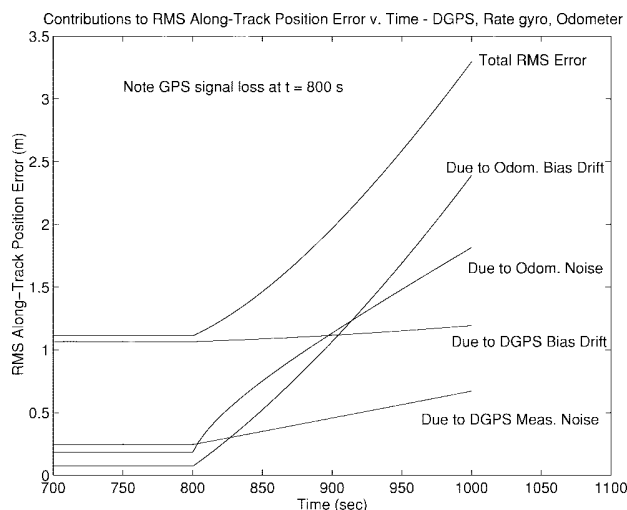


Fig. 12. RMS error in along-track position after a GPS loss (DGPS).

rate of the total position error is still dominated by the gyro bias drift.

VII. IMPLICATIONS FOR VEHICLE NAVIGATION SYSTEM DESIGN

The most glaring implications for sensor selection and system design arise in connection with the type of GPS positioning utilized in the navigation system. The choice of GPS positioning type (if a choice is available) has strong implications for navigation system performance while GPS position fixes are available and during those times when GPS position fixes are unavailable. The positioning accuracy of a system utilizing GPS position fixes (but not utilizing map matching) is virtually the same as that of the GPS position fixes themselves. Hence, statistically, the greatest positioning accuracy that could be expected from a system is roughly that of the GPS position fixes. If, at some point in time, the GPS position fixes become unavailable, the position error will always increase statistically beyond that of the error in the position fixes. The rate at which the position error increases will depend not only on the type of GPS positioning utilized but also on which sensors comprise the dead-reckoning unit and on the drift characteristics of the individual dead-reckoning sensors. Dead-reckoning sensor calibration is substantially better if DGPS position fixes are utilized than if stand-alone GPS position fixes (with SA on) are utilized. In fact, SA has such a strong influence on parameter calibration that improving gyro bias drift by a factor of 3 (from $30^\circ/\text{h}$ to $10^\circ/\text{h}$) was shown to be of little benefit. Hence, if DGPS positioning is utilized, lower-quality dead-reckoning sensors can be employed to achieve a given level of system performance when the position fixes become unavailable for a given duration. Alternately, for a given dead-reckoning unit, positioning error will grow at a much slower rate when GPS position fixes become unavailable if DGPS position fixes are available to calibrate the dead-reckoning sensors in advance.

Once GPS position fixes become unavailable, which sensors comprise the dead-reckoning unit has a strong effect on navigation system performance and on which error parameters dominate the positioning error growth rate. Assuming a dead-reckoning unit comprised of a rate gyro and an odometer, for example, then the rate gyro's bias drift will dominate the position error growth rate (if the rate gyro has the characteristics assumed in this research). The contribution that the gyro's bias drift makes to the cross-track position error is always greater than the contribution made by the random noise in the gyro's output, unless DGPS position fixes are utilized to calibrate the gyro's bias. If a heading measurement from a compass is added to this system, then system performance depends strongly on the compass' magnetic environment [13].

This research also confirmed the intuitively appealing notion that individual dead-reckoning sensors affect each component of position error differently. As one might guess, the heading rate and heading measurement errors contribute only to the cross-track position error, and the odometer errors contribute only to the along-track position error. This simple observation indicates that changes in sensor quality will have little or no impact on "orthogonal" components of the positioning error, e.g., using a high-quality gyro will not improve the along-track positioning error. Furthermore, improvements in one component of position accuracy will improve calibration of only the associated dead-reckoning sensors.

In the future, if SA is turned off and DGPS becomes widely available, then the navigation system design may be altered to take advantage of the improved positioning accuracy. For example, it is likely that if DGPS position fixes are utilized in a land-vehicle navigation system the system designer could significantly relax the constraints on dead-reckoning sensor performance that are now in place for systems that utilize GPS fixes with SA. This can be done because the DGPS fixes cause the calibration of the dead-reckoning sensors and the estimates of the vehicle's kinematic quantities to be estimated much more accurately than when stand-alone GPS (with SA) is used. If SA is turned off, the improvements in system performance will not be as dramatic as when DGPS position fixes are utilized and probably will not warrant significant changes in existing navigation system designs.

VIII. SUMMARY

The key conclusions of this research are the following.

- Positioning accuracy is dominated by GPS position fixes while the fixes are available. Utilizing dead-reckoning sensors that are much better than existing low-cost dead-reckoning sensors does not decrease positioning error significantly below the bias error in the GPS position fixes.
- When stand-alone GPS fixes (with SA on) are utilized, the calibration error induced by SA when the position fixes are available has very significant deleterious effects on system performance after the position fixes

become unavailable. For a system comprised of a GPS receiver, a rate gyro, and an odometer, improving gyro quality significantly does not cause a commensurate improvement in system performance because heading error is strongly affected by SA. For this system, then, calibration error plays a more significant role than sensor quality in determining system performance after position fixes become unavailable.

- When DGPS position fixes are utilized, parameter calibration improves significantly. The improvement in sensor calibration means that lower-quality (and therefore less expensive) dead-reckoning sensors can be used to achieve a given level of system performance when the position fixes become unavailable for a given duration. Furthermore, the use of DGPS improves the estimates of correlated dead-reckoning sensor errors more than it does the estimates of uncorrelated dead-reckoning sensor errors. As a result, uncorrelated errors are much more significant contributors to positioning error when DGPS fixes are utilized than when stand-alone GPS position fixes are utilized.

ACKNOWLEDGMENT

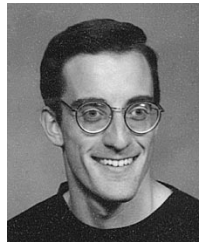
The authors are grateful to several corporations for their generous support of this research, including Etak Inc. and Navigation Technologies, who generously supplied their electronic map databases of the Bay Area. In addition, the authors wish to thank several colleagues—Y. C. Chao, Dr. T. Walter, Dr. R. Gazit, A. Hansen, and P.-Y. Ko.

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