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Review

Fifty Years of the Dynamical Casimir Effect

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Abstract: This is a digest of the main achievements in the wide area, called the Dynamical Casimir Effect nowadays, for the past 50 years, with the emphasis on results obtained after 2010.

Keywords: moving boundaries; photon generation from vacuum; quantum friction; effective Hamiltonians; parametric oscillator; atomic excitations; nonlinear optical materials; cavity and circuit QED; decoherence; entanglement

1. Introduction

Fifty years ago, in June 1969, G.T. Moore finished his PhD thesis (prepared under the guidance of H.N. Pendleton III and submitted to the Brandeis University), part of which was published in the next year as [1]. Using a simplified one-dimensional model, Moore showed that motions of ideal boundaries of a one-dimensional cavity could result in a generation of quanta of the electromagnetic field from the initial *vacuum quantum state*. A few years later, DeWitt [2] demonstrated that moving boundaries could induce particle creation from a vacuum in a single-mirror set-up. A more detailed study was performed by Fulling and Davies [3,4]. Thus, step by step, year by year, more and more authors followed this direction of research. In 1989, two names were suggested for such kinds of phenomena: *dynamic (or non-adiabatic) Casimir effect* [5] and *Nonstationary Casimir Effect* [6]. Being supported in part by the authority of Schwinger [7], the first name gradually acquired the overwhelming popularity, so that now we have an established direction in the theoretical and experimental physics, known under the general name *Dynamical Casimir Effect* (DCE). This area became rather large by now: more than 300 papers containing the words "dynamical Casimir" have been published already, including more than 100 publications during the past decade. Moore's paper [1] has been cited more than 400 times, and some authors use the name "Moore effect" instead of DCE (other names were "Mirror Induced Radiation" or "Motion Induced Radiation").

To combine different studies under the same "roof", it seems reasonable to assume the following definition of the Dynamical Casimir Effect: *Macroscopic phenomena caused by changes of vacuum quantum states of fields due to fast time variations of positions (or properties) of boundaries confining the fields (or other parameters)*. Such phenomena include, in particular, the modification of the Casimir force for moving boundaries. However, the most important manifestation is *the creation of the field quanta (photons) due to the motion of neutral boundaries*. The most important ingredients of the DCE are *quantum vacuum fluctuations* and *macroscopic manifestations*. The reference to vacuum fluctuations explains the appearance of Casimir's name (by analogy with the famous *static* Casimir effect, which is also considered frequently as a manifestation of quantum vacuum fluctuations), although Casimir himself did not write anything on this subject. Therefore, the DCE can be considered as the specific subfield of a much bigger physical area, known nowadays under the name *Casimir Physics*. This whole area is outside the present study, so that we give only a few references to the relevant reviews and books [8–24].

In turn, the subject of the DCE can be divided in several sub-areas. In the strict (narrow) sense, one can think about the "single mirror DCE" or "cavity DCE". In the most wide sense, the DCE can be related to the *amplification of quantum (vacuum) fluctuations in macroscopic systems*, and many authors

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studied analogs of the DCE in parametrically excited media. The "intermediate" area of quantum circuits with time-dependent parameters is also frequently considered as belonging to the DCE. The aim of our review is to describe the main achievements in all those sub-areas for the past 50 years, as well as still unsolved and challenging problems. This is not the first review on the subject. The early history, including studies on the *classical fields* with moving boundaries, can be found in [25–28]. The most recent reviews were published almost a decade ago [29–31]. Therefore, here I tried to collect a more or less complete list of references to the publications of the decade after 2009. As for earlier papers, I have included only a small part of them, to maintain a reasonable length of this review.

2. One-Dimensional Models with Moving Boundaries

One-dimensional "toy" models were considered by many authors because they are much simpler than the realistic 3D ones; therefore, they admit exact and relatively simple analytical solutions, which can provide some insight into the real effects.

2.1. Classical Fields

The history of studies on classical fields in time-dependent domains began almost 100 years ago. The first exact solution of the wave equation (with $c = 1$)

$$\partial^2 A/\partial t^2 - \partial^2 A/\partial x^2 = 0, \tag{1}$$

in a time-dependent domain $0 < x < L(t)$, satisfying the boundary conditions

$$A(0,t) = A(L(t),t) = 0, \tag{2}$$

was obtained by Nicolai [32] for

$$L(t) = L_0(1 + \alpha t). \tag{3}$$

The solution was interpreted in terms of the transverse vibrations of a string with a variable length. A few years later, these results were published in [33], where the extension to the case of electromagnetic field was also made. A similar treatment was given by Havelock [34] in connection with the problem of radiation pressure. However, this subject hardly attracted the attention of researchers for several decades. The splash of new studies happened in the 1960s, probably due to the invention of lasers. We cite here only a few of relevant papers [35–38]. Much more references can be found in [26,39,40].

2.2. Quantum Fields

In the quantum case, one of the most interesting problems is the creation of quanta of different fields from the initial vacuum (ground) state due to the time variation of some parameters. The generation of massive particles by time-dependent gravitational fields was considered for the first time, probably, by Schrödinger [41]. Later on, this problem was treated by Imamura [42], Parker [43,44], and Zel'dovich [45]. However, the main subject of the present paper is the creation of *massless* quanta (photons). In this case, the simplest model is the so-called "scalar electrodynamics", where the main object is the single component of the vector potential operator, perpendicular to the axis x (and parallel to the infinite plane surfaces confining the field). This operator satisfies, in the Heisenberg picture, the same Equation (1) as the classical field vector. If the field is confined between two ideal surfaces (ideal conducting walls) with coordinates $x = 0$ and $x = L(t) > 0$, then the condition of vanishing the electric field at each surface, *in the frame moving with the wall*, results in the boundary conditions (2). It is assumed usually that both walls were at rest for $t \leq 0$, so that initially the field can be written in the form

$$A(x, t < 0) = \sum_{n=0}^{\infty} c_n \sin(n\pi x/L_0) \exp(-i\omega_n t), \quad \omega_n = n\pi/L_0, \tag{4}$$

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where coefficients c_n are complex numbers in the classical case and operators in the quantum case. Then, one has to find the function (operator) $A(x,t)$ for $t > 0$. This problem can be solved in the frameworks of two approaches.

2.2.1. Moore's Approach

The specific feature of the wave equation in the single spatial dimension (1) is the existence of the well known form of the general solution: $A(x,t) = f(x-t) + g(x+t)$, where f and g can be arbitrary functions. How can these functions be chosen to satisfy the additional boundary conditions (2)? Moore [1] has found a complete set of solutions to the problem, satisfying the initial condition (4), in the form

$$A_n(x,t) = C_n \left\{ \exp\left[-i\pi n R(t-x)\right] - \exp\left[-i\pi n R(t+x)\right] \right\}, \tag{5}$$

where function $R(\xi)$ must satisfy the functional equation

$$R(t+L(t)) - R(t-L(t)) = 2. \tag{6}$$

As a matter of fact, a quite similar approach was used in [32–34], although for a *linear* function $L(t)$ only. Independently, Equation (6) was obtained by Vesnitskii [38].

It is easy to find an exact solution to Equation (6) for the uniform law of motion of the boundary (3). This was done by many authors, starting from [32–34]. We give it in the form obtained in [37]:

$$R_\alpha(\xi) = \frac{2\ln|1+\alpha\xi|}{\ln|(1+v)/(1-v)|}, \quad v = \alpha L_0. \tag{7}$$

Evidently, function (7) goes to $R_0(\xi) = \xi/L_0$ if $\alpha \to 0$. For an arbitrary nonrelativistic law of motion, one can find the solution in the form of the expansion over subsequent time derivatives of the wall displacement. Such an idea goes to the papers [1,38] whose results (with some corrections) can be written in the form [26]

$$R(\xi) = \xi\lambda(\xi) - \frac{1}{2}\xi^2\dot{\lambda}(\xi) + \frac{1}{6}\xi\ddot{\lambda}(\xi)\left[\xi^2 - L^2(\xi)\right] + \cdots, \quad \lambda(\xi) \equiv L^{-1}(\xi). \tag{8}$$

In the special case of $L(t) = L_0/(1+\alpha t)$, when $\ddot{\lambda}(\xi) \equiv 0$, Equation (8) yields another exact solution, $R(\xi) = L_0^{-1}\left(\xi + \frac{1}{2}\alpha\xi^2\right)$. Unfortunately, the expansions such as Label (8) cannot be used in the long-time limit $\xi \to \infty$, since the terms proportional to the derivatives of $\lambda(\xi)$ (which are supposed to be small corrections) become bigger than the unperturbed term $\xi\lambda(\xi)$.

Several exact solutions to the Moore equation were found with the aid of the "inverse" method [40,46,47], when one chooses some reasonable function $R(\xi)$ and determines the corresponding law of motion of the boundary $L(t)$ using the consequence of Equation (6),

$$\dot{L}(t) = \frac{R'[t-L(t)] - R'[t+L(t)]}{R'[t-L(t)] + R'[t+L(t)]}. \tag{9}$$

To solve differential Equation (9) with some simple functions, $R(\xi)$ is more easier than solving the functional Equation (6) for the given function $L(t)$. However, the dependence $L(t)$ does not appear to be admissible from the point of view of physics (the velocity may occur greater than the speed of light, or some discontinuities may arise) for any simple function $R(\xi)$. A large list of simple functions $R(\xi)$ (rational, exponential, logarithmic, hyperbolic, trigonometrical, and inverse trigonometrical) and, corresponding to them, functions $L(t)$ can be found in [40]. The cases considered in [47] correspond to some monotonous displacements of the mirror from the initial to final positions. Typical functions $R(\xi)$ used in [47] were some combinations of ξ/L_0 and some trigonometric functions, such as $\sin(m\pi\xi/L_0)$. However, none of these functions can be used in the parametric resonance case. Exact solutions for some specific trajectories of the *single* mirror were found in [48–59]. Two mirrors moving with constant accelerations in opposite directions were considered in [60,61]. Massless spin-1/2 fields in a 1D box with moving boundaries were

studied in [62], while the single mirror case was considered in [63]. The solutions of the 1D Klein–Gordon equation with one uniformly moving boundary (and another boundary at rest) were found in [64,65].

The *asymptotic* solution of the Moore equation for $L(t) = L_0\left[1 + \epsilon \sin(\pi q t/L_0)\right]$, with $q = 1, 2, \ldots$ and $|\epsilon| \ll 1$, was found in [66,67]. For $\epsilon t \gg 1$, it has the form (here $L_0 = 1$)

$$R(t) = t - \frac{2}{\pi q}\,\mathrm{Im}\left\{\ln\left[1 + \zeta + \exp(i\pi q t)(1-\zeta)\right]\right\}, \qquad \zeta = \exp\left[(-1)^{q+1}\pi q \epsilon t\right], \tag{10}$$

which clearly demonstrates that the asymptotic mode structure in the resonance case is quite different from the mode structure inside the cavity with unmoving walls. The solution (10) was improved in [68]. The additional term

$$\Delta R(t) = \frac{2\zeta\epsilon(-1)^q z \sin(q\pi z)}{1 + \zeta^2 + (1-\zeta^2)\cos(q\pi z)}, \qquad -1 \le z = t - 2p \le 1, \quad p = 0, 1, 2, \ldots \tag{11}$$

is negligible in the long-time limit, but it is crucial for the solution to satisfy the correct boundary condition at short times. The case of two moving boundaries, left $L(t)$ and right $R(t)$, was studied in [69]. Then, Equations (5) and (6) can be generalized as follows:

$$A_n(x,t) = C_n\left\{\exp\left[-i\pi n F(t-x)\right] - \exp\left[-i\pi n G(t+x)\right]\right\}, \tag{12}$$

$$G(t + R(t)) - F(t - R(t)) = 2, \qquad G(t + L(t)) - F(t - L(t)) = 0. \tag{13}$$

The solutions generalizing (10) and (11) were found in [69] for the boundaries oscillating with equal resonance frequencies, but different amplitudes and with some phase difference. Numerical solutions of Equations (13) were presented in [70]. The authors of [71,72] pointed out on a possible physical realization of one-dimensional models in the case of TEM modes in cylindrical waveguides. The methods of characteristics and circle maps were applied in [73–75]. The one-dimensional cavity with one and two oscillating mirrors was considered within the framework of the "optical" approach in [76]. Generalizations of Moore's approach to the one-dimensional vibrating cavities were considered in [77]. A one-dimensional *uniformly contracting* cavity was studied in [78]. The computer program for the numerical solution of the Moore equation was presented in [79]. The *Floquet map* was applied in [80].

2.2.2. The Modification of the Casimir Force

How can the function $R(\xi)$ be used? The first application is connected with the modification of the static Casimir force due to the motion of boundaries. It was shown in [3,81] that the force pressing the moving wall (more precisely, the T_{11} component of the energy-momentum tensor of the field) is given by a simple formula (for the initial vacuum state of the field)

$$F = -g[t - L(t)] - g[t + L(t)], \tag{14}$$

where

$$g(\xi) = \frac{1}{24\pi}\left\{\frac{R'''(\xi)}{R'(\xi)} - \frac{3}{2}\left[\frac{R''(\xi)}{R'(\xi)}\right]^2\right\} + \frac{\pi}{48}\left[R'(\xi)\right]^2. \tag{15}$$

For the wall at rest, when $R_0(\xi) = \xi/L_0$, one gets the following expression for the stationary Casimir force in one dimension [3,81] (returning to the dimensional variables):

$$F = -\pi\hbar c/(24L_0^2). \tag{16}$$

If the distance between the boundaries $L(t)$ slowly varies with time (in the sense that $|dL/dt| \ll c$), then the approximate solution (8) yields [6] the following generalization of formula (16):

$$F = -\frac{\pi\hbar c}{24L^2(t)}\left[1 + \left(\frac{\dot{L}}{c}\right)^2\left(\frac{7}{3} - \frac{1}{\pi^2}\right) - \frac{L\ddot{L}}{c^2}\left(\frac{2}{3} - \frac{2}{\pi^2}\right)\right]. \tag{17}$$

The Casimir force in a cavity moving with a constant acceleration was calculated in [82].

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In the parametric resonance case, the force averaged over the period of oscillations was calculated in [67] (in the dimensionless units with $L_0 = 1$):

$$\langle F \rangle = -\frac{\pi}{24}\left[q^2 + \frac{1}{2}\left(1-q^2\right)\left(\zeta + \zeta^{-1}\right)\right], \qquad \zeta = \exp\left[(-1)^{q+1}\pi q \epsilon t\right]. \tag{18}$$

For $q \geq 2$, we do not have attraction, but an exponentially increasing pressure on the oscillating wall due to the creation of real photons in the cavity. An effective action approach to the problem of dynamical Casimir force in 1+1 dimensions was developed in [83].

2.2.3. Generation of Quanta inside the 1D Cavity with Moving Boundary

The knowledge of function $R(\xi)$ also permits one to calculate the mean number of created quanta inside a nonstationary cavity. The general scheme is as follows [1,81]. Taking into account only the electromagnetic modes whose vector potential is directed along z-axis ("scalar electrodynamics"), one can write down the field operator *in the Heisenberg representation* $\hat{A}(x,t)$ at $t \leq 0$ (when both the plates were at rest at the positions $x_{left} = 0$ and $x_{right} = L_0$) as (we assume $c = \hbar = 1$)

$$\hat{A}_{in} = 2\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\sin\frac{n\pi x}{L_0}\hat{b}_n \exp\left(-i\omega_n t\right) + \text{h.c.} \tag{19}$$

where $\hat{b}_n$ means the usual annihilation photon operator and $\omega_n = \pi n/L_0$. The choice of coefficients in Equation (19) corresponds to the standard form of the field Hamiltonian (in the Gaussian units)

$$\hat{H} \equiv \frac{1}{8\pi}\int_0^{L_0} \mathrm{d}x\,\left[(\partial A/\partial t)^2 + (\partial A/\partial x)^2\right] = \sum_{n=1}^{\infty}\omega_n\left(\hat{b}_n^\dagger \hat{b}_n + 1/2\right). \tag{20}$$

For $t > 0$, the field operator can be written as

$$\hat{A}(x,t) = 2\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\left[\hat{b}_n \psi^{(n)}(x,t) + \text{h.c.}\right], \tag{21}$$

where functions $\psi^{(n)}(x,t)$ are given by formula (5) with $C_n = -i/2$ (in order to satisfy the initial conditions). If the wall stops after some time T, then the field operator can be expanded again over the complete set of sine functions, like in Equation (19), but with the "physical" operators $\hat{a}_m$ and $\hat{a}_m^\dagger$ instead of $\hat{b}_m$ and $\hat{b}_m^\dagger$. The two sets of operators are related by means of the Bogoliubov transformation

$$\hat{a}_m = \sum_{n=1}^{\infty}\left(\hat{b}_n \alpha_{nm} + \hat{b}_n^\dagger \beta_{nm}^*\right), \quad m = 1, 2, \ldots, \tag{22}$$

which implies the unitarity conditions

$$\sum_{m=1}^{\infty}\left(\alpha_{nm}^*\alpha_{km} - \beta_{nm}^*\beta_{km}\right) = \delta_{nk}, \quad \sum_{n=1}^{\infty}\left(\alpha_{nm}^*\alpha_{nj} - \beta_{nm}^*\beta_{nj}\right) = \delta_{mj}, \quad \sum_{n=1}^{\infty}\left(\beta_{nm}^*\alpha_{nk} - \beta_{nk}^*\alpha_{nm}\right) = 0. \tag{23}$$

If the final position of moving boundary coincides with initial position L_0, then the Bogoliubov coefficients can be found in the form [84] (here $c = 1$)

$$\left.\begin{matrix}\alpha_{mn}\\ \beta_{mn}\end{matrix}\right\} = \frac{1}{2}\sqrt{m/n}\int_{t/L_0-1}^{t/L_0+1} dx \exp\left\{-i\pi\left[nR\left(L_0 x\right) \mp mx\right]\right\}. \tag{24}$$

The mean number of photons in the mth mode equals the average value of the operator $\hat{a}_m^\dagger \hat{a}_m$ in the initial state $|\text{in}\rangle$, since namely this operator has a physical meaning at $t > T$:

$$\mathcal{N}_m \equiv \langle \text{in}|\hat{a}_m^\dagger \hat{a}_m|\text{in}\rangle = \sum_n |\beta_{nm}|^2 + \sum_{n,k}\left[\left(\alpha_{nm}^*\alpha_{km} + \beta_{nm}^*\beta_{km}\right)\langle \hat{b}_n^\dagger \hat{b}_k\rangle + 2\text{Re}\left(\beta_{nm}\alpha_{km}\langle \hat{b}_n \hat{b}_k\rangle\right)\right]. \tag{25}$$

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The first sum on the right-hand side of (25) describes the effect of the photon creation from vacuum, while the other sums are different from zero only in the case of a non-vacuum initial state of the field.

In the resonance case, solution (10) leads after long calculations to the following simple formula for the rate of photon generation in the mth mode, when the wall vibrates at the twice frequency of the first resonator eigenmode, in the limit $\epsilon t \gg 1$ [67]:

$$d\mathcal{N}_m/dt \approx |\epsilon| \left[1-(-1)^m\right]/(m\pi). \tag{26}$$

The energy density inside the cavity can be calculated as [3,81] $\langle T_{00}(x,t)\rangle = -g(t+x) - g(t-x)$, with the same function $g(\xi)$ (15). It was shown in [68,85–87] that, for $q \geq 2$, the energy density grows exponentially in the form of q traveling wave packets, which become narrower and higher as time increases. The total energy also grows exponentially. For two resonantly oscillating boundaries (with equal frequencies), the asymptotic photon production rate is given by the same formula (26), where ϵ should be replaced with $\tilde{\epsilon} = \epsilon_L + (-1)^{q+1}\epsilon_R \cos(\phi)$, where $\epsilon_{L,R}$ are relative oscillation amplitudes of each boundary and ϕ the phase difference [69].

The influence of initial states of the field and different boundary conditions on the energy density and radiation force was studied analytically and numerically in [88–91]. The case of two relativistic moving mirrors was studied in [92].

2.2.4. Expansions over the Instantaneous Basis

Another approach to the problem consists of expanding the function $\psi^{(n)}(x,t)$ in Equation (21) in a series with respect to the *instantaneous basis* [93–97]:

$$\psi^{(n)}(x,t>0) = \sum_{k=1}^{\infty} Q_k^{(n)}(t)\sqrt{\frac{L_0}{L(t)}}\sin\left(\frac{\pi k[x-u(t)]}{L(t)}\right), \quad n=1,2,\ldots \tag{27}$$

with the initial conditions $Q_k^{(n)}(0) = \delta_{kn}$, $\dot{Q}_k^{(n)}(0) = -i\omega_n\delta_{kn}$, $k,n = 1,2,\ldots$. This way, we satisfy automatically both the boundary conditions, $A(u(t),t) = A(u(t)+L(t),t) = 0$, and the initial condition (4). Putting expression (27) into the wave Equation (1), one can arrive after some algebra at an infinite set of coupled differential equations [97–99]

$$\ddot{Q}_k^{(n)} + \omega_k^2(t)Q_k^{(n)} = 2\sum_{j=1}^{\infty} g_{kj}(t)\dot{Q}_j^{(n)} + \sum_{j=1}^{\infty}\dot{g}_{kj}(t)Q_j^{(n)} + \mathcal{O}\left(g_{kj}^2\right), \tag{28}$$

$$\omega_k(t) = k\pi/L(t), \quad g_{kj}(t) = -g_{jk}(t) = (-1)^{k-j}\frac{2kj\left(\dot{L}+\dot{u}\epsilon_{kj}\right)}{(j^2-k^2)\,L(t)}, \quad \epsilon_{kj} = 1-(-1)^{k-j}, \quad j \neq k. \tag{29}$$

For $u = 0$ (the left wall at rest), the equations like (28)–(29) were derived in [95,100].

If the wall comes back to its initial position L_0 after some interval of time T, then the right-hand side of Equation (28) disappears, so, at $t > T$, one can write

$$Q_k^{(n)}(t) = \xi_k^{(n)}e^{-i\omega_k(t+\delta T)} + \eta_k^{(n)}e^{i\omega_k(t+\delta T)}, \quad k,n = 1,2,\ldots \tag{30}$$

where $\xi_k^{(n)}$ and $\eta_k^{(n)}$ are some constant complex coefficients. To find these coefficients, one has to solve an infinite set of coupled Equation (28) ($k = 1,2,\ldots$) with time-dependent parameters; moreover, each equation also contains an infinite number of terms. However, the problem can be essentially simplified, if the walls perform small oscillations at the frequency ω_w close to some unperturbed field eigenfrequency: $L(t) = L_0\left(1+\varepsilon_L\sin\left[p\omega_1(1+\delta)t\right]\right)$, $u(t) = \varepsilon_u L_0 \sin\left[p\omega_1(1+\delta)t+\varphi\right]$. Assuming $|\varepsilon_L|, |\varepsilon_u| \sim \varepsilon \ll 1$, it is natural to look for the solutions to Equation (28) in the form

$$Q_k^{(n)}(t) = \xi_k^{(n)}e^{-i\omega_k(1+\delta)t} + \eta_k^{(n)}e^{i\omega_k(1+\delta)t}, \tag{31}$$

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allowing now the coefficients $\xi_k^{(n)}$ and $\eta_k^{(n)}$ to be *slowly varying functions of time*. Then, using the standard method of slowly varying amplitudes [101], one can arrive [97] at the following set of equations, containing only three terms with simple *time independent* coefficients in the right-hand sides:

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\xi_k^{(n)} = (-1)^p\left[(k+p)\xi_{k+p}^{(n)} - (k-p)\xi_{k-p}^{(n)}\right] + 2i\gamma k\xi_k^{(n)}, \tag{32}$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\eta_k^{(n)} = (-1)^p\left[(k+p)\eta_{k+p}^{(n)} - (k-p)\eta_{k-p}^{(n)}\right] - 2i\gamma k\eta_k^{(n)}, \tag{33}$$

$$\tau = \varepsilon\omega_1 t/2, \quad \gamma = \delta/\varepsilon, \qquad \xi_k^{(n)}(0) = \delta_{kn}, \quad \eta_k^{(n)}(0) = 0. \tag{34}$$

Equations (32) and (33) correspond to the case of $u = 0$ (the left wall at rest). Actually, the term $\dot{u}$ in $g_{kj}(t)$ does not make any contribution to the simplified equations of motion for *even* values of integer parameter p, so that the rate of change of the cavity length $\dot{L}/L_0$ is only important in this case. On the contrary, if p is an *odd* number, then the field evolution depends on the velocity of the *center of the cavity* $v_c = \dot{u} + \dot{L}/2$, but does not depend on $\dot{L}$ alone [102]. Therefore, one should simply replace $\dot{L}/L_0$ by $2v_c/L_0$, if p is an odd number.

The uncoupled Equations (32) and (33) hold only for $k \geq p$. This means that they describe the evolution of *all* the Bogoliubov coefficients, only if $p = 1$. Then *all* the functions $\eta_k^{(n)}(t)$ are *identically equal to zero* due to the initial conditions (34); consequently, no photon can be created *from vacuum* (in the lowest order approximation with respect to the small parameter ε). This special "semi-resonance" case was studied in detail in [97,103], where exact analytical solutions to Equation (32) were obtained in terms of the Jacobi polynomials. It was demonstrated that the total number of photons is an integral of motion in this specific case. (A similar phenomenon in the classical case was discussed in [46], whereas the quantum case was also considered in [96]). The exact formula for the total energy in all cavity modes for $p = 1$ (normalized by the fundamental cavity mode energy $\hbar\omega_1$) is as follows:

$$\mathcal{E}(\tau) = \mathcal{E}(0) + \frac{2\sinh^2(a\tau)}{a^2}\left[\mathcal{E}(0) - \frac{\gamma}{2}\mathrm{Im}(\mathcal{G}_1)\right] + \mathrm{Re}(\mathcal{G}_1)\frac{\sinh(2a\tau)}{2a}, \tag{35}$$

$$a = \sqrt{1-\gamma^2}, \qquad \mathcal{G}_1 = 2\sum_{n=1}^{\infty}\sqrt{n(n+1)}\langle \mathrm{in}|\hat{b}_n^\dagger \hat{b}_{n+1}|\mathrm{in}\rangle. \tag{36}$$

It can be shown that $|\dot{\mathcal{E}}(0)| \leq \mathcal{E}(0)$. Thus, the total energy grows exponentially, when $\tau \gg 1$ (provided $\gamma < 1$), although it can decrease at $\tau \ll 1$, if $\dot{\mathcal{E}}(0) = \mathrm{Re}(\mathcal{G}_1) < 0$. Since the total number of photons is constant, such a behavior is explained by the effect of pumping the highest modes at the expense of the lowest ones (in the classical case this effect was noticed in 1960s: see [39]). This results in an effective cooling of the lowest electromagnetic modes. The absence of photon creation in the fundamental mode was related to the $SL(2,R)$ symmetry of the problem in Ref. [104].

If $p \geq 2$, we have, in addition to (32) and (33), $p-1$ pairs of *coupled* equations for the coefficients with lower indexes $1 \leq k \leq p-1$,

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\xi_k^{(n)} = (-1)^p\left[(k+p)\xi_{k+p}^{(n)} - (p-k)\eta_{p-k}^{(n)}\right] + 2i\gamma k\xi_k^{(n)}, \tag{37}$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\eta_k^{(n)} = (-1)^p\left[(k+p)\eta_{k+p}^{(n)} - (p-k)\xi_{p-k}^{(n)}\right] - 2i\gamma k\eta_k^{(n)}. \tag{38}$$

In this case, some functions $\eta_k^{(n)}(t)$ are not equal to zero at $t > 0$, thus we have the effect of photon creation from the vacuum. It is convenient to introduce a new set of coefficients $\rho_k^{(n)}$, whose lower indexes run over all integers from $-\infty$ to ∞:

$$\rho_k^{(n)} = \begin{cases} \xi_k^{(n)} & k > 0 \\ 0 & k = 0 \\ -\eta_{-k}^{(n)} & k < 0 \end{cases} \tag{39}$$

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Then, Equations (32), (33), (37) and (38) can be combined into a *single* set of equations ($k = \pm 1, \pm 2, \ldots$) [97]

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\rho_k^{(n)} = (-1)^p \left[(k+p)\rho_{k+p}^{(n)} - (k-p)\rho_{k-p}^{(n)}\right] + 2i\gamma k \rho_k^{(n)}, \qquad \rho_k^{(n)}(0) = \delta_{kn}. \tag{40}$$

A remarkable feature of the set of Equation (40) is that its solutions satisfy *exactly* the unitarity conditions (23), which can be rewritten as ($m, j, n, k = 1, 2, \ldots$)

$$\sum_{m=-\infty}^{\infty} m\rho_m^{(n)*}\rho_m^{(k)} = n\delta_{nk} \quad \sum_{n=1}^{\infty} \frac{m}{n}\left[\rho_m^{(n)*}\rho_j^{(n)} - \rho_{-m}^{(n)*}\rho_{-j}^{(n)}\right] = \delta_{mj} \quad \sum_{n=1}^{\infty} \frac{1}{n}\left[\rho_m^{(n)*}\rho_{-j}^{(n)} - \rho_j^{(n)*}\rho_{-m}^{(n)}\right] = 0.$$

Exact solutions to the set (40) were found in [97] in terms of the Gauss hypergeometric function. They can be expressed also in terms of the complete elliptic integrals, if $p = 2$ [105]. The most interesting consequences of these solutions are as follows.

There is no photon creation in the modes with numbers $p, 2p, \ldots$. In the short-time limit ($\tau \ll 1$), $\dot{\mathcal{N}}_{j+pq}^{(vac)} \sim \tau^{2q+1}$, but the photon generation rate in each mode tends to the constant value in the long-time limit:

$$\frac{d}{d\tau}\mathcal{N}_{j+pq}^{(vac)} \approx \frac{2ap^2 \sin^2(\pi j/p)}{\pi^2(j+pq)}, \quad ap\tau \gg 1. \tag{41}$$

The total number of photons created from vacuum in all modes increases in time approximately quadratically, both in the short-time and in the long-time limits (although with different coefficients):

$$\dot{\mathcal{N}}^{(vac)} = \frac{1}{3}p(p^2-1), \quad |ap\tau| \ll 1, \qquad \dot{\mathcal{N}}^{(vac)} = 2a^2p^3/\pi^2, \quad ap\tau \gg 1, \quad a > 0. \tag{42}$$

At the same time, the total number of "nonvacuum" photons increases in time only *linearly* at $ap\tau \gg 1$.

The total energy in all modes for the initial vacuum state of field is given by the exact formula [97]

$$\mathcal{E}^{(vac)}(\tau) = \frac{p^2-1}{12a^2}\sinh^2(pa\tau)\,. \tag{43}$$

The total energy increases exponentially at $\tau \to \infty$, provided $\gamma < 1$. In the special case of $\gamma = 0$, such a behavior of the total energy was obtained also in the frameworks of other approaches in [100,106–108]. Note that the total "vacuum" and "nonvacuum" energies increase exponentially with time, if $\gamma < 1$, whereas the total number of photons increases only as τ^2 and τ, respectively, under the same conditions. This happens because the number of effectively excited modes increases in time exponentially [109]. The parametric instability in systems with moving boundaries was studied in [110–113].

The solutions found in [97] were used in [114,115] to study the effects of *squeezing* and the formation of narrow packets inside the 1D cavity with oscillating walls. The emission of narrow packets outside the 1D Fabry–Perot cavity was studied in [116–118]. The influence of the dispersion of the reflection coefficient of the fixed mirror (with the ideal moving one) and the frequency detuning on the formation of packets inside the 1D cavity was investigated for different initial conditions in [119–121]. The comparisons of numerical and analytical calculations in the one-dimensional case were made in [122–124]. It was shown that the coincidence of results is excellent for small enough values of parameter $\varepsilon < 10^{-5}$ (remember that the realistic values are expected to be less than 10^{-7}). However, strong differences were observed for big values $\varepsilon > 10^{-2}$. For other approaches to solving equations like (28), see, e.g., [99,125,126]. Recently, these equations were solved numerically in the 1D and 3D cases, for one and two moving boundaries, in the study [127].

The case of mixed boundary conditions—Dirichlet condition $\phi(t, L(t)) = 0$ for the moving wall and Neumann condition $\partial_x\phi(t, 0) = 0$ for the wall at rest—was solved, following the method of [105], in papers [128,129]. In this case, photons can be produced in all modes, differently from the Dirichlet–Dirichlet boundary conditions used in [105].

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The analytical results shown above were obtained in the first order approximation with respect to the small parameter ε. Taking into account the higher order approximations, the resonance frequencies can be made smaller than the fundamental frequency ω_1. For example, it was shown that, in the n-th order approximation, the resonances at the frequency $2\tilde{\omega}_1/n$ are possible [130] (the frequency $\tilde{\omega}_1$ is slightly shifted from ω_1 due to nonlinear effects). In particular, the photon creation from vacuum at the frequency close to ω_1 becomes possible in the second order approximation.

2.2.5. Quantum Regime of the Wall Motion

In all papers cited above, the law of motion of the wall (boundary) was prescribed. The inclusion of the moving boundary as a part of the dynamical system was made by Barton and Calogeracos [131,132]. However, they considered the boundary as a classical particle. The back reaction of the field on the motion of a classical wall was taken into account in [133,134]. The resonant energy exchange between a moving boundary and cavity modes was studied in [135]. Another approach was used in Ref. [136], where the mirror was replaced by an ensemble of electrons and ions, bounded by some effective parabolic potential.

The quantized motion of the boundary was considered in [137–141]. The authors of [139–141] considered a scalar field in a one-dimensional cavity with one fixed and one mobile wall; the latter was bound to an equilibrium position by a harmonic potential and its mechanical degrees of freedom were treated quantum mechanically. The presence of the moving wall yielded an effective interaction between the field modes, described by means of the interaction Hamiltonian

$$\hat{H}_{int} = \left(\hat{b} + \hat{b}^{\dagger}\right) \sum_{jk} C_{kj} \mathcal{N} \left[\left(\hat{a}_k + \hat{a}_k^{\dagger}\right) \left(\hat{a}_j + \hat{a}_j^{\dagger}\right) \right], \tag{44}$$

where $\hat{b}$ and $\hat{a}_j$ are bosonic operators, describing the mobile mirror and field modes, respectively, and $\mathcal{N}$ is the normal ordering operator. Note that a simplified version of (44), $\hat{H}_{int} = \hbar G \left(\hat{b} + \hat{b}^{\dagger}\right) \hat{a}^{\dagger}\hat{a}$, was used to describe the ponderomotive effects of a strong laser field in cavities with oscillating walls, but under the condition that the mechanical oscillator frequency is many orders of magnitude smaller than the cavity frequency, so that the DCE is negligible [142–145]. The recent progress in studies of such fully quantized optomechanical systems in connection with the DCE can be seen in [146–152].

2.3. Partially Transparent Mirrors and General Boundary Conditions in 1D

The DCE with partially reflecting mirrors in the one-dimensional models was considered in [53,102,117,131,132,153–159]. The comparison of the Dirichlet and Neumann boundary conditions in terms of the solutions to Moore's Equation (6) was performed in paper [72]. Another approach to this problem was used in [160]. The so-called Robin boundary conditions of the form

$$\left(\phi - \gamma(t)\partial\phi/\partial x = 0\right)\big|_{\text{at the boundary}} \tag{45}$$

were studied in connection with the DCE in [131,132,161–164]. The simplest model uses the complex reflection and transmission coefficients in the form [131,132]:

$$r = -\frac{i\gamma}{\omega + i\gamma}, \quad t = \frac{\omega}{\omega + i\gamma}. \tag{46}$$

This is equivalent to the model of the boundary as an equivalent delta-potential or a jellium-type plasma sheet of infinitely small thickness. Such kind of models was used e.g., in Refs. [165,166]. Quantum and classical effects produced by thin sheets of electrons, working as relativistic mirrors, were considered in [167–169].

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The further generalization to the $\delta - \delta'$ potential was considered in [170]. In this model, the reflection and transmission coefficients have the form

$$r = \frac{2\omega\lambda - i\gamma}{\omega\left(\lambda^2+1\right)+i\gamma}, \quad t = \frac{\omega\left(\lambda^2+1\right)}{\omega\left(\lambda^2+1\right)+i\gamma}. \tag{47}$$

It was shown that a partially reflecting single moving mirror can produce a larger number of particles in comparison with a perfect one. The interference between the motion of a single mirror and the time-dependent Robin parameter was studied in [171]. The problem of creation of a time-dependent boundary was addressed in [172]. The catastrophic generation of quanta due to instantaneous appearance and disappearance of a wall in a cavity was shown in [173]. The explosive photon production due to instantaneous changes from the Neumann to the Dirichlet BC (or reversely) was considered in [174]. (A similar unphysical behavior was discovered in [175] in the case of instantaneous displacements of boundaries.) Microscopic models of mirrors as oscillators coupled bilinearly to quantum fields were considered in [176–180].

3. Three-Dimensional Models with Moving Boundaries

3.1. Single Mirror DCE

The effect of photon production from vacuum, caused by single mirrors moving in the three-dimensional space with *relativistic* velocities or with great accelerations, was studied in the papers [181–186]. The nonrelativistic motion of the plane mirror was considered in [187–191]. The radiation from dynamically deforming mirrors with different boundary conditions was calculated in [192–194]. Graphene-like mirrors were considered in [195]. Classical analogs of the DCE due to the oscillating motion of the plane surfaces containing dipole layers were considered in [196,197]. For the most recent publications, one can see [198–200].

Various quantum effects arising due to the motion of *dielectric boundaries* in three dimensions, including the modification of the Casimir force and creation of photons, have been studied in [201–208].

Quantum Friction for Moving Surfaces

The first calculations of the forces acting on the single mirrors moving with *nonrelativistic* velocities due to the vacuum or thermal fluctuations of the field were performed in the framework of the spectral approach (using the fluctuation–dissipation theorem) in [209] (three-dimensional case, the force proportional to the fifth-order derivative of the coordinate) and in [210] (one-dimensional model, the force proportional to the third-order derivative of the coordinate). These studies were continued in [25,153,211–217], where it was assumed that the velocity of the boundary is perpendicular to the surface.

The frictional force proportional to the constant relative velocity of two parallel plates, when this velocity is also parallel to the surfaces, was calculated by Teodorovich [218] in the van der Waals regime (i.e., neglecting the retardation effects). More general results were obtained by Levitov [219] and Polevoi [220]. In particular, Levitov had shown that the linear dependence on the velocity disappears at zero absolute temperature, but the friction force is still nonzero: it is proportional to v^3 for a very small separation between the plates, while it becomes velocity independent (like the dry friction) for a big enough separation. Later on, the theory of "Casimir friction" was the subject of studies [221–228], with controversial results. Moreover, the existence of such a friction force was questioned in [229] and especially in [230]. The latter paper triggered hot discussions [231–235]. More recent results can be found in [236–250]. In particular, it was shown in [246] that the friction force between two graphene sheets is nonzero only if the relative velocity is larger than the Fermi velocity of the charge carriers in graphene.

In 1971, Zel'dovich [251,252] predicted that a rotating object can amplify certain incident waves under the condition $\omega - L\Omega < 0$, where ω is the frequency of the field mode, L is its azimuthal quantum number, and Ω is the angular velocity of the rotating body (a cylinder). Furthermore, he conjectured that such an object should spontaneously emit radiation for these selected modes. This idea was further

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developed in [253]. The effect was called "superradiance" in [254,255] and "spontaneous superradiance" in [256]. Recently, a similar phenomenon—the friction force on the rotating bodies due to the interaction with vacuum (or thermal) fluctuations of the EM field—was studied in detail in papers [257–260]. Analogs in superfluids were considered in [261]. The wave instability of the electromagnetic field inside a rotating cylinder, resulting in exponential growth with time, was studied in [262].

3.2. Cavity DCE

3.2.1. Effective Hamiltonians

The instantaneous basis approach, described above, corresponds to the Heisenberg description of the quantum systems. It was believed for some time after Moore [1] that it is the only possible approach to the systems with moving boundaries. However, an equivalent Schrödinger description (although approximate) also exists. It is based on using some *effective Hamiltonians* [94–96,98–100,156,175,263–267]. The general structure of the effective infinite-dimensional quadratic Lagrangians and Hamiltonians arising in the canonical approach to the dynamical Casimir effect was analyzed and classified in [98,263].

Following [268], let us suppose that the set of Maxwell's equation in a medium with *time-independent* parameters and boundaries can be reduced to an equation of the form $\hat{\mathcal{K}}(\{L\})\mathbf{F}_\alpha(\mathbf{r};\{L\}) = \omega_\alpha^2(\{L\})\mathbf{F}_\alpha(\mathbf{r};\{L\})$, where $\{L\}$ means a set of parameters (for example, the distance between the walls or the dielectric permittivity inside the cavity), $\omega_\alpha(\{L\})$ is the eigenfrequency of the field mode labeled by the number (or a set of numbers) α and $\mathbf{F}_\alpha(\mathbf{r};\{L\})$ is some vector function describing the EM field (e.g., the vector potential). In the simplest cases, $\hat{\mathcal{K}}(\{L\})$ is reduced to the Laplace operator. Usually, the operator $\hat{\mathcal{K}}(\{L\})$ is self-adjoint, and the set of functions $\{\mathbf{F}_\alpha(\mathbf{r};\{L\})\}$ is orthonormal and complete in some sense.

Now, suppose that parameters $L_1, L_2, \ldots, L_n$ become time-dependent. If one can still satisfy automatically the boundary conditions, expanding the field $\mathbf{F}(\mathbf{r},t)$ over "instantaneous" eigenfunctions $\mathbf{F}(\mathbf{r},t) = \sum_\alpha q_\alpha(t)\mathbf{F}_\alpha(\mathbf{r};\{L(t)\})$ (this is true, e.g., for the Dirichlet boundary conditions, which are equivalent in some cases to the TE polarization of the field modes), then the dynamics of the field is described completely by the generalized coordinates $q_\alpha(t)$, whose equations of motion can be derived from the *effective time-dependent Hamiltonian* [98]

$$H = \frac{1}{2}\sum_\alpha \left[p_\alpha^2 + \omega_\alpha^2\left(\{L(t)\}\right)q_\alpha^2\right] + \sum_{k=1}^{n}\frac{\dot{L}_k(t)}{L_k(t)}\sum_{\alpha\neq\beta} p_\alpha m_{\alpha\beta}^{(k)} q_\beta, \tag{48}$$

$$m_{\alpha\beta}^{(k)} = -m_{\beta\alpha}^{(k)} = L_k \int dV \frac{\partial \mathbf{F}_\alpha\left(\mathbf{r};\{L\}\right)}{\partial L_k}\mathbf{F}_\beta\left(\mathbf{r};\{L\}\right). \tag{49}$$

Consequently, the field problem can be reduced to studying the dynamics of the infinite set of harmonic oscillators with time-dependent frequencies and bilinear specific (coordinate–momentum) time-dependent coupling. Methods of diagonalization of some specific Hamiltonians like (48) were considered in [269,270].

3.2.2. Parametric Oscillator Model

From the point of view of applications to the DCE, the most important cases are those where the parameters $L_k(t)$ vary in time *periodically*. In the case of small *harmonic* variations at the frequency close to the double unperturbed eigenfrequency of some mode $2\omega_0$, the equations of motion resulting from Hamiltonian (48) can be solved approximately with the aid of the method of slowly varying amplitudes [101]. If the difference $\omega_\alpha - \omega_\beta$ is not close to $2\omega_0$ for all those modes which have nonzero (or not very small) coupling coefficients $m_{\alpha\beta}$, then only the selected mode with label 0 can be excited *in the long-time limit*, and one can consider only this single resonance mode [27,105], whose excitation is described by the Hamiltonian

$$H = \left[p_0^2 + \omega_0^2\left(\{L(t)\}\right)q_0^2\right]/2. \tag{50}$$

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The theory of quantum nonstationary harmonic oscillator has been well developed since its foundation by Husimi in 1953 [271] (see, e.g., [272–274] for the reviews and references). It appears that all dynamical properties of the *quantum* oscillator are determined by the fundamental set of solutions of the *classical* equation of motion

$$\ddot{\varepsilon} + \omega^2(t)\varepsilon = 0. \tag{51}$$

For harmonic variations of the frequency in the form $\omega(t) = \omega_0 \left[1 + 2\kappa \cos(2\omega_0 t)\right]$, with $|\kappa| \ll 1$, Equation (51) can be solved approximately using, e.g., the method of averaging over fast oscillations or the method of slowly varying amplitudes [101]. The final result is a simple formula for the number of quanta created from the initial vacuum state [105,275]:

$$\mathcal{N} = \sinh^2\left(\omega_0 \kappa t\right). \tag{52}$$

Here, one can notice the striking difference between the parabolic dependence on time of the total number of created quanta in the 1D case, according to Equation (42), and the exponential dependence (52) in the 3D case. This is explained by the crucial difference in the eigenmode spectra: it is equidistant in the 1D case, but strongly non-equidistant in the generic 3D case. A smooth transition from one situation to another was demonstrated in [276]. Anharmonic and random periodic displacements of boundaries were studied in [277].

In the literature on the DCE, many authors frequently use an equivalent form of the single-mode effective Hamiltonian (50) in terms of annihilation and creation operators, derived by Law [96],

$$\hat{H} = \omega(t)\hat{a}^\dagger\hat{a} - i\chi_t\left(\hat{a}^2 - \hat{a}^{\dagger 2}\right), \qquad \chi_t = (d\omega/dt)/(4\omega), \quad \hbar = 1. \tag{53}$$

Algebraic methods of solving the Schrödinger equation with Hamiltonian (53) were used in [278]. Recently, the harmonic oscillator model was re-discovered and applied to the circuit QED in [279].

3.2.3. The Role of Intermode Interactions

The single-mode approximation of the generic Hamiltonian (48) does not work, if the frequency difference between specific modes equals (almost) exactly twice the resonance frequency of some mode. For example, such a *resonance coupling* between two modes is possible in cubical cavities [280]. This case was studied in [280,281]. It was shown that the number of photons in both the coupled modes grows exponentially with time in the long time limit $\omega_0 \kappa t \gg 1$, but the rate of photon generation (the argument of the exponential function) turns out to be *two times smaller* than the value of this rate in the absence of the resonance coupling. (Actually, this rate depends on the concrete values of the coupling coefficients $m_{\alpha\beta}$, but in any case it cannot exceed the 'uncoupled' values [281].) This example indicates that the resonance coupling between the modes should be avoided in order to achieve the maximal photon generation rate, at least in the case of TE modes. A detailed numerical study of this case for different sizes of the rectangular cavities was performed in [282]. The authors of paper [166] used numerical methods, taking into account the interaction between 50 lowest coupled modes in the rectangular cavity, bisected by a "plasma sheet" with a periodically varying number of free carriers. Some results of that paper show that the intermode coupling can increase the number of photons in the modes of EM field with the TM polarization.

The parametric excitation of the resonantly coupled modes of the scalar field in the cubic cavity was studied in [283]. The case of the TE polarization of the vector EM field was also considered there. The "swinging" cubic cavity, performing small amplitude periodical rotations along the z-axis, $\theta(t) = \epsilon \sin(\Omega t)$, was considered in [284] (in the model of scalar field). The frequency Ω was chosen in such a way that three modes, $(1,1,1)$, $(1,2,1)$ and $(2,1,1)$, were resonantly coupled.

The role of intermode interactions was also studied in paper [285]. There, the evolution of the classical electromagnetic field inside the cylindrical (rectangular) cavity with ideal boundaries was studied analytically and numerically in the case, when the conductivity σ and dielectric permittivity ϵ

of a thin slab attached to the base of the cylinder can vary with time. It was shown that the single-mode model can be justified, if the perturbation of the field is small due to the smallness of σ and $\delta\epsilon$. However, no amplification can be achieved in this case for microwave fields, if σ and $\delta\epsilon$ are due to the creation of free carriers inside the slab.

3.2.4. Time-Dependent Casimir Force

Remember that the famous Casimir formula [286] for the attraction force per unit area between two ideal infinite plates in three dimensions reads

$$F/S = -\pi^2\hbar c/(240L_0^4). \tag{54}$$

It was generalized, using the Green function method, to the case of uniform motion of one boundary (in the direction perpendiculat to its surface) in [287] for the scalar field and in [288] for the vector electromagnetic field. The results of two models turned out to be qualitatively different. Namely, the scalar model predicted the increase of the absolute value of the Casimir force in the nonrelativistic case:

$$F_{scal} = -\frac{\pi^2}{480L^4(t)}\left[1+\frac{8}{3}\left(\frac{v}{c}\right)^2+\cdots\right], \qquad v/c \ll 1. \tag{55}$$

On the contrary, the account of the TM modes of the electromagnetic field results in the decrease of the absolute value of the Casimir force in the nonrelativistic case:

$$F_{EM} = -\frac{\pi^2}{240L^4(t)}\left[1+\left(\frac{v}{c}\right)^2\left(\frac{2}{3}-\frac{10}{\pi^2}\right)+\cdots\right], \qquad v/c \ll 1. \tag{56}$$

Moreover, as a matter of fact, the force practically does not depend on the velocity (with accuracy about 10%), decreasing monotonously (by the absolute value) to the ultra-relativistic limit

$$F_{EM} = -\frac{3}{8\pi^2L^4(t)}\left[1+\frac{1}{16}\left(1-v^2/c^2\right)^2+\cdots\right], \qquad 1-(v/c)^2 \ll 1. \tag{57}$$

However, the force can be significantly amplified under the resonance conditions, either in the LC-contour [209] or in the Fabry–Perot cavity [210] (see also [84]). The time-dependent force between vibrating plates was considered in [289].

3.2.5. Vector Fields in 3D Cavities

Quantum properties of the electromagnetic field between two infinite plates, moving with constant relative velocity, with account of the field polarization, were considered in [290]. The photon generation in the TE and TM modes of electromagnetic field between two parallel vibrating plates was considered in [265]. It was shown that the photon generation rate in the TM modes is higher by one order of magnitude than that in the TE modes (see also [266]). The case of a three-dimensional rectangular cavity divided into two parts by an ideal mirror, which suddenly disappears, was considered in [291]. TE and TM modes in a resonant cavity bisected by a plasma sheet were considered in [166]. The DCE for the Dirac field inside the 3D cubic box with oscillating walls was studied in [292]. The spherical and cylindrical geometries were considered in [72,293–298]. General boundary conditions in 3D cavities were used in [299,300]. The scattering approach was applied to the DCE problem in [301]. Numerical algorithms were elaborated in [302].

3.2.6. Saturated Regimes

An unlimited growth of the number of generated "Casimir photons" happens in the simplified models only. The maximal number of quanta that could be generated under realistic conditions is limited due to many factors. One such factor is related to unavoidable nonlinearities in real

systems [303–306]. Another mechanism, related to the temporal coherence, was studied in [307]. The saturation due to the finite reflectance spectral band of mirrors was considered in [308].

4. General Parametric DCE

4.1. Circuit DCE

In view of great difficulties in observing weak manifestations of the DCE, several authors came to the idea of simulating (modeling) this effect in more simple arrangements. Probably, the simplest possibility is to use some electrical circuits [309]. The idea to use quantum resonant oscillatory contours or Josephson junctions with time-dependent parameters (capacitance, inductance, magnetic flux, critical current, etc.) was put forward by Man'ko many years ago [310]. More elaborated proposals in the same direction were presented in [311–315]. The idea to use a superconducting coplanar waveguide in combination with a Josephson junction was developed in [316–319], and the experiments were reported in [320–323]. Their success is related to the possibility of achieving the effective velocity of the boundary up to 25% of the light velocity in vacuum (although the analogy with the motion of real boundaries is not perfect). A detailed review of this approach was given in [31]. The experiments [320–323] were performed in the frequency interval of a few GHz. Their arrangements can be considered as realizations of the *one-dimensional* models with time-dependent parameters. The experiment [320] was performed with an open strip-line waveguide, where the photon generation was achieved due to periodical fast changes of the boundary conditions on one side of the line. Although that boundary conditions can be interpreted as equivalent to a high effective velocity of an oscillating boundary, such an interpretation has a limited range of validity. The second experiment [322] used the parametric resonance excitation of quanta due to changes of the effective speed of light. This arrangement can be considered as a one-dimensional system with periodically varying distributed parameters. The results of that experiments stimulated many theoretical papers, suggesting further improvements of the experimental schemes [324–337]. The circuit QED with "artificial atoms" (qubits) was the subject of studies [338–350]. The most recent review on parametric effects in circuit QED can be found in [351].

4.2. Analogs of DCE in Condensed Matter

Analogs of the DCE in Bose–Einstein condensates and ultracold gases with time-dependent parameters were considered in [352–364]. An obvious advantage of replacing EM waves with their sound analogs is a possibility of achieving high ratios of effective velocities to the sound speed, including the supersonic regimes [358,360]. The use of plasmon resonances in metallic nanoparticles surrounded by an amplifying medium, excited by femtosecond lasers, was suggested in [365]. The DCE with polaritons was studied in [366–370], and the DCE with magnons in [371,372]. The DCE for phonons was considered in [373–376].

4.3. DCE and Atomic Excitations

Many papers were devoted to the interaction between the "Casimir photons" and multilevel systems ("atoms" or "qubits") inside cavities or quantum circuits [306,377–408]. In particular, two-level atoms (qubits) were considered in [306,378,380–382,384,386,391,392,394,400,401,403,404,408]. Two two-level atoms were studied in [390,396,405,407], and ensembles of many two-level atoms in [383,385,388,395, 397,398,401,406]. Three-level systems were considered in [379,387,389,408]. The case of N-level atoms in equally spaced, resonant ladder configuration, was investigated in [388]. The circuit DCE with two-level "atoms" was studied in [409]. The case of a two-level atom near a single accelerated mirror was studied in [410]. A possibility of generation of quantum field states with the hyper-Poissonian statistics, as well as some other "exotic" quantum states, due to the DCE in the presence of "atoms", was shown in [411,412]. The "anti-dynamic" Casimir effect (coherent annihilation of system excitations from common initial states due to parametric modulation) was predicted and studied in papers [304,340,413–418]. The effect of generation of pairs of atomic excitations instead of photons ("inverse DCE") was considered in [304].

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A possibility of exciting the Rydberg atoms passing near the boundary with oscillating parameters ("oscillating effective mirror") was studied in [419]. The influence of the oscillating boundary on the spontaneous emission rate of an atom placed nearby this boundary was considered in [420,421]. The influence of the DCE on the fidelity of the quantum state transfer between two qubits in the ultrastrong coupling regime was studied in [405]. The generation of photons by accelerated neutral objects in cavities with static walls was considered, e.g., in [422].

5. Experimental Proposals for the Cavity DCE

5.1. Difficulties with Real Moving Boundaries

According to formula (52), a possibility of experimental verification of the DCE depends on the amplitude of the frequency variation $\Delta\omega = 2\kappa\omega_0$. The main difficulty is due to the very high frequency $2\omega_0$. The most exciting dream is to observe the "Casimir light" in the visible part of the electromagnetic spectrum. However, it seems to be very improbable using real mirrors oscillating at the frequency about 10^{15} Hz, due to the practical impossibility of exciting and maintaining the *high frequency oscillations* of the suspended plate with a large amplitude and for a long time.

It seems that the only possible way to realize the real motion of material boundaries at high frequencies is not to move the whole mirror, but to cause its surface to perform harmonic vibrations with the aid of some mechanism, e.g., using the piezo-effect. The amplitude of such vibrations ΔL is connected with the maximal relative deformation δ in a standing acoustic wave inside the wall as $\delta = \omega_w \Delta L / v_s$, where $v_s \sim 5 \cdot 10^3$ m/s is the sound velocity. Since usual materials cannot bear deformations exceeding the value $\delta_{max} \sim 10^{-2}$, the velocity of the boundary cannot exceed the value $v_{max} \sim \delta_{max} v_s \sim 50$ m/s (independent on the frequency). The maximal possible frequency variation amplitude $\Delta\omega$ can be evaluated as $\Delta\omega \sim v_s \delta_{max}/(2L)$, where L is the cavity size. For the optical frequencies, taking $2L > 1\,\mu$m, we obtain $\Delta\omega < 5 \times 10^7\,\mathrm{s}^{-1}$, whereas, for the microwave frequencies (in the GHz band) with $L \sim 1$ cm, we have $\Delta\omega < 10^3\,\mathrm{s}^{-1}$. Since the time of excitation t must be bigger than $1/\Delta\omega$, the quality factor of the cavity Q must be not less than $Q_{min} \approx \omega_0/\Delta\omega \approx (L/\lambda) 4\pi c/(v_s \delta) \sim 10^8 (L/\lambda)$. Consequently, there are two main challenges: how to excite high frequency surface oscillations and how to maintain the high quality factor in the regime of strong surface vibrations. The excitation of high amplitude surface vibrations at the optical frequencies seems very problematic. Therefore, hardly the "Casimir light" in the visible region can be generated in systems with really moving boundaries. However, this seems to be possible in other schemes, where changes of some parameters can be interpreted as variations of an "effective length" of the cavity.

The GHz frequency band seems more promising. In such a case, the dimensions of cavities must be of the order of few centimeters. Superconducting cavities with the quality factors exceeding 10^{10} in the frequency band from 1 to 50 GHz have been available for a long time. Therefore, the most difficult problem is to excite the surface vibrations. At lower frequencies, it was solved long ago. Only recently, a significant progress was achieved in fabrication of the so called "film bulk acoustic resonators" (FBARs): piezoelectric devices working at the frequencies from 1 to 3 GHz [423]. They consist of an aluminum nitride (AlN) film of thickness corresponding to one half of the acoustic wavelength, sandwiched between two electrodes. It was suggested [424–426] to use such kind of devices to excite the surface vibrations of cavities in order to observe the DCE. However, no experimental results in this direction were reported until now. For the most recent proposals, see [427,428].

5.2. Simulations with Semiconductor Slabs

In view of difficulties of the excitation of oscillating motion of real boundaries, the ideas concerning the *imitation* of this motion attracted more and more attention with the course of time. The first concrete suggestion was made by Yablonovitch [5], who proposed to use a medium with a rapidly decreasing in time refractive index ("plasma window") to simulate the so-called Unruh effect. In addition, he pointed out that fast changes of dielectric properties can be achieved in semiconductors illuminated

by subpicosecond optical pulses and supposed that "the moving plasma front can act as a moving mirror exceeding the speed of light." Similar ideas and different possible schemes based on fast changes of the carrier concentration in semiconductors illuminated by laser pulses were discussed in [429–431]. Yablonovitch [5,429] put emphasis on the excitation of *virtual electron–hole pairs* by optical radiation tuned to the transparent region just below the band gap in a semiconductor photodiode. He showed that big changes of the *real part* of the dielectric permittivity could be achieved in this way.

The key idea of the experiment named "MIR" in the university of Padua [432,433] was to imitate the motion of a boundary, using an effective "plasma mirror" formed by *real electron–hole pairs* in a thin film near the surface of a semiconductor slab, illuminated by a periodical sequence of short laser pulses. If the interval between pulses exceeds the recombination time of carriers in the semiconductor, a highly conducting layer will periodically appear and disappear on the surface of the slab. This can be interpreted as periodical displacements of the boundary. The basic physical idea was nicely explained in [433]: "...this effective motion is much more convenient than a mechanical motion, since in a metal mirror only the conduction electrons reflect the electromagnetic waves, whereas a great amount of power would be wasted in the acceleration of the much heavier nuclei." However, attempts to implement this idea in practice [434] met severe difficulties due to high losses in semiconductor materials. A rough explanation was given in [285,435]. It seems that the necessary condition for the photon generation, in addition to a high value of the slab conductivity, is the big *negative* change of the real part of the dielectric permeability. Probably, this regime was not reached in the experiments. An idea to use the resonance between the field mode and cyclotron transitions inside a semiconductor heterostructure in a strong and rapidly varying magnetic field was suggested in [436].

5.3. Simulations with Linear and Nonlinear Optical Materials

The main mechanism of the DCE is amplification of vacuum fluctuations due to fast variations of instantaneous eigenfrequences of the field modes. These variations can happen either due to change of real dimensions of the cavity confining the field (DCE in narrow sense) or due to changes of the effective (optical) length, when dielectric properties of the medium inside the cavity depend on time (DCE in wide sense). This second possibility was investigated theoretically by many authors [96,275,437–460]. The main problem is how to realize fast variations of the dielectric permeability in real experiments. The idea to use laser beams passing through a material with nonlinear optical properties was considered in [5,429,438–440,448], and concrete experimental schemes were proposed in [461–464]. Although those schemes are rather different (Dezael and Lambrecht [461] and Hizhnyakov [463] considered nonlinear crystals with the second-order nonlinearity, whereas Faccio and Carusotto [462] proposed to exploit the third-order Kerr effect), their common feature is the prediction of generation of infrared [461,462] or even visible [463] photons, whose frequency is of the same order as the frequency of the laser beam. The experiment based on the suggestion [462] was realized recently [465]. A simulation of the DCE in photonic lattices or photonic crystals was suggested in papers [466,467].

The evaluation of a possibility to obtain parametric amplification of the *microwave* vacuum field, using a reentrant cavity enclosing a nonlinear crystal with a strong third-order nonlinearity, whose refractive index is modulated by near infrared high-intensity laser pulses, was performed in [468,469]. Such a configuration seems to be more adequate for the simulation of the DCE because the effective time-varying length of the cavity is created by infrared quanta, whereas an antenna put inside the cavity can select microwave (RF) quanta only, whose frequency is five orders of magnitude smaller. This could help experimentalists to get rid of various spurious effects due to other possible mechanisms of photon generation.

5.4. Interaction with Detectors

An important part of the experiments on the DCE is how to *detect* the "Casimir photons". This problem was considered in [105,411,470–481]. Detectors modeled by harmonic oscillators were studied in [105,471,475–479]. Two-level and multilevel detectors were considered in [471,477,481]. Electron beams as detectors were suggested in [472]. Rydberg atoms as detectors were proposed in [474].

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6. DCE and Other Quantum Phenomena

Various phenomena connected with the DCE were investigated in [482–486]. In particular, the DCE in curved spacetimes or in the presence of gravitational fields was studied in [487–491]. The role of the dynamical Casimir effect in the cosmological problems was studied in [158,492]. The DCE in an infinite uniformly accelerated medium was considered in [493]. The influence of gravitational fields on the DCE was studied in [494]. Many of these studies are connected with the general problem of the quantum radiation produced by time-varying metrics. It takes the origin in the papers published about 50 years ago: [2,4,43–45,495–497]. For the recent studies one can see Refs. [454,498] The differences between the DCE and the *Unruh effect* [499–501] were discussed, e.g., in studies [31,80,500,502–505]. Intersections between the DCE and quantum thermodynamics were discussed in [138,415,417,506]. Interferometers and detectors of weak signals with moving mirrors were considered in [61,507,508]. Possible (although looking fantastic at the moment) applications of the DCE to the space flights were discussed in [509].

6.1. Damping and Decoherence

The necessity of taking into account the effects of damping in the DCE was clear from the very beginning. Various estimations of the influence of these effects on the photon production rate inside the cavity and in quantum circuits were made in [27,102,348,349,510–516].

The closely connected problem of *decoherence* due to the DCE was considered for the first time in [517,518]. For other publications in this direction, see, e,g., [380,519–522].

6.2. Entanglement

The problem of entanglement creation by means of the DCE was put forward in the study [523]. Various aspects of the problem of entanglement in systems with moving mirrors (or their analogs) were considered in Refs. [143,144,333,346,361,363,524–539]. The quantum discord in the DCE was the subject of studies [333,540]. Other aspects of the quantum information theory were investigated in connection with the DCE in [541–543]. Possible applications of the DCE in quantum simulations were discussed in [544].

6.3. Other Dynamical Effects

The *Dynamical Lamb Effect*, introduced in [545], consists of the excitation of atoms inside a cavity with moving walls, without creation of real photons in the cavity. For further studies, one can consult papers [348,349,523,533–535,546–548].

The *Dynamical Casimir–Polder Effects* are related to the influence of the EM field fluctuations on the interaction between atomic-size objects (or between such objects and walls) in non-stationary situations. The term was coined in papers [549,550]. For further developments, one can consult studies [419,551–558].

7. Conclusions

This short review demonstrates an impressive expansion of the front of research related to the DCE (or its analogs) in many different areas that happened during the past decade. Studies in the area of quantum circuits come first (Section 4.1), stimulated by experiments [320,322,323]. They are followed by many results and suggestions that connect the DCE with the condensed matter (Section 4.2) and atomic physics (Section 4.3). Here, the first experiment was reported in [353]. The first experimental results related to the analog of the DCE in an optical fiber were also obtained recently [465]. Thus, the fascinating dynamical Casimir physics continues to attract many scientists. However, an observation of the "real" DCE in cavities with moving boundaries is still a challenge (or dream).

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