

INTRODUCTION TO

ENGINEERING DESIGN

**Modelling, Synthesis
and Problem Solving
Strategies**

ANDREW SAMUEL • JOHN WEIR

***Introduction to Engineering Design:
Modelling, Synthesis and Problem Solving Strategies***

To Eva and Cathy

Introduction to Engineering Design: Modelling, Synthesis and Problem Solving Strategies

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The previous equation for $\tau_{\max}$ is sometimes rewritten as:

$$\tau_{\max} = K_{\text{shear}} \frac{8FD}{\pi d^3}, \quad (3.22)$$

where

$$K_{\text{shear}} = \left(1 + \frac{0.5}{C}\right) = \left(1 + \frac{d}{2D}\right). \quad (3.23)$$

The parameter K_{shear} is the shear-stress multiplication factor for helical springs. It is a useful means of predicting the maximum *nominal* shear stress (unmodified by the effects of stress concentrations). As we shall see in Section 3.3.4 on cyclically loaded springs, stress concentration effects further elevate the actual peak stress in the vicinity of the inner surface of the wire. However these effects are highly localised and tend to disappear when the material of the spring yields locally (*sets*), so they are not critical for statically loaded springs.

3. MODELLING DEFLECTION IN COMPRESSION SPRINGS (STATIC LOADING)

We use the symbol s to denote coordinate distances along the helical centreline traced out by the wire of a coil spring. The relation between torsional shear strain and rotation for a short length, ds , of this wire is just like that for any shaft in torsion:

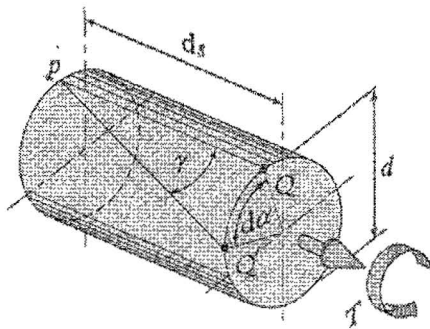


Figure 3.39 Shear strain and rotation in an elemental length of a helical spring

In the exaggerated figure above, torque T produces a small rotation $d\alpha$ in the element of length ds , by means of the shear strain γ in which the longitudinal line PQ rotates to PQ' . Since the small arc length QQ' (the circumferential deflection over the small axial distance ds) is subtended by both γ and $d\alpha$, it must be that

$\gamma \cdot ds = d\alpha \cdot d/2$. Now by applying Hooke's Law for circular bars to the shear stresses and shear strains at the surface of spring wire, we obtain:

$$\gamma = \frac{\tau}{G} = \frac{Td}{2I_p} \cdot \frac{1}{G}$$

or

$$\gamma = \frac{8FD}{\pi G d^3}, \quad (3.24)$$

where

$$G = \frac{E}{2(1+\mu)}. \quad (3.25)$$

The material constant G is the shear modulus of elasticity (approximately 90 GPa for most steels). Integrating:

$$\begin{aligned} \alpha &= \int_0^{n\pi D} \frac{2\gamma}{d} ds = \int_0^{n\pi D} \frac{16FD}{\pi d^4 G} ds \\ &= \frac{16FD^2 n}{d^4 G}. \end{aligned} \quad (3.26)$$

In Equation (3.26), α is the total torsional rotation of the spring wire due to the applied force F . However, any rotation, $d\alpha$, of wire at a radius $D/2$ from the central axis of the coil corresponds to an axial compression of $dx = d\alpha \times D/2$ in the coil. Therefore the overall compression of the coil spring is given by:

$$x = \alpha \cdot D/2 = \frac{8FD^3 n}{d^4 G}. \quad (3.27)$$

The spring rate k (or its *stiffness*) is the force, F , required to cause a deflection x of the force at the centre of the coil. It is trivial now to re-express Equation (3.27) in terms of the spring *stiffness*, k :

$$k = \left(\frac{F}{x}\right) = \frac{d^4 G}{8D^3 n}. \quad (3.28)$$

A similar result may be obtained by energy methods (*Castigliano's theorem*).

3.3.4 Stresses in linear compression springs, dynamic loading

Springs are very often subject to dynamic loading. Frequently the spring deflection will vary cyclically with time over a large number of cycles. Our approach to the prediction of stresses in helical

compression springs in the previous section remains valid for dynamic loading, except that there is a further effect to be considered. Additional account must be taken of the effects of stress concentration due to wire curvature.

We found in Section 3.3.3 that the torsional and direct shear stresses may be superimposed to get the maximum shear stress in the coil wire, and that this occurs at the inner surface of the coil spring. This is precisely the location of a localised stress concentration phenomenon first reported and analysed by A. M. Wahl^{3.3}, who found that the shear stress on the inside surface of a curved bar is increased due to its curvature.

The Wahl correction factor for stress concentration due to curvature is a function of the geometric spring constant $C = D/d$, and is of the form:

$$K_{Wahl} = \frac{4C-1}{4C+1} + \frac{0.615}{C}, \quad (3.29)$$

which is inclusive of the influences of direct shear (F/A) and torsional shear ($Td/2I_p$) as well as wire curvature. For large C , this is only marginally different from the value of K_{shear} , given in Equation (3.23). In practical spring manufacture, C usually takes values ranging from 6 to 12.

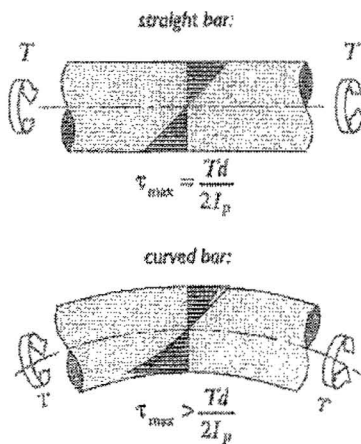


Figure 3.40 Stress concentration due to wire curvature

The Wahl correction factor was originally developed to account for stress concentration due to curvature in round bars under torsional loading. It is a shear-stress multiplication factor for helical springs, and is used in the equation

$$\tau_{peak} = K_{Wahl} \frac{8FD}{\pi d^3}, \quad (3.30)$$

to predict the highly localised peak shear stress near the inner surface of the spring wire. This peak stress disappears due to local yielding once the spring is *set*, so it is not such an important issue under static loading. But consider the situation under dynamic loading (cyclic variation with time). If the initial spring load and deflection is such that τ_{peak} exceeds S_{sy} (the shear stress value corresponding to the onset of yielding) by a small amount, then, naturally enough, the spring wire will yield locally in the vicinity of this peak stress. The metal on the innermost surface of the wire will experience microstructural sliding of its grain boundaries — it will *flow* (invisibly) — until the strains associated with the local peak stress are *relieved*. In this condition the spring is said to be *set*. When the load applied to the spring diminishes again (say to zero), the deflection of the spring away from its *set* configuration will cause new peak stresses to develop at the inner wire surface, this time in the opposite direction to the first set of stresses. If the change in spring deflection is great enough, the new peak stresses may be sufficient to cause yielding again, reversing the first *set*. In the extreme, the spring will be repeatedly *set* and *reset*, suffering localised reversed yielding, and it will very soon fail.

For this reason it is essential for dynamically loaded springs that the value of τ_{peak} remain below S_{sy} . It is also highly desirable that such springs be designed to have stresses below the *shear stress endurance limit* (S'_{se}) so they will have a satisfactory dynamic life.

Unfortunately there is very little data on the value of S'_{se} . The best is that of F.P. Zimmerli^{3.4}, whose results suggest, interestingly, that the shear stress endurance limit is independent of material, size and tensile strength for all spring steels up to 10 mm in diameter. The only relevant issue is whether or not the spring wire has been *peened*. Peening is a process of bombarding the surface with fine steel particles. This process tends to remove surface defects as well as work hardening the surface, but is not always feasible, especially for small springs.

Zimmerli's results may be summarised as follows:

- for unpeened springs, $S'_{se} = 310 \text{ MPa}$;
- for peened springs, $S'_{se} = 465 \text{ MPa}$.

Springs subject to fluctuating loads should be designed against fatigue by means of the approach familiar from Chapter 2.4, but using shear

3.3 See Wahl, A.M. Helical Compression and Tension Springs, ASME Paper A-38, J. Appl. Mech. 2(1):1935

3.4 F.P. Zimmerli, 'Human failures in spring applications', The Mainspring, publication of Associated Spring Corporation, Bristol, Connecticut, No. 17, August-September 1957

properties (S'_{sc} , S_{sy} and S_{sU}) to construct the A - M diagram. Shear stresses associated with dynamic loading, and hence the mean and amplitude of such fluctuating stresses (τ_a and τ_m), should be calculated on the basis of Equation (3.30), using K_{Wahl} .

3.3.5 Other practical issues for compression springs

1. BUCKLING

Compression springs can buckle (like a column), but for $(\ell_0/D) < 4$ this is not a serious problem. Guiding the spring can overcome such buckling problems generally.

2. BOTTOMING

Compression springs can *bottom out* — they become solid when the coils are fully in contact with each other. In this way the stress in the section may be limited.

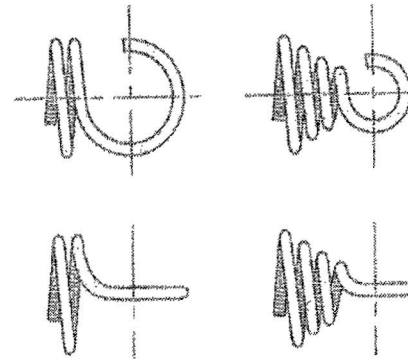
3. BINDING

The average coil diameter, D , increases when a spring is compressed (and likewise decreases when it is stretched). This may lead to problems if the spring is required to operate in a cavity, so the designer must ensure sufficient clearance is provided. The worst case occurs when the spring is fully *bottomed*, in which case the outer diameter is:

$$D_{O,max} = \frac{\sqrt{\pi^2 D^2 + p^2 - d^2}}{\pi} + d.$$

3.3.6 Tension springs

The stress and deflection relationships presented above are generally applicable to helical tension springs also. However, in tension springs the stresses in the end supports may be more significant than in the coils due to stress concentrations. Cunning design of the end loops of wire can alleviate this problem to some degree. Some typical end connections in extension springs are shown in Figure 3.41.



Standard ends

Ends with reduced stress concentration

Figure 3.41 End designs for tension springs

3.3.7 Torsion springs

As discussed in Section 3.3.3, most torsion springs are, from a structural analysis point-of-view, curved beams in bending. Stresses in torsion springs are found from the theory of bending of curved beams. In general

$$\sigma_{B,max} = K_B \frac{M}{Z_c}, \quad (3.31)$$

where K_B is a correction factor to allow for stress concentrations in the bending stress distribution induced by curvature, and Z_c is the section modulus of the curved beam. For round wire,

$$Z_c = \frac{I_{zz}}{c} = \frac{\pi d^4/64}{d/2} = \frac{\pi d^3}{32}.$$

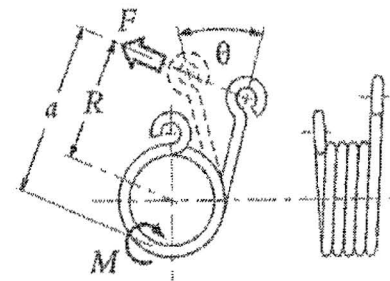


Figure 3.42 Bending moment in a torsion spring

3.4 F. P. Zimmerli, 'Human failures in spring applications', *The Mainspring*, publication of Associated Spring Corporation, Bristol, Connecticut, No. 17, August-September 1957

Torsion springs arise in a wide variety of shapes, and employ many techniques for exerting the torque, $T=FR$, about the torsional axis of the spring. The arrangement in Figure 3.42 is a typical one. As in this figure, it is often convenient to express the maximum bending moment, $M=Fa$, experienced by the wire as a product of some contact force, F , and a maximum perpendicular distance, a , to the wire centreline (a is the *bending moment-arm*, not to be confused with the *torsional moment-arm*, R). The maximum bending stress for a round-wire torsion spring can then be written:

$$\sigma_{B,\max} = K_B \frac{32Fa}{\pi d^3} \quad (3.32)$$

Bending stress concentrations are similar in concept to those described for shear stress in Section 3.3.4. They are again most severe at the inner fibre of the wire cross-section. A conservative approximation to Wahl's formulation of the stress concentration there is given by:

$$K_B = \frac{4C-1}{4C-4} \quad (3.33)$$

The total rotation of a torsion spring (having elastic modulus E , mean coil diameter D , wire diameter d , and n working (active) coils, with a force F acting on it about a torsional moment arm R) can be calculated from strain energy methods to be:

$$\theta = \frac{64FRDn}{d^4E}$$

Some care needs to be taken in applying this equation, since:

- it uses the torsional moment, FR , not the maximum bending moment, Fa ;
- the mean coil diameter decreases by the ratio $n/(n+\theta)$ when the spring is wound;
- the number of active coils should be adjusted to account for the flexural contribution of the spring's straight ends (if any);
- large inaccuracies can arise from inter-coil friction; and
- practical tests show that, by ignoring the effect of wire curvature, the equation underestimates the true rotation by about 6%.

The torsional stiffness, k_T , is the torque per radian of rotation. Introducing a correction factor, C_{cur} ,

to account for wire curvature, k_T may be found from the previous equation.

$$k_T = \frac{T}{\theta} = C_{\text{cur}} \frac{d^4 E}{64 D n} \quad (3.34)$$

Often the stiffness of torsion springs is expressed as a *spring rate*, k'_T , which is the torque per turn (2π radians) of the spring. Combining $C_{\text{cur}} 2\pi/64$ into a single constant, we obtain:

$$k'_T = \frac{d^4 E}{10.8 D n} \quad (3.35)$$

Ex. 3.10 Automatic garage door

An automatic garage door consists of several plate assemblies that are stored horizontally 3.2 m above the garage floor in a roller-track near the ceiling, and can be wound down along the curved track into their vertical *door* position by means of a small, geared electric motor. The motor is connected to the door plates through drive chains (exactly like a bicycle chain), which it drives with a long shaft and sprockets having a pitch diameter of 50 mm, as shown in Figure 3.43.

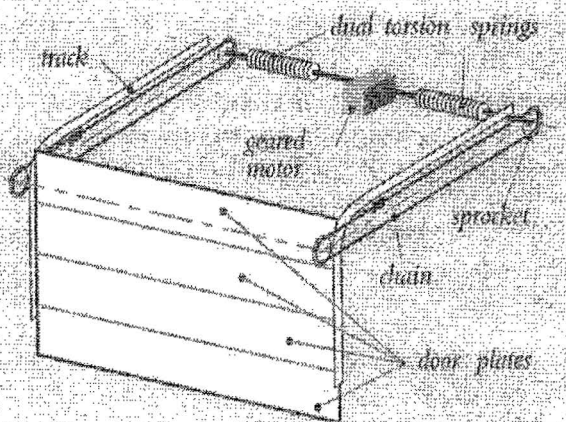


Figure 3.43 Garage door schematic

The plates have a total mass of 160 kg. The geared motor unit has plenty of torque capability to overcome friction in the track system, but not to support the weight of the door plates when they are nearly closed. For this purpose, two large helical torsion springs are fitted around the shaft between the geared motor and the chain's drive sprockets. Design the torsion springs to take the full weight of the door in the *almost-closed* position.

3.3.8 Properties of materials used for springs

Table 3.1 gives the properties for some commonly used spring materials. The following needs to be noted:

- (a) Tensile strength varies with heat treatment and wire diameter. It is higher at small diameters.
- (b) Temperature and shock loading are critical factors in material choice.
- (c) Since failure is by shear, we need a value for yielding in shear: S_{sy}
- (d) But S_{sy} is not always well reported. If tabulated values are not available, designers may use the following approximate values with a suitable safety margin:
 $S_{sy} \approx 0.577 S_y \approx 0.433 S_U$.

3.3.9 Vibrations and natural frequency

Springs are usually the means by which vibration is imparted to other inertial bodies — their restoring force drives the oscillatory motion in free vibration systems. But a spring is itself an inertial

body, and the designer needs to be aware of the dangers of exciting vibratory motions within the spring at resonant frequencies.

For a spring supported between two parallel plates, its natural frequency (in Hz) is:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

The mass m of the spring may be found from:

$$m = \frac{\pi^2 d^2 D n \rho}{4}$$

Where a spring is compressed very rapidly (as in the valve gear of an engine), parts of the spring can *bump* against adjacent coils before the whole spring has time to adjust itself to the average compression. The resulting wave of motion is called *surging* and it can occur at frequencies as high as the 13th harmonic of the spring.

Typically an engine at 5000 rpm (83.3 Hz) has its valve springs operating at 2500 cycles/minute (41.7 Hz). The 13th harmonic of this motion is at $13 \times 41.7 = 542$ Hz. The spring must be designed to have a natural frequency well above this value — at least 15 or 20 times the forcing frequency is recommended.

Table 3.1 Spring material properties

Name	Designation	Description	S_U / MPa (2 mm wire)
Music wire (0.8-0.95%C)	ASTM A228-51	The best and toughest material used for springs; $0^\circ\text{C} < \text{temp} < 120^\circ\text{C}$	2000
Oil tempered wire (0.6-0.7%C)	ASTM A229-41	General purpose spring steel; $0^\circ\text{C} < \text{temp} < 180^\circ\text{C}$	1700
Chrome vanadium	ASTM A231-41	Most popular alloy spring steel — good for shockloading; $0^\circ\text{C} < \text{temp} < 220^\circ\text{C}$	1800
Inconel	ASTM X-750	Clock springs	1300
Phosphor bronze	ASTM B-159	Commutator brush springs	900

Ex. 3.11 Simple compression spring example

The following information is available for a typical helical compression spring:

- material is music wire: $S_u = 2000 \text{ MPa}$, $S'_e = 310 \text{ MPa}$ — unpeened;
- free length $\ell_0 = 105 \text{ mm}$;
- outside diameter $D_o = 14.5 \text{ mm}$;
- wire diameter $d = 2.24$;
- working (active) coils $n = 21$;
- preload $= 45 \text{ N}$; maximum load $= 225 \text{ N}$.

Determine factor of safety against fatigue, natural frequency and surge frequency.

Ex. 3.12

- The recurve bow shown in Figure 3.22 is capable of imparting substantially greater energy per unit extension to a projectile than the simple *longbow*. Explain this using energy principles.
- The ballista (or catapult) shown in Figure 3.23 imparts energy to a projectile by winding up a torsion spring made up from heavy elastic ropes. A movie director has hired you to advise on a realistic ballista to be used in a period drama. Develop a simple mathematical model for the ballista, that will allow you to select a torsion spring necessary to hurl a 10 kg projectile over a distance of 100 m. Clearly explain all your design assumptions.

where

$$K_{\text{shear}} = \left(1 + \frac{0.5}{C} \right); C = \left(\frac{D}{d} \right). \quad (3.23)$$

Require $\tau_{\text{max}} < S_{sy}$.

Dynamic loading:

Model

$$\tau_{\text{peak}} = K_{\text{Wahl}} \frac{8FD}{\pi d^3}, \quad (3.30)$$

where

$$K_{\text{Wahl}} = \frac{4C-1}{4C+1} + \frac{0.615}{C}. \quad (3.29)$$

Require $\tau_{\text{peak}} < S'_e$.

Also check A - M diagram for τ_a and τ_m .

Also check natural vibration frequency for resonance.

Linear stiffness:

Model

$$k = \left(\frac{F}{x} \right) = \frac{d^4 G}{8D^3 n}. \quad (3.28)$$

Also check compression springs for buckling, bottoming, binding.

(b) Torsion springs

Bending stresses:

Model

$$\sigma_{B,\text{max}} = K_B \frac{32Fa}{\pi d^3}, \quad (3.32)$$

where

$$K_B = \frac{4C-1}{4C-4}. \quad (3.33)$$

Require $\sigma_{B,\text{max}} < S_y$ or S'_e — as applicable.

Rotational stiffness ('spring rate'):

Model

$$k'_T = \frac{d^4 E}{10.8 D n} \text{ (per turn)}. \quad (3.35)$$

3.3.10 Summary of design rules for springs

(a) Compression and tension springs

For compression and tension springs, Equations (3.28) and either (3.22) or (3.30) are usually manipulated to achieve the desired spring behaviour.

Static loading:

Model

$$\tau_{\text{max}} = K_{\text{shear}} \frac{8FD}{\pi d^3}, \quad (3.22)$$

3.4 Columns: design for axial loading

In this section, we consider the design of *columns* — structural elements whose purpose is to resist axial compression. For many engineering components we can treat structural integrity for axial compression and tension in similar ways, with some special exceptions. One exception already noted was the difference experienced by structures subjected to dynamic loading (refer to the *A-M* diagram Figure 2.36). *Columns* represent another exception, in that they are designed to resist failure by elastic instability, *buckling*, rather than yielding or fracture.

Buckling is a form of failure due to elastic instability, in which a component might fail at loads significantly below that expected to be carried by the cross section from elementary failure theory.

The first to investigate the elastic deflection curves of beams was Leonhard Euler (1701-1783). While Euler's name remains directly associated with column buckling, it is worth noting that the problem was suggested to him by Daniel Bernoulli (1700-1782), who also contributed to the theory of elastic curves^{3,5}. Bernoulli recognised Euler's mathematical capabilities, specifically his knowledge of the calculus of variations (*isoperimetric method*), which Euler used in deriving the curve shape for a beam under axial loading (*column*). J. L. Lagrange (1736-1813) later extended Euler's work to show that Euler's equation has a sinusoidal solution curve, with half wave length proportional to the length of the beam. Although his direct contribution now appears shrouded in history, it was Lagrange who was actually responsible for the classical equation we now refer to as the *Euler buckling curve*.

As more and more use is made of high strength alloys, the cross sectional areas of components are being reduced, and elastic instability has become an important consideration in the design of engineering components.

The first to study buckling of thin rectangular bars was Ludwig Prandtl, who wrote his doctoral thesis on this subject. (*Kipperscheinungen*, Dissertation Munich, 1899).

The study of this subject was considerably extended in the early part of this century by several workers including Timoshenko (buckling of I-beams, axially loaded curved sheet panels and torsional buckling of members with thin cross

section) and R. von Mises (cylindrical shells under combined axial and lateral pressure; *VDI*, vol 52, p750, 1914).

The buckling of thin tubes under pure bending was studied by W. Flüge, *Ing. Arch.* Vol. 3, p463, 1932.

3.4.1 Identifying columns

Together with beams, columns constitute the earliest and most enduring structural components employed by humans to fabricate their built environment.

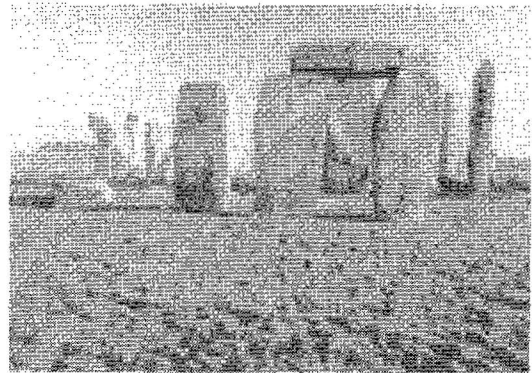


Figure 3.44 Columns and beams in the Neolithic circle at Stonehenge, England

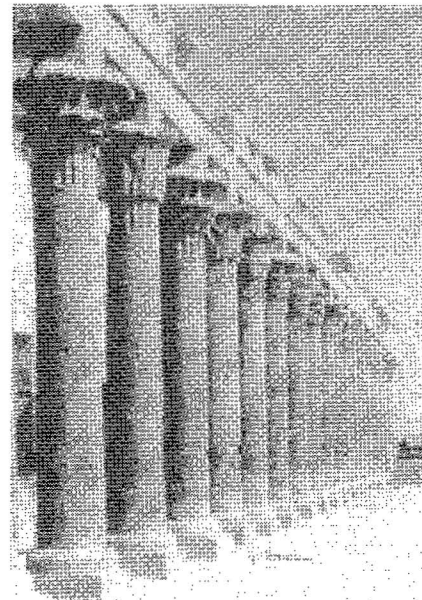


Figure 3.45 Columns of the Temple of Isis at Aswan, Egypt.

3.5 L. Euler, 1759, *Sur la force des colonnes*, *Mem. Acad. Berlin*, V. 13, quoted in Timoshenko 1953