

Mechanical Engineering

Shigley's Mechanical Engineering Design,
Eighth Edition

Budynas–Nisbett



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Preface

Objectives

This text is intended for students beginning the study of mechanical engineering design. The focus is on blending fundamental development of concepts with practical specification of components. Students of this text should find that it inherently directs them into familiarity with both the basis for decisions and the standards of industrial components. For this reason, as students transition to practicing engineers, they will find that this text is indispensable as a reference text. The objectives of the text are to:

- Cover the basics of machine design, including the design process, engineering mechanics and materials, failure prevention under static and variable loading, and characteristics of the principal types of mechanical elements.
- Offer a practical approach to the subject through a wide range of real-world applications and examples.
- Encourage readers to link design and analysis.
- Encourage readers to link fundamental concepts with practical component specification.

New to This Edition

This eighth edition contains the following significant enhancements:

- *New chapter on the Finite Element Method.* In response to many requests from reviewers, this edition presents an introductory chapter on the finite element method. The goal of this chapter is to provide an overview of the terminology, method, capabilities, and applications of this tool in the design environment.
- *New transmission case study.* The traditional separation of topics into chapters sometimes leaves students at a loss when it comes time to integrate dependent topics in a larger design process. A comprehensive case study is incorporated through standalone example problems in multiple chapters, then culminated with a new chapter that discusses and demonstrates the integration of the parts into a complete design process. Example problems relevant to the case study are presented on engineering paper background to quickly identify them as part of the case study.
- *Revised and expanded coverage of shaft design.* Complementing the new transmission case study is a significantly revised and expanded chapter focusing on issues relevant to shaft design. The motivating goal is to provide a meaningful presentation that allows a new designer to progress through the entire shaft design process – from general shaft layout to specifying dimensions. The chapter has been moved to immediately follow the fatigue chapter, providing an opportunity to seamlessly transition from the fatigue coverage to its application in the design of shafts.
- *Availability of information to complete the details of a design.* Additional focus is placed on ensuring the designer can carry the process through to completion.

By assigning larger design problems in class, the authors have identified where the students lack details. For example, information is now provided for such details as specifying keys to transmit torque, stress concentration factors for keyways and retaining ring grooves, and allowable deflections for gears and bearings. The use of internet catalogs and engineering component search engines is emphasized to obtain current component specifications.

- *Streamlining of presentation.* Coverage of material continues to be streamlined to focus on presenting straightforward concept development and a clear design procedure for student designers.

Content Changes and Reorganization

A new Part 4: *Analysis Tools* has been added at the end of the book to include the new chapter on finite elements and the chapter on statistical considerations. Based on a survey of instructors, the consensus was to move these chapters to the end of the book where they are available to those instructors wishing to use them. Moving the statistical chapter from its former location causes the renumbering of the former chapters 2 through 7. Since the shaft chapter has been moved to immediately follow the fatigue chapter, the component chapters (Chapters 8 through 17) maintain their same numbering. The new organization, along with brief comments on content changes, is given below:

Part 1: Basics

Part 1 provides a logical and unified introduction to the background material needed for machine design. The chapters in Part 1 have received a thorough cleanup to streamline and sharpen the focus, and eliminate clutter.

- *Chapter 1, Introduction.* Some outdated and unnecessary material has been removed. A new section on problem specification introduces the transmission case study.
- *Chapter 2, Materials.* New material is included on selecting materials in a design process. The Ashby charts are included and referenced as a design tool.
- *Chapter 3, Load and Stress Analysis.* Several sections have been rewritten to improve clarity. Bending in two planes is specifically addressed, along with an example problem.
- *Chapter 4, Deflection and Stiffness.* Several sections have been rewritten to improve clarity. A new example problem for deflection of a stepped shaft is included. A new section is included on elastic stability of structural members in compression.

Part 2: Failure Prevention

This section covers failure by static and dynamic loading. These chapters have received extensive cleanup and clarification, targeting student designers.

- *Chapter 5, Failures Resulting from Static Loading.* In addition to extensive cleanup for improved clarity, a summary of important design equations is provided at the end of the chapter.
- *Chapter 6, Fatigue Failure Resulting from Variable Loading.* Confusing material on obtaining and using the S-N diagram is clarified. The multiple methods for obtaining notch sensitivity are condensed. The section on combination loading is rewritten for greater clarity. A chapter summary is provided to overview the analysis roadmap and important design equations used in the process of fatigue analysis.

Part 3: Design of Mechanical Elements

Part 3 covers the design of specific machine components. All chapters have received general cleanup. The shaft chapter has been moved to the beginning of the section. The arrangement of chapters, along with any significant changes, is described below:

- *Chapter 7, Shafts and Shaft Components.* This chapter is significantly expanded and rewritten to be comprehensive in designing shafts. Instructors that previously did not specifically cover the shaft chapter are encouraged to use this chapter immediately following the coverage of fatigue failure. The design of a shaft provides a natural progression from the failure prevention section into application toward components. This chapter is an essential part of the new transmission case study. The coverage of setscrews, keys, pins, and retaining rings, previously placed in the chapter on bolted joints, has been moved into this chapter. The coverage of limits and fits, previously placed in the chapter on statistics, has been moved into this chapter.
- *Chapter 8, Screws, Fasteners, and the Design of Nonpermanent Joints.* The section on setscrews, keys, and pins, has been moved from this chapter to Chapter 7. The coverage of bolted and riveted joints loaded in shear has been returned to this chapter.
- *Chapter 9, Welding, Bonding, and the Design of Permanent Joints.* The section on bolted and riveted joints loaded in shear has been moved to Chapter 8.
- *Chapter 10, Mechanical Springs.*
- *Chapter 11, Rolling-Contact Bearings.*
- *Chapter 12, Lubrication and Journal Bearings.*
- *Chapter 13, Gears – General.* New example problems are included to address design of compound gear trains to achieve specified gear ratios. The discussion of the relationship between torque, speed, and power is clarified.
- *Chapter 14, Spur and Helical Gears.* The current AGMA standard (ANSI/AGMA 2001-D04) has been reviewed to ensure up-to-date information in the gear chapters. All references in this chapter are updated to reflect the current standard.
- *Chapter 15, Bevel and Worm Gears.*
- *Chapter 16, Clutches, Brakes, Couplings, and Flywheels.*
- *Chapter 17, Flexible Mechanical Elements.*
- *Chapter 18, Power Transmission Case Study.* This new chapter provides a complete case study of a double reduction power transmission. The focus is on providing an example for student designers of the process of integrating topics from multiple chapters. Instructors are encouraged to include one of the variations of this case study as a design project in the course. Student feedback consistently shows that this type of project is one of the most valuable aspects of a first course in machine design. This chapter can be utilized in a tutorial fashion for students working through a similar design.

Part 4: Analysis Tools

Part 4 includes a new chapter on finite element methods, and a new location for the chapter on statistical considerations. Instructors can reference these chapters as needed.

- *Chapter 19, Finite Element Analysis.* This chapter is intended to provide an introduction to the finite element method, and particularly its application to the machine design process.

- *Chapter 20, Statistical Considerations.* This chapter is relocated and organized as a tool for users that wish to incorporate statistical concepts into the machine design process. This chapter should be reviewed if Secs. 5–13, 6–17, or Chap. 11 are to be covered.

Supplements

The 8th edition of *Shigley's Mechanical Engineering Design* features McGraw-Hill's ARIS (Assessment Review and Instruction System). ARIS makes homework meaningful—and manageable—for instructors and students. Instructors can assign and grade text-specific homework within the industry's most robust and versatile homework management system. Students can access multimedia learning tools and benefit from unlimited practice via algorithmic problems. Go to aris.mhhe.com to learn more and register!
The array of tools available to users of *Shigley's Mechanical Engineering Design* includes:

Student Supplements

- *Tutorials—Presentation of major concepts, with visuals.* Among the topics covered are pressure vessel design, press and shrink fits, contact stresses, and design for static failure.
- *MATLAB® for machine design.* Includes visual simulations and accompanying source code. The simulations are linked to examples and problems in the text and demonstrate the ways computational software can be used in mechanical design and analysis.
- *Fundamentals of engineering (FE) exam questions for machine design.* Interactive problems and solutions serve as effective, self-testing problems as well as excellent preparation for the FE exam.
- *Algorithmic Problems.* Allow step-by-step problem-solving using a recursive computational procedure (algorithm) to create an infinite number of problems.

Instructor Supplements (under password protection)

- *Solutions manual.* The instructor's manual contains solutions to most end-of-chapter nondesign problems.
- *PowerPoint® slides.* Slides of important figures and tables from the text are provided in PowerPoint format for use in lectures.

List of Symbols

This is a list of common symbols used in machine design and in this book. Specialized use in a subject-matter area often attracts fore and post subscripts and superscripts. To make the table brief enough to be useful the symbol kernels are listed. See Table 14–1, pp. 715–716 for spur and helical gearing symbols, and Table 15–1, pp. 769–770 for bevel-gear symbols.

A	Area, coefficient
$\mathbf{A}$	Area variate
a	Distance, regression constant
$\hat{a}$	Regression constant estimate
$\mathbf{a}$	Distance variate
B	Coefficient
Bhn	Brinell hardness
$\mathbf{B}$	Variate
b	Distance, Weibull shape parameter, range number, regression constant, width
$\hat{b}$	Regression constant estimate
$\mathbf{b}$	Distance variate
C	Basic load rating, bolted-joint constant, center distance, coefficient of variation, column end condition, correction factor, specific heat capacity, spring index
c	Distance, viscous damping, velocity coefficient
CDF	Cumulative distribution function
COV	Coefficient of variation
$\mathbf{c}$	Distance variate
D	Helix diameter
d	Diameter, distance
E	Modulus of elasticity, energy, error
e	Distance, eccentricity, efficiency, Naperian logarithmic base
F	Force, fundamental dimension force
f	Coefficient of friction, frequency, function
fom	Figure of merit
G	Torsional modulus of elasticity
g	Acceleration due to gravity, function
H	Heat, power
H_B	Brinell hardness
HRC	Rockwell C-scale hardness
h	Distance, film thickness
$\hbar_{CR}$	Combined overall coefficient of convection and radiation heat transfer
I	Integral, linear impulse, mass moment of inertia, second moment of area
i	Index
$\mathbf{i}$	Unit vector in x-direction

J	Mechanical equivalent of heat, polar second moment of area, geometry factor
j	Unit vector in the y-direction
K	Service factor, stress-concentration factor, stress-augmentation factor, torque coefficient
k	Marin endurance limit modifying factor, spring rate
k	k variate, unit vector in the z-direction
L	Length, life, fundamental dimension length
LN	Lognormal distribution
l	Length
M	Fundamental dimension mass, moment
M	Moment vector, moment variate
m	Mass, slope, strain-strengthening exponent
N	Normal force, number, rotational speed
N	Normal distribution
n	Load factor, rotational speed, safety factor
n_d	Design factor
P	Force, pressure, diametral pitch
PDF	Probability density function
p	Pitch, pressure, probability
Q	First moment of area, imaginary force, volume
q	Distributed load, notch sensitivity
R	Radius, reaction force, reliability, Rockwell hardness, stress ratio
R	Vector reaction force
r	Correlation coefficient, radius
r	Distance vector
S	Sommerfeld number, strength
S	S variate
s	Distance, sample standard deviation, stress
T	Temperature, tolerance, torque, fundamental dimension time
T	Torque vector, torque variate
t	Distance, Student's t-statistic, time, tolerance
U	Strain energy
U	Uniform distribution
u	Strain energy per unit volume
V	Linear velocity, shear force
v	Linear velocity
W	Cold-work factor, load, weight
W	Weibull distribution
w	Distance, gap, load intensity
w	Vector distance
X	Coordinate, truncated number
x	Coordinate, true value of a number, Weibull parameter
x	x variate
Y	Coordinate
y	Coordinate, deflection
y	y variate
Z	Coordinate, section modulus, viscosity
z	Standard deviation of the unit normal distribution
z	Variate of z

α	Coefficient, coefficient of linear thermal expansion, end-condition for springs, thread angle
β	Bearing angle, coefficient
Δ	Change, deflection
δ	Deviation, elongation
ϵ	Eccentricity ratio, engineering (normal) strain
$\boldsymbol{\epsilon}$	Normal distribution with a mean of 0 and a standard deviation of s
ε	True or logarithmic normal strain
Γ	Gamma function
γ	Pitch angle, shear strain, specific weight
λ	Slenderness ratio for springs
$\boldsymbol{\lambda}$	Unit lognormal with a mean of l and a standard deviation equal to COV
μ	Absolute viscosity, population mean
ν	Poisson ratio
ω	Angular velocity, circular frequency
ϕ	Angle, wave length
ψ	Slope integral
ρ	Radius of curvature
σ	Normal stress
σ'	Von Mises stress
$\boldsymbol{\sigma}$	Normal stress variate
$\hat{\sigma}$	Standard deviation
τ	Shear stress
$\boldsymbol{\tau}$	Shear stress variate
θ	Angle, Weibull characteristic parameter
¢	Cost per unit weight
$	Cost

Table 10–8

Maximum Allowable Stresses for ASTM A228 and Type 302 Stainless Steel Helical Extension Springs in Cyclic Applications

Source: From *Design Handbook*, 1987, p. 52. Courtesy of Associated Spring.

	Percent of Tensile Strength		
Number of Cycles	In Torsion		In Bending
	Body	End	End
10^5	36	34	51
10^6	33	30	47
10^7	30	28	45

This information is based on the following conditions: not shot-peened, no surging and ambient environment with a low temperature heat treatment applied. Stress ratio = 0.

as S_r for bending or S_{sr} for torsion. Then for torsion, for example, $S_{sa} = S_{sm} = S_{sr}/2$ and the Gerber ordinate intercept, given by Eq. (6–42) for shear, is

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})^2} = \frac{S_{sr}/2}{1 - \left(\dfrac{S_{sr}/2}{S_{su}}\right)^2} \tag{10–42}$$

So in Ex. 10–7 an estimate for the bending endurance limit from Table 10–8 would be

$$S_r = 0.45 S_{ut} = 0.45(264.7) = 119.1 \text{ kpsi}$$

and from Eq. (10–42)

$$S_e = \frac{S_r/2}{1 - [S_r/(2S_{ut})]^2} = \frac{119.1/2}{1 - \left(\dfrac{119.1/2}{264.7}\right)^2} = 62.7 \text{ kpsi}$$

Using this in place of 67.1 kpsi in Ex. 10–7 results in $(n_f)_A = 1.03$, a reduction of 5 percent.

10–12 Helical Coil Torsion Springs

When a helical coil spring is subjected to end torsion, it is called a *torsion spring*. It is usually close-wound, as is a helical coil extension spring, but with negligible initial tension. There are single-bodied and double-bodied types as depicted in Fig. 10–9. As shown in the figure, torsion springs have ends configured to apply torsion to the coil body in a convenient manner, with short hook, hinged straight offset, straight torsion, and special ends. The ends ultimately connect a force at a distance from the coil axis to apply a torque. The most frequently encountered (and least expensive) end is the straight torsion end. If intercoil friction is to be avoided completely, the spring can be wound with a pitch that just separates the body coils. Helical coil torsion springs are usually used with a rod or arbor for reactive support when ends cannot be built in, to maintain alignment, and to provide buckling resistance if necessary.

The wire in a torsion spring is in bending, in contrast to the torsion encountered in helical coil compression and extension springs. The springs are designed to wind tighter in service. As the applied torque increases, the inside diameter of the coil decreases. Care must be taken so that the coils do not interfere with the pin, rod, or arbor. The bending mode in the coil might seem to invite square- or rectangular-cross-section wire, but cost, range of materials, and availability discourage its use.

Figure 10–9

Torsion springs. *(Courtesy of Associated Spring.)*

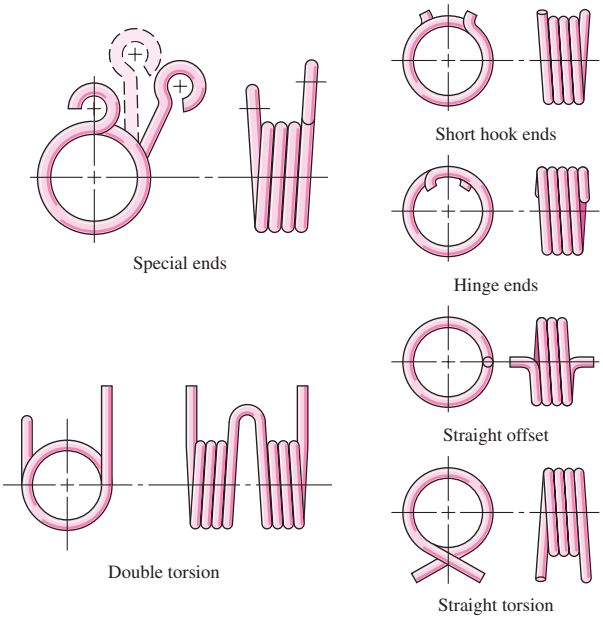


Table 10–9

End Position Tolerances for Helical Coil Torsion Springs (for D/d Ratios up to and Including 16)

Source: From *Design Handbook*, 1987, p. 52. Courtesy of Associated Spring.

Total Coils	Tolerance: ± Degrees*
Up to 3	8
Over 3–10	10
Over 10–20	15
Over 20–30	20
Over 30	25

*Closer tolerances available on request.

Torsion springs are familiar in clothespins, window shades, and animal traps, where they may be seen around the house, and out of sight in counterbalance mechanisms, ratchets, and a variety of other machine components. There are many stock springs that can be purchased off-the-shelf from a vendor. This selection can add economy of scale to small projects, avoiding the cost of custom design and small-run manufacture.

Describing the End Location

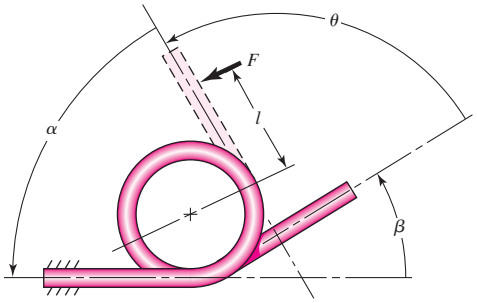
In specifying a torsion spring, the ends must be located relative to each other. Commercial tolerances on these relative positions are listed in Table 10–9. The simplest scheme for expressing the initial unloaded location of one end with respect to the other is in terms of an angle β defining the partial turn present in the coil body as $N_p = \beta/360^\circ$, as shown in Fig. 10–10. For analysis purposes the nomenclature of Fig. 10–10 can be used. Communication with a springmaker is often in terms of the back-angle α.

The number of body turns N_b is the number of turns in the free spring body by count. The body-turn count is related to the initial position angle β by

$$N_b = \text{integer} + \frac{\beta}{360^\circ} = \text{integer} + N_p$$

Figure 10–10

The free-end location angle is β. The rotational coordinate θ is proportional to the product Fl. Its back angle is α. For all positions of the moving end $\theta + \alpha = \Sigma = \text{constant}$.



where N_p is the number of partial turns. The above equation means that N_b takes on noninteger, discrete values such as 5.3, 6.3, 7.3, . . . , with successive differences of 1 as possibilities in designing a specific spring. This consideration will be discussed later.

Bending Stress

A torsion spring has bending induced in the coils, rather than torsion. This means that residual stresses built in during winding are in the same direction but of opposite sign to the working stresses that occur during use. The strain-strengthening locks in residual stresses opposing working stresses *provided* the load is always applied in the winding sense. Torsion springs can operate at bending stresses exceeding the yield strength of the wire from which it was wound.

The bending stress can be obtained from curved-beam theory expressed in the form

$$\sigma = K\frac{Mc}{I}$$

where K is a stress-correction factor. The value of K depends on the shape of the wire cross section and whether the stress sought is at the inner or outer fiber. Wahl analytically determined the values of K to be, for round wire,

$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} \qquad K_o = \frac{4C^2 + C - 1}{4C(C+1)} \tag{10–43}$$

where C is the spring index and the subscripts i and o refer to the inner and outer fibers, respectively. In view of the fact that K_o is always less than unity, we shall use K_i to estimate the stresses. When the bending moment is $M = Fr$ and the section modulus $I/c = d^3/32$, we express the bending equation as

$$\sigma = K_i\frac{32Fr}{\pi d^3} \tag{10–44}$$

which gives the bending stress for a round-wire torsion spring.

Deflection and Spring Rate

For torsion springs, angular deflection can be expressed in radians or revolutions (turns). If a term contains revolution units the term will be expressed with a prime sign. The spring rate k' is expressed in units of torque/revolution (lbf · in/rev or N · mm/rev) and moment is proportional to angle θ' expressed in turns rather than radians. The spring rate, if linear, can be expressed as

$$k' = \frac{M_1}{\theta_1'} = \frac{M_2}{\theta_2'} = \frac{M_2 - M_1}{\theta_2' - \theta_1'} \tag{10–45}$$

where the moment M can be expressed as Fl or Fr.

The angle subtended by the end deflection of a cantilever, when viewed from the built-in ends, is y/l rad. From Table A–9–1,

$$\theta_e = \frac{y}{l} = \frac{Fl^2}{3EI} = \frac{Fl^2}{3E(\pi d^4/64)} = \frac{64Ml}{3\pi d^4 E} \tag{10–46}$$

For a straight torsion end spring, end corrections such as Eq. (10–46) must be added to the body-coil deflection. The strain energy in bending is, from Eq. (4–18),

$$U = \int \frac{M^2\,dx}{2EI}$$

For a torsion spring, $M = Fl = Fr$, and integration must be accomplished over the length of the body-coil wire. The force F will deflect through a distance $r\theta$ where θ is the angular deflection of the coil body, in radians. Applying Castigliano's theorem gives

$$r\theta = \frac{\partial U}{\partial F} = \int_0^{\pi DN_b} \frac{\partial}{\partial F}\left(\frac{F^2r^2\,dx}{2EI}\right) = \int_0^{\pi DN_b} \frac{Fr^2\,dx}{EI}$$

Substituting $I = \pi d^4/64$ for round wire and solving for θ gives

$$\theta = \frac{64FrDN_b}{d^4E} = \frac{64MDN_b}{d^4E}$$

The total angular deflection in radians is obtained by adding Eq. (10–46) for each end of lengths l_1, l_2:

$$\theta_t = \frac{64MDN_b}{d^4E} + \frac{64Ml_1}{3\pi d^4E} + \frac{64Ml_2}{3\pi d^4E} = \frac{64MD}{d^4E}\left(N_b + \frac{l_1 + l_2}{3\pi D}\right) \tag{10–47}$$

The equivalent number of active turns N_a is expressed as

$$N_a = N_b + \frac{l_1 + l_2}{3\pi D} \tag{10–48}$$

The spring rate k in torque per radian is

$$k = \frac{Fr}{\theta_t} = \frac{M}{\theta_t} = \frac{d^4E}{64DN_a} \tag{10–49}$$

The spring rate may also be expressed as torque per turn. The expression for this is obtained by multiplying Eq. (10–49) by 2π rad/turn. Thus spring rate k' (units torque/turn) is

$$k' = \frac{2\pi d^4E}{64DN_a} = \frac{d^4E}{10.2DN_a} \tag{10–50}$$

Tests show that the effect of friction between the coils and arbor is such that the constant 10.2 should be increased to 10.8. The equation above becomes

$$k' = \frac{d^4E}{10.8DN_a} \tag{10–51}$$

(units torque per turn). Equation (10–51) gives better results. Also Eq. (10–47) becomes

$$\theta_t' = \frac{10.8MD}{d^4E}\left(N_b + \frac{l_1 + l_2}{3\pi D}\right) \tag{10–52}$$

Torsion springs are frequently used over a round bar or pin. When the load is applied to a torsion spring, the spring winds up, causing a decrease in the inside diameter of the coil body. It is necessary to ensure that the inside diameter of the coil never becomes equal to or less than the diameter of the pin, in which case loss of spring function would ensue. The helix diameter of the coil D' becomes

$$D' = \frac{N_b D}{N_b + \theta'_c} \tag{10–53}$$

where θ'_c is the angular deflection of the body of the coil in number of turns, given by

$$\theta'_c = \frac{10.8 M D N_b}{d^4 E} \tag{10–54}$$

The new inside diameter $D'_i = D' - d$ makes the diametral clearance Δ between the body coil and the pin of diameter D_p equal to

$$\Delta = D' - d - D_p = \frac{N_b D}{N_b + \theta'_c} - d - D_p \tag{10–55}$$

Equation (10–55) solved for N_b is

$$N_b = \frac{\theta'_c(\Delta + d + D_p)}{D - \Delta - d - D_p} \tag{10–56}$$

which gives the number of body turns corresponding to a specified diametral clearance of the arbor. This angle may not be in agreement with the necessary partial-turn remainder. Thus the diametral clearance may be exceeded but not equaled.

Static Strength

First column entries in Table 10–6 can be divided by 0.577 (from distortion-energy theory) to give

$$S_y = \begin{cases} 0.78 S_{ut} & \text{Music wire and cold-drawn carbon steels} \\ 0.87 S_{ut} & \text{OQ\&T carbon and low-alloy steels} \\ 0.61 S_{ut} & \text{Austenitic stainless steel and nonferrous alloys} \end{cases} \tag{10–57}$$

Fatigue Strength

Since the spring wire is in bending, the Sines equation is not applicable. The Sines model is in the presence of pure torsion. Since Zimmerli's results were for compression springs (wire in pure torsion), we will use the repeated bending stress ($R = 0$) values provided by Associated Spring in Table 10–10. As in Eq. (10–40) we will use the Gerber fatigue-failure criterion incorporating the Associated Spring $R = 0$ fatigue strength S_r:

$$S_e = \frac{S_r/2}{1 - \left(\dfrac{S_r/2}{S_{ut}}\right)^2} \tag{10–58}$$

The value of S_r (and S_e) has been corrected for size, surface condition, and type of loading, but not for temperature or miscellaneous effects. The Gerber fatigue criterion is now defined. The strength-amplitude component is given by Table 6–7, p. 299, as

$$S_a = \frac{r^2 S_{ut}^2}{2 S_e}\left[-1 + \sqrt{1 + \left(\frac{2 S_e}{r S_{ut}}\right)^2}\right] \tag{10–59}$$

Table 10–10

Maximum Recommended Bending Stresses (K_B Corrected) for Helical Torsion Springs in Cyclic Applications as Percent of S_{ut}

Source: Courtesy of Associated Spring.

Fatigue Life, cycles	ASTM A228 and Type 302 Stainless Steel		ASTM A230 and A232	
	Not Shot-Peened	Shot-Peened*	Not Shot-Peened	Shot-Peened*
10^5	53	62	55	64
10^6	50	60	53	62

This information is based on the following conditions: no surging, springs are in the "as-stress-relieved" condition.

*Not always possible.

where the slope of the load line is $r = M_a/M_m$. The load line is radial through the origin of the designer's fatigue diagram. The factor of safety guarding against fatigue failure is

$$n_f = \frac{S_a}{\sigma_a} \tag{10–60}$$

Alternatively, we can find n_f directly by using Table 6–7, p. 299:

$$n_f = \frac{1}{2}\frac{\sigma_a}{S_e}\left(\frac{S_{ut}}{\sigma_m}\right)^2\left[-1+\sqrt{1+\left(2\frac{\sigma_m}{S_{ut}}\frac{S_e}{\sigma_a}\right)^2}\right] \tag{10–61}$$

EXAMPLE 10–8

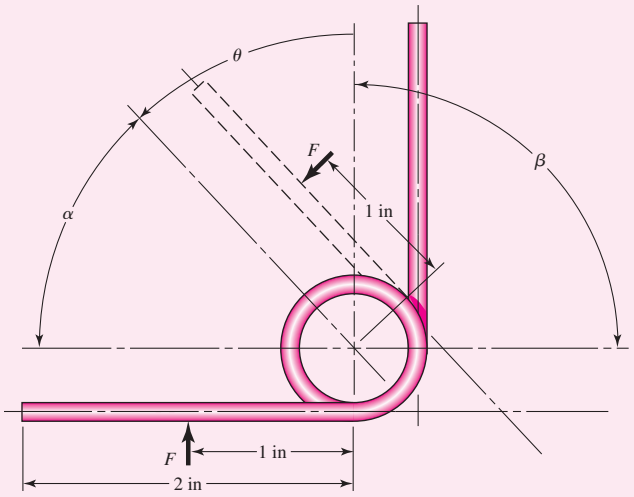
A stock spring is shown in Fig. 10–11. It is made from 0.072-in-diameter music wire and has $4\frac{1}{4}$ body turns with straight torsion ends. It works over a pin of 0.400 in diameter. The coil outside diameter is $\frac{19}{32}$ in.

(*a*) Find the maximum operating torque and corresponding rotation for static loading.

(*b*) Estimate the inside coil diameter and pin diametral clearance when the spring is subjected to the torque in part (*a*).

Figure 10–11

Angles α, β, and θ are measured between the straight-end centerline translated to the coil axis. Coil OD is 19/32 in.



(*c*) Estimate the fatigue factor of safety n_f if the applied moment varies between $M_{\min} = 1$ to $M_{\max} = 5$ lbf · in.

Solution

(*a*) For music wire, from Table 10–4 we find that $A = 201$ kpsi · in^m and $m = 0.145$. Therefore,

$$S_{ut} = \frac{A}{d^m} = \frac{201}{(0.072)^{0.145}} = 294.4 \text{ kpsi}$$

Using Eq. (10–57) gives

$$S_y = 0.78 S_{ut} = 0.78(294.4) = 229.6 \text{ kpsi}$$

The mean coil diameter is $D = 19/32 - 0.072 = 0.5218$ in. The spring index $C = D/d = 0.5218/0.072 = 7.247$. The bending stress correction factor K_i from Eq. (10–43), is

$$K_i = \frac{4(7.247)^2 - 7.247 - 1}{4(7.247)(7.247 - 1)} = 1.115$$

Now rearrange Eq. (10–44), substitute S_y for σ, and solve for the maximum torque Fr to obtain

$$M_{\max} = (Fr)_{\max} = \frac{\pi d^3 S_y}{32 K_i} = \frac{\pi (0.072)^3 229\,600}{32(1.115)} = 7.546 \text{ lbf} \cdot \text{in}$$

Note that no factor of safety has been used. Next, from Eq. (10–54), the number of turns of the coil body θ'_c is

$$\theta'_c = \frac{10.8 M D N_b}{d^4 E} = \frac{10.8(7.546)0.5218(4.25)}{0.072^4 (28.5) 10^6} = 0.236 \text{ turn}$$

Answer

$$(\theta'_c)_{\text{deg}} = 0.236(360^\circ) = 85.0^\circ$$

The active number of turns N_a, from Eq. (10–48), is

$$N_a = N_b + \frac{l_1 + l_2}{3\pi D} = 4.25 + \frac{1 + 1}{3\pi (0.5218)} = 4.657 \text{ turns}$$

The spring rate of the complete spring, from Eq. (10–51), is

$$k' = \frac{0.072^4 (28.5) 10^6}{10.8(0.5218)4.657} = 29.18 \text{ lbf} \cdot \text{in/turn}$$

The number of turns of the complete spring θ' is

$$\theta' = \frac{M}{k'} = \frac{7.546}{29.18} = 0.259 \text{ turn}$$

Answer

$$(\theta'_s)_{\text{deg}} = 0.259(360^\circ) = 93.24^\circ$$

(*b*) With no load, the mean coil diameter of the spring is 0.5218 in. From Eq. (10–53),

$$D' = \frac{N_b D}{N_b + \theta'_c} = \frac{4.25(0.5218)}{4.25 + 0.236} = 0.494 \text{ in}$$

The diametral clearance between the inside of the spring coil and the pin at load is

Answer
$$\Delta = D' - d - D_p = 0.494 - 0.072 - 0.400 = 0.022 \text{ in}$$

(*c*) Fatigue:

$$M_a = (M_{\max} - M_{\min})/2 = (5 - 1)/2 = 2 \text{ lbf} \cdot \text{in}$$

$$M_m = (M_{\max} + M_{\min})/2 = (5 + 1)/2 = 3 \text{ lbf} \cdot \text{in}$$

$$r = \frac{M_a}{M_m} = \frac{2}{3}$$

$$\sigma_a = K_i \frac{32 M_a}{\pi d^3} = 1.115 \frac{32(2)}{\pi 0.072^3} = 60\,857 \text{ psi}$$

$$\sigma_m = \frac{M_m}{M_a} \sigma_a = \frac{3}{2}(60\,857) = 91\,286 \text{ psi}$$

From Table 10–10, $S_r = 0.50 S_{ut} = 0.50(294.4) = 147.2$ kpsi. Then

$$S_e = \frac{147.2/2}{1 - \left(\dfrac{147.2/2}{294.4}\right)^2} = 78.51 \text{ kpsi}$$

The amplitude component of the strength S_a, from Eq. (10–59), is

$$S_a = \frac{(2/3)^2 294.4^2}{2(78.51)} \left[-1 + \sqrt{1 + \left(\frac{2}{2/3} \frac{78.51}{294.4} \right)^2} \right] = 68.85 \text{ kpsi}$$

The fatigue factor of safety is

Answer
$$n_f = \frac{S_a}{\sigma_a} = \frac{68.85}{60.86} = 1.13$$

10–13 Belleville Springs

The inset of Fig. 10–12 shows a coned-disk spring, commonly called a *Belleville spring*. Although the mathematical treatment is beyond the scope of this book, you should at least become familiar with the remarkable characteristics of these springs.

Aside from the obvious advantage that a Belleville spring occupies only a small space, variation in the h/t ratio will produce a wide variety of load-deflection curve shapes, as illustrated in Fig. 10–12. For example, using an h/t ratio of 2.83 or larger gives an S curve that might be useful for snap-acting mechanisms. A reduction of the ratio to a value between 1.41 and 2.1 causes the central portion of the curve to become horizontal, which means that the load is constant over a considerable deflection range.

A higher load for a given deflection may be obtained by nesting, that is, by stacking the springs in parallel. On the other hand, stacking in series provides a larger deflection for the same load, but in this case there is danger of instability.