

Course 1

Modules 1–6

Fundamentals of Light and Lasers

3rd Edition



OP-TEC

Optics and Photonics Series

Course 1
**Fundamentals of Light
and Lasers**

3rd Edition

OPTICS AND PHOTONICS SERIES



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PREFACE

The six instructional modules (chapters) contained in this text are designed for use by students and instructors involved in the preparation of technicians in the areas of optics, electro-optics, lasers, and photonics. The materials can be used as an introductory course in AAS programs in laser/electro-optics and photonics at two-year postsecondary community and technical colleges, as supplementary laser/electro-optics courses in related fields such as biomedical equipment, manufacturing, defense and nanotechnology, as well as in programs designed to retrain or update the skills of engineering technicians who are already employed. These materials can also support dual-credit offerings for high school students in STEM career pathways. The modules are designed to be used as a unit or independently, as long as prerequisites have been met.

Acknowledgments

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Linda J. Vandergriff (Science Applications International Corporation) (Module 1-1)
Raman Kolluri (Camden County College, Blackwood, New Jersey) (Modules 1-2 and 1-6)
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Dan Hull of OP-TEC provided editorial oversight for the 3rd Edition. Workplace scenarios were developed by Gary Beasley (Central Carolina Community College, Lillington, North Carolina) and the Meet a Technician feature was developed by Dan Hull of OP-TEC. Formatting was provided by Rachel Haferkamp of OP-TEC.

FEATURES

Mathematics Videos: Math videos are provided to assist students in performing mathematical operations required in the text. The following is a list of available videos:

1. Scientific Notation
2. Unit Conversion
3. Introductory Algebra
4. Powers and Roots
5. Ratio and Proportion
6. Exponents and Logarithms
7. Graphing in Rectangular Coordinates
8. Geometry
9. Angle measure in Two and Three Dimension
10. Trigonometry
11. Special Graphs

These videos can be accessed at <http://optecvideo.opteccrm.org/> In the Preface of each module, this list of topics is repeated with check marks (✓) by those mathematics topics that will be used in the module. This checklist provides instructors a means of front loading the presentation of material in a module with mathematics instruction. If instructors prefer to provide math instruction using a “just on time delivery” mode, a special icon  appears where the mathematics topics presented in these videos are first used within a module.

Laboratory Videos: Laboratories that support, reinforce, and provide a hands-on means of learning presented material are included in each module of the text. To assist students in preparing for these laboratories, the text provides internet accessibility to videos that present safety issues, equipment use, data collection techniques, required calculations, and expected results related to these laboratories. Instructors can use these videos as pre-lab activities to familiarize students with the laboratories, making their laboratory time more efficient and effective. Laboratory procedures are located at the end of each module; the  icon identifies those that have videos. Students can access these videos at <http://optecvideo.opteccrm.org/>.

Mini-Tutorial Videos: The text contains some photonics concepts that students often require addition instruction to learn. To assist students with these concepts, mini-tutorial videos are available in the text. These videos present instruction by experienced photonics faculty that give new insights into understanding these difficult concepts and help supplement the instruction students are receiving in their classrooms. Instructors will find these videos valuable in providing out-of-class tutoring assistance to students and also in planning lessons involving these concepts. Each module in the text contains a Mini-Tutorial video which is identified by  the icon and these videos are accessible at <http://optecvideo.opteccrm.org/>.

Applets: Some photonics concepts are difficult to visualize. Within each module, applets are referenced that provide students interactive, graphic simulations to improve their understanding of presented concepts. There are over 50 applets distributed throughout the six modules of the text. Applets applicable to certain content within the text are highlighted by a capital **A** with numbers appearing under them. These numbers refer to specific applets that are accessible at <http://optecvideo.optecrm.org/>.

Real World Connections and Applications: Students are interested in how photonics technicians prepare for their careers, what skills they will need to be successful in future jobs, and what kinds of responsibilities they might be assigned in the workplace. The text has three features that address these questions and bring real world context to its content.

Scenarios: Each module in the text begins the presentation of material with a short essay that depicts the role of photonics technicians in the workplace and provides a link between the material contained in the module and how these technicians apply it.

Meet a Technician: This feature contains a bio of actual technicians with workforce experience in the photonics industry. The bio presents details about how these individuals decided to become technicians, what their experiences were in earning a degree in photonics, what responsibilities they have in the workplace, and how their photonics degree led them to their current jobs. A brief bio from a member of the Photonics Alumni Counsel for Technicians (PACT) is located at the end of each module.

Workplace Scenarios: These problem-based activities allow students to work in groups to generate solutions to tasks that can arise in today's photonics organizations. Students take on technician roles and, within teams, use the material in a module to develop strategies and results for meeting assigned tasks. These activities also provide students an opportunity to enhance their writing skills by requiring reports, in the form of e-mails and memorandums. The scenarios are instructor-friendly with full solutions and instructor notes provided. Teacher Notes and Solutions for the Workplace Scenarios can be obtained by instructors only through OP-TEC at www.op-tec.org/laser.

Glossary of Terms: A glossary of terms for Course 1: Fundamentals of Light and Lasers 3rd Edition.

Index: An index of term for Course 1: Fundamentals of Light and Lasers 3rd Edition.

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Basic Geometrical Optics

Module 1-4
of
Course 1, *Fundamentals of Light and Lasers*
3rd Edition

OPTICS AND PHOTONICS SERIES



PREFACE TO MODULE 1-4

This is the fourth module in *Fundamentals of Light and Lasers*. Following are the titles of all six modules in the course:

1. *Nature and Properties of Light*
2. *Optical Handling and Positioning*
3. *Light Sources and Laser Safety*
4. *Basic Geometrical Optics*
5. *Basic Physical Optics*
6. *Principles of Lasers*

The six modules can be used as a unit or independently, as long as prerequisites have been met.

FEATURES

Mathematics Videos: Math videos are provided to assist students in doing mathematical operations required in this module. The math concepts used in this module are checked ✓ in the following list:

1. Scientific Notation
2. Unit Conversion
3. Introductory Algebra ✓
4. Powers and Roots ✓
5. Ratio and Proportion
6. Exponents and Logarithms
7. Graphing in Rectangular Coordinates ✓
8. Geometry
9. Angle Measure in Two and Three Dimension ✓
10. Trigonometry ✓
11. Special Graphs

This checklist provides instructors a means of front-loading the presentation of material in this module with mathematics instruction. If instructors prefer to provide math instruction using a “just on time delivery” mode, a special icon  appears where the checked mathematics topics are first used within this module. All math videos can be accessed at <http://optecvideo.opteccrm.org/>.

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Module 1-4

Basic Geometrical Optics

INTRODUCTION

Optics is the cornerstone of photonics systems and applications. In this module, you will learn about one of the two main divisions of basic optics—*geometrical* (ray) *optics*. In the next module, you will learn about the other—*physical* (wave) *optics*. *Geometrical optics* will help you understand the basics of light reflection and refraction and the use of simple optical elements such as mirrors, prisms, lenses, and fibers. *Physical optics* will help you understand the phenomena of light waves, light wave interference, diffraction, and polarization, the use of thin film coatings on mirrors to enhance or suppress reflection, and the operation of such devices as gratings and quarter-wave plates.

PREREQUISITES

Before you work through this module, you should have completed Module 1-1, *Nature and Properties of Light*; Module 1-2, *Optical Handling and Positioning*; and Module 1-3, *Light Sources and Laser Safety*. In addition, you should be able to manipulate and use algebraic formulas, deal with units, understand the geometry of circles and triangles, and use the basic trigonometric functions (sin, cos, tan) as they apply to the relationships of sides and angles in right triangles.

OBJECTIVES

When you finish this module, you will be able to:

- Distinguish between *light rays* and *light waves*.
- State the *law of reflection* and show with appropriate drawings how it applies to light rays at plane and spherical surfaces.
- State *Snell's law of refraction* and show with appropriate drawings how it applies to light rays at plane and spherical interfaces between different optical media.
- Define *index of refraction* and give typical values for glass, water, and air.
- Calculate the *critical angle* of incidence for the interface between two optical media and describe the process of *total internal reflection*.

- Describe how total internal reflection can be used to redirect light in prisms and trap light in fibers.
- Describe *dispersion* of light and show how a prism disperses white light.
- Calculate the *minimum angle of deviation* for a prism and show how this angle can be used to determine the refractive index of a transparent material shaped into a prism.
- Describe what is meant by *Gaussian* or *paraxial* optics.
- Describe the relationship between *collimated light* and the *focal points* of convex and concave *mirrors*.
- Use *ray-tracing techniques* to locate the images formed by plane and spherical mirrors.
- Use the *mirror equations* to determine location, size, orientation, and nature of images formed with spherical mirrors.
- Distinguish between a *thin lens* and a *thick lens*.
- Describe the shapes of three typical *converging (positive)* thin lenses and three typical *diverging (negative)* thin lenses.
- Describe the *f-number* and *numerical aperture* for a lens and explain how they control image brightness.
- Use *ray-tracing techniques* to locate images formed by *thin lenses*.
- Describe the relationship between *collimated light* and the *focal points* of a *thin lens*.
- Use the *lensmaker's equation* to determine the focal length of a thin lens.
- Use the *thin-lens equations* to determine location, size, orientation, and nature of the images formed by simple lenses.

SCENARIO

Manuel Martinez is a photonics technician hired recently to work for a large optical company that manufactures optical components such as mirrors, lenses, prisms, beam splitters, fibers, and Brewster windows—all to customer specifications. While in school, Manuel studied light imaging with mirrors and lenses, ray tracing, and calculations with simple formulas. After two months on the job, he discovered that he uses those same ideas day in and day out. To be sure, things are much more “high tech” in his company, for now Manuel has access to powerful computers and computer programs that trace rays through complicated optical systems, often containing elements with nonspherical surfaces, something Manuel never had a chance to do at school. He enjoys the challenge of using state-of-the-art lab equipment he’s never seen before, including autocollimators, spectrophotometers, and surface profilers. All in all, he’s really satisfied because all of the optics he had in his “Geo” course back at school really prepared him for his laboratory work here. This month, Manuel is learning how to “grind and polish optical surfaces to spec,” and how to apply the principles of geometrical optics to determine when the surfaces are “near tolerance.” Manuel finds his work fascinating and can hardly wait to get to work each morning. “Geo” was never so much fun.

BASIC CONCEPTS

The Laws of Reflection and Refraction

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The numbers below the **A** that appears in several places in this module reference specific applets that demonstrate the concepts being described in the text. These applets are accessible at <http://optecvideo.opteccrm.org/>.

We begin our study of *basic geometrical optics* by examining how light reflects and refracts at smooth, *plane* interfaces. Figure 4-1a shows ordinary *reflection* of light at a plane surface, and Figure 4-1b shows *refraction* of light at two successive plane surfaces. In each instance, light is pictured simply in terms of straight lines, which we refer to as *light rays*.

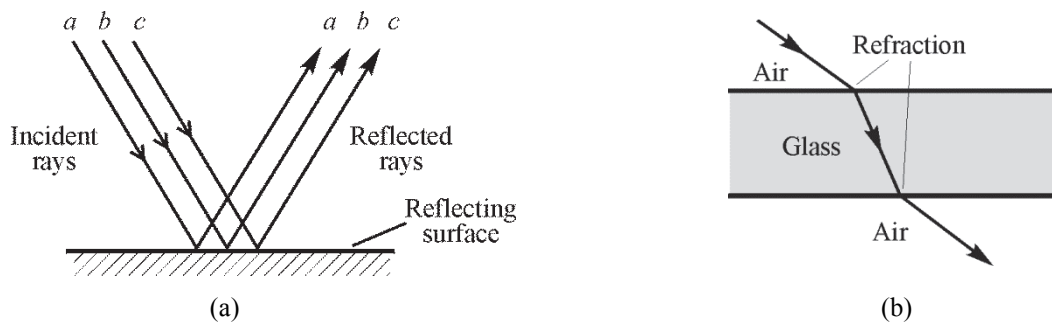


Figure 4-1 Light rays undergoing reflection and refraction at plane surfaces

After a study of how light reflects and refracts at plane surfaces, we extend our analysis to smooth, curved surfaces, thereby setting the stage for light interaction with mirrors and lenses—the basic elements in many optical systems.

In this module, the analysis of how light interacts with plane and a curved surface is carried out with *light rays*. A light ray is nothing more than an imaginary line directed along the path that light energy follows. It is helpful to think of a light ray as a thin pencil of light, very much like a narrow, well-defined laser beam. For example, if you observe the passage of a laser beam in a fish tank filled with milky water and visually trace the path of the beam from entrance into and exit from the tank, you are, in effect, looking at a “light ray” representation of light energy passing through the tank. We all use light rays to explain how shadows are formed.

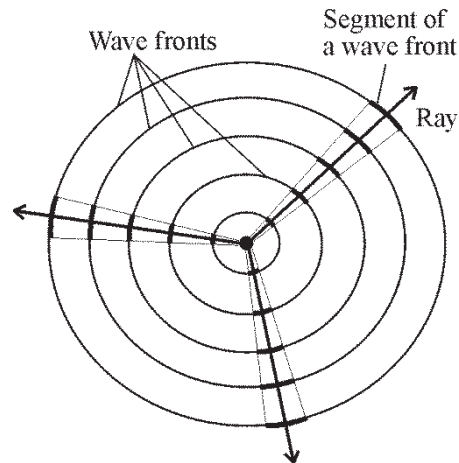
Light rays and light waves

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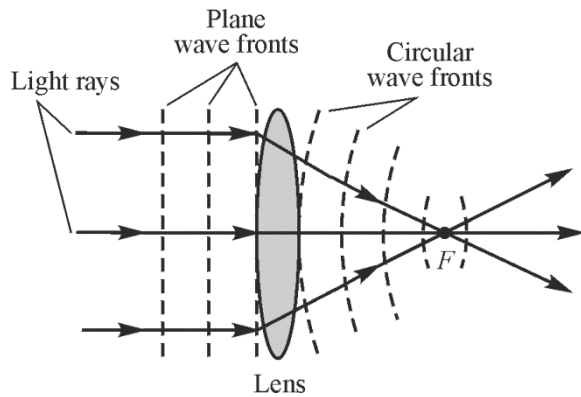
Before we look more closely at the use of *light rays*, we need to say a brief word about *light waves* and the connection between light rays and light waves. For most of us, *wave motion* is easily visualized in terms of water waves—such as those created on a quiet pond by a bobbing cork. See Figure 4-2a. The successive high points (crests) and low points (troughs) on the water surface occur as a train of circular waves moves radially outward from the bobbing cork. Each of the circular waves represents a *wave front*. A wave front is defined here as a series of adjacent points that connect identical wave displacements—that is, points that connect identical positions above or below the normal surface of the quiet pond.



(a) Radiating circular waves from a bobbing cork



(b) Light rays and circular wave fronts



(c) Bending of light rays and changing shapes of wave fronts by a thin lens

Figure 4-2 *Waves and rays*

In Figure 4-2b, circular wave fronts are shown with radial lines drawn perpendicularly to them along several directions. Each of the rays describes the motion of a *restricted part or segment* of the wave front along a particular direction. Geometrically, then, a *ray* is a line perpendicular to a series of successive *wave fronts*. The direction of the ray specifies the *direction of energy flow* in the wave.

Figure 4-2c shows a cross section of plane wave fronts of light bent by a lens into circular wave fronts that then converge onto a focal point F . The same diagram shows three particular *light rays* perpendicular to these wave fronts, undergoing bending by the lens as they head for the focal point F . Figure 4-2c shows the geometrical connection between actual waves and the rays used to represent them. In the study of *geometrical optics*, we find it useful to represent the interaction of light waves with plane and spherical surfaces—such as those found in mirrors and lenses—in terms of the bending of *light rays*.

With the useful geometrical idea of a simple light ray, we can illustrate propagation, reflection, and refraction of light in clear, uncomplicated drawings. For example, in Figure 4-3a, the propagation of light from a “point source” is represented by equally spaced light rays radiating outwardly from the source. Each ray indicates the geometrical path along which light energy moves as it leaves the source. Figure 4-3b shows the reflection of several light rays at a curved mirror surface, and Figure 4-3c shows the refraction (bending) of a single light ray passing through a prism, that is, as it passes from air, through the prism material, and again into air.

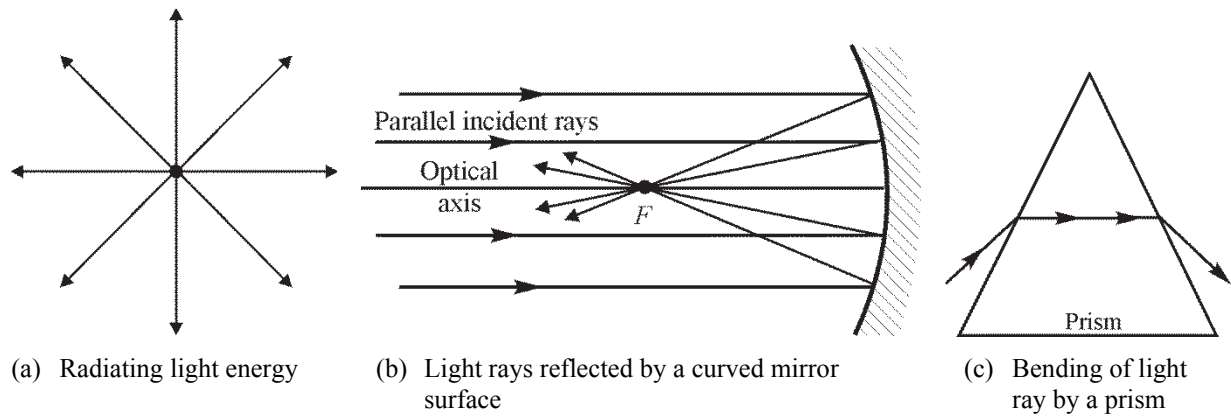


Figure 4-3 Typical light rays in (a) propagation, (b) reflection, and (c) refraction

A Reflection of light from optical surfaces

119,
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When light is incident on an *interface* between two transparent optical media—such as at the surface between air and glass or between water and glass—four things can happen to the incident light.

- It can be partly or totally reflected at the interface.
- It can be scattered in random directions at the interface.
- It can be partly transmitted via refraction at the interface and enter the second medium.
- It can be partly absorbed in either medium.

In our introductory study of geometrical optics, we shall consider only smooth surfaces that give rise to *specular* (regular, geometric) reflections as shown in Figure 4-4a. We shall ignore ragged, uneven surfaces that give rise to *diffuse* (irregular) reflections as shown in Figure 4-4b.

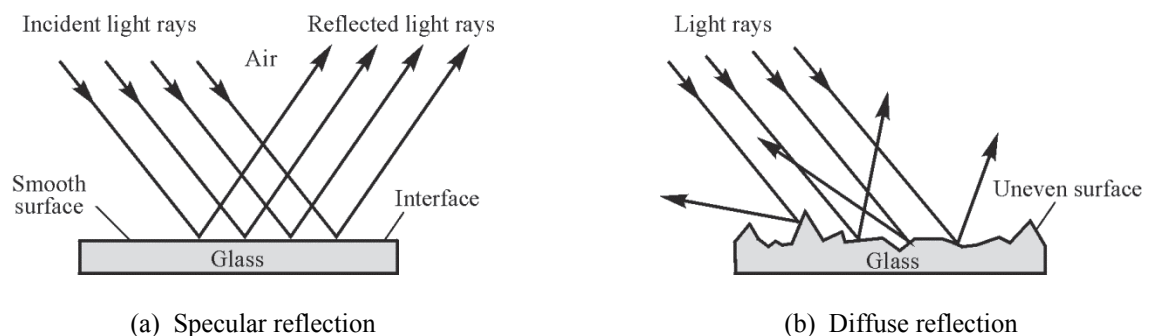


Figure 4-4 Specular and diffuse reflection

In addition, for simplicity, we shall ignore the *absorption* of light energy that most always occurs along a path of travel. But, keep in mind that absorption is an important consideration when the amount of light energy transmitted from source to receiver is a factor of concern, as, for instance, when we want as much light energy as possible to pass through an optical system such as a microscope.

The law of reflection: plane surface—When light reflects from a plane surface, as shown in Figure 4-5a, the angle (B) formed by the *reflected* ray and the *normal* (line perpendicular to the surface) at the point of incidence is *always equal to* the angle (A) formed by the *incident* ray and the same normal. Note carefully that the incident ray, reflected ray, and normal always lie in the *same plane*.

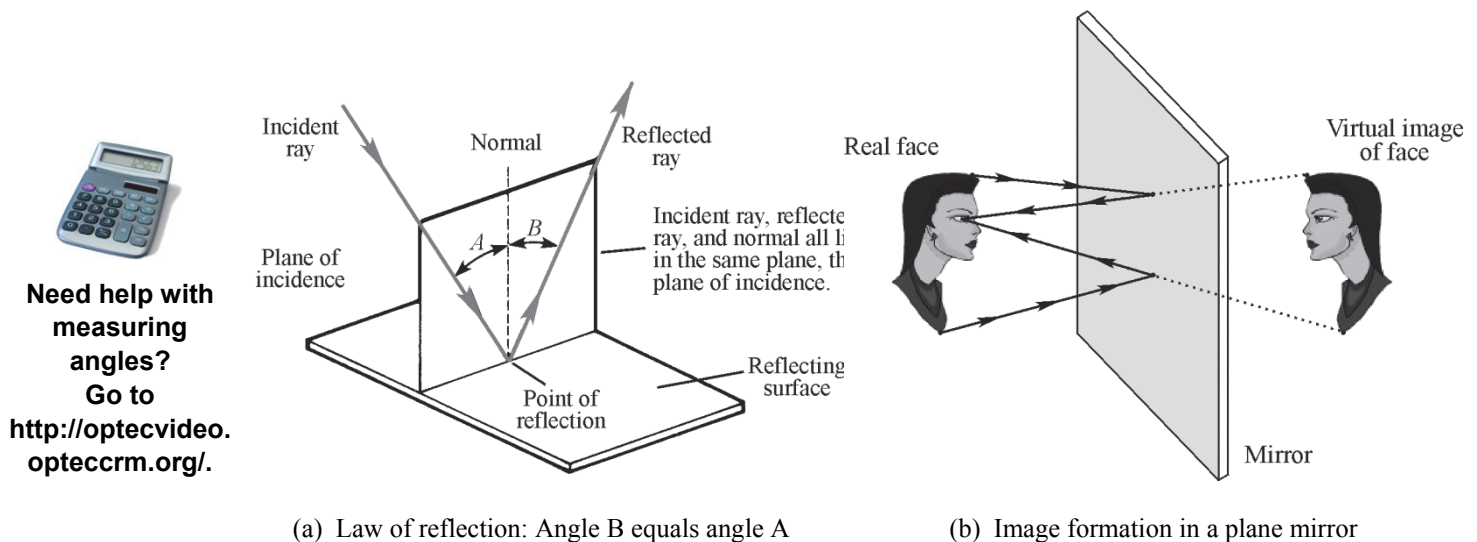


Figure 4-5 Reflection from a plane surface and a mirror

The geometry of Figure 4-5a reminds us that reflection of light rays from a plane, smooth surface is like the geometry of pool shots “banked” off the walls of a billiard table.

With the *law of reflection* in mind, we can see that, for the specular reflection shown earlier in Figure 4-4a, each of the incident, parallel rays reflects off the surface at the same angle, thereby remaining parallel in reflection as a group. In Figure 4-4b, where the surface is made up of many *small*, randomly oriented plane surfaces, each ray reflects in a direction different from its neighbor, even though each ray does obey the *law of reflection* at its own small surface segment.

The image of your face that you see in a mirror each morning is formed by light rays obeying the *law of reflection*. Light rays leave each point on your face, travel to the mirror, and reflect back to your eyes, as shown in Figure 4-5b. The extension of these rays behind the mirror forms a *virtual* image of your face. The image is called “virtual” because it’s **not** formed behind the mirror, even though you “see” it there.

Reflection from a curved surface—With spherical mirrors, reflection of light occurs along a curved surface. The *law of reflection* holds, since at each point on the curved surface one can draw a *surface tangent* (a plane surface) and erect a *normal* to a point *P* on the surface where the light is incident, as shown in Figure 4-6. One then applies the *law of reflection* at point *P* just as was illustrated in Figure 4-5, with the incident and reflected rays making the same angles (*A* and *B*) with the normal to the surface at *P*. Note that successive surface tangents along the curved mirror surface in Figure 4-6 are *ordered* (not random) sections of “plane mirrors” and serve—when connected—as a spherical surface mirror, capable of forming distinct images.

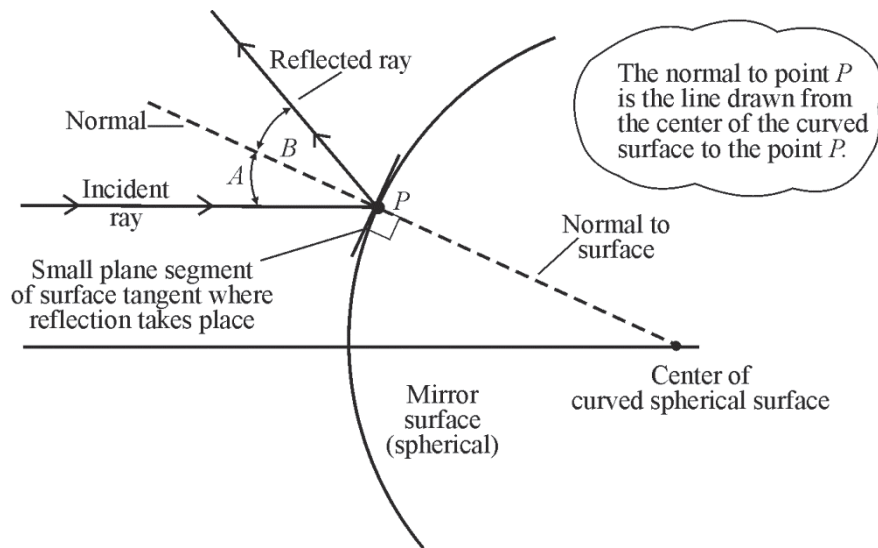
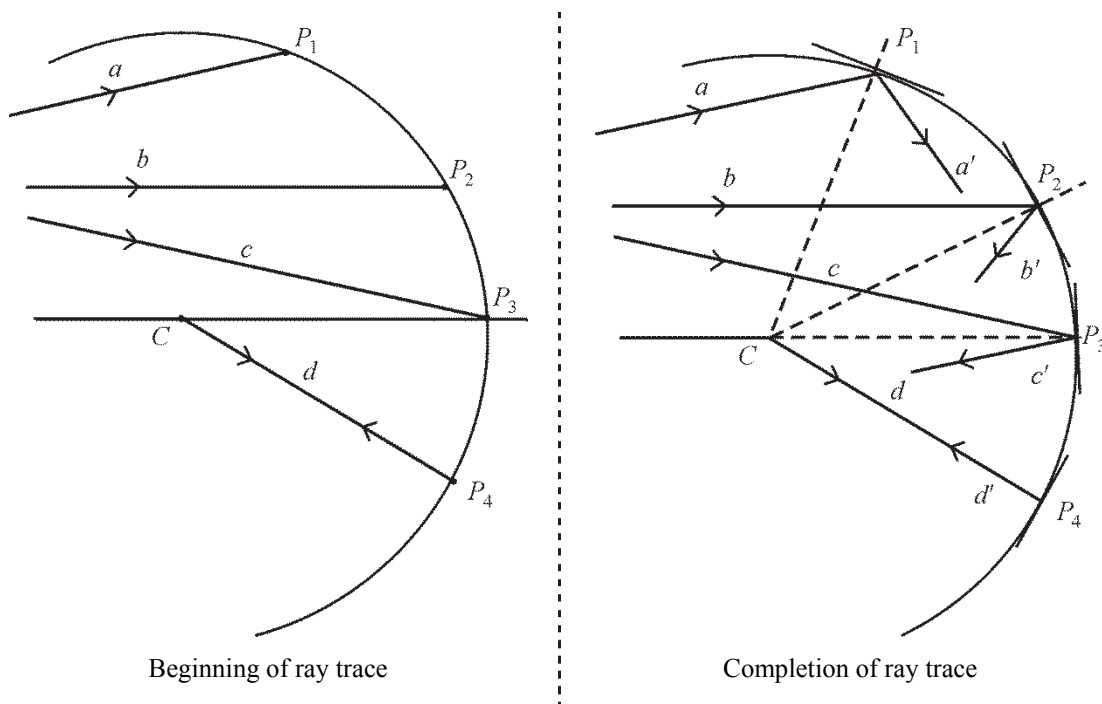


Figure 4-6 Reflection at a curved spherical surface: Angle B equals angle A .

Thus, since point P can be moved anywhere along the curved surface and a normal can be drawn there, we can always find the direction of the reflected ray by applying the *law of reflection*. We shall apply this technique when studying the way mirrors reflect light to form images.

Example 1

Using the *law of reflection*, complete the ray-trace diagram for the four rays (a , b , c , d) incident on the curved surface shown at the left below, given that the center of the curved surface is at point C .



Solution: Draw a normal (shown dashed) from point C to each of the points P_1 , P_2 , P_3 , and P_4 , as shown above in the drawing at the right. At each point, draw the appropriate reflected ray (a' , b' , c' , d') so that each ray makes an angle with its normal equal to the angle made by its incident ray (a , b , c , d) at that point. Note that ray d reflects *back along itself* since it is incident along the line of the normal from C to point P_4 .

Refraction of light from optical interfaces

When light is incident at an *interface*—the geometrical plane that separates one optical medium from another—it will be *partly reflected* and *partly transmitted*. Figure 4-7 shows a three-dimensional view of light incident on a partially reflecting surface (interface), being partly reflected there (according to the *law of reflection*) and partly refracted into the second medium. The bending of light rays at an interface between two optical media is called *refraction*. Before we examine, in detail, the process of refraction, we need to describe optical media in terms of an *index of refraction*.

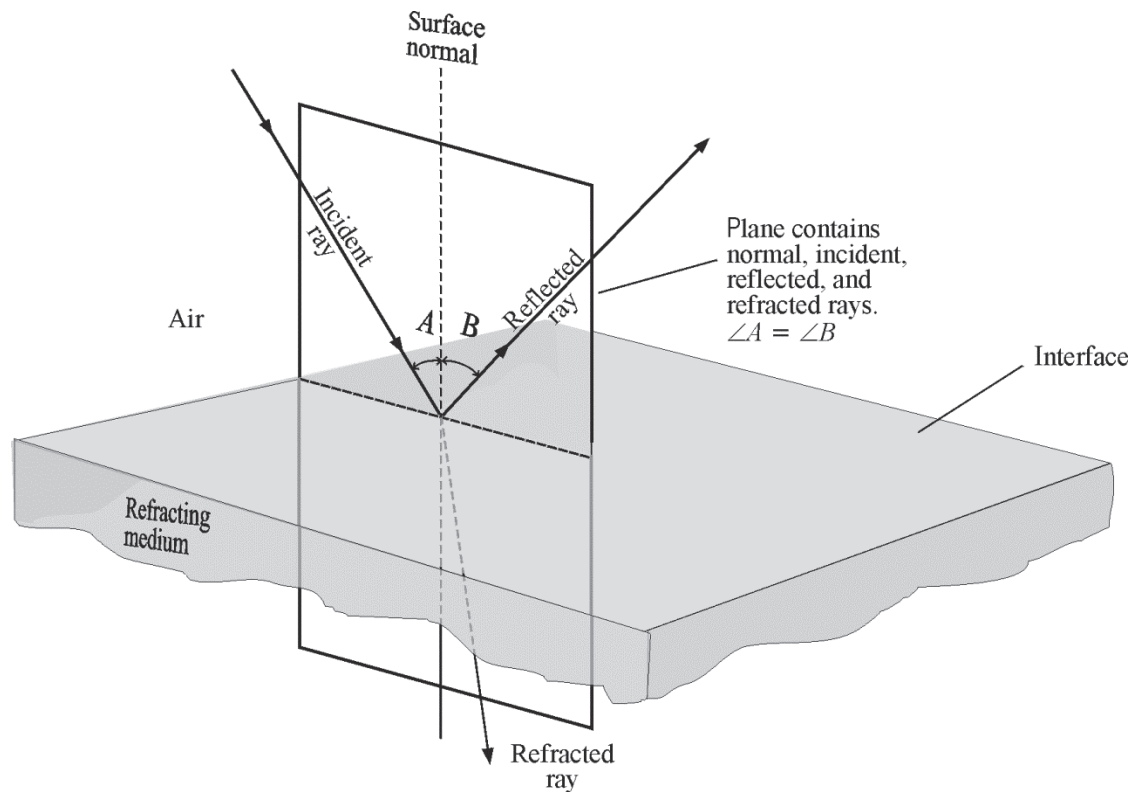


Figure 4-7 Reflection and refraction at an interface between air and a different optical medium

Index of refraction—The two transparent optical media that form an interface are distinguished from one another by a constant called the *index of refraction*, generally labeled with the symbol n . The index of refraction for any transparent optical medium is defined as the ratio of the speed of light in a vacuum to the speed of light in the medium, as given in Equation 4-1.

$$n = \frac{c}{v} \quad (4-1)$$

where c = speed of light in free space (vacuum)

v = speed of light in the medium

n = index of refraction of the medium

The index of refraction for free space (a vacuum) is exactly *one*. For air and most gases, it is very nearly one, so in most calculations it is taken to be 1.0. For other optical materials—such as glass, silicon, diamond, gallium arsenide, and germanium—it has values greater than one. Table 4-1 lists indexes of refraction for some common materials. Note the relatively high refractive indexes for the semiconductor materials gallium arsenide, silicon, and germanium.

Table 4-1. Indexes of Refraction for Various Materials at 589 nm

Substance	n	Substance	n
Air	1.0003	Glass (flint)	1.66
Benzene	1.50	Glycerin	1.47
Carbon disulfide	1.63	Polystyrene	1.49
Corn syrup	2.21	Quartz (fused)	1.46
Diamond	2.42	Sodium chloride	1.54
Ethyl alcohol	1.36	Water	1.33
Gallium arsenide (semiconductor)	3.40	Ice	1.31
Glass (crown)	1.52	Germanium	4.1
Zircon	1.92	Silicon	3.5

The greater the index of refraction of a medium, the slower the speed of light in that medium and the greater the light ray is bent in going from air into the medium. Figure 4-8 shows two general cases, one for light passing from a medium of lower index into a medium of higher index, such as from air into glass, the other from a medium of higher index into a medium of lower index, such as from glass into water. Note that in the first case (lower-to-higher), the light ray is always *bent toward the normal*. In the second case (higher-to-lower), the light ray is always *bent away from the normal*. It is helpful to memorize these effects since they often help one trace light through optical media in a generally correct manner.

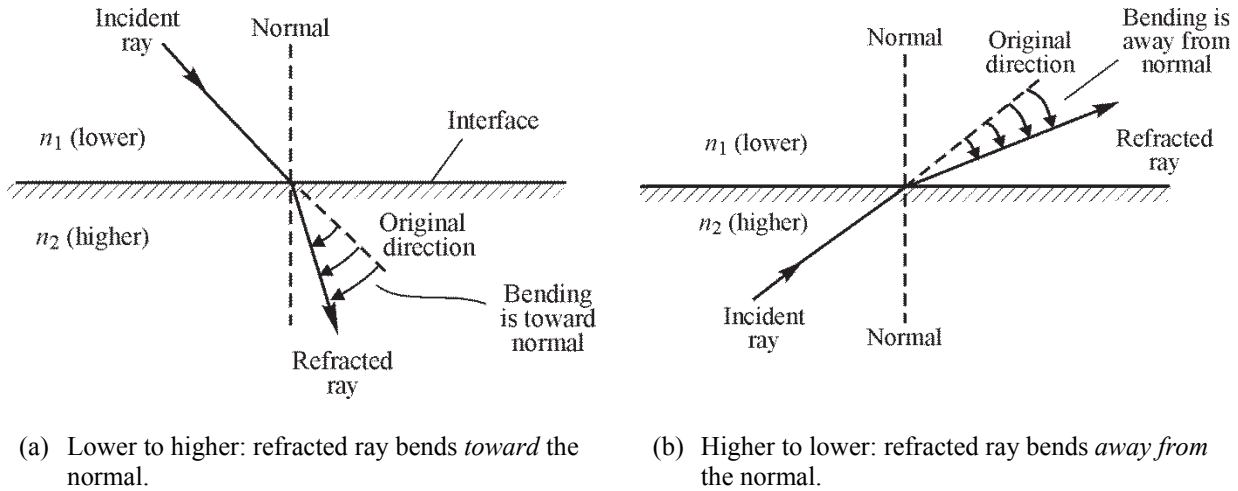


Figure 4-8 Refraction at an interface between media of different refractive indexes n_1 and n_2

Snell's law—*Snell's law of refraction* relates the *sines* of the angles of incidence and refraction at an interface between two optical media to the *indexes of refraction* of the two media. The law is named after a Dutch astronomer, Willebrord Snell, who formulated the law in the 17th century. Snell's law enables us to calculate the direction of the refracted ray if we know the refractive indexes of the two media and the direction of the incident ray. The mathematical expression of Snell's law and an accompanying drawing are given in Figure 4-9.

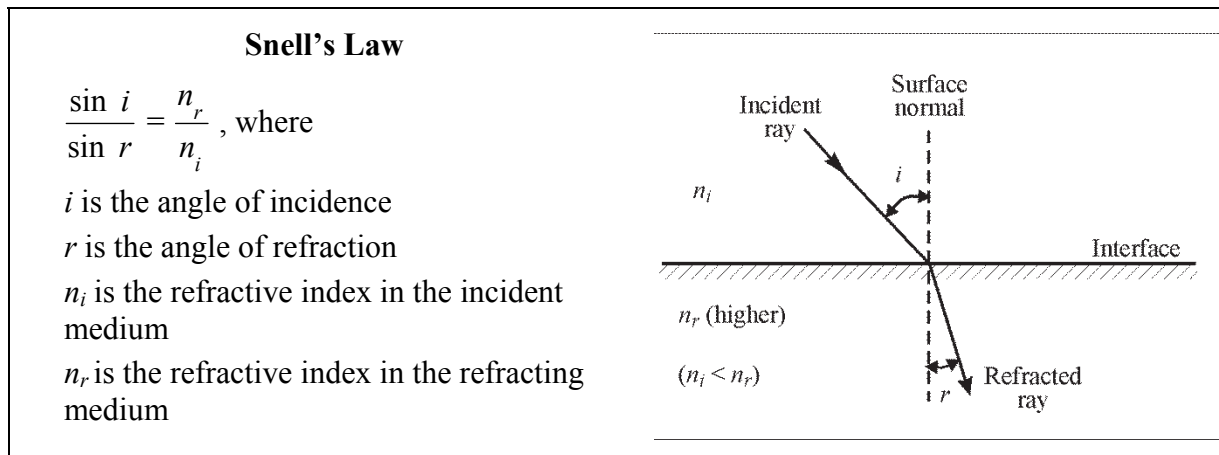


Figure 4-9 Snell's law: formula and geometry

Note carefully that both the angle of incidence (i) and the angle of refraction (r) are measured with respect to the surface normal. Note again that the incident ray, surface normal, and refracted ray all lie in the same geometrical plane.

In practice Snell's law is often written simply as

$$n_i \sin i = n_r \sin r \quad (4-2)$$

Need help with trigonometry?

Go to
<http://optecvideo.opteccrm.org/>



Now let's look at an example that makes use of *Snell's law*.

Example 2

In a handheld optical instrument used under water, light is incident from water onto the plane surface of flint glass at an angle of incidence of 45° .

- (a) What is the angle of reflected light off the flint glass surface?
- (b) Does the refracted ray bend toward or away from the normal?
- (c) What is the angle of refraction at the flint glass surface?

Solution

- (a) From the *law of reflection*, the reflected light must head off at an angle of 45° with the normal. (Note: The angle of reflection is **not** dependent on the refractive indexes of the two media.)
- (b) From Table 4-1, the index of refraction is 1.33 for water and 1.63 for flint glass. Thus, light is moving from a lower to a higher index of refraction and will bend **toward** the normal. Thus, we know right away that the angle of refraction r should be less than 45° .
- (c) From *Snell's law*, Equation 4-2, we have:

$$n_i \sin i = n_r \sin r$$

$$\text{where } n_i = 1.33, i = 45^\circ, \text{ and } n_r = 1.63$$

$$\text{Thus, } \sin r = \frac{1.33 \sin 45^\circ}{1.63} = \frac{(1.33)(0.707)}{1.63} = 0.577$$

$$\text{So } r = \sin^{-1}(0.577) = 35.2^\circ$$

The angle of refraction is about 35° , clearly less than 45° , just as was predicted in part (b).

Note: The function $\sin^{-1}$ is also referred to as the *arcsin*. We will use the $\sin^{-1}$ notation since that is what is found on the keypads of scientific calculators.

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algebra?
Go to
[http://optecvideo
.opteccrm.org/](http://optecvideo.opteccrm.org/).

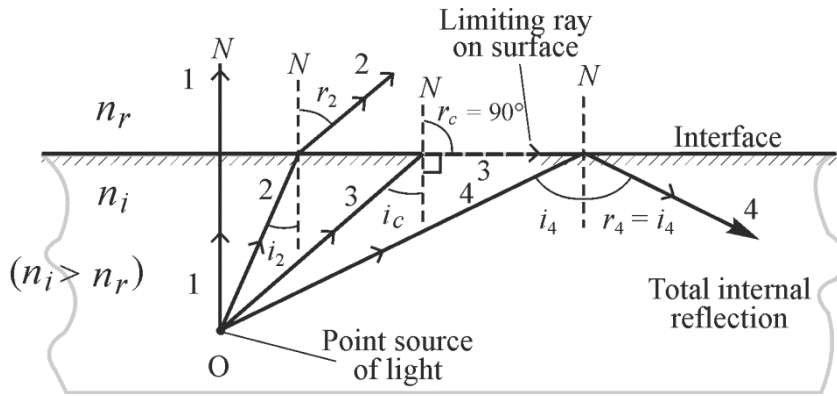


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Critical angle and total internal reflection—When light travels from a medium of *higher index to one of lower index*, we encounter some interesting results. Refer to Figure 4-10, where we see four rays of light originating from point O in the higher-index medium. Each ray is incident on the interface at a different angle of incidence. Ray 1 is incident on the interface at 90° (normal incidence) so there is no bending as it moves into the lower-index medium.

The light in the direction along the normal N at the interface simply speeds up in the second medium (why?) as it continues along the same direction. Ray 2 is incident at angle i_2 and refracts (bends away from the normal N) at angle r_2 . Ray 3 is incident at the so-called *critical angle* i_c , large enough to cause the refracted ray bending away from the normal to bend by 90° , at angle r_c , thereby traveling *along the interface* between the two media. Ray 4 is incident on the interface at an angle *greater than* the critical angle and is *totally reflected* into the same medium from which it came. Ray 4 obeys the *law of reflection* so that its angle of reflection r_4 is exactly equal to its angle of incidence i_4 .



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 for a short lesson
 about Total Internal
 Reflection.

Figure 4-10 Critical angle i_c and total internal reflection

We make use of the phenomenon of total internal reflection when we design light propagation in fibers by “trapping” the light in the fiber through successive internal reflections along the fiber. We do this also when designing “retroreflecting” prisms, i.e., those that redirect the reflected light along the same path as the entering light. Compared with ordinary reflection from mirrors, the sharpness and brightness of totally internally reflected light beams in such prisms is considerably greater.

The calculation of the *critical angle of incidence* for any two optical media—whenever light is incident from a medium of higher index to a medium of lower index—is accomplished with *Snell’s law*. Referring to Ray3 in Figure 4-10 and using Snell’s law in Equation 4-2 appropriately, we have

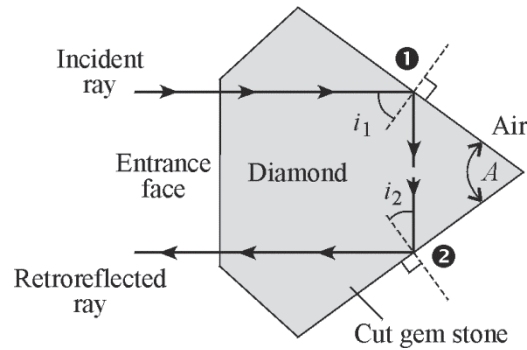
$$n_i \sin i_c = n_r \sin 90^\circ$$

where n_i is the index for the incident medium, i_c is the critical angle of incidence, n_r is the index for the medium of lower index, and $r_c = 90^\circ$ is the angle of refraction at the critical angle. Then, since $\sin 90^\circ = 1$, we obtain for the critical angle,

$$i_c = \sin^{-1} \left(\frac{n_r}{n_i} \right) \quad (4-3)$$

Example 3

The phenomenon of total internal reflection is used by diamond cutters to shape the gem stone in such a way that light incident on the entrance face is totally reflected back out the entrance face. (See accompanying sketch.) Such total reflection causes the diamond to sparkle. What is the *critical angle* for light at the diamond-air interface?



Solution: From Table 4-1, we see that the refractive index for light near wavelength $\lambda = 589 \text{ nm}$ is 2.42. Assume that this value is reasonably the same for all wavelengths in white light. Then at either position ① or ② at the diamond-air interface we can write, using Equation 4-3,

$$i_c = \sin^{-1} \left(\frac{n_{\text{air}}}{n_{\text{diam}}} \right), \text{ where } n_{\text{air}} = 1, n_{\text{diam}} = 2.42$$

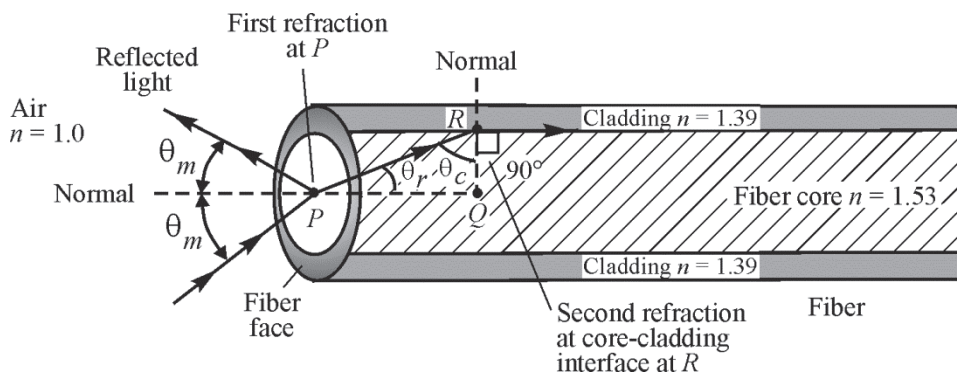
$$\text{Thus } i_c = \sin^{-1} \left(\frac{1}{2.42} \right) = \sin^{-1}(0.4132) = 24.4^\circ$$

Thus, if angle A of the cut stone is 50° or larger, angles i_1 and i_2 will always be larger than 24.4° and total internal reflection will occur. (Can you explain where the value 50° comes from?)

Now, let's use Equation 4-3 in Example 4 to determine the entrance cone for light rays incident on the face of a cladded fiber if the light is to be trapped by total internal reflection at the core-cladding interface in the fiber.

Example 4

A step-index fiber 0.0025 inch in diameter has a core index of 1.53 and a cladding index of 1.39. (See drawing.) Such clad fibers are used frequently in applications involving communication, sensing, and imaging.



What is the maximum acceptance angle θ_m for a narrow beam of light incident on the fiber face such that the refracted ray in the core of the fiber is incident on the cladding at just the critical angle?

Solution: Let's work backwards from the second refraction at the core-cladding interface at R to the first refraction at the fiber face at P . Thus first find θ_c , then θ_m .

- (1) From Equation 4-3, at the core-cladding interface at R ,

$$\theta_c = \sin^{-1} \left(\frac{1.39}{1.53} \right) = 65.3^\circ$$

- (2) In right-triangle PQR, $\theta_r + \theta_c + 90^\circ = 180^\circ$, so $\theta_r = 90^\circ - \theta_c = 90^\circ - 65.3^\circ = 24.7^\circ$

- (3) Then, from Snell's law, at the fiber face at P ,

$$n_{air} \sin \theta_m = n_{core} \sin \theta_r$$

so that
$$\sin \theta_m = \left(\frac{n_{core}}{n_{air}} \right) \sin \theta_r = \left(\frac{1.53}{1.00} \right) \sin (24.7^\circ)$$

from which
$$\sin \theta_m = 0.639$$

Finally, then
$$\theta_m = \sin^{-1} 0.639 = 39.7^\circ$$

Thus, the half acceptance angle is 39.7° and the *acceptance angle* is twice that, or $2\theta_m = 79.4^\circ$. This acceptance angle indicates that any light ray incident on the fiber face within the acceptance angle 79.4° will undergo total internal reflection at the core-cladding face and remain trapped in the fiber as it propagates along the fiber.

Refraction in prisms

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Glass prisms are often used to bend light in a given direction as well as to bend it back again (retroreflection) as we have just seen in the cut diamond. The process of refraction in prisms can be understood with the use of light rays and Snell's law. Look at Figure 4-11a. When a light ray enters a prism at one face and exits at another, the exiting ray is *deviated* from its original direction. The prism shown—with equal base angles—is isosceles in cross section with *apex angle* $A = 30^\circ$ and refractive index $n = 1.50$. The incident angle θ at the left prism face and the *angle of deviation* δ at the right prism face are shown on the diagram.

Figure 4-11b shows how the angle of deviation δ changes as the angle θ of the incident ray changes. The curve shown is for the specific prism drawn in Figure 4-11a. Note that the angle of deviation δ goes through a minimum value as the angle of incidence θ increases. For this prism, δ has a minimum value equal to about 16° when the angle of incidence is near 27° . Each prism shape of refractive index n has its own unique *minimum angle of deviation*.

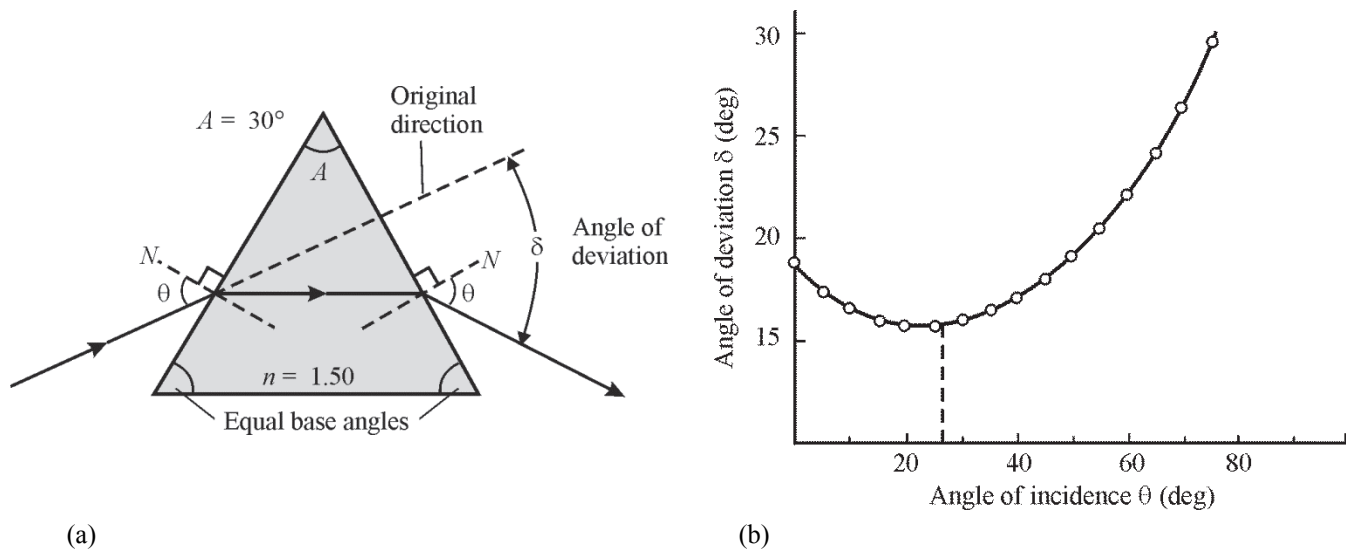


Figure 4-11 *Refraction of light through a prism*

Minimum angle of deviation—It turns out that we can determine the refractive index of a transparent material by shaping it in the form of an isosceles prism and then measuring its *minimum* angle of deviation δ_m . With reference to Figure 4-11a, the proven relationship between the refractive index n , the prism apex angle A , and the minimum angle of deviation δ_m is given by

$$n = \frac{\sin \left(\frac{A + \delta_m}{2} \right)}{\sin \frac{A}{2}} \quad (4-4)$$

where both A and δ_m are measured in degrees.

The derivation of Equation 4-4 is straightforward, but a bit tedious. Details of the derivation—making use of Snell’s law and geometric relations between angles at each refracting surface—can be found in most standard texts on geometrical optics. (See suggested references at the end of the module.) For now, we’ll be content to just use Equation 4-4 in Example 5 to determine the index of refraction of an unknown glass shaped in the form of an isosceles prism.

Example 5

Your supervisor hands you a piece of transparent optical material of unknown index of refraction. He asks you to determine the refractive index. You take the sample to the glass shop and have it shaped in the form of an isosceles prism with an apex angle of 25° . In the laboratory, with the help of a laser beam and a prism table, the *minimum* angle of deviation for this prism is measured carefully to be 15.8° . What must the refractive index of this glass material be?

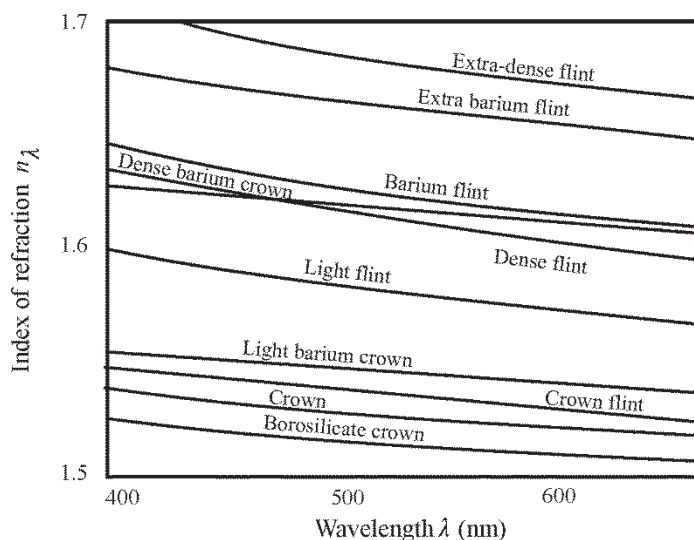
Solution: Given that $\delta_m = 15.8^\circ$ and $A = 25^\circ$, we use Equation 4-4 to calculate the refractive index.

$$n = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{25^\circ + 15.8^\circ}{2}\right)}{\sin\left(\frac{25^\circ}{2}\right)} = \frac{\sin(20.4^\circ)}{\sin(12.5^\circ)} = \frac{0.3486}{0.2164} = 1.61$$

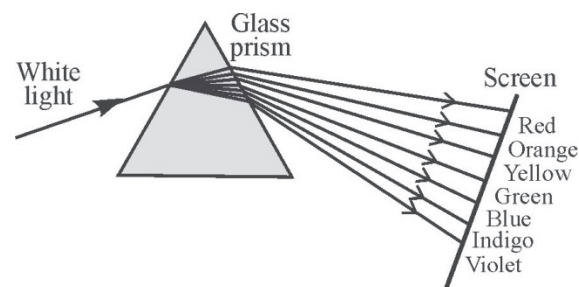
(Comparing this value of $n = 1.61$ with refractive indexes given in Table 4-1, the unknown glass could well be flint glass.)

Dispersion of light—Table 4-1 lists indexes of refraction for various substances at the wavelength of 589 nm. It turns out that the refractive index is slightly wavelength dependent so values of n change a little as λ changes. For example, the index of refraction for flint glass is about 1% higher for blue light (around 450 nm) than for red light (around 650 nm). The variation of refractive index n with wavelength λ is called *dispersion*. Knowing about dispersion, for example, is important in designing prism spectrometers to measure the wavelengths of spectra from different light sources. Figure 4-12a shows a normal dispersion curve of n_λ versus λ for different types of optical glass. Figure 4-12b shows the separation of the individual colors blended together in white light—400 nm to 700 nm—after passing through a prism. Note that n_λ decreases from shorter to longer wavelengths, thus causing the red light to be less deviated than the blue light as it passes through a prism. Dispersion accounts for the colors seen in a rainbow, where the individual raindrops serve as the refracting “prism.”

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(a) Optical glass dispersion curves



(b) Refraction by a prism

Figure 4-12 Typical dispersion curves and separation of white light after refraction by a prism

Special applications of prisms—Prisms that depend on *total internal reflection* are commonly used in optical systems, both to change direction of light travel and to change the orientation of an image. While mirrors can be used to achieve similar ends, the reflecting faces of a prism are easier to keep free of contamination and the process of total internal reflection is capable of higher reflectivity. Some common prisms in use today are shown in Figure 4-13.

Details of light redirection and image reorientation are shown for each one with the help of straight-line symbols labeled with circles and arrows. If, for example, the Dove prism in Figure 4-13b is rotated about its long axis, the image will also be rotated correspondingly.

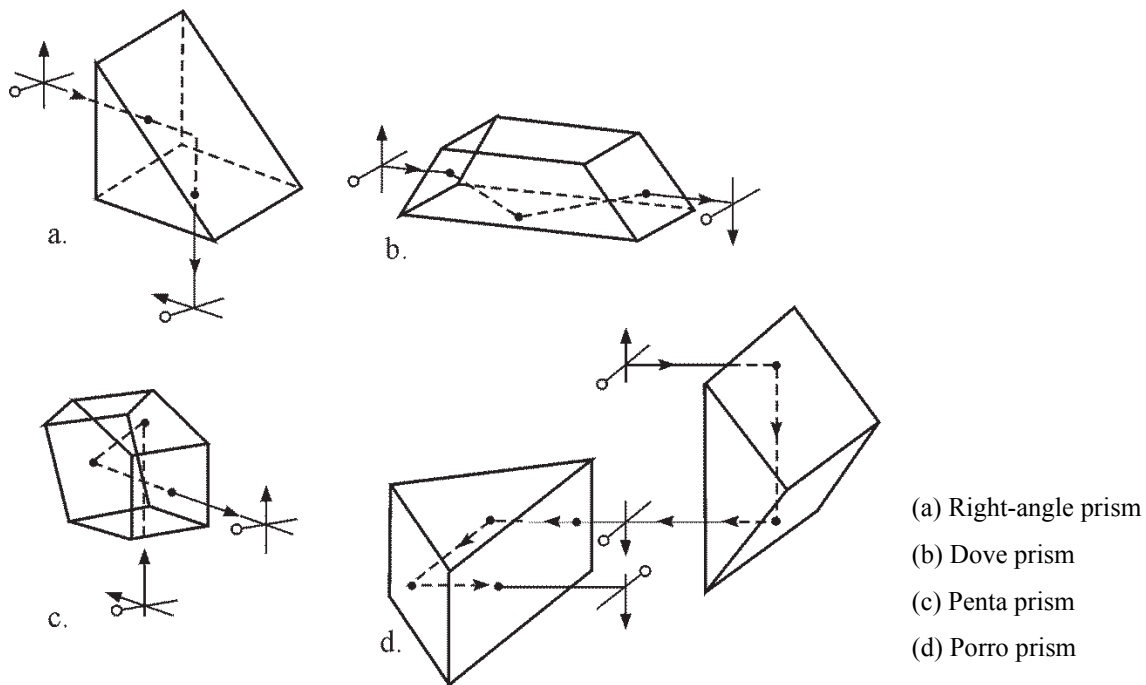


Figure 4-13 Image manipulation with refracting prisms

The Porro prism, consisting of *two right-angle prisms*, is used in *binoculars*, for example, to produce erect final images and, at the same time, permit the distance between the object-viewing lenses to be greater than the normal viewer's eye-to-eye distance, thereby enhancing the stereoscopic effect produced by ordinary binocular vision.

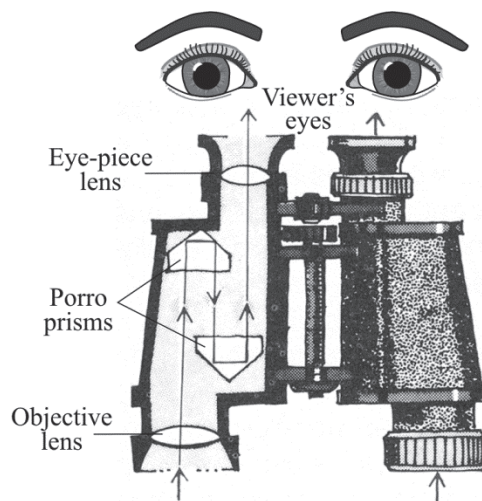


Figure 4-14 The use of Porro prisms in ordinary binoculars

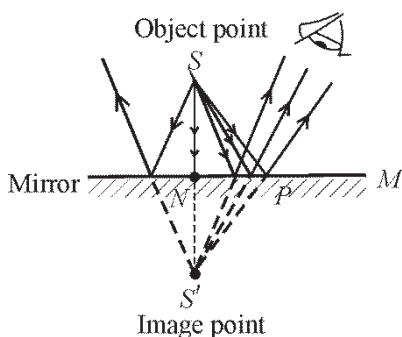
Figure 4-14 shows the light path of a ray entering the objective lens of a set of binoculars, passing through the Porro prisms, and then exiting the eyepiece into the viewer's eyes. Note especially how the presence of the Porro prisms in each tube enables the distance between the two objective lenses to be about twice the distance between the two eye-piece lenses.

Image Formation with Mirrors

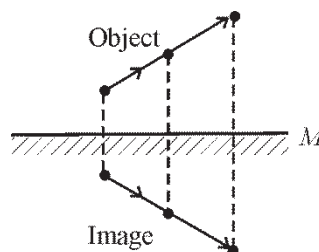
Mirrors, of course, are everywhere—in homes, auto headlamps, astronomical telescopes, and laser cavities—and in many other places. Plane and spherical mirrors are used to form three-dimensional images of three-dimensional objects. If the size, orientation, and location of an object relative to a mirror are known, the *law of reflection* and *ray tracing* can be used to locate the image graphically. Appropriate *mathematical formulas* can also be used to calculate the locations and sizes of the images formed by mirrors. In this section, we shall use both graphical ray tracing and formulas.

Images formed with plane mirrors

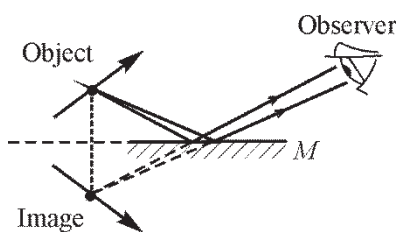
Images with mirrors are formed when many nonparallel rays from a given point on a source are reflected from the mirror surface, converge, and form a corresponding image point. When this happens, point by point for an extended object, an image of the object, point by point, is formed. Image formation in a plane mirror is illustrated in several sketches shown in Figure 4-15.



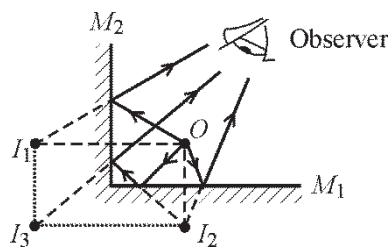
(a) Imaging a point source



(b) Imaging an extended object



(c) Image seen in a plane mirror is same size as object.



(d) Multiple images of point O with inclined mirrors

Figure 4-15 Image formation in a plane mirror

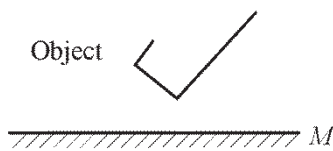
In Figure 4-15a, point object S sends diverging rays toward a plane mirror, which reflects them as shown. The *law of reflection* ensures that pairs of triangles like SNP and $S'NP$ are equal, so that all reflected rays appear to originate at the *image point* S' , which lies along the normal line SN , and at such depth that the *image distance* $S'N$ equals the *object distance* SN . The eye sees a point image at S' in exactly the same way it would see a real point object placed there. Since the actual rays do not exist behind the mirror surface, the image is said to be a *virtual image*, as mentioned earlier. Thus, the image S' cannot be projected onto a screen as in the case of a *real image*.

An extended object, such as the arrow in Figure 4-15b, is imaged point by point by a plane mirror surface in similar fashion. Each object point has its corresponding image point along its normal to the mirror surface and *as far below the reflecting surface as the object point lies above the surface*. Note that image position does not depend on the position of the eye. The construction in Figure 4-15b also makes clear that the image size is identical to the object size, giving a *magnification* of unity. In addition, the transverse orientations of object and image are the same. A right-handed object, however, appears left-handed in its mirror image.

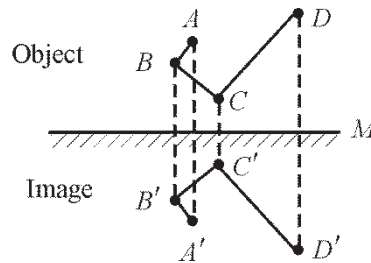
In Figure 4-15c, where the mirror does not lie directly below the object, the mirror plane may be extended, as shown. This extension enables one to determine the position of the image as seen by an eye positioned to receive reflected rays originating at the object. Figure 4-15d illustrates multiple images of a point object O formed by two perpendicular mirrors, M_1 and M_2 . The two images, I_1 and I_2 , result from a single reflection in one of the two mirrors. A third image, I_3 , is formed by sequential reflections involving both mirrors. All parts of Figure 4-14 and the related discussion above should be understood clearly because they are fundamental to the optics of images. Look at Example 6.

Example 6

Making use of the *law of reflection* and the conclusions drawn from Figure 4-14, draw the image of the letter L positioned above a plane mirror as shown below in (a).



(a) Original object and mirror surface M



(b) Point-by-point trace to locate image in mirror M

Solution: Make use of the fact that each point on the image is as far below the mirror—along a line perpendicular to the mirror—as the actual object point is above the mirror. Indicate key points (A , B , C , D) on the object and locate corresponding key points (A' , B' , C' , D') on the image. Sketch in the image as shown in (b).

Summary of ray optics for plane mirrors:

- Each ray of light incident on a mirror from a point on an object obeys the law of reflection at the plane mirror surface.
- A point source of light in front of a mirror forms a virtual image as far behind the mirror as the point source is in front.
- Plane mirrors form virtual images that are the same size as the original objects. Hence, the magnification is unity.

Images formed with spherical mirrors

As we showed earlier in Figure 4-6, the *law of reflection* can be used to determine the direction along which any ray incident on a spherical mirror surface will be reflected. Using the *law of reflection*, we can trace rays from any point on a given object to the mirror, and from there on to the corresponding image point. This is the method of *graphical ray tracing*.

Graphical ray-trace method—To employ the method of ray tracing, we agree on the following:

- Light from an object will be incident on a mirror surface initially from the left.
- The *axis of symmetry* normal to the mirror surface is defined as the *optical axis*.
- The point where the optical axis meets the mirror surface is called the *vertex*, generally denoted by the letter *V*.

To locate an image, we use two important points common to any spherical mirror surface, the *center of curvature* *C* and the *focal point* *F*. They are shown in Figure 4-16, with the mirror vertex *V*, for both a *concave* and a *convex* spherical mirror. The *center of curvature* *C* is the geometrical center for the curved surface of the sphere-shaped mirror. The *focal point* *F* is the point along the *optical axis* at which light rays parallel to the optical axis converge after reflection from the mirror.

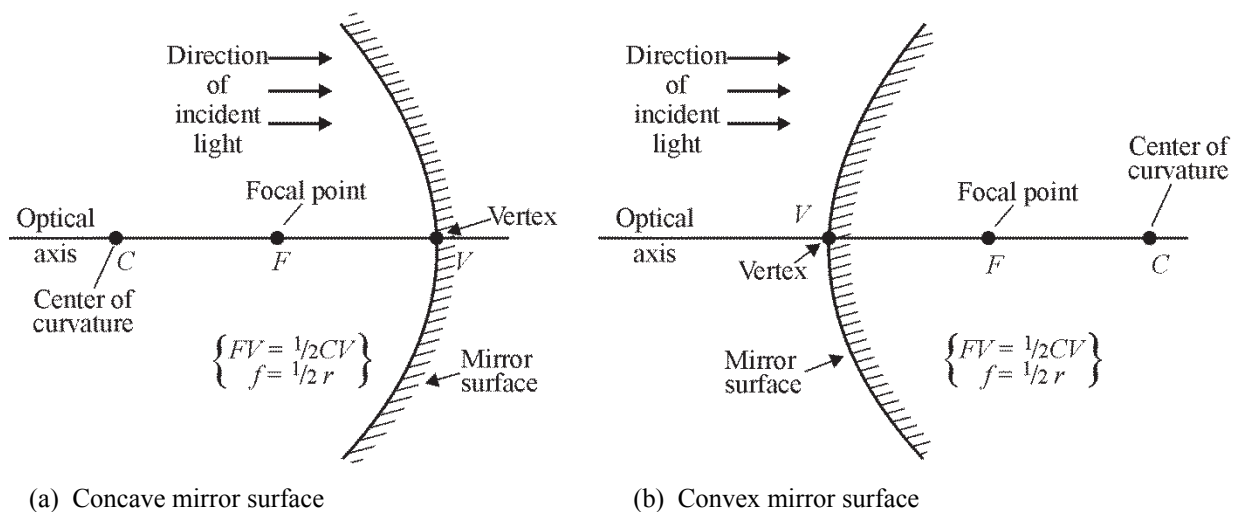


Figure 4-16 Defining points for concave and convex mirrors

The outer edges of *concave* mirrors always curve toward the oncoming light. Such mirrors have their center of curvature *C* and focal point *F* located to the *left* of the vertex as seen in Figure 4-16a. The outer edges of *convex* mirrors always curve away from the oncoming light, and their center of curvature *C* and focal point *F* are located to the *right* of the vertex. See Figure 4-16b.

The important connection between parallel light rays and the focal points for mirror surfaces is shown in Figure 4-17a, b. Parallel rays are light rays coming from a very distant source (such as the sun) or as part of a collimated laser beam. The *law of reflection*, applied to a small tangent segment at each point on the mirror surface where a ray is incident, requires that the ray be reflected so as to pass through a focal point *F* in front of a concave mirror (Figure 4-17a) or be

reflected to appear to come from a focal point F behind a convex mirror (Figure 4-17b). Notice that a line drawn from the center of curvature C to any point on the mirror such as P is a *normal* line and thus bisects the angle between the incident and reflected rays, as shown.

As long as the transverse dimension of the mirror is not too large, simple geometry shows that the point F , for either mirror, is located at the midpoint between C and V , so that the distance FV is one-half the radius of curvature CV . The distance FV is called the *focal length* and is commonly labeled as f . Therefore, one has $f = \frac{1}{2}r$.

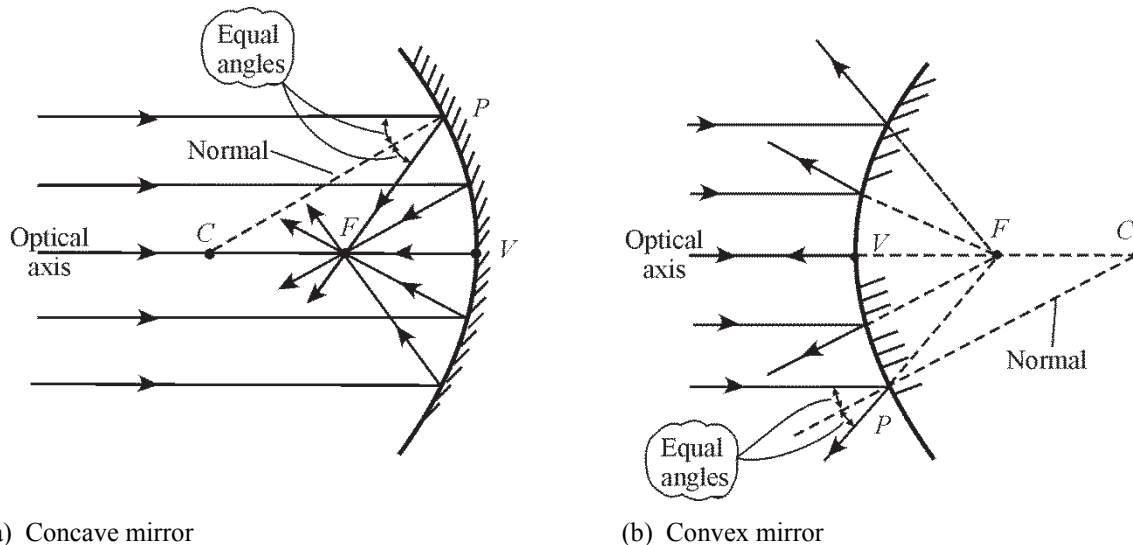


Figure 4-17 Parallel rays and focal points

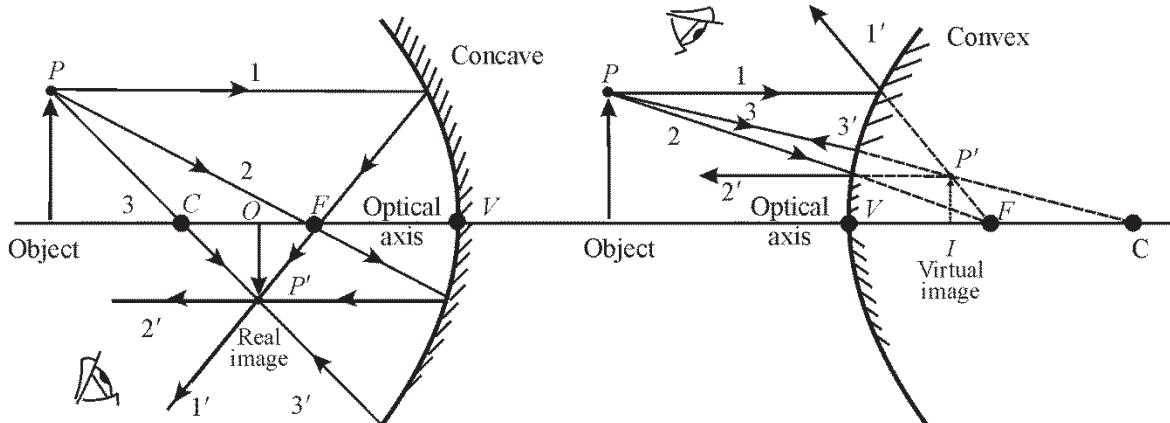
Key rays used in ray tracing—Figures 4-18a,b show three key rays—for each mirror—that are used to locate an image point corresponding to a given object point. They are labeled 1, 2, and 3. After reflection, the corresponding rays are labeled 1', 2', and 3'. Any two rays, drawn from object point P , will locate the corresponding image point P' . In most cases, it is sufficient to locate one point, like P' , to be able to draw the entire image. Note carefully, with reference to Figure 4-18a, b, the following facts:

For a concave mirror:

- An incident ray from object point P , initially *parallel to the axis*, such as ray 1, reflects from the mirror and *passes through the focal point F* (labeled ray 1').
- An incident ray from P passing through the focal point F on its way to the mirror, such as ray 2, reflects from the mirror *as a ray parallel to the axis* (labeled ray 2').
- An incident ray from P passing through the center of curvature C on its way to the mirror, such as ray 3, *reflects back along itself* (labeled ray 3').
- Reflected rays 1', 2', and 3' converge after reflection to locate point P' on the image. The complete image, arrow OP' , is a *real image* that can be formed on a screen located there. It is seen as a real image by an eye located where shown in Figure 4-18a.

For a convex mirror:

- An incident ray from object point P , initially parallel to the axis, such as ray 1, reflects from the mirror *as if it came originally from the focal point F behind the mirror* (labeled ray 1').
- An incident ray from P , such as ray 2, which is headed initially on its way to the mirror in a direction toward the focal point F behind the mirror, reflects from the mirror in a direction parallel to the optical axis (labeled ray 2').
- An incident ray from P , such as ray 3, which is headed initially on its way to the mirror in a direction toward the center of curvature C behind the mirror, reflects back along itself (labeled ray 3').
- Rays 1', 2', and 3' diverge (spread apart) after reflection. A person looking toward the mirror from the left (see eye) intercepts the diverging rays and sees them as if they appear to come from their common intersection point P' , behind the mirror. The complete image (arrow IP') is *virtual* since it cannot be formed on a screen placed there.



(a) Ray tracing from P to P' for a *concave* mirror

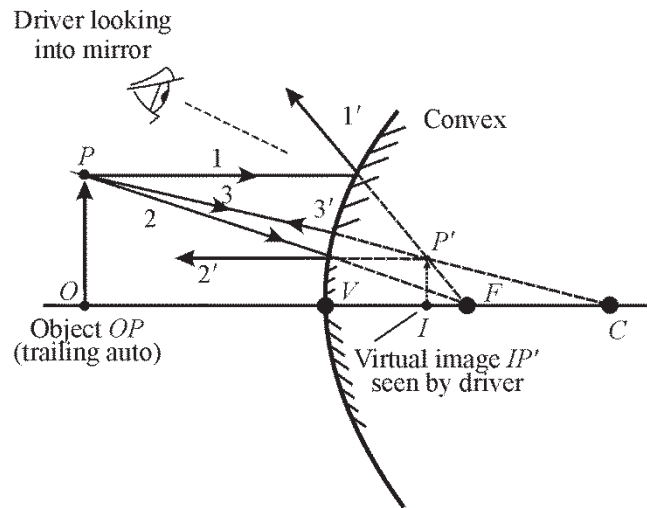
(b) Ray tracing from P to P' for a *convex* mirror

Figure 4-18 Key rays for graphical ray tracing with spherical mirrors

Example 7

The passenger-side mirror on an automobile is a slightly *convex* mirror. It provides the driver with a wider field of view, but a significantly reduced image. Assume that object OP shown in the sketch is part of an automobile trailing the driver's car. Use three key rays to locate the reduced, virtual image of the trailing auto.

Solution: Use key rays 1, 2, and 3 incident on the convex mirror from point P on object OP . Use the key points F and C , and the steps for ray tracing light incident on a convex mirror given earlier, to locate P' .



The three reflected rays 1', 2', and 3' *diverge* after reflection. They appear to come from a common point P' *behind* the mirror. Repeating this process for other points on the object, between O and P , then locates the complete image IP' . Note that virtual image IP' is reduced in size, about one-third as large as object OP . As a result, drivers are always cautioned that images they see in the passenger-side mirror are *physically* NEARER to them than they *appear to be* in the mirror. Thus, drivers are cautioned not to change lanes without checking the rear-view mirror and also making a quick “head-check” over the shoulder to verify the *actual position* of the trailing car.

Mirror formulas for image location

In place of the graphical ray-tracing methods described above, we can use formulas to *calculate* the image location. We shall derive below a “mirror formula” and then use the formula to determine image location. The derivation is typical of those found in geometrical optics, and is instructive in its use of combined algebra, geometry, and trigonometry. *(If the derivation is not of interest to you, you may skip to the next section, where the derived formula is used in typical calculations. Be sure, though, that you learn about the sign convention discussed following.)*

OPTIONAL—Derivation of the mirror formula. The drawing we need to help us carry out the derivation is shown in Figure 4-19. The important quantities are the object distance p , from O to V ; the image distance q , from I to V ; and the radius of curvature r , from C to V or C to P . Thus, both p and q are measured relative to the mirror vertex, as shown. A plus or minus sign on r will indicate whether the mirror is concave or convex. All other quantities in Figure 4-19 are used in the derivation but will not show up in the final “mirror formula.”

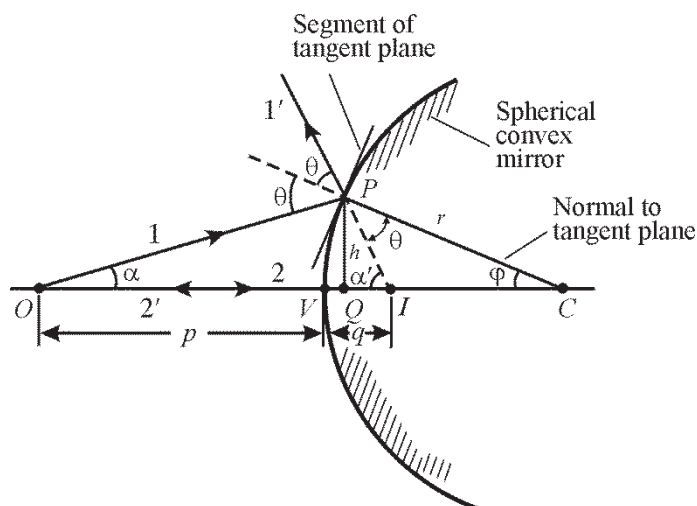


Figure 4-19 Basic drawing for deriving the mirror formula

The mirror shown in Figure 4-19 is convex with center of curvature C on the right. Two rays of light, 1 and 2, originating at object point O —located here on the optical axis—are drawn. One is drawn normal to the convex surface at its vertex V . The other is drawn as an arbitrary ray incident at P . Ray 2 reflects back along itself. Ray 1 reflects at P as if incident on a segment of the plane tangent at P , according to the *law of reflection*. Relative to each other, the two reflected rays ($1'$ and $2'$) diverge as they leave the mirror. The intersection of the two rays (extended backward) determines the image point I corresponding to object point O . Thus, the image is virtual and located behind the mirror surface.

A perpendicular of height h is drawn from P to the axis at Q . We seek a relationship between p and q that depends only on the radius of curvature r of the mirror. As we shall see, such a relation is possible only to a *first-order approximation of the sines and cosines of angles*, such as α and ϕ , made by the object and image rays at various points on the spherical surface. This means that, in the expansions of $\sin \phi$ and $\cos \phi$ in series, as shown here in part,

$$\sin \phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \dots$$

$$\cos \phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \dots$$

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we consider only the *first terms* and write as an approximation.

$$\begin{aligned}\sin \phi &\cong \phi \\ \cos \phi &\cong 1 \\ \text{and } \tan \phi &= \frac{\sin \phi}{\cos \phi} \cong \frac{\phi}{1} \cong \phi\end{aligned}$$

These approximate relations are accurate to 1% or less if the angle ϕ is 10° or smaller. This approximation leads to so-called *first-order (or Gaussian) optics*, after Karl Friedrich Gauss, who in 1841 developed the foundations of this subject. We return now to the problem at hand—that of relating p , q , and r . Notice that two angular relationships may be obtained from Figure 4-19, using the fact that the exterior angle of a triangle equals the sum of its two interior angles. Thus,

$$\text{in } \triangle OPC, \theta = \alpha + \phi \quad \text{and} \quad \text{in } \triangle OPI, 2\theta = \alpha + \alpha'.$$

After eliminating θ between these two equations, we get

$$\alpha - \alpha' = -2\phi$$

Using the small-angle approximations, the angles α , α' , and ϕ above can be replaced by their tangents $\tan \alpha \cong \alpha = \frac{h}{p}$, $\tan \alpha' \cong \alpha' = \frac{h}{q}$, and $\tan \phi \cong \phi = \frac{h}{r}$ so that the relationship

$\alpha - \alpha' = -2\phi$ now becomes

$$\frac{h}{p} - \frac{h}{q} = -2\frac{h}{r}$$

Note that in these tangent relationships, we have neglected the axial distance VQ , which is very small when ϕ is small. Cancellation of h in the above equation produces the desired relationship,

$$\frac{1}{p} - \frac{1}{q} = -\frac{2}{r} \quad (4-5)$$

If the spherical surface in Figure 4-19 is chosen to be concave instead, the center of curvature will be to the left. For certain positions of the object point O , it is then possible to find a real image point, also to the left of the mirror. In these cases, the resulting geometric relationship analogous to Equation 4-5 consists of the same terms, but with different algebraic signs, depending on the *sign convention* employed. **(END OF OPTIONAL MATERIAL)**

With an appropriate sign convention, a *mirror equation* valid for both convex and concave mirrors is given by Equation 4-6.

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r} \quad \begin{array}{l} \text{Spherical} \\ \text{mirror} \\ \text{equation} \end{array} \quad (4-6)$$

Sign convention—The sign convention to be used in conjunction with Equation 4-6 and Figure 4-19 is as follows.

- Object and image distances p and q are both *positive* when located to the *left* of the vertex and both *negative* when located to the *right*.
- The radius of curvature r is *positive* when the center of curvature C is to the *left* of the vertex (concave mirror surface) and *negative* when C is to the *right* (convex mirror surface).
- Vertical dimensions are positive above the optical axis and negative below.
- And since the focal length f is equal to one-half of the radius of curvature r , i.e., $f = \frac{r}{2}$, as shown earlier in connection with Figures 4-16 and 4-17, the focal length f is *positive for concave mirrors* and *negative for convex mirrors*.

In the application of these rules, light is assumed to be directed initially, as we mentioned earlier, from left to right. According to this sign convention, *positive* object and image distances correspond to *real* objects and images, and *negative* object and image distances correspond to *virtual* objects and images. *Virtual objects* occur only with a sequence of two or more reflecting or refracting elements. We shall demonstrate the use of virtual objects in an example that follows later.

Magnification of a mirror image—Figure 4-20 shows a drawing from which the *magnification*—ratio of image height h_i to object height h_o —can be determined. Since angles θ_i , θ_r , and α are equal to each other, it follows that triangles VOP and VIP' are similar triangles. Thus, the corresponding sides of the two similar triangles are proportional and one can write

$$\frac{h_i}{h_o} = \frac{q}{p}$$

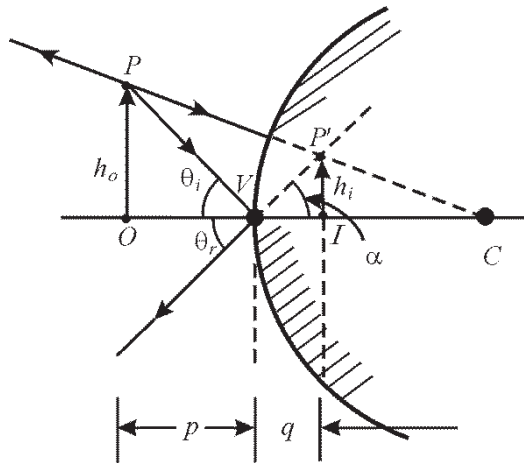


Figure 4-20 Construction for derivation of mirror magnification formula

This gives at once the magnification m —defined as image size divided by the object size—to be

$$m \equiv \frac{h_i}{h_o} = \frac{q}{p}$$

When the *sign convention* for spherical mirrors is taken into account, one has, for the *general case*, a single equation, Equation 4-7, valid for both convex and concave mirrors.

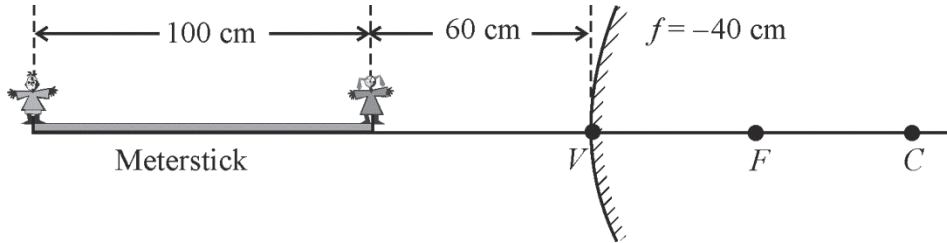
$$m = -\frac{q}{p} \quad \text{Magnification for spherical mirrors} \quad (4-7)$$

If, after calculation, the value of m is positive, the image is erect. If the value is negative, the image is inverted.

Let us now use the spherical mirror formulas in Equations 4-6 and 4-7, and the sign convention, to locate an image and determine its size.

Example 8

A meterstick lies along the optical axis of a convex mirror of focal length 40 cm, with its near end 60 cm from the mirror surface. Five-centimeter toy figures stand erect on both the near and far ends of the meterstick. (a) How long is the virtual image of the meterstick? (b) How tall are the toy figures in the image, and are they erect or inverted?



Solution: Use the mirror equation $\frac{1}{p} + \frac{1}{q} = \frac{2}{r}$ twice, once for the near end and once for the far

end of the meterstick. Use the magnification equation $m = -\frac{q}{p}$ for each toy figure.

- (a) **Near end:** Sign convention gives $p = +60$ cm, $r = 2f = 2 \times (-40) = -80$ cm, since f is negative for a convex mirror. Thus the mirror equation becomes:

$$\therefore \frac{1}{60} + \frac{1}{q_n} = -\frac{2}{80}, \text{ so } q_n = -24 \text{ cm}$$

Negative sign indicates that the image of the near end is *virtual*, 24 cm to the right of V .

Far end: $p = +160$ cm, $r = -80$ cm, and the mirror equation becomes:

$$\frac{1}{160} + \frac{1}{q_f} = -\frac{1}{40}, \text{ so } q_f = -32 \text{ cm}$$

Far-end image is *virtual*, 32 cm to the right of V .

$\therefore$ Therefore, the length of the meterstick image is $32 \text{ cm} - 24 \text{ cm} = 8 \text{ cm}$ long.

- (b) **Near-end toy figure:**

$$m_n = \frac{-q}{p} = \frac{-(-24)}{60} = +0.4 \text{ (Image is erect since } m \text{ is positive.)}$$

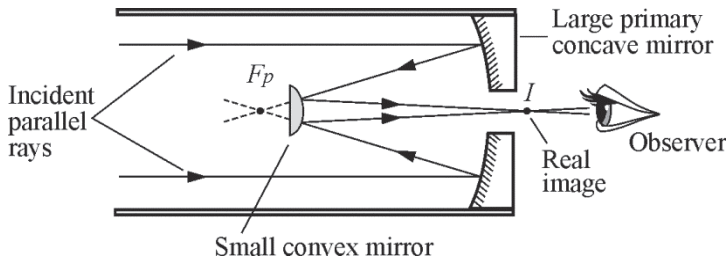
The toy figure is $5 \text{ cm} \times 0.4 = 2 \text{ cm}$ tall, at near end of the meterstick image.

Far-end toy figure:

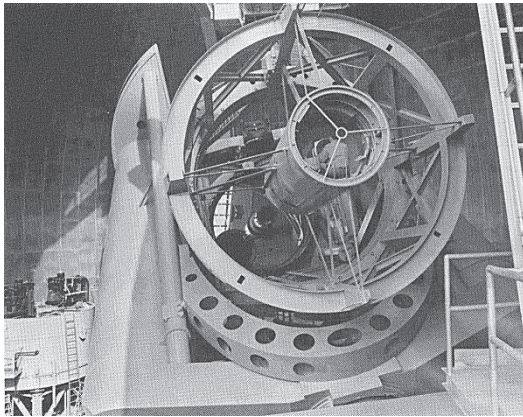
$$m_f = \frac{-q}{p} = \frac{-(-32)}{160} = +0.2 \text{ (Image is erect since } m \text{ is positive.)}$$

The toy figure is $5 \text{ cm} \times 0.2 = 1 \text{ cm}$ tall, at far end of the meterstick image.

Telescopes and mirrors—Mirrors play an important role in the design of reflecting telescopes used in astronomy. One such design, known as a Cassegrain telescope, makes use of both a concave and convex mirror to form a real image of the “target” object. The huge *concave* mirror is used to collect the light incident from distant stars and then focus the parallel light rays at the mirror focal point F_p . See sketch (a). Before this light reaches the point F_p , however, it is intercepted by a much smaller *convex* mirror, which then reflects the light to a real image point I , as shown.



(a) Ray optics for a Cassegrain telescope



(b) Hale telescope (200-in.) showing observer in prime-focus cage and reflecting surface of 200-in. mirror (California Institute of Technology.)

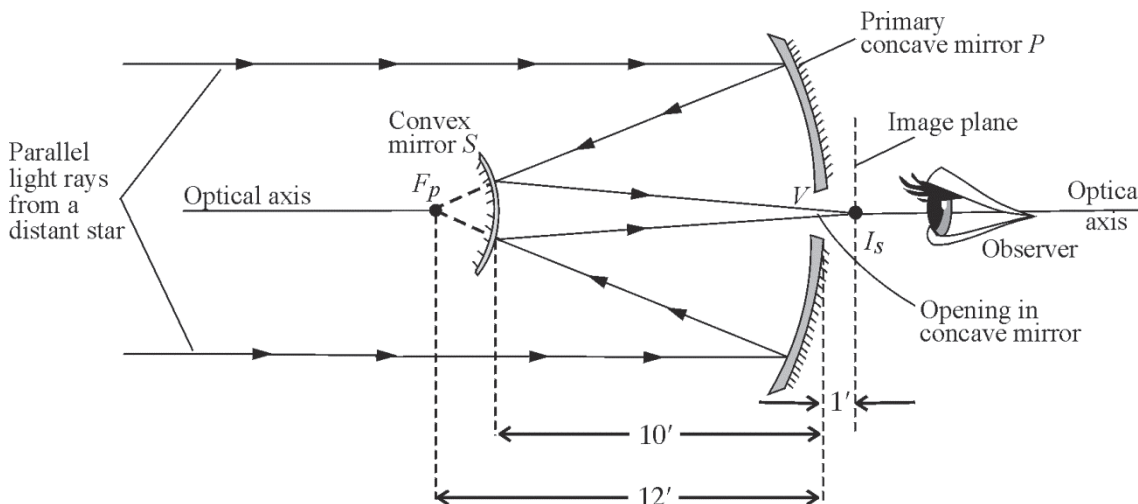
Figure 4-21 *Cassegrain telescope*

One of the more famous telescopes, located at Mount Palomar, is the 200-inch Hale telescope. The insert shows an astronomer, seated in the prime-focus cage of this telescope, where the image from the primary mirror would be formed.

Example 9

The primary concave mirror of a Cassegrain reflecting telescope has a focal length of 12 feet. The secondary convex mirror is positioned 10 feet in front of the concave mirror along the optical axis. See following sketch.

Light reflected from the primary concave mirror P , heading toward the focal point F_P is intercepted by the secondary convex mirror S before it reaches the focal point F_P and again reflected to form a real image at I_S , one foot behind the concave mirror. Here it can be recorded on film or viewed by an observer, as shown.



For this Cassegrain telescope,

- Where is the image of a distant star formed by the primary concave mirror P ?
- What is the radius of curvature r of the concave mirror?
- What should the radius of curvature of the convex mirror S be so that it forms a real image one foot behind the opening in the concave mirror? What is the focal length of this mirror?

Solution

(a) Since light entering the concave mirror from distant stars is made up of parallel light rays, the image of such a star would be formed at the focal point F_P , 12 feet to the left of the mirror vertex V .

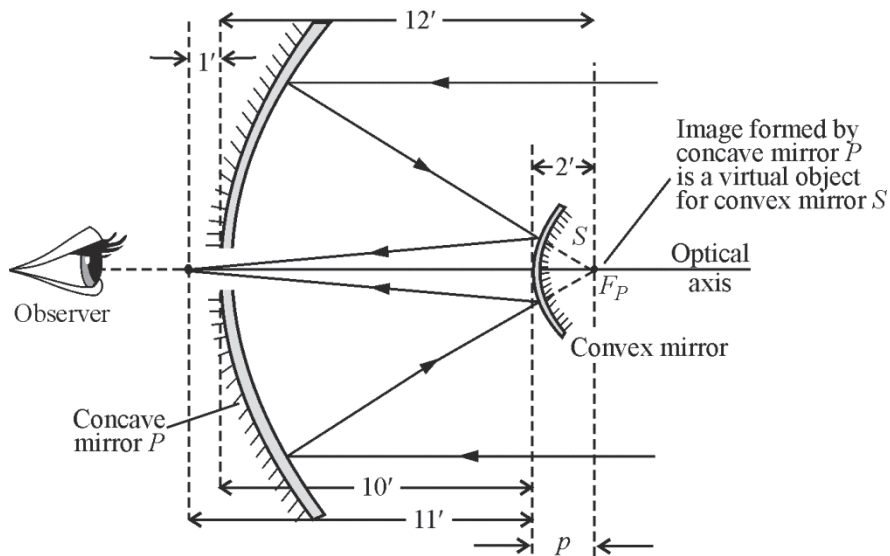
(b) Since $f = \frac{r}{2}$, $r = 2f = 2(12') = 24'$. (Note that r is + according to the sign convention for a concave mirror.)

(c) To find the radius of curvature r_s of the secondary mirror, knowing object and image position for the secondary mirror, we can use the formula $\frac{1}{p} + \frac{1}{q} = \frac{2}{r_s}$ and solve for r_s . To do this correctly,

we **MUST** pay close attention to the **SIGN CONVENTIONS** already defined.

Thus, according to the basis for our sign convention, we want light to be incident on the convex mirror *traveling from left to right*. In the drawing above, light is incident on the convex mirror from *right to left*. So, in our mind, we position ourselves *behind* the drawing and *look out* through the page, seeing the light then moving from our left to our right.

Second, from this position, the object p for the convex mirror is a *virtual object*, since it is the image (virtual) formed by the concave mirror at F_p , a point behind the convex mirror. Thus $p = -(12' - 10') = -2'$, since virtual objects are negative. The “repositioned” system is sketched below.



Third, the image q is positive since it is on the same side of the convex mirror as the approaching light and thus is equal to $10' + 1' = +11'$.

Solving $\frac{1}{p} + \frac{1}{q} = \frac{2}{r_s}$, with $p = -2'$ and $q = +11'$ we have

$$\frac{1}{(-2)} + \frac{1}{11} = \frac{2}{r_s}$$

$$\frac{(11 - 2)}{(-2 \times 11)} = \frac{2}{r_s} \quad \text{or} \quad \frac{9}{-22} = \frac{2}{r_s}$$

$$\text{so that } r_s = \frac{-44}{9} = -4.89'$$

Note that r_s comes out to be negative as it should in accordance with the sign convention, since the mirror facing the left-to-right traveling light in the repositioned drawing is convex in shape.

And since the focal length is always one-half of the radius of curvature,

$$f = \frac{r}{2} = \frac{-4.89'}{2} = -2.44 \text{ feet.}$$

Image Formation with Lenses

Lenses are at the heart of many optical devices, not the least of which are cameras, microscopes, binoculars, and telescopes. Just as the *law of reflection* determines the imaging properties of mirrors, so *Snell's law of refraction* determines the imaging properties of lenses. Lenses are essentially light-controlling elements, used primarily for image formation with visible light, but also for ultraviolet and infrared light. In this section, we shall look first at the types and properties of lenses, then use graphical ray-tracing techniques to locate images, and finally use mathematical formulas to locate the size, orientation, and position of images in simple lens systems.

Function of a lens

A lens is made up of a transparent refracting medium, generally of some type of glass, with spherically shaped surfaces on the front and back. A ray incident on the lens bends (according to Snell's law) at the front surface, propagates through the lens, and bends again at the rear surface (according to Snell's law). Figure 4-22 shows a rather thick lens bending (refracting) rays from an object OP to form an image $O'P'$. The ray-tracing techniques and lens formulas we shall use here are based again on *Gaussian optics*, just as they were for mirrors.

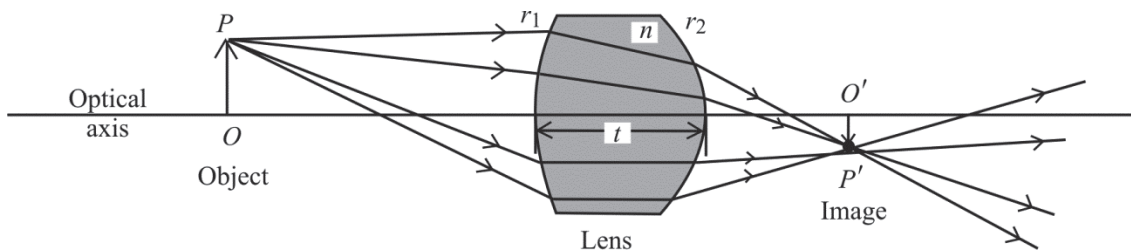


Figure 4-22 Refraction of light rays by a lens

As we have seen, *Gaussian optics*—sometimes also called *paraxial optics*—arises from the basic approximations $\sin \varphi \cong \varphi$, $\tan \varphi \cong \varphi$, and $\cos \varphi \cong 1$. These approximations greatly simplify ray tracing and lens formulas, but they do restrict the angles the light rays make with the optical axis—for practical purposes—to rather small values of 20° or less.

Types of lenses

If the *axial thickness* t of a lens (see Figure 4-22) is small compared with the radii of curvature r_1 and r_2 of its surfaces, it can be treated as a *thin* lens. Ray-tracing techniques and lens formulas are relatively simple for thin lenses. If the thickness of a lens is not negligible compared with the radii of curvature of its faces, it must be treated as a *thick* lens. Ray-tracing techniques and lens-imaging formulas are more complicated for thick lenses. When working with thick lenses, one can make use of available computer programs to trace the rays through the lenses or make surface-by-surface calculations. In this basic introduction of geometrical optics, however, we shall deal only with thin lenses.

Converging and diverging thin lenses—In Figure 4-23, we show the shapes of several common “thin” lenses. Even though a “thickness” is shown, the use of thin lenses assumes that the rays simply refract at the front and rear faces without a translation through the intervening lens medium. The first three lenses are *thicker in the middle than at the edges* and are described as *converging* or *positive* lenses. They cause parallel rays passing through them to bend toward one another and give rise to *positive* focal lengths. The last three lenses are *thinner in the middle than at the edges* and are described as *diverging* or *negative* lenses. They cause parallel rays passing through them to spread as they leave the lens and thereby give rise to *negative* focal lengths. In Figure 4-23, names associated with the different shapes are noted.

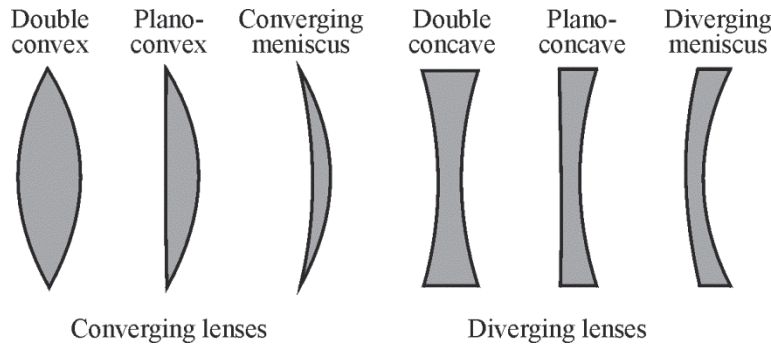


Figure 4-23 Shapes of common thin lenses

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Focal points of thin lenses—Just as for mirrors, the *focal points* of lenses are defined in terms of the effect that lenses have on incident parallel light rays and plane wave fronts. Figure 4-24 shows parallel light rays and their associated plane wave fronts incident on a *positive* lens (Figure 4-24a) and a *negative* lens (Figure 4-24b). For the positive lens, refraction of the light by the lens brings it to a focal point F (real image) to the right of the lens. For the negative lens, refraction of the light by the lens causes it to diverge *as if it were coming from* focal point F located to the left of the lens. Note how the *plane wave fronts* are changed to *converging spherical wave fronts* by the positive lens and to *diverging spherical wave fronts* by the negative lens. This occurs because light travels more slowly in the lens medium than in the surrounding air, so the thicker parts of the lens retard the light along the wave fronts more than do the thinner parts.

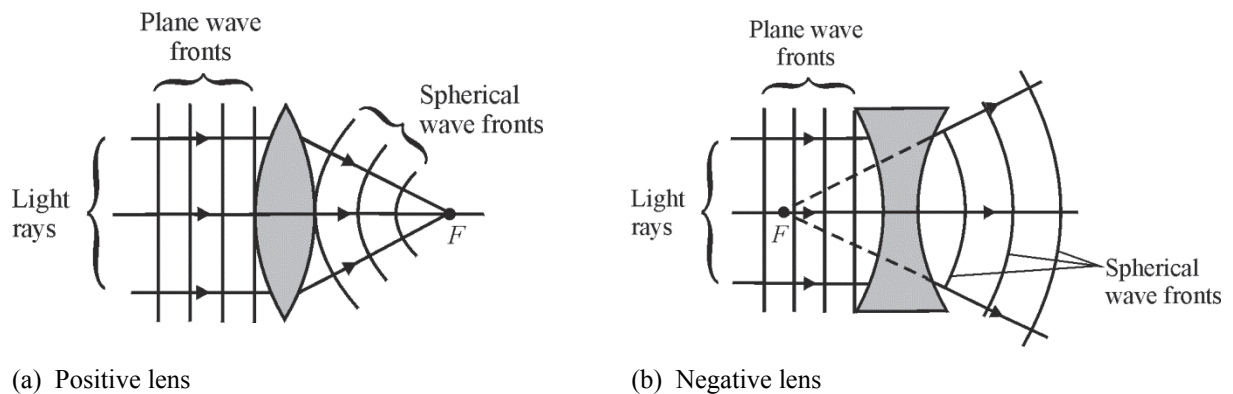


Figure 4-24 Focal points for positive and negative lenses

Recall that, for mirrors, there is but a *single* focal point for each mirror surface since light remains always on the same side of the mirror. For thin lenses, there are *two* focal points, symmetrically located on each side of the lens, since light can approach from either side of the lens. The sketches in Figure 4-25 indicate the role that the two focal points play, for positive lenses (Figure 4-25a) and negative lenses (Figure 4-25b). Study these figures carefully.

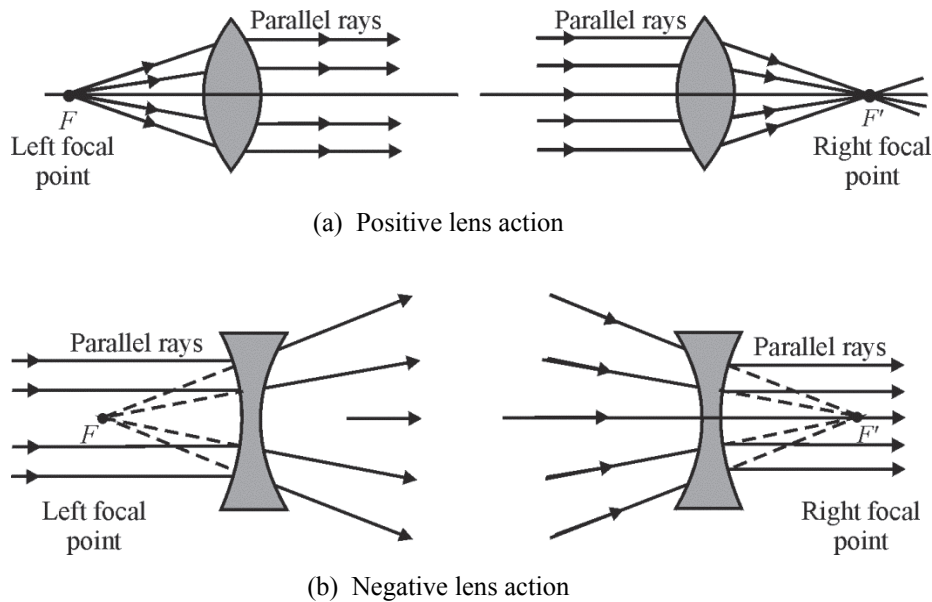


Figure 4-25 Relationship of parallel light rays to right and left focal points in thin lenses

The left sketch in Figure 4-25a shows light rays emerging from the *left focal point* F of the positive lens, striking the positive thin lens and emerging as *parallel rays*. The right sketch in Figure 4-25a shows a set of *parallel rays* incident on a positive thin lens and converging after refraction so as to pass through the right focal point F' .

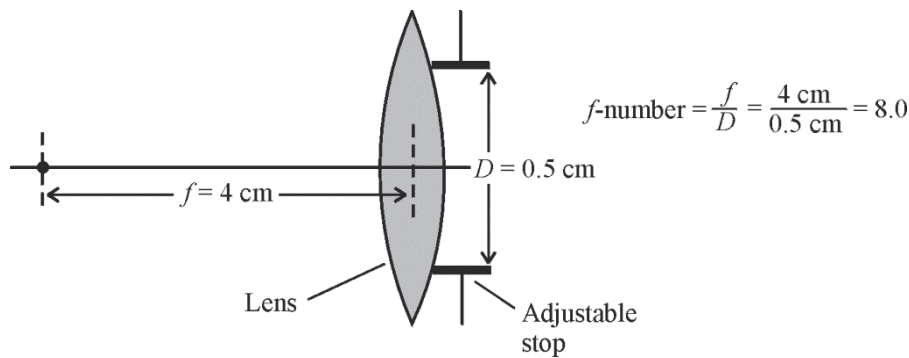
The left sketch of Figure 4-25b shows parallel rays incident on a negative thin lens and diverging after refraction by the lens as if to come from its left focal point F (see dashed lines). The right sketch in Figure 4-25b shows light converging onto a negative thin lens, heading for the right focal point F' (see dashed lines), but being refracted by the lens so as to emerge as parallel rays.

f -number and numerical aperture of a lens—The transverse dimension (size) of a lens determines its light-gathering power and, consequently, the brightness of the image it forms. Two commonly used indicators of this special characteristic of a lens are called the *f -number* and the *numerical aperture*.

The *f -number*, also referred to as the *relative aperture* and the *f /stop*, is defined simply as the ratio of the focal length f of the lens, to its transverse diameter D , as given in Equation 4-8.

$$f\text{-number} = \frac{f}{D} \quad (4-8)$$

For example, a lens of focal length 4 cm *stopped down* to an aperture of diameter 0.5 cm, as shown in the sketch below, has an f -number of $4/0.5 = 8$.



Photographers usually refer to the situation pictured as a lens with an f/stop of $f/8$. Before fully automated—“point and shoot”—cameras came along, a photographer would routinely select an aperture size D for a given camera lens, an appropriate shutter speed, and a proper focus f to achieve both the desired image brightness and sharpness.

Table 4-2 lists the usual choices of f/stops (f -numbers) available on cameras and the corresponding image irradiance or “brightness”—in watts per square meter incident on the film surface. The listing gives the irradiance E_0 as the value for an f/stop of 1 and shows how the image irradiance decreases as the lens is “stopped down,” that is, as the adjustable aperture size behind the camera lens is made smaller so that D decreases. From Equation 4-8, it should be clear that, for a given camera lens of focal length f , the f/stop or f -number increases as D decreases, that is, as the lens is “stopped down.” Clearly, then, *increasing* the f -number of a lens *decreases* its light-gathering power.

Table 4-2. Relative Image Irradiance (Brightness) as a Function of f/stop Setting

f/stop or $f\text{-number}$	Relative Image Irradiance in watts/m^2
1	E_0
1.4	$E_0/2$
2	$E_0/4$
2.8	$E_0/8$
4	$E_0/16$
5.6	$E_0/32$
8	$E_0/64$
11	$E_0/128$
16	$E_0/256$
22	$E_0/512$

Since the *total light energy exposure* in joules/m^2 on the film is the *product* of the irradiance in $\text{joules}/(\text{m}^2\text{-s})$ and the exposure time (shutter speed) in seconds, a desirable film exposure can be obtained in a variety of ways. Thus, if a particular film—whose speed is described by an ASA number such as ASA 200—is perfectly exposed by light from a particular scene with a shutter speed of 1/50 second and an f/stop of $f/8$ (irradiance equals $E_0/64$ from Table 4-2), it will also be

perfectly exposed by any other combination that gives the same total exposure in joules/m^2 . For example, by choosing a shutter speed of $1/100$ second and an f/stop of $f/5.6$, the exposure time is cut in half while the irradiance ($E_0/32$) is doubled, thereby leaving *no net change* in the film exposure in joules/m^2 .

The *numerical aperture* is another important design parameter for a lens, related again directly to how much light the lens gathers. If the focal length of a design lens increases and its diameter decreases, the solid angle (cone) of useful light rays from object to image for such a lens decreases. For example, the concept of a *numerical aperture* finds immediate application in the design of the objective lens (the lens next to the specimen under observation) for a *microscope*, as is shown below. Light-gathering capability is crucial for microscopes.

Figure 4-26 depicts the light-gathering power of a lens relative to a point O on a specimen covered by a glass slide. Lens L is the objective lens of a microscope focused on the specimen. For the *sake of comparison only*, we show an *oil-immersion* system and an *air-immersion* system, side-by-side, left and right of the symmetry axis of the microscope lens. On the right side of the symmetry axis of the lens, the light-gathering power of the lens—with *air between the cover slide and the lens*—is depicted in terms of limiting ray 1 and half-angle α_{air} . On the left side, by contrast, the increased light-gathering power of the lens—with *oil situated between the cover slide and the lens*—is shown in terms of limiting ray 2 and the *larger* half-angle α_{oil} .

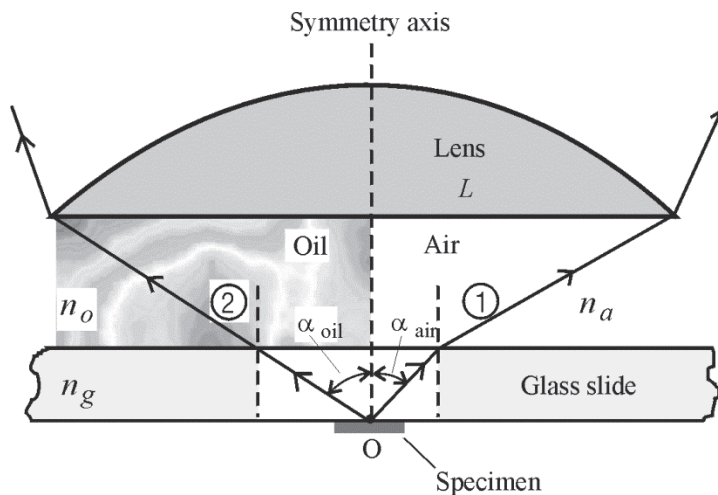


Figure 4-26 Comparison of light-gathering power of an oil-immersion lens system and an air-immersion lens system, showing that the oil-immersion lens system is superior

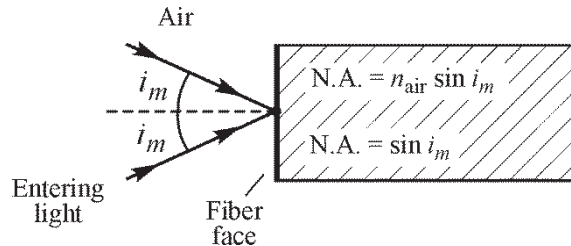
The oil is chosen to have an index of refraction (n_o) very near that of the cover slide (n_g) so that little or no refraction occurs for limiting ray 2 at the glass-oil interface, while considerable refraction does occur at the glass-air interface for limiting ray 1. Consequently, half-angle α_{oil} is greater than half-angle α_{air} . As Figure 4-26 shows, since ray 1 suffers refraction at the glass-air interface, the cone of rays (from point O on the specimen) accepted by the lens is restricted to the smaller half-angle α_{air} . The *numerical aperture* of a lens is defined so as to take into account the difference in solid angles (cones) of light accepted by different optical systems.

The definition of numerical aperture (*N.A.*) is given in Equation 4-9 as

$$N.A. = n \sin \alpha \quad (4-9)$$

where n is the index of refraction of the *intervening medium* between object and lens—such as oil—and α is the half-angle defined by the limiting ray (α_{air} or α_{oil} in Figure 4-26). Thus the “light-gathering” power of the microscope’s objective lens in Figure 4-26 is increased by increasing the refractive index of the intervening oil medium.

The *numerical aperture* (*N.A.*) of an *optical fiber* is an important measure of the fiber’s light capturing ability at its entrance face. In terms of the geometry introduced earlier in Example 4, we see, with the help of the accompanying sketch, that $N.A. = \sin i_m$.



It is shown in most basic books on optics that image brightness is dependent on values of the f -number or numerical aperture, in accordance with the following proportionalities:

$$\text{image brightness} \propto \frac{1}{(f\text{-number})^2} \quad (\text{as involved in Table 4-2})$$

$$\text{image brightness} \propto (N.A.)^2$$

In summary, one can *increase* the light-gathering power of a lens and the brightness of the image formed by a lens by *decreasing* the f -number of the lens (increasing lens diameter) or by *increasing* the numerical aperture of the lens (increasing an appropriate refraction index and thus making possible a larger acceptance angle).

Image location by ray tracing

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To locate the image of an object formed by a thin lens, we make use of three key points related to the lens and associate each of them with a defining ray. The three points are the *left focal point* F , the *right focal point* F' , and the *lens vertex (center)* V . In Figure 4-27, the three rays are shown leaving object point P and passing through the lens to locate a corresponding image point P' , for both a positive and a negative lens. The complete object is labeled OP and the corresponding image IP' . The defining rays are labeled to show clearly their connection to the points F , F' , and V . In practice, of course, only two of the three rays are needed to locate the desired image point. (The third ray is often used as a “check.”) Note also that the location of image point P' is generally sufficient to sketch in the rest of the image IP' , to correspond with the given object OP .

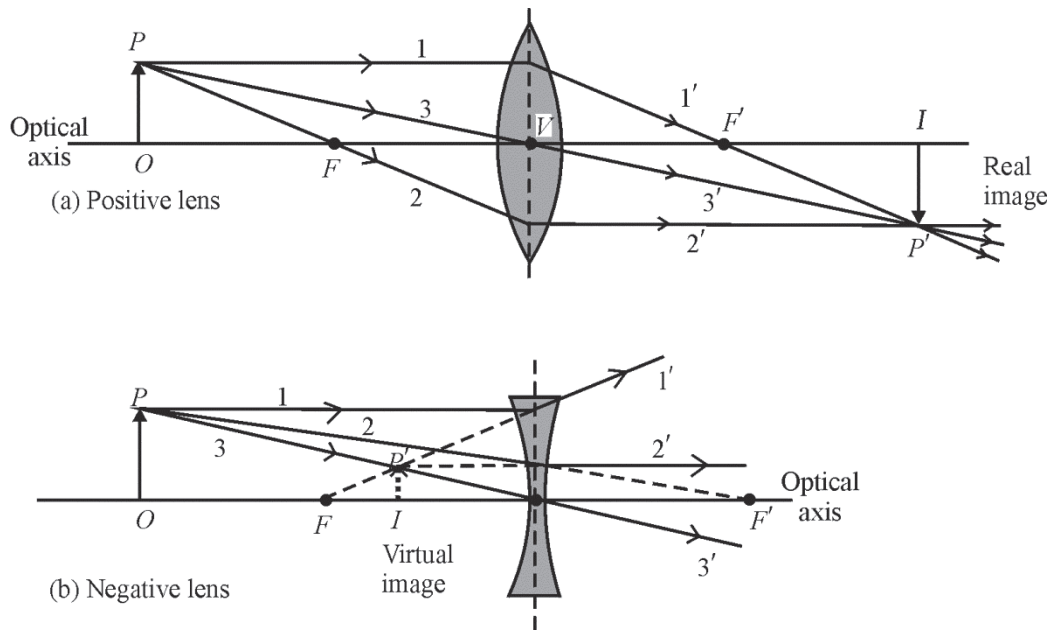


Figure 4-27 Ray diagrams for image formation by positive and negative lenses

The behavior of rays 1 and 2—connected with the left and right focal points for both the positive and negative lenses—should be apparent from a review of Figure 4-25. Thus parallel ray 1 for the *positive* lens is bent by the lens so as to go through the right focal point F' , and ray 2, which passes through the left focal point F on its way to the lens, is bent by the lens so as to emerge as a ray parallel to the optic axis. And for the *negative* lens, parallel ray 1 is bent outward by the lens in a direction such as to appear that it had come from the left focal point F , while ray 2, heading initially toward the right focal point F' , is bent by the lens so as to emerge as a ray parallel to the optic axis. The behavior of ray 3—going straight through either lens at its center V —is a consequence of assuming that the lens has zero thickness. Also, it should be clear in Figure 4-27 that the positive lens forms a real image while the negative lens forms a virtual image.

One can apply the principles of ray tracing illustrated in Figure 4-27 to a train of thin lenses. Figure 4-28 shows a ray trace through an “optical system” made up of a positive and a negative lens. For accuracy in drawing, a common practice (used here) is to show the positive lens as a vertical line with normal arrowheads and the negative lens as a vertical line with inverted arrowheads, and to show all ray bending at the vertical lines. Note that the primary object is labeled RO_1 (real object 1) and its image formed by the positive lens is labeled RI_1 (real image 1). The image RI_1 then serves as a real object (RO_2) for the negative lens, leading finally (in this case) to a virtual image VI_2 .

Test your understanding of ray tracing through thin lenses by accounting for each numbered ray drawn in the figure. What happens to rays 1 and 3 relative to the negative lens? Why are two other rays (4 and 5) introduced in continuing the ray trace from RO_2 to the negative lens? Is it a “correct” procedure to introduce rays 4 and 5, even though they are not extensions of original rays 1, 2, or 3? Can you sketch in where rays 4 and 5 originating from object point P would have had to strike an *enlarged* positive lens so as to become rays 4 and 5, which are shown passing through P' ?

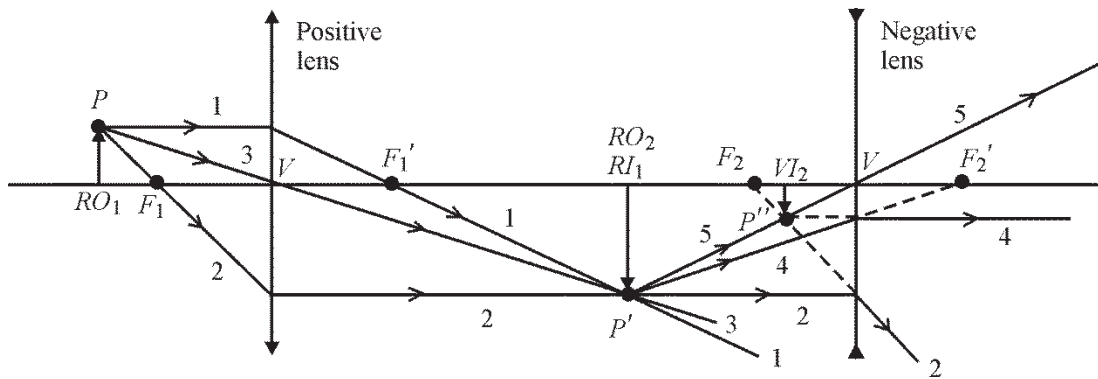


Figure 4-28 Ray diagram for image formation through two lenses

Lens formulas for thin lenses

As with mirrors, convenient formulas can be used to locate the image mathematically. The derivation of such formulas—as was carried out for spherical mirrors in the previous section—can be found in most texts on geometrical optics. The derivation essentially traces an arbitrary ray *geometrically* and *mathematically* from a given object point through the two refracting surfaces of a thin lens to the corresponding image point. *Snell's law* is applied for the ray at each spherical refracting surface. The details of the derivation involve the geometry of triangles and the approximations mentioned earlier— $\sin \phi \cong \phi$, $\tan \phi \cong \phi$, and $\cos \phi \cong 1$ —to simplify the final results. Figure 4-29 shows the essential elements that show up in the final equations, relating object distance p to image distance q , for a lens of focal length f with radii of curvature r_1 and r_2 and refractive index n_g . For generality, the lens is shown situated in an arbitrary medium of refractive index n . If the medium is air, then, of course, $n = 1$.

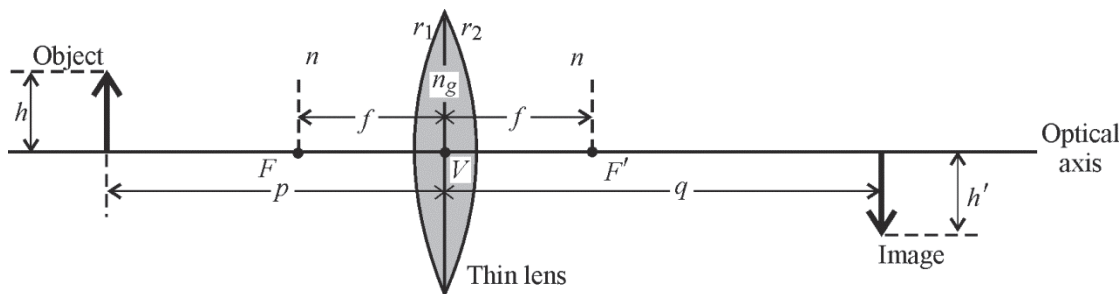


Figure 4-29 Defining quantities for image formation with a thin lens

Equations for thin lens calculations—The *thin lens equation* is given by Equation 4-10.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad (4-10)$$

where p is the object distance (from object to lens vertex V)

q is the image distance (from image to lens vertex V)

f is the focal length (from either focal point F or F' to the lens vertex V)

For a lens of refractive index n_g situated in a medium of refractive index n , the relationship between the parameters n , n_g , r_1 , r_2 and the focal length f is given by the *lensmaker's equation* in Equation 4-11.

$$\frac{1}{f} = \left(\frac{n_g - n}{n} \right) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (4-11)$$

where n is the index of refraction of the medium surrounding the lens

n_g is the index of refraction of the lens materials

r_1 is the radius of curvature of the *front face* of the lens

r_2 is the radius of curvature of the *rear face* of the lens

The magnification m produced by a thin lens is given in Equation 4-12.

$$m = \frac{h_i}{h_o} = -\frac{q}{p} \quad (4-12)$$

where m is the magnification produced by the lens (ratio of image size to object size)

h_i is the vertical transverse size of the image, measured perpendicularly to the optical axis

h_o is the vertical transverse size of the object, measured perpendicularly to the optical axis

p and q are object and image distance respectively

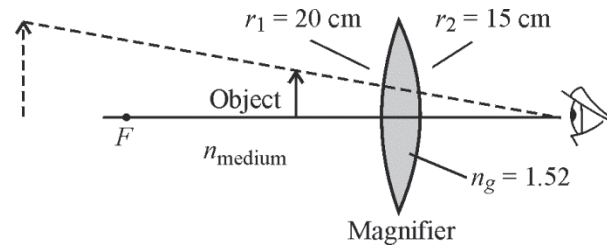
Sign convention for thin lens formulas—Just as for mirrors, we must agree on a sign convention to be used in the application of Equations 4-10, 4-11, and 4-12:

- We draw the “optical system” such that light travels initially from left to right toward the first lens.
- Object distance p is *positive* for *real* objects located to the *left* of the lens and *negative* for *virtual* objects located to the *right* of the lens.
- Image distance q is *positive* for *real* images formed to the *right* of the lens and *negative* for *virtual* images formed to the *left* of the lens.
- The focal length f is *positive* for a *converging* lens, *negative* for a *diverging* lens.
- The radius of curvature r is *positive* for a *convex* surface, *negative* for a *concave* surface. (Note that this is the *reverse* of the sign convention adopted for mirrors.)
- Transverse distances (h_o and h_i) are *positive* *above* the optical axis, *negative* *below*.

Now let's apply Equations 4-10, 4-11, and 4-12 in several examples, where the use of the sign convention is illustrated and where the size, orientation, and location of a final image are determined.

Example 10

A double convex thin lens, such as that shown here, can be used as a simple “magnifier” to read small print. This magnifier has a front surface with a radius of curvature of 20 cm and a rear surface with a radius of curvature of 15 cm. The lens material has a refractive index of 1.52. Answer the following questions to learn more about this simple magnifying lens.



- What is its focal length when used in air?
- What is its focal length if used in water ($n = 1.33$)?
- Does it matter which lens face is turned toward the light?
- How far would you hold an index card from this lens to form a sharp image of the sun on the card?

Solution

- Use the *lensmaker's equation*. With the lens in air and the sign convention given, we have $n_g = 1.52$, $n = 1.00$, $r_1 = +20$ cm, and negative $r_2 = -15$ cm. (Note carefully that with light traveling from left to right, the front face is convex, so r_1 is positive; the rear face is concave, so r_2 is negative.) Then

$$\frac{1}{f} = \left(\frac{n_g - n}{n} \right) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \left(\frac{1.52 - 1}{1} \right) \left(\frac{1}{20} - \frac{1}{-15} \right) = 0.0607, \text{ and thus } f = \frac{1}{0.0607} = 16.5$$

So $f = +16.5$ cm. It is a converging lens so the sign is positive, as it should be.

- With the lens in water and the given sign convention we have $n_g = 1.52$, $n = 1.33$, $r_1 = +20$ cm and $r_2 = -15$ cm. Then

$$\frac{1}{f} = \left(\frac{1.52 - 1.33}{1.33} \right) \left(\frac{1}{20} - \frac{1}{-15} \right) = 0.0167$$

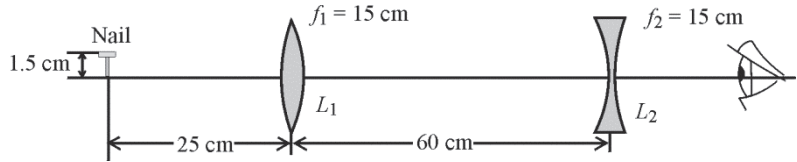
$f = 60$ cm (still a converging but less so than in air)

- No, the magnifying lens behaves the same, having the same focal length, no matter which refracting surface faces the light. You can prove this by reversing the lens and repeating the calculation with Equation 4-11. Results are the same. But **note carefully**, reversing a *thick* lens changes its effect on the light passing through it. The two orientations are *not* equivalent for a thick lens!!
- Since the sun is very far away, its light is collimated (parallel rays) as it strikes the lens and will come to a focus at the lens focal point. Thus, one should hold the lens about 16.5 cm from the index card to form a sharp image on the card. (In fact, this technique of using a lens to focus sunlight on an index card can be used to get a decent estimate of the focal length of the lens. In practice, one can focus the overhead lights in a laboratory or room on the index card with a lens and estimate quite well the focal length of the positive lens.)

Example 11

Multi-lens arrays are used in many optical systems, commonly, as we know, in adjustable or “point-and-shoot” cameras. The following hypothetical two-lens system is made up of a converging (positive) lens followed by a diverging (negative) lens, each of focal

length 15 cm. Suppose that the system is used to form an image of a short nail, 1.5 cm high, standing erect, 25 cm from the first lens. The two lenses are separated by a distance of 60 cm. See diagram. (Refer to Figure 4-28 for a ray-trace diagram of what’s going on in this problem.)



Locate the final image, determine its size, and state whether it is real or virtual, erect or inverted.

Solution: We apply the thin lens equations to each lens in turn, making use of the correct sign convention at each step.

$$\text{Lens } L_1: \frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1} \quad \text{or} \quad \frac{1}{25} + \frac{1}{q_1} = \frac{1}{15} \quad (f_1 \text{ is } + \text{ since lens } L_1 \text{ is converging.})$$

Solving, we obtain $q_1 = +37.5$ cm (Since the sign is positive, the image is real and located 37.5 cm to the right of lens L_1 .)

The first image, at a distance of $q_1 = 37.5$ cm, from L_1 , serves as the object for the lens L_2 . This object is real and to the left of lens L_2 . Thus p_2 is positive and equal to $(60 - 37.5) = 22.5$ cm. The focal length for L_2 is negative ($f_2 = -15$ cm) since it is a diverging lens. So, the thin lens equation becomes

$$\text{Lens } L_2: \frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2} \quad \text{or} \quad \frac{1}{22.5} + \frac{1}{q_2} = \frac{1}{-15}, \text{ giving } q_2 = -9 \text{ cm}$$

Since q_2 is negative, it locates a *virtual* image, 9 cm to the left of lens L_2 (See Figure 4-28). An observer looking into lens L_2 from the right would see an image of the nail 9 cm to the left of the lens.

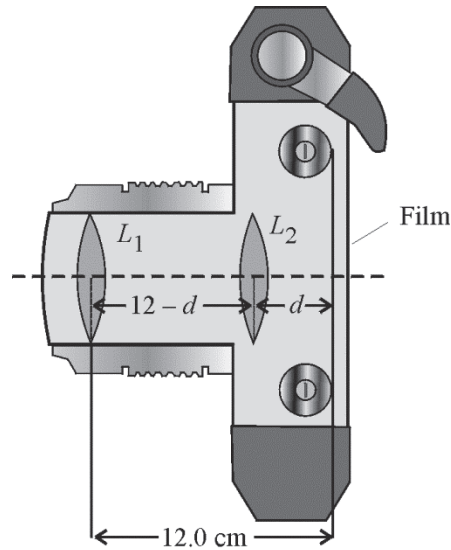
The overall magnification for the two-lens system is given by the combined magnification of the two lenses. Then

$$m_{\text{sys}} = m_1 \times m_2 = \left(-\frac{q_1}{p_1} \right) \left(-\frac{q_2}{p_2} \right) = \left(-\frac{37.5}{25} \right) \left(-\frac{-9}{22.5} \right) = -0.6$$

Thus, the final image is inverted (since overall magnification is negative) and is of final size $(0.6 \times 1.5 \text{ cm}) = 0.9 \text{ cm}$. (See ray trace in Figure 4-28.)

Example 12

An adjustable 2-lens camera, such as the one shown, has a focal length of 15 cm for converging lens L_1 and 13.3 cm for converging lens L_2 . Lens L_1 is fixed in the camera, a distance of 12 cm from the film plane. Lens L_2 is adjustable and is moved to bring objects into a clear focus on the film. Where must L_2 be situated (i.e., what is d) when the camera is focused on an object 1.33 meters away?



Solution

Step 1: Find image distance q_1 for lens L_1 :

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1} \text{ where } p_1 = 133 \text{ cm and } f_1 = 15 \text{ cm.}$$

$$\text{Thus, } \frac{1}{133} + \frac{1}{q_1} = \frac{1}{15}, \text{ giving } q_1 = \frac{(15)(133)}{(133-15)} = 16.9 \text{ cm.}$$

So lens L_1 alone would form a real image, 16.9 cm to its right. This distance is clearly greater than the 12.0 cm dimension of the camera, so this real image, formed to the right of lens L_2 , becomes a *virtual* object for lens L_2 . The virtual object distance p_2 , negative by sign convention, is then $-(16.9 - (12 - d))$. We know that q_2 must be equal to d since the camera is in focus.

Step 2: Find d by using the thin lens equation for lens L_2 :

$$\text{Thus, } \frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2} \text{ where } p_2 = -(16.9 - (12 - d)) = -(4.9 + d), q_2 = d \text{ and } f_2 = 13.3 \text{ cm}$$

Substituting we get,

$$\frac{1}{-(4.9 + d)} + \frac{1}{d} = \frac{1}{13.3} \text{ or } \frac{d - (4.9 + d)}{-(4.9 + d)(d)} = \frac{1}{13.3} \text{ or } \frac{-4.9}{-4.9d - d^2} = \frac{1}{13.3}$$

This gives the quadratic equation $-d^2 - 4.9d = -(4.9)(13.3)$ or $d^2 + 4.9d - 65.17 = 0$. Using

the standard solution for a quadratic equation of form $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

where $a = 1$, $b = 4.9$, $c = -65.17$, and $x = d$, we get,

$$d = \frac{-4.9 \pm \sqrt{(4.9)^2 - 4(1)(-65.17)}}{2} = \frac{4.9 \pm 16.9}{2}, \text{ or } d = \frac{12}{2} = 6 \text{ cm, or } d = \frac{-21.8}{2} = -10.9 \text{ cm.}$$



Need help with roots?

Go to

<http://optecvideo.opteccrm.org/>.

We discard the negative value of $d = -10.9$ cm because it makes no physical sense. Hence, we have, finally, $d = 6$ cm. Thus L_2 is positioned 6 cm in front of the film plane when the 1.33 m distant object is in focus.

LABORATORIES

Laboratory 1-4A: Prisms and Lenses

Go to
<http://optecvideo.opteccrm.org/>
for an introduction to this
activity.



In this laboratory, you will perform the following simple experiments with prisms and lenses:

- Determine the index of refraction of a prism material.
- Demonstrate total internal reflection (TIR) with right-angle prisms and show how to use the prisms to produce (a) 90° bending, (b) retroreflection, and (c) “periscope-type” bending.
- Determine the index of refraction of a thin-lens material.
- Determine the focal lengths of convex and concave lenses.

Equipment

The following equipment is needed to complete this laboratory:

- 1 equilateral prism^a (25-mm faces by 25 mm long works well)
- 2 45° - 45° - 90° prisms^a (25-mm legs, 35-mm hypotenuse, 25 mm thick)
- 2 diode laser pointers^b (5 mW or less)
- 1 spherometer^b
- 1 double-convex lens^a (75-mm diameter by 150-mm focal length)
- 1 double-concave lens^a (75-mm diameter by 150-mm focal length)
- 1 protractor
- 1 white cardboard screen

Index cards, white paper sheet, ($8\frac{1}{2}'' \times 11''$ and $11'' \times 17''$), masking tape, and ruler

^a These items are readily available from Edmund Scientific, Barrington, New Jersey, 609-574-6250, as parts of their “Educational Quality Demonstration Optics,” at reasonable prices. See their Industrial Optics Division catalog.

^b These items are also available from Edmund Scientific but are more expensive.

Procedure

A. Index of refraction of a prism material

1. Arrange the laser pointer, equilateral prism, and white cardboard screen on a flat tabletop as shown in Figure 4-30. Center the prism over a sheet of white paper. Fasten down the white paper, cardboard screen, and laser with tape.

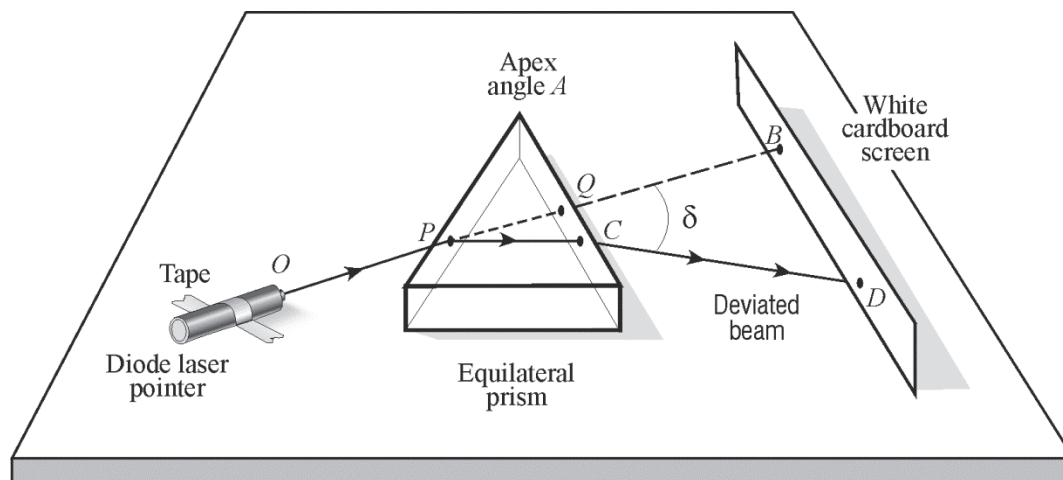


Figure 4-30 Setup for measuring minimum angle of deviation

2. As you rotate the prism relative to the incident laser beam, the laser spot D on the screen moves, so the angle of deviation δ will become larger or smaller. By experimentation, determine the *smallest angle of deviation* (δ_m) between an original beam direction $OPQB$ and the deviated beam CD . (It should be clear that the farther the screen is from the prism the more precise will be your determination of δ_m , since small changes in spot D will then be more exaggerated.)
3. When you have achieved the minimum angle for δ , carefully tape the prism in place. Trace the prism edges on the paper, the straight segments OP and QB along the *original direction*, and the segment CD . (**Note:** Location of laser spots Q , C on the exit face of the prism and B , D on the screen are needed to be able to draw segments QB and CD .) With the line segments drawn, remove the prism and measure the minimum angle δ_m with a protractor. Complete a ray trace of the incident beam through the prism, deviated at angle δ_m . Is the segment DC parallel to the prism base? Should it be?
4. Record the measured angle δ_m and the apex angle A . Use the formula

$$n = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}}$$

to calculate the index of refraction n . Compare your value with values given in Table 4-1. Does it agree with any value given there? What is your best guess for the prism material?

B. Total internal reflection (TIR)

(When you have finished this part, you will have three different traces of laser light interacting with right-angle prisms, all on an 11" × 17" sheet of white tracing paper.)

1. Set a right-angle prism on one of its parallel sides on a sheet of 11" × 17" white tracing paper. Tape the paper and prism in position. Shine a diode laser beam onto an appropriate face of the prism so that it *undergoes total internal reflection* (TIR) inside the prism and exits the prism at 90° to its original direction of entry. Use index cards as a screen to “locate” the laser beam outside the prism. On the paper, trace the edges of the prism, a line along the incident beam, a line along the path through the prism, and a line along the exit beam. Label the angles of incidence and reflection and their values at the face where TIR takes place. What would you need to know to determine the critical angle at this face? Is the incident angle on the face where TIR occurs larger or smaller than the critical angle?
2. Move the right-angle prism to a different position on the 11" × 17" paper and tape it down. Direct the diode laser beam onto an appropriate face so that the beam returns along a direction parallel to its entering direction. Use index cards to “locate” the beam path. When you have achieved this condition of *retroreflection*, trace the edges of the prism, the entering beam, the path through the prism, and the exit beam. Draw appropriate angles at the faces where TIR occurs and give their correct values.
3. Move two right-angle prisms to a new location on the 11" × 17" paper. Arrange them to produce “periscope action.” This action requires, for example, that a horizontal beam that enters at one level be deflected downward 90° and exit horizontally at a different level, as shown in the accompanying sketch. Here the dashed squares indicate the locations of the two prisms. Use index cards to locate the beam through the prism arrangement.

When you have achieved the “periscope” geometry, tape the prisms down. Trace their edges, and trace the laser beam path from initial entry to final exit. Show where TIR occurs and label the incident and reflected angles there correctly, at each position.

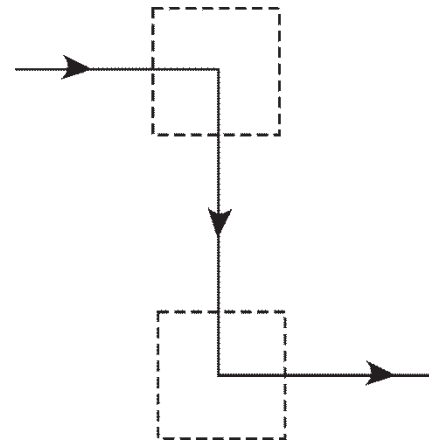


Figure 4-31

C. Index of refraction of a thin lens

Use the lensmaker's equation $\left[\frac{1}{f} = \left(\frac{n_g - n}{n} \right) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \right]$ to determine the value of n_g for the

double-convex lens. Use a ruler, overhead lights, and an index card to obtain a good approximation for the focal length of the lens. (Going outside and imaging the sun would be even better.) Use a spherometer to measure the radii of curvature r_1 and r_2 . (You will have to be especially creative to get r_1 and r_2 if you don't have access to a spherometer.) With the values

of f , r_1 , and r_2 , solve the lensmaker's equation for n_g , the index of refraction of the lens glass. Compare your value with values given in Table 4-1. Do you have a match?

D. Measuring the focal lengths of thin lenses

Set up the two diode lasers on a stand or optical bench so that they emit beams parallel to one another and normal to the plane defining the vertical position of the thin lens. See Figure 4-32. (To see the beams converging on the image side of the lens, you will have to use chalk-dust particles or smoke—some form of cloud—to illuminate the path.) By moving the screen forward and backward, you can locate a position where the beams cross to form the “smallest” spot. This is the *focal point* for the lens. Measuring the optical bench distance from lens to focal point gives the *focal length*. Compare this value to the value you obtained in part C, when you simply imaged a “distant” object on an index card. Which method is more accurate? Which method is easier?

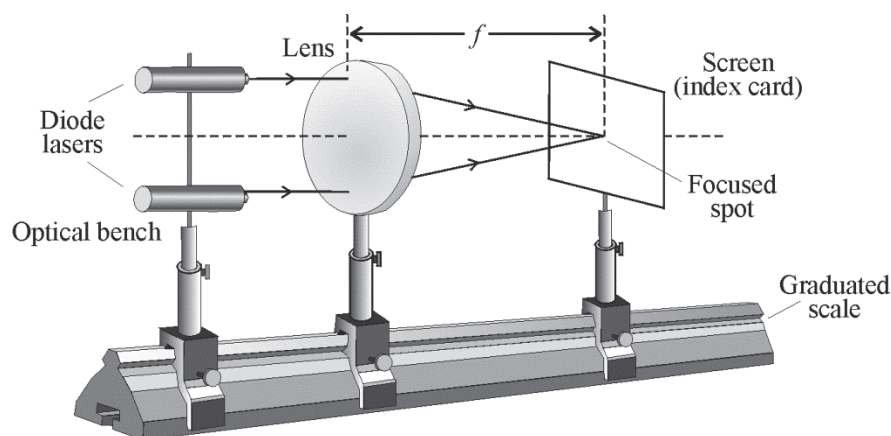


Figure 4-32 Setup for determining focal length of a positive lens

Replace the positive lens in Figure 4-32 with the negative lens. The challenge now is greater since the two laser beams diverge on the right side of the lens—and do not form a real image anywhere. Can you design a method to locate the spots of the two parallel beams at the lens and the spots for the two diverging beams on the right of the lens, then “trace” your way back to locate the focal point on the left side of the lens? If you can locate the focal point on the left, you can then measure its distance from the lens to get the focal length of the negative lens.

Laboratory 1-4B:

Optical Alignment Techniques

Go to
[http://optecvideo
.opteccrm.org/](http://optecvideo.opteccrm.org/)
for an
introduction to
this activity.



In this laboratory, you will complete the following task:

- Practice basic optical alignment techniques using an optical breadboard.

Equipment

Newport Kit: OEK-STD

<u>Part</u>	<u>Qty</u>	<u>Description</u>	<u>Also ...</u>
BSA-I	1	Beam Steering Assembly	Meter Stick
TA-II	1	Target Assembly	Long Section of Paper
LCA	1	Lens Chuck Assembly	
LA	1	Laser Assembly	

Optical Breadboard
Index Card

Procedure

1. Construct the following component assemblies:
 - A. Beam Steering Assembly mounted on a rotational stage, BSA-I
 - B. Target Assembly, TA-II
 - C. Lens Chuck Assembly, LCA
 - D. Laser Assembly, LA
 - E. Keep the BSA-I, LCA, TA-II and the LA setup, as they will be used in the next step.
2. Alignment
 - A. Mount the Laser Assembly near a corner of the breadboard so that the beam follows the outermost row of holes. Center the target (+) in a filter holder attached to a post.
 - B. Count 5 breadboard holes from the laser output along the laser beam axis. Mount a post holder.
 - C. Count 5 holes starting from the far end of the breadboard along the laser axis. Mount a post holder.
 - D. Place the card in the filter holder and position the assembly in the post holder nearest the laser so that the + is centered on the row of holes over which the laser beam travels. Adjust the height so that the beam spot is on the +.
 - E. Place the assembly in the post holder farthest from the laser. Does the beam spot lie on the +? If not, make the necessary adjustments to the laser mount until it does.
 - F. Place the assembly again in the post holder nearest the laser. Is the beam still on the +? If it is, proceed to the next section. If not, make adjustments to the laser mount to center the spot.
 - G. Place the assembly in the post holder farthest from the laser. The beam alignment should be improved. You may have to repeat steps E and F a few times before a horizontal beam is achieved.

Show your work to the instructor and sign off. _____ (Instructor's Initials)

3. Inserting Components into Aligned Axis

When inserting components into the optical axis, you can begin with the first added component and proceed outward, away from the laser; however, the preferred method is to begin with the farthest component and proceed inward, toward the laser. Most components are aligned on-axis, using the method of retroreflection. As a component is added to the system, it is positioned so that the beam reflected back through the system, centering on the laser output aperture. This is possible when a narrow beam of low-divergence is incident on the component's surface precisely at the center, because there it is normal to the surface. Thus, when a component is added to the system, the orientation of the component is adjusted until retroreflection occurs; once the optical axis has been established, the laser's orientation should not be altered when aligning a component.

When you need to use a component in the system that is intended to deflect the beam, such as a mirror or beamsplitter, they must be aligned using part of the method of establishing an optical axis as described previously.

- A. Once the optical axis is established, set up a BSA-I near the corner of the breadboard along the optical axis so that the beam is deflected 90° along a row of holes.
- B. Use the method of establishing an optical axis (see Alignment) to check the beam along the new path.
- C. Place a +100 mm lens in the lens chuck assembly. Use finger cots to prevent fingerprints from contaminating the lens surface.
- D. Place the LCA assembly in the beam path following the mirror. Confirm that the lens is centered on the optical axis either by centering the return beam onto the laser aperture or by observing that the transmitted beam remains centered using the +.

Show your work to the instructor and sign off _____ (Instructor's Initials)

Laboratory 1-4C: Law of Reflection

Go to
<http://optecvideo.opteccrm.org/>
for an introduction to this
activity.



In this laboratory, you will complete the following task:

- Observe that a beam is deflected from a mirror at an angle that is **twice** the angle of rotation of the mirror.

Equipment

Newport Kit: OEK-STD

<u>Part</u>	<u>Qty</u>	<u>Description</u>	<u>Also ...</u>
LA	1	Laser Assembly	Meter Stick
BSA-I	2	Beam Steering Assembly	Long Section of Paper
TA-I	1	Target Assembly	
RSA-IT	1	Rotation Stage	

Procedure

1. The optical breadboard should be located on a table near a wall. Tape a sheet of paper on the wall at the same height you plan to set your laser.
2. Mount the He-Ne laser as described by your instructor and place it at the rear of the breadboard.
3. Mount a Beam Steering Assembly (BSA-I) onto a Rotation Stage (RSA-IT). The Rotation Stage should be attached to the table with screws. The BSA-I should be located so that the incident laser beam is perpendicular to the wall as shown in Figure 4-33.

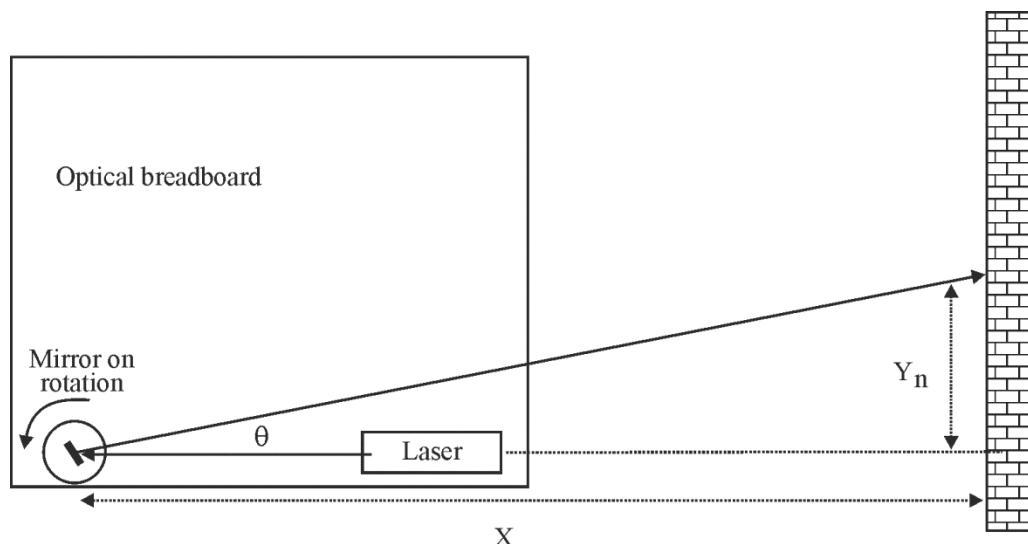


Figure 4-33: *From Laser Beam at Mirror to Point on Wall*

4. Adjust the rotation stage to read 0 degrees. Then, adjust the beam steering mirror to reflect the beam back onto itself without changing the rotation stage reading.

AVOID SHINING THE REFLECTED BEAM ON SPECULAR REFLECTING SURFACES OR OTHER MEMBERS OF THE CLASS.

5. Adjust the vertical adjustment on the mirror to place the beam on a piece of paper taped to the wall above and behind the laser. Mark the location of the spot and call it " Y_0 " in Table 4-3.
6. Rotate the mirror by turning the rotation stage and mark the new location of the spot. Record the angle from the rotation stage. Repeat this for 5 different angles that produce beam positions separated by a foot or more.
7. Measure and record the distance " X " from the mirror to the wall. Measure and record the distances from Y_0 to the marks on the piece of paper. Call these measurements Y_1 , Y_2 , etc. Use these measurements along with your knowledge of trigonometry and a calculator to determine the angle between the laser beam direction and the reflected direction. Record your calculations and these angles next to those for the incident angles in Table 4-3.

8. Compare the total reflected angles to the incident angles. You should find that the total beam deviation angle is twice the angle of incidence, thus confirming the law of reflection.

Table 4-3

Angle of Rotation	X	Y _n	Deviation Angle (Degrees)		Error
(Degrees)	(m)	(m)	Measured	Expected	(%)

Laboratory 1-4D: Lenses

In this laboratory, you will complete the following task:

- Experimentally measure real images formed by converging lenses.

Go to
<http://optecvideo.opteccrm.org/>
 for an introduction to this activity.



Equipment (most from Newport Kit: OEK-STD)

–25 mm Diverging Lens
 1 LCA on a post
 1 LCA on adjustable base
 2 BSA Assemblies
 +50.2 mm Converging Lens
 +75.6 mm Converging Lens
 Filter Holder and Clothespins
 1 mm tall transparent arrow slide

Procedure

1. Install your Laser and two BSA assemblies. Insert the diverging lens as close to the laser as possible. Direct the expanded beam as shown in the diagram.

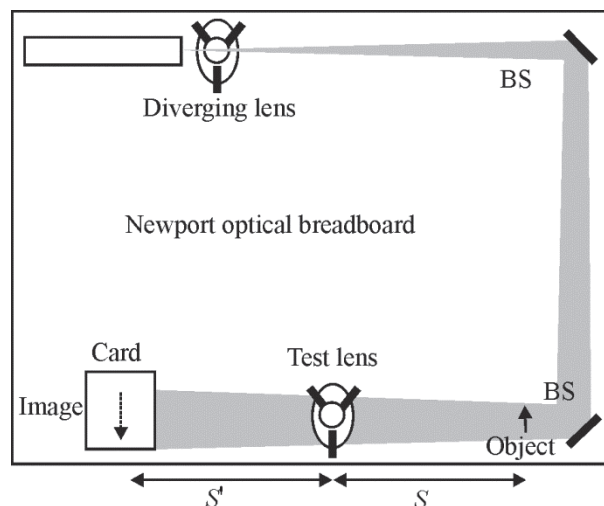


Figure 4-34

2. Mount the object on a filter holder in the middle of the expanded beam. Place this object as close to the mirror as possible.
3. Mount the desired lens into the LCA and place it on an adjustable base. Move the lens to a length of s (the desired object position) from the object.
4. Project the image on a card and record the image position (s') and the image height (y').
5. Make sure to enter all of your data in the Table 4-4.
6. Calculate s' for each measurement and compare to the measured result. Determine % Error.
7. Calculate the magnification (and thus y') for each measurement and compare to the measured result. Determine % Error.

Table 4-4

s (mm)	Y (mm)	f (mm)	s' (mm)	y' (mm)	$s'_{\text{predicted}}$ (mm)	%Error	$y'_{\text{predicted}}$ (mm)	%Error
100	10	50.2						
150	10	50.2						
200	10	50.2						
100	10	75.6						
150	10	75.6						
200	10	75.6						

WORKPLACE SCENARIO

Here is your opportunity to use the concepts learned in this module to solve an actual problem that could arise in a photonics company. Your instructor will provide directions for developing a solution.

Geometrical Optics – Ray Tracing

Scenario

After graduating with a two year associate degree in photonics technology, you joined a local company that designs and builds imaging systems, including cameras. Your job title is Engineering Photonics Technician Level II and one of your main responsibilities includes supporting the engineering research and design team. This responsibility includes building, testing, and performing failure analysis of engineering design product prototype runs^c. Prototype runs are usually small generating approximately 5 to 10 finished products. After a product has completed prototype testing and gone commercial, product volumes may be as high as a million a month. With volumes this high, problems can be extremely costly to correct!

Your main areas of responsibility during prototype testing are to:

1. Assist with obtaining parts and building the prototype.
2. Test the prototype.
3. Collect data on prototype performance.
4. Perform root-cause failure analysis on any prototype failures.
5. Document and report the results of the prototype run.

A new development engineer, Rick, thinks the company should get into the disposable camera business. His reasoning is that there is still exists a market among non-technical consumers, who are not yet into the digital camera age. With permission from his supervisor, Rick designs a disposable camera for taking pictures of distant objects and brings the design to you. With the design in hand, you order the parts specified in the design, and built the prototype units. Next, you tested these units by taking pictures and evaluating the picture quality. During evaluation of the picture quality, you discover that all of the pictures were slightly out of focus. You immediately recognize there is a problem with camera and begin to look for the root-cause of it.

Your first step in determining this root-cause is to review the camera's optical system physical layout drawing to determine the dimensions and locations of the prototype camera's optical components. The following table summarizes your findings:

^c Prototype runs are used to provide initial validation of the design, and hopefully determine any design problems prior to higher volume production runs, where finding problems would be more costly.

Type Data Parameter	Data
Lens Type	Converging Lens
Lens Focal Length	40 mm
Aperture to Film Distance	60 mm
Aperture to Lens Distance	10 mm

Problem and Solution Strategy

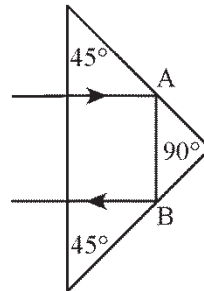
Since a focusing problem was discovered with the new disposable camera during prototype testing, you suspect that one possible cause may be the placement of the film with respect to the lens. To test your suspicion, you decide on two approaches. First, you will use the thin lens equation presented in Module 1-4 of the *Fundamentals of Light and Lasers* text to calculate the distance between the film and lens that will produce a focused image. To double check this calculation, you also decide to use a ray tracing applet that you found on the internet at <http://www.mtholyoke.edu/~mpeterso/classes/phys301/geomopti/twolenses.html>. With the results generated from these two approaches, you will be able to determine the correct placement of the film with respect to the aperture. To document your results, you have decided to prepare a report for Rick that will include your calculations and ray tracing efforts and also explain the reason for the camera's focusing problem.

PROBLEM EXERCISES AND QUESTIONS

Most of the problem exercises that follow are like the examples worked out in the text. Those problems marked with an asterisk (*) may be considered as *challenge* problems.

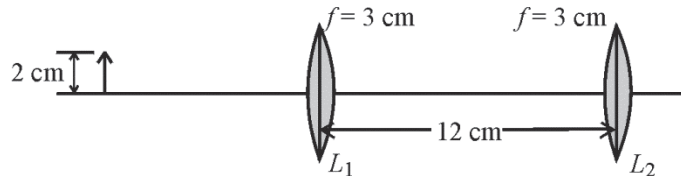
1. The speed of light in a transparent semiconductor material is measured to be 7.37×10^7 m/s. What is the index of refraction of this material? What is a good guess for the identity of this material?
2. A laser beam is incident on a quartz prism of index $n = 1.46$ at an angle of incidence of 60° . What are the angles of reflection and refraction at the air-quartz interface?

3. What is the critical angle for total internal reflection at points A and B in the 45° – 45° – 90° retroreflector prism shown here if the prism has an index of $n = 1.66$?



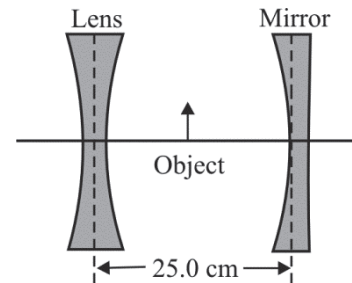
4. An isosceles prism made of zircon with equal base angles of 75° produces an angle of minimum deviation of $\delta_m = 30^\circ$ when a laser beam is passed through it. What is the index of refraction of this material? How does your value compare with that given in Table 4-1?
5. Based on the dispersion curve for light flint glass given in Figure 4-12, estimate to two decimal places the value of n_λ at $\lambda = 400$ nm and $\lambda = 650$ nm.
6. You are handed a concave mirror with a radius of curvature of 36 cm.
 - (a) What is its focal length?
 - (b) Where is the image of an object placed 36 cm in front of the mirror found?
 - (c) Is it real? Is it larger or smaller than the object?
7. Make a ray-trace diagram to locate the position and size of the image formed in Problem 6.
8. A driver sees a virtual image of a trailing motorcycle in the slightly convex, spherical rear-view mirror on the passenger side. The reduced virtual image seen by the driver “appears” to be about 3 car-lengths (~ 60 feet) behind.
 - (a) What is the actual distance of the motorcycle from the mirror if the radius of curvature of the mirror is 130 ft?
 - (b) How does the size of the motorcycle seen in the mirror compare with its actual size?
9. A camera lens with an f -number of 4 and shutter speed of $1/50$ second will give the same film exposure in joules/m² as the lens set at f -number of 5.6 and shutter speed of _____?

10. What is the numerical aperture (N.A.) for a fiber optic strand whose maximum acceptance half-angle at the air-fiber interface is 28° ?
11. A plano-convex lens of focal length 25.0 cm is to be made with crown glass of refractive index 1.52. Use the lensmaker's equation to determine the radius of curvature of the curved face to be ground and polished by the glass shop.
12. Two positive thin lenses, each of focal length $f = 3$ cm, are separated by a distance of 12 cm. An object 2 cm high is located 6 cm to the left of the first lens. See sketch.

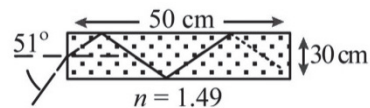


On an $8\frac{1}{2}'' \times 11''$ sheet of paper, make a drawing of the two-lens system, *to scale*. Use ray-tracing techniques to locate the final image and describe its size and nature.

13. Use the thin-lens equations to determine the position and size of the final image for the object specified in the two-lens system of Problem 12. Do your calculated results agree with your ray-tracing results?
14. The object shown in the accompanying sketch is midway between the lens and the mirror. The radius of curvature of the mirror is 20 cm. The concave lens has a focal length of 16.7 cm. (a) Where is the light that travels first to the mirror and then to the lens finally imaged? (b) Where is the light finally imaged that travels first to the lens? (Note: Be especially careful of applying the sign convention!)

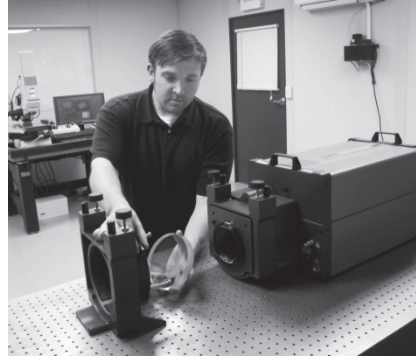


15. A laser beam is incident at the center of the end face of a cylindrical rod of material as shown in the sketch. The refractive index of the rod is 1.49. How many internal reflections does the laser beam experience before it exits the rod?



MEET A TECHNICIAN

John A. Crawford



John studied electro-optics at Indiana University of Pennsylvania (IUP) where he earned an Associate of Science Degree in Electro-Optics, which has led John to a rewarding career as an optical engineer at Prematech Advanced Ceramics.

John recognizes a strong link between math and photonics.

“Math is the tool that makes physics work and physics is the tool that makes photonics work; so one needs to start by having a good understanding of mathematics.”

One particular achievement that John takes pride in is the completion of a nine-inch, conventionally-polished, ceramic, parabolic mirror.

“Using a spindle and lapping tool is not the easiest way to produce an aspheric mirror, but the end result is astonishing.”

In the future, John would like to create larger, more specialized optics. Specifically, he would like to work on optics projects for astronomical telescopes, like The James Web Space Telescope.

John is enjoying his career in photonics and is looking forward to many more years pursuing it.

“I wanted to be part of something that is exciting, interesting, and personally challenging. The industry is young and rapidly growing so the professional possibilities are boundless.”

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